Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/203-7.6.2-Inverse-hyperbolic-cosecant-functions

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December 9, 2023 Compiled on December 9, 2023 at 7:16am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [71]. This is test number [203].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (71)	0.00 (0)
Mathematica	98.59 (70)	1.41 (1)
Maple	74.65 (53)	25.35 (18)
Fricas	74.65 (53)	25.35 (18)
Maxima	59.15 (42)	40.85 (29)
Mupad	57.75 (41)	42.25 (30)
Sympy	50.70 (36)	49.30 (35)
Giac	45.07 (32)	54.93 (39)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

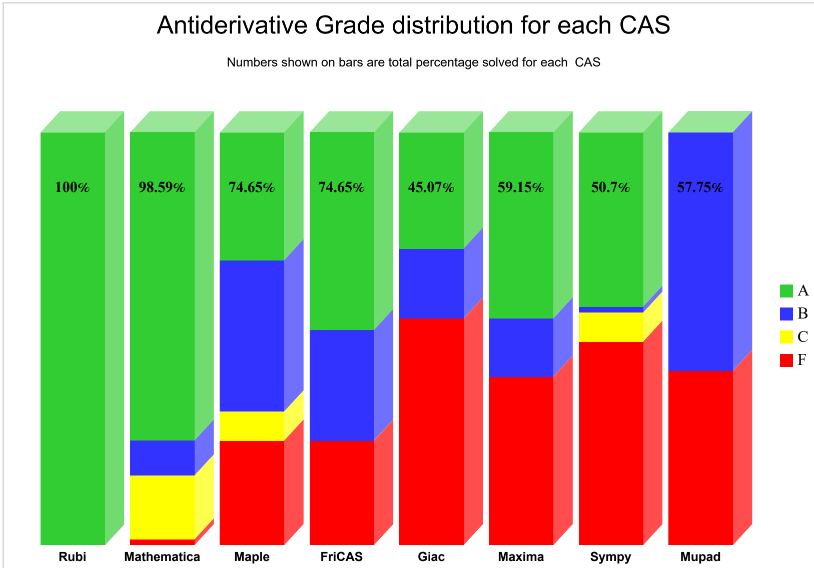
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

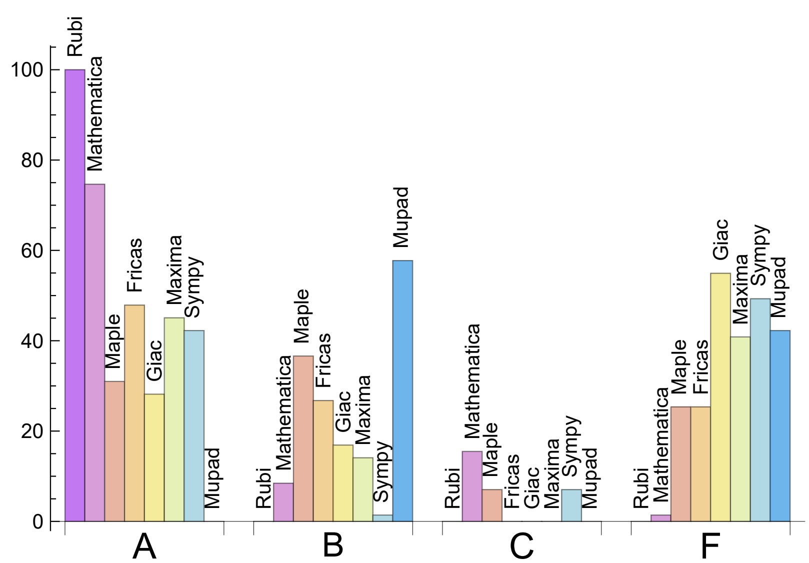
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.732	0.000	11.268	0.000
Mathematica	74.648	8.451	15.493	1.408
Fricas	47.887	26.761	0.000	25.352
Maxima	45.070	14.085	0.000	40.845
Sympy	42.254	1.408	7.042	49.296
Maple	30.986	36.620	7.042	25.352
Giac	28.169	16.901	0.000	54.930
Mupad	0.000	57.746	0.000	42.254

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	18	88.89	0.00	11.11
Maple	18	100.00	0.00	0.00
Maxima	29	93.10	0.00	6.90
Mupad	30	0.00	100.00	0.00
Sympy	35	91.43	8.57	0.00
Giac	39	84.62	0.00	15.38

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Fricas	0.24
Giac	0.28
Maple	0.33
Rubi	0.44
Mathematica	0.83
Sympy	1.67
Mupad	5.15

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	45.49	0.94	42.00	0.93
Sympy	57.83	1.13	50.00	1.18
Maxima	69.24	1.31	59.50	1.21
Giac	78.41	1.65	73.00	1.49
Fricas	104.72	1.78	70.00	1.37
Rubi	107.87	1.05	68.00	1.00
Maple	111.94	1.96	111.00	1.65
Mathematica	256.67	1.31	54.00	1.02

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

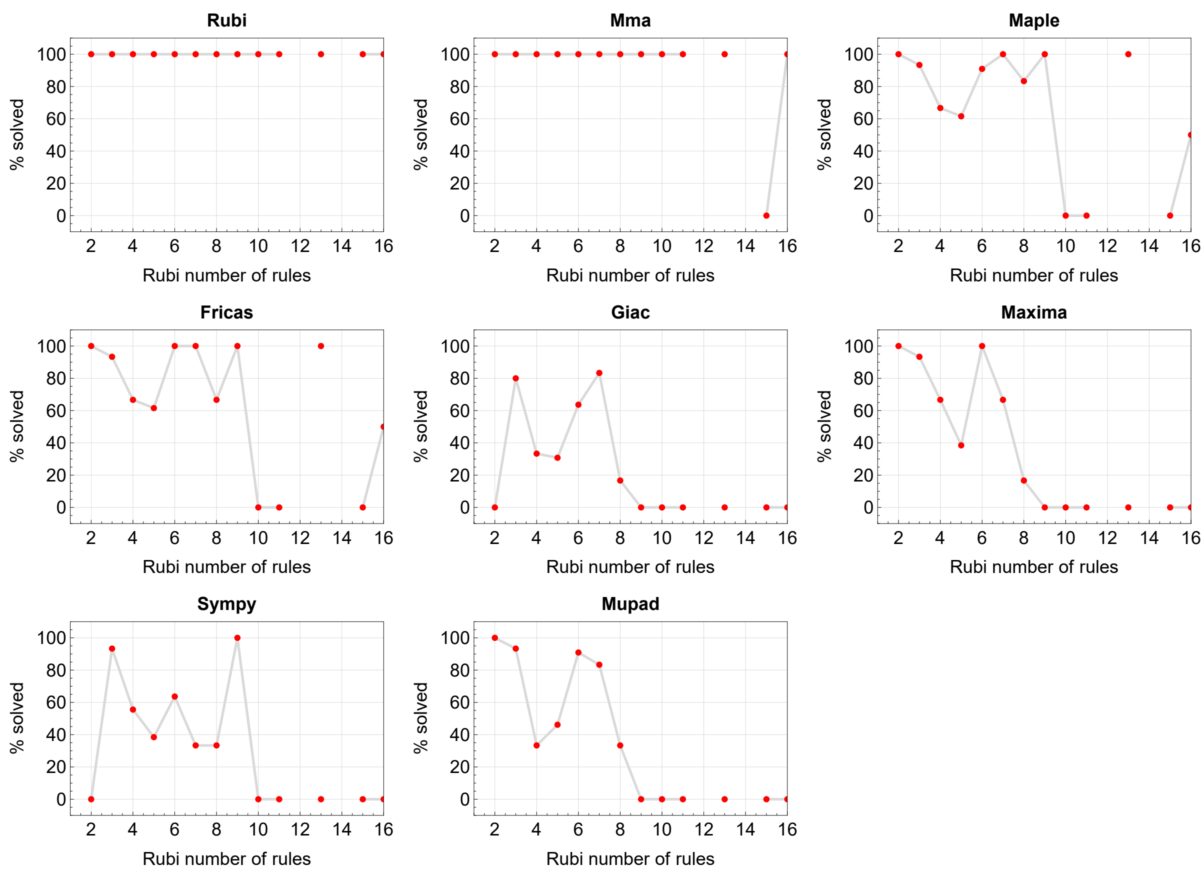


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

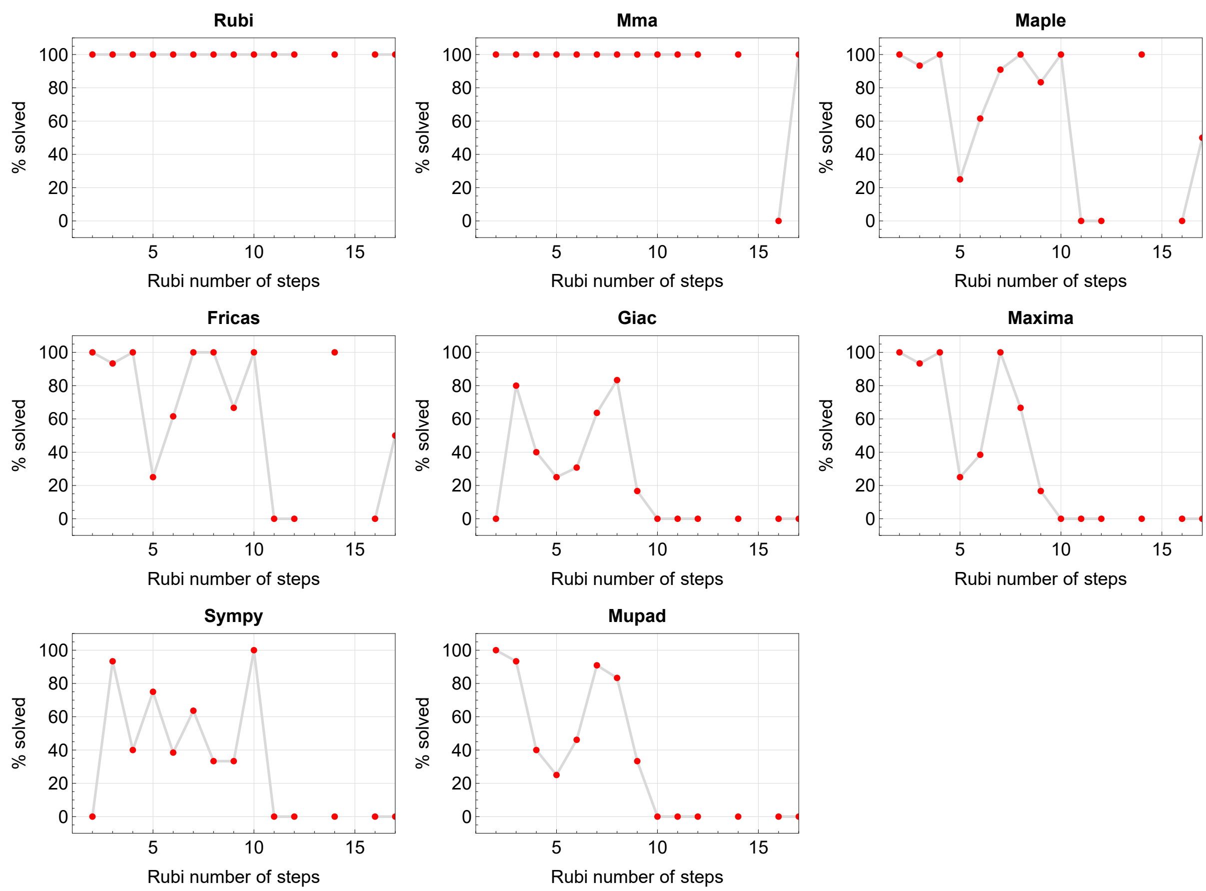


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

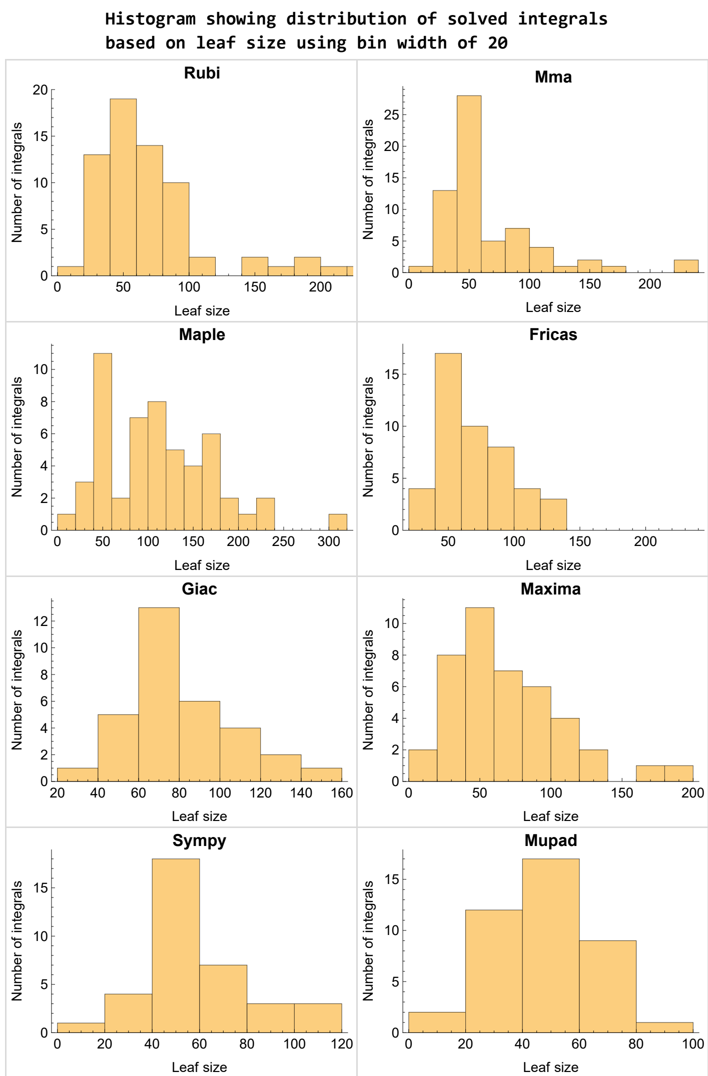


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

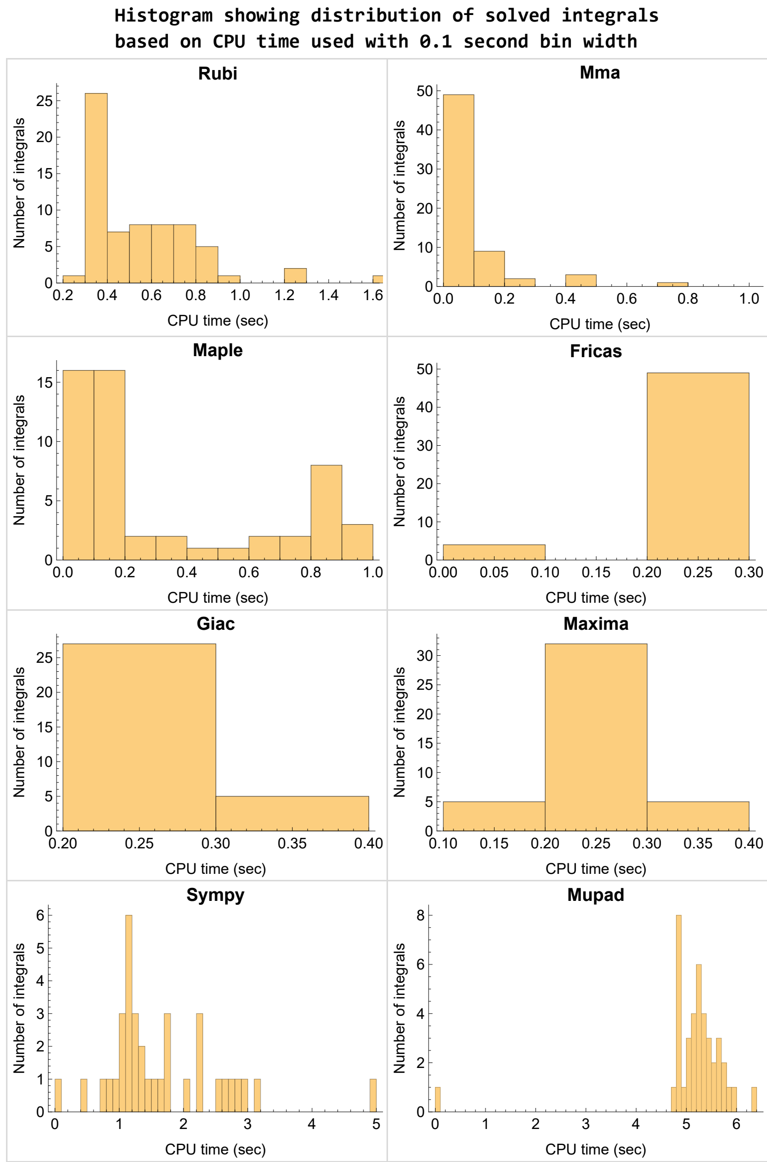


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

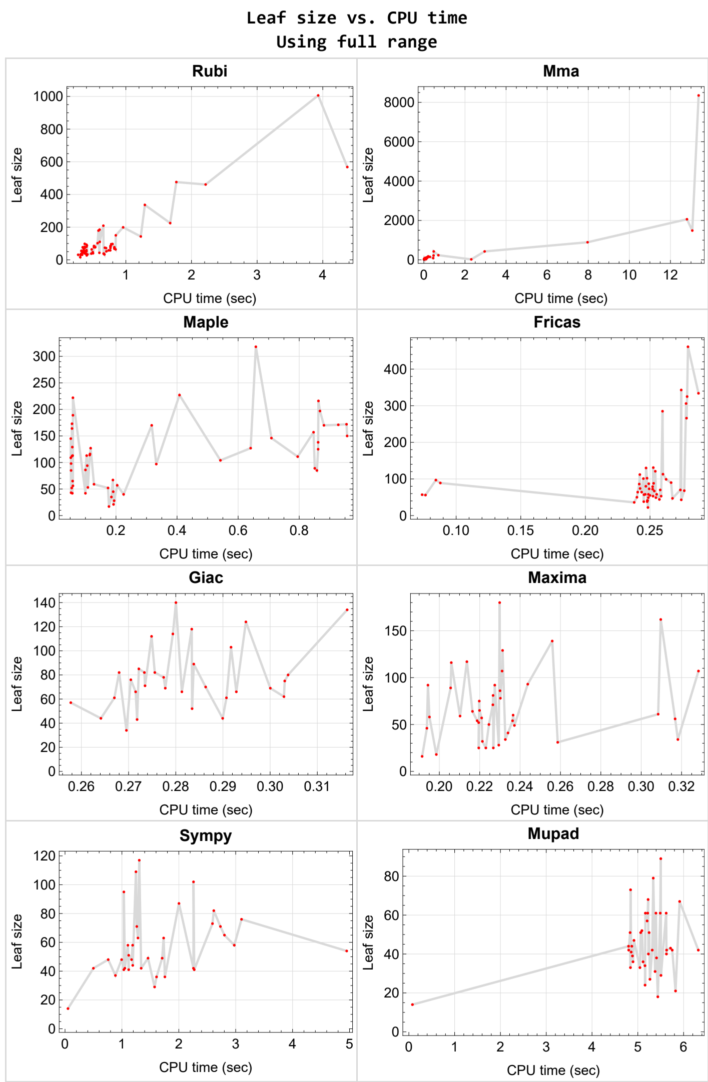


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{10, 18, 23, 24, 25, 41, 45, 55, 69, 70, 71\}$

Mathematica $\{7, 8, 12, 13\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

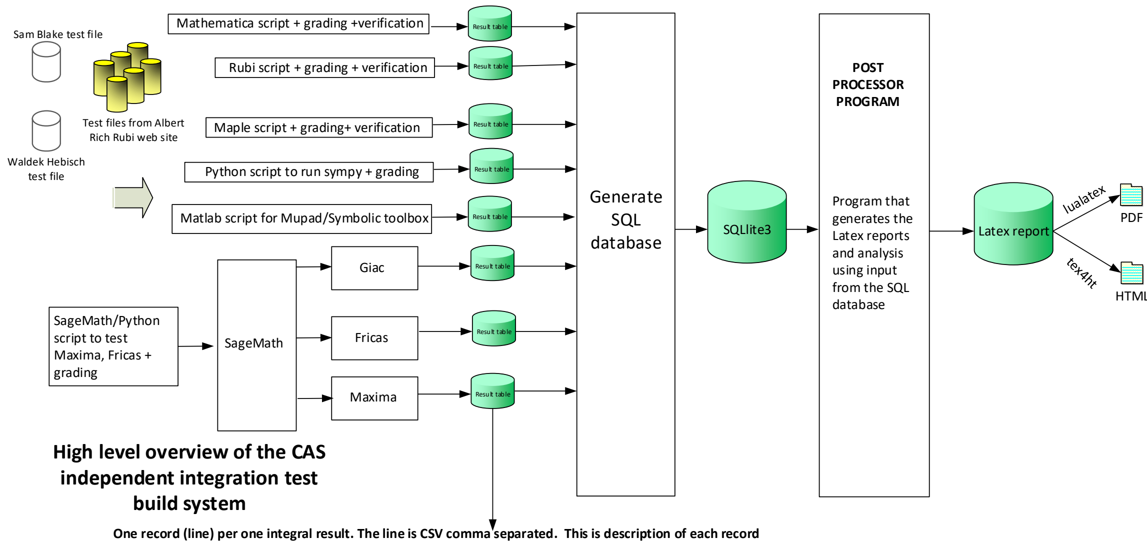
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71 }

B grade { }

C grade { 4, 10, 11, 18, 23, 24, 25, 69 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 6, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69 }

B grade { 5, 9, 10, 25, 70, 71 }

C grade { 4, 7, 8, 12, 13, 23, 38, 40, 42, 44, 46 }

F normal fail { 11 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 34, 36, 47, 49, 50, 55, 57, 70 }

B grade { 5, 6, 30, 31, 32, 33, 35, 39, 41, 43, 45, 51, 52, 53, 54, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68 }

C grade { 38, 40, 42, 44, 46 }

F normal fail { 4, 7, 8, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 37, 48, 59, 69, 71 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 30, 32, 34, 36, 38, 39, 40, 44, 46, 47, 49, 50, 51, 53, 57, 58, 60, 61, 62, 63, 64, 66, 68 }

B grade { 1, 2, 3, 5, 6, 31, 33, 35, 41, 43, 45, 52, 54, 55, 56, 65, 67, 70, 71 }

C grade { }

F normal fail { 4, 7, 8, 9, 10, 11, 12, 13, 18, 24, 26, 37, 42, 48, 59, 69 }

F(-1) timedout fail { }

F(-2) exception fail { 23, 25 }

2.1.5 Maxima

A grade { 14, 15, 16, 17, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 34, 36, 39, 43, 47, 49, 50, 51, 52, 53, 55, 57, 61, 63, 66, 68, 70, 71 }

B grade { 33, 35, 41, 45, 54, 56, 58, 60, 62, 64 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 38, 40, 42, 44, 46, 59, 65, 67, 69 }

F(-1) timedout fail { }

F(-2) exception fail { 37, 48 }

2.1.6 Giac

A grade { 27, 28, 29, 30, 31, 35, 39, 41, 43, 49, 51, 56, 58, 60, 61, 62, 63, 64, 66, 68 }

B grade { 33, 34, 36, 45, 47, 50, 52, 54, 55, 57, 65, 67 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 38, 40, 42, 44, 46, 69, 70, 71 }

F(-1) timedout fail { }

F(-2) exception fail { 26, 32, 37, 48, 53, 59 }

2.1.7 Mupad

A grade { }

B grade { 17, 19, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 41, 42, 43, 44, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 23, 24, 25, 26, 37, 38, 40, 46, 48, 59, 69 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 43, 45, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 66, 68 }

B grade { 54 }

C grade { 38, 40, 42, 44, 46 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 59, 60, 61, 62, 63, 64, 65, 67, 69 }

F(-1) timedout fail { 13, 70, 71 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	150	149	227	0	325	0	0	0
N.S.	1	1.02	1.01	1.54	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.541	0.293	0.408	0.000	0.279	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	129	170	0	306	0	0	0
N.S.	1	1.00	1.17	1.55	0.00	2.78	0.00	0.00	0.00
time (sec)	N/A	0.381	0.179	0.317	0.000	0.278	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	74	110	97	0	285	0	0	0
N.S.	1	0.99	1.47	1.29	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	0.432	0.096	0.332	0.000	0.260	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	225	427	0	0	0	0	0	0
N.S.	1	1.39	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.028	0.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	72	141	127	0	343	0	0	0
N.S.	1	1.14	2.24	2.02	0.00	5.44	0.00	0.00	0.00
time (sec)	N/A	0.420	0.168	0.641	0.000	0.274	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	143	220	318	0	461	0	0	0
N.S.	1	1.25	1.93	2.79	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	0.737	0.472	0.658	0.000	0.279	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	501	476	1487	0	0	0	0	0	0
N.S.	1	0.95	2.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.107	13.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	351	336	893	0	0	0	0	0	0
N.S.	1	0.96	2.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.802	7.967	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	199	427	0	0	0	0	0	0
N.S.	1	1.03	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	2.957	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	84	176	0	0	0	0	0	0
N.S.	1	0.99	2.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	0.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	475	568	0	0	0	0	0	0	0
N.S.	1	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.610	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	448	461	2061	0	0	0	0	0	0
N.S.	1	1.03	4.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.350	12.804	0.000	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1024	1006	8350	0	0	0	0	0	0
N.S.	1	0.98	8.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.428	13.372	0.000	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	79	47	40	58	55	0	0	0
N.S.	1	0.69	0.41	0.35	0.51	0.48	0.00	0.00	0.00
time (sec)	N/A	0.222	0.027	0.225	0.195	0.250	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	68	42	35	46	50	0	0	0
N.S.	1	0.76	0.47	0.39	0.52	0.56	0.00	0.00	0.00
time (sec)	N/A	0.221	0.020	0.185	0.194	0.240	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	55	35	28	34	43	0	0	0
N.S.	1	0.86	0.55	0.44	0.53	0.67	0.00	0.00	0.00
time (sec)	N/A	0.208	0.019	0.194	0.233	0.274	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	21	18	36	0	0	18
N.S.	1	1.00	0.77	0.68	0.58	1.16	0.00	0.00	0.58
time (sec)	N/A	0.184	2.309	0.192	0.199	0.238	0.000	0.000	5.437

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	58	46	0	0	0	0	0	0
N.S.	1	1.26	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	54	42	45	65	44	0	0	33
N.S.	1	0.86	0.67	0.71	1.03	0.70	0.00	0.00	0.52
time (sec)	N/A	0.213	0.017	0.191	0.220	0.257	0.000	0.000	4.839

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	77	47	57	92	53	0	0	0
N.S.	1	0.86	0.52	0.63	1.02	0.59	0.00	0.00	0.00
time (sec)	N/A	0.215	0.028	0.204	0.194	0.259	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	98	52	67	116	58	0	0	0
N.S.	1	0.85	0.45	0.58	1.01	0.50	0.00	0.00	0.00
time (sec)	N/A	0.228	0.032	0.190	0.206	0.249	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	17	16	22	14	0	14
N.S.	1	1.00	1.12	1.06	1.00	1.38	0.88	0.00	0.88
time (sec)	N/A	0.205	0.003	0.177	0.191	0.248	0.052	0.000	0.079

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	77	64	0	0	0	0	0	0
N.S.	1	1.26	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	68	50	0	0	0	0	0	0
N.S.	1	1.26	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.513	0.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	97	236	0	0	0	0	0	0
N.S.	1	1.26	3.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	0.706	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	54	0	0	0	65	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	1.25	0.00	0.00
time (sec)	N/A	0.251	0.034	0.000	0.000	0.000	2.803	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	53	50	53	63	78	41
N.S.	1	1.00	0.91	0.98	0.93	0.98	1.17	1.44	0.76
time (sec)	N/A	0.233	0.033	0.108	0.225	0.252	1.282	0.277	4.856

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	76	112	107	79	73	69	61
N.S.	1	1.07	1.01	1.49	1.43	1.05	0.97	0.92	0.81
time (sec)	N/A	0.246	0.042	0.056	0.231	0.252	2.592	0.300	5.490

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	43	25	41	41	44	33
N.S.	1	1.00	1.23	1.39	0.81	1.32	1.32	1.42	1.06
time (sec)	N/A	0.208	0.030	0.053	0.223	0.248	1.119	0.264	5.046

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	46	47	85	78	64	29	52	39
N.S.	1	0.98	1.00	1.81	1.66	1.36	0.62	1.11	0.83
time (sec)	N/A	0.217	0.023	0.053	0.230	0.244	1.575	0.283	4.883

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	35	35	113	64	86	48	66	36
N.S.	1	1.46	1.46	4.71	2.67	3.58	2.00	2.75	1.50
time (sec)	N/A	0.211	0.016	0.104	0.216	0.242	0.759	0.271	4.895

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	45	42	109	54	64	41	0	34
N.S.	1	1.18	1.11	2.87	1.42	1.68	1.08	0.00	0.89
time (sec)	N/A	0.225	0.028	0.052	0.236	0.240	2.271	0.000	5.156

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	43	145	86	102	37	82	42
N.S.	1	1.10	1.08	3.62	2.15	2.55	0.92	2.05	1.05
time (sec)	N/A	0.215	0.020	0.052	0.230	0.248	0.886	0.268	5.629

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	37	42	25	47	48	69	42
N.S.	1	1.00	1.19	1.35	0.81	1.52	1.55	2.23	1.35
time (sec)	N/A	0.199	0.024	0.057	0.227	0.267	0.995	0.278	4.802

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	74	53	173	129	113	95	103	61
N.S.	1	1.14	0.82	2.66	1.98	1.74	1.46	1.58	0.94
time (sec)	N/A	0.242	0.033	0.056	0.231	0.260	1.035	0.292	5.619

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	46	52	41	58	58	124	61
N.S.	1	1.10	0.90	1.02	0.80	1.14	1.14	2.43	1.20
time (sec)	N/A	0.230	0.026	0.055	0.234	0.246	1.105	0.295	5.165

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	0	71	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.20	0.00	0.00
time (sec)	N/A	0.247	0.047	0.000	0.000	0.000	2.731	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	209	112	150	0	89	48	0	0
N.S.	1	1.03	0.55	0.74	0.00	0.44	0.24	0.00	0.00
time (sec)	N/A	0.391	0.174	0.957	0.000	0.088	1.170	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	53	94	81	70	36	57	42
N.S.	1	0.98	1.02	1.81	1.56	1.35	0.69	1.10	0.81
time (sec)	N/A	0.231	0.038	0.106	0.227	0.273	1.755	0.258	5.315

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	88	113	104	0	56	41	0	0
N.S.	1	1.02	1.31	1.21	0.00	0.65	0.48	0.00	0.00
time (sec)	N/A	0.256	0.164	0.542	0.000	0.076	1.033	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	42	116	71	88	58	61	43
N.S.	1	1.00	1.05	2.90	1.78	2.20	1.45	1.52	1.08
time (sec)	N/A	0.229	0.022	0.115	0.227	0.253	2.975	0.267	5.714

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	179	96	146	0	0	42	0	24
N.S.	1	1.08	0.58	0.88	0.00	0.00	0.25	0.00	0.15
time (sec)	N/A	0.363	0.106	0.709	0.000	0.000	0.499	0.000	5.158

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	22	86	54	74	54	66	36
N.S.	1	1.02	0.48	1.87	1.17	1.61	1.17	1.43	0.78
time (sec)	N/A	0.219	0.038	0.100	0.219	0.242	4.951	0.281	5.115

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	96	111	0	57	42	0	27
N.S.	1	1.02	1.05	1.22	0.00	0.63	0.46	0.00	0.30
time (sec)	N/A	0.251	0.110	0.795	0.000	0.074	1.052	0.000	5.265

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	49	24	114	92	101	42	76	42
N.S.	1	1.17	0.57	2.71	2.19	2.40	1.00	1.81	1.00
time (sec)	N/A	0.237	0.034	0.114	0.227	0.245	1.339	0.270	6.321

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	185	114	171	0	97	44	0	0
N.S.	1	1.02	0.63	0.94	0.00	0.54	0.24	0.00	0.00
time (sec)	N/A	0.364	0.146	0.928	0.000	0.084	1.190	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	42	25	49	49	71	44
N.S.	1	1.00	1.26	1.35	0.81	1.58	1.58	2.29	1.42
time (sec)	N/A	0.201	0.034	0.100	0.220	0.255	1.459	0.273	4.798

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	76	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	1.19	0.00	0.00
time (sec)	N/A	0.532	0.046	0.000	0.000	0.000	3.103	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	84	127	117	87	82	80	73
N.S.	1	1.00	0.99	1.49	1.38	1.02	0.96	0.94	0.86
time (sec)	N/A	0.488	0.044	0.117	0.214	0.249	2.615	0.304	4.842

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	59	32	49	51	66	40
N.S.	1	1.00	1.16	1.55	0.84	1.29	1.34	1.74	1.05
time (sec)	N/A	0.428	0.032	0.128	0.221	0.249	1.120	0.293	5.231

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	57	98	89	72	36	62	51
N.S.	1	1.00	1.10	1.88	1.71	1.38	0.69	1.19	0.98
time (sec)	N/A	0.441	0.025	0.053	0.206	0.252	1.609	0.303	5.063

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	44	129	75	99	63	82	52
N.S.	1	1.00	1.02	3.00	1.74	2.30	1.47	1.91	1.21
time (sec)	N/A	0.368	0.033	0.057	0.220	0.262	1.733	0.276	5.090

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	113	59	73	49	0	47
N.S.	1	1.00	1.11	2.40	1.26	1.55	1.04	0.00	1.00
time (sec)	N/A	0.310	0.029	0.058	0.210	0.249	1.708	0.000	4.916

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	39	164	93	112	87	118	44
N.S.	1	1.00	1.03	4.32	2.45	2.95	2.29	3.11	1.16
time (sec)	N/A	0.409	0.036	0.056	0.244	0.242	2.003	0.283	4.869

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	33	46	56	28	57	58	75	51
N.S.	1	0.97	1.35	1.65	0.82	1.68	1.71	2.21	1.50
time (sec)	N/A	0.429	0.029	0.059	0.229	0.245	1.189	0.303	4.832

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	189	139	121	109	112	68
N.S.	1	1.00	1.00	2.59	1.90	1.66	1.49	1.53	0.93
time (sec)	N/A	0.478	0.034	0.059	0.256	0.254	1.247	0.275	5.225

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	65	52	67	71	134	67
N.S.	1	1.00	0.93	1.12	0.90	1.16	1.22	2.31	1.16
time (sec)	N/A	0.461	0.034	0.058	0.220	0.253	1.262	0.316	5.913

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	74	222	180	131	117	140	89
N.S.	1	1.00	0.77	2.31	1.88	1.36	1.22	1.46	0.93
time (sec)	N/A	0.480	0.050	0.059	0.230	0.253	1.305	0.280	5.502

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	88	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.454	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	102	85	172	162	90	0	89	79
N.S.	1	1.11	0.92	1.87	1.76	0.98	0.00	0.97	0.86
time (sec)	N/A	0.349	0.085	0.955	0.310	0.266	0.000	0.284	5.337

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	64	125	49	58	0	85	61
N.S.	1	1.07	0.89	1.74	0.68	0.81	0.00	1.18	0.85
time (sec)	N/A	0.332	0.058	0.862	0.237	0.255	0.000	0.272	5.216

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	64	54	138	107	68	0	61	51
N.S.	1	1.08	0.92	2.34	1.81	1.15	0.00	1.03	0.86
time (sec)	N/A	0.297	0.065	0.862	0.328	0.277	0.000	0.291	5.247

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	89	31	38	0	44	31
N.S.	1	1.00	0.97	2.47	0.86	1.06	0.00	1.22	0.86
time (sec)	N/A	0.254	0.039	0.851	0.259	0.245	0.000	0.290	5.377

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	85	61	38	0	34	21
N.S.	1	1.00	1.41	3.15	2.26	1.41	0.00	1.26	0.78
time (sec)	N/A	0.259	0.115	0.858	0.308	0.248	0.000	0.270	5.824

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	38	37	170	0	80	0	70	38
N.S.	1	1.15	1.12	5.15	0.00	2.42	0.00	2.12	1.15
time (sec)	N/A	0.243	0.069	0.881	0.000	0.247	0.000	0.286	5.409

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	30	157	34	41	42	43	29
N.S.	1	1.10	1.00	5.23	1.13	1.37	1.40	1.43	0.97
time (sec)	N/A	0.252	0.061	0.847	0.318	0.248	2.257	0.272	5.505

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	81	58	216	0	130	0	114	61
N.S.	1	1.35	0.97	3.60	0.00	2.17	0.00	1.90	1.02
time (sec)	N/A	0.328	0.076	0.863	0.000	0.247	0.000	0.279	5.396

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	77	54	197	56	70	102	82	57
N.S.	1	1.26	0.89	3.23	0.92	1.15	1.67	1.34	0.93
time (sec)	N/A	0.327	0.082	0.868	0.317	0.258	2.259	0.273	5.203

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	73	53	0	0	0	0	0	0
N.S.	1	1.20	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	38	95	52	57	266	0	0	42
N.S.	1	0.83	2.07	1.13	1.24	5.78	0.00	0.00	0.91
time (sec)	N/A	0.297	0.097	0.174	0.221	0.278	0.000	0.000	5.752

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	B	F(-1)	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	40	93	0	60	334	0	0	40
N.S.	1	0.87	2.02	0.00	1.30	7.26	0.00	0.00	0.87
time (sec)	N/A	0.316	0.127	0.000	0.237	0.287	0.000	0.000	5.626

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [3] had the largest ratio of [1.62500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.02	10	0.800
2	A	6	5	1.00	10	0.500
3	A	14	13	0.99	8	1.625
4	C	17	16	1.39	10	1.600
5	A	9	8	1.14	10	0.800
6	A	17	16	1.25	10	1.600
7	A	6	5	0.95	20	0.250
8	A	6	5	0.96	20	0.250
9	A	6	5	1.03	18	0.278
10	C	9	8	0.99	12	0.667
11	C	16	15	1.20	20	0.750
12	A	6	5	1.03	20	0.250
13	A	6	5	0.98	20	0.250
14	A	4	4	0.69	10	0.400
15	A	4	4	0.76	10	0.400
16	A	4	4	0.86	8	0.500
17	A	2	2	1.00	6	0.333
18	C	11	10	1.26	10	1.000
19	A	6	5	0.86	10	0.500
20	A	7	6	0.86	10	0.600
21	A	8	7	0.85	10	0.700
22	A	3	3	1.00	4	0.750

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	C	11	10	1.26	10	1.000
24	C	11	10	1.26	10	1.000
25	C	11	10	1.26	10	1.000
26	A	5	4	1.00	10	0.400
27	A	4	4	1.00	10	0.400
28	A	8	7	1.07	10	0.700
29	A	3	3	1.00	10	0.300
30	A	7	6	0.98	8	0.750
31	A	6	5	1.46	6	0.833
32	A	7	6	1.18	10	0.600
33	A	6	5	1.10	10	0.500
34	A	3	3	1.00	10	0.300
35	A	7	6	1.14	10	0.600
36	A	6	5	1.10	10	0.500
37	A	5	4	1.00	12	0.333
38	A	10	9	1.03	12	0.750
39	A	7	6	0.98	12	0.500
40	A	6	5	1.02	12	0.417
41	A	7	6	1.00	10	0.600
42	A	9	8	1.08	8	1.000
43	A	7	6	1.02	12	0.500
44	A	6	5	1.02	12	0.417
45	A	7	6	1.17	12	0.500
46	A	9	8	1.02	12	0.667
47	A	3	3	1.00	12	0.250
48	A	3	3	1.00	12	0.250
49	A	3	3	1.00	12	0.250
50	A	3	3	1.00	12	0.250
51	A	3	3	1.00	12	0.250
52	A	3	3	1.00	10	0.300
53	A	3	3	1.00	8	0.375
54	A	3	3	1.00	12	0.250
55	A	5	4	0.97	12	0.333
56	A	3	3	1.00	12	0.250

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	12	0.250
58	A	3	3	1.00	12	0.250
59	A	5	4	1.00	23	0.174
60	A	9	8	1.11	21	0.381
61	A	7	6	1.07	21	0.286
62	A	8	7	1.08	21	0.333
63	A	3	3	1.00	21	0.143
64	A	6	5	1.00	19	0.263
65	A	8	7	1.15	18	0.389
66	A	4	4	1.10	21	0.190
67	A	8	7	1.35	21	0.333
68	A	8	7	1.26	21	0.333
69	C	12	11	1.20	19	0.579
70	A	7	6	0.83	12	0.500
71	A	7	6	0.87	14	0.429

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \mathrm{csch}^{-1}(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 147

$$\int x^3 \mathrm{csch}^{-1}(a+bx)\,dx = -\frac{(2-17a^2)(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^4}+\frac{x^2(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{12b^2} -\frac{a(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\mathrm{csch}^{-1}(a+bx)}{4b^4} +\frac{1}{4}x^4\mathrm{csch}^{-1}(a+bx)+\frac{a(1-2a^2)\,\mathrm{arctanh}\left(\sqrt{1+\frac{1}{(a+bx)^2}}\right)}{2b^4}$$

output
```
-1/4*a^4*arccsch(b*x+a)/b^4+1/4*x^4*arccsch(b*x+a)+1/2*a*(-2*a^2+1)*arctan
h((1+1/(b*x+a)^2)^(1/2))/b^4-1/12*(-17*a^2+2)*(b*x+a)*(1+1/(b*x+a)^2)^(1/2
)/b^4+1/12*x^2*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^2-1/3*a*(b*x+a)^2*(1+1/(b*x
+a)^2)^(1/2)/b^4
```

3.1.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int x^3 \mathrm{csch}^{-1}(a+bx)\,dx = \frac{\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}(-2a+13a^3-2bx+9a^2bx-3ab^2x^2+b^3x^3)+3b^4x^4\mathrm{csch}^{-1}(a+bx)-3a^4\mathrm{arcsinh}\left(\frac{1}{a+bx}\right)}{12b^4}$$

input
```
Integrate[x^3*ArcCsch[a + b*x],x]
```

output
```
(Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(-2*a + 13*a^3 - 2*b*x +
9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3) + 3*b^4*x^4*ArcCsch[a + b*x] - 3*a^4*Ar
cSinh[(a + b*x)^(-1)] + 6*a*(1 - 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(1 + a^2 +
 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

3.1.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6876, 25, 5992, 3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6876}$$

$$-\frac{\int b^3x^3(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -b^3x^3(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{5992}$$

$$-\frac{\frac{1}{4}\int b^4x^4d\text{csch}^{-1}(a+bx)-\frac{1}{4}b^4x^4\text{csch}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}b^4x^4\text{csch}^{-1}(a+bx)+\frac{1}{4}\int\left(a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)\right)^4d\text{csch}^{-1}(a+bx)}{b^4}$$

$$\Big\downarrow \text{4269}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\int -bx\left(3a^3+8(a+bx)^2a+\left(2-9a^2\right)(a+bx)\right)d\text{csch}^{-1}(a+bx)-\frac{1}{3}b^2x^2(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}\right)-\frac{1}{4}b^4x^4\text{cs}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}b^4x^4\text{csch}^{-1}(a+bx)+\frac{1}{4}\left(-\frac{1}{3}b^2x^2(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}+\frac{1}{3}\int\left(a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)\right)\left(3a^3-8\csc\left(i\text{csc}\right.\right.\right.}{b^4}$$

$$\Big\downarrow 4536$$

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(6a^4+12\left(1-2a^2\right)(a+bx)a-2\left(2-17a^2\right)(a+bx)^2\right)d\text{csch}^{-1}(a+bx)+4a\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)^2\right)\right.}{b^4}$$

$$\Big\downarrow 2009$$

$$-\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(6a^4\text{csch}^{-1}(a+bx)-12\left(1-2a^2\right)a\text{arctanh}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)+2\left(2-17a^2\right)(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}\right)+4a\right.\right.}{b^4}$$

input
```
Int[x^3*ArcCsch[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCsch[a + b*x]) + (-1/3*(b^2*x^2*(a + b*x)*Sqrt[1 + (a
+ b*x)^(-2)]) + (4*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^(-2)] + (2*(2 - 17*a^2
)*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)] + 6*a^4*ArcCsch[a + b*x] - 12*a*(1 -
2*a^2)*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/2)/3)/4)/b^4)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.1.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{\frac{\operatorname{arccsch}(bx+a)a^4}{4}-\operatorname{arccsch}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccsch}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccsch}(bx+a)a(bx+a)^3+\frac{\operatorname{arccsch}(bx+a)(bx+a)^4}{4}-\cdots}{\cdots}$
default	$\frac{\frac{\operatorname{arccsch}(bx+a)a^4}{4}-\operatorname{arccsch}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccsch}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccsch}(bx+a)a(bx+a)^3+\frac{\operatorname{arccsch}(bx+a)(bx+a)^4}{4}-\cdots}{\cdots}$
parts	$\frac{x^4\operatorname{arccsch}(bx+a)}{4}-\frac{\sqrt{b^2x^2+2abx+a^2+1}\left(3a^4\operatorname{arctanh}\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2+1}}\right)\sqrt{b^2}-x^2\sqrt{b^2x^2+2abx+a^2+1}\,b^2\sqrt{b^2}+4\cdots\right.}{\cdots}$

input
```
int(x^3*arccsch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*arccsch(b*x+a)*a^4-arccsch(b*x+a)*a^3*(b*x+a)+3/2*arccsch(b*x+a
)*a^2*(b*x+a)^2-arccsch(b*x+a)*a*(b*x+a)^3+1/4*arccsch(b*x+a)*(b*x+a)^4-1/
12*((b*x+a)^2+1)^(1/2)*(3*a^4*arctanh(1/((b*x+a)^2+1)^(1/2))+12*a^3*arcsin
h(b*x+a)-18*a^2*((b*x+a)^2+1)^(1/2)+6*a*(b*x+a)*((b*x+a)^2+1)^(1/2)-(b*x+a
)^2*((b*x+a)^2+1)^(1/2)-6*a*arcsinh(b*x+a)+2*((b*x+a)^2+1)^(1/2))/(((b*x+a
)^2+1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.21

$$\int x^3 \text{csch}^{-1}(a+bx)\,dx$$

$$= \frac{3\,b^4x^4\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right) - 3\,a^4\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right) + 3\,a^4\log\left(-b\right.}{}$$

input
```
integrate(x^3*arccsch(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^4*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2
 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(-b*x + (b*x + a)*sqrt((b^2*
x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) + 3*a^4*log(-
b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^
2)) - a - 1) + 6*(2*a^3 - a)*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a) + (b^3*x^3 - 3*a*b^2*x^2 + 13*a
^3 + (9*a^2 - 2)*b*x - 2*a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 +
2*a*b*x + a^2)))/b^4
```

3.1.6 Sympy [F]

$$\int x^3 \text{csch}^{-1}(a+bx)\,dx = \int x^3 \operatorname{acsch}(a+bx)\,dx$$

input
```
integrate(x**3*acsch(b*x+a),x)
```

output
```
Integral(x**3*acsch(a + b*x), x)
```

3.1.7 Maxima [F]

$$\int x^3 \text{csch}^{-1}(a+bx)\,dx = \int x^3 \operatorname{arcsch}(bx+a)\,dx$$

input
```
integrate(x^3*arccsch(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(-I*a^3 + I*a)*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b +
 1))/b^4 + 1/8*(2*b^4*x^4*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) + b^2
*x^2 - 6*a*b*x - (a^4 - 6*a^2 + 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b
^4*x^4 - a^4)*log(b*x + a))/b^4 + integrate(1/4*(b^2*x^5 + a*b*x^4)/(b^2*x
^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 1), x)
```

3.1.8 Giac [F]

$$\int x^3 \text{csch}^{-1}(a+bx)\,dx = \int x^3 \operatorname{arcsch}(bx+a)\,dx$$

input
```
integrate(x^3*arccsch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*arccsch(b*x + a), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{csch}^{-1}(a+bx)\,dx = \int x^3 \operatorname{asinh}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x^3*asinh(1/(a + b*x)),x)
```

output
```
int(x^3*asinh(1/(a + b*x)), x)
```

3.2 $\int x^2 \text{csch}^{-1}(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 110

$$\int x^2 \text{csch}^{-1}(a+bx)\,dx = -\frac{5a(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^3}+\frac{x(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\text{csch}^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{csch}^{-1}(a+bx) - \frac{(1-6a^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{(a+bx)^2}}\right)}{6b^3}$$

output
```
1/3*a^3*arccsch(b*x+a)/b^3+1/3*x^3*arccsch(b*x+a)-1/6*(-6*a^2+1)*arctanh((
1+1/(b*x+a)^2)^(1/2))/b^3-5/6*a*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^3+1/6*x*(b
*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^2
```

3.2.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int x^2 \text{csch}^{-1}(a+bx)\,dx = \frac{(-5a^2-4abx+b^2x^2)\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}+2b^3x^3\text{csch}^{-1}(a+bx)+2a^3\text{arcsinh}\left(\frac{1}{a+bx}\right)+(-1+6a^2)\log\left((a-\right.}{6b^3}$$

input
```
Integrate[x^2*ArcCsch[a + b*x],x]
```

output
```
((-5*a^2 - 4*a*b*x + b^2*x^2)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)
^2] + 2*b^3*x^3*ArcCsch[a + b*x] + 2*a^3*ArcSinh[(a + b*x)^(-1)] + (-1 + 6
*a^2)*Log[(a + b*x)*(1 + Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])]
)/(6*b^3)
```

3.2.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6876, 5992, 3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6876}$$

$$-\frac{\int b^2x^2(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 5992}$$

$$-\frac{-\frac{1}{3}\int -b^3x^3d\text{csch}^{-1}(a+bx)-\frac{1}{3}b^3x^3\text{csch}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{3}b^3x^3\text{csch}^{-1}(a+bx)-\frac{1}{3}\int \left(a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)\right)^3d\text{csch}^{-1}(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 4269}$$

$$-\frac{\frac{1}{3}\left(-\frac{1}{2}\int \left(2a^3+5(a+bx)^2a+\left(1-6a^2\right)(a+bx)\right)d\text{csch}^{-1}(a+bx)-\frac{1}{2}bx\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)\right)-\frac{1}{3}b^3x^3\text{csch}^{-1}}{b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\left(-2a^3\text{csch}^{-1}(a+bx)+\left(1-6a^2\right)\text{arctanh}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)+5a(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}\right)-\frac{1}{2}bx(a+bx)\sqrt{\frac{1}{(a+}}\right.}{b^3}$$

input
```
Int[x^2*ArcCsch[a + b*x],x]
```

output
```
-((-1/3*(b^3*x^3*ArcCsch[a + b*x]) + (-1/2*(b*x*(a + b*x)*Sqrt[1 + (a + b*
x)^(-2)]) + (5*a*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)] - 2*a^3*ArcCsch[a + b*
x] + (1 - 6*a^2)*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/2)/3)/b^3)
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.2.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccsch}(bx+a)a^3}{3}+\operatorname{arccsch}(bx+a)a^2(bx+a)-\operatorname{arccsch}(bx+a)a(bx+a)^2+\frac{\operatorname{arccsch}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{(bx+a)^2+1}\left(2a^3\operatorname{arctanh}\right.}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccsch}(bx+a)a^3}{3}+\operatorname{arccsch}(bx+a)a^2(bx+a)-\operatorname{arccsch}(bx+a)a(bx+a)^2+\frac{\operatorname{arccsch}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{(bx+a)^2+1}\left(2a^3\operatorname{arctanh}\right.}{}}{b^3}$
parts	$\frac{x^3\operatorname{arccsch}(bx+a)}{3}+\frac{\sqrt{b^2x^2+2abx+a^2+1}\left(2a^3\operatorname{arctanh}\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2+1}}\right)\sqrt{b^2}+6\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2+1}\sqrt{b^2+}}{\sqrt{b^2}}\right.\right.}{6b^3\sqrt{\frac{b^2x^2+}{(}}}$

input
```
int(x^2*arccsch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/3*arccsch(b*x+a)*a^3+arccsch(b*x+a)*a^2*(b*x+a)-arccsch(b*x+a)*a
*(b*x+a)^2+1/3*arccsch(b*x+a)*(b*x+a)^3+1/6*((b*x+a)^2+1)^(1/2)*(2*a^3*arc
tanh(1/((b*x+a)^2+1)^(1/2))+6*a^2*arcsinh(b*x+a)-6*a*((b*x+a)^2+1)^(1/2)+(
b*x+a)*((b*x+a)^2+1)^(1/2)-arcsinh(b*x+a))/(((b*x+a)^2+1)/(b*x+a)^2)^(1/2)
/(b*x+a))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.78

$$\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx$$

$$= \frac{2b^3x^3\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)+2a^3\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2abx+a^2+1}{b^2x^2+2abx+a^2}}-a+1\right)-2a^3\log\left(-b\right.}{}$$

input
```
integrate(x^2*arccsch(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) + 2*a^3*log(-b*x + (b*x + a)*sqrt((b^2*x
^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) - 2*a^3*log(-b
*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2
)) - a - 1) - (6*a^2 - 1)*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a
^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a) + (b^2*x^2 - 4*a*b*x - 5*a^2)*sqrt
((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^3
```

3.2.6 Sympy [F]

$$\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx = \int x^2 \operatorname{acsch}(a+bx)\,dx$$

input
```
integrate(x**2*acsch(b*x+a),x)
```

output
```
Integral(x**2*acsch(a + b*x), x)
```

3.2.7 Maxima [F]

$$\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx = \int x^2 \operatorname{arcsch}(bx+a)\,dx$$

input
```
integrate(x^2*arccsch(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(3*I*a^2 - I)*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b +
1))/b^3 + 1/6*(2*b^3*x^3*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) + 2*b*
x + (a^3 - 3*a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b^3*x^3 + a^3)*log(b
*x + a))/b^3 + integrate(1/3*(b^2*x^4 + a*b*x^3)/(b^2*x^2 + 2*a*b*x + a^2
+ (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 1), x)
```

3.2.8 Giac [F]

$$\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx = \int x^2 \operatorname{arcsch}(bx+a)\,dx$$

input
```
integrate(x^2*arccsch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*arccsch(b*x + a), x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{csch}^{-1}(a+bx)\,dx = \int x^2 \operatorname{asinh}\left(\frac{1}{a+b\,x}\right)\,dx$$

input
```
int(x^2*asinh(1/(a + b*x)),x)
```

output
```
int(x^2*asinh(1/(a + b*x)), x)
```

3.3 $\int x\text{csch}^{-1}(a+bx)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 75

$$\int x\text{csch}^{-1}(a+bx)\,dx=\frac{(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2b^2}-\frac{a^2\text{csch}^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{csch}^{-1}(a+bx)-\frac{a\text{arctanh}\left(\sqrt{1+\frac{1}{(a+bx)^2}}\right)}{b^2}$$

output
```
-1/2*a^2*arccsch(b*x+a)/b^2+1/2*x^2*arccsch(b*x+a)-a*arctanh((1+1/(b*x+a)^
2)^(1/2))/b^2+1/2*(b*x+a)*(1+1/(b*x+a)^2)^(1/2)/b^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int x\text{csch}^{-1}(a+bx)\,dx$$
$$=\frac{(a+bx)\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}+b^2x^2\text{csch}^{-1}(a+bx)-a^2\text{arcsinh}\left(\frac{1}{a+bx}\right)-2a\log\left((a+bx)\left(1+\sqrt{\frac{1+a^2+2abx+b^2}{(a+bx)^2}}\right.\right.}{2b^2}$$

input
```
Integrate[x*ArcCsch[a + b*x],x]
```

output
```
((a + b*x)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + b^2*x^2*ArcCs
ch[a + b*x] - a^2*ArcSinh[(a + b*x)^(-1)] - 2*a*Log[(a + b*x)*(1 + Sqrt[(1
 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(2*b^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {6876, 25, 5992, 3042, 4260, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^{-1}(a+bx)\,dx$$

$$\Big\downarrow \text{6876}$$

$$-\frac{\int bx(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -bx(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{5992}$$

$$-\frac{\frac{1}{2}\int b^2x^2d\text{csch}^{-1}(a+bx)-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\int\left(a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)\right)^2d\text{csch}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{4260}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\left(-2ia\int -i(a+bx)d\text{csch}^{-1}(a+bx)-\int -(a+bx)^2d\text{csch}^{-1}(a+bx)+a^2\text{csch}^{-1}(a+\right.}{b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\left(-2ia\int -i(a+bx)d\text{csch}^{-1}(a+bx)+\int(a+bx)^2d\text{csch}^{-1}(a+bx)+a^2\text{csch}^{-1}(a+bx\right.}{b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{1}{2}\left(-2a\int(a+bx)d\text{csch}^{-1}(a+bx)+\int(a+bx)^2d\text{csch}^{-1}(a+bx)+a^2\text{csch}^{-1}(a+bx)\right)-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\Big(-2a\int i\csc\left(i\text{csch}^{-1}(a+bx)\right)d\text{csch}^{-1}(a+bx)+\int -\csc\left(i\text{csch}^{-1}(a+bx)\right)^2d\text{csch}^{-1}}{b^2}$$

$\downarrow$ 25

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\Big(-2a\int i\csc\left(i\text{csch}^{-1}(a+bx)\right)d\text{csch}^{-1}(a+bx)-\int \csc\left(i\text{csch}^{-1}(a+bx)\right)^2d\text{csch}^{-1}(a}{b^2}$$

$\downarrow$ 26

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\Big(-2ia\int \csc\left(i\text{csch}^{-1}(a+bx)\right)d\text{csch}^{-1}(a+bx)-\int \csc\left(i\text{csch}^{-1}(a+bx)\right)^2d\text{csch}^{-1}(a}{b^2}$$

$\downarrow$ 4254

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\left(-i\int 1d\left(-i(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}\right)-2ia\int \csc\left(i\text{csch}^{-1}(a+bx)\right)d\text{csch}^{-1}(a+bx)+\right.}{b^2}$$

$\downarrow$ 24

$$-\frac{-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)+\frac{1}{2}\Big(-2ia\int \csc\left(i\text{csch}^{-1}(a+bx)\right)d\text{csch}^{-1}(a+bx)+a^2\text{csch}^{-1}(a+bx)-(a+bx)\sqrt{\frac{1}{(a+b}}}{b^2}$$

$\downarrow$ 4257

$$-\frac{\frac{1}{2}\left(a^2\text{csch}^{-1}(a+bx)+2a\text{arctanh}\left(\sqrt{\frac{1}{(a+bx)^2}+1}\right)-(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}\right)-\frac{1}{2}b^2x^2\text{csch}^{-1}(a+bx)}{b^2}$$

input
```
Int[x*ArcCsch[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCsch[a + b*x]) + (-((a + b*x)*Sqrt[1 + (a + b*x)^(-2)]
) + a^2*ArcCsch[a + b*x] + 2*a*ArcTanh[Sqrt[1 + (a + b*x)^(-2)]])/2)/b^2)
```

3.3. $\int x\text{csch}^{-1}(a+bx)\,dx$

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.3.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\operatorname{arccsch}(bx+a)a(bx+a)+\frac{\operatorname{arccsch}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{(bx+a)^2+1}\left(2a\operatorname{arcsinh}(bx+a)-\sqrt{(bx+a)^2+1}\right)}{2(bx+a)\sqrt{\frac{(bx+a)^2+1}{(bx+a)^2}}}}{b^2}$
default	$\frac{-\operatorname{arccsch}(bx+a)a(bx+a)+\frac{\operatorname{arccsch}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{(bx+a)^2+1}\left(2a\operatorname{arcsinh}(bx+a)-\sqrt{(bx+a)^2+1}\right)}{2(bx+a)\sqrt{\frac{(bx+a)^2+1}{(bx+a)^2}}}}{b^2}$
parts	$\frac{x^2\operatorname{arccsch}(bx+a)}{2}+\frac{\sqrt{b^2x^2+2abx+a^2+1}\left(-a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2+1}}\right)\sqrt{b^2}-2a\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2+1}\sqrt{b}}{\sqrt{b^2}}\right.\right.}{2b^2\sqrt{\frac{b^2x^2+2abx+a^2+1}{(bx+a)^2}}(bx+a)\sqrt{b^2}}$

input
```
int(x*arccsch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-arccsch(b*x+a)*a*(b*x+a)+1/2*arccsch(b*x+a)*(b*x+a)^2-1/2*((b*x+a)
^2+1)^(1/2)*(2*a*arcsinh(b*x+a)-((b*x+a)^2+1)^(1/2))/(b*x+a)/(((b*x+a)^2+1
)/(b*x+a)^2)^(1/2))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(65) = 130.

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.80

$$\int x\text{csch}^{-1}(a+bx)\,dx$$
$$= \frac{b^2x^2\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)-a^2\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right)+a^2\log\left(-bx+(\right.}{}$$

input
```
integrate(x*arccsch(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 +
2*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(-b*x + (b*x + a)*sqrt((b^2*x^2 +
 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a + 1) + a^2*log(-b*x + (
b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a
 - 1) + 2*a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x
^2 + 2*a*b*x + a^2)) - a) + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

3.3.6 Sympy [F]

$$\int x\text{csch}^{-1}(a+bx)\,dx=\int x\,\text{acsch}\,(a+bx)\,dx$$

input
```
integrate(x*acsch(b*x+a),x)
```

output
```
Integral(x*acsch(a + b*x), x)
```

3.3.7 Maxima [F]

$$\int x\text{csch}^{-1}(a+bx)\,dx=\int x\,\text{arcsch}\,(bx+a)\,\,dx$$

input
```
integrate(x*arccsch(b*x+a),x, algorithm="maxima")
```

output
```
1/2*I*a*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/b^2 + 1
/4*(2*b^2*x^2*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1) - (a^2 - 1)*log(b
^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a))/b^2 + integr
ate(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + 1), x)
```

3.3.8 Giac [F]

$$\int x\text{csch}^{-1}(a+bx)\,dx = \int x\,\text{arcsch}\,(bx+a)\;dx$$

input
```
integrate(x*arccsch(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*arccsch(b*x + a), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^{-1}(a+bx)\,dx = \int x\,\text{asinh}\left(\frac{1}{a+b\,x}\right)\;dx$$

input
```
int(x*asinh(1/(a + b*x)),x)
```

output
```
int(x*asinh(1/(a + b*x)), x)
```

3.4 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx$

3.4.1 Optimal result

Integrand size = 10, antiderivative size = 162

$$\begin{aligned}\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx &= \operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{1+a^2}}\right)\\ &\quad+\operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1+\sqrt{1+a^2}}\right)\\ &\quad-\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{2\operatorname{csch}^{-1}(a+bx)}\right)\\ &\quad+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1-\sqrt{1+a^2}}\right)+\operatorname{PolyLog}\left(2,\frac{ae^{\operatorname{csch}^{-1}(a+bx)}}{1+\sqrt{1+a^2}}\right)\\ &\quad-\frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(a+bx)}\right)\end{aligned}$$

output
```
-arccsch(b*x+a)*ln(1-(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)+arccsch(b*x+a)*l
n(1-a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/(1-(a^2+1)^(1/2)))+arccsch(b*x+a)*
ln(1-a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/(1+(a^2+1)^(1/2)))-1/2*polylog(2,
(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)+polylog(2,a*(1/(b*x+a)+(1+1/(b*x+a)^2
)^(1/2))/(1-(a^2+1)^(1/2)))+polylog(2,a*(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))/
(1+(a^2+1)^(1/2)))
```

3.4.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.64

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx$$
$$= \frac{1}{8}\Bigg(\pi^2 - 4i\pi\operatorname{csch}^{-1}(a+bx) - 8\operatorname{csch}^{-1}(a+bx)^2 - 32\arcsin\left(\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right)\arctan\left(\frac{(1-ia)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(a+bx)\right)\right)}{\sqrt{1+a^2}}\right)$$
$$- 8\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{-2\operatorname{csch}^{-1}(a+bx)}\right) + 4i\pi\log\left(1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$+ 8\operatorname{csch}^{-1}(a+bx)\log\left(1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$+ 16i\arcsin\left(\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(-1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$+ 4i\pi\log\left(1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$+ 8\operatorname{csch}^{-1}(a+bx)\log\left(1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$- 16i\arcsin\left(\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right) - 4i\pi\log\left(\frac{bx}{a+bx}\right)$$
$$+ 4\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(a+bx)}\right) + 8\operatorname{PolyLog}\left(2,\frac{\left(-1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)$$
$$+ 8\operatorname{PolyLog}\left(2,-\frac{\left(1+\sqrt{1+a^2}\right)e^{\operatorname{csch}^{-1}(a+bx)}}{a}\right)\Bigg)$$

input
```
Integrate[ArcCsch[a + b*x]/x,x]
```

output
```
(Pi^2 - (4*I)*Pi*ArcCsch[a + b*x] - 8*ArcCsch[a + b*x]^2 - 32*ArcSin[Sqrt[
(-I + a)/a]/Sqrt[2]]*ArcTan[((1 - I*a)*Cot[(Pi + (2*I)*ArcCsch[a + b*x])/4
])/Sqrt[1 + a^2]] - 8*ArcCsch[a + b*x]*Log[1 - E^(-2*ArcCsch[a + b*x])] +
(4*I)*Pi*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + 8*ArcCsch[
a + b*x]*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] + (16*I)*Arc
Sin[Sqrt[(-I + a)/a]/Sqrt[2]]*Log[1 - ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a +
b*x])/a] + (4*I)*Pi*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] +
8*ArcCsch[a + b*x]*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a] - (
16*I)*ArcSin[Sqrt[(-I + a)/a]/Sqrt[2]]*Log[1 + ((1 + Sqrt[1 + a^2])*E^ArcC
sch[a + b*x])/a] - (4*I)*Pi*Log[(b*x)/(a + b*x)] + 4*PolyLog[2, E^(-2*ArcC
sch[a + b*x])] + 8*PolyLog[2, ((-1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a]
 + 8*PolyLog[2, -(((1 + Sqrt[1 + a^2])*E^ArcCsch[a + b*x])/a)])/8
```

3.4.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.600, Rules used = {6876, 25, 6130, 6103, 25, 3042, 26, 4199, 25, 2620, 2715, 2838, 6095, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}(a+bx)}{x} \, dx$$

$$\Big\downarrow \; 6876$$

$$-\int \frac{(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{bx}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \; 25$$

$$\int -\frac{(a+bx)^2\sqrt{\frac{1}{(a+bx)^2}+1}\text{csch}^{-1}(a+bx)}{bx}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \; 6130$$

$$\int \frac{(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}\text{csch}^{-1}(a+bx)}{\frac{a}{a+bx}-1}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \; 6103$$

$$a\int -\frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)-\int (a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-\int (a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)d\text{csch}^{-1}(a+bx)-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{3042}$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)-\int -i\text{csch}^{-1}(a+bx)\tan\left(i\text{csch}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{26}$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+i\int \text{csch}^{-1}(a+bx)\tan\left(i\text{csch}^{-1}(a+bx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{4199}$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+i\left(2i\int -\frac{e^{2\text{csch}^{-1}(a+bx)}\text{csch}^{-1}(a+bx)}{1-e^{2\text{csch}^{-1}(a+bx)}}d\text{csch}^{-1}(a+bx)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \text{25}$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+i\left(-2i\int \frac{e^{2\text{csch}^{-1}(a+bx)}\text{csch}^{-1}(a+bx)}{1-e^{2\text{csch}^{-1}(a+bx)}}d\text{csch}^{-1}(a+bx)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)d\text{csch}^{-1}(a+bx)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a\right.$$

3.4. $\int \frac{\text{csch}^{-1}(a+bx)}{x}\,dx$

$$\Big\downarrow \; 2715$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+ i\left(-2i\left(\frac{1}{4}\int e^{-2\text{csch}^{-1}(a+bx)}\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)de^{2\text{csch}^{-1}(a+bx)}-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\right.$$

$$\Big\downarrow \; 2838$$

$$-a\int \frac{\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)+ i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(a+bx)}\right)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \; 6095$$

$$-a\left(\int \frac{e^{\text{csch}^{-1}(a+bx)}\text{csch}^{-1}(a+bx)}{-e^{\text{csch}^{-1}(a+bx)}a-\sqrt{a^2+1}+1}d\text{csch}^{-1}(a+bx)+\int \frac{e^{\text{csch}^{-1}(a+bx)}\text{csch}^{-1}(a+bx)}{-e^{\text{csch}^{-1}(a+bx)}a+\sqrt{a^2+1}+1}d\text{csch}^{-1}(a+bx)+\frac{\text{csc}}{}\right. i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(a+bx)}\right)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \; 2620$$

$$-a\left(\frac{\int \log\left(1-\frac{ae^{\text{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)d\text{csch}^{-1}(a+bx)}{a}+\frac{\int \log\left(1-\frac{ae^{\text{csch}^{-1}(a+bx)}}{\sqrt{a^2+1}+1}\right)d\text{csch}^{-1}(a+bx)}{a}-\frac{\text{csch}^{-1}(a+bx)\,\text{lo}}{}\right. i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(a+bx)}\right)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \; 2715$$

$$-a\left(\frac{\int e^{-\text{csch}^{-1}(a+bx)}\log\left(1-\frac{ae^{\text{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)de^{\text{csch}^{-1}(a+bx)}}{a}+\frac{\int e^{-\text{csch}^{-1}(a+bx)}\log\left(1-\frac{ae^{\text{csch}^{-1}(a+bx)}}{\sqrt{a^2+1}+1}\right)de^{\text{csch}^{-1}(a+}}{a}\right. i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(a+bx)}\right)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

$$\Big\downarrow \; 2838$$

3.4. $\int \frac{\text{csch}^{-1}(a+bx)}{x}\,dx$

$$-a\left(-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)}{a}-\frac{\text{PolyLog}\left(2,\frac{ae^{\text{csch}^{-1}(a+bx)}}{\sqrt{a^2+1}+1}\right)}{a}-\frac{\text{csch}^{-1}(a+bx)\log\left(1-\frac{ae^{\text{csch}^{-1}(a+bx)}}{1-\sqrt{a^2+1}}\right)}{a}-\frac{\text{cs}}{}\right.$$

$$i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(a+bx)}\right)-\frac{1}{2}\text{csch}^{-1}(a+bx)\log\left(1-e^{2\text{csch}^{-1}(a+bx)}\right)\right)-\frac{1}{2}i\text{csch}^{-1}(a+bx)^2\right)$$

input
```
Int[ArcCsch[a + b*x]/x,x]
```

output
```
-(a*(ArcCsch[a + b*x]^2/(2*a) - (ArcCsch[a + b*x]*Log[1 - (a*E^ArcCsch[a +
 b*x])/(1 - Sqrt[1 + a^2])])/a - (ArcCsch[a + b*x]*Log[1 - (a*E^ArcCsch[a
+ b*x])/(1 + Sqrt[1 + a^2])])/a - PolyLog[2, (a*E^ArcCsch[a + b*x])/(1 - S
qrt[1 + a^2])]/a - PolyLog[2, (a*E^ArcCsch[a + b*x])/(1 + Sqrt[1 + a^2])]/
a)) + I*((-1/2*I)*ArcCsch[a + b*x]^2 - (2*I)*(-1/2*(ArcCsch[a + b*x]*Log[1
 - E^(2*ArcCsch[a + b*x])]) - PolyLog[2, E^(2*ArcCsch[a + b*x])]/4))
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6130
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csch[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> I
nt[(e + f*x)^m*Sinh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Sinh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.4. $\int \frac{\text{csch}^{-1}(a+bx)}{x} \, dx$

3.4.4 Maple [F]

$$\int \frac{\operatorname{arccsch}(bx+a)}{x}dx$$

input
```
int(arccsch(b*x+a)/x,x)
```

output
```
int(arccsch(b*x+a)/x,x)
```

3.4.5 Fricas [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x}\,dx$$

input
```
integrate(arccsch(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccsch(b*x + a)/x, x)
```

3.4.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{acsch}(a+bx)}{x}\,dx$$

input
```
integrate(acsch(b*x+a)/x,x)
```

output
```
Integral(acsch(a + b*x)/x, x)
```

3.4.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x}\,dx$$

input
```
integrate(arccsch(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arccsch(b*x + a)/x, x)
```

3.4.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x}\,dx$$

input
```
integrate(arccsch(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccsch(b*x + a)/x, x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

input
```
int(asinh(1/(a + b*x))/x,x)
```

output
```
int(asinh(1/(a + b*x))/x, x)
```

3.5 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = -\frac{b\operatorname{csch}^{-1}(a+bx)}{a} - \frac{\operatorname{csch}^{-1}(a+bx)}{x} + \frac{2b\operatorname{arctanh}\left(\frac{a+\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)}{\sqrt{1+a^2}}\right)}{a\sqrt{1+a^2}}$$

output
```
-b*arccsch(b*x+a)/a-arccsch(b*x+a)/x+2*b*arctanh((a+tanh(1/2*arccsch(b*x+a
)))/(a^2+1)^(1/2))/a/(a^2+1)^(1/2)
```

3.5.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(63) = 126.

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = -\frac{\operatorname{csch}^{-1}(a+bx)}{x} - \frac{b\left(\sqrt{1+a^2}\operatorname{arcsinh}\left(\frac{1}{a+bx}\right) + \log(x) - \log\left(1+a^2+abx+a\sqrt{1+a^2}\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}} + \sqrt{1+a^2}bx\sqrt{\frac{1+a^2}{}}\right.\right.}{a\sqrt{1+a^2}}$$

input
```
Integrate[ArcCsch[a + b*x]/x^2,x]
```

output
```
-(ArcCsch[a + b*x]/x) - (b*(Sqrt[1 + a^2]*ArcSinh[(a + b*x)^(-1)] + Log[x]
 - Log[1 + a^2 + a*b*x + a*Sqrt[1 + a^2]*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2
)/(a + b*x)^2] + Sqrt[1 + a^2]*b*x*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a +
 b*x)^2]]))/(a*Sqrt[1 + a^2])
```

3.5.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6876, 5992, 3042, 4270, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6876}$$

$$-b\int \frac{(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{b^2x^2}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{ 5992}$$

$$-b\left(\int -\frac{1}{bx}d\text{csch}^{-1}(a+bx)+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-b\left(\frac{\text{csch}^{-1}(a+bx)}{bx}+\int \frac{1}{a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)}d\text{csch}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{ 4270}$$

$$-b\left(-\frac{\int \frac{1}{1-\frac{a}{a+bx}}d\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-b\left(-\frac{\int \frac{1}{ia\sin\left(i\text{csch}^{-1}(a+bx)\right)+1}d\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{ 3139}$$

$$-b\left(-\frac{2\int \frac{1}{-\tanh^2\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)-2a\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)+1}d\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)}{a}+\frac{\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

$\Big\downarrow$ 1083

$$-b\left(\frac{4\int \frac{1}{4(a^2+1)-\left(-2a-2\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)\right)^2}d\left(-2a-2\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)\right)}{a}+\frac{\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

$\Big\downarrow$ 219

$$-b\left(\frac{2\text{arctanh}\left(\frac{-2\tanh\left(\frac{1}{2}\text{csch}^{-1}(a+bx)\right)-2a}{2\sqrt{a^2+1}}\right)}{a\sqrt{a^2+1}}+\frac{\text{csch}^{-1}(a+bx)}{a}+\frac{\text{csch}^{-1}(a+bx)}{bx}\right)$$

input
```
Int[ArcCsch[a + b*x]/x^2,x]
```

output
```
-(b*(ArcCsch[a + b*x]/a + ArcCsch[a + b*x]/(b*x) + (2*ArcTanh[(-2*a - 2*Ta
nh[ArcCsch[a + b*x]/2])/(2*Sqrt[1 + a^2])])/(a*Sqrt[1 + a^2])))
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.02

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccsch}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)\sqrt{a^2+1}-\ln\left(\frac{2\sqrt{a^2+1}\sqrt{(bx+a)^2+1}+2(bx+a)a+2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2+1}{(bx+a)^2}}(bx+a)a\sqrt{a^2+1}}\right)$
default	$b\left(-\frac{\operatorname{arccsch}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)\sqrt{a^2+1}-\ln\left(\frac{2\sqrt{a^2+1}\sqrt{(bx+a)^2+1}+2(bx+a)a+2}{bx}\right)\right)}{\sqrt{\frac{(bx+a)^2+1}{(bx+a)^2}}(bx+a)a\sqrt{a^2+1}}\right)$
parts	$-\frac{\operatorname{arccsch}(bx+a)}{x}-\frac{b\sqrt{b^2x^2+2abx+a^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2+1}}\right)\sqrt{a^2+1}-\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2}}{x}\right.\right.}{\sqrt{\frac{b^2x^2+2abx+a^2+1}{(bx+a)^2}}(bx+a)a\sqrt{a^2+1}}$

input
```
int(arccsch(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arccsch(b*x+a)-((b*x+a)^2+1)^(1/2)*(arctanh(1/((b*x+a)^2+1)^(1/2
))*(a^2+1)^(1/2)-ln(2*((a^2+1)^(1/2)*((b*x+a)^2+1)^(1/2)+(b*x+a)*a+1)/b/x)
)/((((b*x+a)^2+1)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2+1)^(1/2))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(57) = 114$.

Time = 0.27 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.44

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx =$$

$$-\frac{(a^2+1)bx\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+1}{b^2x^2+2\,abx+a^2}}-a+1\right)-(a^2+1)bx\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx}{b^2x^2+2\,ab}}\right.}{}$$

input
```
integrate(arccsch(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
-((a^2 + 1)*b*x*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
^2*x^2 + 2*a*b*x + a^2)) - a + 1) - (a^2 + 1)*b*x*log(-b*x + (b*x + a)*sqr
t((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a - 1) - sqrt
(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + (a*b*x + a^2 + (a*b*x + a^2)*sqrt((b^2
*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)*sqrt(a^2 + 1) +
(a^3 + (a^2 + 1)*b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*
a*b*x + a^2)) + a)/x) + (a^3 + a)*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)))/((a^3 + a)*x)
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acsch}(a+bx)}{x^2}\,dx$$

input
```
integrate(acsch(b*x+a)/x**2,x)
```

output
```
Integral(acsch(a + b*x)/x**2, x)
```

3.5.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x^2}\,dx$$

input
```
integrate(arccsch(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/2*I*b*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/(a^2 +
 1) - b*log(x)/(a^3 + a) - 1/2*(a^2*b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) -
 2*(a^3 + (a^2*b + b)*x + a)*log(b*x + a) + 2*(a^3 + a)*log(sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1) + 1))/((a^3 + a)*x) - integrate((b^2*x + a*b)/(b^2*x^3
 + 2*a*b*x^2 + (a^2 + 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)), x)
```

3.5.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x^2}\,dx$$

input
```
integrate(arccsch(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(arccsch(b*x + a)/x^2, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^2}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

input
```
int(asinh(1/(a + b*x))/x^2,x)
```

output
```
int(asinh(1/(a + b*x))/x^2, x)
```

3.6 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3} dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 114

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3} dx = \frac{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}{2a(1+a^2)x} + \frac{b^2\operatorname{csch}^{-1}(a+bx)}{2a^2} - \frac{\operatorname{csch}^{-1}(a+bx)}{2x^2} - \frac{(1+2a^2)b^2\operatorname{arctanh}\left(\frac{a+\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)}{\sqrt{1+a^2}}\right)}{a^2(1+a^2)^{3/2}}$$

output
```
1/2*b^2*arccsch(b*x+a)/a^2-1/2*arccsch(b*x+a)/x^2-(2*a^2+1)*b^2*arctanh((a
+tanh(1/2*arccsch(b*x+a)))/(a^2+1)^(1/2))/a^2/(a^2+1)^(3/2)+1/2*b*(b*x+a)*
(1+1/(b*x+a)^2)^(1/2)/a/(a^2+1)/x
```

3.6.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3} dx = \frac{1}{2}\left(\frac{b(a+bx)\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}}{a(1+a^2)x} - \frac{\operatorname{csch}^{-1}(a+bx)}{x^2} + \frac{b^2\operatorname{arcsinh}\left(\frac{1}{a+bx}\right)}{a^2} + \frac{(1+2a^2)b^2\log(x)}{a^2(1+a^2)^{3/2}} - \frac{(1+2a^2)b^2\log\left(1+a^2+abx+a\sqrt{1+a^2}\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}+\sqrt{1+a^2}bx\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{a^2(1+a^2)^{3/2}}\right)$$

input
```
Integrate[ArcCsch[a + b*x]/x^3,x]
```

output
```
((b*(a + b*x)*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*(1 + a^2
)*x) - ArcCsch[a + b*x]/x^2 + (b^2*ArcSinh[(a + b*x)^(-1)])/a^2 + ((1 + 2*
a^2)*b^2*Log[x])/(a^2*(1 + a^2)^(3/2)) - ((1 + 2*a^2)*b^2*Log[1 + a^2 + a*
b*x + a*Sqrt[1 + a^2]*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2] + Sq
rt[1 + a^2]*b*x*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]])/(a^2*(1
+ a^2)^(3/2)))/2
```

3.6.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {6876, 25, 5992, 3042, 4272, 25, 3042, 4407, 26, 3042, 26, 4318, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6876}$$

$$-b^2 \int \frac{(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{b^3x^3}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{25}$$

$$b^2 \int -\frac{(a+bx)^2\sqrt{1+\frac{1}{(a+bx)^2}}\text{csch}^{-1}(a+bx)}{b^3x^3}d\text{csch}^{-1}(a+bx)$$

$$\Big\downarrow \text{5992}$$

$$-b^2\left(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}-\frac{1}{2}\int \frac{1}{b^2x^2}d\text{csch}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-b^2\left(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}-\frac{1}{2}\int \frac{1}{\left(a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)\right)^2}d\text{csch}^{-1}(a+bx)\right)$$

$$\Big\downarrow \text{4272}$$

$$-b^2\left(\frac{1}{2}\left(\frac{\int \frac{a^2+(a+bx)a+1}{bx}d\text{csch}^{-1}(a+bx)}{a\left(a^2+1\right)}-\frac{(a+bx)\sqrt{\frac{1}{(a+bx)^2}+1}}{a\left(a^2+1\right)bx}\right)+\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \; 25$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\int -\frac{a^2+(a+bx)a+1}{bx}d\text{csch}^{-1}(a+bx)}{a\left(a^2+1\right)}-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}\right)+\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-b^2\left(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}-\frac{\int \frac{a^2+i\csc\left(i\text{csch}^{-1}(a+bx)\right)a+1}{a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)}d\text{csch}^{-1}(a+bx)}{a\left(a^2+1\right)}\right)\right)$$

$$\Big\downarrow \; 4407$$

$$-b^2\left(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}-\frac{\frac{\left(a^2+1\right)\text{csch}^{-1}(a+bx)}{a}+\frac{i\left(2a^2+1\right)\int \frac{i(a+bx)}{bx}d\text{csch}^{-1}(a+bx)}{a}}{a\left(a^2+1\right)}\right)\right)$$

$$\Big\downarrow \; 26$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{\left(2a^2+1\right)\int -\frac{a+bx}{bx}d\text{csch}^{-1}(a+bx)}{a}+\frac{\left(a^2+1\right)\text{csch}^{-1}(a+bx)}{a}}{a\left(a^2+1\right)}-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}\right)+\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \; 3042$$

$$-b^2\Bigg(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\Bigg(-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}-\frac{\frac{\left(a^2+1\right)\text{csch}^{-1}(a+bx)}{a}+\frac{\left(2a^2+1\right)\int \frac{i\csc\left(i\text{csch}^{-1}(a+bx)\right)}{a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)}d\text{csch}^{-1}(a}{a}}{a\left(a^2+1\right)}$$

$$\Big\downarrow \; 26$$

$$-b^2\Bigg(\frac{\text{csch}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\Bigg(-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a\left(a^2+1\right)bx}-\frac{\frac{\left(a^2+1\right)\text{csch}^{-1}(a+bx)}{a}+\frac{i\left(2a^2+1\right)\int \frac{\csc\left(i\text{csch}^{-1}(a+bx)\right)}{a-i\csc\left(i\text{csch}^{-1}(a+bx)\right)}d\text{csch}^{-1}(}{a}}{a\left(a^2+1\right)}$$

3.6. $\int \frac{\text{csch}^{-1}(a+bx)}{x^3}\,dx$

$$\Big\downarrow \ 4318$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(a^2+1)\operatorname{csch}^{-1}(a+bx)}{a}-\frac{(2a^2+1)\int \frac{1}{1-\frac{a}{a+bx}}d\operatorname{csch}^{-1}(a+bx)}{a}}{a(a^2+1)}-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a(a^2+1)bx}\right)+\frac{\operatorname{csch}^{-1}(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \ 3042$$

$$-b^2\left(\frac{\operatorname{csch}^{-1}(a+bx)}{2b^2x^2}+\frac{1}{2}\left(-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a(a^2+1)bx}-\frac{\frac{(a^2+1)\operatorname{csch}^{-1}(a+bx)}{a}-\frac{(2a^2+1)\int \frac{1}{ia\sin\left(i\operatorname{csch}^{-1}(a+bx)\right)+1}d\operatorname{csch}^{-1}(\cdots}{a}}{a(a^2+1)}\cdots\right.\right.$$

$$\Big\downarrow \ 3139$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{(a^2+1)\operatorname{csch}^{-1}(a+bx)}{a}-\frac{2(2a^2+1)\int \frac{1}{-\tanh^2\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)-2a\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)+1}d\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)}{a}}{a(a^2+1)}-\sqrt{\cdots}\right.\right.$$

$$\Big\downarrow \ 1083$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{4(2a^2+1)\int \frac{1}{4(a^2+1)-\left(-2a-2\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)\right)^2}d\left(-2a-2\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)\right)}{a}+\frac{(a^2+1)\operatorname{csch}^{-1}(a+bx)}{a}}{a(a^2+1)}-\sqrt{\frac{\cdots}{(a\cdots}}\right.\right.$$

$$\Big\downarrow \ 219$$

$$-b^2\left(\frac{1}{2}\left(-\frac{\frac{2(2a^2+1)\operatorname{arctanh}\left(\frac{-2\tanh\left(\frac{1}{2}\operatorname{csch}^{-1}(a+bx)\right)-2a}{2\sqrt{a^2+1}}\right)}{a\sqrt{a^2+1}}+\frac{(a^2+1)\operatorname{csch}^{-1}(a+bx)}{a}}{a(a^2+1)}-\frac{\sqrt{\frac{1}{(a+bx)^2}+1}(a+bx)}{a(a^2+1)bx}\right)+\frac{\operatorname{csch}^{-1}(\cdots}{2b\cdots}\right.$$

input
```
Int[ArcCsch[a + b*x]/x^3,x]
```

3.6. $\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx$

output
```
-(b^2*(ArcCsch[a + b*x]/(2*b^2*x^2) + (-(((a + b*x)*Sqrt[1 + (a + b*x)^(-2
)])/(a*(1 + a^2)*b*x)) - (((1 + a^2)*ArcCsch[a + b*x])/a + (2*(1 + 2*a^2)*
ArcTanh[(-2*a - 2*Tanh[ArcCsch[a + b*x]/2])/(2*Sqrt[1 + a^2])])/(a*Sqrt[1
+ a^2]))/(a*(1 + a^2)))/2))
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

3.6. $\int \frac{\text{csch}^{-1}(a+bx)}{x^3} \, dx$

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(100) = 200$.

Time = 0.66 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.79

method	result
parts	$-\frac{\operatorname{arccsch}(bx+a)}{2x^2}+\frac{b\sqrt{b^2x^2+2abx+a^2+1}\left((a^2+1)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{1}{\sqrt{b^2x^2+2abx+a^2+1}}\right)a^2bx-2\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}}{x}\right.\right.}{}$
derivativedivides	$b^2\left(-\frac{\operatorname{arccsch}(bx+a)}{2b^2x^2}+\frac{\sqrt{(bx+a)^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)(a^2+1)^{\frac{3}{2}}a^3+\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)(a^2+1)^{\frac{3}{2}}a^2\right.}{}\right.$
default	$b^2\left(-\frac{\operatorname{arccsch}(bx+a)}{2b^2x^2}+\frac{\sqrt{(bx+a)^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)(a^2+1)^{\frac{3}{2}}a^3+\operatorname{arctanh}\left(\frac{1}{\sqrt{(bx+a)^2+1}}\right)(a^2+1)^{\frac{3}{2}}a^2\right.}{}\right.$

input
```
int(arccsch(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccsch(b*x+a)/x^2+1/2*b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*((a^2+1)^(3/2)
*arctanh(1/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))*a^2*b*x-2*ln(2*(a*b*x+(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2+1)/x)*a^4*b*x+b*arctanh(1/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))*x*(a^2+1)^(3/2)+(a^2+1)^(3/2)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a-3*ln(2*(a*b*x+(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2+1)/
x)*a^2*b*x-b*ln(2*(a*b*x+(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2+1
)/x)*x)/((b^2*x^2+2*a*b*x+a^2+1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2+1)^(5/2
)/x
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(100) = 200.

Time = 0.28 (sec) , antiderivative size = 461, normalized size of antiderivative = 4.04

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx$$

$$= \frac{(2a^2+1)\sqrt{a^2+1}b^2x^2\log\left(-\frac{a^2bx+a^3-\left(abx+a^2+(abx+a^2)\sqrt{\frac{b^2x^2+2abx+a^2+1}{b^2x^2+2abx+a^2}}+1\right)\sqrt{a^2+1}+(a^3+(a^2+1)bx+a)\sqrt{\frac{b^2x^2+2abx+a^2\cdots}{b^2x^2+2abx+a^2\cdots}}}{x}\right.}{}$$

input
```
integrate(arccsch(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*((2*a^2 + 1)*sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 - (a*b*x + a^2
+ (a*b*x + a^2)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^
2)) + 1)*sqrt(a^2 + 1) + (a^3 + (a^2 + 1)*b*x + a)*sqrt((b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/x) + (a^4 + 2*a^2 + 1)*b^2*x^2
*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*
x + a^2)) - a + 1) - (a^4 + 2*a^2 + 1)*b^2*x^2*log(-b*x + (b*x + a)*sqrt((
b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) - a - 1) + (a^3 +
a)*b^2*x^2 - (a^6 + 2*a^4 + a^2)*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
a^2 + 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + ((a^3 + a)*b^2*x^2 +
 (a^4 + a^2)*b*x)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + 1)/(b^2*x^2 + 2*a*b*x +
a^2)))/((a^6 + 2*a^4 + a^2)*x^2)
```

3.6.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{acsch}(a+bx)}{x^3}\,dx$$

input
```
integrate(acsch(b*x+a)/x**3,x)
```

output
```
Integral(acsch(a + b*x)/x**3, x)
```

3.6.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x^3}\,dx$$

input
```
integrate(arccsch(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*I*a*b^2*(log(I*(b^2*x + a*b)/b + 1) - log(-I*(b^2*x + a*b)/b + 1))/(a^
4 + 2*a^2 + 1) + 1/2*(3*a^2*b^2 + b^2)*log(x)/(a^6 + 2*a^4 + a^2) + 1/4*((
a^4*b^2 - a^2*b^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^3*b + a*b)*
x + 2*(a^6 + 2*a^4 - (a^4*b^2 + 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b*x + a) -
 2*(a^6 + 2*a^4 + a^2)*log(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1))/((a^6 +
 2*a^4 + a^2)*x^2) - integrate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a
^2 + 1)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)), x)
```

3.6.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{x^3}\,dx$$

input
```
integrate(arccsch(b*x+a)/x^3,x, algorithm="giac")
```

output
```
integrate(arccsch(b*x + a)/x^3, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

input
```
int(asinh(1/(a + b*x))/x^3,x)
```

output
```
int(asinh(1/(a + b*x))/x^3, x)
```

3.7 $\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx$

3.7.1 Optimal result

Integrand size = 20, antiderivative size = 501

$$\int (e+fx)^3 \left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2 dx$$
$$= \frac{b^2f^2(de-cf)x}{d^3} + \frac{b^2f^3(c+dx)^2}{12d^4} - \frac{bf^3(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\operatorname{csch}^{-1}(c+dx)\right)}{3d^4}$$
$$+ \frac{3bf(de-cf)^2(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\operatorname{csch}^{-1}(c+dx)\right)}{d^4}$$
$$+ \frac{bf^2(de-cf)(c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\operatorname{csch}^{-1}(c+dx)\right)}{d^4}$$
$$+ \frac{bf^3(c+dx)^3\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\operatorname{csch}^{-1}(c+dx)\right)}{6d^4}$$
$$- \frac{(de-cf)^4\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{4d^4f} + \frac{(e+fx)^4\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{4f}$$
$$- \frac{2bf^2(de-cf)\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4}$$
$$+ \frac{4b(de-cf)^3\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4} - \frac{b^2f^3\log(c+dx)}{3d^4}$$
$$+ \frac{3b^2f(de-cf)^2\log(c+dx)}{d^4} - \frac{b^2f^2(de-cf)\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4}$$
$$+ \frac{2b^2(de-cf)^3\operatorname{PolyLog}\left(2,-e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4}$$
$$+ \frac{b^2f^2(de-cf)\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4} - \frac{2b^2(de-cf)^3\operatorname{PolyLog}\left(2,e^{\operatorname{csch}^{-1}(c+dx)}\right)}{d^4}$$

output
```
b^2*f^2*(-c*f+d*e)*x/d^3+1/12*b^2*f^3*(d*x+c)^2/d^4-1/4*(-c*f+d*e)^4*(a+b*
arccsch(d*x+c))^2/d^4/f+1/4*(f*x+e)^4*(a+b*arccsch(d*x+c))^2/f-2*b*f^2*(-c
*f+d*e)*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4+
4*b*(-c*f+d*e)^3*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*x+c)^2)^(1
/2))/d^4-1/3*b^2*f^3*ln(d*x+c)/d^4+3*b^2*f*(-c*f+d*e)^2*ln(d*x+c)/d^4-b^2*
f^2*(-c*f+d*e)*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^4+2*b^2*(-c*f
+d*e)^3*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^4+b^2*f^2*(-c*f+d*e)
*polylog(2,1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4-2*b^2*(-c*f+d*e)^3*polylog
(2,1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^4-1/3*b*f^3*(d*x+c)*(a+b*arccsch(d*x
+c))*(1+1/(d*x+c)^2)^(1/2)/d^4+3*b*f*(-c*f+d*e)^2*(d*x+c)*(a+b*arccsch(d*x
+c))*(1+1/(d*x+c)^2)^(1/2)/d^4+b*f^2*(-c*f+d*e)*(d*x+c)^2*(a+b*arccsch(d*x
+c))*(1+1/(d*x+c)^2)^(1/2)/d^4+1/6*b*f^3*(d*x+c)^3*(a+b*arccsch(d*x+c))*(1
+1/(d*x+c)^2)^(1/2)/d^4
```

3.7.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.06 (sec) , antiderivative size = 1487, normalized size of antiderivative = 2.97

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
a^2*e^3*x + (3*a^2*e^2*f*x^2)/2 + a^2*e*f^2*x^3 + (a^2*f^3*x^4)/4 + (a*b*(
3*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCsch[c + d*x] + (f*(c +
 d*x)*Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2]*((-2 + 13*c^2)*f^2 -
 2*c*d*f*(15*e + 2*f*x) + d^2*(18*e^2 + 6*e*f*x + f^2*x^2)) - 3*c*(-4*d^3*
e^3 + 6*c*d^2*e^2*f - 4*c^2*d*e*f^2 + c^3*f^3)*ArcSinh[(c + d*x)^(-1)] + 6
*(2*d^3*e^3 - 6*c*d^2*e^2*f + (-1 + 6*c^2)*d*e*f^2 + c*(1 - 2*c^2)*f^3)*Lo
g[(c + d*x)*(1 + Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2])])/d^4))/
6 - (b^2*e^3*(-(ArcCsch[c + d*x]*((c + d*x)*ArcCsch[c + d*x] - 2*Log[1 - E
^(-ArcCsch[c + d*x])] + 2*Log[1 + E^(-ArcCsch[c + d*x])])) + 2*PolyLog[2,
-E^(-ArcCsch[c + d*x])] - 2*PolyLog[2, E^(-ArcCsch[c + d*x])]))/d - (3*b^2
*d*e^2*f*x*(((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x])/d^2 + ((
c + d*x)^2*ArcCsch[c + d*x]^2)/(2*d^2) - (c*ArcCsch[c + d*x]^2*Coth[ArcCsc
h[c + d*x]/2])/(2*d^2) - Log[(c + d*x)^(-1)]/d^2 - ((2*I)*c*(I*ArcCsch[c +
 d*x]*(Log[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^(-ArcCsch[c + d*x])]) +
I*(PolyLog[2, -E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-ArcCsch[c + d*x])])
))/d^2 + (c*ArcCsch[c + d*x]^2*Tanh[ArcCsch[c + d*x]/2])/(2*d^2)))/((c + d
*x)*(-1 + c/(c + d*x))) - (b^2*e*f^2*(2*(-2 + 12*c*ArcCsch[c + d*x] + ArcC
sch[c + d*x]^2 - 6*c^2*ArcCsch[c + d*x]^2)*Coth[ArcCsch[c + d*x]/2] + 2*Ar
cCsch[c + d*x]*(-1 + 3*c*ArcCsch[c + d*x])*Csch[ArcCsch[c + d*x]/2]^2 - (A
rcCsch[c + d*x]^2*Csch[ArcCsch[c + d*x]/2]^4)/(2*(c + d*x)) - 48*c*(Log...
```

3.7.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6876, 5992, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2dx$$

$$\Big\downarrow \text{6876}$$

$$-\frac{\int (c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}(de-cf+f(c+dx))^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2d\text{csch}^{-1}(c+dx)}{d^4}$$

$$\Big\downarrow \text{5992}$$

$$-\frac{\frac{b\int (de-cf+f(c+dx))^4\left(a+b\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)}{2f}-\frac{(f(c+dx)-cf+de)^4\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{4f}}{d^4}$$

$$\downarrow \text{3042}$$

$$-\frac{-\frac{(f(c+dx)-cf+de)^4\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{4f}+\frac{b\int\left(a+b\text{csch}^{-1}(c+dx)\right)\left(de-cf+if\csc\left(i\text{csch}^{-1}(c+dx)\right)\right)^4d\text{csch}^{-1}(c+dx)}{2f}}{d^4}$$

$$\downarrow \text{4678}$$

$$-\frac{b\int\left(d^4\left(\frac{cf\left(-4d^3e^3+6cd^2fe^2-4c^2df^2e+c^3f^3\right)}{d^4e^4}+1\right)\left(a+b\text{csch}^{-1}(c+dx)\right)e^4+4d^3f\left(1-\frac{cf\left(3d^2e^2-3cdfe+c^2f^2\right)}{d^3e^3}\right)(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)e^3-\right.}{}$$

$$\downarrow \text{2009}$$

$$-\frac{b\left(4f^3(de-cf)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)-8f(de-cf)^3\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)-2f^3(c+dx)^2\right.}{}$$

input
```
Int[(e + f*x)^3*(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
-((-1/4*((d*e - c*f + f*(c + d*x))^4*(a + b*ArcCsch[c + d*x])^2)/f + (b*(-
2*b*f^3*(d*e - c*f)*(c + d*x) - (b*f^4*(c + d*x)^2)/6 + (2*f^4*(c + d*x)*S
qrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/3 - 6*f^2*(d*e - c*f)^2*
(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]) - 2*f^3*(d*e -
 c*f)*(c + d*x)^2*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]) - (f^4
*(c + d*x)^3*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/3 + ((d*e
- c*f)^4*(a + b*ArcCsch[c + d*x])^2)/(2*b) + 4*f^3*(d*e - c*f)*(a + b*ArcC
sch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] - 8*f*(d*e - c*f)^3*(a + b*ArcCs
ch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] - (2*b*f^4*Log[(c + d*x)^(-1)])/3
 + 6*b*f^2*(d*e - c*f)^2*Log[(c + d*x)^(-1)] + 2*b*f^3*(d*e - c*f)*PolyLog
[2, -E^ArcCsch[c + d*x]] - 4*b*f*(d*e - c*f)^3*PolyLog[2, -E^ArcCsch[c + d
*x]] - 2*b*f^3*(d*e - c*f)*PolyLog[2, E^ArcCsch[c + d*x]] + 4*b*f*(d*e - c
*f)^3*PolyLog[2, E^ArcCsch[c + d*x]]))/(2*f))/d^4)
```

3.7. $\int(e+fx)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.7.4 Maple [F]

$$\int (fx+e)^3 (a+b\,\text{arccsch}\,(dx+c))^2\,dx$$

input
```
int((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x)
```

output
```
int((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x)
```

3.7.5 Fricas [F]

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^3 (b\,\text{arcsch}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^3*x^3 + 3*a^2*e*f^2*x^2 + 3*a^2*e^2*f*x + a^2*e^3 + (b^2*f^
3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x + b^2*e^3)*arccsch(d*x + c)^2 + 2*
(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^2*f*x + a*b*e^3)*arccsch(d*x + c)
, x)
```

3.7.6 Sympy [F]

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (a+b\,\text{acsch}\,(c+dx))^2 (e+fx)^3 \, dx$$

input
```
integrate((f*x+e)**3*(a+b*acsch(d*x+c))**2,x)
```

output
```
Integral((a + b*acsch(c + d*x))**2*(e + f*x)**3, x)
```

3.7.7 Maxima [F]

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^3 (b\,\text{arcsch}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*f^3*x^4 + a^2*e*f^2*x^3 + 3/2*a^2*e^2*f*x^2 + a^2*e^3*x + (2*(d*x
+ c)*arccsch(d*x + c) + log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(d*x
 + c)^2 + 1) - 1))*a*b*e^3/d + 1/4*(b^2*f^3*x^4 + 4*b^2*e*f^2*x^3 + 6*b^2*
e^2*f*x^2 + 4*b^2*e^3*x)*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - in
tegrate(-1/2*(2*(b^2*d^2*f^3*x^5 + b^2*c^2*e^3 + b^2*e^3 + (3*b^2*d^2*e*f^
2 + 2*b^2*c*d*f^3)*x^4 + (6*b^2*c*d*e*f^2 + b^2*c^2*f^3 + (3*d^2*e^2*f + f
^3)*b^2)*x^3 + (6*b^2*c*d*e^2*f + 3*b^2*c^2*e*f^2 + (d^2*e^3 + 3*e*f^2)*b^
2)*x^2 + (2*b^2*c*d*e^3 + 3*b^2*c^2*e^2*f + 3*b^2*e^2*f)*x)*log(d*x + c)^2
 - 4*(a*b*d^2*f^3*x^5 + (3*a*b*d^2*e*f^2 + 2*a*b*c*d*f^3)*x^4 + (6*a*b*c*d
*e*f^2 + a*b*c^2*f^3 + (3*d^2*e^2*f + f^3)*a*b)*x^3 + 3*(2*a*b*c*d*e^2*f +
 a*b*c^2*e*f^2 + a*b*e*f^2)*x^2 + 3*(a*b*c^2*e^2*f + a*b*e^2*f)*x)*log(d*x
 + c) + (4*a*b*d^2*f^3*x^5 + 4*(3*a*b*d^2*e*f^2 + 2*a*b*c*d*f^3)*x^4 + 4*(
6*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (3*d^2*e^2*f + f^3)*a*b)*x^3 + 12*(2*a*b*c
*d*e^2*f + a*b*c^2*e*f^2 + a*b*e*f^2)*x^2 + 12*(a*b*c^2*e^2*f + a*b*e^2*f)
*x - 4*(b^2*d^2*f^3*x^5 + b^2*c^2*e^3 + b^2*e^3 + (3*b^2*d^2*e*f^2 + 2*b^2
*c*d*f^3)*x^4 + (6*b^2*c*d*e*f^2 + b^2*c^2*f^3 + (3*d^2*e^2*f + f^3)*b^2)*
x^3 + (6*b^2*c*d*e^2*f + 3*b^2*c^2*e*f^2 + (d^2*e^3 + 3*e*f^2)*b^2)*x^2 +
(2*b^2*c*d*e^3 + 3*b^2*c^2*e^2*f + 3*b^2*e^2*f)*x)*log(d*x + c) + ((4*a*b*
d^2*f^3 - b^2*d^2*f^3)*x^5 + (12*a*b*d^2*e*f^2 - 4*b^2*d^2*e*f^2 + (8*a*b*
d*f^3 - b^2*d*f^3)*c)*x^4 - 2*(3*b^2*d^2*e^2*f - 2*a*b*c^2*f^3 - 2*(3*d...
```

3.7.8 Giac [F]

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx = \int (fx+e)^3 (b\,\text{arcsch}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^3*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*(b*arccsch(d*x + c) + a)^2, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^3 \left(a+b\,\mathrm{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2 \,dx$$

input
```
int((e + f*x)^3*(a + b*asinh(1/(c + d*x)))^2,x)
```

output
```
int((e + f*x)^3*(a + b*asinh(1/(c + d*x)))^2, x)
```

3.8 $\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx$

3.8.1 Optimal result

Integrand size = 20, antiderivative size = 351

$$\begin{aligned}
&\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx \\
&= \frac{b^2f^2x}{3d^2} + \frac{2bf(de-cf)(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^3} \\
&+ \frac{bf^2(c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{3d^3} - \frac{(de-cf)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3d^3f} \\
&+ \frac{(e+fx)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f} - \frac{2bf^2\left(a+b\text{csch}^{-1}(c+dx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{3d^3} \\
&+ \frac{4b(de-cf)^2\left(a+b\text{csch}^{-1}(c+dx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d^3} \\
&+ \frac{2b^2f(de-cf)\log(c+dx)}{d^3} - \frac{b^2f^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{3d^3} \\
&+ \frac{2b^2(de-cf)^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d^3} \\
&+ \frac{b^2f^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{3d^3} - \frac{2b^2(de-cf)^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d^3}
\end{aligned}$$

output
```
1/3*b^2*f^2*x/d^2-1/3*(-c*f+d*e)^3*(a+b*arccsch(d*x+c))^2/d^3/f+1/3*(f*x+e
)^3*(a+b*arccsch(d*x+c))^2/f-2/3*b*f^2*(a+b*arccsch(d*x+c))*arctanh(1/(d*x
+c)+(1+1/(d*x+c)^2)^(1/2))/d^3+4*b*(-c*f+d*e)^2*(a+b*arccsch(d*x+c))*arcta
nh(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^3+2*b^2*f*(-c*f+d*e)*ln(d*x+c)/d^3-1
/3*b^2*f^2*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^3+2*b^2*(-c*f+d*e
)^2*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2))/d^3+1/3*b^2*f^2*polylog(2,
1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d^3-2*b^2*(-c*f+d*e)^2*polylog(2,1/(d*x+c
)+(1+1/(d*x+c)^2)^(1/2))/d^3+2*b*f*(-c*f+d*e)*(d*x+c)*(a+b*arccsch(d*x+c))
*(1+1/(d*x+c)^2)^(1/2)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccsch(d*x+c))*(1+1/(
d*x+c)^2)^(1/2)/d^3
```

3.8.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 893, normalized size of antiderivative = 2.54

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx$$

$$= a^2e^2x + a^2efx^2 + \frac{1}{3}a^2f^2x^3 + \frac{1}{3}ab\Bigg(2x\left(3e^2+3efx+f^2x^2\right)\text{csch}^{-1}(c+dx)$$

$$+ \frac{-f(c+dx)\sqrt{\frac{1+c^2+2cdx+d^2x^2}{(c+dx)^2}}(5cf-d(6e+fx)) + 2c(3d^2e^2-3cdef+c^2f^2)\,\text{arcsinh}\left(\frac{1}{c+dx}\right) + (6d^2e^2 -}{d^3}$$

$$- \frac{b^2e^2\left(-\text{csch}^{-1}(c+dx)\left((c+dx)\text{csch}^{-1}(c+dx) - 2\log\left(1-e^{-\text{csch}^{-1}(c+dx)}\right) + 2\log\left(1+e^{-\text{csch}^{-1}(c+dx)}\right)\right.\right.}{d}$$

$$- \frac{2b^2defx\left(\frac{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\text{csch}^{-1}(c+dx)}{d^2} + \frac{(c+dx)^2\text{csch}^{-1}(c+dx)^2}{2d^2} - \frac{c\text{csch}^{-1}(c+dx)^2\coth\left(\frac{1}{2}\text{csch}^{-1}(c+dx)\right)}{2d^2} - \frac{\log\left(\frac{1}{c+d}\right.}{d^2}\right.}{}$$

$$- \frac{b^2f^2\left(2\left(-2+12c\text{csch}^{-1}(c+dx)+\text{csch}^{-1}(c+dx)^2-6c^2\text{csch}^{-1}(c+dx)^2\right)\coth\left(\frac{1}{2}\text{csch}^{-1}(c+dx)\right) + 2c\right.}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*ArcCsch[c + d*x] + (-(f*(c + d*x)*Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^
2)/(c + d*x)^2]*(5*c*f - d*(6*e + f*x))) + 2*c*(3*d^2*e^2 - 3*c*d*e*f + c^
2*f^2)*ArcSinh[(c + d*x)^(-1)] + (6*d^2*e^2 - 12*c*d*e*f + (-1 + 6*c^2)*f^
2)*Log[(c + d*x)*(1 + Sqrt[(1 + c^2 + 2*c*d*x + d^2*x^2)/(c + d*x)^2])])/d
^3))/3 - (b^2*e^2*(-(ArcCsch[c + d*x]*((c + d*x)*ArcCsch[c + d*x] - 2*Log[
1 - E^(-ArcCsch[c + d*x])] + 2*Log[1 + E^(-ArcCsch[c + d*x])])) + 2*PolyLo
g[2, -E^(-ArcCsch[c + d*x])] - 2*PolyLog[2, E^(-ArcCsch[c + d*x])]))/d - (
2*b^2*d*e*f*x*(((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x])/d^2 +
 ((c + d*x)^2*ArcCsch[c + d*x]^2)/(2*d^2) - (c*ArcCsch[c + d*x]^2*Coth[Arc
Csch[c + d*x]/2])/(2*d^2) - Log[(c + d*x)^(-1)]/d^2 - ((2*I)*c*(I*ArcCsch[
c + d*x]*(Log[1 - E^(-ArcCsch[c + d*x])] - Log[1 + E^(-ArcCsch[c + d*x])])
 + I*(PolyLog[2, -E^(-ArcCsch[c + d*x])] - PolyLog[2, E^(-ArcCsch[c + d*x]
)])))/d^2 + (c*ArcCsch[c + d*x]^2*Tanh[ArcCsch[c + d*x]/2])/(2*d^2)))/((c
+ d*x)*(-1 + c/(c + d*x))) - (b^2*f^2*(2*(-2 + 12*c*ArcCsch[c + d*x] + Arc
Csch[c + d*x]^2 - 6*c^2*ArcCsch[c + d*x]^2)*Coth[ArcCsch[c + d*x]/2] + 2*A
rcCsch[c + d*x]*(-1 + 3*c*ArcCsch[c + d*x])*Csch[ArcCsch[c + d*x]/2]^2 - (
ArcCsch[c + d*x]^2*Csch[ArcCsch[c + d*x]/2]^4)/(2*(c + d*x)) - 48*c*(Log[1
/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + Log[Sqrt[1 + (c + d*x)^(-2)]]) +
8*(-1 + 6*c^2)*(ArcCsch[c + d*x]*(Log[1 - E^(-ArcCsch[c + d*x])] - Log[...
```

3.8.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6876, 5992, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{6876}$$

$$-\frac{\int (c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}(de-cf+f(c+dx))^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2d\text{csch}^{-1}(c+dx)}{d^3}$$

$$\Big\downarrow \text{5992}$$

$$-\frac{\frac{2b\int (de-cf+f(c+dx))^3\left(a+b\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)}{3f}-\frac{(f(c+dx)-cf+de)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f}}{d^3}$$

$\downarrow$ 3042

$$-\frac{-\frac{(f(c+dx)-cf+de)^3\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{3f}+\frac{2b\int\left(a+b\text{csch}^{-1}(c+dx)\right)\left(de-cf+if\csc\left(i\text{csch}^{-1}(c+dx)\right)\right)^3d\text{csch}^{-1}(c+dx)}{3f}}{d^3}$$

$\downarrow$ 4678

$$-\frac{\frac{2b\int\left(d^3\left(1-\frac{cf\left(3d^2e^2-3cdfe+c^2f^2\right)}{d^3e^3}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)e^3+3d^2f\left(\frac{cf(cf-2de)}{d^2e^2}+1\right)(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)e^2+3df^2\left(1-\frac{cf}{de}\right)(c+dx)^2\left(\ldots\right.}{3f}}{d^3}$$

$\downarrow$ 2009

$$-\frac{2b\left(-6f(de-cf)^2\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)+f^3\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)-3f^2(c+dx)\sqrt{\frac{1}{(c+dx)\ldots}}\right.}{\ldots}$$

input
```
Int[(e + f*x)^2*(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
-((-1/3*((d*e - c*f + f*(c + d*x))^3*(a + b*ArcCsch[c + d*x])^2)/f + (2*b*
(-1/2*(b*f^3*(c + d*x)) - 3*f^2*(d*e - c*f)*(c + d*x)*Sqrt[1 + (c + d*x)^(
-2)]*(a + b*ArcCsch[c + d*x]) - (f^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^(-2)]*
(a + b*ArcCsch[c + d*x]))/2 + ((d*e - c*f)^3*(a + b*ArcCsch[c + d*x])^2)/(
2*b) + f^3*(a + b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] - 6*f*(d*e
 - c*f)^2*(a + b*ArcCsch[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] + 3*b*f^2*(
d*e - c*f)*Log[(c + d*x)^(-1)] + (b*f^3*PolyLog[2, -E^ArcCsch[c + d*x]])/2
 - 3*b*f*(d*e - c*f)^2*PolyLog[2, -E^ArcCsch[c + d*x]] - (b*f^3*PolyLog[2,
 E^ArcCsch[c + d*x]])/2 + 3*b*f*(d*e - c*f)^2*PolyLog[2, E^ArcCsch[c + d*x
]]))/(3*f))/d^3)
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.8.4 Maple [F]

$$\int (fx+e)^2 (a+b\operatorname{arccsch}(dx+c))^2\,dx$$

input
```
int((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x)
```

output
```
int((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x)
```

3.8.5 Fricas [F]

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int (fx+e)^2(b\operatorname{arcsch}(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arccsch(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*ar
ccsch(d*x + c), x)
```

3.8.6 Sympy [F]

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int \left(a+b\,\text{acsch}\,(c+dx)\right)^2 (e+fx)^2 \, dx$$

input
```
integrate((f*x+e)**2*(a+b*acsch(d*x+c))**2,x)
```

output
```
Integral((a + b*acsch(c + d*x))**2*(e + f*x)**2, x)
```

3.8.7 Maxima [F]

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b\,\text{arcsch}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*f^2*x^3 + a^2*e*f*x^2 + a^2*e^2*x + (2*(d*x + c)*arccsch(d*x + c)
+ log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(d*x + c)^2 + 1) - 1))*a*b
*e^2/d + 1/3*(b^2*f^2*x^3 + 3*b^2*e*f*x^2 + 3*b^2*e^2*x)*log(sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1) + 1)^2 - integrate(-1/3*(3*(b^2*d^2*f^2*x^4 + b^2*c^2
*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 + (4*b^2*c*d*e*f + b^2*
c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d*e^2 + b^2*c^2*e*f + b^2*e*
f)*x)*log(d*x + c)^2 - 6*(a*b*d^2*f^2*x^4 + 2*(a*b*d^2*e*f + a*b*c*d*f^2)*
x^3 + (4*a*b*c*d*e*f + a*b*c^2*f^2 + a*b*f^2)*x^2 + 2*(a*b*c^2*e*f + a*b*e
*f)*x)*log(d*x + c) + 2*(3*a*b*d^2*f^2*x^4 + 6*(a*b*d^2*e*f + a*b*c*d*f^2)
*x^3 + 3*(4*a*b*c*d*e*f + a*b*c^2*f^2 + a*b*f^2)*x^2 + 6*(a*b*c^2*e*f + a*
b*e*f)*x - 3*(b^2*d^2*f^2*x^4 + b^2*c^2*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b
^2*c*d*f^2)*x^3 + (4*b^2*c*d*e*f + b^2*c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2
+ 2*(b^2*c*d*e^2 + b^2*c^2*e*f + b^2*e*f)*x)*log(d*x + c) + ((3*a*b*d^2*f^
2 - b^2*d^2*f^2)*x^4 + (6*a*b*d^2*e*f - 3*b^2*d^2*e*f + (6*a*b*d*f^2 - b^2
*d*f^2)*c)*x^3 - 3*(b^2*d^2*e^2 - a*b*c^2*f^2 - a*b*f^2 - (4*a*b*d*e*f - b
^2*d*e*f)*c)*x^2 - 3*(b^2*c*d*e^2 - 2*a*b*c^2*e*f - 2*a*b*e*f)*x - 3*(b^2*
d^2*f^2*x^4 + b^2*c^2*e^2 + b^2*e^2 + 2*(b^2*d^2*e*f + b^2*c*d*f^2)*x^3 +
(4*b^2*c*d*e*f + b^2*c^2*f^2 + (d^2*e^2 + f^2)*b^2)*x^2 + 2*(b^2*c*d*e^2 +
 b^2*c^2*e*f + b^2*e*f)*x)*log(d*x + c))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
)*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + 3*sqrt(d^2*x^2 + 2*c*d*x...
```

3.8.8 Giac [F]

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (fx+e)^2 (b\,\text{arcsch}\,(dx+c)+a)^2 \, dx$$

input
```
integrate((f*x+e)^2*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*arccsch(d*x + c) + a)^2, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx = \int (e+f\,x)^2 \left(a+b\,\text{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2 \, dx$$

input
```
int((e + f*x)^2*(a + b*asinh(1/(c + d*x)))^2,x)
```

output
```
int((e + f*x)^2*(a + b*asinh(1/(c + d*x)))^2, x)
```

3.9 $\int (e+fx) \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx$

3.9.1 Optimal result

Integrand size = 18, antiderivative size = 194

$$\begin{aligned}\int (e+fx) \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx &= \frac{bf(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)}{d^2} \\ &- \frac{(de-cf)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2d^2f} + \frac{(e+fx)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f} \\ &+ \frac{4b(de-cf)\left(a+b\text{csch}^{-1}(c+dx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d^2} + \frac{b^2f\log(c+dx)}{d^2} \\ &+ \frac{2b^2(de-cf)\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d^2} - \frac{2b^2(de-cf)\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d^2}\end{aligned}$$

output
```
-1/2*(-c*f+d*e)^2*(a+b*arccsch(d*x+c))^2/d^2/f+1/2*(f*x+e)^2*(a+b*arccsch(
d*x+c))^2/f+4*b*(-c*f+d*e)*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c)+(1+1/(d*
x+c)^2)^(1/2))/d^2+b^2*f*ln(d*x+c)/d^2+2*b^2*(-c*f+d*e)*polylog(2,-1/(d*x+
c)-(1+1/(d*x+c)^2)^(1/2))/d^2-2*b^2*(-c*f+d*e)*polylog(2,1/(d*x+c)+(1+1/(d
*x+c)^2)^(1/2))/d^2+b*f*(d*x+c)*(a+b*arccsch(d*x+c))*(1+1/(d*x+c)^2)^(1/2)
/d^2
```

3.9.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 427 vs. 2(194) = 388.

Time = 2.96 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.20

$$\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$$

$$=\frac{2a^2(de-cf)(c+dx)+a^2f(c+dx)^2+2abf(c+dx)\left(\sqrt{1+\frac{1}{(c+dx)^2}}+(c+dx)\text{csch}^{-1}(c+dx)\right)+2b^2f\Big(\cdots}{\cdots}$$

input
```
Integrate[(e + f*x)*(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
(2*a^2*(d*e - c*f)*(c + d*x) + a^2*f*(c + d*x)^2 + 2*a*b*f*(c + d*x)*(Sqrt
[1 + (c + d*x)^(-2)] + (c + d*x)*ArcCsch[c + d*x]) + 2*b^2*f*((c + d*x)*Sq
rt[1 + (c + d*x)^(-2)]*ArcCsch[c + d*x] + ((c + d*x)^2*ArcCsch[c + d*x]^2)
/2 - Log[(c + d*x)^(-1)]) + 4*a*b*d*e*((c + d*x)*ArcCsch[c + d*x] + Log[Cs
ch[ArcCsch[c + d*x]/2]/(2*(c + d*x))] - Log[Sinh[ArcCsch[c + d*x]/2]]) - 4
*a*b*c*f*((c + d*x)*ArcCsch[c + d*x] + Log[Csch[ArcCsch[c + d*x]/2]/(2*(c
+ d*x))] - Log[Sinh[ArcCsch[c + d*x]/2]]) + 2*b^2*d*e*(ArcCsch[c + d*x]*((
c + d*x)*ArcCsch[c + d*x] - 2*Log[1 - E^(-ArcCsch[c + d*x])] + 2*Log[1 + E
^(-ArcCsch[c + d*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c + d*x])] + 2*PolyLog[
2, E^(-ArcCsch[c + d*x])]) - 2*b^2*c*f*(ArcCsch[c + d*x]*((c + d*x)*ArcCsc
h[c + d*x] - 2*Log[1 - E^(-ArcCsch[c + d*x])] + 2*Log[1 + E^(-ArcCsch[c +
d*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c + d*x])] + 2*PolyLog[2, E^(-ArcCsch[
c + d*x])]))/(2*d^2)
```

3.9.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6876, 5992, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 6876$$

$$-\frac{\int (c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}(de-cf+f(c+dx))\left(a+b\text{csch}^{-1}(c+dx)\right)^2d\text{csch}^{-1}(c+dx)}{d^2}$$

$$\Big\downarrow \; 5992$$

$$-\frac{\frac{b\int (de-cf+f(c+dx))^2\left(a+b\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)}{f}-\frac{(f(c+dx)-cf+de)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f}}{d^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{(f(c+dx)-cf+de)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f}+\frac{b\int \left(a+b\text{csch}^{-1}(c+dx)\right)\left(de-cf+if\csc\left(i\text{csch}^{-1}(c+dx)\right)\right)^2d\text{csch}^{-1}(c+dx)}{f}}{d^2}$$

$$\Big\downarrow \; 4678$$

$$-\frac{\frac{b\int \left(d^2\left(\frac{cf(cf-2de)}{d^2e^2}+1\right)\left(a+b\text{csch}^{-1}(c+dx)\right)e^2+2df\left(1-\frac{cf}{de}\right)(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)e+f^2(c+dx)^2\left(a+b\text{csch}^{-1}(c+dx)\right)\right)d\text{csch}^{-1}(c+d}{f}}{d^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(-4f(de-cf)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)+\frac{(de-cf)^2\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2b}-\left(f^2(c+dx)\sqrt{\frac{1}{(c+dx)^2}+1}\left(a+b\text{csch}^{-1}(c+\right.\right.}{f}$$

input

```
Int[(e + f*x)*(a + b*ArcCsch[c + d*x])^2,x]
```

output

```
-((-1/2*((d*e - c*f + f*(c + d*x))^2*(a + b*ArcCsch[c + d*x])^2)/f + (b*(-
(f^2*(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x])) + ((d*e
- c*f)^2*(a + b*ArcCsch[c + d*x])^2)/(2*b) - 4*f*(d*e - c*f)*(a + b*ArcCsc
h[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] + b*f^2*Log[(c + d*x)^(-1)] - 2*b*
f*(d*e - c*f)*PolyLog[2, -E^ArcCsch[c + d*x]] + 2*b*f*(d*e - c*f)*PolyLog[
2, E^ArcCsch[c + d*x]]))/f)/d^2)
```

3.9. $\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4678
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.9.4 Maple [F]

$$\int (fx+e)(a+b\,\mathrm{arccsch}(dx+c))^2\,dx$$

input
```
int((f*x+e)*(a+b*arccsch(d*x+c))^2,x)
```

output
```
int((f*x+e)*(a+b*arccsch(d*x+c))^2,x)
```

3.9.5 Fricas [F]

$$\int (e+fx) \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx = \int (fx+e)(b\,\text{arcsch}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccsch(d*x + c)^2 + 2*(a*b*f
*x + a*b*e)*arccsch(d*x + c), x)
```

3.9.6 Sympy [F]

$$\int (e+fx) \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx = \int (a+b\,\text{acsch}\,(c+dx))^2\,(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*acsch(d*x+c))**2,x)
```

output
```
Integral((a + b*acsch(c + d*x))**2*(e + f*x), x)
```

3.9.7 Maxima [F]

$$\int (e+fx) \left(a+b\text{csch}^{-1}(c+dx)\right)^2 dx = \int (fx+e)(b\,\text{arcsch}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*f*x^2 + a^2*e*x + (2*(d*x + c)*arccsch(d*x + c) + log(sqrt(1/(d*x
+ c)^2 + 1) + 1) - log(sqrt(1/(d*x + c)^2 + 1) - 1))*a*b*e/d + 1/2*(b^2*f*
x^2 + 2*b^2*e*x)*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - integrate(
-((b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*
b^2*c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^3 + 2*a*
b*c*d*f*x^2 + (a*b*c^2*f + a*b*f)*x)*log(d*x + c) + (2*a*b*d^2*f*x^3 + 4*a
*b*c*d*f*x^2 + 2*(a*b*c^2*f + a*b*f)*x - 2*(b^2*d^2*f*x^3 + b^2*c^2*e + b^
2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*b^2*c*d*e + b^2*c^2*f + b^2*f)*x)
*log(d*x + c) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*a*b*d^2*f - b^2*d^2*
f)*x^3 - (2*b^2*d^2*e - (4*a*b*d*f - b^2*d*f)*c)*x^2 - 2*(b^2*c*d*e - a*b*
c^2*f - a*b*f)*x - 2*(b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b
^2*c*d*f)*x^2 + (2*b^2*c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x + c))*log(sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
(b^2*d^2*f*x^3 + b^2*c^2*e + b^2*e + (b^2*d^2*e + 2*b^2*c*d*f)*x^2 + (2*b^
2*c*d*e + b^2*c^2*f + b^2*f)*x)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^3 + 2*a*b*
c*d*f*x^2 + (a*b*c^2*f + a*b*f)*x)*log(d*x + c)))/(d^2*x^2 + 2*c*d*x + c^2
 + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 1), x)
```

3.9.8 Giac [F]

$$\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int (fx+e)(b\,\text{arcsch}\,(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*arccsch(d*x + c) + a)^2, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int (e+f\,x)\,\left(a+b\,\text{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((e + f*x)*(a + b*asinh(1/(c + d*x)))^2,x)
```

output
```
int((e + f*x)*(a + b*asinh(1/(c + d*x)))^2, x)
```

3.10 $\int (a+b\text{csch}^{-1}(c+dx))^2 \, dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 85

$$\int (a+b\text{csch}^{-1}(c+dx))^2 \, dx = \frac{(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d} + \frac{4b\left(a+b\text{csch}^{-1}(c+dx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)}{d} + \frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(c+dx)}\right)}{d} - \frac{2b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(c+dx)}\right)}{d}$$

output
```
(d*x+c)*(a+b*arccsch(d*x+c))^2/d+4*b*(a+b*arccsch(d*x+c))*arctanh(1/(d*x+c
)+(1+1/(d*x+c)^2)^(1/2))/d+2*b^2*polylog(2,-1/(d*x+c)-(1+1/(d*x+c)^2)^(1/2
))/d-2*b^2*polylog(2,1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))/d
```

3.10.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(85) = 170.

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.07

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx$$
$$= \frac{a^2c+a^2dx+2ab(c+dx)\text{csch}^{-1}(c+dx)+b^2c\text{csch}^{-1}(c+dx)^2+b^2dx\text{csch}^{-1}(c+dx)^2-2b^2\text{csch}^{-1}(c+dx}{}$$

input
```
Integrate[(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
(a^2*c + a^2*d*x + 2*a*b*(c + d*x)*ArcCsch[c + d*x] + b^2*c*ArcCsch[c + d*
x]^2 + b^2*d*x*ArcCsch[c + d*x]^2 - 2*b^2*ArcCsch[c + d*x]*Log[1 - E^(-Arc
Csch[c + d*x])] + 2*b^2*ArcCsch[c + d*x]*Log[1 + E^(-ArcCsch[c + d*x])] +
2*a*b*Log[Cosh[ArcCsch[c + d*x]/2]] - 2*a*b*Log[Sinh[ArcCsch[c + d*x]/2]]
- 2*b^2*PolyLog[2, -E^(-ArcCsch[c + d*x])] + 2*b^2*PolyLog[2, E^(-ArcCsch[
c + d*x])])/d
```

3.10.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6870, 6834, 5975, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2 \, dx$$

$$\Big\downarrow \text{6870}$$

$$\frac{\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2 d(c+dx)}{d}$$

$$\Big\downarrow \text{6834}$$

$$-\frac{\int (c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2 d\text{csch}^{-1}(c+dx)}{d}$$

$$\Big\downarrow \; 5975$$

$$-\frac{2b\int (c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2+2b\int i\left(a+b\text{csch}^{-1}(c+dx)\right)\csc\left(i\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2+2ib\int \left(a+b\text{csch}^{-1}(c+dx)\right)\csc\left(i\text{csch}^{-1}(c+dx)\right)d\text{csch}^{-1}(c+dx)}{d}$$

$$\Big\downarrow \; 4670$$

$$-\frac{-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2+2ib\left(ib\int \log\left(1-e^{\text{csch}^{-1}(c+dx)}\right)d\text{csch}^{-1}(c+dx)-ib\int \log\left(1+e^{\text{csch}^{-1}(c+dx}\right.\right.}{d}$$

$$\Big\downarrow \; 2715$$

$$-\frac{-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2+2ib\left(-ib\int e^{-\text{csch}^{-1}(c+dx)}\log\left(1+e^{\text{csch}^{-1}(c+dx)}\right)de^{\text{csch}^{-1}(c+dx)}+ib\int e^{-\text{csch}^{-}}\right.}{d}$$

$$\Big\downarrow \; 2838$$

$$-\frac{-(c+dx)\left(a+b\text{csch}^{-1}(c+dx)\right)^2+2ib\left(2i\text{arctanh}\left(e^{\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)+ib\,\text{PolyLog}(2,-c-\right.}{d}$$

input
```
Int[(a + b*ArcCsch[c + d*x])^2,x]
```

output
```
-((-((c + d*x)*(a + b*ArcCsch[c + d*x])^2) + (2*I)*b*((2*I)*(a + b*ArcCsch
[c + d*x])*ArcTanh[E^ArcCsch[c + d*x]] - I*b*PolyLog[2, E^ArcCsch[c + d*x]
] + I*b*PolyLog[2, -c - d*x]))/d)
```

3.10. $\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx$

3.10.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6834
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 6870
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
   Subst[Int[(a + b*ArcCsch[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[p, 0]
```

3.10.4 Maple [F]

$$\int (a+b\ \operatorname{arccsch}(dx+c))^2\,dx$$

input
```
int((a+b*arccsch(d*x+c))^2,x)
```

output
```
int((a+b*arccsch(d*x+c))^2,x)
```

3.10.5 Fricas [F]

$$\int \left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2\,dx=\int (b\operatorname{arcsch}(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2, x)
```

3.10.6 Sympy [F]

$$\int \left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2\,dx=\int (a+b\operatorname{acsch}(c+dx))^2\,dx$$

input
```
integrate((a+b*acsch(d*x+c))**2,x)
```

output
```
Integral((a + b*acsch(c + d*x))**2, x)
```

3.10.7 Maxima [F]

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int \left(b\,\text{arcsch}\,(dx+c)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2,x, algorithm="maxima")
```

output
```
(x*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2 - integrate(-((d^2*x^2 + 2
*c*d*x + c^2 + 1)^(3/2)*log(d*x + c)^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log
(d*x + c)^2 - 2*((d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d*x + c) + sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*(d^2*x^2 + c*d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log
(d*x + c)))*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1))/(d^2*x^2 + 2*c*d*x
 + c^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 1), x))*b^2 + a^2*x + (2*(d
*x + c)*arccsch(d*x + c) + log(sqrt(1/(d*x + c)^2 + 1) + 1) - log(sqrt(1/(
d*x + c)^2 + 1) - 1))*a*b/d
```

3.10.8 Giac [F]

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int \left(b\,\text{arcsch}\,(dx+c)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(d*x + c) + a)^2, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\,dx=\int \left(a+b\,\text{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a + b*asinh(1/(c + d*x)))^2,x)
```

output
```
int((a + b*asinh(1/(c + d*x)))^2, x)
```

3.11 $\int \frac{\left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$

3.11.1 Optimal result

Integrand size = 20, antiderivative size = 475

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$$
$$=-\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)}{f}$$
$$+\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1+\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$
$$+\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2\log\left(1+\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$
$$-\frac{b\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)}{f}$$
$$+\frac{2b\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$
$$+\frac{2b\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$
$$+\frac{b^2\,\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(c+dx)}\right)}{2f}-\frac{2b^2\,\text{PolyLog}\left(3,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$
$$-\frac{2b^2\,\text{PolyLog}\left(3,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{f}$$

output
```
-(a+b*arccsch(d*x+c))^2*ln(1-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))^2)/f+(a+b*a
rccsch(d*x+c))^2*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2
*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f+(a+b*arccsch(d*x+c))^2*ln(1+(1/(d*x+
c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1
/2)))/f-b*(a+b*arccsch(d*x+c))*polylog(2,(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))
^2)/f+2*b*(a+b*arccsch(d*x+c))*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2)
)*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f+2*b*(a+b*arccsch
(d*x+c))*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f+1/2*b^2*polylog(3,(1/(d*x+c)+(1+1/(d*x
+c)^2)^(1/2))^2)/f-2*b^2*polylog(3,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*
f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/f-2*b^2*polylog(3,-(1/(d
*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
^(1/2)))/f
```

3.11.2 Mathematica [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x), x]
```

3.11.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6876, 6130, 6103, 3042, 26, 4199, 25, 2620, 3011, 2720, 6095, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$$

$$\Big\downarrow \text{6876}$$

$$-\int \frac{(c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{de-cf+f(c+dx)}d\text{csch}^{-1}(c+dx)$$

$$\Big\downarrow \text{ 6130}$$

$$-\int \frac{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)$$

$$\Big\downarrow \text{ 6103}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}-\frac{\int (c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2d\text{csch}^{-1}(c+dx)}{f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}-\frac{\int -i\left(a+b\text{csch}^{-1}(c+dx)\right)^2\tan\left(i\text{csch}^{-1}(c+dx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(c+dx)}{f}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\int \left(a+b\text{csch}^{-1}(c+dx)\right)^2\tan\left(i\text{csch}^{-1}(c+dx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(c+dx)}{f}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\left(2i\int -\frac{e^{2\text{csch}^{-1}(c+dx)}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{1-e^{2\text{csch}^{-1}(c+dx)}}d\text{csch}^{-1}(c+dx)-\frac{i\left(a+b\text{csch}^{-1}(c+dx)\right)^3}{3b}\right)}{f}$$

$$\Big\downarrow \text{ 25}$$

3.11. $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}dx$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\left(-2i\int \frac{e^{2\text{csch}^{-1}(c+dx)}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{1-e^{2\text{csch}^{-1}(c+dx)}}d\text{csch}^{-1}(c+dx)-\frac{i\left(a+b\text{csch}^{-1}(c+dx)\right)^3}{3b}\right)}{f}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\left(-2i\left(b\int \left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)d\text{csch}^{-1}(c+dx)-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}\right.\right.\right.}{f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\left(-2i\left(b\left(\frac{1}{2}b\int \text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)d\text{csch}^{-1}(c+dx)-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)\right.\right.\right.}{f}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(de-cf)\int \frac{\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+\frac{de-cf}{c+dx}}d\text{csch}^{-1}(c+dx)}{f}+\frac{i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)de^{2\text{csch}^{-1}(c+dx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csc}\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \text{6095}$$

$$\frac{(de-cf)\left(\int \frac{e^{\text{csch}^{-1}(c+dx)}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+e^{\text{csch}^{-1}(c+dx)}(de-cf)-\sqrt{d^2e^2-2cdfe+c^2f^2+f^2}}d\text{csch}^{-1}(c+dx)+\int \frac{e^{\text{csch}^{-1}(c+dx)}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f+e^{\text{csch}^{-1}(c+dx)}(de-cf)+\sqrt{d^2e^2-2cdfe+c^2f}}\right.}{f}$$

$$\frac{i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)de^{2\text{csch}^{-1}(c+dx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csc}\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \text{2620}$$

3.11. $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$

$$(de-cf)\left(-\frac{2b\int\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}+1\right)d\text{csch}^{-1}(c+dx)}{de-cf}-\frac{2b\int\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{e^{\text{csch}^{-1}(}}{f+\sqrt{d^2e^2-}}\right.}{de-c}\right.$$

$$\frac{i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)de^{2\text{csch}^{-1}(c+dx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csc}\right.\right.\right.\right.}{f}$$

$\Big\downarrow$ 3011

$$(de-cf)\left(-\frac{2b\left(b\int\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)d\text{csch}^{-1}(c+dx)-\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+\right.}}\right.\right.}{de-cf}\right.$$

$$\frac{i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)de^{2\text{csch}^{-1}(c+dx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csc}\right.\right.\right.\right.}{f}$$

$\Big\downarrow$ 2720

$$(de-cf)\left(-\frac{2b\left(b\int e^{-\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)de^{\text{csch}^{-1}(c+dx)}-\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,-\frac{e^{\text{csc}}}{f-\sqrt{d}}\right.\right.}{de-cf}\right.$$

$$\frac{i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(c+dx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)de^{2\text{csch}^{-1}(c+dx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csc}\right.\right.\right.\right.}{f}$$

$\Big\downarrow$ 7143

$$(de-cf)\left(-\frac{2b\left(b\,\text{PolyLog}\left(3,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)-\left(a+b\text{csch}^{-1}(c+dx)\right)\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}\right)\right)}{de-cf}-\frac{2b\left(b\right.}{}\right.$$

$$\frac{i\left(-2i\left(b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(c+dx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(c+dx)}\right)\left(a+b\text{csch}^{-1}(c+dx)\right)\right)-\frac{1}{2}\log\left(1-e^{2\text{cs}}\right.\right.\right.}{f}$$

3.11. $\int\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$

input
```
Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x),x]
```

output
```
(I*(((-1/3*I)*(a + b*ArcCsch[c + d*x])^3)/b - (2*I)*(-1/2*((a + b*ArcCsch[
c + d*x])^2*Log[1 - E^(2*ArcCsch[c + d*x])]) + b*(-1/2*((a + b*ArcCsch[c +
 d*x])*PolyLog[2, E^(2*ArcCsch[c + d*x])]) + (b*PolyLog[3, E^(2*ArcCsch[c
+ d*x])])/4))))/f + ((d*e - c*f)*(-1/3*(a + b*ArcCsch[c + d*x])^3/(b*(d*e
- c*f)) + ((a + b*ArcCsch[c + d*x])^2*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c
*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/(d*e - c*f) + ((a +
 b*ArcCsch[c + d*x])^2*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[
d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/(d*e - c*f) - (2*b*(-((a + b*ArcCs
ch[c + d*x])*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2]))]) + b*PolyLog[3, -((E^ArcCsch[c + d*x]*(
d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))]))/(d*e - c*f
) - (2*b*(-((a + b*ArcCsch[c + d*x])*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e
 - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))]) + b*PolyLog[3,
 -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c
^2)*f^2]))]))/(d*e - c*f)))/f
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6095
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

rule 6103
```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[1/a   Int[(e + f*x)^m*Coth[
c + d*x]^n, x], x] - Simp[b/a   Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x
]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 IGtQ[m, 0] && IGtQ[n, 0]
```

rule 6130
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/(Csch[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> I
nt[(e + f*x)^m*Sinh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Sinh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

3.11. $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx$

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.11.4 Maple [F]

$$\int \frac{(a + b\ \text{arccsch}\,(dx + c))^2}{fx + e} dx$$

input
```
int((a+b*arccsch(d*x+c))^2/(f*x+e),x)
```

output
```
int((a+b*arccsch(d*x+c))^2/(f*x+e),x)
```

3.11.5 Fricas [F]

$$\int \frac{\left(a + b\text{csch}^{-1}(c + dx)\right)^2}{e + fx} dx = \int \frac{(b\,\text{arcsch}\,(dx + c) + a)^2}{fx + e} dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f*x + e)
, x)
```

3.11.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{(a+b\operatorname{acsch}(c+dx))^2}{e+fx}\,dx$$

input
```
integrate((a+b*acsch(d*x+c))**2/(f*x+e),x)
```

output
```
Integral((a + b*acsch(c + d*x))**2/(e + f*x), x)
```

3.11.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{(b\operatorname{arcsch}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(b^2*log(sqrt(1/(d*x + c)^2 + 1) + 1/(d*x +
c))^2/(f*x + e) + 2*a*b*log(sqrt(1/(d*x + c)^2 + 1) + 1/(d*x + c))/(f*x +
e), x)
```

3.11.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{e+fx}\,dx=\int \frac{(b\operatorname{arcsch}(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*arccsch(d*x + c) + a)^2/(f*x + e), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{csch}^{-1}(c + dx)\right)^2}{e + fx}\,dx = \int \frac{\left(a + b\,\text{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2}{e + f\,x}\,dx$$

input
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x),x)
```

output
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x), x)
```

3.12 $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

3.12.1 Optimal result

Integrand size = 20, antiderivative size = 448

$$\begin{aligned}
\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx
&= \frac{d\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(de-cf)} - \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(e+fx)} \\
&- \frac{2bd\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)\sqrt{d^2e^2-2cdef+(1+c^2)\,f^2}} \\
&+ \frac{2bd\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)\sqrt{d^2e^2-2cdef+(1+c^2)\,f^2}} \\
&- \frac{2b^2d\,\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)\sqrt{d^2e^2-2cdef+(1+c^2)\,f^2}} \\
&+ \frac{2b^2d\,\text{PolyLog}\left(2,-\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)\sqrt{d^2e^2-2cdef+(1+c^2)\,f^2}}
\end{aligned}$$

output
```
d*(a+b*arccsch(d*x+c))^2/f/(-c*f+d*e)-(a+b*arccsch(d*x+c))^2/f/(f*x+e)-2*b
*d*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/
(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)/(d^2*e^2-2*c*d*e*f+(
c^2+1)*f^2)^(1/2)+2*b*d*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^
2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)
/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)-2*b^2*d*polylog(2,-(1/(d*x+c)+(1+1/
(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-
c*f+d*e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)+2*b^2*d*polylog(2,-(1/(d*x+
c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1
/2)))/(-c*f+d*e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)
```

3.12.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.80 (sec) , antiderivative size = 2061, normalized size of antiderivative = 4.60

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-(a^2/(f*(e + f*x))) - (2*a*b*(c + d*x)^2*(f + (d*e - c*f)/(c + d*x))^2*(A
rcCsch[c + d*x]/(f + (d*e)/(c + d*x) - (c*f)/(c + d*x)) - (2*ArcTan[(d*e -
 c*f - f*Tanh[ArcCsch[c + d*x]/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - (1 + c^2)
*f^2]])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - (1 + c^2)*f^2]))/(d*(-(d*e) + c*f)*(
e + f*x)^2) - (b^2*(c + d*x)^2*(f + (d*e - c*f)/(c + d*x))^2*(ArcCsch[c +
d*x]^2/((-(d*e) + c*f)*(f + (d*e)/(c + d*x) - (c*f)/(c + d*x))) + (2*(((-I
)*Pi*ArcTanh[(-(d*e) + c*f + f*Tanh[ArcCsch[c + d*x]/2])/Sqrt[f^2 + (d*e -
 c*f)^2]])/Sqrt[f^2 + (d*e - c*f)^2] - (2*(Pi/2 - I*ArcCsch[c + d*x])*ArcT
anh[((f - I*(d*e - c*f))*Cot[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^
2) + 2*c*d*e*f - f^2 - c^2*f^2]] - 2*ArcCos[((-I)*f)/(d*e - c*f)]*ArcTanh[
((-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2)
+ 2*c*d*e*f - f^2 - c^2*f^2]] + (ArcCos[((-I)*f)/(d*e - c*f)] - (2*I)*(Arc
Tanh[((f - I*(d*e - c*f))*Cot[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e
^2) + 2*c*d*e*f - f^2 - c^2*f^2]] - ArcTanh[((-f - I*(d*e - c*f))*Tan[(Pi/
2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]])
)*Log[Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 - c^2*f^2]/(Sqrt[2]*E^((I/2)*(Pi/2
 - I*ArcCsch[c + d*x]))*Sqrt[(-I)*(d*e - c*f)]*Sqrt[f + (d*e - c*f)/(c + d
*x)])] + (ArcCos[((-I)*f)/(d*e - c*f)] + (2*I)*(ArcTanh[((f - I*(d*e - c*f
))*Cot[(Pi/2 - I*ArcCsch[c + d*x])/2])/Sqrt[-(d^2*e^2) + 2*c*d*e*f - f^2 -
 c^2*f^2]] - ArcTanh[((-f - I*(d*e - c*f))*Tan[(Pi/2 - I*ArcCsch[c + d*...
```

3.12.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6876, 5992, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}dx$$

$$\Big\downarrow \text{6876}$$

$$-d\int \frac{(c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(de-cf+f(c+dx))^2}d\text{csch}^{-1}(c+dx)$$

$$\Big\downarrow \text{5992}$$

$$-d\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(f(c+dx)-cf+de)}-\frac{2b\int \frac{a+b\text{csch}^{-1}(c+dx)}{de-cf+f(c+dx)}d\text{csch}^{-1}(c+dx)}{f}\right)$$

$$\Big\downarrow \ 3042$$

$$-d\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(f(c+dx)-cf+de)}-\frac{2b\int \frac{a+b\text{csch}^{-1}(c+dx)}{de-cf+if\csc\left(i\text{csch}^{-1}(c+dx)\right)}d\text{csch}^{-1}(c+dx)}{f}\right)$$

$$\Big\downarrow \ 4679$$

$$-d\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(f(c+dx)-cf+de)}-\frac{2b\int \left(\frac{a+b\text{csch}^{-1}(c+dx)}{de-cf}+\frac{f\left(a+b\text{csch}^{-1}(c+dx)\right)}{(cf-de)\left(f+\frac{de\left(1-\frac{cf}{de}\right)}{c+dx}\right)}\right)d\text{csch}^{-1}(c+dx)}{f}\right)$$

$$\Big\downarrow \ 2009$$

$$-d\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{f(f(c+dx)-cf+de)}-\frac{2b\left(-\frac{f\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{(de-cf)e^{\text{csch}^{-1}(c+dx)}}{f-\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}+1\right)}{(de-cf)\sqrt{(c^2+1)f^2-2cdef+d^2e^2}}+\frac{f\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(-\right.}{(de-cf)\sqrt{(c^2+1)}}\right.}{}\right.$$

input
```
Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-(d*((a + b*ArcCsch[c + d*x])^2/(f*(d*e - c*f + f*(c + d*x))) - (2*b*((a +
 b*ArcCsch[c + d*x])^2/(2*b*(d*e - c*f)) - (f*(a + b*ArcCsch[c + d*x])*Log
[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 +
 c^2)*f^2])])/((d*e - c*f)*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]) + (f
*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sq
rt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*f)*Sqrt[d^2*e^2 - 2*c
*d*e*f + (1 + c^2)*f^2]) - (b*f*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*
f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e - c*f)*Sqrt[d
^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]) + (b*f*PolyLog[2, -((E^ArcCsch[c + d*
x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2]))])/((d*e -
 c*f)*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])))/f))
```

3.12. $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.12.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arccsch}\,(dx + c))^2}{(fx + e)^2} dx$$

input
```
int((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x)
```

output
```
int((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x)
```

3.12.5 Fricas [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(b\,\text{arcsch}\,(dx+c)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f^2*x^2
+ 2*e*f*x + e^2), x)
```

3.12.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(a+b\,\text{acsch}\,(c+dx)\right)^2}{\left(e+fx\right)^2}\,dx$$

input
```
integrate((a+b*acsch(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Integral((a + b*acsch(c + d*x))**2/(e + f*x)**2, x)
```

3.12.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx=\int \frac{\left(b\,\text{arcsch}\,(dx+c)+a\right)^2}{\left(fx+e\right)^2}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
-b^2*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2/(f^2*x + e*f) - a^2/(f^2
*x + e*f) - integrate(-((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*
log(d*x + c)^2 - 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d
*x + c) + 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b - (b^2*d^2*f*
x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c) + (b^2*c*d*e + (c^2*f
+ f)*a*b + (a*b*d^2*f + b^2*d^2*f)*x^2 + (2*a*b*c*d*f + (d^2*e + c*d*f)*b^
2)*x - (b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c))*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^
2*f + f)*b^2)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f +
 f)*a*b)*log(d*x + c)))/(d^2*f^3*x^4 + c^2*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)
*x^3 + e^2*f + (d^2*e^2*f + 4*c*d*e*f^2 + c^2*f^3 + f^3)*x^2 + 2*(c*d*e^2*
f + c^2*e*f^2 + e*f^2)*x + (d^2*f^3*x^4 + c^2*e^2*f + 2*(d^2*e*f^2 + c*d*f
^3)*x^3 + e^2*f + (d^2*e^2*f + 4*c*d*e*f^2 + c^2*f^3 + f^3)*x^2 + 2*(c*d*e
^2*f + c^2*e*f^2 + e*f^2)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

3.12.8 Giac [F]

$$\int \frac{(a+b\text{csch}^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\,\text{arcsch}\,(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(d*x + c) + a)^2/(f*x + e)^2, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{csch}^{-1}(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2}{(e+f\,x)^2}\,dx$$

input
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^2,x)
```

output
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^2, x)
```

3.13 $\int \frac{\left(a+b\,\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx$

3.13.1 Optimal result

Integrand size = 20, antiderivative size = 1024

$$
\begin{aligned}
&\int \frac{\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx \\
&= -\frac{bd^2f\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\operatorname{csch}^{-1}(c+dx)\right)}{(de-cf)\left(d^2e^2-2cdef+(1+c^2)f^2\right)\left(f+\frac{de-cf}{c+dx}\right)} \\
&\quad+\frac{d^2\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{2f(de-cf)^2}-\frac{\left(a+b\operatorname{csch}^{-1}(c+dx)\right)^2}{2f(e+fx)^2} \\
&\quad+\frac{bd^2f^2\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\left(d^2e^2-2cdef+(1+c^2)f^2\right)^{3/2}} \\
&\quad-\frac{2bd^2\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\sqrt{d^2e^2-2cdef+(1+c^2)f^2}} \\
&\quad-\frac{bd^2f^2\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\left(d^2e^2-2cdef+(1+c^2)f^2\right)^{3/2}} \\
&\quad+\frac{2bd^2\left(a+b\operatorname{csch}^{-1}(c+dx)\right)\log\left(1+\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\sqrt{d^2e^2-2cdef+(1+c^2)f^2}} \\
&\quad+\frac{b^2d^2f\log\left(f+\frac{de-cf}{c+dx}\right)}{(de-cf)^2\left(d^2e^2-2cdef+(1+c^2)f^2\right)} \\
&\quad+\frac{b^2d^2f^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\left(d^2e^2-2cdef+(1+c^2)f^2\right)^{3/2}} \\
&\quad-\frac{2b^2d^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\sqrt{d^2e^2-2cdef+(1+c^2)f^2}} \\
&\quad-\frac{b^2d^2f^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\left(d^2e^2-2cdef+(1+c^2)f^2\right)^{3/2}} \\
&\quad+\frac{2b^2d^2\operatorname{PolyLog}\left(2,-\frac{e^{\operatorname{csch}^{-1}(c+dx)}(de-cf)}{f+\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}\right)}{(de-cf)^2\sqrt{d^2e^2-2cdef+(1+c^2)f^2}}
\end{aligned}
$$

output
```
1/2*d^2*(a+b*arccsch(d*x+c))^2/f/(-c*f+d*e)^2-1/2*(a+b*arccsch(d*x+c))^2/f
/(f*x+e)^2+b^2*d^2*f*ln(f+(-c*f+d*e)/(d*x+c))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*
e*f+(c^2+1)*f^2)+b*d^2*f^2*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+
c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d
*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(3/2)-b*d^2*f^2*(a+b*arccsch(d*x+c))
*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e*f+(
c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(3/2)+b^2
*d^2*f^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*
f^2)^(3/2)-b^2*d^2*f^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+
d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*
d*e*f+(c^2+1)*f^2)^(3/2)-2*b*d^2*(a+b*arccsch(d*x+c))*ln(1+(1/(d*x+c)+(1+1
/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(
-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)+2*b*d^2*(a+b*arccsch(d*x
+c))*ln(1+(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f+(d^2*e^2-2*c*d*e
*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)
-2*b^2*d^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+d*e)/(f-(d^2
*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2*c*d*e*f+(c^2+1
)*f^2)^(1/2)+2*b^2*d^2*polylog(2,-(1/(d*x+c)+(1+1/(d*x+c)^2)^(1/2))*(-c*f+
d*e)/(f+(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^(1/2)))/(-c*f+d*e)^2/(d^2*e^2-2...
```

3.13.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.37 (sec) , antiderivative size = 8350, normalized size of antiderivative = 8.15

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^3,x]
```

output
```
Result too large to show
```

3.13.3 Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 1006, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6876, 5992, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}dx$$

$$\Big\downarrow \text{ 6876}$$

$$-d^2\int \frac{(c+dx)^2\sqrt{1+\frac{1}{(c+dx)^2}}\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(de-cf+f(c+dx))^3}d\text{csch}^{-1}(c+dx)$$

$$\Big\downarrow \text{ 5992}$$

$$-d^2\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f(f(c+dx)-cf+de)^2}-\frac{b\int \frac{a+b\text{csch}^{-1}(c+dx)}{(de-cf+f(c+dx))^2}d\text{csch}^{-1}(c+dx)}{f}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-d^2\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f(f(c+dx)-cf+de)^2}-\frac{b\int \frac{a+b\text{csch}^{-1}(c+dx)}{\left(de-cf+if\csc\left(i\text{csch}^{-1}(c+dx)\right)\right)^2}d\text{csch}^{-1}(c+dx)}{f}\right)$$

$$\Big\downarrow \text{ 4679}$$

$$-d^2\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f(f(c+dx)-cf+de)^2}-\frac{b\int \left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)f^2}{(de-cf)^2\left(f+\frac{de\left(1-\frac{cf}{de}\right)}{c+dx}\right)^2}+\frac{2\left(a+b\text{csch}^{-1}(c+dx)\right)f}{(de-cf)^2\left(-f-\frac{de\left(1-\frac{cf}{de}\right)}{c+dx}\right)}+\frac{a+b\text{csch}^{-1}(c+dx)}{(de-cf)^2}\right)d\text{csc}}{f}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-d^2\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{2f(de-cf+f(c+dx))^2}-\frac{b\left(\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{e^{\text{csch}^{-1}(c+dx)}(de-cf)}{f-\sqrt{d^2e^2-2cdfe+\left(c^2+1\right)f^2}}+1\right)f^3}{(de-cf)^2(d^2e^2-2cdfe+(c^2+1)f^2)^{3/2}}-\frac{\left(a+b\text{csch}^{-1}(c+dx)\right)\log\left(\frac{-}{f\cdot}\right.}{(de-cf)^2(d^2e^2-2}\right.}{}\right.$$

input
```
Int[(a + b*ArcCsch[c + d*x])^2/(e + f*x)^3,x]
```

output
```
-(d^2*((a + b*ArcCsch[c + d*x])^2/(2*f*(d*e - c*f + f*(c + d*x))^2) - (b*(
-((f^2*Sqrt[1 + (c + d*x)^(-2)]*(a + b*ArcCsch[c + d*x]))/((d*e - c*f)*(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(f + (d*e - c*f)/(c + d*x)))) + (a + b*
ArcCsch[c + d*x])^2/(2*b*(d*e - c*f)^2) + (f^3*(a + b*ArcCsch[c + d*x])*Lo
g[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2])])/((d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2))
 - (2*f*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/
(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*f)^2*Sqrt[d^2*
e^2 - 2*c*d*e*f + (1 + c^2)*f^2]) - (f^3*(a + b*ArcCsch[c + d*x])*Log[1 +
(E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)
*f^2])])/((d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2)) + (2*
f*(a + b*ArcCsch[c + d*x])*Log[1 + (E^ArcCsch[c + d*x]*(d*e - c*f))/(f + S
qrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2])])/((d*e - c*f)^2*Sqrt[d^2*e^2 -
2*c*d*e*f + (1 + c^2)*f^2]) + (b*f^2*Log[f + (d*e - c*f)/(c + d*x)])/((d*e
 - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) + (b*f^3*PolyLog[2, -((E^
ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^
2]))])/((d*e - c*f)^2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^(3/2)) - (2*b*
f*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f - Sqrt[d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2]))])/((d*e - c*f)^2*Sqrt[d^2*e^2 - 2*c*d*e*f + (1 + c^
2)*f^2]) - (b*f^3*PolyLog[2, -((E^ArcCsch[c + d*x]*(d*e - c*f))/(f + Sq...
```

3.13. $\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx$

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4679
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

rule 5992
```
Int[Coth[(c_.) + (d_.)*(x_)]*Csch[(c_.) + (d_.)*(x_)]*(Csch[(c_.) + (d_.)*(
x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Csch[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b
*d*(n + 1)))   Int[(e + f*x)^(m - 1)*(a + b*Csch[c + d*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6876
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[-(d^(m + 1))^(-1)   Subst[Int[(a + b*x)^p*Csch[x]*C
oth[x]*(d*e - c*f + f*Csch[x])^m, x], x, ArcCsch[c + d*x]], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

3.13.4 Maple [F]

$$\int \frac{(a + b\,\text{arccsch}\,(dx + c))^2}{(fx + e)^3} dx$$

input
```
int((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x)
```

output
```
int((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x)
```

3.13.5 Fricas [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = \int \frac{\left(b\,\text{arcsch}\,(dx+c)+a\right)^2}{\left(fx+e\right)^3}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b^2*arccsch(d*x + c)^2 + 2*a*b*arccsch(d*x + c) + a^2)/(f^3*x^3
+ 3*e*f^2*x^2 + 3*e^2*f*x + e^3), x)
```

3.13.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(d*x+c))**2/(f*x+e)**3,x)
```

output
```
Timed out
```

3.13.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(c+dx)\right)^2}{(e+fx)^3}\,dx = \int \frac{\left(b\,\text{arcsch}\,(dx+c)+a\right)^2}{\left(fx+e\right)^3}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)^2/(f^3*x^2 + 2*e*f^2*x
 + e^2*f) - 1/2*a^2/(f^3*x^2 + 2*e*f^2*x + e^2*f) - integrate(-((b^2*d^2*f
*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c)^2 - 2*(a*b*d^2*f*x^2
+ 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d*x + c) + (2*a*b*d^2*f*x^2 + 4*a*b
*c*d*f*x + 2*(c^2*f + f)*a*b - 2*(b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f +
 f)*b^2)*log(d*x + c) + (b^2*c*d*e + 2*(c^2*f + f)*a*b + (2*a*b*d^2*f + b^
2*d^2*f)*x^2 + (4*a*b*c*d*f + (d^2*e + c*d*f)*b^2)*x - 2*(b^2*d^2*f*x^2 +
2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x + c))*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*log(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*((b^2*d^2*f*x^2 + 2*b^2*c*d*f*x + (c^2*f + f)*b^2)*log(d*x +
c)^2 - 2*(a*b*d^2*f*x^2 + 2*a*b*c*d*f*x + (c^2*f + f)*a*b)*log(d*x + c)))/
(d^2*f^4*x^5 + c^2*e^3*f + (3*d^2*e*f^3 + 2*c*d*f^4)*x^4 + e^3*f + (3*d^2*
e^2*f^2 + 6*c*d*e*f^3 + c^2*f^4 + f^4)*x^3 + (d^2*e^3*f + 6*c*d*e^2*f^2 +
3*c^2*e*f^3 + 3*e*f^3)*x^2 + (2*c*d*e^3*f + 3*c^2*e^2*f^2 + 3*e^2*f^2)*x +
 (d^2*f^4*x^5 + c^2*e^3*f + (3*d^2*e*f^3 + 2*c*d*f^4)*x^4 + e^3*f + (3*d^2
*e^2*f^2 + 6*c*d*e*f^3 + c^2*f^4 + f^4)*x^3 + (d^2*e^3*f + 6*c*d*e^2*f^2 +
 3*c^2*e*f^3 + 3*e*f^3)*x^2 + (2*c*d*e^3*f + 3*c^2*e^2*f^2 + 3*e^2*f^2)*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

3.13.8 Giac [F]

$$\int \frac{(a+b\operatorname{csch}^{-1}(c+dx))^2}{(e+fx)^3}\,dx = \int \frac{(b\operatorname{arcsch}(dx+c)+a)^2}{(fx+e)^3}\,dx$$

input
```
integrate((a+b*arccsch(d*x+c))^2/(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(d*x + c) + a)^2/(f*x + e)^3, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{csch}^{-1}(c+dx))^2}{(e+fx)^3}\,dx = \int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{c+d\,x}\right)\right)^2}{(e+f\,x)^3}\,dx$$

input
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^3,x)
```

output
```
int((a + b*asinh(1/(c + d*x)))^2/(e + f*x)^3, x)
```

3.14 $\int x^3 \text{csch}^{-1}(\sqrt{x})\,dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 114

$$\int x^3 \text{csch}^{-1}(\sqrt{x})\,dx = -\frac{\sqrt{-1-x}\sqrt{x}}{4\sqrt{-x}} - \frac{(-1-x)^{3/2}\sqrt{x}}{4\sqrt{-x}} - \frac{3(-1-x)^{5/2}\sqrt{x}}{20\sqrt{-x}} - \frac{(-1-x)^{7/2}\sqrt{x}}{28\sqrt{-x}} + \frac{1}{4}x^4\text{csch}^{-1}(\sqrt{x})$$

output
```
1/4*x^4*arccsch(x^(1/2))-1/4*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)-3/20*(-1-x)^(
5/2)*x^(1/2)/(-x)^(1/2)-1/28*(-1-x)^(7/2)*x^(1/2)/(-x)^(1/2)-1/4*(-1-x)^(1
/2)*x^(1/2)/(-x)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int x^3 \text{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{140}\sqrt{1+\frac{1}{x}}\sqrt{x}(-16+8x-6x^2+5x^3) + \frac{1}{4}x^4\text{csch}^{-1}(\sqrt{x})$$

input
```
Integrate[x^3*ArcCsch[Sqrt[x]],x]
```

output
```
(Sqrt[1 + x^(-1)]*Sqrt[x]*(-16 + 8*x - 6*x^2 + 5*x^3))/140 + (x^4*ArcCsch[
Sqrt[x]])/4
```

3.14.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6900, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{6900}$$

$$\frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x^3}{2\sqrt{-x-1}}dx}{4\sqrt{-x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x^3}{\sqrt{-x-1}}dx}{8\sqrt{-x}}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \left(-(-x-1)^{5/2}-3(-x-1)^{3/2}-3\sqrt{-x-1}-\frac{1}{\sqrt{-x-1}}\right)dx}{8\sqrt{-x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\left(\frac{2}{7}(-x-1)^{7/2}+\frac{6}{5}(-x-1)^{5/2}+2(-x-1)^{3/2}+2\sqrt{-x-1}\right)\sqrt{x}}{8\sqrt{-x}}$$

input
```
Int[x^3*ArcCsch[Sqrt[x]],x]
```

output
```
-1/8*((2*Sqrt[-1 - x] + 2*(-1 - x)^(3/2) + (6*(-1 - x)^(5/2))/5 + (2*(-1 -
 x)^(7/2))/7)*Sqrt[x])/Sqrt[-x] + (x^4*ArcCsch[Sqrt[x]])/4
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.14.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.35

method	result	size
parts	$\frac{x^4 \operatorname{arccsch}\left(\sqrt{x}\right)}{4}+\frac{\sqrt{\frac{1+x}{x}}\,\sqrt{x}\,\left(5x^3-6x^2+8x-16\right)}{140}$	40
derivativedivides	$\frac{x^4 \operatorname{arccsch}\left(\sqrt{x}\right)}{4}+\frac{(1+x)\left(5x^3-6x^2+8x-16\right)}{140\sqrt{\frac{1+x}{x}}\,\sqrt{x}}$	43
default	$\frac{x^4 \operatorname{arccsch}\left(\sqrt{x}\right)}{4}+\frac{(1+x)\left(5x^3-6x^2+8x-16\right)}{140\sqrt{\frac{1+x}{x}}\,\sqrt{x}}$	43

input
```
int(x^3*arccsch(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccsch(x^(1/2))+1/140*((1+x)/x)^(1/2)*x^(1/2)*(5*x^3-6*x^2+8*x-16
)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int x^3 \text{csch}^{-1}(\sqrt{x})\, dx = \frac{1}{4}\, x^4 \log\left(\frac{x\sqrt{\frac{x+1}{x}} + \sqrt{x}}{x}\right) + \frac{1}{140}\left(5\, x^3 - 6\, x^2 + 8\, x - 16\right)\sqrt{x}\sqrt{\frac{x+1}{x}}$$

input
```
integrate(x^3*arccsch(x^(1/2)),x, algorithm="fricas")
```

output
```
1/4*x^4*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/140*(5*x^3 - 6*x^2 + 8*x
- 16)*sqrt(x)*sqrt((x + 1)/x)
```

3.14.6 Sympy [F]

$$\int x^3 \text{csch}^{-1}(\sqrt{x})\, dx = \int x^3 \operatorname{acsch}\left(\sqrt{x}\right) dx$$

input
```
integrate(x**3*acsch(x**(1/2)),x)
```

output
```
Integral(x**3*acsch(sqrt(x)), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int x^3 \text{csch}^{-1}(\sqrt{x})\, dx = \frac{1}{28}\, x^{\frac{7}{2}}\left(\frac{1}{x}+1\right)^{\frac{7}{2}} - \frac{3}{20}\, x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{4}\, x^4 \operatorname{arcsch}\left(\sqrt{x}\right) + \frac{1}{4}\, x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}} - \frac{1}{4}\sqrt{x}\sqrt{\frac{1}{x}+1}$$

input
```
integrate(x^3*arccsch(x^(1/2)),x, algorithm="maxima")
```

output
```
1/28*x^(7/2)*(1/x + 1)^(7/2) - 3/20*x^(5/2)*(1/x + 1)^(5/2) + 1/4*x^4*arcc
sch(sqrt(x)) + 1/4*x^(3/2)*(1/x + 1)^(3/2) - 1/4*sqrt(x)*sqrt(1/x + 1)
```

3.14.8 Giac [F]

$$\int x^3\text{csch}^{-1}\left(\sqrt{x}\right)\,dx=\int x^3\,\text{arcsch}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x^3*arccsch(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^3*arccsch(sqrt(x)), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int x^3\text{csch}^{-1}\left(\sqrt{x}\right)\,dx=\int x^3\,\text{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

input
```
int(x^3*asinh(1/x^(1/2)),x)
```

output
```
int(x^3*asinh(1/x^(1/2)), x)
```

3.15 $\int x^2 \text{csch}^{-1}(\sqrt{x})\,dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int x^2 \text{csch}^{-1}(\sqrt{x})\,dx = \frac{\sqrt{-1-x}\sqrt{x}}{3\sqrt{-x}} + \frac{2(-1-x)^{3/2}\sqrt{x}}{9\sqrt{-x}} + \frac{(-1-x)^{5/2}\sqrt{x}}{15\sqrt{-x}} + \frac{1}{3}x^3\text{csch}^{-1}(\sqrt{x})$$

output
```
1/3*x^3*arccsch(x^(1/2))+2/9*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)+1/15*(-1-x)^(
5/2)*x^(1/2)/(-x)^(1/2)+1/3*(-1-x)^(1/2)*x^(1/2)/(-x)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

$$\int x^2 \text{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{45}\sqrt{1+\frac{1}{x}}\sqrt{x}(8-4x+3x^2) + \frac{1}{3}x^3\text{csch}^{-1}(\sqrt{x})$$

input
```
Integrate[x^2*ArcCsch[Sqrt[x]],x]
```

output
```
(Sqrt[1 + x^(-1)]*Sqrt[x]*(8 - 4*x + 3*x^2))/45 + (x^3*ArcCsch[Sqrt[x]])/3
```

3.15.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6900, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\text{csch}^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{ 6900}$$

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x^2}{2\sqrt{-x-1}}dx}{3\sqrt{-x}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x^2}{\sqrt{-x-1}}dx}{6\sqrt{-x}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \left((-x-1)^{3/2}+2\sqrt{-x-1}+\frac{1}{\sqrt{-x-1}}\right)dx}{6\sqrt{-x}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\left(-\frac{2}{5}(-x-1)^{5/2}-\frac{4}{3}(-x-1)^{3/2}-2\sqrt{-x-1}\right)\sqrt{x}}{6\sqrt{-x}}$$

input
```
Int[x^2*ArcCsch[Sqrt[x]],x]
```

output
```
-1/6*((-2*Sqrt[-1 - x] - (4*(-1 - x)^(3/2))/3 - (2*(-1 - x)^(5/2))/5)*Sqrt
[x])/Sqrt[-x] + (x^3*ArcCsch[Sqrt[x]])/3
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.15.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.39

method	result	size
parts	$\frac{x^{3} \operatorname{arccsch}\left(\sqrt{x}\right)}{3}+\frac{\sqrt{\frac{1+x}{x}}\, \sqrt{x}\, \left(3 x^{2}-4 x +8\right)}{45}$	35
derivativedivides	$\frac{x^{3} \operatorname{arccsch}\left(\sqrt{x}\right)}{3}+\frac{\left(1+x \right) \left(3 x^{2}-4 x +8\right)}{45 \sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	38
default	$\frac{x^{3} \operatorname{arccsch}\left(\sqrt{x}\right)}{3}+\frac{\left(1+x \right) \left(3 x^{2}-4 x +8\right)}{45 \sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	38

input
```
int(x^2*arccsch(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccsch(x^(1/2))+1/45*((1+x)/x)^(1/2)*x^(1/2)*(3*x^2-4*x+8)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int x^2 \operatorname{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{3}\,x^3\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right) + \frac{1}{45}\left(3\,x^2-4\,x+8\right)\sqrt{x}\sqrt{\frac{x+1}{x}}$$

input
```
integrate(x^2*arccsch(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*x^3*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/45*(3*x^2 - 4*x + 8)*sqrt
(x)*sqrt((x + 1)/x)
```

3.15.6 Sympy [F]

$$\int x^2 \operatorname{csch}^{-1}(\sqrt{x})\,dx = \int x^2 \operatorname{acsch}\left(\sqrt{x}\right)dx$$

input
```
integrate(x**2*acsch(x**(1/2)),x)
```

output
```
Integral(x**2*acsch(sqrt(x)), x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int x^2 \operatorname{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{15}\,x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}} + \frac{1}{3}\,x^3\operatorname{arcsch}\left(\sqrt{x}\right) - \frac{2}{9}\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{x}\sqrt{\frac{1}{x}+1}$$

input
```
integrate(x^2*arccsch(x^(1/2)),x, algorithm="maxima")
```

output
```
1/15*x^(5/2)*(1/x + 1)^(5/2) + 1/3*x^3*arccsch(sqrt(x)) - 2/9*x^(3/2)*(1/x
 + 1)^(3/2) + 1/3*sqrt(x)*sqrt(1/x + 1)
```

3.15.8 Giac [F]

$$\int x^2\text{csch}^{-1}\left(\sqrt{x}\right)\,dx = \int x^2\,\text{arcsch}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x^2*arccsch(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^2*arccsch(sqrt(x)), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^2\text{csch}^{-1}\left(\sqrt{x}\right)\,dx = \int x^2\,\text{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

input
```
int(x^2*asinh(1/x^(1/2)),x)
```

output
```
int(x^2*asinh(1/x^(1/2)), x)
```

3.16 $\int x\text{csch}^{-1}(\sqrt{x})\,dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 64

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx=-\frac{\sqrt{-1-x}\sqrt{x}}{2\sqrt{-x}}-\frac{(-1-x)^{3/2}\sqrt{x}}{6\sqrt{-x}}+\frac{1}{2}x^2\text{csch}^{-1}(\sqrt{x})$$

output
```
1/2*x^2*arccsch(x^(1/2))-1/6*(-1-x)^(3/2)*x^(1/2)/(-x)^(1/2)-1/2*(-1-x)^(1
/2)*x^(1/2)/(-x)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.55

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx=\frac{1}{6}\sqrt{1+\frac{1}{x}}(-2+x)\sqrt{x}+\frac{1}{2}x^2\text{csch}^{-1}(\sqrt{x})$$

input
```
Integrate[x*ArcCsch[Sqrt[x]],x]
```

output
```
(Sqrt[1 + x^(-1)]*(-2 + x)*Sqrt[x])/6 + (x^2*ArcCsch[Sqrt[x]])/2
```

3.16.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6900, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\text{csch}^{-1}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 6900}$$

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x}{2\sqrt{-x-1}}dx}{2\sqrt{-x}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{x}{\sqrt{-x-1}}dx}{4\sqrt{-x}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \left(-\sqrt{-x-1}-\frac{1}{\sqrt{-x-1}}\right)dx}{4\sqrt{-x}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2\text{csch}^{-1}\left(\sqrt{x}\right)-\frac{\left(\frac{2}{3}(-x-1)^{3/2}+2\sqrt{-x-1}\right)\sqrt{x}}{4\sqrt{-x}}$$

input
```
Int[x*ArcCsch[Sqrt[x]],x]
```

output
```
-1/4*((2*Sqrt[-1 - x] + (2*(-1 - x)^(3/2))/3)*Sqrt[x])/Sqrt[-x] + (x^2*Arc
Csch[Sqrt[x]])/2
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.16.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

method	result	size
parts	$\frac{x^{2} \operatorname{arccsch}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{\frac{1+x}{x}}\, \sqrt{x}\, \left(x -2\right)}{6}$	28
derivativedivides	$\frac{x^{2} \operatorname{arccsch}\left(\sqrt{x}\right)}{2}+\frac{\left(1+x \right) \left(x -2\right)}{6 \sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	31
default	$\frac{x^{2} \operatorname{arccsch}\left(\sqrt{x}\right)}{2}+\frac{\left(1+x \right) \left(x -2\right)}{6 \sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	31

input
```
int(x*arccsch(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccsch(x^(1/2))+1/6*((1+x)/x)^(1/2)*x^(1/2)*(x-2)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{2}\,x^2\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)+\frac{1}{6}\,(x-2)\sqrt{x}\sqrt{\frac{x+1}{x}}$$

input
```
integrate(x*arccsch(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + 1/6*(x - 2)*sqrt(x)*sqrt((x
 + 1)/x)
```

3.16.6 Sympy [F]

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx = \int x\,\text{acsch}\left(\sqrt{x}\right)dx$$

input
```
integrate(x*acsch(x**(1/2)),x)
```

output
```
Integral(x*acsch(sqrt(x)), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.53

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx = \frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}}+\frac{1}{2}\,x^2\,\text{arcsch}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x}\sqrt{\frac{1}{x}+1}$$

input
```
integrate(x*arccsch(x^(1/2)),x, algorithm="maxima")
```

output
```
1/6*x^(3/2)*(1/x + 1)^(3/2) + 1/2*x^2*arccsch(sqrt(x)) - 1/2*sqrt(x)*sqrt(
1/x + 1)
```

3.16.8 Giac [F]

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx = \int x\,\text{arcsch}\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x*arccsch(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x*arccsch(sqrt(x)), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x\text{csch}^{-1}(\sqrt{x})\,dx = \int x\,\text{asinh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

input
```
int(x*asinh(1/x^(1/2)),x)
```

output
```
int(x*asinh(1/x^(1/2)), x)
```

3.17 $\int \operatorname{csch}^{-1}(\sqrt{x})\,dx$

3.17.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int \operatorname{csch}^{-1}(\sqrt{x})\,dx=\frac{\sqrt{-1-x}\sqrt{x}}{\sqrt{-x}}+x\operatorname{csch}^{-1}(\sqrt{x})$$

output
```
x*arccsch(x^(1/2))+(-1-x)^(1/2)*x^(1/2)/(-x)^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \operatorname{csch}^{-1}(\sqrt{x})\,dx=\sqrt{1+\frac{1}{x}}\sqrt{x}+x\operatorname{csch}^{-1}(\sqrt{x})$$

input
```
Integrate[ArcCsch[Sqrt[x]],x]
```

output
```
Sqrt[1 + x^(-1)]*Sqrt[x] + x*ArcCsch[Sqrt[x]]
```

3.17.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6898, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \ 6898$$

$$x\operatorname{csch}^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}\int \frac{1}{2\sqrt{-x-1}}dx}{\sqrt{-x}}$$

$$\Big\downarrow \ 17$$

$$\frac{\sqrt{-x-1}\sqrt{x}}{\sqrt{-x}}+x\operatorname{csch}^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCsch[Sqrt[x]],x]
```

output
```
(Sqrt[-1 - x]*Sqrt[x])/Sqrt[-x] + x*ArcCsch[Sqrt[x]]
```

3.17.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 6898
```
Int[ArcCsch[u_], x_Symbol] :> Simp[x*ArcCsch[u], x] - Simp[u/Sqrt[-u^2]   I
nt[SimplifyIntegrand[x*(D[u, x]/(u*Sqrt[-1 - u^2])), x], x], x] /; InverseF
unctionFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

3.17.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
parts	$x \,\mathrm{arccsch}\left(\sqrt{x}\right)+\sqrt{\frac{1+x}{x}}\, \sqrt{x}$	21
derivativedivides	$x \,\mathrm{arccsch}\left(\sqrt{x}\right)+\frac{1+x}{\sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	24
default	$x \,\mathrm{arccsch}\left(\sqrt{x}\right)+\frac{1+x}{\sqrt{\frac{1+x}{x}}\, \sqrt{x}}$	24

input
```
int(arccsch(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccsch(x^(1/2))+((1+x)/x)^(1/2)*x^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \operatorname{csch}^{-1}\left(\sqrt{x}\right)\, dx = x \log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)+\sqrt{x}\sqrt{\frac{x+1}{x}}$$

input
```
integrate(arccsch(x^(1/2)),x, algorithm="fricas")
```

output
```
x*log((x*sqrt((x + 1)/x) + sqrt(x))/x) + sqrt(x)*sqrt((x + 1)/x)
```

3.17.6 Sympy [F]

$$\int \operatorname{csch}^{-1}\left(\sqrt{x}\right)\, dx = \int \operatorname{acsch}\left(\sqrt{x}\right) dx$$

input
```
integrate(acsch(x**(1/2)),x)
```

output
```
Integral(acsch(sqrt(x)), x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^{-1}(\sqrt{x})\, dx = x \operatorname{arcsch}(\sqrt{x}) + \sqrt{x}\sqrt{\frac{1}{x}+1}$$

input
```
integrate(arccsch(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccsch(sqrt(x)) + sqrt(x)*sqrt(1/x + 1)
```

3.17.8 Giac [F]

$$\int \operatorname{csch}^{-1}(\sqrt{x})\, dx = \int \operatorname{arcsch}(\sqrt{x})\, dx$$

input
```
integrate(arccsch(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(arccsch(sqrt(x)), x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \operatorname{csch}^{-1}(\sqrt{x})\, dx = x \operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x}\sqrt{\frac{1}{x}+1}$$

input
```
int(asinh(1/x^(1/2)),x)
```

output
```
x*asinh(1/x^(1/2)) + x^(1/2)*(1/x + 1)^(1/2)
```

3.18 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} \, dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} \, dx = \operatorname{csch}^{-1}(\sqrt{x})^2 - 2\operatorname{csch}^{-1}(\sqrt{x}) \log\left(1 - e^{2\operatorname{csch}^{-1}(\sqrt{x})}\right) - \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(\sqrt{x})}\right)$$

output
```
arccsch(x^(1/2))^2-2*arccsch(x^(1/2))*ln(1-(1/x^(1/2)+(1+1/x)^(1/2))^2)-po
lylog(2,(1/x^(1/2)+(1+1/x)^(1/2))^2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} \, dx = \operatorname{csch}^{-1}(\sqrt{x}) \left(\operatorname{csch}^{-1}(\sqrt{x}) - 2\log\left(1 - e^{2\operatorname{csch}^{-1}(\sqrt{x})}\right)\right) - \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(\sqrt{x})}\right)$$

input
```
Integrate[ArcCsch[Sqrt[x]]/x,x]
```

output
```
ArcCsch[Sqrt[x]]*(ArcCsch[Sqrt[x]] - 2*Log[1 - E^(2*ArcCsch[Sqrt[x]])]) -
PolyLog[2, E^(2*ArcCsch[Sqrt[x]])]
```

3.18.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7267, 6836, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}\left(\sqrt{x}\right)}{x} \, dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\text{csch}^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{6836}$$

$$-2\int \frac{\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{6190}$$

$$-2\int \sqrt{1+\frac{1}{x}}\sqrt{x}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-2\int -i\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)\tan\left(i\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{26}$$

$$2i\int \text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)\tan\left(i\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{4199}$$

$$2i\left(2i\int -\frac{e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}}d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)-\frac{ix}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$2i\left(-2i\int \frac{e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}}d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)-\frac{ix}{2}\right)$$

$$\Big\downarrow \; 2620$$

$$2i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)d\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)\right)-\frac{ix}{2}\right)$$

$$\Big\downarrow \; 2715$$

$$2i\left(-2i\left(\frac{1}{4}\int e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)de^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)\right)-\frac{ix}{2}\right)$$

$$\Big\downarrow \; 2838$$

$$2i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}\right)\right)-\frac{ix}{2}\right)$$

input
```
Int[ArcCsch[Sqrt[x]]/x,x]
```

output
```
(2*I)*((-1/2*I)*x - (2*I)*(-1/2*(ArcSinh[1/Sqrt[x]]*Log[1 - E^(2*ArcSinh[1
/Sqrt[x]])]) - PolyLog[2, E^(2*ArcSinh[1/Sqrt[x]])]/4))
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.18.4 Maple [F]

$$\int \frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}dx$$

input
```
int(arccsch(x^(1/2))/x,x)
```

output
```
int(arccsch(x^(1/2))/x,x)
```

3.18.5 Fricas [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} dx = \int \frac{\operatorname{arcsch}(\sqrt{x})}{x} dx$$

input
```
integrate(arccsch(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccsch(sqrt(x))/x, x)
```

3.18.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} dx = \int \frac{\operatorname{acsch}(\sqrt{x})}{x} dx$$

input
```
integrate(acsch(x**(1/2))/x,x)
```

output
```
Integral(acsch(sqrt(x))/x, x)
```

3.18.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x} dx = \int \frac{\operatorname{arcsch}(\sqrt{x})}{x} dx$$

input
```
integrate(arccsch(x^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arccsch(sqrt(x))/x, x)
```

3.18.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccsch(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arccsch(sqrt(x))/x, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

input
```
int(asinh(1/x^(1/2))/x,x)
```

output
```
int(asinh(1/x^(1/2))/x, x)
```

3.19 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx$

3.19.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx=\frac{\sqrt{-1-x}}{2\sqrt{-x}\sqrt{x}}-\frac{\operatorname{csch}^{-1}(\sqrt{x})}{x}-\frac{\sqrt{x}\arctan\left(\sqrt{-1-x}\right)}{2\sqrt{-x}}$$

output
```
-arccsch(x^(1/2))/x+1/2*(-1-x)^(1/2)/(-x)^(1/2)/x^(1/2)-1/2*arctan((-1-x)^
(1/2))*x^(1/2)/(-x)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx=\frac{\sqrt{\frac{1+x}{x}}}{2\sqrt{x}}-\frac{\operatorname{csch}^{-1}(\sqrt{x})}{x}-\frac{1}{2}\operatorname{arcsinh}\left(\frac{1}{\sqrt{x}}\right)$$

input
```
Integrate[ArcCsch[Sqrt[x]]/x^2,x]
```

output
```
Sqrt[(1 + x)/x]/(2*Sqrt[x]) - ArcCsch[Sqrt[x]]/x - ArcSinh[1/Sqrt[x]]/2
```

3.19.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6900, 27, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \; 6900$$

$$\frac{\sqrt{x}\int \frac{1}{2\sqrt{-x-1}x^2}dx}{\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{-x-1}x^2}dx}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \; 52$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{x}-\frac{1}{2}\int \frac{1}{\sqrt{-x-1}x}dx\right)}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{x}\left(\int \frac{1}{x}d\sqrt{-x-1}+\frac{\sqrt{-x-1}}{x}\right)}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{x}-\arctan\left(\sqrt{-x-1}\right)\right)}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCsch[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCsch[Sqrt[x]]/x) + (Sqrt[x]*(Sqrt[-1 - x]/x - ArcTan[Sqrt[-1 - x]]))/
(2*Sqrt[-x])
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.19. $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx$

3.19.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1+x}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x+\sqrt{1+x}\right)}{2\sqrt{\frac{1+x}{x}}\,x^{\frac{3}{2}}}$	45
default	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1+x}\,\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x+\sqrt{1+x}\right)}{2\sqrt{\frac{1+x}{x}}\,x^{\frac{3}{2}}}$	45
parts	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{\frac{1+x}{x}}\,\sqrt{x}\,\left(\ln\left(\sqrt{1+x}-1\right)x-\ln\left(\sqrt{1+x}+1\right)x+2\sqrt{1+x}\right)}{4\sqrt{1+x}\,\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}$	77

input
```
int(arccsch(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccsch(x^(1/2))/x+1/2*(1+x)^(1/2)*(-arctanh(1/(1+x)^(1/2))*x+(1+x)^(1/2)
)/((1+x)/x)^(1/2)/x^(3/2)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{(x+2)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)-\sqrt{x}\sqrt{\frac{x+1}{x}}}{2\,x}$$

input
```
integrate(arccsch(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/2*((x + 2)*log((x*sqrt((x + 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt((x + 1)/
x))/x
```

3.19.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^2}\,dx$$

input
```
integrate(acsch(x**(1/2))/x**2,x)
```

output
```
Integral(acsch(sqrt(x))/x**2, x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{x}\sqrt{\frac{1}{x}+1}}{2\left(x\left(\frac{1}{x}+1\right)-1\right)} - \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x} - \frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)$$

input
```
integrate(arccsch(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(x)*sqrt(1/x + 1)/(x*(1/x + 1) - 1) - arccsch(sqrt(x))/x - 1/4*log
(sqrt(x)*sqrt(1/x + 1) + 1) + 1/4*log(sqrt(x)*sqrt(1/x + 1) - 1)
```

3.19.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\,dx = \int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^2}\,dx$$

input
```
integrate(arccsch(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate(arccsch(sqrt(x))/x^2, x)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^2}\, dx = \frac{\sqrt{\frac{1}{x}+1}}{2\sqrt{x}} - \frac{2\,\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}+\frac{\sqrt{x}}{4}\right)}{\sqrt{x}}$$

input
```
int(asinh(1/x^(1/2))/x^2,x)
```

output
```
(1/x + 1)^(1/2)/(2*x^(1/2)) - (2*asinh(1/x^(1/2))*(1/(2*x^(1/2)) + x^(1/2)
/4))/x^(1/2)
```

3.20 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3} dx$

3.20.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3} dx = \frac{\sqrt{-1-x}}{8\sqrt{-x}x^{3/2}} - \frac{3\sqrt{-1-x}}{16\sqrt{-x}\sqrt{x}} - \frac{\operatorname{csch}^{-1}(\sqrt{x})}{2x^2} + \frac{3\sqrt{x}\arctan\left(\sqrt{-1-x}\right)}{16\sqrt{-x}}$$

output
```
-1/2*arccsch(x^(1/2))/x^2+1/8*(-1-x)^(1/2)/x^(3/2)/(-x)^(1/2)-3/16*(-1-x)^
(1/2)/(-x)^(1/2)/x^(1/2)+3/16*arctan((-1-x)^(1/2))*x^(1/2)/(-x)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3} dx = \frac{\sqrt{1+\frac{1}{x}}(2-3x)\sqrt{x} - 8\operatorname{csch}^{-1}(\sqrt{x}) + 3x^2\operatorname{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}{16x^2}$$

input
```
Integrate[ArcCsch[Sqrt[x]]/x^3,x]
```

output
```
(Sqrt[1 + x^(-1)]*(2 - 3*x)*Sqrt[x] - 8*ArcCsch[Sqrt[x]] + 3*x^2*ArcSinh[1
/Sqrt[x]])/(16*x^2)
```

3.20.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6900, 27, 52, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{ 6900}$$

$$\frac{\sqrt{x}\int \frac{1}{2\sqrt{-x-1}x^3}dx}{2\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{-x-1}x^3}dx}{4\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\int \frac{1}{\sqrt{-x-1}x^2}dx\right)}{4\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\frac{\sqrt{-x-1}}{x}-\frac{1}{2}\int \frac{1}{\sqrt{-x-1}x}dx\right)\right)}{4\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{-x-1}+\frac{\sqrt{-x-1}}{x}\right)\right)}{4\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\frac{\sqrt{-x-1}}{x}-\arctan\left(\sqrt{-x-1}\right)\right)\right)}{4\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCsch[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCsch[Sqrt[x]]/x^2 + (Sqrt[x]*(Sqrt[-1 - x]/(2*x^2) - (3*(Sqrt[-1 -
 x]/x - ArcTan[Sqrt[-1 - x]]))/4))/(4*Sqrt[-x])
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.20.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{1+x}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^{2}-3\sqrt{1+x}\,x+2\sqrt{1+x}\right)}{16\sqrt{\frac{1+x}{x}}\,x^{\frac{5}{2}}}$	57
default	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{1+x}\,\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^{2}-3\sqrt{1+x}\,x+2\sqrt{1+x}\right)}{16\sqrt{\frac{1+x}{x}}\,x^{\frac{5}{2}}}$	57
parts	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{\frac{1+x}{x}}\,\sqrt{x}\,\left(3\ln\left(\sqrt{1+x}+1\right)x^{2}-3\ln\left(\sqrt{1+x}-1\right)x^{2}-6\sqrt{1+x}\,x+4\sqrt{1+x}\right)}{32\sqrt{1+x}\,\left(\sqrt{1+x}+1\right)^{2}\left(\sqrt{1+x}-1\right)^{2}}$	90

input
```
int(arccsch(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccsch(x^(1/2))/x^2+1/16*(1+x)^(1/2)*(3*arctanh(1/(1+x)^(1/2))*x^2-3
*(1+x)^(1/2)*x+2*(1+x)^(1/2))/((1+x)/x)^(1/2)/x^(5/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^{3}}\,dx=-\frac{(3\,x-2)\sqrt{x}\sqrt{\frac{x+1}{x}}-(3\,x^{2}-8)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)}{16\,x^{2}}$$

input
```
integrate(arccsch(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/16*((3*x - 2)*sqrt(x)*sqrt((x + 1)/x) - (3*x^2 - 8)*log((x*sqrt((x + 1)
/x) + sqrt(x))/x))/x^2
```

3.20.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^3}\,dx$$

input
```
integrate(acsch(x**(1/2))/x**3,x)
```

output
```
Integral(acsch(sqrt(x))/x**3, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3}\,dx = -\frac{3\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}} - 5\,\sqrt{x}\sqrt{\frac{1}{x}+1}}{16\left(x^2\left(\frac{1}{x}+1\right)^2 - 2\,x\left(\frac{1}{x}+1\right)+1\right)} - \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{2\,x^2} + \frac{3}{32}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}+1\right) - \frac{3}{32}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)$$

input
```
integrate(arccsch(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/16*(3*x^(3/2)*(1/x + 1)^(3/2) - 5*sqrt(x)*sqrt(1/x + 1))/(x^2*(1/x + 1)
^2 - 2*x*(1/x + 1) + 1) - 1/2*arccsch(sqrt(x))/x^2 + 3/32*log(sqrt(x)*sqrt
(1/x + 1) + 1) - 3/32*log(sqrt(x)*sqrt(1/x + 1) - 1)
```

3.20.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^3}\,dx$$

input
```
integrate(arccsch(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
integrate(arccsch(sqrt(x))/x^3, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^3}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

input
```
int(asinh(1/x^(1/2))/x^3,x)
```

output
```
int(asinh(1/x^(1/2))/x^3, x)
```

3.21 $\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4} \, dx$

3.21.1 Optimal result

Integrand size = 10, antiderivative size = 115

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4} \, dx = \frac{\sqrt{-1-x}}{18\sqrt{-x}x^{5/2}} - \frac{5\sqrt{-1-x}}{72\sqrt{-x}x^{3/2}} + \frac{5\sqrt{-1-x}}{48\sqrt{-x}\sqrt{x}} - \frac{\operatorname{csch}^{-1}(\sqrt{x})}{3x^3} - \frac{5\sqrt{x}\arctan\left(\sqrt{-1-x}\right)}{48\sqrt{-x}}$$

output
```
-1/3*arccsch(x^(1/2))/x^3+1/18*(-1-x)^(1/2)/x^(5/2)/(-x)^(1/2)-5/72*(-1-x)
^(1/2)/x^(3/2)/(-x)^(1/2)+5/48*(-1-x)^(1/2)/(-x)^(1/2)/x^(1/2)-5/48*arctan
((-1-x)^(1/2))*x^(1/2)/(-x)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.45

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4} \, dx = \frac{\sqrt{1+\frac{1}{x}}\sqrt{x}(8-10x+15x^2) - 48\operatorname{csch}^{-1}(\sqrt{x}) - 15x^3\operatorname{arcsinh}\left(\frac{1}{\sqrt{x}}\right)}{144x^3}$$

input
```
Integrate[ArcCsch[Sqrt[x]]/x^4,x]
```

output
```
(Sqrt[1 + x^(-1)]*Sqrt[x]*(8 - 10*x + 15*x^2) - 48*ArcCsch[Sqrt[x]] - 15*x
^3*ArcSinh[1/Sqrt[x]])/(144*x^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6900, 27, 52, 52, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{6900}$$

$$\frac{\sqrt{x}\int \frac{1}{2\sqrt{-x-1}x^4}dx}{3\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}\int \frac{1}{\sqrt{-x-1}x^4}dx}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{3x^3}-\frac{5}{6}\int \frac{1}{\sqrt{-x-1}x^3}dx\right)}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{3x^3}-\frac{5}{6}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\int \frac{1}{\sqrt{-x-1}x^2}dx\right)\right)}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{3x^3}-\frac{5}{6}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\frac{\sqrt{-x-1}}{x}-\frac{1}{2}\int \frac{1}{\sqrt{-x-1}x}dx\right)\right)\right)}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{3x^3}-\frac{5}{6}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\int \frac{1}{x}d\sqrt{-x-1}+\frac{\sqrt{-x-1}}{x}\right)\right)\right)}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{x}\left(\frac{\sqrt{-x-1}}{3x^3}-\frac{5}{6}\left(\frac{\sqrt{-x-1}}{2x^2}-\frac{3}{4}\left(\frac{\sqrt{-x-1}}{x}-\arctan\left(\sqrt{-x-1}\right)\right)\right)\right)}{6\sqrt{-x}}-\frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{3x^3}$$

input
```
Int[ArcCsch[Sqrt[x]]/x^4,x]
```

output
```
-1/3*ArcCsch[Sqrt[x]]/x^3 + (Sqrt[x]*(Sqrt[-1 - x]/(3*x^3) - (5*(Sqrt[-1 -
 x]/(2*x^2) - (3*(Sqrt[-1 - x]/x - ArcTan[Sqrt[-1 - x]]))/4))/6))/(6*Sqrt[
-x])
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 6900
```
Int[((a_.) + ArcCsch[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCsch[u])/(d*(m + 1))), x] - Simp[b*(u/(d*(m
 + 1)*Sqrt[-u^2]))   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sq
rt[-1 - u^2])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && In
verseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Fun
ctionOfExponentialQ[u, x]
```

3.21.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1+x}\,\left(-15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^{3}+15\sqrt{1+x}\,x^{2}-10\sqrt{1+x}\,x+8\sqrt{1+x}\right)}{144\sqrt{\frac{1+x}{x}}\,x^{\frac{7}{2}}}$	67
default	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1+x}\,\left(-15\,\operatorname{arctanh}\left(\frac{1}{\sqrt{1+x}}\right)x^{3}+15\sqrt{1+x}\,x^{2}-10\sqrt{1+x}\,x+8\sqrt{1+x}\right)}{144\sqrt{\frac{1+x}{x}}\,x^{\frac{7}{2}}}$	67
parts	$-\frac{\operatorname{arccsch}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{\frac{1+x}{x}}\,\sqrt{x}\,\left(15\ln\left(\sqrt{1+x}-1\right)x^{3}-15\ln\left(\sqrt{1+x}+1\right)x^{3}+30\sqrt{1+x}\,x^{2}-20\sqrt{1+x}\,x+16\sqrt{1+x}\right)}{288\sqrt{1+x}\,\left(\sqrt{1+x}-1\right)^{3}\left(\sqrt{1+x}+1\right)^{3}}$	100

input
```
int(arccsch(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccsch(x^(1/2))/x^3+1/144*(1+x)^(1/2)*(-15*arctanh(1/(1+x)^(1/2))*x^
3+15*(1+x)^(1/2)*x^2-10*(1+x)^(1/2)*x+8*(1+x)^(1/2))/((1+x)/x)^(1/2)/x^(7/
2)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \frac{\operatorname{csch}^{-1}\left(\sqrt{x}\right)}{x^4}\,dx=\frac{\left(15\,x^{2}-10\,x+8\right)\sqrt{x}\sqrt{\frac{x+1}{x}}-3\left(5\,x^{3}+16\right)\log\left(\frac{x\sqrt{\frac{x+1}{x}}+\sqrt{x}}{x}\right)}{144\,x^{3}}$$

input
```
integrate(arccsch(x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/144*((15*x^2 - 10*x + 8)*sqrt(x)*sqrt((x + 1)/x) - 3*(5*x^3 + 16)*log((x
*sqrt((x + 1)/x) + sqrt(x))/x))/x^3
```

3.21.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{acsch}\left(\sqrt{x}\right)}{x^4}\,dx$$

input
```
integrate(acsch(x**(1/2))/x**4,x)
```

output
```
Integral(acsch(sqrt(x))/x**4, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4}\,dx = \frac{15\,x^{\frac{5}{2}}\left(\frac{1}{x}+1\right)^{\frac{5}{2}} - 40\,x^{\frac{3}{2}}\left(\frac{1}{x}+1\right)^{\frac{3}{2}} + 33\,\sqrt{x}\sqrt{\frac{1}{x}+1}}{144\left(x^3\left(\frac{1}{x}+1\right)^3 - 3\,x^2\left(\frac{1}{x}+1\right)^2 + 3\,x\left(\frac{1}{x}+1\right) - 1\right)} - \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{3\,x^3} - \frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}+1\right) + \frac{5}{96}\log\left(\sqrt{x}\sqrt{\frac{1}{x}+1}-1\right)$$

input
```
integrate(arccsch(x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
1/144*(15*x^(5/2)*(1/x + 1)^(5/2) - 40*x^(3/2)*(1/x + 1)^(3/2) + 33*sqrt(x
)*sqrt(1/x + 1))/(x^3*(1/x + 1)^3 - 3*x^2*(1/x + 1)^2 + 3*x*(1/x + 1) - 1)
 - 1/3*arccsch(sqrt(x))/x^3 - 5/96*log(sqrt(x)*sqrt(1/x + 1) + 1) + 5/96*l
og(sqrt(x)*sqrt(1/x + 1) - 1)
```

3.21.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{arcsch}\left(\sqrt{x}\right)}{x^4}\,dx$$

input
```
integrate(arccsch(x^(1/2))/x^4,x, algorithm="giac")
```

output
```
integrate(arccsch(sqrt(x))/x^4, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(\sqrt{x})}{x^4}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

input

```
int(asinh(1/x^(1/2))/x^4,x)
```

output

```
int(asinh(1/x^(1/2))/x^4, x)
```

3.22 $\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) \, dx$

3.22.1 Optimal result

Integrand size = 4, antiderivative size = 16

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) \, dx = -\sqrt{1+x^2} + x\operatorname{arcsinh}(x)$$

output
```
x*arcsinh(x)-(x^2+1)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) \, dx = -\sqrt{1+x^2} + x\operatorname{csch}^{-1}\left(\frac{1}{x}\right)$$

input
```
Integrate[ArcCsch[x^(-1)],x]
```

output
```
-Sqrt[1 + x^2] + x*ArcCsch[x^(-1)]
```

3.22.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6882, 6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{6882}$$

$$\int \operatorname{arcsinh}(x)dx$$

$$\Big\downarrow \text{6187}$$

$$x\operatorname{arcsinh}(x) - \int \frac{x}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{241}$$

$$x\operatorname{arcsinh}(x) - \sqrt{x^2+1}$$

input
```
Int[ArcCsch[x^(-1)],x]
```

output
```
-Sqrt[1 + x^2] + x*ArcSinh[x]
```

3.22.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 6882
```
Int[ArcCsch[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSinh[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
parts	$x \operatorname{arccsch}\left(\frac{1}{x}\right)-\sqrt{x^{2}+1}$	17
derivativedivides	$x \operatorname{arccsch}\left(\frac{1}{x}\right)-\frac{x^{2}\left(\frac{1}{x^{2}}+1\right)}{\sqrt{\left(\frac{1}{x^{2}}+1\right) x^{2}}}$	29
default	$x \operatorname{arccsch}\left(\frac{1}{x}\right)-\frac{x^{2}\left(\frac{1}{x^{2}}+1\right)}{\sqrt{\left(\frac{1}{x^{2}}+1\right) x^{2}}}$	29

input
```
int(arccsch(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arccsch(1/x)-(x^2+1)^(1/2)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right)\,dx = x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
integrate(arccsch(1/x),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right)\,dx = x\operatorname{acsch}\left(\frac{1}{x}\right)-\sqrt{x^2+1}$$

input
```
integrate(acsch(1/x),x)
```

output
```
x*acsch(1/x) - sqrt(x**2 + 1)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx = x \operatorname{arcsch}\left(\frac{1}{x}\right) - \sqrt{x^2+1}$$

input
```
integrate(arccsch(1/x),x, algorithm="maxima")
```

output
```
x*arccsch(1/x) - sqrt(x^2 + 1)
```

3.22.8 Giac [F]

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx = \int \operatorname{arcsch}\left(\frac{1}{x}\right) dx$$

input
```
integrate(arccsch(1/x),x, algorithm="giac")
```

output
```
integrate(arccsch(1/x), x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{csch}^{-1}\left(\frac{1}{x}\right) dx = x \operatorname{asinh}(x) - \sqrt{x^2+1}$$

input
```
int(asinh(x),x)
```

output
```
x*asinh(x) - (x^2 + 1)^(1/2)
```

3.23 $\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx=\frac{\operatorname{csch}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{csch}^{-1}(ax^n)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{n}-\frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(ax^n)}\right)}{2n}$$

output
```
1/2*arccsch(a*x^n)^2/n-arccsch(a*x^n)*ln(1-(1/a/(x^n)+(1+1/a^2/(x^n)^2)^(1
/2))^2)/n-1/2*polylog(2,(1/a/(x^n)+(1+1/a^2/(x^n)^2)^(1/2))^2)/n
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx=-\frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{x^{-2n}}{a^2}\right)}{an}+\left(\operatorname{csch}^{-1}(ax^n)-\operatorname{arcsinh}\left(\frac{x^{-n}}{a}\right)\right)\log(x)$$

input
```
Integrate[ArcCsch[a*x^n]/x,x]
```

output
```
-(HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, -(1/(a^2*x^(2*n)))]/(a*n*
x^n)) + (ArcCsch[a*x^n] - ArcSinh[1/(a*x^n)])*Log[x]
```

3.23.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7282, 6836, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{ 7282}$$

$$\frac{\int x^{-n}\operatorname{csch}^{-1}(ax^n)\,dx^n}{n}$$

$$\Big\downarrow \text{ 6836}$$

$$-\frac{\int x^{-n}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)dx^{-n}}{n}$$

$$\Big\downarrow \text{ 6190}$$

$$-\frac{\int ax^n\sqrt{\frac{x^{-2n}}{a^2}+1}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -i\text{arcsinh}\left(\frac{x^{-n}}{a}\right)\tan\left(i\text{arcsinh}\left(\frac{x^{-n}}{a}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\int \text{arcsinh}\left(\frac{x^{-n}}{a}\right)\tan\left(i\text{arcsinh}\left(\frac{x^{-n}}{a}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}{n}$$

$$\Big\downarrow \text{ 4199}$$

$$\frac{i\left(2i\int -\frac{e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}{1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}}d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\left(-2i\int \frac{e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}{1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}}d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

$\big\downarrow$ 2620

$$\frac{i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)d\text{arcsinh}\left(\frac{x^{-n}}{a}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

$\big\downarrow$ 2715

$$\frac{i\left(-2i\left(\frac{1}{4}\int e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)de^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)\right)-\right.}{n}$$

$\big\downarrow$ 2838

$$\frac{i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{x^{-n}}{a}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{x^{-n}}{a}\right)}\right)\right)-\frac{1}{2}ix^{2n}\right)}{n}$$

input
```
Int[ArcCsch[a*x^n]/x,x]
```

output
```
(I*((-1/2*I)*x^(2*n) - (2*I)*(-1/2*(ArcSinh[1/(a*x^n)]*Log[1 - E^(2*ArcSin
h[1/(a*x^n)])]) - PolyLog[2, E^(2*ArcSinh[1/(a*x^n)])]/4)))/n
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.23. $\int \frac{\text{csch}^{-1}(ax^n)}{x} \, dx$

3.23.4 Maple [F]

$$\int \frac{\operatorname{arccsch}(a\,x^n)}{x}dx$$

input
```
int(arccsch(a*x^n)/x,x)
```

output
```
int(arccsch(a*x^n)/x,x)
```

3.23.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccsch(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.23.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acsch}(ax^n)}{x}\,dx$$

input
```
integrate(acsch(a*x**n)/x,x)
```

output
```
Integral(acsch(a*x**n)/x, x)
```

3.23.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcsch}(ax^n)}{x}\,dx$$

input
```
integrate(arccsch(a*x^n)/x,x, algorithm="maxima")
```

output
```
a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) + x)*sqrt(a
^2*x^(2*n) + 1) + x), x) + n*integrate(log(x)/(a^2*x*x^(2*n) + x), x) - lo
g(a)*log(x) - log(x)*log(x^n) + log(x)*log(sqrt(a^2*x^(2*n) + 1) + 1)
```

3.23.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arcsch}(ax^n)}{x}\,dx$$

input
```
integrate(arccsch(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccsch(a*x^n)/x, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

input
```
int(asinh(1/(a*x^n))/x,x)
```

output
```
int(asinh(1/(a*x^n))/x, x)
```

3.24 $\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx = \frac{1}{10}\operatorname{csch}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{csch}^{-1}(ax^5)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^5)}\right) - \frac{1}{10}\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(ax^5)}\right)$$

output
```
1/10*arccsch(a*x^5)^2-1/5*arccsch(a*x^5)*ln(1-(1/a/x^5+(1+1/a^2/x^10)^(1/2
))^2)-1/10*polylog(2,(1/a/x^5+(1+1/a^2/x^10)^(1/2))^2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx = \frac{1}{10}\left(\operatorname{csch}^{-1}(ax^5)^2 - 2\operatorname{csch}^{-1}(ax^5)\log\left(1-e^{2\operatorname{csch}^{-1}(ax^5)}\right) - \operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(ax^5)}\right)\right)$$

input
```
Integrate[ArcCsch[a*x^5]/x,x]
```

output
```
(ArcCsch[a*x^5]^2 - 2*ArcCsch[a*x^5]*Log[1 - E^(2*ArcCsch[a*x^5])] - PolyL
og[2, E^(2*ArcCsch[a*x^5])])/10
```

3.24.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7282, 6836, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}(ax^5)}{x} \, dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{1}{5} \int \frac{\text{csch}^{-1}(ax^5)}{x^5} dx^5$$

$$\Big\downarrow \text{6836}$$

$$-\frac{1}{5} \int \frac{\text{arcsinh}\left(\frac{1}{ax^5}\right)}{x^5} d\frac{1}{x^5}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{1}{5} \int a\sqrt{1+\frac{1}{x^{10}a^2}} x^5 \text{arcsinh}\left(\frac{1}{ax^5}\right) d\text{arcsinh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5} \int -i\text{arcsinh}\left(\frac{1}{ax^5}\right) \tan\left(i\text{arcsinh}\left(\frac{1}{ax^5}\right)+\frac{\pi}{2}\right) d\text{arcsinh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{5} i \int \text{arcsinh}\left(\frac{1}{ax^5}\right) \tan\left(i\text{arcsinh}\left(\frac{1}{ax^5}\right)+\frac{\pi}{2}\right) d\text{arcsinh}\left(\frac{1}{ax^5}\right)$$

$$\Big\downarrow \text{4199}$$

$$\frac{1}{5} i \left(2i \int -\frac{e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)} \text{arcsinh}\left(\frac{1}{ax^5}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}} d\text{arcsinh}\left(\frac{1}{ax^5}\right) - \frac{ix^{10}}{2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5} i \left(-2i \int \frac{e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)} \text{arcsinh}\left(\frac{1}{ax^5}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}} d\text{arcsinh}\left(\frac{1}{ax^5}\right) - \frac{ix^{10}}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{5}i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)d\text{arcsinh}\left(\frac{1}{ax^5}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{ax^5}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)\right)-\frac{ix^{10}}{2}\right)$$

$$\Big\downarrow \textbf{ 2715}$$

$$\frac{1}{5}i\left(-2i\left(\frac{1}{4}\int e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)de^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{1}{ax^5}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)\right)\right.$$

$$\Big\downarrow \textbf{ 2838}$$

$$\frac{1}{5}i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{ax^5}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{ax^5}\right)}\right)\right)-\frac{ix^{10}}{2}\right)$$

input
```
Int[ArcCsch[a*x^5]/x,x]
```

output
```
(I/5)*((-1/2*I)*x^10 - (2*I)*(-1/2*(ArcSinh[1/(a*x^5)]*Log[1 - E^(2*ArcSin
h[1/(a*x^5)])]) - PolyLog[2, E^(2*ArcSinh[1/(a*x^5)])]/4))
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.24.4 Maple [F]

$$\int \frac{\operatorname{arccsch}\left(a\,x^{5}\right)}{x}dx$$

input
```
int(arccsch(a*x^5)/x,x)
```

output
```
int(arccsch(a*x^5)/x,x)
```

3.24.5 Fricas [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcsch}(ax^5)}{x}\,dx$$

input
```
integrate(arccsch(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccsch(a*x^5)/x, x)
```

3.24.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acsch}(ax^5)}{x}\,dx$$

input
```
integrate(acsch(a*x**5)/x,x)
```

output
```
Integral(acsch(a*x**5)/x, x)
```

3.24.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{arcsch}(ax^5)}{x}\,dx$$

input
```
integrate(arccsch(a*x^5)/x,x, algorithm="maxima")
```

output
```
5*a^2*integrate(x^9*log(x)/(a^2*x^10 + (a^2*x^10 + 1)^(3/2) + 1), x) - 1/2
*log(a^2*x^10 + 1)*log(x) - log(a)*log(x) - 5/2*log(x)^2 + log(x)*log(sqrt
(a^2*x^10 + 1) + 1) - 1/20*dilog(-a^2*x^10)
```

3.24.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x} \, dx = \int \frac{\operatorname{arcsch}(ax^5)}{x} \, dx$$

input
```
integrate(arccsch(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccsch(a*x^5)/x, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(ax^5)}{x} \, dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a\,x^5}\right)}{x} \, dx$$

input
```
int(asinh(1/(a*x^5))/x,x)
```

output
```
int(asinh(1/(a*x^5))/x, x)
```

3.25 $\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right)\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right)\,dx=\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\operatorname{csch}^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{b}-\frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
1/2*arccsch(c*exp(b*x+a))^2/b-arccsch(c*exp(b*x+a))*ln(1-(1/c/exp(b*x+a)+(
1+1/c^2/exp(b*x+a)^2)^(1/2))^2)/b-1/2*polylog(2,(1/c/exp(b*x+a)+(1+1/c^2/e
xp(b*x+a)^2)^(1/2))^2)/b
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(77) = 154.

Time = 0.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right)\,dx=x\operatorname{csch}^{-1}\left(ce^{a+bx}\right)+\frac{e^{-a-bx}\sqrt{1+c^2e^{2(a+bx)}}\left(\log^2\left(-c^2e^{2(a+bx)}\right)+\operatorname{arctanh}\left(\sqrt{1+c^2e^{2(a+bx)}}\right)\left(-8bx+4\log\left(-c^2e^{2(a+bx)}\right)\right)-\cdots\right.}{\cdots}$$

input
```
Integrate[ArcCsch[c*E^(a + b*x)],x]
```

output
```
x*ArcCsch[c*E^(a + b*x)] + (E^(-a - b*x)*Sqrt[1 + c^2*E^(2*(a + b*x))]*(Lo
g[-(c^2*E^(2*(a + b*x)))]^2 + ArcTanh[Sqrt[1 + c^2*E^(2*(a + b*x))]]*(-8*b
*x + 4*Log[-(c^2*E^(2*(a + b*x)))]) - 4*Log[-(c^2*E^(2*(a + b*x)))]*Log[(1
 + Sqrt[1 + c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqrt[1 + c^2*E^(2*(a + b
*x))])/2]^2 - 4*PolyLog[2, (1 - Sqrt[1 + c^2*E^(2*(a + b*x))])/2]))/(8*b*c
*Sqrt[1 + 1/(c^2*E^(2*(a + b*x)))])
```

3.25.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2720, 6836, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^{-1}\left(ce^{a+bx}\right) dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\text{csch}^{-1}\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{6836}$$

$$-\frac{\int e^{-a-bx}\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)de^{-a-bx}}{b}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{\int ce^{a+bx}\sqrt{1+\frac{e^{-2a-2bx}}{c^2}}\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)\tan\left(i\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)\tan\left(i\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}{b}$$

$$\Big\downarrow \text{4199}$$

$$\frac{i\left(2i\int -\frac{e^{a+bx+2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}}{1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}}d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\left(-2i\int \frac{e^{a+bx+2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}}{1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}}d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)d\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{i\left(-2i\left(\frac{1}{4}\int e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)de^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{e^{-a-bx}}{c}\right)}\right)\right)-\frac{1}{2}ie^{2a+2bx}\right)}{b}$$

input
```
Int[ArcCsch[c*E^(a + b*x)],x]
```

output
```
(I*((-1/2*I)*E^(2*a + 2*b*x) - (2*I)*(-1/2*(ArcSinh[E^(-a - b*x)/c]*Log[1
- E^(2*ArcSinh[E^(-a - b*x)/c])]) - PolyLog[2, E^(2*ArcSinh[E^(-a - b*x)/c
])]/4)))/b
```

3.25. $\int \text{csch}^{-1}\left(ce^{a+bx}\right)\,dx$

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.25.4 Maple [F]

$$\int \operatorname{arccsch}\left(e^{bx+a} c\right) dx$$

input
```
int(arccsch(exp(b*x+a)*c),x)
```

output
```
int(arccsch(exp(b*x+a)*c),x)
```

3.25.5 Fricas [F(-2)]

Exception generated.

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccsch(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.25.6 Sympy [F]

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{acsch}\left(ce^{a+bx}\right) dx$$

input
```
integrate(acsch(c*exp(b*x+a)),x)
```

output
```
Integral(acsch(c*exp(a + b*x)), x)
```

3.25.7 Maxima [F]

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{arcsch}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arccsch(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2
*a) + 1)^(3/2) + 1), x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c^2*e^(2
*b*x + 2*a) + 1) + 1) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c
^2*e^(2*b*x + 2*a)))/b
```

3.25.8 Giac [F]

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{arcsch}\left(ce^{(bx+a)}\right) dx$$

input
```
integrate(arccsch(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccsch(c*e^(b*x + a)), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^{-1}\left(ce^{a+bx}\right) dx = \int \operatorname{asinh}\left(\frac{e^{-a-bx}}{c}\right) dx$$

input
```
int(asinh(exp(- a - b*x)/c),x)
```

output
```
int(asinh(exp(- a - b*x)/c), x)
```

3.26 $\int e^{\operatorname{csch}^{-1}(ax)} x^m\,dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 52

$$\int e^{\operatorname{csch}^{-1}(ax)} x^m\,dx = \frac{x^m}{am} + \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{1}{a^2x^2}\right)}{1+m}$$

output
```
x^m/a/m+x^(1+m)*hypergeom([-1/2, -1/2-1/2*m],[-1/2*m+1/2],-1/a^2/x^2)/(1+m
)
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int e^{\operatorname{csch}^{-1}(ax)} x^m\,dx = \frac{x^m}{am} + \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1-m),1+\frac{1}{2}(-1-m),-\frac{1}{a^2x^2}\right)}{1+m}$$

input
```
Integrate[E^ArcCsch[a*x]*x^m,x]
```

output
```
x^m/(a*m) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, 1 + (-1 - m)/2,
 -(1/(a^2*x^2))])/(1 + m)
```

3.26.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6890, 15, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^m dx+\frac{\int x^{m-1}dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^m dx+\frac{x^m}{am}$$

$$\Big\downarrow \text{ 862}$$

$$\frac{x^m}{am}-\left(\frac{1}{x}\right)^m x^m \int \sqrt{1+\frac{1}{a^2x^2}}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m}{am}$$

input
```
Int[E^ArcCsch[a*x]*x^m,x]
```

output
```
x^m/(a*m) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, (1 - m)/2, -(1/
(a^2*x^2))])/(1 + m)
```

3.26.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.26.4 Maple [F]

$$\int \left(\frac{1}{ax}+\sqrt{1+\frac{1}{a^2x^2}}\right)x^m dx$$

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x)
```

output
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x)
```

3.26.5 Fricas [F]

$$\int e^{\operatorname{csch}^{-1}(ax)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="fricas")
```

output
```
integral((a*x*x^m*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + x^m)/(a*x), x)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int e^{\operatorname{csch}^{-1}(ax)}x^m\,dx = -\frac{a^m a^{-m-1}x^m\Gamma\left(-\frac{m}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\right|a^2x^2e^{i\pi}\right)}{2\Gamma\left(1-\frac{m}{2}\right)} - \frac{\begin{cases}-\log(x) & \text{for } m=0\\ -\frac{x^m}{m} & \text{otherwise}\end{cases}}{a}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**m,x)
```

output
```
-a**m*a**(-m - 1)*x**m*gamma(-m/2)*hyper((-1/2, m/2), (m/2 + 1,), a**2*x**
2*exp_polar(I*pi))/(2*gamma(1 - m/2)) - Piecewise((-log(x), Eq(m, 0)), (-x
**m/m, True))/a
```

3.26.7 Maxima [F]

$$\int e^{\operatorname{csch}^{-1}(ax)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)dx$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)*x^m/x, x)/a + x^m/(a*m)
```

3.26.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{csch}^{-1}(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{csch}^{-1}(ax)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{1}{a\,x}\right)\,dx$$

input
```
int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)), x)
```

3.27 $\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx=-\frac{2\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3}{15a^2}+\frac{x^4}{4a}+\frac{1}{5}\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^5$$

output
```
-2/15*(1+1/a^2/x^2)^(3/2)*x^3/a^2+1/4*x^4/a+1/5*(1+1/a^2/x^2)^(3/2)*x^5
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx=\frac{x^4}{4a}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x(-2+a^2x^2+3a^4x^4)}{15a^4}$$

input
```
Integrate[E^ArcCsch[a*x]*x^4,x]
```

output
```
x^4/(4*a) + (Sqrt[1 + 1/(a^2*x^2)]*x*(-2 + a^2*x^2 + 3*a^4*x^4))/(15*a^4)
```

3.27.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6890, 15, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{\text{csch}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^4dx+\frac{\int x^3dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^4dx+\frac{x^4}{4a}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{2\int \sqrt{1+\frac{1}{a^2x^2}}x^2dx}{5a^2}+\frac{1}{5}x^5\left(\frac{1}{a^2x^2}+1\right)^{3/2}+\frac{x^4}{4a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{1}{5}x^5\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{15a^2}+\frac{x^4}{4a}$$

input
```
Int[E^ArcCsch[a*x]*x^4,x]
```

output
```
(-2*(1 + 1/(a^2*x^2))^(3/2)*x^3)/(15*a^2) + x^4/(4*a) + ((1 + 1/(a^2*x^2))
^(3/2)*x^5)/5
```

3.27.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.27.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\sqrt{\frac{a^{2} x^{2}+1}{a^{2} x^{2}}}\, x \left(a^{2} x^{2}+1\right) \left(3 a^{2} x^{2}-2\right)}{15 a^{4}}+\frac{x^{4}}{4 a}$	53
trager	$\frac{\frac{\left(x^{3}+x^{2}+x+1\right) \left(x-1\right)}{4}+\frac{\left(3 a^{4} x^{4}+a^{2} x^{2}-2\right) x \sqrt{-\frac{-a^{2} x^{2}-1}{a^{2} x^{2}}}}{15 a^{3}}}{a}$	63

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(a^2*x^2+1)/a^4*(3*a^2*x^2-2)+1/4*x^4/a
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{15\,a^3x^4 + 4\,(3\,a^4x^5 + a^2x^3 - 2\,x)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{60\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="fricas")
```

output
```
1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 + a^2*x^3 - 2*x)*sqrt((a^2*x^2 + 1)/(a^2*x
^2)))/a^4
```

3.27.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{x^4\sqrt{a^2x^2+1}}{5a} + \frac{x^4}{4a} + \frac{x^2\sqrt{a^2x^2+1}}{15a^3} - \frac{2\sqrt{a^2x^2+1}}{15a^5}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**4,x)
```

output
```
x**4*sqrt(a**2*x**2 + 1)/(5*a) + x**4/(4*a) + x**2*sqrt(a**2*x**2 + 1)/(15
*a**3) - 2*sqrt(a**2*x**2 + 1)/(15*a**5)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{x^4}{4\,a} + \frac{3\,a^2x^5\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}} - 5\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}}{15\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="maxima")
```

output
```
1/4*x^4/a + 1/15*(3*a^2*x^5*(1/(a^2*x^2) + 1)^(5/2) - 5*x^3*(1/(a^2*x^2) +
 1)^(3/2))/a^2
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = -\frac{a^2x^2+1}{2\,a^5} + \frac{2\,|a|\operatorname{sgn}(x)}{15\,a^6} + \frac{12\,(a^2x^2+1)^{\frac{5}{2}}|a|\operatorname{sgn}(x) - 20\,(a^2x^2+1)^{\frac{3}{2}}|a|\operatorname{sgn}(x) + 15\,(a^2x^2+1)^2a}{60\,a^6}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^4,x, algorithm="giac")
```

output
```
-1/2*(a^2*x^2 + 1)/a^5 + 2/15*abs(a)*sgn(x)/a^6 + 1/60*(12*(a^2*x^2 + 1)^(
5/2)*abs(a)*sgn(x) - 20*(a^2*x^2 + 1)^(3/2)*abs(a)*sgn(x) + 15*(a^2*x^2 +
1)^2*a)/a^6
```

3.27.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int e^{\operatorname{csch}^{-1}(ax)}x^4\,dx = \sqrt{\frac{1}{a^2\,x^2}+1}\left(\frac{x^5}{5}-\frac{2\,x}{15\,a^4}+\frac{x^3}{15\,a^2}\right)+\frac{x^4}{4\,a}$$

input
```
int(x^4*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(1/(a^2*x^2) + 1)^(1/2)*(x^5/5 - (2*x)/(15*a^4) + x^3/(15*a^2)) + x^4/(4*a
)
```

3.28 $\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx$

3.28.1 Optimal result

Integrand size = 10, antiderivative size = 75

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx=\frac{\sqrt{1+\frac{1}{a^2x^2}}x^2}{8a^2}+\frac{x^3}{3a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^2}}x^4-\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{8a^4}$$

output
```
1/3*x^3/a-1/8*arctanh((1+1/a^2/x^2)^(1/2))/a^4+1/8*x^2*(1+1/a^2/x^2)^(1/2)
/a^2+1/4*x^4*(1+1/a^2/x^2)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx=\frac{a^2x^2\left(3\sqrt{1+\frac{1}{a^2x^2}}+8ax+6a^2\sqrt{1+\frac{1}{a^2x^2}}x^2\right)-3\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

input
```
Integrate[E^ArcCsch[a*x]*x^3,x]
```

output
```
(a^2*x^2*(3*Sqrt[1 + 1/(a^2*x^2)] + 8*a*x + 6*a^2*Sqrt[1 + 1/(a^2*x^2)]*x^
2) - 3*Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])/(24*a^4)
```

3.28.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6890, 15, 798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6890}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}} x^3 dx + \frac{\int x^2 dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}} x^3 dx + \frac{x^3}{3a}$$

$$\Big\downarrow \text{798}$$

$$\frac{x^3}{3a} - \frac{1}{2}\int \sqrt{1+\frac{1}{a^2x^2}} x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{\frac{1}{a^2x^2}+1} - \frac{\int \frac{x^4}{\sqrt{1+\frac{1}{a^2x^2}}} d\frac{1}{x^2}}{4a^2}\right) + \frac{x^3}{3a}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{\frac{1}{a^2x^2}+1} - \frac{x^2\left(-\sqrt{\frac{1}{a^2x^2}+1}\right) - \frac{\int \frac{x^2}{\sqrt{1+\frac{1}{a^2x^2}}} d\frac{1}{x^2}}{2a^2}}{4a^2}\right) + \frac{x^3}{3a}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{\frac{1}{a^2x^2}+1} - \frac{x^2\left(-\sqrt{\frac{1}{a^2x^2}+1}\right) - \int \frac{1}{\frac{a^2}{x^4}-a^2} d\sqrt{1+\frac{1}{a^2x^2}}}{4a^2}\right) + \frac{x^3}{3a}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{\frac{1}{a^2x^2}+1}-\frac{\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^2}-x^2\sqrt{\frac{1}{a^2x^2}+1}}{4a^2}\right)+\frac{x^3}{3a}$$

input
```
Int[E^ArcCsch[a*x]*x^3,x]
```

output
```
x^3/(3*a) + ((Sqrt[1 + 1/(a^2*x^2)]*x^4)/2 - (-(Sqrt[1 + 1/(a^2*x^2)]*x^2)
 + ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/a^2)/(4*a^2))/2
```

3.28.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.28.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\, x \left(2x\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}} a^4 - x\sqrt{\frac{a^2x^2+1}{a^2}}\, a^2 - \ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{8\sqrt{\frac{a^2x^2+1}{a^2}}\, a^4} + \frac{x^3}{3a}$	112

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(2*x*((a^2*x^2+1)/a^2)^(3/2)*a^4-x*((a^2
*x^2+1)/a^2)^(1/2)*a^2-ln(x+((a^2*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1
/2)/a^4+1/3*x^3/a
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{8\,a^3x^3 + 3\,(2\,a^4x^4 + a^2x^2)\sqrt{\frac{a^2x^2+1}{a^2x^2}} + 3\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax\right)}{24\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="fricas")
```

output
```
1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 + a^2*x^2)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) +
3*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^4
```

3.28.6 Sympy [A] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{ax^5}{4\sqrt{a^2x^2+1}} + \frac{x^3}{3a} + \frac{3x^3}{8a\sqrt{a^2x^2+1}} + \frac{x}{8a^3\sqrt{a^2x^2+1}} - \frac{\operatorname{asinh}(ax)}{8a^4}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**3,x)
```

output
```
a*x**5/(4*sqrt(a**2*x**2 + 1)) + x**3/(3*a) + 3*x**3/(8*a*sqrt(a**2*x**2 +
 1)) + x/(8*a**3*sqrt(a**2*x**2 + 1)) - asinh(a*x)/(8*a**4)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{x^3}{3\,a} + \frac{\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} + \sqrt{\frac{1}{a^2x^2}+1}}{8\left(a^4\left(\frac{1}{a^2x^2}+1\right)^2 - 2\,a^4\left(\frac{1}{a^2x^2}+1\right) + a^4\right)} - \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{16\,a^4} + \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{16\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="maxima")
```

output
```
1/3*x^3/a + 1/8*((1/(a^2*x^2) + 1)^(3/2) + sqrt(1/(a^2*x^2) + 1))/(a^4*(1/
(a^2*x^2) + 1)^2 - 2*a^4*(1/(a^2*x^2) + 1) + a^4) - 1/16*log(sqrt(1/(a^2*x
^2) + 1) + 1)/a^4 + 1/16*log(sqrt(1/(a^2*x^2) + 1) - 1)/a^4
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{1}{8}\sqrt{a^2x^2+1}\left(\frac{2\,x^2|a|\operatorname{sgn}(x)}{a^2} + \frac{|a|\operatorname{sgn}(x)}{a^4}\right)x + \frac{x^3}{3\,a} + \frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\operatorname{sgn}(x)}{8\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^3,x, algorithm="giac")
```

output
```
1/8*sqrt(a^2*x^2 + 1)*(2*x^2*abs(a)*sgn(x)/a^2 + abs(a)*sgn(x)/a^4)*x + 1/
3*x^3/a + 1/8*log(-x*abs(a) + sqrt(a^2*x^2 + 1))*sgn(x)/a^4
```

3.28.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int e^{\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{x^4\sqrt{\frac{1}{a^2\,x^2}+1}}{4} - \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\right)}{8\,a^4} + \frac{x^3}{3\,a} + \frac{x^2\sqrt{\frac{1}{a^2\,x^2}+1}}{8\,a^2}$$

input
```
int(x^3*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(x^4*(1/(a^2*x^2) + 1)^(1/2))/4 - atanh((1/(a^2*x^2) + 1)^(1/2))/(8*a^4) +
 x^3/(3*a) + (x^2*(1/(a^2*x^2) + 1)^(1/2))/(8*a^2)
```

3.29 $\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{x^2}{2a}+\frac{1}{3}\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3$$

output
```
1/2*x^2/a+1/3*(1+1/a^2/x^2)^(3/2)*x^3
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{3ax^2+2\sqrt{1+\frac{1}{a^2x^2}}(x+a^2x^3)}{6a^2}$$

input
```
Integrate[E^ArcCsch[a*x]*x^2,x]
```

output
```
(3*a*x^2 + 2*Sqrt[1 + 1/(a^2*x^2)]*(x + a^2*x^3))/(6*a^2)
```

3.29.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6890, 15, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\operatorname{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6890}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^2dx+\frac{\int xdx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}x^2dx+\frac{x^2}{2a}$$

$$\Big\downarrow \text{796}$$

$$\frac{1}{3}x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}+\frac{x^2}{2a}$$

input
```
Int[E^ArcCsch[a*x]*x^2,x]
```

output
```
x^2/(2*a) + ((1 + 1/(a^2*x^2))^(3/2)*x^3)/3
```

3.29.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.29.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(a^2x^2+1\right)}{3a^2}+\frac{x^2}{2a}$	43
trager	$\frac{\frac{(x-1)(1+x)}{2}+\frac{\left(a^2x^2+1\right)x\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{3a}}{a}$	49

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(a^2*x^2+1)/a^2+1/2*x^2/a
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{3\,ax^2+2\left(a^2x^3+x\right)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{6\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="fricas")
```

output
```
1/6*(3*a*x^2 + 2*(a^2*x^3 + x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)))/a^2
```

3.29.6 Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{x^2\sqrt{a^2x^2+1}}{3a}+\frac{x^2}{2a}+\frac{\sqrt{a^2x^2+1}}{3a^3}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x**2,x)
```

output
```
x**2*sqrt(a**2*x**2 + 1)/(3*a) + x**2/(2*a) + sqrt(a**2*x**2 + 1)/(3*a**3)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{1}{3}\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+\frac{x^2}{2\,a}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="maxima")
```

output
```
1/3*x^3*(1/(a^2*x^2) + 1)^(3/2) + 1/2*x^2/a
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{(a^2x^2+1)^{\frac{3}{2}}|a|\text{sgn}(x)}{3\,a^4}+\frac{a^2x^2+1}{2\,a^3}-\frac{|a|\text{sgn}(x)}{3\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x^2,x, algorithm="giac")
```

output
```
1/3*(a^2*x^2 + 1)^(3/2)*abs(a)*sgn(x)/a^4 + 1/2*(a^2*x^2 + 1)/a^3 - 1/3*ab
s(a)*sgn(x)/a^4
```

3.29.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int e^{\operatorname{csch}^{-1}(ax)}x^2\,dx = \left(\frac{x}{3\,a^2}+\frac{x^3}{3}\right)\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{x^2}{2\,a}$$

input
```
int(x^2*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
(x/(3*a^2) + x^3/3)*(1/(a^2*x^2) + 1)^(1/2) + x^2/(2*a)
```

3.30 $\int e^{\operatorname{csch}^{-1}(ax)}x\,dx$

3.30.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{x}{a}+\frac{1}{2}\sqrt{1+\frac{1}{a^2x^2}}x^2+\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{2a^2}$$

output
```
x/a+1/2*arctanh((1+1/a^2/x^2)^(1/2))/a^2+1/2*x^2*(1+1/a^2/x^2)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{ax\left(2+a\sqrt{1+\frac{1}{a^2x^2}}x\right)+\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

input
```
Integrate[E^ArcCsch[a*x]*x,x]
```

output
```
(a*x*(2 + a*Sqrt[1 + 1/(a^2*x^2)]*x) + Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])
/(2*a^2)
```

3.30.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6890, 24, 798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\operatorname{csch}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}xdx+\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}xdx+\frac{x}{a}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{x}{a}-\frac{1}{2}\int \sqrt{1+\frac{1}{a^2x^2}}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(x^2\sqrt{\frac{1}{a^2x^2}+1}-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{2a^2}\right)+\frac{x}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(x^2\sqrt{\frac{1}{a^2x^2}+1}-\int \frac{1}{\frac{a^2}{x^4}-a^2}d\sqrt{1+\frac{1}{a^2x^2}}\right)+\frac{x}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^2}+x^2\sqrt{\frac{1}{a^2x^2}+1}\right)+\frac{x}{a}$$

input
```
Int[E^ArcCsch[a*x]*x,x]
```

output
```
x/a + (Sqrt[1 + 1/(a^2*x^2)]*x^2 + ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/a^2)/2
```

3.30.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(39) = 78$.

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{\sqrt{\frac{a^{2} x^{2}+1}{a^{2} x^{2}}}\, x \left(x \sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\, a^{2}+\ln\left(x+\sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\right)\right)}{2 \sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\, a^{2}}+\frac{x}{a}$	85

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(x*((a^2*x^2+1)/a^2)^(1/2)*a^2+ln(x+((a^
2*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1/2)/a^2+x/a
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{a^2x^2\sqrt{\frac{a^2x^2+1}{a^2x^2}}+2\,ax-\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{2\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 2*a*x - log(a*x*sqrt((a^2*x^2
 + 1)/(a^2*x^2)) - a*x))/a^2
```

3.30.6 Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{x\sqrt{a^2x^2+1}}{2a}+\frac{x}{a}+\frac{\operatorname{asinh}(ax)}{2a^2}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))*x,x)
```

output
```
x*sqrt(a**2*x**2 + 1)/(2*a) + x/a + asinh(a*x)/(2*a**2)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{x}{a}+\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2\left(a^2\left(\frac{1}{a^2x^2}+1\right)-a^2\right)}+\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{4\,a^2}-\frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{4\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="maxima")
```

output
```
x/a + 1/2*sqrt(1/(a^2*x^2) + 1)/(a^2*(1/(a^2*x^2) + 1) - a^2) + 1/4*log(sq
rt(1/(a^2*x^2) + 1) + 1)/a^2 - 1/4*log(sqrt(1/(a^2*x^2) + 1) - 1)/a^2
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{\sqrt{a^2x^2+1}x|a|\mathrm{sgn}(x)}{2\,a^2}+\frac{x}{a}-\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\mathrm{sgn}(x)}{2\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))*x,x, algorithm="giac")
```

output
```
1/2*sqrt(a^2*x^2 + 1)*x*abs(a)*sgn(x)/a^2 + x/a - 1/2*log(-x*abs(a) + sqrt
(a^2*x^2 + 1))*sgn(x)/a^2
```

3.30.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int e^{\operatorname{csch}^{-1}(ax)}x\,dx=\frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\right)}{2\,a^2}+\frac{x}{a}+\frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{2}$$

input
```
int(x*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x)),x)
```

output
```
atanh((1/(a^2*x^2) + 1)^(1/2))/(2*a^2) + x/a + (x^2*(1/(a^2*x^2) + 1)^(1/2
))/2
```

3.31 $\int e^{\operatorname{csch}^{-1}(ax)}\,dx$

3.31.1 Optimal result

Integrand size = 6, antiderivative size = 24

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = e^{\operatorname{csch}^{-1}(ax)}x - \frac{\operatorname{csch}^{-1}(ax)}{a} + \frac{\log(x)}{a}$$

output
```
(1/a/x+(1+1/a^2/x^2)^(1/2))*x-arccsch(a*x)/a+ln(x)/a
```

3.31.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = \frac{a\sqrt{1+\frac{1}{a^2x^2}}x - \operatorname{arcsinh}\left(\frac{1}{ax}\right) + \log(ax)}{a}$$

input
```
Integrate[E^ArcCsch[a*x],x]
```

output
```
(a*Sqrt[1 + 1/(a^2*x^2)]*x - ArcSinh[1/(a*x)] + Log[a*x])/a
```

3.31.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6885, 14, 773, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6885}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}dx+\frac{\int \frac{1}{x}dx}{a}$$

$$\Big\downarrow \text{14}$$

$$\int \sqrt{1+\frac{1}{a^2x^2}}dx+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{773}$$

$$\frac{\log(x)}{a}-\int \sqrt{1+\frac{1}{a^2x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$-\frac{\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x}}{a^2}+x\sqrt{\frac{1}{a^2x^2}+1}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{222}$$

$$x\sqrt{\frac{1}{a^2x^2}+1}-\frac{\operatorname{arcsinh}\left(\frac{1}{ax}\right)}{a}+\frac{\log(x)}{a}$$

input
```
Int[E^ArcCsch[a*x],x]
```

output
```
Sqrt[1 + 1/(a^2*x^2)]*x - ArcSinh[1/(a*x)]/a + Log[x]/a
```

3.31.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 6885
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)], x_Symbol] :> Simp[1/a   Int[1/x^p, x], x]
+ Int[Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, p}, x]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

method	result	size
default	$-\frac{\sqrt{\frac{a^{2} x^{2}+1}{a^{2} x^{2}}}\, x \left(-\sqrt{\frac{1}{a^{2}}}\, \sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\, a^{2}+\ln\left(\frac{2 \sqrt{\frac{1}{a^{2}}}\, \sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\, a^{2}+2}{a^{2} x}\right)\right)}{\sqrt{\frac{1}{a^{2}}}\, \sqrt{\frac{a^{2} x^{2}+1}{a^{2}}}\, a^{2}}+\frac{\ln\left(x\right)}{a}$	113

input
```
int(1/a/x+(1+1/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(-(1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2
+ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^2))/(1/a^2)^(1/2)/
((a^2*x^2+1)/a^2)^(1/2)/a^2+ln(x)/a
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.58

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = \frac{ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax + 1\right) + \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax - 1\right) + \log(x)}{a}$$

input
```
integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2))
 - a*x + 1) + log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + log(x))/a
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = \frac{x}{\sqrt{1+\frac{1}{a^2x^2}}} + \frac{\log(x)}{a} - \frac{\operatorname{asinh}\left(\frac{1}{ax}\right)}{a} + \frac{1}{a^2x\sqrt{1+\frac{1}{a^2x^2}}}$$

input
```
integrate(1/a/x+(1+1/a**2/x**2)**(1/2),x)
```

output
```
x/sqrt(1 + 1/(a**2*x**2)) + log(x)/a - asinh(1/(a*x))/a + 1/(a**2*x*sqrt(1
 + 1/(a**2*x**2)))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = x\sqrt{\frac{1}{a^2x^2}+1} - \frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{2\,a} + \frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{2\,a} + \frac{\log(x)}{a}$$

input
```
integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="maxima")
```

output
```
x*sqrt(1/(a^2*x^2) + 1) - 1/2*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1)/a + 1/2*l
og(a*x*sqrt(1/(a^2*x^2) + 1) - 1)/a + log(x)/a
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = -\frac{\left(\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x)-\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x)-2\sqrt{a^2x^2+1}\operatorname{sgn}(x)\right)|a|}{2a^2} + \frac{\log(|x|)}{a}$$

input
```
integrate(1/a/x+(1+1/a^2/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - log(sqrt(a^2*x^2 + 1) - 1)*sgn(x
) - 2*sqrt(a^2*x^2 + 1)*sgn(x))*abs(a)/a^2 + log(abs(x))/a
```

3.31.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int e^{\operatorname{csch}^{-1}(ax)}\,dx = \frac{\ln(x)}{a} + x\sqrt{\frac{1}{a^2x^2}+1} + \frac{\operatorname{asin}\left(\frac{1i}{a\,x}\right)\,1i}{a}$$

input
```
int((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x),x)
```

output
```
log(x)/a + (asin(1i/(a*x))*1i)/a + x*(1/(a^2*x^2) + 1)^(1/2)
```

3.32 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx=-\sqrt{1+\frac{1}{a^2x^2}}-\frac{1}{ax}+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)$$

output
```
-1/a/x+arctanh((1+1/a^2/x^2)^(1/2))-(1+1/a^2/x^2)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx=-\sqrt{1+\frac{1}{a^2x^2}}-\frac{1}{ax}+\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)$$

input
```
Integrate[E^ArcCsch[a*x]/x,x]
```

output
```
-Sqrt[1 + 1/(a^2*x^2)] - 1/(a*x) + Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x]
```

3.32.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6890, 15, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \; 6890$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x}dx+\frac{\int \frac{1}{x^2}dx}{a}$$

$$\Big\downarrow \; 15$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x}dx-\frac{1}{ax}$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{2}\int \sqrt{1+\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}-\frac{1}{ax}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(-\int \frac{x^2}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x^2}-2\sqrt{\frac{1}{a^2x^2}+1}\right)-\frac{1}{ax}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-2a^2\int \frac{1}{\frac{a^2}{x^4}-a^2}d\sqrt{1+\frac{1}{a^2x^2}}-2\sqrt{\frac{1}{a^2x^2}+1}\right)-\frac{1}{ax}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(2\text{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)-2\sqrt{\frac{1}{a^2x^2}+1}\right)-\frac{1}{ax}$$

input
```
Int[E^ArcCsch[a*x]/x,x]
```

output
```
-(1/(a*x)) + (-2*Sqrt[1 + 1/(a^2*x^2)] + 2*ArcTanh[Sqrt[1 + 1/(a^2*x^2)]])
/2
```

3.32.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(34) = 68$.

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

method	result	size
default	$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}-\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2x^2-\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)x\right)}{\sqrt{\frac{a^2x^2+1}{a^2}}}-\frac{1}{ax}$	109

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-((a^2*x^2+1)/a^2/x^2)^(1/2)*(a^2*((a^2*x^2+1)/a^2)^(3/2)-((a^2*x^2+1)/a^2
)^(1/2)*a^2*x^2-ln(x+((a^2*x^2+1)/a^2)^(1/2))*x)/((a^2*x^2+1)/a^2)^(1/2)-1
/a/x
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx = -\frac{ax\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)+ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}+ax+1}{ax}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="fricas")
```

output
```
-(a*x*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x) + a*x*sqrt((a^2*x^2 + 1
)/(a^2*x^2)) + a*x + 1)/(a*x)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx = -\frac{ax}{\sqrt{a^2x^2+1}}+\operatorname{asinh}(ax)-\frac{1}{ax}-\frac{1}{ax\sqrt{a^2x^2+1}}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x,x)
```

output
```
-a*x/sqrt(a**2*x**2 + 1) + asinh(a*x) - 1/(a*x) - 1/(a*x*sqrt(a**2*x**2 +
1))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx = -\sqrt{\frac{1}{a^2x^2}+1}-\frac{1}{ax}+\frac{1}{2}\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="maxima")
```

output
```
-sqrt(1/(a^2*x^2) + 1) - 1/(a*x) + 1/2*log(sqrt(1/(a^2*x^2) + 1) + 1) - 1/
2*log(sqrt(1/(a^2*x^2) + 1) - 1)
```

3.32.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.32.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x}\,dx = \operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\right)-\sqrt{\frac{1}{a^2\,x^2}+1}-\frac{1}{a\,x}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x,x)
```

output
```
atanh((1/(a^2*x^2) + 1)^(1/2)) - (1/(a^2*x^2) + 1)^(1/2) - 1/(a*x)
```

3.33 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{1}{2ax^2}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a\operatorname{csch}^{-1}(ax)$$

output
```
-1/2/a/x^2-1/2*a*arccsch(a*x)-1/2*(1+1/a^2/x^2)^(1/2)/x
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{1+a\sqrt{1+\frac{1}{a^2x^2}}\,x+a^2x^2\operatorname{arcsinh}\left(\frac{1}{ax}\right)}{2ax^2}$$

input
```
Integrate[E^ArcCsch[a*x]/x^2,x]
```

output
```
-1/2*(1 + a*Sqrt[1 + 1/(a^2*x^2)]*x + a^2*x^2*ArcSinh[1/(a*x)])/(a*x^2)
```

3.33.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6890, 15, 858, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax)}}{x^2}dx$$

$$\Big\downarrow \text{6890}$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}dx+\frac{\int \frac{1}{x^3}dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}dx-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{1+\frac{1}{a^2x^2}}d\frac{1}{x}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x}-\frac{1}{2}a\text{arcsinh}\left(\frac{1}{ax}\right)-\frac{1}{2ax^2}$$

input
```
Int[E^ArcCsch[a*x]/x^2,x]
```

output
```
-1/2*1/(a*x^2) - Sqrt[1 + 1/(a^2*x^2)]/(2*x) - (a*ArcSinh[1/(a*x)])/2
```

3.33.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(32) = 64$.

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.62

method	result	size
default	$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}-\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}\,a^2x^2+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^2\right)}{2x\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}}-\frac{1}{2a\,x^2}$	145

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a^2*x^2+1)/a^2/x^2)^(1/2)/x*(a^2*((a^2*x^2+1)/a^2)^(3/2)*(1/a^2)^(1
/2)-((a^2*x^2+1)/a^2)^(1/2)*(1/a^2)^(1/2)*a^2*x^2+ln(2*((1/a^2)^(1/2)*((a^
2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^2)*x^2)/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/
2)-1/2/a/x^2
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(32) = 64$.

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.55

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)+ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}+1}{2\,ax^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^2*x^2*l
og(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + a*x*sqrt((a^2*x^2 + 1)/(
a^2*x^2)) + 1)/(a*x^2)
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -a\left(\frac{\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}}{2ax}\right)-\frac{1}{2ax^2}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**2,x)
```

output
```
-a*(asinh(1/(a*x))/2 + sqrt(1 + 1/(a**2*x**2))/(2*a*x)) - 1/(2*a*x**2)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(32) = 64$.

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{a^2x\sqrt{\frac{1}{a^2x^2}+1}}{2\left(a^2x^2\left(\frac{1}{a^2x^2}+1\right)-1\right)} - \frac{1}{4}a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right) + \frac{1}{4}a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right) - \frac{1}{2\,ax^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*x*sqrt(1/(a^2*x^2) + 1)/(a^2*x^2*(1/(a^2*x^2) + 1) - 1) - 1/4*a*l
og(a*x*sqrt(1/(a^2*x^2) + 1) + 1) + 1/4*a*log(a*x*sqrt(1/(a^2*x^2) + 1) -
1) - 1/2/(a*x^2)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{a^4|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - a^4|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) + \frac{2\left(\sqrt{a^2x^2+1}a^4|a|\operatorname{sgn}(x)+a^5\right)}{a^2x^2}}{4\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^2,x, algorithm="giac")
```

output
```
-1/4*(a^4*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - a^4*abs(a)*log(sqrt(a
^2*x^2 + 1) - 1)*sgn(x) + 2*(sqrt(a^2*x^2 + 1)*a^4*abs(a)*sgn(x) + a^5)/(a
^2*x^2))/a^4
```

3.33.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{2\sqrt{\frac{1}{a^2}}} - \frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{2\,x} - \frac{1}{2\,a\,x^2}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^2,x)
```

output
```
- asinh((1/a^2)^(1/2)/x)/(2*(1/a^2)^(1/2)) - (1/(a^2*x^2) + 1)^(1/2)/(2*x)
 - 1/(2*a*x^2)
```

3.34 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} \, dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} \, dx = -\frac{1}{3}a^2\left(1+\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{3ax^3}$$

output
```
-1/3*a^2*(1+1/a^2/x^2)^(3/2)-1/3/a/x^3
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} \, dx = -\frac{1+a\sqrt{1+\frac{1}{a^2x^2}}x(1+a^2x^2)}{3ax^3}$$

input
```
Integrate[E^ArcCsch[a*x]/x^3,x]
```

output
```
-1/3*(1 + a*Sqrt[1 + 1/(a^2*x^2)]*x*(1 + a^2*x^2))/(a*x^3)
```

3.34.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6890, 15, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \; 6890$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^3}dx+\frac{\int \frac{1}{x^4}dx}{a}$$

$$\Big\downarrow \; 15$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^3}dx-\frac{1}{3ax^3}$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{3}a^2\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{3ax^3}$$

input
```
Int[E^ArcCsch[a*x]/x^3,x]
```

output
```
-1/3*(a^2*(1 + 1/(a^2*x^2))^(3/2)) - 1/(3*a*x^3)
```

3.34.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.34.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,(a^2x^2+1)}{3x^2}-\frac{1}{3a\,x^3}$	42
trager	$\frac{-\frac{1}{3x^3}-\frac{a\left(a^2x^2+1\right)\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{3x^2}}{a}$	46

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^2*(a^2*x^2+1)-1/3/a/x^3
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3}\,dx = -\frac{a^3x^3+(a^3x^3+ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}}+1}{3\,ax^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/3*(a^3*x^3 + (a^3*x^3 + a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 1)/(a*x^3)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} dx = \begin{cases} \frac{-a\left(\begin{cases} \sqrt{1+\frac{1}{a^2x^2}}\left(\frac{a^2}{3}+\frac{1}{3x^2}\right) & \text{for } \frac{1}{a^2} \neq 0 \\ \frac{1}{2x^2} & \text{otherwise} \end{cases}\right) - \frac{1}{3x^3}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**3,x)
```

output
```
Piecewise(((-a*Piecewise((sqrt(1 + 1/(a**2*x**2))*(a**2/3 + 1/(3*x**2)), N
e(a**(-2), 0)), (1/(2*x**2), True)) - 1/(3*x**3))/a, Ne(a, 0)), (0, True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} dx = -\frac{1}{3} a^2 \left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} - \frac{1}{3\,ax^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/3*a^2*(1/(a^2*x^2) + 1)^(3/2) - 1/3/(a*x^3)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3} dx = \frac{2\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^4 a^2 \operatorname{sgn}(x) + a^2 \operatorname{sgn}(x)\right)}{3\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{1}{3\,ax^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^3,x, algorithm="giac")
```

output
```
2/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^4*a^2*sgn(x) + a^2*sgn(x))/((x*abs(a
) - sqrt(a^2*x^2 + 1))^2 - 1)^3 - 1/3/(a*x^3)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^3}\,dx = -\frac{\frac{x\sqrt{\frac{1}{a^2x^2}+1}}{3}+\frac{1}{3a}}{x^3}-\frac{a^2\sqrt{\frac{1}{a^2x^2}+1}}{3}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^3,x)
```

output
```
- ((x*(1/(a^2*x^2) + 1)^(1/2))/3 + 1/(3*a))/x^3 - (a^2*(1/(a^2*x^2) + 1)^(
1/2))/3
```

3.35 $\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx$

3.35.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx=-\frac{1}{4ax^4}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{4x^3}-\frac{a^2\sqrt{1+\frac{1}{a^2x^2}}}{8x}+\frac{1}{8}a^3\operatorname{csch}^{-1}(ax)$$

output
```
-1/4/a/x^4+1/8*a^3*arccsch(a*x)-1/4*(1+1/a^2/x^2)^(1/2)/x^3-1/8*a^2*(1+1/a
^2/x^2)^(1/2)/x
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx=\frac{-2-a\sqrt{1+\frac{1}{a^2x^2}}x(2+a^2x^2)+a^4x^4\operatorname{arcsinh}\left(\frac{1}{ax}\right)}{8ax^4}$$

input
```
Integrate[E^ArcCsch[a*x]/x^4,x]
```

output
```
(-2 - a*Sqrt[1 + 1/(a^2*x^2)]*x*(2 + a^2*x^2) + a^4*x^4*ArcSinh[1/(a*x)])/
(8*a*x^4)
```

3.35.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6890, 15, 858, 248, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}dx$$

$$\Big\downarrow \text{6890}$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^4}dx+\frac{\int \frac{1}{x^5}dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^4}dx-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}d\frac{1}{x}-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{248}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}x^2}d\frac{1}{x}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{4x^3}-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x}-\frac{a^2\sqrt{\frac{1}{a^2x^2}+1}}{2x}\right)-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{4x^3}-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{4x^3}+\frac{1}{4}\left(\frac{1}{2}a^3\text{arcsinh}\left(\frac{1}{ax}\right)-\frac{a^2\sqrt{\frac{1}{a^2x^2}+1}}{2x}\right)-\frac{1}{4ax^4}$$

input
```
Int[E^ArcCsch[a*x]/x^4,x]
```

output
```
-1/4*1/(a*x^4) - Sqrt[1 + 1/(a^2*x^2)]/(4*x^3) + (-1/2*(a^2*Sqrt[1 + 1/(a^
2*x^2)])/x + (a^3*ArcSinh[1/(a*x)])/2)/4
```

3.35.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.35. $\int \frac{e^{\text{csch}^{-1}(ax)}}{x^4}\,dx$

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(53) = 106$.

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.66

method	result	si
default	$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\, a^2 \left(\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}} \sqrt{\frac{1}{a^2}}\, a^2x^2-\sqrt{\frac{a^2x^2+1}{a^2}}\, \sqrt{\frac{1}{a^2}}\, a^2x^4+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\, a^2+2}{a^2x}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}\right)}{8x^3\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}}-\frac{1}{4a\,x^4}$	1'

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^3*a^2*(((a^2*x^2+1)/a^2)^(3/2)*(1/a^2)^(
1/2)*a^2*x^2-((a^2*x^2+1)/a^2)^(1/2)*(1/a^2)^(1/2)*a^2*x^4+ln(2*((1/a^2)^(
1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^2)*x^4-2*((a^2*x^2+1)/a^2)^(3/2)*(
1/a^2)^(1/2))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/2)-1/4/a/x^4
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(53) = 106$.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.74

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\, dx = \frac{a^4x^4 \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax + 1\right) - a^4x^4 \log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}} - ax - 1\right) - (a^3x^3 + 2\,ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}} - 2}{8\,ax^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/8*(a^4*x^4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^4*x^4*lo
g(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - (a^3*x^3 + 2*a*x)*sqrt((a
^2*x^2 + 1)/(a^2*x^2)) - 2)/(a*x^4)
```

3.35.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx = \begin{cases} \frac{-a\left(\begin{cases} -\frac{a^2\log\left(2\sqrt{1+\frac{1}{a^2x^2}}\sqrt{\frac{1}{a^2}+\frac{2}{a^2x}}\right)}{8\sqrt{\frac{1}{a^2}}} + \sqrt{1+\frac{1}{a^2x^2}}\left(\frac{a^2}{8x}+\frac{1}{4x^3}\right) & \text{for } \frac{1}{a^2}\neq 0 \\ \frac{1}{3x^3} & \text{otherwise} \end{cases}\right) - \frac{1}{4x^4}}{a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**4,x)
```

output
```
Piecewise(((-a*Piecewise((-a**2*log(2*sqrt(1 + 1/(a**2*x**2))*sqrt(a**(-2)
) + 2/(a**2*x))/(8*sqrt(a**(-2))) + sqrt(1 + 1/(a**2*x**2))*(a**2/(8*x) +
1/(4*x**3)), Ne(a**(-2), 0)), (1/(3*x**3), True)) - 1/(4*x**4))/a, Ne(a, 0
)), (0, True))
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx = \frac{1}{16}a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right) - \frac{1}{16}a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right) - \frac{a^6x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} + a^4x\sqrt{\frac{1}{a^2x^2}+1}}{8\left(a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2 - 2a^2x^2\left(\frac{1}{a^2x^2}+1\right)+1\right)} - \frac{1}{4ax^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
1/16*a^3*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - 1/16*a^3*log(a*x*sqrt(1/(a^2
*x^2) + 1) - 1) - 1/8*(a^6*x^3*(1/(a^2*x^2) + 1)^(3/2) + a^4*x*sqrt(1/(a^2
*x^2) + 1))/(a^4*x^4*(1/(a^2*x^2) + 1)^2 - 2*a^2*x^2*(1/(a^2*x^2) + 1) + 1
) - 1/4/(a*x^4)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx$$
$$= \frac{a^6|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - a^6|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) - \frac{2\left(\left(a^2x^2+1\right)^{\frac{3}{2}}a^6|a|\operatorname{sgn}(x)+\sqrt{a^2x^2+1}a^6|a|\operatorname{sgn}}{a^4x^4}}{16\,a^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^4,x, algorithm="giac")
```

output
```
1/16*(a^6*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - a^6*abs(a)*log(sqrt(a
^2*x^2 + 1) - 1)*sgn(x) - 2*((a^2*x^2 + 1)^(3/2)*a^6*abs(a)*sgn(x) + sqrt(
a^2*x^2 + 1)*a^6*abs(a)*sgn(x) + 2*a^7)/(a^4*x^4))/a^4
```

3.35.9 Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^4}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{8\left(\frac{1}{a^2}\right)^{3/2}} - \frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,x^3} - \frac{1}{4\,a\,x^4} - \frac{a^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{8\,x}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^4,x)
```

output
```
asinh((1/a^2)^(1/2)/x)/(8*(1/a^2)^(3/2)) - (1/(a^2*x^2) + 1)^(1/2)/(4*x^3)
 - 1/(4*a*x^4) - (a^2*(1/(a^2*x^2) + 1)^(1/2))/(8*x)
```

3.36 $\int \frac{e^{\mathbf{csch}^{-1}(ax)}}{x^5} dx$

3.36.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5} dx = \frac{1}{3}a^4\left(1+\frac{1}{a^2x^2}\right)^{3/2} - \frac{1}{5}a^4\left(1+\frac{1}{a^2x^2}\right)^{5/2} - \frac{1}{5ax^5}$$

output
```
1/3*a^4*(1+1/a^2/x^2)^(3/2)-1/5*a^4*(1+1/a^2/x^2)^(5/2)-1/5/a/x^5
```

3.36.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5} dx = \frac{-3 + a\sqrt{1+\frac{1}{a^2x^2}}x(-3 - a^2x^2 + 2a^4x^4)}{15ax^5}$$

input
```
Integrate[E^ArcCsch[a*x]/x^5,x]
```

output
```
(-3 + a*Sqrt[1 + 1/(a^2*x^2)]*x*(-3 - a^2*x^2 + 2*a^4*x^4))/(15*a*x^5)
```

3.36.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6890, 15, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax)}}{x^5} \, dx$$

$$\Big\downarrow \; 6890$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^5}dx+\frac{\int \frac{1}{x^6}dx}{a}$$

$$\Big\downarrow \; 15$$

$$\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^5}dx-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{2}\int \frac{\sqrt{1+\frac{1}{a^2x^2}}}{x^2}d\frac{1}{x^2}-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 53$$

$$-\frac{1}{2}\int \left(a^2\left(1+\frac{1}{a^2x^2}\right)^{3/2}-a^2\sqrt{1+\frac{1}{a^2x^2}}\right)d\frac{1}{x^2}-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{2}{3}a^4\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{2}{5}a^4\left(\frac{1}{a^2x^2}+1\right)^{5/2}\right)-\frac{1}{5ax^5}$$

input
```
Int[E^ArcCsch[a*x]/x^5,x]
```

output
```
((2*a^4*(1 + 1/(a^2*x^2))^(3/2))/3 - (2*a^4*(1 + 1/(a^2*x^2))^(5/2))/5)/2
- 1/(5*a*x^5)
```

3.36.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.36.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,(a^2x^2+1)(2a^2x^2-3)}{15x^4}-\frac{1}{5a\,x^5}$	52
trager	$\frac{-\frac{1}{5x^5}+\frac{a\left(2a^4x^4-a^2x^2-3\right)\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{15x^4}}{a}$	55

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/15*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^4*(a^2*x^2+1)*(2*a^2*x^2-3)-1/5/a/x^5
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^5x^5 + (2\,a^5x^5 - a^3x^3 - 3\,ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}} - 3}{15\,ax^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="fricas")
```

output
```
1/15*(2*a^5*x^5 + (2*a^5*x^5 - a^3*x^3 - 3*a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^
2)) - 3)/(a*x^5)
```

3.36.6 Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = \begin{cases} \frac{-a\left(\begin{cases}\sqrt{1+\frac{1}{a^2x^2}}\left(-\frac{2a^4}{15}+\frac{a^2}{15x^2}+\frac{1}{5x^4}\right) & \text{for } \frac{1}{a^2}\neq 0 \\ \frac{1}{4x^4} & \text{otherwise}\end{cases}\right)-\frac{1}{5x^5}}{a} & \text{for } a\neq 0 \\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))/x**5,x)
```

output
```
Piecewise(((-a*Piecewise((sqrt(1 + 1/(a**2*x**2))*(-2*a**4/15 + a**2/(15*x
**2) + 1/(5*x**4)), Ne(a**(-2), 0)), (1/(4*x**4), True)) - 1/(5*x**5))/a,
Ne(a, 0)), (0, True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = -\frac{1}{5}\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}} + \frac{1}{3}\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} - \frac{1}{5\,ax^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="maxima")
```

output
```
-1/5*a^4*(1/(a^2*x^2) + 1)^(5/2) + 1/3*a^4*(1/(a^2*x^2) + 1)^(3/2) - 1/5/(
a*x^5)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = \frac{4\left(15\left(x|a|-\sqrt{a^2x^2+1}\right)^6a^4\operatorname{sgn}(x)+5\left(x|a|-\sqrt{a^2x^2+1}\right)^4a^4\operatorname{sgn}(x)+5\left(x|a|-\sqrt{a^2x^2+1}\right)^2a^4\operatorname{sgn}(x)-\right.}{15\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^5} - \frac{1}{5\,ax^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))/x^5,x, algorithm="giac")
```

output
```
4/15*(15*(x*abs(a) - sqrt(a^2*x^2 + 1))^6*a^4*sgn(x) + 5*(x*abs(a) - sqrt(
a^2*x^2 + 1))^4*a^4*sgn(x) + 5*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^4*sgn(x)
 - a^4*sgn(x))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^5 - 1/5/(a*x^5)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{e^{\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = \frac{2\,a^4\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15} - \frac{\frac{x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{5}+\frac{1}{5\,a}}{x^5} - \frac{a^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15\,x^2}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))/x^5,x)
```

output
```
(2*a^4*(1/(a^2*x^2) + 1)^(1/2))/15 - ((x*(1/(a^2*x^2) + 1)^(1/2))/5 + 1/(5
*a))/x^5 - (a^2*(1/(a^2*x^2) + 1)^(1/2))/(15*x^2)
```

3.37 $\int e^{\text{csch}^{-1}(ax^2)} x^m \, dx$

3.37.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int e^{\text{csch}^{-1}(ax^2)} x^m \, dx = -\frac{x^{-1+m}}{a(1-m)} + \frac{x^{1+m} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}(-1-m), \frac{3-m}{4}, -\frac{1}{a^2x^4}\right)}{1+m}$$

output
```
-x^(-1+m)/a/(1-m)+x^(1+m)*hypergeom([-1/2, -1/4-1/4*m],[3/4-1/4*m],-1/a^2/
x^4)/(1+m)
```

3.37.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int e^{\text{csch}^{-1}(ax^2)} x^m \, dx = x^{-1+m} \left(\frac{1}{a(-1+m)} + \frac{x^2 \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{1}{4} - \frac{m}{4}, \frac{3}{4} - \frac{m}{4}, -\frac{1}{a^2x^4}\right)}{1+m} \right)$$

input
```
Integrate[E^ArcCsch[a*x^2]*x^m,x]
```

output
```
x^(-1 + m)*(1/(a*(-1 + m)) + (x^2*Hypergeometric2F1[-1/2, -1/4 - m/4, 3/4
- m/4, -(1/(a^2*x^4))])/(1 + m))
```

3.37.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6890, 15, 862, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\text{csch}^{-1}(ax^2)} \, dx$$

$$\Big\downarrow \text{6890}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^m dx+\frac{\int x^{m-2}dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^m dx-\frac{x^{m-1}}{a(1-m)}$$

$$\Big\downarrow \text{862}$$

$$-\left(\frac{1}{x}\right)^m x^m \int \sqrt{1+\frac{1}{x^4a^2}}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}-\frac{x^{m-1}}{a(1-m)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}(-m-1),\frac{3-m}{4},-\frac{1}{a^2x^4}\right)}{m+1}-\frac{x^{m-1}}{a(1-m)}$$

input
```
Int[E^ArcCsch[a*x^2]*x^m,x]
```

output
```
-(x^(-1 + m)/(a*(1 - m))) + (x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/4,
 (3 - m)/4, -(1/(a^2*x^4))])/(1 + m)
```

3.37.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.37.4 Maple [F]

$$\int \left(\frac{1}{a\,x^2}+\sqrt{1+\frac{1}{a^2x^4}}\right)x^m dx$$

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x)
```

output
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x)
```

3.37.5 Fricas [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="fricas")
```

output
```
integral((a*x^2*x^m*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + x^m)/(a*x^2), x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^m\,dx = -\frac{x^{m+1}\Gamma\left(-\frac{m}{4}-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{m}{4}-\frac{1}{4}\\ \frac{3}{4}-\frac{m}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(\frac{3}{4}-\frac{m}{4}\right)}+\frac{\begin{cases}\frac{x^m}{mx-x} & \text{for } m\neq 1\\ \frac{x^m\log(x)}{x} & \text{otherwise}\end{cases}}{a}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**m,x)
```

output
```
-x**(m + 1)*gamma(-m/4 - 1/4)*hyper((-1/2, -m/4 - 1/4), (3/4 - m/4,), exp_
polar(I*pi)/(a**2*x**4))/(4*gamma(3/4 - m/4)) + Piecewise((x**m/(m*x - x),
 Ne(m, 1)), (x**m*log(x)/x, True))/a
```

3.37.7 Maxima [F(-2)]

Exception generated.

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^m\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.37.8 Giac [F(-2)]

Exception generated.

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^m\,dx = \int x^m\left(\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^m*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^m*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.38 $\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 202

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx = -\frac{2\sqrt{1+\frac{1}{a^2x^4}}}{5a^2\left(a+\frac{1}{x^2}\right)x}+\frac{2\sqrt{1+\frac{1}{a^2x^4}}x}{5a^2}+\frac{x^3}{3a}+\frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}x^5 + \frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}x\right)\middle|\frac{1}{2}\right)}{5a^{7/2}\sqrt{1+\frac{1}{a^2x^4}}} - \frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{5a^{7/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

output
```
1/3*x^3/a-2/5*(1+1/a^2/x^4)^(1/2)/a^2/(a+1/x^2)/x+2/5*x*(1+1/a^2/x^4)^(1/2
)/a^2+1/5*x^5*(1+1/a^2/x^4)^(1/2)+2/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^
2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticE(sin(2*arccot(x*a^(1/2))),1/2*2
^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(7/2)/(1+1/a^2/x^4)^(1/2)-1/5*(a
+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*Ellipt
icF(sin(2*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/
a^(7/2)/(1+1/a^2/x^4)^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.55

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx$$
$$= \frac{4\sqrt{2}e^{-\operatorname{csch}^{-1}(ax^2)}\left(\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{-1+e^{2\operatorname{csch}^{-1}(ax^2)}}\right)^{5/2}x^5\left(-4+7e^{2\operatorname{csch}^{-1}(ax^2)}+4\left(1-e^{2\operatorname{csch}^{-1}(ax^2)}\right)^{5/2}\text{Hypergeometric2F}\right.}{21\,(ax^2)^{5/2}}$$

input
```
Integrate[E^ArcCsch[a*x^2]*x^4,x]
```

output
```
(4*Sqrt[2]*(E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2])))^(5/2)*x^5*(-4 +
7*E^(2*ArcCsch[a*x^2]) + 4*(1 - E^(2*ArcCsch[a*x^2]))^(5/2)*Hypergeometric
2F1[3/4, 7/2, 7/4, E^(2*ArcCsch[a*x^2])]))/(21*E^ArcCsch[a*x^2]*(a*x^2)^(5
/2))
```

3.38.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6890, 15, 858, 809, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4e^{\operatorname{csch}^{-1}(ax^2)}\,dx$$
$$\big\downarrow \; 6890$$
$$\int \sqrt{1+\frac{1}{x^4a^2}}x^4dx+\frac{\int x^2dx}{a}$$
$$\big\downarrow \; 15$$
$$\int \sqrt{1+\frac{1}{x^4a^2}}x^4dx+\frac{x^3}{3a}$$
$$\big\downarrow \; 858$$

$$\frac{x^3}{3a}-\int \sqrt{1+\frac{1}{x^4a^2}}x^6d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$-\frac{2\int \frac{x^2}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2\left(\frac{\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}x^2}d\frac{1}{x}}{a^2}-x\sqrt{\frac{1}{a^2x^4}+1}\right)}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

$$\Big\downarrow \text{834}$$

$$-\frac{2\left(\frac{a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-a\int \frac{a-\frac{1}{x^2}}{a\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}}{a^2}-x\sqrt{\frac{1}{a^2x^4}+1}\right)}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}}{a^2}-x\sqrt{\frac{1}{a^2x^4}+1}\right)}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

$$\Big\downarrow \text{761}$$

$$-\frac{2\left(\frac{\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}}{a^2}-x\sqrt{\frac{1}{a^2x^4}+1}\right)}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

$$\Big\downarrow \text{1510}$$

3.38. $\int e^{\text{csch}^{-1}(ax^2)}x^4\,dx$

$$-\frac{2\left(\frac{\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right)|\frac{1}{2}\right)}{\sqrt{\frac{1}{a^2x^4}+1}}+\frac{a^2\sqrt{\frac{1}{a^2x^4}+1}}{x\left(a+\frac{1}{x^2}\right)}}{a^2}-x\sqrt{\frac{1}{a^2x^4}+1}\right)}{5a^2}+\frac{1}{5}x^5\sqrt{\frac{1}{a^2x^4}+1}+\frac{x^3}{3a}$$

input
```
Int[E^ArcCsch[a*x^2]*x^4,x]
```

output
```
x^3/(3*a) + (Sqrt[1 + 1/(a^2*x^4)]*x^5)/5 - (2*(-(Sqrt[1 + 1/(a^2*x^4)]*x)
 + ((a^2*Sqrt[1 + 1/(a^2*x^4)])/((a + x^(-2))*x) - (Sqrt[a]*Sqrt[(a^2 + x^
(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticE[2*ArcTan[1/(Sqrt[a]*x)], 1/2]
)/Sqrt[1 + 1/(a^2*x^4)] + (Sqrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a
+ x^(-2))*EllipticF[2*ArcTan[1/(Sqrt[a]*x)], 1/2])/(2*Sqrt[1 + 1/(a^2*x^4)
]))/a^2))/(5*a^2)
```

3.38.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.38.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

method	result
default	$\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\, x^2\left(\sqrt{ia}\, a^3x^7+x^3a\sqrt{ia}+2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\ \mathrm{EllipticF}\left(x\sqrt{ia},i\right)-2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\ \mathrm{EllipticE}\left(x\sqrt{ia},i\right)\right)}{5a(x^4a^2+1)\sqrt{ia}}+\frac{x^3}{3a}$

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*((a^2*x^4+1)/x^4/a^2)^(1/2)*x^2*((I*a)^(1/2)*a^3*x^7+x^3*a*(I*a)^(1/2)
+2*I*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*EllipticF(x*(I*a)^(1/2),I)-2*I*(1
-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*EllipticE(x*(I*a)^(1/2),I))/a/(a^2*x^4+1
)/(I*a)^(1/2)+1/3*x^3/a
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx$$
$$= \frac{5\,ax^3 + 6\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - 6\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) + 3\,(a^2x^5 + 2\,x)\sqrt{\frac{a^2x^4\cdots}{a^2\cdots}}}{15\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="fricas")
```

output
```
1/15*(5*a*x^3 + 6*(-1/a^2)^(3/4)*elliptic_e(arcsin((-1/a^2)^(1/4)/x), -1)
- 6*(-1/a^2)^(3/4)*elliptic_f(arcsin((-1/a^2)^(1/4)/x), -1) + 3*(a^2*x^5 +
 2*x)*sqrt((a^2*x^4 + 1)/(a^2*x^4)))/a^2
```

3.38.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx = -\frac{x^5\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(-\frac{1}{4}\right)} + \frac{x^3}{3a}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**4,x)
```

output
```
-x**5*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), exp_polar(I*pi)/(a**2*x**4)
)/(4*gamma(-1/4)) + x**3/(3*a)
```

3.38.7 Maxima [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="maxima")
```

output
```
1/3*x^3/a + integrate(sqrt(a^2*x^4 + 1)*x^2, x)/a
```

3.38.8 Giac [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^4,x, algorithm="giac")
```

output
```
integrate(x^4*(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2)), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^4\,dx = \int x^4\left(\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}\right)dx$$

input
```
int(x^4*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^4*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.39 $\int e^{\text{csch}^{-1}(ax^2)}x^3\,dx$

3.39.1 Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{\text{csch}^{-1}(ax^2)}x^3\,dx=\frac{x^2}{2a}+\frac{1}{4}\sqrt{1+\frac{1}{a^2x^4}}x^4+\frac{\text{arctanh}\left(\sqrt{1+\frac{1}{a^2x^4}}\right)}{4a^2}$$

output
```
1/2*x^2/a+1/4*arctanh((1+1/a^2/x^4)^(1/2))/a^2+1/4*x^4*(1+1/a^2/x^4)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int e^{\text{csch}^{-1}(ax^2)}x^3\,dx=\frac{ax^2\left(2+a\sqrt{1+\frac{1}{a^2x^4}}x^2\right)+\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^4}}\right)x^2\right)}{4a^2}$$

input
```
Integrate[E^ArcCsch[a*x^2]*x^3,x]
```

output
```
(a*x^2*(2 + a*Sqrt[1 + 1/(a^2*x^4)]*x^2) + Log[(1 + Sqrt[1 + 1/(a^2*x^4)])
*x^2])/(4*a^2)
```

3.39.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6890, 15, 798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\text{csch}^{-1}(ax^2)} \, dx$$

$$\Big\downarrow \text{6890}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^3dx+\frac{\int xdx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^3dx+\frac{x^2}{2a}$$

$$\Big\downarrow \text{798}$$

$$\frac{x^2}{2a}-\frac{1}{4}\int \sqrt{1+\frac{1}{x^4a^2}}x^8d\frac{1}{x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(x^4\sqrt{\frac{1}{a^2x^4}+1}-\frac{\int \frac{x^4}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x^4}}{2a^2}\right)+\frac{x^2}{2a}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(x^4\sqrt{\frac{1}{a^2x^4}+1}-\int \frac{1}{\frac{a^2}{x^8}-a^2}d\sqrt{1+\frac{1}{x^4a^2}}\right)+\frac{x^2}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)}{a^2}+x^4\sqrt{\frac{1}{a^2x^4}+1}\right)+\frac{x^2}{2a}$$

input

```
Int[E^ArcCsch[a*x^2]*x^3,x]
```

output
```
x^2/(2*a) + (Sqrt[1 + 1/(a^2*x^4)]*x^4 + ArcTanh[Sqrt[1 + 1/(a^2*x^4)]]/a^
2)/4
```

3.39.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(42) = 84.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{\sqrt{\frac{x^{4} a^{2}+1}{x^{4} a^{2}}}\, x^{2} \left(x^{2} \sqrt{\frac{x^{4} a^{2}+1}{a^{2}}}\, a^{2}+\ln \left(x^{2}+\sqrt{\frac{x^{4} a^{2}+1}{a^{2}}}\right)\right)}{4 \sqrt{\frac{x^{4} a^{2}+1}{a^{2}}}\, a^{2}}+\frac{x^{2}}{2 a}$	94

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((a^2*x^4+1)/x^4/a^2)^(1/2)*x^2*(x^2*(1/a^2*(a^2*x^4+1))^(1/2)*a^2+ln(
x^2+(1/a^2*(a^2*x^4+1))^(1/2)))/(1/a^2*(a^2*x^4+1))^(1/2)/a^2+1/2*x^2/a
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx = \frac{a^2x^4\sqrt{\frac{a^2x^4+1}{a^2x^4}} + 2\,ax^2 - \log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}} - ax^2\right)}{4\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="fricas")
```

output
```
1/4*(a^2*x^4*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 2*a*x^2 - log(a*x^2*sqrt((a^2
*x^4 + 1)/(a^2*x^4)) - a*x^2))/a^2
```

3.39.6 Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx = \frac{x^2\sqrt{a^2x^4+1}}{4a} + \frac{x^2}{2a} + \frac{\operatorname{asinh}(ax^2)}{4a^2}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**3,x)
```

output
```
x**2*sqrt(a**2*x**4 + 1)/(4*a) + x**2/(2*a) + asinh(a*x**2)/(4*a**2)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx = \frac{x^2}{2\,a} + \frac{\sqrt{\frac{1}{a^2x^4}+1}}{4\left(a^2\left(\frac{1}{a^2x^4}+1\right)-a^2\right)} + \frac{\log\left(\sqrt{\frac{1}{a^2x^4}+1}+1\right)}{8\,a^2} - \frac{\log\left(\sqrt{\frac{1}{a^2x^4}+1}-1\right)}{8\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="maxima")
```

output
```
1/2*x^2/a + 1/4*sqrt(1/(a^2*x^4) + 1)/(a^2*(1/(a^2*x^4) + 1) - a^2) + 1/8*
log(sqrt(1/(a^2*x^4) + 1) + 1)/a^2 - 1/8*log(sqrt(1/(a^2*x^4) + 1) - 1)/a^
2
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx = \frac{2\,ax^2 + \left(\sqrt{a^2x^4+1}x^2 - \frac{\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right)}{|a|}\right)|a|}{4\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^3,x, algorithm="giac")
```

output
```
1/4*(2*a*x^2 + (sqrt(a^2*x^4 + 1)*x^2 - log(-x^2*abs(a) + sqrt(a^2*x^4 + 1
))/abs(a))*abs(a))/a^2
```

3.39.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx = \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^4}+1}\right)}{4\,a^2} + \frac{x^4\,\sqrt{\frac{1}{a^2\,x^4}+1}}{4} + \frac{x^2}{2\,a}$$

input
```
int(x^3*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
atanh((1/(a^2*x^4) + 1)^(1/2))/(4*a^2) + (x^4*(1/(a^2*x^4) + 1)^(1/2))/4 +
 x^2/(2*a)
```

3.39. $\int e^{\operatorname{csch}^{-1}(ax^2)}x^3\,dx$

3.40 $\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx$

3.40.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx=\frac{x}{a}+\frac{1}{3}\sqrt{1+\frac{1}{a^2x^4}}x^3-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3a^{5/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

output
```
x/a+1/3*x^3*(1+1/a^2/x^4)^(1/2)-1/3*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)
^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*2^(
1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(5/2)/(1+1/a^2/x^4)^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx=$$
$$-\frac{2\sqrt{2}e^{-\operatorname{csch}^{-1}(ax^2)}\left(\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{-1+e^{2\operatorname{csch}^{-1}(ax^2)}}\right)^{3/2}x\left(1-2e^{2\operatorname{csch}^{-1}(ax^2)}-\left(1-e^{2\operatorname{csch}^{-1}(ax^2)}\right)^{3/2}\text{Hypergeometric2F1}\right.}{3a\sqrt{ax^2}}$$

input
```
Integrate[E^ArcCsch[a*x^2]*x^2,x]
```

output
```
(-2*Sqrt[2]*(E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2])))^(3/2)*x*(1 - 2*
E^(2*ArcCsch[a*x^2]) - (1 - E^(2*ArcCsch[a*x^2]))^(3/2)*Hypergeometric2F1[
1/4, 1/2, 5/4, E^(2*ArcCsch[a*x^2])]))/(3*a*E^ArcCsch[a*x^2]*Sqrt[a*x^2])
```

3.40.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6890, 24, 858, 809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\operatorname{csch}^{-1}(ax^2)}\,dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^2dx+\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}x^2dx+\frac{x}{a}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{x}{a}-\int \sqrt{1+\frac{1}{x^4a^2}}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}}{3a^2}+\frac{1}{3}x^3\sqrt{\frac{1}{a^2x^4}+1}+\frac{x}{a}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{3}x^3\sqrt{\frac{1}{a^2x^4}+1}-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{3a^{5/2}\sqrt{\frac{1}{a^2x^4}+1}}+\frac{x}{a}$$

input
```
Int[E^ArcCsch[a*x^2]*x^2,x]
```

output
```
x/a + (Sqrt[1 + 1/(a^2*x^4)]*x^3)/3 - (Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]
*(a + x^(-2))*EllipticF[2*ArcTan[1/(Sqrt[a]*x)], 1/2])/(3*a^(5/2)*Sqrt[1 +
 1/(a^2*x^4)])
```

3.40.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.40.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\sqrt{\frac{x^{4} a^{2}+1}{x^{4} a^{2}}}\, x^{2} \left(\sqrt{i a}\, a^{2} x^{5}+2 \sqrt{-i a \,x^{2}+1}\, \sqrt{i a \,x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{i a}, i\right)+x \sqrt{i a}\right)}{3 \left(x^{4} a^{2}+1\right) \sqrt{i a}}+\frac{x}{a}$	104

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a^2*x^4+1)/x^4/a^2)^(1/2)*x^2*((I*a)^(1/2)*a^2*x^5+2*(1-I*a*x^2)^(1/
2)*(1+I*a*x^2)^(1/2)*EllipticF(x*(I*a)^(1/2),I)+x*(I*a)^(1/2))/(a^2*x^4+1)
/(I*a)^(1/2)+x/a
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx = \frac{ax^3\sqrt{\frac{a^2x^4+1}{a^2x^4}} + 2\,a\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x}\right) | -1\right) + 3\,x}{3\,a}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="fricas")
```

output
```
1/3*(a*x^3*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 2*a*(-1/a^2)^(3/4)*elliptic_f(a
rcsin((-1/a^2)^(1/4)/x), -1) + 3*x)/a
```

3.40.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx = -\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(\frac{1}{4}\right)} + \frac{x}{a}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x**2,x)
```

output
```
-x**3*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), exp_polar(I*pi)/(a**2*x**4))
/(4*gamma(1/4)) + x/a
```

3.40.7 Maxima [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx=\int x^2\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="maxima")
```

output
```
x/a + integrate(sqrt(a^2*x^4 + 1), x)/a
```

3.40.8 Giac [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx=\int x^2\left(\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\right)\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2)), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x^2\,dx=\int x^2\left(\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}\right)\,dx$$

input
```
int(x^2*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
int(x^2*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.41 $\int e^{\text{csch}^{-1}(ax^2)}x\,dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 40

$$\int e^{\text{csch}^{-1}(ax^2)}x\,dx=\frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}}x^2-\frac{\text{csch}^{-1}(ax^2)}{2a}+\frac{\log(x)}{a}$$

output
```
-1/2*arccsch(a*x^2)/a+ln(x)/a+1/2*x^2*(1+1/a^2/x^4)^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int e^{\text{csch}^{-1}(ax^2)}x\,dx=\frac{a\sqrt{1+\frac{1}{a^2x^4}}x^2-\text{arcsinh}\left(\frac{1}{ax^2}\right)+\log(ax^2)}{2a}$$

input
```
Integrate[E^ArcCsch[a*x^2]*x,x]
```

output
```
(a*Sqrt[1 + 1/(a^2*x^4)]*x^2 - ArcSinh[1/(a*x^2)] + Log[a*x^2])/(2*a)
```

3.41.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6890, 14, 858, 807, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\text{csch}^{-1}(ax^2)}\,dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}xdx+\frac{\int \frac{1}{x}dx}{a}$$

$$\Big\downarrow \text{ 14}$$

$$\int \sqrt{1+\frac{1}{x^4a^2}}xdx+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\log(x)}{a}-\int \sqrt{1+\frac{1}{x^4a^2}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\log(x)}{a}-\frac{1}{2}\int \sqrt{1+\frac{1}{a^2x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{2}\left(x\sqrt{\frac{1}{a^2x^2}+1}-\frac{\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x^2}}{a^2}\right)+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(x\sqrt{\frac{1}{a^2x^2}+1}-\frac{\text{arcsinh}\left(\frac{1}{ax^2}\right)}{a}\right)+\frac{\log(x)}{a}$$

input
```
Int[E^ArcCsch[a*x^2]*x,x]
```

output
```
(Sqrt[1 + 1/(a^2*x^2)]*x - ArcSinh[1/(a*x^2)]/a)/2 + Log[x]/a
```

3.41.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.90

method	result	size
default	$\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\,x^2\left(\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{x^4a^2+1}{a^2}}\,a^2-\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{x^4a^2+1}{a^2}}\,a^2+2}{a^2x^2}\right)\right)}{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{x^4a^2+1}{a^2}}\,a^2}+\frac{\ln(x)}{a}$	116

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a^2*x^4+1)/x^4/a^2)^(1/2)*x^2*((1/a^2)^(1/2)*(1/a^2*(a^2*x^4+1))^(1/
2)*a^2-ln(2*((1/a^2)^(1/2)*(1/a^2*(a^2*x^4+1))^(1/2)*a^2+1)/a^2/x^2))/(1/a
^2)^(1/2)/(1/a^2*(a^2*x^4+1))^(1/2)/a^2+ln(x)/a
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int e^{\operatorname{csch}^{-1}(ax^2)} x\,dx = \frac{2\,ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}} - \log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1\right) + \log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-1\right) + 4\,\log\left(x\right)}{4\,a}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="fricas")
```

output
```
1/4*(2*a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - log(a*x^2*sqrt((a^2*x^4 + 1)/
(a^2*x^4)) + 1) + log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - 1) + 4*log(x))
/a
```

3.41.6 Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{\operatorname{csch}^{-1}(ax^2)} x\,dx = \frac{x^2}{2\sqrt{1+\frac{1}{a^2x^4}}} + \frac{\log\left(x\right)}{a} - \frac{\operatorname{asinh}\left(\frac{1}{ax^2}\right)}{2a} + \frac{1}{2a^2x^2\sqrt{1+\frac{1}{a^2x^4}}}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))*x,x)
```

output
```
x**2/(2*sqrt(1 + 1/(a**2*x**4))) + log(x)/a - asinh(1/(a*x**2))/(2*a) + 1/
(2*a**2*x**2*sqrt(1 + 1/(a**2*x**4)))
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(34) = 68$.

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x\,dx = \frac{1}{2}x^2\sqrt{\frac{1}{a^2x^4}+1} - \frac{\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}+1\right)}{4\,a} + \frac{\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}-1\right)}{4\,a} + \frac{\log(x)}{a}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="maxima")
```

output
```
1/2*x^2*sqrt(1/(a^2*x^4) + 1) - 1/4*log(a*x^2*sqrt(1/(a^2*x^4) + 1) + 1)/a
 + 1/4*log(a*x^2*sqrt(1/(a^2*x^4) + 1) - 1)/a + log(x)/a
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x\,dx = \frac{(a-|a|)\log\left(\sqrt{a^2x^4+1}+1\right)+(a+|a|)\log\left(\sqrt{a^2x^4+1}-1\right)+2\sqrt{a^2x^4+1}|a|}{4\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))*x,x, algorithm="giac")
```

output
```
1/4*((a - abs(a))*log(sqrt(a^2*x^4 + 1) + 1) + (a + abs(a))*log(sqrt(a^2*x
^4 + 1) - 1) + 2*sqrt(a^2*x^4 + 1)*abs(a))/a^2
```

3.41.9 Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int e^{\operatorname{csch}^{-1}(ax^2)}x\,dx = \frac{x^2\sqrt{\frac{1}{a^2x^4}+1}}{2} - \frac{\ln\left(\frac{1}{x^2}\right)}{2a} - \frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x^2}\right)\sqrt{\frac{1}{a^2}}}{2}$$

input
```
int(x*((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2)),x)
```

output
```
(x^2*(1/(a^2*x^4) + 1)^(1/2))/2 - log(1/x^2)/(2*a) - (asinh((1/a^2)^(1/2)/
x^2)*(1/a^2)^(1/2))/2
```

3.42 $\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx$

3.42.1 Optimal result

Integrand size = 8, antiderivative size = 165

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx = -\frac{1}{ax} - \frac{2\sqrt{1+\frac{1}{a^2x^4}}}{\left(a+\frac{1}{x^2}\right)x} + \sqrt{1+\frac{1}{a^2x^4}}\,x + \frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}x\right)\middle|\frac{1}{2}\right)}{a^{3/2}\sqrt{1+\frac{1}{a^2x^4}}} - \frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{a^{3/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

output

```
-1/a/x-2*(1+1/a^2/x^4)^(1/2)/(a+1/x^2)/x+x*(1+1/a^2/x^4)^(1/2)+2*(a+1/x^2)
*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticE(sin
(2*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(3/2)
/(1+1/a^2/x^4)^(1/2)-(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*ar
ccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/2*2^(1/2))*((a^2+1/x
^4)/(a+1/x^2)^2)^(1/2)/a^(3/2)/(1+1/a^2/x^4)^(1/2)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.58

$$\int e^{\text{csch}^{-1}(ax^2)}\,dx$$
$$=\frac{\sqrt{2}e^{\text{csch}^{-1}(ax^2)}\sqrt{\frac{e^{\text{csch}^{-1}(ax^2)}}{-1+e^{2\text{csch}^{-1}(ax^2)}}}x\left(-3+4\sqrt{1-e^{2\text{csch}^{-1}(ax^2)}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},e^{2\text{csch}^{-1}(ax^2)}\right)\right)}{3\sqrt{ax^2}}$$

input
```
Integrate[E^ArcCsch[a*x^2],x]
```

output
```
(Sqrt[2]*E^ArcCsch[a*x^2]*Sqrt[E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2])
)]*x*(-3 + 4*Sqrt[1 - E^(2*ArcCsch[a*x^2])]*Hypergeometric2F1[3/4, 3/2, 7/
4, E^(2*ArcCsch[a*x^2])]))/(3*Sqrt[a*x^2])
```

3.42.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6885, 15, 773, 809, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\text{csch}^{-1}(ax^2)}\,dx$$
$$\Big\downarrow \text{6885}$$
$$\int \sqrt{1+\frac{1}{x^4a^2}}dx+\frac{\int \frac{1}{x^2}dx}{a}$$
$$\Big\downarrow \text{15}$$
$$\int \sqrt{1+\frac{1}{x^4a^2}}dx-\frac{1}{ax}$$
$$\Big\downarrow \text{773}$$
$$-\int \sqrt{1+\frac{1}{x^4a^2}}x^2d\frac{1}{x}-\frac{1}{ax}$$

$$\Big\downarrow \color{blue}{809}$$

$$-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}x^2}d\frac{1}{x}}{a^2}+x\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{ax}$$

$$\Big\downarrow \color{blue}{834}$$

$$-\frac{2\left(a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-a\int \frac{a-\frac{1}{x^2}}{a\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)}{a^2}+x\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{ax}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{2\left(a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)}{a^2}+x\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{ax}$$

$$\Big\downarrow \color{blue}{761}$$

$$-\frac{2\left(\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)}{a^2}+x\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{ax}$$

$$\Big\downarrow \color{blue}{1510}$$

$$-\frac{2\left(\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right)|\frac{1}{2}\right)}{\sqrt{\frac{1}{a^2x^4}+1}}+\frac{a^2\sqrt{\frac{1}{a^2x^4}+1}}{x\left(a+\frac{1}{x^2}\right)}\right)}{a^2}+x\sqrt{\frac{1}{a^2x^4}+1}-\frac{1}{ax}$$

input
```
Int[E^ArcCsch[a*x^2],x]
```

output
```
-(1/(a*x)) + Sqrt[1 + 1/(a^2*x^4)]*x - (2*((a^2*Sqrt[1 + 1/(a^2*x^4)])/((a
 + x^(-2))*x) - (Sqrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a + x^(-2))*
EllipticE[2*ArcTan[1/(Sqrt[a]*x)], 1/2])/Sqrt[1 + 1/(a^2*x^4)] + (Sqrt[a]*
Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticF[2*ArcTan[1/(Sqr
t[a]*x)], 1/2])/(2*Sqrt[1 + 1/(a^2*x^4)])))/a^2
```

3.42. $\int e^{\text{csch}^{-1}(ax^2)}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6885
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)], x_Symbol] :> Simp[1/a   Int[1/x^p, x], x]
+ Int[Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, p}, x]
```

3.42.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\, x\left(-\sqrt{ia}\, a^2x^4+2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x\,\mathrm{EllipticF}\left(x\sqrt{ia},i\right)a-2i\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x\,\mathrm{EllipticE}\left(x\sqrt{ia},i\right)a-\sqrt{ia}\right)}{\left(x^4a^2+1\right)\sqrt{ia}}-$

input
```
int(1/a/x^2+(1+1/a^2/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a^2*x^4+1)/x^4/a^2)^(1/2)*x*(-(I*a)^(1/2)*a^2*x^4+2*I*(1-I*a*x^2)^(1/2)*
(1+I*a*x^2)^(1/2)*x*EllipticF(x*(I*a)^(1/2),I)*a-2*I*(1-I*a*x^2)^(1/2)*(1+
I*a*x^2)^(1/2)*x*EllipticE(x*(I*a)^(1/2),I)*a-(I*a)^(1/2))/(a^2*x^4+1)/(I*
a)^(1/2)-1/a/x
```

3.42.5 Fricas [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\, dx = \int \sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\, dx$$

input
```
integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="fricas")
```

output
```
integral((a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^2), x)
```

3.42.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\, dx = -\frac{x\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{e^{i\pi}}{a^2x^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}-\frac{1}{ax}$$

input
```
integrate(1/a/x**2+(1+1/a**2/x**4)**(1/2),x)
```

output
```
-x*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), exp_polar(I*pi)/(a**2*x**4))/(4
*gamma(3/4)) - 1/(a*x)
```

3.42.7 Maxima [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx = \int \sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\,dx$$

input
```
integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^4 + 1)/x^2, x)/a - 1/(a*x)
```

3.42.8 Giac [F]

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx = \int \sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}\,dx$$

input
```
integrate(1/a/x^2+(1+1/a^2/x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2), x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.15

$$\int e^{\operatorname{csch}^{-1}(ax^2)}\,dx = x\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{1}{a^2\,x^4}\right)-\frac{1}{a\,x}$$

input
```
int((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2),x)
```

output
```
x*hypergeom([-1/2, -1/4], 3/4, -1/(a^2*x^4)) - 1/(a*x)
```

3.43 $\int \frac{e^{\mathbf{csch}^{-1}\left(ax^2\right)}}{x} \, dx$

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{e^{\operatorname{csch}^{-1}\left(ax^2\right)}}{x} \, dx = -\frac{1}{2}\sqrt{1+\frac{1}{a^2x^4}} - \frac{1}{2ax^2} + \frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a^2x^4}}\right)$$

output
```
-1/2/a/x^2+1/2*arctanh((1+1/a^2/x^4)^(1/2))-1/2*(1+1/a^2/x^4)^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{e^{\operatorname{csch}^{-1}\left(ax^2\right)}}{x} \, dx = -\frac{1}{2}e^{\operatorname{csch}^{-1}\left(ax^2\right)} + \operatorname{arctanh}\left(e^{\operatorname{csch}^{-1}\left(ax^2\right)}\right)$$

input
```
Integrate[E^ArcCsch[a*x^2]/x,x]
```

output
```
-1/2*E^ArcCsch[a*x^2] + ArcTanh[E^ArcCsch[a*x^2]]
```

3.43.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6890, 15, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x} \, dx$$

$$\Big\downarrow \; 6890$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x}dx+\frac{\int \frac{1}{x^3}dx}{a}$$

$$\Big\downarrow \; 15$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x}dx-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{4}\int \sqrt{1+\frac{1}{x^4a^2}}x^4d\frac{1}{x^4}-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{4}\left(-\int \frac{x^4}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x^4}-2\sqrt{\frac{1}{a^2x^4}+1}\right)-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(-2a^2\int \frac{1}{\frac{a^2}{x^8}-a^2}d\sqrt{1+\frac{1}{x^4a^2}}-2\sqrt{\frac{1}{a^2x^4}+1}\right)-\frac{1}{2ax^2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(2\text{arctanh}\left(\sqrt{\frac{1}{a^2x^4}+1}\right)-2\sqrt{\frac{1}{a^2x^4}+1}\right)-\frac{1}{2ax^2}$$

input
```
Int[E^ArcCsch[a*x^2]/x,x]
```

output
```
-1/2*1/(a*x^2) + (-2*Sqrt[1 + 1/(a^2*x^4)] + 2*ArcTanh[Sqrt[1 + 1/(a^2*x^4
)]])/4
```

3.43.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.43. $\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x} \, dx$

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

method	result	size
default	$-\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\left(-\ln\left(x^2+\sqrt{\frac{x^4a^2+1}{a^2}}\right)x^2+\sqrt{\frac{x^4a^2+1}{a^2}}\right)}{2\sqrt{\frac{x^4a^2+1}{a^2}}}-\frac{1}{2a\,x^2}$	86

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a^2*x^4+1)/x^4/a^2)^(1/2)*(-ln(x^2+(1/a^2*(a^2*x^4+1))^(1/2))*x^2+(
1/a^2*(a^2*x^4+1))^(1/2))/(1/a^2*(a^2*x^4+1))^(1/2)-1/2/a/x^2
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx = -\frac{ax^2\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-ax^2\right)+ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+ax^2+1}{2\,ax^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="fricas")
```

output
```
-1/2*(a*x^2*log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - a*x^2) + a*x^2*sqrt(
(a^2*x^4 + 1)/(a^2*x^4)) + a*x^2 + 1)/(a*x^2)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx = -\frac{ax^2}{2\sqrt{a^2x^4+1}}+\frac{\operatorname{asinh}(ax^2)}{2}-\frac{1}{2ax^2}-\frac{1}{2ax^2\sqrt{a^2x^4+1}}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x,x)
```

output
```
-a*x**2/(2*sqrt(a**2*x**4 + 1)) + asinh(a*x**2)/2 - 1/(2*a*x**2) - 1/(2*a*
x**2*sqrt(a**2*x**4 + 1))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx = -\frac{1}{2}\sqrt{\frac{1}{a^2x^4}+1} - \frac{1}{2\,ax^2} + \frac{1}{4}\,\log\left(\sqrt{\frac{1}{a^2x^4}+1}+1\right) - \frac{1}{4}\,\log\left(\sqrt{\frac{1}{a^2x^4}+1}-1\right)$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="maxima")
```

output
```
-1/2*sqrt(1/(a^2*x^4) + 1) - 1/2/(a*x^2) + 1/4*log(sqrt(1/(a^2*x^4) + 1) +
 1) - 1/4*log(sqrt(1/(a^2*x^4) + 1) - 1)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx = -\frac{a^2\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right) - \frac{2\,a^2}{\left(x^2|a|-\sqrt{a^2x^4+1}\right)^2-1} + \frac{a}{x^2}}{2\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x,x, algorithm="giac")
```

output
```
-1/2*(a^2*log(-x^2*abs(a) + sqrt(a^2*x^4 + 1)) - 2*a^2/((x^2*abs(a) - sqrt
(a^2*x^4 + 1))^2 - 1) + a/x^2)/a^2
```

3.43.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{a^2\,x^4}+1}\right)}{2} - \frac{\sqrt{\frac{1}{a^2\,x^4}+1}}{2} - \frac{1}{2\,a\,x^2}$$

input
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x,x)
```

output
```
atanh((1/(a^2*x^4) + 1)^(1/2))/2 - (1/(a^2*x^4) + 1)^(1/2)/2 - 1/(2*a*x^2)
```

3.44 $\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^2} dx$

3.44.1 Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^2} dx = -\frac{1}{3ax^3} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{3x} - \frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{3\sqrt{a}\sqrt{1+\frac{1}{a^2x^4}}}$$

output
```
-1/3/a/x^3-1/3*(1+1/a^2/x^4)^(1/2)/x-1/3*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)
))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),1/
2*2^(1/2))*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/a^(1/2)/(1+1/a^2/x^4)^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^2} dx = -\frac{a\sqrt{\frac{e^{\text{csch}^{-1}(ax^2)}}{-2+2e^{2\text{csch}^{-1}(ax^2)}}}x\left(-1+e^{2\text{csch}^{-1}(ax^2)}+4\sqrt{1-e^{2\text{csch}^{-1}(ax^2)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},e^{2\text{csch}^{-1}(ax^2)}\right.\right.}{3\sqrt{ax^2}}$$

input
```
Integrate[E^ArcCsch[a*x^2]/x^2,x]
```

output
```
-1/3*(a*Sqrt[E^ArcCsch[a*x^2]/(-2 + 2*E^(2*ArcCsch[a*x^2]))]*x*(-1 + E^(2*
ArcCsch[a*x^2]) + 4*Sqrt[1 - E^(2*ArcCsch[a*x^2])]*Hypergeometric2F1[1/4,
1/2, 5/4, E^(2*ArcCsch[a*x^2])]))/Sqrt[a*x^2]
```

3.44.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6890, 15, 858, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^2}dx$$

$$\Big\downarrow \text{6890}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^2}dx+\frac{\int \frac{1}{x^4}dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^2}dx-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{1+\frac{1}{x^4a^2}}d\frac{1}{x}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{748}$$

$$-\frac{2}{3}\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{3x}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{3\sqrt{a}\sqrt{\frac{1}{a^2x^4}+1}}-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{3x}-\frac{1}{3ax^3}$$

input
```
Int[E^ArcCsch[a*x^2]/x^2,x]
```

output
```
-1/3*1/(a*x^3) - Sqrt[1 + 1/(a^2*x^4)]/(3*x) - (Sqrt[(a^2 + x^(-4))/(a + x
^(-2))^2]*(a + x^(-2))*EllipticF[2*ArcTan[1/(Sqrt[a]*x)], 1/2])/(3*Sqrt[a]
*Sqrt[1 + 1/(a^2*x^4)])
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.44.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\left(-2\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{ia},i\right)x^3a^2+\sqrt{ia}\,a^2x^4+\sqrt{ia}\right)}{3x(x^4a^2+1)\sqrt{ia}}-\frac{1}{3a\,x^3}$	111

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a^2*x^4+1)/x^4/a^2)^(1/2)*(-2*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)*E
llipticF(x*(I*a)^(1/2),I)*x^3*a^2+(I*a)^(1/2)*a^2*x^4+(I*a)^(1/2))/x/(a^2*
x^4+1)/(I*a)^(1/2)-1/3/a/x^3
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx = -\frac{2\,(-a^2)^{\frac{3}{4}}\,x^3F\left(\arcsin\left((-a^2)^{\frac{1}{4}}\,x\right)\,|-1\right)+ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{3\,ax^3}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(-a^2)^(3/4)*x^3*elliptic_f(arcsin((-a^2)^(1/4)*x), -1) + a*x^2*sq
rt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^3)
```

3.44.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.46

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}-\frac{1}{3ax^3}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**2,x)
```

output
```
-gamma(1/4)*hyper((-1/2, 1/4), (5/4,), exp_polar(I*pi)/(a**2*x**4))/(4*x*g
amma(5/4)) - 1/(3*a*x**3)
```

3.44.7 Maxima [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^2}\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^4 + 1)/x^4, x)/a - 1/3/(a*x^3)
```

3.44.8 Giac [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^2}\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^2,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2))/x^2, x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.30

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^2}\,dx = -\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{1}{a^2\,x^4}\right)}{x} - \frac{1}{3\,a\,x^3}$$

input
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^2,x)
```

output
```
- hypergeom([-1/2, 1/4], 5/4, -1/(a^2*x^4))/x - 1/(3*a*x^3)
```

3.45 $\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx$

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{1}{4ax^4} - \frac{\sqrt{1+\frac{1}{a^2x^4}}}{4x^2} - \frac{1}{4}a\operatorname{csch}^{-1}(ax^2)$$

output
```
-1/4/a/x^4-1/4*a*arccsch(a*x^2)-1/4*(1+1/a^2/x^4)^(1/2)/x^2
```

3.45.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{1}{8}a\left(e^{2\operatorname{csch}^{-1}(ax^2)} + 2\operatorname{csch}^{-1}(ax^2)\right)$$

input
```
Integrate[E^ArcCsch[a*x^2]/x^3,x]
```

output
```
-1/8*(a*(E^(2*ArcCsch[a*x^2]) + 2*ArcCsch[a*x^2]))
```

3.45.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6890, 15, 858, 807, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^3}dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^3}dx+\frac{\int \frac{1}{x^5}dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^3}dx-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x}d\frac{1}{x}-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{1}{2}\int \sqrt{1+\frac{1}{a^2x^2}}d\frac{1}{x^2}-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}d\frac{1}{x^2}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x^2}\right)-\frac{1}{4ax^4}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2x^2}-\frac{1}{2}a\text{arcsinh}\left(\frac{1}{ax^2}\right)\right)-\frac{1}{4ax^4}$$

input
```
Int[E^ArcCsch[a*x^2]/x^3,x]
```

output
```
-1/4*1/(a*x^4) + (-1/2*Sqrt[1 + 1/(a^2*x^2)]/x^2 - (a*ArcSinh[1/(a*x^2)])/
2)/2
```

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.45. $\int \frac{e^{\text{csch}^{-1}(ax^2)}}{x^3}\,dx$

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.71

method	result	size
default	$-\frac{\sqrt{\frac{x^{4}a^{2}+1}{x^{4}a^{2}}}\left(\ln\left(\frac{2\sqrt{\frac{1}{a^{2}}}\sqrt{\frac{x^{4}a^{2}+1}{a^{2}}}a^{2}+2}{a^{2}x^{2}}\right)x^{4}+\sqrt{\frac{x^{4}a^{2}+1}{a^{2}}}\sqrt{\frac{1}{a^{2}}}\right)}{4x^{2}\sqrt{\frac{x^{4}a^{2}+1}{a^{2}}}\sqrt{\frac{1}{a^{2}}}}-\frac{1}{4a\,x^{4}}$	114

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a^2*x^4+1)/x^4/a^2)^(1/2)/x^2*(ln(2*((1/a^2)^(1/2)*(1/a^2*(a^2*x^4+
1))^(1/2)*a^2+1)/a^2/x^2)*x^4+(1/a^2*(a^2*x^4+1))^(1/2)*(1/a^2)^(1/2))/(1/
a^2*(a^2*x^4+1))^(1/2)/(1/a^2)^(1/2)-1/4/a/x^4
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(34) = 68$.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{a^2x^4\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1\right)-a^2x^4\log\left(ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}-1\right)+2\,ax^2\sqrt{\frac{a^2x^4+1}{a^2x^4}}+2}{8\,ax^4}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/8*(a^2*x^4*log(a*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1) - a^2*x^4*log(a
*x^2*sqrt((a^2*x^4 + 1)/(a^2*x^4)) - 1) + 2*a*x^2*sqrt((a^2*x^4 + 1)/(a^2*
x^4)) + 2)/(a*x^4)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{a\left(\frac{\operatorname{asinh}\left(\frac{1}{ax^2}\right)}{2} + \frac{\sqrt{1+\frac{1}{a^2x^4}}}{2ax^2}\right)}{2} - \frac{1}{4ax^4}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**3,x)
```

output
```
-a*(asinh(1/(a*x**2))/2 + sqrt(1 + 1/(a**2*x**4))/(2*a*x**2))/2 - 1/(4*a*x
**4)
```

3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(34) = 68.

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.19

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{a^2x^2\sqrt{\frac{1}{a^2x^4}+1}}{4\left(a^2x^4\left(\frac{1}{a^2x^4}+1\right)-1\right)} - \frac{1}{8}\,a\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}+1\right) + \frac{1}{8}\,a\log\left(ax^2\sqrt{\frac{1}{a^2x^4}+1}-1\right) - \frac{1}{4\,ax^4}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*x^2*sqrt(1/(a^2*x^4) + 1)/(a^2*x^4*(1/(a^2*x^4) + 1) - 1) - 1/8*a
*log(a*x^2*sqrt(1/(a^2*x^4) + 1) + 1) + 1/8*a*log(a*x^2*sqrt(1/(a^2*x^4) +
 1) - 1) - 1/4/(a*x^4)
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{a^4|a|\log\left(\sqrt{a^2x^4+1}+1\right) - a^4|a|\log\left(\sqrt{a^2x^4+1}-1\right) + \frac{2\left(\sqrt{a^2x^4+1}a^4|a|+a^5\right)}{a^2x^4}}{8\,a^4}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/8*(a^4*abs(a)*log(sqrt(a^2*x^4 + 1) + 1) - a^4*abs(a)*log(sqrt(a^2*x^4
+ 1) - 1) + 2*(sqrt(a^2*x^4 + 1)*a^4*abs(a) + a^5)/(a^2*x^4))/a^4
```

3.45.9 Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^3}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x^2}\right)}{4\sqrt{\frac{1}{a^2}}} - \frac{\sqrt{\frac{1}{a^2\,x^4}+1}}{4\,x^2} - \frac{1}{4\,a\,x^4}$$

input
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^3,x)
```

output
```
- asinh((1/a^2)^(1/2)/x^2)/(4*(1/a^2)^(1/2)) - (1/(a^2*x^4) + 1)^(1/2)/(4*
x^2) - 1/(4*a*x^4)
```

3.46 $\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx$

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 181

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx = -\frac{1}{5ax^5}-\frac{\sqrt{1+\frac{1}{a^2x^4}}}{5x^3}-\frac{2a^2\sqrt{1+\frac{1}{a^2x^4}}}{5\left(a+\frac{1}{x^2}\right)x}+\frac{2\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\cot^{-1}\left(\sqrt{a}x\right)|\frac{1}{2}\right)}{5\sqrt{1+\frac{1}{a^2x^4}}}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\sqrt{a}x\right),\frac{1}{2}\right)}{5\sqrt{1+\frac{1}{a^2x^4}}}$$

output
```
-1/5/a/x^5-1/5*(1+1/a^2/x^4)^(1/2)/x^3-2/5*a^2*(1+1/a^2/x^4)^(1/2)/(a+1/x^
2)/x+2/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/2)))^2)^(1/2)/cos(2*arccot(x*a^(1/
2)))*EllipticE(sin(2*arccot(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^2+1/x^4)/
(a+1/x^2)^2)^(1/2)/(1+1/a^2/x^4)^(1/2)-1/5*(a+1/x^2)*(cos(2*arccot(x*a^(1/
2)))^2)^(1/2)/cos(2*arccot(x*a^(1/2)))*EllipticF(sin(2*arccot(x*a^(1/2))),
1/2*2^(1/2))*a^(1/2)*((a^2+1/x^4)/(a+1/x^2)^2)^(1/2)/(1+1/a^2/x^4)^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4} \, dx$$
$$= \frac{(ax^2)^{3/2} \left(3\left(1-e^{2\operatorname{csch}^{-1}(ax^2)}\right)^{3/2}+4e^{2\operatorname{csch}^{-1}(ax^2)}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2\operatorname{csch}^{-1}(ax^2)}\right)\right)}{6\sqrt{2-2e^{2\operatorname{csch}^{-1}(ax^2)}}\sqrt{\frac{e^{\operatorname{csch}^{-1}(ax^2)}}{-1+e^{2\operatorname{csch}^{-1}(ax^2)}}}x^3}$$

input
```
Integrate[E^ArcCsch[a*x^2]/x^4,x]
```

output
```
((a*x^2)^(3/2)*(3*(1 - E^(2*ArcCsch[a*x^2]))^(3/2) + 4*E^(2*ArcCsch[a*x^2]
)*Hypergeometric2F1[-1/2, 3/4, 7/4, E^(2*ArcCsch[a*x^2])]))/(6*Sqrt[2 - 2*
E^(2*ArcCsch[a*x^2])]*Sqrt[E^ArcCsch[a*x^2]/(-1 + E^(2*ArcCsch[a*x^2]))]*x
^3)
```

3.46.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6890, 15, 858, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4} \, dx$$
$$\Big\downarrow \ 6890$$
$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^4}dx+\frac{\int \frac{1}{x^6}dx}{a}$$
$$\Big\downarrow \ 15$$
$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^4}dx-\frac{1}{5ax^5}$$
$$\Big\downarrow \ 858$$

$$-\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^2}d\frac{1}{x}-\frac{1}{5ax^5}$$

$\downarrow$ 811

$$-\frac{2}{5}\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}x^2}d\frac{1}{x}-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{1}{5ax^5}$$

$\downarrow$ 834

$$-\frac{2}{5}\left(a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-a\int \frac{a-\frac{1}{x^2}}{a\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{1}{5ax^5}$$

$\downarrow$ 27

$$-\frac{2}{5}\left(a\int \frac{1}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{1}{5ax^5}$$

$\downarrow$ 761

$$-\frac{2}{5}\left(\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\int \frac{a-\frac{1}{x^2}}{\sqrt{1+\frac{1}{x^4a^2}}}d\frac{1}{x}\right)-\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{1}{5ax^5}$$

$\downarrow$ 1510

$$-\frac{2}{5}\left(\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right),\frac{1}{2}\right)}{2\sqrt{\frac{1}{a^2x^4}+1}}-\frac{\sqrt{a}\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)E\left(2\arctan\left(\frac{1}{\sqrt{a}x}\right)|\frac{1}{2}\right)}{\sqrt{\frac{1}{a^2x^4}+1}}+\frac{a^2}{x}\right.$$
$$\frac{\sqrt{\frac{1}{a^2x^4}+1}}{5x^3}-\frac{1}{5ax^5}$$

input
```
Int[E^ArcCsch[a*x^2]/x^4,x]
```

3.46. $\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^4}\,dx$

output
```
-1/5*1/(a*x^5) - Sqrt[1 + 1/(a^2*x^4)]/(5*x^3) - (2*((a^2*Sqrt[1 + 1/(a^2*
x^4)])/((a + x^(-2))*x) - (Sqrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a
+ x^(-2))*EllipticE[2*ArcTan[1/(Sqrt[a]*x)], 1/2])/Sqrt[1 + 1/(a^2*x^4)] +
 (Sqrt[a]*Sqrt[(a^2 + x^(-4))/(a + x^(-2))^2]*(a + x^(-2))*EllipticF[2*Arc
Tan[1/(Sqrt[a]*x)], 1/2])/(2*Sqrt[1 + 1/(a^2*x^4)])))/5
```

3.46.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.46.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

method	result
default	$\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\left(-2\sqrt{ia}\,a^4x^8+2ia^3\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x^5\,\mathrm{EllipticF}\left(x\sqrt{ia},i\right)-2ia^3\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,x^5\,\mathrm{EllipticE}\left(x\sqrt{ia},i\right)-3\sqrt{ia}\right.}{5x^3(x^4a^2+1)\sqrt{ia}}$

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*((a^2*x^4+1)/x^4/a^2)^(1/2)*(-2*(I*a)^(1/2)*a^4*x^8+2*I*a^3*(1-I*a*x^2
)^(1/2)*(1+I*a*x^2)^(1/2)*x^5*EllipticF(x*(I*a)^(1/2),I)-2*I*a^3*(1-I*a*x^
2)^(1/2)*(1+I*a*x^2)^(1/2)*x^5*EllipticE(x*(I*a)^(1/2),I)-3*(I*a)^(1/2)*a^
2*x^4-(I*a)^(1/2))/x^3/(a^2*x^4+1)/(I*a)^(1/2)-1/5/a/x^5
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.54

$$\int \frac{e^{\mathrm{csch}^{-1}(ax^2)}}{x^4}\,dx =$$
$$-\frac{2\,(-a^2)^{\frac{3}{4}}\,a^2x^5E(\arcsin\left((-a^2)^{\frac{1}{4}}\,x\right)\,|-1)-2\,(-a^2)^{\frac{3}{4}}\,a^2x^5F(\arcsin\left((-a^2)^{\frac{1}{4}}\,x\right)\,|-1)+(2\,a^3x^6+ax^2)\sqrt{}}{5\,ax^5}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
-1/5*(2*(-a^2)^(3/4)*a^2*x^5*elliptic_e(arcsin((-a^2)^(1/4)*x), -1) - 2*(-
a^2)^(3/4)*a^2*x^5*elliptic_f(arcsin((-a^2)^(1/4)*x), -1) + (2*a^3*x^6 + a
*x^2)*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^5)
```

3.46.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.24

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{a^2x^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)} - \frac{1}{5ax^5}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**4,x)
```

output
```
-gamma(3/4)*hyper((-1/2, 3/4), (7/4,), exp_polar(I*pi)/(a**2*x**4))/(4*x**
3*gamma(7/4)) - 1/(5*a*x**5)
```

3.46.7 Maxima [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^4}\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^4 + 1)/x^6, x)/a - 1/5/(a*x^5)
```

3.46.8 Giac [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1}{a^2x^4}+1}+\frac{1}{ax^2}}{x^4}\,dx$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^4,x, algorithm="giac")
```

output
```
integrate((sqrt(1/(a^2*x^4) + 1) + 1/(a*x^2))/x^4, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1}{a^2\,x^4}+1}+\frac{1}{a\,x^2}}{x^4}\,dx$$

input
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^4,x)
```

output
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^4, x)
```

3.47 $\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^5}\,dx$

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^5}\,dx=-\frac{1}{6}a^2\left(1+\frac{1}{a^2x^4}\right)^{3/2}-\frac{1}{6ax^6}$$

output
```
-1/6*a^2*(1+1/a^2/x^4)^(3/2)-1/6/a/x^6
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{e^{\text{csch}^{-1}\left(ax^2\right)}}{x^5}\,dx=-\frac{1+a\sqrt{1+\frac{1}{a^2x^4}}x^2(1+a^2x^4)}{6ax^6}$$

input
```
Integrate[E^ArcCsch[a*x^2]/x^5,x]
```

output
```
-1/6*(1 + a*Sqrt[1 + 1/(a^2*x^4)]*x^2*(1 + a^2*x^4))/(a*x^6)
```

3.47.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6890, 15, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}dx$$

$$\Big\downarrow \text{ 6890}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^5}dx+\frac{\int \frac{1}{x^7}dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$\int \frac{\sqrt{1+\frac{1}{x^4a^2}}}{x^5}dx-\frac{1}{6ax^6}$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{1}{6}a^2\left(\frac{1}{a^2x^4}+1\right)^{3/2}-\frac{1}{6ax^6}$$

input
```
Int[E^ArcCsch[a*x^2]/x^5,x]
```

output
```
-1/6*(a^2*(1 + 1/(a^2*x^4))^(3/2)) - 1/(6*a*x^6)
```

3.47.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6890
```
Int[E^ArcCsch[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[1/a   Int[x^(
m - p), x], x] + Int[x^m*Sqrt[1 + 1/(a^2*x^(2*p))], x] /; FreeQ[{a, m, p},
x]
```

3.47.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\sqrt{\frac{x^4a^2+1}{x^4a^2}}\,(x^4a^2+1)}{6x^4}-\frac{1}{6a\,x^6}$	42
trager	$\frac{-\frac{1}{6x^6}-\frac{a\left(x^4a^2+1\right)\sqrt{-\frac{-x^4a^2-1}{x^4a^2}}}{6x^4}}{a}$	46

input
```
int((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((a^2*x^4+1)/x^4/a^2)^(1/2)/x^4*(a^2*x^4+1)-1/6/a/x^6
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}\,dx=-\frac{a^3x^6+(a^3x^6+ax^2)\sqrt{\frac{a^2x^4+1}{a^2x^4}}+1}{6\,ax^6}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^6 + (a^3*x^6 + a*x^2)*sqrt((a^2*x^4 + 1)/(a^2*x^4)) + 1)/(a*x^
6)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}\,dx = \begin{cases} \frac{-a\left(\begin{cases} \sqrt{1+\frac{1}{a^2x^4}}\left(\frac{a^2}{3}+\frac{1}{3x^4}\right) & \text{for } \frac{1}{a^2}\neq 0 \\ \frac{1}{2x^4} & \text{otherwise} \end{cases}\right)-\frac{1}{3x^6}}{2a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x**2+(1+1/a**2/x**4)**(1/2))/x**5,x)
```

output
```
Piecewise(((-a*Piecewise((sqrt(1 + 1/(a**2*x**4))*(a**2/3 + 1/(3*x**4)), N
e(a**(-2), 0)), (1/(2*x**4), True)) - 1/(3*x**6))/(2*a), Ne(a, 0)), (0, Tr
ue))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}\,dx = -\frac{1}{6}\,a^2\left(\frac{1}{a^2x^4}+1\right)^{\frac{3}{2}} - \frac{1}{6\,ax^6}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="maxima")
```

output
```
-1/6*a^2*(1/(a^2*x^4) + 1)^(3/2) - 1/6/(a*x^6)
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.29

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}\,dx = \frac{\frac{2\left(3\left(x^2|a|-\sqrt{a^2x^4+1}\right)^4a^4+a^4\right)}{\left(\left(x^2|a|-\sqrt{a^2x^4+1}\right)^2-1\right)^3} - \frac{a}{x^6}}{6\,a^2}$$

input
```
integrate((1/a/x^2+(1+1/a^2/x^4)^(1/2))/x^5,x, algorithm="giac")
```

output
```
1/6*(2*(3*(x^2*abs(a) - sqrt(a^2*x^4 + 1))^4*a^4 + a^4)/((x^2*abs(a) - sqr
t(a^2*x^4 + 1))^2 - 1)^3 - a/x^6)/a^2
```

3.47.9 Mupad [B] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{e^{\operatorname{csch}^{-1}(ax^2)}}{x^5}\,dx = -\frac{\frac{1}{6\,a}+\frac{x^2\,\sqrt{\frac{1}{a^2\,x^4}+1}}{6}}{x^6}-\frac{a^2\,\sqrt{\frac{1}{a^2\,x^4}+1}}{6}$$

input
```
int(((1/(a^2*x^4) + 1)^(1/2) + 1/(a*x^2))/x^5,x)
```

output
```
- (1/(6*a) + (x^2*(1/(a^2*x^4) + 1)^(1/2))/6)/x^6 - (a^2*(1/(a^2*x^4) + 1)
^(1/2))/6
```

3.48 $\int e^{2\text{csch}^{-1}(ax)} x^m \, dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int e^{2\text{csch}^{-1}(ax)} x^m \, dx = -\frac{2x^{-1+m}}{a^2(1-m)} + \frac{x^{1+m}}{1+m} + \frac{2x^m \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, -\frac{1}{a^2x^2}\right)}{am}$$

output
```
-2*x^(-1+m)/a^2/(1-m)+x^(1+m)/(1+m)+2*x^m*hypergeom([-1/2, -1/2*m],[1-1/2*
m],-1/a^2/x^2)/a/m
```

3.48.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int e^{2\text{csch}^{-1}(ax)} x^m \, dx = x^m \left(\frac{2}{a^2(-1+m)x} + \frac{x}{1+m} + \frac{2 \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{m}{2}, 1-\frac{m}{2}, -\frac{1}{a^2x^2}\right)}{am} \right)$$

input
```
Integrate[E^(2*ArcCsch[a*x])*x^m,x]
```

output
```
x^m*(2/(a^2*(-1 + m)*x) + x/(1 + m) + (2*Hypergeometric2F1[-1/2, -1/2*m, 1
 - m/2, -(1/(a^2*x^2))])/(a*m))
```

3.48.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{2\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \; 6892$$

$$\int \left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2 x^m dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2x^{m-2}}{a^2}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}x^{m-1}}{a}+x^m\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2x^m \text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},-\frac{1}{a^2x^2}\right)}{am}-\frac{2x^{m-1}}{a^2(1-m)}+\frac{x^{m+1}}{m+1}$$

input
```
Int[E^(2*ArcCsch[a*x])*x^m,x]
```

output
```
(-2*x^(-1 + m))/(a^2*(1 - m)) + x^(1 + m)/(1 + m) + (2*x^m*Hypergeometric2
F1[-1/2, -1/2*m, 1 - m/2, -(1/(a^2*x^2))])/(a*m)
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.48.4 Maple [F]

$$\int \left(\frac{1}{ax}+\sqrt{1+\frac{1}{a^2x^2}}\right)^2 x^m dx$$

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x)
```

output
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x)
```

3.48.5 Fricas [F]

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2 dx$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="fricas")
```

output
```
integral((2*a*x*x^m*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + (a^2*x^2 + 2)*x^m)/(a^
2*x^2), x)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^m\,dx=\begin{cases}\frac{x^{m+1}}{m+1} & \text{for } m\neq -1\\ \log(x) & \text{otherwise}\end{cases}-\frac{x^m\Gamma\left(-\frac{m}{2}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{m}{2}\\ 1-\frac{m}{2}\end{matrix}\right|\frac{e^{i\pi}}{a^2x^2}\right)}{a\Gamma\left(1-\frac{m}{2}\right)}+\frac{2\left(\begin{cases}\frac{x^m}{mx-x} & \text{for } m\neq 1\\ \frac{x^m\log(x)}{x} & \text{otherwise}\end{cases}\right)}{a^2}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**m,x)
```

output
```
Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) - x**m*gamma(-m
/2)*hyper((-1/2, -m/2), (1 - m/2,), exp_polar(I*pi)/(a**2*x**2))/(a*gamma(
1 - m/2)) + 2*Piecewise((x**m/(m*x - x), Ne(m, 1)), (x**m*log(x)/x, True))
/a**2
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int e^{2\operatorname{csch}^{-1}(ax)} x^m\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

3.48.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\operatorname{csch}^{-1}(ax)} x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^m\,dx=\int x^m\left(\sqrt{\frac{1}{a^2\,x^2}+1}+\frac{1}{a\,x}\right)^2\,dx$$

input
```
int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
int(x^m*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2, x)
```

3.49 $\int e^{2\text{csch}^{-1}(ax)}x^4\,dx$

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 85

$$\int e^{2\text{csch}^{-1}(ax)}x^4\,dx=\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{4a^3}+\frac{2x^3}{3a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^4}{2a}+\frac{x^5}{5}-\frac{\text{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{4a^5}$$

output
```
2/3*x^3/a^2+1/5*x^5-1/4*arctanh((1+1/a^2/x^2)^(1/2))/a^5+1/4*x^2*(1+1/a^2/
x^2)^(1/2)/a^3+1/2*x^4*(1+1/a^2/x^2)^(1/2)/a
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int e^{2\text{csch}^{-1}(ax)}x^4\,dx$$
$$=\frac{a^2x^2\left(15\sqrt{1+\frac{1}{a^2x^2}}+40ax+30a^2\sqrt{1+\frac{1}{a^2x^2}}\,x^2+12a^3x^3\right)-15\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)}{60a^5}$$

input
```
Integrate[E^(2*ArcCsch[a*x])*x^4,x]
```

output
```
(a^2*x^2*(15*Sqrt[1 + 1/(a^2*x^2)] + 40*a*x + 30*a^2*Sqrt[1 + 1/(a^2*x^2)]
*x^2 + 12*a^3*x^3) - 15*Log[(1 + Sqrt[1 + 1/(a^2*x^2)])*x])/(60*a^5)
```

3.49.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{2\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{ 6892}$$

$$\int x^4 \left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2 dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2x^2}{a^2}+\frac{2x^3\sqrt{\frac{1}{a^2x^2}+1}}{a}+x^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x^3}{3a^2}+\frac{x^4\sqrt{\frac{1}{a^2x^2}+1}}{2a}-\frac{\text{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{4a^5}+\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{4a^3}+\frac{x^5}{5}$$

input
```
Int[E^(2*ArcCsch[a*x])*x^4,x]
```

output
```
(Sqrt[1 + 1/(a^2*x^2)]*x^2)/(4*a^3) + (2*x^3)/(3*a^2) + (Sqrt[1 + 1/(a^2*x
^2)]*x^4)/(2*a) + x^5/5 - ArcTanh[Sqrt[1 + 1/(a^2*x^2)]]/(4*a^5)
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.49.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\frac{1}{5}a^2x^5+\frac{1}{3}x^3}{a^2}-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(-2x\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}a^4+x\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{4a^5\sqrt{\frac{a^2x^2+1}{a^2}}}+\frac{x^3}{3a^2}$	127

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/5*a^2*x^5+1/3*x^3)-1/4/a^5*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(-2*x*((
a^2*x^2+1)/a^2)^(3/2)*a^4+x*((a^2*x^2+1)/a^2)^(1/2)*a^2+ln(x+((a^2*x^2+1)/
a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1/2)+1/3*x^3/a^2
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^4\,dx$$
$$=\frac{12\,a^5x^5+40\,a^3x^3+15\,(2\,a^4x^4+a^2x^2)\sqrt{\frac{a^2x^2+1}{a^2x^2}}+15\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{60\,a^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="fricas")
```

output
```
1/60*(12*a^5*x^5 + 40*a^3*x^3 + 15*(2*a^4*x^4 + a^2*x^2)*sqrt((a^2*x^2 + 1
)/(a^2*x^2)) + 15*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^5
```

3.49.6 Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{x^5}{5} + \frac{x^5}{2\sqrt{a^2x^2+1}} + \frac{2x^3}{3a^2} + \frac{3x^3}{4a^2\sqrt{a^2x^2+1}} + \frac{x}{4a^4\sqrt{a^2x^2+1}} - \frac{\operatorname{asinh}(ax)}{4a^5}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**4,x)
```

output
```
x**5/5 + x**5/(2*sqrt(a**2*x**2 + 1)) + 2*x**3/(3*a**2) + 3*x**3/(4*a**2*s
qrt(a**2*x**2 + 1)) + x/(4*a**4*sqrt(a**2*x**2 + 1)) - asinh(a*x)/(4*a**5)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{1}{5}\,x^5 + \frac{2\,x^3}{3\,a^2} + \frac{\frac{2\left(\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^4\left(\frac{1}{a^2x^2}+1\right)^2-2\,a^4\left(\frac{1}{a^2x^2}+1\right)+a^4} - \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{a^4} + \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a^4}}{8\,a}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="maxima")
```

output
```
1/5*x^5 + 2/3*x^3/a^2 + 1/8*(2*((1/(a^2*x^2) + 1)^(3/2) + sqrt(1/(a^2*x^2)
 + 1))/(a^4*(1/(a^2*x^2) + 1)^2 - 2*a^4*(1/(a^2*x^2) + 1) + a^4) - log(sqr
t(1/(a^2*x^2) + 1) + 1)/a^4 + log(sqrt(1/(a^2*x^2) + 1) - 1)/a^4)/a
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{1}{4}\,\sqrt{a^2x^2+1}x\left(\frac{2\,x^2|a|\mathrm{sgn}(x)}{a^3} + \frac{|a|\mathrm{sgn}(x)}{a^5}\right) + \frac{3\,a^2x^5+10\,x^3}{15\,a^2} + \frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\mathrm{sgn}(x)}{4\,a^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="giac")
```

output
```
1/4*sqrt(a^2*x^2 + 1)*x*(2*x^2*abs(a)*sgn(x)/a^3 + abs(a)*sgn(x)/a^5) + 1/
15*(3*a^2*x^5 + 10*x^3)/a^2 + 1/4*log(-x*abs(a) + sqrt(a^2*x^2 + 1))*sgn(x
)/a^5
```

3.49.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^4\,dx = \frac{x^5}{5} + \frac{2\,x^3}{3\,a^2} + \frac{x^4\,\sqrt{\frac{1}{a^2\,x^2}+1}}{2\,a} + \frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,a^3} + \frac{\operatorname{atan}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a^5}$$

input
```
int(x^4*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(atan((1/(a^2*x^2) + 1)^(1/2)*1i)*1i)/(4*a^5) + x^5/5 + (2*x^3)/(3*a^2) +
(x^4*(1/(a^2*x^2) + 1)^(1/2))/(2*a) + (x^2*(1/(a^2*x^2) + 1)^(1/2))/(4*a^3
)
```

3.50 $\int e^{2\text{csch}^{-1}(ax)}x^3\,dx$

3.50.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int e^{2\text{csch}^{-1}(ax)}x^3\,dx=\frac{x^2}{a^2}+\frac{2\left(1+\frac{1}{a^2x^2}\right)^{3/2}x^3}{3a}+\frac{x^4}{4}$$

output
```
x^2/a^2+2/3*(1+1/a^2/x^2)^(3/2)*x^3/a+1/4*x^4
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int e^{2\text{csch}^{-1}(ax)}x^3\,dx=\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2\sqrt{1+\frac{1}{a^2x^2}}(x+a^2x^3)}{3a^3}$$

input
```
Integrate[E^(2*ArcCsch[a*x])*x^3,x]
```

output
```
x^2/a^2 + x^4/4 + (2*Sqrt[1 + 1/(a^2*x^2)]*(x + a^2*x^3))/(3*a^3)
```

3.50.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \text{6892}$$

$$\int x^3 \left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2 dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2x^2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2x}{a^2}+x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{a^2}+\frac{2x^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}}{3a}+\frac{x^4}{4}$$

input
```
Int[E^(2*ArcCsch[a*x])*x^3,x]
```

output
```
x^2/a^2 + (2*(1 + 1/(a^2*x^2))^(3/2)*x^3)/(3*a) + x^4/4
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.50.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{(a^2x^2+1)^2}{4a^4}+\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x(a^2x^2+1)}{3a^3}+\frac{x^2}{2a^2}$	59
trager	$\frac{\frac{(a^2x^3+a^2x^2+a^2x+a^2+4x+4)(x-1)}{4}+\frac{2(a^2x^2+1)x\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{3a}}{a^2}$	73

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^4*(a^2*x^2+1)^2+2/3/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(a^2*x^2+1)+1/
2*x^2/a^2
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^3\,dx=\frac{3\,a^3x^4+12\,ax^2+8\,(a^2x^3+x)\sqrt{\frac{a^2x^2+1}{a^2x^2}}}{12\,a^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^3*x^4 + 12*a*x^2 + 8*(a^2*x^3 + x)*sqrt((a^2*x^2 + 1)/(a^2*x^2))
)/a^3
```

3.50.6 Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^3\,dx=\frac{x^4}{4}+\frac{2x^2\sqrt{a^2x^2+1}}{3a^2}+\frac{x^2}{a^2}+\frac{2\sqrt{a^2x^2+1}}{3a^4}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**3,x)
```

output
```
x**4/4 + 2*x**2*sqrt(a**2*x**2 + 1)/(3*a**2) + x**2/a**2 + 2*sqrt(a**2*x**
2 + 1)/(3*a**4)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{1}{4}\,x^4 + \frac{2\,x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}}{3\,a} + \frac{x^2}{a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="maxima")
```

output
```
1/4*x^4 + 2/3*x^3*(1/(a^2*x^2) + 1)^(3/2)/a + x^2/a^2
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^3\,dx = \frac{a^2x^2+1}{2\,a^4} - \frac{2\,|a|\text{sgn}(x)}{3\,a^5} + \frac{8\,(a^2x^2+1)^{\frac{3}{2}}a^2|a|\text{sgn}(x) + 3\,(a^2x^2+1)^2a^3}{12\,a^7}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="giac")
```

output
```
1/2*(a^2*x^2 + 1)/a^4 - 2/3*abs(a)*sgn(x)/a^5 + 1/12*(8*(a^2*x^2 + 1)^(3/2
)*a^2*abs(a)*sgn(x) + 3*(a^2*x^2 + 1)^2*a^3)/a^7
```

3.50.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^3\,dx = \sqrt{\frac{1}{a^2\,x^2}+1}\,\left(\frac{2\,x}{3\,a^3} + \frac{2\,x^3}{3\,a}\right) + \frac{x^4}{4} + \frac{x^2}{a^2}$$

input
```
int(x^3*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(1/(a^2*x^2) + 1)^(1/2)*((2*x)/(3*a^3) + (2*x^3)/(3*a)) + x^4/4 + x^2/a^2
```

3.51 $\int e^{2\text{csch}^{-1}(ax)}x^2\,dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{\sqrt{1+\frac{1}{a^2x^2}}\,x^2}{a}+\frac{x^3}{3}+\frac{\text{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{a^3}$$

output
```
2*x/a^2+1/3*x^3+arctanh((1+1/a^2/x^2)^(1/2))/a^3+x^2*(1+1/a^2/x^2)^(1/2)/a
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx=\frac{ax\left(6+3a\sqrt{1+\frac{1}{a^2x^2}}\,x+a^2x^2\right)+3\log\left(\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)}{3a^3}$$

input
```
Integrate[E^(2*ArcCsch[a*x])*x^2,x]
```

output
```
(a*x*(6 + 3*a*Sqrt[1 + 1/(a^2*x^2)]*x + a^2*x^2) + 3*Log[(1 + Sqrt[1 + 1/(
a^2*x^2)])*x])/(3*a^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2\text{csch}^{-1}(ax)} \, dx$$

$$\Big\downarrow \; 6892$$

$$\int x^2 \left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2 dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2x\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2}{a^2}+x^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2x}{a^2}+\frac{\text{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^3}+\frac{x^3}{3}$$

input
```
Int[E^(2*ArcCsch[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + (Sqrt[1 + 1/(a^2*x^2)]*x^2)/a + x^3/3 + ArcTanh[Sqrt[1 + 1/(a^
2*x^2)]]/a^3
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.51.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{\frac{1}{3}a^2x^3+x}{a^2}+\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(x\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+\ln\left(x+\sqrt{\frac{a^2x^2+1}{a^2}}\right)\right)}{a^3\sqrt{\frac{a^2x^2+1}{a^2}}}+\frac{x}{a^2}$	98

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/3*a^2*x^3+x)+1/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*(x*((a^2*x^2+1)/
a^2)^(1/2)*a^2+ln(x+((a^2*x^2+1)/a^2)^(1/2)))/((a^2*x^2+1)/a^2)^(1/2)+x/a^
2
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

$$\int e^{2\operatorname{csch}^{-1}(ax)}x^2\,dx=\frac{a^3x^3+3\,a^2x^2\sqrt{\frac{a^2x^2+1}{a^2x^2}}+6\,ax-3\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax\right)}{3\,a^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^3*x^3 + 3*a^2*x^2*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 6*a*x - 3*log(a*x
*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x))/a^3
```

3.51.6 Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx = \frac{x^3}{3} + \frac{x\sqrt{a^2x^2+1}}{a^2} + \frac{2x}{a^2} + \frac{\operatorname{asinh}(ax)}{a^3}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x**2,x)
```

output
```
x**3/3 + x*sqrt(a**2*x**2 + 1)/a**2 + 2*x/a**2 + asinh(a*x)/a**3
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx = \frac{1}{3}\,x^3 + \frac{\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a^2\left(\frac{1}{a^2x^2}+1\right)-a^2} + \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{a^2} - \frac{\log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a^2}}{2\,a} + \frac{2\,x}{a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*(2*sqrt(1/(a^2*x^2) + 1)/(a^2*(1/(a^2*x^2) + 1) - a^2) + log
(sqrt(1/(a^2*x^2) + 1) + 1)/a^2 - log(sqrt(1/(a^2*x^2) + 1) - 1)/a^2)/a +
2*x/a^2
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx = \frac{\sqrt{a^2x^2+1}x|a|\text{sgn}(x)}{a^3} + \frac{a^2x^3+6\,x}{3\,a^2} - \frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)\text{sgn}(x)}{a^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="giac")
```

output
```
sqrt(a^2*x^2 + 1)*x*abs(a)*sgn(x)/a^3 + 1/3*(a^2*x^3 + 6*x)/a^2 - log(-x*a
bs(a) + sqrt(a^2*x^2 + 1))*sgn(x)/a^3
```

3.51.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int e^{2\text{csch}^{-1}(ax)}x^2\,dx = \frac{2\,x}{a^2} + \frac{x^3}{3} + \frac{x^2\,\sqrt{\frac{1}{a^2\,x^2}+1}}{a} - \frac{\text{atan}\left(\sqrt{\frac{1}{a^2\,x^2}+1}\,1\text{i}\right)\,1\text{i}}{a^3}$$

input
```
int(x^2*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
(2*x)/a^2 - (atan((1/(a^2*x^2) + 1)^(1/2)*1i)*1i)/a^3 + x^3/3 + (x^2*(1/(a
^2*x^2) + 1)^(1/2))/a
```

3.52 $\int e^{2\text{csch}^{-1}(ax)}x\,dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 43

$$\int e^{2\text{csch}^{-1}(ax)}x\,dx=\frac{2\sqrt{1+\frac{1}{a^2x^2}}\,x}{a}+\frac{x^2}{2}-\frac{2\text{csch}^{-1}(ax)}{a^2}+\frac{2\log(x)}{a^2}$$

output
```
1/2*x^2-2*arccsch(a*x)/a^2+2*ln(x)/a^2+2*x*(1+1/a^2/x^2)^(1/2)/a
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int e^{2\text{csch}^{-1}(ax)}x\,dx=\frac{ax\left(4\sqrt{1+\frac{1}{a^2x^2}}+ax\right)-4\text{arcsinh}\left(\frac{1}{ax}\right)+4\log(x)}{2a^2}$$

input
```
Integrate[E^(2*ArcCsch[a*x])*x,x]
```

output
```
(a*x*(4*Sqrt[1 + 1/(a^2*x^2)] + a*x) - 4*ArcSinh[1/(a*x)] + 4*Log[x])/(2*a
^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2\text{csch}^{-1}(ax)}\,dx$$

$$\Big\downarrow \ 6892$$

$$\int x\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2}{a^2x}+x\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2x\sqrt{\frac{1}{a^2x^2}+1}}{a}+\frac{2\log(x)}{a^2}-\frac{2\text{csch}^{-1}(ax)}{a^2}+\frac{x^2}{2}$$

input
```
Int[E^(2*ArcCsch[a*x])*x,x]
```

output
```
(2*Sqrt[1 + 1/(a^2*x^2)]*x)/a + x^2/2 - (2*ArcCsch[a*x])/a^2 + (2*Log[x])/
a^2
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

method	result	size
default	$\frac{\frac{a^2x^2}{2}+\ln(x)}{a^2}+\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,x\left(\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2-\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)\right)}{a^3\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}}+\frac{\ln(x)}{a^2}$	129

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*a^2*x^2+ln(x))+2/a^3*((a^2*x^2+1)/a^2/x^2)^(1/2)*x*((1/a^2)^(1/
2)*((a^2*x^2+1)/a^2)^(1/2)*a^2-ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)
*a^2+1)/x/a^2))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/2)+ln(x)/a^2
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(39) = 78$.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx = \frac{a^2x^2+4\,ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-4\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)+4\,\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)+4\,\log(x)}{2\,a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 + 4*a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - 4*log(a*x*sqrt((a^2*x
^2 + 1)/(a^2*x^2)) - a*x + 1) + 4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) -
a*x - 1) + 4*log(x))/a^2
```

3.52.6 Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx=\frac{x^2}{2}+\frac{2x}{a\sqrt{1+\frac{1}{a^2x^2}}}+\frac{2\log{(x)}}{a^2}-\frac{2\operatorname{asinh}{\left(\frac{1}{ax}\right)}}{a^2}+\frac{2}{a^3x\sqrt{1+\frac{1}{a^2x^2}}}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2*x,x)
```

output
```
x**2/2 + 2*x/(a*sqrt(1 + 1/(a**2*x**2))) + 2*log(x)/a**2 - 2*asinh(1/(a*x)
)/a**2 + 2/(a**3*x*sqrt(1 + 1/(a**2*x**2)))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx=\frac{1}{2}\,x^2+\frac{2\,x\sqrt{\frac{1}{a^2x^2}+1}-\frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)}{a}+\frac{\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a}}{a}+\frac{2\,\log\left(x\right)}{a^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="maxima")
```

output
```
1/2*x^2 + (2*x*sqrt(1/(a^2*x^2) + 1) - log(a*x*sqrt(1/(a^2*x^2) + 1) + 1)/
a + log(a*x*sqrt(1/(a^2*x^2) + 1) - 1)/a)/a + 2*log(x)/a^2
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx$$
$$=\frac{4\sqrt{a^2x^2+1}|a|\mathrm{sgn}(x)+(a^2x^2+1)a-2\left(|a|\mathrm{sgn}(x)-a\right)\log\left(\sqrt{a^2x^2+1}+1\right)+2\left(|a|\mathrm{sgn}(x)+a\right)\log\left(\sqrt{a^2\right.}$$
$$2\,a^3$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="giac")
```

output
```
1/2*(4*sqrt(a^2*x^2 + 1)*abs(a)*sgn(x) + (a^2*x^2 + 1)*a - 2*(abs(a)*sgn(x
) - a)*log(sqrt(a^2*x^2 + 1) + 1) + 2*(abs(a)*sgn(x) + a)*log(sqrt(a^2*x^2
 + 1) - 1))/a^3
```

3.52.9 Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int e^{2\operatorname{csch}^{-1}(ax)}x\,dx=\frac{x^2}{2}-\frac{2\,\ln\left(\frac{1}{x}\right)}{a^2}+\frac{2\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{a}-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{a^3\,\sqrt{\frac{1}{a^2}}}$$

input
```
int(x*((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output
```
x^2/2 - (2*log(1/x))/a^2 + (2*x*(1/(a^2*x^2) + 1)^(1/2))/a - (2*asinh((1/a
^2)^(1/2)/x))/(a^3*(1/a^2)^(1/2))
```

3.53 $\int e^{2\text{csch}^{-1}(ax)}\,dx$

3.53.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int e^{2\text{csch}^{-1}(ax)}\,dx=-\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}-\frac{2}{a^2x}+x+\frac{2\text{arctanh}\left(\sqrt{1+\frac{1}{a^2x^2}}\right)}{a}$$

output
```
-2/a^2/x+x+2*arctanh((1+1/a^2/x^2)^(1/2))/a-2*(1+1/a^2/x^2)^(1/2)/a
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int e^{2\text{csch}^{-1}(ax)}\,dx=-\frac{2\sqrt{1+\frac{1}{a^2x^2}}}{a}-\frac{2}{a^2x}+x+\frac{2\log\left(a\left(1+\sqrt{1+\frac{1}{a^2x^2}}\right)x\right)}{a}$$

input
```
Integrate[E^(2*ArcCsch[a*x]),x]
```

output
```
(-2*Sqrt[1 + 1/(a^2*x^2)])/a - 2/(a^2*x) + x + (2*Log[a*(1 + Sqrt[1 + 1/(a
^2*x^2)])*x])/a
```

3.53.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6887, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\text{csch}^{-1}(ax)}\,dx$$

$$\Big\downarrow \text{6887}$$

$$\int \left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{ax}+\frac{2}{a^2x^2}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(\sqrt{\frac{1}{a^2x^2}+1}\right)}{a}-\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a}-\frac{2}{a^2x}+x$$

input
```
Int[E^(2*ArcCsch[a*x]),x]
```

output
```
(-2*Sqrt[1 + 1/(a^2*x^2)])/a - 2/(a^2*x) + x + (2*ArcTanh[Sqrt[1 + 1/(a^2*
x^2)]])/a
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6887
```
Int[E^(ArcCsch[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[1 + 1/u^2])^n, x] /
; IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

method	result	size
default	$x-\frac{2}{x\,a^{2}}-\frac{2\sqrt{\frac{a^{2}x^{2}+1}{a^{2}x^{2}}}\,\left(a^{2}\left(\frac{a^{2}x^{2}+1}{a^{2}}\right)^{\frac{3}{2}}-\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\,a^{2}x^{2}-\ln\left(x+\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}\right)x\right)}{a\sqrt{\frac{a^{2}x^{2}+1}{a^{2}}}}$	113

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x-2/x/a^2-2/a*((a^2*x^2+1)/a^2/x^2)^(1/2)*(a^2*((a^2*x^2+1)/a^2)^(3/2)-((a
^2*x^2+1)/a^2)^(1/2)*a^2*x^2-ln(x+((a^2*x^2+1)/a^2)^(1/2))*x)/((a^2*x^2+1)
/a^2)^(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int e^{2\operatorname{csch}^{-1}(ax)}\,dx=\frac{a^{2}x^{2}-2\,ax\log\left(ax\sqrt{\frac{a^{2}x^{2}+1}{a^{2}x^{2}}}-ax\right)-2\,ax\sqrt{\frac{a^{2}x^{2}+1}{a^{2}x^{2}}}-2\,ax-2}{a^{2}x}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 - 2*a*x*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x) - 2*a*x*sqrt
((a^2*x^2 + 1)/(a^2*x^2)) - 2*a*x - 2)/(a^2*x)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int e^{2\text{csch}^{-1}(ax)}\,dx = x - \frac{2x}{\sqrt{a^2x^2+1}} + \frac{2\,\text{asinh}\,(ax)}{a} - \frac{2}{a^2x} - \frac{2}{a^2x\sqrt{a^2x^2+1}}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2,x)
```

output
```
x - 2*x/sqrt(a**2*x**2 + 1) + 2*asinh(a*x)/a - 2/(a**2*x) - 2/(a**2*x*sqrt
(a**2*x**2 + 1))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int e^{2\text{csch}^{-1}(ax)}\,dx = x - \frac{2\sqrt{\frac{1}{a^2x^2}+1} - \log\left(\sqrt{\frac{1}{a^2x^2}+1}+1\right) + \log\left(\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{a} - \frac{2}{a^2x}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
x - (2*sqrt(1/(a^2*x^2) + 1) - log(sqrt(1/(a^2*x^2) + 1) + 1) + log(sqrt(1
/(a^2*x^2) + 1) - 1))/a - 2/(a^2*x)
```

3.53.8 Giac [F(-2)]

Exception generated.

$$\int e^{2\text{csch}^{-1}(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.53.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{2\text{csch}^{-1}(ax)}\,dx = x - \frac{2\sqrt{\frac{1}{a^2x^2}+1}}{a} - \frac{2}{a^2x} - \frac{\text{atan}\left(\sqrt{\frac{1}{a^2x^2}+1}\,1\text{i}\right)\,2\text{i}}{a}$$

input

```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2,x)
```

output

```
x - (atan((1/(a^2*x^2) + 1)^(1/2)*1i)*2i)/a - (2*(1/(a^2*x^2) + 1)^(1/2))/
a - 2/(a^2*x)
```

3.54 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x}\,dx$

3.54.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x}\,dx = -\frac{1}{a^2x^2} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{ax} - \text{csch}^{-1}(ax) + \log(x)$$

output
```
-1/a^2/x^2-arccsch(a*x)+ln(x)-(1+1/a^2/x^2)^(1/2)/a/x
```

3.54.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x}\,dx = -\frac{1+a\sqrt{1+\frac{1}{a^2x^2}}x}{a^2x^2} - \text{arcsinh}\left(\frac{1}{ax}\right) + \log(x)$$

input
```
Integrate[E^(2*ArcCsch[a*x])/x,x]
```

output
```
-((1 + a*Sqrt[1 + 1/(a^2*x^2)]*x)/(a^2*x^2)) - ArcSinh[1/(a*x)] + Log[x]
```

3.54.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x} \, dx$$

$$\Big\downarrow \text{ 6892}$$

$$\int \frac{\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2}{x}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2}{a^2x^3}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{ax^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{ax}-\frac{1}{a^2x^2}-\text{csch}^{-1}(ax)+\log(x)$$

input
```
Int[E^(2*ArcCsch[a*x])/x,x]
```

output
```
-(1/(a^2*x^2)) - Sqrt[1 + 1/(a^2*x^2)]/(a*x) - ArcCsch[a*x] + Log[x]
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(36) = 72.

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

method	result	size
default	$\frac{a^2\ln(x)-\frac{1}{2x^2}}{a^2}-\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(a^2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}-\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}\,a^2x^2+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^2\right)}{ax\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}}-\frac{1}{2a^2x^2}$	164

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a^2*ln(x)-1/2/x^2)-1/a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x*(a^2*((a^2*x^2
+1)/a^2)^(3/2)*(1/a^2)^(1/2)-((a^2*x^2+1)/a^2)^(1/2)*(1/a^2)^(1/2)*a^2*x^2
+ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^2)*x^2)/(1/a^2)^(1
/2)/((a^2*x^2+1)/a^2)^(1/2)-1/2/a^2/x^2
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.95

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x}\,dx = -\frac{a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-a^2x^2\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)-a^2x^2\log(x)+ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}+1}{a^2x^2}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
-(a^2*x^2*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^2*x^2*log(a
*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - a^2*x^2*log(x) + a*x*sqrt((a
^2*x^2 + 1)/(a^2*x^2)) + 1)/(a^2*x^2)
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.

Time = 2.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x}\,dx = \begin{cases} \frac{a^2\log(x) - 2a\left(\begin{cases} \frac{\log\left(2\sqrt{1+\frac{1}{a^2x^2}}\sqrt{\frac{1}{a^2}}+\frac{2}{a^2x}\right)}{2\sqrt{\frac{1}{a^2}}} + \frac{\sqrt{1+\frac{1}{a^2x^2}}}{2x} & \text{for } \frac{1}{a^2} \neq 0 \\ \frac{1}{x} & \text{otherwise} \end{cases}\right) - \frac{1}{x^2}}{a^2} & \text{for } a^2 \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x,x)
```

output
```
Piecewise(((a**2*log(x) - 2*a*Piecewise((log(2*sqrt(1 + 1/(a**2*x**2))*sqr
t(a**(-2)) + 2/(a**2*x))/(2*sqrt(a**(-2))) + sqrt(1 + 1/(a**2*x**2))/(2*x)
, Ne(a**(-2), 0)), (1/x, True)) - 1/x**2)/a**2, Ne(a**2, 0)), (nan, True))
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x}\,dx = -\frac{\frac{2\,a^2x\sqrt{\frac{1}{a^2x^2}+1}}{a^2x^2\left(\frac{1}{a^2x^2}+1\right)-1} + a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right) - a\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)}{2\,a} - \frac{1}{a^2x^2} + \log(x)$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
-1/2*(2*a^2*x*sqrt(1/(a^2*x^2) + 1)/(a^2*x^2*(1/(a^2*x^2) + 1) - 1) + a*lo
g(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - a*log(a*x*sqrt(1/(a^2*x^2) + 1) - 1))/a
 - 1/(a^2*x^2) + log(x)
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.11

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x}\,dx =$$

$$-\frac{\left(a^4|a|\mathrm{sgn}(x)-a^5\right)\log\left(\sqrt{a^2x^2+1}+1\right)-\left(a^4|a|\mathrm{sgn}(x)+a^5\right)\log\left(\sqrt{a^2x^2+1}-1\right)+\frac{2\left(\sqrt{a^2x^2+1}a^4|a|\mathrm{sgn}(x)\right.}{\left(\sqrt{a^2x^2+1}+1\right)\left(\sqrt{a^2x^2+\right.}}}{2\,a^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="giac")
```

output
```
-1/2*((a^4*abs(a)*sgn(x) - a^5)*log(sqrt(a^2*x^2 + 1) + 1) - (a^4*abs(a)*s
gn(x) + a^5)*log(sqrt(a^2*x^2 + 1) - 1) + 2*(sqrt(a^2*x^2 + 1)*a^4*abs(a)*
sgn(x) + a^5)/((sqrt(a^2*x^2 + 1) + 1)*(sqrt(a^2*x^2 + 1) - 1)))/a^5
```

3.54.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x}\,dx = -\ln\left(\frac{1}{x}\right) - \operatorname{asinh}\left(\frac{1}{a\,x}\right) - \frac{1}{a^2\,x^2} - \frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{a\,x}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x,x)
```

output
```
- log(1/x) - asinh(1/(a*x)) - 1/(a^2*x^2) - (1/(a^2*x^2) + 1)^(1/2)/(a*x)
```

3.55 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^2} \, dx$

3.55.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^2} \, dx = -\frac{2}{3}a\left(1+\frac{1}{a^2x^2}\right)^{3/2} - \frac{2}{3a^2x^3} - \frac{1}{x}$$

output
```
-2/3*a*(1+1/a^2/x^2)^(3/2)-2/3/a^2/x^3-1/x
```

3.55.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^2} \, dx = -\frac{2+3a^2x^2+2a\sqrt{1+\frac{1}{a^2x^2}}x(1+a^2x^2)}{3a^2x^3}$$

input
```
Integrate[E^(2*ArcCsch[a*x])/x^2,x]
```

output
```
-1/3*(2 + 3*a^2*x^2 + 2*a*Sqrt[1 + 1/(a^2*x^2)]*x*(1 + a^2*x^2))/(a^2*x^3)
```

3.55.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6892, 7266, 2542, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^2} \, dx$$

$$\Big\downarrow \; 6892$$

$$\int \frac{\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2}{x^2}dx$$

$$\Big\downarrow \; 7266$$

$$-\int \left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)^2 d\frac{1}{x}$$

$$\Big\downarrow \; 2542$$

$$-\frac{1}{2}a\int \left(1+\frac{1}{x^2}\right)d\left(\sqrt{1+\frac{1}{a^2x^2}}+\frac{1}{ax}\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}a\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}+\frac{1}{3x^3}\right)$$

input
```
Int[E^(2*ArcCsch[a*x])/x^2,x]
```

output
```
-1/2*(a*(Sqrt[1 + 1/(a^2*x^2)] + 1/(3*x^3) + 1/(a*x)))
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.55.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

method	result	size
trager	$\frac{-\frac{3a^2x^2+2}{3x^3}-\frac{2a\left(a^2x^2+1\right)\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{3x^2}}{a^2}$	56
default	$\frac{-\frac{1}{3x^3}-\frac{a^2}{x}}{a^2}-\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,(a^2x^2+1)}{3a\,x^2}-\frac{1}{3a^2x^3}$	63

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3*(3*a^2*x^2+2)/x^3-2/3/x^2*a*(a^2*x^2+1)*(-(-a^2*x^2-1)/a^2/x^2
)^(1/2))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{2\,a^3x^3 + 3\,a^2x^2 + 2\,(a^3x^3 + ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}} + 2}{3\,a^2x^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a^3*x^3 + 3*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^
2)) + 2)/(a^2*x^3)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = \begin{cases} \frac{-\frac{a^2}{x} - 2a\left(\begin{cases} \sqrt{1+\frac{1}{a^2x^2}}\left(\frac{a^2}{3}+\frac{1}{3x^2}\right) & \text{for } \frac{1}{a^2} \neq 0 \\ \frac{1}{2x^2} & \text{otherwise} \end{cases}\right) - \frac{2}{3x^3}}{a^2} & \text{for } a^2 \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**2,x)
```

output
```
Piecewise(((-a**2/x - 2*a*Piecewise((sqrt(1 + 1/(a**2*x**2))*(a**2/3 + 1/(
3*x**2)), Ne(a**(-2), 0)), (1/(2*x**2), True)) - 2/(3*x**3))/a**2, Ne(a**2
, 0)), (nan, True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{2}{3}\,a\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}} - \frac{1}{x} - \frac{2}{3\,a^2x^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
-2/3*a*(1/(a^2*x^2) + 1)^(3/2) - 1/x - 2/3/(a^2*x^3)
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(28) = 56$.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = \frac{4\left(3\left(x|a|-\sqrt{a^2x^2+1}\right)^4 a\operatorname{sgn}(x)+a\operatorname{sgn}(x)\right)}{3\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^3}-\frac{3\,a^2x^2+2}{3\,a^2x^3}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
4/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^4*a*sgn(x) + a*sgn(x))/((x*abs(a) -
sqrt(a^2*x^2 + 1))^2 - 1)^3 - 1/3*(3*a^2*x^2 + 2)/(a^2*x^3)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2}\,dx = -\frac{\frac{2}{3\,a^2}+\frac{2\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{3\,a}}{x^3}-\frac{\frac{2\,a\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{3}+1}{x}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^2,x)
```

output
```
- (2/(3*a^2) + (2*x*(1/(a^2*x^2) + 1)^(1/2))/(3*a))/x^3 - ((2*a*x*(1/(a^2*
x^2) + 1)^(1/2))/3 + 1)/x
```

3.56 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3}\,dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3}\,dx = -\frac{1}{2a^2x^4} - \frac{\sqrt{1+\frac{1}{a^2x^2}}}{2ax^3} - \frac{1}{2x^2} - \frac{a\sqrt{1+\frac{1}{a^2x^2}}}{4x} + \frac{1}{4}a^2\text{csch}^{-1}(ax)$$

output
```
-1/2/a^2/x^4-1/2/x^2+1/4*a^2*arccsch(a*x)-1/2*(1+1/a^2/x^2)^(1/2)/a/x^3-1/
4*a*(1+1/a^2/x^2)^(1/2)/x
```

3.56.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3}\,dx = -\frac{1}{2a^2x^4} - \frac{1}{2x^2} + \left(-\frac{1}{2ax^3} - \frac{a}{4x}\right)\sqrt{\frac{1+a^2x^2}{a^2x^2}} + \frac{1}{4}a^2\text{arcsinh}\left(\frac{1}{ax}\right)$$

input
```
Integrate[E^(2*ArcCsch[a*x])/x^3,x]
```

output
```
-1/2*1/(a^2*x^4) - 1/(2*x^2) + (-1/2*1/(a*x^3) - a/(4*x))*Sqrt[(1 + a^2*x^
2)/(a^2*x^2)] + (a^2*ArcSinh[1/(a*x)])/4
```

3.56.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3} \, dx$$

$$\Big\downarrow \text{ 6892}$$

$$\int \frac{\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2}{a^2x^5}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{ax^4}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2a^2x^4}-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{4x}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{2ax^3}+\frac{1}{4}a^2\text{csch}^{-1}(ax)-\frac{1}{2x^2}$$

input
```
Int[E^(2*ArcCsch[a*x])/x^3,x]
```

output
```
-1/2*1/(a^2*x^4) - Sqrt[1 + 1/(a^2*x^2)]/(2*a*x^3) - 1/(2*x^2) - (a*Sqrt[1
 + 1/(a^2*x^2)])/(4*x) + (a^2*ArcCsch[a*x])/4
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(59) = 118$.

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.59

method	result
default	$\frac{-\frac{1}{4x^4}-\frac{a^2}{2x^2}}{a^2}+\frac{a\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}\,a^2x^2-\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}\,a^2x^4+\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+2}{a^2x}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{a^2}}\right)}{4x^3\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}}$

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4/x^4-1/2*a^2/x^2)+1/4*a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^3*(((a^2*
x^2+1)/a^2)^(3/2)*(1/a^2)^(1/2)*a^2*x^2-((a^2*x^2+1)/a^2)^(1/2)*(1/a^2)^(1
/2)*a^2*x^4+ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^2)*x^4-
2*((a^2*x^2+1)/a^2)^(3/2)*(1/a^2)^(1/2))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(
1/2)-1/4/a^2/x^4
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(59) = 118$.

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-a^4x^4\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)-2\,a^2x^2-(a^3x^3+2\,ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}}-}{4\,a^2x^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(a^4*x^4*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - a^4*x^4*lo
g(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) - 2*a^2*x^2 - (a^3*x^3 + 2*
a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - 2)/(a^2*x^4)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3}\,dx = \begin{cases} \frac{-\frac{a^2}{2x^2}-2a\left(\begin{cases} -\frac{a^2\log\left(2\sqrt{1+\frac{1}{a^2x^2}}\sqrt{\frac{1}{a^2}}+\frac{2}{a^2x}\right)}{8\sqrt{\frac{1}{a^2}}}+\sqrt{1+\frac{1}{a^2x^2}}\left(\frac{a^2}{8x}+\frac{1}{4x^3}\right) & \text{for } \frac{1}{a^2}\neq 0 \\ \frac{1}{3x^3} & \text{otherwise} \end{cases}\right)-\frac{1}{2x^4}}{a^2} & \text{for } a^2\neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**3,x)
```

output
```
Piecewise(((-a**2/(2*x**2) - 2*a*Piecewise((-a**2*log(2*sqrt(1 + 1/(a**2*x
**2))*sqrt(a**(-2)) + 2/(a**2*x))/(8*sqrt(a**(-2))) + sqrt(1 + 1/(a**2*x**
2))*(a**2/(8*x) + 1/(4*x**3)), Ne(a**(-2), 0)), (1/(3*x**3), True)) - 1/(2
*x**4))/a**2, Ne(a**2, 0)), (nan, True))
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(59) = 118.

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3}\,dx = \frac{a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-a^3\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)-\frac{2\left(a^6x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}+a^4x\sqrt{\frac{1}{a^2x^2}+1}\right)}{a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2-2a^2x^2\left(\frac{1}{a^2x^2}+1\right)+1}}{8a} - \frac{1}{2x^2} - \frac{1}{2a^2x^4}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
1/8*(a^3*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - a^3*log(a*x*sqrt(1/(a^2*x^2)
 + 1) - 1) - 2*(a^6*x^3*(1/(a^2*x^2) + 1)^(3/2) + a^4*x*sqrt(1/(a^2*x^2) +
 1))/(a^4*x^4*(1/(a^2*x^2) + 1)^2 - 2*a^2*x^2*(1/(a^2*x^2) + 1) + 1))/a -
1/2/x^2 - 1/2/(a^2*x^4)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3}\,dx$$
$$= \frac{a^6|a|\log\left(\sqrt{a^2x^2+1}+1\right)\text{sgn}(x) - a^6|a|\log\left(\sqrt{a^2x^2+1}-1\right)\text{sgn}(x) - \frac{2\left(\left(a^2x^2+1\right)^{\frac{3}{2}}a^6|a|\text{sgn}(x)+\sqrt{a^2x^2+1}a^6|a|\text{sgn}}{a^4x^4}}{8\,a^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
1/8*(a^6*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - a^6*abs(a)*log(sqrt(a^
2*x^2 + 1) - 1)*sgn(x) - 2*((a^2*x^2 + 1)^(3/2)*a^6*abs(a)*sgn(x) + sqrt(a
^2*x^2 + 1)*a^6*abs(a)*sgn(x) + 2*(a^2*x^2 + 1)*a^7)/(a^4*x^4))/a^5
```

3.56.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^3}\,dx = \frac{a\,\text{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{4\sqrt{\frac{1}{a^2}}} - \frac{1}{2\,a^2\,x^4} - \frac{a\,\sqrt{\frac{1}{a^2\,x^2}+1}}{4\,x} - \frac{1}{2\,x^2} - \frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{2\,a\,x^3}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^3,x)
```

output
```
(a*asinh((1/a^2)^(1/2)/x))/(4*(1/a^2)^(1/2)) - 1/(2*a^2*x^4) - (a*(1/(a^2*
x^2) + 1)^(1/2))/(4*x) - 1/(2*x^2) - (1/(a^2*x^2) + 1)^(1/2)/(2*a*x^3)
```

3.57 $\int \frac{e^{2\mathrm{csch}^{-1}(ax)}}{x^4}\,dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{e^{2\mathrm{csch}^{-1}(ax)}}{x^4}\,dx = \frac{2}{3}a^3\left(1+\frac{1}{a^2x^2}\right)^{3/2} - \frac{2}{5}a^3\left(1+\frac{1}{a^2x^2}\right)^{5/2} - \frac{2}{5a^2x^5} - \frac{1}{3x^3}$$

output
```
2/3*a^3*(1+1/a^2/x^2)^(3/2)-2/5*a^3*(1+1/a^2/x^2)^(5/2)-2/5/a^2/x^5-1/3/x^
3
```

3.57.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\mathrm{csch}^{-1}(ax)}}{x^4}\,dx = -\frac{6+5a^2x^2+2a\sqrt{1+\frac{1}{a^2x^2}}\,x(3+a^2x^2-2a^4x^4)}{15a^2x^5}$$

input
```
Integrate[E^(2*ArcCsch[a*x])/x^4,x]
```

output
```
-1/15*(6 + 5*a^2*x^2 + 2*a*Sqrt[1 + 1/(a^2*x^2)]*x*(3 + a^2*x^2 - 2*a^4*x^
4))/(a^2*x^5)
```

3.57.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4} \, dx$$

$$\Big\downarrow \text{6892}$$

$$\int \frac{\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2}{x^4}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2}{a^2x^6}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{ax^5}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5a^2x^5}-\frac{2}{5}a^3\left(\frac{1}{a^2x^2}+1\right)^{5/2}+\frac{2}{3}a^3\left(\frac{1}{a^2x^2}+1\right)^{3/2}-\frac{1}{3x^3}$$

input
```
Int[E^(2*ArcCsch[a*x])/x^4,x]
```

output
```
(2*a^3*(1 + 1/(a^2*x^2))^(3/2))/3 - (2*a^3*(1 + 1/(a^2*x^2))^(5/2))/5 - 2/
(5*a^2*x^5) - 1/(3*x^3)
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.57.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

method	result	size
trager	$\frac{-\frac{5a^2x^2+6}{15x^5}+\frac{2a\left(2a^4x^4-a^2x^2-3\right)\sqrt{-\frac{-a^2x^2-1}{a^2x^2}}}{15x^4}}{a^2}$	65
default	$\frac{-\frac{a^2}{3x^3}-\frac{1}{5x^5}}{a^2}+\frac{2\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,(a^2x^2+1)\,(2a^2x^2-3)}{15a\,x^4}-\frac{1}{5a^2x^5}$	73

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/15*(5*a^2*x^2+6)/x^5+2/15/x^4*a*(2*a^4*x^4-a^2*x^2-3)*(-(-a^2*x^
2-1)/a^2/x^2)^(1/2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^4}\,dx = \frac{4\,a^5x^5-5\,a^2x^2+2\,(2\,a^5x^5-a^3x^3-3\,ax)\sqrt{\frac{a^2x^2+1}{a^2x^2}}-6}{15\,a^2x^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="fricas")
```

output
```
1/15*(4*a^5*x^5 - 5*a^2*x^2 + 2*(2*a^5*x^5 - a^3*x^3 - 3*a*x)*sqrt((a^2*x^
2 + 1)/(a^2*x^2)) - 6)/(a^2*x^5)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx = \begin{cases} \frac{-\frac{a^2}{3x^3} - 2a\left(\begin{cases} \sqrt{1+\frac{1}{a^2x^2}}\left(-\frac{2a^4}{15} + \frac{a^2}{15x^2} + \frac{1}{5x^4}\right) & \text{for } \frac{1}{a^2} \neq 0 \\ \frac{1}{4x^4} & \text{otherwise} \end{cases}\right) - \frac{2}{5x^5}}{a^2} & \text{for } a^2 \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**4,x)
```

output
```
Piecewise(((-a**2/(3*x**3) - 2*a*Piecewise((sqrt(1 + 1/(a**2*x**2))*(-2*a*
*4/15 + a**2/(15*x**2) + 1/(5*x**4)), Ne(a**(-2), 0)), (1/(4*x**4), True))
 - 2/(5*x**5))/a**2, Ne(a**2, 0)), (nan, True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx = -\frac{2\left(3\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}} - 5\,a^4\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}\right)}{15\,a} - \frac{1}{3\,x^3} - \frac{2}{5\,a^2x^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="maxima")
```

output
```
-2/15*(3*a^4*(1/(a^2*x^2) + 1)^(5/2) - 5*a^4*(1/(a^2*x^2) + 1)^(3/2))/a -
1/3/x^3 - 2/5/(a^2*x^5)
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx = \frac{8\left(15\left(x|a|-\sqrt{a^2x^2+1}\right)^6a^3\text{sgn}(x)+5\left(x|a|-\sqrt{a^2x^2+1}\right)^4a^3\text{sgn}(x)+5\left(x|a|-\sqrt{a^2x^2+1}\right)^2a^3\text{sgn}(x)-\right.}{15\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^5} - \frac{5\,a^2x^2+6}{15\,a^2x^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="giac")
```

output
```
8/15*(15*(x*abs(a) - sqrt(a^2*x^2 + 1))^6*a^3*sgn(x) + 5*(x*abs(a) - sqrt(
a^2*x^2 + 1))^4*a^3*sgn(x) + 5*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^3*sgn(x)
 - a^3*sgn(x))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^5 - 1/15*(5*a^2*x^2
+ 6)/(a^2*x^5)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^4}\,dx = \frac{4\,a^3\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15} - \frac{\frac{2\,a\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{15}+\frac{1}{3}}{x^3} - \frac{\frac{2}{5\,a^2}+\frac{2\,x\,\sqrt{\frac{1}{a^2\,x^2}+1}}{5\,a}}{x^5}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^4,x)
```

output
```
(4*a^3*(1/(a^2*x^2) + 1)^(1/2))/15 - ((2*a*x*(1/(a^2*x^2) + 1)^(1/2))/15 +
 1/3)/x^3 - (2/(5*a^2) + (2*x*(1/(a^2*x^2) + 1)^(1/2))/(5*a))/x^5
```

3.58 $\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx$

3.58.1 Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx=-\frac{1}{3a^2x^6}-\frac{\sqrt{1+\frac{1}{a^2x^2}}}{3ax^5}-\frac{1}{4x^4}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{12x^3}+\frac{a^3\sqrt{1+\frac{1}{a^2x^2}}}{8x}-\frac{1}{8}a^4\text{csch}^{-1}(ax)$$

output
```
-1/3/a^2/x^6-1/4/x^4-1/8*a^4*arccsch(a*x)-1/3*(1+1/a^2/x^2)^(1/2)/a/x^5-1/
12*a*(1+1/a^2/x^2)^(1/2)/x^3+1/8*a^3*(1+1/a^2/x^2)^(1/2)/x
```

3.58.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx=\frac{\frac{(4+3a^2x^2)\left(-2-2a\sqrt{1+\frac{1}{a^2x^2}}x+a^3\sqrt{1+\frac{1}{a^2x^2}}x^3\right)}{x^6}-3a^6\text{arcsinh}\left(\frac{1}{ax}\right)}{24a^2}$$

input
```
Integrate[E^(2*ArcCsch[a*x])/x^5,x]
```

output
```
(((4 + 3*a^2*x^2)*(-2 - 2*a*Sqrt[1 + 1/(a^2*x^2)]*x + a^3*Sqrt[1 + 1/(a^2*
x^2)]*x^3))/x^6 - 3*a^6*ArcSinh[1/(a*x)])/(24*a^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6892, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5} dx$$

$$\Big\downarrow \text{6892}$$

$$\int \frac{\left(\sqrt{\frac{1}{a^2x^2}+1}+\frac{1}{ax}\right)^2}{x^5}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2}{a^2x^7}+\frac{2\sqrt{\frac{1}{a^2x^2}+1}}{ax^6}+\frac{1}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}a^4\text{csch}^{-1}(ax)-\frac{1}{3a^2x^6}-\frac{\sqrt{\frac{1}{a^2x^2}+1}}{3ax^5}-\frac{a\sqrt{\frac{1}{a^2x^2}+1}}{12x^3}+\frac{a^3\sqrt{\frac{1}{a^2x^2}+1}}{8x}-\frac{1}{4x^4}$$

input
```
Int[E^(2*ArcCsch[a*x])/x^5,x]
```

output
```
-1/3*1/(a^2*x^6) - Sqrt[1 + 1/(a^2*x^2)]/(3*a*x^5) - 1/(4*x^4) - (a*Sqrt[1
 + 1/(a^2*x^2)])/(12*x^3) + (a^3*Sqrt[1 + 1/(a^2*x^2)])/(8*x) - (a^4*ArcCs
ch[a*x])/8
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6892
```
Int[E^(ArcCsch[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[1 +
1/u^2])^n, x] /; FreeQ[m, x] && IntegerQ[n]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(78) = 156$.

Time = 0.06 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.31

method	result
default	$\frac{-\frac{a^2}{4x^4}-\frac{1}{6x^6}}{a^2} - \frac{a\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(3\sqrt{\frac{1}{a^2}}\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}a^4x^4-3\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}a^4x^6+3\ln\left(\frac{2\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}a^2+2}{a^2x}\right)a^2x^6-6\left(\frac{a^2x^2+1}{a^2}\right)^{\frac{3}{2}}\cdots\right.}{24x^5\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}}$

input
```
int((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4*a^2/x^4-1/6/x^6)-1/24*a*((a^2*x^2+1)/a^2/x^2)^(1/2)/x^5*(3*(1/
a^2)^(1/2)*((a^2*x^2+1)/a^2)^(3/2)*a^4*x^4-3*(1/a^2)^(1/2)*((a^2*x^2+1)/a^
2)^(1/2)*a^4*x^6+3*ln(2*((1/a^2)^(1/2)*((a^2*x^2+1)/a^2)^(1/2)*a^2+1)/x/a^
2)*a^2*x^6-6*((a^2*x^2+1)/a^2)^(3/2)*(1/a^2)^(1/2)*a^2*x^2+8*((a^2*x^2+1)/
a^2)^(3/2)*(1/a^2)^(1/2))/((a^2*x^2+1)/a^2)^(1/2)/(1/a^2)^(1/2)-1/6/a^2/x^
6
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

$$\int \frac{e^{2\text{csch}^{-1}(ax)}}{x^5}\,dx = -\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{a^2x^2+1}{a^2x^2}}-ax-1\right)+6\,a^2x^2-(3\,a^5x^5-2\,a^3x^3-\cdots}{24\,a^2x^6}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="fricas")
```

output
```
-1/24*(3*a^6*x^6*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x + 1) - 3*a^6*
x^6*log(a*x*sqrt((a^2*x^2 + 1)/(a^2*x^2)) - a*x - 1) + 6*a^2*x^2 - (3*a^5*
x^5 - 2*a^3*x^3 - 8*a*x)*sqrt((a^2*x^2 + 1)/(a^2*x^2)) + 8)/(a^2*x^6)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^5}\,dx$$
$$= \begin{cases} \frac{-\frac{a^2}{4x^4}-2a\left(\begin{cases} \frac{a^4\log\left(2\sqrt{1+\frac{1}{a^2x^2}}\sqrt{\frac{1}{a^2}}+\frac{2}{a^2x}\right)}{16\sqrt{\frac{1}{a^2}}}+\sqrt{1+\frac{1}{a^2x^2}}\left(-\frac{a^4}{16x}+\frac{a^2}{24x^3}+\frac{1}{6x^5}\right) & \text{for } \frac{1}{a^2}\neq 0 \\ \frac{1}{5x^5} & \text{otherwise} \end{cases}\right)-\frac{1}{3x^6}}{a^2} & \text{for } a^2\neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate((1/a/x+(1+1/a**2/x**2)**(1/2))**2/x**5,x)
```

output
```
Piecewise(((-a**2/(4*x**4) - 2*a*Piecewise((a**4*log(2*sqrt(1 + 1/(a**2*x*
*2))*sqrt(a**(-2)) + 2/(a**2*x))/(16*sqrt(a**(-2))) + sqrt(1 + 1/(a**2*x**
2))*(-a**4/(16*x) + a**2/(24*x**3) + 1/(6*x**5)), Ne(a**(-2), 0)), (1/(5*x
**5), True)) - 1/(3*x**6))/a**2, Ne(a**2, 0)), (nan, True))
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(78) = 156.

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^5}\,dx =$$
$$-\frac{3\,a^5\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}+1\right)-3\,a^5\log\left(ax\sqrt{\frac{1}{a^2x^2}+1}-1\right)-\frac{2\left(3\,a^{10}x^5\left(\frac{1}{a^2x^2}+1\right)^{\frac{5}{2}}-8\,a^8x^3\left(\frac{1}{a^2x^2}+1\right)^{\frac{3}{2}}-3\,a^6x\sqrt{\frac{1}{a^2 \cdots}}\cdots\right.}{a^6x^6\left(\frac{1}{a^2x^2}+1\right)^3-3\,a^4x^4\left(\frac{1}{a^2x^2}+1\right)^2+3\,a^2x^2\left(\frac{1}{a^2x^2}+\cdots\right.}}{48\,a}$$
$$-\frac{1}{4\,x^4}-\frac{1}{3\,a^2x^6}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="maxima")
```

output
```
-1/48*(3*a^5*log(a*x*sqrt(1/(a^2*x^2) + 1) + 1) - 3*a^5*log(a*x*sqrt(1/(a^
2*x^2) + 1) - 1) - 2*(3*a^10*x^5*(1/(a^2*x^2) + 1)^(5/2) - 8*a^8*x^3*(1/(a
^2*x^2) + 1)^(3/2) - 3*a^6*x*sqrt(1/(a^2*x^2) + 1))/(a^6*x^6*(1/(a^2*x^2)
+ 1)^3 - 3*a^4*x^4*(1/(a^2*x^2) + 1)^2 + 3*a^2*x^2*(1/(a^2*x^2) + 1) - 1))
/a - 1/4/x^4 - 1/3/(a^2*x^6)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.46

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^5}\,dx =$$

$$-\frac{3\,a^8|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x) - 3\,a^8|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x) - \frac{2\left(3\,(a^2x^2+1)^{\frac{5}{2}}a^8|a|\operatorname{sgn}(x) - 8\,(a^2x^2+\cdots\right.}{\cdots}}{48\,a^5}$$

input
```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(3*a^8*abs(a)*log(sqrt(a^2*x^2 + 1) + 1)*sgn(x) - 3*a^8*abs(a)*log(s
qrt(a^2*x^2 + 1) - 1)*sgn(x) - 2*(3*(a^2*x^2 + 1)^(5/2)*a^8*abs(a)*sgn(x)
- 8*(a^2*x^2 + 1)^(3/2)*a^8*abs(a)*sgn(x) - 3*sqrt(a^2*x^2 + 1)*a^8*abs(a)
*sgn(x) - 6*(a^2*x^2 + 1)*a^9 - 2*a^9)/(a^6*x^6))/a^5
```

3.58.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^5}\,dx = \frac{a^3\sqrt{\frac{1}{a^2\,x^2}+1}}{8\,x} - \frac{1}{3\,a^2\,x^6} - \frac{a\sqrt{\frac{1}{a^2\,x^2}+1}}{12\,x^3} - \frac{1}{4\,x^4} - \frac{\sqrt{\frac{1}{a^2\,x^2}+1}}{3\,a\,x^5} - \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{a^2}}}{x}\right)}{8\sqrt{\frac{1}{a^2}}}$$

input
```
int(((1/(a^2*x^2) + 1)^(1/2) + 1/(a*x))^2/x^5,x)
```

output
```
(a^3*(1/(a^2*x^2) + 1)^(1/2))/(8*x) - 1/(3*a^2*x^6) - (a*(1/(a^2*x^2) + 1)
^(1/2))/(12*x^3) - 1/(4*x^4) - (1/(a^2*x^2) + 1)^(1/2)/(3*a*x^5) - (a^3*as
inh((1/a^2)^(1/2)/x))/(8*(1/a^2)^(1/2))
```

3.59 $\int \frac{e^{\text{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx$

3.59.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{e^{\text{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = -\frac{d(dx)^{-1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},-\frac{1}{c^2x^2}\right)}{c^2(1-m)} + \frac{(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},-c^2x^2\right)}{cm}$$

output
```
-d*(d*x)^(-1+m)*hypergeom([1/2, -1/2*m+1/2],[3/2-1/2*m],-1/c^2/x^2)/c^2/(1
-m)+(d*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],-c^2*x^2)/c/m
```

3.59.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int \frac{e^{\text{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \frac{(dx)^m\left(\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},1+\frac{m}{2},-c^2x^2\right)}{\sqrt{1+c^2x^2}}+\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},1+\frac{m}{2},-c^2x^2\right)}{c}\right)}{m}$$

input
```
Integrate[(E^ArcCsch[c*x]*(d*x)^m)/(1 + c^2*x^2),x]
```

output
```
((d*x)^m*((Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, m/2, 1 + m/2, -(
c^2*x^2)])/Sqrt[1 + c^2*x^2] + Hypergeometric2F1[1, m/2, 1 + m/2, -(c^2*x^
2)]/c))/m
```

3.59.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6896, 278, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(cx)}(dx)^m}{c^2x^2+1} \, dx$$

$$\Big\downarrow \text{ 6896}$$

$$\frac{d^2\int \frac{(dx)^{m-2}}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{d\int \frac{(dx)^{m-1}}{c^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{d^2\int \frac{(dx)^{m-2}}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},-c^2x^2\right)}{cm}$$

$$\Big\downarrow \text{ 862}$$

$$\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},-c^2x^2\right)}{cm}-\frac{d\left(\frac{1}{x}\right)^{m-1}(dx)^{m-1}\int \frac{\left(\frac{1}{x}\right)^{-m}}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},-c^2x^2\right)}{cm}-\frac{d(dx)^{m-1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},-\frac{1}{c^2x^2}\right)}{c^2(1-m)}$$

input
```
Int[(E^ArcCsch[c*x]*(d*x)^m)/(1 + c^2*x^2),x]
```

output
```
-((d*(d*x)^(-1 + m)*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, -(1/(c^2*
x^2))])/(c^2*(1 - m))) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -(c
^2*x^2)])/(c*m)
```

3.59.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.59.4 Maple [F]

$$\int \frac{\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^{2}x^{2}}}\right)(dx)^{m}}{c^{2}x^{2}+1}dx$$

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x)
```

output
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x)
```

3.59.5 Fricas [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \int \frac{(dx)^m\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{c^2x^2+1}\,dx$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(((d*x)^m*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + (d*x)^m)/(c^3*x^3 +
c*x), x)
```

3.59.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \frac{\int \frac{(dx)^m}{c^2x^3+x}\,dx + \int \frac{cx(dx)^m\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^3+x}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*(d*x)**m/(c**2*x**2+1),x)
```

output
```
(Integral((d*x)**m/(c**2*x**3 + x), x) + Integral(c*x*(d*x)**m*sqrt(1 + 1/
(c**2*x**2))/(c**2*x**3 + x), x))/c
```

3.59.7 Maxima [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \int \frac{(dx)^m\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{c^2x^2+1}\,dx$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="ma
xima")
```

output
```
integrate((d*x)^m*(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(c^2*x^2 + 1), x)
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*(d*x)^m/(c^2*x^2+1),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}(dx)^m}{1+c^2x^2}\,dx = \int \frac{\left(\sqrt{\frac{1}{c^2\,x^2}+1}+\frac{1}{c\,x}\right)\,{\left(d\,x\right)}^m}{c^2\,x^2+1}\,dx$$

input
```
int((((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2 + 1),x)
```

output
```
int((((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2 + 1), x)
```

3.60 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = -\frac{x}{c^5} - \frac{3\sqrt{1+\frac{1}{c^2x^2}}x^2}{8c^4} + \frac{x^3}{3c^3} + \frac{\sqrt{1+\frac{1}{c^2x^2}}x^4}{4c^2} + \frac{\arctan(cx)}{c^6} + \frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{8c^6}$$

output
```
-x/c^5+1/3*x^3/c^3+arctan(c*x)/c^6+3/8*arctanh((1+1/c^2/x^2)^(1/2))/c^6-3/
8*x^2*(1+1/c^2/x^2)^(1/2)/c^4+1/4*x^4*(1+1/c^2/x^2)^(1/2)/c^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{cx\left(-24-9c\sqrt{1+\frac{1}{c^2x^2}}x+8c^2x^2+6c^3\sqrt{1+\frac{1}{c^2x^2}}x^3\right)+24\arctan(cx)+9\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{24c^6}$$

input
```
Integrate[(E^ArcCsch[c*x]*x^5)/(1 + c^2*x^2),x]
```

output
```
(c*x*(-24 - 9*c*Sqrt[1 + 1/(c^2*x^2)]*x + 8*c^2*x^2 + 6*c^3*Sqrt[1 + 1/(c^
2*x^2)]*x^3) + 24*ArcTan[c*x] + 9*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(24*
c^6)
```

3.60.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6896, 254, 798, 52, 52, 73, 221, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 e^{\text{csch}^{-1}(cx)}}{c^2x^2+1}dx$$

$$\Big\downarrow \text{6896}$$

$$\frac{\int \frac{x^4}{c^2x^2+1}dx}{c}+\frac{\int \frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}-\frac{\int \frac{x^6}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}-\frac{-\frac{3\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}-\frac{-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}-\frac{-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int \frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c}-\frac{-\frac{3\left(\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}}{c}-\frac{-\frac{3\left(\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

input
```
Int[(E^ArcCsch[c*x]*x^5)/(1 + c^2*x^2),x]
```

output
```
(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c^5)/c - (-1/2*(Sqrt[1 + 1/(c^2*x^2)
]*x^4) - (3*(-(Sqrt[1 + 1/(c^2*x^2)]*x^2) + ArcTanh[Sqrt[1 + 1/(c^2*x^2)]]
/c^2))/(4*c^2))/(2*c^2)
```

3.60.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(78) = 156$.

Time = 0.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\, x\left(2x\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}} c^4-5x\sqrt{\frac{c^2x^2+1}{c^2}}\, c^2-5\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)+8\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)\right)}{8\sqrt{\frac{c^2x^2+1}{c^2}}\, c^6}+\frac{\frac{\frac{1}{3}c^2x^3-x}{c^4}+}{c}$

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(2*x*(1/c^2*(c^2*x^2+1))^(3/2)*c^4-5*x*(
1/c^2*(c^2*x^2+1))^(1/2)*c^2-5*ln(x+(1/c^2*(c^2*x^2+1))^(1/2))+8*ln(x+(-(-
c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)))/(1/c^2*(c^2*x^2+1))^
(1/2)/c^6+1/c*(1/c^4*(1/3*c^2*x^3-x)+1/c^5*arctan(c*x))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{8\,c^3x^3 - 24\,cx + 3\,(2\,c^4x^4 - 3\,c^2x^2)\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 24\,\arctan(cx) - 9\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)}{24\,c^6}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/24*(8*c^3*x^3 - 24*c*x + 3*(2*c^4*x^4 - 3*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 24*arctan(c*x) - 9*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x)
)/c^6
```

3.60.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{\int \frac{x^4}{c^2x^2+1}\,dx + \int \frac{cx^5\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**5/(c**2*x**2+1),x)
```

output
```
(Integral(x**4/(c**2*x**2 + 1), x) + Integral(c*x**5*sqrt(1 + 1/(c**2*x**2
))/(c**2*x**2 + 1), x))/c
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(78) = 156.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{c^2x^3-3\,x}{3\,c^5} - \frac{\frac{2\left(\frac{5\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-\frac{3\left(\frac{c^2x^2+1}{x^2}\right)^{\frac{3}{2}}}{c^3}\right)}{\frac{2\left(c^2x^2+1\right)}{c^2x^2}-\frac{\left(c^2x^2+1\right)^2}{c^4x^4}-1} - 3\,\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}+1\right) + 3\,\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-1\right)}{16\,c^6} + \frac{\arctan(cx)}{c^6}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
1/3*(c^2*x^3 - 3*x)/c^5 - 1/16*(2*(5*sqrt((c^2*x^2 + 1)/x^2)/c - 3*((c^2*x
^2 + 1)/x^2)^(3/2)/c^3)/(2*(c^2*x^2 + 1)/(c^2*x^2) - (c^2*x^2 + 1)^2/(c^4*
x^4) - 1) - 3*log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) + 3*log(sqrt((c^2*x^2 + 1
)/x^2)/c - 1))/c^6 + arctan(c*x)/c^6
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{1}{8}\sqrt{c^2x^2+1}x\left(\frac{2\,x^2|c|\mathrm{sgn}(x)}{c^4}-\frac{3\,|c|\mathrm{sgn}(x)}{c^6}\right) - \frac{3\,\log\left(-x|c|+\sqrt{c^2x^2+1}\right)\mathrm{sgn}(x)}{8\,c^6} + \frac{\arctan(cx)}{c^6} + \frac{c^6x^3-3\,c^4x}{3\,c^9}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^5/(c^2*x^2+1),x, algorithm="giac")
```

output
```
1/8*sqrt(c^2*x^2 + 1)*x*(2*x^2*abs(c)*sgn(x)/c^4 - 3*abs(c)*sgn(x)/c^6) -
3/8*log(-x*abs(c) + sqrt(c^2*x^2 + 1))*sgn(x)/c^6 + arctan(c*x)/c^6 + 1/3*
(c^6*x^3 - 3*c^4*x)/c^9
```

3.60.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{e^{\text{csch}^{-1}(cx)}x^5}{1+c^2x^2}\,dx = \frac{3\,\text{atanh}\left(\sqrt{\frac{1}{c^2\,x^2}+1}\right)}{8\,c^6}+\frac{3\,\text{atan}(c\,x)-3\,c\,x+c^3\,x^3}{3\,c^6}+\frac{x^4\,\sqrt{\frac{1}{c^2\,x^2}+1}}{4\,c^2}-\frac{3\,x^2\,\sqrt{\frac{1}{c^2\,x^2}+1}}{8\,c^4}$$

input
```
int((x^5*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

output
```
(3*atanh((1/(c^2*x^2) + 1)^(1/2)))/(8*c^6) + (3*atan(c*x) - 3*c*x + c^3*x^
3)/(3*c^6) + (x^4*(1/(c^2*x^2) + 1)^(1/2))/(4*c^2) - (3*x^2*(1/(c^2*x^2) +
 1)^(1/2))/(8*c^4)
```

3.61 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx=-\frac{2\sqrt{1+\frac{1}{c^2x^2}}\,x}{3c^4}+\frac{x^2}{2c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{3c^2}-\frac{\log\left(1+c^2x^2\right)}{2c^5}$$

output
```
1/2*x^2/c^3-1/2*ln(c^2*x^2+1)/c^5-2/3*x*(1+1/c^2/x^2)^(1/2)/c^4+1/3*x^3*(1
+1/c^2/x^2)^(1/2)/c^2
```

3.61.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx=\frac{cx\left(-4\sqrt{1+\frac{1}{c^2x^2}}+3cx+2c^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2\right)-3\log\left(1+c^2x^2\right)}{6c^5}$$

input
```
Integrate[(E^ArcCsch[c*x]*x^4)/(1 + c^2*x^2),x]
```

output
```
(c*x*(-4*Sqrt[1 + 1/(c^2*x^2)] + 3*c*x + 2*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2)
- 3*Log[1 + c^2*x^2])/(6*c^5)
```

3.61.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6896, 243, 49, 803, 746, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 e^{\operatorname{csch}^{-1}(cx)}}{c^2x^2+1}dx$$

$$\Big\downarrow \text{ 6896}$$

$$\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \frac{x^3}{c^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \frac{x^2}{c^2x^2+1}dx^2}{2c}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2c}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2c}+\frac{\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3c^2}}{c^2}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{2c}+\frac{\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}}{c^2}+\frac{\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}}{2c}$$

input
```
Int[(E^ArcCsch[c*x]*x^4)/(1 + c^2*x^2),x]
```

output
```
((-2*Sqrt[1 + 1/(c^2*x^2)]*x)/(3*c^2) + (Sqrt[1 + 1/(c^2*x^2)]*x^3)/3)/c^2
 + (x^2/c^2 - Log[1 + c^2*x^2]/c^4)/(2*c)
```

3.61.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.61. $\int \frac{e^{\text{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx$

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(60) = 120.

Time = 0.86 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.74

method	result	size
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^2-3\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)}{3c^4\sqrt{\frac{c^2x^2+1}{c^2}}}+\frac{\frac{x^2}{2c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c^4}}{c}$	125

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*((c^2*x^2+1)/c^2/x^2)^(1/2)*x/c^4*((1/c^2*(c^2*x^2+1))^(3/2)*c^2-3*(-(
-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2))/(1/c^2*(c^2*x^2+1))^
(1/2)+1/c*(1/2*x^2/c^2-1/2/c^4*ln(c^2*x^2+1))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx=\frac{3\,c^2x^2+2\,(c^3x^3-2\,cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-3\,\log\left(c^2x^2+1\right)}{6\,c^5}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/6*(3*c^2*x^2 + 2*(c^3*x^3 - 2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 3*log
(c^2*x^2 + 1))/c^5
```

3.61.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx=\frac{\int \frac{x^3}{c^2x^2+1}\,dx+\int \frac{cx^4\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x***4/(c**2*x**2+1),x)
```

output
```
(Integral(x**3/(c**2*x**2 + 1), x) + Integral(c*x**4*sqrt(1 + 1/(c**2*x**2
))/(c**2*x**2 + 1), x))/c
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx = \frac{x^2}{2\,c^3} + \frac{\sqrt{c^2x^2+1}(c^2x^2-2)}{3\,c^5} - \frac{\log\left(c^2x^2+1\right)}{2\,c^5}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
1/2*x^2/c^3 + 1/3*sqrt(c^2*x^2 + 1)*(c^2*x^2 - 2)/c^5 - 1/2*log(c^2*x^2 +
1)/c^5
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx = -\frac{\log\left(c^2x^2+1\right)}{2\,c^5} + \frac{2\,|c|\mathrm{sgn}(x)}{3\,c^6} + \frac{2\,(c^2x^2+1)^{\frac{3}{2}}c^{12}|c|\mathrm{sgn}(x) - 6\,\sqrt{c^2x^2+1}c^{12}|c|\mathrm{sgn}(x) + 3\,(c^2x^2+1)c^{13}}{6\,c^{18}}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^4/(c^2*x^2+1),x, algorithm="giac")
```

output
```
-1/2*log(c^2*x^2 + 1)/c^5 + 2/3*abs(c)*sgn(x)/c^6 + 1/6*(2*(c^2*x^2 + 1)^(
3/2)*c^12*abs(c)*sgn(x) - 6*sqrt(c^2*x^2 + 1)*c^12*abs(c)*sgn(x) + 3*(c^2*
x^2 + 1)*c^13)/c^18
```

3.61.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{e^{\text{csch}^{-1}(cx)}x^4}{1+c^2x^2}\,dx = \frac{x^3\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3\,c^2} - \frac{2\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{3\,c^4} - \frac{\ln\left(c^2\,x^2+1\right)-c^2\,x^2}{2\,c^5}$$

input
```
int((x^4*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

output
```
(x^3*(1/(c^2*x^2) + 1)^(1/2))/(3*c^2) - (2*x*(1/(c^2*x^2) + 1)^(1/2))/(3*c
^4) - (log(c^2*x^2 + 1) - c^2*x^2)/(2*c^5)
```

3.62 $\int \frac{e^{\mathbf{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = \frac{x}{c^3}+\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c^2}-\frac{\arctan(cx)}{c^4}-\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^4}$$

output
```
x/c^3-arctan(c*x)/c^4-1/2*arctanh((1+1/c^2/x^2)^(1/2))/c^4+1/2*x^2*(1+1/c^
2/x^2)^(1/2)/c^2
```

3.62.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = -\frac{-cx\left(2+c\sqrt{1+\frac{1}{c^2x^2}}\,x\right)+2\arctan(cx)+\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{2c^4}$$

input
```
Integrate[(E^ArcCsch[c*x]*x^3)/(1 + c^2*x^2),x]
```

output
```
-1/2*(-(c*x*(2 + c*Sqrt[1 + 1/(c^2*x^2)]*x)) + 2*ArcTan[c*x] + Log[(1 + Sq
rt[1 + 1/(c^2*x^2)])*x])/c^4
```

3.62.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6896, 262, 216, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{\operatorname{csch}^{-1}(cx)}}{c^2x^2+1}dx$$

$$\Big\downarrow \text{6896}$$

$$\frac{\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \frac{x^2}{c^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{262}$$

$$\frac{\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}-\frac{\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}-\frac{x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}}{2c^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}-\frac{x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int \frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}}{2c^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}-\frac{\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

input
```
Int[(E^ArcCsch[c*x]*x^3)/(1 + c^2*x^2),x]
```

output
```
(x/c^2 - ArcTan[c*x]/c^3)/c - (-(Sqrt[1 + 1/(c^2*x^2)]*x^2) + ArcTanh[Sqrt
[1 + 1/(c^2*x^2)]]/c^2)/(2*c^2)
```

3.62.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(51) = 102$.

Time = 0.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

method	result	size
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\, x \left(x\sqrt{\frac{c^2x^2+1}{c^2}}\, c^2+\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)-2\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)\right)}{2\sqrt{\frac{c^2x^2+1}{c^2}}\, c^4}+\frac{\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}}{c}$	138

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(x*(1/c^2*(c^2*x^2+1))^(1/2)*c^2+ln(x+(1
/c^2*(c^2*x^2+1))^(1/2))-2*ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2
))/c^4)^(1/2)))/(1/c^2*(c^2*x^2+1))^(1/2)/c^4+1/c*(x/c^2-1/c^3*arctan(c*x)
)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\, dx = \frac{c^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\, cx - 2\, \arctan\left(cx\right) + \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)}{2\, c^4}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/2*(c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*c*x - 2*arctan(c*x) + log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x))/c^4
```

3.62.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = \frac{\int \frac{x^2}{c^2x^2+1}\,dx + \int \frac{cx^3\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**3/(c**2*x**2+1),x)
```

output
```
(Integral(x**2/(c**2*x**2 + 1), x) + Integral(c*x**3*sqrt(1 + 1/(c**2*x**2
))/(c**2*x**2 + 1), x))/c
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.81

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = \frac{x}{c^3} + \frac{\frac{2\sqrt{\frac{c^2x^2+1}{x^2}}}{c\left(\frac{c^2x^2+1}{c^2x^2}-1\right)} - \log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}+1\right) + \log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c}-1\right)}{4\,c^4} - \frac{\arctan(cx)}{c^4}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
x/c^3 + 1/4*(2*sqrt((c^2*x^2 + 1)/x^2)/(c*((c^2*x^2 + 1)/(c^2*x^2) - 1)) -
 log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) + log(sqrt((c^2*x^2 + 1)/x^2)/c - 1))/
c^4 - arctan(c*x)/c^4
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = \frac{\sqrt{c^2x^2+1}x|c|\operatorname{sgn}(x)}{2\,c^4} + \frac{x}{c^3} + \frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)\operatorname{sgn}(x)}{2\,c^4} - \frac{\arctan(cx)}{c^4}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^3/(c^2*x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(c^2*x^2 + 1)*x*abs(c)*sgn(x)/c^4 + x/c^3 + 1/2*log(-x*abs(c) + sq
rt(c^2*x^2 + 1))*sgn(x)/c^4 - arctan(c*x)/c^4
```

3.62.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^3}{1+c^2x^2}\,dx = \frac{x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c^2} - \frac{\operatorname{atan}(c\,x)-c\,x}{c^4} - \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^4}$$

input
```
int((x^3*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

output
```
(x^2*(1/(c^2*x^2) + 1)^(1/2))/(2*c^2) - (atan(c*x) - c*x)/c^4 - atanh((1/(
c^2*x^2) + 1)^(1/2))/(2*c^4)
```

3.63 $\int \frac{e^{\text{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{e^{\text{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{\sqrt{1+\frac{1}{c^2x^2}}\,x}{c^2} + \frac{\log(1+c^2x^2)}{2c^3}$$

output
```
1/2*ln(c^2*x^2+1)/c^3+x*(1+1/c^2/x^2)^(1/2)/c^2
```

3.63.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e^{\text{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{2c\sqrt{1+\frac{1}{c^2x^2}}\,x + \log(1+c^2x^2)}{2c^3}$$

input
```
Integrate[(E^ArcCsch[c*x]*x^2)/(1 + c^2*x^2),x]
```

output
```
(2*c*Sqrt[1 + 1/(c^2*x^2)]*x + Log[1 + c^2*x^2])/(2*c^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6896, 240, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{\text{csch}^{-1}(cx)}}{c^2x^2+1} \, dx$$

$$\Big\downarrow \text{ 6896}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\int \frac{x}{c^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c^2}+\frac{\log\left(c^2x^2+1\right)}{2c^3}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c^2}+\frac{\log\left(c^2x^2+1\right)}{2c^3}$$

input
```
Int[(E^ArcCsch[c*x]*x^2)/(1 + c^2*x^2),x]
```

output
```
(Sqrt[1 + 1/(c^2*x^2)]*x)/c^2 + Log[1 + c^2*x^2]/(2*c^3)
```

3.63.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(32) = 64$.

Time = 0.85 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}}{\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2}+\frac{\ln\left(c^2x^2+1\right)}{2c^3}$	89

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))
/c^4)^(1/2)/(1/c^2*(c^2*x^2+1))^(1/2)/c^2+1/2*ln(c^2*x^2+1)/c^3
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx=\frac{2\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\log\left(c^2x^2+1\right)}{2\,c^3}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/2*(2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + log(c^2*x^2 + 1))/c^3
```

3.63.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{\int \frac{x}{c^2x^2+1}\,dx + \int \frac{cx^2\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x**2/(c**2*x**2+1),x)
```

output
```
(Integral(x/(c**2*x**2 + 1), x) + Integral(c*x**2*sqrt(1 + 1/(c**2*x**2))/
(c**2*x**2 + 1), x))/c
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{\log(c^3x^2+c)}{2\,c^3} + \frac{\sqrt{c^2x^2+1}}{c^3}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
1/2*log(c^3*x^2 + c)/c^3 + sqrt(c^2*x^2 + 1)/c^3
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{\sqrt{c^2x^2+1}|c|\operatorname{sgn}(x)}{c^4} + \frac{\log(c^2x^2+1)}{2\,c^3} - \frac{|c|\operatorname{sgn}(x)}{c^4}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x^2/(c^2*x^2+1),x, algorithm="giac")
```

output
```
sqrt(c^2*x^2 + 1)*abs(c)*sgn(x)/c^4 + 1/2*log(c^2*x^2 + 1)/c^3 - abs(c)*sg
n(x)/c^4
```

3.63.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x^2}{1+c^2x^2}\,dx = \frac{\ln\left(c^2\,x^2+1\right)+2\,c\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c^3}$$

input
```
int((x^2*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

output
```
(log(c^2*x^2 + 1) + 2*c*x*(1/(c^2*x^2) + 1)^(1/2))/(2*c^3)
```

3.64 $\int \frac{e^{\text{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \frac{e^{\text{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx=\frac{\arctan(cx)}{c^2}+\frac{\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^2}$$

output
```
arctan(c*x)/c^2+arctanh((1+1/c^2/x^2)^(1/2))/c^2
```

3.64.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{e^{\text{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx=\frac{\arctan(cx)}{c^2}+\frac{\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{c^2}$$

input
```
Integrate[(E^ArcCsch[c*x]*x)/(1 + c^2*x^2),x]
```

output
```
ArcTan[c*x]/c^2 + Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])]/c^2
```

3.64.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6896, 216, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{\operatorname{csch}^{-1}(cx)}}{c^2x^2+1}dx$$

$$\Big\downarrow \text{6896}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}dx}{c^2}+\frac{\int \frac{1}{c^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x}dx}{c^2}+\frac{\arctan(cx)}{c^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{\arctan(cx)}{c^2}-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\arctan(cx)}{c^2}-\int \frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\arctan(cx)}{c^2}+\frac{\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}$$

input
```
Int[(E^ArcCsch[c*x]*x)/(1 + c^2*x^2),x]
```

output
```
ArcTan[c*x]/c^2 + ArcTanh[Sqrt[1 + 1/(c^2*x^2)]]/c^2
```

3.64.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(25) = 50$.

Time = 0.86 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

method	result	size
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2}+\frac{\arctan(cx)}{c^2}$	85

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
((c^2*x^2+1)/c^2/x^2)^(1/2)*x*ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(
1/2))/c^4)^(1/2))/(1/c^2*(c^2*x^2+1))^(1/2)/c^2+arctan(c*x)/c^2
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = \frac{\arctan(cx) - \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)}{c^2}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="fricas")
```

output
```
(arctan(c*x) - log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x))/c^2
```

3.64.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = \frac{\int \frac{cx\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx + \int \frac{1}{c^2x^2+1}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))*x/(c**2*x**2+1),x)
```

output
```
(Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**2 + 1), x) + Integral(1/(c*
*2*x**2 + 1), x))/c
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = \frac{\log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c} + 1\right) - \log\left(\frac{\sqrt{\frac{c^2x^2+1}{x^2}}}{c} - 1\right)}{2\,c^2} + \frac{\arctan(cx)}{c^2}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*(log(sqrt((c^2*x^2 + 1)/x^2)/c + 1) - log(sqrt((c^2*x^2 + 1)/x^2)/c -
1))/c^2 + arctan(c*x)/c^2
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = -\frac{\log\left(-x|c|+\sqrt{c^2x^2+1}\right)\operatorname{sgn}(x)}{c^2}+\frac{\arctan(cx)}{c^2}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))*x/(c^2*x^2+1),x, algorithm="giac")
```

output
```
-log(-x*abs(c) + sqrt(c^2*x^2 + 1))*sgn(x)/c^2 + arctan(c*x)/c^2
```

3.64.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}x}{1+c^2x^2}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\frac{1}{c^2\,x^2}+1}\right)+\operatorname{atan}(c\,x)}{c^2}$$

input
```
int((x*((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 + 1),x)
```

output
```
(atanh((1/(c^2*x^2) + 1)^(1/2)) + atan(c*x))/c^2
```

3.65 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx$

3.65.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = -\frac{\operatorname{csch}^{-1}(cx)}{c}+\frac{\log(x)}{c}-\frac{\log(1+c^2x^2)}{2c}$$

output
```
-arccsch(c*x)/c+ln(x)/c-1/2*ln(c^2*x^2+1)/c
```

3.65.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{c}+\frac{\log(x)}{c}-\frac{\log(1+c^2x^2)}{2c}$$

input
```
Integrate[E^ArcCsch[c*x]/(1 + c^2*x^2),x]
```

output
```
-(ArcSinh[1/(c*x)]/c) + Log[x]/c - Log[1 + c^2*x^2]/(2*c)
```

3.65.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6894, 243, 47, 14, 16, 858, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{c^2x^2+1}dx$$

$$\big\downarrow \text{ 6894}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c^2}+\frac{\int \frac{1}{x(c^2x^2+1)}dx}{c}$$

$$\big\downarrow \text{ 243}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c^2}+\frac{\int \frac{1}{x^2(c^2x^2+1)}dx^2}{2c}$$

$$\big\downarrow \text{ 47}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c^2}+\frac{\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2}{2c}$$

$$\big\downarrow \text{ 14}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c^2}+\frac{\log(x^2)-c^2\int \frac{1}{c^2x^2+1}dx^2}{2c}$$

$$\big\downarrow \text{ 16}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c^2}+\frac{\log(x^2)-\log(c^2x^2+1)}{2c}$$

$$\big\downarrow \text{ 858}$$

$$\frac{\log(x^2)-\log(c^2x^2+1)}{2c}-\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c^2}$$

$$\big\downarrow \text{ 222}$$

$$\frac{\log(x^2)-\log(c^2x^2+1)}{2c}-\frac{\text{arcsinh}\left(\frac{1}{cx}\right)}{c}$$

input
```
Int[E^ArcCsch[c*x]/(1 + c^2*x^2),x]
```

output
```
-(ArcSinh[1/(c*x)]/c) + (Log[x^2] - Log[1 + c^2*x^2])/(2*c)
```

3.65.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6894
```
Int[E^ArcCsch[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[1/(a*c^2
)   Int[1/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Simp[1/c   Int[1/(x*(a + b*
x^2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b - a*c^2, 0]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(31) = 62$.

Time = 0.88 (sec) , antiderivative size = 170, normalized size of antiderivative = 5.15

method	result	s
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(\sqrt{\frac{1}{c^2}}\,\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2-\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\,c^2\sqrt{\frac{1}{c^2}}-\ln\left(\frac{2\sqrt{\frac{1}{c^2}}\,\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+2}{c^2x}\right)\right)}{\sqrt{\frac{1}{c^2}}\,\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2}+\frac{\ln(x)-\frac{\ln\left(c^2x^2+1\right)}{2}}{c}$	1

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
((c^2*x^2+1)/c^2/x^2)^(1/2)*x*((1/c^2)^(1/2)*(1/c^2*(c^2*x^2+1))^(1/2)*c^2
-(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)*c^2*(1/c^2)^(1/2)
-ln(2*((1/c^2)^(1/2)*(1/c^2*(c^2*x^2+1))^(1/2)*c^2+1)/c^2/x))/(1/c^2)^(1/2
)/(1/c^2*(c^2*x^2+1))^(1/2)/c^2+1/c*(ln(x)-1/2*ln(c^2*x^2+1))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = -\frac{\log\left(c^2x^2+1\right)+2\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-2\,\log\left(x\right)}{2\,c}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(log(c^2*x^2 + 1) + 2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - 2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 2*log(x))/c
```

3.65.6 Sympy [F]

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = \frac{\int \frac{cx\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^3+x}\,dx + \int \frac{1}{c^2x^3+x}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/(c**2*x**2+1),x)
```

output
```
(Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**3 + x), x) + Integral(1/(c*
*2*x**3 + x), x))/c
```

3.65.7 Maxima [F]

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = \int \frac{\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}}{c^2x^2+1}\,dx$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*log(c^2*x^2 + 1)/c + log(x)/c + integrate(sqrt(c^2*x^2 + 1)/(c^3*x^3
+ c*x), x)
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(31) = 62$.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = -\frac{\log(c^2x^2+1)}{2\,c} - \frac{(|c|\text{sgn}(x)-c)\log\left(\sqrt{c^2x^2+1}+1\right)}{2\,c^2} + \frac{(|c|\text{sgn}(x)+c)\log\left(\sqrt{c^2x^2+1}-1\right)}{2\,c^2}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/(c^2*x^2+1),x, algorithm="giac")
```

output
```
-1/2*log(c^2*x^2 + 1)/c - 1/2*(abs(c)*sgn(x) - c)*log(sqrt(c^2*x^2 + 1) +
1)/c^2 + 1/2*(abs(c)*sgn(x) + c)*log(sqrt(c^2*x^2 + 1) - 1)/c^2
```

3.65.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{1+c^2x^2}\,dx = -\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{c^2}}}{x}\right)\sqrt{\frac{1}{c^2}} - \frac{\ln\left(c^2\,x^2+1\right)-2\,\ln\left(x\right)}{2\,c}$$

input
```
int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(c^2*x^2 + 1),x)
```

output
```
- asinh((1/c^2)^(1/2)/x)*(1/c^2)^(1/2) - (log(c^2*x^2 + 1) - 2*log(x))/(2*
c)
```

3.66 $\int \frac{e^{\text{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx = -\sqrt{1+\frac{1}{c^2x^2}}-\frac{1}{cx}-\arctan(cx)$$

output
```
-1/c/x-arctan(c*x)-(1+1/c^2/x^2)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx = -\sqrt{1+\frac{1}{c^2x^2}}-\frac{1}{cx}-\arctan(cx)$$

input
```
Integrate[E^ArcCsch[c*x]/(x*(1 + c^2*x^2)),x]
```

output
```
-Sqrt[1 + 1/(c^2*x^2)] - 1/(c*x) - ArcTan[c*x]
```

3.66.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6896, 264, 216, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x\left(c^2x^2+1\right)}dx$$

$$\Big\downarrow \ 6896$$

$$\frac{\int \frac{1}{x^2(c^2x^2+1)}dx}{c}+\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}dx}{c^2}$$

$$\Big\downarrow \ 264$$

$$\frac{c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}}{c}+\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}dx}{c^2}$$

$$\Big\downarrow \ 216$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}dx}{c^2}+\frac{-c\arctan(cx)-\frac{1}{x}}{c}$$

$$\Big\downarrow \ 793$$

$$\frac{-c\arctan(cx)-\frac{1}{x}}{c}-\sqrt{\frac{1}{c^2x^2}+1}$$

input
```
Int[E^ArcCsch[c*x]/(x*(1 + c^2*x^2)),x]
```

output
```
-Sqrt[1 + 1/(c^2*x^2)] + (-x^(-1) - c*ArcTan[c*x])/c
```

3.66.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(28) = 56$.

Time = 0.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 5.23

method	result
default	$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^2-\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2x^2-\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)x+\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)x\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}}+\frac{-\frac{1}{x}-c\arctan}{c}$

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-((c^2*x^2+1)/c^2/x^2)^(1/2)*((1/c^2*(c^2*x^2+1))^(3/2)*c^2-(1/c^2*(c^2*x^
2+1))^(1/2)*c^2*x^2-ln(x+(1/c^2*(c^2*x^2+1))^(1/2))*x+ln(x+(-(-c^2*x+(-c^2
)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2))*x)/(1/c^2*(c^2*x^2+1))^(1/2)+1/c
*(-1/x-c*arctan(c*x))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx = -\frac{cx\arctan(cx) + cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + cx + 1}{cx}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="fricas")
```

output
```
-(c*x*arctan(c*x) + c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c*x + 1)/(c*x)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx = -c\left(\begin{cases}\mathrm{NaN} & \text{for } c=0\\ \frac{\sqrt{1+\frac{1}{c^2x^2}}}{c} & \text{otherwise}\end{cases}\right) + \frac{c\,\mathrm{atan}\left(\frac{1}{x\sqrt{c^2}}\right)}{\sqrt{c^2}} - \frac{1}{cx}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x/(c**2*x**2+1),x)
```

output
```
-c*Piecewise((nan, Eq(c, 0)), (sqrt(1 + 1/(c**2*x**2))/c, True)) + c*atan(
1/(x*sqrt(c**2)))/sqrt(c**2) - 1/(c*x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x(1+c^2x^2)}\,dx = -\frac{\sqrt{c^2x^2+1}}{cx} - \frac{1}{cx} - \arctan(cx)$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="maxima")
```

output
```
-sqrt(c^2*x^2 + 1)/(c*x) - 1/(c*x) - arctan(c*x)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x\left(1+c^2x^2\right)}\,dx=\frac{2\operatorname{sgn}(x)}{\left(x|c|-\sqrt{c^2x^2+1}\right)^2-1}-\frac{1}{cx}-\arctan\left(cx\right)$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x/(c^2*x^2+1),x, algorithm="giac")
```

output
```
2*sgn(x)/((x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1) - 1/(c*x) - arctan(c*x)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x\left(1+c^2x^2\right)}\,dx=-\operatorname{atan}(c\,x)-\frac{x\,\sqrt{\frac{1}{c^2\,x^2}+1}+\frac{1}{c}}{x}$$

input
```
int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x*(c^2*x^2 + 1)),x)
```

output
```
- atan(c*x) - (x*(1/(c^2*x^2) + 1)^(1/2) + 1/c)/x
```

3.67 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = -\frac{1}{2cx^2}-\frac{\sqrt{1+\frac{1}{c^2x^2}}}{2x}+\frac{1}{2}c\operatorname{csch}^{-1}(cx)-c\log(x)+\frac{1}{2}c\log\left(1+c^2x^2\right)$$

output
```
-1/2/c/x^2+1/2*c*arccsch(c*x)-c*ln(x)+1/2*c*ln(c^2*x^2+1)-1/2*(1+1/c^2/x^2
)^(1/2)/x
```

3.67.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = \frac{1}{2}\left(-\frac{1}{cx^2}-\frac{\sqrt{1+\frac{1}{c^2x^2}}}{x}+c\operatorname{arcsinh}\left(\frac{1}{cx}\right)-2c\log(x)+c\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[E^ArcCsch[c*x]/(x^2*(1 + c^2*x^2)),x]
```

output
```
(-(1/(c*x^2)) - Sqrt[1 + 1/(c^2*x^2)]/x + c*ArcSinh[1/(c*x)] - 2*c*Log[x]
+ c*Log[1 + c^2*x^2])/2
```

3.67.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6896, 243, 54, 858, 262, 222, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{x^2 (c^2x^2+1)} \, dx$$

$$\Big\downarrow \ 6896$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}dx}{c^2}+\frac{\int \frac{1}{x^3(c^2x^2+1)}dx}{c}$$

$$\Big\downarrow \ 243$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}dx}{c^2}+\frac{\int \frac{1}{x^4(c^2x^2+1)}dx^2}{2c}$$

$$\Big\downarrow \ 54$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}dx}{c^2}+\frac{\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{2c}$$

$$\Big\downarrow \ 858$$

$$\frac{\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{2c}-\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \ 262$$

$$\frac{\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{2c}-\frac{\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c^2}$$

$$\Big\downarrow \ 222$$

$$\frac{\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{2c}-\frac{\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^3\text{arcsinh}\left(\frac{1}{cx}\right)}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{c^2(-\log(x^2))+c^2\log(c^2x^2+1)-\frac{1}{x^2}}{2c}-\frac{\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^3\text{arcsinh}\left(\frac{1}{cx}\right)}{c^2}$$

input
```
Int[E^ArcCsch[c*x]/(x^2*(1 + c^2*x^2)),x]
```

output
```
-(((c^2*Sqrt[1 + 1/(c^2*x^2)])/(2*x) - (c^3*ArcSinh[1/(c*x)])/2)/c^2) + (-
x^(-2) - c^2*Log[x^2] + c^2*Log[1 + c^2*x^2])/(2*c)
```

3.67.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(50) = 100$.

Time = 0.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.60

method	result
default	$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(c^2\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c^2}}+\sqrt{\frac{c^2x^2+1}{c^2}}\sqrt{\frac{1}{c^2}}\,c^2x^2-2\sqrt{\frac{1}{c^2}}\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\,c^2x^2-\ln\left(\frac{2\sqrt{\frac{1}{c^2}}\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+2}{c^2x}\right)\right.}{2x\sqrt{\frac{c^2x^2+1}{c^2}}\sqrt{\frac{1}{c^2}}}$

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((c^2*x^2+1)/c^2/x^2)^(1/2)/x*(c^2*(1/c^2*(c^2*x^2+1))^(3/2)*(1/c^2)^
(1/2)+(1/c^2*(c^2*x^2+1))^(1/2)*(1/c^2)^(1/2)*c^2*x^2-2*(1/c^2)^(1/2)*(-(-
c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2)*c^2*x^2-ln(2*((1/c^2)^
(1/2)*(1/c^2*(c^2*x^2+1))^(1/2)*c^2+1)/c^2/x)*x^2)/(1/c^2*(c^2*x^2+1))^(1/
2)/(1/c^2)^(1/2)+1/c*(-1/2/x^2-c^2*ln(x)+1/2*c^2*ln(c^2*x^2+1))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.17

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = \frac{c^2x^2\log(c^2x^2+1)+c^2x^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-c^2x^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-2\,c^2x^2\log(x)-\cdots}{2\,cx^2}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/2*(c^2*x^2*log(c^2*x^2 + 1) + c^2*x^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) - c*x + 1) - c^2*x^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1)
- 2*c^2*x^2*log(x) - c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 1)/(c*x^2)
```

3.67.6 Sympy [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = \frac{\int \frac{cx\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^5+x^3}\,dx + \int \frac{1}{c^2x^5+x^3}\,dx}{c}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x**2/(c**2*x**2+1),x)
```

output
```
(Integral(c*x*sqrt(1 + 1/(c**2*x**2))/(c**2*x**5 + x**3), x) + Integral(1/
(c**2*x**5 + x**3), x))/c
```

3.67.7 Maxima [F]

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = \int \frac{\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}}{(c^2x^2+1)x^2}\,dx$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
1/2*c*log(c^2*x^2 + 1) - c*log(x) - 1/2/(c*x^2) + integrate(sqrt(c^2*x^2 +
 1)/(c^3*x^5 + c*x^3), x)
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx =& \frac{1}{2}c\log\left(c^2x^2+1\right) + \frac{1}{4}\left(|c|\operatorname{sgn}(x)-2c\right)\log\left(\sqrt{c^2x^2+1}+1\right) \\ &- \frac{1}{4}\left(|c|\operatorname{sgn}(x)+2c\right)\log\left(\sqrt{c^2x^2+1}-1\right) \\ &- \frac{\sqrt{c^2x^2+1}|c|\operatorname{sgn}(x)+c}{2\left(\sqrt{c^2x^2+1}+1\right)\left(\sqrt{c^2x^2+1}-1\right)}\end{aligned}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^2/(c^2*x^2+1),x, algorithm="giac")
```

output
```
1/2*c*log(c^2*x^2 + 1) + 1/4*(abs(c)*sgn(x) - 2*c)*log(sqrt(c^2*x^2 + 1) +
 1) - 1/4*(abs(c)*sgn(x) + 2*c)*log(sqrt(c^2*x^2 + 1) - 1) - 1/2*(sqrt(c^2
*x^2 + 1)*abs(c)*sgn(x) + c)/((sqrt(c^2*x^2 + 1) + 1)*(sqrt(c^2*x^2 + 1) -
 1))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^2(1+c^2x^2)}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{\frac{1}{c^2}}}{x}\right)}{2\sqrt{\frac{1}{c^2}}} + \frac{c\,\ln\left(-c^2\,x^2-1\right)}{2} - c\,\ln\left(x\right) - \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,x} - \frac{1}{2\,c\,x^2}$$

input
```
int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x^2*(c^2*x^2 + 1)),x)
```

output
```
asinh((1/c^2)^(1/2)/x)/(2*(1/c^2)^(1/2)) + (c*log(- c^2*x^2 - 1))/2 - c*lo
g(x) - (1/(c^2*x^2) + 1)^(1/2)/(2*x) - 1/(2*c*x^2)
```

3.68 $\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx = c^2\sqrt{1+\frac{1}{c^2x^2}} - \frac{1}{3}c^2\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{3cx^3} + \frac{c}{x} + c^2\arctan(cx)$$

output
```
-1/3*c^2*(1+1/c^2/x^2)^(3/2)-1/3/c/x^3+c/x+c^2*arctan(c*x)+c^2*(1+1/c^2/x^
2)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx = -\frac{1}{3cx^3} + \frac{c}{x} + \frac{\sqrt{1+\frac{1}{c^2x^2}}(-1+2c^2x^2)}{3x^2} + c^2\arctan(cx)$$

input
```
Integrate[E^ArcCsch[c*x]/(x^3*(1 + c^2*x^2)),x]
```

output
```
-1/3*1/(c*x^3) + c/x + (Sqrt[1 + 1/(c^2*x^2)]*(-1 + 2*c^2*x^2))/(3*x^2) +
c^2*ArcTan[c*x]
```

3.68.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6896, 264, 264, 216, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\text{csch}^{-1}(cx)}}{x^3 (c^2x^2+1)}dx$$

$$\Big\downarrow \text{6896}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}dx}{c^2}+\frac{\int \frac{1}{x^4(c^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{264}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}dx}{c^2}+\frac{c^2\left(-\int \frac{1}{x^2(c^2x^2+1)}dx\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{264}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}dx}{c^2}+\frac{-\left(c^2\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}dx}{c^2}+\frac{-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}}{c}$$

$$\Big\downarrow \text{798}$$

$$\frac{-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}}{c}-\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x^2}}{2c^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}}{c}-\frac{\int \left(c^2\sqrt{1+\frac{1}{c^2x^2}}-\frac{c^2}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}}{c}-\frac{\frac{2}{3}c^4\left(\frac{1}{c^2x^2}+1\right)^{3/2}-2c^4\sqrt{\frac{1}{c^2x^2}+1}}{2c^2}$$

input
```
Int[E^ArcCsch[c*x]/(x^3*(1 + c^2*x^2)),x]
```

output
```
-1/2*(-2*c^4*Sqrt[1 + 1/(c^2*x^2)] + (2*c^4*(1 + 1/(c^2*x^2))^(3/2))/3)/c^
2 + (-1/3*1/x^3 - c^2*(-x^(-1) - c*ArcTan[c*x]))/c
```

3.68.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6896
```
Int[(E^ArcCsch[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Simp[d^2/(a*c^2)   Int[(d*x)^(m - 2)/Sqrt[1 + 1/(c^2*x^2)], x], x]
+ Simp[d/c   Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, m}
, x] && EqQ[b - a*c^2, 0]
```

3.68. $\int \frac{e^{\text{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx$

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(53) = 106$.

Time = 0.87 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.23

method	result
default	$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2\left(3\left(\frac{c^2x^2+1}{c^2}\right)^{\frac{3}{2}}c^2x^2-3\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2x^4-3\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)x^3+3\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)x^3-\left(\frac{c^2x^2+1}{c^2}\right.}{3x^2\sqrt{\frac{c^2x^2+1}{c^2}}}$

input
```
int((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*((c^2*x^2+1)/c^2/x^2)^(1/2)/x^2*c^2*(3*(1/c^2*(c^2*x^2+1))^(3/2)*c^2*x
^2-3*(1/c^2*(c^2*x^2+1))^(1/2)*c^2*x^4-3*ln(x+(1/c^2*(c^2*x^2+1))^(1/2))*x
^3+3*ln(x+(-(-c^2*x+(-c^2)^(1/2))*(c^2*x+(-c^2)^(1/2))/c^4)^(1/2))*x^3-(1/
c^2*(c^2*x^2+1))^(3/2))/(1/c^2*(c^2*x^2+1))^(1/2)+1/c*(-1/3/x^3+c^2/x+c^3*
arctan(c*x))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3\left(1+c^2x^2\right)}\,dx = \frac{3\,c^3x^3\arctan\left(cx\right)+2\,c^3x^3+3\,c^2x^2+\left(2\,c^3x^3-cx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-1}{3\,cx^3}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="fricas
")
```

output
```
1/3*(3*c^3*x^3*arctan(c*x) + 2*c^3*x^3 + 3*c^2*x^2 + (2*c^3*x^3 - c*x)*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - 1)/(c*x^3)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx = -\frac{c^3\operatorname{atan}\left(\frac{1}{x\sqrt{c^2}}\right)}{\sqrt{c^2}} - c\left(\begin{cases} 2c^4\left(\frac{\left(1+\frac{1}{c^2x^2}\right)^{\frac{3}{2}}}{6c^3} - \frac{\sqrt{1+\frac{1}{c^2x^2}}}{2c^3}\right) & \text{for } \frac{1}{c^2}\neq 0 \\ -\frac{c\log\left(c^2+\frac{1}{x^2}\right)}{2} + \frac{1}{2cx^2} & \text{otherwise}\end{cases}\right) + \frac{c}{x} - \frac{1}{3cx^3}$$

input
```
integrate((1/c/x+(1+1/c**2/x**2)**(1/2))/x**3/(c**2*x**2+1),x)
```

output
```
-c**3*atan(1/(x*sqrt(c**2)))/sqrt(c**2) - c*Piecewise((2*c**4*((1 + 1/(c**
2*x**2))**(3/2)/(6*c**3) - sqrt(1 + 1/(c**2*x**2))/(2*c**3)), Ne(c**(-2),
0)), (-c*log(c**2 + x**(-2))/2 + 1/(2*c*x**2), True)) + c/x - 1/(3*c*x**3)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3(1+c^2x^2)}\,dx = c^2\arctan(cx) + \frac{(2\,c^2x^2-1)\sqrt{c^2x^2+1}}{3\,cx^3} + \frac{3\,c^2x^2-1}{3\,cx^3}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="maxima
")
```

output
```
c^2*arctan(c*x) + 1/3*(2*c^2*x^2 - 1)*sqrt(c^2*x^2 + 1)/(c*x^3) + 1/3*(3*c
^2*x^2 - 1)/(c*x^3)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3 (1+c^2x^2)} \, dx = c^2 \arctan(cx) + \frac{4 \left(3 \left(x|c| - \sqrt{c^2x^2+1}\right)^2 - 1\right) c^2 \operatorname{sgn}(x)}{3 \left(\left(x|c| - \sqrt{c^2x^2+1}\right)^2 - 1\right)^3} + \frac{3 c^2x^2 - 1}{3 cx^3}$$

input
```
integrate((1/c/x+(1+1/c^2/x^2)^(1/2))/x^3/(c^2*x^2+1),x, algorithm="giac")
```

output
```
c^2*arctan(c*x) + 4/3*(3*(x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1)*c^2*sgn(x)/
((x*abs(c) - sqrt(c^2*x^2 + 1))^2 - 1)^3 + 1/3*(3*c^2*x^2 - 1)/(c*x^3)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{e^{\operatorname{csch}^{-1}(cx)}}{x^3 (1+c^2x^2)} \, dx = \frac{c + \frac{2 c^2 x \sqrt{\frac{1}{c^2 x^2}+1}}{3}}{x} - \frac{\frac{x \sqrt{\frac{1}{c^2 x^2}+1}}{3} + \frac{1}{3 c}}{x^3} + c^2 \operatorname{atan}(c x)$$

input
```
int(((1/(c^2*x^2) + 1)^(1/2) + 1/(c*x))/(x^3*(c^2*x^2 + 1)),x)
```

output
```
(c + (2*c^2*x*(1/(c^2*x^2) + 1)^(1/2))/3)/x - ((x*(1/(c^2*x^2) + 1)^(1/2))
/3 + 1/(3*c))/x^3 + c^2*atan(c*x)
```

3.69 $\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.69.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\operatorname{csch}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{2d}$$

output
```
1/2*arccsch(b*x+a)^2/d-arccsch(b*x+a)*ln(1-(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2
))^2)/d-1/2*polylog(2,(1/(b*x+a)+(1+1/(b*x+a)^2)^(1/2))^2)/d
```

3.69.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{\operatorname{csch}^{-1}(a+bx)^2-2\operatorname{csch}^{-1}(a+bx)\log\left(1-e^{2\operatorname{csch}^{-1}(a+bx)}\right)-\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(a+bx)}\right)}{2d}$$

input
```
Integrate[ArcCsch[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(ArcCsch[a + b*x]^2 - 2*ArcCsch[a + b*x]*Log[1 - E^(2*ArcCsch[a + b*x])] -
 PolyLog[2, E^(2*ArcCsch[a + b*x])])/(2*d)
```

3.69.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {6874, 27, 6836, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}dx$$

$$\Big\downarrow \text{6874}$$

$$\frac{\int \frac{b\text{csch}^{-1}(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\text{csch}^{-1}(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{6836}$$

$$-\frac{\int (a+bx)\text{arcsinh}\left(\frac{1}{a+bx}\right)d\frac{1}{a+bx}}{d}$$

$$\Big\downarrow \text{6190}$$

$$-\frac{\int (a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}\text{arcsinh}\left(\frac{1}{a+bx}\right)d\text{arcsinh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i\text{arcsinh}\left(\frac{1}{a+bx}\right)\tan\left(i\text{arcsinh}\left(\frac{1}{a+bx}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \text{arcsinh}\left(\frac{1}{a+bx}\right)\tan\left(i\text{arcsinh}\left(\frac{1}{a+bx}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{1}{a+bx}\right)}{d}$$

$$\Big\downarrow \; 4199$$

$$\frac{i\left(2i\int -\frac{e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\text{arcsinh}\left(\frac{1}{a+bx}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}}d\text{arcsinh}\left(\frac{1}{a+bx}\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{i\left(-2i\int \frac{e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\text{arcsinh}\left(\frac{1}{a+bx}\right)}{1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}}d\text{arcsinh}\left(\frac{1}{a+bx}\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2620$$

$$\frac{i\left(-2i\left(\frac{1}{2}\int \log\left(1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\right)d\text{arcsinh}\left(\frac{1}{a+bx}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\right)\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2715$$

$$\frac{i\left(-2i\left(\frac{1}{4}\int (a+bx)\log(-a-bx+1)de^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}-\frac{1}{2}\text{arcsinh}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\right)\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

$$\Big\downarrow \; 2838$$

$$\frac{i\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{1}{a+bx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{a+bx}\right)}\right)\right)-\frac{i}{2(a+bx)^2}\right)}{d}$$

input
```
Int[ArcCsch[a + b*x]/((a*d)/b + d*x),x]
```

output
```
(I*((-1/2*I)/(a + b*x)^2 - (2*I)*(-1/2*(ArcSinh[(a + b*x)^(-1)]*Log[1 - E^
(2*ArcSinh[(a + b*x)^(-1)])]) - PolyLog[2, E^(2*ArcSinh[(a + b*x)^(-1)])]/
4)))/d
```

3.69. $\int \frac{\text{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

3.69. $\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

rule 6874
```
Int[((a_.) + ArcCsch[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcCsch[x])^p, x
], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0]
&& IGtQ[p, 0]
```

3.69.4 Maple [F]

$$\int \frac{\operatorname{arccsch}(bx+a)}{\frac{ad}{b}+dx}dx$$

input
```
int(arccsch(b*x+a)/(a*d/b+d*x),x)
```

output
```
int(arccsch(b*x+a)/(a*d/b+d*x),x)
```

3.69.5 Fricas [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\operatorname{arcsch}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccsch(b*x + a)/(b*d*x + a*d), x)
```

3.69.6 Sympy [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{b\int \frac{\operatorname{acsch}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(acsch(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acsch(a + b*x)/(a + b*x), x)/d
```

3.69.7 Maxima [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
-1/4*(2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a) + dilog(-b^2*x^2 - 2
*a*b*x - a^2))/d - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1) + 1))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b^2*d*
x^2 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d + d)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) + d), x)
```

3.69.8 Giac [F]

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{arcsch}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccsch(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arccsch(b*x + a)/(d*x + a*d/b), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\operatorname{asinh}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(asinh(1/(a + b*x))/(d*x + (a*d)/b),x)
```

output
```
int(asinh(1/(a + b*x))/(d*x + (a*d)/b), x)
```

3.70 $\int x^3 \text{csch}^{-1}(a+bx^4)\,dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int x^3 \text{csch}^{-1}(a+bx^4)\,dx=\frac{(a+bx^4)\,\text{csch}^{-1}(a+bx^4)}{4b}+\frac{\text{arctanh}\left(\sqrt{1+\frac{1}{(a+bx^4)^2}}\right)}{4b}$$

output
```
1/4*(b*x^4+a)*arccsch(b*x^4+a)/b+1/4*arctanh((1+1/(b*x^4+a)^2)^(1/2))/b
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int x^3 \text{csch}^{-1}(a+bx^4)\,dx=\frac{(a+bx^4)\,\text{csch}^{-1}(a+bx^4)}{4b}-\frac{\sqrt{1+(a+bx^4)^2}\log\left(-a-bx^4+\sqrt{1+(a+bx^4)^2}\right)}{4b\,(a+bx^4)\sqrt{1+\frac{1}{(a+bx^4)^2}}}$$

input
```
Integrate[x^3*ArcCsch[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcCsch[a + b*x^4])/(4*b) - (Sqrt[1 + (a + b*x^4)^2]*Log[-a -
 b*x^4 + Sqrt[1 + (a + b*x^4)^2]])/(4*b*(a + b*x^4)*Sqrt[1 + (a + b*x^4)^(
-2)])
```

3.70.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7266, 6868, 895, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\text{csch}^{-1}\left(a+bx^4\right)\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4}\int \text{csch}^{-1}\left(bx^4+a\right)dx^4$$

$$\Big\downarrow \text{6868}$$

$$\frac{1}{4}\left(\int \frac{1}{\left(bx^4+a\right)\sqrt{1+\frac{1}{\left(bx^4+a\right)^2}}}dx^4+\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{b}\right)$$

$$\Big\downarrow \text{895}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{1+\frac{1}{x^8}}x^4}d\left(bx^4+a\right)}{b}+\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{b}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{b}-\frac{\int \frac{1}{\sqrt{1+\frac{1}{x^8}}x^4}d\frac{1}{x^8}}{2b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{b}-\frac{\int \frac{1}{x^8-1}d\sqrt{1+\frac{1}{x^8}}}{b}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)\text{csch}^{-1}\left(a+bx^4\right)}{b}+\frac{\text{arctanh}\left(\sqrt{\frac{1}{x^8}+1}\right)}{b}\right)$$

input
```
Int[x^3*ArcCsch[a + b*x^4],x]
```

output
```
(((a + b*x^4)*ArcCsch[a + b*x^4])/b + ArcTanh[Sqrt[1 + x^(-8)]]/b)/4
```

3.70.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 6868
```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsch[c + d*
x]/d), x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.70.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arccsch}(b\,x^4+a)+\ln\left(b\,x^4+a+(b\,x^4+a)\sqrt{1+\frac{1}{(b\,x^4+a)^2}}\right)}{4b}$	52
default	$\frac{(b\,x^4+a)\,\mathrm{arccsch}(b\,x^4+a)+\ln\left(b\,x^4+a+(b\,x^4+a)\sqrt{1+\frac{1}{(b\,x^4+a)^2}}\right)}{4b}$	52

input
```
int(x^3*arccsch(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arccsch(b*x^4+a)+ln(b*x^4+a+(b*x^4+a)*(1+1/(b*x^4+a)^2)^(
1/2)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 5.78

$$\int x^3 \mathrm{csch}^{-1}(a+bx^4)\,dx$$
$$= \frac{bx^4\log\left(\frac{(bx^4+a)\sqrt{\frac{b^2x^8+2\,abx^4+a^2+1}{b^2x^8+2\,abx^4+a^2}}+1}{bx^4+a}\right)+a\log\left(-bx^4+(bx^4+a)\sqrt{\frac{b^2x^8+2\,abx^4+a^2+1}{b^2x^8+2\,abx^4+a^2}}-a+1\right)-a\log\left(-bx^4\right.}{4\,b}$$

input
```
integrate(x^3*arccsch(b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(b*x^4*log(((b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8
+ 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(-b*x^4 + (b*x^4 + a)*sqrt((b
^2*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) - a + 1) - a*lo
g(-b*x^4 + (b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x^4 + a^2 + 1)/(b^2*x^8 + 2*a
*b*x^4 + a^2)) - a - 1) - log(-b*x^4 + (b*x^4 + a)*sqrt((b^2*x^8 + 2*a*b*x
^4 + a^2 + 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) - a))/b
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \operatorname{csch}^{-1}(a+bx^4)\,dx = \text{Timed out}$$

input
```
integrate(x**3*acsch(b*x**4+a),x)
```

output
```
Timed out
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int x^3 \operatorname{csch}^{-1}(a+bx^4)\,dx = \frac{2(bx^4+a)\operatorname{arcsch}(bx^4+a)+\log\left(\sqrt{\frac{1}{(bx^4+a)^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{(bx^4+a)^2}+1}-1\right)}{8b}$$

input
```
integrate(x^3*arccsch(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arccsch(b*x^4 + a) + log(sqrt(1/(b*x^4 + a)^2 + 1) + 1)
 - log(sqrt(1/(b*x^4 + a)^2 + 1) - 1))/b
```

3.70.8 Giac [F]

$$\int x^3 \operatorname{csch}^{-1}(a+bx^4)\,dx = \int x^3 \operatorname{arcsch}(bx^4+a)\,dx$$

input
```
integrate(x^3*arccsch(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(x^3*arccsch(b*x^4 + a), x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x^3 \text{csch}^{-1}\left(a+bx^4\right)\,dx=\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{{\left(b\,x^4+a\right)}^2}+1}\right)}{4\,b}+\frac{\mathrm{asinh}\left(\frac{1}{b\,x^4+a}\right)\,\left(b\,x^4+a\right)}{4\,b}$$

input
```
int(x^3*asinh(1/(a + b*x^4)),x)
```

output
```
atanh((1/(a + b*x^4)^2 + 1)^(1/2))/(4*b) + (asinh(1/(a + b*x^4))*(a + b*x^
4))/(4*b)
```

3.71 $\int x^{-1+n}\operatorname{csch}^{-1}(a+bx^n)\,dx$

3.71.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int x^{-1+n}\operatorname{csch}^{-1}(a+bx^n)\,dx=\frac{(a+bx^n)\operatorname{csch}^{-1}(a+bx^n)}{bn}+\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

output
```
(a+b*x^n)*arccsch(a+b*x^n)/b/n+arctanh((1+1/(a+b*x^n)^2)^(1/2))/b/n
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(46) = 92.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int x^{-1+n}\operatorname{csch}^{-1}(a+bx^n)\,dx=\frac{(a+bx^n)^2\operatorname{csch}^{-1}(a+bx^n)-\frac{\sqrt{1+(a+bx^n)^2}\log\left(-a-bx^n+\sqrt{1+(a+bx^n)^2}\right)}{\sqrt{1+\frac{1}{(a+bx^n)^2}}}}{bn(a+bx^n)}$$

input
```
Integrate[x^(-1 + n)*ArcCsch[a + b*x^n],x]
```

output
```
((a + b*x^n)^2*ArcCsch[a + b*x^n] - (Sqrt[1 + (a + b*x^n)^2]*Log[-a - b*x^
n + Sqrt[1 + (a + b*x^n)^2]])/Sqrt[1 + (a + b*x^n)^(-2)])/(b*n*(a + b*x^n)
)
```

3.71.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7266, 6868, 895, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\text{csch}^{-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{\int \text{csch}^{-1}(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{ 6868}$$

$$\frac{\int \frac{1}{(bx^n+a)\sqrt{1+\frac{1}{(bx^n+a)^2}}}dx^n+\frac{(a+bx^n)\text{csch}^{-1}(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \text{ 895}$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{x^{-2n}+1}}d(bx^n+a)}{b}+\frac{(a+bx^n)\text{csch}^{-1}(a+bx^n)}{b}}{n}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\frac{(a+bx^n)\text{csch}^{-1}(a+bx^n)}{b}-\frac{\int \frac{x^{-n}}{\sqrt{x^{-2n}+1}}dx^{-2n}}{2b}}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{(a+bx^n)\text{csch}^{-1}(a+bx^n)}{b}-\frac{\int \frac{1}{x^{2n}-1}d\sqrt{x^{-2n}+1}}{b}}{n}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\frac{(a+bx^n)\text{csch}^{-1}(a+bx^n)}{b}+\frac{\text{arctanh}\left(\sqrt{x^{-2n}+1}\right)}{b}}{n}$$

input
```
Int[x^(-1 + n)*ArcCsch[a + b*x^n],x]
```

output
```
(((a + b*x^n)*ArcCsch[a + b*x^n])/b + ArcTanh[Sqrt[1 + x^(-2*n)]]/b)/n
```

3.71.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 6868
```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsch[c + d*
x]/d), x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.71.4 Maple [F]

$$\int x^{-1+n} \operatorname{arccsch}(a+b x^n) \, dx$$

input
```
int(x^(-1+n)*arccsch(a+b*x^n),x)
```

output
```
int(x^(-1+n)*arccsch(a+b*x^n),x)
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(44) = 88.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 7.26

$$\int x^{-1+n}\text{csch}^{-1}(a+bx^n)\,dx$$

$$= \frac{a\log\left(-b\cosh\left(n\log\left(x\right)\right)-b\sinh\left(n\log\left(x\right)\right)-a+\sqrt{\frac{2\,ab+(a^2+b^2+1)\cosh(n\log(x))-(a^2-b^2+1)\sinh(n\log(x))}{\cosh(n\log(x))-\sinh(n\log(x))}}+1\right)-}{}$$

input
```
integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="fricas")
```

output
```
(a*log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a + sqrt((2*a*b + (a^2 + b^2
 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x)) - s
inh(n*log(x)))) + 1) - a*log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a + sq
rt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)
))/(cosh(n*log(x)) - sinh(n*log(x)))) - 1) + (b*cosh(n*log(x)) + b*sinh(n*
log(x)))*log((sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 +
1)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) + 1)/(b*cosh(n*log(x
)) + b*sinh(n*log(x)) + a)) - log(-b*cosh(n*log(x)) - b*sinh(n*log(x)) - a
 + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*l
og(x)))/(cosh(n*log(x)) - sinh(n*log(x))))))/(b*n)
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}\text{csch}^{-1}(a+bx^n)\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+n)*acsch(a+b*x**n),x)
```

output
```
Timed out
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int x^{-1+n}\text{csch}^{-1}(a+bx^n)\,dx = \frac{2\,(bx^n+a)\,\text{arcsch}\,(bx^n+a)+\log\left(\sqrt{\frac{1}{(bx^n+a)^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{(bx^n+a)^2}+1}-1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arccsch(b*x^n + a) + log(sqrt(1/(b*x^n + a)^2 + 1) + 1)
 - log(sqrt(1/(b*x^n + a)^2 + 1) - 1))/(b*n)
```

3.71.8 Giac [F]

$$\int x^{-1+n}\text{csch}^{-1}(a+bx^n)\,dx = \int x^{n-1}\,\text{arcsch}\,(bx^n+a)\,dx$$

input
```
integrate(x^(-1+n)*arccsch(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)*arccsch(b*x^n + a), x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x^{-1+n}\text{csch}^{-1}(a+bx^n)\,dx = \frac{\text{atanh}\left(\sqrt{\frac{1}{(a+b\,x^n)^2}+1}\right)+\text{asinh}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

input
```
int(x^(n - 1)*asinh(1/(a + b*x^n)),x)
```

output
```
(atanh((1/(a + b*x^n)^2 + 1)^(1/2)) + asinh(1/(a + b*x^n))*(a + b*x^n))/(b
*n)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(exppn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```