Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

8-Special-functions/205-8.2-Fresnel-integral-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	94.50 (206)	5.50 (12)
Mathematica	87.16 (190)	12.84 (28)
Fricas	87.16 (190)	12.84 (28)
Maple	70.64 (154)	29.36 (64)
Maxima	55.05 (120)	44.95 (98)
Sympy	54.13 (118)	45.87 (100)
Mupad	27.52 (60)	72.48 (158)
Giac	27.52 (60)	72.48 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

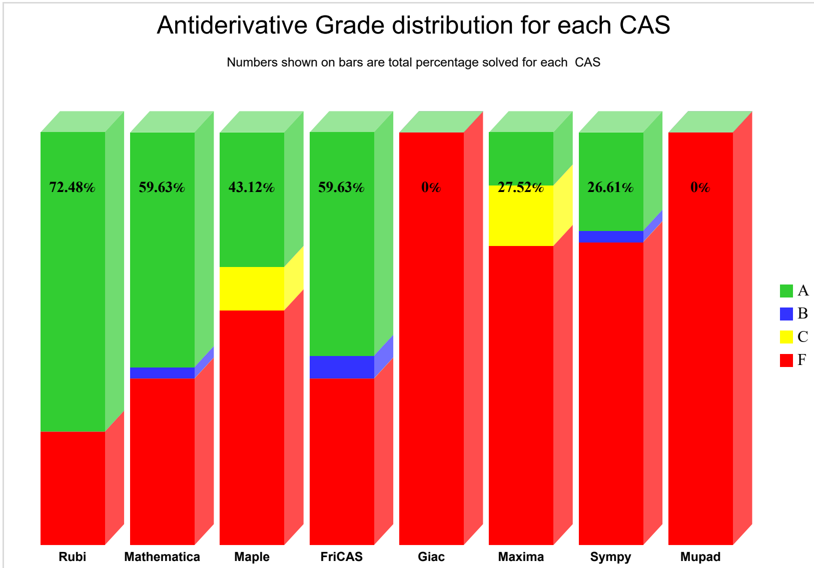
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

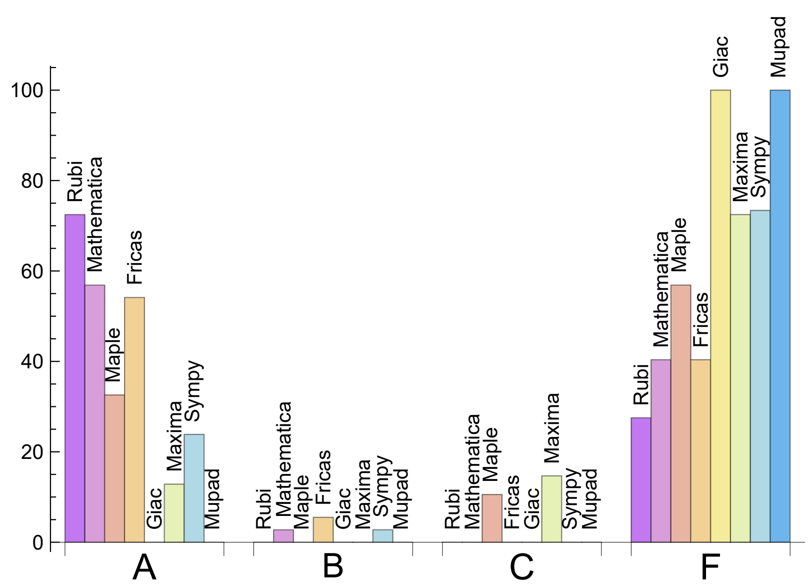
System	% A grade	% B grade	% C grade	% F grade
Rubi	65.138	1.835	0.000	33.028
Mathematica	56.881	2.752	0.000	40.367
Fricas	54.128	5.505	0.000	40.367
Maple	32.569	0.000	10.550	56.881
Sympy	23.853	2.752	0.000	73.394
Maxima	12.844	0.000	14.679	72.477
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	12	100.00	0.00	0.00
Mathematica	28	100.00	0.00	0.00
Fricas	28	100.00	0.00	0.00
Maple	64	100.00	0.00	0.00
Maxima	98	100.00	0.00	0.00
Sympy	100	100.00	0.00	0.00
Mupad	158	0.00	100.00	0.00
Giac	158	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.26
Giac	0.27
Maxima	0.31
Mathematica	0.31
Maple	0.57
Rubi	0.69
Sympy	4.35
Mupad	4.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	16.90	1.08	19.50	1.00
Giac	16.90	1.08	19.50	1.00
Sympy	49.28	1.06	20.00	1.00
Maxima	51.11	1.10	20.00	1.00
Maple	57.58	0.88	24.50	0.91
Mathematica	86.49	1.04	59.00	1.00
Fricas	97.03	1.06	53.50	1.00
Rubi	109.03	1.10	64.50	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

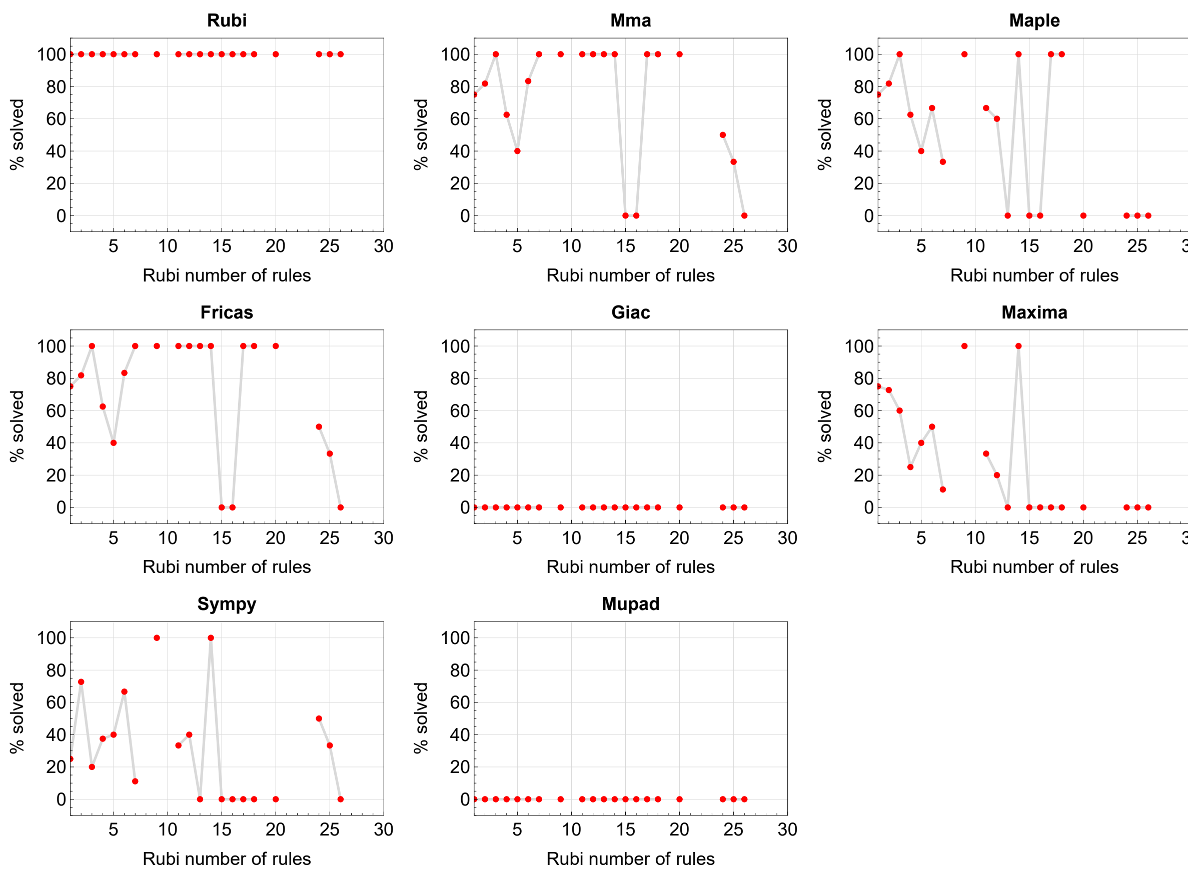


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

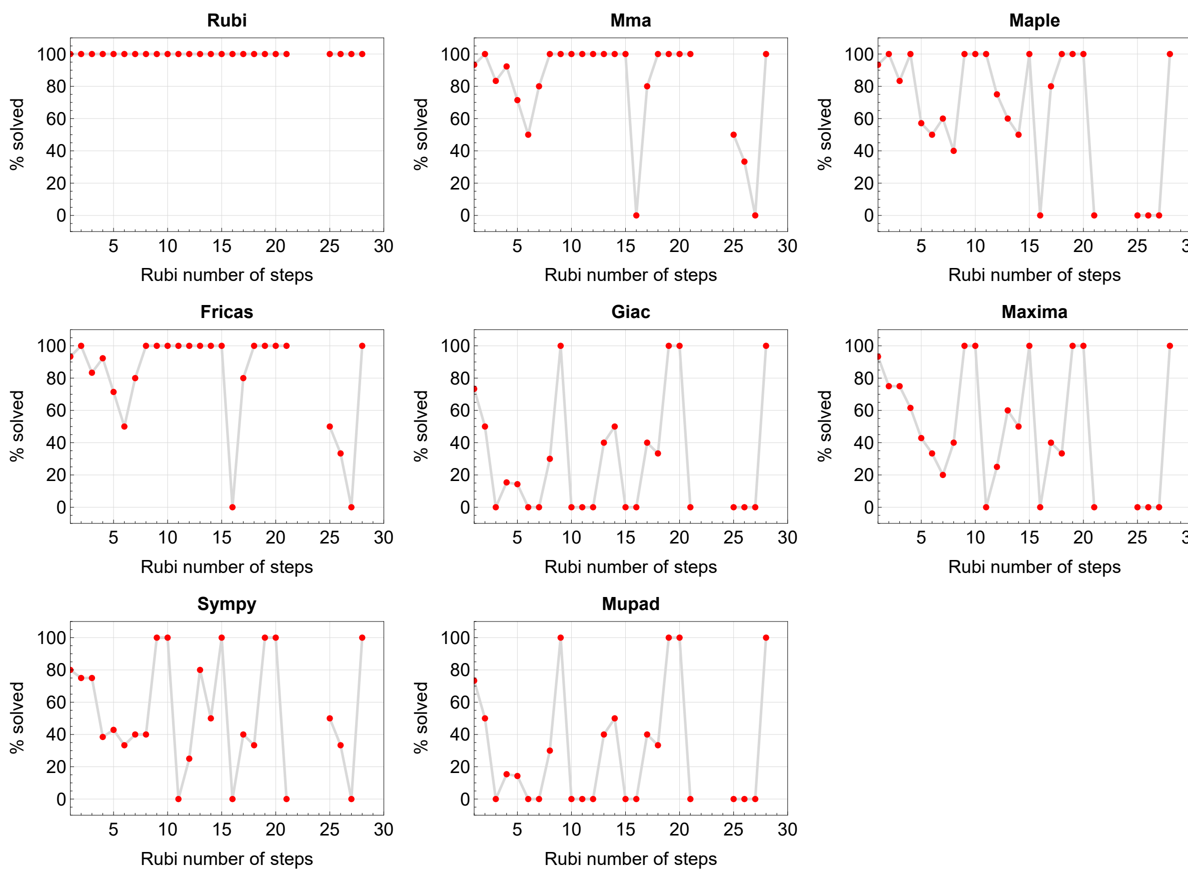


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

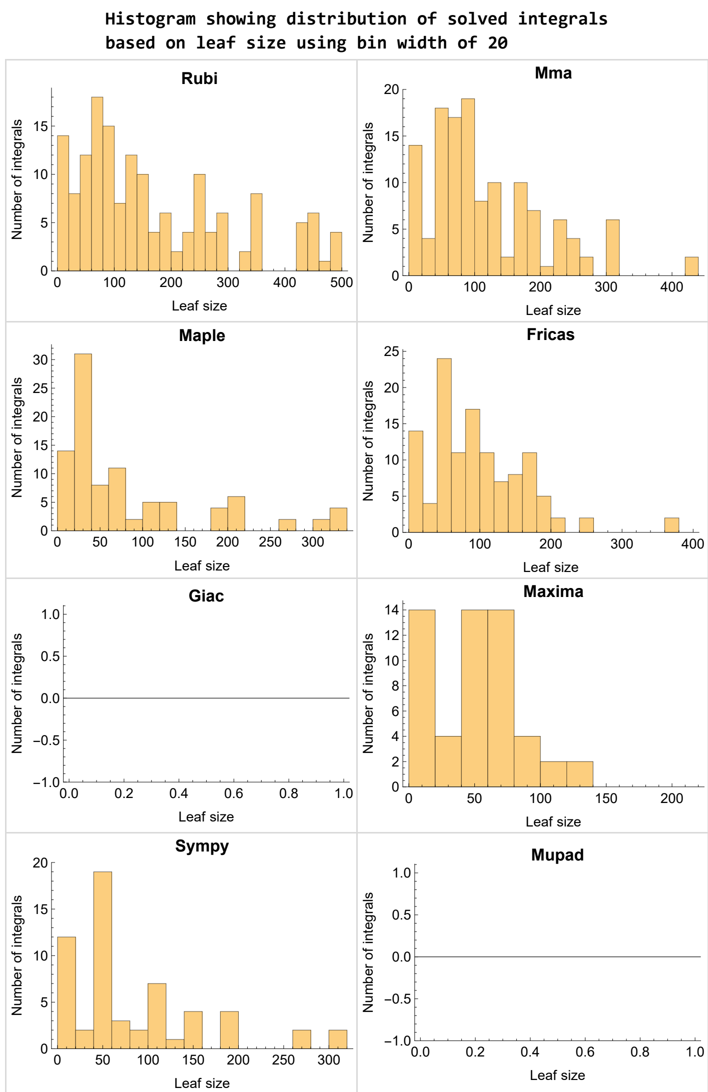


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

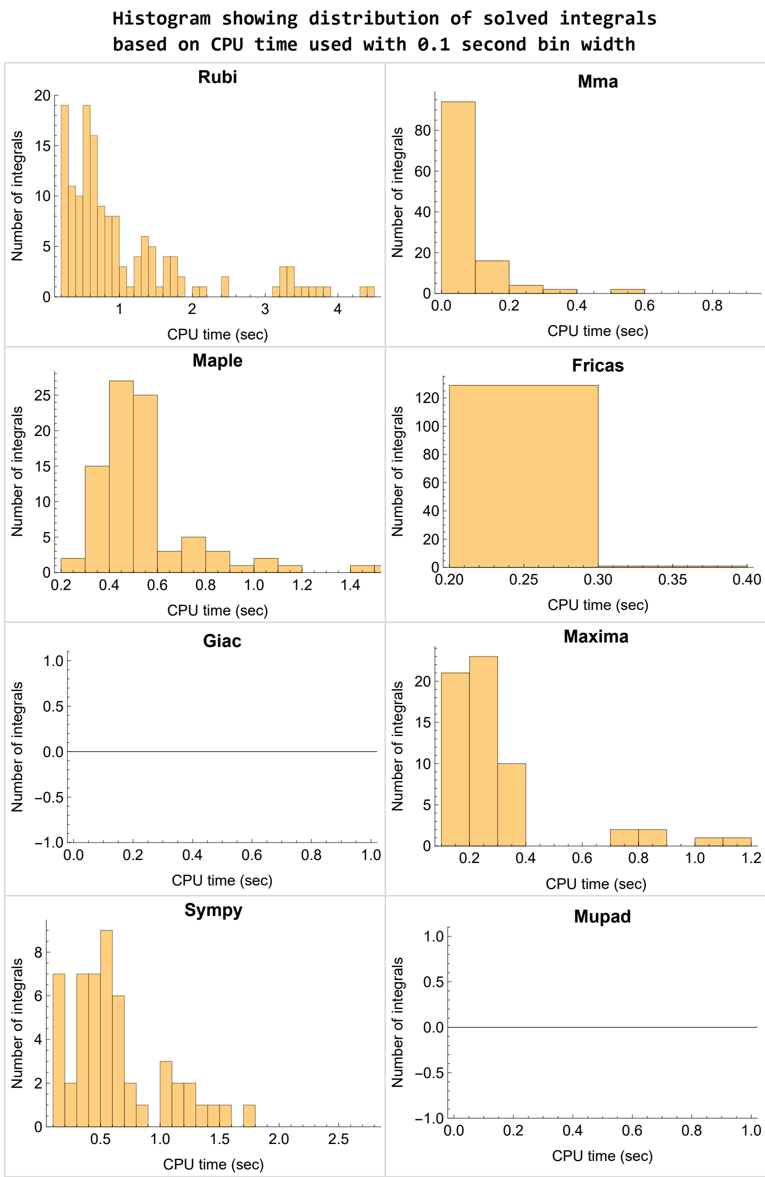


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

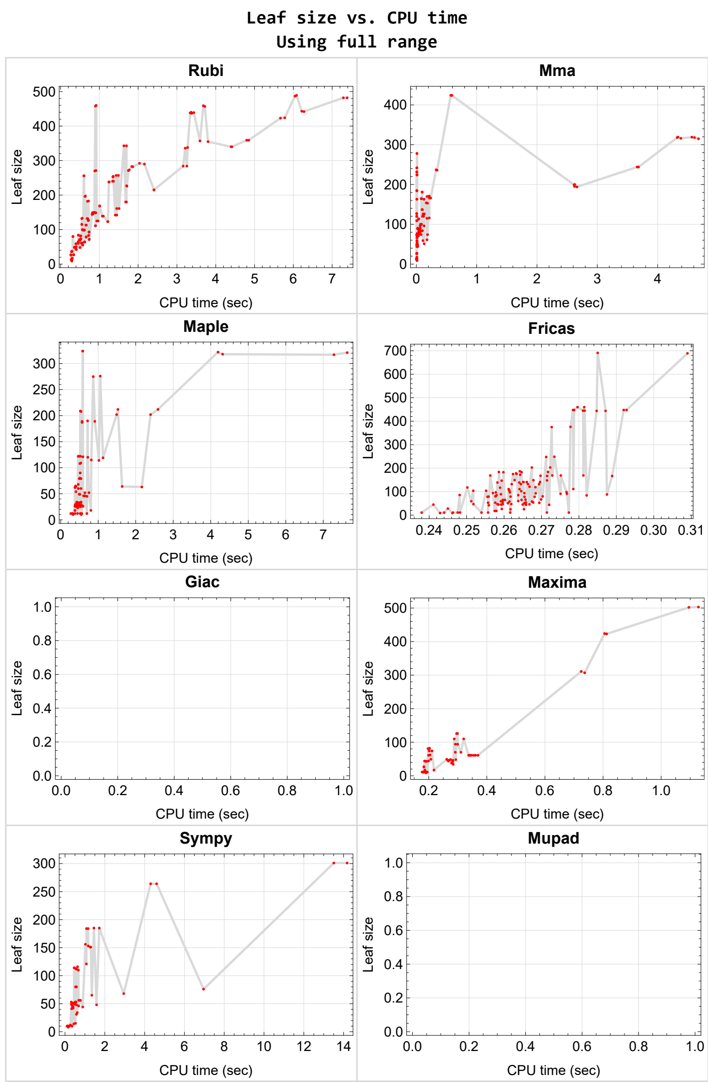


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

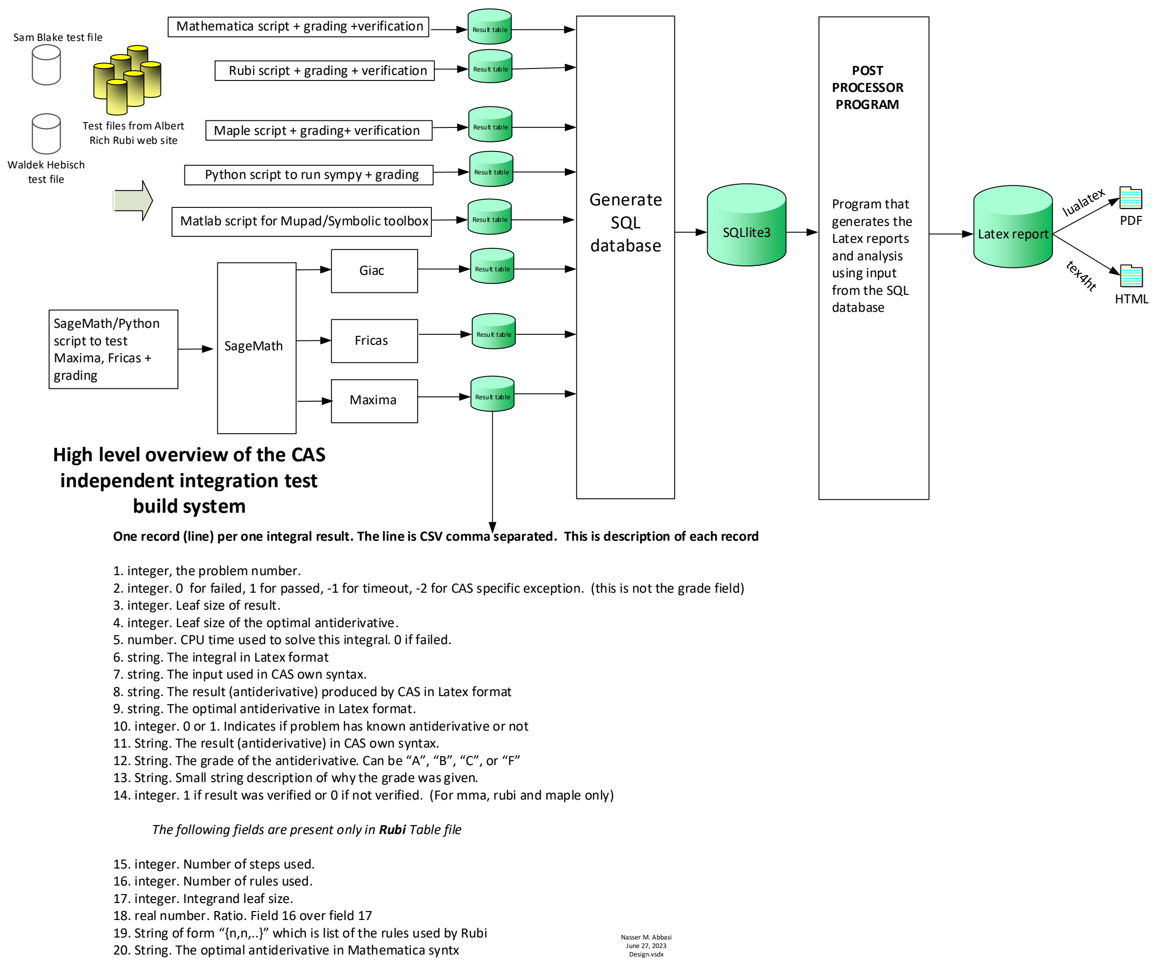
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 43, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 83, 93, 94, 95, 96, 97, 98, 99, 101, 105, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 152, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 192, 202, 203, 204, 205, 206, 207, 208, 210, 214 }

B grade { 72, 92, 181, 201 }

C grade { }

F normal fail { 31, 47, 71, 87, 91, 109, 140, 156, 180, 196, 200, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 27, 31, 32, 34, 35, 36, 38, 43, 47, 51, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 83, 87, 92, 93, 94, 96, 97, 98, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 134, 135, 136, 140, 141, 143, 144, 145, 147, 152, 156, 160, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 192, 196, 201, 202, 203, 205, 206, 207, 210, 214, 218 }

B grade { 22, 28, 57, 131, 137, 166 }

C grade { }

F normal fail { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 5, 6, 8, 9, 10, 12, 13, 19, 20, 21, 22, 25, 26, 27, 28, 32, 34, 36, 38, 51, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 92, 94, 96, 98, 114, 117, 118, 119, 122, 123, 128, 129, 130, 131, 134, 135, 136, 137, 141, 143, 145, 147, 160, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 201, 203, 205, 207 }

B grade { }

C grade { 1, 2, 3, 4, 7, 11, 14, 15, 16, 17, 18, 110, 111, 112, 113, 115, 116, 120, 121, 124, 125, 126, 127 }

F normal fail { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 34, 35, 36, 38, 43, 47, 51, 57, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 83, 87, 92, 93, 94, 96, 97, 98, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 143, 144, 145, 147, 152, 156, 160, 166, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 192, 196, 201, 202, 203, 205, 206, 207, 210, 214, 218 }

B grade { 54, 55, 56, 58, 59, 60, 163, 164, 165, 167, 168, 169 }

C grade { }

F normal fail { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 2, 4, 6, 8, 22, 28, 57, 65, 66, 67, 68, 69, 70, 79, 111, 113, 115, 117, 131, 137, 166, 174, 175, 176, 177, 178, 179, 188 }

B grade { }

C grade { 1, 3, 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 26, 27, 110, 112, 114, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136 }

F normal fail { 9, 19, 20, 21, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 118, 128, 129, 130, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 181, 182, 183, 184, 185, 186, 187, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 65, 66, 67, 68, 69, 71, 75, 79, 93, 97, 110, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 124, 125, 126, 127, 174, 175, 176, 177, 178, 180, 184, 188, 202, 206 }

B grade { 8, 10, 70, 117, 119, 179 }

C grade { }

F normal fail { 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 72, 73, 74, 76, 77, 78, 83, 87, 91, 92, 94, 95, 96, 98, 99, 101, 105, 109, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 181, 182, 183, 185, 186, 187, 192, 196, 200, 201, 203, 204, 205, 207, 208, 210, 214, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	145	88	29	126	84	184	0	0
N.S.	1	1.17	0.71	0.23	1.02	0.68	1.48	0.00	0.00
time (sec)	N/A	0.508	0.053	0.438	0.296	0.282	1.161	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	125	83	29	74	78	156	0	0
N.S.	1	1.15	0.76	0.27	0.68	0.72	1.43	0.00	0.00
time (sec)	N/A	0.597	0.043	0.399	0.211	0.256	1.028	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	113	79	29	110	85	53	0	0
N.S.	1	1.14	0.80	0.29	1.11	0.86	0.54	0.00	0.00
time (sec)	N/A	0.410	0.056	0.437	0.320	0.264	0.511	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	93	71	29	62	65	121	0	0
N.S.	1	1.11	0.85	0.35	0.74	0.77	1.44	0.00	0.00
time (sec)	N/A	0.468	0.032	0.455	0.205	0.260	1.073	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	82	74	62	94	58	112	0	0
N.S.	1	1.11	1.00	0.84	1.27	0.78	1.51	0.00	0.00
time (sec)	N/A	0.334	0.013	0.383	0.292	0.267	0.566	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	61	59	54	49	53	80	0	0
N.S.	1	1.03	1.00	0.92	0.83	0.90	1.36	0.00	0.00
time (sec)	N/A	0.351	0.010	0.408	0.262	0.258	0.528	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	50	49	29	70	51	53	0	0
N.S.	1	1.02	1.00	0.59	1.43	1.04	1.08	0.00	0.00
time (sec)	N/A	0.260	0.009	0.434	0.311	0.265	0.316	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	26	26	48	0	0
N.S.	1	1.00	1.00	0.96	1.00	1.00	1.85	0.00	0.00
time (sec)	N/A	0.177	0.005	0.525	0.184	0.259	0.572	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	46	0	0
N.S.	1	1.00	0.00	0.40	0.00	0.00	0.63	0.00	0.00
time (sec)	N/A	0.334	0.000	0.549	0.000	0.000	0.368	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	25	42	0	0
N.S.	1	1.00	1.00	0.89	1.41	0.93	1.56	0.00	0.00
time (sec)	N/A	0.219	0.010	0.444	0.278	0.265	0.360	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	42	44	27	61	45	51	0	0
N.S.	1	0.95	1.00	0.61	1.39	1.02	1.16	0.00	0.00
time (sec)	N/A	0.257	0.010	0.392	0.370	0.265	0.421	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	55	52	46	42	44	56	0	0
N.S.	1	1.06	1.00	0.88	0.81	0.85	1.08	0.00	0.00
time (sec)	N/A	0.360	0.012	0.611	0.283	0.266	0.775	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	69	65	61	54	110	0	0
N.S.	1	1.03	1.00	0.94	0.88	0.78	1.59	0.00	0.00
time (sec)	N/A	0.318	0.012	0.396	0.355	0.266	0.670	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	83	77	29	48	65	46	0	0
N.S.	1	1.08	1.00	0.38	0.62	0.84	0.60	0.00	0.00
time (sec)	N/A	0.451	0.016	0.467	0.274	0.265	0.682	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	98	76	29	61	80	56	0	0
N.S.	1	1.04	0.81	0.31	0.65	0.85	0.60	0.00	0.00
time (sec)	N/A	0.379	0.046	0.388	0.361	0.270	0.748	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	111	85	79	46	78	68	0	0
N.S.	1	1.09	0.83	0.77	0.45	0.76	0.67	0.00	0.00
time (sec)	N/A	0.568	0.048	0.498	0.268	0.268	2.950	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	127	84	29	61	80	185	0	0
N.S.	1	1.07	0.71	0.24	0.51	0.67	1.55	0.00	0.00
time (sec)	N/A	0.454	0.045	0.506	0.337	0.258	1.720	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	139	96	29	48	91	48	0	0
N.S.	1	1.09	0.76	0.23	0.38	0.72	0.38	0.00	0.00
time (sec)	N/A	0.692	0.129	0.454	0.276	0.275	1.584	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	271	424	276	0	376	0	0	0
N.S.	1	0.92	1.43	0.93	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.561	0.589	1.055	0.000	0.278	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	183	236	189	0	248	0	0	0
N.S.	1	0.95	1.22	0.98	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.445	0.344	0.911	0.000	0.271	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	115	61	109	0	132	0	0	0
N.S.	1	0.95	0.50	0.90	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.343	0.176	0.537	0.000	0.264	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	0
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	0.00
time (sec)	N/A	0.185	0.026	0.493	0.189	0.241	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.189	0.021	0.344	0.796	0.253	0.466	0.263	4.941

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.190	2.304	0.310	0.458	0.271	0.582	0.276	4.817

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	197	166	189	503	175	0	0	0
N.S.	1	0.86	0.72	0.83	2.20	0.76	0.00	0.00	0.00
time (sec)	N/A	0.396	0.238	0.571	1.129	0.262	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	133	115	121	424	147	0	0	0
N.S.	1	0.90	0.78	0.82	2.88	1.00	0.00	0.00	0.00
time (sec)	N/A	0.351	0.174	0.593	0.805	0.259	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	84	51	80	307	104	0	0	0
N.S.	1	0.88	0.53	0.83	3.20	1.08	0.00	0.00	0.00
time (sec)	N/A	0.292	0.130	0.510	0.737	0.255	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	0
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	0.00
time (sec)	N/A	0.186	0.019	0.423	0.192	0.259	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.180	0.015	0.171	0.780	0.253	0.378	0.256	4.650

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.182	1.498	0.157	0.445	0.268	0.283	0.262	4.675

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	0	181	0	0	183	0	0	0
N.S.	1	0.00	0.72	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.000	0.096	0.000	0.000	0.260	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	459	171	324	0	184	0	0	0
N.S.	1	1.92	0.72	1.36	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	2.225	0.213	0.585	0.000	0.272	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	265	357	0	0	0	0	0	0	0
N.S.	1	1.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.180	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	292	137	208	0	149	0	0	0
N.S.	1	1.65	0.77	1.18	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.224	0.101	0.544	0.000	0.263	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	180	113	0	0	117	0	0	0
N.S.	1	1.29	0.81	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.037	0.038	0.000	0.000	0.259	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	169	100	122	0	111	0	0	0
N.S.	1	1.36	0.81	0.98	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.629	0.088	0.466	0.000	0.279	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	143	149	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	61	55	49	0	60	0	0	0
N.S.	1	1.11	1.00	0.89	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.290	0.012	0.513	0.000	0.264	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.184	0.014	0.025	0.227	0.247	1.082	0.260	4.655

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.261	0.019	0.070	0.221	0.265	1.185	0.262	4.673

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.265	0.017	0.066	0.226	0.256	1.066	0.257	4.833

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.506	0.021	0.091	0.225	0.250	1.123	0.253	4.805

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	142	127	0	0	111	0	0	0
N.S.	1	1.12	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.881	0.005	0.000	0.000	0.261	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.951	0.021	0.075	0.214	0.257	1.401	0.259	4.802

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	1.407	0.016	0.068	0.214	0.259	1.327	0.261	4.848

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	1.629	0.025	0.066	0.221	0.261	1.618	0.257	4.746

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	0	242	0	0	187	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.000	0.008	0.000	0.000	0.264	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	2.629	0.024	0.072	0.213	0.247	2.499	0.255	4.848

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	497	460	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.561	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	279	256	0	0	0	0	0	0	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	67	60	0	89	0	0	0
N.S.	1	1.03	0.96	0.86	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.454	0.011	0.470	0.000	0.259	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.205	0.027	0.189	0.222	0.249	0.422	0.263	4.877

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	29	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.81	0.94	1.12	1.12
time (sec)	N/A	0.201	0.062	0.261	0.227	0.255	0.596	0.262	4.847

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	257	319	0	0	448	0	0	0
N.S.	1	1.11	1.38	0.00	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.905	4.568	0.000	0.000	0.292	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	253	319	0	0	448	0	0	0
N.S.	1	1.11	1.41	0.00	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.834	4.337	0.000	0.000	0.279	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	238	316	0	0	445	0	0	0
N.S.	1	1.11	1.48	0.00	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.779	4.386	0.000	0.000	0.281	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	64	164	63	81	119	0	0	0
N.S.	1	0.98	2.52	0.97	1.25	1.83	0.00	0.00	0.00
time (sec)	N/A	0.283	0.084	2.163	0.197	0.268	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	240	195	0	0	444	0	0	0
N.S.	1	1.11	0.90	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.833	2.622	0.000	0.000	0.285	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	254	200	0	0	460	0	0	0
N.S.	1	1.11	0.88	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.828	2.626	0.000	0.000	0.281	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	343	244	0	0	691	0	0	0
N.S.	1	1.22	0.87	0.00	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	1.025	3.683	0.000	0.000	0.285	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	99	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	99	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.185	0.005	0.296	0.188	0.238	0.220	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.177	0.003	0.270	0.180	0.256	0.155	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	0	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.89	0.00	0.00
time (sec)	N/A	0.179	0.012	0.332	0.191	0.243	0.162	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	0	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	0.00	0.00
time (sec)	N/A	0.185	0.004	0.384	0.184	0.248	0.351	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	14	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.08	0.00	0.00
time (sec)	N/A	0.183	0.005	0.535	0.177	0.244	0.443	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	18	31	0	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	1.82	0.00	0.00
time (sec)	N/A	0.188	0.007	0.381	0.187	0.257	0.573	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	A	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	0	232	0	0	169	301	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.73	1.30	0.00	0.00
time (sec)	N/A	0.000	0.010	0.000	0.000	0.256	13.508	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	440	153	318	0	167	0	0	0
N.S.	1	2.04	0.71	1.47	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	2.023	0.185	4.317	0.000	0.289	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	248	338	0	0	0	0	0	0	0
N.S.	1	1.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.985	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	273	120	202	0	132	0	0	0
N.S.	1	1.73	0.76	1.28	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.083	0.119	1.492	0.000	0.269	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	161	120	0	0	105	151	0	0
N.S.	1	1.34	1.00	0.00	0.00	0.88	1.26	0.00	0.00
time (sec)	N/A	0.938	0.006	0.000	0.000	0.261	1.270	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	150	83	115	0	94	0	0	0
N.S.	1	1.43	0.79	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.528	0.076	0.818	0.000	0.265	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	137	132	0	0	0	0	0	0	0
N.S.	1	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	47	0	0	0
N.S.	1	1.00	0.90	0.94	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.223	0.019	0.645	0.000	0.260	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.170	0.000	0.309	0.191	0.262	0.104	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.185	0.023	0.137	0.281	0.274	1.065	0.275	4.784

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.185	0.023	0.133	0.275	0.258	1.009	0.274	4.875

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.409	0.024	0.147	0.275	0.250	1.265	0.286	4.837

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	123	109	0	0	98	0	0	0
N.S.	1	1.13	1.00	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.750	0.007	0.000	0.000	0.277	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.796	0.026	0.135	0.270	0.251	3.134	0.279	4.821

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.253	0.026	0.125	0.280	0.258	5.694	0.286	4.831

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.449	0.023	0.137	0.263	0.251	11.356	0.281	4.774

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	0	224	0	0	172	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.000	0.014	0.000	0.000	0.264	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.419	0.030	0.125	0.279	0.252	36.284	0.264	4.976

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.190	0.027	0.141	0.262	0.256	66.992	0.270	4.938

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.180	0.053	0.151	0.290	0.258	1.106	0.298	4.832

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	307	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	439	163	321	0	169	0	0	0
N.S.	1	2.02	0.75	1.48	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	1.994	0.127	7.636	0.000	0.281	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	284	184	0	0	141	264	0	0
N.S.	1	1.54	1.00	0.00	0.00	0.77	1.43	0.00	0.00
time (sec)	N/A	1.882	0.008	0.000	0.000	0.265	4.604	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	283	126	212	0	139	0	0	0
N.S.	1	1.70	0.76	1.28	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.111	0.133	2.594	0.000	0.258	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	195	227	0	0	0	0	0	0	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	149	90	119	0	97	0	0	0
N.S.	1	1.38	0.83	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.562	0.064	1.129	0.000	0.258	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	78	73	0	0	58	114	0	0
N.S.	1	1.07	1.00	0.00	0.00	0.79	1.56	0.00	0.00
time (sec)	N/A	0.414	0.005	0.000	0.000	0.261	0.474	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	53	0	0	0
N.S.	1	1.00	0.81	0.88	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.254	0.021	0.754	0.000	0.256	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.185	0.026	0.136	0.266	0.242	1.077	0.273	4.795

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	46	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.313	0.005	0.000	0.000	0.267	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.406	0.026	0.139	0.276	0.254	1.191	0.269	4.787

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.558	0.026	0.145	0.271	0.263	1.680	0.280	4.748

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.794	0.026	0.137	0.267	0.273	3.082	0.289	4.977

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	215	163	0	0	141	0	0	0
N.S.	1	1.32	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	1.460	0.008	0.000	0.000	0.265	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.415	0.025	0.138	0.268	0.258	11.227	0.283	4.717

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.263	0.026	0.155	0.283	0.265	21.230	0.271	4.763

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.426	0.027	0.135	0.289	0.247	38.007	0.282	4.701

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	0	278	0	0	203	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.000	0.009	0.000	0.000	0.267	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	144	89	26	126	85	184	0	0
N.S.	1	1.16	0.72	0.21	1.02	0.69	1.48	0.00	0.00
time (sec)	N/A	0.491	0.053	0.598	0.299	0.258	1.080	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	125	83	26	74	79	153	0	0
N.S.	1	1.15	0.76	0.24	0.68	0.72	1.40	0.00	0.00
time (sec)	N/A	0.574	0.041	0.579	0.202	0.256	1.165	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	113	80	26	110	86	49	0	0
N.S.	1	1.14	0.81	0.26	1.11	0.87	0.49	0.00	0.00
time (sec)	N/A	0.402	0.048	0.394	0.288	0.248	0.485	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	93	71	26	61	66	116	0	0
N.S.	1	1.11	0.85	0.31	0.73	0.79	1.38	0.00	0.00
time (sec)	N/A	0.454	0.031	0.401	0.200	0.266	0.628	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	81	74	62	94	59	112	0	0
N.S.	1	1.09	1.00	0.84	1.27	0.80	1.51	0.00	0.00
time (sec)	N/A	0.323	0.013	0.382	0.300	0.251	0.555	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	61	59	26	49	54	80	0	0
N.S.	1	1.03	1.00	0.44	0.83	0.92	1.36	0.00	0.00
time (sec)	N/A	0.345	0.011	0.380	0.207	0.268	0.576	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	50	49	26	70	51	49	0	0
N.S.	1	1.02	1.00	0.53	1.43	1.04	1.00	0.00	0.00
time (sec)	N/A	0.254	0.010	0.389	0.292	0.261	0.319	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	27	28	44	0	0
N.S.	1	1.00	1.00	0.96	1.00	1.04	1.63	0.00	0.00
time (sec)	N/A	0.172	0.004	0.476	0.183	0.245	0.405	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	41	0	0
N.S.	1	1.00	0.00	0.33	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.315	0.000	0.457	0.000	0.000	0.336	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	34	25	53	0	0
N.S.	1	1.00	1.00	0.89	1.26	0.93	1.96	0.00	0.00
time (sec)	N/A	0.210	0.010	0.438	0.284	0.262	0.551	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	43	44	26	61	42	51	0	0
N.S.	1	0.98	1.00	0.59	1.39	0.95	1.16	0.00	0.00
time (sec)	N/A	0.252	0.010	0.443	0.339	0.256	0.358	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	55	52	26	44	44	42	0	0
N.S.	1	1.06	1.00	0.50	0.85	0.85	0.81	0.00	0.00
time (sec)	N/A	0.355	0.014	0.428	0.268	0.272	0.403	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	69	64	61	56	110	0	0
N.S.	1	1.01	1.00	0.93	0.88	0.81	1.59	0.00	0.00
time (sec)	N/A	0.302	0.012	0.410	0.342	0.260	0.663	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	83	77	68	46	65	65	0	0
N.S.	1	1.08	1.00	0.88	0.60	0.84	0.84	0.00	0.00
time (sec)	N/A	0.439	0.014	0.469	0.265	0.260	1.346	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	99	74	26	61	79	56	0	0
N.S.	1	1.05	0.79	0.28	0.65	0.84	0.60	0.00	0.00
time (sec)	N/A	0.358	0.079	0.394	0.342	0.262	0.688	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	111	84	26	48	78	44	0	0
N.S.	1	1.09	0.82	0.25	0.47	0.76	0.43	0.00	0.00
time (sec)	N/A	0.547	0.100	0.424	0.293	0.265	0.889	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	126	85	26	61	81	185	0	0
N.S.	1	1.06	0.71	0.22	0.51	0.68	1.55	0.00	0.00
time (sec)	N/A	0.430	0.045	0.428	0.348	0.258	1.453	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	139	96	90	46	91	76	0	0
N.S.	1	1.09	0.76	0.71	0.36	0.72	0.60	0.00	0.00
time (sec)	N/A	0.673	0.062	0.521	0.283	0.277	6.963	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	270	424	275	0	375	0	0	0
N.S.	1	0.91	1.42	0.92	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.534	0.571	0.868	0.000	0.273	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	182	237	190	0	249	0	0	0
N.S.	1	0.94	1.22	0.98	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.432	0.327	0.718	0.000	0.273	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	115	74	108	0	132	0	0	0
N.S.	1	0.94	0.61	0.89	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.348	0.185	0.512	0.000	0.266	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	0
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	0.00
time (sec)	N/A	0.185	0.023	0.544	0.199	0.258	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.190	0.020	0.314	0.748	0.261	0.381	0.270	4.652

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.185	1.500	0.283	0.456	0.250	0.577	0.264	4.665

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	196	166	187	502	176	0	0	0
N.S.	1	0.86	0.73	0.82	2.21	0.78	0.00	0.00	0.00
time (sec)	N/A	0.390	0.215	0.571	1.096	0.263	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	132	116	122	423	148	0	0	0
N.S.	1	0.89	0.78	0.82	2.86	1.00	0.00	0.00	0.00
time (sec)	N/A	0.345	0.214	0.527	0.812	0.271	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	84	59	79	311	104	0	0	0
N.S.	1	0.88	0.62	0.83	3.27	1.09	0.00	0.00	0.00
time (sec)	N/A	0.286	0.109	0.542	0.725	0.252	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	0
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	0.00
time (sec)	N/A	0.192	0.020	0.434	0.186	0.252	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.181	0.017	0.175	0.767	0.262	0.314	0.263	4.587

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.185	0.941	0.128	0.438	0.247	0.260	0.258	4.637

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	0	181	0	0	183	0	0	0
N.S.	1	0.00	0.72	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.000	0.098	0.000	0.000	0.265	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	457	170	324	0	184	0	0	0
N.S.	1	1.91	0.71	1.36	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	2.194	0.177	0.589	0.000	0.259	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	265	355	0	0	0	0	0	0	0
N.S.	1	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.228	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	290	137	209	0	149	0	0	0
N.S.	1	1.64	0.77	1.18	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.187	0.093	0.524	0.000	0.268	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	180	114	0	0	118	0	0	0
N.S.	1	1.29	0.81	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.028	0.039	0.000	0.000	0.250	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	168	100	122	0	111	0	0	0
N.S.	1	1.35	0.81	0.98	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.622	0.068	0.495	0.000	0.264	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	144	149	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.507	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	60	54	49	0	59	0	0	0
N.S.	1	1.11	1.00	0.91	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.280	0.011	0.493	0.000	0.259	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.178	0.016	0.022	0.219	0.262	1.110	0.261	4.788

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.255	0.020	0.069	0.218	0.262	1.039	0.267	4.907

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.253	0.015	0.063	0.215	0.249	1.085	0.274	4.918

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.506	0.023	0.063	0.217	0.257	1.110	0.256	4.959

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	142	127	0	0	102	0	0	0
N.S.	1	1.12	1.00	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.866	0.006	0.000	0.000	0.265	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.925	0.022	0.073	0.214	0.251	1.197	0.268	4.902

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	1.367	0.018	0.069	0.222	0.260	1.294	0.260	4.952

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	1.594	0.022	0.063	0.225	0.256	1.494	0.250	5.111

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	0	242	0	0	178	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.000	0.009	0.000	0.000	0.263	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	2.610	0.021	0.062	0.256	0.257	2.175	0.263	4.719

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	495	458	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	279	256	0	0	0	0	0	0	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	71	66	60	0	88	0	0	0
N.S.	1	1.03	0.96	0.87	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.453	0.010	0.543	0.000	0.287	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.203	0.032	0.171	0.224	0.278	0.436	0.250	4.751

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	29	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.81	0.94	1.12	1.12
time (sec)	N/A	0.198	0.057	0.239	0.230	0.263	0.571	0.273	4.641

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	257	318	0	0	448	0	0	0
N.S.	1	1.11	1.38	0.00	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	0.864	4.612	0.000	0.000	0.293	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	253	318	0	0	448	0	0	0
N.S.	1	1.11	1.40	0.00	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.816	4.324	0.000	0.000	0.278	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	238	315	0	0	445	0	0	0
N.S.	1	1.11	1.47	0.00	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.763	4.677	0.000	0.000	0.282	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	65	165	64	82	121	0	0	0
N.S.	1	0.98	2.50	0.97	1.24	1.83	0.00	0.00	0.00
time (sec)	N/A	0.281	0.081	1.638	0.203	0.267	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	240	194	0	0	444	0	0	0
N.S.	1	1.11	0.89	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.815	2.661	0.000	0.000	0.287	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	254	199	0	0	460	0	0	0
N.S.	1	1.11	0.87	0.00	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.812	2.615	0.000	0.000	0.280	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	343	244	0	0	689	0	0	0
N.S.	1	1.22	0.87	0.00	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.980	3.661	0.000	0.000	0.309	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	99	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	100	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.181	0.009	0.522	0.188	0.277	0.181	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.180	0.006	0.451	0.187	0.271	0.110	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	0.00
time (sec)	N/A	0.179	0.010	0.551	0.190	0.246	0.113	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	12	0	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.09	0.00	0.00
time (sec)	N/A	0.179	0.006	0.564	0.183	0.248	0.272	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.00	0.00
time (sec)	N/A	0.183	0.006	0.700	0.195	0.246	0.527	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	18	34	0	0
N.S.	1	1.00	1.00	1.06	1.00	1.06	2.00	0.00	0.00
time (sec)	N/A	0.186	0.010	0.806	0.218	0.258	0.615	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	A	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	0	231	0	0	169	301	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.73	1.30	0.00	0.00
time (sec)	N/A	0.000	0.008	0.000	0.000	0.275	14.179	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	438	154	317	0	167	0	0	0
N.S.	1	2.04	0.72	1.47	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	1.959	0.164	7.284	0.000	0.272	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	247	336	0	0	0	0	0	0	0
N.S.	1	1.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.908	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	271	120	202	0	132	0	0	0
N.S.	1	1.73	0.76	1.29	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	1.057	0.100	2.396	0.000	0.262	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	161	120	0	0	105	151	0	0
N.S.	1	1.34	1.00	0.00	0.00	0.88	1.26	0.00	0.00
time (sec)	N/A	0.885	0.008	0.000	0.000	0.261	1.294	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	149	83	114	0	94	0	0	0
N.S.	1	1.43	0.80	1.10	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.529	0.066	1.020	0.000	0.257	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	136	132	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	47	0	0	0
N.S.	1	1.00	0.92	0.94	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.226	0.023	0.702	0.000	0.262	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	0.00
time (sec)	N/A	0.171	0.000	0.324	0.191	0.254	0.113	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.184	0.023	0.134	0.278	0.261	0.963	0.279	4.781

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.182	0.023	0.126	0.283	0.253	0.931	0.277	4.667

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.413	0.023	0.129	0.290	0.258	1.169	0.289	4.649

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	123	109	0	0	93	0	0	0
N.S.	1	1.13	1.00	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.747	0.008	0.000	0.000	0.268	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.808	0.024	0.132	0.292	0.253	2.956	0.280	4.641

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.206	0.024	0.131	0.289	0.260	5.583	0.286	4.703

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.444	0.025	0.126	0.282	0.267	11.051	0.277	4.610

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	0	224	0	0	168	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.000	0.008	0.000	0.000	0.270	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.394	0.028	0.126	0.294	0.257	36.966	0.281	4.620

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.160	0.023	0.125	0.296	0.257	65.464	0.281	4.715

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	19	19	19	19	19
N.S.	1	1.00	1.11	0.89	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.173	0.052	0.129	0.284	0.257	1.129	0.306	4.700

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	308	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	437	163	322	0	169	0	0	0
N.S.	1	2.00	0.75	1.48	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	1.988	0.125	4.194	0.000	0.273	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	284	185	0	0	141	264	0	0
N.S.	1	1.54	1.00	0.00	0.00	0.76	1.43	0.00	0.00
time (sec)	N/A	1.951	0.008	0.000	0.000	0.266	4.301	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	282	126	212	0	139	0	0	0
N.S.	1	1.69	0.75	1.27	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	1.114	0.111	1.525	0.000	0.264	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	196	226	0	0	0	0	0	0	0
N.S.	1	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.052	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	148	90	120	0	97	0	0	0
N.S.	1	1.36	0.83	1.10	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.555	0.064	0.724	0.000	0.262	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	79	74	0	0	67	114	0	0
N.S.	1	1.07	1.00	0.00	0.00	0.91	1.54	0.00	0.00
time (sec)	N/A	0.429	0.006	0.000	0.000	0.270	0.458	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	59	48	52	0	52	0	0	0
N.S.	1	0.98	0.80	0.87	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.252	0.021	0.647	0.000	0.269	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	19	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	1.00	1.00
time (sec)	N/A	0.187	0.023	0.121	0.309	0.268	1.178	0.274	4.715

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	45	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.312	0.007	0.000	0.000	0.259	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.402	0.028	0.128	0.277	0.264	1.277	0.275	4.821

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.558	0.024	0.119	0.267	0.252	1.801	0.272	4.814

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.802	0.026	0.126	0.276	0.264	3.027	0.286	4.805

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	215	163	0	0	141	0	0	0
N.S.	1	1.32	1.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	1.439	0.009	0.000	0.000	0.270	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	1.406	0.024	0.138	0.274	0.248	11.018	0.297	4.775

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.262	0.026	0.135	0.262	0.262	20.384	0.292	4.781

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	20	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	2.431	0.024	0.130	0.271	0.250	36.669	0.285	4.769

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	0	278	0	0	203	0	0	0
N.S.	1	0.00	1.00	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.000	0.010	0.000	0.000	0.272	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [142] had the largest ratio of [2.600000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.17	8	0.750
2	A	12	11	1.15	8	1.375
3	A	5	5	1.14	8	0.625
4	A	10	9	1.11	8	1.125
5	A	4	4	1.11	8	0.500
6	A	7	6	1.03	8	0.750
7	A	3	3	1.02	6	0.500
8	A	1	1	1.00	4	0.250
9	A	4	4	1.00	8	0.500
10	A	2	2	1.00	8	0.250
11	A	3	3	0.95	8	0.375
12	A	7	6	1.06	8	0.750
13	A	4	4	1.03	8	0.500
14	A	10	9	1.08	8	1.125
15	A	5	5	1.04	8	0.625
16	A	12	11	1.09	8	1.375
17	A	6	6	1.07	8	0.750
18	A	15	14	1.09	8	1.750
19	A	4	3	0.92	14	0.214
20	A	4	3	0.95	14	0.214
21	A	4	3	0.95	12	0.250
22	A	1	1	1.00	6	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	N/A	1	0	1.00	14	0.000
24	N/A	1	0	1.00	14	0.000
25	A	4	3	0.86	10	0.300
26	A	4	3	0.90	10	0.300
27	A	4	3	0.88	8	0.375
28	A	1	1	1.00	6	0.167
29	N/A	1	0	1.00	10	0.000
30	N/A	1	0	1.00	10	0.000
31	F	0	0	N/A	0.000	N/A
32	A	18	18	1.92	10	1.800
33	A	26	25	1.35	10	2.500
34	A	12	12	1.65	10	1.200
35	A	14	13	1.29	10	1.300
36	A	7	7	1.36	10	0.700
37	A	7	6	1.04	8	0.750
38	A	4	4	1.11	6	0.667
39	N/A	1	0	1.00	10	0.000
40	N/A	2	0	1.00	10	0.000
41	N/A	2	0	1.00	10	0.000
42	N/A	5	0	1.00	10	0.000
43	A	13	12	1.12	10	1.200
44	N/A	9	0	1.00	10	0.000
45	N/A	18	0	1.00	10	0.000
46	N/A	14	0	1.00	10	0.000
47	F	0	0	N/A	0.000	N/A
48	N/A	20	0	1.00	10	0.000
49	A	3	2	0.93	16	0.125
50	A	3	2	0.92	14	0.143
51	A	5	4	1.03	8	0.500
52	N/A	1	0	1.00	16	0.000
53	N/A	1	0	1.00	16	0.000
54	A	8	7	1.11	17	0.412

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
55	A	8	7	1.11	15	0.467
56	A	8	7	1.11	13	0.538
57	A	4	3	0.98	17	0.176
58	A	8	7	1.11	17	0.412
59	A	8	7	1.11	17	0.412
60	A	8	7	1.22	19	0.368
61	A	6	5	1.00	22	0.227
62	A	6	5	1.00	22	0.227
63	A	5	4	0.98	19	0.211
64	A	5	4	0.98	19	0.211
65	A	3	2	1.00	19	0.105
66	A	3	2	1.00	17	0.118
67	A	3	2	1.00	19	0.105
68	A	3	2	1.00	19	0.105
69	A	3	2	1.00	19	0.105
70	A	3	2	1.00	19	0.105
71	F	0	0	N/A	0.000	N/A
72	B	17	17	2.04	20	0.850
73	A	25	24	1.36	20	1.200
74	A	11	11	1.73	20	0.550
75	A	13	12	1.34	20	0.600
76	A	6	6	1.43	20	0.300
77	A	6	5	0.96	20	0.250
78	A	2	2	1.00	18	0.111
79	A	3	2	1.00	17	0.118
80	N/A	1	0	1.00	20	0.000
81	N/A	1	0	1.00	20	0.000
82	N/A	4	0	1.00	20	0.000
83	A	12	11	1.13	20	0.550
84	N/A	8	0	1.00	20	0.000
85	N/A	17	0	1.00	20	0.000
86	N/A	13	0	1.00	20	0.000
87	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	N/A	19	0	1.00	20	0.000
89	N/A	31	0	1.00	20	0.000
90	N/A	1	0	1.00	19	0.000
91	F	0	0	N/A	0.000	N/A
92	B	18	18	2.02	20	0.900
93	A	25	24	1.54	20	1.200
94	A	12	12	1.70	20	0.600
95	A	16	15	1.16	20	0.750
96	A	7	7	1.38	20	0.350
97	A	7	6	1.07	20	0.300
98	A	3	3	1.00	18	0.167
99	A	1	1	1.00	17	0.059
100	N/A	1	0	1.00	20	0.000
101	A	5	4	1.00	20	0.200
102	N/A	4	0	1.00	20	0.000
103	N/A	8	0	1.00	20	0.000
104	N/A	8	0	1.00	20	0.000
105	A	21	20	1.32	20	1.000
106	N/A	13	0	1.00	20	0.000
107	N/A	28	0	1.00	20	0.000
108	N/A	19	0	1.00	20	0.000
109	F	0	0	N/A	0.000	N/A
110	A	6	6	1.16	8	0.750
111	A	13	12	1.15	8	1.500
112	A	5	5	1.14	8	0.625
113	A	10	9	1.11	8	1.125
114	A	4	4	1.09	8	0.500
115	A	8	7	1.03	8	0.875
116	A	3	3	1.02	6	0.500
117	A	1	1	1.00	4	0.250
118	A	4	4	1.00	8	0.500
119	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	0.98	8	0.375
121	A	8	7	1.06	8	0.875
122	A	4	4	1.01	8	0.500
123	A	10	9	1.08	8	1.125
124	A	5	5	1.05	8	0.625
125	A	13	12	1.09	8	1.500
126	A	6	6	1.06	8	0.750
127	A	15	14	1.09	8	1.750
128	A	4	3	0.91	14	0.214
129	A	4	3	0.94	14	0.214
130	A	4	3	0.94	12	0.250
131	A	1	1	1.00	6	0.167
132	N/A	1	0	1.00	14	0.000
133	N/A	1	0	1.00	14	0.000
134	A	4	3	0.86	10	0.300
135	A	4	3	0.89	10	0.300
136	A	4	3	0.88	8	0.375
137	A	1	1	1.00	6	0.167
138	N/A	1	0	1.00	10	0.000
139	N/A	1	0	1.00	10	0.000
140	F	0	0	N/A	0.000	N/A
141	A	18	18	1.91	10	1.800
142	A	27	26	1.34	10	2.600
143	A	12	12	1.64	10	1.200
144	A	14	13	1.29	10	1.300
145	A	7	7	1.35	10	0.700
146	A	7	6	1.03	8	0.750
147	A	4	4	1.11	6	0.667
148	N/A	1	0	1.00	10	0.000
149	N/A	2	0	1.00	10	0.000
150	N/A	2	0	1.00	10	0.000
151	N/A	5	0	1.00	10	0.000
152	A	13	12	1.12	10	1.200

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	N/A	9	0	1.00	10	0.000
154	N/A	18	0	1.00	10	0.000
155	N/A	14	0	1.00	10	0.000
156	F	0	0	N/A	0.000	N/A
157	N/A	20	0	1.00	10	0.000
158	A	3	2	0.93	16	0.125
159	A	3	2	0.92	14	0.143
160	A	5	4	1.03	8	0.500
161	N/A	1	0	1.00	16	0.000
162	N/A	1	0	1.00	16	0.000
163	A	8	7	1.11	17	0.412
164	A	8	7	1.11	15	0.467
165	A	8	7	1.11	13	0.538
166	A	4	3	0.98	17	0.176
167	A	8	7	1.11	17	0.412
168	A	8	7	1.11	17	0.412
169	A	8	7	1.22	19	0.368
170	A	6	5	1.00	22	0.227
171	A	6	5	1.00	22	0.227
172	A	5	4	0.98	19	0.211
173	A	5	4	0.99	19	0.211
174	A	3	2	1.00	19	0.105
175	A	3	2	1.00	17	0.118
176	A	3	2	1.00	19	0.105
177	A	3	2	1.00	19	0.105
178	A	3	2	1.00	19	0.105
179	A	3	2	1.00	19	0.105
180	F	0	0	N/A	0.000	N/A
181	B	17	17	2.04	20	0.850
182	A	26	25	1.36	20	1.250
183	A	11	11	1.73	20	0.550
184	A	13	12	1.34	20	0.600
185	A	6	6	1.43	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
186	A	6	5	0.97	20	0.250
187	A	2	2	1.00	18	0.111
188	A	3	2	1.00	17	0.118
189	N/A	1	0	1.00	20	0.000
190	N/A	1	0	1.00	20	0.000
191	N/A	4	0	1.00	20	0.000
192	A	12	11	1.13	20	0.550
193	N/A	8	0	1.00	20	0.000
194	N/A	17	0	1.00	20	0.000
195	N/A	13	0	1.00	20	0.000
196	F	0	0	N/A	0.000	N/A
197	N/A	19	0	1.00	20	0.000
198	N/A	31	0	1.00	20	0.000
199	N/A	1	0	1.00	19	0.000
200	F	0	0	N/A	0.000	N/A
201	B	18	18	2.00	20	0.900
202	A	26	25	1.54	20	1.250
203	A	12	12	1.69	20	0.600
204	A	17	16	1.15	20	0.800
205	A	7	7	1.36	20	0.350
206	A	7	6	1.07	20	0.300
207	A	3	3	0.98	18	0.167
208	A	1	1	1.00	17	0.059
209	N/A	1	0	1.00	20	0.000
210	A	5	4	1.00	20	0.200
211	N/A	4	0	1.00	20	0.000
212	N/A	8	0	1.00	20	0.000
213	N/A	8	0	1.00	20	0.000
214	A	21	20	1.32	20	1.000
215	N/A	13	0	1.00	20	0.000
216	N/A	28	0	1.00	20	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
217	N/A	19	0	1.00	20	0.000
218	F	0	0	N/A	0.000	N/A

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^7 \text{FresnelS}(bx)\,dx$

3.1.1 Optimal result

Integrand size = 8, antiderivative size = 124

$$\int x^7 \text{FresnelS}(bx)\,dx = -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\,\text{FresnelS}(bx)}{8b^8\pi^4} + \frac{1}{8}x^8\,\text{FresnelS}(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2}$$

output
```
-35/8*x^3*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/8*x^7*cos(1/2*b^2*Pi*x^2)/b/Pi-10
5/8*FresnelS(b*x)/b^8/Pi^4+1/8*x^8*FresnelS(b*x)+105/8*x*sin(1/2*b^2*Pi*x^
2)/b^7/Pi^4-7/8*x^5*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

3.1.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int x^7 \text{FresnelS}(bx)\,dx = \frac{b^3\pi x^3(-35+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + (-105+b^8\pi^4x^8)\,\text{FresnelS}(bx) - 7bx(-15+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

input
```
Integrate[x^7*FresnelS[b*x],x]
```

output
```
(b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^
8)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*
Pi^4)
```

3.1.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6980, 3866, 3867, 3866, 3867, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \text{FresnelS}(bx)\,dx$$

$$\Big\downarrow \text{ 6980}$$

$$\frac{1}{8}x^8\text{FresnelS}(bx)-\frac{1}{8}b\int x^8\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{8}x^8\text{FresnelS}(bx)-\frac{1}{8}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3867}$$

$$\frac{1}{8}x^8\text{FresnelS}(bx)-\frac{1}{8}b\left(\frac{7\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{8}x^8\text{FresnelS}(bx)-\frac{1}{8}b\left(\frac{7\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3867}$$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)-\frac{1}{8}b\left(\frac{7\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3832$$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)-\frac{1}{8}b\left(\frac{7\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\text{FresnelS}(bx)}{\pi b^3}\right)}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^7*FresnelS[b*x],x]
```

output
```
(x^8*FresnelS[b*x])/8 - (b*(-((x^7*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + (7*((x
^5*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (5*(-((x^3*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi
)) + (3*(-(FresnelS[b*x]/(b^3*Pi)) + (x*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b
^2*Pi)))/(b^2*Pi)))/(b^2*Pi)))/8
```

3.1. $\int x^7\,\text{FresnelS}(bx)\,dx$

3.1.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.1.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

method	result
meijerg	$\frac{\pi b^{3} x^{11} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{11}{4}\right],\left[\frac{3}{2}, \frac{7}{4}, \frac{15}{4}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{66}$
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx) b^{8} x^{8}}{8}+\frac{b^{7} x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{8 \pi}-\frac{7\left(\frac{b^{5} x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{5\left(-\frac{b^{3} x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{\frac{3 b x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{3 \operatorname{FresnelS}(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8 \pi}}{b^{8}}$
default	$\frac{\frac{\operatorname{FresnelS}(bx) b^{8} x^{8}}{8}+\frac{b^{7} x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{8 \pi}-\frac{7\left(\frac{b^{5} x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{5\left(-\frac{b^{3} x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{\frac{3 b x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{3 \operatorname{FresnelS}(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8 \pi}}{b^{8}}$
parts	$\frac{x^{8} \operatorname{FresnelS}(bx)}{8}-\frac{b\left(-\frac{x^{7} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{\frac{7 x^{5} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{35\left(-\frac{x^{3} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{\frac{3 x \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{3 \operatorname{FresnelS}\left(\frac{\sqrt{\pi} b^{2} x}{\sqrt{b^{2} \pi}}\right)}{b^{2} \sqrt{\pi} \sqrt{b^{2} \pi}}}{b^{2} \pi}\right)}{b^{2} \pi}}{b^{2} \pi}\right)}{8}$

input
```
int(x^7*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/66*Pi*b^3*x^11*hypergeom([3/4,11/4],[3/2,7/4,15/4],-1/16*x^4*Pi^2*b^4)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int x^7 \operatorname{FresnelS}(bx)\, dx = \frac{\left(\pi^{3} b^{7} x^{7}-35\, \pi b^{3} x^{3}\right) \cos\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)+\left(\pi^{4} b^{8} x^{8}-105\right) \mathrm{S}\left(bx\right)-7\left(\pi^{2} b^{5} x^{5}-15\, bx\right) \sin\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)}{8\, \pi^{4} b^{8}}$$

input
```
integrate(x^7*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/8*((pi^3*b^7*x^7 - 35*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 -
105)*fresnel_sin(b*x) - 7*(pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/(pi
^4*b^8)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \frac{231x^8S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\Gamma\left(\frac{15}{4}\right)} + \frac{231x^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi b\Gamma\left(\frac{15}{4}\right)} - \frac{1617x^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{15}{4}\right)} - \frac{8085x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{15}{4}\right)} + \frac{24255x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{15}{4}\right)} - \frac{24255S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**7*fresnels(b*x),x)
```

output
```
231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2
*x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)
*gamma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*g
amma(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma
(3/4)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*p
i**4*b**8*gamma(15/4))
```

3.1.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \frac{1}{8}x^8\,\mathrm{S}\,(bx) - \frac{\sqrt{\frac{1}{2}}\left(\left(105i+105\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - \left(105i-105\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right) - 4\left(\sqrt{\frac{1}{2}}\pi^4b^7x^7 - 35\right.\right.}{16\,\pi^5b^8}$$

input
```
integrate(x^7*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/8*x^8*fresnel_sin(b*x) - 1/16*sqrt(1/2)*((105*I + 105)*(1/4)^(1/4)*pi*er
f(sqrt(1/2*I*pi)*b*x) - (105*I - 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b
*x) - 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*cos(1/2*pi*b^
2*x^2) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*
x^2))/(pi^5*b^8)
```

3.1.8 Giac [F]

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \int x^7 \mathrm{S}\,(bx)\ dx$$

input
```
integrate(x^7*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_sin(b*x), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelS}(bx)\,dx = \int x^7 \operatorname{FresnelS}(b\,x)\ dx$$

input
```
int(x^7*FresnelS(b*x),x)
```

output
```
int(x^7*FresnelS(b*x), x)
```

3.2 $\int x^6 \operatorname{FresnelS}(bx)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7\operatorname{FresnelS}(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}$$

output
```
-24/7*x^2*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/7*x^6*cos(1/2*b^2*Pi*x^2)/b/Pi+1/
7*x^7*FresnelS(b*x)+48/7*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*sin(1/2*b^2*
Pi*x^2)/b^3/Pi^2
```

3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{1}{7}x^7\operatorname{FresnelS}(bx) - \frac{6(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}$$

input
```
Integrate[x^6*FresnelS[b*x],x]
```

output
```
(x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*Fresnel
S[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

3.2.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {6980, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{FresnelS}(bx) \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{7}b\int x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\int x^6 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\int x^6 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\left(\frac{6\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2} - \frac{2x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\left(\frac{6\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{2x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\left(\frac{6\left(\frac{4\int -x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2} + \frac{2x^4 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{2x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}x^7 \text{FresnelS}(bx) - \frac{1}{14}b\left(\frac{6\left(\frac{2x^4 \sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{4\int x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2} - \frac{2x^6 \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)-\frac{1}{14}b\left(\frac{6\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{2x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3777

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)-\frac{1}{14}b\left(\frac{6\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{2x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3042

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)-\frac{1}{14}b\left(\frac{6\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\int\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{2x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3117

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)-\frac{1}{14}b\left(\frac{6\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{2x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^6*FresnelS[b*x],x]
```

output
```
(x^7*FresnelS[b*x])/7 - (b*((-2*x^6*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi) + (6*((2
*x^4*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (4*((-2*x^2*Cos[(b^2*Pi*x^2)/2])/(b^2
*Pi) + (4*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)))/(b^2*Pi)))/(b^2*Pi)))/14
```

3.2. $\int x^6\,\text{FresnelS}(bx)\,dx$

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.2.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{\pi\, b^{3} x^{10} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{3}{2}, \frac{7}{4}, \frac{7}{2}\right], -\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{60}$	29
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx) b^{7} x^{7}}{7}+\frac{b^{6} x^{6} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{7 \pi}-\frac{6\left(\frac{b^{4} x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{4\left(-\frac{b^{2} x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{2 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi^{2}}\right)}{\pi}\right)}{7 \pi}}{b^{7}}$	107
default	$\frac{\frac{\operatorname{FresnelS}(bx) b^{7} x^{7}}{7}+\frac{b^{6} x^{6} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{7 \pi}-\frac{6\left(\frac{b^{4} x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}-\frac{4\left(-\frac{b^{2} x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi}+\frac{2 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{\pi^{2}}\right)}{\pi}\right)}{7 \pi}}{b^{7}}$	107
parts	$\frac{x^{7} \operatorname{FresnelS}(bx)}{7}-\frac{b\left(-\frac{x^{6} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{6 x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{24\left(-\frac{x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{2 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{4} \pi^{2}}\right)}{b^{2} \pi}\right)}{7}$	113

input
```
int(x^6*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/60*Pi*b^3*x^10*hypergeom([3/4,5/2],[3/2,7/4,7/2],-1/16*x^4*Pi^2*b^4)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^{6} \operatorname{FresnelS}(bx)\, dx = \frac{\pi^{4} b^{7} x^{7} \operatorname{S}(bx) + (\pi^{3} b^{6} x^{6} - 24\, \pi b^{2} x^{2}) \cos\left(\frac{1}{2}\, \pi b^{2} x^{2}\right) - 6\, (\pi^{2} b^{4} x^{4} - 8) \sin\left(\frac{1}{2}\, \pi b^{2} x^{2}\right)}{7\, \pi^{4} b^{7}}$$

input
```
integrate(x^6*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/7*(pi^4*b^7*x^7*fresnel_sin(b*x) + (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/
2*pi*b^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{3x^7 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)} + \frac{3x^6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{28\pi b\Gamma\left(\frac{7}{4}\right)} - \frac{9x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{7}{4}\right)} - \frac{18x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{7}{4}\right)} + \frac{36\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**6*fresnels(b*x),x)
```

output
```
3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/
2)*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)
/(14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi
**3*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gam
ma(7/4))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \frac{1}{7}x^7\,\mathrm{S}\left(bx\right) + \frac{\left(\pi^3b^6x^6 - 24\,\pi b^2x^2\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right) - 6\left(\pi^2b^4x^4 - 8\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{7\,\pi^4b^7}$$

input
```
integrate(x^6*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/7*x^7*fresnel_sin(b*x) + 1/7*((pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/2*pi*
b^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.2.8 Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \int x^6 \operatorname{S}(bx)\,dx$$

input
```
integrate(x^6*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_sin(b*x), x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx)\,dx = \int x^6 \operatorname{FresnelS}(b\,x)\,dx$$

input
```
int(x^6*FresnelS(b*x),x)
```

output
```
int(x^6*FresnelS(b*x), x)
```

3.3 $\int x^5 \operatorname{FresnelS}(bx) \, dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x^5 \operatorname{FresnelS}(bx) \, dx = -\frac{5x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5 \operatorname{FresnelC}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 \operatorname{FresnelS}(bx) - \frac{5x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}$$

output
```
-5/2*x*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/6*x^5*cos(1/2*b^2*Pi*x^2)/b/Pi+5/2*F
resnelC(b*x)/b^6/Pi^3+1/6*x^6*FresnelS(b*x)-5/6*x^3*sin(1/2*b^2*Pi*x^2)/b^
3/Pi^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int x^5 \operatorname{FresnelS}(bx) \, dx = \frac{bx(-15 + b^4\pi^2x^4) \cos\left(\frac{1}{2}b^2\pi x^2\right) + 15 \operatorname{FresnelC}(bx) + b^6\pi^3x^6 \operatorname{FresnelS}(bx) - 5b^3\pi x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

input
```
Integrate[x^5*FresnelS[b*x],x]
```

output
```
(b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^
3*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6980, 3866, 3867, 3866, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{FresnelS}(bx) \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{6} x^6 \text{FresnelS}(bx) - \frac{1}{6} b \int x^6 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{6} x^6 \text{FresnelS}(bx) - \frac{1}{6} b \left(\frac{5 \int x^4 \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{\pi b^2} - \frac{x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3867}$$

$$\frac{1}{6} x^6 \text{FresnelS}(bx) - \frac{1}{6} b \left(\frac{5 \left(\frac{x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2} - \frac{3 \int x^2 \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx}{\pi b^2}\right)}{\pi b^2} - \frac{x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{6} x^6 \text{FresnelS}(bx) - \frac{1}{6} b \left(\frac{5 \left(\frac{x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2} - \frac{3 \left(\frac{\int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx}{\pi b^2} - \frac{x \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{6} x^6 \text{FresnelS}(bx) - \frac{1}{6} b \left(\frac{5 \left(\frac{x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2} - \frac{3 \left(\frac{\text{FresnelC}(bx)}{\pi b^3} - \frac{x \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^5 \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^5*FresnelS[b*x],x]
```

output
```
(x^6*FresnelS[b*x])/6 - (b*(-((x^5*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + (5*((-
3*(-((x*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + FresnelC[b*x]/(b^3*Pi)))/(b^2*Pi)
 + (x^3*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/6
```

3.3.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.3.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{\pi b^3 x^9 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{9}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{13}{4}\right],-\frac{x^4 \pi^2 b^4}{16}\right)}{54}$	29
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx) b^6 x^6}{6}+\frac{b^5 x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3 x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96
default	$\frac{\frac{\operatorname{FresnelS}(bx) b^6 x^6}{6}+\frac{b^5 x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3 x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96
parts	$\frac{x^6 \operatorname{FresnelS}(bx)}{6}-\frac{b\left(-\frac{x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}+\frac{\frac{5x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}-\frac{15\left(-\frac{x \cos\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}+\frac{\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\, b^2 x}{\sqrt{b^2 \pi}}\right)}{b^2 \sqrt{\pi}\, \sqrt{b^2 \pi}}\right)}{b^2 \pi}}{b^2 \pi}\right)}{6}$	123

input
```
int(x^5*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/54*Pi*b^3*x^9*hypergeom([3/4,9/4],[3/2,7/4,13/4],-1/16*x^4*Pi^2*b^4)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x^5 \operatorname{FresnelS}(bx)\, dx$$
$$= \frac{\pi^3 b^7 x^6 \operatorname{S}(bx) - 5\,\pi b^4 x^3 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + (\pi^2 b^6 x^5 - 15\, b^2 x) \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 15\,\sqrt{b^2}\operatorname{C}\left(\sqrt{b^2} x\right)}{6\,\pi^3 b^7}$$

input
```
integrate(x^5*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/6*(pi^3*b^7*x^6*fresnel_sin(b*x) - 5*pi*b^4*x^3*sin(1/2*pi*b^2*x^2) + (p
i^2*b^6*x^5 - 15*b^2*x)*cos(1/2*pi*b^2*x^2) + 15*sqrt(b^2)*fresnel_cos(sqr
t(b^2)*x))/(pi^3*b^7)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{\pi b^3 x^9 \Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right) {}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{3}{2},\frac{7}{4},\frac{13}{4}\end{matrix}\right| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**5*fresnels(b*x),x)
```

output
```
pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi
**2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))
```

3.3.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \frac{1}{6}x^6 \operatorname{S}(bx) - \frac{\sqrt{\frac{1}{2}}\left(20\sqrt{\frac{1}{2}}\pi^2 b^3 x^3 \sin\left(\frac{1}{2}\pi b^2 x^2\right) + (15i-15)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi}bx\right) - (15i+15)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi}b\right.\right.}{12\pi^4 b^6}$$

input
```
integrate(x^5*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/6*x^6*fresnel_sin(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*sin(1
/2*pi*b^2*x^2) + (15*I - 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*
I + 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) - 4*(sqrt(1/2)*pi^3*b^5*x^
5 - 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

3.3.8 Giac [F]

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \int x^5 \operatorname{S}(bx)\,dx$$

input
```
integrate(x^5*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_sin(b*x), x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx)\,dx = \int x^5 \operatorname{FresnelS}(b\,x)\,dx$$

input
```
int(x^5*FresnelS(b*x),x)
```

output
```
int(x^5*FresnelS(b*x), x)
```

3.4 $\int x^4 \operatorname{FresnelS}(bx)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5\operatorname{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

output
```
-8/5*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/5*x^4*cos(1/2*b^2*Pi*x^2)/b/Pi+1/5*x^5
*FresnelS(b*x)-4/5*x^2*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{1}{5}x^5\operatorname{FresnelS}(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

input
```
Integrate[x^4*FresnelS[b*x],x]
```

output
```
((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x
])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

3.4.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6980, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{FresnelS}(bx) \, dx$$

$$\Big\downarrow \text{ 6980}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{5}b \int x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b \int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b \int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b\left(\frac{4\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2} - \frac{2x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b\left(\frac{4\int x^2 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right) dx^2}{\pi b^2} - \frac{2x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b\left(\frac{4\left(\frac{2\int -\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2} + \frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{2x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx) - \frac{1}{10}b\left(\frac{4\left(\frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{2\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2} - \frac{2x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}x^5\,\text{FresnelS}(bx)-\frac{1}{10}b\left(\frac{4\left(\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{5}x^5\,\text{FresnelS}(bx)-\frac{1}{10}b\left(\frac{4\left(\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^4*FresnelS[b*x],x]
```

output
```
(x^5*FresnelS[b*x])/5 - (b*((-2*x^4*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi) + (4*((4
*Cos[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (2*x^2*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/
(b^2*Pi)))/10
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.4.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{\pi b^3 x^8 \operatorname{hypergeom}\left(\left[\frac{3}{4},2\right],\left[\frac{3}{2},\frac{7}{4},3\right],-\frac{x^4\pi^2 b^4}{16}\right)}{48}$	29
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(bx) b^5 x^5}{5}+\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2 x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80
default	$\frac{\frac{\operatorname{FresnelS}(bx) b^5 x^5}{5}+\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2 x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80
parts	$\frac{x^5 \operatorname{FresnelS}(bx)}{5}-\frac{b\left(-\frac{x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{4x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}}{b^2\pi}\right)}{5}$	83

input
```
int(x^4*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/48*Pi*b^3*x^8*hypergeom([3/4,2],[3/2,7/4,3],-1/16*x^4*Pi^2*b^4)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{\pi^3 b^5 x^5 \operatorname{S}(bx) - 4\pi b^2 x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) + (\pi^2 b^4 x^4 - 8)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{5\pi^3 b^5}$$

input
```
integrate(x^4*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/5*(pi^3*b^5*x^5*fresnel_sin(b*x) - 4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + (p
i^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{3x^5 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)} + \frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{20\pi b\Gamma\left(\frac{7}{4}\right)} - \frac{3x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{7}{4}\right)} - \frac{6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**4*fresnels(b*x),x)
```

output
```
3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/
2)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)
/(5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**
5*gamma(7/4))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \frac{1}{5}\,x^5\,\mathrm{S}\left(bx\right) - \frac{4\,\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - \left(\pi^2b^4x^4 - 8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

input
```
integrate(x^4*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/5*x^5*fresnel_sin(b*x) - 1/5*(4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - (pi^2*b
^4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.4.8 Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx)\,dx = \int x^4\,\mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x^4*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_sin(b*x), x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx)\, dx = \int x^4 \operatorname{FresnelS}(b\,x)\, dx$$

input
```
int(x^4*FresnelS(b*x),x)
```

output
```
int(x^4*FresnelS(b*x), x)
```

3.5 $\int x^3 \operatorname{FresnelS}(bx)\,dx$

3.5.1 Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\operatorname{FresnelS}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

output
```
1/4*x^3*cos(1/2*b^2*Pi*x^2)/b/Pi+3/4*FresnelS(b*x)/b^4/Pi^2+1/4*x^4*Fresne
lS(b*x)-3/4*x*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\operatorname{FresnelS}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelS}(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

input
```
Integrate[x^3*FresnelS[b*x],x]
```

output
```
(x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

3.5.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6980, 3866, 3867, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{FresnelS}(bx)\, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx) - \frac{1}{4}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3867}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx) - \frac{1}{4}b\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)}{\pi b^2} - \frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx) - \frac{1}{4}b\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\text{FresnelS}(bx)}{\pi b^3}\right)}{\pi b^2} - \frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^3*FresnelS[b*x],x]
```

output
```
(x^4*FresnelS[b*x])/4 - (b*(-((x^3*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + (3*(-(
FresnelS[b*x]/(b^3*Pi)) + (x*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/4
```

3.5.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{\frac{\pi x^3 b^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4}-\frac{3 \sin\left(\frac{b^2 \pi x^2}{2}\right) b x}{4}+\frac{\left(21 x^4 \pi^2 b^4+63\right) \operatorname{FresnelS}(b x)}{84}}{\pi^2 b^4}$	62
derivativedivides	$\frac{\frac{\operatorname{FresnelS}(b x) b^4 x^4}{4}+\frac{b^3 x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4\pi}-\frac{3\left(\frac{b x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{\operatorname{FresnelS}(b x)}{\pi}\right)}{4\pi}}{b^4}$	70
default	$\frac{\frac{\operatorname{FresnelS}(b x) b^4 x^4}{4}+\frac{b^3 x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{4\pi}-\frac{3\left(\frac{b x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi}-\frac{\operatorname{FresnelS}(b x)}{\pi}\right)}{4\pi}}{b^4}$	70
parts	$\frac{x^4 \operatorname{FresnelS}(b x)}{4}-\frac{b\left(-\frac{x^3 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}+\frac{\frac{3 x \sin\left(\frac{b^2 \pi x^2}{2}\right)}{b^2 \pi}-\frac{3 \operatorname{FresnelS}\left(\frac{\sqrt{\pi}\, b^2 x}{\sqrt{b^2 \pi}}\right)}{b^2 \sqrt{\pi}\, \sqrt{b^2 \pi}}}{b^2 \pi}\right)}{4}$	94

input
```
int(x^3*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
2/Pi^2/b^4*(1/8*Pi*x^3*b^3*cos(1/2*b^2*Pi*x^2)-3/8*sin(1/2*b^2*Pi*x^2)*b*x
+1/168*(21*Pi^2*b^4*x^4+63)*FresnelS(b*x))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right) - 3\,bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + \left(\pi^2b^4x^4+3\right)\operatorname{S}\left(bx\right)}{4\,\pi^2b^4}$$

input
```
integrate(x^3*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/4*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - 3*b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^
4*x^4 + 3)*fresnel_sin(b*x))/(pi^2*b^4)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{21x^4S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\Gamma\left(\frac{11}{4}\right)} + \frac{21x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi b\Gamma\left(\frac{11}{4}\right)} - \frac{63x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{11}{4}\right)} + \frac{63S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**3*fresnels(b*x),x)
```

output
```
21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x*
*2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/
4)/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**
4*gamma(11/4))
```

3.5.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \frac{1}{4}x^4 \operatorname{S}(bx) + \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi^2b^3x^3\cos\left(\frac{1}{2}\pi b^2x^2\right) - 12\sqrt{\frac{1}{2}}\pi bx\sin\left(\frac{1}{2}\pi b^2x^2\right) + (3i+3)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi}bx\right) - (3i-3)}{8\pi^3b^4}$$

input
```
integrate(x^3*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/4*x^4*fresnel_sin(b*x) + 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*cos(1/2
*pi*b^2*x^2) - 12*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^2*x^2) + (3*I + 3)*(1/4)^(
1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I - 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I
*pi)*b*x))/(pi^3*b^4)
```

3.5.8 Giac [F]

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \int x^3 \operatorname{S}(bx)\,dx$$

input
```
integrate(x^3*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_sin(b*x), x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)\,dx = \int x^3 \operatorname{FresnelS}(b\,x)\,dx$$

input
```
int(x^3*FresnelS(b*x),x)
```

output
```
int(x^3*FresnelS(b*x), x)
```

3.6 $\int x^2 \text{FresnelS}(bx) \, dx$

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \text{FresnelS}(bx) \, dx = \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

output
```
1/3*x^2*cos(1/2*b^2*Pi*x^2)/b/Pi+1/3*x^3*FresnelS(b*x)-2/3*sin(1/2*b^2*Pi*
x^2)/b^3/Pi^2
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2 \text{FresnelS}(bx) \, dx = \frac{x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 \text{FresnelS}(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

input
```
Integrate[x^2*FresnelS[b*x],x]
```

output
```
(x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*P
i*x^2)/2])/(3*b^3*Pi^2)
```

3.6.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6980, 3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelS}(bx) \, dx$$

$$\Big\downarrow \; 6980$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{6}b\int x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{6}b\int x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2$$

$$\Big\downarrow \; 3777$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{6}b\left(\frac{2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2} - \frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{6}b\left(\frac{2\int \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3117$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx) - \frac{1}{6}b\left(\frac{4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[x^2*FresnelS[b*x],x]
```

output
```
(x^3*FresnelS[b*x])/3 - (b*((-2*x^2*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi) + (4*Sin
[(b^2*Pi*x^2)/2])/(b^4*Pi^2)))/6
```

3.6.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
default	$\frac{\frac{\mathrm{FresnelS}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
parts	$\frac{x^3\,\mathrm{FresnelS}(bx)}{3}-\frac{b\left(-\frac{x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}\right)}{3}$	54
meijerg	$\frac{\frac{\sqrt{\pi}\,x^2b^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\sqrt{\pi}}+\frac{\pi^{\frac{3}{2}}x^3b^3\,\mathrm{FresnelS}(bx)}{3}}{b^3\pi^{\frac{3}{2}}}$	60

input
```
int(x^2*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*FresnelS(b*x)*b^3*x^3+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi
^2*sin(1/2*b^2*Pi*x^2))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \frac{\pi^2 b^3 x^3\,\mathrm{S}\left(bx\right) + \pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\,\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{3\,\pi^2 b^3}$$

input
```
integrate(x^2*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/3*(pi^2*b^3*x^3*fresnel_sin(b*x) + pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*si
n(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \frac{x^3 S(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x^2\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)} - \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi^2 b^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*fresnels(b*x),x)
```

output
```
x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*ga
mma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**
3*gamma(7/4))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \frac{1}{3}\,x^3\,\mathrm{S}\left(bx\right) + \frac{\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\,\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{3\,\pi^2 b^3}$$

input
```
integrate(x^2*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/3*x^3*fresnel_sin(b*x) + 1/3*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*sin(1/2
*pi*b^2*x^2))/(pi^2*b^3)
```

3.6.8 Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \int x^2\,\mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x^2*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_sin(b*x), x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx)\,dx = \int x^2 \operatorname{FresnelS}(b\,x)\,dx$$

input
```
int(x^2*FresnelS(b*x),x)
```

output
```
int(x^2*FresnelS(b*x), x)
```

3.7 $\int x \operatorname{FresnelS}(bx) \, dx$

3.7.1 Optimal result

Integrand size = 6, antiderivative size = 49

$$\int x \operatorname{FresnelS}(bx) \, dx = \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\operatorname{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2 \operatorname{FresnelS}(bx)$$

output
```
1/2*x*cos(1/2*b^2*Pi*x^2)/b/Pi-1/2*FresnelC(b*x)/b^2/Pi+1/2*x^2*FresnelS(b
*x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x \operatorname{FresnelS}(bx) \, dx = \frac{x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\operatorname{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2 \operatorname{FresnelS}(bx)$$

input
```
Integrate[x*FresnelS[b*x],x]
```

output
```
(x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*Fresnel
S[b*x])/2
```

3.7.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6980, 3866, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{FresnelS}(bx) \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{2}x^2 \operatorname{FresnelS}(bx) - \frac{1}{2}b \int x^2 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{2}x^2 \operatorname{FresnelS}(bx) - \frac{1}{2}b\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} - \frac{x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{2}x^2 \operatorname{FresnelS}(bx) - \frac{1}{2}b\left(\frac{\operatorname{FresnelC}(bx)}{\pi b^3} - \frac{x\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

input
```
Int[x*FresnelS[b*x],x]
```

output
```
-1/2*(b*(-((x*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + FresnelC[b*x]/(b^3*Pi))) +
(x^2*FresnelS[b*x])/2
```

3.7.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

method	result	size
meijerg	$\frac{b^{3}\pi\,x^{5}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{9}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{30}$	29
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)b^{2}x^{2}}{2}+\frac{bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx)}{2\pi}}{b^{2}}$	44
default	$\frac{\frac{\mathrm{FresnelS}(bx)b^{2}x^{2}}{2}+\frac{bx\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx)}{2\pi}}{b^{2}}$	44
parts	$\frac{x^{2}\,\mathrm{FresnelS}(bx)}{2}-\frac{b\left(-\frac{x\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right)}{2}$	64

input
```
int(x*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/30*b^3*Pi*x^5*hypergeom([3/4,5/4],[3/2,7/4,9/4],-1/16*x^4*Pi^2*b^4)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int x\,\mathrm{FresnelS}(bx)\,dx=\frac{\pi b^{3}x^{2}\,\mathrm{S}\left(bx\right)+b^{2}x\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-\sqrt{b^{2}}\,\mathrm{C}\left(\sqrt{b^{2}}x\right)}{2\,\pi b^{3}}$$

input
```
integrate(x*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
1/2*(pi*b^3*x^2*fresnel_sin(b*x) + b^2*x*cos(1/2*pi*b^2*x^2) - sqrt(b^2)*f
resnel_cos(sqrt(b^2)*x))/(pi*b^3)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int x\,\text{FresnelS}(bx)\,dx = \frac{\pi b^3x^5\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*fresnels(b*x),x)
```

output
```
pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))
```

3.7.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int x\,\text{FresnelS}(bx)\,dx = \frac{1}{2}x^2\,\text{S}\,(bx) + \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right) + (i-1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) - (i+1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right)\right)}{4\,\pi^2b^2}$$

input
```
integrate(x*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
1/2*x^2*fresnel_sin(b*x) + 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^
2*x^2) + (I - 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (I + 1)*(1/4)^(1
/4)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

3.7.8 Giac [F]

$$\int x\,\mathrm{FresnelS}(bx)\,dx=\int x\,\mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x*fresnel_sin(b*x), x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelS}(bx)\,dx=\int x\,\mathrm{FresnelS}(b\,x)\,dx$$

input
```
int(x*FresnelS(b*x),x)
```

output
```
int(x*FresnelS(b*x), x)
```

3.8 $\int \text{FresnelS}(bx)\,dx$

3.8.1 Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \text{FresnelS}(bx)\,dx = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + x\,\text{FresnelS}(bx)$$

output
```
cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + x\,\text{FresnelS}(bx)$$

input
```
Integrate[FresnelS[b*x],x]
```

output
```
Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6972}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\, dx$$

$$\Big\downarrow \text{6972}$$

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\,\text{FresnelS}(bx)$$

input
```
Int[FresnelS[b*x],x]
```

output
```
Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]
```

3.8.3.1 Defintions of rubi rules used

rule 6972
```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b\pi}+x\,\text{FresnelS}\left(bx\right)$	25
derivativedivides	$\frac{\text{FresnelS}(bx)bx+\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$	27
default	$\frac{\text{FresnelS}(bx)bx+\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$	27
meijerg	$\frac{b^{3}\pi\,x^{4}\,\text{hypergeom}\left(\left[\frac{3}{4},1\right],\left[\frac{3}{2},\frac{7}{4},2\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{24}$	29

input
```
int(FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{\pi bx\,\text{S}\,(bx) + \cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi b}$$

input
```
integrate(fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
(pi*b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2))/(pi*b)
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \text{FresnelS}(bx)\,dx = \frac{3xS(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x),x)
```

output
```
3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(
3/4)/(4*pi*b*gamma(7/4))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\,dx = \frac{bx\,\text{S}\,(bx) + \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
(b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2)/pi)/b
```

3.8.8 Giac [F]

$$\int \text{FresnelS}(bx)\,dx = \int \text{S}\left(bx\right)\,dx$$

input
```
integrate(fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\,dx = \int \text{FresnelS}(b\,x)\,dx$$

input
```
int(FresnelS(b*x),x)
```

output
```
int(FresnelS(b*x), x)
```

3.9 $\int \frac{\text{FresnelS}(bx)}{x} \, dx$

3.9.1 Optimal result

Integrand size = 8, antiderivative size = 73

$$\int \frac{\text{FresnelS}(bx)}{x} \, dx = \frac{1}{2} ibx \, {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{1}{2} ib^2 \pi x^2\right) - \frac{1}{2} ibx \, {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{1}{2} ib^2 \pi x^2\right)$$

output
```
1/2*I*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)-1/2*I*b*x*hyp
ergeom([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

3.9.2 Mathematica [F]

$$\int \frac{\text{FresnelS}(bx)}{x} \, dx = \int \frac{\text{FresnelS}(bx)}{x} \, dx$$

input
```
Integrate[FresnelS[b*x]/x,x]
```

output
```
Integrate[FresnelS[b*x]/x, x]
```

3.9.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6978, 26, 6912, 6914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x} \, dx$$

$$\Big\downarrow \text{6978}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right) \int \frac{\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right) b\sqrt{\pi} x\right)}{x}dx+\left(\frac{1}{4}-\frac{i}{4}\right) \int -\frac{i\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right) b\sqrt{\pi} x\right)}{x}dx$$

$$\Big\downarrow \text{26}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right) \int \frac{\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right) b\sqrt{\pi} x\right)}{x}dx-\left(\frac{1}{4}+\frac{i}{4}\right) \int \frac{\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right) b\sqrt{\pi} x\right)}{x}dx$$

$$\Big\downarrow \text{6912}$$

$$\frac{1}{2} i b x \, _2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2} i b^2 \pi x^2\right)-\left(\frac{1}{4}+\frac{i}{4}\right) \int \frac{\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right) b\sqrt{\pi} x\right)}{x}dx$$

$$\Big\downarrow \text{6914}$$

$$\frac{1}{2} i b x \, _2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2} i b^2 \pi x^2\right)-\frac{1}{2} i b x \, _2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2} i b^2 \pi x^2\right)$$

input
```
Int[FresnelS[b*x]/x,x]
```

output
```
(I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2] -
 (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

3.9.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6912
```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

rule 6914
```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

rule 6978
```
Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(1 + I)/4   Int[Erf[(Sqrt[
Pi]/2)*(1 + I)*b*x]/x, x], x] + Simp[(1 - I)/4   Int[Erf[(Sqrt[Pi]/2)*(1 -
I)*b*x]/x, x], x] /; FreeQ[b, x]
```

3.9.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

method	result	size
meijerg	$\frac{\pi\, x^{3} b^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{4}\right], \left[\frac{3}{2}, \frac{7}{4}, \frac{7}{4}\right], -\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{18}$	29

input
```
int(FresnelS(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)
```

3.9.5 Fricas [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\, dx = \int \frac{S\left(bx\right)}{x}\, dx$$

input
```
integrate(fresnel_sin(b*x)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)/x, x)
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \frac{\pi b^3 x^3 \Gamma^2\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{3}{2}, \frac{7}{4}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma^2\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x,x)
```

output
```
pi*b**3*x**3*gamma(3/4)**2*hyper((3/4, 3/4), (3/2, 7/4, 7/4), -pi**2*b**4*
x**4/16)/(32*gamma(7/4)**2)
```

3.9.7 Maxima [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{S}\,(bx)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)/x, x)
```

3.9.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{S}\,(bx)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x}\,dx$$

input
```
int(FresnelS(b*x)/x,x)
```

output
```
int(FresnelS(b*x)/x, x)
```

3.10 $\int \frac{\text{FresnelS}(bx)}{x^2}\,dx$

3.10.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\text{FresnelS}(bx)}{x} + \frac{1}{2}b\,\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

output
```
-FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\text{FresnelS}(bx)}{x} + \frac{1}{2}b\,\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

input
```
Integrate[FresnelS[b*x]/x^2,x]
```

output
```
-(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

3.10.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6980, 3856}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^2} \, dx$$

$$\Big\downarrow \; 6980$$

$$b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx - \frac{\text{FresnelS}(bx)}{x}$$

$$\Big\downarrow \; 3856$$

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{x}$$

input
```
Int[FresnelS[b*x]/x^2,x]
```

output
```
-(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

3.10.3.1 Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\text{FresnelS}(bx)}{x}+\frac{b\,\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}$	24
derivativedivides	$b\left(-\frac{\text{FresnelS}(bx)}{bx}+\frac{\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{\text{FresnelS}(bx)}{bx}+\frac{\text{Si}\left(\frac{b^2\pi\,x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{b^3\pi\,x^2\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{12}$	29

input
```
int(FresnelS(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \frac{bx\,\text{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,\text{S}\left(bx\right)}{2\,x}$$

input
```
integrate(fresnel_sin(b*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x*sin_integral(1/2*pi*b^2*x^2) - 2*fresnel_sin(b*x))/x
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \frac{\pi b^3x^2\Gamma\left(\frac{3}{4}\right)\,{}_2F_3\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**2,x)
```

output
```
pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**
4/16)/(16*gamma(7/4))
```

3.10.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = -\frac{1}{4}\,b\left(i\,\text{Ei}\left(\frac{1}{2}i\,\pi b^2x^2\right) - i\,\text{Ei}\left(-\frac{1}{2}i\,\pi b^2x^2\right)\right) - \frac{\text{S}\,(bx)}{x}$$

input
```
integrate(fresnel_sin(b*x)/x^2,x, algorithm="maxima")
```

output
```
-1/4*b*(I*Ei(1/2*I*pi*b^2*x^2) - I*Ei(-1/2*I*pi*b^2*x^2)) - fresnel_sin(b*
x)/x
```

3.10.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{S}\,(bx)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^2, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^2}\,dx$$

input
```
int(FresnelS(b*x)/x^2,x)
```

output
```
int(FresnelS(b*x)/x^2, x)
```

3.11 $\int \frac{\text{FresnelS}(bx)}{x^3}\,dx$

3.11.1 Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \frac{1}{2}b^2\pi\,\text{FresnelC}(bx) - \frac{\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

output
```
1/2*b^2*Pi*FresnelC(b*x)-1/2*FresnelS(b*x)/x^2-1/2*b*sin(1/2*b^2*Pi*x^2)/x
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \frac{1}{2}b^2\pi\,\text{FresnelC}(bx) - \frac{\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

input
```
Integrate[FresnelS[b*x]/x^3,x]
```

output
```
(b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])
/(2*x)
```

3.11.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6980, 3868, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^3} \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{2}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)}{2x^2}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{2}b\left(\pi b^2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)}{2x^2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{2}b\left(\pi b\,\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)}{2x^2}$$

input
```
Int[FresnelS[b*x]/x^3,x]
```

output
```
-1/2*FresnelS[b*x]/x^2 + (b*(b*Pi*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/x))/
2
```

3.11.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.11.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
meijerg	$\frac{\pi b^3 x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{6}$	27
derivativedivides	$b^2\left(-\frac{\mathrm{FresnelS}(bx)}{2b^2x^2}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2bx}+\frac{\pi\,\mathrm{FresnelC}(bx)}{2}\right)$	43
default	$b^2\left(-\frac{\mathrm{FresnelS}(bx)}{2b^2x^2}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2bx}+\frac{\pi\,\mathrm{FresnelC}(bx)}{2}\right)$	43
parts	$-\frac{\mathrm{FresnelS}(bx)}{2x^2}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}+\frac{b^2\pi^{\frac{3}{2}}\,\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{2}$	60

input
```
int(FresnelS(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*Pi*b^3*x*hypergeom([1/4,3/4],[5/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\mathrm{FresnelS}(bx)}{x^3}\,dx = \frac{\pi\sqrt{b^2}bx^2\,\mathrm{C}\left(\sqrt{b^2}x\right) - bx\sin\left(\frac{1}{2}\pi b^2x^2\right) - \mathrm{S}\left(bx\right)}{2\,x^2}$$

input
```
integrate(fresnel_sin(b*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(pi*sqrt(b^2)*b*x^2*fresnel_cos(sqrt(b^2)*x) - b*x*sin(1/2*pi*b^2*x^2)
 - fresnel_sin(b*x))/x^2
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \frac{\pi b^3 x \Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{5}{4}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**3,x)
```

output
```
pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*
b**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))
```

3.11.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{\pi x^2}\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, \frac{1}{2}i\pi b^2x^2\right) - (i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -\frac{1}{2}i\pi b^2x^2\right)\right)b^2}{16\,x} - \frac{\mathrm{S}\left(bx\right)}{2\,x^2}$$

input
```
integrate(fresnel_sin(b*x)/x^3,x, algorithm="maxima")
```

output
```
-1/16*sqrt(1/2)*sqrt(pi*x^2)*((I - 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2
) - (I + 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_si
n(b*x)/x^2
```

3.11.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{S(bx)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^3, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^3}\,dx$$

input
```
int(FresnelS(b*x)/x^3,x)
```

output
```
int(FresnelS(b*x)/x^3, x)
```

3.12 $\int \frac{\text{FresnelS}(bx)}{x^4} dx$

3.12.1 Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{\text{FresnelS}(bx)}{x^4} dx = \frac{1}{12} b^3 \pi \operatorname{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3} - \frac{b \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6x^2}$$

output
```
1/12*b^3*Pi*Ci(1/2*b^2*Pi*x^2)-1/3*FresnelS(b*x)/x^3-1/6*b*sin(1/2*b^2*Pi*
x^2)/x^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^4} dx = \frac{1}{12} b^3 \pi \operatorname{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelS}(bx)}{3x^3} - \frac{b \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{6x^2}$$

input
```
Integrate[FresnelS[b*x]/x^4,x]
```

output
```
(b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(
b^2*Pi*x^2)/2])/(6*x^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6980, 3860, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^4} \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{3} b \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^3} dx - \frac{\text{FresnelS}(bx)}{3x^3}$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{6} b \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\text{FresnelS}(bx)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6} b \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\text{FresnelS}(bx)}{3x^3}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{6} b \left(\frac{1}{2} \pi b^2 \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelS}(bx)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6} b \left(\frac{1}{2} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2 + \frac{\pi}{2}\right)}{x^2} dx^2 - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelS}(bx)}{3x^3}$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{6} b \left(\frac{1}{2} \pi b^2 \text{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelS}(bx)}{3x^3}$$

input
```
Int[FresnelS[b*x]/x^4,x]
```

output
```
-1/3*FresnelS[b*x]/x^3 + (b*((b^2*Pi*CosIntegral[(b^2*Pi*x^2)/2])/2 - Sin[
(b^2*Pi*x^2)/2]/x^2))/6
```

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

method	result	size
parts	$-\frac{\text{FresnelS}(bx)}{3x^3}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi\ \text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{3}$	46
derivativedivides	$b^3\left(-\frac{\text{FresnelS}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\ \text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\text{FresnelS}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\ \text{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
meijerg	$\frac{\pi^{\frac{3}{2}}b^3\left(-\frac{\pi^{\frac{3}{2}}x^4b^4\,\text{hypergeom}\left(\left[1,1,\frac{7}{4}\right],\left[2,2,\frac{5}{2},\frac{11}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{21}+\frac{\frac{16\gamma}{3}-\frac{16\ln(2)}{3}-\frac{80}{9}+\frac{32\ln(x)}{3}+\frac{16\ln(\pi)}{3}+\frac{32\ln(b)}{3}}{\sqrt{\pi}}\right)}{64}$	68

input
```
int(FresnelS(b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*FresnelS(b*x)/x^3+1/3*b*(-1/2*sin(1/2*b^2*Pi*x^2)/x^2+1/4*b^2*Pi*Ci(1
/2*b^2*Pi*x^2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \frac{\pi b^3x^3\,\text{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-4\,\text{S}\left(bx\right)}{12\,x^3}$$

input
```
integrate(fresnel_sin(b*x)/x^4,x, algorithm="fricas")
```

output
```
1/12*(pi*b^3*x^3*cos_integral(1/2*pi*b^2*x^2) - 2*b*x*sin(1/2*pi*b^2*x^2)
- 4*fresnel_sin(b*x))/x^3
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = -\frac{\pi^3b^7x^4\Gamma\left(\frac{7}{4}\right){}_3F_4\left(\begin{matrix}1,1,\frac{7}{4}\\2,2,\frac{5}{2},\frac{11}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{768\Gamma\left(\frac{11}{4}\right)}+\frac{\pi b^3\log\left(b^4x^4\right)}{24}$$

input
```
integrate(fresnels(b*x)/x**4,x)
```

output
```
-pi**3*b**7*x**4*gamma(7/4)*hyper((1, 1, 7/4), (2, 2, 5/2, 11/4), -pi**2*b
**4*x**4/16)/(768*gamma(11/4)) + pi*b**3*log(b**4*x**4)/24
```

3.12.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \frac{1}{24}\left(\pi\Gamma\left(-1, \frac{1}{2}i\,\pi b^2x^2\right) + \pi\Gamma\left(-1, -\frac{1}{2}i\,\pi b^2x^2\right)\right)b^3 - \frac{\mathrm{S}\,(bx)}{3\,x^3}$$

input
```
integrate(fresnel_sin(b*x)/x^4,x, algorithm="maxima")
```

output
```
1/24*(pi*gamma(-1, 1/2*I*pi*b^2*x^2) + pi*gamma(-1, -1/2*I*pi*b^2*x^2))*b^
3 - 1/3*fresnel_sin(b*x)/x^3
```

3.12.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\mathrm{S}\,(bx)}{x^4}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^4, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^4} \, dx = \int \frac{\text{FresnelS}(b\,x)}{x^4} \, dx$$

input
```
int(FresnelS(b*x)/x^4,x)
```

output
```
int(FresnelS(b*x)/x^4, x)
```

3.13 $\int \frac{\text{FresnelS}(bx)}{x^5} dx$

3.13.1 Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelS}(bx)}{x^5} dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}$$

output
```
-1/12*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x-1/12*b^4*Pi^2*FresnelS(b*x)-1/4*Fresnel
S(b*x)/x^4-1/12*b*sin(1/2*b^2*Pi*x^2)/x^3
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^5} dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x} - \frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx) - \frac{\text{FresnelS}(bx)}{4x^4} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}$$

input
```
Integrate[FresnelS[b*x]/x^5,x]
```

output
```
-1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresn
elS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

3.13.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6980, 3868, 3869, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^5} \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{4}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)}{4x^4}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{4}b\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)}{4x^4}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{4}b\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)}{4x^4}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{4}b\left(\frac{1}{3}\pi b^2\left(-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}-\pi b\,\text{FresnelS}(bx)\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)}{4x^4}$$

input
```
Int[FresnelS[b*x]/x^5,x]
```

output
```
-1/4*FresnelS[b*x]/x^4 + (b*((b^2*Pi*(-(Cos[(b^2*Pi*x^2)/2]/x) - b*Pi*Fres
nelS[b*x]))/3 - Sin[(b^2*Pi*x^2)/2]/(3*x^3)))/4
```

3.13.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$b^4\left(-\frac{\mathrm{FresnelS}(bx)}{4b^4x^4}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{FresnelS}(bx)\right)}{12}\right)$	65
default	$b^4\left(-\frac{\mathrm{FresnelS}(bx)}{4b^4x^4}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{FresnelS}(bx)\right)}{12}\right)$	65
meijerg	$\frac{\pi^2b^4\left(-\frac{32\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi xb}-\frac{32\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2x^3b^3}-\frac{32\left(x^4\pi^2b^4+3\right)\mathrm{FresnelS}(bx)}{3\pi^2x^4b^4}\right)}{128}$	79
parts	$-\frac{\mathrm{FresnelS}(bx)}{4x^4}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{x}-\frac{b^2\pi^{\frac{3}{2}}\,\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{3}\right)}{4}$	83

input
```
int(FresnelS(b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
b^4*(-1/4*FresnelS(b*x)/b^4/x^4-1/12/b^3/x^3*sin(1/2*b^2*Pi*x^2)+1/12*Pi*(
-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{\mathrm{FresnelS}(bx)}{x^5}\,dx = -\frac{\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4+3\right)\mathrm{S}\left(bx\right)}{12\,x^4}$$

input
```
integrate(fresnel_sin(b*x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) + b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^
4*x^4 + 3)*fresnel_sin(b*x))/x^4
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = \frac{\pi^2 b^4 S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64\Gamma\left(\frac{7}{4}\right)} + \frac{\pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x\Gamma\left(\frac{7}{4}\right)} + \frac{b \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x^3\Gamma\left(\frac{7}{4}\right)} + \frac{3S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64x^4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**5,x)
```

output
```
pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2
*x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)
/(64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

3.13.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{3}{2}}\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2}, \frac{1}{2}i\,\pi b^2 x^2\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{3}{2}, -\frac{1}{2}i\,\pi b^2 x^2\right)\right)b^4}{64\,x^3} - \frac{\mathrm{S}\,(bx)}{4\,x^4}$$

input
```
integrate(fresnel_sin(b*x)/x^5,x, algorithm="maxima")
```

output
```
-1/64*sqrt(1/2)*(pi*x^2)^(3/2)*(-(I + 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*
x^2) + (I - 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresn
el_sin(b*x)/x^4
```

3.13.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{S(bx)}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^5, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^5}\,dx$$

input
```
int(FresnelS(b*x)/x^5,x)
```

output
```
int(FresnelS(b*x)/x^5, x)
```

3.14 $\int \frac{\text{FresnelS}(bx)}{x^6} dx$

3.14.1 Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\text{FresnelS}(bx)}{x^6} dx = -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{\text{FresnelS}(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

output
```
-1/40*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^2-1/5*FresnelS(b*x)/x^5-1/80*b^5*Pi^2*S
i(1/2*b^2*Pi*x^2)-1/20*b*sin(1/2*b^2*Pi*x^2)/x^4
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)}{x^6} dx = -\frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2} - \frac{\text{FresnelS}(bx)}{5x^5} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

input
```
Integrate[FresnelS[b*x]/x^6,x]
```

output
```
-1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b
^2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

3.14.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6980, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^6} \, dx$$

$$\Big\downarrow \text{ 6980}$$

$$\frac{1}{5} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^5} dx - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{10} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx^2 - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx^2 - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{10} b \left(\frac{1}{4} \pi b^2 \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10} b \left(\frac{1}{4} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2 + \frac{\pi}{2}\right)}{x^4} dx^2 - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{10} b \left(\frac{1}{4} \pi b^2 \left(\frac{1}{2} \pi b^2 \int -\frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{10} b \left(\frac{1}{4} \pi b^2 \left(-\frac{1}{2} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)}{5x^5}$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{10}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)}{5x^5}$$

input
```
Int[FresnelS[b*x]/x^6,x]
```

output
```
-1/5*FresnelS[b*x]/x^5 + (b*(-1/2*Sin[(b^2*Pi*x^2)/2]/x^4 + (b^2*Pi*(-(Cos
[(b^2*Pi*x^2)/2]/x^2) - (b^2*Pi*SinIntegral[(b^2*Pi*x^2)/2])/2))/4))/10
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

method	result	size
meijerg	$-\frac{\pi b^{3} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{1}{2}, \frac{3}{2}, \frac{7}{4}\right], -\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{12 x^{2}}$	29
parts	$-\frac{\operatorname{FresnelS}(bx)}{5x^{5}}+\frac{b\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4x^{4}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2x^{2}}-\frac{b^{2}\pi \operatorname{Si}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{5}$	68
derivativedivides	$b^{5}\left(-\frac{\operatorname{FresnelS}(bx)}{5b^{5}x^{5}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{20b^{4}x^{4}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}-\frac{\pi \operatorname{Si}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{20}\right)$	71
default	$b^{5}\left(-\frac{\operatorname{FresnelS}(bx)}{5b^{5}x^{5}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{20b^{4}x^{4}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}-\frac{\pi \operatorname{Si}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{20}\right)$	71

input
```
int(FresnelS(b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/12*Pi*b^3/x^2*hypergeom([-1/2,3/4],[1/2,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{\pi^2 b^5 x^5 \,\text{Si}\left(\frac{1}{2}\,\pi b^2 x^2\right) + 2\,\pi b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,bx \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 16\,\text{S}\,(bx)}{80\,x^5}$$

input
```
integrate(fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/80*(pi^2*b^5*x^5*sin_integral(1/2*pi*b^2*x^2) + 2*pi*b^3*x^3*cos(1/2*pi
*b^2*x^2) + 4*b*x*sin(1/2*pi*b^2*x^2) + 16*fresnel_sin(b*x))/x^5
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x^2 \Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**6,x)
```

output
```
-pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/1
6)/(16*x**2*gamma(7/4))
```

3.14.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{\text{FresnelS}(bx)}{x^6}\,dx = -\frac{1}{80}\left(-i\,\pi^2\Gamma\left(-2, \frac{1}{2}i\,\pi b^2 x^2\right) + i\,\pi^2\Gamma\left(-2, -\frac{1}{2}i\,\pi b^2 x^2\right)\right)b^5 - \frac{\text{S}\,(bx)}{5\,x^5}$$

input
```
integrate(fresnel_sin(b*x)/x^6,x, algorithm="maxima")
```

output
```
-1/80*(-I*pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + I*pi^2*gamma(-2, -1/2*I*pi*b^
2*x^2))*b^5 - 1/5*fresnel_sin(b*x)/x^5
```

3.14.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^6} \, dx = \int \frac{\text{S}\left(bx\right)}{x^6} \, dx$$

input
```
integrate(fresnel_sin(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^6, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^6} \, dx = \int \frac{\text{FresnelS}(b\,x)}{x^6} \, dx$$

input
```
int(FresnelS(b*x)/x^6,x)
```

output
```
int(FresnelS(b*x)/x^6, x)
```

3.15 $\int \frac{\text{FresnelS}(bx)}{x^7}\,dx$

3.15.1 Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3} - \frac{1}{90}b^6\pi^3\,\text{FresnelC}(bx) - \frac{\text{FresnelS}(bx)}{6x^6} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}$$

output
```
-1/90*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^3-1/90*b^6*Pi^3*FresnelC(b*x)-1/6*Fresn
elS(b*x)/x^6-1/30*b*sin(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*sin(1/2*b^2*Pi*x
^2)/x
```

3.15.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = \frac{1}{90}\left(-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} - b^6\pi^3\,\text{FresnelC}(bx) - \frac{15\,\text{FresnelS}(bx)}{x^6} + \frac{b(-3+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\right)$$

input
```
Integrate[FresnelS[b*x]/x^7,x]
```

output
```
(-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*Fresne
lS[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

3.15.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6980, 3868, 3869, 3868, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^7} \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{6} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx - \frac{\text{FresnelS}(bx)}{6x^6}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6} b \left(\frac{1}{5} \pi b^2 \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)}{6x^6}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{6} b \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)}{6x^6}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6} b \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \left(\pi b^2 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)}{6x^6}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{6} b \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \left(\pi b \, \text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)}{6x^6}$$

input
```
Int[FresnelS[b*x]/x^7,x]
```

output
```
-1/6*FresnelS[b*x]/x^6 + (b*(-1/5*Sin[(b^2*Pi*x^2)/2]/x^5 + (b^2*Pi*(-1/3*
Cos[(b^2*Pi*x^2)/2]/x^3 - (b^2*Pi*(b*Pi*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2
]/x))/3))/5))/6
```

3.15.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.31

method	result	siz
meijerg	$-\frac{\pi b^3 \operatorname{hypergeom}\left(\left[-\frac{3}{4},\frac{3}{4}\right],\left[\frac{1}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{18x^3}$	29
derivativedivides	$b^6\left(-\frac{\operatorname{FresnelS}(bx)}{6b^6x^6}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\operatorname{FresnelC}(bx)\right)}{3}\right)}{30}\right)$	86
default	$b^6\left(-\frac{\operatorname{FresnelS}(bx)}{6b^6x^6}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\operatorname{FresnelC}(bx)\right)}{3}\right)}{30}\right)$	86
parts	$-\frac{\operatorname{FresnelS}(bx)}{6x^6}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5x^5}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}+\frac{b^2\pi^{\frac{3}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{3}\right)}{5}\right)}{6}$	10

input
```
int(FresnelS(b*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/18*Pi*b^3/x^3*hypergeom([-3/4,3/4],[1/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

3.15. $\int \frac{\operatorname{FresnelS}(bx)}{x^7}\,dx$

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = -\frac{\pi^3\sqrt{b^2}b^5x^6\,\text{C}\left(\sqrt{b^2}x\right)+\pi b^3x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)-\left(\pi^2b^5x^5-3\,bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)+15\,\text{S}\left(bx\right)}{90\,x^6}$$

input
```
integrate(fresnel_sin(b*x)/x^7,x, algorithm="fricas")
```

output
```
-1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_cos(sqrt(b^2)*x) + pi*b^3*x^3*cos(1/
2*pi*b^2*x^2) - (pi^2*b^5*x^5 - 3*b*x)*sin(1/2*pi*b^2*x^2) + 15*fresnel_si
n(b*x))/x^6
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = \frac{\pi b^3\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**7,x)
```

output
```
pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*
b**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))
```

3.15.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{\text{FresnelS}(bx)}{x^7}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(\pi x^2\right)^{\frac{5}{2}}\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{1}{2}i\pi b^2x^2\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^6}{192\,x^5}-\frac{\text{S}\left(bx\right)}{6\,x^6}$$

input
```
integrate(fresnel_sin(b*x)/x^7,x, algorithm="maxima")
```

output
```
-1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I - 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2
*x^2) + (I + 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fres
nel_sin(b*x)/x^6
```

3.15.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^7} \, dx = \int \frac{S(bx)}{x^7} \, dx$$

input
```
integrate(fresnel_sin(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^7, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^7} \, dx = \int \frac{\text{FresnelS}(b\,x)}{x^7} \, dx$$

input
```
int(FresnelS(b*x)/x^7,x)
```

output
```
int(FresnelS(b*x)/x^7, x)
```

3.16 $\int \frac{\text{FresnelS}(bx)}{x^8} dx$

3.16.1 Optimal result

Integrand size = 8, antiderivative size = 102

$$\int \frac{\text{FresnelS}(bx)}{x^8} dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} - \frac{1}{672}b^7\pi^3\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelS}(bx)}{7x^7} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}$$

output
```
-1/672*b^7*Pi^3*Ci(1/2*b^2*Pi*x^2)-1/168*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^4-1/
7*FresnelS(b*x)/x^7-1/42*b*sin(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*sin(1/2*
b^2*Pi*x^2)/x^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{\text{FresnelS}(bx)}{x^8} dx = \frac{1}{672}\left(-\frac{4b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} - b^7\pi^3\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{96\,\text{FresnelS}(bx)}{x^7} + \frac{2b(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\right)$$

input
```
Integrate[FresnelS[b*x]/x^8,x]
```

output
```
((-4*b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - b^7*Pi^3*CosIntegral[(b^2*Pi*x^2)/2
] - (96*FresnelS[b*x])/x^7 + (2*b*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])
/x^6)/672
```

3.16.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {6980, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx$$

$$\Big\downarrow \text{ 6980}$$

$$\frac{1}{7}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{14}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{14}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2 - \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right) - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2 - \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right) - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int -\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2 - \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right) - \frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)}{7x^7}$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)}{7x^7}$$

input
```
Int[FresnelS[b*x]/x^8,x]
```

output
```
-1/7*FresnelS[b*x]/x^7 + (b*(-1/3*Sin[(b^2*Pi*x^2)/2]/x^6 + (b^2*Pi*(-1/2*
Cos[(b^2*Pi*x^2)/2]/x^4 - (b^2*Pi*((b^2*Pi*CosIntegral[(b^2*Pi*x^2)/2])/2
- Sin[(b^2*Pi*x^2)/2]/x^2))/4))/6))/14
```

3.16. $\int \frac{\text{FresnelS}(bx)}{x^8}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result	s
meijerg	$\frac{\pi^{\frac{7}{2}} b^{7} \left(\frac{\pi^{\frac{3}{2}} x^{4} b^{4} \,\mathrm{hypergeom}\left(\left[1,1,\frac{11}{4}\right],\left[2,3,\frac{7}{2},\frac{15}{4}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{165}-\frac{16\left(-\frac{89}{21}+2\gamma-2\ln(2)+4\ln(x)+2\ln(\pi)+4\ln(b)\right)}{21\sqrt{\pi}}-\frac{128}{3\pi^{\frac{5}{2}} x^{4} b^{4}}\right)}{1024}$	7
parts	$-\frac{\mathrm{FresnelS}(bx)}{7x^{7}}+\frac{b\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{6x^{6}}+\frac{b^{2}\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4x^{4}}-\frac{b^{2}\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2x^{2}}+\frac{b^{2}\pi\,\mathrm{Ci}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{7}$	9
derivativedivides	$b^{7}\left(-\frac{\mathrm{FresnelS}(bx)}{7b^{7}x^{7}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{42b^{6}x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4b^{4}x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	9
default	$b^{7}\left(-\frac{\mathrm{FresnelS}(bx)}{7b^{7}x^{7}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{42b^{6}x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4b^{4}x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}+\frac{\pi\,\mathrm{Ci}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	9

input
```
int(FresnelS(b*x)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/1024*Pi^(7/2)*b^7*(1/165*Pi^(3/2)*x^4*b^4*hypergeom([1,1,11/4],[2,3,7/2,
15/4],-1/16*x^4*Pi^2*b^4)-16/21*(-89/21+2*gamma-2*ln(2)+4*ln(x)+2*ln(Pi)+4
*ln(b))/Pi^(1/2)-128/3/Pi^(5/2)/x^4/b^4)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = -\frac{\pi^3 b^7 x^7 \,\text{Ci}\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,\pi b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 2\left(\pi^2 b^5 x^5 - 8\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 96\,\text{S}\left(bx\right)}{672\,x^7}$$

input
```
integrate(fresnel_sin(b*x)/x^8,x, algorithm="fricas")
```

output
```
-1/672*(pi^3*b^7*x^7*cos_integral(1/2*pi*b^2*x^2) + 4*pi*b^3*x^3*cos(1/2*p
i*b^2*x^2) - 2*(pi^2*b^5*x^5 - 8*b*x)*sin(1/2*pi*b^2*x^2) + 96*fresnel_sin
(b*x))/x^7
```

3.16.6 Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(\begin{matrix} 1,1,\frac{11}{4} \\ 2,3,\frac{7}{2},\frac{15}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{61440\Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log\left(b^4 x^4\right)}{1344} - \frac{\pi b^3}{24 x^4}$$

input
```
integrate(fresnels(b*x)/x**8,x)
```

output
```
pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2
*b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b
**3/(24*x**4)
```

3.16.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.45

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = -\frac{1}{224}\left(\pi^3\Gamma\left(-3, \frac{1}{2} i\,\pi b^2 x^2\right) + \pi^3\Gamma\left(-3, -\frac{1}{2} i\,\pi b^2 x^2\right)\right) b^7 - \frac{\text{S}\left(bx\right)}{7\,x^7}$$

input
```
integrate(fresnel_sin(b*x)/x^8,x, algorithm="maxima")
```

output
```
-1/224*(pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + pi^3*gamma(-3, -1/2*I*pi*b^2*x^
2))*b^7 - 1/7*fresnel_sin(b*x)/x^7
```

3.16.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{S(bx)}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^8, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^8}\,dx$$

input
```
int(FresnelS(b*x)/x^8,x)
```

output
```
int(FresnelS(b*x)/x^8, x)
```

3.17 $\int \frac{\text{FresnelS}(bx)}{x^9}\,dx$

3.17.1 Optimal result

Integrand size = 8, antiderivative size = 119

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)-\frac{\text{FresnelS}(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}$$

output
```
-1/280*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^5+1/840*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x
+1/840*b^8*Pi^4*FresnelS(b*x)-1/8*FresnelS(b*x)/x^8-1/56*b*sin(1/2*b^2*Pi*
x^2)/x^7+1/840*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^3
```

3.17.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = \frac{b^3\pi x^3(-3+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\,\text{FresnelS}(bx)+bx(-15+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

input
```
Integrate[FresnelS[b*x]/x^9,x]
```

output
```
(b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8
)*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

3.17.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6980, 3868, 3869, 3868, 3869, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^9} \, dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{8} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^8} dx - \frac{\text{FresnelS}(bx)}{8x^8}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{8} b \left(\frac{1}{7} \pi b^2 \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)}{8x^8}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{8} b \left(\frac{1}{7} \pi b^2 \left(-\frac{1}{5} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)}{8x^8}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{8} b \left(\frac{1}{7} \pi b^2 \left(-\frac{1}{5} \pi b^2 \left(\frac{1}{3} \pi b^2 \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)}{8x^8}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{8} b \left(\frac{1}{7} \pi b^2 \left(-\frac{1}{5} \pi b^2 \left(\frac{1}{3} \pi b^2 \left(-\pi b^2 \int \sin\left(\frac{1}{2} b^2 \pi x^2\right) dx - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \text{sin} \ldots \frac{\text{FresnelS}(bx)}{8x^8}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{8} b \left(\frac{1}{7} \pi b^2 \left(-\frac{1}{5} \pi b^2 \left(\frac{1}{3} \pi b^2 \left(-\frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{x} - \pi b \, \text{FresnelS}(bx)\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{7x^7} \ldots \frac{\text{FresnelS}(bx)}{8x^8}$$

input
```
Int[FresnelS[b*x]/x^9,x]
```

output
```
-1/8*FresnelS[b*x]/x^8 + (b*(-1/7*Sin[(b^2*Pi*x^2)/2]/x^7 + (b^2*Pi*(-1/5*
Cos[(b^2*Pi*x^2)/2]/x^5 - (b^2*Pi*((b^2*Pi*(-(Cos[(b^2*Pi*x^2)/2]/x) - b*P
i*FresnelS[b*x]))/3 - Sin[(b^2*Pi*x^2)/2]/(3*x^3)))/5))/7))/8
```

3.17.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

method	result
meijerg	$-\frac{\pi b^3 \operatorname{hypergeom}\left(\left[-\frac{5}{4},\frac{3}{4}\right],\left[-\frac{1}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{30x^5}$
derivativedivides	$b^8\left(-\frac{\operatorname{FresnelS}(bx)}{8b^8x^8}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\operatorname{FresnelS}(bx)\right)}{3}\right)}{5}\right.}{56}\right.$
default	$b^8\left(-\frac{\operatorname{FresnelS}(bx)}{8b^8x^8}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\operatorname{FresnelS}(bx)\right)}{3}\right)}{5}\right.}{56}\right.$

$b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{7x^7}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{5x^5}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{x}-\frac{b^2\pi^{\frac{3}{2}}\operatorname{FresnelS}\left(\ldots\right)}{\sqrt{b^2\pi}}\right.}{3}\right.}{5}\right.}{7}\right.$

3.17. $\int \frac{\operatorname{FresnelS}(bx)}{x^9}\,dx$

input
```
int(FresnelS(b*x)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/30*Pi*b^3/x^5*hypergeom([-5/4,3/4],[-1/4,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{\mathrm{FresnelS}(bx)}{x^9}\,dx = \frac{(\pi^3 b^7 x^7 - 3\,\pi b^3 x^3)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + (\pi^4 b^8 x^8 - 105)\,\mathrm{S}\,(bx) + (\pi^2 b^5 x^5 - 15\,bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{840\,x^8}$$

input
```
integrate(fresnel_sin(b*x)/x^9,x, algorithm="fricas")
```

output
```
1/840*((pi^3*b^7*x^7 - 3*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 -
 105)*fresnel_sin(b*x) + (pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/x^8
```

3.17.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{\mathrm{FresnelS}(bx)}{x^9}\,dx = \frac{\pi^4 b^8 S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{3584\Gamma\left(\frac{7}{4}\right)} + \frac{\pi^3 b^7 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x\Gamma\left(\frac{7}{4}\right)} + \frac{\pi^2 b^5 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^3\Gamma\left(\frac{7}{4}\right)} - \frac{3\pi b^3 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^5\Gamma\left(\frac{7}{4}\right)} - \frac{15b \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^7\Gamma\left(\frac{7}{4}\right)} - \frac{15S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{512x^8\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**9,x)
```

output
```
pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi
*b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**
2/2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*ga
mma(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3
584*x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))
```

3.17.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{7}{2}}\left((i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{1}{2}i\pi b^2x^2\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^8}{512\,x^7} - \frac{\text{S}\,(bx)}{8\,x^8}$$

input
```
integrate(fresnel_sin(b*x)/x^9,x, algorithm="maxima")
```

output
```
-1/512*sqrt(1/2)*(pi*x^2)^(7/2)*((I + 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2*
x^2) - (I - 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fresn
el_sin(b*x)/x^8
```

3.17.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\text{S}\,(bx)}{x^9}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^9, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^9}\,dx$$

input
```
int(FresnelS(b*x)/x^9,x)
```

output
```
int(FresnelS(b*x)/x^9, x)
```

3.18 $\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx$

3.18.1 Optimal result

Integrand size = 8, antiderivative size = 127

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}-\frac{\text{FresnelS}(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

output
```
-1/432*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^6+1/3456*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/
x^2-1/9*FresnelS(b*x)/x^9+1/6912*b^9*Pi^4*Si(1/2*b^2*Pi*x^2)-1/72*b*sin(1/
2*b^2*Pi*x^2)/x^8+1/1728*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^4
```

3.18.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{\frac{2b^3\pi(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}-\frac{768\,\text{FresnelS}(bx)}{x^9}+\frac{4b(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}+b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

input
```
Integrate[FresnelS[b*x]/x^10,x]
```

output
```
((2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (768*FresnelS[b*
x])/x^9 + (4*b*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*Si
nIntegral[(b^2*Pi*x^2)/2])/6912
```

3.18.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {6980, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}dx$$

$$\Big\downarrow \text{6980}$$

$$\frac{1}{9}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{18}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int -\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 3778

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 3042

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 3778

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\right.\right.$$
$$\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 25

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\right.\right.$$
$$\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 3042

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\right.\right.$$
$$\frac{\text{FresnelS}(bx)}{9x^9}$$

$\big\downarrow$ 3780

$$\frac{1}{18}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\frac{1}{2}\pi\right.}{4x^8}\right.$$
$$\frac{\text{FresnelS}(bx)}{9x^9}$$

3.18. $\int\frac{\text{FresnelS}(bx)}{x^{10}}\,dx$

input
```
Int[FresnelS[b*x]/x^10,x]
```

output
```
-1/9*FresnelS[b*x]/x^9 + (b*(-1/4*Sin[(b^2*Pi*x^2)/2]/x^8 + (b^2*Pi*(-1/3*
Cos[(b^2*Pi*x^2)/2]/x^6 - (b^2*Pi*(-1/2*Sin[(b^2*Pi*x^2)/2]/x^4 + (b^2*Pi*
(-(Cos[(b^2*Pi*x^2)/2]/x^2) - (b^2*Pi*SinIntegral[(b^2*Pi*x^2)/2])/2))/4))
/6))/8))/18
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6980
```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Sin[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

method	result
meijerg	$-\frac{\pi b^3 \,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{36x^6}$
parts	$-\frac{\mathrm{FresnelS}(bx)}{9x^9}+\frac{b\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{8x^8}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{8}\right)}{9}$
derivativedivides	$b^9\left(-\frac{\mathrm{FresnelS}(bx)}{9b^9x^9}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$
default	$b^9\left(-\frac{\mathrm{FresnelS}(bx)}{9b^9x^9}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$

3.18. $\int \frac{\mathrm{FresnelS}(bx)}{x^{10}}\,dx$

input
```
int(FresnelS(b*x)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/36*Pi*b^3/x^6*hypergeom([-3/2,3/4],[-1/2,3/2,7/4],-1/16*x^4*Pi^2*b^4)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{\pi^4 b^9 x^9 \,\text{Si}\left(\frac{1}{2}\pi b^2 x^2\right) + 2\left(\pi^3 b^7 x^7 - 8\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) + 4\left(\pi^2 b^5 x^5 - 24\,bx\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right) - 768\,\text{S}\,(bx)}{6912\,x^9}$$

input
```
integrate(fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

output
```
1/6912*(pi^4*b^9*x^9*sin_integral(1/2*pi*b^2*x^2) + 2*(pi^3*b^7*x^7 - 8*pi
*b^3*x^3)*cos(1/2*pi*b^2*x^2) + 4*(pi^2*b^5*x^5 - 24*b*x)*sin(1/2*pi*b^2*x
^2) - 768*fresnel_sin(b*x))/x^9
```

3.18.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{\pi b^3 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{48 x^6 \Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(fresnels(b*x)/x**10,x)
```

output
```
-pi*b**3*gamma(3/4)*hyper((-3/2, 3/4), (-1/2, 3/2, 7/4), -pi**2*b**4*x**4/
16)/(48*x**6*gamma(7/4))
```

3.18.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = -\frac{1}{576}\left(i\,\pi^4\Gamma\left(-4,\frac{1}{2}i\,\pi b^2x^2\right) - i\,\pi^4\Gamma\left(-4,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^9 - \frac{\mathrm{S}\,(bx)}{9\,x^9}$$

input
```
integrate(fresnel_sin(b*x)/x^10,x, algorithm="maxima")
```

output
```
-1/576*(I*pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) - I*pi^4*gamma(-4, -1/2*I*pi*b^
2*x^2))*b^9 - 1/9*fresnel_sin(b*x)/x^9
```

3.18.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\mathrm{S}\,(bx)}{x^{10}}\,dx$$

input
```
integrate(fresnel_sin(b*x)/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)/x^10, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(b\,x)}{x^{10}}\,dx$$

input
```
int(FresnelS(b*x)/x^10,x)
```

output
```
int(FresnelS(b*x)/x^10, x)
```

3.19 $\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 296

$$\begin{aligned}\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx =& \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &+ \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\ &+ \frac{d^2(bc-ad)(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} \\ &+ \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} \\ &- \frac{3d(bc-ad)^2\operatorname{FresnelC}(a+bx)}{2b^4\pi} \\ &- \frac{(bc-ad)^4\operatorname{FresnelS}(a+bx)}{4b^4d} \\ &+ \frac{3d^3\operatorname{FresnelS}(a+bx)}{4b^4\pi^2} + \frac{(c+dx)^4\operatorname{FresnelS}(a+bx)}{4d} \\ &- \frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} \\ &- \frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}\end{aligned}$$

output
```
(-a*d+b*c)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*d*(-a*d+b*c)^2*(b*x+a)*cos(1
/2*Pi*(b*x+a)^2)/b^4/Pi+d^2*(-a*d+b*c)*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4
/Pi+1/4*d^3*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)^2*Fres
nelC(b*x+a)/b^4/Pi-1/4*(-a*d+b*c)^4*FresnelS(b*x+a)/b^4/d+3/4*d^3*FresnelS
(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelS(b*x+a)/d-2*d^2*(-a*d+b*c)*sin(1/2*
Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

3.19.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.43

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx$$
$$= \frac{4b^3c^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-6ab^2c^2d\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+4a^2bcd^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-a^3d^3\pi\cos\left(\frac{1}{2}\pi(a-\right.}{}$$

input
```
Integrate[(c + d*x)^3*FresnelS[a + b*x],x]
```

output
```
(4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)
^2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a
+ b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi
*x*Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^
3*c*d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x
)^2)/2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*Fr
esnelC[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2
*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4)
)*FresnelS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*
(a + b*x)^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

3.19.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \text{6982}$$

$$\frac{(c+dx)^4 \operatorname{FresnelS}(a+bx)}{4d}-\frac{b\int (c+dx)^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{4d}$$

$$\Big\downarrow \text{3914}$$

$$\frac{(c+dx)^4 \operatorname{FresnelS}(a+bx)}{4d}-$$

$$\frac{\int \left(\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^4+4d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^3+6d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-\right.}{4b^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^4 \operatorname{FresnelS}(a+bx)}{4d}-$$

$$\frac{\frac{8d^3(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2}-\frac{4d^3(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}+\frac{6d^2(bc-ad)^2\operatorname{FresnelC}(a+bx)}{\pi}-\frac{6d^2(a+bx)(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{}$$

input
```
Int[(c + d*x)^3*FresnelS[a + b*x],x]
```

output
```
((c + d*x)^4*FresnelS[a + b*x])/(4*d) - ((-4*d*(b*c - a*d)^3*Cos[(Pi*(a +
b*x)^2)/2]])/Pi - (6*d^2*(b*c - a*d)^2*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]])/P
i - (4*d^3*(b*c - a*d)*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2]])/Pi - (d^4*(a +
 b*x)^3*Cos[(Pi*(a + b*x)^2)/2]])/Pi + (6*d^2*(b*c - a*d)^2*FresnelC[a + b*
x])/Pi + (b*c - a*d)^4*FresnelS[a + b*x] - (3*d^4*FresnelS[a + b*x])/Pi^2
+ (8*d^3*(b*c - a*d)*Sin[(Pi*(a + b*x)^2)/2]])/Pi^2 + (3*d^4*(a + b*x)*Sin[
(Pi*(a + b*x)^2)/2]])/Pi^2)/(4*b^4*d)
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.19.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\mathrm{FresnelS}(bx+a)}{\pi}\right)}{\pi}+\frac{4(ad-bc)}{}}{}$
default	$\frac{\frac{\mathrm{FresnelS}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\mathrm{FresnelS}(bx+a)}{\pi}\right)}{\pi}+\frac{4(ad-bc)}{}}{}$
parts	Expression too large to display

input
```
int((d*x+c)^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*FresnelS(b*x+a)*(a*d-b*c-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b
*x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)
-1/Pi*FresnelS(b*x+a))+4*(a*d-b*c)*d^3/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-
8*(a*d-b*c)*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-6*(a*d-b*c)^2*d^2/Pi*(b*x+a)*co
s(1/2*Pi*(b*x+a)^2)+6*(a*d-b*c)^2*d^2/Pi*FresnelC(b*x+a)+4*(a*d-b*c)^3*d/P
i*cos(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^4*FresnelS(b*x+a)))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.27

$$\int (c+dx)^3 \,\mathrm{FresnelS}(a+bx)\,dx =$$

$$-\frac{6\,\pi(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-(\pi^2(4\,ab^3c^3-6\,a^2b^2c^2d+4\,a^3bcd^2-a^4d^3)+3\,d^3)\sqrt{b^2}\,\mathrm{S}}{}$$

input
```
integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_cos(sqrt(
b^2)*(b*x + a)/b) - (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 -
 a^4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - (pi*b^4*
d^3*x^3 + pi*(4*b^4*c*d^2 - a*b^3*d^3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d
^2 + a^2*b^2*d^3)*x + pi*(4*b^4*c^3 - 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^
3*b*d^3))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi^2*b^5*d^3*x^4
+ 4*pi^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^2*b^5*c^3*x)*fresnel_
sin(b*x + a) + (3*b^2*d^3*x + 8*b^2*c*d^2 - 5*a*b*d^3)*sin(1/2*pi*b^2*x^2
+ pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

3.19.6 Sympy [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (c+dx)^3\, S(a+bx)\; dx$$

input
```
integrate((d*x+c)**3*fresnels(b*x+a),x)
```

output
```
Integral((c + d*x)**3*fresnels(a + b*x), x)
```

3.19.7 Maxima [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^3\, \mathrm{S}\,(bx+a)\; dx$$

input
```
integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

3.19.8 Giac [F]

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^3 \operatorname{S}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \operatorname{FresnelS}(a+bx)\,dx = \int \operatorname{FresnelS}(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(FresnelS(a + b*x)*(c + d*x)^3,x)
```

output
```
int(FresnelS(a + b*x)*(c + d*x)^3, x)
```

3.20 $\int (c+dx)^2 \text{FresnelS}(a+bx)\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 193

$$\begin{aligned}\int (c+dx)^2 \text{FresnelS}(a+bx)\,dx = {} & \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\ & + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\ & + \frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} \\ & - \frac{d(bc-ad)\,\text{FresnelC}(a+bx)}{b^3\pi} \\ & - \frac{(bc-ad)^3\,\text{FresnelS}(a+bx)}{3b^3d} \\ & + \frac{(c+dx)^3\,\text{FresnelS}(a+bx)}{3d} - \frac{2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}\end{aligned}$$

output
```
(-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*
(b*x+a)^2)/b^3/Pi+1/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b
*c)*FresnelC(b*x+a)/b^3/Pi-1/3*(-a*d+b*c)^3*FresnelS(b*x+a)/b^3/d+1/3*(d*x
+c)^3*FresnelS(b*x+a)/d-2/3*d^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2
```

3.20.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int (c+dx)^2 \text{FresnelS}(a+bx)\,dx$$
$$= \frac{3b^2c^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 3abcd\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + a^2d^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 3b^2cd\pi x\cos\left(\frac{1}{2}\pi(a+b\right.}{}$$

input
```
Integrate[(c + d*x)^2*FresnelS[a + b*x],x]
```

output
```
(3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/
2] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)
^2)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a
+ b*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2
 - 3*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a +
 b*x] - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

3.20.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \text{FresnelS}(a+bx)\,dx$$
$$\Big\downarrow \text{6982}$$
$$\frac{(c+dx)^3 \text{FresnelS}(a+bx)}{3d} - \frac{b\int (c+dx)^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$
$$\Big\downarrow \text{3914}$$
$$\frac{(c+dx)^3 \text{FresnelS}(a+bx)}{3d} - \frac{\int \left(\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^3 + 3d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^2 + 3d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-\right.}{3b^3d}$$
$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^3 \operatorname{FresnelS}(a+bx)}{3d} - \frac{\frac{3d^2(bc-ad)\operatorname{FresnelC}(a+bx)}{\pi} - \frac{3d^2(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + (bc-ad)^3 \operatorname{FresnelS}(a+bx) - \frac{3d(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + }{3b^3d}$$

input
```
Int[(c + d*x)^2*FresnelS[a + b*x],x]
```

output
```
((c + d*x)^3*FresnelS[a + b*x])/(3*d) - ((-3*d*(b*c - a*d)^2*Cos[(Pi*(a +
b*x)^2)/2])/Pi - (3*d^2*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi
- (d^3*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/Pi + (3*d^2*(b*c - a*d)*Fresne
lC[a + b*x])/Pi + (b*c - a*d)^3*FresnelS[a + b*x] + (2*d^3*Sin[(Pi*(a + b*
x)^2)/2])/Pi^2)/(3*b^3*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.20.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\mathrm{FresnelS}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{3(ad-bc)d^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+}{3b^2d}}{b}$
default	$\frac{-\frac{\mathrm{FresnelS}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{3(ad-bc)d^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+}{3b^2d}}{b}$
parts	$\frac{\mathrm{FresnelS}(bx+a)d^2x^3}{3}+\mathrm{FresnelS}(bx+a)\,dc\,x^2+\mathrm{FresnelS}(bx+a)\,c^2x+\frac{\mathrm{FresnelS}(bx+a)c^3}{3d}-\ \ b$

input
```
int((d*x+c)^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*FresnelS(b*x+a)*(a*d-b*c-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(d^3/Pi*(b
*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-3*(a*d-b*c)
*d^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+3*(a*d-b*c)*d^2/Pi*FresnelC(b*x+a)+3
*(a*d-b*c)^2*d/Pi*cos(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^3*FresnelS(b*x+a)))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.28

$$\int (c+dx)^2\,\mathrm{FresnelS}(a+bx)\,dx$$
$$=\frac{\pi^2(3\,ab^2c^2-3\,a^2bcd+a^3d^2)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-2\,bd^2\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-3\,\pi(bcd-ad^2)\sqrt{b^2}}{}$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_sin(sqrt
(b^2)*(b*x + a)/b) - 2*b*d^2*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) -
 3*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (pi*b
^3*d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^2*c*d +
a^2*b*d^2))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi^2*b^4*d^2*x^
3 + 3*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_sin(b*x + a))/(pi^2*b^4
)
```

3.20.6 Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (c+dx)^2 S(a+bx)\ dx$$

input
```
integrate((d*x+c)**2*fresnels(b*x+a),x)
```

output
```
Integral((c + d*x)**2*fresnels(a + b*x), x)
```

3.20.7 Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)\ dx$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

3.20.8 Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int (dx+c)^2 \operatorname{S}(bx+a)\ dx$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)\,dx = \int \operatorname{FresnelS}(a+b\,x)\ (c+d\,x)^2\,dx$$

input
```
int(FresnelS(a + b*x)*(c + d*x)^2,x)
```

output
```
int(FresnelS(a + b*x)*(c + d*x)^2, x)
```

3.21 $\int (c+dx) \text{FresnelS}(a+bx) \, dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 121

$$\int (c+dx) \text{FresnelS}(a+bx) \, dx = \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{d\,\text{FresnelC}(a+bx)}{2b^2\pi} - \frac{(bc-ad)^2\,\text{FresnelS}(a+bx)}{2b^2d} + \frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d}$$

output
```
(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2
)/b^2/Pi-1/2*d*FresnelC(b*x+a)/b^2/Pi-1/2*(-a*d+b*c)^2*FresnelS(b*x+a)/b^2
/d+1/2*(d*x+c)^2*FresnelS(b*x+a)/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int (c+dx) \text{FresnelS}(a+bx) \, dx = \frac{-d\,\text{FresnelC}(a+bx) + (2bc-ad+bdx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + \pi(a+bx)\,\text{FresnelS}(a+bx)\right)}{2b^2\pi}$$

input
```
Integrate[(c + d*x)*FresnelS[a + b*x],x]
```

output
```
(-(d*FresnelC[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2)/2] +
 Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)
```

3.21.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \; 6982$$

$$\frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d}-\frac{b\int (c+dx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}$$

$$\Big\downarrow \; 3914$$

$$\frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d}-\frac{\int \left(\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^2+2d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)+d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\right)d(a+b}{2b^2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{(c+dx)^2\,\text{FresnelS}(a+bx)}{2d}-\frac{(bc-ad)^2\,\text{FresnelS}(a+bx)-\frac{2d(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}+\frac{d^2\,\text{FresnelC}(a+bx)}{\pi}-\frac{d^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{2b^2d}$$

input
```
Int[(c + d*x)*FresnelS[a + b*x],x]
```

output
```
((c + d*x)^2*FresnelS[a + b*x])/(2*d) - ((-2*d*(b*c - a*d)*Cos[(Pi*(a + b*
x)^2)/2])/Pi - (d^2*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi + (d^2*FresnelC[
a + b*x])/Pi + (b*c - a*d)^2*FresnelS[a + b*x])/(2*b^2*d)
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{\mathrm{FresnelS}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\,\mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
default	$\frac{-\frac{\mathrm{FresnelS}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\,\mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
parts	$\frac{\mathrm{FresnelS}(bx+a)d\,x^2}{2}+\mathrm{FresnelS}\left(bx+a\right)cx-\frac{b\left(-\frac{dx\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{da\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\ldots\right)}{b^2\pi}\ldots\right)}{\ldots}\right)}{\ldots}$

input
```
int((d*x+c)*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-FresnelS(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(d
/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-d/Pi*FresnelC(b*x+a)-(2*a*d-2*b*c)/Pi*co
s(1/2*Pi*(b*x+a)^2)))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx$$
$$=\frac{\pi(2\,abc-a^2d)\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-\sqrt{b^2}d\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(b^2dx+2\,b^2c-abd)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a}{2\,\pi b^3}$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - s
qrt(b^2)*d*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (b^2*d*x + 2*b^2*c - a*b*d
)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi*b^3*d*x^2 + 2*pi*b^3*c
*x)*fresnel_sin(b*x + a))/(pi*b^3)
```

3.21.6 Sympy [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int (c+dx)\,S(a+bx)\,dx$$

input
```
integrate((d*x+c)*fresnels(b*x+a),x)
```

output
```
Integral((c + d*x)*fresnels(a + b*x), x)
```

3.21.7 Maxima [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)\,dx=\int (dx+c)\,\text{S}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

3.21.8 Giac [F]

$$\int (c+dx) \operatorname{FresnelS}(a+bx)\, dx = \int (dx+c) \operatorname{S}(bx+a)\, dx$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx) \operatorname{FresnelS}(a+bx)\, dx = \int \operatorname{FresnelS}(a+b\,x)\ (c+d\,x)\, dx$$

input
```
int(FresnelS(a + b*x)*(c + d*x),x)
```

output
```
int(FresnelS(a + b*x)*(c + d*x), x)
```

3.22 $\int \text{FresnelS}(a+bx)\,dx$

3.22.1 Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

output
```
cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

3.22.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}+\frac{a\,\text{FresnelS}(a+bx)}{b}+x\,\text{FresnelS}(a+bx)-\frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelS[a + b*x],x]
```

output
```
(Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a +
b*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^
2)/2])/(b*Pi)
```

3.22.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6972}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6972}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

input
```
Int[FresnelS[a + b*x],x]
```

output
```
Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

3.22.3.1 Defintions of rubi rules used

rule 6972
```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
parts	$x\ \text{FresnelS}\left(bx+a\right)-b\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	86

input
```
int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \text{FresnelS}(a+bx)\,dx=\frac{(\pi bx+\pi a)\,\text{S}\left(bx+a\right)+\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/
2*pi*a^2))/(pi*b)
```

3.22.6 Sympy [F]

$$\int \text{FresnelS}(a+bx)\,dx=\int S(a+bx)\;dx$$

input
```
integrate(fresnels(b*x+a),x)
```

output
```
Integral(fresnels(a + b*x), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \text{FresnelS}(a+bx)\,dx=\frac{(bx+a)\,\text{S}\left(bx+a\right)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a
^2)/pi)/b
```

3.22.8 Giac [F]

$$\int \text{FresnelS}(a+bx)\,dx=\int \text{S}\left(bx+a\right)\,dx$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)\,dx=\int \text{FresnelS}(a+b\,x)\,dx$$

input
```
int(FresnelS(a + b*x),x)
```

output
```
int(FresnelS(a + b*x), x)
```

3.23 $\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)/(d*x+c),x)
```

3.23.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[FresnelS[a + b*x]/(c + d*x),x]
```

output
```
Integrate[FresnelS[a + b*x]/(c + d*x), x]
```

3.23.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \; 6988$$
$$\int \frac{\text{FresnelS}(a+bx)}{c+dx} dx$$

input
```
Int[FresnelS[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

3.23.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.23.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)}{dx+c} dx$$

input
```
int(FresnelS(b*x+a)/(d*x+c),x)
```

output
```
int(FresnelS(b*x+a)/(d*x+c),x)
```

3.23.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{S}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)/(d*x + c), x)
```

3.23.6 Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{S(a+bx)}{c+dx}\,dx$$

input
```
integrate(fresnels(b*x+a)/(d*x+c),x)
```

output
```
Integral(fresnels(a + b*x)/(c + d*x), x)
```

3.23.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{S}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

3.23.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{S}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

3.23.9 Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(FresnelS(a + b*x)/(c + d*x),x)
```

output
```
int(FresnelS(a + b*x)/(c + d*x), x)
```

3.24 $\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} \, dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)/(d*x+c)^2,x)
```

3.24.2 Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]
```

3.24.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 6988}$$

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}dx$$

input
```
Int[FresnelS[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.24.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.24.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

output
```
int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

3.24.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.24.6 Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{S(a+bx)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(fresnels(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(fresnels(a + b*x)/(c + d*x)**2, x)
```

3.24.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

3.24.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{S}\,(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

3.24.9 Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelS}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(FresnelS(a + b*x)/(c + d*x)^2,x)
```

output
```
int(FresnelS(a + b*x)/(c + d*x)^2, x)
```

3.25 $\int x^3 \operatorname{FresnelS}(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 229

$$\begin{aligned}\int x^3 \operatorname{FresnelS}(a+bx)\,dx &= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi}\\ &\quad-\frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}\\ &\quad-\frac{3a^2\operatorname{FresnelC}(a+bx)}{2b^4\pi}-\frac{a^4\operatorname{FresnelS}(a+bx)}{4b^4}\\ &\quad+\frac{3\operatorname{FresnelS}(a+bx)}{4b^4\pi^2}+\frac{1}{4}x^4\operatorname{FresnelS}(a+bx)\\ &\quad+\frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2}-\frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}\end{aligned}$$

output
```
-a^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*a^2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^
4/Pi-a*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+1/4*(b*x+a)^3*cos(1/2*Pi*(b*
x+a)^2)/b^4/Pi-3/2*a^2*FresnelC(b*x+a)/b^4/Pi-1/4*a^4*FresnelS(b*x+a)/b^4+
3/4*FresnelS(b*x+a)/b^4/Pi^2+1/4*x^4*FresnelS(b*x+a)+2*a*sin(1/2*Pi*(b*x+a
)^2)/b^4/Pi^2-3/4*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

3.25.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx$$
$$= \frac{-a^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+a^2b\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-ab^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+b^3\pi x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{}$$

input

```
Integrate[x^3*FresnelS[a + b*x],x]
```

output

```
(-(a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] -
a*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2]
- 6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a +
b*x] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4
*Pi^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \text{6982}$$

$$\frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) - \frac{\int \left(\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a^4 - 4(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a^3 + 6(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a^2 - 4(a+bx)^3\sin\left(\frac{1}{2}\pi(a\right.\right.}{4b^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4 \operatorname{FresnelS}(a+bx) - \frac{a^4 \operatorname{FresnelS}(a+bx) + \frac{4a^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + \frac{6a^2 \operatorname{FresnelC}(a+bx)}{\pi} - \frac{6a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{3 \operatorname{FresnelS}(a+bx)}{\pi^2} - \frac{8a \sin\left(\frac{1}{2}\pi(a+\right.}{\pi^2}}{4b^4}$$

input
```
Int[x^3*FresnelS[a + b*x],x]
```

output
```
(x^4*FresnelS[a + b*x])/4 - ((4*a^3*Cos[(Pi*(a + b*x)^2)/2])/Pi - (6*a^2*(
a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi + (4*a*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2])/Pi - ((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/Pi + (6*a^2*FresnelC[a +
 b*x])/Pi + a^4*FresnelS[a + b*x] - (3*FresnelS[a + b*x])/Pi^2 - (8*a*Sin[
(Pi*(a + b*x)^2)/2])/Pi^2 + (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi^2)/(4
*b^4)
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\text{FresnelS}(bx+a)b^4x^4}{4} - \frac{a^4\,\text{FresnelS}(bx+a)}{4} - \frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi} - \frac{3a^2\,\text{FresnelC}(bx+a)}{2\pi} - \frac{a(bx+a)^2\,\text{co}}{}}{b^4}$
default	$\frac{\frac{\text{FresnelS}(bx+a)b^4x^4}{4} - \frac{a^4\,\text{FresnelS}(bx+a)}{4} - \frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi} - \frac{3a^2\,\text{FresnelC}(bx+a)}{2\pi} - \frac{a(bx+a)^2\,\text{co}}{}}{b^4}$
parts	$\frac{x^4\,\text{FresnelS}(bx+a)}{4} - \frac{b\left(-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi} - \frac{a\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx\right.}{b^2\pi}\right.}{}\right.}{}\right.}{}$

input
```
int(x^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4-1/4*a^4*FresnelS(b*x+a)-a^3/Pi*cos(1/2*
Pi*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC
(b*x+a)-a/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2
)+1/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(
b*x+a)^2)-1/Pi*FresnelS(b*x+a)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \frac{\pi^2 b^5 x^4 \operatorname{S}(bx+a) - 6\pi a^2\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2a^4-3)\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^4x^3 - \pi ab^3x^2 + \pi a^2b^2x -}{4\pi^2 b^5}$$

input
```
integrate(x^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(pi^2*b^5*x^4*fresnel_sin(b*x + a) - 6*pi*a^2*sqrt(b^2)*fresnel_cos(sq
rt(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x
 + a)/b) + (pi*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*cos(1/2*p
i*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (3*b^2*x - 5*a*b)*sin(1/2*pi*b^2*x^2
+ pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

3.25.6 Sympy [F]

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \int x^3 S(a+bx)\,dx$$

input
```
integrate(x**3*fresnels(b*x+a),x)
```

output
```
Integral(x**3*fresnels(a + b*x), x)
```

3.25.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.20

$$\int x^3 \operatorname{FresnelS}(a+bx)\,dx = \frac{1}{4}x^4\operatorname{S}(bx+a) - \frac{\left(16\left(\pi^2 e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)} + \pi^2 e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^4 + 32\left(-i\pi\Gamma\left(2, \frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right.\right.\right.}{}$$

input
```
integrate(x^3*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*fresnel_sin(b*x + a) - 1/32*(16*(pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a
*b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a
^2))*a^4 + 32*(-I*pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2
) + I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16
*((pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*
pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(-I*pi*gamma(2, 1/2*I*pi*
b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I
*pi*a*b*x - 1/2*I*pi*a^2))*a)*b*x - ((-(I + 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(
1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(
5/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 -
 12*((I - 1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) - (I + 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/
2*I*pi*a^2))*a^2 - (4*I + 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) + (4*I - 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*p
i*a*b*x - 1/2*I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi
^3*b^6*x + pi^3*a*b^5)
```

3.25.8 Giac [F]

$$\int x^3 \text{FresnelS}(a+bx)\,dx = \int x^3\,\text{S}\,(bx+a)\,dx$$

input
```
integrate(x^3*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_sin(b*x + a), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \text{FresnelS}(a+bx)\,dx = \int x^3\,\text{FresnelS}(a+b\,x)\,dx$$

input
```
int(x^3*FresnelS(a + b*x),x)
```

output
```
int(x^3*FresnelS(a + b*x), x)
```

3.26 $\int x^2 \text{FresnelS}(a+bx)\,dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 147

$$\begin{aligned}\int x^2 \text{FresnelS}(a+bx)\,dx = {} & \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\ & + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} \\ & + \frac{a\,\text{FresnelC}(a+bx)}{b^3\pi} + \frac{a^3\,\text{FresnelS}(a+bx)}{3b^3} \\ & + \frac{1}{3}x^3\,\text{FresnelS}(a+bx) - \frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}\end{aligned}$$

output
```
a^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-a*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+1/
3*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*FresnelC(b*x+a)/b^3/Pi+1/3*a^3*
FresnelS(b*x+a)/b^3+1/3*x^3*FresnelS(b*x+a)-2/3*sin(1/2*Pi*(b*x+a)^2)/b^3/
Pi^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int x^2 \text{FresnelS}(a+bx)\,dx$$
$$= \frac{a^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - ab\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + b^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 3a\pi\,\text{FresnelC}(a+bx) + \pi^2(}{3b^3\pi^2}$$

input
```
Integrate[x^2*FresnelS[a + b*x],x]
```

output
```
(a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*P
i*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3
*x^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \text{6982}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(a+bx)-\frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(a+bx)- \frac{\int \left(-\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a^3+3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a^2-3(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)a+(a+bx)^3\sin\left(\frac{1}{2}\pi(a\right.\right.}{3b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(a+bx)- \frac{a^3(-\operatorname{FresnelS}(a+bx))-\frac{3a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}-\frac{3a\operatorname{FresnelC}(a+bx)}{\pi}+\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2}+\frac{3a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}-\frac{(a+bx)^2 c}{}}{3b^3}$$

input
```
Int[x^2*FresnelS[a + b*x],x]
```

output
```
(x^3*FresnelS[a + b*x])/3 - ((-3*a^2*Cos[(Pi*(a + b*x)^2)/2])/Pi + (3*a*(a
 + b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi - ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2]
)/Pi - (3*a*FresnelC[a + b*x])/Pi - a^3*FresnelS[a + b*x] + (2*Sin[(Pi*(a
+ b*x)^2)/2])/Pi^2)/(3*b^3)
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelS}(bx+a)b^3x^3}{3}+\frac{a^3\,\text{FresnelS}(bx+a)}{3}+\frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a\,\text{FresnelC}(bx+a)}{\pi}+\frac{(bx+a)^2\cos\left(\frac{\pi(b\cdots}{3\pi}}{b^3}$
default	$\frac{\frac{\text{FresnelS}(bx+a)b^3x^3}{3}+\frac{a^3\,\text{FresnelS}(bx+a)}{3}+\frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a\,\text{FresnelC}(bx+a)}{\pi}+\frac{(bx+a)^2\cos\left(\frac{\pi(b\cdots}{3\pi}}{b^3}$
parts	$\frac{x^3\,\text{FresnelS}(bx+a)}{3}-\frac{b\left(-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\cdots\right)}{b^2\pi}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$

input
```
int(x^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*FresnelS(b*x+a)*b^3*x^3+1/3*a^3*FresnelS(b*x+a)+a^2/Pi*cos(1/2*
Pi*(b*x+a)^2)-a/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+a/Pi*FresnelC(b*x+a)+1/3/
Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*sin(1/2*Pi*(b*x+a)^2))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{\pi^2 b^4 x^3 \operatorname{S}(bx+a) + \pi^2 a^3 \sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + 3\pi a\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + (\pi b^3 x^2 - \pi a b^2 x + \pi a^2 b)\cos\left(\frac{1}{2}\pi b^2 x^2\right.}{3\pi^2 b^4}$$

input
```
integrate(x^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(pi^2*b^4*x^3*fresnel_sin(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_sin(sq
rt(b^2)*(b*x + a)/b) + 3*pi*a*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b)
 + (pi*b^3*x^2 - pi*a*b^2*x + pi*a^2*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/
2*pi*a^2) - 2*b*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

3.26.6 Sympy [F]

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 S(a+bx)\,dx$$

input
```
integrate(x**2*fresnels(b*x+a),x)
```

output
```
Integral(x**2*fresnels(a + b*x), x)
```

3.26.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.88

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \frac{1}{3}x^3 \operatorname{S}(bx+a) + \frac{\left(12\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^3+4\left(3\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\pi b^2\right.}\right.\right.}{}$$

input
```
integrate(x^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*fresnel_sin(b*x + a) + 1/24*(12*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b
*x + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
*a^3 + 4*(3*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-
1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - 2*I*gamma(2, 1/2*I*pi
*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*I*gamma(2, -1/2*I*pi*b^2*x^2 - I
*pi*a*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(-I*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*
a*b*x + 1/2*I*pi*a^2) + I*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*
pi*a^2)) - sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*p
i^(3/2)*(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I
 - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi
*a^2)) - 1))*a^3 - 6*((I - 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a
*b*x + 1/2*I*pi*a^2) - (I + 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi
*a*b*x - 1/2*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

3.26.8 Giac [F]

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 \operatorname{S}(bx+a)\,dx$$

input
```
integrate(x^2*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_sin(b*x + a), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(a+bx)\,dx = \int x^2 \operatorname{FresnelS}(a+b\,x)\,dx$$

input
```
int(x^2*FresnelS(a + b*x),x)
```

output
```
int(x^2*FresnelS(a + b*x), x)
```

3.27 $\int x\,\text{FresnelS}(a+bx)\,dx$

3.27.1 Optimal result

Integrand size = 8, antiderivative size = 96

$$\int x\,\text{FresnelS}(a+bx)\,dx = -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{\text{FresnelC}(a+bx)}{2b^2\pi}-\frac{a^2\,\text{FresnelS}(a+bx)}{2b^2}+\frac{1}{2}x^2\,\text{FresnelS}(a+bx)$$

output
```
-a*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi-1
/2*FresnelC(b*x+a)/b^2/Pi-1/2*a^2*FresnelS(b*x+a)/b^2+1/2*x^2*FresnelS(b*x
+a)
```

3.27.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int x\,\text{FresnelS}(a+bx)\,dx = -\frac{\text{FresnelC}(a+bx)+(a-bx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi(a+bx)\,\text{FresnelS}(a+bx)\right)}{2b^2\pi}$$

input
```
Integrate[x*FresnelS[a + b*x],x]
```

output
```
-1/2*(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2)/2] + Pi*(a + b*x
)*FresnelS[a + b*x]))/(b^2*Pi)
```

3.27.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6982, 3914, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \,\text{FresnelS}(a + bx)\, dx$$

$$\Big\downarrow \text{6982}$$

$$\frac{1}{2}x^2 \,\text{FresnelS}(a + bx) - \frac{1}{2}b \int x^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right) dx$$

$$\Big\downarrow \text{3914}$$

$$\frac{1}{2}x^2 \,\text{FresnelS}(a + bx) - \frac{\int \left(\sin\left(\frac{1}{2}\pi(a + bx)^2\right) a^2 - 2(a + bx)\sin\left(\frac{1}{2}\pi(a + bx)^2\right) a + (a + bx)^2 \sin\left(\frac{1}{2}\pi(a + bx)^2\right)\right) d(a + bx)}{2b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2 \,\text{FresnelS}(a+bx) - \frac{a^2 \,\text{FresnelS}(a + bx) + \frac{\text{FresnelC}(a+bx)}{\pi} + \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{2b^2}$$

input
```
Int[x*FresnelS[a + b*x],x]
```

output
```
(x^2*FresnelS[a + b*x])/2 - ((2*a*Cos[(Pi*(a + b*x)^2)/2])/Pi - ((a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/Pi + FresnelC[a + b*x]/Pi + a^2*FresnelS[a + b*x
])/(2*b^2)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3914
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6982
```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\operatorname{FresnelS}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\operatorname{FresnelC}(bx+a)}{2\pi}}{b^2}$
default	$\frac{\operatorname{FresnelS}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\operatorname{FresnelC}(bx+a)}{2\pi}}{b^2}$
parts	$\frac{x^2\operatorname{FresnelS}(bx+a)}{2}-\frac{b\left(-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(-\frac{\cos\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\operatorname{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)}{b}\right.}{2}$

input
```
int(x*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(FresnelS(b*x+a)*(-(b*x+a)*a+1/2*(b*x+a)^2)-a/Pi*cos(1/2*Pi*(b*x+a)^
2)+1/2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int x\operatorname{FresnelS}(a+bx)\,dx$$
$$=\frac{\pi b^3x^2\operatorname{S}(bx+a)-\pi a^2\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(b^2x-ab)\cos\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)-\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^3}$$

input
```
integrate(x*fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(pi*b^3*x^2*fresnel_sin(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_sin(sqrt(b
^2)*(b*x + a)/b) + (b^2*x - a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2) - sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)
```

3.27.6 Sympy [F]

$$\int x \operatorname{FresnelS}(a+bx)\,dx = \int xS(a+bx)\,dx$$

input
```
integrate(x*fresnels(b*x+a),x)
```

output
```
Integral(x*fresnels(a + b*x), x)
```

3.27.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.20

$$\int x \operatorname{FresnelS}(a+bx)\,dx = \frac{1}{2}x^2\,\mathrm{S}\left(bx+a\right) - \frac{\left(8\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)abx+8\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-\right.}\right.}{}$$

input
```
integrate(x*fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*fresnel_sin(b*x + a) - 1/16*(8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*
x + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*
a*b*x + 8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-1/
2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2*x^2 + 4*p
i*a*b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*pi*b^2*x^2
 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(-
1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 - (2*I - 2)*sqrt(
2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + (2*I + 2)*sq
rt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)))*b/(pi^2*
b^4*x + pi^2*a*b^3)
```

3.27.8 Giac [F]

$$\int x \operatorname{FresnelS}(a+bx) \, dx = \int x \operatorname{S}(bx+a) \, dx$$

input
```
integrate(x*fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*fresnel_sin(b*x + a), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelS}(a+bx) \, dx = \int x \operatorname{FresnelS}(a+b\,x) \, dx$$

input
```
int(x*FresnelS(a + b*x),x)
```

output
```
int(x*FresnelS(a + b*x), x)
```

3.28 $\int \text{FresnelS}(a+bx)\,dx$

3.28.1 Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}$$

output
```
cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

$$\int \text{FresnelS}(a+bx)\,dx=\frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}+\frac{a\,\text{FresnelS}(a+bx)}{b}+x\,\text{FresnelS}(a+bx)-\frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelS[a + b*x],x]
```

output
```
(Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a +
b*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^
2)/2])/(b*Pi)
```

3.28.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6972}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(a+bx)\,dx$$

$$\Big\downarrow \text{6972}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

input
```
Int[FresnelS[a + b*x],x]
```

output
```
Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

3.28.3.1 Defintions of rubi rules used

rule 6972
```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.28.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\text{FresnelS}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
parts	$x\ \text{FresnelS}\left(bx+a\right)-b\left(-\frac{\cos\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelS}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	86

input
```
int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(\pi bx+\pi a)\,\text{S}\,(bx+a)+\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/
2*pi*a^2))/(pi*b)
```

3.28.6 Sympy [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int S(a+bx)\;dx$$

input
```
integrate(fresnels(b*x+a),x)
```

output
```
Integral(fresnels(a + b*x), x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \text{FresnelS}(a+bx)\,dx = \frac{(bx+a)\,\text{S}\,(bx+a)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a
^2)/pi)/b
```

3.28.8 Giac [F]

$$\int \text{FresnelS}(a+bx)\,dx = \int \text{S}\left(bx+a\right)\,dx$$

input
```
integrate(fresnel_sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)\,dx = \int \text{FresnelS}(a+b\,x)\,dx$$

input
```
int(FresnelS(a + b*x),x)
```

output
```
int(FresnelS(a + b*x), x)
```

3.29 $\int \frac{\text{FresnelS}(a+bx)}{x} \, dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(a+bx)}{x} \, dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{x}, x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)/x,x)
```

3.29.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x} \, dx = \int \frac{\text{FresnelS}(a+bx)}{x} \, dx$$

input
```
Integrate[FresnelS[a + b*x]/x,x]
```

output
```
Integrate[FresnelS[a + b*x]/x, x]
```

3.29.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx$$
$$\Big\downarrow \ 6988$$
$$\int \frac{\text{FresnelS}(a+bx)}{x}dx$$

input
```
Int[FresnelS[a + b*x]/x,x]
```

output
```
$Aborted
```

3.29.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.29.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)}{x}dx$$

input
```
int(FresnelS(b*x+a)/x,x)
```

output
```
int(FresnelS(b*x+a)/x,x)
```

3.29.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{S}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)/x, x)
```

3.29.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{S(a+bx)}{x}\,dx$$

input
```
integrate(fresnels(b*x+a)/x,x)
```

output
```
Integral(fresnels(a + b*x)/x, x)
```

3.29.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{S}\left(bx+a\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)/x, x)
```

3.29.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{S}\,(bx+a)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)/x, x)
```

3.29.9 Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{x}\,dx$$

input
```
int(FresnelS(a + b*x)/x,x)
```

output
```
int(FresnelS(a + b*x)/x, x)
```

3.30 $\int \frac{\text{FresnelS}(a+bx)}{x^2} \, dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(a+bx)}{x^2} \, dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)}{x^2}, x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)/x^2,x)
```

3.30.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2} \, dx = \int \frac{\text{FresnelS}(a+bx)}{x^2} \, dx$$

input
```
Integrate[FresnelS[a + b*x]/x^2,x]
```

output
```
Integrate[FresnelS[a + b*x]/x^2, x]
```

3.30.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \; 6988$$

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}dx$$

input
```
Int[FresnelS[a + b*x]/x^2,x]
```

output
```
$Aborted
```

3.30.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.30.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx+a)}{x^2}dx$$

input
```
int(FresnelS(b*x+a)/x^2,x)
```

output
```
int(FresnelS(b*x+a)/x^2,x)
```

3.30.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\mathrm{S}\,(bx+a)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)/x^2, x)
```

3.30.6 Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{S(a+bx)}{x^2}\,dx$$

input
```
integrate(fresnels(b*x+a)/x**2,x)
```

output
```
Integral(fresnels(a + b*x)/x**2, x)
```

3.30.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\mathrm{S}\,(bx+a)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)/x^2, x)
```

3.30.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{S}\,(bx+a)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)/x^2, x)
```

3.30.9 Mupad [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(a+b\,x)}{x^2}\,dx$$

input
```
int(FresnelS(a + b*x)/x^2,x)
```

output
```
int(FresnelS(a + b*x)/x^2, x)
```

3.31 $\int x^7 \operatorname{FresnelS}(bx)^2 \, dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 253

$$\begin{aligned}\int x^7 \operatorname{FresnelS}(bx)^2\,dx = &-\frac{105x^2}{16b^6\pi^4}+\frac{7x^6}{48b^2\pi^2}-\frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} \\ &+\frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2}-\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{4b^5\pi^3} \\ &+\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{4b\pi}-\frac{105\operatorname{FresnelS}(bx)^2}{8b^8\pi^4} \\ &+\frac{1}{8}x^8\operatorname{FresnelS}(bx)^2+\frac{105x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} \\ &-\frac{7x^5\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}+\frac{10\sin(b^2\pi x^2)}{b^8\pi^5}-\frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}\end{aligned}$$

output
```
-105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2-55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4+
1/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2-35/4*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*
x)/b^5/Pi^3+1/4*x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-105/8*FresnelS(
b*x)^2/b^8/Pi^4+1/8*x^8*FresnelS(b*x)^2+105/4*x*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/b^7/Pi^4-7/4*x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2+10*sin
(b^2*Pi*x^2)/b^8/Pi^5-5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

3.31.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelS}(bx)^2\,dx$$
$$= \frac{-315b^2\pi x^2 + 7b^6\pi^3x^6 + 3b^2\pi x^2(-55 + b^4\pi^2x^4)\cos(b^2\pi x^2) + 6\pi(-105 + b^8\pi^4x^8)\operatorname{FresnelS}(bx)^2 + 12b\pi x \,\mathrm{F}}{}$$

input
```
Integrate[x^7*FresnelS[b*x]^2,x]
```

output
```
(-315*b^2*Pi*x^2 + 7*b^6*Pi^3*x^6 + 3*b^2*Pi*x^2*(-55 + b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 6*Pi*(-105 + b^8*Pi^4*x^8)*FresnelS[b*x]^2 + 12*b*Pi*x*Fresn
elS[b*x]*(b^2*Pi*x^2*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 7*(-15 + b
^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 480*Sin[b^2*Pi*x^2] - 30*b^4*Pi^2*x^4*
Sin[b^2*Pi*x^2])/(48*b^8*Pi^5)
```

3.31.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \operatorname{FresnelS}(bx)^2\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{1}{8}x^8\operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\int x^8\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \; 7008$$

$$\frac{1}{8}x^8\operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^7\sin\left(b^2\pi x^2\right)dx}{2\pi b} - \frac{x^7\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{8}x^8\operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b} - \frac{x^7\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^4\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^4\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos}{\pi b^2}\right.$$

$\downarrow$ 25

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos(}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2-\frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos(}{\pi b^2}\right.$$

$\downarrow$ 3777

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2} - \frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} - \frac{x^7\,\text{Fres}\ldots\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2} - \frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} - \frac{x^7\,\text{Fr}\ldots\right.$$

$\downarrow$ 3117

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} - \frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2} - \frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\ldots\right.}{\pi b^2}\ldots\right.}{\pi b^2}\ldots}{4\pi b}\ldots\right.$$

$\downarrow$ 7016

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x\ldots\right.}{\pi b^2}\ldots\right.$$

$\downarrow$ 3860

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\mathrm{FresnelS}(bx)^2-$$
$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\mathrm{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\mathrm{FresnelS}(bx)^2-$$
$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^5\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\mathrm{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3790

$$\frac{1}{8}x^8\,\mathrm{FresnelS}(bx)^2-$$
$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\mathrm{FresnelS}(bx)\,\mathrm{co}}{\pi b^2}\right.$$

$\downarrow$ 15

$$\frac{1}{8}x^8\,\mathrm{FresnelS}(bx)^2-$$
$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^6}{6}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\mathrm{FresnelS}(bx)\cos\left(\frac{1}{2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

3.31. $\int x^7\,\mathrm{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{x^6}{6}-\frac{1}{2}\int x^4 \sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b} + \frac{x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^7 \operatorname{FresnelS}(bx)\cos}{\pi b^2}\right.$$

$\downarrow$ 3777

$$\frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{1}{2}\left(-\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b} + \frac{x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^7}{}\right.$$

$\downarrow$ 25

$$\frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{1}{2}\left(\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b} + \frac{x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^7 \operatorname{Fre}}{}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{1}{2}\left(\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b} + \frac{x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^7 \operatorname{Fre}}{}\right.$$

$\downarrow$ 3777

3.31. $\int x^7 \operatorname{FresnelS}(bx)^2 \, dx$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{1}{2}\left(\frac{2\left(\frac{\int \cos\left(b^2\pi x^2\right)dx^2}{\pi b^2} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right) + \frac{x^6}{6}}{2\pi b} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x\right.}{\pi b^2}}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{1}{2}\left(\frac{2\left(\frac{\int \sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right) + \frac{x^6}{6}}{2\pi b} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi\right.}{\pi b^2}}{\pi b^2}\right.$$

$\downarrow$ 3117

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}\right)}{\pi b^2}\right.$$

$\downarrow$ 7008

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\sin\right.}{}\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3860

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 -$$

$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(2\left(\text{si}\cdots\right.\right.}{\cdots}\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 -$$

$$\frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(2\left(\text{si}\cdots\right.\right.}{\cdots}\right.}{\pi b^2}\right.$$

$\downarrow$ 3777

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right.}{\pi b^2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b\right.}{\pi b^2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3117

3.31. $\int x^7\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{8}x^8 \operatorname{FresnelS}(bx)^2 - \frac{1}{4}b\left(\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2} - \frac{x^3 \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2 b^4} - \frac{x^2\cos\left(\pi b^2 x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2} + \frac{x^5 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \right.}{\pi b^2}\right.$$

input
```
Int[x^7*FresnelS[b*x]^2,x]
```

output
```
$Aborted
```

3.31.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.31.4 Maple [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx$$

input
```
int(x^7*FresnelS(b*x)^2,x)
```

output
```
int(x^7*FresnelS(b*x)^2,x)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx$$
$$= \frac{2\,\pi^3 b^6 x^6 - 75\,\pi b^2 x^2 + 3\left(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\left(\pi^4 b^7 x^7 - 35\,\pi^2 b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) -}{24\,\pi}$$

input
```
integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/24*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^
2)*fresnel_sin(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_sin(b*x)^2 - 6*(5*
(pi^2*b^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fre
snel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

3.31.6 Sympy [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 S^2(bx) \, dx$$

input
```
integrate(x**7*fresnels(b*x)**2,x)
```

output
```
Integral(x**7*fresnels(b*x)**2, x)
```

3.31.7 Maxima [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2 \, dx = \int x^7 \,\mathrm{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^7*fresnel_sin(b*x)^2, x)
```

3.31.8 Giac [F]

$$\int x^7 \operatorname{FresnelS}(bx)^2\,dx = \int x^7\,S\,(bx)^2\,dx$$

input
```
integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_sin(b*x)^2, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelS}(bx)^2\,dx = \int x^7 \operatorname{FresnelS}(b\,x)^2\,dx$$

input
```
int(x^7*FresnelS(b*x)^2,x)
```

output
```
int(x^7*FresnelS(b*x)^2, x)
```

3.32 $\int x^6 \operatorname{FresnelS}(bx)^2 \, dx$

3.32.1 Optimal result

Integrand size = 10, antiderivative size = 239

$$\begin{aligned}\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = &-\frac{48x}{7b^6\pi^4}+\frac{6x^5}{35b^2\pi^2}-\frac{21x\cos(b^2\pi x^2)}{8b^6\pi^4}+\frac{x^5\cos(b^2\pi x^2)}{14b^2\pi^2}\\&+\frac{531\operatorname{FresnelC}(\sqrt{2}bx)}{56\sqrt{2}b^7\pi^4}-\frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{7b^5\pi^3}\\&+\frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{7b\pi}\\&+\frac{1}{7}x^7\operatorname{FresnelS}(bx)^2+\frac{96\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}\\&-\frac{12x^4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}-\frac{17x^3\sin(b^2\pi x^2)}{28b^4\pi^3}\end{aligned}$$

output
```
-48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2-21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4+1/14*x^
5*cos(b^2*Pi*x^2)/b^2/Pi^2-48/7*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/
Pi^3+2/7*x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/7*x^7*FresnelS(b*x)^
2+96/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-12/7*x^4*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/b^3/Pi^2-17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3+531/112*Fre
snelC(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int x^6 \operatorname{FresnelS}(bx)^2\,dx$$
$$= \frac{2655\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right) + 80b^7\pi^4x^7\operatorname{FresnelS}(bx)^2 + 160\operatorname{FresnelS}(bx)\left(b^2\pi x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x\right.\right.}{}$$

input
```
Integrate[x^6*FresnelS[b*x]^2,x]
```

output
```
(2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 16
0*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(
-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)
*Cos[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2
])))/(560*b^7*Pi^4)
```

3.32.3 Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.92, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.800$, Rules used = {6984, 7008, 3866, 3867, 3866, 3833, 7016, 3872, 15, 3867, 3866, 3833, 7008, 3866, 3833, 7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \operatorname{FresnelS}(bx)^2\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{1}{7}x^7\operatorname{FresnelS}(bx)^2 - \frac{2}{7}b\int x^7\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \; 7008$$

$$\frac{1}{7}x^7\operatorname{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{2\pi b} - \frac{x^6\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3866$$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{5\int x^4\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3867

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\int x^2\sin(b^2\pi x^2)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\cdots\right.}{\pi b^2}\cdots\right.$$

$\big\downarrow$ 3866

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\int\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}-\frac{x^6\,\text{Fresne}\cdots}{\cdots}\right.$$

$\big\downarrow$ 3833

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}-\frac{x\cos(\pi b^2 x\cdots}{2\pi b^2}\cdots\right.}{2\pi b^2}\cdots\right.}{2\pi b^2}\cdots}{2\pi b}\cdots\right.$$

$\big\downarrow$ 7016

3.32. $\int x^6\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x\right.}{\pi b^2}\right.$$

$\downarrow$ 3872

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos}{\pi b^2}\right.$$

$\downarrow$ 15

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^5}{10}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3867

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{Fres}}{}\right.$$

$\downarrow$ 3866

3.32. $\int x^6\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3833

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}\right.}{\pi b^2}\right.$$

$\downarrow$ 7008

3.32. $\int x^6\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(3\left(\frac{\text{Fresn}\cdots}{\cdots}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\pi b^2}\cdots\right)$$

$\downarrow$ 3866

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2-\frac{2}{7}b\left(\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\cdots\right)}{\pi b^2}\cdots\right)$$

$\downarrow$ 3833

3.32. $\int x^6\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} - \frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3} - \frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2} + \frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \cdots\right)}{\pi b^2}\cdots\right)$$

$$\Big\downarrow \text{ 7014}$$

$$\frac{1}{7}x^7\,\text{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}\right)}{\pi b^2} - \frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3} - \frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2} + \frac{x^4\,\text{FresnelS}(b\cdots}{\cdots}\cdots\right)}{\pi b^2}\cdots\right)$$

$$\Big\downarrow \text{ 3838}$$

3.32. $\int x^6\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{7}x^7 \operatorname{FresnelS}(bx)^2 - \frac{2}{7}b\left(\frac{6\left(-\frac{4\left(\frac{2\left(\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4 \operatorname{Fresne}}{}\right.}{\pi b^2}\right.$$

↓ 2009

$$\frac{1}{7}x^7 \operatorname{FresnelS}(bx)^2 - \frac{2}{7}b\left(-\frac{x^6\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{6\left(\frac{x^4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\left(\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}\right)}{\pi b^2}-\frac{x^2}{}\right.}{\pi b^2}\right.}{}\right.$$

input `Int[x^6*FresnelS[b*x]^2,x]`

3.32. $\int x^6 \operatorname{FresnelS}(bx)^2 \, dx$

output
```
(x^7*FresnelS[b*x]^2)/7 - (2*b*(-((x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/
(b^2*Pi)) + (-1/2*(x^5*Cos[b^2*Pi*x^2])/(b^2*Pi) + (5*((-3*(-1/2*(x*Cos[b^
2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi
) + (x^3*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) + (6*((x^4*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (4*((-1/2*(x*Cos[b^2*Pi*x^2])/(
b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(2*b*Pi) - (x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + (2*(-((x/2 - FresnelC[Sqrt[2]*b*x]
/(2*Sqrt[2]*b))/(b*Pi)) + (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(
b^2*Pi)))/(b^2*Pi) - (x^5/10 + ((3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + Fr
esnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) - (x^3*Sin[b^2*Pi*x^2]
)/(2*b^2*Pi))/2)/(b*Pi)))/(b^2*Pi)))/7
```

3.32.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.32.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\text{FresnelS}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelS}(bx)\left(-\frac{b^6 x^6 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} + \frac{\frac{6b^4 x^4 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{24\left(-\frac{b^2 x^2 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}\right.}{}$
default	$\frac{\frac{\text{FresnelS}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelS}(bx)\left(-\frac{b^6 x^6 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} + \frac{\frac{6b^4 x^4 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{24\left(-\frac{b^2 x^2 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}\right.}{}$

input
```
int(x^6*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^7*(1/7*FresnelS(b*x)^2*b^7*x^7-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/
2*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2
*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*b^5*x^5*P
i^2-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x
*2^(1/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^
3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(
1/2))))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int x^6 \,\text{FresnelS}(bx)^2 \, dx$$

$$= \frac{80\,\pi^4 b^8 x^7\, \text{S}\,(bx)^2 + 56\,\pi^2 b^6 x^5 - 2370\, b^2 x + 20\,(4\,\pi^2 b^6 x^5 - 147\, b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 160\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x}{}$$

input
```
integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/560*(80*pi^4*b^8*x^7*fresnel_sin(b*x)^2 + 56*pi^2*b^6*x^5 - 2370*b^2*x +
 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6
 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2655*sqrt(2)*sqrt
(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*
x^2) + 24*(pi^2*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^
4*b^8)
```

3.32.6 Sympy [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 S^2(bx) \, dx$$

input
```
integrate(x**6*fresnels(b*x)**2,x)
```

output
```
Integral(x**6*fresnels(b*x)**2, x)
```

3.32.7 Maxima [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^6*fresnel_sin(b*x)^2, x)
```

3.32.8 Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_sin(b*x)^2, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx)^2 \, dx = \int x^6 \operatorname{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x^6*FresnelS(b*x)^2,x)
```

output
```
int(x^6*FresnelS(b*x)^2, x)
```

3.33 $\int x^5 \text{FresnelS}(bx)^2 \, dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 265

$$\begin{aligned}\int x^5 \text{FresnelS}(bx)^2 \, dx &= \frac{5x^4}{24b^2\pi^2} - \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} + \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} \\ &\quad - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3b\pi} \\ &\quad + \frac{5\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^6\pi^3} + \frac{1}{6}x^6\,\text{FresnelS}(bx)^2 \\ &\quad - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\ &\quad - \frac{5x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3}\end{aligned}$$

output
```
5/24*x^4/b^2/Pi^2-11/6*cos(b^2*Pi*x^2)/b^6/Pi^4+1/12*x^4*cos(b^2*Pi*x^2)/b
^2/Pi^2-5*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi^3+1/3*x^5*cos(1/2*b^2
*Pi*x^2)*FresnelS(b*x)/b/Pi+5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3+1/6*x
^6*FresnelS(b*x)^2-5/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/
b^4/Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^4/Pi^3-5/
3*x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-7/12*x^2*sin(b^2*Pi*x^2)/
b^4/Pi^3
```

3.33.2 Mathematica [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2\,dx = \int x^5 \operatorname{FresnelS}(bx)^2\,dx$$

input
```
Integrate[x^5*FresnelS[b*x]^2,x]
```

output
```
Integrate[x^5*FresnelS[b*x]^2, x]
```

3.33.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.35, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.500, Rules used = {6984, 7008, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3118, 7016, 3860, 3042, 3790, 15, 3042, 3777, 25, 3042, 3118, 7008, 3860, 3042, 3118, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \operatorname{FresnelS}(bx)^2\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \; 7008$$

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\int x^2\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3042

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\int x^2\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\right.}{\pi b^2}\right.$$

$\downarrow$ 25

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{6}x^6 \operatorname{FresnelS}(bx)^2-\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi\right.}{\pi b^2}\right.$$

$\downarrow$ 3118

3.33. $\int x^5 \operatorname{FresnelS}(bx)^2\,dx$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$
$$\frac{1}{3}b\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}\right.$$

$\downarrow$ 7016

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$
$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x\right.}{\pi b^2}\right.$$

$\downarrow$ 3860

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$
$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$
$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b\right.}{\pi b^2}\right.$$

$\downarrow$ 3790

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$
$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\,\text{co}}{\pi b^2}\right.$$

$\downarrow$ 15

3.33. $\int x^5\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\right.}{\pi b^2}\right.$$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos}{\pi b^2}\right.$$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{Fre}}{}\right.$$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{Fresne}}{}\right.$$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2-$$

$$\Big\downarrow \text{3042}$$

3.33. $\int x^5\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{Fresne}\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{ 7008}$$

$$\frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x\cdots\right.\right.}{\pi b^2}\cdots}{\cdots}\right.}{\pi b^2}\cdots\right.$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{6}x^6 \text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x\cdots\right.\right.}{\pi b^2}\cdots}{\cdots}\right.}{\pi b^2}\cdots\right.$$

$$\Big\downarrow \text{ 3042}$$

3.33. $\int x^5 \text{FresnelS}(bx)^2\, dx$

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x\right.}{\pi b^2}\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3118

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\right.}{2\pi b}\right.}{\pi b^2}\right.$$

$\downarrow$ 7000

$$\frac{1}{6}x^6\,\text{FresnelS}(bx)^2 - \frac{1}{3}b\left(\frac{5\left(-\frac{3\left(\frac{-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\right.}{\pi b^2}\right.$$

input
```
Int[x^5*FresnelS[b*x]^2,x]
```

output
```
(x^6*FresnelS[b*x]^2)/6 - (b*(-((x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b
^2*Pi)) + (-((x^4*Cos[b^2*Pi*x^2])/(b^2*Pi)) + (2*(Cos[b^2*Pi*x^2]/(b^4*Pi
^2) + (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi)))/(b^2*Pi))/(4*b*Pi) + (5*((-3*(-1/4*
Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi
) + ((FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{
1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi)))/(b^2*Pi) + (x^3*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^4/4 + (-(Cos[b^2*Pi*x^2]/(b^4*Pi^2)) -
(x^2*Sin[b^2*Pi*x^2])/(b^2*Pi))/2)/(2*b*Pi)))/(b^2*Pi)))/3
```

3.33. $\int x^5\,\text{FresnelS}(bx)^2\,dx$

3.33.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.33.4 Maple [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2\,dx$$

input
```
int(x^5*FresnelS(b*x)^2,x)
```

output
```
int(x^5*FresnelS(b*x)^2,x)
```

3.33.5 Fricas [F]

$$\int x^5 \operatorname{FresnelS}(bx)^2\,dx = \int x^5 \operatorname{S}(bx)^2\,dx$$

input
```
integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^5*fresnel_sin(b*x)^2, x)
```

3.33.6 Sympy [F]

$$\int x^5 \text{FresnelS}(bx)^2 \, dx = \int x^5 S^2(bx) \, dx$$

input
```
integrate(x**5*fresnels(b*x)**2,x)
```

output
```
Integral(x**5*fresnels(b*x)**2, x)
```

3.33.7 Maxima [F]

$$\int x^5 \text{FresnelS}(bx)^2 \, dx = \int x^5 \text{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*fresnel_sin(b*x)^2, x)
```

3.33.8 Giac [F]

$$\int x^5 \text{FresnelS}(bx)^2 \, dx = \int x^5 \text{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_sin(b*x)^2, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx)^2 \, dx = \int x^5 \operatorname{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x^5*FresnelS(b*x)^2,x)
```

output
```
int(x^5*FresnelS(b*x)^2, x)
```

3.34 $\int x^4 \operatorname{FresnelS}(bx)^2 \, dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 177

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \frac{4x^3}{15b^2\pi^2} + \frac{x^3 \cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{16 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{5b^5\pi^3} + \frac{2x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{5b\pi} + \frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 + \frac{43 \operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} - \frac{8x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} - \frac{11x \sin(b^2\pi x^2)}{20b^4\pi^3}$$

output
```
4/15*x^3/b^2/Pi^2+1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-16/5*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b^5/Pi^3+2/5*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1
/5*x^5*FresnelS(b*x)^2-8/5*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-
11/20*x*sin(b^2*Pi*x^2)/b^4/Pi^3+43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1
/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int x^4 \text{FresnelS}(bx)^2 \, dx$$
$$= \frac{32b^3\pi x^3 + 12b^3\pi x^3 \cos(b^2\pi x^2) + 24b^5\pi^3 x^5 \text{FresnelS}(bx)^2 + 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 48\,\text{FresnelS}(bx)\left((-\right.}{120b^5\pi^3}$$

input
```
Integrate[x^4*FresnelS[b*x]^2,x]
```

output
```
(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[
b*x]^2 + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*P
i^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*
Sin[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

3.34.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6984, 7008, 3866, 3867, 3832, 7016, 3872, 15, 3867, 3832, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{FresnelS}(bx)^2 \, dx$$
$$\Big\downarrow \; 6984$$
$$\frac{1}{5}x^5 \text{FresnelS}(bx)^2 - \frac{2}{5}b \int x^5 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$\Big\downarrow \; 7008$$
$$\frac{1}{5}x^5 \text{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^4 \sin\left(b^2\pi x^2\right) dx}{2\pi b} - \frac{x^4 \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$
$$\Big\downarrow \; 3866$$

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^2\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x^3\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}-\frac{x^4\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3867

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin(\pi b^2x^2)}{2\pi b^2}-\frac{\int\sin(b^2\pi x^2)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}-\frac{x^4\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3832

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}-\frac{x^4\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin(\pi b^2x^2)}{2\pi b^2}-\frac{\operatorname{FresnelS}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos(\pi b}{2\pi b^2}}{2\pi b}\right.$$

$\downarrow$ 7016

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\int x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x\right.}{\pi b^2}\right.$$

$\downarrow$ 3872

$$\frac{1}{5}x^5 \operatorname{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\int x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx}{2}-\frac{1}{2}\int x^2\cos(b^2\pi x^2)dx}{\pi b}+\frac{x^2\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\operatorname{FresnelS}(bx)\cos\left(\right.}{\pi b^2}\right.$$

$\downarrow$ 15

3.34. $\int x^4 \operatorname{FresnelS}(bx)^2\,dx$

$$\frac{1}{5}x^5 \,\text{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi\right.}{\pi b^2}\right.$$

$\downarrow$ 3867

$$\frac{1}{5}x^5 \,\text{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}}{}\right.$$

$\downarrow$ 3832

$$\frac{1}{5}x^5 \,\text{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}}{}\right.$$

$\downarrow$ 7006

$$\frac{1}{5}x^5 \,\text{FresnelS}(bx)^2 - \frac{2}{5}b\left(\frac{4\left(-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4}{}\right.$$

$\downarrow$ 3832

3.34. $\int x^4\,\text{FresnelS}(bx)^2\,dx$

$$\frac{1}{5}x^5\,\text{FresnelS}(bx)^2 - \frac{2}{5}b\left(-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{3\left(\frac{x\sin(\pi b^2x^2)}{2\pi b^2}-\frac{\text{FresnelS}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2} - \frac{x^3\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{4\left(\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \right.}{}\right.$$

input
```
Int[x^4*FresnelS[b*x]^2,x]
```

output
```
(x^5*FresnelS[b*x]^2)/5 - (2*b*(-((x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/
(b^2*Pi)) + (-1/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[
2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2
*b*Pi) + (4*((-2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + Fresne
lS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi)))/(b^2*Pi) + (x^2*FresnelS[b*x]*Sin[(b^
2*Pi*x^2)/2])/(b^2*Pi) - (x^3/6 + (FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi
) - (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b^2*Pi)))/5
```

3.34.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.34.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right.}{2\pi}\right.}{}}{b^5}$
default	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right.}{2\pi}\right.}{}}{b^5}$

input
```
int(x^4*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*FresnelS(b*x)^2*b^5*x^5-2*FresnelS(b*x)*(-1/5/Pi*b^4*x^4*cos(1/
2*b^2*Pi*x^2)+4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*
Pi*x^2)))+4/15/Pi^2*b^3*x^3-4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^
(1/2)*FresnelS(b*x*2^(1/2)))-1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2
*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^
(1/2)*FresnelS(b*x*2^(1/2))))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^4 \operatorname{FresnelS}(bx)^2\,dx$$
$$= \frac{24\,\pi^3 b^6 x^5\,\mathrm{S}\,(bx)^2 + 24\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 20\,\pi b^4 x^3 + 48\,(\pi^2 b^5 x^4 - 8\,b)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\,(bx) + 129\,\sqrt{2}\sqrt{}}{120\,\pi^3 b^6}$$

input
```
integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/120*(24*pi^3*b^6*x^5*fresnel_sin(b*x)^2 + 24*pi*b^4*x^3*cos(1/2*pi*b^2*x
^2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresne
l_sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(
16*pi*b^3*x^2*fresnel_sin(b*x) + 11*b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*
b^2*x^2))/(pi^3*b^6)
```

3.34.6 Sympy [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2\,dx = \int x^4 S^2(bx)\;dx$$

input
```
integrate(x**4*fresnels(b*x)**2,x)
```

output
```
Integral(x**4*fresnels(b*x)**2, x)
```

3.34.7 Maxima [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4*fresnel_sin(b*x)^2, x)
```

3.34.8 Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_sin(b*x)^2, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx)^2 \, dx = \int x^4 \operatorname{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x^4*FresnelS(b*x)^2,x)
```

output
```
int(x^4*FresnelS(b*x)^2, x)
```

3.35 $\int x^3 \operatorname{FresnelS}(bx)^2 \, dx$

3.35.1 Optimal result

Integrand size = 10, antiderivative size = 140

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{2b\pi} + \frac{3\operatorname{FresnelS}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelS}(bx)^2 - \frac{3x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} - \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

output
```
3/8*x^2/b^2/Pi^2+1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^3*cos(1/2*b^2*Pi*x
^2)*FresnelS(b*x)/b/Pi+3/4*FresnelS(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelS(b*x)^
2-3/2*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-1/2*sin(b^2*Pi*x^2)/b^4
/Pi^3
```

3.35.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \frac{3b^2\pi x^2 + b^2\pi x^2\cos(b^2\pi x^2) + 2\pi(3+b^4\pi^2x^4)\operatorname{FresnelS}(bx)^2 + 4b\pi x\operatorname{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 3\right.}{8b^4\pi^3}$$

input
```
Integrate[x^3*FresnelS[b*x]^2,x]
```

output
```
(3*b^2*Pi*x^2 + b^2*Pi*x^2*Cos[b^2*Pi*x^2] + 2*Pi*(3 + b^4*Pi^2*x^4)*Fresn
elS[b*x]^2 + 4*b*Pi*x*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 3*Si
n[(b^2*Pi*x^2)/2]) - 4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

3.35.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6984, 7008, 3860, 3042, 3777, 3042, 3117, 7016, 3860, 3042, 3114, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{FresnelS}(bx)^2 \, dx$$

$$\Big\downarrow \text{6984}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx)^2 - \frac{1}{2}b\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b} - \frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b} - \frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4 \text{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b} - \frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int \cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3042

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int \sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3117

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)dx}{\pi b^2}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}\right)$$

$\downarrow$ 7016

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\left(-\frac{\int \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\right.$$

$\downarrow$ 3860

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\left(-\frac{\int \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\right.$$

$\downarrow$ 3042

$$\frac{1}{4}x^4 \operatorname{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\left(-\frac{\int \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\right.$$

$\downarrow$ 3114

3.35. $\int x^3 \operatorname{FresnelS}(bx)^2\, dx$

$$\frac{1}{4}x^4\,\text{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\cdots\right.$$

$\downarrow$ 6994

$$\frac{1}{4}x^4\,\text{FresnelS}(bx)^2 - \frac{1}{2}b\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\cdots\right.$$

$\downarrow$ 15

$$\frac{1}{4}x^4\,\text{FresnelS}(bx)^2 - \frac{1}{2}b\left(-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{3\left(-\frac{\text{FresnelS}(bx)^2}{2\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\cdots}{\cdots}\right)}{\pi b^2}\right.$$

input
```
Int[x^3*FresnelS[b*x]^2,x]
```

output
```
(x^4*FresnelS[b*x]^2)/4 - (b*(-((x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b
^2*Pi)) + (-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*x^2]/(b^4*Pi^2))
/(4*b*Pi) + (3*(-1/2*FresnelS[b*x]^2/(b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^2*Pi) - (x^2/2 - Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)))/(b^
2*Pi)))/2
```

3.35. $\int x^3\,\text{FresnelS}(bx)^2\,dx$

3.35.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.35.4 Maple [F]

$$\int x^3 \operatorname{FresnelS}(bx)^2\, dx$$

input
```
int(x^3*FresnelS(b*x)^2,x)
```

output
```
int(x^3*FresnelS(b*x)^2,x)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{FresnelS}(bx)^2\, dx$$
$$= \frac{2\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + \pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + \pi b^2 x^2 + \left(3\,\pi + \pi^3 b^4 x^4\right) \mathrm{S}\left(bx\right)^2 - 2\left(3\,\pi b x\, \mathrm{S}\left(bx\right) + 2\right.}{4\,\pi^3 b^4}$$

input
```
integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos(
1/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + (3*pi + pi^3*b^4*x^4)*fresnel_sin(b*x)^2
- 2*(3*pi*b*x*fresnel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2
))/(pi^3*b^4)
```

3.35.6 Sympy [F]

$$\int x^3 \text{FresnelS}(bx)^2 \, dx = \int x^3 S^2(bx) \, dx$$

input
```
integrate(x**3*fresnels(b*x)**2,x)
```

output
```
Integral(x**3*fresnels(b*x)**2, x)
```

3.35.7 Maxima [F]

$$\int x^3 \text{FresnelS}(bx)^2 \, dx = \int x^3 \mathrm{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*fresnel_sin(b*x)^2, x)
```

3.35.8 Giac [F]

$$\int x^3 \text{FresnelS}(bx)^2 \, dx = \int x^3 \mathrm{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_sin(b*x)^2, x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)^2 \, dx = \int x^3 \operatorname{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x^3*FresnelS(b*x)^2,x)
```

output
```
int(x^3*FresnelS(b*x)^2, x)
```

3.36 $\int x^2 \operatorname{FresnelS}(bx)^2\,dx$

3.36.1 Optimal result

Integrand size = 10, antiderivative size = 124

$$\begin{aligned}\int x^2 \operatorname{FresnelS}(bx)^2\,dx &= \frac{2x}{3b^2\pi^2}+\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2}\\&\quad+\frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{3b\pi}\\&\quad+\frac{1}{3}x^3\operatorname{FresnelS}(bx)^2-\frac{4\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}\end{aligned}$$

output
```
2/3*x/b^2/Pi^2+1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2+2/3*x^2*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/b/Pi+1/3*x^3*FresnelS(b*x)^2-4/3*FresnelS(b*x)*sin(1/2*b^2*P
i*x^2)/b^3/Pi^2-5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^2 \operatorname{FresnelS}(bx)^2\,dx$$
$$=\frac{2bx(4+\cos\left(b^2\pi x^2\right))-5\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right)+4b^3\pi^2x^3\operatorname{FresnelS}(bx)^2+8\operatorname{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi\right.\right.}{12b^3\pi^2}$$

input
```
Integrate[x^2*FresnelS[b*x]^2,x]
```

output
```
(2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^
2*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] -
2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

3.36.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6984, 7008, 3866, 3833, 7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelS}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6984}$$

$$\frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{2}{3}b \int x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

$$\Big\downarrow \text{ 7008}$$

$$\frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^2 \sin\left(b^2\pi x^2\right)dx}{2\pi b} - \frac{x^2 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\frac{\int \cos(b^2\pi x^2)dx}{2\pi b^2} - \frac{x\cos(\pi b^2 x^2)}{2\pi b^2}}{2\pi b} - \frac{x^2 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3833}$$

$$\frac{1}{3}x^3 \text{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} - \frac{x^2 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3} - \frac{x\cos(\pi b^2 x^2)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{ 7014}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\left(\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{3838}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\left(\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \operatorname{FresnelS}(bx)^2 - \frac{2}{3}b\left(\frac{2\left(\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}\right)}{\pi b^2}-\frac{x^2\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

input
```
Int[x^2*FresnelS[b*x]^2,x]
```

output
```
(x^3*FresnelS[b*x]^2)/3 - (2*b*((-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + Fresn
elC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(2*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(b^2*Pi) + (2*(-((x/2 - FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))
/(b*Pi)) + (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/3
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.36.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx \cos\left(b^2 \pi x^2\right)}{2\pi} +}{}}{b^3}$
default	$\frac{\frac{\mathrm{FresnelS}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelS}(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx \cos\left(b^2 \pi x^2\right)}{2\pi} +}{}}{b^3}$

input
```
int(x^2*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*FresnelS(b*x)^2*b^3*x^3-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/
2*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2-1/3/Pi^2*2^(1/2)*
FresnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*F
resnelC(b*x*2^(1/2))))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx$$
$$= \frac{4\,\pi^2 b^4 x^3\,\mathrm{S}\left(bx\right)^2 + 8\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) + 4\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\,b^2 x - 16\,b\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) -}{12\,\pi^2 b^4}$$

input
```
integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi^2*b^4*x^3*fresnel_sin(b*x)^2 + 8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)
*fresnel_sin(b*x) + 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel
_sin(b*x)*sin(1/2*pi*b^2*x^2) - 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sq
rt(b^2)*x))/(pi^2*b^4)
```

3.36.6 Sympy [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = \int x^2 S^2(bx) \, dx$$

input
```
integrate(x**2*fresnels(b*x)**2,x)
```

output
```
Integral(x**2*fresnels(b*x)**2, x)
```

3.36.7 Maxima [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = \int x^2 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_sin(b*x)^2, x)
```

3.36.8 Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = \int x^2 \operatorname{S}(bx)^2 \, dx$$

input
```
integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_sin(b*x)^2, x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx)^2 \, dx = \int x^2 \operatorname{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x^2*FresnelS(b*x)^2,x)
```

output
```
int(x^2*FresnelS(b*x)^2, x)
```

3.37 $\int x \operatorname{FresnelS}(bx)^2 \, dx$

3.37.1 Optimal result

Integrand size = 8, antiderivative size = 143

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{b\pi} - \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^2\pi} + \frac{1}{2}x^2\operatorname{FresnelS}(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}$$

output
```
1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-1/2*
FresnelC(b*x)*FresnelS(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*x)^2+1/8*I*x^2*hyper
geom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2
, 2],1/2*I*b^2*Pi*x^2)/Pi
```

3.37.2 Mathematica [F]

$$\int x \operatorname{FresnelS}(bx)^2 \, dx = \int x \operatorname{FresnelS}(bx)^2 \, dx$$

input
```
Integrate[x*FresnelS[b*x]^2,x]
```

output
```
Integrate[x*FresnelS[b*x]^2, x]
```

3.37.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6984, 7008, 3860, 3042, 3118, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{FresnelS}(bx)^2\,dx$$

$$\Big\downarrow \text{6984}$$

$$\frac{1}{2}x^2\,\text{FresnelS}(bx)^2-b\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{1}{2}x^2\,\text{FresnelS}(bx)^2-b\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}x^2\,\text{FresnelS}(bx)^2-b\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\,\text{FresnelS}(bx)^2-b\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{2}x^2\,\text{FresnelS}(bx)^2-b\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)$$

$$\Big\downarrow \text{7000}$$

$$\frac{1}{2}x^2 \operatorname{FresnelS}(bx)^2 - b\left(\frac{-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b}}{\pi b^2} - \frac{x\operatorname{FresnelS}(bx)\cos\left(\right.}{\pi b^2}\right.$$

input
```
Int[x*FresnelS[b*x]^2,x]
```

output
```
(x^2*FresnelS[b*x]^2)/2 - b*(-1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2
*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + ((FresnelC[b*x]*FresnelS[b*x])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] +
(I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi
))
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

3.37.4 Maple [F]

$$\int x \,\text{FresnelS}\,(bx)^2\,dx$$

input
```
int(x*FresnelS(b*x)^2,x)
```

output
```
int(x*FresnelS(b*x)^2,x)
```

3.37.5 Fricas [F]

$$\int x\,\text{FresnelS}(bx)^2\,dx = \int x\,\text{S}\,(bx)^2\;dx$$

input
```
integrate(x*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x*fresnel_sin(b*x)^2, x)
```

3.37.6 Sympy [F]

$$\int x\,\text{FresnelS}(bx)^2\,dx = \int xS^2(bx)\;dx$$

input
```
integrate(x*fresnels(b*x)**2,x)
```

output
```
Integral(x*fresnels(b*x)**2, x)
```

3.37.7 Maxima [F]

$$\int x \,\text{FresnelS}(bx)^2 \, dx = \int x \,\text{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*fresnel_sin(b*x)^2, x)
```

3.37.8 Giac [F]

$$\int x \,\text{FresnelS}(bx)^2 \, dx = \int x \,\text{S}\left(bx\right)^2 \, dx$$

input
```
integrate(x*fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*fresnel_sin(b*x)^2, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int x \,\text{FresnelS}(bx)^2 \, dx = \int x \,\text{FresnelS}(b\,x)^2 \, dx$$

input
```
int(x*FresnelS(b*x)^2,x)
```

output
```
int(x*FresnelS(b*x)^2, x)
```

3.38 $\int \text{FresnelS}(bx)^2\,dx$

3.38.1 Optimal result

Integrand size = 6, antiderivative size = 55

$$\int \text{FresnelS}(bx)^2\,dx=\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi}+x\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}$$

output
```
2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+x*FresnelS(b*x)^2-1/2*FresnelS(b*
x*2^(1/2))/b/Pi*2^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^2\,dx=\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b\pi}+x\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi}$$

input
```
Integrate[FresnelS[b*x]^2,x]
```

output
```
(2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - Fresnel
S[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

3.38.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6974, 27, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)^2\,dx$$

$$\Big\downarrow \text{6974}$$

$$x\,\text{FresnelS}(bx)^2-2\int bx\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{27}$$

$$x\,\text{FresnelS}(bx)^2-2b\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{7006}$$

$$x\,\text{FresnelS}(bx)^2-2b\left(\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3832}$$

$$x\,\text{FresnelS}(bx)^2-2b\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

input
```
Int[FresnelS[b*x]^2,x]
```

output
```
x*FresnelS[b*x]^2 - 2*b*(-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) +
 FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi))
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 6974
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Simp[2   Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[
a + b*x], x], x] /; FreeQ[{a, b}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.38.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^2 bx+\frac{2\,\text{FresnelS}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\text{FresnelS}(bx)^2 bx+\frac{2\,\text{FresnelS}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

input
```
int(FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelS(b*x)^2*b*x+2*FresnelS(b*x)/Pi*cos(1/2*b^2*Pi*x^2)-1/2/Pi*2^(
1/2)*FresnelS(b*x*2^(1/2)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \text{FresnelS}(bx)^2\,dx = \frac{2\,\pi b^2 x\,\text{S}\left(bx\right)^2 + 4\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\left(bx\right) - \sqrt{2}\sqrt{b^2}\,\text{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{2\,\pi b^2}$$

input
```
integrate(fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*pi*b^2*x*fresnel_sin(b*x)^2 + 4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b
*x) - sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

3.38.6 Sympy [F]

$$\int \text{FresnelS}(bx)^2\,dx = \int S^2(bx)\,dx$$

input
```
integrate(fresnels(b*x)**2,x)
```

output
```
Integral(fresnels(b*x)**2, x)
```

3.38.7 Maxima [F]

$$\int \text{FresnelS}(bx)^2\,dx = \int \text{S}\left(bx\right)^2\,dx$$

input
```
integrate(fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2, x)
```

3.38.8 Giac [F]

$$\int \text{FresnelS}(bx)^2\,dx = \int S(bx)^2\,dx$$

input
```
integrate(fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)^2\,dx = \int \text{FresnelS}(b\,x)^2\,dx$$

input
```
int(FresnelS(b*x)^2,x)
```

output
```
int(FresnelS(b*x)^2, x)
```

3.39 $\int \frac{\text{FresnelS}(bx)^2}{x}\,dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x}\,dx = \text{Int}\left(\frac{\text{FresnelS}(bx)^2}{x},x\right)$$

output
```
Unintegrable(FresnelS(b*x)^2/x,x)
```

3.39.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x}\,dx$$

input
```
Integrate[FresnelS[b*x]^2/x,x]
```

output
```
Integrate[FresnelS[b*x]^2/x, x]
```

3.39.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx$$
$$\Big\downarrow \; 6988$$
$$\int \frac{\text{FresnelS}(bx)^2}{x} dx$$

input
```
Int[FresnelS[b*x]^2/x,x]
```

output
```
$Aborted
```

3.39.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.39.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^{2}}{x}dx$$

input
```
int(FresnelS(b*x)^2/x,x)
```

output
```
int(FresnelS(b*x)^2/x,x)
```

3.39.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{S}(bx)^2}{x} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x, x)
```

3.39.6 Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{S^2(bx)}{x} \, dx$$

input
```
integrate(fresnels(b*x)**2/x,x)
```

output
```
Integral(fresnels(b*x)**2/x, x)
```

3.39.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{S}(bx)^2}{x} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x, x)
```

3.39.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x, x)
```

3.39.9 Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x} \, dx$$

input
```
int(FresnelS(b*x)^2/x,x)
```

output
```
int(FresnelS(b*x)^2/x, x)
```

3.40 $\int \frac{\text{FresnelS}(bx)^2}{x^2} dx$

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} dx = -\frac{\text{FresnelS}(bx)^2}{x} + 2b\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)$$

output
```
-FresnelS(b*x)^2/x+2*b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} dx = \int \frac{\text{FresnelS}(bx)^2}{x^2} dx$$

input
```
Integrate[FresnelS[b*x]^2/x^2,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^2, x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6984, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 6984}$$

$$2b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)^2}{x}$$

$$\Big\downarrow \text{ 7012}$$

$$2b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)^2}{x}$$

input
```
Int[FresnelS[b*x]^2/x^2,x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^2} dx$$

input
```
int(FresnelS(b*x)^2/x^2,x)
```

output
```
int(FresnelS(b*x)^2/x^2,x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{\text{S}\,(bx)^2}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^2, x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^2}\,dx = \int \frac{S^2(bx)}{x^2}\,dx$$

input
```
integrate(fresnels(b*x)**2/x**2,x)
```

output
```
Integral(fresnels(b*x)**2/x**2, x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} \, dx = \int \frac{S(bx)^2}{x^2} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^2, x)
```

3.40.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} \, dx = \int \frac{S(bx)^2}{x^2} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^2, x)
```

3.40.9 Mupad [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^2} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^2} \, dx$$

input
```
int(FresnelS(b*x)^2/x^2,x)
```

output
```
int(FresnelS(b*x)^2/x^2, x)
```

3.41 $\int \frac{\text{FresnelS}(bx)^2}{x^3} dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} dx = -\frac{\text{FresnelS}(bx)^2}{2x^2} + b\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}, x\right)$$

output
```
-1/2*FresnelS(b*x)^2/x^2+b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/
x^2,x)
```

3.41.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} dx = \int \frac{\text{FresnelS}(bx)^2}{x^3} dx$$

input
```
Integrate[FresnelS[b*x]^2/x^3,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^3, x]
```

3.41.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6984, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^3} \, dx$$
$$\Big\downarrow \text{ 6984}$$
$$b \int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\text{FresnelS}(bx)^2}{2x^2}$$
$$\Big\downarrow \text{ 7012}$$
$$b \int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\text{FresnelS}(bx)^2}{2x^2}$$

input
```
Int[FresnelS[b*x]^2/x^3,x]
```

output
```
$Aborted
```

3.41.3.1 Defintions of rubi rules used

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.41.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^3}dx$$

input
```
int(FresnelS(b*x)^2/x^3,x)
```

output
```
int(FresnelS(b*x)^2/x^3,x)
```

3.41.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{S}\,(bx)^2}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^3, x)
```

3.41.6 Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{S^2(bx)}{x^3}\,dx$$

input
```
integrate(fresnels(b*x)**2/x**3,x)
```

output
```
Integral(fresnels(b*x)**2/x**3, x)
```

3.41.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{S}\,(bx)^2}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^3, x)
```

3.41.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{S}\,(bx)^2}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^3, x)
```

3.41.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^3}\,dx$$

input
```
int(FresnelS(b*x)^2/x^3,x)
```

output
```
int(FresnelS(b*x)^2/x^3, x)
```

3.42 $\int \frac{\text{FresnelS}(bx)^2}{x^4} dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} dx = -\frac{b^2}{6x} + \frac{b^2\cos(b^2\pi x^2)}{6x} - \frac{\text{FresnelS}(bx)^2}{3x^3} + \frac{b^3\pi \,\text{FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2} + \frac{1}{3}b^3\pi \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}, x\right)$$

output
```
-1/6*b^2/x+1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*FresnelS(b*x)^2/x^3-1/3*b*Fresnel
S(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/
3*b^3*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4} dx = \int \frac{\text{FresnelS}(bx)^2}{x^4} dx$$

input
```
Integrate[FresnelS[b*x]^2/x^4,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^4, x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6984, 7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{2}{3}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)^2}{3x^3}$$

$$\Big\downarrow \; 7010$$

$$\frac{2}{3}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}\right)-\frac{\text{FresnelS}(bx)^2}{3x^3}$$

$$\Big\downarrow \; 3869$$

$$\frac{2}{3}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi\right.}{2x^2}\right.\frac{\text{FresnelS}(bx)^2}{3x^3}$$

$$\Big\downarrow \; 3832$$

$$\frac{2}{3}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{\right.\right.\right.\frac{\text{FresnelS}(bx)^2}{3x^3}$$

$$\Big\downarrow \; 7020$$

$$\frac{2}{3}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{\right.\right.\right.\frac{\text{FresnelS}(bx)^2}{3x^3}$$

input
```
Int[FresnelS[b*x]^2/x^4,x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^4} dx$$

input
```
int(FresnelS(b*x)^2/x^4,x)
```

output
```
int(FresnelS(b*x)^2/x^4,x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\, dx = \int \frac{\text{S}\,(bx)^2}{x^4}\, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^4, x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\, dx = \int \frac{S^2(bx)}{x^4}\, dx$$

input
```
integrate(fresnels(b*x)**2/x**4,x)
```

output
```
Integral(fresnels(b*x)**2/x**4, x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^4, x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^4}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^4, x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^4}\,dx$$

input
```
int(FresnelS(b*x)^2/x^4,x)
```

output
```
int(FresnelS(b*x)^2/x^4, x)
```

3.43 $\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 127

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = -\frac{b^2}{24x^2}+\frac{b^2\cos(b^2\pi x^2)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} -\frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)^2}{4x^4} -\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}+\frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

output
```
-1/24*b^2/x^2+1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b^3*Pi*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x-1/12*b^4*Pi^2*FresnelS(b*x)^2-1/4*FresnelS(b*x)^2/x^4+1/12
*b^4*Pi*Si(b^2*Pi*x^2)-1/6*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = -\frac{b^2}{24x^2}+\frac{b^2\cos(b^2\pi x^2)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x} -\frac{1}{12}b^4\pi^2\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)^2}{4x^4} -\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}+\frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[FresnelS[b*x]^2/x^5,x]
```

output
```
-1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/
2]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/
(4*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinInteg
ral[b^2*Pi*x^2])/12
```

3.43.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6984, 7010, 3861, 3042, 3778, 25, 3042, 3780, 7018, 3856, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}dx$$

$$\Big\downarrow \text{ 6984}$$

$$\frac{1}{2}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{ 7010}$$

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 3778

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\right.}{3x^3}\right.\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 25

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\right.}{3x^3}\right.\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 3042

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\right.}{3x^3}\right.\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 3780

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 7018

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\right.\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\big\downarrow$ 3856

3.43. $\int\frac{\text{FresnelS}(bx)^2}{x^5}\,dx$

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelS}(bx)}{3x}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\downarrow$ 6994

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\left(-\pi b\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{3x^3}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{4x^4}$$

$\downarrow$ 15

$$\frac{1}{2}b\left(\frac{1}{3}\pi b^2\left(-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi b\,\text{FresnelS}(bx)^2\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{4x^4}$$

input
```
Int[FresnelS[b*x]^2/x^5,x]
```

output
```
-1/4*FresnelS[b*x]^2/x^4 + (b*(-1/12*b/x^2 - (FresnelS[b*x]*Sin[(b^2*Pi*x^
2)/2])/(3*x^3) + (b^2*Pi*(-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi
*FresnelS[b*x]^2)/2 + (b*SinIntegral[b^2*Pi*x^2])/4))/3 - (b*(-(Cos[b^2*Pi
*x^2]/x^2) - b^2*Pi*SinIntegral[b^2*Pi*x^2]))/12))/2
```

3.43.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.43. $\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx$

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.43.4 Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^5}dx$$

input
```
int(FresnelS(b*x)^2/x^5,x)
```

output
```
int(FresnelS(b*x)^2/x^5,x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \frac{\pi b^4x^4\,\text{Si}\,(\pi b^2x^2) - 2\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx) + b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 - b^2x^2 - 2\,bx\,\text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - (}{12\,x^4}$$

input
```
integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*cos(1/2*pi*b^2*x^
2)*fresnel_sin(b*x) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - b^2*x^2 - 2*b*x*fres
nel_sin(b*x)*sin(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 + 3)*fresnel_sin(b*x)^2)/
x^4
```

3.43.6 Sympy [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{S^2(bx)}{x^5}\,dx$$

input
```
integrate(fresnels(b*x)**2/x**5,x)
```

output
```
Integral(fresnels(b*x)**2/x**5, x)
```

3.43.7 Maxima [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^5, x)
```

3.43.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^5, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)^2}{x^5} dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^5} dx$$

input
```
int(FresnelS(b*x)^2/x^5,x)
```

output
```
int(FresnelS(b*x)^2/x^5, x)
```

3.44 $\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = &-\frac{b^2}{60x^3}+\frac{b^2\cos(b^2\pi x^2)}{60x^3}+\frac{7b^5\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}}\\ &-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{20x^2}-\frac{\text{FresnelS}(bx)^2}{5x^5}\\ &-\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{7b^4\pi\sin(b^2\pi x^2)}{120x}\\ &-\frac{1}{20}b^5\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

output
```
-1/60*b^2/x^3+1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/20*b^3*Pi*cos(1/2*b^2*Pi*x^2)
*FresnelS(b*x)/x^2-1/5*FresnelS(b*x)^2/x^5-1/10*b*FresnelS(b*x)*sin(1/2*b^
2*Pi*x^2)/x^4-7/120*b^4*Pi*sin(b^2*Pi*x^2)/x+7/120*b^5*Pi^2*FresnelC(b*x*2
^(1/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^
2)/x,x)
```

3.44.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6} \, dx = \int \frac{\text{FresnelS}(bx)^2}{x^6} \, dx$$

input
```
Integrate[FresnelS[b*x]^2/x^6,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^6, x]
```

3.44.3 Rubi [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6984, 7010, 3869, 3868, 3833, 7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^6} \, dx$$

$$\Big\downarrow \text{6984}$$

$$\frac{2}{5} b \int \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^5} dx - \frac{\text{FresnelS}(bx)^2}{5x^5}$$

$$\Big\downarrow \text{7010}$$

$$\frac{2}{5} b \left(\frac{1}{4} \pi b^2 \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelS}(bx)}{x^3} dx - \frac{1}{8} b \int \frac{\cos \left(b^2 \pi x^2\right)}{x^4} dx - \frac{\text{FresnelS}(bx) \sin \left(\frac{1}{2} \pi b^2 x^2\right)}{4x^4} - \frac{b}{24x^3}\right) - \frac{\text{FresnelS}(bx)^2}{5x^5}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi\right.}{4x^4}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 3868

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 3833

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 7018

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{1}{8}b\left(-\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 3868

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\,c}{2x}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 3833

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)}{2}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

$\big\downarrow$ 7012

3.44. $\int\frac{\text{FresnelS}(bx)^2}{x^6}\,dx$

$$\frac{2}{5}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)}{2}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{5x^5}$$

input
```
Int[FresnelS[b*x]^2/x^6,x]
```

output
```
$Aborted
```

3.44.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.44.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{FresnelS}(bx)^2}{x^6}dx$$

input
```
int(FresnelS(b*x)^2/x^6,x)
```

output
```
int(FresnelS(b*x)^2/x^6,x)
```

3.44.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\operatorname{S}(bx)^2}{x^6}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^6, x)
```

3.44.6 Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^6} dx = \int \frac{S^2(bx)}{x^6} dx$$

input
```
integrate(fresnels(b*x)**2/x**6,x)
```

output
```
Integral(fresnels(b*x)**2/x**6, x)
```

3.44.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6} dx = \int \frac{\text{S}(bx)^2}{x^6} dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^6, x)
```

3.44.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6} dx = \int \frac{\text{S}(bx)^2}{x^6} dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^6, x)
```

3.44.9 Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^6}\,dx$$

input
```
int(FresnelS(b*x)^2/x^6,x)
```

output
```
int(FresnelS(b*x)^2/x^6, x)
```

3.45 $\int \frac{\text{FresnelS}(bx)^2}{x^7} \, dx$

3.45.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^7} \, dx = &-\frac{b^2}{120x^4} + \frac{b^2 \cos(b^2\pi x^2)}{120x^4} + \frac{1}{72}b^6\pi^2 \operatorname{CosIntegral}(b^2\pi x^2) \\ &- \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{45x^3} - \frac{\text{FresnelS}(bx)^2}{6x^6} \\ &- \frac{b \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5} - \frac{b^4\pi \sin(b^2\pi x^2)}{72x^2} \\ &- \frac{1}{45}b^5\pi^2 \text{Int}\left(\frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}, x\right)\end{aligned}$$

output
```
-1/120*b^2/x^4+1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)+1/120*b^2*cos(b^2*Pi*x^2)/x^4-
1/45*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/6*FresnelS(b*x)^2/x^6-
1/15*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/72*b^4*Pi*sin(b^2*Pi*x^2)/x
^2-1/45*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.45.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$$

input
```
Integrate[FresnelS[b*x]^2/x^7,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^7, x]
```

3.45.3 Rubi [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6984, 7010, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7018, 3860, 3042, 3778, 3042, 3783, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{1}{3}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx - \frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 7010$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx - \frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}dx - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5} - \frac{b}{40x^4}\right) - \frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \; 3861$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{5x^5}\right.\cdots-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{5x^5}\right.\cdots-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{5x^5}\right.\cdots-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right.\cdots-\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3042}$$

3.45. $\int\frac{\text{FresnelS}(bx)^2}{x^7}\,dx$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 3783

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 7018

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\left(\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 3860

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 3042

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 3778

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)}{3}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\big\downarrow$ 3042

3.45. $\int\frac{\text{FresnelS}(bx)^2}{x^7}dx$

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}}{}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\downarrow$ 3783

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(b}{}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

$\downarrow$ 7012

$$\frac{1}{3}b\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(b}{}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{6x^6}$$

input
```
Int[FresnelS[b*x]^2/x^7,x]
```

output
```
$Aborted
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.45. $\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx$

3.45.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{FresnelS}\,(bx)^2}{x^7}dx$$

input
```
int(FresnelS(b*x)^2/x^7,x)
```

output
```
int(FresnelS(b*x)^2/x^7,x)
```

3.45.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\mathrm{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\mathrm{S}\,(bx)^2}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^7, x)
```

3.45.6 Sympy [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{S^2(bx)}{x^7}\,dx$$

input
```
integrate(fresnels(b*x)**2/x**7,x)
```

output
```
Integral(fresnels(b*x)**2/x**7, x)
```

3.45.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{S(bx)^2}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^7, x)
```

3.45.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{S(bx)^2}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^7, x)
```

3.45.9 Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^7}\,dx$$

input
```
int(FresnelS(b*x)^2/x^7,x)
```

output
```
int(FresnelS(b*x)^2/x^7, x)
```

3.46 $\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx$

3.46.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx &= -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos(b^2\pi x^2)}{210x^5}-\frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\ &\quad -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{84x^4}-\frac{\text{FresnelS}(bx)^2}{7x^7} \\ &\quad -\frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}}-\frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right) \\ &\quad -\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6}+\frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^2} \\ &\quad -\frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3}-\frac{1}{168}b^7\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)\end{aligned}$$

output
```
-1/210*b^2/x^5+1/336*b^6*Pi^2/x+1/210*b^2*cos(b^2*Pi*x^2)/x^5-67/5040*b^6*
Pi^2*cos(b^2*Pi*x^2)/x-1/84*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1
/7*FresnelS(b*x)^2/x^7-1/21*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/168*
b^5*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-13/2520*b^4*Pi*sin(b^2*Pi*x
^2)/x^3-67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)-1/168*b^7*Pi^3*Unin
tegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.46.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx=\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx$$

input
```
Integrate[FresnelS[b*x]^2/x^8,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^8, x]
```

3.46.3 Rubi [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6984, 7010, 3869, 3868, 3869, 3832, 7018, 3868, 3869, 3832, 7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx$$

$$\Big\downarrow \text{6984}$$

$$\frac{2}{7}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$$\Big\downarrow \text{7010}$$

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}\right)-\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\right.}{6x^6}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 3868

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 3869

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 3832

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 7018

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right)-\frac{\text{Fresn}}{}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 3868

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)}{4x}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\downarrow$ 3869

3.46. $\int\frac{\text{FresnelS}(bx)^2}{x^8}\,dx$

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin(\pi}{3}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\big\downarrow$ 3832

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\big\downarrow$ 7010

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\big\downarrow$ 3869

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{Fresn}}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\big\downarrow$ 3832

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

$\big\downarrow$ 7020

$$\frac{2}{7}b\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{7x^7}$$

input
```
Int[FresnelS[b*x]^2/x^8,x]
```

3.46. $\int\frac{\text{FresnelS}(bx)^2}{x^8}\,dx$

output
```
$Aborted
```

3.46.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.46.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^8} dx$$

input
```
int(FresnelS(b*x)^2/x^8,x)
```

output
```
int(FresnelS(b*x)^2/x^8,x)
```

3.46.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\,(bx)^2}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^8, x)
```

3.46.6 Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{S^2(bx)}{x^8}\,dx$$

input
```
integrate(fresnels(b*x)**2/x**8,x)
```

output
```
Integral(fresnels(b*x)**2/x**8, x)
```

3.46.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\,(bx)^2}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^8, x)
```

3.46.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8}\,dx = \int \frac{\text{S}\,(bx)^2}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^8, x)
```

3.46.9 Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^8} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^8} \, dx$$

input
```
int(FresnelS(b*x)^2/x^8,x)
```

output
```
int(FresnelS(b*x)^2/x^8, x)
```

3.47 $\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx$

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 242

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = &-\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos(b^2\pi x^2)}{336x^6}-\frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2}\\ &-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x}\\ &+\frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)^2}{8x^8}\\ &-\frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}+\frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3}\\ &-\frac{b^4\pi\sin(b^2\pi x^2)}{420x^4}-\frac{1}{280}b^8\pi^3\,\text{Si}(b^2\pi x^2)\end{aligned}$$

output
```
-1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2+1/336*b^2*cos(b^2*Pi*x^2)/x^6-1/336*b^6
*Pi^2*cos(b^2*Pi*x^2)/x^2-1/140*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x
^5+1/420*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/840*b^8*Pi^4*Fresn
elS(b*x)^2-1/8*FresnelS(b*x)^2/x^8-1/280*b^8*Pi^3*Si(b^2*Pi*x^2)-1/28*b*Fr
esnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/420*b^5*Pi^2*FresnelS(b*x)*sin(1/2*b
^2*Pi*x^2)/x^3-1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos(b^2\pi x^2)}{336x^6}-\frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{420x} + \frac{1}{840}b^8\pi^4\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7}+\frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} - \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4}-\frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

input
```
Integrate[FresnelS[b*x]^2/x^9,x]
```

output
```
-1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
 (b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*Fresne
lS[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x)
+ (b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^
2)/2])/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinInteg
ral[b^2*Pi*x^2])/280
```

3.47.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx$$

$$\Big\downarrow \text{ 6984}$$

$$\frac{1}{4}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$$\Big\downarrow \text{ 7010}$$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{14}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3861

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{7x^7}\right.-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 25

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{7x^7}\right.-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\right.}{7x^7}\right.-\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

3.47. $\int\frac{\text{FresnelS}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right.}{3x^6}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi\right.}{3x}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 25

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3780

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 7018

3.47. $\int\frac{\text{FresnelS}(bx)^2}{x^9}dx$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fres}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3860

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fre}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fre}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{FresnelS}(b\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{Fresne}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{sin}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 25

3.47. $\int\frac{\text{FresnelS}(bx)^2}{x^9}dx$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3780

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 7010

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3861

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{3x^3}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$\downarrow$ 3778

3.47. $\int\frac{\text{FresnelS}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fre}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fre}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fre}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$$\Big\downarrow \; 3780$$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

$$\Big\downarrow \; 7018$$

$$\frac{1}{4}b\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b\right.}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{8x^8}$$

input
```
Int[FresnelS[b*x]^2/x^9,x]
```

output
```
$Aborted
```

3.47. $\int\frac{\text{FresnelS}(bx)^2}{x^9}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.47.4 Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)^2}{x^9}dx$$

input
```
int(FresnelS(b*x)^2/x^9,x)
```

output
```
int(FresnelS(b*x)^2/x^9,x)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = -\frac{3\,\pi^3b^8x^8\,\text{Si}\,(\pi b^2x^2) - 3\,\pi^2b^6x^6 + 5\,b^2x^2 + 5\,(\pi^2b^6x^6 - b^2x^2)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 - 2\,(\pi^3b^7x^7 - 3\,\pi b^3x^3)\cos\left(\frac{1}{2}\right.}{84}$$

input
```
integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="fricas")
```

output
```
-1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 3*pi^2*b^6*x^6 + 5*b^2*x
^2 + 5*(pi^2*b^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 - 2*(pi^3*b^7*x^7 -
3*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - (pi^4*b^8*x^8 - 105)*
fresnel_sin(b*x)^2 + 2*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^2*b^5*x^5 -
 15*b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

3.47.6 Sympy [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{S^2(bx)}{x^9} \, dx$$

input
```
integrate(fresnels(b*x)**2/x**9,x)
```

output
```
Integral(fresnels(b*x)**2/x**9, x)
```

3.47.7 Maxima [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^9} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^9, x)
```

3.47.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^9} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^9, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)^2}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^9}\,dx$$

input
```
int(FresnelS(b*x)^2/x^9,x)
```

output
```
int(FresnelS(b*x)^2/x^9, x)
```

3.48 $\int \frac{\text{FresnelS}(bx)^2}{x^{10}} dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)^2}{x^{10}} dx = & -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos(b^2\pi x^2)}{504x^7} \\ & - \frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3} - \frac{853b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}} \\ & - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{216x^6} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{1728x^2} \\ & - \frac{\text{FresnelS}(bx)^2}{9x^9} - \frac{b\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} \\ & + \frac{b^5\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{864x^4} - \frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5} \\ & + \frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x} + \frac{b^9\pi^4\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)}{1728}\end{aligned}$$

output
```
-1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3+1/504*b^2*cos(b^2*Pi*x^2)/x^7-187/18144
0*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/216*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b
*x)/x^6+1/1728*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/9*FresnelS
(b*x)^2/x^9-1/36*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/864*b^5*Pi^2*Fr
esnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5+85
3/362880*b^8*Pi^3*sin(b^2*Pi*x^2)/x-853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/
2))*2^(1/2)+1/1728*b^9*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)
/x,x)
```

3.48.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$$

input
```
Integrate[FresnelS[b*x]^2/x^10,x]
```

output
```
Integrate[FresnelS[b*x]^2/x^10, x]
```

3.48.3 Rubi [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6984, 7010, 3869, 3868, 3869, 3868, 3833, 7018, 3868, 3869, 3868, 3833, 7010, 3869, 3868, 3833, 7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$$

$$\Big\downarrow \; 6984$$

$$\frac{2}{9}b\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx - \frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 7010$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8} - \frac{b}{112x^7}\right) - \frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 3869$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\cos\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\right.}{8x^8}\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 3868

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b^2x^2\right)}{7x^7}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 3869

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\text{sin}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 3868

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 3833

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 7018

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}\right)-\frac{1}{16}b\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$\downarrow$ 3868

3.48. $\int\frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx}{6}\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3868}$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{7010}$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{Fres}}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3868}$$

3.48. $\int\frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 3833$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 7018$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b\right.}{2x^2}\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 3868$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 3833$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

$$\Big\downarrow \; 7012$$

$$\frac{2}{9}b\left(\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)^2}{9x^9}$$

input
```
Int[FresnelS[b*x]^2/x^10,x]
```

3.48. $\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx$

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6984
```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Sin[(Pi/2)*b^2*x
^2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.48.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx\right)^{2}}{x^{10}}dx$$

input
```
int(FresnelS(b*x)^2/x^10,x)
```

output
```
int(FresnelS(b*x)^2/x^10,x)
```

3.48.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}}\,dx = \int \frac{\mathrm{S}\left(bx\right)^{2}}{x^{10}}\,dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^2/x^10, x)
```

3.48.6 Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{S^2(bx)}{x^{10}} \, dx$$

input
```
integrate(fresnels(b*x)**2/x**10,x)
```

output
```
Integral(fresnels(b*x)**2/x**10, x)
```

3.48.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^{10}} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^2/x^10, x)
```

3.48.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{S}\left(bx\right)^2}{x^{10}} \, dx$$

input
```
integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2/x^10, x)
```

3.48.9 Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelS}(bx)^2}{x^{10}} \, dx = \int \frac{\text{FresnelS}(b\,x)^2}{x^{10}} \, dx$$

input
```
int(FresnelS(b*x)^2/x^10,x)
```

output
```
int(FresnelS(b*x)^2/x^10, x)
```

3.49 $\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx$

3.49.1 Optimal result

Integrand size = 16, antiderivative size = 497

$$\begin{aligned}\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx &= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}\\
&-\frac{5d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2}+\frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^3\pi}\\
&+\frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^3\pi}\\
&+\frac{2d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{3b^3\pi}\\
&-\frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{b^3\pi}\\
&+\frac{(bc-ad)^2(a+bx)\operatorname{FresnelS}(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2\operatorname{FresnelS}(a+bx)^2}{b^3}\\
&+\frac{d^2(a+bx)^3\operatorname{FresnelS}(a+bx)^2}{3b^3}-\frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi}\\
&+\frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}\\
&-\frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi}\\
&-\frac{4d^2\operatorname{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}\end{aligned}$$

output
```
2/3*d^2*x/b^2/Pi^2+1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+1/6*d^2*(b*
x+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+2*(-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)*Fresn
elS(b*x+a)/b^3/Pi+2*d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*
x+a)/b^3/Pi+2/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^3/Pi
-d*(-a*d+b*c)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+(-a*d+b*c)^2*(b*x+a)*
FresnelS(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^2*FresnelS(b*x+a)^2/b^3+1/3*d^2
*(b*x+a)^3*FresnelS(b*x+a)^2/b^3+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hypergeom([1
, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hyp
ergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-4/3*d^2*FresnelS(b*x+a)*
sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2
*2^(1/2)-1/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

3.49.2 Mathematica [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx$$

input
```
Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]
```

3.49.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6986, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6986$$

$$\frac{\int \left((bc-ad)^2 \operatorname{FresnelS}(a+bx)^2 + d^2(a+bx)^2 \operatorname{FresnelS}(a+bx)^2 + 2d(bc-ad)(a+bx)\operatorname{FresnelS}(a+bx)^2\right) d(a+

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi}-\frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{\pi}+$$

input
```
Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

output
```
((2*d^2*(a + b*x))/(3*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*Pi^2)
 + (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*Pi^2) - (5*d^2*FresnelC[Sqrt[2]*
(a + b*x)])/(6*Sqrt[2]*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2]*Fr
esnelS[a + b*x])/Pi + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*F
resnelS[a + b*x])/Pi + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS
[a + b*x])/(3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a + b*x])/Pi
 + (b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2 + d*(b*c - a*d)*(a + b*x)^2
*FresnelS[a + b*x]^2 + (d^2*(a + b*x)^3*FresnelS[a + b*x]^2)/3 - ((b*c - a
*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*Pi) + ((I/4)*d*(b*c - a*d)*(a
+ b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/Pi
- ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/
2)*Pi*(a + b*x)^2])/Pi - (4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])
/(3*Pi^2))/b^3
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6986
```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*
x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

3.49.4 Maple [F]

$$\int (dx+c)^2\,\text{FresnelS}\,(bx+a)^2\,dx$$

input
```
int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

output
```
int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

3.49.5 Fricas [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{S}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_sin(b*x + a)^2, x)
```

3.49.6 Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2 \, dx = \int (c+dx)^2 S^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

3.49.7 Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{S}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

3.49.8 Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int (dx+c)^2\,\mathrm{S}\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelS}(a+bx)^2\,dx = \int \operatorname{FresnelS}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(FresnelS(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(FresnelS(a + b*x)^2*(c + d*x)^2, x)
```

3.50 $\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx$

3.50.1 Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int (c+dx) \operatorname{FresnelS}(a+bx)^2 \, dx = {} & \frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} \\ & + \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^2\pi} \\ & + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelS}(a+bx)}{b^2\pi} \\ & - \frac{d\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{2b^2\pi} \\ & + \frac{(bc-ad)(a+bx)\operatorname{FresnelS}(a+bx)^2}{b^2} \\ & + \frac{d(a+bx)^2\operatorname{FresnelS}(a+bx)^2}{2b^2} \\ & - \frac{(bc-ad)\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} \\ & + \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ & - \frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi}\end{aligned}$$

output
```
1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)*Fresne
lS(b*x+a)/b^2/Pi+d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^2/Pi-1/
2*d*FresnelC(b*x+a)*FresnelS(b*x+a)/b^2/Pi+(-a*d+b*c)*(b*x+a)*FresnelS(b*x
+a)^2/b^2+1/2*d*(b*x+a)^2*FresnelS(b*x+a)^2/b^2+1/8*I*d*(b*x+a)^2*hypergeo
m([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/8*I*d*(b*x+a)^2*hypergeom(
[1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/2*(-a*d+b*c)*FresnelS((b*x+a)
*2^(1/2))/b^2/Pi*2^(1/2)
```

3.50.2 Mathematica [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx$$

input
```
Integrate[(c + d*x)*FresnelS[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

3.50.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6986, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6986$$

$$\frac{\int \left((bc-ad)\,\text{FresnelS}(a+bx)^2+d(a+bx)\,\text{FresnelS}(a+bx)^2\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi}+(a+bx)(bc-ad)\,\text{FresnelS}(a+bx)^2-\frac{(bc-ad)\,\text{Fre}}{}}{}$$

input
```
Int[(c + d*x)*FresnelS[a + b*x]^2,x]
```

output
```
((d*Cos[Pi*(a + b*x)^2])/(4*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2]
*FresnelS[a + b*x])/Pi + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a +
 b*x])/Pi - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*Pi) + (b*c - a*d)*(
a + b*x)*FresnelS[a + b*x]^2 + (d*(a + b*x)^2*FresnelS[a + b*x]^2)/2 - ((b
*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*Pi) + ((I/8)*d*(a + b*x)^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/Pi - ((I/8)
*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/
Pi)/b^2
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6986
```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*
x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

3.50.4 Maple [F]

$$\int (dx + c) \operatorname{FresnelS}(bx + a)^2 \, dx$$

input
```
int((d*x+c)*FresnelS(b*x+a)^2,x)
```

output
```
int((d*x+c)*FresnelS(b*x+a)^2,x)
```

3.50.5 Fricas [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int (dx+c)\,\text{S}\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

3.50.6 Sympy [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int (c+dx)\,S^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*fresnels(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

3.50.7 Maxima [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int (dx+c)\,\text{S}\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

3.50.8 Giac [F]

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int (dx+c)\,\text{S}\,(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelS}(a+bx)^2\,dx=\int \text{FresnelS}(a+b\,x)^2\,(c+d\,x)\,dx$$

input
```
int(FresnelS(a + b*x)^2*(c + d*x),x)
```

output
```
int(FresnelS(a + b*x)^2*(c + d*x), x)
```

3.51 $\int \text{FresnelS}(a+bx)^2\,dx$

3.51.1 Optimal result

Integrand size = 8, antiderivative size = 70

$$\int \text{FresnelS}(a+bx)^2\,dx=\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)}{b\pi}+\frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b}-\frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}$$

output
```
2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b/Pi+(b*x+a)*FresnelS(b*x+a)^2/b-1
/2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \text{FresnelS}(a+bx)^2\,dx=\frac{4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelS}(a+bx)+2\pi(a+bx)\,\text{FresnelS}(a+bx)^2-\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{2b\pi}$$

input
```
Integrate[FresnelS[a + b*x]^2,x]
```

output
```
(4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a +
 b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)
```

3.51.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6974, 7281, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{6974}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - 2\int (a+bx)\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{2\int (a+bx)\,\text{FresnelS}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)d(a+bx)}{b}$$

$$\Big\downarrow \text{7006}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{2\left(\frac{\int \sin(\pi(a+bx)^2)d(a+bx)}{2\pi} - \frac{\text{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}\right)}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{(a+bx)\,\text{FresnelS}(a+bx)^2}{b} - \frac{2\left(\frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{2\sqrt{2}\pi} - \frac{\text{FresnelS}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}\right)}{b}$$

input
```
Int[FresnelS[a + b*x]^2,x]
```

output
```
((a + b*x)*FresnelS[a + b*x]^2)/b - (2*(-((Cos[(Pi*(a + b*x)^2)/2]*Fresnel
S[a + b*x])/Pi) + FresnelS[Sqrt[2]*(a + b*x)]/(2*Sqrt[2]*Pi)))/b
```

3.51.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 6974
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Simp[2   Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[
a + b*x], x], x] /; FreeQ[{a, b}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.51.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\operatorname{FresnelS}(bx+a)^2(bx+a)+\frac{2\operatorname{FresnelS}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\operatorname{FresnelS}(bx+a)^2(bx+a)+\frac{2\operatorname{FresnelS}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

input
```
int(FresnelS(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelS(b*x+a)^2*(b*x+a)+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-
1/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int \text{FresnelS}(a+bx)^2\,dx$$
$$=\frac{4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)\text{S}\left(bx+a\right)+2\left(\pi b^2x+\pi ab\right)\text{S}\left(bx+a\right)^2-\sqrt{2}\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{2\,\pi b^2}$$

input
```
integrate(fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)*fresnel_sin(b*x + a)
+ 2*(pi*b^2*x + pi*a*b)*fresnel_sin(b*x + a)^2 - sqrt(2)*sqrt(b^2)*fresnel
_sin(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

3.51.6 Sympy [F]

$$\int \text{FresnelS}(a+bx)^2\,dx=\int S^2(a+bx)\;dx$$

input
```
integrate(fresnels(b*x+a)**2,x)
```

output
```
Integral(fresnels(a + b*x)**2, x)
```

3.51.7 Maxima [F]

$$\int \text{FresnelS}(a+bx)^2\,dx=\int \text{S}\left(bx+a\right)^2\;dx$$

input
```
integrate(fresnel_sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)^2, x)
```

3.51.8 Giac [F]

$$\int \text{FresnelS}(a+bx)^2\,dx = \int \text{S}\left(bx+a\right)^2\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)^2, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(a+bx)^2\,dx = \int \text{FresnelS}(a+b\,x)^2\,dx$$

input
```
int(FresnelS(a + b*x)^2,x)
```

output
```
int(FresnelS(a + b*x)^2, x)
```

3.52 $\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} dx$

3.52.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)^2}{c+dx}, x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)^2/(d*x+c),x)
```

3.52.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx$$

input
```
Integrate[FresnelS[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[FresnelS[a + b*x]^2/(c + d*x), x]
```

3.52.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx} \, dx$$
$$\Big\downarrow \ 6988$$
$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}dx$$

input
```
Int[FresnelS[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.52.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.52.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)^{2}}{dx+c}dx$$

input
```
int(FresnelS(b*x+a)^2/(d*x+c),x)
```

output
```
int(FresnelS(b*x+a)^2/(d*x+c),x)
```

3.52.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{S}\,(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

3.52.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{S^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(fresnels(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

3.52.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{S}\,(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

3.52.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{S}\,(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

3.52.9 Mupad [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{FresnelS}(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(FresnelS(a + b*x)^2/(c + d*x),x)
```

output
```
int(FresnelS(a + b*x)^2/(c + d*x), x)
```

3.53 $\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx$

3.53.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2},x\right)$$

output
```
Unintegrable(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

3.53.2 Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx = \int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} dx$$

input
```
Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

3.53.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 6988$$

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}dx$$

input
```
Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.53.3.1 Defintions of rubi rules used

rule 6988
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.53.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

3.53.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.53.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{S^2(a+bx)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)
```

3.53.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

3.53.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{S}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

3.53.9 Mupad [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelS}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{FresnelS}(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(FresnelS(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(FresnelS(a + b*x)^2/(c + d*x)^2, x)
```

3.54 $\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$

3.54.1 Optimal result

Integrand size = 17, antiderivative size = 231

$$\begin{aligned}&\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx\\&=\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\frac{1}{3}x^3\operatorname{FresnelS}(d(a+b\log(cx^n)))\end{aligned}$$

output
```
(1/12-1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+
I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+(1/12-1
/12*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*
d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*Fresn
elS(d*(a+b*ln(c*x^n)))
```

3.54.2 Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.38

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{1}{12}x^3\Bigg(4\operatorname{FresnelS}(d(a+b\log(cx^n)))$$
$$+\sqrt[4]{-1}\sqrt{2}e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\Bigg(e^{\frac{9i}{b^2d^2n^2\pi}}\operatorname{erfi}\Bigg(\frac{\left(\frac{1}{2}\right.}{}$$

input
```
Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b
*n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] -
 Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^((9*I)/(b^
2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c
*x^n]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d
^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b
*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))
/(c*x^n)^(3/n)))/12
```

3.54.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7025, 5128, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{ 7025}$$
$$\frac{1}{3}x^3\operatorname{FresnelS}(d(a+b\log(cx^n)))-\frac{1}{3}bdn\int x^2\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx$$
$$\Big\downarrow \text{ 5128}$$
$$\frac{1}{3}x^3\operatorname{FresnelS}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{1}{2}i\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2dx-\frac{1}{2}i\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2dx\right)$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{3}x^3 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{3}bdn\left(\frac{1}{2}ix^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{2-iabd^2n\pi}dx - \frac{1}{2}ix^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int\right.$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{3}x^3 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{3}bdn\left(\frac{ix^3(cx^n)^{-3/n}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{(3-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n} - \frac{ix^3(cx^n)^{-3/n}\int \text{ex}}{}\right.$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{3}x^3 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{3}bdn\left(\frac{ix^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\int \exp\left(\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{3}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n} - \frac{ix^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{3}x^3 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{3}bdn\left(-\frac{ix^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\int \exp\left(-\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{3}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}}}{}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{3}x^3 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{3}bdn\left(-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)}{bdn} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{}{2\pi b^2}}}{}\right.$$

input
```
Int[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

3.54. $\int x^2 \text{FresnelS}(d(a+b\log(cx^n)))\,dx$

output
```
-1/3*(b*d*n*(((-1/4 + I/4)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erf[((1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(b*d*n*(c*x^n)^(3/n)) - ((1/4 - I/4)*E^((-3*a)/(b*n) - ((9*I)/2
)/(b^2*d^2*n^2*Pi))*x^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*
Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*n*(c*x^n)^(3/n)))) + (x^3*FresnelS[d
*(a + b*Log[c*x^n])])/3
```

3.54.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5128
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Simp[I/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] -
Simp[I/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7025
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.54.4 Maple [F]

$$\int x^2 \,\mathrm{FresnelS}\left(d(a+b\ln\left(c\,x^n\right))\right) dx$$

input
```
int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

3.54.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.94

$$\int x^2 \,\mathrm{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{1}{3}x^3\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)$$
$$-\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$+\frac{1}{6}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$-\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$-\frac{1}{6}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$

input
```
integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/3*x^3*fresnel_sin(b*d*log(c*x^n) + a*d) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(
-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*
n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(
pi*b^2*d^2*n^2)) + 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) +
 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n
*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi
*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fr
esnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*
I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*lo
g(c)/n - 3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2))
```

3.54.6 Sympy [F]

$$\int x^2 \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 S(ad+bd\log\left(cx^n\right))\,dx$$

input
```
integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.54.7 Maxima [F]

$$\int x^2 \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2\,\text{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.54.8 Giac [F]

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx=\int x^2 \operatorname{S}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx=\int x^2 \operatorname{FresnelS}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x^2*FresnelS(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.55 $\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$

3.55.1 Optimal result

Integrand size = 15, antiderivative size = 227

$$\begin{aligned}&\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx\\&=\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\frac{1}{2}x^2\operatorname{FresnelS}(d(a+b\log(cx^n)))\end{aligned}$$

output
```
(1/8-1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(
2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))+(1/
8-1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/
d/Pi^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2
*FresnelS(d*(a+b*ln(c*x^n)))
```

3.55.2 Mathematica [A] (verified)

Time = 4.34 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.41

$$\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{1}{8}x^2\Bigg(4\operatorname{FresnelS}(d(a+b\log(cx^n)))$$
$$+\sqrt[4]{-1}\sqrt{2}e^{-\frac{2a}{bn}-\frac{2i}{b^2d^2n^2\pi}-\frac{1}{2}ia^2d^2\pi+iabd^2\pi(n\log(x)-\log(cx^n))-\frac{1}{2}ib^2d^2\pi(-n\log(x)+\log(cx^n))^2}(cx^n)^{-2/n}\Bigg(e^{\frac{4i}{b^2d^2n^2\pi}}\operatorname{erfi}\Bigg(\frac{(\frac{1}{2}+$$

input
```
Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*
n) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] -
Log[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^
2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c
*x^n]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d
^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b
*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))
/(c*x^n)^(2/n)))/8
```

3.55.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {7025, 5128, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 7025$$

$$\frac{1}{2}x^2\operatorname{FresnelS}(d(a+b\log(cx^n)))-\frac{1}{2}bdn\int x\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx$$

$$\Big\downarrow \; 5128$$

$$\frac{1}{2}x^2\operatorname{FresnelS}(d(a+b\log(cx^n)))-$$
$$\frac{1}{2}bdn\left(\frac{1}{2}i\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}xdx-\frac{1}{2}i\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}xdx\right)$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{2}x^2 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{1}{2}ix^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{1-iabd^2n\pi}dx-\frac{1}{2}ix^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int\right.$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{2}x^2 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{ix^2(cx^n)^{-2/n}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{(2-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}-\frac{ix^2(cx^n)^{-2/n}\int \text{ex}}{}\right.$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{2}x^2 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{ix^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n+i)}{\pi b^2d^2n^2}}\int \exp\left(\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{2}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}-\frac{ix^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \text{ex}}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}x^2 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(-\frac{ix^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \exp\left(-\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{2}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{-\frac{2(\pi ab}{\pi b^2}}}{}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2 \text{FresnelS}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{bdn}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n+i}{\pi b^2d^2n^2}}}{}\right.$$

input
```
Int[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

3.55. $\int x \,\text{FresnelS}(d(a+b\log(cx^n)))\,dx$

output
```
-1/2*(b*d*n*(((-1/4 + I/4)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2
*Erf[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])]]/(b*d*n*(c*x^n)^(2/n)) - ((1/4 - I/4)*x^2*Erfi[((1/2 + I/2)*(2/n -
 I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*(I +
 a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n*(c*x^n)^(2/n)))) + (x^2*FresnelS[d*(a
+ b*Log[c*x^n])])/2
```

3.55.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5128
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Simp[I/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] -
Simp[I/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7025
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.55.4 Maple [F]

$$\int x \operatorname{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

3.55.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.97

$$\begin{aligned}&\int x \operatorname{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx =\\
&-\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&+\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&-\frac{1}{4}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&-\frac{1}{4}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)\\
&+\frac{1}{2}x^2\operatorname{S}\left(bd\log\left(cx^{n}\right)+ad\right)\end{aligned}$$

input
```
integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n
^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2
*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)
*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^
2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)
/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) -
 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*l
og(c) + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*s
qrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresne
l_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*s
qrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_sin(b*d*log(c*x^n) +
a*d)
```

3.55.6 Sympy [F]

$$\int x \operatorname{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int xS(ad+bd\log\left(cx^n\right))\,dx$$

input
```
integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.55.7 Maxima [F]

$$\int x \operatorname{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\,\mathrm{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.55.8 Giac [F]

$$\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx=\int x\,\mathrm{S}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx=\int x \operatorname{FresnelS}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*FresnelS(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.56 $\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx$

3.56.1 Optimal result

Integrand size = 13, antiderivative size = 214

$$\begin{aligned}&\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx\\&=\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)\\&\quad+\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x\left(cx^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right)\\&\quad+x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\end{aligned}$$

output
```
(1/4-1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/
d/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+(1/4-1/4*I
)*x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2
))/exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelS(d*(a+b*l
n(c*x^n)))
```

3.56.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right) + \frac{\sqrt[4]{-1}e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(e^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)}{}\right.\right.}{}$$

input
```
Integrate[FresnelS[d*(a + b*Log[c*x^n])],x]
```

output
```
x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d
^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b
^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[
((1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi
])] + I*Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*
d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*S
in[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-
1))
```

3.56.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7022, 5126, 2710, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{7022}$$

$$x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-bdn\int \sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)dx$$

$$\Big\downarrow \text{5126}$$

$$x\,\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)-bdn\left(\frac{1}{2}i\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx-\frac{1}{2}i\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx\right)$$

$$\Big\downarrow \text{2710}$$

$$x\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-$$
$$bdn\left(\frac{1}{2}ix^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)\,d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabd^2n\pi}dx-\frac{1}{2}ix^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int\text{ex}\right.$$

$\downarrow$ 2706

$$x\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-$$
$$bdn\left(\frac{ix(cx^n)^{-1/n}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)\,d^2-\frac{1}{2}ia^2\pi d^2+\frac{(1-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}-\frac{ix(cx^n)^{-1/n}\int\exp\left(\right.}{}\right.$$

$\downarrow$ 2664

$$x\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-$$
$$bdn\left(\frac{ix(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}-\frac{ix(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\right.}{}\right.$$

$\downarrow$ 2633

$$x\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-$$
$$bdn\left(-\frac{ix(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}}{}\right.$$

$\downarrow$ 2634

$$x\,\text{FresnelS}\left(d(a+b\log(cx^n))\right)-$$
$$bdn\left(-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{bdn}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\right.}{bdr}\right.$$

input
```
Int[FresnelS[d*(a + b*Log[c*x^n])],x]
```

output
```
-(b*d*n*(((-1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d
^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*
n^2))*n*(c*x^n)^n^(-1)) - ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b
*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*a*b*n + I/
(d^2*Pi))/(2*b^2*n^2))*n*(c*x^n)^n^(-1)))) + x*FresnelS[d*(a + b*Log[c*x^n
])]
```

3.56. $\int \text{FresnelS}\left(d(a+b\log(cx^n))\right)\,dx$

3.56.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 5126
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Simp[I
/2   Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] - Simp[I/2   Int[E^(I*d*(a
 + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

rule 7022
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelS[d*(a + b*Log[c*x^n])], x] - Simp[b*d*n   Int[Sin[(Pi/2)*(d*(a
+ b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

3.56.4 Maple [F]

$$\int \text{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

output
```
int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

3.56.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(176) = 352.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.08

$$\begin{aligned}
&\int \text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx = \\
&-\frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&+\frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{2}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\text{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{2}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\text{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&+x\,\text{S}\left(bd\log\left(cx^{n}\right)+ad\right)
\end{aligned}$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2
))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(
-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2
*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b
^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi
*a*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*
n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*
d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)
/(pi*b^2*d^2*n^2)) + x*fresnel_sin(b*d*log(c*x^n) + a*d)
```

3.56.6 Sympy [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int S(d(a+b\log\left(cx^n\right)))\,dx$$

input
```
integrate(fresnels(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

3.56.7 Maxima [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \text{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.56.8 Giac [F]

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \text{S}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \text{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

input
```
int(FresnelS(d*(a + b*log(c*x^n))),x)
```

output
```
int(FresnelS(d*(a + b*log(c*x^n))), x)
```

3.57 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}\,dx$

3.57.1 Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{bdn\pi} + \frac{\text{FresnelS}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn}$$

output
```
cos(1/2*d^2*Pi*(a+b*ln(c*x^n))^2)/b/d/n/Pi+FresnelS(d*(a+b*ln(c*x^n)))*(a+
b*ln(c*x^n))/b/n
```

3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(65) = 130.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}\,dx = \frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\cos\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi} + \frac{a\,\text{FresnelS}(d(a+b\log(cx^n)))}{bn} + \frac{\text{FresnelS}(d(a+b\log(cx^n)))\log(cx^n)}{n} - \frac{\sin\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi}$$

input
```
Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)
/2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(
a + b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log
[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

3.57.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3039, 7281, 6972}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \text{FresnelS}(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \; 7281$$

$$\frac{\int \text{FresnelS}(ad+b\log(cx^n)d)d(ad+b\log(cx^n)d)}{bdn}$$

$$\Big\downarrow \; 6972$$

$$\frac{(ad+bd\log(cx^n))\text{FresnelS}(ad+b\log(cx^n)d)+\frac{\cos\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{\pi}}{bdn}$$

input
```
Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(Cos[(Pi*(a*d + b*d*Log[c*x^n])^2)/2]/Pi + FresnelS[a*d + b*d*Log[c*x^n]]*
(a*d + b*d*Log[c*x^n]))/(b*d*n)
```

3.57.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 6972
```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.57.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\text{FresnelS}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	63
default	$\frac{\text{FresnelS}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{ndb}$	63

input
```
int(FresnelS(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/d/b*(FresnelS(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))+1/Pi*cos(1/2*Pi*(
a*d+b*d*ln(c*x^n))^2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{\text{FresnelS}\,(d(a+b\log{(cx^n)}))}{x}\,dx$$
$$= \frac{(\pi bdn\log{(x)} + \pi bd\log{(c)} + \pi ad)\,\mathrm{S}\,(bd\log{(cx^n)} + ad) + \cos\left(\frac{1}{2}\,\pi b^2d^2n^2\log{(x)}^2 + \pi b^2d^2n\log{(c)}\log{(x)} + \right.}{\pi bdn}$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_sin(b*d*log(c*x^n) + a
*d) + cos(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*p
i*b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*
d^2))/(pi*b*d*n)
```

3.57.6 Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \int \frac{S(ad+bd\log\left(cx^n\right))}{x}\,dx$$

input
```
integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(b\log\left(cx^n\right)+a)d\,\mathrm{S}\left((b\log\left(cx^n\right)+a)d\right)+\frac{\cos\left(\frac{1}{2}\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\pi a^2d^2\right)}{\pi}}{bdn}$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*fresnel_sin((b*log(c*x^n) + a)*d) + cos(1/2*pi*b^2*d
^2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

3.57.8 Giac [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \int \frac{\text{S}\left((b\log\left(cx^n\right)+a)d\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x, x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \int \frac{\text{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x}\,dx$$

input
```
int(FresnelS(d*(a + b*log(c*x^n)))/x,x)
```

output
```
int(FresnelS(d*(a + b*log(c*x^n)))/x, x)
```

3.58 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^2}\,dx$

3.58.1 Optimal result

Integrand size = 17, antiderivative size = 217

$$\begin{aligned}\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx &= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}\\ &\quad+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}\\ &\quad-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}\end{aligned}$$

output
```
(1/4-1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2
*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x+(1/4-1/4*I)*
exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*
a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelS(d*(a+b*ln(c*x^
n)))/x
```

3.58.2 Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx =$$

$$-\frac{\sqrt[4]{-1}\sqrt{2}e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}\left(cx^n\right)^{\frac{1}{n}}\left(i\text{erfi}\left(\frac{(-1)^{3/4}\left(-i+abd^2n\pi+b^2d^2n\pi\log\left(cx^n\right)\right)}{bdn\sqrt{2\pi}}\right)+e^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i+abd^2n\pi+b^2d^2n\pi\log(cx\right.}{bdn\sqrt{\pi}}\right.}{4x}$$

input
```
Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)*(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b
*d*n*Sqrt[2*Pi])] + E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*
n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelS[d*(a + b*
Log[c*x^n])])/x
```

3.58.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {7025, 5128, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}dx$$

$$\Big\downarrow \text{7025}$$

$$bdn\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)}{x^2}dx-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}$$

$$\Big\downarrow \text{5128}$$

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+bdn\left(\frac{1}{2}i\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}dx-\frac{1}{2}i\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}dx\right)$$

$$\Big\downarrow \text{2712}$$

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{1}{2}ix^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabn\pi d^2-2}dx-\frac{1}{2}ix^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int\right.$$

$\big\downarrow$ 2706

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{i(cx^n)^{\frac{1}{n}}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2-\frac{(iabn\pi d^2+1)\log(cx^n)}{n}\right)d\log\left(cx^n\right)}{2nx}-\frac{i(cx^n)^{\frac{1}{n}}\int\exp\left(\frac{1}{2}ib^2\pi\log\right.}{}\right.$$

$\big\downarrow$ 2664

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{i(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log\left(cx^n\right)}{2nx}-\frac{i(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\frac{i\left(-ib^2\pi\log(cx^n\right.}{2b}\right.}{2nx}\right.$$

$\big\downarrow$ 2633

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx}-\frac{i(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\frac{i\left(-ib^2\pi\log(cx^n)d^2\right.}{2b^2d^2}\right.}{2nx}\right.$$

$\big\downarrow$ 2634

$$-\frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd\right.}{}\right.}{bdnx}\right.$$

input
```
Int[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

3.58. $\int\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^2}\,dx$

output
```
b*d*n*(((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*
Erf[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(b*d*n*x) + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))
*(c*x^n)^n^(-1)*Erfi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Lo
g[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*n*x)) - FresnelS[d*(a + b*Log[c*x^n])]/x
```

3.58.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5128
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Simp[I/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] -
Simp[I/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7025
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.58.4 Maple [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

3.58.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(177) = 354.

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx$$
$$= \frac{-i\,\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\right.}}{}$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
1/2*(-I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n
^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2
*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x*e^(
log(c)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*
d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d
^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b
*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e
^(log(c)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2
*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2)) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))/x
```

3.58.6 Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{S(ad+bd\log\left(cx^n\right))}{x^2}\,dx$$

input
```
integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)
```

3.58.7 Maxima [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{S}\left((b\log\left(cx^n\right)+a)d\right)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

3.58.8 Giac [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{S}\left((b\log\left(cx^n\right)+a)d\right)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^2}\,dx$$

input
```
int(FresnelS(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(FresnelS(d*(a + b*log(c*x^n)))/x^2, x)
```

3.59 $\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx$

3.59.1 Optimal result

Integrand size = 17, antiderivative size = 228

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

output
```
(1/8-1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/
2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2+(1/8-
1/8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*
x^n))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*Fresnel
S(d*(a+b*ln(c*x^n)))/x^2
```

3.59.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\sqrt[4]{-1}e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(i\,\text{erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+e^{\frac{4i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)}{4\sqrt{2}} - \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*
x^n])))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c
*x^n]))/(b*d*n*Sqrt[2*Pi])] + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*
(2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt
[2] - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

3.59.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7025, 5128, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx$$

$$\Big\downarrow \text{ 7025}$$

$$\frac{1}{2}bdn\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^3}dx - \frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{ 5128}$$

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{1}{2}i\int\frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}dx-\frac{1}{2}i\int\frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}dx\right)$$

$\big\downarrow$ 2712

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{1}{2}ix^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabn\pi d^2-3}dx-\frac{1}{2}ix^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\right.$$

$\big\downarrow$ 2706

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{i(cx^n)^{2/n}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2-\frac{(iabn\pi d^2+2)\log(cx^n)}{n}\right)d\log(cx^n)}{2nx^2}-\frac{i(cx^n)^{2/n}\int\exp\left(\frac{1}{2}ib^2\right.}{}\right.$$

$\big\downarrow$ 2664

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{i(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\int\exp\left(\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{2}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2nx^2}-\frac{i(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int\exp\left(-\frac{i(-\right.}{}}{2}\right.$$

$\big\downarrow$ 2633

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx^2}-\frac{i(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int\exp\left(-\frac{i(-ib^2\right.}{}}{2n}\right.$$

$\big\downarrow$ 2634

$$-\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}bdn\left(\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx^2}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{2/n}e^{-\frac{2(-\pi abd^2n+i)}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{(\right.}{}\right)}{bdnx^2}\right.$$

input
```
Int[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

3.59. $\int\frac{\text{FresnelS}(d(a+b\log(cx^n)))}{x^3}\,dx$

output
```
(b*d*n*(((1/4 - I/4)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(
2/n)*Erf[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d
*Sqrt[Pi])])/(b*d*n*x^2) + ((1/4 - I/4)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2
/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*
(I - a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n*x^2)))/2 - FresnelS[d*(a + b*Log[c
*x^n])]/(2*x^2)
```

3.59.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5128
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Simp[I/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] -
Simp[I/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7025
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.59.4 Maple [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

3.59.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(183) = 366.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.02

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \frac{-i\,\pi\sqrt{b^{2}d^{2}n^{2}}x^{2}e^{\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+\frac{2i}{\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log(x)+\pi b^{2}d^{2}n\log(c)+\pi abd^{2}n+2i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)+i\,\pi\sqrt{b^{2}d^{2}n^{2}}x^{2}e^{\left(\frac{2\,\log(c)}{n}\right.}}{}$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
1/4*(-I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d
^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b
*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)
*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2
*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n
^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n)
 + 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n
*log(c) + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqr
t(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresn
el_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*
sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))
/x^2
```

3.59.6 Sympy [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{S(ad+bd\log\left(cx^n\right))}{x^3}\,dx$$

input
```
integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)
```

3.59.7 Maxima [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\text{S}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

3.59.8 Giac [F]

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\text{S}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\text{FresnelS}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

input
```
int(FresnelS(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(FresnelS(d*(a + b*log(c*x^n)))/x^3, x)
```

3.60 $\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$

3.60.1 Optimal result

Integrand size = 19, antiderivative size = 280

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)}{1+m} + \frac{(ex)^{1+m}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(1+m)}$$

output
```
(1/4-1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)
^m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^
(1/2))/(1+m)/((c*x^n)^((1+m)/n))+(1/4-1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1
+m-I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m
)*(1+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(
1+m)*FresnelS(d*(a+b*ln(c*x^n)))/e/(1+m)
```

3.60.2 Mathematica [A] (verified)

Time = 3.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$= \frac{(ex)^m \left(-\sqrt[4]{-1}\sqrt{2}e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+ \right.\right.}{4(1+m)}$$

input
```
Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(-(((-1)^(1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi
+ b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + E^((I*(1 + m)^2)/(b^2*d^2*
n^2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2
*d^2*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m)) + 4*x*Fre
snelS[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

3.60.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {7025, 5128, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \ 7025$$
$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)} - \frac{bdn\int (ex)^m \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx}{m+1}$$
$$\Big\downarrow \ 5128$$
$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)} - \frac{bdn\left(\frac{1}{2}i\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m dx - \frac{1}{2}i\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m dx\right)}{m+1}$$

$\downarrow$ 2712

$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}-$$
$$\frac{bdn\left(\frac{1}{2}i(ex)^m(cx^n)^{-i\pi abd^2}x^{-m+i\pi abd^2n}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{m-iabd^2n\pi}dx-\frac{1}{2}i(ex)^m(cx^n)^{i\pi abd}\right.}{m+1}$$

$\downarrow$ 2706

$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}-$$
$$\frac{bdn\left(\frac{ix(ex)^m(cx^n)^{-\frac{-i\pi abd^2n+m+1}{n}-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{(-iabn\pi d^2+m+1)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}-\frac{ix(ex)^m(cx^n)^{i\pi}}{}\right.}{m+1}$$

$\downarrow$ 2664

$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}-$$
$$\frac{bdn\left(\frac{ix(ex)^m\exp\left(-\frac{i(m+1)(-2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)(cx^n)^{-\frac{-i\pi abd^2n+m+1}{n}-i\pi abd^2}\int\exp\left(\frac{i(-ib^2n\pi\log(cx^n)d^2-iabn\pi d^2+m+1)^2}{2b^2d^2n^2\pi}\right)d\log(cx^n)}{2n}-\frac{ix}{}\right.}{m+1}$$

$\downarrow$ 2633

$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}-$$
$$\frac{bdn\left(-\frac{ix(ex)^m\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)(cx^n)^{i\pi abd^2-\frac{i\pi abd^2n+m+1}{n}}\int\exp\left(-\frac{i(ib^2n\pi\log(cx^n)d^2+iabn\pi d^2+m+1)^2}{2b^2d^2n^2\pi}\right)d\log(cx^n)}{2n}-\frac{(\frac{1}{4}-\frac{i}{4}}{}\right.}{m+1}$$

$\downarrow$ 2634

$$\frac{(ex)^{m+1}\operatorname{FresnelS}(d(a+b\log(cx^n)))}{e(m+1)}-$$
$$\frac{bdn\left(-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\exp\left(\frac{i(m+1)(2i\pi abd^2n+m+1)}{2\pi b^2d^2n^2}\right)(cx^n)^{i\pi abd^2-\frac{i\pi abd^2n+m+1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1)}{\sqrt{\pi}bdn}\right)}{bdn}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x}{}\right.}{m+1}$$

input
```
Int[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

3.60. $\int(ex)^m\operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx$

output
```
-((b*d*n*(((-1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b
^2*d^2*n^2*Pi))*x*(e*x)^m*(c*x^n)^(I*a*b*d^2*Pi - (1 + m + I*a*b*d^2*n*Pi)
/n)*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[Pi])])/(b*d*n) - ((1/4 - I/4)*x*(e*x)^m*(c*x^n)^((-I)*a*b*d^2
*Pi - (1 + m - I*a*b*d^2*n*Pi)/n)*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*P
i - I*b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])])/(b*d*E^(((I/2)*(1 + m)*
(1 + m - (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n)))/(1 + m)) + ((e*x)^(1
+ m)*FresnelS[d*(a + b*Log[c*x^n])])/(e*(1 + m))
```

3.60.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5128
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Simp[I/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] -
Simp[I/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7025
```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.60.4 Maple [F]

$$\int (ex)^m \operatorname{FresnelS}\left(d(a + b\ln\left(c\,x^n\right))\right) dx$$

input
```
int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

3.60.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(310) = 620$.

Time = 0.29 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.47

$$\int (ex)^m \operatorname{FresnelS}\left(d(a + b\log\left(cx^n\right))\right)\,dx$$
$$= \frac{-i\,\pi\sqrt{b^2d^2n^2}e^{\left(m\log(e) - \frac{m\log(c)}{n} - \frac{am}{bn} - \frac{\log(c)}{n} - \frac{a}{bn} - \frac{i\,m^2}{2\,\pi b^2d^2n^2} - \frac{i\,m}{\pi b^2d^2n^2} - \frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log(x) + \pi b^2d^2n\log(c) + \pi abd^2n + i\,m\cdots}{\pi b^2d^2n^2}\right.}{}$$

input
```
integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/2*(-I*pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)
/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(
pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c)
+ pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(
b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1
/2*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))
*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n -
 I*m - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - pi*sqrt(b^2*d^2*n^2)*e^(m*
log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d
^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b
^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2
*d^2*n^2)/(pi*b^2*d^2*n^2)) - pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/
n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*
b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x)
+ pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*
d^2*n^2)) + 2*x*e^(m*log(e) + m*log(x))*fresnel_sin(b*d*log(c*x^n) + a*d))
/(m + 1)
```

3.60.6 Sympy [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m S(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)
```

3.60.7 Maxima [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{S}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.60.8 Giac [F]

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{S}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m \operatorname{FresnelS}(d(a+b\log(cx^n)))\,dx = \int \operatorname{FresnelS}(d\,(a+b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelS}(bx)\,dx$

3.61.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelS}(bx)\,dx=-\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/8*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*I*b*exp(c)*x^2*hypergeom(
[1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

3.61.2 Mathematica [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelS}(bx)\,dx=\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelS}(bx)\,dx$$

input
```
Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

output
```
Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

3.61.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6990, 26, 6929, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+\frac{1}{2}i\pi b^2x^2}\text{FresnelS}(bx)\,dx$$

$$\Big\downarrow \text{6990}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int -ie^{\frac{1}{2}ib^2\pi x^2+c}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{26}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{6929}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{e^c\int \text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)d\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

$$\Big\downarrow \text{6930}$$

$$\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

input
```
Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

output
```
-1/8*(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (I/4)*b*E^c*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```

3.61.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6990
```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(1 + I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Simp[(1 - I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b,
c, d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

3.61.4 Maple [F]

$$\int e^{c+\frac{ib^2\pi x^2}{2}} \operatorname{FresnelS}(bx)\,dx$$

input
```
int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.61.5 Fricas [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.61.6 Sympy [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=e^c\int e^{\frac{i\pi b^2x^2}{2}}S(bx)\,dx$$

input
```
integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.61.7 Maxima [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.61.8 Giac [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx = \int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx = \int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\text{FresnelS}(b\,x)\,dx$$

input
```
int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
```

output
```
int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$

3.62.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/8*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b-1/4*I*b*exp(c)*x^2*hypergeom(
[1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

3.62.2 Mathematica [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx$$

input
```
Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

output
```
Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

3.62.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6990, 26, 6927, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-\frac{1}{2}i\pi b^2x^2}\text{FresnelS}(bx)\,dx$$

$$\Big\downarrow \text{6990}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int -ie^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{26}$$

$$\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{6927}$$

$$\frac{e^c\int \text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)d\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{4b}-\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{15}$$

$$\frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}-\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{6932}$$

$$\frac{e^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

input
```
Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

output
```
(E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]
```

3.62.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6990
```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[(1 + I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Simp[(1 - I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b,
c, d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

3.62.4 Maple [F]

$$\int e^{c-\frac{ib^2\pi x^2}{2}} \operatorname{FresnelS}(bx)\,dx$$

input
```
int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.62.5 Fricas [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.62.6 Sympy [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=e^c\int e^{-\frac{i\pi b^2x^2}{2}}S(bx)\,dx$$

input
```
integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.62.7 Maxima [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.62.8 Giac [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{S}\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelS}(bx)\,dx=\int \mathrm{e}^{c-\frac{\Pi\,b^2\,x^2\,1\mathrm{i}}{2}}\,\text{FresnelS}(b\,x)\,dx$$

input
```
int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
```

output
```
int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.63 $\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

3.63.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b}+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)$$

output
```
1/2*cos(c)*FresnelS(b*x)^2/b+1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*
I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hy
pergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

3.63.2 Mathematica [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

input
```
Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

output
```
Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

3.63.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6996, 6994, 15, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2+c\right)dx$$

$$\Big\downarrow \text{6996}$$

$$\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx+\cos(c)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{6994}$$

$$\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx+\frac{\cos(c)\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx+\frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b}$$

$$\Big\downarrow \text{7000}$$

$$\frac{\cos(c)\,\text{FresnelS}(bx)^2}{2b}+\sin(c)\left(-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}\right)$$

input
```
Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

output
```
(Cos[c]*FresnelS[b*x]^2)/(2*b) + ((FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I
/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)
*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])*Sin[c]
```

3.63.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 6996
```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[Sin[c]
    Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[Cos[c]   Int[Sin[d*x^2]*Fres
nelS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

3.63.4 Maple [F]

$$\int \text{FresnelS}\left(bx\right)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)dx$$

input
```
int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

output
```
int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

3.63.5 Fricas [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.63.6 Sympy [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int\sin\left(\frac{\pi b^2x^2}{2}+c\right)S(bx)\,dx$$

input
```
integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)
```

output
```
Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

3.63.7 Maxima [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2+c\right)\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.63.8 Giac [F]

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2+c\right)\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \sin\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\text{FresnelS}(b\,x)\,dx$$

input
```
int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
```

output
```
int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.64 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{\text{FresnelS}(bx)^2\sin(c)}{2b}$$

output
```
1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*cos(c)*hypergeom([1,
1],[3/2, 2],-1/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)-1/2*FresnelS(b*x)^2*sin(c)/b
```

3.64.2 Mathematica [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$$

input
```
Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

3.64.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {7002, 6994, 15, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2+c\right)dx$$

$$\Big\downarrow \text{7002}$$

$$\cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx-\sin(c)\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{6994}$$

$$\cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx-\frac{\sin(c)\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx-\frac{\sin(c)\,\text{FresnelS}(bx)^2}{2b}$$

$$\Big\downarrow \text{7000}$$

$$-\frac{\sin(c)\,\text{FresnelS}(bx)^2}{2b}+\cos(c)\left(-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}\right)$$

input
```
Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
Cos[c]*((FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPF
Q[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{
1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]) - (FresnelS[b*x]^2*Sin[c])/(2*b)
```

3.64.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7002
```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[Cos[c]
   Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Simp[Sin[c]   Int[Sin[d*x^2]*Fres
nelS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.64.4 Maple [F]

$$\int \cos\left(c+\frac{b^2\pi\, x^2}{2}\right) \mathrm{FresnelS}\left(bx\right) dx$$

input
```
int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.64.5 Fricas [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2 + c\right) \mathrm{S}\left(bx\right)\, dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.64.6 Sympy [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{\pi b^2x^2}{2}+c\right)S(bx)\,dx$$

input
```
integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

3.64.7 Maxima [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{S}(bx)\,dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.64.8 Giac [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{S}(bx)\,dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int \cos\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\text{FresnelS}(b\,x)\,dx$$

input
```
int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
```

output
```
int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.65 $\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^3}{3b}$$

output
```
1/3*FresnelS(b*x)^3/b
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^3}{3b}$$

input
```
Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^3/(3*b)
```

3.65.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right) \, dx$$

$$\Big\downarrow \text{ 6994}$$

$$\frac{\int \text{FresnelS}(bx)^2 d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{FresnelS}(bx)^3}{3b}$$

input
```
Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^3/(3*b)
```

3.65.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.65.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{FresnelS}(bx)^3}{3b}$	12
default	$\frac{\operatorname{FresnelS}(bx)^3}{3b}$	12

input
```
int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*FresnelS(b*x)^3/b
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\mathrm{S}\,(bx)^3}{3\,b}$$

input
```
integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/3*fresnel_sin(b*x)^3/b
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \operatorname{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \begin{cases} \frac{S^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = \frac{S\,(bx)^3}{3\,b}$$

input
```
integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
1/3*fresnel_sin(b*x)^3/b
```

3.65.8 Giac [F]

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = \int S\,(bx)^2 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\, dx$$

input
```
integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = \int \text{FresnelS}(b\,x)^2\, \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\, dx$$

input
```
int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2), x)
```

3.66 $\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.66.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^2}{2b}$$

output
```
1/2*FresnelS(b*x)^2/b
```

3.66.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{FresnelS}(bx)^2}{2b}$$

input
```
Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^2/(2*b)
```

3.66.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \text{ 6994}$$

$$\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{FresnelS}(bx)^2}{2b}$$

input
```
Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^2/(2*b)
```

3.66.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.66.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelS}(bx)^2}{2b}$	12

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*FresnelS(b*x)^2/b
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\mathrm{S}\left(bx\right)^2}{2\,b}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/2*fresnel_sin(b*x)^2/b
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\,(bx)^2}{2\,b}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
1/2*fresnel_sin(b*x)^2/b
```

3.66.8 Giac [F]

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.67 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx$

3.67.1 Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log(\text{FresnelS}(bx))}{b}$$

output
```
ln(FresnelS(b*x))/b
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log(\text{FresnelS}(bx))}{b}$$

input
```
Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

output
```
Log[FresnelS[b*x]]/b
```

3.67.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6994, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{FresnelS}(bx)}\,dx$$

$$\Big\downarrow \text{ 6994}$$

$$\frac{\int \frac{1}{\text{FresnelS}(bx)}d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log(\text{FresnelS}(bx))}{b}$$

input
```
Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

output
```
Log[FresnelS[b*x]]/b
```

3.67.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.67.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\mathrm{FresnelS}(bx))}{b}$	10
default	$\frac{\ln(\mathrm{FresnelS}(bx))}{b}$	10

input
```
int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(FresnelS(b*x))/b
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)}\,dx = \frac{\log\left(\mathrm{S}\left(bx\right)\right)}{b}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
log(fresnel_sin(b*x))/b
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)}\,dx = \begin{cases} \frac{\log\left(S(bx)\right)}{b} & \text{for } b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
```

output
```
Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \frac{\log\left(\text{S}\left(bx\right)\right)}{b}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
log(fresnel_sin(b*x))/b
```

3.67.8 Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \int \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\text{S}\left(bx\right)}\,dx$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\left(b\,x\right)}\,dx$$

input
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x),x)
```

output
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x), x)
```

3.68 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx$

3.68.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelS}(bx)}$$

output
```
-1/b/FresnelS(b*x)
```

3.68.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelS}(bx)}$$

input
```
Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

output
```
-(1/(b*FresnelS[b*x]))
```

3.68.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\text{FresnelS}(bx)^2}dx$$

$$\Big\downarrow \text{6994}$$

$$\frac{\int \frac{1}{\text{FresnelS}(bx)^2}d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{b\,\text{FresnelS}(bx)}$$

input
```
Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

output
```
-(1/(b*FresnelS[b*x]))
```

3.68.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.68.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\ \mathrm{FresnelS}(bx)}$	12
default	$-\frac{1}{b\ \mathrm{FresnelS}(bx)}$	12

input
```
int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/FresnelS(b*x)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\mathrm{S}\,(bx)}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="fricas")
```

output
```
-1/(b*fresnel_sin(b*x))
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)^2}\,dx = \begin{cases} -\frac{1}{bS(bx)} & \text{for } b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
```

output
```
Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = -\frac{1}{b\,\text{S}\,(bx)}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="maxima")
```

output
```
-1/(b*fresnel_sin(b*x))
```

3.68.8 Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = \int \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{S}\,(bx)^2}\,dx$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="giac")
```

output
```
integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^2, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^2}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\,(b\,x)^2}\,dx$$

input
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2,x)
```

output
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2, x)
```

3.69 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx$

3.69.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

output
```
-1/2/b/FresnelS(b*x)^2
```

3.69.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

input
```
Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

output
```
-1/2*1/(b*FresnelS[b*x]^2)
```

3.69.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{FresnelS}(bx)^3}dx$$
$$\Big\downarrow \ 6994$$
$$\frac{\int \frac{1}{\text{FresnelS}(bx)^3}d\,\text{FresnelS}(bx)}{b}$$
$$\Big\downarrow \ 15$$
$$-\frac{1}{2b\,\text{FresnelS}(bx)^2}$$

input
```
Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

output
```
-1/2*1/(b*FresnelS[b*x]^2)
```

3.69.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.69.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\,\mathrm{FresnelS}(bx)^{2}}$	12
default	$-\frac{1}{2b\,\mathrm{FresnelS}(bx)^{2}}$	12

input
```
int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/FresnelS(b*x)^2
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)^3}\,dx = -\frac{1}{2\,b\,\mathrm{S}\left(bx\right)^{2}}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="fricas")
```

output
```
-1/2/(b*fresnel_sin(b*x)^2)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelS}(bx)^3}\,dx = \begin{cases} -\frac{1}{2bS^{2}(bx)} & \text{for}\ b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
```

output
```
Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = -\frac{1}{2\,b\,\text{S}\,(bx)^2}$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="maxima")
```

output
```
-1/2/(b*fresnel_sin(b*x)^2)
```

3.69.8 Giac [F]

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = \int \frac{\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\text{S}\,(bx)^3}\,dx$$

input
```
integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="giac")
```

output
```
integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^3, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelS}(bx)^3}\,dx = \int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelS}\,(b\,x)^3}\,dx$$

input
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3,x)
```

output
```
int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3, x)
```

3.70 $\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$$

output
```
FresnelS(b*x)^(1+n)/b/(1+n)
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$$

input
```
Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

3.70.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\frac{1}{2}\pi b^2 x^2\right) \text{FresnelS}(bx)^n \, dx$$

$$\Big\downarrow \text{ 6994}$$

$$\frac{\int \text{FresnelS}(bx)^n d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{FresnelS}(bx)^{n+1}}{b(n+1)}$$

input
```
Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

3.70.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.70.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\text{FresnelS}(bx)^{1+n}}{b(1+n)}$	18

input
```
int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
FresnelS(b*x)^(1+n)/b/(1+n)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\,(bx)^n\,\text{S}\,(bx)}{bn+b}$$

input
```
integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
fresnel_sin(b*x)^n*fresnel_sin(b*x)/(b*n + b)
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \begin{cases} 0 & \text{for } b=0 \wedge (b=0 \vee n=-1) \\ \frac{\log(S(bx))}{b} & \text{for } n=-1 \\ \frac{S(bx)S^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, E
q(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{\text{S}(bx)^{n+1}}{b(n+1)}$$

input
```
integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
fresnel_sin(b*x)^(n + 1)/(b*(n + 1))
```

3.70.8 Giac [F]

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{S}(bx)^n \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelS}(b\,x)^n \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.71 $\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 232

$$\begin{aligned}\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = {} & \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} \\ & - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} \\ & - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{105\,\text{FresnelS}(bx)^2}{2b^9\pi^4} \\ & - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} \\ & + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\ & - \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}\end{aligned}$$

output
```
105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2+55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4
*x^6*cos(b^2*Pi*x^2)/b^3/Pi^2+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6
/Pi^3-x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+105/2*FresnelS(b*x)^2/b
^9/Pi^4-105*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelS(b*
x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2-40*sin(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*sin(b^
2*Pi*x^2)/b^5/Pi^3
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00

$$\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} - \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{105\,\text{FresnelS}(bx)^2}{2b^9\pi^4} - \frac{105x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

input
```
Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/
(4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/
(b^2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^
4*Pi^2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5
*Pi^3)
```

3.71.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{ 7008}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^7\sin(b^2\pi x^2)\,dx}{2\pi b} - \frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^4\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^4\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\Big\downarrow$ 3042

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\Big\downarrow$ 3117

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 7016

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3860

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3042

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^7 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3790

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^7 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 15

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^6}{6}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^7 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3042

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^6}{6}-\frac{1}{2}\int x^4\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^7 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3777

3.71. $\int x^8 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3777$$

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\big\downarrow$ 3042

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\big\downarrow$ 3117

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}\right)}{\pi b^2}-$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\big\downarrow$ 7008

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3860

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3042

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3777

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\cdots\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3042

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\cdots\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3117

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}+\frac{x^5\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\cdots\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 7016

3.71. $\int x^8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\left(\frac{3\left(-\frac{\int \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b\right.}{\pi b^2}}{4\pi b}}{\pi b^2}\right.}{\pi b^2}$$

$$\frac{\frac{x^7\operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}}{}$$

input
```
Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
$Aborted
```

3.71.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.71.4 Maple [F]

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

input
```
int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

$$\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx =$$
$$-\frac{2\,\pi^3b^6x^6 - 75\,\pi b^2x^2 + 3\left(\pi^3b^6x^6 - 55\,\pi b^2x^2\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 6\left(\pi^4b^7x^7 - 35\,\pi^2b^3x^3\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right.}{6\,\pi^5b^9}$$

input
```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/6*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^
2)*fresnel_sin(b*x) - 315*pi*fresnel_sin(b*x)^2 - 6*(5*(pi^2*b^4*x^4 - 16)
*cos(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fresnel_sin(b*x))*sin(
1/2*pi*b^2*x^2))/(pi^5*b^9)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int x^8 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \begin{cases} -\frac{x^7\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} - \frac{x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{3\pi^2b^3} - \frac{5x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{6\pi^2b^3} + \frac{7x^5\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{5x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} + \frac{35x^3\cos}{} \\ 0 \end{cases}$$

input
```
integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((-x**7*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**6*sin(pi
*b**2*x**2/2)**2/(3*pi**2*b**3) - 5*x**6*cos(pi*b**2*x**2/2)**2/(6*pi**2*b
**3) + 7*x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 5*x**4*sin(
pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) + 35*x**3*cos(pi*b**2*x**
2/2)*fresnels(b*x)/(pi**3*b**6) + 25*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**4*
b**7) + 40*x**2*cos(pi*b**2*x**2/2)**2/(pi**4*b**7) - 105*x*sin(pi*b**2*x*
*2/2)*fresnels(b*x)/(pi**4*b**8) - 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2
/2)/(pi**5*b**9) + 105*fresnels(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, Tru
e))
```

3.71.7 Maxima [F]

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.71.8 Giac [F]

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelS}(b\,x) \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

input
```
int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.72 $\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 216

$$\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos(b^2\pi x^2)}{16b^7\pi^4} - \frac{x^5\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{531\,\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{48\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{6x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{17x^3\sin(b^2\pi x^2)}{8b^5\pi^3}$$

output
```
24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2+147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^5*c
os(b^2*Pi*x^2)/b^3/Pi^2+24*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-
x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi-48*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/b^8/Pi^4+6*x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+17/8*x^3
*sin(b^2*Pi*x^2)/b^5/Pi^3-531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int x^7 \operatorname{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) dx$$
$$= \frac{-2655\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right) - 160 \operatorname{FresnelS}(bx) \left(b^2\pi x^2(-24 + b^4\pi^2x^4) \cos\left(\frac{1}{2}b^2\pi x^2\right) - 6(-8 + b^4\pi^2x^4) \sin}{160b^8\pi^4}$$

input
```
Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24
+ b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^
2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*
Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

3.72.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 440 vs. 2(216) = 432.

Time = 2.02 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.850, Rules used = {7008, 3866, 3867, 3866, 3833, 7016, 3872, 15, 3867, 3866, 3833, 7008, 3866, 3833, 7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \operatorname{FresnelS}(bx) \sin \left(\frac{1}{2} \pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{6 \int x^5 \cos \left(\frac{1}{2} b^2 \pi x^2\right) \operatorname{FresnelS}(bx) dx}{\pi b^2} + \frac{\int x^6 \sin \left(b^2 \pi x^2\right) dx}{2\pi b} - \frac{x^6 \operatorname{FresnelS}(bx) \cos \left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$\frac{6 \int x^5 \cos \left(\frac{1}{2} b^2 \pi x^2\right) \operatorname{FresnelS}(bx) dx}{\pi b^2} + \frac{\frac{5 \int x^4 \cos(b^2 \pi x^2) dx}{2\pi b^2} - \frac{x^5 \cos(\pi b^2 x^2)}{2\pi b^2}}{2\pi b} - \frac{x^6 \operatorname{FresnelS}(bx) \cos \left(\frac{1}{2} \pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^6\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3866

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^6\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3833

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^6\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 7016

$$\frac{6\left(-\frac{4\int x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3872

$$\frac{6\left(-\frac{4\int x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 15

3.72. $\int x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^5}{10}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 3867

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 3866

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 3833

3.72. $\int x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}\right)}{\pi b^2}-\cdots$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 7008

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{\cdots}\right)}{2\sqrt{2}\pi b^{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3866

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\cdots\right.}{\cdots}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3833

3.72. $\int x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\cdots\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 7014

$$\frac{6\left(-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin(\cdots}{\pi b^2}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3838

$$\frac{6\left(-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx\cdots}{\pi b\cdots}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 2009

3.72. $\int x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$-\frac{x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+$$

$$\frac{6\left(\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi \cdots\right.}{2\pi \cdots}}{2\pi b}\cdots\right.}{\pi b^2}\cdots\right.}{\pi b^2}$$

$$\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

input
```
Int[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + (-1/2*(x^5*Cos[b^2*P
i*x^2])/(b^2*Pi) + (5*((-3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[S
qrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) + (x^3*Sin[b^2*Pi*x^2])/(2*b^2
*Pi)))/(2*b^2*Pi))/(2*b*Pi) + (6*((x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (4*((-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/
(2*Sqrt[2]*b^3*Pi))/(2*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (2*(-((x/2 - FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)) + (Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/(b^2*Pi) - (x^5/10 + (
(3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b
^3*Pi)))/(2*b^2*Pi) - (x^3*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b^2*P
i)
```

3.72.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.72. $\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.72.4 Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.47

method	result
default	$\frac{\text{FresnelS}(bx)\left(-\frac{b^{6}x^{6}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{6b^{4}x^{4}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{24\left(-\frac{b^{2}x^{2}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}\right)}{\pi}\right)}{b^{7}}-\frac{\frac{\frac{3}{5}b^{5}x^{5}\pi^{2}-24bx}{\pi^{4}}-\frac{3\left(\frac{\pi b^{3}x^{3}\sin\left(b^{2}\right.}{2}\right.}{}}{}$

input
```
int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*s
in(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*
b^2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*b^5*x^5*Pi^2-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x
^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*
x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi
*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(
b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int x^{7}\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)\,dx =$$

$$-\frac{56\,\pi^{2}b^{6}x^{5}-2370\,b^{2}x+20\left(4\,\pi^{2}b^{6}x^{5}-147\,b^{2}x\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)^{2}+160\left(\pi^{3}b^{7}x^{6}-24\,\pi b^{3}x^{2}\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\,\mathrm{S}}{160}$$

input
```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/160*(56*pi^2*b^6*x^5 - 2370*b^2*x + 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos
(1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2
)*fresnel_sin(b*x) + 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*
x) - 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 24*(pi^2*b^5*x^4 - 8*b)*fresn
el_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

3.72.6 Sympy [F]

$$\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

input
```
integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**7*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.72.7 Maxima [F]

$$\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.72.8 Giac [F]

$$\int x^7 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.73 $\int x^6 \text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) \, dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 248

$$\int x^6 \text{FresnelS}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) \, dx = -\frac{5x^4}{8b^3\pi^2} + \frac{11 \cos (b^2\pi x^2)}{2b^7\pi^4} - \frac{x^4 \cos (b^2\pi x^2)}{4b^3\pi^2} + \frac{15x \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^5 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{b^2\pi} - \frac{15 \text{FresnelC}(bx) \text{FresnelS}(bx)}{2b^7\pi^3} + \frac{15ix^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} - \frac{15ix^2 \, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3} + \frac{5x^3 \text{FresnelS}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{7x^2 \sin (b^2\pi x^2)}{4b^5\pi^3}$$

output
```
-5/8*x^4/b^3/Pi^2+11/2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^4*cos(b^2*Pi*x^2)/b^
3/Pi^2+15*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*
x^2)*FresnelS(b*x)/b^2/Pi-15/2*FresnelC(b*x)*FresnelS(b*x)/b^7/Pi^3+15/8*I
*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^5/Pi^3-15/8*I*x^2*hype
rgeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3+5*x^3*FresnelS(b*x)*sin(1
/2*b^2*Pi*x^2)/b^4/Pi^2+7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

3.73.2 Mathematica [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

input
```
Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

3.73.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.36, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {7008, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3118, 7016, 3860, 3042, 3790, 15, 3042, 3777, 25, 3042, 3118, 7008, 3860, 3042, 3118, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \; 7008$$

$$\frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^5 \sin\left(b^2\pi x^2\right) dx}{2\pi b} - \frac{x^5 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3860$$

$$\frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^4 \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} - \frac{x^5 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x^4 \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} - \frac{x^5 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$\frac{5\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)dx}{\pi b^2} + \frac{\frac{2\int x^2 \cos(b^2\pi x^2)dx^2}{\pi b^2} - \frac{x^4 \cos(\pi b^2 x^2)}{\pi b^2}}{4\pi b} - \frac{x^5 \operatorname{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3777

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 25

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3042

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3118

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$\big\downarrow$ 7016

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$\big\downarrow$ 3860

3.73. $\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3790}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{15}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3777}$$

3.73. $\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3118$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 7008$$

3.73. $\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\text{co}\right.}{2\pi b}\right.$$

$$\frac{}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3860

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\text{co}\right.}{2\pi b}\right.$$

$$\frac{}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3042

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\text{co}\right.}{2\pi b}\right.$$

$$\frac{}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3118

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b\right.}{\pi^2 b}\right.}{2\pi b}\right.$$

$$\frac{}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 7000

3.73. $\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{5\left(-\frac{3\left(\frac{-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\frac{x^3\,\text{F}\cdots}{}\right.}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

input
```
Int[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + (-((x^4*Cos[b^2*Pi*x
^2])/(b^2*Pi)) + (2*(Cos[b^2*Pi*x^2]/(b^4*Pi^2) + (x^2*Sin[b^2*Pi*x^2])/(b
^2*Pi)))/(b^2*Pi))/(4*b*Pi) + (5*((-3*(-1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (
x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + ((FresnelC[b*x]*FresnelS[b
*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*
Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2
])/(b^2*Pi)))/(b^2*Pi) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
 - (x^4/4 + (-(Cos[b^2*Pi*x^2]/(b^4*Pi^2)) - (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi)
)/2)/(2*b*Pi)))/(b^2*Pi)
```

3.73.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.73. $\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.73.4 Maple [F]

$$\int x^{6} \operatorname{FresnelS}\left(b x\right) \sin\left(\frac{b^{2} \pi \,x^{2}}{2}\right)d x$$

input
```
int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.73.5 Fricas [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^{6} \operatorname{S}\left(b x\right) \sin\left(\frac{1}{2} \, \pi b^{2} x^{2}\right) \,d x$$

input
```
integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.73.6 Sympy [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^{6} \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} S\left(b x\right)\, dx$$

input
```
integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**6*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.73.7 Maxima [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.73.8 Giac [F]

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.74 $\int x^5 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 158

$$\int x^5 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{2x^3}{3b^3\pi^2} - \frac{x^3\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} - \frac{43\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{11x\sin(b^2\pi x^2)}{8b^5\pi^3}$$

output
```
-2/3*x^3/b^3/Pi^2-1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+8*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+4*x^2*F
resnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3-
43/16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int x^5 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx =$$

$$-\frac{32b^3\pi x^3 + 12b^3\pi x^3\cos(b^2\pi x^2) + 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 48\,\text{FresnelS}(bx)\left((-8 + b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x\right.\right.}{48b^6\pi^3}$$

input
```
Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-1/48*(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*Fresnel
S[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2]
 - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(b^6*Pi^3)
```

3.74.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.73, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {7008, 3866, 3867, 3832, 7016, 3872, 15, 3867, 3832, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{4\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4 \sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^4 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$\frac{4\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^4 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$\frac{4\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^4 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{4\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^4 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 7016$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 3872$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 15$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 3867$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 3832$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \ 7006$$

3.74. $\int x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{4\left(-\frac{2\left(\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 3832$$

$$-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{4\left(\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}$$

input
```
Int[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + (-1/2*(x^3*Cos[b^2*P
i*x^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Si
n[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) + (4*((-2*(-((Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*P
i)))/(b^2*Pi) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^3/6
+ (FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi) - (x*Sin[b^2*Pi*x^2])/(2*b^2*P
i))/2)/(b*Pi)))/(b^2*Pi)
```

3.74.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.74. $\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.74.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{\mathrm{FresnelS}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}-\frac{2\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}\right)}{\pi^2}-\frac{-\frac{\pi b^3x^3\cos}{}}{}}{}}{b}$

input
```
int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*s
in(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/
Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2
/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-
1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x^5\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx =$$
$$-\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+20\,\pi b^4x^3+48\left(\pi^2b^5x^4-8\,b\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)+129\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{48\,\pi^3b^7}$$

input
```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*
x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fr
esnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(16*pi*b^3*x^2*fresnel_sin(b*x) + 11*b
^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

3.74.6 Sympy [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \sin\left(\frac{\pi b^2x^2}{2}\right) S(bx) \, dx$$

input
```
integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.74.7 Maxima [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

input
```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.74.8 Giac [F]

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

input
```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^5 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.75 $\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.75.1 Optimal result

Integrand size = 20, antiderivative size = 120

$$\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{3x^2}{4b^3\pi^2}-\frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}-\frac{3\,\text{FresnelS}(bx)^2}{2b^5\pi^2}+\frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

output
```
-3/4*x^2/b^3/Pi^2-1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2-x^3*cos(1/2*b^2*Pi*x^2)
*FresnelS(b*x)/b^2/Pi-3/2*FresnelS(b*x)^2/b^5/Pi^2+3*x*FresnelS(b*x)*sin(1
/2*b^2*Pi*x^2)/b^4/Pi^2+sin(b^2*Pi*x^2)/b^5/Pi^3
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{3x^2}{4b^3\pi^2}-\frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}-\frac{3\,\text{FresnelS}(bx)^2}{2b^5\pi^2}+\frac{3x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

input
```
Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2
*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (
3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*P
i^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7008, 3860, 3042, 3777, 3042, 3117, 7016, 3860, 3042, 3114, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3860}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{7016}$$

$$\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{3860}$$

$$\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{3042}$$

$$\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{3114}$$

$$\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{6994}$$

$$\frac{3\left(-\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ \textbf{15}$$

$$-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{3\left(-\frac{\text{FresnelS}(bx)^2}{2\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}$$

3.75. $\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

input
```
Int[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + (-((x^2*Cos[b^2*Pi*x
^2])/(b^2*Pi)) + Sin[b^2*Pi*x^2]/(b^4*Pi^2))/(4*b*Pi) + (3*(-1/2*FresnelS[
b*x]^2/(b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^2/2
- Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)))/(b^2*Pi)
```

3.75.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.75.4 Maple [F]

$$\int x^{4} \operatorname{FresnelS}\left(b x\right) \sin\left(\frac{b^{2} \pi\, x^{2}}{2}\right)dx$$

input
```
int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^4 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = -\frac{2\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + \pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + \pi b^2 x^2 + 3\,\pi\, \mathrm{S}\left(bx\right)^2 - 2\,\left(3\,\pi bx\, \mathrm{S}\left(bx\right) + 2\, \cos\left(\frac{1}{2}\,\pi\right.\right.}{2\,\pi^3 b^5}$$

input
```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/2*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos
(1/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + 3*pi*fresnel_sin(b*x)^2 - 2*(3*pi*b*x*fr
esnel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \begin{cases} -\frac{x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} - \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3} + \frac{3x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} - \frac{3S^2(bx)}{2\pi^2b^5} & \text{fo} \\ 0 & \text{o} \end{cases}$$

input
```
integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi
*b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3)
 + 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2
/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5),
Ne(b, 0)), (0, True))
```

3.75.7 Maxima [F]

$$\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^4\,\mathrm{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.75.8 Giac [F]

$$\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^4\,\mathrm{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^4\,\operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.76 $\int x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 105

$$\int x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{x}{b^3\pi^2} - \frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

output
```
-x/b^3/Pi^2-1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2-x^2*cos(1/2*b^2*Pi*x^2)*Fresnel
S(b*x)/b^2/Pi+2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+5/8*FresnelC(b*
x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{-2bx(4+\cos(b^2\pi x^2)) + 5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) - 8\,\text{FresnelS}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}$$

input
```
Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-2*b*x*(4 + Cos[b^2*Pi*x^2]) + 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 8*Fresne
lS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*P
i^2)
```

3.76.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {7008, 3866, 3833, 7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^2\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^2\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^2\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{7014}$$

$$\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{3838}$$

$$\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

input
```
Int[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*
Pi))/(2*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + (2*(-((
x/2 - FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)) + (FresnelS[b*x]*Sin[(b
^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.76.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelS}(bx)\left(-\frac{b^{2}x^{2}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}\right)}{b^{3}}-\frac{\frac{bx}{\pi^{2}}-\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^{2}}-\frac{-\frac{bx\cos\left(b^{2}\pi x^{2}\right)}{2\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^{3}}}{b}$	115

input
```
int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*P
i*x^2))-1/b^3*(b*x/Pi^2-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/
2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int x^{3}\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)\,dx =$$
$$-\frac{8\,\pi b^{3}x^{2}\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\text{S}\left(bx\right)+4\,b^{2}x\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)^{2}+6\,b^{2}x-16\,b\,\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-5\,\sqrt{2}\sqrt{b^{2}}\,\text{C}\left(\sqrt{2}\right.}{8\,\pi^{2}b^{5}}$$

input
```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/8*(8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 4*b^2*x*cos(1/2*
pi*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - 5*sq
rt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

3.76.6 Sympy [F]

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

input
```
integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.76.7 Maxima [F]

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.76.8 Giac [F]

$$\int x^3 \operatorname{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{S}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^3\,\operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.77 $\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi} + \frac{\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}$$

output
```
-1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1
/2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],
-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x
^2)/b/Pi
```

3.77.2 Mathematica [F]

$$\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

input
```
Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

3.77.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7008, 3860, 3042, 3118, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelS}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7008}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int x \sin\left(b^2\pi x^2\right) dx}{2\pi b} - \frac{x \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3860}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} - \frac{x \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2} + \frac{\int \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} - \frac{x \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)dx}{\pi b^2} - \frac{x \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

$$\Big\downarrow \text{7000}$$

$$\frac{-\frac{1}{8}ibx^2\, {}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\, {}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b}}{\pi b^2} - \frac{x \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

input
```
Int[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b
^2*Pi) + ((FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Hypergeometric
PFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ
[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi)
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

3.77.4 Maple [F]

$$\int x^2 \operatorname{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

input
```
int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.77.5 Fricas [F]

$$\int x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.77.6 Sympy [F]

$$\int x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

input
```
integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.77.7 Maxima [F]

$$\int x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^2\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.77.8 Giac [F]

$$\int x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\int x^2\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\int x^2\operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.78 $\int x \,\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.78.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^2\pi}+\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}$$

output
```
-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/4*FresnelS(b*x*2^(1/2))/b^2/Pi
*2^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{-4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)+\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4b^2\pi}$$

input
```
Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*
b^2*Pi)
```

3.78.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \text{7006}$$

$$\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

input
```
Int[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2
*Sqrt[2]*b^2*Pi)
```

3.78.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.78.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelS}(bx)}{b\pi}+\frac{\operatorname{FresnelS}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	46

input
```
int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^
(1/2))/b
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)-\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

input
```
integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
-1/4*(4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - sqrt(2)*sqrt(b^2)*fresnel
_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

3.78.6 Sympy [F]

$$\int x\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

input
```
integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.78.7 Maxima [F]

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x\,\text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

input
```
integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.78.8 Giac [F]

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x\,\text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

input
```
integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x\,\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.79 $\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.79.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

output
```
1/2*FresnelS(b*x)^2/b
```

3.79.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelS}(bx)^2}{2b}$$

input
```
Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^2/(2*b)
```

3.79.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \; 6994$$

$$\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\text{FresnelS}(bx)^2}{2b}$$

input
```
Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
FresnelS[b*x]^2/(2*b)
```

3.79.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.79.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelS}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelS}(bx)^2}{2b}$	12

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*FresnelS(b*x)^2/b
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{\text{S}\left(bx\right)^2}{2\,b}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/2*fresnel_sin(b*x)^2/b
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\begin{cases}\frac{S^2(bx)}{2b} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{S}\,(bx)^2}{2\,b}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
1/2*fresnel_sin(b*x)^2/b
```

3.79.8 Giac [F]

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.80 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

3.80.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx=\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.80.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

3.80.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\,dx$$
$$\Big\downarrow \text{ 7012}$$
$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

output
```
$Aborted
```

3.80.3.1 Defintions of rubi rules used

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.80.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.80.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.80.6 Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

3.80.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.80.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{S\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.80.9 Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelS}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.81 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

3.81.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

output
```
Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.81.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

3.81.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$

$$\Big\downarrow \text{7012}$$

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}dx$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

output
```
$Aborted
```

3.81.3.1 Defintions of rubi rules used

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.81.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^2}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.81.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.81.6 Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^2}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

3.81.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.81.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.81.9 Mupad [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

3.82 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

3.82.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}+\frac{b^2\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} + \frac{1}{2}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)$$

output
```
-1/4*b/x+1/4*b*cos(b^2*Pi*x^2)/x-1/2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2
+1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*
b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.82.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

3.82.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{b}{4x}$$

$$\Big\downarrow \text{7020}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{b}{4x}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

output
```
$Aborted
```

3.82.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.82.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^3}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

3.82.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.82.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^3}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

3.82.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.82.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{S\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.82.9 Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.83 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

3.83.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{1}{6}b^3\pi^2\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b^3\pi\,\text{Si}\left(b^2\pi x^2\right)$$

output
```
-1/12*b/x^2+1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fres
nelS(b*x)/x-1/6*b^3*Pi^2*FresnelS(b*x)^2+1/6*b^3*Pi*Si(b^2*Pi*x^2)-1/3*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

3.83.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x} - \frac{1}{6}b^3\pi^2\,\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} + \frac{1}{6}b^3\pi\,\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

output
```
-1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

3.83.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {7010, 3861, 3042, 3778, 25, 3042, 3780, 7018, 3856, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{7018}$$

$$\frac{1}{3}\pi b^2\left(-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3856}$$

$$\frac{1}{3}\pi b^2\left(-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{6994}$$

$$\frac{1}{3}\pi b^2\left(-\pi b\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{15}$$

3.83. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx$

$$\frac{1}{3}\pi b^2\left(-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi b\,\text{FresnelS}(bx)^2\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

output
```
-1/12*b/x^2 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x^3) + (b^2*Pi*(-((Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*SinInt
egral[b^2*Pi*x^2])/4))/3 - (b*(-(Cos[b^2*Pi*x^2]/x^2) - b^2*Pi*SinIntegral
[b^2*Pi*x^2]))/12
```

3.83.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.83.4 Maple [F]

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{4}}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx =$$
$$-\frac{\pi^2 b^3 x^3\,\text{S}\,(bx)^2 - \pi b^3 x^3\,\text{Si}\,(\pi b^2 x^2) + 2\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{S}\,(bx) - bx\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + bx + 2\,\text{S}\,(bx)\sin}{6\,x^3}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(pi^2*b^3*x^3*fresnel_sin(b*x)^2 - pi*b^3*x^3*sin_integral(pi*b^2*x^2
) + 2*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - b*x*cos(1/2*pi*b^2
*x^2)^2 + b*x + 2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2))/x^3
```

3.83.6 Sympy [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^4}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

3.83.7 Maxima [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^4}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.83.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.84 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

3.84.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2\,\text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^2} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}b^4\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
-1/24*b/x^3+1/24*b*cos(b^2*Pi*x^2)/x^3-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fres
nelS(b*x)/x^2-1/4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-7/48*b^3*Pi*sin(b^
2*Pi*x^2)/x+7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unint
egrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.84.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

3.84.3 Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3869, 3868, 3833, 7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}\,dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$\big\downarrow$ 3833

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$\big\downarrow$ 7018

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$\big\downarrow$ 3868

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$\big\downarrow$ 3833

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$\big\downarrow$ 7012

3.84. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx$

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos}{2x^2}\right.$$
$$\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

output
```
$Aborted
```

3.84.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.84.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^5}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

3.84.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.84.6 Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

3.84.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.84.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.84.9 Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} \, dx = \int \frac{\text{FresnelS}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5} \, dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

3.85 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

3.85.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}+\frac{1}{24}b^5\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x^3} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2} - \frac{1}{15}b^4\pi^2\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

output
```
-1/40*b/x^4+1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)+1/40*b*cos(b^2*Pi*x^2)/x^4-1/15*b
^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/5*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/x^5-1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.85.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]
```

3.85.3 Rubi [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7018, 3860, 3042, 3778, 3042, 3783, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}\,dx$$

$$\Big\downarrow \ 7010$$

$$\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3861$$

$$\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3778$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3778$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3783$$

$$\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 7018$$

3.85. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 3860

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 3042

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 3778

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\cos}{3x^3}\right.$$
$$-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 3042

3.85. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)}{3x\ldots}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 3783

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\,\text{co}\ldots}{3x^3}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\downarrow$ 7012

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\,\text{co}\ldots}{3x^3}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

output
```
$Aborted
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.85. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

3.85.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi \,x^{2}}{2}\right)}{x^{6}}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

3.85.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx=\int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2} x^{2}\right)}{x^{6}}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.85.6 Sympy [N/A]

Not integrable

Time = 5.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx=\int \frac{\sin\left(\frac{\pi b^{2} x^{2}}{2}\right) S\left(bx\right)}{x^{6}}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

3.85.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.85.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.85.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.86 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

3.86.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = &-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}\\ &-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{24x^4}\\ &-\frac{7b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)\\ &-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}\\ &+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2}-\frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3}\\ &-\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x},x\right)\end{aligned}$$

output
```
-1/60*b/x^5+1/96*b^5*Pi^2/x+1/60*b*cos(b^2*Pi*x^2)/x^5-67/1440*b^5*Pi^2*co
s(b^2*Pi*x^2)/x-1/24*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/6*Fres
nelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/x^2-13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3-67/1440*b^6*Pi^3*FresnelS(b*
x*2^(1/2))*2^(1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS
(b*x)/x,x)
```

3.86.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx=\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

3.86.3 Rubi [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3869, 3868, 3869, 3832, 7018, 3868, 3869, 3832, 7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx - \frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{b}{60x^5}$$

$$\Big\downarrow \text{ 3869}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx - \frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx - \frac{\cos\left(\pi b^2x^2\right)}{x}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{b}{60x^5}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{b}{60x^5}$$

$$\Big\downarrow \text{ 7018}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{b}{60x^5}$$

$$\Big\downarrow \text{ 3868}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\text{FresnelS}(bx)\cos\left(\right.}{4x^4} - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} - \frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{b}{60x^5}$$

3.86. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

$$\big\downarrow \ 3869$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\big\downarrow \ 3832$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{F}\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\big\downarrow \ 7010$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}\right)\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\big\downarrow \ 3869$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(}{}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

3.86. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{F}\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\Big\downarrow \text{7020}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{F}\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

output
```
$Aborted
```

3.86.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.86.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{7}}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

3.86.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.86.6 Sympy [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^7}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

3.86.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.86.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.86.9 Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.87 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

3.87.1 Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = & -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}\\ & -\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5}\\ & +\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x}\\ & +\frac{1}{210}b^7\pi^4\,\text{FresnelS}(bx)^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}\\ & +\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}\\ & -\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{70}b^7\pi^3\,\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

output
```
-1/84*b/x^6+1/420*b^5*Pi^2/x^2+1/84*b*cos(b^2*Pi*x^2)/x^6-1/84*b^5*Pi^2*co
s(b^2*Pi*x^2)/x^2-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5+1/105*
b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/210*b^7*Pi^4*FresnelS(b*x)^
2-1/70*b^7*Pi^3*Si(b^2*Pi*x^2)-1/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1
/105*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/105*b^3*Pi*sin(b^2*P
i*x^2)/x^4
```

3.87.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{35x^5} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x} + \frac{1}{210}b^7\pi^4\text{FresnelS}(bx)^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} - \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{70}b^7\pi^3\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

output
```
-1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*P
i^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])
/(35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi
^4*FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b
^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*
x^2])/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

3.87.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

$$\Big\downarrow \text{ 7010}$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx - \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}dx - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7} - \frac{b}{84x^6}$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx - \frac{1}{28}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7} - \frac{b}{84x^6}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3778$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3778$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3778$$

3.87. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$\big\downarrow$ 25

$$\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$\big\downarrow$ 3780

$$\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 7018

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 3860

3.87. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3042

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3778

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\text{co}}{5x^5}\right.$$
$$-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3042

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{FresnelS}(bx\right.$$
$$-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3778

3.87. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2 \ldots\right.}{2x^4}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 25

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2 \ldots\right.}{2x^4}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3042

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2 \ldots\right.}{2x^4}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3780

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\ldots\right.}{}\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 7010

3.87. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3861$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12\ldots}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\ldots\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3778$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelS}\ldots\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 25$$

3.87. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelS}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3042

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelS}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3780

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\cos\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 7018

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3856

3.87. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)\right)-\text{Fr}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

output
```
$Aborted
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.87.4 Maple [F]

$$\int \frac{\text{FresnelS}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^8}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \frac{\pi^4b^7x^7\,\text{S}\,(bx)^2 - 3\,\pi^3b^7x^7\,\text{Si}\,(\pi b^2x^2) + 3\,\pi^2b^5x^5 - 5\,(\pi^2b^5x^5 - bx)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 2\,(\pi^3b^6x^6 - 3\,\pi b^2x^2)\,\text{co}}{210\,x^7}$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

output
```
1/210*(pi^4*b^7*x^7*fresnel_sin(b*x)^2 - 3*pi^3*b^7*x^7*sin_integral(pi*b^
2*x^2) + 3*pi^2*b^5*x^5 - 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2
*(pi^3*b^6*x^6 - 3*pi*b^2*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 5*b*
x - 2*(2*pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 - 15)*fresnel_sin(
b*x))*sin(1/2*pi*b^2*x^2))/x^7
```

3.87.6 Sympy [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^8}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

3.87.7 Maxima [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.87.8 Giac [F]

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.88 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

3.88.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx &= -\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}\\ &\quad-\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{853b^8\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}\\ &\quad-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^6}\\ &\quad+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{384x^2}\\ &\quad-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}\\ &\quad+\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}\\ &\quad-\frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5}+\frac{853b^7\pi^3\sin\left(b^2\pi x^2\right)}{80640x}\\ &\quad+\frac{1}{384}b^8\pi^4\,\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

output
```
-1/112*b/x^7+1/1152*b^5*Pi^2/x^3+1/112*b*cos(b^2*Pi*x^2)/x^7-187/40320*b^5
*Pi^2*cos(b^2*Pi*x^2)/x^3-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^
6+1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/8*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^
4-19/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5+853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x-
853/80640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrab
le(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.88.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

3.88.3 Rubi [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3869, 3868, 3869, 3868, 3833, 7018, 3868, 3869, 3868, 3833, 7010, 3869, 3868, 3833, 7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{8}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx-\frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\left(-\frac{2}{7}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2 \int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$\downarrow$ 3869

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2 \int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2\right.}{7x^7}\right. \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$\downarrow$ 3833

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx - \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b\right.}{7x}\right. \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$\downarrow$ 7018

3.88. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}\right)-$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos}{6x^6}\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3869

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2\right.}{5x^5}\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{c}{}\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3833

3.88. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$$\Big\downarrow \text{ 7010}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$$\Big\downarrow \text{ 3869}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\text{FresnelS}\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$$\Big\downarrow \text{ 3868}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\text{co}\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$$\Big\downarrow \text{ 3833}$$

3.88. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 7018

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right.\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{Fr}}{}\right.\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 3833

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{F}}{}\right.\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

$\downarrow$ 7012

3.88. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\text{F}\right.\right.\right.$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

output
```
$Aborted
```

3.88.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.88.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{9}}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

3.88.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)}{x^{9}}\,dx=\int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{9}}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.88.6 Sympy [N/A]

Not integrable

Time = 36.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^9}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

3.88.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.88.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.88.9 Mupad [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.89 $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = & -\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{144x^8} \\ & -\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4}-\frac{5b^9\pi^4\,\text{CosIntegral}\left(b^2\pi x^2\right)}{2016} \\ & -\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{63x^7} \\ & +\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x^3} \\ & -\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} \\ & +\frac{b^4\pi^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5} \\ & -\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2} \\ & +\frac{1}{945}b^8\pi^4\,\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)\end{aligned}$$

output
```
-1/144*b/x^8+1/2520*b^5*Pi^2/x^4-5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)+1/144*b*co
s(b^2*Pi*x^2)/x^8-67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/63*b^2*Pi*cos(1/
2*b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*Fresnel
S(b*x)/x^3-1/9*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6+5/2016*
b^7*Pi^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(FresnelS(b*x)*sin
(1/2*b^2*Pi*x^2)/x^2,x)
```

3.89.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

input
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

output
```
Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]
```

3.89.3 Rubi [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7010, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7018, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7010, 3861, 3042, 3778, 25}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

$$\Big\downarrow \text{7010}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3778

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 25

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3778

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx-\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

3.89. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx - \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2 \int -\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx - \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx - \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx - \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2 \int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\sin\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi b^2\right.}{4x^8} \right. - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx - \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2 \int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2 - \frac{\sin\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right) - \cos \right. - \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$$\Big\downarrow \text{ 3783}$$

3.89. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx -$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 7018

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{14}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}\right)-$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3860

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}\right)-$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}\right)-$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\,\text{co}}{7x^7}\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

3.89. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx}{}\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b\right.}{3x}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 25

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b\right.}{3x}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b\right.}{3x}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

3.89. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow\ 3042$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow\ 3783$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow\ 7010$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow\ 3861$$

3.89. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{Fresne}}{}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 25

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{Fresne}}{}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x\right.$$
$$\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

input
```
Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

output
```
$Aborted
```

3.89. $\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.89. $\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.89.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelS}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{10}}dx$$

input
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

output
```
int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

3.89.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^{2}\pi x^{2}\right)}{x^{10}}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{10}}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.89.6 Sympy [N/A]

Not integrable

Time = 66.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^{10}}\,dx$$

input
```
integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

3.89.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.89.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{S}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

input
```
integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.89.9 Mupad [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

input
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

output
```
int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

3.90 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx$

3.90.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \text{Int}\left(\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n, x\right)$$

output
```
Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

3.90.2 Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
```

output
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]
```

3.90.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7004}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{FresnelS}(bx)^n\,dx$$

$$\Big\downarrow \text{7004}$$

$$\int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{FresnelS}(bx)^ndx$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
```

output
```
$Aborted
```

3.90.3.1 Defintions of rubi rules used

rule 7004
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol]
 :> Unintegrable[Cos[c + d*x^2]*FresnelS[a + b*x]^n, x] /; FreeQ[{a, b, c,
d, n}, x]
```

3.90.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)^n\,dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

3.90.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \int \text{S}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="fricas")
```

output
```
integral(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.90.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \int \cos\left(\frac{\pi b^2x^2}{2}\right) S^n(bx)\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)
```

3.90.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)^n\,dx = \int \text{S}\,(bx)^n \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="maxima")
```

output
```
integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.90.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \text{S}\left(bx\right)^n\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="giac")
```

output
```
integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.90.9 Mupad [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)^n\,dx = \int \text{FresnelS}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 307

$$\begin{aligned}\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx =& \frac{35x^4}{8b^5\pi^3}-\frac{x^8}{16b\pi}-\frac{40\cos(b^2\pi x^2)}{b^9\pi^5}+\frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3}\\&-\frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4}\\&+\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2}\\&+\frac{105\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^9\pi^4}\\&-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4}\\&+\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4}\\&-\frac{35x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}\\&+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\\&-\frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4}+\frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

output
```
35/8*x^4/b^5/Pi^3-1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^
2*Pi*x^2)/b^5/Pi^3-105*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+7*x^5*
cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)*FresnelS(b*
x)/b^9/Pi^4-105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/P
i^4+105/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-35*x^
3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelS(b*x)*sin(1/2*b^2
*Pi*x^2)/b^2/Pi-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)/
b^3/Pi^2
```

3.91.2 Mathematica [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$$

input
```
Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

3.91.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{ 7016}$$

$$-\frac{7\int x^6 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^7\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3860}$$

$$-\frac{7\int x^6 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{7\int x^6 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3790}$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^6dx^2}{2}-\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 15$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^8}{8}-\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3042$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^8}{8}-\frac{1}{2}\int x^6\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3777$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{3\int -x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 25$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\int x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3042$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\int x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3777$$

$$-\frac{7\int x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\int x^2\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3042$$

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{7\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{7\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{7\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\int x^6 \operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}+\frac{x^7\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)\,dx$

$$\Big\downarrow 3118$$

$$-\frac{7\int x^6\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^7\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow 7008$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^5\sin(b^2\pi x^2)dx}{2\pi b}-\frac{x^5\,\mathrm{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow 3860$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4\sin(b^2\pi x^2)dx^2}{4\pi b}-\frac{x^5\,\mathrm{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\mathrm{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow 3042$$

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelS}(bx)\,dx$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3777

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\int x^2\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3042

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3777

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^5\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 25$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^5\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^5\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 3118$$

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}+$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \text{7016}$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\left(\frac{x^2\sin\left(\pi b\right.}{\pi b^2}\right.}{}\cdots\right)}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\left(\frac{x^2\sin\left(\pi\right.}{\pi b}\right.}{}\cdots\right)}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \text{3042}$$

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\left(\frac{x^2\sin(\pi \cdots}{\pi b \cdots}\right.}{\cdots}\cdots\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3790

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\cdots\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 15

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\left(\frac{x^2 \cdots}{\cdots}\right.}{\cdots}\cdots\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3042

3.91. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\left(\cdots\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

input
```
Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
$Aborted
```

3.91.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.91.4 Maple [F]

$$\int x^8 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \text{FresnelS}\,(bx)\, dx$$

input
```
int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.91.5 Fricas [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.91.6 Sympy [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

input
```
integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x**8*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.91.7 Maxima [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.91.8 Giac [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^8 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^8\, \text{FresnelS}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 217

$$\begin{aligned}\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx &= \frac{4x^3}{b^5\pi^3} - \frac{x^7}{14b\pi} + \frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} \\ &\quad - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^8\pi^4} \\ &\quad + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} \\ &\quad + \frac{531\,\text{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ &\quad - \frac{24x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ &\quad + \frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &\quad - \frac{147x\sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

output
```
4*x^3/b^5/Pi^3-1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3-48*cos(1/2*
b^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+6*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/b^4/Pi^2-24*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^6*FresnelS(b
*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*s
in(b^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx$$
$$= \frac{896b^3\pi x^3 - 16b^7\pi^3x^7 + 476b^3\pi x^3\cos(b^2\pi x^2) + 3717\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 224\,\text{FresnelS}(bx)\left(6(-8+b^4\pi\right.}{224b^8\pi}$$

input
```
Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*C
os[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])
- 2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^
4)
```

3.92.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 439 vs. $2(217) = 434$.

Time = 1.99 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.02, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {7016, 3872, 15, 3867, 3866, 3867, 3832, 7008, 3866, 3867, 3832, 7016, 3872, 15, 3867, 3832, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)\, dx$$
$$\Big\downarrow \; 7016$$
$$-\frac{6\int x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} + \frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$
$$\Big\downarrow \; 3872$$
$$-\frac{6\int x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{\int x^6dx}{2} - \frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx}{\pi b} + \frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{6\int x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^7}{14}-\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^6\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{6\int x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\int x^4\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}+\frac{x^6\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$-\frac{6\int x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}+\frac{x^6\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{6\int x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}+\frac{x^6\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{6\int x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^6\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{7008}$$

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 3866

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 3867

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 3832

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+$$

$$\frac{\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 7016

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x\sin\left(\pi b^2x \cdots\right.}{2\pi b^2}\cdots\right.}{\cdots}\cdots\right)}{\pi b^2}$$

$$\frac{\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 3872

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\cdots\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 15

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{3\left(\frac{x\sin\left(\cdots\right.}{2\pi\cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 3867

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x\right.}{\pi b^2}}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \; 3832$$

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x\right.}{\pi b^2}}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \; 7006$$

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{6\left(\frac{4\left(-\frac{2\left(\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelS}}{}\right.}{\pi b^2}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{x^6\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-$$

$$\frac{6\left(-\frac{x^4\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{4\left(\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\right.}{}\right.}{}\right.}{\pi b^2}$$

$$\frac{\frac{1}{2}\left(\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

input
```
Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

3.92. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

output
```
(x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (6*(-((x^4*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + (-1/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) +
(3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b
^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) + (4*((-2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi)))/(b^2*Pi) + (x^
2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^3/6 + (FresnelS[Sqrt[2]
*b*x]/(2*Sqrt[2]*b^3*Pi) - (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b^
2*Pi)))/(b^2*Pi) - (x^7/14 + (-1/2*(x^5*Sin[b^2*Pi*x^2])/(b^2*Pi) + (5*(-1
/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2
]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi)))/(2*b^2*Pi))/2)/(
b*Pi)
```

3.92.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.92.4 Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.48

method	result
default	$\frac{\text{FresnelS}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^2\pi\right.}{2}}{}}{}$

input
```
int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*c
os(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b
^2*Pi*x^2))))-1/b^7*(1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi
*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)
*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*
b^5*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/
2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*
sin(b^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx =$$

$$-\frac{16\,\pi^3 b^8 x^7 - 952\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 420\,\pi b^4 x^3 - 1344\,(\pi^2 b^5 x^4 - 8\,b)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right) - 3717\,\sqrt{2}}{224}$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/224*(16*pi^3*b^8*x^7 - 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 420*pi*b^
4*x^3 - 1344*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 3
717*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 28*((4*pi^2*b^6*x
^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 8*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fre
snel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

3.92.6 Sympy [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

input
```
integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x**7*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.92.7 Maxima [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.92.8 Giac [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^7\, \text{FresnelS}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.93.1 Optimal result

Integrand size = 20, antiderivative size = 184

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} + \frac{15\,\text{FresnelS}(bx)^2}{2b^7\pi^3} - \frac{15x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
15/4*x^2/b^5/Pi^3-1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+5*x^3*cos
(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+15/2*FresnelS(b*x)^2/b^7/Pi^3-15*x
*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/b^2/Pi-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/P
i^2
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\frac{15x^2}{4b^5\pi^3}-\frac{x^6}{12b\pi}+\frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2}+\frac{15\,\text{FresnelS}(bx)^2}{2b^7\pi^3}-\frac{15x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{11\sin(b^2\pi x^2)}{2b^7\pi^4}+\frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

input
```
Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b
*x]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3)
+ (x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/
(2*b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

3.93.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {7016, 3860, 3042, 3790, 15, 3042, 3777, 25, 3042, 3777, 3042, 3117, 7008, 3860, 3042, 3777, 3042, 3117, 7016, 3860, 3042, 3114, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{7016}$$

$$-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^5\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3042

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3790

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 15

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^6}{6}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3042

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^6}{6}-\frac{1}{2}\int x^4\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3777

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{2\int -x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 25

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\int x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3042

$$-\frac{5\int x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\int x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3777

3.93. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3042

$$-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3117

$$-\frac{5\int x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$\big\downarrow$ 7008

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$\big\downarrow$ 3860

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$\big\downarrow$ 3042

3.93. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{7016}$$

3.93. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\cdots}{\cdots}}{4\pi b}\cdots\right.}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 3860$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\cdots}{\cdots}}{4\pi b}\cdots\right.}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\cdots}{\cdots}}{4\pi b}\cdots\right.}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 3114$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\cdots\right)}{\pi b\cdots}}{4\pi b}\cdots\right.}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

3.93. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$\Big\downarrow \text{6994}$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2\right.}{\pi b^2}}{4\pi b}\right)}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^5\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{5\left(-\frac{x^3\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{3\left(-\frac{\text{FresnelS}(bx)^2}{2\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}\right)}{\pi b^2}$$

input
```
Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^6/6 + (-((x^4*Sin[b^
2*Pi*x^2])/(b^2*Pi)) + (2*(-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*
x^2]/(b^4*Pi^2)))/(b^2*Pi))/2)/(2*b*Pi) - (5*(-((x^3*Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(b^2*Pi)) + (-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*
x^2]/(b^4*Pi^2))/(4*b*Pi) + (3*(-1/2*FresnelS[b*x]^2/(b^3*Pi) + (x*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^2/2 - Sin[b^2*Pi*x^2]/(2*b^2*Pi)
)/(2*b*Pi)))/(b^2*Pi)))/(b^2*Pi)
```

3.93. $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

3.93.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.93.4 Maple [F]

$$\int x^6 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \operatorname{FresnelS}(bx)\, dx$$

input
```
int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\, dx =$$
$$-\frac{\pi^3 b^6 x^6 - 60\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) - 42\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 24\,\pi b^2 x^2 - 90\,\pi\, \mathrm{S}\left(bx\right)^2 - 6\left(\left(\pi^2 b^4 x\right.\right.}{12\,\pi^4 b^7}$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/12*(pi^3*b^6*x^6 - 60*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)
 - 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 24*pi*b^2*x^2 - 90*pi*fresnel_sin
(b*x)^2 - 6*((pi^2*b^4*x^4 - 22)*cos(1/2*pi*b^2*x^2) + 2*(pi^3*b^5*x^5 - 1
5*pi*b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \begin{cases}-\frac{x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b}-\frac{x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{12\pi b}+\frac{x^5\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2}+\frac{x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3}+\frac{5x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4}+\frac{2x^2\sin^2\left(\frac{\pi}{}\right.}{\pi^3b^5} \\ 0\end{cases}$$

input
```
integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/
2)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*
sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**
2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**
3*b**5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2
*x**2/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x
**2/2)/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, T
rue))
```

3.93.7 Maxima [F]

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.93.8 Giac [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^6 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^6\, \text{FresnelS}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 166

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos(b^2\pi x^2)}{8b^5\pi^3} - \frac{43\,\text{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{8\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^3\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
4*x/b^5/Pi^3-1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+4*x^2*cos(1/2*b
^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2-8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/
Pi^3+x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x^3*sin(b^2*Pi*x^2)/
b^3/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.76

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \frac{-215\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 80\,\text{FresnelS}(bx)\left(4b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) + \left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx(}{80b^6\pi^3}$$

input
```
Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(
b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4
*b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b
^6*Pi^3)
```

3.94.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.70, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7016, 3872, 15, 3867, 3866, 3833, 7008, 3866, 3833, 7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{7016}$$

$$-\frac{4\int x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3872}$$

$$-\frac{4\int x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^4dx}{2}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{4\int x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^5}{10}-\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{4\int x^3 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\big\downarrow$ 3833

$$-\frac{4\int x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$\big\downarrow$ 7008

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$\big\downarrow$ 3866

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$\big\downarrow$ 3833

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$\big\downarrow$ 7014

3.94. $\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$$\Big\downarrow \; 3838$$

$$-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

input
```
Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

3.94. $\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

output
```
(x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (4*((-1/2*(x*Cos[b^2*Pi
*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(2*b*Pi) - (x^
2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) + (2*(-((x/2 - FresnelC[Sqrt
[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)) + (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2
*Pi)))/(b^2*Pi)))/(b^2*Pi) - (x^5/10 + ((3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*
Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) - (x^3*Sin[b^2
*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)
```

3.94.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.94.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.28

method	result
default	$\frac{\operatorname{FresnelS}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{\frac{1}{5}b^5x^5\pi^2-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\operatorname{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}-\cdots}{b}$

input
```
int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*b^5*x
^5*Pi^2-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC
(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*
x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC
(b*x*2^(1/2)))))/b
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\,dx = -\frac{8\,\pi^2 b^6 x^5 - 320\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) - 220\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 210\,b^2 x + 215\,\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{80\,\pi^3 b^7}$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/80*(8*pi^2*b^6*x^5 - 320*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x
) - 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 210*b^2*x + 215*sqrt(2)*sqrt(b^2)*fr
esnel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 2*(p
i^2*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

3.94.6 Sympy [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\,dx = \int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

input
```
integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.94.7 Maxima [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)\,dx = \int x^5 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.94.8 Giac [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^5 \, \text{FresnelS}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.95 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 195

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{x^4}{8b\pi} + \frac{\cos(b^2\pi x^2)}{b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} + \frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x^2\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
-1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3+3*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*
x)/b^4/Pi^2-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2+3/8*I*x^2*hypergeom([
1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2-3/8*I*x^2*hypergeom([1, 1],[3/2
, 2],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/
Pi+1/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

3.95.2 Mathematica [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$$

input
```
Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

3.95.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {7016, 3860, 3042, 3790, 15, 3042, 3777, 25, 3042, 3118, 7008, 3860, 3042, 3118, 7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \ 7016$$

$$-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3860$$

$$-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3790$$

$$-\frac{3\int x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 15$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^4}{4}-\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{\int-\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{3\int x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \text{ 7008}$$

$$-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \text{ 3860}$$

3.95. $\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

$$-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}+\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \; 3118$$

$$-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)dx}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \; 7000$$

$$-\frac{3\left(\frac{-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}-\frac{x\,\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(-\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

input
```
Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(-3*(-1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(b^2*Pi) + ((FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi)))/(b^2*Pi) + (x^3*Fres
nelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (x^4/4 + (-(Cos[b^2*Pi*x^2]/(b^4
*Pi^2)) - (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi))/2)/(2*b*Pi)
```

3.95. $\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

3.95.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

rule 7008
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Simp[1/(2*b*Pi)   Int[x^(m - 1
)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IG
tQ[m, 1]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.95.4 Maple [F]

$$\int x^4 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{FresnelS}\left(bx\right) dx$$

input
```
int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.95.5 Fricas [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\, dx = \int x^4 \cos\left(\frac{1}{2}\pi b^2x^2\right) \mathrm{S}\left(bx\right)\, dx$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.95.6 Sympy [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

input
```
integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.95.7 Maxima [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.95.8 Giac [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelS}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{S}\left(bx\right)\,dx$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int x^4\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx$

3.96.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = -\frac{x^3}{6b\pi} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{b^4\pi^2} - \frac{5\,\text{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

output
```
-1/6*x^3/b/Pi+2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+x^2*FresnelS(b*
x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(
b*x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \frac{-4b^3\pi x^3 - 15\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 24\,\text{FresnelS}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 6bx\sin\left(b^2\pi\right.}{24b^4\pi^2}$$

input
```
Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(-4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 24*FresnelS[b*x]*(2*Co
s[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2
])/(24*b^4*Pi^2)
```

3.96.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {7016, 3872, 15, 3867, 3832, 7006, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{FresnelS}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7016}$$

$$-\frac{2\int x \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3872}$$

$$-\frac{2\int x \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int x^2dx}{2}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\int x \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{x^3}{6}-\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx}{\pi b}+\frac{x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{2\int x \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\int \sin(b^2\pi x^2)dx}{2\pi b^2}-\frac{x\sin(\pi b^2x^2)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}+\frac{x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2\int x \text{FresnelS}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2 \text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin(\pi b^2x^2)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}$$

$$\Big\downarrow \text{7006}$$

$$-\frac{2\left(\frac{\int \sin(b^2\pi x^2)dx}{2\pi b}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin(\pi b^2x^2)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{x^2\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{1}{2}\left(\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\sin(\pi b^2x^2)}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}$$

input
```
Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(-2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*
x]/(2*Sqrt[2]*b^2*Pi)))/(b^2*Pi) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/(b^2*Pi) - (x^3/6 + (FresnelS[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi) - (x*Sin[b^
2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)
```

3.96.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3872
```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Simp[1/2
Int[x^m, x], x] - Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7006
```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Simp[1/(2*b*Pi)   Int[Sin[2*d*x^2], x], x
] /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.96.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\operatorname{FresnelS}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\operatorname{FresnelS}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\operatorname{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	119

input
```
int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6/Pi*b^3*x^3-1/2/Pi
*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx =$$

$$-\frac{4\,\pi b^4x^3 - 48\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right) \mathrm{S}\left(bx\right) + 15\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right) - 12\left(2\,\pi b^3x^2\,\mathrm{S}\left(bx\right) + b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right) \mathrm{s}}{24\,\pi^2b^5}$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/24*(4*pi*b^4*x^3 - 48*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 15*sqrt(
2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(2*pi*b^3*x^2*fresnel_s
in(b*x) + b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

3.96.6 Sympy [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^3 \cos\left(\frac{\pi b^2x^2}{2}\right) S(bx)\, dx$$

input
```
integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.96.7 Maxima [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^3 \cos\left(\frac{1}{2}\,\pi b^2x^2\right) \mathrm{S}\left(bx\right)\, dx$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.96.8 Giac [F]

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^3 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x^3\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.97 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.97.1 Optimal result

Integrand size = 20, antiderivative size = 73

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{x^2}{4b\pi} - \frac{\text{FresnelS}(bx)^2}{2b^3\pi} + \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
-1/4*x^2/b/Pi-1/2*FresnelS(b*x)^2/b^3/Pi+x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^
2)/b^2/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

3.97.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = -\frac{x^2}{4b\pi} - \frac{\text{FresnelS}(bx)^2}{2b^3\pi} + \frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

input
```
Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
-1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*P
i*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

3.97.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7016, 3860, 3042, 3114, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{ 7016}$$

$$-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3860}$$

$$-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)^2dx^2}{2\pi b}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3114}$$

$$-\frac{\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{ 6994}$$

$$-\frac{\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)}{\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\text{FresnelS}(bx)^2}{2\pi b^3}+\frac{x\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x^2}{2}-\frac{\sin(\pi b^2x^2)}{2\pi b^2}}{2\pi b}$$

input
```
Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
-1/2*FresnelS[b*x]^2/(b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2
*Pi) - (x^2/2 - Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)
```

3.97.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7016
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Simp[1/(Pi*b)   Int[x^(m -
1)*Sin[d*x^2]^2, x], x] - Simp[(m - 1)/(2*d)   Int[x^(m - 2)*Sin[d*x^2]*Fre
snelS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m
, 1]
```

3.97.4 Maple [F]

$$\int x^2 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{FresnelS}\,(bx)\, dx$$

input
```
int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$
$$= -\frac{\pi b^2x^2 + 2\,\pi\,\mathrm{S}\,(bx)^2 - 2\left(2\,\pi bx\,\mathrm{S}\,(bx) + \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/4*(pi*b^2*x^2 + 2*pi*fresnel_sin(b*x)^2 - 2*(2*pi*b*x*fresnel_sin(b*x)
+ cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$
$$= \begin{cases} -\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{S^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2
)**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**
2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3
), Ne(b, 0)), (0, True))
```

3.97.7 Maxima [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.97.8 Giac [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{S}\left(bx\right)\, dx$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\, dx = \int x^2\, \text{FresnelS}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx$

3.98.1 Optimal result

Integrand size = 18, antiderivative size = 59

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=-\frac{x}{2b\pi}+\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}+\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

output
```
-1/2*x/b/Pi+FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*FresnelC(b*x*2^(1
/2))/b^2/Pi*2^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\frac{-2bx+\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)+4\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

input
```
Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)
/2])/(4*b^2*Pi)
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7014, 3838, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \ 7014$$

$$\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}$$

$$\Big\downarrow \ 3838$$

$$\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)dx}{\pi b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{x}{2}-\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}}{\pi b}$$

input
```
Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
-((x/2 - FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)) + (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3838
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 7014
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Simp[1/(Pi*b)   Int[Sin[d*x^2]^2, x], x] /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.98.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\frac{\operatorname{FresnelS}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}-\frac{\frac{bx}{2}-\frac{\sqrt{2}\,\operatorname{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

input
```
int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelS(b*x)/b/Pi*sin(1/2*b^2*Pi*x^2)-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*Fresne
lC(b*x*2^(1/2))))/b
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)\,dx$$
$$= -\frac{2\,b^2x-4\,b\,\mathrm{S}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x - 4*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^
2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

3.98.6 Sympy [F]

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\,dx$$

input
```
integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.98.7 Maxima [F]

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.98.8 Giac [F]

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{S}\left(bx\right)\,dx$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx=\int x\,\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$

3.99.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/2*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1
/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

3.99.2 Mathematica [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

3.99.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {7000}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \text{7000}$$

$$-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

output
```
(FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (I/2)*b^2*Pi*x^2]
```

3.99.3.1 Defintions of rubi rules used

rule 7000
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

3.99.4 Maple [F]

$$\int \cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelS}\left(bx\right)dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

3.99.5 Fricas [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.99.6 Sympy [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

3.99.7 Maxima [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)\,dx = \int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.99.8 Giac [F]

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int \cos \left(\frac{1}{2}\pi b^2x^2\right) \text{S}\left(bx\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)\,dx = \int \text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.100 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx$

3.100.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx = \operatorname{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}, x\right)$$

output
```
Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.100.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelS}(bx)}{x}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

3.100.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$
$$\Big\downarrow \ 7020$$
$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
```

output
```
$Aborted
```

3.100.3.1 Defintions of rubi rules used

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.100.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.100.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

3.100.6 Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

3.100.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

3.100.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

3.100.9 Mupad [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.101 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx$

3.101.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} - \frac{1}{2}b\pi\,\text{FresnelS}(bx)^2 + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

output
```
-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x-1/2*b*Pi*FresnelS(b*x)^2+1/4*b*Si(b^2
*Pi*x^2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x} - \frac{1}{2}b\pi\,\text{FresnelS}(bx)^2 + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

output
```
-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*S
inIntegral[b^2*Pi*x^2])/4
```

3.101.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7018, 3856, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \ 7018$$

$$-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}$$

$$\Big\downarrow \ 3856$$

$$-\pi b^2\int \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

$$\Big\downarrow \ 6994$$

$$-\pi b\int \text{FresnelS}(bx)d\,\text{FresnelS}(bx)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

$$\Big\downarrow \ 15$$

$$-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi b\,\text{FresnelS}(bx)^2$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

output
```
-((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*S
inIntegral[b^2*Pi*x^2])/4
```

3.101.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.101.4 Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}\left(bx\right)}{x^2}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = -\frac{2\,\pi bx\,\text{S}\left(bx\right)^2 - bx\,\text{Si}\left(\pi b^2x^2\right) + 4\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{4\,x}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*pi*b*x*fresnel_sin(b*x)^2 - b*x*sin_integral(pi*b^2*x^2) + 4*cos(1
/2*pi*b^2*x^2)*fresnel_sin(b*x))/x
```

3.101.6 Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

3.101.7 Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}(bx)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)
```

3.101.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}(bx)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

3.102 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx$

3.102.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{2x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{4x} - \frac{1}{2}b^2\pi\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
-1/2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2
*Pi*FresnelC(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelS(b*x)*si
n(1/2*b^2*Pi*x^2)/x,x)
```

3.102.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]
```

3.102.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}dx$$

$$\Big\downarrow \ 7018$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \ 3868$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \ 3833$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \ 7012$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

output
```
$Aborted
```

3.102.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.102.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelS}(bx)}{x^3}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

3.102.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right) \text{S}\left(bx\right)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

3.102.6 Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

3.102.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right) \text{S}\left(bx\right)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

3.102.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

3.102.9 Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

3.103 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{3x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{12x^2} - \frac{1}{3}b^2\pi\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

output
```
1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/12*
b*sin(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^2,x)
```

3.103.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]
```

3.103.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3860, 3042, 3778, 3042, 3783, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}dx$$

$$\Big\downarrow \text{7018}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

3.103. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx$

$$\Big\downarrow \text{7012}$$

$$-\frac{1}{3}\pi b^2 \int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx + \frac{1}{12}b\left(\pi b^2 \,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

output
```
$Aborted
```

3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.103. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx$

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.103.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos \left(\frac{b^2 \pi x^2}{2}\right) \operatorname{FresnelS}(bx)}{x^4} dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

3.103.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \operatorname{FresnelS}(bx)}{x^4} dx = \int \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right) S(bx)}{x^4} dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

3.103.6 Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

3.103.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}(bx)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

3.103.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}(bx)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

3.103.9 Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.104 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^5} \, dx$

3.104.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^5} \, dx = {} & \frac{b^3\pi}{16x} - \frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{4x^4} \\ & - \frac{7b^4\pi^2 \text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}} \\ & + \frac{b^2\pi \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{24x^3} \\ & - \frac{1}{8}b^4\pi^2 \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x}, x\right)\end{aligned}$$

output
```
1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/4*cos(1/2*b^2*Pi*x^2)*Fresne
lS(b*x)/x^4+1/8*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/24*b*sin(b^
2*Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Uni
ntegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.104.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]
```

3.104.3 Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3868, 3869, 3832, 7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}dx$$

$$\Big\downarrow \; 7018$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \; 3868$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \; 3869$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{7010}$$

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(bx)\sin}{2x^2}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{Fresnel}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{7020}$$

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{Fresnel}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

output
```
$Aborted
```

3.104. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx$

3.104.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.104.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}\,(bx)}{x^5}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

3.104.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\,(bx)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

3.104.6 Sympy [N/A]

Not integrable

Time = 3.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

3.104.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

3.104.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

3.104.9 Mupad [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.105 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^6} dx$

3.105.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^6} dx &= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{5x^5} \\ &\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{15x} \\ &\quad + \frac{1}{30}b^5\pi^3 \text{FresnelS}(bx)^2 + \frac{b^2\pi \text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} \\ &\quad - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

output
```
1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/5*cos(1/2*b^2*Pi*x^2)*Fr
esnelS(b*x)/x^5+1/15*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/30*b^5
*Pi^3*FresnelS(b*x)^2-7/120*b^5*Pi^2*Si(b^2*Pi*x^2)+1/15*b^2*Pi*FresnelS(b
*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/40*b*sin(b^2*Pi*x^2)/x^4
```

3.105.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{15x} + \frac{1}{30}b^5\pi^3\text{FresnelS}(bx)^2 + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

output
```
(b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/
2]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
15*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral
[b^2*Pi*x^2])/120
```

3.105.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7018, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3780, 7010, 3861, 3042, 3778, 25, 3042, 3780, 7018, 3856, 6994, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}\,dx$$

$$\Big\downarrow \ 7018$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}dx - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \ 3860$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx + \frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2 - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{7010}$$

3.105. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3861}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin}{3x^3}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin}{3x^3}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelS}(bx)\sin}{3x^3}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

3.105. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$

$$\Big\downarrow \text{ 3780}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{1}{12}b\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 7018}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\text{F}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3856}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)\right)-\frac{\text{FresnelS}(}{}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 6994}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b\int\text{FresnelS}(bx)d\,\text{FresnelS}(bx)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{3x}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)-\frac{1}{2}\pi b\,\text{FresnelS}(bx)^2\right)-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

3.105. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$

output
```
-1/5*(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5 - (b^2*Pi*(-1/12*b/x^2 - (Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*x^3) + (b^2*Pi*(-((Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*SinIntegral[b^2*Pi*x^2])
/4))/3 - (b*(-(Cos[b^2*Pi*x^2]/x^2) - b^2*Pi*SinIntegral[b^2*Pi*x^2]))/12)
)/5 + (b*(-1/2*Sin[b^2*Pi*x^2]/x^4 + (b^2*Pi*(-(Cos[b^2*Pi*x^2]/x^2) - b^2
*Pi*SinIntegral[b^2*Pi*x^2]))/2))/20
```

3.105.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.105. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6994
```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.105.4 Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)}{x^6}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx$$
$$= \frac{4\,\pi^3b^5x^5\,\mathrm{S}\left(bx\right)^2 - 7\,\pi^2b^5x^5\,\mathrm{Si}\left(\pi b^2x^2\right) - 10\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 7\,\pi b^3x^3 + 8\left(\pi^2b^4x^4 - 3\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}}{120\,x^5}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

output
```
1/120*(4*pi^3*b^5*x^5*fresnel_sin(b*x)^2 - 7*pi^2*b^5*x^5*sin_integral(pi*
b^2*x^2) - 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 + 7*pi*b^3*x^3 + 8*(pi^2*b^
4*x^4 - 3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2*(4*pi*b^2*x^2*fresnel_
sin(b*x) - 3*b*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/x^5
```

3.105.6 Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

3.105.7 Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

3.105.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)S(bx)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.106 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx$

3.106.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx &= \frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}}\\ &-\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)\\ &-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{6x^6}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{48x^2}\\ &+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4}\\ &-\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{1440x}\\ &+\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

output
```
1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/6*cos(1/2*b^2*Pi*x^2)
*FresnelS(b*x)/x^6+1/48*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2+1/2
4*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/60*b*sin(b^2*Pi*x^2)/x^5+
67/1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*
2^(1/2)+1/48*b^6*Pi^3*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.106.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx=\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]
```

3.106.3 Rubi [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3868, 3869, 3868, 3833, 7010, 3869, 3868, 3833, 7018, 3868, 3833, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}dx$$

$$\Big\downarrow \text{ 7018}$$

$$-\frac{1}{6}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{ 3868}$$

$$-\frac{1}{6}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{ 3869}$$

$$-\frac{1}{6}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$\Big\downarrow$ 3868

$$-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$\Big\downarrow$ 3833

$$-\frac{1}{6}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$\Big\downarrow$ 7010

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}\right)+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$\Big\downarrow$ 3869

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}(bx)\sin}{4x^4}\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$\Big\downarrow$ 3868

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x\right.}{3x^3}\right.\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

3.106. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx$

$$\Big\downarrow \text{3833}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^3}dx-\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2\right.}{3x^3}\right.\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{7018}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{1}{8}\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}(}{}\right.\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}}{}\right.\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{7012}$$

3.106. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}dx$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelS}}{}\right.\right.$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

output
```
$Aborted
```

3.106.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.106.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \operatorname{FresnelS}(bx)}{x^7}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

3.106.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \operatorname{S}(bx)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

3.106.6 Sympy [N/A]

Not integrable

Time = 11.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

3.106.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

3.106.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

3.106.9 Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^7}\,dx=\int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.107 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$

3.107.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{105x^3} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} - \frac{b\sin\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\sin\left(b^2\pi x^2\right)}{84x^2} + \frac{1}{105}b^6\pi^3\,\text{Int}\left(\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

output
```
1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)
/x^4-1/7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/105*b^4*Pi^2*cos(1/2*b^2*
Pi*x^2)*FresnelS(b*x)/x^3+1/35*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^
5-1/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2+1/105*b^6*P
i^3*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.107.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]
```

3.107.3 Rubi [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7010, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7018, 3860, 3042, 3778, 3042, 3783, 7012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

$$\Big\downarrow \text{7018}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

3.107. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx$

$\Big\downarrow$ 3042

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3783

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 7010

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3861

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3042

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

3.107. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx$

$$\Big\downarrow \; 3778$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \; 3778$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \; 3042$$

3.107. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{cos}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3783

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^4}dx-\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos(\pi}{2x}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 7018

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{2}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3860

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3042

3.107. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3778

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelS}\ldots\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3042

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fres}\ldots\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 3783

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresne}\ldots\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\Big\downarrow$ 7012

3.107. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresne}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

output
```
$Aborted
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7012
```
Int[FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelS[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.107.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}\left(bx\right)}{x^8}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

3.107.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

3.107.6 Sympy [N/A]

Not integrable

Time = 21.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

3.107.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

3.107.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

3.107.9 Mupad [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^8}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.108 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^9} \, dx$

3.108.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^9} \, dx &= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} \\ &\quad + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{8x^8} \\ &\quad + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{192x^4} \\ &\quad + \frac{853b^8\pi^4\,\text{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}} \\ &\quad + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} \\ &\quad - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2} \\ &\quad - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3} \\ &\quad + \frac{1}{384}b^8\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}, x\right)\end{aligned}$$

output
```
1/480*b^3*Pi/x^5-1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/8
0640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8+
1/192*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4+1/48*b^2*Pi*FresnelS(
b*x)*sin(1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/x^2-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3
+853/80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegra
ble(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

3.108.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]
```

3.108.3 Rubi [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7018, 3868, 3869, 3868, 3869, 3832, 7010, 3869, 3868, 3869, 3832, 7018, 3868, 3869, 3832, 7010, 3869, 3832, 7020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

$$\Big\downarrow \text{7018}$$

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$\big\downarrow$ 3869

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$\big\downarrow$ 3868

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$\big\downarrow$ 3869

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$\big\downarrow$ 3832

$$-\frac{1}{8}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)$$

$\big\downarrow$ 7010

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right)$$

$\big\downarrow$ 3869

3.108. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelS}(bx)\,\text{si}}{6x^6}\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2\right.}{5x^5}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^5}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{7018}$$

3.108. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right)-\frac{\text{F}}{}\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelS}}{}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{s}}{}\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$$\Big\downarrow \text{7010}$$

3.108. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \ 3869$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \ 3832$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \ 7020$$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

output
```
$Aborted
```

3.108. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

rule 7020
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelS[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelS[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.108. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx$

3.108.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}\left(bx\right)}{x^9}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

3.108.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

3.108.6 Sympy [N/A]

Not integrable

Time = 38.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^9}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

3.108.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

3.108.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^9}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

3.108.9 Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^9}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.109 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx$

3.109.1 Optimal result

Integrand size = 20, antiderivative size = 278

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx &= \frac{b^3\pi}{756x^6}-\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}\\ &\quad+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9}\\ &\quad+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5}\\ &\quad-\frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x}\\ &\quad-\frac{b^9\pi^5\text{FresnelS}(bx)^2}{1890}+\frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}\\ &\quad-\frac{b^6\pi^3\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}\\ &\quad+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4}+\frac{83b^9\pi^4\text{Si}\left(b^2\pi x^2\right)}{30240}\end{aligned}$$

output
```
1/756*b^3*Pi/x^6-1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/
2016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/9*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^
9+1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5-1/945*b^8*Pi^4*cos(
1/2*b^2*Pi*x^2)*FresnelS(b*x)/x-1/1890*b^9*Pi^5*FresnelS(b*x)^2+83/30240*b
^9*Pi^4*Si(b^2*Pi*x^2)+1/63*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1
/945*b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/144*b*sin(b^2*Pi*x^2
)/x^8+67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

3.109.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{945x} - \frac{b^9\pi^5\,\text{FresnelS}(bx)^2}{1890} + \frac{b^2\pi\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3\,\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4} + \frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

output
```
(b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]
*FresnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(31
5*x^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*
FresnelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7
) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi
*x^2])/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^
4*SinIntegral[b^2*Pi*x^2])/30240
```

3.109.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

$$\Big\downarrow \text{ 7018}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}dx - \frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

3.109. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2\right.}{4x^8}\right.-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\downarrow \text{25}$$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2\right.}{4x^8}\right.-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\downarrow \text{3042}$$

3.109. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\downarrow$ 3780

$$-\frac{1}{9}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 7010

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{14}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3861

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3778

3.109. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{7x}\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 25

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{7x}\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelS}(bx)\,\text{s}}{7x}\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3778

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b\right.}{3x}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

3.109. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{co}\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \text{ 3780}$$

3.109. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^6}dx-\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2\right.\right.\right.\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{\sin(\pi b^2x^2)}{4x^8}\right)$$

$\downarrow$ 7018

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int\frac{\sin(b^2\pi x^2)}{x^5}dx-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{\sin(\pi b^2x^2)}{4x^8}\right)$$

$\downarrow$ 3860

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin(b^2\pi x^2)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{\sin(\pi b^2x^2)}{4x^8}\right)$$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin(b^2\pi x^2)}{x^6}dx^2-\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\right.$$
$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{\sin(\pi b^2x^2)}{4x^8}\right)$$

$\downarrow$ 3778

3.109. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelS}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{Fresne}\right.\right.$$

$$\frac{\text{FresnelS}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

output
```
$Aborted
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7010
```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Simp[d*(x^(m + 2)/(Pi*b*(
m + 1)*(m + 2))), x] - Simp[2*(d/(m + 1))   Int[x^(m + 2)*Cos[d*x^2]*Fresne
lS[b*x], x], x] + Simp[d/(Pi*b*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x
]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7018
```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Simp[2*(d/(m + 1))   Int[x
^(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Simp[d/(Pi*b*(m + 1))   Int[x^(
m + 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
&& ILtQ[m, -1]
```

3.109.4 Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}\left(bx\right)}{x^{10}}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelS}(bx)}{x^{10}}\, dx =$$
$$-\frac{16\,\pi^5 b^9 x^9\, \mathrm{S}\left(bx\right)^2 - 83\,\pi^4 b^9 x^9\, \mathrm{Si}\left(\pi b^2 x^2\right) + 83\,\pi^3 b^7 x^7 - 150\,\pi b^3 x^3 - 10\left(15\,\pi^3 b^7 x^7 - 22\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2\right.}{}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

output
```
-1/30240*(16*pi^5*b^9*x^9*fresnel_sin(b*x)^2 - 83*pi^4*b^9*x^9*sin_integra
l(pi*b^2*x^2) + 83*pi^3*b^7*x^7 - 150*pi*b^3*x^3 - 10*(15*pi^3*b^7*x^7 - 2
2*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 +
105)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 2*((67*pi^2*b^5*x^5 - 210*b*x)
*cos(1/2*pi*b^2*x^2) - 16*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*fresnel_sin(b*x))
*sin(1/2*pi*b^2*x^2))/x^9
```

3.109.6 Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

3.109.7 Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{S}\left(bx\right)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

3.109.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{S}\left(bx\right)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelS}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

input
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
```

output
```
int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.110 $\int x^7 \text{FresnelC}(bx) \, dx$

3.110.1 Optimal result

Integrand size = 8, antiderivative size = 124

$$\int x^7 \text{FresnelC}(bx) \, dx = \frac{105x \cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105 \,\text{FresnelC}(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 \text{FresnelC}(bx) + \frac{35x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}$$

output
```
105/8*x*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-7/8*x^5*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2-
105/8*FresnelC(b*x)/b^8/Pi^4+1/8*x^8*FresnelC(b*x)+35/8*x^3*sin(1/2*b^2*Pi
*x^2)/b^5/Pi^3-1/8*x^7*sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.110.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int x^7 \text{FresnelC}(bx) \, dx = \frac{-7bx(-15 + b^4\pi^2x^4) \cos\left(\frac{1}{2}b^2\pi x^2\right) + (-105 + b^8\pi^4x^8) \,\text{FresnelC}(bx) + b^3\pi x^3(35 - b^4\pi^2x^4) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

input
```
Integrate[x^7*FresnelC[b*x],x]
```

output
```
(-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*F
resnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*
Pi^4)
```

3.110.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6981, 3867, 3866, 3867, 3866, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \operatorname{FresnelC}(bx)\,dx$$

$$\Big\downarrow \text{ 6981}$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)-\frac{1}{8}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{ 3867}$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)-\frac{1}{8}b\left(\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{7\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)-\frac{1}{8}b\left(\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3867}$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)-\frac{1}{8}b\left(\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{7\left(\frac{5\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)-\frac{1}{8}b\left(\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{7\left(\frac{5\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)-\frac{1}{8}b\left(\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{7\left(\frac{5\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{\text{FresnelC}(bx)}{\pi b^3}-\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

input
```
Int[x^7*FresnelC[b*x],x]
```

output
```
(x^8*FresnelC[b*x])/8 - (b*((x^7*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (7*(-((x^
5*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + (5*((-3*(-((x*Cos[(b^2*Pi*x^2)/2])/(b^2
*Pi)) + FresnelC[b*x]/(b^3*Pi)))/(b^2*Pi) + (x^3*Sin[(b^2*Pi*x^2)/2])/(b^2
*Pi)))/(b^2*Pi)))/(b^2*Pi)))/8
```

3.110. $\int x^7\,\text{FresnelC}(bx)\,dx$

3.110.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.21

method	result
meijerg	$\frac{b\,x^9\,\mathrm{hypergeom}\left([\frac{1}{4},\frac{9}{4}],[\frac{1}{2},\frac{5}{4},\frac{13}{4}],-\frac{x^4\pi^2b^4}{16}\right)}{9}$
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^8x^8}{8}-\frac{b^7x^7\sin\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}+\frac{-\frac{7b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}+\frac{7\left(\frac{5b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{15\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^8}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^8x^8}{8}-\frac{b^7x^7\sin\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}+\frac{-\frac{7b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}+\frac{7\left(\frac{5b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{15\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^8}$
parts	$\frac{x^8\,\mathrm{FresnelC}(bx)}{8}-\frac{b\left(\frac{x^7\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{7\left(-\frac{x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{5x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{15\left(-\frac{x\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{b^2\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b^2\pi}}{b^2\pi}\right)}{b^2\pi}\right)}{8}$

input
```
int(x^7*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*b*x^9*hypergeom([1/4,9/4],[1/2,5/4,13/4],-1/16*x^4*Pi^2*b^4)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int x^7\,\mathrm{FresnelC}(bx)\,dx = -\frac{7\left(\pi^2b^5x^5-15\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^4b^8x^8-105\right)\mathrm{C}\left(bx\right)+\left(\pi^3b^7x^7-35\,\pi b^3x^3\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{8\,\pi^4b^8}$$

input
```
integrate(x^7*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
-1/8*(7*(pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) - (pi^4*b^8*x^8 - 105)
*fresnel_cos(b*x) + (pi^3*b^7*x^7 - 35*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/(p
i^4*b^8)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int x^7 \,\text{FresnelC}(bx)\,dx = \frac{45x^8C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)} - \frac{45x^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi b\Gamma\left(\frac{13}{4}\right)} - \frac{315x^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{13}{4}\right)} + \frac{1575x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{13}{4}\right)} + \frac{4725x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{13}{4}\right)} - \frac{4725C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**7*fresnelc(b*x),x)
```

output
```
45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x
**2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*ga
mma(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamm
a(1/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4
)/(512*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*
b**8*gamma(13/4))
```

3.110.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int x^7 \,\text{FresnelC}(bx)\,dx = \frac{1}{8}x^8\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(-(105i-105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}bx\right) + (105i+105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}bx\right) + 28\left(\sqrt{\frac{1}{2}}\pi^3b^5x^5 - \right.\right.}{16\,\pi^5b^8}$$

input
```
integrate(x^7*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/8*x^8*fresnel_cos(b*x) - 1/16*sqrt(1/2)*(-(105*I - 105)*(1/4)^(1/4)*pi*e
rf(sqrt(1/2*I*pi)*b*x) + (105*I + 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*
b*x) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^
2) + 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*sin(1/2*pi*b^2
*x^2))/(pi^5*b^8)
```

3.110.8 Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \int x^7 \operatorname{C}(bx)\,dx$$

input
```
integrate(x^7*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_cos(b*x), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx)\,dx = \int x^7 \operatorname{FresnelC}(b\,x)\,dx$$

input
```
int(x^7*FresnelC(b*x),x)
```

output
```
int(x^7*FresnelC(b*x), x)
```

3.111 $\int x^6 \, \text{FresnelC}(bx) \, dx$

3.111.1 Optimal result

Integrand size = 8, antiderivative size = 109

$$\int x^6 \, \text{FresnelC}(bx) \, dx = \frac{48 \cos \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b^7 \pi^4} - \frac{6 x^4 \cos \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b^3 \pi^2} + \frac{1}{7} x^7 \, \text{FresnelC}(bx) + \frac{24 x^2 \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b^5 \pi^3} - \frac{x^6 \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b \pi}$$

output
```
48/7*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/7
*x^7*FresnelC(b*x)+24/7*x^2*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/7*x^6*sin(1/2*b
^2*Pi*x^2)/b/Pi
```

3.111.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int x^6 \, \text{FresnelC}(bx) \, dx = -\frac{6(-8 + b^4 \pi^2 x^4) \cos \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b^7 \pi^4} + \frac{1}{7} x^7 \, \text{FresnelC}(bx) - \frac{x^2(-24 + b^4 \pi^2 x^4) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{7 b^5 \pi^3}$$

input
```
Integrate[x^6*FresnelC[b*x],x]
```

output
```
(-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[
b*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

3.111.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6981, 3861, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{FresnelC}(bx) \, dx$$

$$\Big\downarrow \text{ 6981}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{7}b \int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b \int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b \int x^6 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right) dx^2$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b\left(\frac{6\int -x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2} + \frac{2x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b\left(\frac{2x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{6\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b\left(\frac{2x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{6\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{7}x^7 \text{FresnelC}(bx) - \frac{1}{14}b\left(\frac{2x^6 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{6\left(\frac{4\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2} - \frac{2x^4 \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)-\frac{1}{14}b\left(\frac{2x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{6\left(\frac{4\int x^2\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)-\frac{1}{14}b\left(\frac{2x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{6\left(\frac{4\left(\frac{2\int-\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$\downarrow$ 25

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)-\frac{1}{14}b\left(\frac{2x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{6\left(\frac{4\left(\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$\downarrow$ 3042

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)-\frac{1}{14}b\left(\frac{2x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{6\left(\frac{4\left(\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$\downarrow$ 3118

3.111. $\int x^6\,\text{FresnelC}(bx)\,dx$

$$\frac{1}{7}x^7\,\mathrm{FresnelC}(bx) - \frac{1}{14}b\left(\frac{2x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{6\left(\frac{4\left(\frac{2x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2} - \frac{2x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

input
```
Int[x^6*FresnelC[b*x],x]
```

output
```
(x^7*FresnelC[b*x])/7 - (b*((2*x^6*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (6*((-2
*x^4*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi) + (4*((4*Cos[(b^2*Pi*x^2)/2])/(b^4*Pi^2
) + (2*x^2*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/(b^2*Pi)))/14
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{b\,x^8 \operatorname{hypergeom}\left(\left[\frac{1}{4},2\right],\left[\frac{1}{2},\frac{5}{4},3\right],-\frac{x^4\pi^2b^4}{16}\right)}{8}$	26
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107
default	$\frac{\frac{\operatorname{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107
parts	$\frac{x^7\operatorname{FresnelC}(bx)}{7}-\frac{b\left(\frac{x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{6\left(-\frac{x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\frac{4x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2}}{b^2\pi}\right)}{b^2\pi}\right)}{7}$	112

input
```
int(x^6*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*b*x^8*hypergeom([1/4,2],[1/2,5/4,3],-1/16*x^4*Pi^2*b^4)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{\pi^4 b^7 x^7 \operatorname{C}(bx) - 6\,(\pi^2 b^4 x^4 - 8)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^3 b^6 x^6 - 24\,\pi b^2 x^2)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{7\,\pi^4 b^7}$$

input
```
integrate(x^6*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/7*(pi^4*b^7*x^7*fresnel_cos(b*x) - 6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x
^2) - (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{x^7 C(bx)\,\Gamma\left(\frac{1}{4}\right)}{28\Gamma\left(\frac{5}{4}\right)} - \frac{x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{28\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{3x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{14\pi^2 b^3\Gamma\left(\frac{5}{4}\right)} + \frac{6x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^3 b^5\Gamma\left(\frac{5}{4}\right)} + \frac{12\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^4 b^7\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(x**6*fresnelc(b*x),x)
```

output
```
x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*g
amma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14
*pi**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b
**5*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/
4))
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \frac{1}{7}x^7 \operatorname{C}(bx) - \frac{6(\pi^2b^4x^4 - 8)\cos\left(\frac{1}{2}\pi b^2x^2\right) + (\pi^3b^6x^6 - 24\pi b^2x^2)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}$$

input
```
integrate(x^6*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/7*x^7*fresnel_cos(b*x) - 1/7*(6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2) +
 (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.111.8 Giac [F]

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \int x^6 \operatorname{C}(bx)\,dx$$

input
```
integrate(x^6*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_cos(b*x), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelC}(bx)\,dx = \int x^6 \operatorname{FresnelC}(b\,x)\,dx$$

input
```
int(x^6*FresnelC(b*x),x)
```

output
```
int(x^6*FresnelC(b*x), x)
```

3.112 $\int x^5 \operatorname{FresnelC}(bx)\,dx$

3.112.1 Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 \operatorname{FresnelC}(bx) - \frac{5\operatorname{FresnelS}(bx)}{2b^6\pi^3} + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}$$

output
```
-5/6*x^3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/6*x^6*FresnelC(b*x)-5/2*FresnelS(b
*x)/b^6/Pi^3+5/2*x*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/6*x^5*sin(1/2*b^2*Pi*x^2
)/b/Pi
```

3.112.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{-5b^3\pi x^3\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^6\pi^3x^6 \operatorname{FresnelC}(bx) - 15\operatorname{FresnelS}(bx) + bx(15 - b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

input
```
Integrate[x^5*FresnelC[b*x],x]
```

output
```
(-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*Fresn
elS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

3.112.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6981, 3867, 3866, 3867, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{FresnelC}(bx)\,dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{6}x^6 \text{FresnelC}(bx)-\frac{1}{6}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{3867}$$

$$\frac{1}{6}x^6 \text{FresnelC}(bx)-\frac{1}{6}b\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{6}x^6 \text{FresnelC}(bx)-\frac{1}{6}b\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3867}$$

$$\frac{1}{6}x^6 \text{FresnelC}(bx)-\frac{1}{6}b\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{6}x^6 \text{FresnelC}(bx)-\frac{1}{6}b\left(\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\text{FresnelS}(bx)}{\pi b^3}\right)}{\pi b^2}-\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

input
```
Int[x^5*FresnelC[b*x],x]
```

output
```
(x^6*FresnelC[b*x])/6 - (b*((x^5*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (5*(-((x^
3*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + (3*(-(FresnelS[b*x]/(b^3*Pi)) + (x*Sin[
(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi)))/(b^2*Pi)))/6
```

3.112.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

method	result	size
meijerg	$\frac{b\,x^{7}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{7}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{11}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{7}$	26
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^{6}x^{6}}{6}-\frac{b^{5}x^{5}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6\pi}+\frac{-\frac{5b^{3}x^{3}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}-\frac{3\,\mathrm{FresnelS}(bx)}{\pi}\right)}{6\pi}}{\pi}}{b^{6}}$	97
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^{6}x^{6}}{6}-\frac{b^{5}x^{5}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6\pi}+\frac{-\frac{5b^{3}x^{3}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}-\frac{3\,\mathrm{FresnelS}(bx)}{\pi}\right)}{6\pi}}{\pi}}{b^{6}}$	97
parts	$\frac{x^{6}\,\mathrm{FresnelC}(bx)}{6}-\frac{b\left(\frac{x^{5}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}-\frac{5\left(-\frac{x^{3}\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{\frac{3x\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}-\frac{3\,\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}}{b^{2}\pi}\right)}{b^{2}\pi}\right)}{6}$	123

input
```
int(x^5*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*b*x^7*hypergeom([1/4,7/4],[1/2,5/4,11/4],-1/16*x^4*Pi^2*b^4)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int x^5\,\mathrm{FresnelC}(bx)\,dx$$
$$=\frac{\pi^3b^7x^6\,\mathrm{C}\left(bx\right)-5\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^6x^5-15\,b^2x\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-15\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{b^2}x\right)}{6\,\pi^3b^7}$$

input
```
integrate(x^5*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/6*(pi^3*b^7*x^6*fresnel_cos(b*x) - 5*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - (p
i^2*b^6*x^5 - 15*b^2*x)*sin(1/2*pi*b^2*x^2) - 15*sqrt(b^2)*fresnel_sin(sqr
t(b^2)*x))/(pi^3*b^7)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{bx^7\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)\,{}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**5*fresnelc(b*x),x)
```

output
```
b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b*
*4*x**4/16)/(16*gamma(5/4)*gamma(11/4))
```

3.112.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \frac{1}{6}x^6\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(20\sqrt{\frac{1}{2}}\pi^2b^3x^3\cos\left(\frac{1}{2}\pi b^2x^2\right) + (15i+15)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi}bx\right) - (15i-15)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi}b\right.\right.}{12\pi^4b^6}$$

input
```
integrate(x^5*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/6*x^6*fresnel_cos(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*cos(1
/2*pi*b^2*x^2) + (15*I + 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*
I - 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) + 4*(sqrt(1/2)*pi^3*b^5*x^
5 - 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

3.112.8 Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \int x^5 \operatorname{C}(bx)\,dx$$

input
```
integrate(x^5*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_cos(b*x), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx)\,dx = \int x^5 \operatorname{FresnelC}(b\,x)\,dx$$

input
```
int(x^5*FresnelC(b*x),x)
```

output
```
int(x^5*FresnelC(b*x), x)
```

3.113 $\int x^4 \operatorname{FresnelC}(bx)\,dx$

3.113.1 Optimal result

Integrand size = 8, antiderivative size = 84

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\operatorname{FresnelC}(bx) + \frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}$$

output
```
-4/5*x^2*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)+8/5*sin(1/2*b^
2*Pi*x^2)/b^5/Pi^3-1/5*x^4*sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.113.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\operatorname{FresnelC}(bx) - \frac{(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}$$

input
```
Integrate[x^4*FresnelC[b*x],x]
```

output
```
(-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 +
 b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

3.113.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6981, 3861, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{FresnelC}(bx)\,dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{5}b\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{4\int -x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{4\left(\frac{2\int\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3117$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx) - \frac{1}{10}b\left(\frac{2x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{4\left(\frac{4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} - \frac{2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

input
```
Int[x^4*FresnelC[b*x],x]
```

output
```
(x^5*FresnelC[b*x])/5 - (b*((2*x^4*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (4*((-2
*x^2*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi) + (4*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)))/
(b^2*Pi)))/10
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{b\,x^{6} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{2}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{6}$	26
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx) b^{5} x^{5}}{5}-\frac{b^{4} x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi}+\frac{-\frac{4 b^{2} x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi}+\frac{8 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi^{2}}}{\pi}}{b^{5}}$	81
default	$\frac{\frac{\operatorname{FresnelC}(bx) b^{5} x^{5}}{5}-\frac{b^{4} x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi}+\frac{-\frac{4 b^{2} x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi}+\frac{8 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{5 \pi^{2}}}{\pi}}{b^{5}}$	81
parts	$\frac{x^{5} \operatorname{FresnelC}(bx)}{5}-\frac{b\left(\frac{x^{4} \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}-\frac{4\left(-\frac{x^{2} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{2} \pi}+\frac{2 \sin\left(\frac{b^{2} \pi x^{2}}{2}\right)}{b^{4} \pi^{2}}\right)}{b^{2} \pi}\right)}{5}$	83

input
```
int(x^4*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*x^6*hypergeom([1/4,3/2],[1/2,5/4,5/2],-1/16*x^4*Pi^2*b^4)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int x^4 \operatorname{FresnelC}(bx)\,dx = \frac{\pi^3 b^5 x^5 \operatorname{C}(bx) - 4\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^2 b^4 x^4 - 8) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{5\,\pi^3 b^5}$$

input
```
integrate(x^4*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/5*(pi^3*b^5*x^5*fresnel_cos(b*x) - 4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - (p
i^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.38

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \frac{x^5 C(bx)\,\Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)} - \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{20\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^2 b^3\Gamma\left(\frac{5}{4}\right)} + \frac{2 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^3 b^5\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(x**4*fresnelc(b*x),x)
```

output
```
x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*g
amma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi
**2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamm
a(5/4))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \frac{1}{5}\, x^5\, C\left(bx\right) - \frac{4\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + \left(\pi^2 b^4 x^4 - 8\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{5\,\pi^3 b^5}$$

input
```
integrate(x^4*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/5*x^5*fresnel_cos(b*x) - 1/5*(4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) + (pi^2*b
^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.113.8 Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx)\, dx = \int x^4\, C\left(bx\right)\, dx$$

input
```
integrate(x^4*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_cos(b*x), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx) \, dx = \int x^4 \operatorname{FresnelC}(b\,x) \, dx$$

input
```
int(x^4*FresnelC(b*x),x)
```

output
```
int(x^4*FresnelC(b*x), x)
```

3.114 $\int x^3 \operatorname{FresnelC}(bx)\,dx$

3.114.1 Optimal result

Integrand size = 8, antiderivative size = 74

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

output
```
-3/4*x*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+3/4*FresnelC(b*x)/b^4/Pi^2+1/4*x^4*Fre
snelC(b*x)-1/4*x^3*sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

input
```
Integrate[x^3*FresnelC[b*x],x]
```

output
```
(-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
 (x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

3.114.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6981, 3867, 3866, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{FresnelC}(bx)\,dx$$

$$\Big\downarrow \; 6981$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \; 3867$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{1}{4}b\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)$$

$$\Big\downarrow \; 3866$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{1}{4}b\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx) - \frac{1}{4}b\left(\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{3\left(\frac{\operatorname{FresnelC}(bx)}{\pi b^3} - \frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)$$

input
```
Int[x^3*FresnelC[b*x],x]
```

output
```
(x^4*FresnelC[b*x])/4 - (b*((-3*(-((x*Cos[(b^2*Pi*x^2)/2])/(b^2*Pi)) + Fre
snelC[b*x]/(b^3*Pi)))/(b^2*Pi) + (x^3*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/4
```

3.114.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.114.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{-\frac{3\cos\left(\frac{b^2\pi x^2}{2}\right)bx}{4}-\frac{b^3x^3\pi\sin\left(\frac{b^2\pi x^2}{2}\right)}{4}+\frac{\left(5x^4\pi^2b^4+15\right)\operatorname{FresnelC}(bx)}{20}}{b^4\pi^2}$	62
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx)b^4x^4}{4}-\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{3\operatorname{FresnelC}(bx)}{4\pi}}{\pi}}{b^4}$	70
default	$\frac{\frac{\operatorname{FresnelC}(bx)b^4x^4}{4}-\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{3\operatorname{FresnelC}(bx)}{4\pi}}{\pi}}{b^4}$	70
parts	$\frac{x^4\operatorname{FresnelC}(bx)}{4}-\frac{b\left(\frac{x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}-\frac{3\left(-\frac{x\cos\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi}+\frac{\operatorname{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{b^2\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b^2\pi}\right)}{4}$	93

input
```
int(x^3*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
2/Pi^2/b^4*(-3/8*cos(1/2*b^2*Pi*x^2)*b*x-1/8*b^3*x^3*Pi*sin(1/2*b^2*Pi*x^2
)+1/40*(5*Pi^2*b^4*x^4+15)*FresnelC(b*x))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = -\frac{\pi b^3x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)+3\,bx\cos\left(\frac{1}{2}\pi b^2x^2\right)-\left(\pi^2b^4x^4+3\right)\mathrm{C}\left(bx\right)}{4\,\pi^2b^4}$$

input
```
integrate(x^3*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
-1/4*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) + 3*b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b
^4*x^4 + 3)*fresnel_cos(b*x))/(pi^2*b^4)
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = \frac{5x^4C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)} - \frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi b\Gamma\left(\frac{9}{4}\right)} - \frac{15x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{9}{4}\right)} + \frac{15C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**3*fresnelc(b*x),x)
```

output
```
5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/
2)*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(
64*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gam
ma(9/4))
```

3.114.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = \frac{1}{4}x^4\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi^2b^3x^3\sin\left(\frac{1}{2}\pi b^2x^2\right) + 12\sqrt{\frac{1}{2}}\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right) + (3i-3)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi}bx\right) - (3i+3)}{8\pi^3b^4}$$

input
```
integrate(x^3*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/4*x^4*fresnel_cos(b*x) - 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*sin(1/2
*pi*b^2*x^2) + 12*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^2*x^2) + (3*I - 3)*(1/4)^(
1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I + 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I
*pi)*b*x))/(pi^3*b^4)
```

3.114.8 Giac [F]

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = \int x^3\,\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(x^3*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_cos(b*x), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx)\,dx = \int x^3\operatorname{FresnelC}(b\,x)\,dx$$

input
```
int(x^3*FresnelC(b*x),x)
```

output
```
int(x^3*FresnelC(b*x), x)
```

3.115 $\int x^2 \operatorname{FresnelC}(bx)\,dx$

3.115.1 Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x^2 \operatorname{FresnelC}(bx)\,dx=-\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}+\frac{1}{3}x^3\operatorname{FresnelC}(bx)-\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

output
```
-2/3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)-1/3*x^2*sin(1/2*b^
2*Pi*x^2)/b/Pi
```

3.115.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{FresnelC}(bx)\,dx=-\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}+\frac{1}{3}x^3\operatorname{FresnelC}(bx)-\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

input
```
Integrate[x^2*FresnelC[b*x],x]
```

output
```
(-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(
b^2*Pi*x^2)/2])/(3*b*Pi)
```

3.115.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6981, 3861, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelC}(bx) \, dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{3}b \int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b \int x^2 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right) dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b\left(\frac{2\int -\sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2} + \frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b\left(\frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{2\int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b\left(\frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{2\int \sin\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{\pi b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{3}x^3 \text{FresnelC}(bx) - \frac{1}{6}b\left(\frac{2x^2 \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{4\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi^2 b^4}\right)$$

input
```
Int[x^2*FresnelC[b*x],x]
```

output
```
(x^3*FresnelC[b*x])/3 - (b*((4*Cos[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (2*x^2*Si
n[(b^2*Pi*x^2)/2])/(b^2*Pi)))/6
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.44

method	result	size
meijerg	$\frac{b\,x^{4}\,\text{hypergeom}\left(\left[\frac{1}{4},1\right],\left[\frac{1}{2},\frac{5}{4},2\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{4}$	26
parts	$\frac{x^{3}\,\text{FresnelC}(bx)}{3}-\frac{b\left(\frac{x^{2}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{2}\pi}+\frac{2\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{b^{4}\pi^{2}}\right)}{3}$	53
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^{3}x^{3}}{3}-\frac{b^{2}x^{2}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi}-\frac{2\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi^{2}}}{b^{3}}$	54
default	$\frac{\frac{\text{FresnelC}(bx)b^{3}x^{3}}{3}-\frac{b^{2}x^{2}\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi}-\frac{2\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{3\pi^{2}}}{b^{3}}$	54

input
```
int(x^2*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*x^4*hypergeom([1/4,1],[1/2,5/4,2],-1/16*x^4*Pi^2*b^4)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^2\,\text{FresnelC}(bx)\,dx=\frac{\pi^2b^3x^3\,\text{C}\,(bx)-\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

input
```
integrate(x^2*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/3*(pi^2*b^3*x^3*fresnel_cos(b*x) - pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - 2*co
s(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \frac{x^3 C(bx)\,\Gamma\left(\frac{1}{4}\right)}{12\Gamma\left(\frac{5}{4}\right)} - \frac{x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{6\pi^2b^3\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(x**2*fresnelc(b*x),x)
```

output
```
x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*g
amma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b
**3*gamma(5/4))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \frac{1}{3}\,x^3\,\mathrm{C}\left(bx\right) - \frac{\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + 2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

input
```
integrate(x^2*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/3*x^3*fresnel_cos(b*x) - 1/3*(pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + 2*cos(1/2
*pi*b^2*x^2))/(pi^2*b^3)
```

3.115.8 Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx)\,dx = \int x^2\,\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(x^2*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_cos(b*x), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx)\, dx = \int x^2 \operatorname{FresnelC}(b\,x)\, dx$$

input
```
int(x^2*FresnelC(b*x),x)
```

output
```
int(x^2*FresnelC(b*x), x)
```

3.116 $\int x \operatorname{FresnelC}(bx)\,dx$

3.116.1 Optimal result

Integrand size = 6, antiderivative size = 49

$$\int x \operatorname{FresnelC}(bx)\,dx = \frac{1}{2}x^2 \operatorname{FresnelC}(bx) + \frac{\operatorname{FresnelS}(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

output
```
1/2*x^2*FresnelC(b*x)+1/2*FresnelS(b*x)/b^2/Pi-1/2*x*sin(1/2*b^2*Pi*x^2)/b
/Pi
```

3.116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x \operatorname{FresnelC}(bx)\,dx = \frac{1}{2}x^2 \operatorname{FresnelC}(bx) + \frac{\operatorname{FresnelS}(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

input
```
Integrate[x*FresnelC[b*x],x]
```

output
```
(x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])
/(2*b*Pi)
```

3.116.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6981, 3867, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{FresnelC}(bx)\,dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{3867}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(bx) - \frac{1}{2}b\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\int\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}\right)$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(bx) - \frac{1}{2}b\left(\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\text{FresnelS}(bx)}{\pi b^3}\right)$$

input
```
Int[x*FresnelC[b*x],x]
```

output
```
(x^2*FresnelC[b*x])/2 - (b*(-(FresnelS[b*x]/(b^3*Pi)) + (x*Sin[(b^2*Pi*x^2
)/2])/(b^2*Pi)))/2
```

3.116.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.116.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

method	result	size
meijerg	$\frac{b\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{7}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{3}$	26
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^{2}x^{2}}{2}-\frac{bx\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2\pi}+\frac{\mathrm{FresnelS}(bx)}{2\pi}}{b^{2}}$	44
default	$\frac{\frac{\mathrm{FresnelC}(bx)b^{2}x^{2}}{2}-\frac{bx\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2\pi}+\frac{\mathrm{FresnelS}(bx)}{2\pi}}{b^{2}}$	44
parts	$\frac{x^{2}\,\mathrm{FresnelC}(bx)}{2}-\frac{b\left(\frac{x\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{b^{2}\pi}-\frac{\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{b^{2}\sqrt{\pi}\,\sqrt{b^{2}\pi}}\right)}{2}$	64

input
```
int(x*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*hypergeom([1/4,3/4],[1/2,5/4,7/4],-1/16*x^4*Pi^2*b^4)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int x\,\mathrm{FresnelC}(bx)\,dx=\frac{\pi b^{3}x^{2}\,\mathrm{C}\left(bx\right)-b^{2}x\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)+\sqrt{b^{2}}\,\mathrm{S}\left(\sqrt{b^{2}}x\right)}{2\,\pi b^{3}}$$

input
```
integrate(x*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/2*(pi*b^3*x^2*fresnel_cos(b*x) - b^2*x*sin(1/2*pi*b^2*x^2) + sqrt(b^2)*f
resnel_sin(sqrt(b^2)*x))/(pi*b^3)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x\,\mathrm{FresnelC}(bx)\,dx = \frac{bx^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x*fresnelc(b*x),x)
```

output
```
b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**
4*x**4/16)/(16*gamma(5/4)*gamma(7/4))
```

3.116.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int x\,\mathrm{FresnelC}(bx)\,dx = \frac{1}{2}x^2\,\mathrm{C}\left(bx\right) - \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi bx\sin\left(\frac{1}{2}\pi b^2x^2\right) - (i+1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi}bx\right) + (i-1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi}bx\right)\right)}{4\pi^2b^2}$$

input
```
integrate(x*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/2*x^2*fresnel_cos(b*x) - 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^
2*x^2) - (I + 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) + (I - 1)*(1/4)^(1
/4)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

3.116.8 Giac [F]

$$\int x \,\text{FresnelC}(bx)\, dx = \int x\, C(bx)\, dx$$

input
```
integrate(x*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x*fresnel_cos(b*x), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int x \,\text{FresnelC}(bx)\, dx = \int x\, \text{FresnelC}(b\,x)\, dx$$

input
```
int(x*FresnelC(b*x),x)
```

output
```
int(x*FresnelC(b*x), x)
```

3.117 $\int \text{FresnelC}(bx)\,dx$

3.117.1 Optimal result

Integrand size = 4, antiderivative size = 27

$$\int \text{FresnelC}(bx)\,dx = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

output
```
x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.117.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)\,dx = x\,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelC[b*x],x]
```

output
```
x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

3.117.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6973}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\,dx$$
$$\Big\downarrow \text{6973}$$
$$x\,\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

input
```
Int[FresnelC[b*x],x]
```

output
```
x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)
```

3.117.3.1 Defintions of rubi rules used

rule 6973
```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.117.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$x\ \text{FresnelC}\left(bx\right)-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{b\pi}$	26
derivativedivides	$\frac{\text{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}}{b}$	28
default	$\frac{\text{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\pi}}{b}$	28
meijerg	$\frac{-\frac{4\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{\sqrt{\pi}}+4\sqrt{\pi}\,bx\ \text{FresnelC}(bx)}{4b\sqrt{\pi}}$	36

input
```
int(FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \text{FresnelC}(bx)\,dx = \frac{\pi bx\,\mathrm{C}\left(bx\right) - \sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi b}$$

input
```
integrate(fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
(pi*b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2))/(pi*b)
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \text{FresnelC}(bx)\,dx = \frac{xC(bx)\,\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{4\pi b\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x),x)
```

output
```
x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)
/(4*pi*b*gamma(5/4))
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)\,dx = \frac{bx\,\mathrm{C}\,(bx) - \frac{\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
(b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2)/pi)/b
```

3.117.8 Giac [F]

$$\int \text{FresnelC}(bx)\,dx = \int \mathrm{C}\,(bx)\ dx$$

input
```
integrate(fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)\,dx = \int \text{FresnelC}(b\,x)\ dx$$

input
```
int(FresnelC(b*x),x)
```

output
```
int(FresnelC(b*x), x)
```

3.118 $\int \frac{\text{FresnelC}(bx)}{x} dx$

3.118.1 Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelC}(bx)}{x} dx = \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right) + \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)+1/2*b*x*hyperge
om([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

3.118.2 Mathematica [F]

$$\int \frac{\text{FresnelC}(bx)}{x} dx = \int \frac{\text{FresnelC}(bx)}{x} dx$$

input
```
Integrate[FresnelC[b*x]/x,x]
```

output
```
Integrate[FresnelC[b*x]/x, x]
```

3.118.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6979, 26, 6912, 6914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x} \, dx$$

$$\Big\downarrow \ 6979$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int -\frac{i\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}dx$$

$$\Big\downarrow \ 26$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}dx$$

$$\Big\downarrow \ 6912$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{x}dx+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)$$

$$\Big\downarrow \ 6914$$

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

input
```
Int[FresnelC[b*x]/x,x]
```

output
```
(b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2])/2 + (
b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

3.118.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6912
```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

rule 6914
```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

rule 6979
```
Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[(1 - I)/4   Int[Erf[(Sqrt[
Pi]/2)*(1 + I)*b*x]/x, x], x] + Simp[(1 + I)/4   Int[Erf[(Sqrt[Pi]/2)*(1 -
I)*b*x]/x, x], x] /; FreeQ[b, x]
```

3.118.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.33

method	result	size
meijerg	$bx \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, \frac{5}{4}, \frac{5}{4}\right], -\frac{x^4 \pi^2 b^4}{16}\right)$	23

input
```
int(FresnelC(b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)
```

3.118.5 Fricas [F]

$$\int \frac{\mathrm{FresnelC}(bx)}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)/x, x)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx = \frac{bx\Gamma^2\left(\frac{1}{4}\right)\,{}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{5}{4},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma^2\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x,x)
```

output
```
b*x*gamma(1/4)**2*hyper((1/4, 1/4), (1/2, 5/4, 5/4), -pi**2*b**4*x**4/16)/
(16*gamma(5/4)**2)
```

3.118.7 Maxima [F]

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx = \int \frac{\text{C}(bx)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)/x, x)
```

3.118.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx = \int \frac{\text{C}(bx)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x}\,dx$$

input
```
int(FresnelC(b*x)/x,x)
```

output
```
int(FresnelC(b*x)/x, x)
```

3.119 $\int \frac{\text{FresnelC}(bx)}{x^2} dx$

3.119.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\text{FresnelC}(bx)}{x^2} dx = \frac{1}{2} b \,\text{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

output
```
1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^2} dx = \frac{1}{2} b \,\text{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

input
```
Integrate[FresnelC[b*x]/x^2,x]
```

output
```
(b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

3.119.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6981, 3857}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^2} \, dx$$

$$\Big\downarrow \text{ 6981}$$

$$b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x} dx - \frac{\text{FresnelC}(bx)}{x}$$

$$\Big\downarrow \text{ 3857}$$

$$\frac{1}{2} b \,\text{CosIntegral}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\text{FresnelC}(bx)}{x}$$

input
```
Int[FresnelC[b*x]/x^2,x]
```

output
```
(b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

3.119.3.1 Defintions of rubi rules used

rule 3857
```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.119.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{b\,\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}-\frac{\mathrm{FresnelC}(bx)}{x}$	24
derivativedivides	$b\left(-\frac{\mathrm{FresnelC}(bx)}{bx}+\frac{\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{\mathrm{FresnelC}(bx)}{bx}+\frac{\mathrm{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{b\sqrt{\pi}\left(-\frac{\pi^{\frac{3}{2}}x^4b^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2,\frac{9}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{10}+\frac{8\gamma-8\ln(2)-16+16\ln(x)+8\ln(\pi)+16\ln(b)}{\sqrt{\pi}}\right)}{16}$	66

input
```
int(FresnelC(b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\mathrm{FresnelC}(bx)}{x^2}\,dx=\frac{bx\,\mathrm{Ci}\left(\frac{1}{2}\pi b^2x^2\right)-2\,\mathrm{C}\left(bx\right)}{2\,x}$$

input
```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x*cos_integral(1/2*pi*b^2*x^2) - 2*fresnel_cos(b*x))/x
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = -\frac{\pi^2 b^5 x^4 \Gamma\left(\frac{5}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{5}{4} \\ \frac{3}{2}, 2, 2, \frac{9}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{128\Gamma\left(\frac{9}{4}\right)} + \frac{b\log\left(b^4x^4\right)}{4}$$

input
```
integrate(fresnelc(b*x)/x**2,x)
```

output
```
-pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b*
*4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4
```

3.119.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \frac{1}{4}b\left(\text{Ei}\left(\frac{1}{2}i\,\pi b^2x^2\right) + \text{Ei}\left(-\frac{1}{2}i\,\pi b^2x^2\right)\right) - \frac{\text{C}\,(bx)}{x}$$

input
```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="maxima")
```

output
```
1/4*b*(Ei(1/2*I*pi*b^2*x^2) + Ei(-1/2*I*pi*b^2*x^2)) - fresnel_cos(b*x)/x
```

3.119.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\text{C}\,(bx)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^2, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^2} \, dx = \int \frac{\text{FresnelC}(b\,x)}{x^2} \, dx$$

input
```
int(FresnelC(b*x)/x^2,x)
```

output
```
int(FresnelC(b*x)/x^2, x)
```

3.120 $\int \frac{\text{FresnelC}(bx)}{x^3} \, dx$

3.120.1 Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \frac{\text{FresnelC}(bx)}{x^3} \, dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi \,\text{FresnelS}(bx)$$

output
```
-1/2*b*cos(1/2*b^2*Pi*x^2)/x-1/2*FresnelC(b*x)/x^2-1/2*b^2*Pi*FresnelS(b*x
)
```

3.120.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^3} \, dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi \,\text{FresnelS}(bx)$$

input
```
Integrate[FresnelC[b*x]/x^3,x]
```

output
```
-1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[
b*x])/2
```

3.120.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6981, 3869, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^3} dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx - \frac{\text{FresnelC}(bx)}{2x^2}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{2}b\left(-\pi b^2\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx - \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right) - \frac{\text{FresnelC}(bx)}{2x^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{2}b\left(-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x} - \pi b\,\text{FresnelS}(bx)\right) - \frac{\text{FresnelC}(bx)}{2x^2}$$

input
```
Int[FresnelC[b*x]/x^3,x]
```

output
```
-1/2*FresnelC[b*x]/x^2 + (b*(-(Cos[(b^2*Pi*x^2)/2]/x) - b*Pi*FresnelS[b*x]
))/2
```

3.120.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],-\frac{x^{4}\pi^{2}b^{4}}{16}\right)}{x}$	26
derivativedivides	$b^{2}\left(-\frac{\mathrm{FresnelC}(bx)}{2b^{2}x^{2}}-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2bx}-\frac{\pi\,\mathrm{FresnelS}(bx)}{2}\right)$	43
default	$b^{2}\left(-\frac{\mathrm{FresnelC}(bx)}{2b^{2}x^{2}}-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{2bx}-\frac{\pi\,\mathrm{FresnelS}(bx)}{2}\right)$	43
parts	$-\frac{\mathrm{FresnelC}(bx)}{2x^{2}}+\frac{b\left(-\frac{\cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x}-\frac{b^{2}\pi^{\frac{3}{2}}\,\mathrm{FresnelS}\left(\frac{\sqrt{\pi}\,b^{2}x}{\sqrt{b^{2}\pi}}\right)}{\sqrt{b^{2}\pi}}\right)}{2}$	61

input
```
int(FresnelC(b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-b/x*hypergeom([-1/4,1/4],[1/2,3/4,5/4],-1/16*x^4*Pi^2*b^4)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{FresnelC}(bx)}{x^3}\,dx = -\frac{\pi\sqrt{b^2}bx^2\,\mathrm{S}\left(\sqrt{b^2}x\right)+bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\mathrm{C}\left(bx\right)}{2\,x^2}$$

input
```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(pi*sqrt(b^2)*b*x^2*fresnel_sin(sqrt(b^2)*x) + b*x*cos(1/2*pi*b^2*x^2
) + fresnel_cos(b*x))/x^2
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = \frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**3,x)
```

output
```
b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x
**4/16)/(16*x*gamma(3/4)*gamma(5/4))
```

3.120.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx$$
$$= -\frac{\sqrt{\frac{1}{2}}\sqrt{\pi x^2}\left((i+1)\sqrt{2}\Gamma\left(-\frac{1}{2}, \frac{1}{2}i\pi b^2x^2\right) - (i-1)\sqrt{2}\Gamma\left(-\frac{1}{2}, -\frac{1}{2}i\pi b^2x^2\right)\right)b^2}{16\,x} - \frac{\text{C}\,(bx)}{2\,x^2}$$

input
```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="maxima")
```

output
```
-1/16*sqrt(1/2)*sqrt(pi*x^2)*((I + 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2
) - (I - 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_co
s(b*x)/x^2
```

3.120.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{C}\,(bx)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^3, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^3}\,dx$$

input
```
int(FresnelC(b*x)/x^3,x)
```

output
```
int(FresnelC(b*x)/x^3, x)
```

3.121 $\int \frac{\text{FresnelC}(bx)}{x^4}\,dx$

3.121.1 Optimal result

Integrand size = 8, antiderivative size = 52

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}-\frac{1}{12}b^3\pi\,\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

output
```
-1/6*b*cos(1/2*b^2*Pi*x^2)/x^2-1/3*FresnelC(b*x)/x^3-1/12*b^3*Pi*Si(1/2*b^
2*Pi*x^2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}-\frac{1}{12}b^3\pi\,\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

input
```
Integrate[FresnelC[b*x]/x^4,x]
```

output
```
-1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInte
gral[(b^2*Pi*x^2)/2])/12
```

3.121.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6981, 3861, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^4} \, dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{3} b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^3} dx - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{6} b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6} b \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2 + \frac{\pi}{2}\right)}{x^4} dx^2 - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{6} b \left(\frac{1}{2} \pi b^2 \int -\frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6} b \left(-\frac{1}{2} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6} b \left(-\frac{1}{2} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelC}(bx)}{3x^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{6} b \left(-\frac{1}{2} \pi b^2 \text{Si}\left(\frac{1}{2} b^2 \pi x^2\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\text{FresnelC}(bx)}{3x^3}$$

input
```
Int[FresnelC[b*x]/x^4,x]
```

output
```
-1/3*FresnelC[b*x]/x^3 + (b*(-(Cos[(b^2*Pi*x^2)/2]/x^2) - (b^2*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/2))/6
```

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{2x^2}$	26
parts	$-\frac{\mathrm{FresnelC}(bx)}{3x^3}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{3}$	46
derivativedivides	$b^3\left(-\frac{\mathrm{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\mathrm{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49

input
```
int(FresnelC(b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x^2*hypergeom([-1/2,1/4],[1/2,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\mathrm{FresnelC}(bx)}{x^4}\,dx = -\frac{\pi b^3x^3\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,\mathrm{C}\left(bx\right)}{12\,x^3}$$

input
```
integrate(fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(pi*b^3*x^3*sin_integral(1/2*pi*b^2*x^2) + 2*b*x*cos(1/2*pi*b^2*x^2)
 + 4*fresnel_cos(b*x))/x^3
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b\Gamma\left(\frac{1}{4}\right) {}_2F_3\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2}, \frac{1}{2}, \frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{8x^2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**4,x)
```

output
```
-b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*
x**2*gamma(5/4))
```

3.121.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = -\frac{1}{24}\left(i\pi\Gamma\left(-1, \frac{1}{2}i\pi b^2x^2\right) - i\pi\Gamma\left(-1, -\frac{1}{2}i\pi b^2x^2\right)\right)b^3 - \frac{C(bx)}{3x^3}$$

input
```
integrate(fresnel_cos(b*x)/x^4,x, algorithm="maxima")
```

output
```
-1/24*(I*pi*gamma(-1, 1/2*I*pi*b^2*x^2) - I*pi*gamma(-1, -1/2*I*pi*b^2*x^2
))*b^3 - 1/3*fresnel_cos(b*x)/x^3
```

3.121.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^4}\,dx = \int \frac{C(bx)}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^4, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^4} dx = \int \frac{\text{FresnelC}(b\,x)}{x^4} dx$$

input
```
int(FresnelC(b*x)/x^4,x)
```

output
```
int(FresnelC(b*x)/x^4, x)
```

3.122 $\int \frac{\text{FresnelC}(bx)}{x^5}\,dx$

3.122.1 Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}$$

output
```
-1/12*b*cos(1/2*b^2*Pi*x^2)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)-1/4*FresnelC(b
*x)/x^4+1/12*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x
```

3.122.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^5}\,dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{4x^4} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}$$

input
```
Integrate[FresnelC[b*x]/x^5,x]
```

output
```
-1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC
[b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

3.122.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6981, 3869, 3868, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{4} b \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx - \frac{\text{FresnelC}(bx)}{4x^4}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{4} b \left(-\frac{1}{3} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)}{4x^4}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{4} b \left(-\frac{1}{3} \pi b^2 \left(\pi b^2 \int \cos\left(\frac{1}{2} b^2 \pi x^2\right) dx - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)}{4x^4}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{4} b \left(-\frac{1}{3} \pi b^2 \left(\pi b \,\text{FresnelC}(bx) - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)}{4x^4}$$

input
```
Int[FresnelC[b*x]/x^5,x]
```

output
```
-1/4*FresnelC[b*x]/x^4 + (b*(-1/3*Cos[(b^2*Pi*x^2)/2]/x^3 - (b^2*Pi*(b*Pi*
FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/x))/3))/4
```

3.122.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.122.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$b^4\left(-\frac{\mathrm{FresnelC}(bx)}{4b^4x^4}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{12}\right)$	64
default	$b^4\left(-\frac{\mathrm{FresnelC}(bx)}{4b^4x^4}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{12}\right)$	64
meijerg	$\frac{\pi^2b^4\left(-\frac{32\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2x^3b^3}+\frac{32\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi xb}-\frac{32\left(x^4\pi^2b^4+3\right)\mathrm{FresnelC}(bx)}{3\pi^2x^4b^4}\right)}{128}$	79
parts	$-\frac{\mathrm{FresnelC}(bx)}{4x^4}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}+\frac{b^2\pi^{\frac{3}{2}}\mathrm{FresnelC}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{3}\right)}{4}$	82

input
```
int(FresnelC(b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
b^4*(-1/4*FresnelC(b*x)/b^4/x^4-1/12/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/12*Pi*(
-1/b/x*sin(1/2*b^2*Pi*x^2)+Pi*FresnelC(b*x)))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{\mathrm{FresnelC}(bx)}{x^5}\,dx=\frac{\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^4x^4+3\right)\mathrm{C}\left(bx\right)}{12\,x^4}$$

input
```
integrate(fresnel_cos(b*x)/x^5,x, algorithm="fricas")
```

output
```
1/12*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) - b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b^4
*x^4 + 3)*fresnel_cos(b*x))/x^4
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx = \frac{\pi^2 b^4 C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64\Gamma\left(\frac{5}{4}\right)} - \frac{\pi b^3 \sin\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x\Gamma\left(\frac{5}{4}\right)} + \frac{b\cos\left(\frac{\pi b^2 x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64x^4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**5,x)
```

output
```
pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2
*x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)
/(64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

3.122.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{3}{2}}\left((i-1)\,\sqrt{2}\Gamma\left(-\frac{3}{2}, \frac{1}{2}i\,\pi b^2 x^2\right) - (i+1)\,\sqrt{2}\Gamma\left(-\frac{3}{2}, -\frac{1}{2}i\,\pi b^2 x^2\right)\right)b^4}{64\,x^3} - \frac{\text{C}\,(bx)}{4\,x^4}$$

input
```
integrate(fresnel_cos(b*x)/x^5,x, algorithm="maxima")
```

output
```
-1/64*sqrt(1/2)*(pi*x^2)^(3/2)*((I - 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*x
^2) - (I + 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresne
l_cos(b*x)/x^4
```

3.122.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx = \int \frac{C(bx)}{x^5} \, dx$$

input
```
integrate(fresnel_cos(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^5, x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^5} \, dx = \int \frac{\text{FresnelC}(b\,x)}{x^5} \, dx$$

input
```
int(FresnelC(b*x)/x^5,x)
```

output
```
int(FresnelC(b*x)/x^5, x)
```

3.123 $\int \frac{\text{FresnelC}(bx)}{x^6} dx$

3.123.1 Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\text{FresnelC}(bx)}{x^6} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

output
```
-1/80*b^5*Pi^2*Ci(1/2*b^2*Pi*x^2)-1/20*b*cos(1/2*b^2*Pi*x^2)/x^4-1/5*Fresn
elC(b*x)/x^5+1/40*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x^2
```

3.123.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)}{x^6} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4} - \frac{1}{80}b^5\pi^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)}{5x^5} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

input
```
Integrate[FresnelC[b*x]/x^6,x]
```

output
```
-1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])
/80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

3.123.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {6981, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx$$

$$\Big\downarrow \text{ 6981}$$

$$\frac{1}{5} b \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^5} dx - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{10} b \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx^2 - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10} b \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2 + \frac{\pi}{2}\right)}{x^6} dx^2 - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{10} b \left(\frac{1}{4} \pi b^2 \int -\frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{10} b \left(-\frac{1}{4} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10} b \left(-\frac{1}{4} \pi b^2 \int \frac{\sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{10} b \left(-\frac{1}{4} \pi b^2 \left(\frac{1}{2} \pi b^2 \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx^2 - \frac{\sin\left(\frac{1}{2} \pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\frac{1}{2} \pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{10}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)}{5x^5}$$

$$\Big\downarrow 3783$$

$$\frac{1}{10}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)}{5x^5}$$

input
```
Int[FresnelC[b*x]/x^6,x]
```

output
```
-1/5*FresnelC[b*x]/x^5 + (b*(-1/2*Cos[(b^2*Pi*x^2)/2]/x^4 - (b^2*Pi*((b^2*
Pi*CosIntegral[(b^2*Pi*x^2)/2])/2 - Sin[(b^2*Pi*x^2)/2]/x^2))/4))/10
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.123. $\int \frac{\text{FresnelC}(bx)}{x^6}\,dx$

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.123.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
parts	$-\frac{\operatorname{FresnelC}(bx)}{5x^5}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{5}$	68
derivativedivides	$b^5\left(-\frac{\operatorname{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
default	$b^5\left(-\frac{\operatorname{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
meijerg	$\frac{\pi^{\frac{5}{2}}b^5\left(\frac{\pi^{\frac{3}{2}}x^4b^4\operatorname{hypergeom}\left(\left[1,1,\frac{9}{4}\right],\left[2,\frac{5}{2},3,\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{54}-\frac{8\left(-\frac{19}{5}+2\gamma-2\ln(2)+4\ln(x)+2\ln(\pi)+4\ln(b)\right)}{5\sqrt{\pi}}-\frac{64}{\pi^{\frac{5}{2}}x^4b^4}\right)}{256}$	79

input
```
int(FresnelC(b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*FresnelC(b*x)/x^5+1/5*b*(-1/4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*b^2*Pi*(-1/
2*sin(1/2*b^2*Pi*x^2)/x^2+1/4*b^2*Pi*Ci(1/2*b^2*Pi*x^2)))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = -\frac{\pi^2 b^5 x^5 \operatorname{Ci}\left(\frac{1}{2} \pi b^2 x^2\right) - 2 \pi b^3 x^3 \sin\left(\frac{1}{2} \pi b^2 x^2\right) + 4 bx \cos\left(\frac{1}{2} \pi b^2 x^2\right) + 16 \operatorname{C}(bx)}{80 x^5}$$

input
```
integrate(fresnel_cos(b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/80*(pi^2*b^5*x^5*cos_integral(1/2*pi*b^2*x^2) - 2*pi*b^3*x^3*sin(1/2*pi
*b^2*x^2) + 4*b*x*cos(1/2*pi*b^2*x^2) + 16*fresnel_cos(b*x))/x^5
```

3.123.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \frac{\pi^4 b^9 x^4 \Gamma\left(\frac{9}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{9}{4} \\ 2, \frac{5}{2}, 3, \frac{13}{4} \end{matrix} \middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{6144 \Gamma\left(\frac{13}{4}\right)} - \frac{\pi^2 b^5 \log\left(b^4 x^4\right)}{160} - \frac{b}{4x^4}$$

input
```
integrate(fresnelc(b*x)/x**6,x)
```

output
```
pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b*
*4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4
)
```

3.123.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \frac{1}{80} \left(\pi^2 \Gamma\left(-2, \frac{1}{2} i \pi b^2 x^2\right) + \pi^2 \Gamma\left(-2, -\frac{1}{2} i \pi b^2 x^2\right)\right) b^5 - \frac{\operatorname{C}(bx)}{5 x^5}$$

input
```
integrate(fresnel_cos(b*x)/x^6,x, algorithm="maxima")
```

output
```
1/80*(pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + pi^2*gamma(-2, -1/2*I*pi*b^2*x^2)
)*b^5 - 1/5*fresnel_cos(b*x)/x^5
```

3.123.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \int \frac{C(bx)}{x^6} \, dx$$

input
```
integrate(fresnel_cos(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^6, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^6} \, dx = \int \frac{\text{FresnelC}(b\,x)}{x^6} \, dx$$

input
```
int(FresnelC(b*x)/x^6,x)
```

output
```
int(FresnelC(b*x)/x^6, x)
```

3.124 $\int \frac{\text{FresnelC}(bx)}{x^7} dx$

3.124.1 Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \frac{\text{FresnelC}(bx)}{x^7} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x} - \frac{\text{FresnelC}(bx)}{6x^6} + \frac{1}{90}b^6\pi^3\,\text{FresnelS}(bx) + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}$$

output
```
-1/30*b*cos(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x-1/6*Fr
esnelC(b*x)/x^6+1/90*b^6*Pi^3*FresnelS(b*x)+1/90*b^3*Pi*sin(1/2*b^2*Pi*x^2
)/x^3
```

3.124.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{\text{FresnelC}(bx)}{x^7} dx = \frac{1}{90}\left(\frac{b(-3+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5} - \frac{15\,\text{FresnelC}(bx)}{x^6} + b^6\pi^3\,\text{FresnelS}(bx) + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\right)$$

input
```
Integrate[FresnelC[b*x]/x^7,x]
```

output
```
((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6
+ b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

3.124.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6981, 3869, 3868, 3869, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^7} \, dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{6}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)}{6x^6}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{6}b\left(-\frac{1}{5}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)}{6x^6}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)}{6x^6}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{6}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\pi b^2\int \sin\left(\frac{1}{2}b^2\pi x^2\right)dx-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)}{6x^6}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{6}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}-\pi b\,\text{FresnelS}(bx)\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)}{6x^6}$$

input
```
Int[FresnelC[b*x]/x^7,x]
```

output
```
-1/6*FresnelC[b*x]/x^6 + (b*(-1/5*Cos[(b^2*Pi*x^2)/2]/x^5 - (b^2*Pi*((b^2*
Pi*(-(Cos[(b^2*Pi*x^2)/2]/x) - b*Pi*FresnelS[b*x]))/3 - Sin[(b^2*Pi*x^2)/2
]/(3*x^3)))/5))/6
```

3.124.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.124.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

method	result	siz
meijerg	$-\frac{b\,\text{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{4}\right],\left[-\frac{1}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{5x^5}$	26
derivativedivides	$b^6\left(-\frac{\text{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\text{FresnelS}(bx)\right)}{3}\right)}{30}\right)$	87
default	$b^6\left(-\frac{\text{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\text{FresnelS}(bx)\right)}{3}\right)}{30}\right)$	87
parts	$-\frac{\text{FresnelC}(bx)}{6x^6}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{5x^5}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{x}-\frac{b^2\pi^{\frac{3}{2}}\,\text{FresnelS}\left(\frac{\sqrt{\pi}\,b^2x}{\sqrt{b^2\pi}}\right)}{\sqrt{b^2\pi}}\right)}{3}\right)}{5}\right)}{6}$	10

input
```
int(FresnelC(b*x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b/x^5*hypergeom([-5/4,1/4],[-1/4,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

3.124. $\int \frac{\text{FresnelC}(bx)}{x^7}\,dx$

3.124.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \frac{\pi^3\sqrt{b^2}b^5x^6\,\text{S}\left(\sqrt{b^2}x\right) + \pi b^3x^3\sin\left(\frac{1}{2}\pi b^2x^2\right) + \left(\pi^2b^5x^5 - 3\,bx\right)\cos\left(\frac{1}{2}\pi b^2x^2\right) - 15\,\text{C}\left(bx\right)}{90\,x^6}$$

input
```
integrate(fresnel_cos(b*x)/x^7,x, algorithm="fricas")
```

output
```
1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_sin(sqrt(b^2)*x) + pi*b^3*x^3*sin(1/2
*pi*b^2*x^2) + (pi^2*b^5*x^5 - 3*b*x)*cos(1/2*pi*b^2*x^2) - 15*fresnel_cos
(b*x))/x^6
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}-\frac{5}{4},\frac{1}{4}\\-\frac{1}{4},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16x^5\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**7,x)
```

output
```
b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*
x**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))
```

3.124.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{5}{2}}\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{1}{2}i\,\pi b^2x^2\right) + (i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^6}{192\,x^5} - \frac{\text{C}\left(bx\right)}{6\,x^6}$$

input
```
integrate(fresnel_cos(b*x)/x^7,x, algorithm="maxima")
```

output
```
-1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I + 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2
*x^2) + (I - 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fres
nel_cos(b*x)/x^6
```

3.124.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{C}\,(bx)}{x^7}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^7, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^7}\,dx$$

input
```
int(FresnelC(b*x)/x^7,x)
```

output
```
int(FresnelC(b*x)/x^7, x)
```

3.125 $\int \frac{\text{FresnelC}(bx)}{x^8} dx$

3.125.1 Optimal result

Integrand size = 8, antiderivative size = 102

$$\int \frac{\text{FresnelC}(bx)}{x^8} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2} - \frac{\text{FresnelC}(bx)}{7x^7} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4} + \frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

output
```
-1/42*b*cos(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^2-1/7
*FresnelC(b*x)/x^7+1/672*b^7*Pi^3*Si(1/2*b^2*Pi*x^2)+1/168*b^3*Pi*sin(1/2*
b^2*Pi*x^2)/x^4
```

3.125.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\text{FresnelC}(bx)}{x^8} dx = \frac{1}{672}\left(\frac{2b(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} - \frac{96\,\text{FresnelC}(bx)}{x^7} + \frac{4b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} + b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\right)$$

input
```
Integrate[FresnelC[b*x]/x^8,x]
```

output
```
((2*b*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (96*FresnelC[b*x])/x^
7 + (4*b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^4 + b^7*Pi^3*SinIntegral[(b^2*Pi*x^2)
/2])/672
```

3.125.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {6981, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{14}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{14}b\left(\frac{1}{6}\pi b^2\int -\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$\Big\downarrow$ 3042

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$\Big\downarrow$ 3778

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$\Big\downarrow$ 25

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$\Big\downarrow$ 3042

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

$\Big\downarrow$ 3780

$$\frac{1}{14}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x^7}$$

input
```
Int[FresnelC[b*x]/x^8,x]
```

output
```
-1/7*FresnelC[b*x]/x^7 + (b*(-1/3*Cos[(b^2*Pi*x^2)/2]/x^6 - (b^2*Pi*(-1/2*
Sin[(b^2*Pi*x^2)/2]/x^4 + (b^2*Pi*(-(Cos[(b^2*Pi*x^2)/2]/x^2) - (b^2*Pi*Si
nIntegral[(b^2*Pi*x^2)/2])/2))/4))/6))/14
```

3.125. $\int \frac{\text{FresnelC}(bx)}{x^8}\,dx$

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.125. $\int \frac{\text{FresnelC}(bx)}{x^8} \, dx$

3.125.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.25

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{6x^6}$	26
parts	$-\frac{\mathrm{FresnelC}(bx)}{7x^7}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}-\frac{b^2\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{7}$	90
derivativedivides	$b^7\left(-\frac{\mathrm{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93
default	$b^7\left(-\frac{\mathrm{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\mathrm{Si}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93

input
```
int(FresnelC(b*x)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b/x^6*hypergeom([-3/2,1/4],[-1/2,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \frac{\pi^3 b^7 x^7\,\text{Si}\left(\frac{1}{2}\,\pi b^2 x^2\right) + 4\,\pi b^3 x^3 \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + 2\left(\pi^2 b^5 x^5 - 8\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 96\,\text{C}\left(bx\right)}{672\,x^7}$$

input
```
integrate(fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

output
```
1/672*(pi^3*b^7*x^7*sin_integral(1/2*pi*b^2*x^2) + 4*pi*b^3*x^3*sin(1/2*pi
*b^2*x^2) + 2*(pi^2*b^5*x^5 - 8*b*x)*cos(1/2*pi*b^2*x^2) - 96*fresnel_cos(
b*x))/x^7
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = -\frac{b\Gamma\left(\frac{1}{4}\right)\,{}_2F_3\left(\begin{matrix} -\frac{3}{2},\frac{1}{4} \\ -\frac{1}{2},\frac{1}{2},\frac{5}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**8,x)
```

output
```
-b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(2
4*x**6*gamma(5/4))
```

3.125.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.47

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = -\frac{1}{224}\left(-i\,\pi^3\Gamma\left(-3,\frac{1}{2}i\,\pi b^2 x^2\right) + i\,\pi^3\Gamma\left(-3,-\frac{1}{2}i\,\pi b^2 x^2\right)\right)b^7 - \frac{\text{C}\left(bx\right)}{7\,x^7}$$

input
```
integrate(fresnel_cos(b*x)/x^8,x, algorithm="maxima")
```

output
```
-1/224*(-I*pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + I*pi^3*gamma(-3, -1/2*I*pi*b
^2*x^2))*b^7 - 1/7*fresnel_cos(b*x)/x^7
```

3.125.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{C(bx)}{x^8}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^8, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^8}\,dx$$

input
```
int(FresnelC(b*x)/x^8,x)
```

output
```
int(FresnelC(b*x)/x^8, x)
```

3.126 $\int \frac{\text{FresnelC}(bx)}{x^9} dx$

3.126.1 Optimal result

Integrand size = 8, antiderivative size = 119

$$\int \frac{\text{FresnelC}(bx)}{x^9} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3} + \frac{1}{840}b^8\pi^4\text{FresnelC}(bx) - \frac{\text{FresnelC}(bx)}{8x^8} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}$$

output
```
-1/56*b*cos(1/2*b^2*Pi*x^2)/x^7+1/840*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^3+1/8
40*b^8*Pi^4*FresnelC(b*x)-1/8*FresnelC(b*x)/x^8+1/280*b^3*Pi*sin(1/2*b^2*P
i*x^2)/x^5-1/840*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x
```

3.126.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\text{FresnelC}(bx)}{x^9} dx = \frac{bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + (-105+b^8\pi^4x^8)\text{FresnelC}(bx) + b^3\pi x^3(3-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

input
```
Integrate[FresnelC[b*x]/x^9,x]
```

output
```
(b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fres
nelC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

3.126.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6981, 3869, 3868, 3869, 3868, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^9} \, dx$$

$$\Big\downarrow \; 6981$$

$$\frac{1}{8} b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^8} dx - \frac{\text{FresnelC}(bx)}{8 x^8}$$

$$\Big\downarrow \; 3869$$

$$\frac{1}{8} b \left(-\frac{1}{7} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{7 x^7}\right) - \frac{\text{FresnelC}(bx)}{8 x^8}$$

$$\Big\downarrow \; 3868$$

$$\frac{1}{8} b \left(-\frac{1}{7} \pi b^2 \left(\frac{1}{5} \pi b^2 \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^4} dx - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{5 x^5}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{7 x^7}\right) - \frac{\text{FresnelC}(bx)}{8 x^8}$$

$$\Big\downarrow \; 3869$$

$$\frac{1}{8} b \left(-\frac{1}{7} \pi b^2 \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{3 x^3}\right) - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{5 x^5}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{7 x^7}\right) - \frac{\text{FresnelC}(bx)}{8 x^8}$$

$$\Big\downarrow \; 3868$$

$$\frac{1}{8} b \left(-\frac{1}{7} \pi b^2 \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \left(\pi b^2 \int \cos \left(\frac{1}{2} b^2 \pi x^2\right) dx - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{3 x^3}\right) - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{5 x^5}\right) - \text{cos} \right. \\ \frac{\text{FresnelC}(bx)}{8 x^8}$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{8} b \left(-\frac{1}{7} \pi b^2 \left(\frac{1}{5} \pi b^2 \left(-\frac{1}{3} \pi b^2 \left(\pi b \, \text{FresnelC}(bx) - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{x}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{3 x^3}\right) - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{5 x^5}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x\right.}{7 x^7} \right. \\ \frac{\text{FresnelC}(bx)}{8 x^8}$$

input
```
Int[FresnelC[b*x]/x^9,x]
```

output
```
-1/8*FresnelC[b*x]/x^8 + (b*(-1/7*Cos[(b^2*Pi*x^2)/2]/x^7 - (b^2*Pi*(-1/5*
Sin[(b^2*Pi*x^2)/2]/x^5 + (b^2*Pi*(-1/3*Cos[(b^2*Pi*x^2)/2]/x^3 - (b^2*Pi*
(b*Pi*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/x))/3))/5))/7))/8
```

3.126.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.126.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

method	result
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{7}{4},\frac{1}{4}\right],\left[-\frac{3}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{7x^7}$
derivativedivides	$b^8\left(-\frac{\mathrm{FresnelC}(bx)}{8b^8x^8}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\ \mathrm{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\cdots\right.$
default	$b^8\left(-\frac{\mathrm{FresnelC}(bx)}{8b^8x^8}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\ \mathrm{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\cdots\right.$
	$b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{7x^7}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5x^5}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3x^3}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}+\frac{b^2\pi^{\frac{3}{2}}\ \mathrm{FresnelC}(\cdots)}{\sqrt{b^2\pi}\cdots}\cdots\right)}{3}\right)}{5}\right)}{7}\cdots\right.$

3.126. $\int\frac{\mathrm{FresnelC}(bx)}{x^9}\,dx$

input
```
int(FresnelC(b*x)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b/x^7*hypergeom([-7/4,1/4],[-3/4,1/2,5/4],-1/16*x^4*Pi^2*b^4)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \frac{(\pi^2b^5x^5 - 15\,bx)\cos\left(\frac{1}{2}\,\pi b^2x^2\right) + (\pi^4b^8x^8 - 105)\,\mathrm{C}\,(bx) - (\pi^3b^7x^7 - 3\,\pi b^3x^3)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{840\,x^8}$$

input
```
integrate(fresnel_cos(b*x)/x^9,x, algorithm="fricas")
```

output
```
1/840*((pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 - 105)*
fresnel_cos(b*x) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/x^8
```

3.126.6 Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \frac{\pi^4b^8C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{2560\Gamma\left(\frac{5}{4}\right)} - \frac{\pi^3b^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x\Gamma\left(\frac{5}{4}\right)} + \frac{\pi^2b^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^3\Gamma\left(\frac{5}{4}\right)} + \frac{3\pi b^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^5\Gamma\left(\frac{5}{4}\right)} - \frac{3b\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{512x^7\Gamma\left(\frac{5}{4}\right)} - \frac{21C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{512x^8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(fresnelc(b*x)/x**9,x)
```

output
```
pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi
*b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**
2/2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*ga
mma(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(51
2*x**7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))
```

3.126.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = -\frac{\sqrt{\frac{1}{2}}(\pi x^2)^{\frac{7}{2}}\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{1}{2}i\pi b^2x^2\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^8}{512\,x^7} - \frac{C\,(bx)}{8\,x^8}$$

input
```
integrate(fresnel_cos(b*x)/x^9,x, algorithm="maxima")
```

output
```
-1/512*sqrt(1/2)*(pi*x^2)^(7/2)*(-(I - 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2
*x^2) + (I + 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fres
nel_cos(b*x)/x^8
```

3.126.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{C\,(bx)}{x^9}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^9, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^9}\,dx$$

input
```
int(FresnelC(b*x)/x^9,x)
```

output
```
int(FresnelC(b*x)/x^9, x)
```

3.127 $\int \frac{\text{FresnelC}(bx)}{x^{10}} dx$

3.127.1 Optimal result

Integrand size = 8, antiderivative size = 127

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} dx = -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4} + \frac{b^9\pi^4\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)}{6912} - \frac{\text{FresnelC}(bx)}{9x^9} + \frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6} - \frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}$$

output
```
1/6912*b^9*Pi^4*Ci(1/2*b^2*Pi*x^2)-1/72*b*cos(1/2*b^2*Pi*x^2)/x^8+1/1728*b
^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^4-1/9*FresnelC(b*x)/x^9+1/432*b^3*Pi*sin(1/2
*b^2*Pi*x^2)/x^6-1/3456*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x^2
```

3.127.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} dx = \frac{\frac{4b(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8} + b^9\pi^4\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right) - \frac{768\,\text{FresnelC}(bx)}{x^9} - \frac{2b^3\pi(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}}{6912}$$

input
```
Integrate[FresnelC[b*x]/x^10,x]
```

output
```
((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral
[(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)
*Sin[(b^2*Pi*x^2)/2])/x^6)/6912
```

3.127.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {6981, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)}{x^{10}} dx$$

$$\Big\downarrow \text{6981}$$

$$\frac{1}{9} b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^9} dx - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{18} b \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^{10}} dx^2 - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18} b \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2 + \frac{\pi}{2}\right)}{x^{10}} dx^2 - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18} b \left(\frac{1}{8} \pi b^2 \int -\frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^8} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{4 x^8}\right) - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18} b \left(-\frac{1}{8} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^8} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{4 x^8}\right) - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18} b \left(-\frac{1}{8} \pi b^2 \int \frac{\sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^8} dx^2 - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{4 x^8}\right) - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18} b \left(-\frac{1}{8} \pi b^2 \left(\frac{1}{6} \pi b^2 \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right)}{x^6} dx^2 - \frac{\sin \left(\frac{1}{2} \pi b^2 x^2\right)}{3 x^6}\right) - \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right)}{4 x^8}\right) - \frac{\text{FresnelC}(bx)}{9 x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)-\right.$$
$$\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)\right.$$
$$\frac{\text{FresnelC}(bx)}{9x^9}$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{18}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\,\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^6}\right)\right.$$
$$\frac{\text{FresnelC}(bx)}{9x^9}$$

3.127. $\int\frac{\text{FresnelC}(bx)}{x^{10}}\,dx$

input
```
Int[FresnelC[b*x]/x^10,x]
```

output
```
-1/9*FresnelC[b*x]/x^9 + (b*(-1/4*Cos[(b^2*Pi*x^2)/2]/x^8 - (b^2*Pi*(-1/3*
Sin[(b^2*Pi*x^2)/2]/x^6 + (b^2*Pi*(-1/2*Cos[(b^2*Pi*x^2)/2]/x^4 - (b^2*Pi*
((b^2*Pi*CosIntegral[(b^2*Pi*x^2)/2])/2 - Sin[(b^2*Pi*x^2)/2]/x^2))/4))/6)
)/8))/18
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6981
```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Simp[b/(d*(m + 1))   Int[(d*x)^(m + 1)*
Cos[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

3.127.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{\pi^{\frac{9}{2}} b^9 \left(-\frac{\pi^{\frac{3}{2}} x^4 b^4 \operatorname{hypergeom}\left(\left[1,1,\frac{13}{4}\right],\left[2,\frac{7}{2},4,\frac{17}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{585}+\frac{-\frac{332}{243}+\frac{16\gamma}{27}-\frac{16\ln(2)}{27}+\frac{32\ln(x)}{27}+\frac{16\ln(\pi)}{27}+\frac{32\ln(b)}{27}}{\sqrt{\pi}}-\frac{512}{\pi^{\frac{9}{2}}x^8b^8}+ \cdots\right.}{4096}$
parts	$-\frac{\mathrm{FresnelC}(bx)}{9x^9}+\frac{b\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{8x^8}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6x^6}+\frac{b^2\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4x^4}-\frac{b^2\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2x^2}+\frac{b^2\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4} \cdots\right.}{4}\right.}{6}\right.}{8}\right.}{9}$
derivativedivides	$b^9\left(-\frac{\mathrm{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72} \cdots\right.$
default	$b^9\left(-\frac{\mathrm{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\operatorname{Ci}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72} \cdots\right.$

3.127 $\int \frac{\mathrm{FresnelC}(bx)}{x^{10}}dx$

input
```
int(FresnelC(b*x)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/4096*Pi^(9/2)*b^9*(-1/585*Pi^(3/2)*x^4*b^4*hypergeom([1,1,13/4],[2,7/2,4
,17/4],-1/16*x^4*Pi^2*b^4)+8/27*(-83/18+2*gamma-2*ln(2)+4*ln(x)+2*ln(Pi)+4
*ln(b))/Pi^(1/2)-512/Pi^(9/2)/x^8/b^8+128/5/Pi^(5/2)/x^4/b^4)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \frac{\pi^4 b^9 x^9\,\text{Ci}\left(\frac{1}{2}\pi b^2x^2\right) + 4\left(\pi^2b^5x^5 - 24\,bx\right)\cos\left(\frac{1}{2}\pi b^2x^2\right) - 2\left(\pi^3b^7x^7 - 8\,\pi b^3x^3\right)\sin\left(\frac{1}{2}\pi b^2x^2\right) - 768\,\text{C}\left(bx\right)}{6912\,x^9}$$

input
```
integrate(fresnel_cos(b*x)/x^10,x, algorithm="fricas")
```

output
```
1/6912*(pi^4*b^9*x^9*cos_integral(1/2*pi*b^2*x^2) + 4*(pi^2*b^5*x^5 - 24*b
*x)*cos(1/2*pi*b^2*x^2) - 2*(pi^3*b^7*x^7 - 8*pi*b^3*x^3)*sin(1/2*pi*b^2*x
^2) - 768*fresnel_cos(b*x))/x^9
```

3.127.6 Sympy [A] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{\pi^6 b^{13} x^4 \Gamma\left(\frac{13}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{13}{4} \\ 2, \frac{7}{2}, 4, \frac{17}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{737280\Gamma\left(\frac{17}{4}\right)} + \frac{\pi^4 b^9 \log\left(b^4x^4\right)}{13824} + \frac{\pi^2 b^5}{160x^4} - \frac{b}{8x^8}$$

input
```
integrate(fresnelc(b*x)/x**10,x)
```

output
```
-pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**
2*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + p
i**2*b**5/(160*x**4) - b/(8*x**8)
```

3.127.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = -\frac{1}{576}\left(\pi^4\Gamma\left(-4,\frac{1}{2}i\,\pi b^2x^2\right)+\pi^4\Gamma\left(-4,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^9-\frac{\mathrm{C}\,(bx)}{9\,x^9}$$

input
```
integrate(fresnel_cos(b*x)/x^10,x, algorithm="maxima")
```

output
```
-1/576*(pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) + pi^4*gamma(-4, -1/2*I*pi*b^2*x^
2))*b^9 - 1/9*fresnel_cos(b*x)/x^9
```

3.127.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\mathrm{C}\,(bx)}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)/x^10, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)}{x^{10}}\,dx$$

input
```
int(FresnelC(b*x)/x^10,x)
```

output
```
int(FresnelC(b*x)/x^10, x)
```

3.128 $\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx$

3.128.1 Optimal result

Integrand size = 14, antiderivative size = 298

$$\begin{aligned}\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = &-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2}\\ &-\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}\\ &-\frac{(bc-ad)^4\operatorname{FresnelC}(a+bx)}{4b^4d}\\ &+\frac{3d^3\operatorname{FresnelC}(a+bx)}{4b^4\pi^2}+\frac{(c+dx)^4\operatorname{FresnelC}(a+bx)}{4d}\\ &+\frac{3d(bc-ad)^2\operatorname{FresnelS}(a+bx)}{2b^4\pi}\\ &-\frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}\\ &-\frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi}\\ &-\frac{d^2(bc-ad)(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}\\ &-\frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}\end{aligned}$$

output
```
-2*d^2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*cos(1/2*P
i*(b*x+a)^2)/b^4/Pi^2-1/4*(-a*d+b*c)^4*FresnelC(b*x+a)/b^4/d+3/4*d^3*Fresn
elC(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelC(b*x+a)/d+3/2*d*(-a*d+b*c)^2*Fre
snelS(b*x+a)/b^4/Pi-(-a*d+b*c)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+
b*c)^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-d^2*(-a*d+b*c)*(b*x+a)^2*sin(1
/2*Pi*(b*x+a)^2)/b^4/Pi-1/4*d^3*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

3.128.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.42

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \frac{-8bcd^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+5ad^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-3bd^3x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+(4b^3c^3\pi^2(a+bx)+6b^2c^2}{}$$

input
```
Integrate[(c + d*x)^3*FresnelC[a + b*x],x]
```

output
```
(-8*b*c*d^2*Cos[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Cos[(Pi*(a + b*x)^2)/2] - 3*
b*d^3*x*Cos[(Pi*(a + b*x)^2)/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*
Pi^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2
 + b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*
x] - 4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2)/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a +
b*x)^2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + a^3*d^3*Pi*Sin[(Pi
*(a + b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^
2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] -
4*b^3*c*d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a +
 b*x)^2)/2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

3.128.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\, dx$$

$$\Big\downarrow \; 6983$$

$$\frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{b\int (c+dx)^4 \cos\left(\frac{1}{2}\pi(a+bx)^2\right) dx}{4d}$$

$$\Big\downarrow \; 3915$$

$$\frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{\int \left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^4 + 4d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^3 + 6d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-\right.}{4b^4d}$$

$$\Big\downarrow \; 2009$$

$$\frac{(c+dx)^4 \operatorname{FresnelC}(a+bx)}{4d} - \frac{\frac{4d^3(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + \frac{8d^3(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2} - \frac{6d^2(bc-ad)^2 \operatorname{FresnelS}(a+bx)}{\pi} + \frac{6d^2(a+bx)(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{}$$

input
```
Int[(c + d*x)^3*FresnelC[a + b*x],x]
```

output
```
((c + d*x)^4*FresnelC[a + b*x])/(4*d) - ((8*d^3*(b*c - a*d)*Cos[(Pi*(a + b
*x)^2)/2])/Pi^2 + (3*d^4*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi^2 + (b*c -
a*d)^4*FresnelC[a + b*x] - (3*d^4*FresnelC[a + b*x])/Pi^2 - (6*d^2*(b*c -
a*d)^2*FresnelS[a + b*x])/Pi + (4*d*(b*c - a*d)^3*Sin[(Pi*(a + b*x)^2)/2])
/Pi + (6*d^2*(b*c - a*d)^2*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi + (4*d^3*
(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/Pi + (d^4*(a + b*x)^3*Sin
[(Pi*(a + b*x)^2)/2])/Pi)/(4*b^4*d)
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.128.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\operatorname{FresnelC}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx+a)}{\pi}\right)}{\pi}-\frac{4(ad-bc)}{}}{}}{}$
default	$\frac{\frac{\operatorname{FresnelC}(bx+a)(ad-bc-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\operatorname{FresnelC}(bx+a)}{\pi}\right)}{\pi}-\frac{4(ad-bc)}{}}{}}{}$
parts	Expression too large to display

input
```
int((d*x+c)^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*FresnelC(b*x+a)*(a*d-b*c-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*
x+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)
+1/Pi*FresnelC(b*x+a))-4*(a*d-b*c)*d^3/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-
8*(a*d-b*c)*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+6*(a*d-b*c)^2*d^2/Pi*(b*x+a)*si
n(1/2*Pi*(b*x+a)^2)-6*(a*d-b*c)^2*d^2/Pi*FresnelS(b*x+a)-4*(a*d-b*c)^3*d/P
i*sin(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^4*FresnelC(b*x+a)))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx$$
$$=\frac{6\,\pi(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(\pi^2(4\,ab^3c^3-6\,a^2b^2c^2d+4\,a^3bcd^2-a^4d^3)+3\,d^3)\sqrt{b^2}\,\mathrm{C}\,(}{}$$

input
```
integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b
^2)*(b*x + a)/b) + (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 -
a^4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) - (3*b^2*d^
3*x + 8*b^2*c*d^2 - 5*a*b*d^3)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)
 + (pi^2*b^5*d^3*x^4 + 4*pi^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^
2*b^5*c^3*x)*fresnel_cos(b*x + a) - (pi*b^4*d^3*x^3 + pi*(4*b^4*c*d^2 - a*
b^3*d^3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d^2 + a^2*b^2*d^3)*x + pi*(4*b^
4*c^3 - 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^3*b*d^3))*sin(1/2*pi*b^2*x^2 +
 pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

3.128.6 Sympy [F]

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = \int (c+dx)^3\, C(a+bx)\, dx$$

input
```
integrate((d*x+c)**3*fresnelc(b*x+a),x)
```

output
```
Integral((c + d*x)**3*fresnelc(a + b*x), x)
```

3.128.7 Maxima [F]

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = \int (dx+c)^3\, \mathrm{C}\,(bx+a)\, dx$$

input
```
integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

3.128.8 Giac [F]

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = \int (dx+c)^3 \operatorname{C}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \operatorname{FresnelC}(a+bx)\,dx = \int \operatorname{FresnelC}(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(FresnelC(a + b*x)*(c + d*x)^3,x)
```

output
```
int(FresnelC(a + b*x)*(c + d*x)^3, x)
```

3.129 $\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx$

3.129.1 Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = & -\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}-\frac{(bc-ad)^3\operatorname{FresnelC}(a+bx)}{3b^3d}\\ &+\frac{(c+dx)^3\operatorname{FresnelC}(a+bx)}{3d}\\ &+\frac{d(bc-ad)\operatorname{FresnelS}(a+bx)}{b^3\pi}\\ &-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\ &-\frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\ &-\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}\end{aligned}$$

output
```
-2/3*d^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-1/3*(-a*d+b*c)^3*FresnelC(b*x+a)/b
^3/d+1/3*(d*x+c)^3*FresnelC(b*x+a)/d+d*(-a*d+b*c)*FresnelS(b*x+a)/b^3/Pi-(
-a*d+b*c)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c)*(b*x+a)*sin(1/2*Pi*(
b*x+a)^2)/b^3/Pi-1/3*d^2*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi
```

3.129.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.22

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \frac{-2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi^2(3ab^2c^2-3a^2bcd+a^3d^2+b^3x(3c^2+3cdx+d^2x^2))\operatorname{FresnelC}(a+bx)+3d(b}{}$$

input
```
Integrate[(c + d*x)^2*FresnelC[a + b*x],x]
```

output
```
(-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^
2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)
*Pi*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*P
i*Sin[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d
*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2
*d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

3.129.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx$$
$$\Big\downarrow \; 6983$$
$$\frac{(c+dx)^3 \operatorname{FresnelC}(a+bx)}{3d}-\frac{b\int (c+dx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{3d}$$
$$\Big\downarrow \; 3915$$
$$\frac{(c+dx)^3 \operatorname{FresnelC}(a+bx)}{3d}-$$
$$\frac{\int \left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^3+3d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^2+3d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-\right.}{3b^3d}$$
$$\Big\downarrow \; 2009$$

$$\frac{(c+dx)^3 \operatorname{FresnelC}(a+bx)}{3d} - \frac{-\frac{3d^2(bc-ad)\operatorname{FresnelS}(a+bx)}{\pi} + \frac{3d^2(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + (bc-ad)^3 \operatorname{FresnelC}(a+bx) + \frac{3d(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} +}{3b^3d}$$

input
```
Int[(c + d*x)^2*FresnelC[a + b*x],x]
```

output
```
((c + d*x)^3*FresnelC[a + b*x])/(3*d) - ((2*d^3*Cos[(Pi*(a + b*x)^2)/2])/P
i^2 + (b*c - a*d)^3*FresnelC[a + b*x] - (3*d^2*(b*c - a*d)*FresnelS[a + b*
x])/Pi + (3*d*(b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/Pi + (3*d^2*(b*c - a*
d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi + (d^3*(a + b*x)^2*Sin[(Pi*(a + b
*x)^2)/2])/Pi)/(3*b^3*d)
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\text{FresnelC}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{3(ad-bc)d^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{3b^2d}}{b}$
default	$\frac{-\frac{\text{FresnelC}(bx+a)(ad-bc-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{3(ad-bc)d^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{3b^2d}}{b}$
parts	$\frac{\text{FresnelC}(bx+a)d^2x^3}{3}+\text{FresnelC}\left(bx+a\right)dc\,x^2+\text{FresnelC}\left(bx+a\right)c^2x+\frac{\text{FresnelC}(bx+a)c^3}{3d}-$

input
```
int((d*x+c)^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*FresnelC(b*x+a)*(a*d-b*c-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(-d^3/Pi*(
b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+3*(a*d-b*c
)*d^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-3*(a*d-b*c)*d^2/Pi*FresnelS(b*x+a)-
3*(a*d-b*c)^2*d/Pi*sin(1/2*Pi*(b*x+a)^2)+(a*d-b*c)^3*FresnelC(b*x+a)))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.28

$$\int (c+dx)^2\,\text{FresnelC}(a+bx)\,dx$$
$$=\frac{\pi^2(3\,ab^2c^2-3\,a^2bcd+a^3d^2)\sqrt{b^2}\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-2\,bd^2\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+3\,\pi(bcd-ad^2)\sqrt{b^2}}{}$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_cos(sqrt
(b^2)*(b*x + a)/b) - 2*b*d^2*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) +
 3*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi^2
*b^4*d^2*x^3 + 3*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_cos(b*x + a)
 - (pi*b^3*d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^
2*c*d + a^2*b*d^2))*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4
)
```

3.129.6 Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\, dx = \int (c+dx)^2 C(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

output
```
Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

3.129.7 Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\, dx = \int (dx+c)^2 \operatorname{C}(bx+a)\, dx$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

3.129.8 Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\, dx = \int (dx+c)^2 \operatorname{C}(bx+a)\, dx$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)\,dx = \int \operatorname{FresnelC}(a+b\,x)\;(c+d\,x)^2\,dx$$

input
```
int(FresnelC(a + b*x)*(c + d*x)^2,x)
```

output
```
int(FresnelC(a + b*x)*(c + d*x)^2, x)
```

3.130 $\int (c + dx) \text{FresnelC}(a + bx) \, dx$

3.130.1 Optimal result

Integrand size = 12, antiderivative size = 122

$$\int (c+dx) \text{FresnelC}(a+bx) \, dx = -\frac{(bc-ad)^2 \text{FresnelC}(a+bx)}{2b^2 d} + \frac{(c+dx)^2 \text{FresnelC}(a+bx)}{2d} + \frac{d \, \text{FresnelS}(a+bx)}{2b^2 \pi} - \frac{(bc-ad) \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2 \pi} - \frac{d(a+bx) \sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2 \pi}$$

output
```
-1/2*(-a*d+b*c)^2*FresnelC(b*x+a)/b^2/d+1/2*(d*x+c)^2*FresnelC(b*x+a)/d+1/
2*d*FresnelS(b*x+a)/b^2/Pi-(-a*d+b*c)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*d*(
b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi
```

3.130.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.61

$$\int (c+dx) \text{FresnelC}(a+bx) \, dx = \frac{-\pi(a+bx)(ad-b(2c+dx)) \text{FresnelC}(a+bx) + d \, \text{FresnelS}(a+bx) + (-2bc+ad-bdx) \sin\left(\frac{1}{2}\pi(a+bx\right.}{2b^2\pi}$$

input
```
Integrate[(c + d*x)*FresnelC[a + b*x],x]
```

output
```
(-(Pi*(a + b*x)*(a*d - b*(2*c + d*x))*FresnelC[a + b*x]) + d*FresnelS[a +
b*x] + (-2*b*c + a*d - b*d*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

3.130.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \; 6983$$

$$\frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d}-\frac{b\int (c+dx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx}{2d}$$

$$\Big\downarrow \; 3915$$

$$\frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d}-\frac{\int \left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)^2+2d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)(bc-ad)+d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)d(a+bx)}{2b^2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{(c+dx)^2\,\text{FresnelC}(a+bx)}{2d}-\frac{(bc-ad)^2\,\text{FresnelC}(a+bx)+\frac{2d(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}-\frac{d^2\,\text{FresnelS}(a+bx)}{\pi}+\frac{d^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{2b^2d}$$

input
```
Int[(c + d*x)*FresnelC[a + b*x],x]
```

output
```
((c + d*x)^2*FresnelC[a + b*x])/(2*d) - ((b*c - a*d)^2*FresnelC[a + b*x] -
 (d^2*FresnelS[a + b*x])/Pi + (2*d*(b*c - a*d)*Sin[(Pi*(a + b*x)^2)/2])/Pi
 + (d^2*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi)/(2*b^2*d)
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\text{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\text{FresnelS}(bx+a)}{\pi}+\frac{(2ad-2bc)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
default	$\frac{-\frac{\text{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\text{FresnelS}(bx+a)}{\pi}+\frac{(2ad-2bc)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$
parts	$\frac{\text{FresnelC}(bx+a)\,d\,x^2}{2}+\text{FresnelC}\left(bx+a\right)cx-\frac{b\left(\frac{dx\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{da\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a\right.}{b^2\pi}\right.}{}\right.}{}$

input
```
int((d*x+c)*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-FresnelC(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(-
d/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+d/Pi*FresnelS(b*x+a)+(2*a*d-2*b*c)/Pi*s
in(1/2*Pi*(b*x+a)^2)))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx$$
$$=\frac{\pi(2\,abc-a^2d)\sqrt{b^2}\,\text{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+\sqrt{b^2}d\,\text{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)+(\pi b^3dx^2+2\,\pi b^3cx)\,\text{C}\,(bx+a)-(b^2dx+2\,b^2c-}{2\,\pi b^3}$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + s
qrt(b^2)*d*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi*b^3*d*x^2 + 2*pi*b^3*c
*x)*fresnel_cos(b*x + a) - (b^2*d*x + 2*b^2*c - a*b*d)*sin(1/2*pi*b^2*x^2
+ pi*a*b*x + 1/2*pi*a^2))/(pi*b^3)
```

3.130.6 Sympy [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (c+dx)\,C(a+bx)\;dx$$

input
```
integrate((d*x+c)*fresnelc(b*x+a),x)
```

output
```
Integral((c + d*x)*fresnelc(a + b*x), x)
```

3.130.7 Maxima [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (dx+c)\,\text{C}\,(bx+a)\;dx$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

3.130.8 Giac [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int (dx+c)\,\text{C}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelC}(a+bx)\,dx=\int \text{FresnelC}(a+b\,x)\,\,(c+d\,x)\,\,dx$$

input
```
int(FresnelC(a + b*x)*(c + d*x),x)
```

output
```
int(FresnelC(a + b*x)*(c + d*x), x)
```

3.131 $\int \text{FresnelC}(a+bx)\,dx$

3.131.1 Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \text{FresnelC}(a+bx)\,dx=\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

output
```
(b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

3.131.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \text{FresnelC}(a+bx)\,dx=\frac{a\,\text{FresnelC}(a+bx)}{b}+x\,\text{FresnelC}(a+bx)-\frac{\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)\sin\left(\frac{a^2\pi}{2}\right)}{b\pi}-\frac{\cos\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelC[a + b*x],x]
```

output
```
(a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^
2)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^
2)/2])/(b*Pi)
```

3.131.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6973}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \; 6973$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

input
```
Int[FresnelC[a + b*x],x]
```

output
```
((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

3.131.3.1 Defintions of rubi rules used

rule 6973
```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.131.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
default	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
parts	$x\ \text{FresnelC}\left(bx+a\right)-b\left(\frac{\sin\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	85

input
```
int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \text{FresnelC}(a+bx)\,dx=\frac{(\pi bx+\pi a)\,\text{C}\,(bx+a)-\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/
2*pi*a^2))/(pi*b)
```

3.131.6 Sympy [F]

$$\int \text{FresnelC}(a+bx)\,dx=\int C(a+bx)\;dx$$

input
```
integrate(fresnelc(b*x+a),x)
```

output
```
Integral(fresnelc(a + b*x), x)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \text{FresnelC}(a+bx)\,dx=\frac{(bx+a)\,\text{C}\,(bx+a)-\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a
^2)/pi)/b
```

3.131.8 Giac [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int C\,(bx+a)\;dx$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)\,dx = \int \text{FresnelC}(a+b\,x)\;dx$$

input
```
int(FresnelC(a + b*x),x)
```

output
```
int(FresnelC(a + b*x), x)
```

3.132 $\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx$

3.132.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx=\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)/(d*x+c),x)
```

3.132.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx=\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[FresnelC[a + b*x]/(c + d*x),x]
```

output
```
Integrate[FresnelC[a + b*x]/(c + d*x), x]
```

3.132.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{6989}$$
$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} dx$$

input
```
Int[FresnelC[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.132.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{dx+c} dx$$

input
```
int(FresnelC(b*x+a)/(d*x+c),x)
```

output
```
int(FresnelC(b*x+a)/(d*x+c),x)
```

3.132.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{C}\,(bx+a)}{dx+c}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)/(d*x + c), x)
```

3.132.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{C(a+bx)}{c+dx}\,dx$$

input
```
integrate(fresnelc(b*x+a)/(d*x+c),x)
```

output
```
Integral(fresnelc(a + b*x)/(c + d*x), x)
```

3.132.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx = \int \frac{\text{C}\,(bx+a)}{dx+c}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

3.132.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx=\int \frac{C\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

3.132.9 Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx}\,dx=\int \frac{\text{FresnelC}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(FresnelC(a + b*x)/(c + d*x),x)
```

output
```
int(FresnelC(a + b*x)/(c + d*x), x)
```

3.133 $\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx$

3.133.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx=\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)/(d*x+c)^2,x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx=\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[FresnelC[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{6989}$$
$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

input
```
Int[FresnelC[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{\left(dx+c\right)^{2}} dx$$

input
```
int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

output
```
int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\text{C}\left(bx+a\right)}{\left(dx+c\right)^2} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.133.6 Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} \, dx = \int \frac{C(a+bx)}{\left(c+dx\right)^2} \, dx$$

input
```
integrate(fresnelc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(fresnelc(a + b*x)/(c + d*x)**2, x)
```

3.133.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\text{C}\left(bx+a\right)}{\left(dx+c\right)^2} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

3.133.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{FresnelC}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\mathrm{C}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

3.133.9 Mupad [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{FresnelC}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{FresnelC}(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(FresnelC(a + b*x)/(c + d*x)^2,x)
```

output
```
int(FresnelC(a + b*x)/(c + d*x)^2, x)
```

3.134 $\int x^3 \operatorname{FresnelC}(a+bx)\,dx$

3.134.1 Optimal result

Integrand size = 10, antiderivative size = 227

$$\begin{aligned}\int x^3 \operatorname{FresnelC}(a+bx)\,dx =& \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} \\ &- \frac{a^4 \operatorname{FresnelC}(a+bx)}{4b^4} + \frac{3\operatorname{FresnelC}(a+bx)}{4b^4\pi^2} \\ &+ \frac{1}{4}x^4 \operatorname{FresnelC}(a+bx) + \frac{3a^2 \operatorname{FresnelS}(a+bx)}{2b^4\pi} \\ &+ \frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} \\ &+ \frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi}\end{aligned}$$

output
```
2*a*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4/P
i^2-1/4*a^4*FresnelC(b*x+a)/b^4+3/4*FresnelC(b*x+a)/b^4/Pi^2+1/4*x^4*Fresn
elC(b*x+a)+3/2*a^2*FresnelS(b*x+a)/b^4/Pi+a^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
-3/2*a^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi+a*(b*x+a)^2*sin(1/2*Pi*(b*x+
a)^2)/b^4/Pi-1/4*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

3.134.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.73

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \frac{5a\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bx\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + (3-a^4\pi^2+b^4\pi^2x^4)\operatorname{FresnelC}(a+bx) + 6a^2\pi\operatorname{FresnelS}(a$$

input
```
Integrate[x^3*FresnelC[a + b*x],x]
```

output
```
(5*a*Cos[(Pi*(a + b*x)^2)/2] - 3*b*x*Cos[(Pi*(a + b*x)^2)/2] + (3 - a^4*Pi
^2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi
*Sin[(Pi*(a + b*x)^2)/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*Pi*x
^2*Sin[(Pi*(a + b*x)^2)/2] - b^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi
^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \; 6983$$

$$\frac{1}{4}x^4\operatorname{FresnelC}(a+bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \; 3915$$

$$\frac{1}{4}x^4\operatorname{FresnelC}(a+bx) - \frac{\int \left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^4 - 4(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^3 + 6(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^2 - 4(a+bx)^3\cos\left(\frac{1}{2}\pi(\right.\right.}{4b^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}x^4\,\mathrm{FresnelC}(a+bx) - \frac{a^4\,\mathrm{FresnelC}(a+bx) - \frac{4a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{6a^2\,\mathrm{FresnelS}(a+bx)}{\pi} + \frac{6a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{3\,\mathrm{FresnelC}(a+bx)}{\pi^2} - \frac{4a(a+bx)^2\sin}{}}{4b^4}$$

input
```
Int[x^3*FresnelC[a + b*x],x]
```

output
```
(x^4*FresnelC[a + b*x])/4 - ((-8*a*Cos[(Pi*(a + b*x)^2)/2])/Pi^2 + (3*(a +
 b*x)*Cos[(Pi*(a + b*x)^2)/2])/Pi^2 + a^4*FresnelC[a + b*x] - (3*FresnelC[
a + b*x])/Pi^2 - (6*a^2*FresnelS[a + b*x])/Pi - (4*a^3*Sin[(Pi*(a + b*x)^2
)/2])/Pi + (6*a^2*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi - (4*a*(a + b*x)^2
*Sin[(Pi*(a + b*x)^2)/2])/Pi + ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/Pi)/(
4*b^4)
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)b^4x^4}{4}-\frac{a^4\,\text{FresnelC}(bx+a)}{4}+\frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{3a^2\,\text{FresnelS}(bx+a)}{2\pi}+\frac{a(bx+a)^2\,\text{si}}{}}{b^4}$
default	$\frac{\frac{\text{FresnelC}(bx+a)b^4x^4}{4}-\frac{a^4\,\text{FresnelC}(bx+a)}{4}+\frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{3a^2\,\text{FresnelS}(bx+a)}{2\pi}+\frac{a(bx+a)^2\,\text{si}}{}}{b^4}$
parts	$\frac{x^4\,\text{FresnelC}(bx+a)}{4}-\frac{b\left(\frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi\right.}{b^2\pi}\right.}{}\right.}{}\right.}{}$

input

```
int(x^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4*a^4*FresnelC(b*x+a)+a^3/Pi*sin(1/2*
Pi*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS
(b*x+a)+a/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2
)-1/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*
(b*x+a)^2)+1/Pi*FresnelC(b*x+a)))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.78

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \frac{\pi^2 b^5 x^4 \operatorname{C}(bx+a) + 6\,\pi a^2\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (\pi^2 a^4 - 3)\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - (3\,b^2 x - 5\,ab)\cos\left(\frac{1}{2}\,\pi b^2 x^2 -\right.}{4\,\pi^2 b^5}$$

input
```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(pi^2*b^5*x^4*fresnel_cos(b*x + a) + 6*pi*a^2*sqrt(b^2)*fresnel_sin(sq
rt(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x
 + a)/b) - (3*b^2*x - 5*a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) -
 (pi*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*sin(1/2*pi*b^2*x^2
+ pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

3.134.6 Sympy [F]

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3 C(a+bx)\,dx$$

input
```
integrate(x**3*fresnelc(b*x+a),x)
```

output
```
Integral(x**3*fresnelc(a + b*x), x)
```

3.134.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.21

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \frac{1}{4}\,x^4 \operatorname{C}(bx+a)$$
$$+ \frac{\left(16\left(-i\,\pi^2 e^{\left(\frac{1}{2}i\,\pi b^2 x^2 + i\,\pi abx + \frac{1}{2}i\,\pi a^2\right)} + i\,\pi^2 e^{\left(-\frac{1}{2}i\,\pi b^2 x^2 - i\,\pi abx - \frac{1}{2}i\,\pi a^2\right)}\right)a^4 + 32\left(\pi\Gamma\left(2, \frac{1}{2}i\,\pi b^2 x^2 + i\,\pi abx + \frac{1}{2}i\,\pi\right.\right.\right.}{}$$

input
```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*fresnel_cos(b*x + a) + 1/32*(16*(-I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*p
i*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I
*pi*a^2))*a^4 + 32*(pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a
^2) + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16
*((-I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1
/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(pi*gamma(2, 1/2*I*p
i*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I
*pi*a*b*x - 1/2*I*pi*a^2))*a)*b*x + (((I - 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(1
/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I + 1)*sqrt(2)*pi^(5
/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 +
12*(-(I + 1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) + (I - 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/
2*I*pi*a^2))*a^2 + (4*I - 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) - (4*I + 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*p
i*a*b*x - 1/2*I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi
^3*b^6*x + pi^3*a*b^5)
```

3.134.8 Giac [F]

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3\, C\left(bx+a\right)\,dx$$

input
```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_cos(b*x + a), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(a+bx)\,dx = \int x^3 \operatorname{FresnelC}(a+b\,x)\,dx$$

input
```
int(x^3*FresnelC(a + b*x),x)
```

output
```
int(x^3*FresnelC(a + b*x), x)
```

3.135 $\int x^2 \operatorname{FresnelC}(a+bx)\,dx$

3.135.1 Optimal result

Integrand size = 10, antiderivative size = 148

$$\begin{aligned}\int x^2 \operatorname{FresnelC}(a+bx)\,dx = &-\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}+\frac{a^3 \operatorname{FresnelC}(a+bx)}{3b^3}\\ &+\frac{1}{3}x^3 \operatorname{FresnelC}(a+bx)-\frac{a \operatorname{FresnelS}(a+bx)}{b^3\pi}\\ &-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}\\ &-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}\end{aligned}$$

output
```
-2/3*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2+1/3*a^3*FresnelC(b*x+a)/b^3+1/3*x^3*Fr
esnelC(b*x+a)-a*FresnelS(b*x+a)/b^3/Pi-a^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*
(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-1/3*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b
^3/Pi
```

3.135.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx = -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-\pi^2(a^3+b^3x^3)\operatorname{FresnelC}(a+bx)+3a\pi\operatorname{FresnelS}(a+bx)+a^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

input
```
Integrate[x^2*FresnelC[a + b*x],x]
```

output
```
-1/3*(2*Cos[(Pi*(a + b*x)^2)/2] - Pi^2*(a^3 + b^3*x^3)*FresnelC[a + b*x] +
 3*a*Pi*FresnelS[a + b*x] + a^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Sin[
(Pi*(a + b*x)^2)/2] + b^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi^2)
```

3.135.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \text{6983}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(a+bx) - \frac{1}{3}b\int x^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \text{3915}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(a+bx) - \frac{\int \left(-\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^3 + 3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^2 - 3(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a + (a+bx)^3\cos\left(\frac{1}{2}\pi(\right.\right.}{3b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(a+bx) - \frac{a^3(-\operatorname{FresnelC}(a+bx)) + \frac{3a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + \frac{3a\operatorname{FresnelS}(a+bx)}{\pi} - \frac{3a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + \frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} + \frac{2\,\mathrm{co}}{}}{3b^3}$$

input
```
Int[x^2*FresnelC[a + b*x],x]
```

output
```
(x^3*FresnelC[a + b*x])/3 - ((2*Cos[(Pi*(a + b*x)^2)/2])/Pi^2 - a^3*Fresne
lC[a + b*x] + (3*a*FresnelS[a + b*x])/Pi + (3*a^2*Sin[(Pi*(a + b*x)^2)/2])
/Pi - (3*a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/Pi + ((a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/Pi)/(3*b^3)
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.135.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx+a)b^3x^3}{3}+\frac{a^3\,\mathrm{FresnelC}(bx+a)}{3}-\frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{a\,\mathrm{FresnelS}(bx+a)}{\pi}-\frac{(bx+a)^2\sin\left(\frac{\pi(b\cdots}{}\right)}{3\pi}\cdots}{b^3}$
default	$\frac{\frac{\mathrm{FresnelC}(bx+a)b^3x^3}{3}+\frac{a^3\,\mathrm{FresnelC}(bx+a)}{3}-\frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{a\,\mathrm{FresnelS}(bx+a)}{\pi}-\frac{(bx+a)^2\sin\left(\frac{\pi(b\cdots}{}\right)}{3\pi}\cdots}{b^3}$
parts	$\frac{x^3\,\mathrm{FresnelC}(bx+a)}{3}-\frac{b\left(\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots\right)}{\cdots}$

input
```
int(x^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*FresnelC(b*x+a)*b^3*x^3+1/3*a^3*FresnelC(b*x+a)-a^2/Pi*sin(1/2*
Pi*(b*x+a)^2)+a/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-a/Pi*FresnelS(b*x+a)-1/3/
Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*cos(1/2*Pi*(b*x+a)^2))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx$$
$$= \frac{\pi^2 b^4 x^3\,\mathrm{C}\left(bx+a\right) + \pi^2 a^3 \sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 3\,\pi a \sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) - 2\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right) -}{3\,\pi^2 b^4}$$

input
```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(pi^2*b^4*x^3*fresnel_cos(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_cos(sq
rt(b^2)*(b*x + a)/b) - 3*pi*a*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b)
 - 2*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi*b^3*x^2 - pi*a*b^
2*x + pi*a^2*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

3.135.6 Sympy [F]

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx = \int x^2 C(a+bx)\;dx$$

input
```
integrate(x**2*fresnelc(b*x+a),x)
```

output
```
Integral(x**2*fresnelc(a + b*x), x)
```

3.135.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.86

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx = \frac{1}{3}x^3\operatorname{C}(bx+a) - \frac{\left(12\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+i\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^3+4\left(3\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+i\pi\right.\right.\right.}{}$$

input
```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*fresnel_cos(b*x + a) - 1/24*(12*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*
a*b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*
a^2))*a^3 + 4*(3*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) +
 I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 2*gamma(2,
1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*gamma(2, -1/2*I*pi*b^2*x
^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(gamma(2, 1/2*I*pi*b^2*x^2 + I*
pi*a*b*x + 1/2*I*pi*a^2) + gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I
*pi*a^2)) + sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*(((I - 1)*sqrt(2)*p
i^(3/2)*(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I
 + 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi
*a^2)) - 1))*a^3 + 6*(-(I + 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*
a*b*x + 1/2*I*pi*a^2) + (I - 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*p
i*a*b*x - 1/2*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

3.135.8 Giac [F]

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx = \int x^2\operatorname{C}(bx+a)\,dx$$

input
```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_cos(b*x + a), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(a+bx)\,dx = \int x^2 \operatorname{FresnelC}(a+b\,x)\ dx$$

input
```
int(x^2*FresnelC(a + b*x),x)
```

output
```
int(x^2*FresnelC(a + b*x), x)
```

3.136 $\int x \operatorname{FresnelC}(a+bx)\,dx$

3.136.1 Optimal result

Integrand size = 8, antiderivative size = 95

$$\int x \operatorname{FresnelC}(a+bx)\,dx = -\frac{a^2 \operatorname{FresnelC}(a+bx)}{2b^2}+\frac{1}{2}x^2 \operatorname{FresnelC}(a+bx)+\frac{\operatorname{FresnelS}(a+bx)}{2b^2\pi}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

output
```
-1/2*a^2*FresnelC(b*x+a)/b^2+1/2*x^2*FresnelC(b*x+a)+1/2*FresnelS(b*x+a)/b
^2/Pi+a*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2
/Pi
```

3.136.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int x \operatorname{FresnelC}(a+bx)\,dx = \frac{(-a^2\pi+b^2\pi x^2)\operatorname{FresnelC}(a+bx)+\operatorname{FresnelS}(a+bx)+(a-bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

input
```
Integrate[x*FresnelC[a + b*x],x]
```

output
```
((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x
)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

3.136.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6983, 3915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6983}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{1}{2}b\int x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)dx$$

$$\Big\downarrow \text{ 3915}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{\int\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a^2-2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)a+(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\right)d(a+bx)}{2b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(a+bx)-\frac{a^2\,\text{FresnelC}(a+bx)-\frac{\text{FresnelS}(a+bx)}{\pi}-\frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}+\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi}}{2b^2}$$

input
```
Int[x*FresnelC[a + b*x],x]
```

output
```
(x^2*FresnelC[a + b*x])/2 - (a^2*FresnelC[a + b*x] - FresnelS[a + b*x]/Pi
- (2*a*Sin[(Pi*(a + b*x)^2)/2])/Pi + ((a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/P
i)/(2*b^2)
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3915
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Simp[k/f^(m + 1)   Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x
^(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x
]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

rule 6983
```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Simp[b/(d*(m +
1))   Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
 c, d}, x] && IGtQ[m, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\text{FresnelC}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\text{FresnelS}(bx+a)}{2\pi}}{b^2}$
default	$\frac{\text{FresnelC}(bx+a)\left(-(bx+a)a+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\text{FresnelS}(bx+a)}{2\pi}}{b^2}$
parts	$\frac{x^2\,\text{FresnelC}(bx+a)}{2}-\frac{b\left(\frac{x\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{a\left(\frac{\sin\left(\frac{1}{2}b^2\pi x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\,\text{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)}{b}-\cdots\right.}{2}$

input
```
int(x*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(FresnelC(b*x+a)*(-(b*x+a)*a+1/2*(b*x+a)^2)+a/Pi*sin(1/2*Pi*(b*x+a)^
2)-1/2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a))
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int x\,\text{FresnelC}(a+bx)\,dx$$
$$=\frac{\pi b^3x^2\,\mathrm{C}\left(bx+a\right)-\pi a^2\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)-\left(b^2x-ab\right)\sin\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)+\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{2\pi b^3}$$

input
```
integrate(x*fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(pi*b^3*x^2*fresnel_cos(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_cos(sqrt(b
^2)*(b*x + a)/b) - (b^2*x - a*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2) + sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)
```

3.136.6 Sympy [F]

$$\int x \,\text{FresnelC}(a+bx)\,dx = \int xC(a+bx)\,dx$$

input
```
integrate(x*fresnelc(b*x+a),x)
```

output
```
Integral(x*fresnelc(a + b*x), x)
```

3.136.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.27

$$\int x \,\text{FresnelC}(a+bx)\,dx = \frac{1}{2}x^2\,\text{C}\,(bx+a) + \frac{\left(8\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+i\,\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)abx+8\left(-i\,\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+i\,\pi e^{(}\right.\right.}{}$$

input
```
integrate(x*fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*fresnel_cos(b*x + a) + 1/16*(8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a
*b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a
^2))*a*b*x + 8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I
*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2
*x^2 + 4*pi*a*b*x + 2*pi*a^2)*((-(I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*
pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I + 1)*sqrt(2)*pi^(3/2)*(
erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 + (2*I
+ 2)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) - (2
*I - 2)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
)*b/(pi^2*b^4*x + pi^2*a*b^3)
```

3.136.8 Giac [F]

$$\int x\,\text{FresnelC}(a+bx)\,dx=\int x\,C\left(bx+a\right)\,dx$$

input
```
integrate(x*fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*fresnel_cos(b*x + a), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int x\,\text{FresnelC}(a+bx)\,dx=\int x\,\text{FresnelC}(a+b\,x)\,dx$$

input
```
int(x*FresnelC(a + b*x),x)
```

output
```
int(x*FresnelC(a + b*x), x)
```

3.137 $\int \text{FresnelC}(a + bx) \, dx$

3.137.1 Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \text{FresnelC}(a + bx) \, dx = \frac{(a + bx) \text{FresnelC}(a + bx)}{b} - \frac{\sin \left(\frac{1}{2}\pi(a + bx)^2\right)}{b\pi}$$

output
```
(b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

3.137.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \text{FresnelC}(a + bx) \, dx = \frac{a \text{FresnelC}(a + bx)}{b} + x \text{FresnelC}(a + bx) - \frac{\cos \left(ab\pi x + \frac{1}{2}b^2\pi x^2\right) \sin \left(\frac{a^2\pi}{2}\right)}{b\pi} - \frac{\cos \left(\frac{a^2\pi}{2}\right) \sin \left(ab\pi x + \frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelC[a + b*x],x]
```

output
```
(a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^
2)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^
2)/2])/(b*Pi)
```

3.137.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6973}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(a+bx)\,dx$$

$$\Big\downarrow \text{ 6973}$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

input
```
Int[FresnelC[a + b*x],x]
```

output
```
((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

3.137.3.1 Defintions of rubi rules used

rule 6973
```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

3.137.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
default	$\frac{\text{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
parts	$x\ \text{FresnelC}\left(bx+a\right)-b\left(\frac{\sin\left(\frac{1}{2}b^2\pi\,x^2+\pi abx+\frac{1}{2}\pi\,a^2\right)}{b^2\pi}-\frac{\sqrt{\pi}\,a\ \text{FresnelC}\left(\frac{b^2\pi x+\pi ba}{\sqrt{\pi}\,\sqrt{b^2\pi}}\right)}{b\sqrt{b^2\pi}}\right)$	85

input
```
int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(\pi bx+\pi a)\,\text{C}\,(bx+a)-\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
```

output
```
((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/
2*pi*a^2))/(pi*b)
```

3.137.6 Sympy [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int C(a+bx)\;dx$$

input
```
integrate(fresnelc(b*x+a),x)
```

output
```
Integral(fresnelc(a + b*x), x)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \text{FresnelC}(a+bx)\,dx = \frac{(bx+a)\,\text{C}\,(bx+a)-\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a
^2)/pi)/b
```

3.137.8 Giac [F]

$$\int \text{FresnelC}(a+bx)\,dx = \int \text{C}\left(bx+a\right)\,dx$$

input
```
integrate(fresnel_cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)\,dx = \int \text{FresnelC}(a+b\,x)\,dx$$

input
```
int(FresnelC(a + b*x),x)
```

output
```
int(FresnelC(a + b*x), x)
```

3.138 $\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$

3.138.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x}, x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)/x,x)
```

3.138.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx = \int \frac{\text{FresnelC}(a+bx)}{x} \, dx$$

input
```
Integrate[FresnelC[a + b*x]/x,x]
```

output
```
Integrate[FresnelC[a + b*x]/x, x]
```

3.138.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx$$
$$\Big\downarrow \text{ 6989}$$
$$\int \frac{\text{FresnelC}(a+bx)}{x}dx$$

input
```
Int[FresnelC[a + b*x]/x,x]
```

output
```
$Aborted
```

3.138.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.138.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{x}dx$$

input
```
int(FresnelC(b*x+a)/x,x)
```

output
```
int(FresnelC(b*x+a)/x,x)
```

3.138.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx = \int \frac{\text{C}\,(bx+a)}{x} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)/x, x)
```

3.138.6 Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx = \int \frac{C(a+bx)}{x} \, dx$$

input
```
integrate(fresnelc(b*x+a)/x,x)
```

output
```
Integral(fresnelc(a + b*x)/x, x)
```

3.138.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x} \, dx = \int \frac{\text{C}\,(bx+a)}{x} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)/x, x)
```

3.138.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx = \int \frac{\text{C}\,(bx+a)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)/x, x)
```

3.138.9 Mupad [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx = \int \frac{\text{FresnelC}(a+b\,x)}{x}\,dx$$

input
```
int(FresnelC(a + b*x)/x,x)
```

output
```
int(FresnelC(a + b*x)/x, x)
```

3.139 $\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$

3.139.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x^2},x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)/x^2,x)
```

3.139.2 Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx$$

input
```
Integrate[FresnelC[a + b*x]/x^2,x]
```

output
```
Integrate[FresnelC[a + b*x]/x^2, x]
```

3.139.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \; 6989$$

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}dx$$

input
```
Int[FresnelC[a + b*x]/x^2,x]
```

output
```
$Aborted
```

3.139.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.139.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{x^2}dx$$

input
```
int(FresnelC(b*x+a)/x^2,x)
```

output
```
int(FresnelC(b*x+a)/x^2,x)
```

3.139.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx = \int \frac{\text{C}\,(bx+a)}{x^2} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)/x^2, x)
```

3.139.6 Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx = \int \frac{C(a+bx)}{x^2} \, dx$$

input
```
integrate(fresnelc(b*x+a)/x**2,x)
```

output
```
Integral(fresnelc(a + b*x)/x**2, x)
```

3.139.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} \, dx = \int \frac{\text{C}\,(bx+a)}{x^2} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)/x^2, x)
```

3.139.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx = \int \frac{\text{C}\left(bx+a\right)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)/x^2, x)
```

3.139.9 Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(a+bx)}{x^2}\,dx = \int \frac{\text{FresnelC}(a+b\,x)}{x^2}\,dx$$

input
```
int(FresnelC(a + b*x)/x^2,x)
```

output
```
int(FresnelC(a + b*x)/x^2, x)
```

3.140 $\int x^7 \operatorname{FresnelC}(bx)^2 \, dx$

3.140.1 Optimal result

Integrand size = 10, antiderivative size = 253

$$\begin{aligned}\int x^7 \operatorname{FresnelC}(bx)^2 \, dx = &-\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4} \\ &- \frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{4b^7\pi^4} \\ &- \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{4b^3\pi^2} - \frac{105\operatorname{FresnelC}(bx)^2}{8b^8\pi^4} \\ &+ \frac{1}{8}x^8\operatorname{FresnelC}(bx)^2 + \frac{35x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} \\ &- \frac{x^7\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} - \frac{10\sin(b^2\pi x^2)}{b^8\pi^5} + \frac{5x^4\sin(b^2\pi x^2)}{8b^4\pi^3}\end{aligned}$$

output
```
-105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2+55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4-
1/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2+105/4*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^7/Pi^4-7/4*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2-105/8*Fresne
lC(b*x)^2/b^8/Pi^4+1/8*x^8*FresnelC(b*x)^2+35/4*x^3*FresnelC(b*x)*sin(1/2*
b^2*Pi*x^2)/b^5/Pi^3-1/4*x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-10*sin
(b^2*Pi*x^2)/b^8/Pi^5+5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

3.140.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx$$
$$= \frac{-315b^2\pi x^2 + 7b^6\pi^3x^6 - 3b^2\pi x^2(-55 + b^4\pi^2x^4)\cos(b^2\pi x^2) + 6\pi(-105 + b^8\pi^4x^8)\operatorname{FresnelC}(bx)^2 - 12b\pi x}{}$$

input
```
Integrate[x^7*FresnelC[b*x]^2,x]
```

output
```
(-315*b^2*Pi*x^2 + 7*b^6*Pi^3*x^6 - 3*b^2*Pi*x^2*(-55 + b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 6*Pi*(-105 + b^8*Pi^4*x^8)*FresnelC[b*x]^2 - 12*b*Pi*x*Fresn
elC[b*x]*(7*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-35 + b
^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 480*Sin[b^2*Pi*x^2] + 30*b^4*Pi^2*x^4*
Sin[b^2*Pi*x^2])/(48*b^8*Pi^5)
```

3.140.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx$$
$$\Big\downarrow \; 6985$$
$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$
$$\Big\downarrow \; 7009$$
$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^7 \sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$
$$\Big\downarrow \; 3860$$
$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^6 \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$
$$\Big\downarrow \; 3042$$

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^6 \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\int x^4\cos(b^2\pi x^2)dx^2}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3042

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\int x^4\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3777

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\,\text{s}}{\pi b^2}\right.$$

$\downarrow$ 25

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\,\text{si}}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \text{FresnelC}(bx)\,\text{si}}{\pi b^2}\right.$$

$\downarrow$ 3777

3.140. $\int x^7 \text{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2} - \frac{2\left(\frac{\int\cos(b^2\pi x^2)dx^2}{\pi b^2} - \frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7 \operatorname{Fr}\ldots\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2} - \frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2} - \frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b} + \frac{x^7\ldots\right.$$

$\downarrow$ 3117

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} + \frac{x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\frac{3\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2} - \frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4} - \frac{x^2\cos(\pi b^2 x\ldots}{\pi b^2}\right.}{\pi b^2}\ldots\right.}{\pi b^2}\ldots}{4\pi b}\ldots\right.$$

$\downarrow$ 7017

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} - \frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} + \frac{x^7 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2\ldots\right.}{\pi b^2}\ldots\right.$$

$\downarrow$ 3861

3.140. $\int x^7 \operatorname{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\right.}{\pi b^2}\right.$$

$\downarrow$ 3790

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx}{\pi b}\right.$$

$\downarrow$ 15

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^6}{6}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\downarrow$ 25

3.140. $\int x^7\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\operatorname{FresnelC}(bx)\sin\left(\right.}{\pi b^2}\right.$$

$$\downarrow 3042$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\operatorname{FresnelC}(bx)\operatorname{si}}{\pi b^2}\right.$$

$$\downarrow 3777$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\mathrm{F}}{}\right.$$

$$\downarrow 25$$

$$\frac{1}{8}x^8 \operatorname{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\mathrm{Fr}}{}\right.$$

$$\downarrow 3042$$

3.140. $\int x^7 \operatorname{FresnelC}(bx)^2\, dx$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2-\frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{Fr}\ldots}{}\right.$$

$\Big\downarrow$ 3777

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2-\frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2\ldots\right.}{\pi b^2}\ldots}{\pi b^2}\right.$$

$\Big\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2-\frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\ldots\right.}{\pi b^2}\ldots}{\pi b^2}\right.$$

$\Big\downarrow$ 3117

3.140. $\int x^7\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2} - \frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2} - \frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right) + \frac{x^6}{6}}{2\pi b}\right)}{\pi b^2}\right.$$

$\downarrow$ 7009

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{1}{2}\left(\frac{x^4 s}{}\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3860

3.140. $\int x^7\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 -$$

$$\frac{1}{4}b\left(-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(x^4\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8\,\text{FresnelC}(bx)^2 -$$

$$\frac{1}{4}b\left(-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(x^4\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3777

3.140. $\int x^7\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{8}x^8 \text{FresnelC}(bx)^2 - \frac{1}{4}b\left(-\frac{7\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\right.}{\pi b^2}}{\pi b^2}\right.$$

input
```
Int[x^7*FresnelC[b*x]^2,x]
```

output
```
$Aborted
```

3.140. $\int x^7 \text{FresnelC}(bx)^2 \, dx$

3.140.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.140.4 Maple [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx$$

input
```
int(x^7*FresnelC(b*x)^2,x)
```

output
```
int(x^7*FresnelC(b*x)^2,x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx$$
$$= \frac{5\,\pi^3 b^6 x^6 - 240\,\pi b^2 x^2 - 3\left(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 42\left(\pi^3 b^5 x^5 - 15\,\pi b x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) -}{24}$$

input
```
integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/24*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*c
os(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*f
resnel_cos(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_cos(b*x)^2 + 6*(5*(pi^
2*b^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fre
snel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

3.140.6 Sympy [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7 C^2(bx)\;dx$$

input
```
integrate(x**7*fresnelc(b*x)**2,x)
```

output
```
Integral(x**7*fresnelc(b*x)**2, x)
```

3.140.7 Maxima [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7\,\mathrm{C}\left(bx\right)^2\;dx$$

input
```
integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^7*fresnel_cos(b*x)^2, x)
```

3.140.8 Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx)^2\,dx = \int x^7\,\mathrm{C}\left(bx\right)^2\;dx$$

input
```
integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_cos(b*x)^2, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx)^2 \, dx = \int x^7 \operatorname{FresnelC}(b\,x)^2 \, dx$$

input
```
int(x^7*FresnelC(b*x)^2,x)
```

output
```
int(x^7*FresnelC(b*x)^2, x)
```

3.141 $\int x^6 \operatorname{FresnelC}(bx)^2 \, dx$

3.141.1 Optimal result

Integrand size = 10, antiderivative size = 239

$$\begin{aligned}\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = & -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x \cos(b^2\pi x^2)}{8b^6\pi^4} \\ & - \frac{x^5 \cos(b^2\pi x^2)}{14b^2\pi^2} + \frac{96 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{7b^7\pi^4} \\ & - \frac{12x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{7b^3\pi^2} + \frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 \\ & - \frac{531 \operatorname{FresnelC}\left(\sqrt{2}bx\right)}{56\sqrt{2}b^7\pi^4} + \frac{48x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} \\ & - \frac{2x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{17x^3 \sin(b^2\pi x^2)}{28b^4\pi^3}\end{aligned}$$

output
```
-48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2+21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4-1/14*x^
5*cos(b^2*Pi*x^2)/b^2/Pi^2+96/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^7/Pi^4
-12/7*x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/7*x^7*FresnelC(b*x)
^2+48/7*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-2/7*x^6*FresnelC(b*
x)*sin(1/2*b^2*Pi*x^2)/b/Pi+17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3-531/112*Fre
snelC(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx$$
$$= \frac{80b^7\pi^4x^7 \operatorname{FresnelC}(bx)^2 - 2655\sqrt{2} \operatorname{FresnelC}\left(\sqrt{2}bx\right) - 160 \operatorname{FresnelC}(bx) \left(6(-8 + b^4\pi^2x^4) \cos\left(\frac{1}{2}b^2\pi x^2\right) + \right.}{}$$

input
```
Integrate[x^6*FresnelC[b*x]^2,x]
```

output
```
(80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 16
0*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-
24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*C
os[b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2]
)))/(560*b^7*Pi^4)
```

3.141.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.91, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {6985, 7009, 3866, 3867, 3866, 3833, 7017, 3873, 15, 3867, 3866, 3833, 7009, 3866, 3833, 7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6985}$$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b \int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \text{ 7009}$$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 -$$
$$\frac{2}{7}b\left(-\frac{6\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} - \frac{\int x^6 \sin\left(b^2\pi x^2\right) dx}{2\pi b} + \frac{x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{5\int x^4\cos(b^2\pi x^2)dx}{2\pi b^2} - \frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3867

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2} - \frac{3\int x^2\sin(b^2\pi x^2)dx}{2\pi b^2}\right)}{2\pi b^2} - \frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3866

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2} - \frac{3\left(\frac{\int\cos(b^2\pi x^2)dx}{2\pi b^2} - \frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2} - \frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3833

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} + \frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2} - \frac{3\left(\frac{\operatorname{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3} - \frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2} - \frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}\right)$$

$\downarrow$ 7017

3.141. $\int x^6 \operatorname{FresnelC}(bx)^2\, dx$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\big\downarrow$ 3873

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx+\frac{\int x^4dx}{2}}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\big\downarrow$ 15

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx+\frac{x^5}{10}}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\right.}{\pi b^2}\right.$$

$\big\downarrow$ 3867

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{Fre}}{}\right.$$

$\big\downarrow$ 3866

3.141. $\int x^6 \operatorname{FresnelC}(bx)^2\,dx$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right.}{\pi b^2}\right.$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{1\cdots}}{\pi b}\right.}{\pi b^2}\right.$$

$$\Big\downarrow \; 7009$$

3.141. $\int x^6 \operatorname{FresnelC}(bx)^2 \, dx$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\left(-\frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{\pi b^2}\cdots\right.$$

$$\Big\downarrow \; 3866$$

$$\frac{1}{7}x^7 \operatorname{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\left(-\frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\cdots\right)}{\pi b^2}\cdots\right.$$

$$\Big\downarrow \; 3833$$

3.141. $\int x^6 \operatorname{FresnelC}(bx)^2\, dx$

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\cdots\right)}{\pi b^2}\cdots\right)$$

$\downarrow$ 7015

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\left(-\frac{2\left(\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}\cdots}{\cdots}\right)}{\pi b^2}\cdots\right)$$

$\downarrow$ 3839

3.141. $\int x^6\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2}{7}b\left(-\frac{6\left(\frac{4\left(-\frac{2\left(\frac{\int\left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{Fre}}{}\right.}{\pi b^2}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{7}x^7\,\text{FresnelC}(bx)^2 - \frac{2}{7}b\left(\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}-\frac{6\left(-\frac{x^4\,\text{FresnelC}}{}\right.}{}\right.$$

input
```
Int[x^6*FresnelC[b*x]^2,x]
```

3.141. $\int x^6\,\text{FresnelC}(bx)^2\,dx$

output
```
(x^7*FresnelC[b*x]^2)/7 - (2*b*((x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^2*Pi) - (-1/2*(x^5*Cos[b^2*Pi*x^2])/(b^2*Pi) + (5*((-3*(-1/2*(x*Cos[b^2*P
i*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) +
 (x^3*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) - (6*(-((x^4*Cos[
(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (4*(-1/2*(-1/2*(x*Cos[b^2*Pi*x^
2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(b*Pi) - (2*(-((C
os[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (x/2 + FresnelC[Sqrt[2]*b*x]
/(2*Sqrt[2]*b))/(b*Pi)))/(b^2*Pi) + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(b^2*Pi)))/(b^2*Pi) + (x^5/10 + ((-3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi)
+ FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) + (x^3*Sin[b^2*Pi*
x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b^2*Pi)))/7
```

3.141.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.141.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^7 x^7}{7}-2\,\mathrm{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{7\pi}\right)}{}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^7 x^7}{7}-2\,\mathrm{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{7\pi}\right)}{}$

input
```
int(x^6*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^7*(1/7*FresnelC(b*x)^2*b^7*x^7-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2
*b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*
sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*b^5*x^5*Pi
^2-8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos
(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*
2^(1/2)))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3
*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresne
lC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1
/2))))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int x^6 \,\mathrm{FresnelC}(bx)^2\,dx$$
$$= \frac{80\,\pi^4 b^8 x^7\,\mathrm{C}\left(bx\right)^2+136\,\pi^2 b^6 x^5-5310\,b^2 x-20\left(4\,\pi^2 b^6 x^5-147\,b^2 x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2-960\left(\pi^2 b^5 x^4-8\,b\right)\mathrm{c}}{}$$

input
```
integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/560*(80*pi^4*b^8*x^7*fresnel_cos(b*x)^2 + 136*pi^2*b^6*x^5 - 5310*b^2*x
- 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^
4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 2655*sqrt(2)*sqrt(b^2)*fre
snel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*
(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^
4*b^8)
```

3.141.6 Sympy [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 C^2(bx) \, dx$$

input
```
integrate(x**6*fresnelc(b*x)**2,x)
```

output
```
Integral(x**6*fresnelc(b*x)**2, x)
```

3.141.7 Maxima [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^6*fresnel_cos(b*x)^2, x)
```

3.141.8 Giac [F]

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_cos(b*x)^2, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelC}(bx)^2 \, dx = \int x^6 \operatorname{FresnelC}(b\,x)^2 \, dx$$

input
```
int(x^6*FresnelC(b*x)^2,x)
```

output
```
int(x^6*FresnelC(b*x)^2, x)
```

3.142 $\int x^5 \operatorname{FresnelC}(bx)^2 \, dx$

3.142.1 Optimal result

Integrand size = 10, antiderivative size = 265

$$\begin{aligned}\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = {} & \frac{5x^4}{24b^2\pi^2} + \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} - \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} \\ & - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{6}x^6\operatorname{FresnelC}(bx)^2 \\ & - \frac{5\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} \\ & + \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^4\pi^3} + \frac{5x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} \\ & - \frac{x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3}\end{aligned}$$

output
```
5/24*x^4/b^2/Pi^2+11/6*cos(b^2*Pi*x^2)/b^6/Pi^4-1/12*x^4*cos(b^2*Pi*x^2)/b
^2/Pi^2-5/3*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/6*x^6*Fresnel
C(b*x)^2-5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3-5/8*I*x^2*hypergeom([1,
1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^
3-1/3*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+7/12*x^2*sin(b^2*Pi*x^2)/
b^4/Pi^3
```

3.142.2 Mathematica [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx = \int x^5 \operatorname{FresnelC}(bx)^2\,dx$$

input
```
Integrate[x^5*FresnelC[b*x]^2,x]
```

output
```
Integrate[x^5*FresnelC[b*x]^2, x]
```

3.142.3 Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.34, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.600, Rules used = {6985, 7009, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3118, 7017, 3861, 3042, 3790, 15, 25, 3042, 3777, 25, 3042, 3118, 7009, 3860, 3042, 3118, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx$$

$$\Big\downarrow \; 6985$$

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2-\frac{1}{3}b\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \; 7009$$

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^5\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3777

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\int x^2\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3042

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\int x^2\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\big\downarrow$ 3777

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{\int-\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\big\downarrow$ 25

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin(}{\pi b^2}\right.$$

$\big\downarrow$ 3042

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin(}{\pi b^2}\right.$$

$\big\downarrow$ 3118

3.142. $\int x^5\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$$\frac{1}{3}b\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b}{\pi b^2}}{4\pi b}\right.$$

$\downarrow$ 7017

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3861

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\right.}{\pi b^2}\right.$$

$\downarrow$ 3790

$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx}{\pi b}\right.$$

$\downarrow$ 15

3.142. $\int x^5\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^4}{4}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\downarrow$ 25

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin(}{\pi b^2}\right.$$

$\downarrow$ 3042

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\downarrow$ 3777

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{Fres}}{}\right.$$

$\downarrow$ 25

$$\frac{1}{6}x^6 \operatorname{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{Fresn}}{}\right.$$

3.142. $\int x^5 \operatorname{FresnelC}(bx)^2\, dx$

$\downarrow$ 3042

$$\frac{1}{6}x^6 \text{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{Fresn}\right.$$

$\downarrow$ 3118

$$\frac{1}{6}x^6 \text{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}\right.$$

$\downarrow$ 7009

$$\frac{1}{6}x^6 \text{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\left(-\frac{\int\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2\right.}{\pi b^2}\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3860

$$\frac{1}{6}x^6 \text{FresnelC}(bx)^2 - \frac{1}{3}b\left(-\frac{5\left(\frac{3\left(-\frac{\int\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2\right.}{\pi b^2}\right.}{}\right.}{\pi b^2}\right.$$

$\downarrow$ 3042

3.142. $\int x^5 \text{FresnelC}(bx)^2\,dx$

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2\right.}{\pi b^2}\right.}{}\right.}{\pi b^2}\right.$$
$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$\downarrow$ 3118

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\right.}{2\pi}\right.}{\pi b^2}\right.$$
$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

$\downarrow$ 7001

$$\frac{1}{3}b\left(-\frac{5\left(\frac{3\left(-\frac{\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\right.}{\pi b^2}\right.$$
$$\frac{1}{6}x^6\,\text{FresnelC}(bx)^2-$$

input
```
Int[x^5*FresnelC[b*x]^2,x]
```

output
```
(x^6*FresnelC[b*x]^2)/6 - (b*((x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2
*Pi) - (-((x^4*Cos[b^2*Pi*x^2])/(b^2*Pi)) + (2*(Cos[b^2*Pi*x^2]/(b^4*Pi^2)
 + (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi)))/(b^2*Pi))/(4*b*Pi) - (5*(-((x^3*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (3*(Cos[b^2*Pi*x^2]/(4*b^3*Pi^2)
- ((FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi) + (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(b^2*Pi)))/(b^2*Pi) + (x^4/4 + (Cos[b^2*Pi*x^2]/(b^4*Pi^2) + (x^2*Si
n[b^2*Pi*x^2])/(b^2*Pi))/2)/(2*b*Pi)))/(b^2*Pi)))/3
```

3.142. $\int x^5\,\text{FresnelC}(bx)^2\,dx$

3.142.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.142.4 Maple [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx$$

input
```
int(x^5*FresnelC(b*x)^2,x)
```

output
```
int(x^5*FresnelC(b*x)^2,x)
```

3.142.5 Fricas [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x^5*fresnel_cos(b*x)^2, x)
```

3.142.6 Sympy [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 C^2(bx) \, dx$$

input
```
integrate(x**5*fresnelc(b*x)**2,x)
```

output
```
Integral(x**5*fresnelc(b*x)**2, x)
```

3.142.7 Maxima [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2 \, dx = \int x^5 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*fresnel_cos(b*x)^2, x)
```

3.142.8 Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx = \int x^5\, \mathrm{C}\left(bx\right)^2\,dx$$

input
```
integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_cos(b*x)^2, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx)^2\,dx = \int x^5 \operatorname{FresnelC}(b\,x)^2\,dx$$

input
```
int(x^5*FresnelC(b*x)^2,x)
```

output
```
int(x^5*FresnelC(b*x)^2, x)
```

3.143 $\int x^4 \operatorname{FresnelC}(bx)^2 \, dx$

3.143.1 Optimal result

Integrand size = 10, antiderivative size = 177

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos(b^2\pi x^2)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{5b^3\pi^2} + \frac{1}{5}x^5\operatorname{FresnelC}(bx)^2 - \frac{43\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{20\sqrt{2}b^5\pi^3} + \frac{16\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{11x\sin(b^2\pi x^2)}{20b^4\pi^3}$$

output
```
4/15*x^3/b^2/Pi^2-1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-8/5*x^2*cos(1/2*b^2*Pi
*x^2)*FresnelC(b*x)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)^2+16/5*FresnelC(b*x)*si
n(1/2*b^2*Pi*x^2)/b^5/Pi^3-2/5*x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+
11/20*x*sin(b^2*Pi*x^2)/b^4/Pi^3-43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1
/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx$$
$$= \frac{32b^3\pi x^3 - 12b^3\pi x^3 \cos\left(b^2\pi x^2\right) + 24b^5\pi^3x^5 \operatorname{FresnelC}(bx)^2 - 129\sqrt{2}\operatorname{FresnelS}\left(\sqrt{2}bx\right) - 48\operatorname{FresnelC}(bx)\,(4}{120b^5\pi^3}$$

input
```
Integrate[x^4*FresnelC[b*x]^2,x]
```

output
```
(32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[
b*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^
2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*
Sin[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

3.143.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.64, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6985, 7009, 3866, 3867, 3832, 7017, 3873, 15, 3867, 3832, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \text{7009}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^4 \sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3866}$$

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\int x^2\cos(b^2\pi x^2)dx}{2\pi b^2} - \frac{x^3\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\downarrow$ 3867

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{3\left(\frac{x\sin(\pi b^2x^2)}{2\pi b^2} - \frac{\int \sin(b^2\pi x^2)dx}{2\pi b^2}\right)}{2\pi b^2} - \frac{x^3\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} + \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{\pi b^2}\right.$$

$\downarrow$ 3832

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} + \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\frac{3\left(\frac{x\sin(\pi b^2x^2)}{2\pi b^2} - \frac{\operatorname{FresnelS}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2} - \frac{x^3\cos(}{2\pi}}{2\pi b}\right.$$

$\downarrow$ 7017

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} - \frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} + \frac{x^4 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2\right.}{\pi b^2}\right.$$

$\downarrow$ 3873

$$\frac{1}{5}x^5 \operatorname{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\frac{1}{2}\int x^2\cos(b^2\pi x^2)dx + \frac{\int x^2dx}{2}}{\pi b} - \frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} + \frac{x^4 \operatorname{FresnelC}(bx)\sin}{\pi b^2}\right.$$

$\downarrow$ 15

3.143. $\int x^4 \operatorname{FresnelC}(bx)^2\,dx$

$$\frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{x^3}{6}}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\right.}{\pi b^2}\right.$$

$\downarrow$ 3867

$$\frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}}{}\right.$$

$\downarrow$ 3832

$$\frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}}{}\right.$$

$\downarrow$ 7007

$$\frac{1}{5}x^5\,\text{FresnelC}(bx)^2 - \frac{2}{5}b\left(-\frac{4\left(\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x}{}\right.$$

$\downarrow$ 3832

3.143. $\int x^4\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{5}x^5\,\mathrm{FresnelC}(bx)^2 - \frac{2}{5}b\left(\frac{x^4\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{4\left(\frac{2\left(\frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\mathrm{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}\right)}{\pi b^2} - \frac{x^2\,\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{1}{2}\left(\frac{x\sin(\cdots}{2\pi\cdots}\cdots\right)}{\cdots}\right)}{\pi b^2}\cdots\right.$$

input
```
Int[x^4*FresnelC[b*x]^2,x]
```

output
```
(x^5*FresnelC[b*x]^2)/5 - (2*b*((x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^2*Pi) - (-1/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[2]*
b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2*b*
Pi) - (4*(-((x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (2*(-1/2*F
resnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(b^2*Pi)))/(b^2*Pi) + (x^3/6 + (-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*
Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b^2*Pi)))/5
```

3.143.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.143.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right) + \frac{4b^3x^3}{15\pi^2} + \frac{2bx\sin\left(b^2\pi \ldots\right.}{5\pi}\ldots}{b^5}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right) + \frac{4b^3x^3}{15\pi^2} + \frac{2bx\sin\left(b^2\pi \ldots\right.}{5\pi}\ldots}{b^5}$

input
```
int(x^4*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*FresnelC(b*x)^2*b^5*x^5-2*FresnelC(b*x)*(1/5/Pi*b^4*x^4*sin(1/2
*b^2*Pi*x^2)-4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*
Pi*x^2)))+4/15/Pi^2*b^3*x^3+4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^
(1/2)*FresnelS(b*x*2^(1/2)))+1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2
*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^
(1/2)*FresnelS(b*x*2^(1/2))))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int x^4\,\mathrm{FresnelC}(bx)^2\,dx$$
$$= \frac{24\,\pi^3 b^6 x^5\,C\,(bx)^2 - 24\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 44\,\pi b^4 x^3 - 192\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) C\,(bx) - 129\,\sqrt{2}\sqrt{b^2}\,S\,\left(\ldots\right.}{120\,\pi^3 b^6}$$

input
```
integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/120*(24*pi^3*b^6*x^5*fresnel_cos(b*x)^2 - 24*pi*b^4*x^3*cos(1/2*pi*b^2*x
^2)^2 + 44*pi*b^4*x^3 - 192*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x
) - 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(11*b^2*x*
cos(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4 - 8*b)*fresnel_cos(b*x))*sin(1/2*pi*
b^2*x^2))/(pi^3*b^6)
```

3.143.6 Sympy [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 C^2(bx) \, dx$$

input
```
integrate(x**4*fresnelc(b*x)**2,x)
```

output
```
Integral(x**4*fresnelc(b*x)**2, x)
```

3.143.7 Maxima [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4*fresnel_cos(b*x)^2, x)
```

3.143.8 Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_cos(b*x)^2, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx)^2 \, dx = \int x^4 \operatorname{FresnelC}(b\,x)^2 \, dx$$

input
```
int(x^4*FresnelC(b*x)^2,x)
```

output
```
int(x^4*FresnelC(b*x)^2, x)
```

3.144 $\int x^3 \operatorname{FresnelC}(bx)^2 \, dx$

3.144.1 Optimal result

Integrand size = 10, antiderivative size = 140

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos(b^2\pi x^2)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{2b^3\pi^2} + \frac{3\operatorname{FresnelC}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\operatorname{FresnelC}(bx)^2 - \frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

output
```
3/8*x^2/b^2/Pi^2-1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2-3/2*x*cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/b^3/Pi^2+3/4*FresnelC(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelC(b*x
)^2-1/2*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*sin(b^2*Pi*x^2)/b^4
/Pi^3
```

3.144.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \frac{3b^2\pi x^2 - b^2\pi x^2\cos(b^2\pi x^2) + 2\pi(3 + b^4\pi^2x^4)\operatorname{FresnelC}(bx)^2 - 4b\pi x\operatorname{FresnelC}(bx)\left(3\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2 \right.}{8b^4\pi^3}$$

input
```
Integrate[x^3*FresnelC[b*x]^2,x]
```

output
```
(3*b^2*Pi*x^2 - b^2*Pi*x^2*Cos[b^2*Pi*x^2] + 2*Pi*(3 + b^4*Pi^2*x^4)*Fresn
elC[b*x]^2 - 4*b*Pi*x*FresnelC[b*x]*(3*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Si
n[(b^2*Pi*x^2)/2]) + 4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

3.144.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {6985, 7009, 3860, 3042, 3777, 3042, 3117, 7017, 3861, 3042, 3114, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6985}$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b \int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \text{ 7009}$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(-\frac{3\int x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^3 \sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(-\frac{3\int x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^2 \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(-\frac{3\int x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^2 \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$\Big\downarrow$ 3117

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}\right)$$

$\Big\downarrow$ 7017

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\right.$$

$\Big\downarrow$ 3861

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\right.$$

$\Big\downarrow$ 3042

$$\frac{1}{4}x^4\,\text{FresnelC}(bx)^2-\frac{1}{2}b\left(-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right.$$

$\Big\downarrow$ 3114

3.144. $\int x^3\,\text{FresnelC}(bx)^2\,dx$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} - \frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2} + \frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \right.$$

$$\Big\downarrow \; 6995$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(-\frac{3\left(\frac{\int \operatorname{FresnelC}(bx)d\operatorname{FresnelC}(bx)}{\pi b^3} - \frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2} + \frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \right.$$

$$\Big\downarrow \; 15$$

$$\frac{1}{4}x^4 \operatorname{FresnelC}(bx)^2 - \frac{1}{2}b\left(\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4} - \frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b} - \frac{3\left(\frac{\operatorname{FresnelC}(bx)^2}{2\pi b^3} - \frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}\right.$$

input
```
Int[x^3*FresnelC[b*x]^2,x]
```

output
```
(x^4*FresnelC[b*x]^2)/4 - (b*((x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2
*Pi) - (-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*x^2]/(b^4*Pi^2))/(4
*b*Pi) - (3*(-((x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + FresnelC[
b*x]^2/(2*b^3*Pi) + (x^2/2 + Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)))/(b^2*P
i)))/2
```

3.144. $\int x^3 \operatorname{FresnelC}(bx)^2 \, dx$

3.144.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.144.4 Maple [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx$$

input
```
int(x^3*FresnelC(b*x)^2,x)
```

output
```
int(x^3*FresnelC(b*x)^2,x)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = -\frac{\pi b^2 x^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 - 2\pi b^2 x^2 + 6\pi b x \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{C}(bx) - \left(3\pi + \pi^3 b^4 x^4\right) \mathrm{C}(bx)^2 + 2\left(\pi^2 b^3 x^3 \mathrm{C}(bx)\right.}{4\pi^3 b^4}$$

input
```
integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*p
i*b^2*x^2)*fresnel_cos(b*x) - (3*pi + pi^3*b^4*x^4)*fresnel_cos(b*x)^2 + 2
*(pi^2*b^3*x^3*fresnel_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^
2))/(pi^3*b^4)
```

3.144.6 Sympy [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 C^2(bx) \, dx$$

input
```
integrate(x**3*fresnelc(b*x)**2,x)
```

output
```
Integral(x**3*fresnelc(b*x)**2, x)
```

3.144.7 Maxima [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*fresnel_cos(b*x)^2, x)
```

3.144.8 Giac [F]

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{C}(bx)^2 \, dx$$

input
```
integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_cos(b*x)^2, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx)^2 \, dx = \int x^3 \operatorname{FresnelC}(b\,x)^2 \, dx$$

input
```
int(x^3*FresnelC(b*x)^2,x)
```

output
```
int(x^3*FresnelC(b*x)^2, x)
```

3.145 $\int x^2 \operatorname{FresnelC}(bx)^2 \, dx$

3.145.1 Optimal result

Integrand size = 10, antiderivative size = 124

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{2x}{3b^2\pi^2} - \frac{x\cos(b^2\pi x^2)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{3b^3\pi^2} + \frac{1}{3}x^3\operatorname{FresnelC}(bx)^2 + \frac{5\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{6\sqrt{2}b^3\pi^2} - \frac{2x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

output
```
2/3*x/b^2/Pi^2-1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2-4/3*cos(1/2*b^2*Pi*x^2)*Fres
nelC(b*x)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)^2-2/3*x^2*FresnelC(b*x)*sin(1/2*b
^2*Pi*x^2)/b/Pi+5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{-2bx(-4+\cos(b^2\pi x^2)) + 4b^3\pi^2x^3\operatorname{FresnelC}(bx)^2 + 5\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right) - 8\operatorname{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x\right.\right.}{12b^3\pi^2}$$

input
```
Integrate[x^2*FresnelC[b*x]^2,x]
```

output
```
(-2*b*x*(-4 + Cos[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*FresnelC[b*x]^2 + 5*Sqrt[2
]*FresnelC[Sqrt[2]*b*x] - 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*
x^2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

3.145.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6985, 7009, 3866, 3833, 7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{ 6985}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2}{3}b\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \text{ 7009}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2}{3}b\left(-\frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^2 \sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3866}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2}{3}b\left(-\frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\frac{\int \cos\left(b^2\pi x^2\right)dx}{2\pi b^2} - \frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b} + \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{ 3833}$$

$$\frac{1}{3}x^3 \operatorname{FresnelC}(bx)^2 - \frac{2}{3}b\left(-\frac{2\int x \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} + \frac{x^2 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3} - \frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{ 7015}$$

$$\frac{1}{3}x^3\,\mathrm{FresnelC}(bx)^2 - \frac{2}{3}b\left(-\frac{2\left(\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\mathrm{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{ 3839}$$

$$\frac{1}{3}x^3\,\mathrm{FresnelC}(bx)^2 - \frac{2}{3}b\left(-\frac{2\left(\frac{\int\left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right)dx}{\pi b}-\frac{\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\mathrm{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\,\mathrm{FresnelC}(bx)^2 - \frac{2}{3}b\left(\frac{x^2\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\frac{\mathrm{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}+\frac{x}{2}}{\pi b}-\frac{\mathrm{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{\mathrm{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)$$

input
```
Int[x^2*FresnelC[b*x]^2,x]
```

output
```
(x^3*FresnelC[b*x]^2)/3 - (2*b*(-1/2*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) +
FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(b*Pi) - (2*(-((Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(b^2*Pi)) + (x/2 + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))
/(b*Pi)))/(b^2*Pi) + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/3
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.145.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^2 x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} + \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} + \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} + \cdots}{3\cdots}}{b^3}$
default	$\frac{\frac{\mathrm{FresnelC}(bx)^2 b^3 x^3}{3} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^2 x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} + \frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} + \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} + \cdots}{3\cdots}}{b^3}$

input
```
int(x^2*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*FresnelC(b*x)^2*b^3*x^3-2*FresnelC(b*x)*(1/3/Pi*b^2*x^2*sin(1/2
*b^2*Pi*x^2)+2/3/Pi^2*cos(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2+1/3/Pi^2*2^(1/2)*F
resnelC(b*x*2^(1/2))+1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fr
esnelC(b*x*2^(1/2))))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \frac{4\,\pi^2 b^4 x^3\,\mathrm{C}\,(bx)^2 - 8\,\pi b^3 x^2\,\mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\,b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 10\,b^2 x - 16\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\,(bx)}{12\,\pi^2 b^4}$$

input
```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/12*(4*pi^2*b^4*x^3*fresnel_cos(b*x)^2 - 8*pi*b^3*x^2*fresnel_cos(b*x)*si
n(1/2*pi*b^2*x^2) - 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 10*b^2*x - 16*b*cos(1/
2*pi*b^2*x^2)*fresnel_cos(b*x) + 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*s
qrt(b^2)*x))/(pi^2*b^4)
```

3.145.6 Sympy [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2 \, dx = \int x^2 C^2(bx) \, dx$$

input
```
integrate(x**2*fresnelc(b*x)**2,x)
```

output
```
Integral(x**2*fresnelc(b*x)**2, x)
```

3.145.7 Maxima [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2\,dx = \int x^2\,C\,(bx)^2\,dx$$

input
```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_cos(b*x)^2, x)
```

3.145.8 Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx)^2\,dx = \int x^2\,C\,(bx)^2\,dx$$

input
```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_cos(b*x)^2, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx)^2\,dx = \int x^2 \operatorname{FresnelC}(b\,x)^2\,dx$$

input
```
int(x^2*FresnelC(b*x)^2,x)
```

output
```
int(x^2*FresnelC(b*x)^2, x)
```

3.146 $\int x \,\text{FresnelC}(bx)^2 \, dx$

3.146.1 Optimal result

Integrand size = 8, antiderivative size = 144

$$\int x \,\text{FresnelC}(bx)^2 \, dx = -\frac{\cos(b^2\pi x^2)}{4b^2\pi^2} + \frac{1}{2}x^2 \,\text{FresnelC}(bx)^2 + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^2\pi} + \frac{ix^2 \,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2 \,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{x \,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

output
```
-1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^2*FresnelC(b*x)^2+1/2*FresnelC(b*x)*Fr
esnelS(b*x)/b^2/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/
Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/Pi-x*FresnelC(b*x
)*sin(1/2*b^2*Pi*x^2)/b/Pi
```

3.146.2 Mathematica [F]

$$\int x \,\text{FresnelC}(bx)^2 \, dx = \int x \,\text{FresnelC}(bx)^2 \, dx$$

input
```
Integrate[x*FresnelC[b*x]^2,x]
```

output
```
Integrate[x*FresnelC[b*x]^2, x]
```

3.146.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6985, 7009, 3860, 3042, 3118, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 - b\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx$$

$$\Big\downarrow \text{7009}$$

$$\frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 - b\left(-\frac{\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b} + \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 - b\left(-\frac{\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 - b\left(-\frac{\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b} + \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{2}x^2 \operatorname{FresnelC}(bx)^2 - b\left(-\frac{\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} + \frac{x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)$$

$$\Big\downarrow \text{7001}$$

$$\frac{1}{2}x^2\,\text{FresnelC}(bx)^2 - b\left(-\frac{\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin(}{\pi b^2}\right.$$

input
```
Int[x*FresnelC[b*x]^2,x]
```

output
```
(x^2*FresnelC[b*x]^2)/2 - b*(Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - ((FresnelC[b*x
]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (
-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2
)*b^2*Pi*x^2])/(b^2*Pi) + (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi))
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

3.146.4 Maple [F]

$$\int x\,\mathrm{FresnelC}\,(bx)^2\,dx$$

input
```
int(x*FresnelC(b*x)^2,x)
```

output
```
int(x*FresnelC(b*x)^2,x)
```

3.146.5 Fricas [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{C}\,(bx)^2\;dx$$

input
```
integrate(x*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
integral(x*fresnel_cos(b*x)^2, x)
```

3.146.6 Sympy [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int xC^2(bx)\;dx$$

input
```
integrate(x*fresnelc(b*x)**2,x)
```

output
```
Integral(x*fresnelc(b*x)**2, x)
```

3.146.7 Maxima [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{C}\left(bx\right)^2\,dx$$

input
```
integrate(x*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(x*fresnel_cos(b*x)^2, x)
```

3.146.8 Giac [F]

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{C}\left(bx\right)^2\,dx$$

input
```
integrate(x*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(x*fresnel_cos(b*x)^2, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int x\,\mathrm{FresnelC}(bx)^2\,dx = \int x\,\mathrm{FresnelC}(b\,x)^2\,dx$$

input
```
int(x*FresnelC(b*x)^2,x)
```

output
```
int(x*FresnelC(b*x)^2, x)
```

3.147 $\int \text{FresnelC}(bx)^2 \, dx$

3.147.1 Optimal result

Integrand size = 6, antiderivative size = 54

$$\int \text{FresnelC}(bx)^2\,dx = x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

output
```
x*FresnelC(b*x)^2-2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*FresnelS(b*
x*2^(1/2))/b/Pi*2^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^2\,dx = x\,\text{FresnelC}(bx)^2 + \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{\sqrt{2}b\pi} - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

input
```
Integrate[FresnelC[b*x]^2,x]
```

output
```
x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x
]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

3.147.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6975, 27, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)^2 \, dx$$

$$\Big\downarrow \text{6975}$$

$$x\,\text{FresnelC}(bx)^2 - 2\int bx \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx$$

$$\Big\downarrow \text{27}$$

$$x\,\text{FresnelC}(bx)^2 - 2b\int x \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx$$

$$\Big\downarrow \text{7007}$$

$$x\,\text{FresnelC}(bx)^2 - 2b\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}\right)$$

$$\Big\downarrow \text{3832}$$

$$x\,\text{FresnelC}(bx)^2 - 2b\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}\right)$$

input
```
Int[FresnelC[b*x]^2,x]
```

output
```
x*FresnelC[b*x]^2 - 2*b*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (Fr
esnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi))
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 6975
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Simp[2   Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[
a + b*x], x], x] /; FreeQ[{a, b}, x]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.147.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\operatorname{FresnelC}(bx)^2 bx-\frac{2\operatorname{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\sqrt{2}\operatorname{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\operatorname{FresnelC}(bx)^2 bx-\frac{2\operatorname{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\sqrt{2}\operatorname{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

input
```
int(FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelC(b*x)^2*b*x-2*FresnelC(b*x)/Pi*sin(1/2*b^2*Pi*x^2)+1/2/Pi*2^(
1/2)*FresnelS(b*x*2^(1/2)))
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \text{FresnelC}(bx)^2\,dx = \frac{2\,\pi b^2 x\,\text{C}\left(bx\right)^2 - 4\,b\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + \sqrt{2}\sqrt{b^2}\,\text{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{2\,\pi b^2}$$

input
```
integrate(fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*pi*b^2*x*fresnel_cos(b*x)^2 - 4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x
^2) + sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

3.147.6 Sympy [F]

$$\int \text{FresnelC}(bx)^2\,dx = \int C^2(bx)\ dx$$

input
```
integrate(fresnelc(b*x)**2,x)
```

output
```
Integral(fresnelc(b*x)**2, x)
```

3.147.7 Maxima [F]

$$\int \text{FresnelC}(bx)^2\,dx = \int \text{C}\left(bx\right)^2\ dx$$

input
```
integrate(fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2, x)
```

3.147.8 Giac [F]

$$\int \text{FresnelC}(bx)^2\,dx = \int \text{C}\left(bx\right)^2\,dx$$

input
```
integrate(fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2, x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)^2\,dx = \int \text{FresnelC}(b\,x)^2\,dx$$

input
```
int(FresnelC(b*x)^2,x)
```

output
```
int(FresnelC(b*x)^2, x)
```

3.148 $\int \frac{\text{FresnelC}(bx)^2}{x} dx$

3.148.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x} dx = \text{Int}\left(\frac{\text{FresnelC}(bx)^2}{x}, x\right)$$

output
```
Unintegrable(FresnelC(b*x)^2/x,x)
```

3.148.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x} dx = \int \frac{\text{FresnelC}(bx)^2}{x} dx$$

input
```
Integrate[FresnelC[b*x]^2/x,x]
```

output
```
Integrate[FresnelC[b*x]^2/x, x]
```

3.148.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

$$\Big\downarrow \text{6989}$$

$$\int \frac{\text{FresnelC}(bx)^2}{x} dx$$

input
```
Int[FresnelC[b*x]^2/x,x]
```

output
```
$Aborted
```

3.148.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.148.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^{2}}{x}dx$$

input
```
int(FresnelC(b*x)^2/x,x)
```

output
```
int(FresnelC(b*x)^2/x,x)
```

3.148.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x, x)
```

3.148.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{C^2(bx)}{x}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x,x)
```

output
```
Integral(fresnelc(b*x)**2/x, x)
```

3.148.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x, x)
```

3.148.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\mathrm{C}\,(bx)^2}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x, x)
```

3.148.9 Mupad [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x}\,dx$$

input
```
int(FresnelC(b*x)^2/x,x)
```

output
```
int(FresnelC(b*x)^2/x, x)
```

3.149 $\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx$

3.149.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = -\frac{\text{FresnelC}(bx)^2}{x} + 2b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

output
```
-FresnelC(b*x)^2/x+2*b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.149.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx$$

input
```
Integrate[FresnelC[b*x]^2/x^2,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^2, x]
```

3.149.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6985, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 6985}$$

$$2b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx - \frac{\text{FresnelC}(bx)^2}{x}$$

$$\Big\downarrow \text{ 7013}$$

$$2b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx - \frac{\text{FresnelC}(bx)^2}{x}$$

input
```
Int[FresnelC[b*x]^2/x^2,x]
```

output
```
$Aborted
```

3.149.3.1 Defintions of rubi rules used

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.149.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^2}dx$$

input
```
int(FresnelC(b*x)^2/x^2,x)
```

output
```
int(FresnelC(b*x)^2/x^2,x)
```

3.149.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^2}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^2, x)
```

3.149.6 Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^2}\,dx = \int \frac{C^2(bx)}{x^2}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**2,x)
```

output
```
Integral(fresnelc(b*x)**2/x**2, x)
```

3.149.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^2} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^2, x)
```

3.149.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^2} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^2, x)
```

3.149.9 Mupad [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^2} \, dx$$

input
```
int(FresnelC(b*x)^2/x^2,x)
```

output
```
int(FresnelC(b*x)^2/x^2, x)
```

3.150 $\int \frac{\text{FresnelC}(bx)^2}{x^3} dx$

3.150.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = -\frac{\text{FresnelC}(bx)^2}{2x^2} + b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}, x\right)$$

output
```
-1/2*FresnelC(b*x)^2/x^2+b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/
x^2,x)
```

3.150.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} dx = \int \frac{\text{FresnelC}(bx)^2}{x^3} dx$$

input
```
Integrate[FresnelC[b*x]^2/x^3,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^3, x]
```

3.150.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6985, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^3} \, dx$$
$$\Big\downarrow \; 6985$$
$$b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx - \frac{\text{FresnelC}(bx)^2}{2x^2}$$
$$\Big\downarrow \; 7013$$
$$b \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx - \frac{\text{FresnelC}(bx)^2}{2x^2}$$

input
```
Int[FresnelC[b*x]^2/x^3,x]
```

output
```
$Aborted
```

3.150.3.1 Defintions of rubi rules used

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.150.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^3}dx$$

input
```
int(FresnelC(b*x)^2/x^3,x)
```

output
```
int(FresnelC(b*x)^2/x^3,x)
```

3.150.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx = \int \frac{\text{C}\,(bx)^2}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^3, x)
```

3.150.6 Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx = \int \frac{C^2(bx)}{x^3}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**3,x)
```

output
```
Integral(fresnelc(b*x)**2/x**3, x)
```

3.150.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^3, x)
```

3.150.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^3, x)
```

3.150.9 Mupad [N/A]

Not integrable

Time = 4.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^3}\,dx$$

input
```
int(FresnelC(b*x)^2/x^3,x)
```

output
```
int(FresnelC(b*x)^2/x^3, x)
```

3.151 $\int \frac{\text{FresnelC}(bx)^2}{x^4} dx$

3.151.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx = -\frac{b^2}{6x} - \frac{b^2 \cos(b^2\pi x^2)}{6x} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{3x^2} - \frac{\text{FresnelC}(bx)^2}{3x^3} - \frac{b^3\pi \text{ FresnelS}\left(\sqrt{2}bx\right)}{3\sqrt{2}} - \frac{1}{3}b^3\pi \text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)$$

output
```
-1/6*b^2/x-1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x^2-1/3*FresnelC(b*x)^2/x^3-1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/
3*b^3*Pi*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.151.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} dx = \int \frac{\text{FresnelC}(bx)^2}{x^4} dx$$

input
```
Integrate[FresnelC[b*x]^2/x^4,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^4, x]
```

3.151.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6985, 7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^4} \, dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{2}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx-\frac{\text{FresnelC}(bx)^2}{3x^3}$$

$$\Big\downarrow \text{7011}$$

$$\frac{2}{3}b\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}\right)-\frac{\text{FresnelC}(bx)^2}{3x^3}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{3}b\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\cos\left(\right.}{2x^2}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{3x^3}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2}{3}b\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{3x^3}$$

$$\Big\downarrow \text{7021}$$

$$\frac{2}{3}b\left(-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{3x^3}$$

3.151. $\int \frac{\text{FresnelC}(bx)^2}{x^4} \, dx$

input
```
Int[FresnelC[b*x]^2/x^4,x]
```

output
```
$Aborted
```

3.151.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.151. $\int \frac{\text{FresnelC}(bx)^2}{x^4} dx$

3.151.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^4}dx$$

input
```
int(FresnelC(b*x)^2/x^4,x)
```

output
```
int(FresnelC(b*x)^2/x^4,x)
```

3.151.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\,(bx)^2}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^4, x)
```

3.151.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{C^2(bx)}{x^4}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**4,x)
```

output
```
Integral(fresnelc(b*x)**2/x**4, x)
```

3.151.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\,(bx)^2}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^4, x)
```

3.151.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{C}\,(bx)^2}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^4, x)
```

3.151.9 Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^4}\,dx$$

input
```
int(FresnelC(b*x)^2/x^4,x)
```

output
```
int(FresnelC(b*x)^2/x^4, x)
```

3.152 $\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$

3.152.1 Optimal result

Integrand size = 10, antiderivative size = 127

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx = -\frac{b^2}{24x^2} - \frac{b^2\cos(b^2\pi x^2)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{4x^4} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{12}b^4\pi\,\text{Si}(b^2\pi x^2)$$

output
```
-1/24*b^2/x^2-1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)^2-1/4*FresnelC(b*x)^2/x^4-1/12*b^
4*Pi*Si(b^2*Pi*x^2)+1/6*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx = -\frac{b^2}{24x^2} - \frac{b^2\cos(b^2\pi x^2)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3} - \frac{1}{12}b^4\pi^2\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{4x^4} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{12}b^4\pi\,\text{Si}(b^2\pi x^2)$$

input
```
Integrate[FresnelC[b*x]^2/x^5,x]
```

output
```
-1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fr
esnelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*
x^4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinInteg
ral[b^2*Pi*x^2])/12
```

3.152.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6985, 7011, 3861, 3042, 3778, 25, 3042, 3780, 7019, 3856, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{1}{2} b \int \frac{\cos\left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^4} dx - \frac{\text{FresnelC}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{7011}$$

$$\frac{1}{2} b \left(-\frac{1}{3} \pi b^2 \int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx + \frac{1}{6} b \int \frac{\cos\left(b^2 \pi x^2\right)}{x^3} dx - \frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3} - \frac{b}{12x^2}\right) - \frac{\text{FresnelC}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{2} b \left(-\frac{1}{3} \pi b^2 \int \frac{\text{FresnelC}(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)}{x^2} dx + \frac{1}{12} b \int \frac{\cos\left(b^2 \pi x^2\right)}{x^4} dx^2 - \frac{\text{FresnelC}(bx) \cos\left(\frac{1}{2} \pi b^2 x^2\right)}{3x^3} - \frac{b}{12x^2}\right) - \frac{\text{FresnelC}(bx)^2}{4x^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{7019}$$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\text{Fresn}}{}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{3856}$$

3.152. $\int\frac{\text{FresnelC}(bx)^2}{x^5}\,dx$

$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)\right)-\frac{\text{FresnelC}(bx)}{3x}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{6995}$$
$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\left(\pi b\int\text{FresnelC}(bx)d\,\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)\right)-\frac{\text{FresnelC}(bx)\cos}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{2}b\left(-\frac{1}{3}\pi b^2\left(-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}(b^2\pi x^2)+\frac{1}{2}\pi b\,\text{FresnelC}(bx)^2\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\right.$$
$$\frac{\text{FresnelC}(bx)^2}{4x^4}$$

input
```
Int[FresnelC[b*x]^2/x^5,x]
```

output
```
-1/4*FresnelC[b*x]^2/x^4 + (b*(-1/12*b/x^2 - (Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(3*x^3) - (b^2*Pi*((b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/x + (b*SinIntegral[b^2*Pi*x^2])/4))/3 + (b*(-(Cos[b^2*Pi*x^
2]/x^2) - b^2*Pi*SinIntegral[b^2*Pi*x^2]))/12))/2
```

3.152.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.152. $\int\frac{\text{FresnelC}(bx)^2}{x^5}\,dx$

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.152. $\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.152.4 Maple [F]

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^5}dx$$

input
```
int(FresnelC(b*x)^2/x^5,x)
```

output
```
int(FresnelC(b*x)^2/x^5,x)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = -\frac{\pi b^4x^4\,\text{Si}\left(\pi b^2x^2\right) - 2\,\pi b^3x^3\,\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx) + (\pi^2 b}{12\,x^4}$$

input
```
integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*fresnel_cos(b*x)
*sin(1/2*pi*b^2*x^2) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 + 2*b*x*cos(1/2*pi*b^
2*x^2)*fresnel_cos(b*x) + (pi^2*b^4*x^4 + 3)*fresnel_cos(b*x)^2)/x^4
```

3.152.6 Sympy [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = \int \frac{C^2(bx)}{x^5}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**5,x)
```

output
```
Integral(fresnelc(b*x)**2/x**5, x)
```

3.152.7 Maxima [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^5, x)
```

3.152.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^5}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^5, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^5} \, dx$$

input
```
int(FresnelC(b*x)^2/x^5,x)
```

output
```
int(FresnelC(b*x)^2/x^5, x)
```

3.153 $\int \frac{\text{FresnelC}(bx)^2}{x^6} dx$

3.153.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} dx = -\frac{b^2}{60x^3} - \frac{b^2 \cos(b^2\pi x^2)}{60x^3} - \frac{b \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{10x^4} - \frac{\text{FresnelC}(bx)^2}{5x^5} - \frac{7b^5\pi^2 \text{FresnelC}\left(\sqrt{2}bx\right)}{60\sqrt{2}} + \frac{b^3\pi \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2} + \frac{7b^4\pi \sin(b^2\pi x^2)}{120x} - \frac{1}{20}b^5\pi^2 \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x}, x\right)$$

output
```
-1/60*b^2/x^3-1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/10*b*cos(1/2*b^2*Pi*x^2)*Fres
nelC(b*x)/x^4-1/5*FresnelC(b*x)^2/x^5+1/20*b^3*Pi*FresnelC(b*x)*sin(1/2*b^
2*Pi*x^2)/x^2+7/120*b^4*Pi*sin(b^2*Pi*x^2)/x-7/120*b^5*Pi^2*FresnelC(b*x*2
^(1/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x,x)
```

3.153.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$$

input
```
Integrate[FresnelC[b*x]^2/x^6,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^6, x]
```

3.153.3 Rubi [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6985, 7011, 3869, 3868, 3833, 7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{2}{5}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx-\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$$\Big\downarrow \text{7011}$$

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}\right)-\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelC}(bx)\cos\left(\right.}{4x^4}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 3868

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 3833

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 7019

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\frac{\text{Fresn}}{}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 3868

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)}{2x}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 3833

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx}{2}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

$\big\downarrow$ 7013

3.153. $\int\frac{\text{FresnelC}(bx)^2}{x^6}\,dx$

$$\frac{2}{5}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx}{2}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{5x^5}$$

input
```
Int[FresnelC[b*x]^2/x^6,x]
```

output
```
$Aborted
```

3.153.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.153. $\int\frac{\text{FresnelC}(bx)^2}{x^6}\,dx$

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.153.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^6}dx$$

input
```
int(FresnelC(b*x)^2/x^6,x)
```

output
```
int(FresnelC(b*x)^2/x^6,x)
```

3.153.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{C}\,(bx)^2}{x^6}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^6, x)
```

3.153.6 Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{C^2(bx)}{x^6}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**6,x)
```

output
```
Integral(fresnelc(b*x)**2/x**6, x)
```

3.153.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^6, x)
```

3.153.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^6}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^6, x)
```

3.153.9 Mupad [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^6}\,dx$$

input
```
int(FresnelC(b*x)^2/x^6,x)
```

output
```
int(FresnelC(b*x)^2/x^6, x)
```

3.154 $\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

3.154.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = &-\frac{b^2}{120x^4}-\frac{b^2\cos(b^2\pi x^2)}{120x^4}-\frac{1}{72}b^6\pi^2\,\text{CosIntegral}\left(b^2\pi x^2\right)\\&-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5}-\frac{\text{FresnelC}(bx)^2}{6x^6}\\&+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}+\frac{b^4\pi\sin(b^2\pi x^2)}{72x^2}\\&-\frac{1}{45}b^5\pi^2\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)\end{aligned}$$

output
```
-1/120*b^2/x^4-1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)-1/120*b^2*cos(b^2*Pi*x^2)/x^4-
1/15*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/6*FresnelC(b*x)^2/x^6+1/45*
b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3+1/72*b^4*Pi*sin(b^2*Pi*x^2)/x
^2-1/45*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

3.154.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

input
```
Integrate[FresnelC[b*x]^2/x^7,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^7, x]
```

3.154.3 Rubi [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6985, 7011, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7019, 3860, 3042, 3778, 3042, 3783, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx - \frac{\text{FresnelC}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{7011}$$

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx + \frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5} - \frac{b}{40x^4}\right) - \frac{\text{FresnelC}(bx)^2}{6x^6}$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)-\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3042

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3778

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\text{ co}}{5x^5}\right.\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 25

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\text{ co}}{5x^5}\right.\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3042

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\text{ co}}{5x^5}\right.\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3778

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right.}{2x^4}\right.\right.\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3042

3.154. $\int\frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi\right.}{2x}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 3783

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 7019

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}b\left(\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 3860

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 3778

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx}{3}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\Big\downarrow$ 3042

3.154. $\int\frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresnel}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 3783

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelC}(\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

$\big\downarrow$ 7013

$$\frac{1}{3}b\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{FresnelC}(\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{6x^6}$$

input
```
Int[FresnelC[b*x]^2/x^7,x]
```

output
```
$Aborted
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.154. $\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$

3.154.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\,(bx)^2}{x^7}dx$$

input
```
int(FresnelC(b*x)^2/x^7,x)
```

output
```
int(FresnelC(b*x)^2/x^7,x)
```

3.154.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int \frac{\text{C}\,(bx)^2}{x^7}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^7, x)
```

3.154.6 Sympy [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx = \int \frac{C^2(bx)}{x^7}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**7,x)
```

output
```
Integral(fresnelc(b*x)**2/x**7, x)
```

3.154.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^7} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^7, x)
```

3.154.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^7} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^7, x)
```

3.154.9 Mupad [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^7} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^7} \, dx$$

input
```
int(FresnelC(b*x)^2/x^7,x)
```

output
```
int(FresnelC(b*x)^2/x^7, x)
```

3.155 $\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$

3.155.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = & -\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}-\frac{b^2\cos(b^2\pi x^2)}{210x^5}+\frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} \\ & -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{168x^2} \\ & -\frac{\text{FresnelC}(bx)^2}{7x^7}+\frac{b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)}{72\sqrt{2}} \\ & +\frac{2}{315}\sqrt{2}b^7\pi^3\,\text{FresnelS}\left(\sqrt{2}bx\right)+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} \\ & +\frac{13b^4\pi\sin(b^2\pi x^2)}{2520x^3}+\frac{1}{168}b^7\pi^3\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

output
```
-1/210*b^2/x^5+1/336*b^6*Pi^2/x-1/210*b^2*cos(b^2*Pi*x^2)/x^5+67/5040*b^6*
Pi^2*cos(b^2*Pi*x^2)/x-1/21*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/168*
b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/7*FresnelC(b*x)^2/x^7+1/8
4*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4+13/2520*b^4*Pi*sin(b^2*Pi*x
^2)/x^3+67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)+1/168*b^7*Pi^3*Unin
tegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.155.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

input
```
Integrate[FresnelC[b*x]^2/x^8,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^8, x]
```

3.155.3 Rubi [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6985, 7011, 3869, 3868, 3869, 3832, 7019, 3868, 3869, 3832, 7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

$$\Big\downarrow \text{ 6985}$$

$$\frac{2}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx-\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$$\Big\downarrow \text{ 7011}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}\right)-\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$$\Big\downarrow \text{ 3869}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos}{6x^6}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{3868}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{3869}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{3832}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{7019}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right)-\frac{\text{Fresn}}{}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{3868}$$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelC}(bx}{4}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$
$$\Big\downarrow \text{3869}$$

3.155. $\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin(}{3}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$\big\downarrow$ 3832

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$\big\downarrow$ 7011

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$\big\downarrow$ 3869

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{Fre}}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$\big\downarrow$ 3832

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

$\big\downarrow$ 7021

$$\frac{2}{7}b\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{7x^7}$$

input
```
Int[FresnelC[b*x]^2/x^8,x]
```

3.155. $\int\frac{\text{FresnelC}(bx)^2}{x^8}\,dx$

output
```
$Aborted
```

3.155.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.155. $\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.155.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^{2}}{x^{8}}dx$$

input
```
int(FresnelC(b*x)^2/x^8,x)
```

output
```
int(FresnelC(b*x)^2/x^8,x)
```

3.155.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{\text{C}\left(bx\right)^{2}}{x^{8}}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^8, x)
```

3.155.6 Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx = \int \frac{C^{2}(bx)}{x^{8}}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**8,x)
```

output
```
Integral(fresnelc(b*x)**2/x**8, x)
```

3.155.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^8} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^8, x)
```

3.155.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{\text{C}\left(bx\right)^2}{x^8} \, dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^8, x)
```

3.155.9 Mupad [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} \, dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^8} \, dx$$

input
```
int(FresnelC(b*x)^2/x^8,x)
```

output
```
int(FresnelC(b*x)^2/x^8, x)
```

3.156 $\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

3.156.1 Optimal result

Integrand size = 10, antiderivative size = 242

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos(b^2\pi x^2)}{336x^6}+\frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} + \frac{1}{840}b^8\pi^4\,\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5}-\frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4}+\frac{1}{280}b^8\pi^3\text{Si}(b^2\pi x^2)$$

output
```
-1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2-1/336*b^2*cos(b^2*Pi*x^2)/x^6+1/336*b^6
*Pi^2*cos(b^2*Pi*x^2)/x^2-1/28*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/4
20*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/840*b^8*Pi^4*FresnelC(
b*x)^2-1/8*FresnelC(b*x)^2/x^8+1/280*b^8*Pi^3*Si(b^2*Pi*x^2)+1/140*b^3*Pi*
FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/420*b^7*Pi^3*FresnelC(b*x)*sin(1/2
*b^2*Pi*x^2)/x+1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos(b^2\pi x^2)}{336x^6}+\frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3} + \frac{1}{840}b^8\pi^4\,\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5}-\frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x} + \frac{b^4\pi\sin(b^2\pi x^2)}{420x^4}+\frac{1}{280}b^8\pi^3\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[FresnelC[b*x]^2/x^9,x]
```

output
```
-1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
 (b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b
^8*Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[
b*x]*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinInteg
ral[b^2*Pi*x^2])/280
```

3.156.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx$$

$$\Big\downarrow \; 6985$$

$$\frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx-\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$$\Big\downarrow \; 7011$$

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 3861

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)-\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}\right)$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 3778

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\,\text{co}}{7x^7}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 25

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\,\text{co}}{7x^7}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\,\text{co}}{7x^7}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\Big\downarrow$ 3778

3.156. $\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x\right.}{3x^6}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\right.}{3}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{si}}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 25

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{si}}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{si}}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3780

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 7019

3.156. $\int\frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fres}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3860

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fr}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{Fr}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{FresnelC}(\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{Fresne}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3778

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{sin}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 25

3.156. $\int\frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin}{}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 3780

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\right.\right.\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 7011

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 3861

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right.}{3x^3}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi\right.}{3x^3}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\big\downarrow$ 3778

3.156. $\int\frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{F}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 25

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{F}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3042

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{F}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 3780

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

$\downarrow$ 7019

$$\frac{1}{4}b\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{8x^8}$$

input
```
Int[FresnelC[b*x]^2/x^9,x]
```

output
```
$Aborted
```

3.156. $\int\frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.156. $\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.156.4 Maple [F]

$$\int \frac{\operatorname{FresnelC}(bx)^2}{x^9}\,dx$$

input
```
int(FresnelC(b*x)^2/x^9,x)
```

output
```
int(FresnelC(b*x)^2/x^9,x)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \frac{\operatorname{FresnelC}(bx)^2}{x^9}\,dx = \frac{3\,\pi^3 b^8 x^8\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^6 x^6 + 5\left(\pi^2 b^6 x^6 - b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^5 x^5 - 15\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right.}{840\,x^8}$$

input
```
integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="fricas")
```

output
```
1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 2*pi^2*b^6*x^6 + 5*(pi^2*
b^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 + 2*(pi^2*b^5*x^5 - 15*b*x)*cos(1
/2*pi*b^2*x^2)*fresnel_cos(b*x) + (pi^4*b^8*x^8 - 105)*fresnel_cos(b*x)^2
+ 2*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*fres
nel_cos(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

3.156.6 Sympy [F]

$$\int \frac{\mathrm{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{C^2(bx)}{x^9}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**9,x)
```

output
```
Integral(fresnelc(b*x)**2/x**9, x)
```

3.156.7 Maxima [F]

$$\int \frac{\mathrm{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\mathrm{C}\left(bx\right)^2}{x^9}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^9, x)
```

3.156.8 Giac [F]

$$\int \frac{\mathrm{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\mathrm{C}\left(bx\right)^2}{x^9}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^9, x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)^2}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^9}\,dx$$

input
```
int(FresnelC(b*x)^2/x^9,x)
```

output
```
int(FresnelC(b*x)^2/x^9, x)
```

3.157 $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

3.157.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = &-\frac{b^2}{504x^7}+\frac{b^6\pi^2}{5184x^3}-\frac{b^2\cos(b^2\pi x^2)}{504x^7}\\&+\frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8}\\&+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{864x^4}-\frac{\text{FresnelC}(bx)^2}{9x^9}\\&+\frac{853b^9\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{181440\sqrt{2}}+\frac{b^3\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6}\\&-\frac{b^7\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^2}+\frac{19b^4\pi\sin(b^2\pi x^2)}{15120x^5}\\&-\frac{853b^8\pi^3\sin(b^2\pi x^2)}{362880x}+\frac{b^9\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)}{1728}\end{aligned}$$

output
```
-1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3-1/504*b^2*cos(b^2*Pi*x^2)/x^7+187/18144
0*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/36*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^
8+1/864*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4-1/9*FresnelC(b*x)^2
/x^9+1/216*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6-1/1728*b^7*Pi^3*Fr
esnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5-85
3/362880*b^8*Pi^3*sin(b^2*Pi*x^2)/x+853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/
2))*2^(1/2)+1/1728*b^9*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)
/x,x)
```

3.157.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

input
```
Integrate[FresnelC[b*x]^2/x^10,x]
```

output
```
Integrate[FresnelC[b*x]^2/x^10, x]
```

3.157.3 Rubi [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6985, 7011, 3869, 3868, 3869, 3868, 3833, 7019, 3868, 3869, 3868, 3833, 7011, 3869, 3868, 3833, 7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

$$\Big\downarrow \text{6985}$$

$$\frac{2}{9}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}dx-\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{7011}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}-\frac{b}{112x^7}\right)-\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\cos\left(\pi b^2x^2\right)}{7x^7}\right)-\frac{\text{FresnelC}(bx)\cos}{8x^8}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{3868}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b^2x^2\right)}{7x^7}\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{s}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{3868}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx+\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{3833}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\mathrm{I}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{7019}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}\right)-\frac{\text{Fres}}{}\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$
$$\Big\downarrow \text{3868}$$

3.157. $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(b\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin}{}\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3868}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{Fr}\right.\right.\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{7011}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3869}$$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{Fr}}{}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$$\Big\downarrow \text{3868}$$

3.157. $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$\downarrow$ 3833

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{Fr}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$\downarrow$ 7019

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi\right.}{2x^2}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$\downarrow$ 3868

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$\downarrow$ 3833

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

$\downarrow$ 7013

$$\frac{2}{9}b\left(-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)^2}{9x^9}$$

input
```
Int[FresnelC[b*x]^2/x^10,x]
```

3.157. $\int\frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

output
```
$Aborted
```

3.157.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 6985
```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Simp[2*(b/(m + 1))   Int[x^(m + 1)*Cos[(Pi/2)*b^2*x
^2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.157. $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.157.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^{10}}dx$$

input
```
int(FresnelC(b*x)^2/x^10,x)
```

output
```
int(FresnelC(b*x)^2/x^10,x)
```

3.157.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\left(bx\right)^2}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^2/x^10, x)
```

3.157.6 Sympy [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{C^2(bx)}{x^{10}}\,dx$$

input
```
integrate(fresnelc(b*x)**2/x**10,x)
```

output
```
Integral(fresnelc(b*x)**2/x**10, x)
```

3.157.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\,(bx)^2}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^2/x^10, x)
```

3.157.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{C}\,(bx)^2}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^2/x^10, x)
```

3.157.9 Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)^2}{x^{10}}\,dx$$

input

```
int(FresnelC(b*x)^2/x^10,x)
```

output

```
int(FresnelC(b*x)^2/x^10, x)
```

3.158 $\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx$

3.158.1 Optimal result

Integrand size = 16, antiderivative size = 495

$$\begin{aligned}
\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx
&= \frac{2d^2x}{3b^2\pi^2} - \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2} - \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2} \\
&- \frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\operatorname{FresnelC}(a+bx)}{3b^3\pi^2} \\
&+ \frac{(bc-ad)^2(a+bx)\operatorname{FresnelC}(a+bx)^2}{b^3} + \frac{d(bc-ad)(a+bx)^2\operatorname{FresnelC}(a+bx)^2}{b^3} \\
&+ \frac{d^2(a+bx)^3\operatorname{FresnelC}(a+bx)^2}{3b^3} + \frac{5d^2\operatorname{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}b^3\pi^2} \\
&+ \frac{d(bc-ad)\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{b^3\pi} + \frac{(bc-ad)^2\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^3\pi} \\
&+ \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{id(bc-ad)(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4b^3\pi} \\
&- \frac{2(bc-ad)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&- \frac{2d(bc-ad)(a+bx)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} \\
&- \frac{2d^2(a+bx)^2\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}
\end{aligned}$$

output
```
2/3*d^2*x/b^2/Pi^2-1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-1/6*d^2*(b*
x+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-4/3*d^2*cos(1/2*Pi*(b*x+a)^2)*FresnelC(b*x
+a)/b^3/Pi^2+(-a*d+b*c)^2*(b*x+a)*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+
a)^2*FresnelC(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*FresnelC(b*x+a)^2/b^3+d*(-a*d
+b*c)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*
hypergeom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(
b*x+a)^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-2*(-a*d+b*c)
^2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2*d*(-a*d+b*c)*(b*x+a)*Fre
snelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2/3*d^2*(b*x+a)^2*FresnelC(b*x+a
)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2
*2^(1/2)+1/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

3.158.2 Mathematica [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx = \int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx$$

input
```
Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]
```

3.158.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6987, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx$$

$$\Big\downarrow \; 6987$$

$$\frac{\int \left((bc-ad)^2 \operatorname{FresnelC}(a+bx)^2 + d^2(a+bx)^2 \operatorname{FresnelC}(a+bx)^2 + 2d(bc-ad)(a+bx)\operatorname{FresnelC}(a+bx)^2\right) d(a+$$
$$b^3$$

$$\Big\downarrow \; 2009$$

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi} - \frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi} + \frac{d(bc-ad)\,\text{FresnelC}(a+bx)\,\text{FresnelS}(a+bx)}{\pi} +$$

input
```
Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

output
```
((2*d^2*(a + b*x))/(3*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*Pi^2)
 - (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x)
^2)/2]*FresnelC[a + b*x])/(3*Pi^2) + (b*c - a*d)^2*(a + b*x)*FresnelC[a +
b*x]^2 + d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2 + (d^2*(a + b*x)^3*
FresnelC[a + b*x]^2)/3 + (5*d^2*FresnelC[Sqrt[2]*(a + b*x)])/(6*Sqrt[2]*Pi
^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a + b*x])/Pi + ((b*c - a*d
)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*Pi) + ((I/4)*d*(b*c - a*d)*(a +
b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/Pi -
((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)
*Pi*(a + b*x)^2])/Pi - (2*(b*c - a*d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x
)^2)/2])/Pi - (2*d*(b*c - a*d)*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*
x)^2)/2])/Pi - (2*d^2*(a + b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2
])/(3*Pi))/b^3
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6987
```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*
x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

3.158.4 Maple [F]

$$\int (dx+c)^2 \,\text{FresnelC}\,(bx+a)^2\, dx$$

input
```
int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

output
```
int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

3.158.5 Fricas [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{C}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_cos(b*x + a)^2, x)
```

3.158.6 Sympy [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (c+dx)^2 C^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)
```

3.158.7 Maxima [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c)^2 \operatorname{C}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

3.158.8 Giac [F]

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx=\int (dx+c)^2\,\mathrm{C}\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \operatorname{FresnelC}(a+bx)^2\,dx=\int \operatorname{FresnelC}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(FresnelC(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(FresnelC(a + b*x)^2*(c + d*x)^2, x)
```

3.159 $\int (c+dx) \operatorname{FresnelC}(a+bx)^2 \, dx$

3.159.1 Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int (c+dx) \operatorname{FresnelC}(a+bx)^2 \, dx = &-\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2} \\ &+\frac{(bc-ad)(a+bx)\operatorname{FresnelC}(a+bx)^2}{b^2} \\ &+\frac{d(a+bx)^2\operatorname{FresnelC}(a+bx)^2}{2b^2} \\ &+\frac{d\operatorname{FresnelC}(a+bx)\operatorname{FresnelS}(a+bx)}{2b^2\pi} \\ &+\frac{(bc-ad)\operatorname{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b^2\pi} \\ &+\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ &-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8b^2\pi} \\ &-\frac{2(bc-ad)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} \\ &-\frac{d(a+bx)\operatorname{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}\end{aligned}$$

output
```
-1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+(-a*d+b*c)*(b*x+a)*FresnelC(b*x+a)^2/b^2
+1/2*d*(b*x+a)^2*FresnelC(b*x+a)^2/b^2+1/2*d*FresnelC(b*x+a)*FresnelS(b*x+
a)/b^2/Pi+1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)
/b^2/Pi-1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^
2/Pi-2*(-a*d+b*c)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-d*(b*x+a)*F
resnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(-a*d+b*c)*FresnelS((b*x+a
)*2^(1/2))/b^2/Pi*2^(1/2)
```

3.159.2 Mathematica [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx=\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx$$

input
```
Integrate[(c + d*x)*FresnelC[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

3.159.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6987, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx$$

$$\Big\downarrow \text{6987}$$

$$\frac{\int \left((bc-ad)\,\text{FresnelC}(a+bx)^2+d(a+bx)\,\text{FresnelC}(a+bx)^2\right)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi}+(a+bx)(bc-ad)\,\text{FresnelC}(a+bx)^2-\frac{2(bc-ad)\,\text{F}}{}}{}$$

input
```
Int[(c + d*x)*FresnelC[a + b*x]^2,x]
```

output
```
(-1/4*(d*Cos[Pi*(a + b*x)^2])/Pi^2 + (b*c - a*d)*(a + b*x)*FresnelC[a + b*
x]^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/2 + (d*FresnelC[a + b*x]*Fresne
lS[a + b*x])/(2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*P
i) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*Pi*
(a + b*x)^2])/Pi - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (I/2)*Pi*(a + b*x)^2])/Pi - (2*(b*c - a*d)*FresnelC[a + b*x]*Sin[(Pi*(a
+ b*x)^2)/2])/Pi - (d*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])
/Pi)/b^2
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6987
```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[1/b^(m + 1)   Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*
x)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

3.159.4 Maple [F]

$$\int (dx + c) \operatorname{FresnelC}(bx + a)^2 \, dx$$

input
```
int((d*x+c)*FresnelC(b*x+a)^2,x)
```

output
```
int((d*x+c)*FresnelC(b*x+a)^2,x)
```

3.159.5 Fricas [F]

$$\int (c+dx) \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c) \operatorname{C}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

3.159.6 Sympy [F]

$$\int (c+dx) \operatorname{FresnelC}(a+bx)^2 \, dx = \int (c+dx) \, C^2(a+bx) \, dx$$

input
```
integrate((d*x+c)*fresnelc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

3.159.7 Maxima [F]

$$\int (c+dx) \operatorname{FresnelC}(a+bx)^2 \, dx = \int (dx+c) \operatorname{C}(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

3.159.8 Giac [F]

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx=\int (dx+c)\,\text{C}\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\,\text{FresnelC}(a+bx)^2\,dx=\int \text{FresnelC}(a+b\,x)^2\,(c+d\,x)\,dx$$

input
```
int(FresnelC(a + b*x)^2*(c + d*x),x)
```

output
```
int(FresnelC(a + b*x)^2*(c + d*x), x)
```

3.160 $\int \text{FresnelC}(a+bx)^2\,dx$

3.160.1 Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \text{FresnelC}(a+bx)^2\,dx=\frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b}+\frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}b\pi}-\frac{2\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

output
```
(b*x+a)*FresnelC(b*x+a)^2/b-2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b/Pi+1
/2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \text{FresnelC}(a+bx)^2\,dx=\frac{2\pi(a+bx)\,\text{FresnelC}(a+bx)^2+\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}(a+bx)\right)-4\,\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b\pi}$$

input
```
Integrate[FresnelC[a + b*x]^2,x]
```

output
```
(2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)]
- 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(2*b*Pi)
```

3.160.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6975, 7281, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(a+bx)^2 \, dx$$

$$\Big\downarrow \text{6975}$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - 2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{7007}$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\left(\frac{\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{\int \sin\left(\pi(a+bx)^2\right)d(a+bx)}{2\pi}\right)}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{(a+bx)\,\text{FresnelC}(a+bx)^2}{b} - \frac{2\left(\frac{\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi} - \frac{\text{FresnelS}\left(\sqrt{2}(a+bx)\right)}{2\sqrt{2}\pi}\right)}{b}$$

input
```
Int[FresnelC[a + b*x]^2,x]
```

output
```
((a + b*x)*FresnelC[a + b*x]^2)/b - (2*(-1/2*FresnelS[Sqrt[2]*(a + b*x)]/(
Sqrt[2]*Pi) + (FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/Pi))/b
```

3.160.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 6975
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Simp[2   Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[
a + b*x], x], x] /; FreeQ[{a, b}, x]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.160.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\operatorname{FresnelC}(bx+a)^2(bx+a)-\frac{2\operatorname{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\operatorname{FresnelC}(bx+a)^2(bx+a)-\frac{2\operatorname{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\sqrt{2}\operatorname{FresnelS}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

input
```
int(FresnelC(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(FresnelC(b*x+a)^2*(b*x+a)-2*FresnelC(b*x+a)/Pi*sin(1/2*Pi*(b*x+a)^2)+
1/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \text{FresnelC}(a+bx)^2\,dx = \frac{2\,(\pi b^2x+\pi ab)\,\text{C}\,(bx+a)^2-4\,b\,\text{C}\,(bx+a)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+\sqrt{2}\sqrt{b^2}\,\text{S}\left(\frac{\sqrt{2}\sqrt{b^2}(bx+a)}{b}\right)}{2\,\pi b^2}$$

input
```
integrate(fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(pi*b^2*x + pi*a*b)*fresnel_cos(b*x + a)^2 - 4*b*fresnel_cos(b*x +
a)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + sqrt(2)*sqrt(b^2)*fresnel
_sin(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

3.160.6 Sympy [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int C^2(a+bx)\;dx$$

input
```
integrate(fresnelc(b*x+a)**2,x)
```

output
```
Integral(fresnelc(a + b*x)**2, x)
```

3.160.7 Maxima [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int \text{C}\,(bx+a)^2\;dx$$

input
```
integrate(fresnel_cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)^2, x)
```

3.160.8 Giac [F]

$$\int \text{FresnelC}(a+bx)^2\,dx = \int C\left(bx+a\right)^2\,dx$$

input
```
integrate(fresnel_cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)^2, x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(a+bx)^2\,dx = \int \text{FresnelC}(a+b\,x)^2\,dx$$

input
```
int(FresnelC(a + b*x)^2,x)
```

output
```
int(FresnelC(a + b*x)^2, x)
```

3.161 $\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$

3.161.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{c+dx},x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)^2/(d*x+c),x)
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx = \int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx$$

input
```
Integrate[FresnelC[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[FresnelC[a + b*x]^2/(c + d*x), x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} \, dx$$
$$\Big\downarrow \; 6989$$
$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$$

input
```
Int[FresnelC[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.161.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^{2}}{dx+c} dx$$

input
```
int(FresnelC(b*x+a)^2/(d*x+c),x)
```

output
```
int(FresnelC(b*x+a)^2/(d*x+c),x)
```

3.161.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{C}\,(bx+a)^2}{dx+c} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

3.161.6 Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} \, dx = \int \frac{C^2(a+bx)}{c+dx} \, dx$$

input
```
integrate(fresnelc(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(fresnelc(a + b*x)**2/(c + d*x), x)
```

3.161.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} \, dx = \int \frac{\text{C}\,(bx+a)^2}{dx+c} \, dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

3.161.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{C}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx}\,dx=\int \frac{\text{FresnelC}(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(FresnelC(a + b*x)^2/(c + d*x),x)
```

output
```
int(FresnelC(a + b*x)^2/(c + d*x), x)
```

3.162 $\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$

3.162.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx = \text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

3.162.2 Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx = \int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

input
```
Integrate[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

3.162.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} \, dx$$
$$\Big\downarrow \; 6989$$
$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}dx$$

input
```
Int[FresnelC[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.162.3.1 Defintions of rubi rules used

rule 6989
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Unintegrable[(c + d*x)^m*FresnelC[a + b*x]^n, x] /; FreeQ[{a, b, c, d,
 m, n}, x]
```

3.162.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

3.162.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{C}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.162.6 Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{C^2(a+bx)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)
```

3.162.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{C}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

3.162.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{C}\,(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

3.162.9 Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{\text{FresnelC}(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(FresnelC(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(FresnelC(a + b*x)^2/(c + d*x)^2, x)
```

3.163 $\int x^2 \text{FresnelC}(d(a+b\log(cx^n)))\,dx$

3.163.1 Optimal result

Integrand size = 17, antiderivative size = 231

$$\int x^2 \text{FresnelC}(d(a+b\log(cx^n)))\,dx = \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) - \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right) + \frac{1}{3}x^3\,\text{FresnelC}(d(a+b\log(cx^n)))$$

output
```
(1/12+1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+
I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))-(1/12+1
/12*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*
d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*Fresn
elC(d*(a+b*ln(c*x^n)))
```

3.163.2 Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.38

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \frac{1}{12}x^3\Bigg(4\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

$$+\sqrt[4]{-1}\sqrt{2}e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\Bigg(ie^{\frac{9i}{b^2d^2n^2\pi}}\operatorname{erfi}\Bigg(\frac{(\frac{1}{2}$$

input
```
Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b
*n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] -
 Log[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(
b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d
^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b
*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))
/(c*x^n)^(3/n)))/12
```

3.163.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7026, 5129, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 7026$$

$$\frac{1}{3}x^3\operatorname{FresnelC}(d(a+b\log(cx^n)))-\frac{1}{3}bdn\int x^2\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx$$

$$\Big\downarrow \; 5129$$

$$\frac{1}{3}x^3\operatorname{FresnelC}(d(a+b\log(cx^n)))-\\ \frac{1}{3}bdn\left(\frac{1}{2}\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2dx+\frac{1}{2}\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2dx\right)$$

$\downarrow$ 2712

$$\frac{1}{3}x^3 \text{FresnelC}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{1}{2}x^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{2-iabd^2n\pi}dx+\frac{1}{2}x^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int \mathrm{e}$$

$\downarrow$ 2706

$$\frac{1}{3}x^3 \text{FresnelC}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{x^3(cx^n)^{-3/n}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{(3-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}+\frac{x^3(cx^n)^{-3/n}\int \exp}{}\right.$$

$\downarrow$ 2664

$$\frac{1}{3}x^3 \text{FresnelC}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\int \exp\left(\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{3}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\int}{}\right.$$

$\downarrow$ 2633

$$\frac{1}{3}x^3 \text{FresnelC}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\int \exp\left(-\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{3}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{}{2}}}{}\right.$$

$\downarrow$ 2634

$$\frac{1}{3}x^3 \text{FresnelC}(d(a+b\log(cx^n)))-$$
$$\frac{1}{3}bdn\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)}{bdn}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{}{2\pi b}}}{}\right.$$

input
```
Int[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

3.163. $\int x^2 \text{FresnelC}(d(a+b\log(cx^n)))\,dx$

output
```
-1/3*(b*d*n*(((-1/4 - I/4)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erf[((1/2 + I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(b*d*n*(c*x^n)^(3/n)) + ((1/4 + I/4)*E^((-3*a)/(b*n) - ((9*I)/2
)/(b^2*d^2*n^2*Pi))*x^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*
Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*n*(c*x^n)^(3/n)))) + (x^3*FresnelC[d
*(a + b*Log[c*x^n])])/3
```

3.163.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5129
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Simp[1/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] +
Simp[1/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7026
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.163.4 Maple [F]

$$\int x^2 \operatorname{FresnelC}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

3.163.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.94

$$\int x^2 \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{1}{3}x^3\operatorname{C}\left(bd\log\left(cx^n\right)+ad\right)$$
$$-\frac{1}{6}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$-\frac{1}{6}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$+\frac{1}{6}i\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$
$$-\frac{1}{6}i\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$

input
```
integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
1/3*x^3*fresnel_cos(b*d*log(c*x^n) + a*d) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3
*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(pi
*b^2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) + 9/2
*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log
(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/6*I*pi*s
qrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fres
nel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)
*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*lo
g(c)/n - 3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2))
```

3.163.6 Sympy [F]

$$\int x^2 \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 C(ad+bd\log\left(cx^n\right))\,dx$$

input
```
integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.163.7 Maxima [F]

$$\int x^2 \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2\,\mathrm{C}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.163.8 Giac [F]

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int x^2 \operatorname{C}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int x^2 \operatorname{FresnelC}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x^2*FresnelC(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.164 $\int x \,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right) \,dx$

3.164.1 Optimal result

Integrand size = 15, antiderivative size = 227

$$\begin{aligned}
&\int x \,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right) \,dx \\
&= \left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad -\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2\left(cx^n\right)^{-2/n}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log\left(cx^n\right)\right)}{bd\sqrt{\pi}}\right) \\
&\quad +\frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)
\end{aligned}$$

output
```
(1/8+1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(
2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))-(1/
8+1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/
d/Pi^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2
*FresnelC(d*(a+b*ln(c*x^n)))
```

3.164.2 Mathematica [A] (verified)

Time = 4.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.40

$$\int x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx=\frac{1}{8}x^2\Bigg(4\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)$$

$$+\sqrt[4]{-1}\sqrt{2}e^{-\frac{2a}{bn}-\frac{2i}{b^2d^2n^2\pi}-\frac{1}{2}ia^2d^2\pi+iabd^2\pi(n\log(x)-\log(cx^n))-\frac{1}{2}ib^2d^2\pi(-n\log(x)+\log(cx^n))^2}\left(cx^n\right)^{-2/n}\Bigg(ie^{\frac{4i}{b^2d^2n^2\pi}}\text{erfi}\Bigg(\frac{\left(\frac{1}{2}\right.}{}$$

input
```
Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*
n) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] -
Log[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(
b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d
^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b
*Log[c*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))
/(c*x^n)^(2/n)))/8
```

3.164.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.467$, Rules used = {7026, 5129, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{7026}$$

$$\frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-\frac{1}{2}bdn\int x\cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)dx$$

$$\Big\downarrow \text{5129}$$

$$\frac{1}{2}x^2\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)-$$
$$\frac{1}{2}bdn\left(\frac{1}{2}\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}xdx+\frac{1}{2}\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}xdx\right)$$

$$\Big\downarrow \text{2712}$$

$$\frac{1}{2}x^2 \text{FresnelC}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{1}{2}x^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{1-iabd^2n\pi}dx+\frac{1}{2}x^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int \text{e}\right.$$

$$\Big\downarrow \text{2706}$$

$$\frac{1}{2}x^2 \text{FresnelC}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{x^2(cx^n)^{-2/n}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{(2-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}+\frac{x^2(cx^n)^{-2/n}\int \exp}{}\right.$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{2}x^2 \text{FresnelC}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{x^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n+i)}{\pi b^2d^2n^2}}\int \exp\left(\frac{i(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{2}{n})^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \exp}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$\frac{1}{2}x^2 \text{FresnelC}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \exp\left(-\frac{i(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{2}{n})^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n}{\pi b^2d^2n}}}{}\right.$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{2}x^2 \text{FresnelC}(d(a+b\log(cx^n))) - \frac{1}{2}bdn\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{-\frac{2(\pi abd^2n+i)}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n})}{\sqrt{\pi}bd}\right)}{bdn}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+}{\pi b^2d^2n^2}}}{}\right.$$

input
```
Int[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

3.164. $\int x \, \text{FresnelC}(d(a+b\log(cx^n)))\,dx$

output
```
-1/2*(b*d*n*(((-1/4 - I/4)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2
*Erf[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/(b*d*n*(c*x^n)^(2/n)) + ((1/4 + I/4)*x^2*Erfi[((1/2 + I/2)*(2/n -
 I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*(I +
 a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n*(c*x^n)^(2/n)))) + (x^2*FresnelC[d*(a
+ b*Log[c*x^n])])/2
```

3.164.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5129
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Simp[1/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] +
Simp[1/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7026
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.164.4 Maple [F]

$$\int x \,\mathrm{FresnelC}\left(d(a + b\ln\left(c\,x^{n}\right))\right) dx$$

input
```
int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

3.164.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int x\,\mathrm{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \\
&-\frac{1}{4}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{4}\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&+\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&-\frac{1}{4}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-2i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) \\
&+\frac{1}{2}\,x^2\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)
\end{aligned}$$

input
```
integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2
))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(
-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi
*b^2*d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2
*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log
(c) + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*I*pi*s
qrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresne
l_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*s
qrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_cos(b*d*log(c*x^n) +
a*d)
```

3.164.6 Sympy [F]

$$\int x \operatorname{FresnelC}(d(a + b\log(cx^n)))\,dx = \int xC(ad + bd\log(cx^n))\,dx$$

input
```
integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.164.7 Maxima [F]

$$\int x \operatorname{FresnelC}(d(a + b\log(cx^n)))\,dx = \int x\,\mathrm{C}((b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.164.8 Giac [F]

$$\int x \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int x\,\mathrm{C}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int x \operatorname{FresnelC}(d\,(a+b\,\ln(c\,x^n)))\,dx$$

input
```
int(x*FresnelC(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.165 $\int \text{FresnelC}(d(a+b\log(cx^n)))\,dx$

3.165.1 Optimal result

Integrand size = 13, antiderivative size = 214

$$\begin{aligned}\int \text{FresnelC}(d(a+b\log(cx^n)))\,dx &= \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad-\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\\&\quad+x\,\text{FresnelC}(d(a+b\log(cx^n)))\end{aligned}$$

output
```
(1/4+1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/
d/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))-(1/4+1/4*I
)*x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2
))/exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelC(d*(a+b*l
n(c*x^n)))
```

3.165.2 Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right) + \frac{\sqrt[4]{-1}e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(ie^{\frac{i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

input
```
Integrate[FresnelC[d*(a + b*Log[c*x^n])],x]
```

output
```
x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d
^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b
^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erf
i[((1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[
Pi])] + Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*
d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*S
in[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-
1))
```

3.165.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {7023, 5127, 2710, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{7023}$$

$$x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right) - bdn\int \cos\left(\frac{1}{2}d^2\pi(a+b\log\left(cx^n\right))^2\right)dx$$

$$\Big\downarrow \text{5127}$$

$$x\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right) - bdn\left(\frac{1}{2}\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx + \frac{1}{2}\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}dx\right)$$

$$\Big\downarrow \text{2710}$$

$$x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-bdn\left(\frac{1}{2}x^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)\,d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabd^2n\pi}dx+\frac{1}{2}x^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int\exp$$

$$\Big\downarrow \text{2706}$$

$$x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-bdn\left(\frac{x(cx^n)^{-1/n}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)\,d^2-\frac{1}{2}ia^2\pi d^2+\frac{(1-iabd^2n\pi)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}+\frac{x(cx^n)^{-1/n}\int\exp\left(\frac{1}{2}\right.}{}\right.$$

$$\Big\downarrow \text{2664}$$

$$x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-bdn\left(\frac{x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\frac{i(}{}\right.}{}\right.$$

$$\Big\downarrow \text{2633}$$

$$x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-bdn\left(\frac{x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int\exp\left(-\frac{i\left(ib^2\pi\log(cx^n)d^2+iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log(cx^n)}{2n}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}}{b}\right.$$

$$\Big\downarrow \text{2634}$$

$$x\,\text{FresnelC}\left(d(a+b\log(cx^n))\right)-bdn\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{bdn}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}\left(\right.}{bdn}\right.$$

input
```
Int[FresnelC[d*(a + b*Log[c*x^n])],x]
```

output
```
-(b*d*n*(((-1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d
^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*
n^2))*n*(c*x^n)^n^(-1)) + ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b
*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*a*b*n + I/
(d^2*Pi))/(2*b^2*n^2))*n*(c*x^n)^n^(-1)))) + x*FresnelC[d*(a + b*Log[c*x^n
])]
```

3.165. $\int \text{FresnelC}\left(d(a+b\log(cx^n))\right)\,dx$

3.165.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 5127
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Simp[1
/2   Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] + Simp[1/2   Int[E^(I*d*(a
 + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

rule 7023
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelC[d*(a + b*Log[c*x^n])], x] - Simp[b*d*n   Int[Cos[(Pi/2)*(d*(a
+ b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

3.165.4 Maple [F]

$$\int \text{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

output
```
int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

3.165.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.08

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx = -\frac{1}{2}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) - \frac{1}{2}\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) + \frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\pi b^2d^2n^2}\right)}\,\text{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) - \frac{1}{2}i\,\pi\sqrt{b^2d^2n^2}e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\pi b^2d^2n^2}\right)}\,\text{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) + x\,\text{C}\left(bd\log\left(cx^{n}\right)+ad\right)$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))
*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
 I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log
(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(
x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2
*n^2)) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2
*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a
*b*d^2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*I*pi*sqrt(b^2*d^2*
n^2)*e^(-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*
d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)
/(pi*b^2*d^2*n^2)) + x*fresnel_cos(b*d*log(c*x^n) + a*d)
```

3.165.6 Sympy [F]

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int C(d(a+b\log\left(cx^n\right)))\,dx$$

input
```
integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

3.165.7 Maxima [F]

$$\int \text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \text{C}\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.165.8 Giac [F]

$$\int \text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right) \, dx = \int \text{C}\left((b\log\left(cx^n\right) + a)d\right) \, dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right) \, dx = \int \text{FresnelC}(d\left(a + b\,\ln\left(c\,x^n\right)\right)) \, dx$$

input
```
int(FresnelC(d*(a + b*log(c*x^n))),x)
```

output
```
int(FresnelC(d*(a + b*log(c*x^n))), x)
```

3.166 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx$

3.166.1 Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx=\frac{\text{FresnelC}(d(a+b\log(cx^n)))(a+b\log(cx^n))}{bn}-\frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{bdn\pi}$$

output
```
FresnelC(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sin(1/2*d^2*Pi*(a+b*ln(c*x
^n))^2)/b/d/n/Pi
```

3.166.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\begin{aligned}\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}\,dx &= \frac{a\,\text{FresnelC}(d(a+b\log(cx^n)))}{bn}+\frac{\text{FresnelC}(d(a+b\log(cx^n)))\log(cx^n)}{n}\\&\quad-\frac{\cos\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)\sin\left(\frac{1}{2}a^2d^2\pi\right)}{bdn\pi}\\&\quad-\frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi}\end{aligned}$$

input
```
Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]
*Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]
*Sin[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log
[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

3.166.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 7281, 6973}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \text{FresnelC}(d(a+b\log(cx^n)))\, d\log(cx^n)}{n}$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int \text{FresnelC}(ad+b\log(cx^n)\,d)\, d(ad+b\log(cx^n)\,d)}{bdn}$$

$$\Big\downarrow \text{6973}$$

$$\frac{(ad+bd\log(cx^n))\,\text{FresnelC}(ad+b\log(cx^n)\,d) - \frac{\sin\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{\pi}}{bdn}$$

input
```
Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
(FresnelC[a*d + b*d*Log[c*x^n]]*(a*d + b*d*Log[c*x^n]) - Sin[(Pi*(a*d + b*
d*Log[c*x^n])^2)/2]/Pi)/(b*d*n)
```

3.166.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 6973
```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.166.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^{n}))^{2}}{2}\right)}{\pi}}{ndb}$	64
default	$\frac{\mathrm{FresnelC}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^{n}))^{2}}{2}\right)}{\pi}}{ndb}$	64

input
```
int(FresnelC(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/d/b*(FresnelC(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))-1/Pi*sin(1/2*Pi*(
a*d+b*d*ln(c*x^n))^2))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.83

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)}{x}\,dx$$

$$= \frac{(\pi bdn\log\left(x\right)+\pi bd\log\left(c\right)+\pi ad)\,\mathrm{C}\left(bd\log\left(cx^{n}\right)+ad\right)-\sin\left(\frac{1}{2}\pi b^{2}d^{2}n^{2}\log\left(x\right)^{2}+\pi b^{2}d^{2}n\log\left(c\right)\log\left(x\right)-\right.}{\pi bdn}$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_cos(b*d*log(c*x^n) + a
*d) - sin(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*p
i*b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*
d^2))/(pi*b*d*n)
```

3.166.6 Sympy [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\int \frac{C(ad+bd\log\left(cx^n\right))}{x}\,dx$$

input
```
integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(b\log\left(cx^n\right)+a)d\,\mathrm{C}\left((b\log\left(cx^n\right)+a)d\right)-\frac{\sin\left(\frac{1}{2}\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\pi a^2d^2\right)}{\pi}}{bdn}$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
((b*log(c*x^n) + a)*d*fresnel_cos((b*log(c*x^n) + a)*d) - sin(1/2*pi*b^2*d
^2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

3.166.8 Giac [F]

$$\int \frac{\text{FresnelC}\,(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{C\,((b\log(cx^n)+a)d)}{x}\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\,(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\text{FresnelC}(d\,(a+b\,\ln(c\,x^n)))}{x}\,dx$$

input
```
int(FresnelC(d*(a + b*log(c*x^n)))/x,x)
```

output
```
int(FresnelC(d*(a + b*log(c*x^n)))/x, x)
```

3.167 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

3.167.1 Optimal result

Integrand size = 17, antiderivative size = 217

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}$$

output
```
(1/4+1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2
*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-(1/4+1/4*I)*
exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*
a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelC(d*(a+b*ln(c*x^
n)))/x
```

3.167.2 Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{\sqrt[4]{-1}\sqrt{2}e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\left(\operatorname{erfi}\left(\frac{(-1)^{3/4}(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{i}{b^2d^2n^2\pi}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+abd^2n\pi+b^2d^2n\pi\log(c}{bdn\sqrt{\pi}}\right.\right.}{4x}$$

input
```
Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)*(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d
*n*Sqrt[2*Pi])] + I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*
n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelC[d*(a + b*
Log[c*x^n])])/x
```

3.167.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7026, 5129, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$$

$$\Big\downarrow \; 7026$$

$$bdn\int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^2}dx - \frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x}$$

$$\Big\downarrow \; 5129$$

$$-\frac{\operatorname{FresnelC}(d(a+b\log(cx^n)))}{x} + bdn\left(\frac{1}{2}\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}dx + \frac{1}{2}\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}dx\right)$$

$$\Big\downarrow \; 2712$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{1}{2}x^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabn\pi d^2-2}dx+\frac{1}{2}x^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int \text{ex}\right.$$

$\Big\downarrow$ 2706

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{(cx^n)^{\frac{1}{n}}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2-\frac{(iabn\pi d^2+1)\log(cx^n)}{n}\right)d\log\left(cx^n\right)}{2nx}+\frac{(cx^n)^{\frac{1}{n}}\int \exp\left(\frac{1}{2}ib^2\pi\log^2\right.}{}\right.$$

$\Big\downarrow$ 2664

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int \exp\left(-\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log\left(cx^n\right)}{2nx}+\frac{(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\int \exp\left(\frac{i(ib^2\pi\log(cx^n)d}{2b^2}\right.}{2nx}\right.$$

$\Big\downarrow$ 2633

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\int \exp\left(-\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{1}{n}\right)^2}{2b^2d^2\pi}\right)d\log\left(cx^n\right)}{2nx}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i}{}\right.}{bdnx}\right.$$

$\Big\downarrow$ 2634

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x}+$$
$$bdn\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i\pi abd}{}\right.}{bdnx}\right.$$

input
```
Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

3.167. $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

output
```
b*d*n*(((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*
Erf[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(b*d*n*x) - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))
*(c*x^n)^n^(-1)*Erfi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Lo
g[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*n*x)) - FresnelC[d*(a + b*Log[c*x^n])]/x
```

3.167.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5129
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Simp[1/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] +
Simp[1/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

3.167. $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

rule 7026
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.167.4 Maple [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

3.167.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx$$
$$= \frac{\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\text{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2 d}\right.}}{}$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
1/2*(pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2)
)*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
+ I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c
)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x)
 + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n
^2)) + I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*
n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^
2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*x*e^
(log(c)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2
*d^2*n^2)) - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/x
```

3.167.6 Sympy [F]

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{C(ad + bd\log\left(cx^n\right))}{x^2}\,dx$$

input
```
integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)
```

3.167.7 Maxima [F]

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

3.167.8 Giac [F]

$$\int \frac{\text{FresnelC}\left(d(a + b\log\left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\text{C}\left((b\log\left(cx^n\right) + a)d\right)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\text{FresnelC}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^2}\,dx$$

input
```
int(FresnelC(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(FresnelC(d*(a + b*log(c*x^n)))/x^2, x)
```

3.168 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

3.168.1 Optimal result

Integrand size = 17, antiderivative size = 228

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}\left(cx^n\right)^{2/n}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} - \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}$$

output
```
(1/8+1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/
2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2-(1/8+
1/8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*
x^n))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*Fresnel
C(d*(a+b*ln(c*x^n)))/x^2
```

3.168.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{\sqrt[4]{-1}e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(\text{erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{4i}{b^2d^2n^2\pi}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2i+abd^2n\ldots}{bdn\ldots}\right.\right.}{4\sqrt{2}} - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}$$

input
```
Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
-1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*
x^n])))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[2*Pi])] + I*E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*
(2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt
[2] - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

3.168.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {7026, 5129, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$$

$$\Big\downarrow \text{7026}$$

$$\frac{1}{2}bdn\int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^3}dx - \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}$$

$$\Big\downarrow \text{5129}$$

$$-\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{2}bdn\left(\frac{1}{2}\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}dx + \frac{1}{2}\int \frac{e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}dx\right)$$

$$\Big\downarrow \text{2712}$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}bdn\left(\frac{1}{2}x^{i\pi abd^2n}(cx^n)^{-i\pi abd^2}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{-iabn\pi d^2-3}dx+\frac{1}{2}x^{-i\pi abd^2n}(cx^n)^{i\pi abd^2}\int$$

$$\Big\downarrow \text{2706}$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}bdn\left(\frac{(cx^n)^{2/n}\int \exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2-\frac{(iabn\pi d^2+2)\log(cx^n)}{n}\right)d\log\left(cx^n\right)}{2nx^2}+\frac{(cx^n)^{2/n}\int \exp\left(\frac{1}{2}ib^2\pi\right.}{}\right.$$

$$\Big\downarrow \text{2664}$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}bdn\left(\frac{(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \exp\left(-\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{2}{n}\right)^2}{2b^2d^2\pi}\right)d\log\left(cx^n\right)}{2nx^2}+\frac{(cx^n)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\int \exp\left(\frac{i(ib}{}\right.}{2n}\right.$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}bdn\left(\frac{(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\int \exp\left(-\frac{i\left(-ib^2\pi\log(cx^n)d^2-iab\pi d^2+\frac{2}{n}\right)^2}{2b^2d^2\pi}\right)d\log\left(cx^n\right)}{2nx^2}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\text{erf}}{bdn}\right.$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{2x^2}+\frac{1}{2}bdn\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)}{bdnx^2}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\text{erfi}\left(\frac{(\frac{1}{2}}{}\right.}{bdnx^2}\right.$$

input
```
Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

3.168. $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

output
```
(b*d*n*(((1/4 + I/4)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(
2/n)*Erf[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d
*Sqrt[Pi])])/(b*d*n*x^2) - ((1/4 + I/4)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2
/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(b*d*E^((2*
(I - a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n*x^2)))/2 - FresnelC[d*(a + b*Log[c
*x^n])]/(2*x^2)
```

3.168.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5129
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Simp[1/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] +
Simp[1/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

3.168. $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

rule 7026
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.168.4 Maple [F]

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

input
```
int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

3.168.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(183) = 366.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.02

$$\int \frac{\mathrm{FresnelC}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx = \frac{\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}-\right.}}{}$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
1/4*(pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d^2*
n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^
2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*
e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*
n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(
pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) +
2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*lo
g(c) + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt
(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresne
l_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*s
qrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/
x^2
```

3.168.6 Sympy [F]

$$\int \frac{\operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{C\left(ad+bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

input
```
integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)
```

3.168.7 Maxima [F]

$$\int \frac{\operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\mathrm{C}\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

3.168.8 Giac [F]

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\text{C}\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\text{FresnelC}(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

input
```
int(FresnelC(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(FresnelC(d*(a + b*log(c*x^n)))/x^3, x)
```

3.169 $\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx$

3.169.1 Optimal result

Integrand size = 19, antiderivative size = 280

$$\begin{aligned}\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx &= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n)\right)}{bdn\sqrt{\pi}}\right)}{1+m}\\ &\quad-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m-iabd^2n\pi-ib^2d^2n\pi\log(cx^n)\right)}{bdn\sqrt{\pi}}\right)}{1+m}\\ &\quad+\frac{(ex)^{1+m}\operatorname{FresnelC}(d(a+b\log(cx^n)))}{e(1+m)}\end{aligned}$$

output
```
(1/4+1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)
^m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^
(1/2))/(1+m)/((c*x^n)^((1+m)/n))-(1/4+1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1
+m-I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m
)*(1+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(
1+m)*FresnelC(d*(a+b*ln(c*x^n)))/e/(1+m)
```

3.169.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int (ex)^m \text{FresnelC}(d(a+b\log(cx^n)))\,dx$$
$$= \frac{(ex)^m\left((-1)^{3/4}\sqrt{2}e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)-\right.\right.}{4(1+m)}$$

input
```
Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^m*(((-1)^(3/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi +
b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] - E^((I*(1 + m)^2)/(b^2*d^2*n^
2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d
^2*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*Fresne
lC[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

3.169.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {7026, 5129, 2712, 2706, 2664, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \text{FresnelC}(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{7026}$$
$$\frac{(ex)^{m+1}\text{FresnelC}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bdn\int (ex)^m\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx}{m+1}$$
$$\Big\downarrow \text{5129}$$
$$\frac{(ex)^{m+1}\text{FresnelC}(d(a+b\log(cx^n)))}{e(m+1)}-\frac{bdn\left(\frac{1}{2}\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^mdx+\frac{1}{2}\int e^{\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^mdx\right)}{m+1}$$

$$\Big\downarrow \text{2712}$$

$$\frac{(ex)^{m+1}\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(m+1)}-$$
$$\frac{bdn\left(\frac{1}{2}(ex)^m\left(cx^n\right)^{-i\pi abd^2}x^{-m+i\pi abd^2n}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2\left(cx^n\right)d^2-\frac{1}{2}ia^2\pi d^2\right)x^{m-iabd^2n\pi}dx+\frac{1}{2}(ex)^m\left(cx^n\right)^{i\pi abd^2}\right.}{m+1}$$

$$\Big\downarrow \text{2706}$$

$$\frac{(ex)^{m+1}\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(m+1)}-$$
$$\frac{bdn\left(\frac{x(ex)^m(cx^n)^{-\frac{-i\pi abd^2n+m+1}{n}-i\pi abd^2}\int\exp\left(-\frac{1}{2}ib^2\pi\log^2(cx^n)d^2-\frac{1}{2}ia^2\pi d^2+\frac{\left(-iabn\pi d^2+m+1\right)\log(cx^n)}{n}\right)d\log(cx^n)}{2n}+\frac{x(ex)^m(cx^n)^{i\pi a}}{}\right.}{m+1}$$

$$\Big\downarrow \text{2664}$$

$$\frac{(ex)^{m+1}\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(m+1)}-$$
$$\frac{bdn\left(\frac{x(ex)^m\exp\left(-\frac{i(m+1)\left(-2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)(cx^n)^{-\frac{-i\pi abd^2n+m+1}{n}-i\pi abd^2}\int\exp\left(\frac{i\left(-ib^2n\pi\log(cx^n)d^2-iabn\pi d^2+m+1\right)^2}{2b^2d^2n^2\pi}\right)d\log(cx^n)}{2n}+\frac{x(e}{}\right.}{m+1}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(ex)^{m+1}\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(m+1)}-$$
$$\frac{bdn\left(\frac{x(ex)^m\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)(cx^n)^{i\pi abd^2-\frac{i\pi abd^2n+m+1}{n}}\int\exp\left(-\frac{i\left(ib^2n\pi\log(cx^n)d^2+iabn\pi d^2+m+1\right)^2}{2b^2d^2n^2\pi}\right)d\log(cx^n)}{2n}+\frac{(\frac{1}{4}+\frac{i}{4})x(}{}\right.}{m+1}$$

$$\Big\downarrow \text{2634}$$

$$\frac{(ex)^{m+1}\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)}{e(m+1)}-$$
$$\frac{bdn\left(\frac{(\frac{1}{4}+\frac{i}{4})x(ex)^m\exp\left(-\frac{i(m+1)\left(-2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)(cx^n)^{-\frac{-i\pi abd^2n+m+1}{n}-i\pi abd^2}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2n-i\pi b^2d^2n\log(cx^n)+m+1\right)}{\sqrt{\pi}bdn}\right)}{bdn}-\frac{(\frac{1}{4}}{}\right.}{m+1}$$

input
```
Int[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

3.169. $\int(ex)^m\,\text{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx$

output
```
-((b*d*n*(((-1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b
^2*d^2*n^2*Pi))*x*(e*x)^m*(c*x^n)^(I*a*b*d^2*Pi - (1 + m + I*a*b*d^2*n*Pi)
/n)*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[Pi])])/(b*d*n) + ((1/4 + I/4)*x*(e*x)^m*(c*x^n)^((-I)*a*b*d^2
*Pi - (1 + m - I*a*b*d^2*n*Pi)/n)*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*P
i - I*b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])])/(b*d*E^(((I/2)*(1 + m)*
(1 + m - (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*Pi))*n)))/(1 + m)) + ((e*x)^(1
+ m)*FresnelC[d*(a + b*Log[c*x^n])])/(e*(1 + m))
```

3.169.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 5129
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Simp[1/2   Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] +
Simp[1/2   Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b,
 c, d, e, m, n}, x]
```

rule 7026
```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Simp[b*d*(n/(m + 1))   Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^
n]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

3.169.4 Maple [F]

$$\int (ex)^m \operatorname{FresnelC}\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

3.169.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(310) = 620$.

Time = 0.31 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.46

$$\int (ex)^m \operatorname{FresnelC}\left(d(a+b\log\left(cx^n\right))\right)\,dx =$$

$$-\frac{\pi\sqrt{b^2d^2n^2}e^{\left(m\log(e)-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i\,m^2}{2\,\pi b^2d^2n^2}-\frac{i\,m}{\pi b^2d^2n^2}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\,m+}{\pi b^2d^2n^2}\right.}{}$$

input
```
integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-1/2*(pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n
 - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi
*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*
d^2*n^2)*e^(m*log(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I
*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fre
snel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m
 - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*e^(m*lo
g(e) - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2
*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2
*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d
^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*e^(m*log(e) - m*log(c)/
n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*
b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x)
+ pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*
d^2*n^2)) - 2*x*e^(m*log(e) + m*log(x))*fresnel_cos(b*d*log(c*x^n) + a*d))
/(m + 1)
```

3.169.6 Sympy [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int (ex)^m C(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)
```

3.169.7 Maxima [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \operatorname{C}((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.169.8 Giac [F]

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int (ex)^m C((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m \operatorname{FresnelC}(d(a+b\log(cx^n)))\,dx = \int \operatorname{FresnelC}(d(a+b\ln(c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx$

3.170.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=-\frac{ie^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/8*I*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom(
[1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

3.170.2 Mathematica [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx$$

input
```
Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

output
```
Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

3.170.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6991, 26, 6929, 15, 6930}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+\frac{1}{2}i\pi b^2x^2}\,\text{FresnelC}(bx)\,dx$$

$$\Big\downarrow \; 6991$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int -ie^{\frac{1}{2}ib^2\pi x^2+c}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \; 26$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \; 6929$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{ie^c\int \text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)d\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{4b}$$

$$\Big\downarrow \; 15$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{\frac{1}{2}ib^2\pi x^2+c}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{ie^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

$$\Big\downarrow \; 6930$$

$$\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

input
```
Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

output
```
((-1/8*I)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4
```

3.170.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6929
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*
(Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d,
n}, x] && EqQ[d, b^2]
```

rule 6930
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

rule 6991
```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Simp[(1 - I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Simp[(1 + I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b,
c, d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

3.170.4 Maple [F]

$$\int e^{c+\frac{i b^2 \pi x^2}{2}} \text{FresnelC}(bx)\, dx$$

input
```
int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.170.5 Fricas [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = \int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.170.6 Sympy [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = e^c \int e^{\frac{i\pi b^2x^2}{2}}C(bx)\,dx$$

input
```
integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.170.7 Maxima [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2} \operatorname{FresnelC}(bx)\,dx = \int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.170.8 Giac [F]

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int e^{\left(\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{C}\left(bx\right)\,dx$$

input
```
integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\text{FresnelC}(b\,x)\,dx$$

input
```
int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
```

output
```
int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$

3.171.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx=-\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

output
```
-1/8*I*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom
([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

3.171.2 Mathematica [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx=\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$$

input
```
Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

output
```
Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

3.171.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6991, 26, 6927, 15, 6932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-\frac{1}{2}i\pi b^2x^2}\text{FresnelC}(bx)\,dx$$

$$\Big\downarrow \text{6991}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int -ie^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{26}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx$$

$$\Big\downarrow \text{6927}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{ie^c\int \text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)d\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}x\right)dx-\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

$$\Big\downarrow \text{6932}$$

$$\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}bx\right)^2}{8b}$$

input
```
Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

output
```
((-1/8*I)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/4
```

3.171.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 6927
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[E^c*(
Sqrt[Pi]/(2*b))   Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n}
, x] && EqQ[d, -b^2]
```

rule 6932
```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

rule 6991
```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Simp[(1 - I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Simp[(1 + I
)/4   Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b,
c, d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

3.171.4 Maple [F]

$$\int e^{c-\frac{ib^2\pi x^2}{2}} \operatorname{FresnelC}(bx)\, dx$$

input
```
int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.171.5 Fricas [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{C}\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.171.6 Sympy [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=e^c\int e^{-\frac{i\pi b^2x^2}{2}}C(bx)\,dx$$

input
```
integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.171.7 Maxima [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx=\int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,\text{C}\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.171.8 Giac [F]

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = \int e^{\left(-\frac{1}{2}i\,\pi b^2x^2+c\right)}\,C\left(bx\right)\,dx$$

input
```
integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\,\text{FresnelC}(bx)\,dx = \int \mathrm{e}^{c-\frac{\Pi\,b^2\,x^2\,1\mathrm{i}}{2}}\,\text{FresnelC}(b\,x)\,dx$$

input
```
int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
```

output
```
int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.172 $\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\cos(c)\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\text{FresnelC}(bx)^2\sin(c)}{2b}$$

output
```
1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*cos(c)*hypergeom([1,
1],[3/2, 2],-1/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)+1/2*FresnelC(b*x)^2*sin(c)/b
```

3.172.2 Mathematica [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

input
```
Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

output
```
Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

3.172.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {7003, 6995, 15, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2+c\right)dx$$

$$\Big\downarrow \text{7003}$$

$$\sin(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\cos(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{6995}$$

$$\cos(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{\sin(c)\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\cos(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx+\frac{\sin(c)\,\text{FresnelC}(bx)^2}{2b}$$

$$\Big\downarrow \text{7001}$$

$$\frac{\sin(c)\,\text{FresnelC}(bx)^2}{2b}+\cos(c)\left(\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}\right)$$

input
```
Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

output
```
Cos[c]*((FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPF
Q[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{
1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]) + (FresnelC[b*x]^2*Sin[c])/(2*b)
```

3.172.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7003
```
Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[Sin[c]
   Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[Cos[c]   Int[Sin[d*x^2]*Fres
nelC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.172.4 Maple [F]

$$\int \text{FresnelC}\,(bx) \sin\left(c + \frac{b^2\pi\, x^2}{2}\right) dx$$

input
```
int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

output
```
int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

3.172.5 Fricas [F]

$$\int \text{FresnelC}(bx) \sin\left(c + \frac{1}{2}b^2\pi x^2\right) dx = \int \text{C}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2 + c\right) dx$$

input
```
integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.172.6 Sympy [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \sin\left(\frac{\pi b^2x^2}{2}+c\right)C(bx)\,dx$$

input
```
integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)
```

output
```
Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

3.172.7 Maxima [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.172.8 Giac [F]

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2+c\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\int \sin\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\text{FresnelC}(b\,x)\,dx$$

input
```
int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
```

output
```
int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.173 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx$

3.173.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx = \frac{\cos(c)\operatorname{FresnelC}(bx)^2}{2b} - \frac{\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)\sin(c)$$

output
```
1/2*cos(c)*FresnelC(b*x)^2/b-1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*
I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hy
pergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

3.173.2 Mathematica [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx = \int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx$$

input
```
Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

3.173.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6997, 6995, 15, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2+c\right)dx$$

$$\Big\downarrow \text{6997}$$

$$\cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\sin(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\cos(c)\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{b}-\sin(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{15}$$

$$\frac{\cos(c)\,\text{FresnelC}(bx)^2}{2b}-\sin(c)\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

$$\Big\downarrow \text{7001}$$

$$\frac{\cos(c)\,\text{FresnelC}(bx)^2}{2b}-\sin(c)\left(\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}\right)$$

input
```
Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(Cos[c]*FresnelC[b*x]^2)/(2*b) - ((FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I
/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)
*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])*Sin[c]
```

3.173.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 6997
```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] :> Simp[Cos[c]
   Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[Sin[c]   Int[Sin[d*x^2]*Fres
nelC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.173.4 Maple [F]

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)dx$$

input
```
int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.173.5 Fricas [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int \cos\left(\frac{1}{2}\,\pi b^2x^2+c\right)\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.173.6 Sympy [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int \cos\left(\frac{\pi b^2x^2}{2}+c\right)C(bx)\,dx$$

input
```
integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

3.173.7 Maxima [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{C}\,(bx)\,dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.173.8 Giac [F]

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int \cos\left(\frac{1}{2}\pi b^2x^2+c\right)\text{C}\,(bx)\,dx$$

input
```
integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int \cos\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\text{FresnelC}(b\,x)\,dx$$

input
```
int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
```

output
```
int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.174 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx$

3.174.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \frac{\text{FresnelC}(bx)^3}{3b}$$

output
```
1/3*FresnelC(b*x)^3/b
```

3.174.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \frac{\text{FresnelC}(bx)^3}{3b}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

output
```
FresnelC[b*x]^3/(3*b)
```

3.174.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)^2 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \, dx$$

$$\Big\downarrow \text{ 6995}$$

$$\frac{\int \text{FresnelC}(bx)^2 d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

output
```
FresnelC[b*x]^3/(3*b)
```

3.174.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.174.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^3}{3b}$	12
default	$\frac{\text{FresnelC}(bx)^3}{3b}$	12

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*FresnelC(b*x)^3/b
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^2\,dx = \frac{\text{C}\,(bx)^3}{3\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
1/3*fresnel_cos(b*x)^3/b
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^2\,dx = \begin{cases} \frac{C^3(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
```

output
```
Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \frac{\text{C}\,(bx)^3}{3\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
1/3*fresnel_cos(b*x)^3/b
```

3.174.8 Giac [F]

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \int \cos \left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\,(bx)^2\;dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)^2, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)^2\,dx = \int \text{FresnelC}(b\,x)^2\,\cos \left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2), x)
```

3.175 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.175.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

output
```
1/2*FresnelC(b*x)^2/b
```

3.175.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
FresnelC[b*x]^2/(2*b)
```

3.175.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
FresnelC[b*x]^2/(2*b)
```

3.175.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.175.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelC}(bx)^2}{2b}$	12

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*FresnelC(b*x)^2/b
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{C}\,(bx)^2}{2\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/2*fresnel_cos(b*x)^2/b
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{C}\,(bx)^2}{2\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/2*fresnel_cos(b*x)^2/b
```

3.175.8 Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\,(bx)\;dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int \text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.176 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx$

3.176.1 Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{FresnelC}(bx))}{b}$$

output
```
ln(FresnelC(b*x))/b
```

3.176.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log(\text{FresnelC}(bx))}{b}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

output
```
Log[FresnelC[b*x]]/b
```

3.176.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6995, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{FresnelC}(bx)}\,dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \frac{1}{\text{FresnelC}(bx)}d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

output
```
Log[FresnelC[b*x]]/b
```

3.176.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.176.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\text{FresnelC}(bx))}{b}$	10
default	$\frac{\ln(\text{FresnelC}(bx))}{b}$	10

input
```
int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(FresnelC(b*x))/b
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log\left(\text{C}\left(bx\right)\right)}{b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
log(fresnel_cos(b*x))/b
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \begin{cases} \frac{\log\left(C\left(bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)
```

output
```
Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \frac{\log\left(\text{C}\left(bx\right)\right)}{b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
log(fresnel_cos(b*x))/b
```

3.176.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\text{C}\left(bx\right)}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelC}\left(b\,x\right)}\,dx$$

input
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x),x)
```

output
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x), x)
```

3.177 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx$

3.177.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelC}(bx)}$$

output
```
-1/b/FresnelC(b*x)
```

3.177.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{FresnelC}(bx)}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

output
```
-(1/(b*FresnelC[b*x]))
```

3.177.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{FresnelC}(bx)^2}dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \frac{1}{\text{FresnelC}(bx)^2}d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{b\,\text{FresnelC}(bx)}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

output
```
-(1/(b*FresnelC[b*x]))
```

3.177.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.177.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\ \mathrm{FresnelC}(bx)}$	12
default	$-\frac{1}{b\ \mathrm{FresnelC}(bx)}$	12

input
```
int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/FresnelC(b*x)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\mathrm{C}\,(bx)}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="fricas")
```

output
```
-1/(b*fresnel_cos(b*x))
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelC}(bx)^2}\,dx = \begin{cases} -\frac{1}{bC(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
```

output
```
Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = -\frac{1}{b\,\text{C}\,(bx)}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="maxima")
```

output
```
-1/(b*fresnel_cos(b*x))
```

3.177.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{C}\,(bx)^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^2, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelC}\,(b\,x)^2}\,dx$$

input
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2,x)
```

output
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2, x)
```

3.178 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx$

3.178.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

output
```
-1/2/b/FresnelC(b*x)^2
```

3.178.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

output
```
-1/2*1/(b*FresnelC[b*x]^2)
```

3.178.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\text{FresnelC}(bx)^3} dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \frac{1}{\text{FresnelC}(bx)^3} d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2b\,\text{FresnelC}(bx)^2}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

output
```
-1/2*1/(b*FresnelC[b*x]^2)
```

3.178.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.178.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\,\mathrm{FresnelC}(bx)^2}$	12
default	$-\frac{1}{2b\,\mathrm{FresnelC}(bx)^2}$	12

input
```
int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/FresnelC(b*x)^2
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelC}(bx)^3}\,dx = -\frac{1}{2\,b\,\mathrm{C}\left(bx\right)^2}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="fricas")
```

output
```
-1/2/(b*fresnel_cos(b*x)^2)
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathrm{FresnelC}(bx)^3}\,dx = \begin{cases} -\frac{1}{2bC^2(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)
```

output
```
Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = -\frac{1}{2\,b\,\text{C}\,(bx)^2}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="maxima")
```

output
```
-1/2/(b*fresnel_cos(b*x)^2)
```

3.178.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{\text{C}\,(bx)^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^3, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^3}\,dx = \int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\text{FresnelC}\,(b\,x)^3}\,dx$$

input
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3,x)
```

output
```
int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3, x)
```

3.179 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx$

3.179.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

output
```
FresnelC(b*x)^(1+n)/b/(1+n)
```

3.179.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

output
```
FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

3.179.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(\frac{1}{2}\pi b^2x^2\right)\text{FresnelC}(bx)^n\,dx$$

$$\Big\downarrow \text{ 6995}$$

$$\frac{\int \text{FresnelC}(bx)^n d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

output
```
FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

3.179.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.179.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$	18

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x,method=_RETURNVERBOSE)
```

output
```
FresnelC(b*x)^(1+n)/b/(1+n)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\mathrm{C}\,(bx)^n\,\mathrm{C}\,(bx)}{bn+b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="fricas")
```

output
```
fresnel_cos(b*x)^n*fresnel_cos(b*x)/(b*n + b)
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \begin{cases} \tilde{\infty}x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \frac{\log(C(bx))}{b} & \text{for } n = -1 \\ \frac{C(bx)C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)
```

output
```
Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc
(b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \frac{\text{C}\,(bx)^{n+1}}{b(n+1)}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="maxima")
```

output
```
fresnel_cos(b*x)^(n + 1)/(b*(n + 1))
```

3.179.8 Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \int \text{C}\,(bx)^n\cos\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)^n\,dx = \int \text{FresnelC}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$

3.180.1 Optimal result

Integrand size = 20, antiderivative size = 231

$$\begin{aligned}\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx &= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} \\ &+ \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} \\ &+ \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\ &+ \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} \\ &- \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ &+ \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} \\ &+ \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}\end{aligned}$$

output
```
105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2-55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4
*x^6*cos(b^2*Pi*x^2)/b^3/Pi^2-105*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/
Pi^4+7*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)^
2/b^9/Pi^4-35*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelC(
b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+40*sin(b^2*Pi*x^2)/b^9/Pi^5-5/2*x^4*sin(b^
2*Pi*x^2)/b^5/Pi^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{105\,\text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{40\sin(b^2\pi x^2)}{b^9\pi^5} - \frac{5x^4\sin(b^2\pi x^2)}{2b^5\pi^3}$$

input
```
Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/
(4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(b^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*S
in[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
b^2*Pi) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5
*Pi^3)
```

3.180.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{ 7009}$$

$$-\frac{7\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2} - \frac{\int x^7\sin(b^2\pi x^2)\,dx}{2\pi b} + \frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3860$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^6\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3777$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\int x^4\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\int x^4\sin(b^2\pi x^2+\frac{\pi}{2})dx^2}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3777$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{7\int x^6 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3777$$

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{7\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3042

$$-\frac{7\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$\downarrow$ 3117

$$-\frac{7\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 7017

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3861

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\downarrow$ 3042

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ 3790$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ 15$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^6}{6}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ 25$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow\ 3042$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$\Big\downarrow \; 3777$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{2\int -x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 25$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\int x^2\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3777$$

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3117$$

$$-\frac{7\left(\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}\right)}{\pi b^2}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 7009$$

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2 \cdots\right.}{\pi b^2}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3860$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b \cdots\right.}{\pi b^2}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b \cdots\right.}{\pi b^2}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3777$$

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right.}{\pi b^2}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3117$$

$$-\frac{7\left(\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\right.}{}\right.}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

input
```
Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

3.180. $\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

output
```
$Aborted
```

3.180.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.180.4 Maple [F]

$$\int x^{8} \cos\left(\frac{b^{2} \pi x^{2}}{2}\right) \operatorname{FresnelC}\left(b x\right)dx$$

input
```
int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\,dx =$$
$$-\frac{5\,\pi^3 b^6 x^6 - 240\,\pi b^2 x^2 - 3\,(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 42\,(\pi^3 b^5 x^5 - 15\,\pi b x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\left(bx\right.}{6\,\pi^5 b^9}$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
-1/6*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*c
os(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*f
resnel_cos(b*x) - 315*pi*fresnel_cos(b*x)^2 + 6*(5*(pi^2*b^4*x^4 - 16)*cos
(1/2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fresnel_cos(b*x))*sin(
1/2*pi*b^2*x^2))/(pi^5*b^9)
```

3.180.6 Sympy [A] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$$
$$= \begin{cases} \frac{x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} - \frac{5x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{6\pi^2 b^3} - \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{3\pi^2 b^3} + \frac{7x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{5x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} - \frac{35x^3 \sin\left(\right.}{\pi} \\ 0 \end{cases}$$

input
```
integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Piecewise((x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - 5*x**6*sin(p
i*b**2*x**2/2)**2/(6*pi**2*b**3) - x**6*cos(pi*b**2*x**2/2)**2/(3*pi**2*b*
*3) + 7*x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 5*x**4*sin(p
i*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 35*x**3*sin(pi*b**2*x**2
/2)*fresnelc(b*x)/(pi**3*b**6) + 40*x**2*sin(pi*b**2*x**2/2)**2/(pi**4*b**
7) + 25*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**4*b**7) - 105*x*cos(pi*b**2*x**
2/2)*fresnelc(b*x)/(pi**4*b**8) + 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/
2)/(pi**5*b**9) + 105*fresnelc(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, True
))
```

3.180.7 Maxima [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}(bx)\,dx$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.180.8 Giac [F]

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^8 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathrm{FresnelC}(bx)\,dx = \int x^8\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.181.1 Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = {} & \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos(b^2\pi x^2)}{16b^7\pi^4} \\ & + \frac{x^5\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} \\ & + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} \\ & + \frac{531\,\text{FresnelC}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} \\ & - \frac{24x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} \\ & + \frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{17x^3\sin(b^2\pi x^2)}{8b^5\pi^3}\end{aligned}$$

output
```
24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2-147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*c
os(b^2*Pi*x^2)/b^3/Pi^2-48*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi^4+6*x^
4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2-24*x^2*FresnelC(b*x)*sin(1/2*
b^2*Pi*x^2)/b^6/Pi^3+x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-17/8*x^3
*sin(b^2*Pi*x^2)/b^5/Pi^3+531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$
$$= \frac{2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)+160\,\text{FresnelC}(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^2\pi x^2(-24+b^4\pi^2x^4)\sin\left(\right.\right.}{160b^8\pi^4}$$

input
```
Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*
x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2
)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(-960 + 24*b^
4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

3.181.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 438 vs. 2(215) = 430.

Time = 1.96 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.850, Rules used = {7009, 3866, 3867, 3866, 3833, 7017, 3873, 15, 3867, 3866, 3833, 7009, 3866, 3833, 7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \; 7009$$

$$-\frac{6\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^6\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3866$$

$$-\frac{6\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{5\int x^4\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}+\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3867$$

$$-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3866$$

$$-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3833$$

$$-\frac{6\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 7017$$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 3873$$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx+\frac{\int x^4dx}{2}}{\pi b}-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 15$$

3.181. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx+\frac{x^5}{10}}{\pi b}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 3867$$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 3866$$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \; 3833$$

3.181. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{6\left(\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}\right)}{\pi b^2}+\cdots$$

$$\frac{\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 7009

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x\cdots\right.}{2\pi b^2}\cdots\right.}{\cdots}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 3866

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\cdots}{\cdots}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\Big\downarrow$ 3833

3.181. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{6\left(\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 7015

$$-\frac{6\left(\frac{4\left(-\frac{2\left(\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cdots}{\pi b^2}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3839

$$-\frac{6\left(\frac{4\left(-\frac{2\left(\frac{\int\left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(\cdots}{\cdots}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{5\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 2009

3.181. $\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$\frac{x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{\frac{5\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2} - \frac{3\left(\frac{\operatorname{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3} - \frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)}{2\pi b^2} - \frac{x^5\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b} - \frac{6\left(-\frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{4\left(\frac{x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} - \frac{2\left(\frac{\frac{\operatorname{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}b}+\frac{x}{2}}{\pi b} - \frac{\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{\frac{\operatorname{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3} - \frac{x\cos \cdots}{\cdots}}{2\pi b}\cdots\right)}{\pi b^2}\cdots\right)}{\pi b^2}\cdots$$

input
```
Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (-1/2*(x^5*Cos[b^2*Pi*x
^2])/(b^2*Pi) + (5*((-3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt
[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) + (x^3*Sin[b^2*Pi*x^2])/(2*b^2*Pi
)))/(2*b^2*Pi))/(2*b*Pi) - (6*(-((x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(
b^2*Pi)) + (4*(-1/2*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*
b*x]/(2*Sqrt[2]*b^3*Pi))/(b*Pi) - (2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(b^2*Pi)) + (x/2 + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)))/(b^2*Pi
) + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x^5/10
+ ((-3*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[
2]*b^3*Pi)))/(2*b^2*Pi) + (x^3*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)))/(b
^2*Pi)
```

3.181.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.181. $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.181.4 Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.47

method	result
default	$\frac{\frac{\operatorname{FresnelC}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{3}{5}b^5x^5\pi^2-24bx}{\pi^4}+\frac{\frac{3\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}}{}}{}$

input
```
int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*c
os(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b
^2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*b^5*x^5*Pi^2-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^
3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x
^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*
b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b
^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx =$$
$$-\frac{136\,\pi^2b^6x^5-5310\,b^2x-20\,(4\,\pi^2b^6x^5-147\,b^2x)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-960\,(\pi^2b^5x^4-8\,b)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\,(bx}{160}$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
-1/160*(136*pi^2*b^6*x^5 - 5310*b^2*x - 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*co
s(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel
_cos(b*x) - 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(
17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresn
el_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

3.181.6 Sympy [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\, dx = \int x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x**7*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.181.7 Maxima [F]

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\, dx = \int x^7 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \mathrm{C}\left(bx\right)\, dx$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.181.8 Giac [F]

$$\int x^7 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^7 \cos \left(\frac{1}{2}\pi b^2 x^2\right) \text{C}\left(bx\right)\, dx$$

input
```
integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^7 \,\text{FresnelC}(b\,x)\, \cos \left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

input
```
int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.182.1 Optimal result

Integrand size = 20, antiderivative size = 247

$$\begin{aligned}\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = &-\frac{5x^4}{8b^3\pi^2}-\frac{11\cos(b^2\pi x^2)}{2b^7\pi^4}+\frac{x^4\cos(b^2\pi x^2)}{4b^3\pi^2}\\&+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}\\&+\frac{15\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^7\pi^3}\\&+\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3}\\&-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^5\pi^3}\\&-\frac{15x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}\\&+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{7x^2\sin(b^2\pi x^2)}{4b^5\pi^3}\end{aligned}$$

output
```
-5/8*x^4/b^3/Pi^2-11/2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*cos(b^2*Pi*x^2)/b^
3/Pi^2+5*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+15/2*FresnelC(b*x)
*FresnelS(b*x)/b^7/Pi^3+15/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi
*x^2)/b^5/Pi^3-15/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/
Pi^3-15*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelC(b*x)*sin
(1/2*b^2*Pi*x^2)/b^2/Pi-7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

3.182.2 Mathematica [F]

$$\int x^6 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^6 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$$

input
```
Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

3.182.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.36, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {7009, 3860, 3042, 3777, 3042, 3777, 25, 3042, 3118, 7017, 3861, 3042, 3790, 15, 25, 3042, 3777, 25, 3042, 3118, 7009, 3860, 3042, 3118, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \text{FresnelC}(bx) \cos \left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \; 7009$$

$$-\frac{5\int x^4 \text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\int x^5 \sin \left(b^2\pi x^2\right) dx}{2\pi b}+\frac{x^5 \text{FresnelC}(bx) \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3860$$

$$-\frac{5\int x^4 \text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\int x^4 \sin \left(b^2\pi x^2\right) dx^2}{4\pi b}+\frac{x^5 \text{FresnelC}(bx) \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\int x^4 \text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\int x^4 \sin \left(b^2\pi x^2\right) dx^2}{4\pi b}+\frac{x^5 \text{FresnelC}(bx) \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$-\frac{5\int x^4 \text{FresnelC}(bx) \sin \left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\frac{2\int x^2 \cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4 \cos\left(\pi b^2 x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5 \text{FresnelC}(bx) \sin \left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3118$$

$$-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 7017$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \; 3861$$

3.182. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3042

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3790

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 15

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^4}{4}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 25

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$\Big\downarrow$ 3042

3.182. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3777$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{\int-\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 25$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3042$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3118$$

$$-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}\right)}{\pi b^2}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

3.182. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$\downarrow 7009$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\cdots\right)}{2\pi b}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3860$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\cdots\right)}{2\pi b}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3042$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\cdots\right)}{2\pi b}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 3118$$

$$-\frac{5\left(\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi \cdots\right)}{\pi^2\cdots}\cdots\right)}{2\pi b}\cdots\right)}{\pi b^2}\cdots$$

$$\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}$$

$$\downarrow 7001$$

3.182. $\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{5\left(\frac{3\left(-\frac{-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\frac{x^3\,\text{F}\cdots}{\cdots}\right.}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

input
```
Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (-((x^4*Cos[b^2*Pi*x^2]
)/(b^2*Pi)) + (2*(Cos[b^2*Pi*x^2]/(b^4*Pi^2) + (x^2*Sin[b^2*Pi*x^2])/(b^2*
Pi)))/(b^2*Pi))/(4*b*Pi) - (5*(-((x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(
b^2*Pi)) + (3*(Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - ((FresnelC[b*x]*FresnelS[b*x
])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi
*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])
/(b^2*Pi) + (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x
^4/4 + (Cos[b^2*Pi*x^2]/(b^4*Pi^2) + (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi))/2)/(2
*b*Pi)))/(b^2*Pi)
```

3.182.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.182. $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.182.4 Maple [F]

$$\int x^{6} \cos \left(\frac{b^{2} \pi \,x^{2}}{2}\right) \operatorname{FresnelC}\left(b x \right)d x$$

input
```
int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.182.5 Fricas [F]

$$\int x^6 \cos \left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\, dx = \int x^{6} \cos \left(\frac{1}{2} \, \pi b^{2} x^{2}\right) \operatorname{C}\left(b x\right) \,d x$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
integral(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.182.6 Sympy [F]

$$\int x^6 \cos \left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)\, dx = \int x^{6} \cos{\left(\frac{\pi b^{2} x^{2}}{2} \right)} C\left(b x\right)\, dx$$

input
```
integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x**6*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.182.7 Maxima [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.182.8 Giac [F]

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\,dx$$

input
```
integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^6\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.183.1 Optimal result

Integrand size = 20, antiderivative size = 157

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{2x^3}{3b^3\pi^2} + \frac{x^3\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{43\,\text{FresnelS}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} - \frac{8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11x\sin(b^2\pi x^2)}{8b^5\pi^3}$$

output
```
-2/3*x^3/b^3/Pi^2+1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+4*x^2*cos(1/2*b^2*Pi*x^
2)*FresnelC(b*x)/b^4/Pi^2-8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^4
*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3+
43/16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{-32b^3\pi x^3 + 12b^3\pi x^3\cos(b^2\pi x^2) + 129\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) + 48\,\text{FresnelC}(bx)\left(4b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) + (-\right.}{48b^6\pi^3}$$

input
```
Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqr
t[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4
*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

3.183.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.73, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {7009, 3866, 3867, 3832, 7017, 3873, 15, 3867, 3832, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{7009}$$

$$-\frac{4\int x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$-\frac{4\int x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{4\int x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{4\int x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{7017}$$

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{3873}$$

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{\int x^2dx}{2}}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{x^3}{6}}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{3867}$$

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{7007}$$

3.183. $\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

$$-\frac{4\left(\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

$\downarrow$ 3832

$$\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}$$

input
```
Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (-1/2*(x^3*Cos[b^2*Pi*x
^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b
^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) - (4*(-((x^2*Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/(b^2*Pi)) + (2*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^
2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x^3/6 +
 (-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2
*Pi))/2)/(b*Pi)))/(b^2*Pi)
```

3.183.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.183. $\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.183. $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$

3.183.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}+\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{FresnelS}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}+\frac{-\frac{\pi b^3x^3\cos}{2}}{}}{}}{b}$

input
```
int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3+2/
Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))+1/2
/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-
1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx$$
$$=\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-44\,\pi b^4x^3+192\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)+129\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)-12\,(11\,b}{48\,\pi^3b^7}$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 44*pi*b^4*x^3 + 192*pi*b^3*x^2
*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(
sqrt(2)*sqrt(b^2)*x) - 12*(11*b^2*x*cos(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4
- 8*b)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

3.183.6 Sympy [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

input
```
integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.183.7 Maxima [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}(bx)\,dx$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.183.8 Giac [F]

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^5 \cos\left(\frac{1}{2}\pi b^2 x^2\right) \text{C}(bx)\,dx$$

input
```
integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^5\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.184.1 Optimal result

Integrand size = 20, antiderivative size = 120

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

output
```
-3/4*x^2/b^3/Pi^2+1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2+3*x*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/b^4/Pi^2-3/2*FresnelC(b*x)^2/b^5/Pi^2+x^3*FresnelC(b*x)*sin
(1/2*b^2*Pi*x^2)/b^2/Pi-sin(b^2*Pi*x^2)/b^5/Pi^3
```

3.184.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{3\,\text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

input
```
Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2
*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) +
 (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*P
i^3)
```

3.184.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7009, 3860, 3042, 3777, 3042, 3117, 7017, 3861, 3042, 3114, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \; 7009$$

$$-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3860$$

$$-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{3\int x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{7017}$$

$$-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3861}$$

$$-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{3114}$$

$$-\frac{3\left(\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}-\frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{6995}$$

$$-\frac{3\left(\frac{\int\operatorname{FresnelC}(bx)d\operatorname{FresnelC}(bx)}{\pi b^3}-\frac{x\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}+\frac{x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}$$

$$\Big\downarrow \text{15}$$

3.184. $\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)\,dx$

$$\frac{x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\frac{\sin(\pi b^2 x^2)}{\pi^2 b^4} - \frac{x^2 \cos(\pi b^2 x^2)}{\pi b^2}}{4\pi b} - \frac{3\left(\frac{\operatorname{FresnelC}(bx)^2}{2\pi b^3} - \frac{x \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\frac{\sin(\pi b^2 x^2)}{2\pi b^2} + \frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}$$

input
```
Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (-((x^2*Cos[b^2*Pi*x^2]
)/(b^2*Pi)) + Sin[b^2*Pi*x^2]/(b^4*Pi^2))/(4*b*Pi) - (3*(-((x*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + FresnelC[b*x]^2/(2*b^3*Pi) + (x^2/2 + S
in[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)))/(b^2*Pi)
```

3.184.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.184.4 Maple [F]

$$\int x^4 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)\, dx$$

input
```
int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$$
$$= \frac{\pi b^2x^2 \cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 - 2\,\pi b^2x^2 + 6\,\pi bx \cos\left(\frac{1}{2}\,\pi b^2x^2\right) \mathrm{C}\left(bx\right) - 3\,\pi\,\mathrm{C}\left(bx\right)^2 + 2\,\left(\pi^2b^3x^3\,\mathrm{C}\left(bx\right) - 2\,\cos\left(\frac{1}{2}\,\pi b\right.\right.}{2\,\pi^3b^5}$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/2*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*pi
*b^2*x^2)*fresnel_cos(b*x) - 3*pi*fresnel_cos(b*x)^2 + 2*(pi^2*b^3*x^3*fre
snel_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx$$
$$= \begin{cases} \frac{x^3 \sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} - \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{3x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^2b^4} - \frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} - \frac{3C^2(bx)}{2\pi^2b^5} & \text{for} \\ 0 & \text{oth} \end{cases}$$

input
```
integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*
b**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3)
+ 3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/
2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), N
e(b, 0)), (0, True))
```

3.184.7 Maxima [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\pi b^2x^2\right) C(bx)\,dx$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.184.8 Giac [F]

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^4 \cos\left(\frac{1}{2}\pi b^2x^2\right) C(bx)\,dx$$

input
```
integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^4\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.185.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{x}{b^3\pi^2} + \frac{x\cos(b^2\pi x^2)}{4b^3\pi^2} + \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{5\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

output
```
-x/b^3/Pi^2+1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2+2*cos(1/2*b^2*Pi*x^2)*FresnelC(
b*x)/b^4/Pi^2+x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-5/8*FresnelC(b*
x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{2bx(-4+\cos(b^2\pi x^2)) - 5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right) + 8\,\text{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}$$

input
```
Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
(2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*Fresne
lC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*P
i^2)
```

3.185.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7009, 3866, 3833, 7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7009}$$

$$-\frac{2\int x \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\int x^2 \sin\left(b^2\pi x^2\right) dx}{2\pi b}+\frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$-\frac{2\int x \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}-\frac{\frac{\int \cos\left(b^2\pi x^2\right) dx}{2\pi b^2}-\frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{2\int x \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2}+\frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{7015}$$

$$-\frac{2\left(\frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{3839}$$

$$-\frac{2\left(\frac{\int \left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right) dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2 x^2\right)}{2\pi b^2}}{2\pi b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{2\left(\frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}+\frac{x}{2}}{\pi b} - \frac{\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}\right)}{\pi b^2} - \frac{\frac{\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3} - \frac{x\cos(\pi b^2 x^2)}{2\pi b^2}}{2\pi b}$$

input
```
Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
-1/2*(-1/2*(x*Cos[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]
*b^3*Pi))/(b*Pi) - (2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (
x/2 + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)))/(b^2*Pi) + (x^2*Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.185.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	114

input
```
int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2))-1/b^3*(b*x/Pi^2+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2
/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx$$
$$=\frac{8\,\pi b^3x^2\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-10\,b^2x+16\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)-5\,\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\right.}{8\,\pi^2b^5}$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/8*(8*pi*b^3*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + 4*b^2*x*cos(1/2*p
i*b^2*x^2)^2 - 10*b^2*x + 16*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 5*sq
rt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

3.185.6 Sympy [F]

$$\int x^3 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^3 \cos \left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.185.7 Maxima [F]

$$\int x^3 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^3 \cos \left(\frac{1}{2}\pi b^2 x^2\right) \text{C}\left(bx\right)\, dx$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.185.8 Giac [F]

$$\int x^3 \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^3 \cos \left(\frac{1}{2}\pi b^2 x^2\right) \text{C}\left(bx\right)\, dx$$

input
```
integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^3\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.186.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\cos(b^2\pi x^2)}{4b^3\pi^2} - \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^3\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi} + \frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

output
```
1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-1/2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*
x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom(
[1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b/Pi+x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/b^2/Pi
```

3.186.2 Mathematica [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$$

input
```
Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

3.186.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7009, 3860, 3042, 3118, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7009}$$

$$-\frac{\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} - \frac{\int x \sin\left(b^2\pi x^2\right) dx}{2\pi b} + \frac{x\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3860}$$

$$-\frac{\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} - \frac{\int \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} + \frac{x\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} - \frac{\int \sin\left(b^2\pi x^2\right) dx^2}{4\pi b} + \frac{x\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b^2} + \frac{x\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

$$\Big\downarrow \text{7001}$$

$$-\frac{\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) + \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2} + \frac{x\,\text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} + \frac{\cos\left(\pi b^2 x^2\right)}{4\pi^2 b^3}$$

input
```
Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - ((FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8
)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b
*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi) + (x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

3.186.4 Maple [F]

$$\int x^{2} \cos \left(\frac{b^{2} \pi \,x^{2}}{2}\right) \operatorname{FresnelC}\left(b x \right)d x$$

input
```
int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

output
```
int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

3.186.5 Fricas [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\, dx$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
integral(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.186.6 Sympy [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.186.7 Maxima [F]

$$\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\, dx = \int x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right)\, dx$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.186.8 Giac [F]

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)\,dx$$

input
```
integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx=\int x^2\,\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.187.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi} + \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

output
```
FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*
2^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = -\frac{\sqrt{2}\,\text{FresnelS}\left(\sqrt{2}bx\right) - 4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

input
```
Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
-1/4*(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(b^2*Pi)
```

3.187.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \; 7007$$

$$\frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\int \sin\left(b^2 \pi x^2\right) dx}{2\pi b}$$

$$\Big\downarrow \; 3832$$

$$\frac{\operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2} - \frac{\operatorname{FresnelS}\left(\sqrt{2} bx\right)}{2\sqrt{2}\pi b^2}$$

input
```
Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi)
```

3.187.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.187.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{\mathrm{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}-\frac{\mathrm{FresnelS}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	45

input
```
int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(
1/2))/b
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\frac{4\,b\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/4*(4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^2)*fresnel_
sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

3.187.6 Sympy [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\;dx$$

input
```
integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.187.7 Maxima [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.187.8 Giac [F]

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)\,dx$$

input
```
integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)\,dx=\int x\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.188 $\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

3.188.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

output
```
1/2*FresnelC(b*x)^2/b
```

3.188.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos \left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx = \frac{\text{FresnelC}(bx)^2}{2b}$$

input
```
Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
FresnelC[b*x]^2/(2*b)
```

3.188.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

input
```
Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

output
```
FresnelC[b*x]^2/(2*b)
```

3.188.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

3.188.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\text{FresnelC}(bx)^2}{2b}$	12

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*FresnelC(b*x)^2/b
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{C}\,(bx)^2}{2\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

output
```
1/2*fresnel_cos(b*x)^2/b
```

3.188.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

output
```
Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \frac{\text{C}\,(bx)^2}{2\,b}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

output
```
1/2*fresnel_cos(b*x)^2/b
```

3.188.8 Giac [F]

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\text{C}\,(bx)\;dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx = \int \text{FresnelC}(b\,x)\;\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.189 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$

3.189.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

output
```
Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.189.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]
```

3.189.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$

$$\Big\downarrow \ 7013$$

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]
```

output
```
$Aborted
```

3.189.3.1 Defintions of rubi rules used

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.189.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.189.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{C}(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

3.189.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

3.189.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{C}(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

3.189.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\,(bx)}{x}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

3.189.9 Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.190 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$

3.190.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)$$

output
```
Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

3.190.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

3.190.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$
$$\Big\downarrow \ 7013$$
$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}dx$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
```

output
```
$Aborted
```

3.190.3.1 Defintions of rubi rules used

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.190.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^2}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

3.190.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

3.190.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx=\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

3.190.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx=\int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

3.190.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)C\left(bx\right)}{x^2}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

3.190.9 Mupad [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^2}\,dx = \int \frac{\operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

3.191 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$

3.191.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = -\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2x^2} - \frac{b^2\pi\,\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{1}{2}b^2\pi\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
-1/4*b/x-1/4*b*cos(b^2*Pi*x^2)/x-1/2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2
-1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelC
(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.191.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]
```

3.191.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}dx$$

$$\Big\downarrow \text{7011}$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{b}{4x}$$

$$\Big\downarrow \text{7021}$$

$$-\frac{1}{2}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{b}{4x}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

output
```
$Aborted
```

3.191.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.191.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\operatorname{FresnelC}(bx)}{x^3}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

3.191.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)C(bx)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

3.191.6 Sympy [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

3.191.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)C(bx)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

3.191.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}\left(bx\right)}{x^3}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

3.191.9 Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

3.192 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx$

3.192.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3}-\frac{1}{6}b^3\pi^2\,\text{FresnelC}(bx)^2 + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)$$

output
```
-1/12*b/x^2-1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x^3-1/6*b^3*Pi^2*FresnelC(b*x)^2-1/6*b^3*Pi*Si(b^2*Pi*x^2)+1/3*b^2*Pi*F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

3.192.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3}-\frac{1}{6}b^3\pi^2\,\text{FresnelC}(bx)^2 + \frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

output
```
-1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

3.192.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {7011, 3861, 3042, 3778, 25, 3042, 3780, 7019, 3856, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}dx$$

$$\Big\downarrow \text{ 7011}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{ 3861}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{3}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{7019}$$

$$-\frac{1}{3}\pi b^2\left(\pi b^2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{3856}$$

$$-\frac{1}{3}\pi b^2\left(\pi b^2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{6995}$$

$$-\frac{1}{3}\pi b^2\left(\pi b\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

$$\Big\downarrow \text{15}$$

3.192. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx$

$$-\frac{1}{3}\pi b^2\left(-\frac{\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\operatorname{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\operatorname{FresnelC}(bx)^2\right)-\frac{\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\operatorname{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{b}{12x^2}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

output
```
-1/12*b/x^2 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*x^3) - (b^2*Pi*((b*Pi
*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIntegr
al[b^2*Pi*x^2])/4))/3 + (b*(-(Cos[b^2*Pi*x^2]/x^2) - b^2*Pi*SinIntegral[b^
2*Pi*x^2]))/12
```

3.192.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.192.4 Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}(bx)}{x^4}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = -\frac{\pi^2 b^3 x^3\,\mathrm{C}\left(bx\right)^2 + \pi b^3 x^3\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi b^2 x^2\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) + bx\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\,\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\,\mathrm{C}}{6\,x^3}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(pi^2*b^3*x^3*fresnel_cos(b*x)^2 + pi*b^3*x^3*sin_integral(pi*b^2*x^2
) - 2*pi*b^2*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + b*x*cos(1/2*pi*b^2
*x^2)^2 + 2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x))/x^3
```

3.192.6 Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

3.192.7 Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\,\mathrm{C}\left(bx\right)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

3.192.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^4}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}(bx)}{x^4}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{x^4}\,dx = \int \frac{\operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.193 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^5} \, dx$

3.193.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^5} \, dx = -\frac{b}{24x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{24x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{4x^4} - \frac{7b^4\pi^2 \text{FresnelC}\left(\sqrt{2}bx\right)}{24\sqrt{2}} + \frac{b^2\pi \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2} + \frac{7b^3\pi\sin\left(b^2\pi x^2\right)}{48x} - \frac{1}{8}b^4\pi^2 \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x}, x\right)$$

output
```
-1/24*b/x^3-1/24*b*cos(b^2*Pi*x^2)/x^3-1/4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*
x)/x^4+1/8*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+7/48*b^3*Pi*sin(b^
2*Pi*x^2)/x-7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unint
egrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.193.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]
```

3.193.3 Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7011, 3869, 3868, 3833, 7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}\,dx$$

$$\Big\downarrow \ 7011$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$$\Big\downarrow \ 3869$$

$$-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{b}{24x^3}$$

$$\Big\downarrow \ 3868$$

$$-\frac{1}{4}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} - \frac{b}{24x^3}$$

$\downarrow$ 3833

$$-\frac{1}{4}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{b}{24x^3}$$

$\downarrow$ 7019

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx + \frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right) - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{b}{24x^3}$$

$\downarrow$ 3868

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx + \frac{1}{4}b\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\text{FresnelC}(bx)\sin\left(\right.}{2x^2}\right. \qquad \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{b}{24x^3}$$

$\downarrow$ 3833

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx + \frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\text{FresnelC}(bx)\sin}{2x^2}\right. \qquad \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{b}{24x^3}$$

$\downarrow$ 7013

3.193. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx$

$$-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\sin}{2x^2}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+$$
$$\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{b}{24x^3}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

output
```
$Aborted
```

3.193.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.193. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.193.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^5}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

3.193.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

3.193.6 Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

3.193.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

3.193.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^5}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

3.193.9 Mupad [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.194 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^6}\,dx$

3.194.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^6}\,dx = & -\frac{b}{40x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{40x^4} - \frac{1}{24}b^5\pi^2 \text{CosIntegral}\left(b^2\pi x^2\right) \\ & - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{5x^5} \\ & + \frac{b^2\pi \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{b^3\pi\sin\left(b^2\pi x^2\right)}{24x^2} \\ & - \frac{1}{15}b^4\pi^2 \text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^2}, x\right)\end{aligned}$$

output
```
-1/40*b/x^4-1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)-1/40*b*cos(b^2*Pi*x^2)/x^4-1/5*co
s(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+1/15*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x^3+1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

3.194.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]
```

3.194.3 Rubi [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7011, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7019, 3860, 3042, 3778, 3042, 3783, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}\,dx$$

$$\Big\downarrow \text{7011}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3861}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

3.194. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx$

$$\Big\downarrow \ 7019$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3860$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3778$$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\sin}{3x^3}\right.\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$$\Big\downarrow \ 3042$$

3.194. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx}{3}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\Big\downarrow$ 3783

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\,\text{si}}{3x^3}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

$\Big\downarrow$ 7013

$$-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\,\text{si}}{3x^3}\right.$$
$$\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

output
```
$Aborted
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.194. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)}{x^6} \, dx$

3.194.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^6}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

3.194.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

3.194.6 Sympy [N/A]

Not integrable

Time = 5.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

3.194.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

3.194.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^6}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

3.194.9 Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.195 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7} dx$

3.195.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7} dx = & -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} \\ & + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{6x^6} \\ & + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{48x^2} \\ & + \frac{7b^6\pi^3 \text{FresnelS}\left(\sqrt{2}bx\right)}{144\sqrt{2}} + \frac{1}{45}\sqrt{2}b^6\pi^3 \text{FresnelS}\left(\sqrt{2}bx\right) \\ & + \frac{b^2\pi \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} + \frac{13b^3\pi\sin\left(b^2\pi x^2\right)}{720x^3} \\ & + \frac{1}{48}b^6\pi^3 \text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}, x\right)\end{aligned}$$

output
```
-1/60*b/x^5+1/96*b^5*Pi^2/x-1/60*b*cos(b^2*Pi*x^2)/x^5+67/1440*b^5*Pi^2*co
s(b^2*Pi*x^2)/x-1/6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/48*b^4*Pi^2*co
s(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2+1/24*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x^4+13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3+67/1440*b^6*Pi^3*FresnelS(b*
x*2^(1/2))*2^(1/2)+1/48*b^6*Pi^3*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi
*x^2)/x,x)
```

3.195.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]
```

3.195.3 Rubi [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7011, 3869, 3868, 3869, 3832, 7019, 3868, 3869, 3832, 7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{7011}$$

$$-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{6}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$\Big\downarrow$ 3869

$$-\frac{1}{6}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}-\frac{b}{60x^5}$$

$\Big\downarrow$ 3832

$$-\frac{1}{6}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{b}{60x^5}$$

$\Big\downarrow$ 7019

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{b}{60x^5}$$

$\Big\downarrow$ 3868

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{FresnelC}(bx)\sin}{4x^4}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{b}{60x^5}$$

3.195. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\frac{\sin\left(\pi b^2 x\right.}{3x^3}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\Big\downarrow \text{7011}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}-\frac{b}{4x}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\text{Fresnel}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

3.195. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$

$\Big\downarrow$ 3832

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

$\Big\downarrow$ 7021

$$-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-$$

$$\frac{b}{60x^5}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

output
```
$Aborted
```

3.195.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.195.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^7}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

3.195.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right) \text{C}\left(bx\right)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

3.195.6 Sympy [N/A]

Not integrable

Time = 11.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

3.195.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right) \text{C}\left(bx\right)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

3.195.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}\left(bx\right)}{x^7}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

3.195.9 Mupad [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.196 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

3.196.1 Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = &-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}\\ &+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3}\\ &+\frac{1}{210}b^7\pi^4\,\text{FresnelC}(bx)^2\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}\\ &-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x}\\ &+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}+\frac{1}{70}b^7\pi^3\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

output
```
-1/84*b/x^6+1/420*b^5*Pi^2/x^2-1/84*b*cos(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*co
s(b^2*Pi*x^2)/x^2-1/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/105*b^4*Pi^2
*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/210*b^7*Pi^4*FresnelC(b*x)^2+1/70
*b^7*Pi^3*Si(b^2*Pi*x^2)+1/35*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5
-1/105*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x+1/105*b^3*Pi*sin(b^2*P
i*x^2)/x^4
```

3.196.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{105x^3}+\frac{1}{210}b^7\pi^4\,\text{FresnelC}(bx)^2+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x}+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}+\frac{1}{70}b^7\pi^3\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

output
```
-1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*P
i^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7
) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fre
snelC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) -
(b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*
x^2])/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

3.196.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

$$\Big\downarrow \; 7011$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3861$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3042}$$

3.196. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \text{7019}$$

3.196. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 3860

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 3042

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 3778

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)\,\text{si}}{5x^5}\right.$$
$$-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\big\downarrow$ 3042

3.196. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(b}{}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \text{3780}$$

3.196. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\cos\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 7011

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\cos(b^2\pi x^2)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{}{12}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3861

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\cos(b^2\pi x^2)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3042

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right.}{3x^3}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$
$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos(\pi b^2x^2)}{x^2}\right)-\frac{\sin(\pi b^2x^2)}{2x^4}\right)-\frac{\cos(\pi b^2x^2)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3778

3.196. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresne}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresne}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\text{Fresne}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 3780$$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\text{co}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$$\Big\downarrow \; 7019$$

3.196. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

$\downarrow$ 3856

$$-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\text{Fr}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+$$

$$\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{b}{84x^6}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

output
```
$Aborted
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.196.4 Maple [F]

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelC}\,(bx)}{x^8}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

3.196. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)}{x^8} \, dx$

3.196.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx$$
$$= \frac{\pi^4 b^7 x^7\,\mathrm{C}\left(bx\right)^2 + 3\,\pi^3 b^7 x^7\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^5 x^5 + 5\left(\pi^2 b^5 x^5 - bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\right.}{210\,x^7}$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

output
```
1/210*(pi^4*b^7*x^7*fresnel_cos(b*x)^2 + 3*pi^3*b^7*x^7*sin_integral(pi*b^
2*x^2) - 2*pi^2*b^5*x^5 + 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2
*(pi^2*b^4*x^4 - 15)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(2*pi*b^3*x^
3*cos(1/2*pi*b^2*x^2) - (pi^3*b^6*x^6 - 3*pi*b^2*x^2)*fresnel_cos(b*x))*si
n(1/2*pi*b^2*x^2))/x^7
```

3.196.6 Sympy [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

3.196.7 Maxima [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

3.196.8 Giac [F]

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)\text{C}\left(bx\right)}{x^8}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.197 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

3.197.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx = &-\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}\\ &+\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{192x^4}\\ &+\frac{853b^8\pi^4\,\text{FresnelC}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}\\ &-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2}\\ &+\frac{19b^3\pi\sin\left(b^2\pi x^2\right)}{3360x^5}-\frac{853b^7\pi^3\sin\left(b^2\pi x^2\right)}{80640x}\\ &+\frac{1}{384}b^8\pi^4\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

output
```
-1/112*b/x^7+1/1152*b^5*Pi^2/x^3-1/112*b*cos(b^2*Pi*x^2)/x^7+187/40320*b^5
*Pi^2*cos(b^2*Pi*x^2)/x^3-1/8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+1/192*
b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4+1/48*b^2*Pi*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^
2+19/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5-853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x+
853/80640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrab
le(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.197.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^9} \, dx = \int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^9} \, dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]
```

3.197.3 Rubi [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7011, 3869, 3868, 3869, 3868, 3833, 7019, 3868, 3869, 3868, 3833, 7011, 3869, 3868, 3833, 7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx) \cos \left(\frac{1}{2} \pi b^2 x^2\right)}{x^9} \, dx$$

$$\Big\downarrow \; 7011$$

$$-\frac{1}{8} \pi b^2 \int \frac{\text{FresnelC}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right)}{x^7} dx + \frac{1}{16} b \int \frac{\cos \left(b^2 \pi x^2\right)}{x^8} dx - \frac{\text{FresnelC}(bx) \cos \left(\frac{1}{2} \pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{8}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx + \frac{1}{16}b\left(-\frac{2}{7}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{8}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx + \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2 \int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$$\Big\downarrow \text{3869}$$

$$-\frac{1}{8}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx + \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2 \int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$$\Big\downarrow \text{3868}$$

$$-\frac{1}{8}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx + \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2 \int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b^2\right.}{7x^7} \quad \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} - \frac{b}{112x^7}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{1}{8}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} + \frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\cos\left(\pi b\right.}{7x} \quad \frac{b}{112x^7}$$

$$\Big\downarrow \text{7019}$$

3.197. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3868

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\sin}{6x^6}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3869

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b\right.}{5x^5}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3868

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3833

3.197. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{Fresnel}\right.\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$

$$\frac{b}{112x^7}$$

$\downarrow$ 7011

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}-\frac{}{24}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$

$$\frac{b}{112x^7}$$

$\downarrow$ 3869

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\text{Fresne}}{}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$

$$\frac{b}{112x^7}$$

$\downarrow$ 3868

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$

$$\frac{b}{112x^7}$$

$\downarrow$ 3833

3.197. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{Fresne}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$
$$\frac{b}{112x^7}$$

$\downarrow$ 7019

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right.}{2x^2}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3868

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{Fr}}{}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$
$$\frac{b}{112x^7}$$

$\downarrow$ 3833

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi b\right.}{7x}\right.$$
$$\frac{b}{112x^7}$$

$\downarrow$ 7013

3.197. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}dx$

$$-\frac{1}{8}\pi b^2\left(\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$

$$\frac{1}{16}b\left(-\frac{2}{7}\pi b^2\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\cos\left(\pi\right.}{7x}\right.$$

$$\frac{b}{112x^7}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.197. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.197.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^9}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

3.197.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\text{C}\left(bx\right)}{x^9}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

3.197.6 Sympy [N/A]

Not integrable

Time = 36.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^9} \, dx = \int \frac{\cos \left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^9} \, dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

3.197.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^9} \, dx = \int \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right) \text{C}(bx)}{x^9} \, dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

3.197.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\frac{1}{2} b^2 \pi x^2\right) \text{FresnelC}(bx)}{x^9} \, dx = \int \frac{\cos \left(\frac{1}{2} \pi b^2 x^2\right) \text{C}(bx)}{x^9} \, dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

3.197.9 Mupad [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.198 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

3.198.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = &-\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4}\\ &+\frac{5b^9\pi^4\,\text{CosIntegral}\left(b^2\pi x^2\right)}{2016}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{9x^9}\\ &+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{315x^5}\\ &+\frac{b^2\pi\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}\\ &-\frac{b^6\pi^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3}\\ &+\frac{11b^3\pi\sin\left(b^2\pi x^2\right)}{3024x^6}-\frac{5b^7\pi^3\sin\left(b^2\pi x^2\right)}{2016x^2}\\ &+\frac{1}{945}b^8\pi^4\,\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)\end{aligned}$$

output

```
-1/144*b/x^8+1/2520*b^5*Pi^2/x^4+5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)-1/144*b*co
s(b^2*Pi*x^2)/x^8+67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/9*cos(1/2*b^2*Pi
*x^2)*FresnelC(b*x)/x^9+1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
^5+1/63*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*b^6*Pi^3*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3+11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6-5/2016*
b^7*Pi^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^
2)*FresnelC(b*x)/x^2,x)
```

3.198.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

input
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

output
```
Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]
```

3.198.3 Rubi [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7011, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7019, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7011, 3861, 3042, 3778, 25}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

$$\Big\downarrow \; 7011$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \; 3861$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^{10}}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3778}$$

3.198. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi b^2\right.}{4x^8}\right.-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{9}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx+\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\text{cos}\right.-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$$\Big\downarrow \text{3783}$$

3.198. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}dx$

$$-\frac{1}{9}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx + \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi\right.}{4x}\right. \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$\downarrow$ 7019

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7}\right) + \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi\right.}{4x}\right. \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$\downarrow$ 3860

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx + \frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7}\right) + \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi\right.}{4x}\right. \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx + \frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{7x^7}\right) + \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi\right.}{4x}\right. \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} - \frac{b}{144x^8}$$

$\downarrow$ 3778

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx + \frac{1}{28}b\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2 - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\text{FresnelC}(bx)\,\text{si}}{7x^7}\right. \frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\sin\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^6}\right) - \frac{\cos\left(\pi\right.}{4x}\right. \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{9x^9} - \frac{b}{144x^8}$$

3.198. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(b$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi\right.}{3x}$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 25

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi\right.}{3x}$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi\right.}{3x}$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\downarrow$ 3778

3.198. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3783

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 7011

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{}{4}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}\right.$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\Big\downarrow$ 3861

3.198. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\big\downarrow$ 3042

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right.}{5x^5}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\big\downarrow$ 3778

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\text{Fres}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

$\big\downarrow$ 25

$$-\frac{1}{9}\pi b^2\left(\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\text{Fres}\right.\right.$$
$$\frac{1}{36}b\left(-\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\cos\left(\pi\right.}{4x}$$
$$\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{b}{144x^8}$$

input
```
Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

output
```
$Aborted
```

3.198. $\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

3.198. $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.198.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^{10}}dx$$

input
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

output
```
int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

3.198.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="fricas")
```

output
```
integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

3.198.6 Sympy [N/A]

Not integrable

Time = 65.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)
```

output
```
Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

3.198.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{C}(bx)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="maxima")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

3.198.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\cos\left(\frac{1}{2}\pi b^2 x^2\right)\text{C}(bx)}{x^{10}}\,dx$$

input
```
integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="giac")
```

output
```
integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

3.198.9 Mupad [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

input
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
```

output
```
int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.199 $\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.199.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \text{Int}\left(\text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right), x\right)$$

output
```
Unintegrable(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

3.199.2 Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

input
```
Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

3.199.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7005}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\frac{1}{2}\pi b^2x^2\right) \text{FresnelC}(bx)^n\,dx$$
$$\Big\downarrow \text{ 7005}$$
$$\int \sin\left(\frac{1}{2}\pi b^2x^2\right) \text{FresnelC}(bx)^n dx$$

input
```
Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
$Aborted
```

3.199.3.1 Defintions of rubi rules used

rule 7005
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*Sin[(c_.) + (d_.)*(x_)^2], x_Symbol]
 :> Unintegrable[FresnelC[a + b*x]^n*Sin[c + d*x^2], x] /; FreeQ[{a, b, c,
d, n}, x]
```

3.199.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \text{FresnelC}\left(bx\right)^{n} \sin\left(\frac{b^{2}\pi\, x^{2}}{2}\right) dx$$

input
```
int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

3.199.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{C}\left(bx\right)^n \sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.199.6 Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \sin\left(\frac{\pi b^2x^2}{2}\right)C^n(bx)\,dx$$

input
```
integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)
```

3.199.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{C}\left(bx\right)^n \sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.199.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{C}(bx)^n \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

input
```
integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

3.199.9 Mupad [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(b\,x)^n \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.200 $\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.200.1 Optimal result

Integrand size = 20, antiderivative size = 308

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} + \frac{105\operatorname{FresnelC}(bx)\operatorname{FresnelS}(bx)}{2b^9\pi^4} + \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^7\pi^4} - \frac{105x\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
-35/8*x^4/b^5/Pi^3+1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b
^2*Pi*x^2)/b^5/Pi^3+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^7*
cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+105/2*FresnelC(b*x)*FresnelS(b*x)
/b^9/Pi^4+105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/Pi^
4-105/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-105*x*F
resnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/b^4/Pi^2-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)
/b^3/Pi^2
```

3.200.2 Mathematica [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

input
```
Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

3.200.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{ 7017}$$

$$\frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^7 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b} - \frac{x^7 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^6 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{2\pi b} - \frac{x^7 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^6 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right)^2 dx^2}{2\pi b} - \frac{x^7 \operatorname{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2 x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3790}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^6dx^2}{2}-\frac{1}{2}\int -x^6\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 15$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^8}{8}-\frac{1}{2}\int -x^6\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx^2+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^6\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{3\int -x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\int x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\int x^4\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3777$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\int x^2\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

3.200. $\int x^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{\int -\sin(b^2\pi x^2)dx^2}{\pi b^2}+\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}-\frac{\int\sin(b^2\pi x^2)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

3.200. $\int x^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}-\frac{\int\sin(b^2\pi x^2)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}-\frac{x^7\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^7\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \text{7009}$$

$$\frac{7\left(-\frac{5\int x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^5\sin(b^2\pi x^2)dx}{2\pi b}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \text{3860}$$

$$\frac{7\left(-\frac{5\int x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin(b^2\pi x^2)dx^2}{4\pi b}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

3.200. $\int x^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 3777$$

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\int x^2\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(-\frac{5\int x^4 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7 \text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$$\Big\downarrow \; 3777$$

3.200. $\int x^8 \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 25

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3042

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3118

3.200. $\int x^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}-\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 7017

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{x^2\sin(\pi b}{\pi b^2}\right.\cdots}{\cdots}\right)}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3861

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{x^2\sin(\pi}{\pi b^2}\right.\cdots}{\cdots}\right)}{\pi b^2}$$

$$\frac{x^7\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin(\pi b^2x^2)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3042

3.200. $\int x^8\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(x^2\sin\cdots\right.}{\cdots}\cdots\right.}{\pi b^2}$$

$$\frac{x^7\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 3790

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^2dx^2}{2}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\cdots\right.}{\pi b^2}$$

$$\frac{x^7\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

$\downarrow$ 15

$$\frac{7\left(-\frac{5\left(\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^4}{4}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^5\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(x^2\cdots\right.}{\cdots}\cdots\right.}{\pi b^2}$$

$$\frac{x^7\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^6\sin(\pi b^2x^2)}{\pi b^2}-\frac{3\left(\frac{2\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)}{\pi b^2}-\frac{x^4\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^8}{8}}{2\pi b}$$

3.200. $\int x^8\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

input
```
Int[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
$Aborted
```

3.200.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.200.4 Maple [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

input
```
int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.200.5 Fricas [F]

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.200.6 Sympy [F]

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^8 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

input
```
integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**8*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.200.7 Maxima [F]

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^8 \,\mathrm{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

input
```
integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.200.8 Giac [F]

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^8 \,\mathrm{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

input
```
integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^8 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.201 $\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.201.1 Optimal result

Integrand size = 20, antiderivative size = 218

$$\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{531\,\text{FresnelS}\left(\sqrt{2}bx\right)}{16\sqrt{2}b^8\pi^4} - \frac{48\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{6x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{147x\sin(b^2\pi x^2)}{16b^7\pi^4} + \frac{x^5\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
-4*x^3/b^5/Pi^3+1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3+24*x^2*cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/b^2/Pi-48*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+6*x^4*FresnelC(b*
x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*
sin(b^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \frac{-896b^3\pi x^3 + 16b^7\pi^3x^7 + 476b^3\pi x^3\cos(b^2\pi x^2) + 3717\sqrt{2}\operatorname{FresnelS}(\sqrt{2}bx) - 224\operatorname{FresnelC}(bx)\left(b^2\pi x^2(-2\right.}{224b^8}$$

input
```
Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717
*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*
Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])
 - 2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi
^4)
```

3.201.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 437 vs. 2(218) = 436.

Time = 1.99 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {7017, 3873, 15, 3867, 3866, 3867, 3832, 7009, 3866, 3867, 3832, 7017, 3873, 15, 3867, 3832, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$
$$\Big\downarrow \; 7017$$
$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^6\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} - \frac{x^6\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$
$$\Big\downarrow \; 3873$$
$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx + \frac{\int x^6dx}{2}}{\pi b} - \frac{x^6\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 15$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^6\cos\left(b^2\pi x^2\right)dx+\frac{x^7}{14}}{\pi b}-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3867$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\int x^4\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3866$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3867$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3832$$

$$\frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \; 7009$$

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^4\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{3866}$$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\int x^2\cos\left(b^2\pi x^2\right)dx}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{3867}$$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{3832}$$

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\int x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 7017

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{x\sin\left(\pi b^2x\right.}{2\pi b^2}\right.}{}\right.}{\pi b^2}$$

$$\frac{\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 3873

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{\int x^2dx}{2}}{\pi b}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{x \cdots}{\cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 15

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{x^3}{6}}{\pi b}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{3\left(\frac{x\sin\left(\pi \cdots\right.}{2\pi \cdots}\right.}{\cdots}\right.}{\pi b^2}$$

$$\frac{\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\downarrow$ 3867

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right.}{\pi b^2}$$

$$\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{6\left(-\frac{4\left(\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right.}{\pi b^2}$$

$$\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$$\Big\downarrow \text{7007}$$

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$6\left(-\frac{4\left(\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}\right)}{\pi b^2}+\frac{x^4\,\text{FresnelC}(\ldots}{\pi b^2}\right)$$

$$\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

$\big\downarrow$ 3832

$$-\frac{x^6\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+$$

$$\frac{6\left(\frac{x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{4\left(\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}(\ldots}{2\sqrt{2}\pi b\ldots}\right)\ldots}{\pi b}\right)}{\pi b^2}\right)}{\pi b^2}$$

$$\frac{\frac{1}{2}\left(\frac{x^5\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{5\left(\frac{3\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)}{2\pi b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^7}{14}}{\pi b}$$

input
```
Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

3.201. $\int x^7\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

output
```
-((x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (6*((x^4*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (-1/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) +
(3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b
^2*Pi)))/(2*b^2*Pi))/(2*b*Pi) - (4*(-((x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(b^2*Pi)) + (2*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (Fresnel
C[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x^3/6 + (-1/2*FresnelS[
Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi))
)/(b^2*Pi)))/(b^2*Pi) + (x^7/14 + ((x^5*Sin[b^2*Pi*x^2])/(2*b^2*Pi) - (5*(
-1/2*(x^3*Cos[b^2*Pi*x^2])/(b^2*Pi) + (3*(-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt
[2]*b^3*Pi) + (x*Sin[b^2*Pi*x^2])/(2*b^2*Pi)))/(2*b^2*Pi)))/(2*b^2*Pi))/2)
/(b*Pi)
```

3.201.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

3.201. $\int x^7 \operatorname{FresnelC}(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) \, dx$

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.201.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^{6}x^{6}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{\frac{6b^{4}x^{4}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}-\frac{24\left(-\frac{b^{2}x^{2}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}\right)}{\pi}}{\pi}\right)}{b^{7}}-\frac{-\frac{\frac{1}{7}\pi^{2}b^{7}x^{7}-8b^{3}x^{3}}{2\pi^{3}}+\frac{-\frac{3\pi b^{3}x^{3}\cos}{2}}{}}{}}{}$

input
```
int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*s
in(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*
b^2*Pi*x^2))))-1/b^7*(-1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*
Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*P
i*b^5*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(
1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*
x*sin(b^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \frac{16\,\pi^3 b^8 x^7 + 952\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 1372\,\pi b^4 x^3 - 224\left(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \text{C}\left(bx\right) + 3717}{224\,\pi}$$

input
```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/224*(16*pi^3*b^8*x^7 + 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 1372*pi*b^
4*x^3 - 224*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_cos
(b*x) + 3717*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 28*((4*p
i^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 48*(pi^2*b^5*x^4 - 8*b)*fre
snel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

3.201.6 Sympy [F]

$$\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.201.7 Maxima [F]

$$\int x^7 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^7 \text{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

input
```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.201.8 Giac [F]

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7\, C\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\,dx$$

input
```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int x^7 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^7 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.202 $\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.202.1 Optimal result

Integrand size = 20, antiderivative size = 185

$$\begin{aligned}\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx = & -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} \\ & +\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} \\ & -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi}-\frac{15\operatorname{FresnelC}(bx)^2}{2b^7\pi^3} \\ & +\frac{5x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} \\ & -\frac{11\sin(b^2\pi x^2)}{2b^7\pi^4}+\frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

output
```
-15/4*x^2/b^5/Pi^3+1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+15*x*cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/b^2/Pi-15/2*FresnelC(b*x)^2/b^7/Pi^3+5*x^3*FresnelC(b*x)*sin(1/2*b^2*P
i*x^2)/b^4/Pi^2-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/
Pi^2
```

3.202.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00

$$\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos(b^2\pi x^2)}{4b^5\pi^3} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} - \frac{15\operatorname{FresnelC}(bx)^2}{2b^7\pi^3} + \frac{5x^3\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{11\sin(b^2\pi x^2)}{2b^7\pi^4} + \frac{x^4\sin(b^2\pi x^2)}{4b^3\pi^2}$$

input
```
Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi
^3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (
5*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])
/(2*b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

3.202.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {7017, 3861, 3042, 3790, 15, 25, 3042, 3777, 25, 3042, 3777, 3042, 3117, 7009, 3860, 3042, 3777, 3042, 3117, 7017, 3861, 3042, 3114, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \; 7017$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^5\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3861$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^4\sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3790}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{\int x^4dx^2}{2}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^6}{6}-\frac{1}{2}\int -x^4\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^4\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{2\int -x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}+\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\int x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\int x^2\sin(b^2\pi x^2)dx^2}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

3.202. $\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\Big\downarrow \; 3777$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\int\cos(b^2\pi x^2)dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \; 3117$$

$$\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 7009$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^3\sin(b^2\pi x^2)dx}{2\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 3860$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin(b^2\pi x^2)dx^2}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \; 3042$$

3.202. $\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos\left(b^2\pi x^2\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2}{\pi b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{5\left(-\frac{3\int x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}\right)}{\pi b^2}-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{ 7017}$$

3.202. $\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos\ldots}{\ldots}}{4\pi b}\ldots\right.$$
$$\overline{\pi b^2}$$
$$\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3861}$$

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos\ldots}{\ldots}}{4\pi b}\ldots\right.$$
$$\overline{\pi b^2}$$
$$\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3042}$$

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\ldots}{4\pi b}\ldots\right.$$
$$\overline{\pi b^2}$$
$$\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{3114}$$

$$5\left(-\frac{3\left(\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\ldots}{\pi b^2}}{4\pi b}\ldots\right.$$
$$\overline{\pi b^2}$$
$$\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin(\pi b^2x^2)}{\pi b^2}-\frac{2\left(\frac{\sin(\pi b^2x^2)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

3.202. $\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\Big\downarrow \text{6995}$$

$$\frac{5\left(-\frac{3\left(\frac{\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{\pi b^3}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}+\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2 \right.}{\pi b^2}}{4\pi b}\right.}{\pi b^2}$$

$$\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{x^5\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^4\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}\right)+\frac{x^6}{6}}{2\pi b}+\frac{5\left(\frac{x^3\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\sin\left(\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos\left(\pi b^2x^2\right)}{\pi b^2}}{4\pi b}-\frac{3\left(\frac{\text{FresnelC}(bx)^2}{2\pi b^3}-\frac{x\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin\left(\pi b^2x^2\right)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}\right)}{\pi b^2}\right)}{\pi b^2}$$

input
```
Int[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (x^6/6 + ((x^4*Sin[b
^2*Pi*x^2])/(b^2*Pi) - (2*(-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*
x^2]/(b^4*Pi^2)))/(b^2*Pi))/2)/(2*b*Pi) + (5*((x^3*FresnelC[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^2*Pi) - (-((x^2*Cos[b^2*Pi*x^2])/(b^2*Pi)) + Sin[b^2*Pi*x^2
]/(b^4*Pi^2))/(4*b*Pi) - (3*(-((x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*
Pi)) + FresnelC[b*x]^2/(2*b^3*Pi) + (x^2/2 + Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(
2*b*Pi)))/(b^2*Pi)))/(b^2*Pi)
```

3.202. $\int x^6\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.202.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.202.4 Maple [F]

$$\int x^6 \,\mathrm{FresnelC}\,(bx) \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

input
```
int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.76

$$\int x^6 \,\mathrm{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = \frac{\pi^3 b^6 x^6 + 42\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 66\,\pi b^2 x^2 - 12\left(\pi^3 b^5 x^5 - 15\,\pi b x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\,(bx) - 90\,\pi\, \mathrm{C}\,(bx)^2 +}{12\,\pi^4 b^7}$$

input
```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/12*(pi^3*b^6*x^6 + 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 66*pi*b^2*x^2 -
 12*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 90*p
i*fresnel_cos(b*x)^2 + 6*(10*pi^2*b^3*x^3*fresnel_cos(b*x) + (pi^2*b^4*x^4
 - 22)*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

3.202.6 Sympy [A] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \begin{cases} \frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) \cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{11x^2 \sin^2\left(\frac{\pi \right.}{2\pi^3 b^5} \\ 0 \end{cases}$$

input
```
integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2
)**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*s
in(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2
*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi
**3*b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*
x**2/2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x*
*2/2)/(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, Tr
ue))
```

3.202.7 Maxima [F]

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.202.8 Giac [F]

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^6 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.203 $\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.203.1 Optimal result

Integrand size = 20, antiderivative size = 167

$$\int x^5 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos(b^2\pi x^2)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} - \frac{43\operatorname{FresnelC}\left(\sqrt{2}bx\right)}{8\sqrt{2}b^6\pi^3} + \frac{4x^2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^3\sin(b^2\pi x^2)}{4b^3\pi^2}$$

output
```
-4*x/b^5/Pi^3+1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+8*cos(1/2*b^2*
Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/P
i+4*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x^3*sin(b^2*Pi*x^2)
/b^3/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{-215\sqrt{2}\operatorname{FresnelC}\left(\sqrt{2}bx\right) - 80\operatorname{FresnelC}(bx)\left(\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 4b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx(}{80b^6\pi^3}$$

input
```
Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4
)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 +
4*b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*
b^6*Pi^3)
```

3.203.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.69, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7017, 3873, 15, 3867, 3866, 3833, 7009, 3866, 3833, 7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \text{7017}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b} - \frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3873}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx + \frac{\int x^4dx}{2}}{\pi b} - \frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2} + \frac{\frac{1}{2}\int x^4\cos\left(b^2\pi x^2\right)dx + \frac{x^5}{10}}{\pi b} - \frac{x^4\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\int x^2\sin(b^2\pi x^2)dx}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3866}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\int\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$$\Big\downarrow \text{7009}$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x^2\sin(b^2\pi x^2)dx}{2\pi b}+\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$$\Big\downarrow \text{3866}$$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\frac{\int\cos(b^2\pi x^2)dx}{2\pi b^2}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}}{2\pi b}+\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin(\pi b^2x^2)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}(\sqrt{2}bx)}{2\sqrt{2}\pi b^3}-\frac{x\cos(\pi b^2x^2)}{2\pi b^2}\right)}{2\pi b^2}\right)+\frac{x^5}{10}}{\pi b}$$

$$\Big\downarrow \text{3833}$$

3.203. $\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\frac{4\left(-\frac{2\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{\pi b}+\frac{x^5}{10}$$

$$\Big\downarrow \text{ 7015}$$

$$\frac{4\left(-\frac{2\left(\frac{\int\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{\pi b}+\frac{x^5}{10}$$

$$\Big\downarrow \text{ 3839}$$

$$\frac{4\left(-\frac{2\left(\frac{\int\left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}+\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{\pi b}+\frac{x^5}{10}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^4\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{4\left(\frac{x^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{2\left(\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}+\frac{x}{2}}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}}{2\pi b}\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^3\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{3\left(\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}-\frac{x\cos\left(\pi b^2x^2\right)}{2\pi b^2}\right)}{2\pi b^2}\right)}{\pi b}+\frac{x^5}{10}$$

3.203. $\int x^5\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

input
```
Int[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (4*(-1/2*(-1/2*(x*Co
s[b^2*Pi*x^2])/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi))/(b*Pi)
 - (2*(-((Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (x/2 + FresnelC[S
qrt[2]*b*x]/(2*Sqrt[2]*b))/(b*Pi)))/(b^2*Pi) + (x^2*FresnelC[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x^5/10 + ((-3*(-1/2*(x*Cos[b^2*Pi*x^2]
)/(b^2*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^3*Pi)))/(2*b^2*Pi) + (x^3*
Sin[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)
```

3.203.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 3866
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Simp[e^n*((m - n +
1)/(d*n))   Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
 && IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.203.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{-\frac{\frac{1}{5}b^5x^5\pi^2-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}-\frac{\pi b \cdots}{\cdots}}{\cdots}}{b}$

input
```
int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*s
in(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(-1/2/Pi^3*(1/5*b^5*
x^5*Pi^2-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresnel
C(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b
*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*Fresnel
C(b*x*2^(1/2)))))/b
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{8\pi^2 b^6x^5 + 220\,b^2x\cos\left(\frac{1}{2}\pi b^2x^2\right)^2 - 430\,b^2x - 80\left(\pi^2b^5x^4 - 8\,b\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right) - 215\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\right.}{80\,\pi^3 b^7}$$

input
```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/80*(8*pi^2*b^6*x^5 + 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 430*b^2*x - 80*(p
i^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 215*sqrt(2)*sqrt
(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2
) + 8*pi*b^3*x^2*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

3.203.6 Sympy [F]

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^5\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

input
```
integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.203.7 Maxima [F]

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^5\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.203.8 Giac [F]

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^5\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x^5 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.204 $\int x^4 \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.204.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\begin{aligned}\int x^4 \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)dx = {} & \frac{x^4}{8b\pi} + \frac{\cos(b^2\pi x^2)}{b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} \\ & - \frac{3\,\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b^5\pi^2} \\ & - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\ & + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} \\ & + \frac{3x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x^2\sin(b^2\pi x^2)}{4b^3\pi^2}\end{aligned}$$

output
```
1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3-x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2-3/8*I*x^2*hypergeom([1,
1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^
2+1/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

3.204.2 Mathematica [F]

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

input
```
Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

3.204.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {7017, 3861, 3042, 3790, 15, 25, 3042, 3777, 25, 3042, 3118, 7009, 3860, 3042, 3118, 7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

$$\Big\downarrow \text{7017}$$

$$\frac{3\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^3 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx}{\pi b} - \frac{x^3 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3861}$$

$$\frac{3\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^2 \cos^2\left(\frac{1}{2}b^2\pi x^2\right) dx^2}{2\pi b} - \frac{x^3 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)dx}{\pi b^2} + \frac{\int x^2 \sin\left(\frac{1}{2}b^2\pi x^2 + \frac{\pi}{2}\right)^2 dx^2}{2\pi b} - \frac{x^3 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3790}$$

$$\frac{3\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)dx}{\pi b^2} + \frac{\frac{\int x^2dx^2}{2} - \frac{1}{2}\int -x^2 \cos\left(b^2\pi x^2\right) dx^2}{2\pi b} - \frac{x^3 \text{FresnelC}(bx) \cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{x^4}{4}-\frac{1}{2}\int -x^2\cos\left(b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 25$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3042$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)dx^2+\frac{x^4}{4}}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3777$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{\int -\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}+\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 25$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3042$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx^2}{\pi b^2}\right)+\frac{x^4}{4}}{2\pi b}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow\ 3118$$

$$\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow\ 7009$$

$$\frac{3\left(-\frac{\int\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int x\sin\left(b^2\pi x^2\right)dx}{2\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

3.204. $\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

$$\Big\downarrow \text{3860}$$

$$\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx^2}{4\pi b}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{3\left(-\frac{\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

$$\Big\downarrow \text{7001}$$

$$\frac{3\left(-\frac{\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}}{\pi b^2}+\frac{x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}\right)}{\pi b^2}-\frac{x^3\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x^2\sin\left(\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^2b^4}\right)+\frac{x^4}{4}}{2\pi b}$$

input
```
Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (3*(Cos[b^2*Pi*x^2]/
(4*b^3*Pi^2) - ((FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^2*Pi) + (x*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(b^2*Pi)))/(b^2*Pi) + (x^4/4 + (Cos[b^2*Pi*x^2]/(b^4*Pi
^2) + (x^2*Sin[b^2*Pi*x^2])/(b^2*Pi))/2)/(2*b*Pi)
```

3.204. $\int x^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.204.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3790
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Simp[1/2   Int[(c + d*x)^m, x], x] - Simp[1/2   Int[(c + d*x)^m*Cos[2*e + f
*x], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7009
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Simp[(m - 1)/(2*d)   Int[x^
(m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(4*d)   Int[x^(m - 1)*Sin
[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.204.4 Maple [F]

$$\int x^4 \,\mathrm{FresnelC}\,(bx) \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

input
```
int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.204.5 Fricas [F]

$$\int x^4 \,\mathrm{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx = \int x^4 \,\mathrm{C}\,(bx) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)\, dx$$

input
```
integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.204.6 Sympy [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.204.7 Maxima [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4\, \mathrm{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

input
```
integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.204.8 Giac [F]

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4\, \mathrm{C}\left(bx\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right) dx$$

input
```
integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^4 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.205 $\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.205.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{x^3}{6b\pi} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} - \frac{5\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{4\sqrt{2}b^4\pi^2} + \frac{2\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

output
```
1/6*x^3/b/Pi-x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+2*FresnelC(b*x)*
sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b
*x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3 \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{4b^3\pi x^3 - 15\sqrt{2}\operatorname{FresnelS}\left(\sqrt{2}bx\right) - 24\operatorname{FresnelC}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 6bx\sin\left(b^2\pi x\right.}{24b^4\pi^2}$$

input
```
Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*P
i*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2]
)/(24*b^4*Pi^2)
```

3.205.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {7017, 3873, 15, 3867, 3832, 7007, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

$$\Big\downarrow \text{7017}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3873}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{\int x^2dx}{2}}{\pi b}-\frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\int x^2\cos\left(b^2\pi x^2\right)dx+\frac{x^3}{6}}{\pi b}-\frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3867}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\int\sin\left(b^2\pi x^2\right)dx}{2\pi b^2}\right)+\frac{x^3}{6}}{\pi b}-\frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)dx}{\pi b^2}-\frac{x^2\operatorname{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\operatorname{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}$$

$$\Big\downarrow \text{7007}$$

$$\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\int \sin\left(b^2\pi x^2\right)dx}{2\pi b}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}$$

$$\Big\downarrow \; 3832$$

$$\frac{2\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^2}\right)}{\pi b^2}-\frac{x^2\,\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{1}{2}\left(\frac{x\sin\left(\pi b^2x^2\right)}{2\pi b^2}-\frac{\text{FresnelS}\left(\sqrt{2}bx\right)}{2\sqrt{2}\pi b^3}\right)+\frac{x^3}{6}}{\pi b}$$

input
```
Int[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (2*(-1/2*FresnelS[Sq
rt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
))/(b^2*Pi) + (x^3/6 + (-1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^3*Pi) + (x*S
in[b^2*Pi*x^2])/(2*b^2*Pi))/2)/(b*Pi)
```

3.205.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3867
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Simp[e^n*((m - n + 1)/
(d*n))   Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
 IGtQ[n, 0] && LtQ[n, m + 1]
```

rule 3873
```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Simp[1/2
Int[x^m, x], x] + Simp[1/2   Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a,
b, m, n}, x]
```

rule 7007
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Simp[b/(4*d)   Int[Sin[2*d*x^2], x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.205.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\text{FresnelC}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{\sqrt{2}\ \text{FresnelS}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{b^3x^3}{6\pi}-\frac{\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\ \text{FresnelS}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	120

input
```
int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*P
i*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6/Pi*b^3*x^3-1/2/P
i*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$
$$= \frac{4\,\pi b^4x^3 - 24\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right) - 15\,\sqrt{2}\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\sqrt{b^2}x\right) + 12\,\left(b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right) + 4\,b\,\mathrm{C}\left(bx\right)\right)\sin}{24\,\pi^2b^5}$$

input
```
integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/24*(4*pi*b^4*x^3 - 24*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) -
15*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(b^2*x*cos(1/2*
pi*b^2*x^2) + 4*b*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

3.205.6 Sympy [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \sin\left(\frac{\pi b^2x^2}{2}\right) C(bx)\, dx$$

input
```
integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.205.7 Maxima [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right) dx$$

input
```
integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.205.8 Giac [F]

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2x^2\right) dx$$

input
```
integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^3 \operatorname{FresnelC}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.206 $\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.206.1 Optimal result

Integrand size = 20, antiderivative size = 74

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x^2}{4b\pi} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} + \frac{\operatorname{FresnelC}(bx)^2}{2b^3\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

output
```
1/4*x^2/b/Pi-x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/2*FresnelC(b*x)^
2/b^3/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

3.206.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x^2}{4b\pi} - \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\operatorname{FresnelC}(bx)}{b^2\pi} + \frac{\operatorname{FresnelC}(bx)^2}{2b^3\pi} + \frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

input
```
Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b
*x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

3.206.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7017, 3861, 3042, 3114, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)dx$$

$$\Big\downarrow \text{7017}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3861}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx^2}{2\pi b}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}+\frac{\int \sin\left(\frac{1}{2}b^2\pi x^2+\frac{\pi}{2}\right)^2dx^2}{2\pi b}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \text{3114}$$

$$\frac{\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx}{\pi b^2}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}$$

$$\Big\downarrow \text{6995}$$

$$\frac{\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)}{\pi b^3}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\text{FresnelC}(bx)^2}{2\pi b^3}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\frac{\sin(\pi b^2x^2)}{2\pi b^2}+\frac{x^2}{2}}{2\pi b}$$

input
```
Int[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + FresnelC[b*x]^2/(2*b^3
*Pi) + (x^2/2 + Sin[b^2*Pi*x^2]/(2*b^2*Pi))/(2*b*Pi)
```

3.206.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3114
```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7017
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Simp[(m - 1)/(2*d)   Int[
x^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Simp[b/(2*d)   Int[x^(m - 1)*C
os[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[
m, 1]
```

3.206.4 Maple [F]

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right) dx$$

input
```
int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\pi b^2x^2 - 4\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right) + 2\,\pi\,\mathrm{C}\left(bx\right)^2 + 2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

input
```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/4*(pi*b^2*x^2 - 4*pi*b*x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*pi*fre
snel_cos(b*x)^2 + 2*cos(1/2*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

3.206.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \begin{cases} \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2
*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3)
, Ne(b, 0)), (0, True))
```

3.206.7 Maxima [F]

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.206.8 Giac [F]

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{C}(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int x^2 \operatorname{FresnelC}(b\,x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

input
```
int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.207 $\int x \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

3.207.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} + \frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b^2\pi}$$

output
```
1/2*x/b/Pi-cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/4*FresnelC(b*x*2^(1/
2))/b^2/Pi*2^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int x \,\text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \frac{2bx - 4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx) + \sqrt{2}\,\text{FresnelC}\left(\sqrt{2}bx\right)}{4b^2\pi}$$

input
```
Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*
x])/(4*b^2*Pi)
```

3.207.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7015, 3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \,\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \ 7015$$

$$\frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 3839$$

$$\frac{\int \left(\frac{1}{2}\cos\left(b^2\pi x^2\right)+\frac{1}{2}\right)dx}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}b}+\frac{x}{2}}{\pi b}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

input
```
Int[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
-((Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi)) + (x/2 + FresnelC[Sqrt[2]*
b*x]/(2*Sqrt[2]*b))/(b*Pi)
```

3.207.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

rule 7015
```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Simp[b/(2*d)   Int[Cos[d*x^2]^2, x], x] /
; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.207.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{\mathrm{FresnelC}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}+\frac{\frac{bx}{2}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

input
```
int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

output
```
(-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*Fresn
elC(b*x*2^(1/2))))/b
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\,\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{2\,b^2x-4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\,(bx)+\sqrt{2}\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\sqrt{b^2}x\right)}{4\,\pi b^3}$$

input
```
integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x - 4*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + sqrt(2)*sqrt(b^2
)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

3.207.6 Sympy [F]

$$\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\int x\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

input
```
integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.207.7 Maxima [F]

$$\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\int x\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.207.8 Giac [F]

$$\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\int x\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int x\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int x\,\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

input
```
int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.208 $\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

3.208.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b} + \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right) - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

output
```
1/2*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1
/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

3.208.2 Mathematica [F]

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

input
```
Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

3.208.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {7001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)\,dx$$

$$\Big\downarrow \; 7001$$

$$\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)\,\text{FresnelS}(bx)}{2b}$$

input
```
Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

output
```
(FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (I/2)*b^2*Pi*x^2]
```

3.208.3.1 Defintions of rubi rules used

rule 7001
```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

3.208.4 Maple [F]

$$\int \text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

3.208.5 Fricas [F]

$$\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.208.6 Sympy [F]

$$\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

3.208.7 Maxima [F]

$$\int \operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = \int \mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.208.8 Giac [F]

$$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int C(bx) \sin\left(\frac{1}{2}\pi b^2 x^2\right) dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int \text{FresnelC}(b\,x) \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

input
```
int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

output
```
int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.209 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

3.209.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.209.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

3.209.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$
$$\Big\downarrow \text{7021}$$
$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}dx$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

output
```
$Aborted
```

3.209.3.1 Defintions of rubi rules used

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.209.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.209.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.209.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

3.209.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.209.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

3.209.9 Mupad [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.210 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

3.210.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \frac{1}{2}b\pi\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

output
```
1/2*b*Pi*FresnelC(b*x)^2+1/4*b*Si(b^2*Pi*x^2)-FresnelC(b*x)*sin(1/2*b^2*Pi
*x^2)/x
```

3.210.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \frac{1}{2}b\pi\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x} + \frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

output
```
(b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinI
ntegral[b^2*Pi*x^2])/4
```

3.210.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7019, 3856, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}dx$$

$$\Big\downarrow \ 7019$$

$$\pi b^2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}$$

$$\Big\downarrow \ 3856$$

$$\pi b^2\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

$$\Big\downarrow \ 6995$$

$$\pi b\int \text{FresnelC}(bx)d\,\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

$$\Big\downarrow \ 15$$

$$-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\,\text{FresnelC}(bx)^2$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

output
```
(b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinI
ntegral[b^2*Pi*x^2])/4
```

3.210.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.210.4 Maple [F]

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^2}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \frac{2\,\pi bx\,\text{C}\,(bx)^2 + bx\,\text{Si}\,(\pi b^2x^2) - 4\,\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,x}$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*pi*b*x*fresnel_cos(b*x)^2 + b*x*sin_integral(pi*b^2*x^2) - 4*fresne
l_cos(b*x)*sin(1/2*pi*b^2*x^2))/x
```

3.210.6 Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^2}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

3.210.7 Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.210.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^2}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

3.211 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

3.211.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \frac{b^2\pi\,\text{FresnelC}\left(\sqrt{2}bx\right)}{2\sqrt{2}} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2} - \frac{b\sin\left(b^2\pi x^2\right)}{4x} + \frac{1}{2}b^2\pi\,\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

output
```
-1/2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2
*Pi*FresnelC(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x
^2)*FresnelC(b*x)/x,x)
```

3.211.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

3.211.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^3}dx$$

$$\Big\downarrow \text{7019}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

$$\Big\downarrow \text{7013}$$

$$\frac{1}{2}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

output
```
$Aborted
```

3.211.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.211.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^3}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

3.211.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.211.6 Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^3}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

3.211.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.211.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^3}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

3.211.9 Mupad [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.212 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

3.212.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \frac{1}{12}b^3\pi\,\text{CosIntegral}\left(b^2\pi x^2\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{12x^2} + \frac{1}{3}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)$$

output
```
1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/12*
b*sin(b^2*Pi*x^2)/x^2+1/3*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC
(b*x)/x^2,x)
```

3.212.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]
```

3.212.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7019, 3860, 3042, 3778, 3042, 3783, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^4}dx$$

$$\Big\downarrow \ 7019$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \ 3860$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \ 3778$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \ 3783$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

$$\Big\downarrow \text{7013}$$

$$\frac{1}{3}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

output
```
$Aborted
```

3.212.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.212.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^4}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

3.212.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.212.6 Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^4}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

3.212.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.212.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^4}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

3.212.9 Mupad [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.213 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

3.213.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = -\frac{b^3\pi}{16x} - \frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2} - \frac{7b^4\pi^2\,\text{FresnelS}\left(\sqrt{2}bx\right)}{24\sqrt{2}} - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4} - \frac{b\sin\left(b^2\pi x^2\right)}{24x^3} - \frac{1}{8}b^4\pi^2\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

output
```
-1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/x^2-1/4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/24*b*sin(b
^2*Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Un
integrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.213.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

3.213.3 Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7019, 3868, 3869, 3832, 7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^5}\,dx$$

$$\Big\downarrow \text{7019}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx + \frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{4}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx - \frac{\cos\left(\pi b^2x^2\right)}{x}\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{4}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{7011}$$

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx + \frac{1}{4}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2} - \frac{b}{4x}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx + \frac{1}{4}b\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx - \frac{\cos\left(\pi b^2x^2\right)}{x}\right) - \frac{\text{FresnelC}(bx)\,\text{cos}}{2x^2}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2} + \frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{Fresnel}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

$$\Big\downarrow \text{7021}$$

$$\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2 \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2} + \frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{Fresnel}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} + \frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

output
```
$Aborted
```

3.213. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.213. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

3.213.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{5}}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

3.213.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{5}}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.213.6 Sympy [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{5}}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

3.213.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.213.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^5}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

3.213.9 Mupad [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

3.214 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

3.214.1 Optimal result

Integrand size = 20, antiderivative size = 163

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx &= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos(b^2\pi x^2)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} \\ &\quad - \frac{1}{30}b^5\pi^3\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} \\ &\quad + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} \\ &\quad - \frac{b\sin(b^2\pi x^2)}{40x^4} - \frac{7}{120}b^5\pi^2\,\text{Si}\left(b^2\pi x^2\right)\end{aligned}$$

output
```
-1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/15*b^2*Pi*cos(1/2*b^2*P
i*x^2)*FresnelC(b*x)/x^3-1/30*b^5*Pi^3*FresnelC(b*x)^2-7/120*b^5*Pi^2*Si(b
^2*Pi*x^2)-1/5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5+1/15*b^4*Pi^2*Fresnel
C(b*x)*sin(1/2*b^2*Pi*x^2)/x-1/40*b*sin(b^2*Pi*x^2)/x^4
```

3.214.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{1}{30}b^5\pi^3\,\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

output
```
-1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*
Pi*x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegra
l[b^2*Pi*x^2])/120
```

3.214.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7019, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3780, 7011, 3861, 3042, 3778, 25, 3042, 3780, 7019, 3856, 6995, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^6}dx$$

$$\Big\downarrow \text{7019}$$

$$\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{5}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2 - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2 - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{1}{5}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{5x^5} + \frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right) - \frac{\cos\left(\pi b^2 x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 7011}$$

3.214. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b}{12x^2}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\,\text{c}}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\,\text{c}}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx+\frac{1}{12}b\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(bx)\,\text{c}}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

3.214. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}+\frac{1}{12}b\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{7019}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx+\frac{1}{2}b\int\frac{\sin\left(b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}\right)-\text{Fre}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{3856}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b^2\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\text{FresnelC}(b\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{6995}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\pi b\int\text{FresnelC}(bx)d\,\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)\right)-\frac{\text{FresnelC}(bx)\,\text{c}}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{5}\pi b^2\left(-\frac{1}{3}\pi b^2\left(-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\,\text{FresnelC}(bx)^2\right)-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

3.214. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx$

output
```
-1/5*(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5 + (b^2*Pi*(-1/12*b/x^2 - (Cos
[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*x^3) - (b^2*Pi*((b*Pi*FresnelC[b*x]^2)/
2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIntegral[b^2*Pi*x^2])/4)
)/3 + (b*(-(Cos[b^2*Pi*x^2]/x^2) - b^2*Pi*SinIntegral[b^2*Pi*x^2]))/12))/5
 + (b*(-1/2*Sin[b^2*Pi*x^2]/x^4 + (b^2*Pi*(-(Cos[b^2*Pi*x^2]/x^2) - b^2*Pi
*SinIntegral[b^2*Pi*x^2]))/2))/20
```

3.214.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.214. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 6995
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Simp[Pi*(b/(
2*d))   Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] &&
EqQ[d^2, (Pi^2/4)*b^4]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.214.4 Maple [F]

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^6}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx =$$
$$-\frac{4\,\pi^3b^5x^5\,\mathrm{C}\left(bx\right)^2+7\,\pi^2b^5x^5\,\mathrm{Si}\left(\pi b^2x^2\right)+10\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-3\,\pi b^3x^3+8\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right.}{120\,x^5}$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

output
```
-1/120*(4*pi^3*b^5*x^5*fresnel_cos(b*x)^2 + 7*pi^2*b^5*x^5*sin_integral(pi
*b^2*x^2) + 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 - 3*pi*b^3*x^3 + 8*pi*b^2*
x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(3*b*x*cos(1/2*pi*b^2*x^2) -
4*(pi^2*b^4*x^4 - 3)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/x^5
```

3.214.6 Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^6}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

3.214.7 Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.214.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{C\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^6}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = \int \frac{\text{FresnelC}(b\,x)\ \sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.215 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

3.215.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = &-\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}\\ &-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4}\\ &-\frac{7b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}b^6\pi^3\,\text{FresnelC}\left(\sqrt{2}bx\right)\\ &-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6}\\ &+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^2}\\ &-\frac{b\sin\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{1440x}\\ &-\frac{1}{48}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)\end{aligned}$$

output
```
-1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/24*b^2*Pi*cos(1/2*b^
2*Pi*x^2)*FresnelC(b*x)/x^4-1/6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48
*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/60*b*sin(b^2*Pi*x^2)/x^5
+67/1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))
*2^(1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

3.215.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

3.215.3 Rubi [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7019, 3868, 3869, 3868, 3833, 7011, 3869, 3868, 3833, 7019, 3868, 3833, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{ 7019}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx + \frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{ 3868}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{ 3869}$$

$$\frac{1}{6}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2 x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{6}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$$\Big\downarrow \text{7011}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4} - \frac{b}{24x^3}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\text{FresnelC}(bx)\,\text{co}}{4x^4}\right. \quad \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx + \frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(2\pi b^2\int \cos\left(b^2\pi x^2\right)dx - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2 x\right.}{3x^3}\right.\right. \quad \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6} + \frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right) - \frac{\sin\left(\pi b^2x^2\right)}{x}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right) - \frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$$\Big\downarrow \text{3833}$$

3.215. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}b\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$\big\downarrow$ 7019

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}\right)-\text{Fr}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$\big\downarrow$ 3868

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(2\pi b^2\int\cos\left(b^2\pi x^2\right)dx-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\text{FresnelC}(b\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$\big\downarrow$ 3833

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\text{FresnelC}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$
$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

$\big\downarrow$ 7013

3.215. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}dx$

$$\frac{1}{6}\pi b^2\left(-\frac{1}{4}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}dx+\frac{1}{4}b\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\text{FresnelC}}{}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+$$

$$\frac{1}{12}b\left(\frac{2}{5}\pi b^2\left(-\frac{2}{3}\pi b^2\left(\sqrt{2}\pi b\,\text{FresnelC}\left(\sqrt{2}bx\right)-\frac{\sin\left(\pi b^2x^2\right)}{x}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\sin\left(\pi b^2x^2\right)}{5x^5}\right)$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

output
```
$Aborted
```

3.215.3.1 Defintions of rubi rules used

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.215. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.215.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^7}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

3.215.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\operatorname{C}(bx)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.215.6 Sympy [N/A]

Not integrable

Time = 11.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^7}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

3.215.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.215.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^7}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

3.215.9 Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.216 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

3.216.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = & -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\,\text{CosIntegral}\left(b^2\pi x^2\right) \\ & - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5} \\ & - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} \\ & + \frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3} \\ & - \frac{b\sin\left(b^2\pi x^2\right)}{84x^6} + \frac{b^5\pi^2\sin\left(b^2\pi x^2\right)}{84x^2} \\ & - \frac{1}{105}b^6\pi^3\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}, x\right)\end{aligned}$$

output
```
-1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2
)/x^4-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/7*FresnelC(b*x)*
sin(1/2*b^2*Pi*x^2)/x^7+1/105*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
^3-1/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2-1/105*b^6*
Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

3.216.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]
```

3.216.3 Rubi [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7019, 3860, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7011, 3861, 3042, 3778, 25, 3042, 3778, 3042, 3783, 7019, 3860, 3042, 3778, 3042, 3783, 7013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^8}\,dx$$

$$\Big\downarrow \ 7019$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3860$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3778$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3778

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 25

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\big\downarrow$ 3778

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

3.216. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3783$$

$$\frac{1}{7}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 7011$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{10}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3861$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{b}{40x^4}\right)+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

3.216. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\text{FresnelC}(bx)}{5x}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2\right.}{2x^4}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{3042}$$

3.216. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\text{cos}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{ CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3783}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}dx+\frac{1}{20}b\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{ CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi\right.}{2x}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{ CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 7019}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{6}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\frac{1}{20}\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{ CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\text{ CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$$\Big\downarrow \text{ 3042}$$

3.216. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}\right)+\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\downarrow$ 3778

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}(\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\downarrow$ 3042

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^2}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{Fresn}}{}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\downarrow$ 3783

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}}{}\right.\right.$$
$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

$\downarrow$ 7013

3.216. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

$$\frac{1}{7}\pi b^2\left(-\frac{1}{5}\pi b^2\left(\frac{1}{3}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}dx+\frac{1}{12}b\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\text{FresnelC}}{}\right.\right.$$

$$\frac{1}{28}b\left(\frac{1}{3}\pi b^2\left(-\frac{1}{2}\pi b^2\left(\pi b^2\,\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\sin\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\cos\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^6}\right)-$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.216. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7013
```
Int[Cos[(c_.) + (d_.)*(x_)^2]*FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_
))^(m_.), x_Symbol] :> Unintegrable[(e*x)^m*Cos[c + d*x^2]*FresnelC[a + b*x
]^n, x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.216.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^8}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

3.216.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.216.6 Sympy [N/A]

Not integrable

Time = 20.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^8}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

3.216.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.216.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{C\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^8}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

3.216.9 Mupad [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.217 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

3.217.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = &-\frac{b^3\pi}{480x^5}+\frac{b^7\pi^3}{768x}-\frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5}\\&+\frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6}\\&+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{384x^2}\\&+\frac{853b^8\pi^4\,\text{FresnelS}\left(\sqrt{2}bx\right)}{40320\sqrt{2}}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}\\&+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}\\&-\frac{b\sin\left(b^2\pi x^2\right)}{112x^7}+\frac{187b^5\pi^2\sin\left(b^2\pi x^2\right)}{40320x^3}\\&+\frac{1}{384}b^8\pi^4\,\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)\end{aligned}$$

output

```
-1/480*b^3*Pi/x^5+1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/
80640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(
b*x)/x^6+1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/8*FresnelC
(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x^4-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^
3+853/80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegr
able(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

3.217.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

output
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

3.217.3 Rubi [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7019, 3868, 3869, 3868, 3869, 3832, 7011, 3869, 3868, 3869, 3832, 7019, 3868, 3869, 3832, 7011, 3869, 3832, 7021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^9}\,dx$$

$$\Big\downarrow \ 7019$$

$$\frac{1}{8}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$$\Big\downarrow \ 3868$$

$$\frac{1}{8}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx - \frac{\sin\left(\pi b^2x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}$$

$$\Big\downarrow \ 3869$$

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx - \frac{\cos\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx - \frac{\sin\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{7x^7}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}$$

$\downarrow$ 3869

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx - \frac{\cos\left(\pi b^2 x^2\right)}{x}\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\pi b^2\right.}{7x^7} \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8}$$

$\downarrow$ 3832

$$\frac{1}{8}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2 x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\pi b\right.}{7x^7}$$

$\downarrow$ 7011

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx + \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{6x^6} - \frac{b}{60x^5}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8x^8} + \frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2 x^2\right)}{x} - \sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right) - \frac{\sin\left(\pi b^2 x^2\right)}{3x^3}\right) - \frac{\cos\left(\pi b^2 x^2\right)}{5x^5}\right) - \frac{\sin\left(\pi b\right.}{7x^7}$$

$\downarrow$ 3869

3.217. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\text{FresnelC}(bx)\,\text{c}}{6x^{6}}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \text{3868}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b\right.}{5x^5}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \text{3869}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int \sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^6}+\frac{1}{12}b\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^{7}}\right.$$

$$\Big\downarrow \text{7019}$$

3.217. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^4}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}\right)-\text{Fr}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 3868

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\text{FresnelC}(\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 3869

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\text{si}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 3832

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4x^4}+\frac{1}{8}b\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 7011

3.217. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}dx$

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right.}{2x^2}\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 3869

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx+\frac{1}{4}b\left(-2\pi b^2\int\sin\left(b^2\pi x^2\right)dx-\frac{\cos\left(\pi b^2x^2\right)}{x}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 3832

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

$\downarrow$ 7021

$$\frac{1}{8}\pi b^2\left(-\frac{1}{6}\pi b^2\left(\frac{1}{4}\pi b^2\left(-\frac{1}{2}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x^2}+\frac{1}{4}b\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}\right.\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8x^8}+$$
$$\frac{1}{16}b\left(\frac{2}{7}\pi b^2\left(-\frac{2}{5}\pi b^2\left(\frac{2}{3}\pi b^2\left(-\frac{\cos\left(\pi b^2x^2\right)}{x}-\sqrt{2}\pi b\,\text{FresnelS}\left(\sqrt{2}bx\right)\right)-\frac{\sin\left(\pi b^2x^2\right)}{3x^3}\right)-\frac{\cos\left(\pi b^2x^2\right)}{5x^5}\right)-\frac{\sin\left(\pi b\right.}{7x^7}\right.$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

output
```
$Aborted
```

3.217. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

3.217.3.1 Defintions of rubi rules used

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3868
```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 3869
```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Simp[d*(n/(e^n*(m + 1)))   Int[
(e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &
& LtQ[m, -1]
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

rule 7021
```
Int[FresnelC[(a_.) + (b_.)*(x_)]^(n_.)*((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)
*(x_)^2], x_Symbol] :> Unintegrable[(e*x)^m*FresnelC[a + b*x]^n*Sin[c + d*x
^2], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

3.217. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

3.217.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{9}}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

3.217.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{x^{9}}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
```

output
```
integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.217.6 Sympy [N/A]

Not integrable

Time = 36.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{9}}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

3.217.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.217.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^9}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

3.217.9 Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.218 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

3.218.1 Optimal result

Integrand size = 20, antiderivative size = 278

$$\begin{aligned}\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx =& -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}\\&+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}\\&+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3}\\&+\frac{b^9\pi^5\,\text{FresnelC}(bx)^2}{1890}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}\\&+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}\\&-\frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x}-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}\\&+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4}+\frac{83b^9\pi^4\text{Si}\left(b^2\pi x^2\right)}{30240}\end{aligned}$$

output
```
-1/756*b^3*Pi/x^6+1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5
/2016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/63*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresnel
C(b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/1890*b^9
*Pi^5*FresnelC(b*x)^2+83/30240*b^9*Pi^4*Si(b^2*Pi*x^2)-1/9*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^
5-1/945*b^8*Pi^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x-1/144*b*sin(b^2*Pi*x^
2)/x^8+67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

3.218.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos(b^2\pi x^2)}{3024x^6}+\frac{5b^7\pi^3\cos(b^2\pi x^2)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3}+\frac{b^9\pi^5\,\text{FresnelC}(bx)^2}{1890}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{b^8\pi^4\,\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x}-\frac{b\sin(b^2\pi x^2)}{144x^8}+\frac{67b^5\pi^2\sin(b^2\pi x^2)}{30240x^4}+\frac{83b^9\pi^4\text{Si}(b^2\pi x^2)}{30240}$$

input
```
Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

output
```
-1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/
(3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^
2*Pi*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*
x^5) - (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*P
i*x^2])/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi
^4*SinIntegral[b^2*Pi*x^2])/30240
```

3.218.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

$$\Big\downarrow \ 7019$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{18}b\int \frac{\sin(b^2\pi x^2)}{x^9}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^{10}}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(\frac{1}{3}\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{9}\pi b^2\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx+\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$$\Big\downarrow \text{3042}$$

3.218. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\Big\downarrow$ 3778

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2 - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2 - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\Big\downarrow$ 3778

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int -\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2\right.}{4x^8}$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\Big\downarrow$ 25

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2\right.}{4x^8}$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\Big\downarrow$ 3042

3.218. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx$

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2 - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2\right.}{4x^8}$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}$$

$\downarrow$ 3780

$$\frac{1}{9}\pi b^2 \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^8}dx - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right) - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 7011

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx + \frac{1}{14}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}dx - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7} - \frac{b}{84x^6}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right) - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3861

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx + \frac{1}{28}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}dx^2 - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7} - \frac{b}{84x^6}\right) - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right) - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx + \frac{1}{28}b\int \frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^8}dx^2 - \frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7} - \frac{b}{84x^6}\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9} + \frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right) - \frac{\cos\left(\pi b^2x^2\right)}{x^2}\right) - \frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right) - \frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right) - \frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3778

3.218. $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(\frac{1}{3}\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 25

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\text{FresnelC}(bx)}{7x}\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3778

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi\right.}{3x}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

3.218. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\int\frac{\sin\left(b^2\pi x^2+\frac{\pi}{2}\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{co}\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3778

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(\pi b^2\int-\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 25

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3042

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\int\frac{\sin\left(b^2\pi x^2\right)}{x^2}dx^2-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\right.\right.\right.$$
$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$
$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}\left(b^2\pi x^2\right)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$\downarrow$ 3780

3.218. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}dx-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{1}{28}b\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x\right.\right.\right.\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \; 7019$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{10}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^5}dx-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\text{F}\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \; 3860$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^6}dx^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}\right)-\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

$$\Big\downarrow \; 3778$$

3.218. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

$$\frac{1}{9}\pi b^2\left(-\frac{1}{7}\pi b^2\left(\frac{1}{5}\pi b^2\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^4}dx+\frac{1}{20}b\left(\frac{1}{2}\pi b^2\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}dx^2-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\text{FresnelC}\right.\right.$$

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+$$

$$\frac{1}{36}b\left(\frac{1}{4}\pi b^2\left(-\frac{1}{3}\pi b^2\left(\frac{1}{2}\pi b^2\left(-\pi b^2\text{Si}(b^2\pi x^2)-\frac{\cos\left(\pi b^2x^2\right)}{x^2}\right)-\frac{\sin\left(\pi b^2x^2\right)}{2x^4}\right)-\frac{\cos\left(\pi b^2x^2\right)}{3x^6}\right)-\frac{\sin\left(\pi b^2x^2\right)}{4x^8}\right)$$

input
```
Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

output
```
$Aborted
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.218. $\int\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 7011
```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[b*(x^(m + 2)/(2*(m +
 1)*(m + 2))), x] + Simp[2*(d/(m + 1))   Int[x^(m + 2)*Sin[d*x^2]*FresnelC[
b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m + 1)*Cos[2*d*x^2], x], x]) /;
FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

rule 7019
```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Simp[2*(d/(m + 1))   Int[
x^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Simp[b/(2*(m + 1))   Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

3.218.4 Maple [F]

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{10}}dx$$

input
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

output
```
int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$
$$= \frac{16\,\pi^5 b^9 x^9\,\text{C}\left(bx\right)^2 + 83\,\pi^4 b^9 x^9\,\text{Si}\left(\pi b^2 x^2\right) - 67\,\pi^3 b^7 x^7 + 70\,\pi b^3 x^3 + 10\left(15\,\pi^3 b^7 x^7 - 22\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)}{}$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

output
```
1/30240*(16*pi^5*b^9*x^9*fresnel_cos(b*x)^2 + 83*pi^4*b^9*x^9*sin_integral
(pi*b^2*x^2) - 67*pi^3*b^7*x^7 + 70*pi*b^3*x^3 + 10*(15*pi^3*b^7*x^7 - 22*
pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*cos(
1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*((67*pi^2*b^5*x^5 - 210*b*x)*cos(1/2*
pi*b^2*x^2) - 16*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 + 105)*fresnel_cos(b*x))*s
in(1/2*pi*b^2*x^2))/x^9
```

3.218.6 Sympy [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^{10}}\,dx$$

input
```
integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

output
```
Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

3.218.7 Maxima [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.218.8 Giac [F]

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{C(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x^{10}}\,dx$$

input
```
integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

output
```
integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = \int \frac{\text{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

input
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

output
```
int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```