Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

8-Special-functions/208-8.8-Polylogarithm-function

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [198]. This is test number [208].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (198)	0.00 (0)
Mathematica	98.48 (195)	1.52 (3)
Maple	76.26 (151)	23.74 (47)
Maxima	63.64 (126)	36.36 (72)
Fricas	52.02 (103)	47.98 (95)
Mupad	35.35 (70)	64.65 (128)
Sympy	23.23 (46)	76.77 (152)
Giac	7.58 (15)	92.42 (183)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

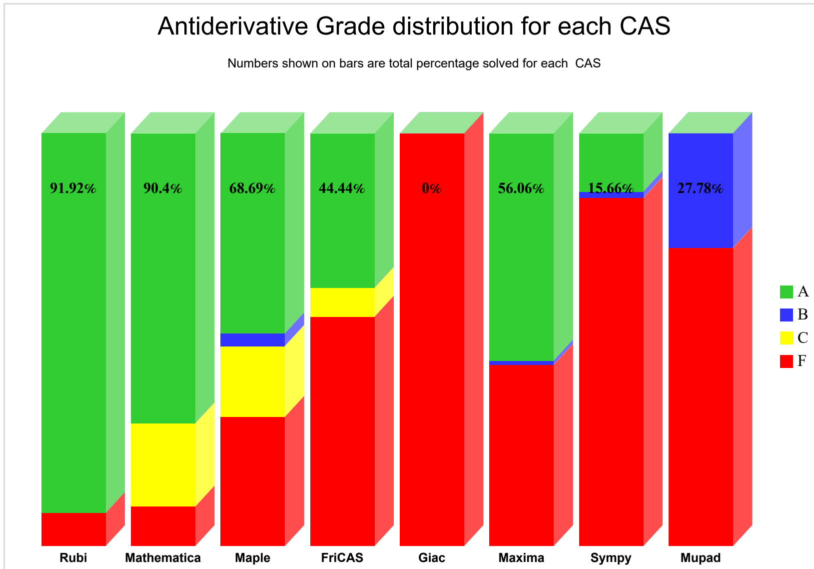
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

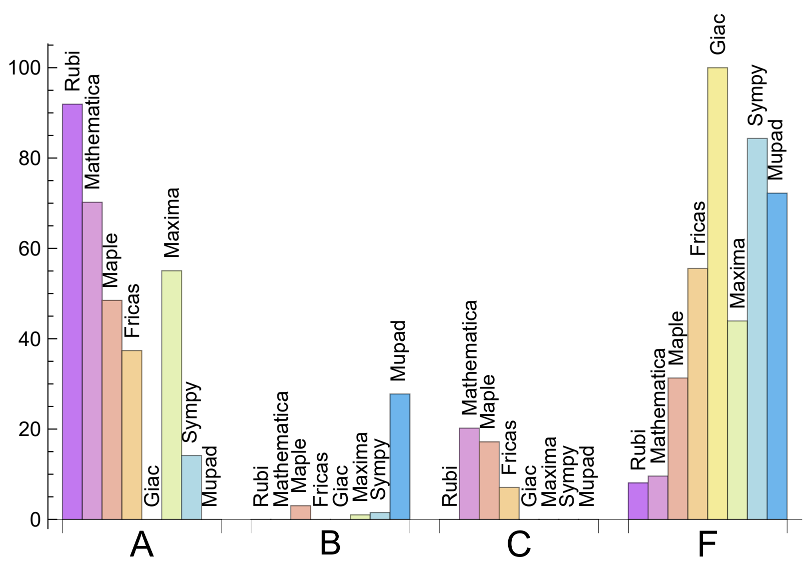
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.919	0.000	0.000	8.081
Mathematica	70.202	0.000	20.202	9.596
Maxima	55.051	1.010	0.000	43.939
Maple	48.485	3.030	17.172	31.313
Fricas	37.374	0.000	7.071	55.556
Sympy	14.141	1.515	0.000	84.343
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	27.778	0.000	72.222

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	47	100.00	0.00	0.00
Maxima	72	100.00	0.00	0.00
Fricas	95	100.00	0.00	0.00
Mupad	128	0.00	100.00	0.00
Sympy	152	83.55	16.45	0.00
Giac	183	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Fricas	0.26
Giac	0.28
Mathematica	0.41
Rubi	0.56
Maple	1.06
Mupad	5.03
Sympy	10.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	11.20	1.14	9.00	1.18
Mupad	41.01	0.94	46.00	0.90
Sympy	86.57	1.12	41.50	0.91
Maple	102.28	1.15	88.00	1.07
Fricas	105.26	1.26	72.00	1.15
Maxima	132.07	1.02	79.00	1.00
Mathematica	179.62	0.84	65.00	0.87
Rubi	230.57	1.07	100.00	1.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

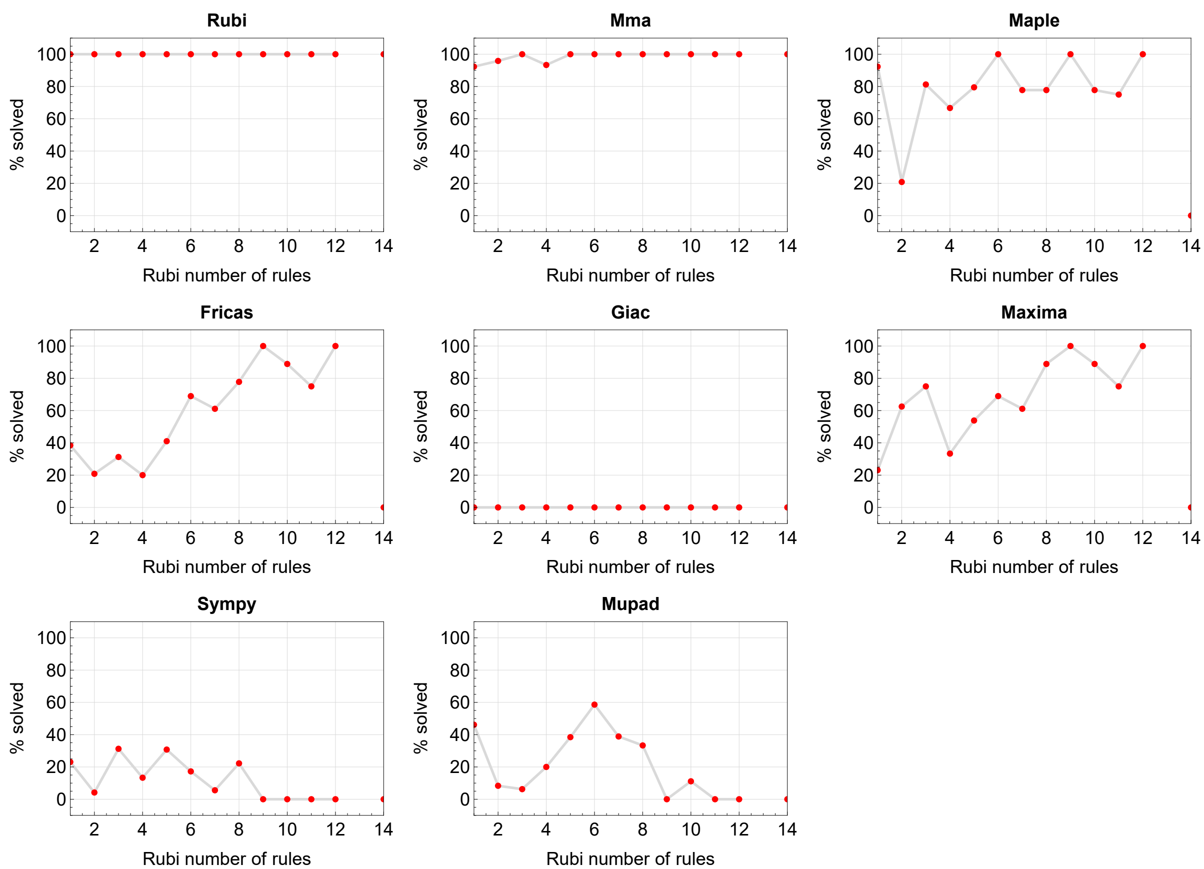


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

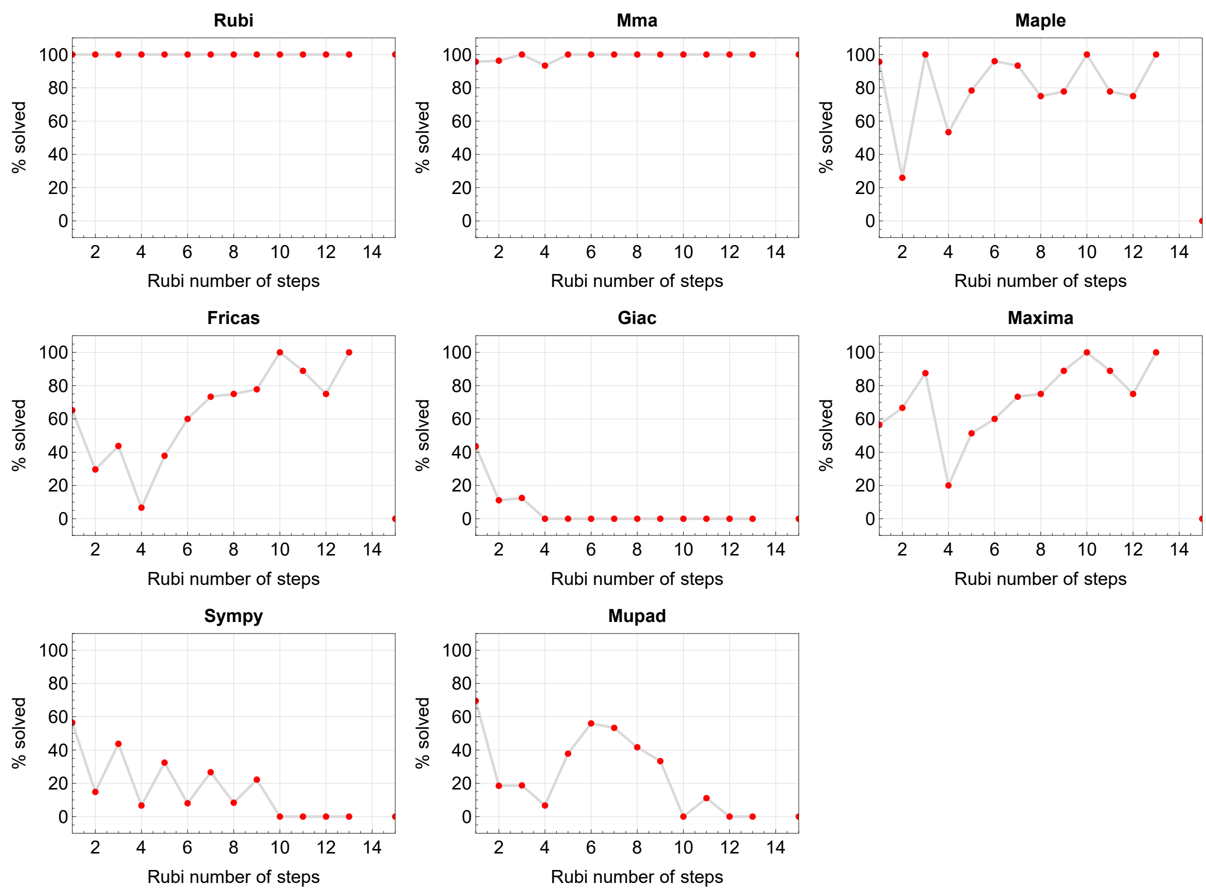


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

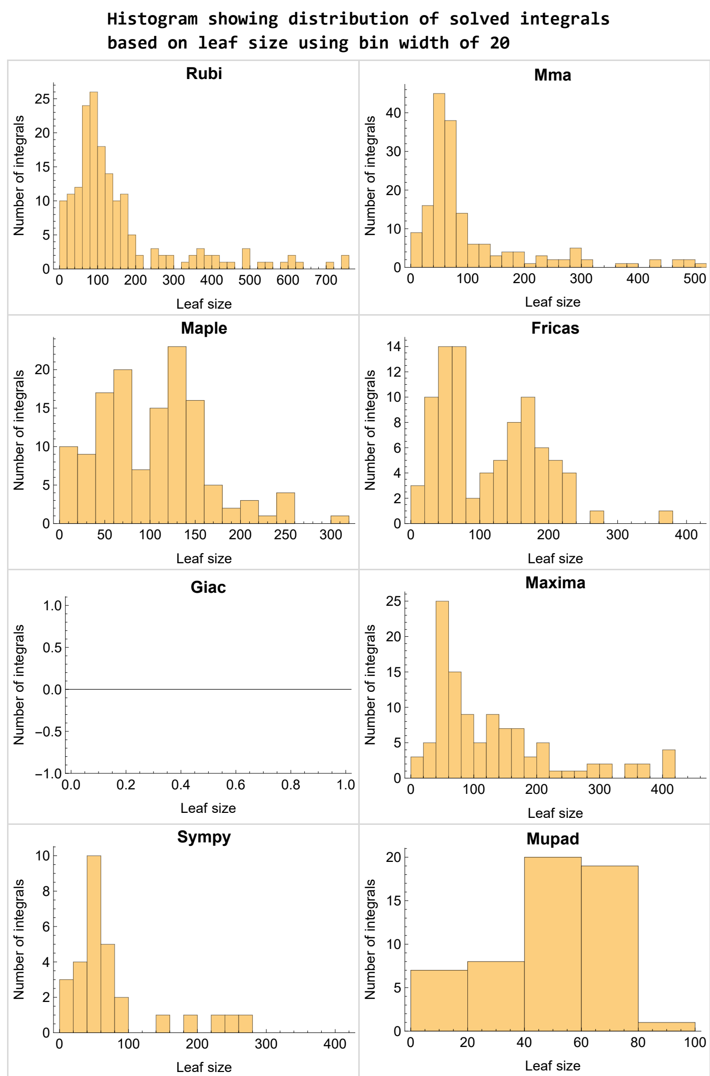


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

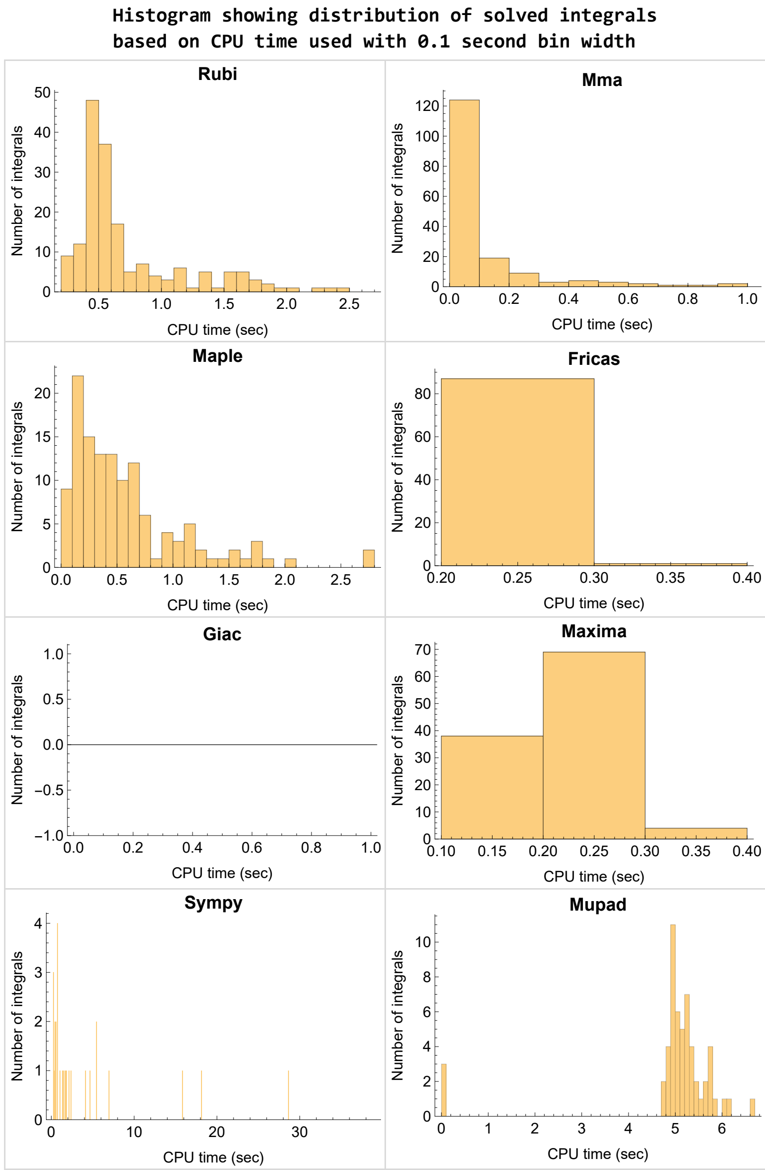


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

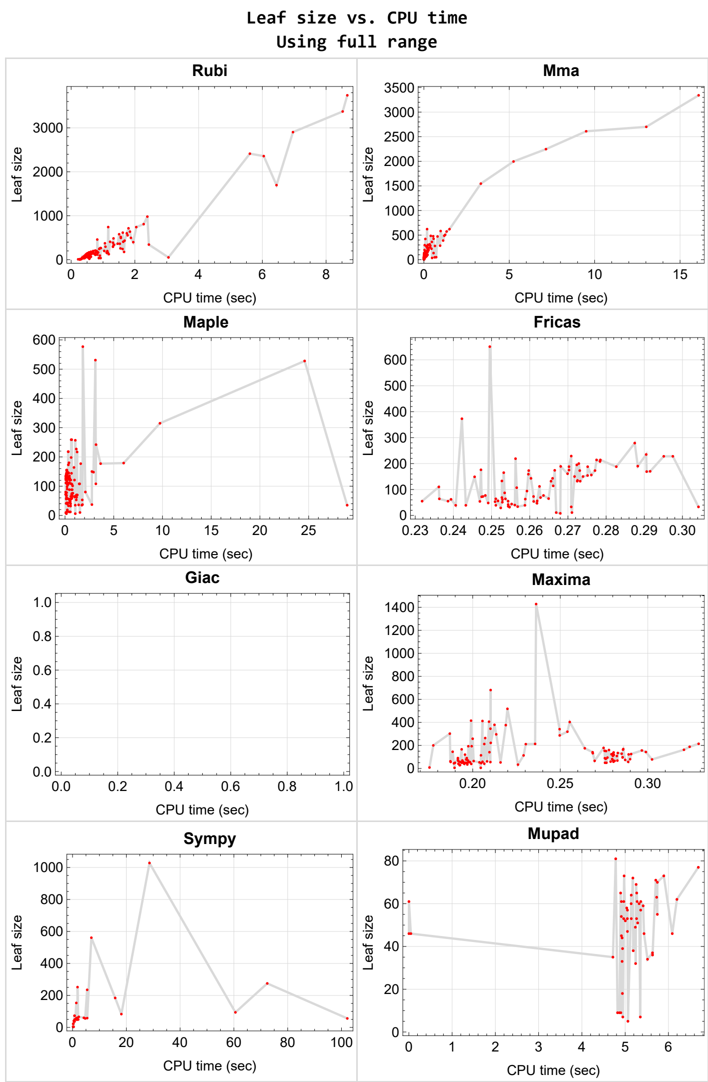


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{17, 18, 37, 38\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

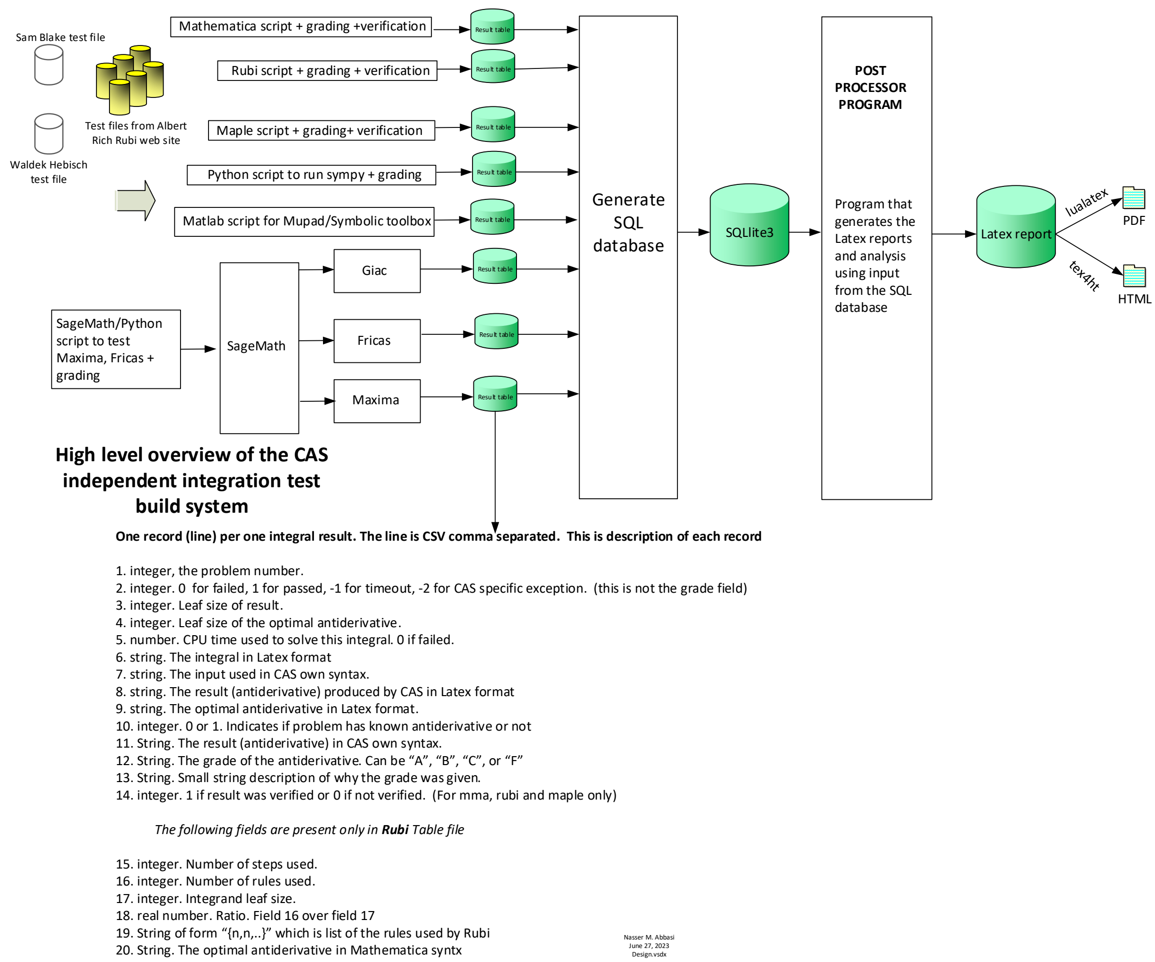
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 87, 102, 105, 108, 111, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198 }

B grade { }

C grade { 17, 18, 30, 31, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113 }

F normal fail { 101, 152, 196 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 116, 121, 124, 125, 126, 128, 129, 130, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 165, 173 }

B grade { 39, 40, 41, 42, 43, 44 }

C grade { 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F normal fail { 101, 127, 131, 132, 133, 134, 135, 136, 141, 156, 157, 158, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 124, 125, 126, 131, 132, 133, 137, 138, 139, 140, 145, 146, 150, 151, 152, 153, 154, 155, 165 }

B grade { }

C grade { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

F normal fail { 6, 15, 22, 35, 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 128, 129, 130, 134, 135, 136, 141, 142, 143, 144, 147, 148, 149, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 124, 125, 126, 128, 129, 130, 131, 132, 133, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 153, 154, 161, 162, 163, 164, 165, 166, 167, 168, 169, 184, 185, 186, 189, 190, 191, 192, 193, 197, 198 }

B grade { 144, 155 }

C grade { }

F normal fail { 22, 35, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 134, 135, 136, 141, 149, 150, 151, 152, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 187, 188, 194, 195, 196 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 101, 116, 126, 133, 140, 145, 146, 153, 154, 155, 165, 173 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 121, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 19, 20, 21, 23, 24, 25, 26, 27, 28, 116, 124, 125, 126, 137, 138, 139, 140 }

B grade { 29, 30, 153 }

C grade { }

F normal fail { 11, 12, 13, 14, 16, 17, 18, 22, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 197 }

F(-1) timedout fail { 31, 64, 72, 76, 77, 85, 86, 90, 108, 143, 144, 151, 152, 154, 155, 177, 178, 179, 181, 182, 183, 184, 191, 192, 198 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	96	73	72	72	72	66	0	69
N.S.	1	1.12	0.85	0.84	0.84	0.84	0.77	0.00	0.80
time (sec)	N/A	0.275	0.032	0.674	0.192	0.247	2.326	0.000	5.254

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	86	65	65	64	64	58	0	61
N.S.	1	1.13	0.86	0.86	0.84	0.84	0.76	0.00	0.80
time (sec)	N/A	0.276	0.026	0.597	0.192	0.236	1.401	0.000	5.361

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	76	57	56	56	56	49	0	53
N.S.	1	1.15	0.86	0.85	0.85	0.85	0.74	0.00	0.80
time (sec)	N/A	0.261	0.023	0.517	0.193	0.252	0.790	0.000	5.263

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	66	48	49	48	48	41	0	46
N.S.	1	1.18	0.86	0.88	0.86	0.86	0.73	0.00	0.82
time (sec)	N/A	0.252	0.021	0.495	0.197	0.249	0.473	0.000	5.436

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	32	26	33	29	29	22	0	32
N.S.	1	1.10	0.90	1.14	1.00	1.00	0.76	0.00	1.10
time (sec)	N/A	0.209	0.013	0.352	0.193	0.252	0.230	0.000	5.241

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	0	3	0	5
N.S.	1	1.00	1.00	1.20	1.00	0.00	0.60	0.00	1.00
time (sec)	N/A	0.179	0.002	1.052	0.190	0.000	0.409	0.000	5.064

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	40	28	34	24	0	34
N.S.	1	1.00	1.00	1.11	0.78	0.94	0.67	0.00	0.94
time (sec)	N/A	0.222	0.011	0.569	0.193	0.257	0.317	0.000	5.517

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	57	50	57	40	47	42	0	51
N.S.	1	0.98	0.86	0.98	0.69	0.81	0.72	0.00	0.88
time (sec)	N/A	0.254	0.023	0.669	0.192	0.262	0.580	0.000	5.289

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	52	68	49	56	51	0	57
N.S.	1	1.01	0.76	1.00	0.72	0.82	0.75	0.00	0.84
time (sec)	N/A	0.263	0.025	0.727	0.196	0.262	1.034	0.000	5.359

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	60	76	58	65	60	0	60
N.S.	1	1.01	0.77	0.97	0.74	0.83	0.77	0.00	0.77
time (sec)	N/A	0.268	0.028	0.802	0.194	0.265	1.799	0.000	5.323

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	103	86	78	77	77	0	0	71
N.S.	1	1.17	0.98	0.89	0.88	0.88	0.00	0.00	0.81
time (sec)	N/A	0.341	0.011	0.168	0.196	0.248	0.000	0.000	5.708

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	93	78	69	69	69	0	0	63
N.S.	1	1.19	1.00	0.88	0.88	0.88	0.00	0.00	0.81
time (sec)	N/A	0.333	0.010	0.152	0.194	0.253	0.000	0.000	5.728

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	83	69	62	61	61	0	0	55
N.S.	1	1.22	1.01	0.91	0.90	0.90	0.00	0.00	0.81
time (sec)	N/A	0.313	0.008	0.154	0.208	0.253	0.000	0.000	5.744

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	39	39	41	39	39	0	0	37
N.S.	1	1.15	1.15	1.21	1.15	1.15	0.00	0.00	1.09
time (sec)	N/A	0.266	0.011	0.109	0.199	0.241	0.000	0.000	5.632

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	0	3	0	0
N.S.	1	1.00	1.00	1.20	1.00	0.00	0.60	0.00	0.00
time (sec)	N/A	0.174	0.002	0.145	0.205	0.000	0.231	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	57	33	39	0	0	36
N.S.	1	1.00	0.96	1.24	0.72	0.85	0.00	0.00	0.78
time (sec)	N/A	0.271	0.026	0.121	0.226	0.251	0.000	0.000	5.633

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	70	74	25	90	47	54	0	0	46
N.S.	1	1.06	0.36	1.29	0.67	0.77	0.00	0.00	0.66
time (sec)	N/A	0.306	0.009	0.215	0.197	0.261	0.000	0.000	6.089

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	25	106	56	63	0	0	62
N.S.	1	1.08	0.31	1.32	0.70	0.79	0.00	0.00	0.78
time (sec)	N/A	0.310	0.011	0.231	0.187	0.259	0.000	0.000	6.196

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	84	65	65	62	62	56	0	61
N.S.	1	1.14	0.88	0.88	0.84	0.84	0.76	0.00	0.82
time (sec)	N/A	0.285	0.016	0.658	0.187	0.239	4.685	0.000	4.913

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	74	56	57	54	54	48	0	53
N.S.	1	1.16	0.88	0.89	0.84	0.84	0.75	0.00	0.83
time (sec)	N/A	0.292	0.013	0.295	0.198	0.247	1.623	0.000	5.051

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	43	44	40	40	39	0	45
N.S.	1	1.02	0.93	0.96	0.87	0.87	0.85	0.00	0.98
time (sec)	N/A	0.250	0.008	0.445	0.195	0.256	0.589	0.000	4.913

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0	9
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.175	0.003	1.534	0.000	0.000	0.000	0.000	4.871

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	49	43	34	44	37	0	44
N.S.	1	1.04	1.00	0.88	0.69	0.90	0.76	0.00	0.90
time (sec)	N/A	0.261	0.013	0.226	0.197	0.255	0.769	0.000	4.929

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	51	52	46	55	49	0	53
N.S.	1	1.02	0.80	0.81	0.72	0.86	0.77	0.00	0.83
time (sec)	N/A	0.282	0.025	0.523	0.205	0.254	2.111	0.000	4.966

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	68	62	55	64	58	0	61
N.S.	1	1.04	0.92	0.84	0.74	0.86	0.78	0.00	0.82
time (sec)	N/A	0.297	0.023	1.185	0.210	0.251	5.449	0.000	4.968

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	82	65	63	80	159	94	0	60
N.S.	1	1.12	0.89	0.86	1.10	2.18	1.29	0.00	0.82
time (sec)	N/A	0.275	0.061	0.987	0.290	0.260	60.570	0.000	5.139

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	72	57	55	68	143	83	0	52
N.S.	1	1.14	0.90	0.87	1.08	2.27	1.32	0.00	0.83
time (sec)	N/A	0.269	0.037	0.413	0.283	0.260	18.156	0.000	5.004

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	48	39	43	49	107	60	0	39
N.S.	1	1.20	0.98	1.08	1.22	2.68	1.50	0.00	0.98
time (sec)	N/A	0.220	0.023	0.176	0.276	0.257	4.199	0.000	4.940

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	45	41	39	49	94	184	0	38
N.S.	1	1.07	0.98	0.93	1.17	2.24	4.38	0.00	0.90
time (sec)	N/A	0.232	0.015	0.250	0.276	0.259	15.850	0.000	5.186

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	47	47	57	114	275	0	47
N.S.	1	1.09	0.84	0.84	1.02	2.04	4.91	0.00	0.84
time (sec)	N/A	0.240	0.014	0.529	0.289	0.266	72.400	0.000	5.053

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	71	47	53	65	132	0	0	58
N.S.	1	1.08	0.71	0.80	0.98	2.00	0.00	0.00	0.88
time (sec)	N/A	0.253	0.016	1.152	0.281	0.273	0.000	0.000	5.037

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	103	88	80	77	77	0	0	73
N.S.	1	1.17	1.00	0.91	0.88	0.88	0.00	0.00	0.83
time (sec)	N/A	0.369	0.013	0.066	0.196	0.264	0.000	0.000	4.976

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	93	79	72	69	69	0	0	65
N.S.	1	1.19	1.01	0.92	0.88	0.88	0.00	0.00	0.83
time (sec)	N/A	0.363	0.012	0.055	0.197	0.263	0.000	0.000	4.895

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	52	56	53	53	0	0	57
N.S.	1	1.02	0.87	0.93	0.88	0.88	0.00	0.00	0.95
time (sec)	N/A	0.315	0.011	0.133	0.204	0.251	0.000	0.000	5.053

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0	9
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.178	0.003	0.085	0.000	0.000	0.000	0.000	4.824

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	60	68	41	51	0	0	54
N.S.	1	1.03	0.95	1.08	0.65	0.81	0.00	0.00	0.86
time (sec)	N/A	0.315	0.022	0.138	0.207	0.253	0.000	0.000	4.911

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	78	84	30	98	55	64	0	0	65
N.S.	1	1.08	0.38	1.26	0.71	0.82	0.00	0.00	0.83
time (sec)	N/A	0.352	0.011	0.206	0.198	0.254	0.000	0.000	5.267

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	96	30	115	64	73	0	0	73
N.S.	1	1.09	0.34	1.31	0.73	0.83	0.00	0.00	0.83
time (sec)	N/A	0.359	0.015	0.193	0.201	0.261	0.000	0.000	5.893

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	101	77	144	95	189	0	0	72
N.S.	1	1.16	0.89	1.66	1.09	2.17	0.00	0.00	0.83
time (sec)	N/A	0.333	0.131	0.173	0.286	0.268	0.000	0.000	5.179

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	91	69	136	81	173	0	0	64
N.S.	1	1.18	0.90	1.77	1.05	2.25	0.00	0.00	0.83
time (sec)	N/A	0.317	0.118	0.187	0.280	0.260	0.000	0.000	5.141

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	60	50	119	59	133	0	0	49
N.S.	1	1.20	1.00	2.38	1.18	2.66	0.00	0.00	0.98
time (sec)	N/A	0.270	0.078	0.180	0.279	0.253	0.000	0.000	5.236

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	60	50	112	58	112	0	0	53
N.S.	1	1.11	0.93	2.07	1.07	2.07	0.00	0.00	0.98
time (sec)	N/A	0.282	0.073	0.187	0.280	0.263	0.000	0.000	5.142

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	80	61	125	66	132	0	0	59
N.S.	1	1.14	0.87	1.79	0.94	1.89	0.00	0.00	0.84
time (sec)	N/A	0.301	0.073	0.196	0.270	0.266	0.000	0.000	5.416

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	90	69	138	74	150	0	0	70
N.S.	1	1.12	0.86	1.72	0.92	1.88	0.00	0.00	0.88
time (sec)	N/A	0.310	0.081	0.201	0.287	0.272	0.000	0.000	5.744

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	69	108	0	0	0	0	0
N.S.	1	1.04	0.97	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.037	1.455	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	69	108	0	0	0	0	0
N.S.	1	1.04	0.97	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.033	0.658	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	88	0	0	0	0	0
N.S.	1	1.00	0.94	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.037	0.443	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	11	0	0	0
N.S.	1	1.00	1.00	1.09	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.003	0.550	0.000	0.271	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	60	106	0	0	0	0	0
N.S.	1	1.01	0.87	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.043	0.722	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	81	61	108	0	0	0	0	0
N.S.	1	1.04	0.78	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.044	1.500	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	79	61	108	0	0	0	0	0
N.S.	1	1.04	0.80	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.043	3.164	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	94	41	132	0	0	0	0	0
N.S.	1	1.07	0.47	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.023	0.327	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	94	41	132	0	0	0	0	0
N.S.	1	1.07	0.47	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.010	0.289	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	67	39	105	0	0	0	0	0
N.S.	1	0.97	0.57	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.009	0.300	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	11	0	0	0
N.S.	1	1.00	1.00	1.09	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.004	0.428	0.000	0.267	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	37	129	0	0	0	0	0
N.S.	1	1.01	0.44	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.009	0.294	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	101	41	132	0	0	0	0	0
N.S.	1	1.06	0.43	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.011	0.313	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	99	41	132	0	0	0	0	0
N.S.	1	1.06	0.44	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.009	0.338	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	138	90	101	128	190	0	0	0
N.S.	1	1.18	0.77	0.86	1.09	1.62	0.00	0.00	0.00
time (sec)	N/A	0.294	0.094	0.778	0.280	0.288	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	118	75	88	109	143	0	0	0
N.S.	1	1.16	0.74	0.86	1.07	1.40	0.00	0.00	0.00
time (sec)	N/A	0.283	0.068	0.683	0.280	0.266	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	90	63	74	83	135	0	0	0
N.S.	1	1.12	0.79	0.92	1.04	1.69	0.00	0.00	0.00
time (sec)	N/A	0.265	0.076	0.709	0.276	0.273	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	51	59	71	132	0	0	0
N.S.	1	1.04	0.75	0.87	1.04	1.94	0.00	0.00	0.00
time (sec)	N/A	0.256	0.064	0.697	0.280	0.272	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	98	57	75	89	150	0	0	0
N.S.	1	1.10	0.64	0.84	1.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.267	0.078	0.699	0.275	0.274	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	118	65	88	108	170	0	0	0
N.S.	1	1.11	0.61	0.83	1.02	1.60	0.00	0.00	0.00
time (sec)	N/A	0.285	0.080	0.736	0.282	0.292	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	182	98	149	156	279	0	0	0
N.S.	1	1.19	0.64	0.97	1.02	1.82	0.00	0.00	0.00
time (sec)	N/A	0.384	0.216	0.242	0.297	0.288	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	162	88	141	143	229	0	0	0
N.S.	1	1.19	0.65	1.04	1.05	1.68	0.00	0.00	0.00
time (sec)	N/A	0.381	0.181	0.119	0.299	0.271	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	142	73	133	122	173	0	0	0
N.S.	1	1.17	0.60	1.10	1.01	1.43	0.00	0.00	0.00
time (sec)	N/A	0.352	0.165	0.076	0.289	0.277	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	110	57	127	94	161	0	0	0
N.S.	1	1.13	0.59	1.31	0.97	1.66	0.00	0.00	0.00
time (sec)	N/A	0.339	0.115	0.071	0.277	0.270	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	91	58	111	78	156	0	0	0
N.S.	1	1.07	0.68	1.31	0.92	1.84	0.00	0.00	0.00
time (sec)	N/A	0.329	0.099	0.085	0.302	0.276	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	122	64	122	97	175	0	0	0
N.S.	1	1.13	0.59	1.13	0.90	1.62	0.00	0.00	0.00
time (sec)	N/A	0.347	0.095	0.075	0.286	0.270	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	142	72	135	118	195	0	0	0
N.S.	1	1.14	0.58	1.08	0.94	1.56	0.00	0.00	0.00
time (sec)	N/A	0.352	0.118	0.074	0.281	0.272	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	168	101	135	160	188	0	0	0
N.S.	1	1.20	0.72	0.96	1.14	1.34	0.00	0.00	0.00
time (sec)	N/A	0.340	0.091	0.457	0.278	0.275	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	149	91	127	139	173	0	0	0
N.S.	1	1.19	0.73	1.02	1.11	1.38	0.00	0.00	0.00
time (sec)	N/A	0.328	0.063	0.459	0.280	0.273	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	137	57	127	128	154	0	0	0
N.S.	1	1.19	0.50	1.10	1.11	1.34	0.00	0.00	0.00
time (sec)	N/A	0.314	0.072	0.484	0.285	0.275	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	117	62	111	123	174	0	0	0
N.S.	1	1.14	0.60	1.08	1.19	1.69	0.00	0.00	0.00
time (sec)	N/A	0.310	0.074	0.470	0.279	0.267	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	125	62	111	125	200	0	0	0
N.S.	1	1.13	0.56	1.00	1.13	1.80	0.00	0.00	0.00
time (sec)	N/A	0.305	0.077	0.421	0.290	0.273	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	144	70	122	151	214	0	0	0
N.S.	1	1.14	0.56	0.97	1.20	1.70	0.00	0.00	0.00
time (sec)	N/A	0.326	0.079	0.431	0.276	0.278	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	198	89	155	178	235	0	0	0
N.S.	1	1.23	0.55	0.96	1.11	1.46	0.00	0.00	0.00
time (sec)	N/A	0.430	0.101	0.197	0.275	0.291	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	194	89	155	175	208	0	0	0
N.S.	1	1.20	0.55	0.96	1.09	1.29	0.00	0.00	0.00
time (sec)	N/A	0.420	0.086	0.152	0.264	0.278	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	175	68	147	153	188	0	0	0
N.S.	1	1.20	0.47	1.01	1.05	1.29	0.00	0.00	0.00
time (sec)	N/A	0.400	0.086	0.168	0.275	0.270	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	159	68	147	141	169	0	0	0
N.S.	1	1.19	0.51	1.10	1.05	1.26	0.00	0.00	0.00
time (sec)	N/A	0.395	0.079	0.242	0.268	0.291	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	71	131	132	188	0	0	0
N.S.	1	1.14	0.58	1.07	1.08	1.54	0.00	0.00	0.00
time (sec)	N/A	0.380	0.082	0.276	0.269	0.283	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	151	71	131	134	214	0	0	0
N.S.	1	1.14	0.54	0.99	1.02	1.62	0.00	0.00	0.00
time (sec)	N/A	0.376	0.089	0.285	0.283	0.279	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	170	79	142	163	228	0	0	0
N.S.	1	1.16	0.54	0.97	1.11	1.55	0.00	0.00	0.00
time (sec)	N/A	0.390	0.090	0.157	0.286	0.295	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	172	84	142	168	228	0	0	0
N.S.	1	1.17	0.57	0.97	1.14	1.55	0.00	0.00	0.00
time (sec)	N/A	0.402	0.090	0.161	0.286	0.297	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	104	82	121	0	0	0	0	0
N.S.	1	1.03	0.81	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	0.104	0.493	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	103	82	121	0	0	0	0	0
N.S.	1	1.03	0.82	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	0.097	0.505	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	94	48	109	0	0	0	0	0
N.S.	1	1.01	0.52	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	0.027	0.572	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	99	48	121	0	0	0	0	0
N.S.	1	1.02	0.49	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.033	0.592	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	109	48	121	0	0	0	0	0
N.S.	1	1.04	0.46	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	0.036	0.972	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	131	50	145	0	0	0	0	0
N.S.	1	1.05	0.40	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.041	0.356	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	50	145	0	0	0	0	0
N.S.	1	1.05	0.40	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.033	0.379	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	117	50	133	0	0	0	0	0
N.S.	1	1.02	0.43	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.028	0.372	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	122	50	145	0	0	0	0	0
N.S.	1	1.03	0.42	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.036	0.344	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	136	50	145	0	0	0	0	0
N.S.	1	1.05	0.39	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.034	0.346	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	5	7	7	7	7	7
N.S.	1	1.00	1.29	0.71	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.256	0.007	0.019	0.196	0.244	0.232	0.270	5.195

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	5	7	7	7	7	7
N.S.	1	1.00	1.29	0.71	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.209	0.008	0.020	0.208	0.261	0.224	0.283	5.182

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	5	7	7	8	7	7
N.S.	1	1.00	1.29	0.71	1.00	1.00	1.14	1.00	1.00
time (sec)	N/A	0.159	0.007	0.020	0.189	0.246	0.233	0.270	4.964

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	5	7	7	8	7	7
N.S.	1	1.00	1.29	0.71	1.00	1.00	1.14	1.00	1.00
time (sec)	N/A	0.206	0.007	0.020	0.180	0.234	0.210	0.271	4.962

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	5	7	7	8	7	7
N.S.	1	1.00	1.29	0.71	1.00	1.00	1.14	1.00	1.00
time (sec)	N/A	0.251	0.008	0.020	0.197	0.226	0.246	0.273	5.027

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	B
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	0	0	0	0	0	0	7
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.144	0.000	0.000	0.000	0.000	0.000	0.000	4.946

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	85	53	144	0	0	0	0	0
N.S.	1	1.09	0.68	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.041	0.664	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	114	88	173	0	0	0	0	0
N.S.	1	1.12	0.86	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.055	0.337	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	143	119	198	0	0	0	0	0
N.S.	1	1.18	0.98	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	0.062	0.626	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	101	72	177	0	0	0	0	0
N.S.	1	1.07	0.77	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.042	1.609	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	132	126	218	0	0	0	0	0
N.S.	1	1.12	1.07	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.067	0.342	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	163	166	259	0	0	0	0	0
N.S.	1	1.15	1.17	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	0.083	0.684	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	101	72	177	0	0	0	0	0
N.S.	1	1.07	0.77	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.042	3.652	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	132	126	218	0	0	0	0	0
N.S.	1	1.12	1.07	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.069	0.320	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	163	166	259	0	0	0	0	0
N.S.	1	1.15	1.17	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	0.087	0.631	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	106	80	148	0	0	0	0	0
N.S.	1	1.05	0.79	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.062	2.919	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	138	50	180	0	0	0	0	0
N.S.	1	1.06	0.38	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.025	0.404	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	170	52	217	0	0	0	0	0
N.S.	1	1.10	0.34	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.513	0.026	1.240	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	7	9	9
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.00	1.29	1.29
time (sec)	N/A	0.172	0.020	0.047	0.184	0.233	0.296	0.274	5.176

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	5	5	7	5	7	7	5	7	7
N.S.	1	1.00	1.40	1.00	1.40	1.40	1.00	1.40	1.40
time (sec)	N/A	0.162	0.004	0.046	0.186	0.228	0.234	0.274	5.264

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	0	0	5	0	7
N.S.	1	1.00	1.00	1.14	0.00	0.00	0.71	0.00	1.00
time (sec)	N/A	0.184	0.003	0.088	0.000	0.000	0.216	0.000	5.351

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	11	11	8	11	11
N.S.	1	1.00	1.22	1.00	1.22	1.22	0.89	1.22	1.22
time (sec)	N/A	0.178	0.023	0.042	0.218	0.224	0.304	0.275	5.180

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	11	11	8	11	11
N.S.	1	1.00	1.22	1.00	1.22	1.22	0.89	1.22	1.22
time (sec)	N/A	0.178	0.018	0.045	0.212	0.233	0.347	0.272	5.287

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	11	11	8	11	11
N.S.	1	1.00	1.22	1.00	1.22	1.22	0.89	1.22	1.22
time (sec)	N/A	0.176	0.021	0.040	0.184	0.234	0.423	0.282	5.161

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	7	9	9
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.00	1.29	1.29
time (sec)	N/A	0.166	0.005	0.026	0.180	0.234	0.312	0.274	5.312

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	0	0	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.003	0.229	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	11	13	11	13	13	10	13	13
N.S.	1	1.00	1.18	1.00	1.18	1.18	0.91	1.18	1.18
time (sec)	N/A	0.185	0.019	0.028	0.202	0.231	0.494	0.281	4.651

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	11	13	11	13	13	10	13	13
N.S.	1	1.00	1.18	1.00	1.18	1.18	0.91	1.18	1.18
time (sec)	N/A	0.185	0.019	0.026	0.200	0.234	0.613	0.285	4.761

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	265	144	257	200	165	235	0	0
N.S.	1	1.02	0.55	0.99	0.77	0.63	0.90	0.00	0.00
time (sec)	N/A	0.580	0.153	1.056	0.177	0.253	5.440	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	156	96	164	145	110	153	0	0
N.S.	1	1.03	0.63	1.08	0.95	0.72	1.01	0.00	0.00
time (sec)	N/A	0.416	0.074	0.917	0.189	0.236	1.396	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	53	63	90	55	73	0	61
N.S.	1	1.12	0.88	1.05	1.50	0.92	1.22	0.00	1.02
time (sec)	N/A	0.377	0.014	0.675	0.192	0.232	0.733	0.000	5.277

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	459	422	0	0	0	0	0	0
N.S.	1	1.14	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	82	73	80	114	0	0	0	0
N.S.	1	0.98	0.87	0.95	1.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.362	0.042	2.089	0.205	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	155	131	150	193	0	0	0	0
N.S.	1	0.90	0.76	0.87	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.125	2.761	0.197	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	260	222	242	302	0	0	0	0
N.S.	1	0.94	0.80	0.88	1.09	0.00	0.00	0.00	0.00
time (sec)	N/A	0.564	0.236	3.181	0.187	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	347	340	296	0	264	219	0	0	0
N.S.	1	0.98	0.85	0.00	0.76	0.63	0.00	0.00	0.00
time (sec)	N/A	0.843	0.049	0.000	0.207	0.256	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	197	198	0	193	149	0	0	0
N.S.	1	0.99	1.00	0.00	0.97	0.75	0.00	0.00	0.00
time (sec)	N/A	0.493	0.039	0.000	0.199	0.246	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	93	66	0	120	73	0	0	77
N.S.	1	1.11	0.79	0.00	1.43	0.87	0.00	0.00	0.92
time (sec)	N/A	0.556	0.020	0.000	0.196	0.248	0.000	0.000	6.691

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	15	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	486	520	477	0	0	0	0	0	0
N.S.	1	1.07	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.963	0.546	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	629	601	573	0	0	0	0	0	0
N.S.	1	0.96	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.095	1.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	580	485	531	681	651	1028	0	0
N.S.	1	0.96	0.80	0.88	1.13	1.08	1.70	0.00	0.00
time (sec)	N/A	0.913	0.403	3.119	0.210	0.250	28.613	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	374	274	577	406	373	561	0	0
N.S.	1	0.97	0.71	1.50	1.05	0.97	1.46	0.00	0.00
time (sec)	N/A	0.644	0.138	1.826	0.209	0.242	6.999	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	161	227	212	176	252	0	0
N.S.	1	1.00	0.77	1.08	1.01	0.84	1.20	0.00	0.00
time (sec)	N/A	0.465	0.086	1.156	0.207	0.247	1.858	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	53	63	90	55	73	0	61
N.S.	1	1.12	0.88	1.05	1.50	0.92	1.22	0.00	1.02
time (sec)	N/A	0.379	0.007	0.709	0.197	0.239	0.765	0.000	0.002

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	591	743	622	0	0	0	0	0	0
N.S.	1	1.26	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	136	108	179	166	0	0	0	0
N.S.	1	0.99	0.78	1.30	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.111	6.017	0.194	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	244	190	315	379	0	0	0	0
N.S.	1	0.88	0.68	1.13	1.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	0.199	9.742	0.212	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	408	313	528	1428	0	0	0	0
N.S.	1	0.91	0.70	1.18	3.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.730	0.342	24.588	0.236	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	46	65	44	39	0	0	46
N.S.	1	1.02	1.00	1.41	0.96	0.85	0.00	0.00	1.00
time (sec)	N/A	0.370	0.040	1.159	0.195	0.243	0.000	0.000	0.046

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	46	65	44	39	0	0	46
N.S.	1	1.02	1.00	1.41	0.96	0.85	0.00	0.00	1.00
time (sec)	N/A	0.374	0.002	0.970	0.189	0.259	0.000	0.000	0.002

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	70	49	0	0	0	0
N.S.	1	1.00	1.00	1.37	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.074	1.260	0.194	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	70	49	0	0	0	0
N.S.	1	1.00	1.00	1.37	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	0.002	1.167	0.190	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	0	0	0	0	0
N.S.	1	1.00	0.97	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.017	1.792	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	35	0	33	0	0	0
N.S.	1	1.00	0.97	1.06	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.007	1.730	0.000	0.304	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	35	0	33	0	0	0
N.S.	1	1.00	0.97	1.06	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.008	28.952	0.000	0.271	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	0	37	0	32	0	0	0
N.S.	1	1.00	0.00	1.12	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.220	0.000	2.758	0.000	0.255	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	48	35	58	35	56	0	33
N.S.	1	1.00	1.33	0.97	1.61	0.97	1.56	0.00	0.92
time (sec)	N/A	0.553	0.633	1.053	0.206	0.254	102.172	0.000	4.929

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	52	42	0	0	35
N.S.	1	1.00	1.00	1.00	1.44	1.17	0.00	0.00	0.97
time (sec)	N/A	0.573	0.481	1.396	0.216	0.254	0.000	0.000	4.717

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	51	53	211	87	0	0	81
N.S.	1	1.00	0.98	1.02	4.06	1.67	0.00	0.00	1.56
time (sec)	N/A	2.015	0.723	1.770	0.230	0.253	0.000	0.000	4.782

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	165	135	0	0	0	0	0	0
N.S.	1	1.22	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.674	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	115	100	0	0	0	0	0	0
N.S.	1	1.15	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.006	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	0	0	0	0	0
N.S.	1	1.00	1.00	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.006	0.570	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	20	22	17	21	21
N.S.	1	1.00	1.11	1.00	1.05	1.16	0.89	1.11	1.11
time (sec)	N/A	0.283	0.032	0.042	0.209	0.249	2.654	0.284	4.762

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	300	433	223	0	376	0	0	0	0
N.S.	1	1.44	0.74	0.00	1.25	0.00	0.00	0.00	0.00
time (sec)	N/A	1.003	0.446	0.000	0.219	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	364	192	0	296	0	0	0	0
N.S.	1	1.41	0.74	0.00	1.15	0.00	0.00	0.00	0.00
time (sec)	N/A	0.891	0.272	0.000	0.213	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	296	160	0	222	0	0	0	0
N.S.	1	1.13	0.61	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.674	0.226	0.000	0.210	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	162	119	0	141	0	0	0	0
N.S.	1	1.23	0.90	0.00	1.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.027	0.000	0.210	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	8	8	0	0	9
N.S.	1	1.00	1.00	0.91	0.73	0.73	0.00	0.00	0.82
time (sec)	N/A	0.195	0.021	0.159	0.175	0.268	0.000	0.000	4.899

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	110	115	0	113	0	0	0	0
N.S.	1	0.99	1.04	0.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	0.101	0.000	0.229	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	191	200	185	0	162	0	0	0	0
N.S.	1	1.05	0.97	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.652	0.208	0.000	0.321	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	294	246	0	188	0	0	0	0
N.S.	1	1.20	1.00	0.00	0.77	0.00	0.00	0.00	0.00
time (sec)	N/A	0.794	0.178	0.000	0.324	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	363	277	0	214	0	0	0	0
N.S.	1	1.26	0.97	0.00	0.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.933	0.190	0.000	0.329	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	423	408	252	0	0	0	0	0	0
N.S.	1	0.96	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.977	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	211	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.831	0.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	196	149	0	0	0	0	0	0
N.S.	1	1.17	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	0	0	18
N.S.	1	1.00	1.00	0.95	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.350	0.038	0.236	0.000	0.000	0.000	0.000	4.936

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	154	150	0	0	0	0	0	0
N.S.	1	0.99	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	0.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	243	238	0	0	0	0	0	0
N.S.	1	0.91	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.952	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	344	301	0	0	0	0	0	0
N.S.	1	1.01	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.450	0.235	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2995	3373	2610	0	0	0	0	0	0
N.S.	1	1.13	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.422	9.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2252	2358	1996	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.770	5.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1653	1697	1546	0	0	0	0	0	0
N.S.	1	1.03	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.011	3.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	28	39	26	28	29
N.S.	1	1.00	1.07	1.00	1.04	1.44	0.96	1.04	1.07
time (sec)	N/A	0.209	0.377	0.108	0.296	0.276	126.623	0.306	8.408

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2498	2413	2247	0	0	0	0	0	0
N.S.	1	0.97	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.472	7.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3119	2903	2700	0	0	0	0	0	0
N.S.	1	0.93	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.489	13.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3733	3741	3341	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.343	16.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	661	742	425	0	415	0	0	0	0
N.S.	1	1.12	0.64	0.00	0.63	0.00	0.00	0.00	0.00
time (sec)	N/A	1.256	0.570	0.000	0.199	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	546	616	362	0	345	0	0	0	0
N.S.	1	1.13	0.66	0.00	0.63	0.00	0.00	0.00	0.00
time (sec)	N/A	0.979	0.470	0.000	0.210	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	390	485	285	0	258	0	0	0	0
N.S.	1	1.24	0.73	0.00	0.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.816	0.379	0.000	0.200	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	177	137	0	0	0	0	0	0
N.S.	1	1.16	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.017	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	125	135	0	0	0	0	0	0
N.S.	1	0.95	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.739	0.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	331	387	285	0	213	0	0	0	0
N.S.	1	1.17	0.86	0.00	0.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.817	0.952	0.000	0.236	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	460	495	389	0	287	0	0	0	0
N.S.	1	1.08	0.85	0.00	0.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.948	1.128	0.000	0.250	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	584	638	505	0	341	0	0	0	0
N.S.	1	1.09	0.86	0.00	0.58	0.00	0.00	0.00	0.00
time (sec)	N/A	1.134	1.247	0.000	0.250	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	900	981	583	0	518	0	0	0	0
N.S.	1	1.09	0.65	0.00	0.58	0.00	0.00	0.00	0.00
time (sec)	N/A	1.455	0.983	0.000	0.220	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	645	715	472	0	412	0	0	0	0
N.S.	1	1.11	0.73	0.00	0.64	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	0.802	0.000	0.206	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	497	298	0	0	0	0	0	0
N.S.	1	1.24	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.149	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	257	280	0	0	0	0	0	0
N.S.	1	1.18	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.973	0.649	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	343	399	0	0	0	0	0	0	0
N.S.	1	1.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.192	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	515	555	488	0	319	0	0	0	0
N.S.	1	1.08	0.95	0.00	0.62	0.00	0.00	0.00	0.00
time (sec)	N/A	1.107	1.193	0.000	0.254	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	767	809	621	0	403	0	0	0	0
N.S.	1	1.05	0.81	0.00	0.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.404	1.501	0.000	0.255	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [133] had the largest ratio of [1.11111000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.12	9	0.556
2	A	5	5	1.13	9	0.556
3	A	5	5	1.15	9	0.556
4	A	5	5	1.18	7	0.714
5	A	5	4	1.10	5	0.800
6	A	1	1	1.00	9	0.111
7	A	6	6	1.00	9	0.667
8	A	5	5	0.98	9	0.556
9	A	5	5	1.01	9	0.556
10	A	5	5	1.01	9	0.556
11	A	6	6	1.17	9	0.667
12	A	6	6	1.19	9	0.667
13	A	6	6	1.22	7	0.857
14	A	6	5	1.15	5	1.000
15	A	1	1	1.00	9	0.111
16	A	7	7	1.00	9	0.778
17	A	6	6	1.06	9	0.667
18	A	6	6	1.08	9	0.667
19	A	7	6	1.14	11	0.545
20	A	7	6	1.16	11	0.545
21	A	6	5	1.02	9	0.556
22	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	7	1.04	11	0.636
24	A	7	6	1.02	11	0.545
25	A	7	6	1.04	11	0.545
26	A	5	5	1.12	11	0.455
27	A	5	5	1.14	11	0.455
28	A	5	5	1.20	7	0.714
29	A	4	4	1.07	11	0.364
30	A	5	5	1.09	11	0.455
31	A	6	6	1.08	11	0.545
32	A	8	7	1.17	11	0.636
33	A	8	7	1.19	11	0.636
34	A	7	6	1.02	9	0.667
35	A	1	1	1.00	11	0.091
36	A	9	8	1.03	11	0.727
37	A	8	7	1.08	11	0.636
38	A	8	7	1.09	11	0.636
39	A	6	6	1.16	11	0.545
40	A	6	6	1.18	11	0.545
41	A	6	6	1.20	7	0.857
42	A	5	5	1.11	11	0.455
43	A	6	6	1.14	11	0.545
44	A	7	7	1.12	11	0.636
45	A	4	4	1.04	11	0.364
46	A	4	4	1.04	9	0.444
47	A	4	4	1.00	7	0.571
48	A	1	1	1.00	11	0.091
49	A	4	4	1.01	11	0.364
50	A	4	4	1.04	11	0.364
51	A	4	4	1.04	11	0.364
52	A	5	5	1.07	11	0.455
53	A	5	5	1.07	9	0.556
54	A	5	5	0.97	7	0.714
55	A	1	1	1.00	11	0.091
56	A	5	5	1.01	11	0.455

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
57	A	5	5	1.06	11	0.455
58	A	5	5	1.06	11	0.455
59	A	9	8	1.18	13	0.615
60	A	8	7	1.16	13	0.538
61	A	7	6	1.12	13	0.462
62	A	6	5	1.04	13	0.385
63	A	7	6	1.10	13	0.462
64	A	8	7	1.11	13	0.538
65	A	11	10	1.19	13	0.769
66	A	10	9	1.19	13	0.692
67	A	9	8	1.17	13	0.615
68	A	8	7	1.13	13	0.538
69	A	7	6	1.07	13	0.462
70	A	8	7	1.13	13	0.538
71	A	9	8	1.14	13	0.615
72	A	11	10	1.20	15	0.667
73	A	11	10	1.19	15	0.667
74	A	10	9	1.19	15	0.600
75	A	10	9	1.14	15	0.600
76	A	9	8	1.13	15	0.533
77	A	11	10	1.14	15	0.667
78	A	13	12	1.23	15	0.800
79	A	12	11	1.20	15	0.733
80	A	12	11	1.20	15	0.733
81	A	11	10	1.19	15	0.667
82	A	11	10	1.14	15	0.667
83	A	10	9	1.14	15	0.600
84	A	12	11	1.16	15	0.733
85	A	11	10	1.17	15	0.667
86	A	5	5	1.03	15	0.333
87	A	5	5	1.03	15	0.333
88	A	5	5	1.01	15	0.333
89	A	5	5	1.02	15	0.333
90	A	5	5	1.04	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	6	1.05	15	0.400
92	A	6	6	1.05	15	0.400
93	A	6	6	1.02	15	0.400
94	A	6	6	1.03	15	0.400
95	A	6	6	1.05	15	0.400
96	N/A	3	0	1.00	7	0.000
97	N/A	2	0	1.00	7	0.000
98	N/A	1	0	1.00	7	0.000
99	N/A	2	0	1.00	7	0.000
100	N/A	3	0	1.00	7	0.000
101	A	1	1	1.00	15	0.067
102	A	4	4	1.09	11	0.364
103	A	5	5	1.12	11	0.455
104	A	6	6	1.18	11	0.545
105	A	5	5	1.07	13	0.385
106	A	6	6	1.12	13	0.462
107	A	7	7	1.15	13	0.538
108	A	5	5	1.07	13	0.385
109	A	6	6	1.12	13	0.462
110	A	7	7	1.15	13	0.538
111	A	5	5	1.05	13	0.385
112	A	6	6	1.06	13	0.462
113	A	7	7	1.10	13	0.538
114	N/A	1	0	1.00	7	0.000
115	N/A	1	0	1.00	5	0.000
116	A	1	1	1.00	9	0.111
117	N/A	1	0	1.00	9	0.000
118	N/A	1	0	1.00	9	0.000
119	N/A	1	0	1.00	9	0.000
120	N/A	1	0	1.00	7	0.000
121	A	1	1	1.00	11	0.091
122	N/A	1	0	1.00	11	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	N/A	1	0	1.00	11	0.000
124	A	3	3	1.02	13	0.231
125	A	3	3	1.03	11	0.273
126	A	9	8	1.12	9	0.889
127	A	4	3	1.14	13	0.231
128	A	3	3	0.98	13	0.231
129	A	3	3	0.90	13	0.231
130	A	3	3	0.94	13	0.231
131	A	2	2	0.98	13	0.154
132	A	2	2	0.99	11	0.182
133	A	11	10	1.11	9	1.111
134	N/A	2	0	1.00	13	0.000
135	A	2	2	1.07	13	0.154
136	A	2	2	0.96	13	0.154
137	A	3	3	0.96	17	0.176
138	A	3	3	0.97	17	0.176
139	A	3	3	1.00	15	0.200
140	A	9	8	1.12	9	0.889
141	A	4	3	1.26	17	0.176
142	A	3	3	0.99	17	0.176
143	A	3	3	0.88	17	0.176
144	A	3	3	0.91	17	0.176
145	A	6	5	1.02	9	0.556
146	A	7	6	1.02	12	0.500
147	A	2	2	1.00	12	0.167
148	A	3	3	1.00	15	0.200
149	A	1	1	1.00	34	0.029
150	A	1	1	1.00	34	0.029
151	A	1	1	1.00	34	0.029
152	A	2	2	1.00	37	0.054
153	A	2	2	1.00	53	0.038
154	A	2	2	1.00	53	0.038
155	A	5	4	1.00	76	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	6	5	1.22	19	0.263
157	A	5	4	1.15	19	0.211
158	A	4	3	1.00	17	0.176
159	A	3	2	1.00	15	0.133
160	N/A	2	0	1.00	19	0.000
161	A	5	5	1.44	16	0.312
162	A	5	5	1.41	16	0.312
163	A	4	4	1.13	14	0.286
164	A	9	8	1.23	13	0.615
165	A	1	1	1.00	16	0.062
166	A	4	4	0.99	16	0.250
167	A	5	5	1.05	16	0.312
168	A	5	5	1.20	16	0.312
169	A	5	5	1.26	16	0.312
170	A	8	7	0.96	20	0.350
171	A	8	7	1.00	18	0.389
172	A	9	8	1.17	17	0.471
173	A	3	3	1.00	20	0.150
174	A	8	7	0.99	20	0.350
175	A	11	10	0.91	20	0.500
176	A	15	14	1.01	20	0.700
177	A	2	2	1.13	27	0.074
178	A	2	2	1.05	25	0.080
179	A	5	5	1.03	24	0.208
180	N/A	1	0	1.00	27	0.000
181	A	2	2	0.97	27	0.074
182	A	2	2	0.93	27	0.074
183	A	2	2	1.00	27	0.074
184	A	2	2	1.12	21	0.095
185	A	2	2	1.13	19	0.105
186	A	2	2	1.24	18	0.111
187	A	12	11	1.16	21	0.524
188	A	7	7	0.95	21	0.333

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
189	A	2	2	1.17	21	0.095
190	A	2	2	1.08	21	0.095
191	A	2	2	1.09	21	0.095
192	A	2	2	1.09	24	0.083
193	A	2	2	1.11	23	0.087
194	A	5	5	1.24	26	0.192
195	A	4	4	1.18	26	0.154
196	A	4	4	1.16	26	0.154
197	A	2	2	1.08	26	0.077
198	A	2	2	1.05	26	0.077

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^4 \operatorname{PolyLog}(2, ax)\, dx$

3.1.1 Optimal result

Integrand size = 9, antiderivative size = 86

$$\int x^4 \operatorname{PolyLog}(2, ax)\, dx = -\frac{x}{25a^4} - \frac{x^2}{50a^3} - \frac{x^3}{75a^2} - \frac{x^4}{100a} - \frac{x^5}{125} - \frac{\log(1-ax)}{25a^5} + \frac{1}{25}x^5\log(1-ax) + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax)$$

output
```
-1/25*x/a^4-1/50*x^2/a^3-1/75*x^3/a^2-1/100*x^4/a-1/125*x^5-1/25*ln(-a*x+1
)/a^5+1/25*x^5*ln(-a*x+1)+1/5*x^5*polylog(2,a*x)
```

3.1.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{PolyLog}(2, ax)\, dx = \frac{-ax(60 + 30ax + 20a^2x^2 + 15a^3x^3 + 12a^4x^4) + 60(-1 + a^5x^5)\log(1-ax) + 300a^5x^5 \operatorname{PolyLog}(2, ax)}{1500a^5}$$

input
```
Integrate[x^4*PolyLog[2, a*x],x]
```

output
```
(-(a*x*(60 + 30*a*x + 20*a^2*x^2 + 15*a^3*x^3 + 12*a^4*x^4)) + 60*(-1 + a^
5*x^5)*Log[1 - a*x] + 300*a^5*x^5*PolyLog[2, a*x])/(1500*a^5)
```

3.1.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(2,ax)-\frac{1}{5}\int -x^4\log(1-ax)dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}\int x^4\log(1-ax)dx+\frac{1}{5}x^5 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{5}\left(\frac{1}{5}a\int \frac{x^5}{1-ax}dx+\frac{1}{5}x^5\log(1-ax)\right)+\frac{1}{5}x^5 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{5}\left(\frac{1}{5}a\int \left(-\frac{x^4}{a}-\frac{x^3}{a^2}-\frac{x^2}{a^3}-\frac{x}{a^4}-\frac{1}{a^5(ax-1)}-\frac{1}{a^5}\right)dx+\frac{1}{5}x^5\log(1-ax)\right)+\frac{1}{5}x^5 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}\left(\frac{1}{5}a\left(-\frac{\log(1-ax)}{a^6}-\frac{x}{a^5}-\frac{x^2}{2a^4}-\frac{x^3}{3a^3}-\frac{x^4}{4a^2}-\frac{x^5}{5a}\right)+\frac{1}{5}x^5\log(1-ax)\right)+\frac{1}{5}x^5 \operatorname{PolyLog}(2,ax)$$

input
```
Int[x^4*PolyLog[2, a*x],x]
```

output
```
((x^5*Log[1 - a*x])/5 + (a*(-(x/a^5) - x^2/(2*a^4) - x^3/(3*a^3) - x^4/(4*
a^2) - x^5/(5*a) - Log[1 - a*x]/a^6))/5)/5 + (x^5*PolyLog[2, a*x])/5
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

method	result
meijerg	$\frac{-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{1500}-\frac{\left(-6a^5x^5+6\right)\ln(-ax+1)}{150}+\frac{a^5x^5\,\mathrm{polylog}(2,ax)}{5}}{a^5}$
parallelrisch	$\frac{300a^5x^5\,\mathrm{polylog}(2,ax)+60\ln(-ax+1)x^5a^5-12a^5x^5-15a^4x^4-60-20a^3x^3-30a^2x^2-60ax-60\ln(-ax+1)}{1500a^5}$
parts	$\frac{x^5\,\mathrm{polylog}(2,ax)}{5}-\frac{\frac{(-ax+1)^5\ln(-ax+1)}{5}-\frac{(-ax+1)^5}{25}-\ln(-ax+1)(-ax+1)^4+\frac{(-ax+1)^4}{4}+2\ln(-ax+1)(-ax+1)^3-\frac{2(-}{}}{5a^5}$
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{polylog}(2,ax)}{5}-\frac{(-ax+1)^5\ln(-ax+1)}{25}+\frac{(-ax+1)^5}{125}+\frac{\ln(-ax+1)(-ax+1)^4}{5}-\frac{(-ax+1)^4}{20}-\frac{2\ln(-ax+1)(-ax+1)^3}{5}+\frac{2(-ax+1)^3}{15}}{a^5}$
default	$\frac{\frac{a^5x^5\,\mathrm{polylog}(2,ax)}{5}-\frac{(-ax+1)^5\ln(-ax+1)}{25}+\frac{(-ax+1)^5}{125}+\frac{\ln(-ax+1)(-ax+1)^4}{5}-\frac{(-ax+1)^4}{20}-\frac{2\ln(-ax+1)(-ax+1)^3}{5}+\frac{2(-ax+1)^3}{15}}{a^5}$

input
```
int(x^4*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-1/1500*a*x*(12*a^4*x^4+15*a^3*x^3+20*a^2*x^2+30*a*x+60)-1/150*(-6*
a^5*x^5+6)*ln(-a*x+1)+1/5*a^5*x^5*polylog(2,a*x))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx = \frac{300\,a^5x^5\operatorname{Li}_2(ax) - 12\,a^5x^5 - 15\,a^4x^4 - 20\,a^3x^3 - 30\,a^2x^2 - 60\,ax + 60\,(a^5x^5 - 1)\log(-ax+1)}{1500\,a^5}$$

input
```
integrate(x^4*polylog(2,a*x),x, algorithm="fricas")
```

output
```
1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30
*a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

3.1.6 Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx = \begin{cases} -\frac{x^5\operatorname{Li}_1(ax)}{25} + \frac{x^5\operatorname{Li}_2(ax)}{5} - \frac{x^5}{125} - \frac{x^4}{100a} - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} + \frac{\operatorname{Li}_1(ax)}{25a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*polylog(2,a*x),x)
```

output
```
Piecewise((-x**5*polylog(1, a*x)/25 + x**5*polylog(2, a*x)/5 - x**5/125 -
x**4/(100*a) - x**3/(75*a**2) - x**2/(50*a**3) - x/(25*a**4) + polylog(1,
a*x)/(25*a**5), Ne(a, 0)), (0, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx = \frac{300\,a^5x^5\operatorname{Li}_2(ax) - 12\,a^5x^5 - 15\,a^4x^4 - 20\,a^3x^3 - 30\,a^2x^2 - 60\,ax + 60\,(a^5x^5-1)\log(-ax+1)}{1500\,a^5}$$

input
```
integrate(x^4*polylog(2,a*x),x, algorithm="maxima")
```

output
```
1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30
*a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

3.1.8 Giac [F]

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx = \int x^4\operatorname{Li}_2(ax)\;dx$$

input
```
integrate(x^4*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(x^4*dilog(a*x), x)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int x^4 \operatorname{PolyLog}(2,ax)\,dx = \frac{x^5\,\ln(1-a\,x)}{25} - \frac{\ln(a\,x-1)}{25\,a^5} - \frac{x}{25\,a^4} - \frac{x^5}{125} + \frac{x^5\operatorname{polylog}(2,a\,x)}{5} - \frac{x^4}{100\,a} - \frac{x^3}{75\,a^2} - \frac{x^2}{50\,a^3}$$

input
```
int(x^4*polylog(2, a*x),x)
```

output
```
(x^5*log(1 - a*x))/25 - log(a*x - 1)/(25*a^5) - x/(25*a^4) - x^5/125 + (x^
5*polylog(2, a*x))/5 - x^4/(100*a) - x^3/(75*a^2) - x^2/(50*a^3)
```

3.2 $\int x^3 \operatorname{PolyLog}(2,ax)\,dx$

3.2.1 Optimal result

Integrand size = 9, antiderivative size = 76

$$\int x^3 \operatorname{PolyLog}(2,ax)\,dx = -\frac{x}{16a^3} - \frac{x^2}{32a^2} - \frac{x^3}{48a} - \frac{x^4}{64} - \frac{\log(1-ax)}{16a^4} + \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)$$

output
```
-1/16*x/a^3-1/32*x^2/a^2-1/48*x^3/a-1/64*x^4-1/16*ln(-a*x+1)/a^4+1/16*x^4*
ln(-a*x+1)+1/4*x^4*polylog(2,a*x)
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int x^3 \operatorname{PolyLog}(2,ax)\,dx = \frac{-ax(12+6ax+4a^2x^2+3a^3x^3)+12(-1+a^4x^4)\log(1-ax)+48a^4x^4 \operatorname{PolyLog}(2,ax)}{192a^4}$$

input
```
Integrate[x^3*PolyLog[2, a*x],x]
```

output
```
(-(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3)) + 12*(-1 + a^4*x^4)*Log[1 - a
*x] + 48*a^4*x^4*PolyLog[2, a*x])/(192*a^4)
```

3.2.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{PolyLog}(2,ax)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)-\frac{1}{4}\int -x^3\log(1-ax)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int x^3\log(1-ax)dx+\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{4}\left(\frac{1}{4}a\int \frac{x^4}{1-ax}dx+\frac{1}{4}x^4\log(1-ax)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\left(\frac{1}{4}a\int \left(-\frac{x^3}{a}-\frac{x^2}{a^2}-\frac{x}{a^3}-\frac{1}{a^4(ax-1)}-\frac{1}{a^4}\right)dx+\frac{1}{4}x^4\log(1-ax)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{4}a\left(-\frac{\log(1-ax)}{a^5}-\frac{x}{a^4}-\frac{x^2}{2a^3}-\frac{x^3}{3a^2}-\frac{x^4}{4a}\right)+\frac{1}{4}x^4\log(1-ax)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax)$$

input
```
Int[x^3*PolyLog[2, a*x],x]
```

output
```
((x^4*Log[1 - a*x])/4 + (a*(-(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4/(4*
a) - Log[1 - a*x]/a^5))/4)/4 + (x^4*PolyLog[2, a*x])/4
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result
meijerg	$-\frac{\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{960}+\frac{\left(-5a^4x^4+5\right)\ln(-ax+1)}{80}-\frac{a^4x^4\operatorname{polylog}(2,ax)}{4}}{a^4}$
parallelrisch	$\frac{48a^4x^4\operatorname{polylog}(2,ax)+12\ln(-ax+1)a^4x^4-3a^4x^4-12-4a^3x^3-6a^2x^2-12ax-12\ln(-ax+1)}{192a^4}$
parts	$\frac{x^4\operatorname{polylog}(2,ax)}{4}+\frac{\frac{\ln(-ax+1)(-ax+1)^4}{4}-\frac{(-ax+1)^4}{16}-\ln(-ax+1)(-ax+1)^3+\frac{(-ax+1)^3}{3}+\frac{3\ln(-ax+1)(-ax+1)^2}{2}-\frac{3(-ax}{4}}{4a^4}$
derivativedivides	$\frac{\frac{a^4x^4\operatorname{polylog}(2,ax)}{4}+\frac{\ln(-ax+1)(-ax+1)^4}{16}-\frac{(-ax+1)^4}{64}-\frac{\ln(-ax+1)(-ax+1)^3}{4}+\frac{(-ax+1)^3}{12}+\frac{3\ln(-ax+1)(-ax+1)^2}{8}-\frac{3(-ax+1)^2}{16}}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{polylog}(2,ax)}{4}+\frac{\ln(-ax+1)(-ax+1)^4}{16}-\frac{(-ax+1)^4}{64}-\frac{\ln(-ax+1)(-ax+1)^3}{4}+\frac{(-ax+1)^3}{12}+\frac{3\ln(-ax+1)(-ax+1)^2}{8}-\frac{3(-ax+1)^2}{16}}{a^4}$

input
```
int(x^3*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*(1/960*a*x*(15*a^3*x^3+20*a^2*x^2+30*a*x+60)+1/80*(-5*a^4*x^4+5)*ln
(-a*x+1)-1/4*a^4*x^4*polylog(2,a*x))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{PolyLog}(2,ax)\,dx = \frac{48\,a^4x^4\operatorname{Li}_2(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4 - 1)\log\left(-ax + 1\right)}{192\,a^4}$$

input
```
integrate(x^3*polylog(2,a*x),x, algorithm="fricas")
```

output
```
1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x
+ 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

3.2.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int x^3 \operatorname{PolyLog}(2,ax)\,dx = \begin{cases} -\frac{x^4\operatorname{Li}_1(ax)}{16} + \frac{x^4\operatorname{Li}_2(ax)}{4} - \frac{x^4}{64} - \frac{x^3}{48a} - \frac{x^2}{32a^2} - \frac{x}{16a^3} + \frac{\operatorname{Li}_1(ax)}{16a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*polylog(2,a*x),x)
```

output
```
Piecewise((-x**4*polylog(1, a*x)/16 + x**4*polylog(2, a*x)/4 - x**4/64 - x
**3/(48*a) - x**2/(32*a**2) - x/(16*a**3) + polylog(1, a*x)/(16*a**4), Ne(
a, 0)), (0, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{PolyLog}(2, ax)\, dx = \frac{48\, a^4 x^4 \operatorname{Li}_2(ax) - 3\, a^4 x^4 - 4\, a^3 x^3 - 6\, a^2 x^2 - 12\, ax + 12\, (a^4 x^4 - 1) \log(-ax + 1)}{192\, a^4}$$

input
```
integrate(x^3*polylog(2,a*x),x, algorithm="maxima")
```

output
```
1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x
+ 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

3.2.8 Giac [F]

$$\int x^3 \operatorname{PolyLog}(2, ax)\, dx = \int x^3 \operatorname{Li}_2(ax)\, dx$$

input
```
integrate(x^3*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(x^3*dilog(a*x), x)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int x^3 \operatorname{PolyLog}(2, ax)\, dx = \frac{x^4 \ln(1 - a\, x)}{16} - \frac{\ln(a\, x - 1)}{16\, a^4} - \frac{x}{16\, a^3} - \frac{x^4}{64} + \frac{x^4 \operatorname{polylog}(2, a\, x)}{4} - \frac{x^3}{48\, a} - \frac{x^2}{32\, a^2}$$

input
```
int(x^3*polylog(2, a*x),x)
```

output
```
(x^4*log(1 - a*x))/16 - log(a*x - 1)/(16*a^4) - x/(16*a^3) - x^4/64 + (x^4
*polylog(2, a*x))/4 - x^3/(48*a) - x^2/(32*a^2)
```

3.3 $\int x^2 \operatorname{PolyLog}(2, ax)\,dx$

3.3.1 Optimal result

Integrand size = 9, antiderivative size = 66

$$\int x^2 \operatorname{PolyLog}(2, ax)\,dx = -\frac{x}{9a^2} - \frac{x^2}{18a} - \frac{x^3}{27} - \frac{\log(1-ax)}{9a^3} + \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3 \operatorname{PolyLog}(2, ax)$$

output
```
-1/9*x/a^2-1/18*x^2/a-1/27*x^3-1/9*ln(-a*x+1)/a^3+1/9*x^3*ln(-a*x+1)+1/3*x
^3*polylog(2,a*x)
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int x^2 \operatorname{PolyLog}(2, ax)\,dx = \frac{-ax(6+3ax+2a^2x^2) + 6(-1+a^3x^3)\log(1-ax) + 18a^3x^3 \operatorname{PolyLog}(2, ax)}{54a^3}$$

input
```
Integrate[x^2*PolyLog[2, a*x],x]
```

output
```
(-(a*x*(6 + 3*a*x + 2*a^2*x^2)) + 6*(-1 + a^3*x^3)*Log[1 - a*x] + 18*a^3*x
^3*PolyLog[2, a*x])/(54*a^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(2, ax)\, dx$$

$$\Big\downarrow \; 7145$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(2, ax) - \frac{1}{3}\int -x^2 \log(1-ax)dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\int x^2 \log(1-ax)dx + \frac{1}{3}x^3 \operatorname{PolyLog}(2, ax)$$

$$\Big\downarrow \; 2842$$

$$\frac{1}{3}\left(\frac{1}{3}a\int \frac{x^3}{1-ax}dx + \frac{1}{3}x^3 \log(1-ax)\right) + \frac{1}{3}x^3 \operatorname{PolyLog}(2, ax)$$

$$\Big\downarrow \; 49$$

$$\frac{1}{3}\left(\frac{1}{3}a\int \left(-\frac{x^2}{a} - \frac{x}{a^2} - \frac{1}{a^3(ax-1)} - \frac{1}{a^3}\right)dx + \frac{1}{3}x^3 \log(1-ax)\right) + \frac{1}{3}x^3 \operatorname{PolyLog}(2, ax)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{1}{3}a\left(-\frac{\log(1-ax)}{a^4} - \frac{x}{a^3} - \frac{x^2}{2a^2} - \frac{x^3}{3a}\right) + \frac{1}{3}x^3 \log(1-ax)\right) + \frac{1}{3}x^3 \operatorname{PolyLog}(2, ax)$$

input
```
Int[x^2*PolyLog[2, a*x],x]
```

output
```
((x^3*Log[1 - a*x])/3 + (a*(-(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1 - a
*x]/a^4))/3)/3 + (x^3*PolyLog[2, a*x])/3
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

method	result
meijerg	$\frac{-\frac{a x \left(4 a^{2} x^{2}+6 a x +12\right)}{108}-\frac{\left(-4 a^{3} x^{3}+4\right) \ln \left(-a x +1\right)}{36}+\frac{a^{3} x^{3} \operatorname{polylog}\left(2, a x \right)}{3}}{a^{3}}$
parallelrisch	$\frac{18 a^{3} x^{3} \operatorname{polylog}\left(2, a x \right)+6 \ln \left(-a x +1\right) x^{3} a^{3}-2 a^{3} x^{3}-6-3 a^{2} x^{2}-6 a x -6 \ln \left(-a x +1\right)}{54 a^{3}}$
parts	$\frac{x^{3} \operatorname{polylog}\left(2, a x \right)}{3}-\frac{\frac{\ln \left(-a x +1\right) \left(-a x +1\right)^{3}}{3}-\frac{\left(-a x +1\right)^{3}}{9}-\ln \left(-a x +1\right) \left(-a x +1\right)^{2}+\frac{\left(-a x +1\right)^{2}}{2}+\ln \left(-a x +1\right) \left(-a x +1\right)+a x -1}{3 a^{3}}$
derivativedivides	$\frac{\frac{a^{3} x^{3} \operatorname{polylog}\left(2, a x \right)}{3}-\frac{\ln \left(-a x +1\right) \left(-a x +1\right)^{3}}{9}+\frac{\left(-a x +1\right)^{3}}{27}+\frac{\ln \left(-a x +1\right) \left(-a x +1\right)^{2}}{3}-\frac{\left(-a x +1\right)^{2}}{6}-\frac{\ln \left(-a x +1\right) \left(-a x +1\right)}{3}+\frac{1}{3}-\frac{a x}{3}}{a^{3}}$
default	$\frac{\frac{a^{3} x^{3} \operatorname{polylog}\left(2, a x \right)}{3}-\frac{\ln \left(-a x +1\right) \left(-a x +1\right)^{3}}{9}+\frac{\left(-a x +1\right)^{3}}{27}+\frac{\ln \left(-a x +1\right) \left(-a x +1\right)^{2}}{3}-\frac{\left(-a x +1\right)^{2}}{6}-\frac{\ln \left(-a x +1\right) \left(-a x +1\right)}{3}+\frac{1}{3}-\frac{a x}{3}}{a^{3}}$

input
```
int(x^2*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/108*a*x*(4*a^2*x^2+6*a*x+12)-1/36*(-4*a^3*x^3+4)*ln(-a*x+1)+1/3*
a^3*x^3*polylog(2,a*x))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2 \operatorname{PolyLog}(2,ax)\,dx = \frac{18\,a^3x^3\mathrm{Li}_2(ax) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\,(a^3x^3 - 1)\log(-ax+1)}{54\,a^3}$$

input
```
integrate(x^2*polylog(2,a*x),x, algorithm="fricas")
```

output
```
1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
 1)*log(-a*x + 1))/a^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int x^2 \operatorname{PolyLog}(2,ax)\,dx = \begin{cases} -\frac{x^3\,\mathrm{Li}_1(ax)}{9} + \frac{x^3\,\mathrm{Li}_2(ax)}{3} - \frac{x^3}{27} - \frac{x^2}{18a} - \frac{x}{9a^2} + \frac{\mathrm{Li}_1(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*polylog(2,a*x),x)
```

output
```
Piecewise((-x**3*polylog(1, a*x)/9 + x**3*polylog(2, a*x)/3 - x**3/27 - x*
*2/(18*a) - x/(9*a**2) + polylog(1, a*x)/(9*a**3), Ne(a, 0)), (0, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2 \operatorname{PolyLog}(2, ax) \, dx$$
$$= \frac{18 a^3 x^3 \operatorname{Li}_2(ax) - 2 a^3 x^3 - 3 a^2 x^2 - 6 ax + 6 (a^3 x^3 - 1) \log(-ax + 1)}{54 a^3}$$

input
```
integrate(x^2*polylog(2,a*x),x, algorithm="maxima")
```

output
```
1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
 1)*log(-a*x + 1))/a^3
```

3.3.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(2, ax) \, dx = \int x^2 \operatorname{Li}_2(ax) \, dx$$

input
```
integrate(x^2*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*dilog(a*x), x)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int x^2 \operatorname{PolyLog}(2, ax) \, dx = \frac{x^3 \ln(1 - a x)}{9} - \frac{\ln(a x - 1)}{9 a^3} - \frac{x}{9 a^2} - \frac{x^3}{27} + \frac{x^3 \operatorname{polylog}(2, a x)}{3} - \frac{x^2}{18 a}$$

input
```
int(x^2*polylog(2, a*x),x)
```

output
```
(x^3*log(1 - a*x))/9 - log(a*x - 1)/(9*a^3) - x/(9*a^2) - x^3/27 + (x^3*po
lylog(2, a*x))/3 - x^2/(18*a)
```

3.4 $\int x \operatorname{PolyLog}(2, ax)\, dx$

3.4.1 Optimal result

Integrand size = 7, antiderivative size = 56

$$\int x \operatorname{PolyLog}(2, ax)\, dx = -\frac{x}{4a} - \frac{x^2}{8} - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2\log(1-ax) + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax)$$

output
```
-1/4*x/a-1/8*x^2-1/4*ln(-a*x+1)/a^2+1/4*x^2*ln(-a*x+1)+1/2*x^2*polylog(2,a
*x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x \operatorname{PolyLog}(2, ax)\, dx = \frac{-ax(2+ax) + 2(-1+a^2x^2)\log(1-ax) + 4a^2x^2 \operatorname{PolyLog}(2, ax)}{8a^2}$$

input
```
Integrate[x*PolyLog[2, a*x],x]
```

output
```
(-(a*x*(2 + a*x)) + 2*(-1 + a^2*x^2)*Log[1 - a*x] + 4*a^2*x^2*PolyLog[2, a
*x])/(8*a^2)
```

3.4.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(2,ax)\,dx$$

$$\Big\downarrow \; 7145$$

$$\frac{1}{2}x^2\operatorname{PolyLog}(2,ax)-\frac{1}{2}\int -x\log(1-ax)dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\int x\log(1-ax)dx+\frac{1}{2}x^2\operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \; 2842$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{x^2}{1-ax}dx+\frac{1}{2}x^2\log(1-ax)\right)+\frac{1}{2}x^2\operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\left(\frac{1}{2}a\int\left(-\frac{x}{a}-\frac{1}{a^2(ax-1)}-\frac{1}{a^2}\right)dx+\frac{1}{2}x^2\log(1-ax)\right)+\frac{1}{2}x^2\operatorname{PolyLog}(2,ax)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{1}{2}a\left(-\frac{\log(1-ax)}{a^3}-\frac{x}{a^2}-\frac{x^2}{2a}\right)+\frac{1}{2}x^2\log(1-ax)\right)+\frac{1}{2}x^2\operatorname{PolyLog}(2,ax)$$

input
```
Int[x*PolyLog[2, a*x],x]
```

output
```
((x^2*Log[1 - a*x])/2 + (a*(-(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3))/2)/2
 + (x^2*PolyLog[2, a*x])/2
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
meijerg	$-\frac{\frac{ax(3ax+6)}{24}+\frac{\left(-3a^2x^2+3\right)\ln(-ax+1)}{12}-\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}}{a^2}$	49
parallelrisch	$\frac{4a^2x^2\,\mathrm{polylog}(2,ax)+2\ln(-ax+1)a^2x^2-a^2x^2-2ax-2\ln(-ax+1)}{8a^2}$	56
parts	$\frac{x^2\,\mathrm{polylog}(2,ax)}{2}+\frac{\frac{\ln(-ax+1)(-ax+1)^2}{2}-\frac{(-ax+1)^2}{4}-\ln(-ax+1)(-ax+1)-ax+1}{2a^2}$	65
derivativedivides	$\frac{\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}+\frac{\ln(-ax+1)(-ax+1)^2}{4}-\frac{(-ax+1)^2}{8}-\frac{\ln(-ax+1)(-ax+1)}{2}+\frac{1}{2}-\frac{ax}{2}}{a^2}$	66
default	$\frac{\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}+\frac{\ln(-ax+1)(-ax+1)^2}{4}-\frac{(-ax+1)^2}{8}-\frac{\ln(-ax+1)(-ax+1)}{2}+\frac{1}{2}-\frac{ax}{2}}{a^2}$	66

input
```
int(x*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(1/24*a*x*(3*a*x+6)+1/12*(-3*a^2*x^2+3)*ln(-a*x+1)-1/2*a^2*x^2*poly
log(2,a*x))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x \,\text{PolyLog}(2, ax)\, dx = \frac{4\, a^2 x^2 \text{Li}_2(ax) - a^2 x^2 - 2\, ax + 2\, (a^2 x^2 - 1) \log(-ax + 1)}{8\, a^2}$$

input
```
integrate(x*polylog(2,a*x),x, algorithm="fricas")
```

output
```
1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1
))/a^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int x \,\text{PolyLog}(2, ax)\, dx = \begin{cases} -\frac{x^2 \text{Li}_1(ax)}{4} + \frac{x^2 \text{Li}_2(ax)}{2} - \frac{x^2}{8} - \frac{x}{4a} + \frac{\text{Li}_1(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*polylog(2,a*x),x)
```

output
```
Piecewise((-x**2*polylog(1, a*x)/4 + x**2*polylog(2, a*x)/2 - x**2/8 - x/(
4*a) + polylog(1, a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x \,\text{PolyLog}(2, ax)\, dx = \frac{4\, a^2 x^2 \text{Li}_2(ax) - a^2 x^2 - 2\, ax + 2\, (a^2 x^2 - 1) \log(-ax + 1)}{8\, a^2}$$

input
```
integrate(x*polylog(2,a*x),x, algorithm="maxima")
```

output
```
1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1
))/a^2
```

3.4.8 Giac [F]

$$\int x \operatorname{PolyLog}(2, ax) \, dx = \int x \operatorname{Li}_2(ax) \, dx$$

input
```
integrate(x*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(x*dilog(a*x), x)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x \operatorname{PolyLog}(2, ax) \, dx = \frac{x^2 \ln(1 - a\,x)}{4} - \frac{\ln(1 - a\,x)}{4\,a^2} - \frac{x}{4\,a} - \frac{x^2}{8} + \frac{x^2 \operatorname{polylog}(2, a\,x)}{2}$$

input
```
int(x*polylog(2, a*x),x)
```

output
```
(x^2*log(1 - a*x))/4 - log(1 - a*x)/(4*a^2) - x/(4*a) - x^2/8 + (x^2*polyl
og(2, a*x))/2
```

3.5 $\int \text{PolyLog}(2, ax)\,dx$

3.5.1 Optimal result

Integrand size = 5, antiderivative size = 29

$$\int \text{PolyLog}(2, ax)\,dx = -x - \frac{(1-ax)\log(1-ax)}{a} + x\,\text{PolyLog}(2, ax)$$

output
```
-x-(-a*x+1)*ln(-a*x+1)/a+x*polylog(2,a*x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \text{PolyLog}(2, ax)\,dx = -x + \left(-\frac{1}{a} + x\right)\log(1-ax) + x\,\text{PolyLog}(2, ax)$$

input
```
Integrate[PolyLog[2, a*x],x]
```

output
```
-x + (-a^(-1) + x)*Log[1 - a*x] + x*PolyLog[2, a*x]
```

3.5.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {7140, 25, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2, ax)\, dx$$

$$\Big\downarrow \; 7140$$

$$x\,\text{PolyLog}(2, ax) - \int -\log(1-ax)dx$$

$$\Big\downarrow \; 25$$

$$\int \log(1-ax)dx + x\,\text{PolyLog}(2, ax)$$

$$\Big\downarrow \; 2836$$

$$x\,\text{PolyLog}(2, ax) - \frac{\int \log(1-ax)d(1-ax)}{a}$$

$$\Big\downarrow \; 2732$$

$$x\,\text{PolyLog}(2, ax) - \frac{ax + (1-ax)\log(1-ax) - 1}{a}$$

input
```
Int[PolyLog[2, a*x],x]
```

output
```
-((-1 + a*x + (1 - a*x)*Log[1 - a*x])/a) + x*PolyLog[2, a*x]
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

method	result	size
meijerg	$\frac{-ax-\frac{(-2ax+2)\ln(-ax+1)}{2}+ax\,\text{polylog}(2,ax)}{a}$	33
parts	$x\,\text{polylog}(2,ax)-\frac{\ln(-ax+1)(-ax+1)+ax-1}{a}$	33
derivativedivides	$\frac{ax\,\text{polylog}(2,ax)-\ln(-ax+1)(-ax+1)+1-ax}{a}$	34
default	$\frac{ax\,\text{polylog}(2,ax)-\ln(-ax+1)(-ax+1)+1-ax}{a}$	34
parallelrisch	$\frac{-1+ax\,\text{polylog}(2,ax)+a\ln(-ax+1)x-ax-\ln(-ax+1)}{a}$	38

input
```
int(polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-a*x-1/2*(-2*a*x+2)*ln(-a*x+1)+a*x*polylog(2,a*x))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}(2,ax)\,dx=\frac{ax\text{Li}_2(ax)-ax+(ax-1)\log(-ax+1)}{a}$$

input
```
integrate(polylog(2,a*x),x, algorithm="fricas")
```

output
```
(a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \text{PolyLog}(2,ax)\,dx = \begin{cases} -x\,\text{Li}_1(ax) + x\,\text{Li}_2(ax) - x + \frac{\text{Li}_1(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(polylog(2,a*x),x)
```

output
```
Piecewise((-x*polylog(1, a*x) + x*polylog(2, a*x) - x + polylog(1, a*x)/a,
 Ne(a, 0)), (0, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}(2,ax)\,dx = \frac{ax\text{Li}_2(ax) - ax + (ax-1)\log(-ax+1)}{a}$$

input
```
integrate(polylog(2,a*x),x, algorithm="maxima")
```

output
```
(a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

3.5.8 Giac [F]

$$\int \text{PolyLog}(2,ax)\,dx = \int \text{Li}_2(ax)\,dx$$

input
```
integrate(polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(dilog(a*x), x)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \mathrm{PolyLog}(2,ax)\,dx = x\,\mathrm{polylog}(2,a\,x) - \frac{\ln(1-a\,x)}{a} - x + x\,\ln(1-a\,x)$$

input
```
int(polylog(2, a*x),x)
```

output
```
x*polylog(2, a*x) - log(1 - a*x)/a - x + x*log(1 - a*x)
```

3.6 $\int \frac{\text{PolyLog}(2,ax)}{x} dx$

3.6.1 Optimal result

Integrand size = 9, antiderivative size = 5

$$\int \frac{\text{PolyLog}(2,ax)}{x} dx = \text{PolyLog}(3,ax)$$

output
```
polylog(3,a*x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax)}{x} dx = \text{PolyLog}(3,ax)$$

input
```
Integrate[PolyLog[2, a*x]/x,x]
```

output
```
PolyLog[3, a*x]
```

3.6.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{x} \, dx$$

$$\Big\downarrow \text{7143}$$

$$\text{PolyLog}(3,ax)$$

input
```
Int[PolyLog[2, a*x]/x,x]
```

output
```
PolyLog[3, a*x]
```

3.6.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.6.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\text{polylog}(3, ax)$	6
default	$\text{polylog}(3, ax)$	6
meijerg	$\text{polylog}(3, ax)$	6
parts	$\text{polylog}(3, ax)$	6

input
```
int(polylog(2,a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(3,a*x)
```

3.6.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, ax)}{x} \, dx = \int \frac{\text{Li}_2(ax)}{x} \, dx$$

input
```
integrate(polylog(2,a*x)/x,x, algorithm="fricas")
```

output
```
integral(dilog(a*x)/x, x)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{\text{PolyLog}(2, ax)}{x} \, dx = \text{Li}_3(ax)$$

input
```
integrate(polylog(2,a*x)/x,x)
```

output
```
polylog(3, a*x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2, ax)}{x} \, dx = \text{Li}_3(ax)$$

input
```
integrate(polylog(2,a*x)/x,x, algorithm="maxima")
```

output
```
polylog(3, a*x)
```

3.6.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{x}\,dx = \int \frac{\text{Li}_2(ax)}{x}\,dx$$

input
```
integrate(polylog(2,a*x)/x,x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/x, x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax)}{x}\,dx = \text{polylog}(3,a\,x)$$

input
```
int(polylog(2, a*x)/x,x)
```

output
```
polylog(3, a*x)
```

3.7 $\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx$

3.7.1 Optimal result

Integrand size = 9, antiderivative size = 36

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{PolyLog}(2,ax)}{x}$$

output
```
a*ln(x)-a*ln(-a*x+1)+ln(-a*x+1)/x-polylog(2,a*x)/x
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{PolyLog}(2,ax)}{x}$$

input
```
Integrate[PolyLog[2, a*x]/x^2,x]
```

output
```
a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

3.7.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 25, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx$$

$$\Big\downarrow \text{7145}$$

$$\int -\frac{\log(1-ax)}{x^2}dx-\frac{\text{PolyLog}(2,ax)}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\log(1-ax)}{x^2}dx-\frac{\text{PolyLog}(2,ax)}{x}$$

$$\Big\downarrow \text{2842}$$

$$a\int \frac{1}{x(1-ax)}dx-\frac{\text{PolyLog}(2,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$a\left(a\int \frac{1}{1-ax}dx+\int \frac{1}{x}dx\right)-\frac{\text{PolyLog}(2,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a\left(a\int \frac{1}{1-ax}dx+\log(x)\right)-\frac{\text{PolyLog}(2,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\text{PolyLog}(2,ax)}{x}+a(\log(x)-\log(1-ax))+\frac{\log(1-ax)}{x}$$

input
```
Int[PolyLog[2, a*x]/x^2,x]
```

output
```
a*(Log[x] - Log[1 - a*x]) + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

3.7.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

method	result	size
parts	$-\frac{\text{polylog}(2,ax)}{x}+a\left(\ln(-ax)+\frac{\ln(-ax+1)(-ax+1)}{ax}\right)$	40
derivativedivides	$a\left(-\frac{\text{polylog}(2,ax)}{ax}+\ln(-ax)+\frac{\ln(-ax+1)(-ax+1)}{ax}\right)$	42
default	$a\left(-\frac{\text{polylog}(2,ax)}{ax}+\ln(-ax)+\frac{\ln(-ax+1)(-ax+1)}{ax}\right)$	42
meijerg	$a\left(\frac{(-4ax+4)\ln(-ax+1)}{4ax}-\frac{\text{polylog}(2,ax)}{ax}+\ln(x)+\ln(-a)\right)$	44
parallelrisch	$\frac{a^2\ln(x)x-a^2\ln(-ax+1)x-a\,\text{polylog}(2,ax)+a\ln(-ax+1)}{xa}$	46

input
```
int(polylog(2,a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-polylog(2,a*x)/x+a*(ln(-a*x)+ln(-a*x+1)*(-a*x+1)/a/x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = -\frac{ax\log(ax-1)-ax\log(x)+\text{Li}_2(ax)-\log(-ax+1)}{x}$$

input
```
integrate(polylog(2,a*x)/x^2,x, algorithm="fricas")
```

output
```
-(a*x*log(a*x - 1) - a*x*log(x) + dilog(a*x) - log(-a*x + 1))/x
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = a\log(x)+a\,\text{Li}_1(ax)-\frac{\text{Li}_1(ax)}{x}-\frac{\text{Li}_2(ax)}{x}$$

input
```
integrate(polylog(2,a*x)/x**2,x)
```

output
```
a*log(x) + a*polylog(1, a*x) - polylog(1, a*x)/x - polylog(2, a*x)/x
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = a\log(x) - \frac{(ax-1)\log(-ax+1) + \text{Li}_2(ax)}{x}$$

input
```
integrate(polylog(2,a*x)/x^2,x, algorithm="maxima")
```

output
```
a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x))/x
```

3.7.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = \int \frac{\text{Li}_2(ax)}{x^2}\,dx$$

input
```
integrate(polylog(2,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/x^2, x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}\,dx = \frac{\ln(1-a\,x) - \text{polylog}(2,a\,x)}{x} + a\ \ln(x) - a\ \ln(1-a\,x)$$

input
```
int(polylog(2, a*x)/x^2,x)
```

output
```
(log(1 - a*x) - polylog(2, a*x))/x + a*log(x) - a*log(1 - a*x)
```

3.8 $\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx$

3.8.1 Optimal result

Integrand size = 9, antiderivative size = 58

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = -\frac{a}{4x}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

output
```
-1/4*a/x+1/4*a^2*ln(x)-1/4*a^2*ln(-a*x+1)+1/4*ln(-a*x+1)/x^2-1/2*polylog(2
,a*x)/x^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = \frac{-ax+a^2x^2\log(x)+\log(1-ax)-a^2x^2\log(1-ax)-2\,\text{PolyLog}(2,ax)}{4x^2}$$

input
```
Integrate[PolyLog[2, a*x]/x^3,x]
```

output
```
(-(a*x) + a^2*x^2*Log[x] + Log[1 - a*x] - a^2*x^2*Log[1 - a*x] - 2*PolyLog
[2, a*x])/(4*x^2)
```

3.8.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{x^3} \, dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{2}\int -\frac{\log(1-ax)}{x^3}dx-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \frac{\log(1-ax)}{x^3}dx-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

$$\Big\downarrow \ 2842$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{x^2(1-ax)}dx+\frac{\log(1-ax)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \left(-\frac{a^2}{ax-1}+\frac{a}{x}+\frac{1}{x^2}\right)dx+\frac{\log(1-ax)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\log(1-ax)}{2x^2}+\frac{1}{2}a\left(a\log(x)-a\log(1-ax)-\frac{1}{x}\right)\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

input
```
Int[PolyLog[2, a*x]/x^3,x]
```

output
```
(Log[1 - a*x]/(2*x^2) + (a*(-x^(-1) + a*Log[x] - a*Log[1 - a*x]))/2)/2 - P
olyLog[2, a*x]/(2*x^2)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
parallelrisch	$\frac{a^2\ln(x)x^2-\ln(-ax+1)a^2x^2-a^2x^2-ax-2\,\mathrm{polylog}(2,ax)+\ln(-ax+1)}{4x^2}$	57
parts	$-\frac{\mathrm{polylog}(2,ax)}{2x^2}-\frac{a^2\left(-\frac{\ln(-ax)}{2}+\frac{1}{2ax}+\frac{\ln(-ax+1)(-ax+1)(-ax-1)}{2a^2x^2}\right)}{2}$	60
derivativedivides	$a^2\left(-\frac{\mathrm{polylog}(2,ax)}{2a^2x^2}+\frac{\ln(-ax)}{4}-\frac{1}{4ax}-\frac{\ln(-ax+1)(-ax+1)(-ax-1)}{4a^2x^2}\right)$	61
default	$a^2\left(-\frac{\mathrm{polylog}(2,ax)}{2a^2x^2}+\frac{\ln(-ax)}{4}-\frac{1}{4ax}-\frac{\ln(-ax+1)(-ax+1)(-ax-1)}{4a^2x^2}\right)$	61
meijerg	$-a^2\left(-\frac{9ax+27}{36ax}-\frac{(-9a^2x^2+9)\ln(-ax+1)}{36a^2x^2}+\frac{\mathrm{polylog}(2,ax)}{2a^2x^2}+\frac{1}{4}-\frac{\ln(x)}{4}-\frac{\ln(-a)}{4}+\frac{1}{ax}\right)$	77

input
```
int(polylog(2,a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2*ln(x)*x^2-ln(-a*x+1)*a^2*x^2-a^2*x^2-a*x-2*polylog(2,a*x)+ln(-a*x
+1))/x^2
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = -\frac{a^2x^2\log(ax-1) - a^2x^2\log(x) + ax + 2\,\text{Li}_2(ax) - \log(-ax+1)}{4\,x^2}$$

input
```
integrate(polylog(2,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*dilog(a*x) - log(-a*
x + 1))/x^2
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = \frac{a^2\log(x)}{4} + \frac{a^2\,\text{Li}_1(ax)}{4} - \frac{a}{4x} - \frac{\text{Li}_1(ax)}{4x^2} - \frac{\text{Li}_2(ax)}{2x^2}$$

input
```
integrate(polylog(2,a*x)/x**3,x)
```

output
```
a**2*log(x)/4 + a**2*polylog(1, a*x)/4 - a/(4*x) - polylog(1, a*x)/(4*x**2
) - polylog(2, a*x)/(2*x**2)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = \frac{1}{4}\,a^2\log(x) - \frac{ax + (a^2x^2-1)\log(-ax+1) + 2\,\text{Li}_2(ax)}{4\,x^2}$$

input
```
integrate(polylog(2,a*x)/x^3,x, algorithm="maxima")
```

output
```
1/4*a^2*log(x) - 1/4*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x))/x^
2
```

3.8.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = \int \frac{\text{Li}_2(ax)}{x^3}\,dx$$

input
```
integrate(polylog(2,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/x^3, x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{\text{PolyLog}(2,ax)}{x^3}\,dx = \frac{a^2\,\ln(x)}{2} + \frac{\ln(1-a\,x)}{4\,x^2} - \frac{a^2\,\ln(a\,x^2-x)}{4} - \frac{a}{4\,x} - \frac{\text{polylog}(2,a\,x)}{2\,x^2}$$

input
```
int(polylog(2, a*x)/x^3,x)
```

output
```
(a^2*log(x))/2 + log(1 - a*x)/(4*x^2) - (a^2*log(a*x^2 - x))/4 - a/(4*x) -
 polylog(2, a*x)/(2*x^2)
```

3.9 $\int \frac{\text{PolyLog}(2,ax)}{x^4}\,dx$

3.9.1 Optimal result

Integrand size = 9, antiderivative size = 68

$$\int \frac{\text{PolyLog}(2,ax)}{x^4}\,dx = -\frac{a}{18x^2} - \frac{a^2}{9x} + \frac{1}{9}a^3\log(x) - \frac{1}{9}a^3\log(1-ax) + \frac{\log(1-ax)}{9x^3} - \frac{\text{PolyLog}(2,ax)}{3x^3}$$

output
```
-1/18*a/x^2-1/9*a^2/x+1/9*a^3*ln(x)-1/9*a^3*ln(-a*x+1)+1/9*ln(-a*x+1)/x^3-
1/3*polylog(2,a*x)/x^3
```

3.9.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{\text{PolyLog}(2,ax)}{x^4}\,dx = -\frac{ax(1+2ax) - 2a^3x^3\log(x) + 2(-1+a^3x^3)\log(1-ax) + 6\,\text{PolyLog}(2,ax)}{18x^3}$$

input
```
Integrate[PolyLog[2, a*x]/x^4,x]
```

output
```
-1/18*(a*x*(1 + 2*a*x) - 2*a^3*x^3*Log[x] + 2*(-1 + a^3*x^3)*Log[1 - a*x]
+ 6*PolyLog[2, a*x])/x^3
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{x^4}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\int -\frac{\log(1-ax)}{x^4}dx-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{\log(1-ax)}{x^4}dx-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(\frac{1}{3}a\int \frac{1}{x^3(1-ax)}dx+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\left(\frac{1}{3}a\int \left(-\frac{a^3}{ax-1}+\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{3}a\left(a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}\right)+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

input
```
Int[PolyLog[2, a*x]/x^4,x]
```

output
```
(Log[1 - a*x]/(3*x^3) + (a*(-1/2*1/x^2 - a/x + a^2*Log[x] - a^2*Log[1 - a*
x]))/3)/3 - PolyLog[2, a*x]/(3*x^3)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{2a^3\ln(x)x^3-2\ln(-ax+1)x^3a^3-2a^3x^3-2a^2x^2-ax-6\,\mathrm{polylog}(2,ax)+2\ln(-ax+1)}{18x^3}$
parts	$-\frac{\mathrm{polylog}(2,ax)}{3x^3}+\frac{a^3\left(\frac{\ln(-ax)}{3}-\frac{1}{3ax}-\frac{1}{6a^2x^2}+\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^2+3ax\right)}{3a^3x^3}\right)}{3}$
derivativedivides	$a^3\left(-\frac{\mathrm{polylog}(2,ax)}{3a^3x^3}+\frac{\ln(-ax)}{9}-\frac{1}{9ax}-\frac{1}{18a^2x^2}+\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^2+3ax\right)}{9a^3x^3}\right)$
default	$a^3\left(-\frac{\mathrm{polylog}(2,ax)}{3a^3x^3}+\frac{\ln(-ax)}{9}-\frac{1}{9ax}-\frac{1}{18a^2x^2}+\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^2+3ax\right)}{9a^3x^3}\right)$
meijerg	$a^3\left(\frac{32a^2x^2+60ax+192}{432a^2x^2}+\frac{(-16a^3x^3+16)\ln(-ax+1)}{144a^3x^3}-\frac{\mathrm{polylog}(2,ax)}{3a^3x^3}-\frac{2}{27}+\frac{\ln(x)}{9}+\frac{\ln(-a)}{9}-\frac{1}{2a^2x^2}-\right.$

input
```
int(polylog(2,a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/18*(2*a^3*ln(x)*x^3-2*ln(-a*x+1)*x^3*a^3-2*a^3*x^3-2*a^2*x^2-a*x-6*polyl
og(2,a*x)+2*ln(-a*x+1))/x^3
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{\text{PolyLog}(2,ax)}{x^4}\,dx = -\frac{2\,a^3x^3\log(ax-1) - 2\,a^3x^3\log(x) + 2\,a^2x^2 + ax + 6\,\text{Li}_2(ax) - 2\,\log(-ax+1)}{18\,x^3}$$

input
```
integrate(polylog(2,a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/18*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dil
og(a*x) - 2*log(-a*x + 1))/x^3
```

3.9.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\text{PolyLog}(2,ax)}{x^4}\,dx = \frac{a^3\log(x)}{9} + \frac{a^3\,\text{Li}_1(ax)}{9} - \frac{a^2}{9x} - \frac{a}{18x^2} - \frac{\text{Li}_1(ax)}{9x^3} - \frac{\text{Li}_2(ax)}{3x^3}$$

input
```
integrate(polylog(2,a*x)/x**4,x)
```

output
```
a**3*log(x)/9 + a**3*polylog(1, a*x)/9 - a**2/(9*x) - a/(18*x**2) - polylo
g(1, a*x)/(9*x**3) - polylog(2, a*x)/(3*x**3)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\text{PolyLog}(2, ax)}{x^4} \, dx = \frac{1}{9} a^3 \log(x) - \frac{2 a^2 x^2 + ax + 2 (a^3 x^3 - 1) \log(-ax + 1) + 6 \operatorname{Li}_2(ax)}{18 x^3}$$

input
```
integrate(polylog(2,a*x)/x^4,x, algorithm="maxima")
```

output
```
1/9*a^3*log(x) - 1/18*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) + 6
*dilog(a*x))/x^3
```

3.9.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax)}{x^4} \, dx = \int \frac{\operatorname{Li}_2(ax)}{x^4} \, dx$$

input
```
integrate(polylog(2,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/x^4, x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{\text{PolyLog}(2, ax)}{x^4} \, dx = \frac{2 a^3 \ln(x)}{9} - \frac{\frac{a x}{18} - \frac{\ln(1 - a x)}{9} + \frac{\operatorname{polylog}(2, a x)}{3} + \frac{a^2 x^2}{9}}{x^3} - \frac{a^3 \ln(a x^2 - x)}{9}$$

input
```
int(polylog(2, a*x)/x^4,x)
```

output
```
(2*a^3*log(x))/9 - ((a*x)/18 - log(1 - a*x)/9 + polylog(2, a*x)/3 + (a^2*x
^2)/9)/x^3 - (a^3*log(a*x^2 - x))/9
```

3.10 $\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx$

3.10.1 Optimal result

Integrand size = 9, antiderivative size = 78

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = -\frac{a}{48x^3}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x) -\frac{1}{16}a^4\log(1-ax)+\frac{\log(1-ax)}{16x^4}-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

output
```
-1/48*a/x^3-1/32*a^2/x^2-1/16*a^3/x+1/16*a^4*ln(x)-1/16*a^4*ln(-a*x+1)+1/1
6*ln(-a*x+1)/x^4-1/4*polylog(2,a*x)/x^4
```

3.10.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = -\frac{ax(2+3ax+6a^2x^2)-6a^4x^4\log(x)+6(-1+a^4x^4)\log(1-ax)+24\,\text{PolyLog}(2,ax)}{96x^4}$$

input
```
Integrate[PolyLog[2, a*x]/x^5,x]
```

output
```
-1/96*(a*x*(2 + 3*a*x + 6*a^2*x^2) - 6*a^4*x^4*Log[x] + 6*(-1 + a^4*x^4)*L
og[1 - a*x] + 24*PolyLog[2, a*x])/x^4
```

3.10.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{x^5} \, dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{4}\int -\frac{\log(1-ax)}{x^5}dx-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{4}\int \frac{\log(1-ax)}{x^5}dx-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

$$\Big\downarrow \ 2842$$

$$\frac{1}{4}\left(\frac{1}{4}a\int \frac{1}{x^4(1-ax)}dx+\frac{\log(1-ax)}{4x^4}\right)-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{4}\left(\frac{1}{4}a\int \left(-\frac{a^4}{ax-1}+\frac{a^3}{x}+\frac{a^2}{x^2}+\frac{a}{x^3}+\frac{1}{x^4}\right)dx+\frac{\log(1-ax)}{4x^4}\right)-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{1}{4}a\left(a^3\log(x)-a^3\log(1-ax)-\frac{a^2}{x}-\frac{a}{2x^2}-\frac{1}{3x^3}\right)+\frac{\log(1-ax)}{4x^4}\right)-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

input
```
Int[PolyLog[2, a*x]/x^5,x]
```

output
```
(Log[1 - a*x]/(4*x^4) + (a*(-1/3*1/x^3 - a/(2*x^2) - a^2/x + a^3*Log[x] -
a^3*Log[1 - a*x]))/4)/4 - PolyLog[2, a*x]/(4*x^4)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.10.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{6\ln(x)x^4a^4-6\ln(-ax+1)a^4x^4-6a^4x^4-6a^3x^3-3a^2x^2-2ax-24\operatorname{polylog}(2,ax)+6\ln(-ax+1)}{96x^4}$
parts	$-\frac{\operatorname{polylog}(2,ax)}{4x^4}-\frac{a^4\left(\frac{1}{12a^3x^3}+\frac{1}{4ax}-\frac{\ln(-ax)}{4}+\frac{1}{8a^2x^2}+\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^3-4(-ax+1)^2+2-6ax\right)}{4a^4x^4}\right)}{4}$
derivativedivides	$a^4\left(-\frac{\operatorname{polylog}(2,ax)}{4a^4x^4}-\frac{1}{48a^3x^3}+\frac{\ln(-ax)}{16}-\frac{1}{16ax}-\frac{1}{32a^2x^2}-\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^3-4(-ax+1)^2\right.}{16a^4x^4}\right.$
default	$a^4\left(-\frac{\operatorname{polylog}(2,ax)}{4a^4x^4}-\frac{1}{48a^3x^3}+\frac{\ln(-ax)}{16}-\frac{1}{16ax}-\frac{1}{32a^2x^2}-\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^3-4(-ax+1)^2\right.}{16a^4x^4}\right.$
meijerg	$-a^4\left(-\frac{225a^3x^3+350a^2x^2+675ax+2250}{7200a^3x^3}-\frac{(-25a^4x^4+25)\ln(-ax+1)}{400a^4x^4}+\frac{\operatorname{polylog}(2,ax)}{4a^4x^4}+\frac{1}{32}-\frac{\ln(x)}{16}-\frac{\ln(-}{16}\right.$

input
```
int(polylog(2,a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/96*(6*ln(x)*x^4*a^4-6*ln(-a*x+1)*a^4*x^4-6*a^4*x^4-6*a^3*x^3-3*a^2*x^2-2
*a*x-24*polylog(2,a*x)+6*ln(-a*x+1))/x^4
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = -\frac{6\,a^4x^4\log{(ax-1)}-6\,a^4x^4\log{(x)}+6\,a^3x^3+3\,a^2x^2+2\,ax+24\,\text{Li}_2(ax)-6\,\log{(-ax+1)}}{96\,x^4}$$

input
```
integrate(polylog(2,a*x)/x^5,x, algorithm="fricas")
```

output
```
-1/96*(6*a^4*x^4*log(a*x - 1) - 6*a^4*x^4*log(x) + 6*a^3*x^3 + 3*a^2*x^2 +
 2*a*x + 24*dilog(a*x) - 6*log(-a*x + 1))/x^4
```

3.10.6 Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = \frac{a^4\log{(x)}}{16}+\frac{a^4\,\text{Li}_1\,(ax)}{16}-\frac{a^3}{16x}-\frac{a^2}{32x^2}-\frac{a}{48x^3}-\frac{\text{Li}_1\,(ax)}{16x^4}-\frac{\text{Li}_2\,(ax)}{4x^4}$$

input
```
integrate(polylog(2,a*x)/x**5,x)
```

output
```
a**4*log(x)/16 + a**4*polylog(1, a*x)/16 - a**3/(16*x) - a**2/(32*x**2) -
a/(48*x**3) - polylog(1, a*x)/(16*x**4) - polylog(2, a*x)/(4*x**4)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = \frac{1}{16}a^4\log(x) - \frac{6\,a^3x^3 + 3\,a^2x^2 + 2\,ax + 6\,(a^4x^4-1)\log(-ax+1) + 24\,\text{Li}_2(ax)}{96\,x^4}$$

input
```
integrate(polylog(2,a*x)/x^5,x, algorithm="maxima")
```

output
```
1/16*a^4*log(x) - 1/96*(6*a^3*x^3 + 3*a^2*x^2 + 2*a*x + 6*(a^4*x^4 - 1)*lo
g(-a*x + 1) + 24*dilog(a*x))/x^4
```

3.10.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = \int \frac{\text{Li}_2(ax)}{x^5}\,dx$$

input
```
integrate(polylog(2,a*x)/x^5,x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/x^5, x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx = \frac{\ln(1-a\,x)}{16\,x^4} - \frac{\text{polylog}(2,a\,x)}{4\,x^4} - \frac{a^3\,x^2 + \frac{a^2\,x}{2} + \frac{a}{3}}{16\,x^3} - \frac{a^4\,\text{atan}(a\,x\,2\mathrm{i} - \mathrm{i})\;1\mathrm{i}}{8}$$

input
```
int(polylog(2, a*x)/x^5,x)
```

output
```
log(1 - a*x)/(16*x^4) - (a^4*atan(a*x*2i - 1i)*1i)/8 - polylog(2, a*x)/(4*
x^4) - (a/3 + (a^2*x)/2 + a^3*x^2)/(16*x^3)
```

3.11 $\int x^3 \operatorname{PolyLog}(3, ax) \, dx$

3.11.1 Optimal result

Integrand size = 9, antiderivative size = 88

$$\int x^3 \operatorname{PolyLog}(3, ax) \, dx = \frac{x}{64a^3} + \frac{x^2}{128a^2} + \frac{x^3}{192a} + \frac{x^4}{256} + \frac{\log(1-ax)}{64a^4} - \frac{1}{64}x^4 \log(1-ax) - \frac{1}{16}x^4 \operatorname{PolyLog}(2, ax) + \frac{1}{4}x^4 \operatorname{PolyLog}(3, ax)$$

output
```
1/64*x/a^3+1/128*x^2/a^2+1/192*x^3/a+1/256*x^4+1/64*ln(-a*x+1)/a^4-1/64*x^
4*ln(-a*x+1)-1/16*x^4*polylog(2,a*x)+1/4*x^4*polylog(3,a*x)
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{PolyLog}(3, ax) \, dx = \frac{12ax + 6a^2x^2 + 4a^3x^3 + 3a^4x^4 + 12\log(1-ax) - 12a^4x^4 \log(1-ax) - 48a^4x^4 \operatorname{PolyLog}(2, ax) + 192a^4x}{768a^4}$$

input
```
Integrate[x^3*PolyLog[3, a*x],x]
```

output
```
(12*a*x + 6*a^2*x^2 + 4*a^3*x^3 + 3*a^4*x^4 + 12*Log[1 - a*x] - 12*a^4*x^4
*Log[1 - a*x] - 48*a^4*x^4*PolyLog[2, a*x] + 192*a^4*x^4*PolyLog[3, a*x])/
(768*a^4)
```

3.11.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \text{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{4}x^4 \text{PolyLog}(3,ax)-\frac{1}{4}\int x^3 \text{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{4}\left(\frac{1}{4}\int -x^3\log(1-ax)dx-\frac{1}{4}x^4 \text{PolyLog}(2,ax)\right)+\frac{1}{4}x^4 \text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(-\frac{1}{4}\int x^3\log(1-ax)dx-\frac{1}{4}x^4 \text{PolyLog}(2,ax)\right)+\frac{1}{4}x^4 \text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{4}a\int \frac{x^4}{1-ax}dx-\frac{1}{4}x^4\log(1-ax)\right)-\frac{1}{4}x^4 \text{PolyLog}(2,ax)\right)+\frac{1}{4}x^4 \text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{4}a\int \left(-\frac{x^3}{a}-\frac{x^2}{a^2}-\frac{x}{a^3}-\frac{1}{a^4(ax-1)}-\frac{1}{a^4}\right)dx-\frac{1}{4}x^4\log(1-ax)\right)-\frac{1}{4}x^4 \text{PolyLog}(2,ax)\right)+\frac{1}{4}x^4 \text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{4}a\left(-\frac{\log(1-ax)}{a^5}-\frac{x}{a^4}-\frac{x^2}{2a^3}-\frac{x^3}{3a^2}-\frac{x^4}{4a}\right)-\frac{1}{4}x^4\log(1-ax)\right)-\frac{1}{4}x^4 \text{PolyLog}(2,ax)\right)+\frac{1}{4}x^4 \text{PolyLog}(3,ax)$$

input
```
Int[x^3*PolyLog[3, a*x],x]
```

output
```
((-1/4*(x^4*Log[1 - a*x]) - (a*(-(x/a^4) - x^2/(2*a^3) - x^3/(3*a^2) - x^4
/(4*a) - Log[1 - a*x]/a^5))/4)/4 - (x^4*PolyLog[2, a*x])/4)/4 + (x^4*PolyL
og[3, a*x])/4
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
meijerg	$-\frac{-\frac{a x \left(15 a^{3} x^{3}+20 a^{2} x^{2}+30 a x +60\right)}{3840}-\frac{\left(-5 a^{4} x^{4}+5\right) \ln \left(-a x +1\right)}{320}+\frac{a^{4} x^{4} \operatorname{polylog}\left(2, a x \right)}{16}-\frac{a^{4} x^{4} \operatorname{polylog}\left(3, a x \right)}{4}}{a^{4}}$	78

input
```
int(x^3*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/a^4*(-1/3840*a*x*(15*a^3*x^3+20*a^2*x^2+30*a*x+60)-1/320*(-5*a^4*x^4+5)
*ln(-a*x+1)+1/16*a^4*x^4*polylog(2,a*x)-1/4*a^4*x^4*polylog(3,a*x))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{PolyLog}(3,ax)\,dx = -\frac{48\,a^4x^4\mathrm{Li}_2(ax) - 192\,a^4x^4\mathrm{polylog}(3,ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4 - 1)\log(-ax +}{768\,a^4}$$

input
```
integrate(x^3*polylog(3,a*x),x, algorithm="fricas")
```

output
```
-1/768*(48*a^4*x^4*dilog(a*x) - 192*a^4*x^4*polylog(3, a*x) - 3*a^4*x^4 -
4*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

3.11.6 Sympy [F]

$$\int x^3 \operatorname{PolyLog}(3,ax)\,dx = \int x^3 \operatorname{Li}_3(ax)\,dx$$

input
```
integrate(x**3*polylog(3,a*x),x)
```

output
```
Integral(x**3*polylog(3, a*x), x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{PolyLog}(3,ax)\,dx = -\frac{48\,a^4x^4\mathrm{Li}_2(ax) - 192\,a^4x^4\mathrm{Li}_3(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4 - 1)\log(-ax + 1)}{768\,a^4}$$

input
```
integrate(x^3*polylog(3,a*x),x, algorithm="maxima")
```

output
```
-1/768*(48*a^4*x^4*dilog(a*x) - 192*a^4*x^4*polylog(3, a*x) - 3*a^4*x^4 -
4*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

3.11.8 Giac [F]

$$\int x^3 \operatorname{PolyLog}(3, ax)\, dx = \int x^3 \operatorname{Li}_3(ax)\, dx$$

input
```
integrate(x^3*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate(x^3*polylog(3, a*x), x)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int x^3 \operatorname{PolyLog}(3, ax)\, dx = \frac{\ln(a\,x-1)}{64\,a^4} - \frac{x^4\,\ln(1-a\,x)}{64} + \frac{x}{64\,a^3} + \frac{x^4}{256} - \frac{x^4\,\mathrm{polylog}(2,a\,x)}{16} + \frac{x^4\,\mathrm{polylog}(3,a\,x)}{4} + \frac{x^3}{192\,a} + \frac{x^2}{128\,a^2}$$

input
```
int(x^3*polylog(3, a*x),x)
```

output
```
log(a*x - 1)/(64*a^4) - (x^4*log(1 - a*x))/64 + x/(64*a^3) + x^4/256 - (x^
4*polylog(2, a*x))/16 + (x^4*polylog(3, a*x))/4 + x^3/(192*a) + x^2/(128*a
^2)
```

3.12 $\int x^2 \operatorname{PolyLog}(3, ax)\, dx$

3.12.1 Optimal result

Integrand size = 9, antiderivative size = 78

$$\int x^2 \operatorname{PolyLog}(3, ax)\, dx = \frac{x}{27a^2} + \frac{x^2}{54a} + \frac{x^3}{81} + \frac{\log(1-ax)}{27a^3} - \frac{1}{27}x^3 \log(1-ax) - \frac{1}{9}x^3 \operatorname{PolyLog}(2, ax) + \frac{1}{3}x^3 \operatorname{PolyLog}(3, ax)$$

output
```
1/27*x/a^2+1/54*x^2/a+1/81*x^3+1/27*ln(-a*x+1)/a^3-1/27*x^3*ln(-a*x+1)-1/9
*x^3*polylog(2,a*x)+1/3*x^3*polylog(3,a*x)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int x^2 \operatorname{PolyLog}(3, ax)\, dx = \frac{6ax + 3a^2x^2 + 2a^3x^3 + 6\log(1-ax) - 6a^3x^3 \log(1-ax) - 18a^3x^3 \operatorname{PolyLog}(2, ax) + 54a^3x^3 \operatorname{PolyLog}(3, a}{162a^3}$$

input
```
Integrate[x^2*PolyLog[3, a*x],x]
```

output
```
(6*a*x + 3*a^2*x^2 + 2*a^3*x^3 + 6*Log[1 - a*x] - 6*a^3*x^3*Log[1 - a*x] -
 18*a^3*x^3*PolyLog[2, a*x] + 54*a^3*x^3*PolyLog[3, a*x])/(162*a^3)
```

3.12.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)-\frac{1}{3}\int x^2 \operatorname{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\left(\frac{1}{3}\int -x^2\log(1-ax)dx-\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int x^2\log(1-ax)dx-\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-\frac{1}{3}a\int \frac{x^3}{1-ax}dx-\frac{1}{3}x^3\log(1-ax)\right)-\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-\frac{1}{3}a\int \left(-\frac{x^2}{a}-\frac{x}{a^2}-\frac{1}{a^3(ax-1)}-\frac{1}{a^3}\right)dx-\frac{1}{3}x^3\log(1-ax)\right)-\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(-\frac{1}{3}a\left(-\frac{\log(1-ax)}{a^4}-\frac{x}{a^3}-\frac{x^2}{2a^2}-\frac{x^3}{3a}\right)-\frac{1}{3}x^3\log(1-ax)\right)-\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax)$$

input
```
Int[x^2*PolyLog[3, a*x],x]
```

output
```
((-1/3*(x^3*Log[1 - a*x]) - (a*(-(x/a^3) - x^2/(2*a^2) - x^3/(3*a) - Log[1
 - a*x]/a^4))/3)/3 - (x^3*PolyLog[2, a*x])/3)/3 + (x^3*PolyLog[3, a*x])/3
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{\frac{ax\left(4a^2x^2+6ax+12\right)}{324}+\frac{\left(-4a^3x^3+4\right)\ln(-ax+1)}{108}-\frac{a^3x^3\operatorname{polylog}(2,ax)}{9}+\frac{a^3x^3\operatorname{polylog}(3,ax)}{3}}{a^3}$	69

input
```
int(x^2*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/324*a*x*(4*a^2*x^2+6*a*x+12)+1/108*(-4*a^3*x^3+4)*ln(-a*x+1)-1/9*
a^3*x^3*polylog(2,a*x)+1/3*a^3*x^3*polylog(3,a*x))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int x^2 \operatorname{PolyLog}(3, ax)\, dx = -\frac{18\, a^3x^3\mathrm{Li}_2(ax) - 54\, a^3x^3\mathrm{polylog}(3, ax) - 2\, a^3x^3 - 3\, a^2x^2 - 6\, ax + 6\, (a^3x^3 - 1)\log\left(-ax + 1\right)}{162\, a^3}$$

input
```
integrate(x^2*polylog(3,a*x),x, algorithm="fricas")
```

output
```
-1/162*(18*a^3*x^3*dilog(a*x) - 54*a^3*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3
*a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(-a*x + 1))/a^3
```

3.12.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}(3, ax)\, dx = \int x^2 \mathrm{Li}_3\left(ax\right)\, dx$$

input
```
integrate(x**2*polylog(3,a*x),x)
```

output
```
Integral(x**2*polylog(3, a*x), x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int x^2 \operatorname{PolyLog}(3, ax)\, dx = -\frac{18\, a^3x^3\mathrm{Li}_2(ax) - 54\, a^3x^3\mathrm{Li}_3(ax) - 2\, a^3x^3 - 3\, a^2x^2 - 6\, ax + 6\, (a^3x^3 - 1)\log\left(-ax + 1\right)}{162\, a^3}$$

input
```
integrate(x^2*polylog(3,a*x),x, algorithm="maxima")
```

output
```
-1/162*(18*a^3*x^3*dilog(a*x) - 54*a^3*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3
*a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(-a*x + 1))/a^3
```

3.12.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(3, ax) \, dx = \int x^2 \operatorname{Li}_3(ax) \, dx$$

input
```
integrate(x^2*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate(x^2*polylog(3, a*x), x)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int x^2 \operatorname{PolyLog}(3, ax) \, dx = \frac{\ln (a\,x-1)}{27\,a^3} - \frac{x^3\,\ln (1-a\,x)}{27} + \frac{x}{27\,a^2} + \frac{x^3}{81} - \frac{x^3\,\operatorname{polylog}(2, a\,x)}{9} + \frac{x^3\,\operatorname{polylog}(3, a\,x)}{3} + \frac{x^2}{54\,a}$$

input
```
int(x^2*polylog(3, a*x),x)
```

output
```
log(a*x - 1)/(27*a^3) - (x^3*log(1 - a*x))/27 + x/(27*a^2) + x^3/81 - (x^3
*polylog(2, a*x))/9 + (x^3*polylog(3, a*x))/3 + x^2/(54*a)
```

3.13 $\int x \,\text{PolyLog}(3,ax)\,dx$

3.13.1 Optimal result

Integrand size = 7, antiderivative size = 68

$$\int x \,\text{PolyLog}(3,ax)\,dx = \frac{x}{8a}+\frac{x^2}{16}+\frac{\log(1-ax)}{8a^2}-\frac{1}{8}x^2\log(1-ax) -\frac{1}{4}x^2\,\text{PolyLog}(2,ax)+\frac{1}{2}x^2\,\text{PolyLog}(3,ax)$$

output
```
1/8*x/a+1/16*x^2+1/8*ln(-a*x+1)/a^2-1/8*x^2*ln(-a*x+1)-1/4*x^2*polylog(2,a
*x)+1/2*x^2*polylog(3,a*x)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int x \,\text{PolyLog}(3,ax)\,dx = \frac{2ax+a^2x^2+2\log(1-ax)-2a^2x^2\log(1-ax)-4a^2x^2\,\text{PolyLog}(2,ax)+8a^2x^2\,\text{PolyLog}(3,ax)}{16a^2}$$

input
```
Integrate[x*PolyLog[3, a*x],x]
```

output
```
(2*a*x + a^2*x^2 + 2*Log[1 - a*x] - 2*a^2*x^2*Log[1 - a*x] - 4*a^2*x^2*Pol
yLog[2, a*x] + 8*a^2*x^2*PolyLog[3, a*x])/(16*a^2)
```

3.13.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {7145, 7145, 25, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)-\frac{1}{2}\int x \operatorname{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -x\log(1-ax)dx-\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int x\log(1-ax)dx-\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}a\int \frac{x^2}{1-ax}dx-\frac{1}{2}x^2\log(1-ax)\right)-\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax)\right)+\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}a\int \left(-\frac{x}{a}-\frac{1}{a^2(ax-1)}-\frac{1}{a^2}\right)dx-\frac{1}{2}x^2\log(1-ax)\right)-\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax)\right)+ \frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}a\left(-\frac{\log(1-ax)}{a^3}-\frac{x}{a^2}-\frac{x^2}{2a}\right)-\frac{1}{2}x^2\log(1-ax)\right)-\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax)\right)+ \frac{1}{2}x^2 \operatorname{PolyLog}(3,ax)$$

input
```
Int[x*PolyLog[3, a*x],x]
```

output
```
((-1/2*(x^2*Log[1 - a*x]) - (a*(-(x/a^2) - x^2/(2*a) - Log[1 - a*x]/a^3))/
2)/2 - (x^2*PolyLog[2, a*x])/2)/2 + (x^2*PolyLog[3, a*x])/2
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result	size
meijerg	$-\frac{-\frac{ax(3ax+6)}{48}-\frac{\left(-3a^2x^2+3\right)\ln(-ax+1)}{24}+\frac{a^2x^2\operatorname{polylog}(2,ax)}{4}-\frac{a^2x^2\operatorname{polylog}(3,ax)}{2}}{a^2}$	62

input
```
int(x*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-1/48*a*x*(3*a*x+6)-1/24*(-3*a^2*x^2+3)*ln(-a*x+1)+1/4*a^2*x^2*pol
ylog(2,a*x)-1/2*a^2*x^2*polylog(3,a*x))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int x\,\text{PolyLog}(3,ax)\,dx = -\frac{4\,a^2x^2\text{Li}_2(ax) - 8\,a^2x^2\text{polylog}(3,ax) - a^2x^2 - 2\,ax + 2\,(a^2x^2-1)\log(-ax+1)}{16\,a^2}$$

input
```
integrate(x*polylog(3,a*x),x, algorithm="fricas")
```

output
```
-1/16*(4*a^2*x^2*dilog(a*x) - 8*a^2*x^2*polylog(3, a*x) - a^2*x^2 - 2*a*x
+ 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

3.13.6 Sympy [F]

$$\int x\,\text{PolyLog}(3,ax)\,dx = \int x\,\text{Li}_3\,(ax)\;dx$$

input
```
integrate(x*polylog(3,a*x),x)
```

output
```
Integral(x*polylog(3, a*x), x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int x\,\text{PolyLog}(3,ax)\,dx = -\frac{4\,a^2x^2\text{Li}_2(ax) - 8\,a^2x^2\text{Li}_3(ax) - a^2x^2 - 2\,ax + 2\,(a^2x^2-1)\log(-ax+1)}{16\,a^2}$$

input
```
integrate(x*polylog(3,a*x),x, algorithm="maxima")
```

output
```
-1/16*(4*a^2*x^2*dilog(a*x) - 8*a^2*x^2*polylog(3, a*x) - a^2*x^2 - 2*a*x
+ 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

3.13.8 Giac [F]

$$\int x\,\text{PolyLog}(3,ax)\,dx = \int x\text{Li}_3(ax)\,dx$$

input
```
integrate(x*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate(x*polylog(3, a*x), x)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int x\,\text{PolyLog}(3,ax)\,dx = \frac{\ln(a\,x-1)}{8\,a^2} - \frac{x^2\,\ln(1-a\,x)}{8} + \frac{x}{8\,a} + \frac{x^2}{16} - \frac{x^2\,\text{polylog}(2,a\,x)}{4} + \frac{x^2\,\text{polylog}(3,a\,x)}{2}$$

input
```
int(x*polylog(3, a*x),x)
```

output
```
log(a*x - 1)/(8*a^2) - (x^2*log(1 - a*x))/8 + x/(8*a) + x^2/16 - (x^2*poly
log(2, a*x))/4 + (x^2*polylog(3, a*x))/2
```

3.14 $\int \text{PolyLog}(3, ax)\, dx$

3.14.1 Optimal result

Integrand size = 5, antiderivative size = 34

$$\int \text{PolyLog}(3, ax)\, dx = x + \frac{(1-ax)\log(1-ax)}{a} - x\,\text{PolyLog}(2, ax) + x\,\text{PolyLog}(3, ax)$$

output
```
x+(-a*x+1)*ln(-a*x+1)/a-x*polylog(2,a*x)+x*polylog(3,a*x)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \text{PolyLog}(3, ax)\, dx = x\left(1-\log(1-ax)+\frac{\log(1-ax)}{ax}-\text{PolyLog}(2, ax)+\text{PolyLog}(3, ax)\right)$$

input
```
Integrate[PolyLog[3, a*x],x]
```

output
```
x*(1 - Log[1 - a*x] + Log[1 - a*x]/(a*x) - PolyLog[2, a*x] + PolyLog[3, a*
x])
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {7140, 7140, 25, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}(3,ax)-\int \text{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{7140}$$

$$\int -\log(1-ax)dx-x\,\text{PolyLog}(2,ax)+x\,\text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{25}$$

$$-\int \log(1-ax)dx-x\,\text{PolyLog}(2,ax)+x\,\text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log(1-ax)d(1-ax)}{a}-x\,\text{PolyLog}(2,ax)+x\,\text{PolyLog}(3,ax)$$

$$\Big\downarrow \text{2732}$$

$$-x\,\text{PolyLog}(2,ax)+x\,\text{PolyLog}(3,ax)+\frac{ax+(1-ax)\log(1-ax)-1}{a}$$

input
```
Int[PolyLog[3, a*x],x]
```

output
```
(-1 + a*x + (1 - a*x)*Log[1 - a*x])/a - x*PolyLog[2, a*x] + x*PolyLog[3, a
*x]
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
meijerg	$\frac{ax+\frac{(-2ax+2)\ln(-ax+1)}{2}-ax\,\mathrm{polylog}(2,ax)+ax\,\mathrm{polylog}(3,ax)}{a}$	41

input
```
int(polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x+1/2*(-2*a*x+2)*ln(-a*x+1)-a*x*polylog(2,a*x)+a*x*polylog(3,a*x))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \mathrm{PolyLog}(3,ax)\,dx = -\frac{ax\mathrm{Li}_2(ax) - ax\mathrm{polylog}(3,ax) - ax + (ax-1)\log(-ax+1)}{a}$$

input
```
integrate(polylog(3,a*x),x, algorithm="fricas")
```

output
```
-(a*x*dilog(a*x) - a*x*polylog(3, a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

3.14.6 Sympy [F]

$$\int \text{PolyLog}(3, ax)\, dx = \int \text{Li}_3\,(ax)\ dx$$

input
```
integrate(polylog(3,a*x),x)
```

output
```
Integral(polylog(3, a*x), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \text{PolyLog}(3, ax)\, dx = -\frac{ax\text{Li}_2(ax) - ax\text{Li}_3(ax) - ax + (ax - 1)\log\left(-ax + 1\right)}{a}$$

input
```
integrate(polylog(3,a*x),x, algorithm="maxima")
```

output
```
-(a*x*dilog(a*x) - a*x*polylog(3, a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

3.14.8 Giac [F]

$$\int \text{PolyLog}(3, ax)\, dx = \int \text{Li}_3(ax)\, dx$$

input
```
integrate(polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x), x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \text{PolyLog}(3, ax)\, dx = x + \frac{\ln(a\,x-1)}{a} - x\,\text{polylog}(2, a\,x) + x\,\text{polylog}(3, a\,x) - x\,\ln(1-a\,x)$$

input
```
int(polylog(3, a*x),x)
```

output
```
x + log(a*x - 1)/a - x*polylog(2, a*x) + x*polylog(3, a*x) - x*log(1 - a*x
)
```

3.15 $\int \frac{\text{PolyLog}(3,ax)}{x}\,dx$

3.15.1 Optimal result

Integrand size = 9, antiderivative size = 5

$$\int \frac{\text{PolyLog}(3,ax)}{x}\,dx = \text{PolyLog}(4,ax)$$

output
```
polylog(4,a*x)
```

3.15.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(3,ax)}{x}\,dx = \text{PolyLog}(4,ax)$$

input
```
Integrate[PolyLog[3, a*x]/x,x]
```

output
```
PolyLog[4, a*x]
```

3.15.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{x}\,dx$$

$$\Big\downarrow \text{ 7143}$$

$$\text{PolyLog}(4,ax)$$

input
```
Int[PolyLog[3, a*x]/x,x]
```

output
```
PolyLog[4, a*x]
```

3.15.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.15.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\text{polylog}(4, ax)$	6
default	$\text{polylog}(4, ax)$	6
meijerg	$\text{polylog}(4, ax)$	6

input
```
int(polylog(3,a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(4,a*x)
```

3.15.5 Fricas [F]

$$\int \frac{\mathrm{PolyLog}(3,ax)}{x}\,dx = \int \frac{\mathrm{Li}_3(ax)}{x}\,dx$$

input
```
integrate(polylog(3,a*x)/x,x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x)/x, x)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{\mathrm{PolyLog}(3,ax)}{x}\,dx = \mathrm{Li}_4\left(ax\right)$$

input
```
integrate(polylog(3,a*x)/x,x)
```

output
```
polylog(4, a*x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{PolyLog}(3,ax)}{x}\,dx = \mathrm{Li}_4(ax)$$

input
```
integrate(polylog(3,a*x)/x,x, algorithm="maxima")
```

output
```
polylog(4, a*x)
```

3.15.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax)}{x}\, dx = \int \frac{\text{Li}_3(ax)}{x}\, dx$$

input
```
integrate(polylog(3,a*x)/x,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/x, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax)}{x}\, dx = \int \frac{\text{polylog}(3, a\, x)}{x}\, dx$$

input
```
int(polylog(3, a*x)/x,x)
```

output
```
int(polylog(3, a*x)/x, x)
```

3.16 $\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx$

3.16.1 Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx = a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{PolyLog}(2,ax)}{x} - \frac{\text{PolyLog}(3,ax)}{x}$$

output
```
a*ln(x)-a*ln(-a*x+1)+ln(-a*x+1)/x-polylog(2,a*x)/x-polylog(3,a*x)/x
```

3.16.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx = -\frac{-ax\log(-ax) - \log(1-ax) + ax\log(1-ax) + \text{PolyLog}(2,ax) + \text{PolyLog}(3,ax)}{x}$$

input
```
Integrate[PolyLog[3, a*x]/x^2,x]
```

output
```
-((-(a*x*Log[-(a*x)]) - Log[1 - a*x] + a*x*Log[1 - a*x] + PolyLog[2, a*x]
+ PolyLog[3, a*x])/x)
```

3.16.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {7145, 7145, 25, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}dx$$

$$\Big\downarrow \text{7145}$$

$$\int \frac{\text{PolyLog}(2,ax)}{x^2}dx-\frac{\text{PolyLog}(3,ax)}{x}$$

$$\Big\downarrow \text{7145}$$

$$\int -\frac{\log(1-ax)}{x^2}dx-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\log(1-ax)}{x^2}dx-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}$$

$$\Big\downarrow \text{2842}$$

$$a\int \frac{1}{x(1-ax)}dx-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$a\left(a\int \frac{1}{1-ax}dx+\int \frac{1}{x}dx\right)-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$a\left(a\int \frac{1}{1-ax}dx+\log(x)\right)-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+\frac{\log(1-ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+a(\log(x)-\log(1-ax))+\frac{\log(1-ax)}{x}$$

input
```
Int[PolyLog[3, a*x]/x^2,x]
```

output
```
a*(Log[x] - Log[1 - a*x]) + Log[1 - a*x]/x - PolyLog[2, a*x]/x - PolyLog[3
, a*x]/x
```

3.16.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

method	result	size
meijerg	$a\left(\frac{(-8ax+8)\ln(-ax+1)}{8ax}-\frac{\mathrm{polylog}(2,ax)}{ax}-\frac{\mathrm{polylog}(3,ax)}{ax}+\ln(x)+\ln(-a)\right)$	57

input
```
int(polylog(3,a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/8/a/x*(-8*a*x+8)*ln(-a*x+1)-1/a/x*polylog(2,a*x)-1/a/x*polylog(3,a*x)
+ln(x)+ln(-a))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\mathrm{PolyLog}(3,ax)}{x^2}\,dx = -\frac{ax\log(ax-1)-ax\log(x)+\mathrm{Li}_2(ax)-\log(-ax+1)+\mathrm{polylog}(3,ax)}{x}$$

input
```
integrate(polylog(3,a*x)/x^2,x, algorithm="fricas")
```

output
```
-(a*x*log(a*x - 1) - a*x*log(x) + dilog(a*x) - log(-a*x + 1) + polylog(3,
a*x))/x
```

3.16.6 Sympy [F]

$$\int \frac{\mathrm{PolyLog}(3,ax)}{x^2}\,dx = \int \frac{\mathrm{Li}_3(ax)}{x^2}\,dx$$

input
```
integrate(polylog(3,a*x)/x**2,x)
```

output
```
Integral(polylog(3, a*x)/x**2, x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx = a\log(x) - \frac{(ax-1)\log(-ax+1) + \text{Li}_2(ax) + \text{Li}_3(ax)}{x}$$

input
```
integrate(polylog(3,a*x)/x^2,x, algorithm="maxima")
```

output
```
a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x) + polylog(3, a*x))/x
```

3.16.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx = \int \frac{\text{Li}_3(ax)}{x^2}\,dx$$

input
```
integrate(polylog(3,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/x^2, x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(3,ax)}{x^2}\,dx = 2\,a\,\text{atanh}(2\,a\,x-1) - \frac{\text{polylog}(2,a\,x) - \ln(1-a\,x) + \text{polylog}(3,a\,x)}{x}$$

input
```
int(polylog(3, a*x)/x^2,x)
```

output
```
2*a*atanh(2*a*x - 1) - (polylog(2, a*x) - log(1 - a*x) + polylog(3, a*x))/
x
```

3.17 $\int \frac{\text{PolyLog}(3,ax)}{x^3} \, dx$

3.17.1 Optimal result

Integrand size = 9, antiderivative size = 70

$$\int \frac{\text{PolyLog}(3,ax)}{x^3} \, dx = -\frac{a}{8x} + \frac{1}{8}a^2 \log(x) - \frac{1}{8}a^2 \log(1-ax) + \frac{\log(1-ax)}{8x^2} - \frac{\text{PolyLog}(2,ax)}{4x^2} - \frac{\text{PolyLog}(3,ax)}{2x^2}$$

output
```
-1/8*a/x+1/8*a^2*ln(x)-1/8*a^2*ln(-a*x+1)+1/8*ln(-a*x+1)/x^2-1/4*polylog(2
,a*x)/x^2-1/2*polylog(3,a*x)/x^2
```

3.17.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{\text{PolyLog}(3,ax)}{x^3} \, dx = \frac{G_{5,5}^{2,4}\left(-ax \middle| \begin{array}{c} 1,1,1,1,3 \\ 1,2,0,0,0 \end{array}\right)}{x^2}$$

input
```
Integrate[PolyLog[3, a*x]/x^3,x]
```

output
```
MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x)]/x^2
```

3.17.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{x^3}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\int \frac{\text{PolyLog}(2,ax)}{x^3}dx-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{\log(1-ax)}{x^3}dx-\frac{\text{PolyLog}(2,ax)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{\log(1-ax)}{x^3}dx-\frac{\text{PolyLog}(2,ax)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{x^2(1-ax)}dx+\frac{\log(1-ax)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}a\int \left(-\frac{a^2}{ax-1}+\frac{a}{x}+\frac{1}{x^2}\right)dx+\frac{\log(1-ax)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\log(1-ax)}{2x^2}+\frac{1}{2}a\left(a\log(x)-a\log(1-ax)-\frac{1}{x}\right)\right)-\frac{\text{PolyLog}(2,ax)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

input
```
Int[PolyLog[3, a*x]/x^3,x]
```

output
```
((Log[1 - a*x]/(2*x^2) + (a*(-x^(-1) + a*Log[x] - a*Log[1 - a*x]))/2)/2 -
PolyLog[2, a*x]/(2*x^2))/2 - PolyLog[3, a*x]/(2*x^2)
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	s
meijerg	$-a^{2}\left(-\frac{81ax+378}{432ax}-\frac{\left(-27a^{2}x^{2}+27\right)\ln\left(-ax+1\right)}{216a^{2}x^{2}}+\frac{\operatorname{polylog}\left(2,ax\right)}{4a^{2}x^{2}}+\frac{\operatorname{polylog}\left(3,ax\right)}{2a^{2}x^{2}}+\frac{3}{16}-\frac{\ln\left(x\right)}{8}-\frac{\ln\left(-a\right)}{8}+\frac{1}{ax}\right)$	9

input
```
int(polylog(3,a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-1/432/a/x*(81*a*x+378)-1/216/a^2/x^2*(-27*a^2*x^2+27)*ln(-a*x+1)+1/
4/a^2/x^2*polylog(2,a*x)+1/2/a^2/x^2*polylog(3,a*x)+3/16-1/8*ln(x)-1/8*ln(
-a)+1/a/x)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{\text{PolyLog}(3,ax)}{x^3}\,dx = -\frac{a^2x^2\log(ax-1) - a^2x^2\log(x) + ax + 2\,\text{Li}_2(ax) - \log(-ax+1) + 4\,\text{polylog}(3,ax)}{8\,x^2}$$

input
```
integrate(polylog(3,a*x)/x^3,x, algorithm="fricas")
```

output
```
-1/8*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*dilog(a*x) - log(-a*
x + 1) + 4*polylog(3, a*x))/x^2
```

3.17.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{x^3}\,dx = \int \frac{\text{Li}_3(ax)}{x^3}\,dx$$

input
```
integrate(polylog(3,a*x)/x**3,x)
```

output
```
Integral(polylog(3, a*x)/x**3, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int \frac{\text{PolyLog}(3,ax)}{x^3}\,dx = \frac{1}{8}\,a^2\log(x) - \frac{ax + (a^2x^2-1)\log(-ax+1) + 2\,\text{Li}_2(ax) + 4\,\text{Li}_3(ax)}{8\,x^2}$$

input
```
integrate(polylog(3,a*x)/x^3,x, algorithm="maxima")
```

output
```
1/8*a^2*log(x) - 1/8*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x) + 4
*polylog(3, a*x))/x^2
```

3.17.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax)}{x^3} dx = \int \frac{\text{Li}_3(ax)}{x^3} dx$$

input
```
integrate(polylog(3,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/x^3, x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int \frac{\text{PolyLog}(3, ax)}{x^3} dx = \frac{a^2 \operatorname{atanh}(2\,a\,x-1)}{4} - \frac{\frac{a\,x}{8} - \frac{\ln(1-a\,x)}{8} + \frac{\text{polylog}(2,a\,x)}{4} + \frac{\text{polylog}(3,a\,x)}{2}}{x^2}$$

input
```
int(polylog(3, a*x)/x^3,x)
```

output
```
(a^2*atanh(2*a*x - 1))/4 - ((a*x)/8 - log(1 - a*x)/8 + polylog(2, a*x)/4 +
 polylog(3, a*x)/2)/x^2
```

3.18 $\int \frac{\text{PolyLog}(3,ax)}{x^4} dx$

3.18.1 Optimal result

Integrand size = 9, antiderivative size = 80

$$\int \frac{\text{PolyLog}(3,ax)}{x^4} dx = -\frac{a}{54x^2} - \frac{a^2}{27x} + \frac{1}{27}a^3\log(x) - \frac{1}{27}a^3\log(1-ax) + \frac{\log(1-ax)}{27x^3} - \frac{\text{PolyLog}(2,ax)}{9x^3} - \frac{\text{PolyLog}(3,ax)}{3x^3}$$

output
```
-1/54*a/x^2-1/27*a^2/x+1/27*a^3*ln(x)-1/27*a^3*ln(-a*x+1)+1/27*ln(-a*x+1)/
x^3-1/9*polylog(2,a*x)/x^3-1/3*polylog(3,a*x)/x^3
```

3.18.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.31

$$\int \frac{\text{PolyLog}(3,ax)}{x^4} dx = \frac{G_{5,5}^{2,4}\left(-ax \middle| \begin{array}{c} 1,1,1,1,4 \\ 1,3,0,0,0 \end{array}\right)}{x^3}$$

input
```
Integrate[PolyLog[3, a*x]/x^4,x]
```

output
```
MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x)]/x^3
```

3.18.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7145, 7145, 25, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{x^4} \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\int \frac{\text{PolyLog}(2,ax)}{x^4}dx-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\left(\frac{1}{3}\int -\frac{\log(1-ax)}{x^4}dx-\frac{\text{PolyLog}(2,ax)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{\log(1-ax)}{x^4}dx-\frac{\text{PolyLog}(2,ax)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{3}a\int \frac{1}{x^3(1-ax)}dx+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{3}a\int \left(-\frac{a^3}{ax-1}+\frac{a^2}{x}+\frac{a}{x^2}+\frac{1}{x^3}\right)dx+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{3}a\left(a^2\log(x)-a^2\log(1-ax)-\frac{a}{x}-\frac{1}{2x^2}\right)+\frac{\log(1-ax)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

input
```
Int[PolyLog[3, a*x]/x^4,x]
```

output
```
((Log[1 - a*x]/(3*x^3) + (a*(-1/2*1/x^2 - a/x + a^2*Log[x] - a^2*Log[1 - a
*x]))/3)/3 - PolyLog[2, a*x]/(3*x^3))/3 - PolyLog[3, a*x]/(3*x^3)
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

method	result
meijerg	$a^3\left(\frac{64a^2x^2+152ax+832}{1728a^2x^2}+\frac{\left(-64a^3x^3+64\right)\ln\left(-ax+1\right)}{1728a^3x^3}-\frac{\operatorname{polylog}(2,ax)}{9a^3x^3}-\frac{\operatorname{polylog}(3,ax)}{3a^3x^3}-\frac{1}{27}+\frac{\ln(x)}{27}+\frac{\ln(-a)}{27}-\frac{1}{2a^2x}\right.$

input
```
int(polylog(3,a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/1728/a^2/x^2*(64*a^2*x^2+152*a*x+832)+1/1728/a^3/x^3*(-64*a^3*x^3+6
4)*ln(-a*x+1)-1/9/a^3/x^3*polylog(2,a*x)-1/3/a^3/x^3*polylog(3,a*x)-1/27+1
/27*ln(x)+1/27*ln(-a)-1/2/a^2/x^2-1/8/a/x)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx = -\frac{2\,a^3x^3\log(ax-1) - 2\,a^3x^3\log(x) + 2\,a^2x^2 + ax + 6\,\text{Li}_2(ax) - 2\,\log(-ax+1) + 18\,\text{polylog}(3,ax)}{54\,x^3}$$

input
```
integrate(polylog(3,a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/54*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dil
og(a*x) - 2*log(-a*x + 1) + 18*polylog(3, a*x))/x^3
```

3.18.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx = \int \frac{\text{Li}_3(ax)}{x^4}\,dx$$

input
```
integrate(polylog(3,a*x)/x**4,x)
```

output
```
Integral(polylog(3, a*x)/x**4, x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx = \frac{1}{27}\,a^3\log(x) - \frac{2\,a^2x^2 + ax + 2\,(a^3x^3-1)\log(-ax+1) + 6\,\text{Li}_2(ax) + 18\,\text{Li}_3(ax)}{54\,x^3}$$

input
```
integrate(polylog(3,a*x)/x^4,x, algorithm="maxima")
```

output
```
1/27*a^3*log(x) - 1/54*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) +
6*dilog(a*x) + 18*polylog(3, a*x))/x^3
```

3.18.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx = \int \frac{\text{Li}_3(ax)}{x^4}\,dx$$

input
```
integrate(polylog(3,a*x)/x^4,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/x^4, x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx = \frac{\ln(1-a\,x)}{27\,x^3} - \frac{\text{polylog}(2,a\,x)}{9\,x^3} - \frac{\text{polylog}(3,a\,x)}{3\,x^3} - \frac{x\,a^2+\frac{a}{2}}{27\,x^2} - \frac{a^3\,\text{atan}(a\,x\,2\text{i}-\text{i})\,2\text{i}}{27}$$

input
```
int(polylog(3, a*x)/x^4,x)
```

output
```
log(1 - a*x)/(27*x^3) - (a^3*atan(a*x*2i - 1i)*2i)/27 - polylog(2, a*x)/(9
*x^3) - polylog(3, a*x)/(3*x^3) - (a/2 + a^2*x)/(27*x^2)
```

3.19 $\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx$

3.19.1 Optimal result

Integrand size = 11, antiderivative size = 74

$$\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx = -\frac{x^2}{18a^2} - \frac{x^4}{36a} - \frac{x^6}{54} - \frac{\log(1-ax^2)}{18a^3} + \frac{1}{18}x^6 \log(1-ax^2) + \frac{1}{6}x^6 \operatorname{PolyLog}(2, ax^2)$$

output
```
-1/18*x^2/a^2-1/36*x^4/a-1/54*x^6-1/18*ln(-a*x^2+1)/a^3+1/18*x^6*ln(-a*x^2
+1)+1/6*x^6*polylog(2,a*x^2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx = \frac{-ax^2(6 + 3ax^2 + 2a^2x^4) + 6(-1 + a^3x^6) \log(1-ax^2) + 18a^3x^6 \operatorname{PolyLog}(2, ax^2)}{108a^3}$$

input
```
Integrate[x^5*PolyLog[2, a*x^2],x]
```

output
```
(-(a*x^2*(6 + 3*a*x^2 + 2*a^2*x^4)) + 6*(-1 + a^3*x^6)*Log[1 - a*x^2] + 18
*a^3*x^6*PolyLog[2, a*x^2])/(108*a^3)
```

3.19.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 25, 2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \text{PolyLog}\left(2,ax^2\right) dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)-\frac{1}{3}\int -x^5 \log \left(1-ax^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int x^5 \log \left(1-ax^2\right)dx+\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{6}\int x^4 \log \left(1-ax^2\right)dx^2+\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{6}\left(\frac{1}{3}a\int \frac{x^6}{1-ax^2}dx^2+\frac{1}{3}x^6 \log \left(1-ax^2\right)\right)+\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}\left(\frac{1}{3}a\int \left(-\frac{x^4}{a}-\frac{x^2}{a^2}-\frac{1}{a^3\left(ax^2-1\right)}-\frac{1}{a^3}\right)dx^2+\frac{1}{3}x^6 \log \left(1-ax^2\right)\right)+\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{1}{3}a\left(-\frac{\log \left(1-ax^2\right)}{a^4}-\frac{x^2}{a^3}-\frac{x^4}{2a^2}-\frac{x^6}{3a}\right)+\frac{1}{3}x^6 \log \left(1-ax^2\right)\right)+\frac{1}{6}x^6 \text{PolyLog}\left(2,ax^2\right)$$

input
```
Int[x^5*PolyLog[2, a*x^2],x]
```

output
```
((x^6*Log[1 - a*x^2])/3 + (a*(-(x^2/a^3) - x^4/(2*a^2) - x^6/(3*a) - Log[1
 - a*x^2]/a^4))/3)/6 + (x^6*PolyLog[2, a*x^2])/6
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.19.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{-\frac{a\,x^{2}\left(4x^{4}a^{2}+6a\,x^{2}+12\right)}{108}-\frac{\left(-4a^{3}x^{6}+4\right)\ln\left(-a\,x^{2}+1\right)}{36}+\frac{a^{3}x^{6}\operatorname{polylog}\left(2,a\,x^{2}\right)}{3}}{2a^{3}}$	65
default	$\frac{x^{6}\operatorname{polylog}(2,a\,x^{2})}{6}+\frac{x^{6}\ln(-a\,x^{2}+1)}{18}+\frac{a\left(-\frac{\frac{1}{3}a^{2}x^{6}+\frac{1}{2}a\,x^{4}+x^{2}}{2a^{3}}-\frac{\ln\left(a\,x^{2}-1\right)}{2a^{4}}\right)}{9}$	68
parts	$\frac{x^{6}\operatorname{polylog}(2,a\,x^{2})}{6}+\frac{x^{6}\ln(-a\,x^{2}+1)}{18}+\frac{a\left(-\frac{\frac{1}{3}a^{2}x^{6}+\frac{1}{2}a\,x^{4}+x^{2}}{2a^{3}}-\frac{\ln\left(a\,x^{2}-1\right)}{2a^{4}}\right)}{9}$	68
parallelrisch	$\frac{18a^{3}x^{6}\operatorname{polylog}(2,a\,x^{2})+6\ln(-a\,x^{2}+1)x^{6}a^{3}-2a^{3}x^{6}-3x^{4}a^{2}-6-6a\,x^{2}-6\ln(-a\,x^{2}+1)}{108a^{3}}$	73

input
```
int(x^5*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(-1/108*a*x^2*(4*a^2*x^4+6*a*x^2+12)-1/36*(-4*a^3*x^6+4)*ln(-a*x^2
+1)+1/3*a^3*x^6*polylog(2,a*x^2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x^5\,\text{PolyLog}\left(2,ax^2\right)\,dx = \frac{18\,a^3x^6\text{Li}_2(ax^2)-2\,a^3x^6-3\,a^2x^4-6\,ax^2+6\,(a^3x^6-1)\log\left(-ax^2+1\right)}{108\,a^3}$$

input
```
integrate(x^5*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*
x^6 - 1)*log(-a*x^2 + 1))/a^3
```

3.19.6 Sympy [A] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx = \begin{cases} -\frac{x^6 \operatorname{Li}_1(ax^2)}{18} + \frac{x^6 \operatorname{Li}_2(ax^2)}{6} - \frac{x^6}{54} - \frac{x^4}{36a} - \frac{x^2}{18a^2} + \frac{\operatorname{Li}_1(ax^2)}{18a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*polylog(2,a*x**2),x)
```

output
```
Piecewise((-x**6*polylog(1, a*x**2)/18 + x**6*polylog(2, a*x**2)/6 - x**6/
54 - x**4/(36*a) - x**2/(18*a**2) + polylog(1, a*x**2)/(18*a**3), Ne(a, 0)
), (0, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx = \frac{18 a^3 x^6 \operatorname{Li}_2(ax^2) - 2 a^3 x^6 - 3 a^2 x^4 - 6 a x^2 + 6 (a^3 x^6 - 1) \log(-ax^2 + 1)}{108 a^3}$$

input
```
integrate(x^5*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*
x^6 - 1)*log(-a*x^2 + 1))/a^3
```

3.19.8 Giac [F]

$$\int x^5 \operatorname{PolyLog}(2, ax^2) \, dx = \int x^5 \operatorname{Li}_2(ax^2) \, dx$$

input
```
integrate(x^5*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*dilog(a*x^2), x)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int x^5 \operatorname{PolyLog}\left(2,ax^2\right)\,dx=\frac{x^6\,\mathrm{polylog}(2,a\,x^2)}{6}-\frac{\ln\left(a\,x^2-1\right)}{18\,a^3}+\frac{x^6\,\ln\left(1-a\,x^2\right)}{18}-\frac{x^6}{54}-\frac{x^2}{18\,a^2}-\frac{x^4}{36\,a}$$

input

```
int(x^5*polylog(2, a*x^2),x)
```

output

```
(x^6*polylog(2, a*x^2))/6 - log(a*x^2 - 1)/(18*a^3) + (x^6*log(1 - a*x^2))
/18 - x^6/54 - x^2/(18*a^2) - x^4/(36*a)
```

3.20 $\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx$

3.20.1 Optimal result

Integrand size = 11, antiderivative size = 64

$$\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx = -\frac{x^2}{8a} - \frac{x^4}{16} - \frac{\log(1-ax^2)}{8a^2} + \frac{1}{8}x^4\log(1-ax^2) + \frac{1}{4}x^4\operatorname{PolyLog}(2, ax^2)$$

output
```
-1/8*x^2/a-1/16*x^4-1/8*ln(-a*x^2+1)/a^2+1/8*x^4*ln(-a*x^2+1)+1/4*x^4*poly
log(2,a*x^2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx = \frac{-ax^2(2+ax^2) + 2(-1+a^2x^4)\log(1-ax^2) + 4a^2x^4\operatorname{PolyLog}(2, ax^2)}{16a^2}$$

input
```
Integrate[x^3*PolyLog[2, a*x^2],x]
```

output
```
(-(a*x^2*(2 + a*x^2)) + 2*(-1 + a^2*x^4)*Log[1 - a*x^2] + 4*a^2*x^4*PolyLo
g[2, a*x^2])/(16*a^2)
```

3.20.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 25, 2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{PolyLog}\left(2,ax^2\right) dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)-\frac{1}{2}\int -x^3 \log \left(1-ax^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int x^3 \log \left(1-ax^2\right)dx+\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{4}\int x^2 \log \left(1-ax^2\right)dx^2+\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{4}\left(\frac{1}{2}a\int \frac{x^4}{1-ax^2}dx^2+\frac{1}{2}x^4 \log \left(1-ax^2\right)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\left(\frac{1}{2}a\int \left(-\frac{x^2}{a}-\frac{1}{a^2\left(ax^2-1\right)}-\frac{1}{a^2}\right)dx^2+\frac{1}{2}x^4 \log \left(1-ax^2\right)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}a\left(-\frac{\log \left(1-ax^2\right)}{a^3}-\frac{x^2}{a^2}-\frac{x^4}{2a}\right)+\frac{1}{2}x^4 \log \left(1-ax^2\right)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}\left(2,ax^2\right)$$

input
```
Int[x^3*PolyLog[2, a*x^2],x]
```

output
```
((x^4*Log[1 - a*x^2])/2 + (a*(-(x^2/a^2) - x^4/(2*a) - Log[1 - a*x^2]/a^3)
)/2)/4 + (x^4*PolyLog[2, a*x^2])/4
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.20.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
meijerg	$-\frac{\frac{a\,x^{2}\left(3a\,x^{2}+6\right)}{24}+\frac{\left(-3x^{4}a^{2}+3\right)\ln\left(-a\,x^{2}+1\right)}{12}-\frac{a^{2}x^{4}\operatorname{polylog}\left(2,a\,x^{2}\right)}{2}}{2a^{2}}$	57
default	$\frac{x^{4}\operatorname{polylog}(2,a\,x^{2})}{4}+\frac{x^{4}\ln(-a\,x^{2}+1)}{8}+\frac{a\left(-\frac{\frac{1}{2}a\,x^{4}+x^{2}}{2a^{2}}-\frac{\ln\left(a\,x^{2}-1\right)}{2a^{3}}\right)}{4}$	60
parts	$\frac{x^{4}\operatorname{polylog}(2,a\,x^{2})}{4}+\frac{x^{4}\ln(-a\,x^{2}+1)}{8}+\frac{a\left(-\frac{\frac{1}{2}a\,x^{4}+x^{2}}{2a^{2}}-\frac{\ln\left(a\,x^{2}-1\right)}{2a^{3}}\right)}{4}$	60
parallelrisch	$\frac{4a^{2}x^{4}\operatorname{polylog}(2,a\,x^{2})+2\ln(-a\,x^{2}+1)x^{4}a^{2}-x^{4}a^{2}-2a\,x^{2}-2\ln(-a\,x^{2}+1)}{16a^{2}}$	64

input
```
int(x^3*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(1/24*a*x^2*(3*a*x^2+6)+1/12*(-3*a^2*x^4+3)*ln(-a*x^2+1)-1/2*a^2*
x^4*polylog(2,a*x^2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int x^3\operatorname{PolyLog}\left(2,ax^2\right)\,dx=\frac{4\,a^2x^4\mathrm{Li}_2(ax^2)-a^2x^4-2\,ax^2+2\,(a^2x^4-1)\log\left(-ax^2+1\right)}{16\,a^2}$$

input
```
integrate(x^3*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*
x^2 + 1))/a^2
```

3.20.6 Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx = \begin{cases} -\frac{x^4 \operatorname{Li}_1(ax^2)}{8} + \frac{x^4 \operatorname{Li}_2(ax^2)}{4} - \frac{x^4}{16} - \frac{x^2}{8a} + \frac{\operatorname{Li}_1(ax^2)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*polylog(2,a*x**2),x)
```

output
```
Piecewise((-x**4*polylog(1, a*x**2)/8 + x**4*polylog(2, a*x**2)/4 - x**4/1
6 - x**2/(8*a) + polylog(1, a*x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx = \frac{4\,a^2x^4\operatorname{Li}_2(ax^2) - a^2x^4 - 2\,ax^2 + 2\,(a^2x^4 - 1)\log(-ax^2 + 1)}{16\,a^2}$$

input
```
integrate(x^3*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*
x^2 + 1))/a^2
```

3.20.8 Giac [F]

$$\int x^3 \operatorname{PolyLog}(2, ax^2)\, dx = \int x^3 \operatorname{Li}_2(ax^2)\, dx$$

input
```
integrate(x^3*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*dilog(a*x^2), x)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int x^3 \operatorname{PolyLog}\left(2,ax^2\right)\,dx=\frac{x^4\operatorname{polylog}(2,a\,x^2)}{4}-\frac{\ln\left(a\,x^2-1\right)}{8\,a^2}+\frac{x^4\,\ln\left(1-a\,x^2\right)}{8}-\frac{x^4}{16}-\frac{x^2}{8\,a}$$

input

```
int(x^3*polylog(2, a*x^2),x)
```

output

```
(x^4*polylog(2, a*x^2))/4 - log(a*x^2 - 1)/(8*a^2) + (x^4*log(1 - a*x^2))/
8 - x^4/16 - x^2/(8*a)
```

3.21 $\int x \operatorname{PolyLog}(2, ax^2)\, dx$

3.21.1 Optimal result

Integrand size = 9, antiderivative size = 46

$$\int x \operatorname{PolyLog}(2, ax^2)\, dx = -\frac{x^2}{2} - \frac{(1-ax^2)\log(1-ax^2)}{2a} + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2)$$

output
```
-1/2*x^2-1/2*(-a*x^2+1)*ln(-a*x^2+1)/a+1/2*x^2*polylog(2,a*x^2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int x \operatorname{PolyLog}(2, ax^2)\, dx = \frac{-ax^2 + (-1+ax^2)\log(1-ax^2) + ax^2 \operatorname{PolyLog}(2, ax^2)}{2a}$$

input
```
Integrate[x*PolyLog[2, a*x^2],x]
```

output
```
(-(a*x^2) + (-1 + a*x^2)*Log[1 - a*x^2] + a*x^2*PolyLog[2, a*x^2])/(2*a)
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7145, 25, 2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}\left(2, a x^2\right) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{2} x^2 \operatorname{PolyLog}\left(2, a x^2\right)-\int -x \log \left(1-a x^2\right) dx$$

$$\Big\downarrow \text{ 25}$$

$$\int x \log \left(1-a x^2\right) dx+\frac{1}{2} x^2 \operatorname{PolyLog}\left(2, a x^2\right)$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{2} \int \log \left(1-a x^2\right) dx^2+\frac{1}{2} x^2 \operatorname{PolyLog}\left(2, a x^2\right)$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{1}{2} x^2 \operatorname{PolyLog}\left(2, a x^2\right)-\frac{\int \log \left(1-a x^2\right) d\left(1-a x^2\right)}{2 a}$$

$$\Big\downarrow \text{ 2732}$$

$$\frac{1}{2} x^2 \operatorname{PolyLog}\left(2, a x^2\right)-\frac{a x^2+\left(1-a x^2\right) \log \left(1-a x^2\right)-1}{2 a}$$

input
```
Int[x*PolyLog[2, a*x^2],x]
```

output
```
-1/2*(-1 + a*x^2 + (1 - a*x^2)*Log[1 - a*x^2])/a + (x^2*PolyLog[2, a*x^2])
/2
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result	size
meijerg	$\frac{-a\,x^2-\frac{\left(-2a\,x^2+2\right)\ln\left(-a\,x^2+1\right)}{2}+a\,x^2\,\operatorname{polylog}\left(2,a\,x^2\right)}{2a}$	44
parts	$\frac{x^2\operatorname{polylog}\left(2,a\,x^2\right)}{2}-\frac{\left(-a\,x^2+1\right)\ln\left(-a\,x^2+1\right)+a\,x^2-1}{2a}$	44
derivativedivides	$\frac{a\,x^2\operatorname{polylog}\left(2,a\,x^2\right)-\left(-a\,x^2+1\right)\ln\left(-a\,x^2+1\right)+1-a\,x^2}{2a}$	45
default	$\frac{a\,x^2\operatorname{polylog}\left(2,a\,x^2\right)-\left(-a\,x^2+1\right)\ln\left(-a\,x^2+1\right)+1-a\,x^2}{2a}$	45
parallelrisch	$\frac{a\,x^2\operatorname{polylog}\left(2,a\,x^2\right)+\ln\left(-a\,x^2+1\right)x^2a-a\,x^2-\ln\left(-a\,x^2+1\right)}{2a}$	50

input
```
int(x*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-a*x^2-1/2*(-2*a*x^2+2)*ln(-a*x^2+1)+a*x^2*polylog(2,a*x^2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x \operatorname{PolyLog}\left(2, a x^2\right) dx = \frac{a x^2 \operatorname{Li}_2(a x^2) - a x^2 + (a x^2 - 1) \log\left(-a x^2 + 1\right)}{2 a}$$

input
```
integrate(x*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x \operatorname{PolyLog}\left(2, a x^2\right) dx = \begin{cases} -\frac{x^2 \operatorname{Li}_1(a x^2)}{2} + \frac{x^2 \operatorname{Li}_2(a x^2)}{2} - \frac{x^2}{2} + \frac{\operatorname{Li}_1(a x^2)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*polylog(2,a*x**2),x)
```

output
```
Piecewise((-x**2*polylog(1, a*x**2)/2 + x**2*polylog(2, a*x**2)/2 - x**2/2
 + polylog(1, a*x**2)/(2*a), Ne(a, 0)), (0, True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x \operatorname{PolyLog}\left(2, a x^2\right) dx = \frac{a x^2 \operatorname{Li}_2(a x^2) - a x^2 + (a x^2 - 1) \log\left(-a x^2 + 1\right)}{2 a}$$

input
```
integrate(x*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

3.21.8 Giac [F]

$$\int x \,\text{PolyLog}\left(2,ax^2\right)\,dx = \int x\text{Li}_2\left(ax^2\right)\,dx$$

input
```
integrate(x*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(x*dilog(a*x^2), x)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x \,\text{PolyLog}\left(2,ax^2\right)\,dx = \frac{x^2\,\text{polylog}(2,a\,x^2)}{2} - \frac{\ln\left(a\,x^2-1\right)}{2\,a} + \frac{x^2\,\ln\left(1-a\,x^2\right)}{2} - \frac{x^2}{2}$$

input
```
int(x*polylog(2, a*x^2),x)
```

output
```
(x^2*polylog(2, a*x^2))/2 - log(a*x^2 - 1)/(2*a) + (x^2*log(1 - a*x^2))/2
- x^2/2
```

3.22 $\int \frac{\text{PolyLog}(2,ax^2)}{x}\,dx$

3.22.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(2,ax^2)}{x}\,dx = \frac{\text{PolyLog}(3,ax^2)}{2}$$

output
```
1/2*polylog(3,a*x^2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax^2)}{x}\,dx = \frac{\text{PolyLog}(3,ax^2)}{2}$$

input
```
Integrate[PolyLog[2, a*x^2]/x,x]
```

output
```
PolyLog[3, a*x^2]/2
```

3.22.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2, ax^2)}{x} \, dx$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\text{PolyLog}(3, ax^2)}{2}$$

input
```
Int[PolyLog[2, a*x^2]/x,x]
```

output
```
PolyLog[3, a*x^2]/2
```

3.22.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.22.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\operatorname{polylog}(3,a\,x^{2})}{2}$	10
meijerg	$\frac{\operatorname{polylog}(3,a\,x^{2})}{2}$	10
parts	$\frac{\operatorname{polylog}(3,a\,x^{2})}{2}$	10

input
```
int(polylog(2,a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*polylog(3,a*x^2)
```

3.22.5 Fricas [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x}\,dx = \int \frac{\text{Li}_2(ax^2)}{x}\,dx$$

input
```
integrate(polylog(2,a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(dilog(a*x^2)/x, x)
```

3.22.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x}\,dx = \int \frac{\text{Li}_2\left(ax^2\right)}{x}\,dx$$

input
```
integrate(polylog(2,a*x**2)/x,x)
```

output
```
Integral(polylog(2, a*x**2)/x, x)
```

3.22.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x}\,dx = \int \frac{\text{Li}_2(ax^2)}{x}\,dx$$

input
```
integrate(polylog(2,a*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(dilog(a*x^2)/x, x)
```

3.22.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^2)}{x} \, dx = \int \frac{\text{Li}_2(ax^2)}{x} \, dx$$

input
```
integrate(polylog(2,a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x, x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\text{PolyLog}(2, ax^2)}{x} \, dx = \frac{\text{polylog}(3, a\,x^2)}{2}$$

input
```
int(polylog(2, a*x^2)/x,x)
```

output
```
polylog(3, a*x^2)/2
```

3.23 $\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx$

3.23.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx = a\log(x) - \frac{1}{2}a\log\left(1-ax^2\right) + \frac{\log(1-ax^2)}{2x^2} - \frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

output
```
a*ln(x)-1/2*a*ln(-a*x^2+1)+1/2*ln(-a*x^2+1)/x^2-1/2*polylog(2,a*x^2)/x^2
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx = a\log(x) - \frac{1}{2}a\log\left(1-ax^2\right) + \frac{\log(1-ax^2)}{2x^2} - \frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^3,x]
```

output
```
a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^
2]/(2*x^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 25, 2904, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}dx$$

$$\Big\downarrow \text{7145}$$

$$\int -\frac{\log(1-ax^2)}{x^3}dx-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\log(1-ax^2)}{x^3}dx-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{2}\int \frac{\log(1-ax^2)}{x^4}dx^2-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{1-ax^2}dx^2+\int \frac{1}{x^2}dx^2\right)+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{1-ax^2}dx^2+\log(x^2)\right)+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(a(\log(x^2)-\log(1-ax^2))+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

input
```
Int[PolyLog[2, a*x^2]/x^3,x]
```

output
```
(a*(Log[x^2] - Log[1 - a*x^2]) + Log[1 - a*x^2]/x^2)/2 - PolyLog[2, a*x^2]
/(2*x^2)
```

3.23.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.23.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\operatorname{polylog}(2,a\,x^{2})}{2x^{2}}+\frac{\ln(-a\,x^{2}+1)}{2x^{2}}+a\left(-\frac{\ln(a\,x^{2}-1)}{2}+\ln(x)\right)$	43
parts	$-\frac{\operatorname{polylog}(2,a\,x^{2})}{2x^{2}}+\frac{\ln(-a\,x^{2}+1)}{2x^{2}}+a\left(-\frac{\ln(a\,x^{2}-1)}{2}+\ln(x)\right)$	43
meijerg	$\frac{a\left(\frac{(-4a\,x^{2}+4)\ln(-a\,x^{2}+1)}{4a\,x^{2}}-\frac{\operatorname{polylog}(2,a\,x^{2})}{a\,x^{2}}+2\ln(x)+\ln(-a)\right)}{2}$	53
parallelrisch	$\frac{2a^{2}\ln(x)x^{2}-x^{2}\ln(-a\,x^{2}+1)a^{2}-a\operatorname{polylog}(2,a\,x^{2})+a\ln(-a\,x^{2}+1)}{2x^{2}a}$	58

input
```
int(polylog(2,a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*polylog(2,a*x^2)/x^2+1/2*ln(-a*x^2+1)/x^2+a*(-1/2*ln(a*x^2-1)+ln(x))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx = -\frac{ax^2\log(ax^2-1)-2\,ax^2\log(x)+\text{Li}_2(ax^2)-\log(-ax^2+1)}{2\,x^2}$$

input
```
integrate(polylog(2,a*x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + dilog(a*x^2) - log(-a*x^2 +
1))/x^2
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx = a\log(x)+\frac{a\,\text{Li}_1(ax^2)}{2}-\frac{\text{Li}_1(ax^2)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}$$

input
```
integrate(polylog(2,a*x**2)/x**3,x)
```

output
```
a*log(x) + a*polylog(1, a*x**2)/2 - polylog(1, a*x**2)/(2*x**2) - polylog(
2, a*x**2)/(2*x**2)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{x^3}\, dx = a\log\left(x\right) - \frac{\left(ax^2-1\right)\log\left(-ax^2+1\right) + \text{Li}_2(ax^2)}{2\,x^2}$$

input
```
integrate(polylog(2,a*x^2)/x^3,x, algorithm="maxima")
```

output
```
a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2))/x^2
```

3.23.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{x^3}\, dx = \int \frac{\text{Li}_2(ax^2)}{x^3}\, dx$$

input
```
integrate(polylog(2,a*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^3, x)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{x^3}\, dx = \frac{3\,a\,\ln\left(x\right)}{2} + \frac{\frac{\ln\left(1-a\,x^2\right)}{2} - \frac{\mathrm{polylog}\left(2,a\,x^2\right)}{2}}{x^2} - \frac{a\,\ln\left(a\,x^3-x\right)}{2}$$

input
```
int(polylog(2, a*x^2)/x^3,x)
```

output
```
(3*a*log(x))/2 + (log(1 - a*x^2)/2 - polylog(2, a*x^2)/2)/x^2 - (a*log(a*x
^3 - x))/2
```

3.24 $\int \frac{\text{PolyLog}(2,ax^2)}{x^5}\,dx$

3.24.1 Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^5}\,dx = -\frac{a}{8x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{8}a^2\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{8x^4}-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

output
```
-1/8*a/x^2+1/4*a^2*ln(x)-1/8*a^2*ln(-a*x^2+1)+1/8*ln(-a*x^2+1)/x^4-1/4*pol
ylog(2,a*x^2)/x^4
```

3.24.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^5}\,dx = -\frac{ax^2-2a^2x^4\log(x)+(-1+a^2x^4)\log\left(1-ax^2\right)+2\,\text{PolyLog}(2,ax^2)}{8x^4}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^5,x]
```

output
```
-1/8*(a*x^2 - 2*a^2*x^4*Log[x] + (-1 + a^2*x^4)*Log[1 - a*x^2] + 2*PolyLog
[2, a*x^2])/x^4
```

3.24.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 25, 2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^5}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\int -\frac{\log(1-ax^2)}{x^5}dx-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{\log(1-ax^2)}{x^5}dx-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{4}\int \frac{\log(1-ax^2)}{x^6}dx^2-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{4}\left(\frac{1}{2}a\int \frac{1}{x^4(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{2x^4}\right)-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\left(\frac{1}{2}a\int \left(-\frac{a^2}{ax^2-1}+\frac{a}{x^2}+\frac{1}{x^4}\right)dx^2+\frac{\log(1-ax^2)}{2x^4}\right)-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}a\left(a\log(x^2)-a\log(1-ax^2)-\frac{1}{x^2}\right)+\frac{\log(1-ax^2)}{2x^4}\right)-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

input
```
Int[PolyLog[2, a*x^2]/x^5,x]
```

output
```
(Log[1 - a*x^2]/(2*x^4) + (a*(-x^(-2) + a*Log[x^2] - a*Log[1 - a*x^2]))/2)
/4 - PolyLog[2, a*x^2]/(4*x^4)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.24.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\operatorname{polylog}(2,a\,x^{2})}{4x^{4}}+\frac{\ln(-a\,x^{2}+1)}{8x^{4}}+\frac{a\left(-\frac{a\ln\left(a\,x^{2}-1\right)}{2}-\frac{1}{2x^{2}}+a\ln(x)\right)}{4}$	52
parts	$-\frac{\operatorname{polylog}(2,a\,x^{2})}{4x^{4}}+\frac{\ln(-a\,x^{2}+1)}{8x^{4}}+\frac{a\left(-\frac{a\ln\left(a\,x^{2}-1\right)}{2}-\frac{1}{2x^{2}}+a\ln(x)\right)}{4}$	52
parallelrisch	$\frac{2a^{2}\ln(x)x^{4}-\ln(-a\,x^{2}+1)x^{4}a^{2}-x^{4}a^{2}-a\,x^{2}-2\operatorname{polylog}(2,a\,x^{2})+\ln(-a\,x^{2}+1)}{8x^{4}}$	66
meijerg	$-\frac{a^{2}\left(-\frac{9a\,x^{2}+27}{36a\,x^{2}}-\frac{\left(-9x^{4}a^{2}+9\right)\ln\left(-a\,x^{2}+1\right)}{36a^{2}x^{4}}+\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{2a^{2}x^{4}}+\frac{1}{4}-\frac{\ln(x)}{2}-\frac{\ln(-a)}{4}+\frac{1}{a\,x^{2}}\right)}{2}$	83

input
```
int(polylog(2,a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*polylog(2,a*x^2)/x^4+1/8*ln(-a*x^2+1)/x^4+1/4*a*(-1/2*a*ln(a*x^2-1)-1
/2/x^2+a*ln(x))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x^5}\,dx$$
$$= -\frac{a^2x^4\log\left(ax^2-1\right)-2\,a^2x^4\log\left(x\right)+ax^2+2\,\text{Li}_2(ax^2)-\log\left(-ax^2+1\right)}{8\,x^4}$$

input
```
integrate(polylog(2,a*x^2)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*dilog(a*x^2) -
 log(-a*x^2 + 1))/x^4
```

3.24.6 Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{PolyLog}(2, ax^2)}{x^5}\, dx = \frac{a^2\log(x)}{4} + \frac{a^2\operatorname{Li}_1(ax^2)}{8} - \frac{a}{8x^2} - \frac{\operatorname{Li}_1(ax^2)}{8x^4} - \frac{\operatorname{Li}_2(ax^2)}{4x^4}$$

input
```
integrate(polylog(2,a*x**2)/x**5,x)
```

output
```
a**2*log(x)/4 + a**2*polylog(1, a*x**2)/8 - a/(8*x**2) - polylog(1, a*x**2
)/(8*x**4) - polylog(2, a*x**2)/(4*x**4)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{PolyLog}(2, ax^2)}{x^5}\, dx = \frac{1}{4}a^2\log(x) - \frac{ax^2 + (a^2x^4 - 1)\log(-ax^2 + 1) + 2\operatorname{Li}_2(ax^2)}{8\,x^4}$$

input
```
integrate(polylog(2,a*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/4*a^2*log(x) - 1/8*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x^
2))/x^4
```

3.24.8 Giac [F]

$$\int \frac{\operatorname{PolyLog}(2, ax^2)}{x^5}\, dx = \int \frac{\operatorname{Li}_2(ax^2)}{x^5}\, dx$$

input
```
integrate(polylog(2,a*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^5, x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{\text{PolyLog}\,(2,ax^2)}{x^5}\,dx = \frac{a^2\,\ln{(x)}}{4} - \frac{\text{polylog}(2,a\,x^2)}{4\,x^4} - \frac{a^2\,\ln{(a\,x^2-1)}}{8} - \frac{a}{8\,x^2} + \frac{\ln{(1-a\,x^2)}}{8\,x^4}$$

input
```
int(polylog(2, a*x^2)/x^5,x)
```

output
```
(a^2*log(x))/4 - polylog(2, a*x^2)/(4*x^4) - (a^2*log(a*x^2 - 1))/8 - a/(8
*x^2) + log(1 - a*x^2)/(8*x^4)
```

3.25 $\int \frac{\text{PolyLog}(2,ax^2)}{x^7}\,dx$

3.25.1 Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^7}\,dx = -\frac{a}{36x^4}-\frac{a^2}{18x^2}+\frac{1}{9}a^3\log(x)-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{\log(1-ax^2)}{18x^6}-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

output
```
-1/36*a/x^4-1/18*a^2/x^2+1/9*a^3*ln(x)-1/18*a^3*ln(-a*x^2+1)+1/18*ln(-a*x^
2+1)/x^6-1/6*polylog(2,a*x^2)/x^6
```

3.25.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^7}\,dx = -\frac{ax^2+2a^2x^4-4a^3x^6\log(x)-2\log(1-ax^2)+2a^3x^6\log(1-ax^2)+6\,\text{PolyLog}(2,ax^2)}{36x^6}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^7,x]
```

output
```
-1/36*(a*x^2 + 2*a^2*x^4 - 4*a^3*x^6*Log[x] - 2*Log[1 - a*x^2] + 2*a^3*x^6
*Log[1 - a*x^2] + 6*PolyLog[2, a*x^2])/x^6
```

3.25.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 25, 2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^7}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\int -\frac{\log(1-ax^2)}{x^7}dx-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{\log(1-ax^2)}{x^7}dx-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{6}\int \frac{\log(1-ax^2)}{x^8}dx^2-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{6}\left(\frac{1}{3}a\int \frac{1}{x^6(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{3x^6}\right)-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{6}\left(\frac{1}{3}a\int \left(-\frac{a^3}{ax^2-1}+\frac{a^2}{x^2}+\frac{a}{x^4}+\frac{1}{x^6}\right)dx^2+\frac{\log(1-ax^2)}{3x^6}\right)-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{1}{3}a\left(a^2\log(x^2)-a^2\log(1-ax^2)-\frac{a}{x^2}-\frac{1}{2x^4}\right)+\frac{\log(1-ax^2)}{3x^6}\right)-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

input
```
Int[PolyLog[2, a*x^2]/x^7,x]
```

output
```
(Log[1 - a*x^2]/(3*x^6) + (a*(-1/2*1/x^4 - a/x^2 + a^2*Log[x^2] - a^2*Log[
1 - a*x^2]))/3)/6 - PolyLog[2, a*x^2]/(6*x^6)
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.25.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\operatorname{polylog}(2,a\,x^2)}{6x^6}+\frac{\ln(-a\,x^2+1)}{18x^6}+\frac{a\left(-\frac{a^2\ln\left(a\,x^2-1\right)}{2}-\frac{1}{4x^4}-\frac{a}{2x^2}+a^2\ln(x)\right)}{9}$	62
parts	$-\frac{\operatorname{polylog}(2,a\,x^2)}{6x^6}+\frac{\ln(-a\,x^2+1)}{18x^6}+\frac{a\left(-\frac{a^2\ln\left(a\,x^2-1\right)}{2}-\frac{1}{4x^4}-\frac{a}{2x^2}+a^2\ln(x)\right)}{9}$	62
parallelrisch	$\frac{4a^3\ln(x)x^6-2\ln(-a\,x^2+1)x^6a^3-2a^3x^6-2x^4a^2-a\,x^2-6\operatorname{polylog}(2,a\,x^2)+2\ln(-a\,x^2+1)}{36x^6}$	76
meijerg	$\frac{a^3\left(\frac{32x^4a^2+60a\,x^2+192}{432a^2x^4}+\frac{\left(-16a^3x^6+16\right)\ln\left(-a\,x^2+1\right)}{144a^3x^6}-\frac{\operatorname{polylog}\left(2,a\,x^2\right)}{3a^3x^6}-\frac{2}{27}+\frac{2\ln(x)}{9}+\frac{\ln(-a)}{9}-\frac{1}{2a^2x^4}-\frac{1}{4a\,x^2}\right)}{2}$	100

input
```
int(polylog(2,a*x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*polylog(2,a*x^2)/x^6+1/18*ln(-a*x^2+1)/x^6+1/9*a*(-1/2*a^2*ln(a*x^2-1
)-1/4/x^4-1/2*a/x^2+a^2*ln(x))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^7}\,dx$$
$$= -\frac{2\,a^3x^6\log\left(ax^2-1\right)-4\,a^3x^6\log\left(x\right)+2\,a^2x^4+ax^2+6\,\mathrm{Li}_2(ax^2)-2\,\log\left(-ax^2+1\right)}{36\,x^6}$$

input
```
integrate(polylog(2,a*x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/36*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 + 6
*dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^6
```

3.25.6 Sympy [A] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^7} dx = \frac{a^3 \log(x)}{9} + \frac{a^3 \operatorname{Li}_1(ax^2)}{18} - \frac{a^2}{18x^2} - \frac{a}{36x^4} - \frac{\operatorname{Li}_1(ax^2)}{18x^6} - \frac{\operatorname{Li}_2(ax^2)}{6x^6}$$

input
```
integrate(polylog(2,a*x**2)/x**7,x)
```

output
```
a**3*log(x)/9 + a**3*polylog(1, a*x**2)/18 - a**2/(18*x**2) - a/(36*x**4)
- polylog(1, a*x**2)/(18*x**6) - polylog(2, a*x**2)/(6*x**6)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^7} dx$$
$$= \frac{1}{9} a^3 \log(x) - \frac{2a^2x^4 + ax^2 + 2(a^3x^6 - 1)\log(-ax^2 + 1) + 6\operatorname{Li}_2(ax^2)}{36x^6}$$

input
```
integrate(polylog(2,a*x^2)/x^7,x, algorithm="maxima")
```

output
```
1/9*a^3*log(x) - 1/36*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 + 1)
 + 6*dilog(a*x^2))/x^6
```

3.25.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^7} dx = \int \frac{\operatorname{Li}_2(ax^2)}{x^7} dx$$

input
```
integrate(polylog(2,a*x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^7, x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^7} dx = \frac{a^3 \ln(x)}{9} - \frac{\text{polylog}(2, a x^2)}{6 x^6} - \frac{a^3 \ln(a x^2 - 1)}{18} - \frac{a}{36 x^4} + \frac{\ln(1 - a x^2)}{18 x^6} - \frac{a^2}{18 x^2}$$

input
```
int(polylog(2, a*x^2)/x^7,x)
```

output
```
(a^3*log(x))/9 - polylog(2, a*x^2)/(6*x^6) - (a^3*log(a*x^2 - 1))/18 - a/(
36*x^4) + log(1 - a*x^2)/(18*x^6) - a^2/(18*x^2)
```

3.26 $\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx$

3.26.1 Optimal result

Integrand size = 11, antiderivative size = 73

$$\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx = -\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{4\operatorname{arctanh}(\sqrt{a}x)}{25a^{5/2}} + \frac{2}{25}x^5 \log(1-ax^2) + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2)$$

output
```
-4/25*x/a^2-4/75*x^3/a-4/125*x^5+4/25*arctanh(x*a^(1/2))/a^(5/2)+2/25*x^5*
ln(-a*x^2+1)+1/5*x^5*polylog(2,a*x^2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx = \frac{1}{375}\left(-\frac{60x}{a^2} - \frac{20x^3}{a} - 12x^5 + \frac{60\operatorname{arctanh}(\sqrt{a}x)}{a^{5/2}} + 30x^5 \log(1-ax^2) + 75x^5 \operatorname{PolyLog}(2, ax^2)\right)$$

input
```
Integrate[x^4*PolyLog[2, a*x^2],x]
```

output
```
((-60*x)/a^2 - (20*x^3)/a - 12*x^5 + (60*ArcTanh[Sqrt[a]*x])/a^(5/2) + 30*
x^5*Log[1 - a*x^2] + 75*x^5*PolyLog[2, a*x^2])/375
```

3.26.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 25, 2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \text{PolyLog}\left(2,ax^2\right) \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{5}x^5 \text{PolyLog}\left(2,ax^2\right)-\frac{2}{5}\int -x^4 \log \left(1-ax^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}\int x^4 \log \left(1-ax^2\right)dx+\frac{1}{5}x^5 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{5}\left(\frac{2}{5}a\int \frac{x^6}{1-ax^2}dx+\frac{1}{5}x^5 \log \left(1-ax^2\right)\right)+\frac{1}{5}x^5 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{2}{5}\left(\frac{2}{5}a\int \left(-\frac{x^4}{a}-\frac{x^2}{a^2}+\frac{1}{a^3\left(1-ax^2\right)}-\frac{1}{a^3}\right)dx+\frac{1}{5}x^5 \log \left(1-ax^2\right)\right)+\frac{1}{5}x^5 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}\left(\frac{2}{5}a\left(\frac{\text{arctanh}\left(\sqrt{a}x\right)}{a^{7/2}}-\frac{x}{a^3}-\frac{x^3}{3a^2}-\frac{x^5}{5a}\right)+\frac{1}{5}x^5 \log \left(1-ax^2\right)\right)+\frac{1}{5}x^5 \text{PolyLog}\left(2,ax^2\right)$$

input
```
Int[x^4*PolyLog[2, a*x^2],x]
```

output
```
(2*((2*a*(-(x/a^3) - x^3/(3*a^2) - x^5/(5*a) + ArcTanh[Sqrt[a]*x]/a^(7/2))
)/5 + (x^5*Log[1 - a*x^2])/5))/5 + (x^5*PolyLog[2, a*x^2])/5
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

method	result	siz
default	$\frac{x^{5} \operatorname{polylog}\left(2, a\,x^{2}\right)}{5}+\frac{2 x^{5} \ln \left(-a\,x^{2}+1\right)}{25}+\frac{4 a \left(-\frac{\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a\,x^{3}+x}{a^{3}}+\frac{\operatorname{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{7}{2}}}\right)}{25}$	63
parts	$\frac{x^{5} \operatorname{polylog}\left(2, a\,x^{2}\right)}{5}+\frac{2 x^{5} \ln \left(-a\,x^{2}+1\right)}{25}+\frac{4 a \left(-\frac{\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a\,x^{3}+x}{a^{3}}+\frac{\operatorname{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{7}{2}}}\right)}{25}$	63
meijerg	$-\frac{-\frac{2 x \left(-a\right)^{\frac{7}{2}} \left(84 x^{4} a^{2}+140 a\,x^{2}+420\right)}{2625 a^{3}}-\frac{4 x \left(-a\right)^{\frac{7}{2}} \left(\ln \left(1-\sqrt{a\,x^{2}}\right)-\ln \left(1+\sqrt{a\,x^{2}}\right)\right)}{25 a^{3} \sqrt{a\,x^{2}}}+\frac{4 x^{5} \left(-a\right)^{\frac{7}{2}} \ln \left(-a\,x^{2}+1\right)}{25 a}+\frac{2 x^{5} \left(-a\right)^{\frac{7}{2}} \operatorname{polylog}\left(2, a\,x^{2}\right)}{5 a}}{2 a^{2} \sqrt{-a}}$	12

input
```
int(x^4*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*polylog(2,a*x^2)+2/25*x^5*ln(-a*x^2+1)+4/25*a*(-1/a^3*(1/5*a^2*x^5
+1/3*a*x^3+x)+1/a^(7/2)*arctanh(x*a^(1/2)))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.18

$$\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx$$
$$= \left[\frac{75\, a^3x^5\mathrm{Li}_2(ax^2) + 30\, a^3x^5 \log(-ax^2+1) - 12\, a^3x^5 - 20\, a^2x^3 - 60\, ax + 30\,\sqrt{a} \log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{375\, a^3}, \frac{75\, a^3 \right.$$

input
```
integrate(x^4*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
[1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12*a^3*x^5
- 20*a^2*x^3 - 60*a*x + 30*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 -
1)))/a^3, 1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12
*a^3*x^5 - 20*a^2*x^3 - 60*a*x - 60*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

3.26.6 Sympy [A] (verification not implemented)

Time = 60.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx$$
$$= \begin{cases} -\frac{2x^5 \operatorname{Li}_1(ax^2)}{25} + \frac{x^5 \operatorname{Li}_2(ax^2)}{5} - \frac{4x^5}{125} - \frac{4x^3}{75a} - \frac{4x}{25a^2} - \frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{25a^3\sqrt{\frac{1}{a}}} - \frac{2\operatorname{Li}_1(ax^2)}{25a^3\sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*polylog(2,a*x**2),x)
```

output
```
Piecewise((-2*x**5*polylog(1, a*x**2)/25 + x**5*polylog(2, a*x**2)/5 - 4*x
**5/125 - 4*x**3/(75*a) - 4*x/(25*a**2) - 4*log(x - sqrt(1/a))/(25*a**3*sq
rt(1/a)) - 2*polylog(1, a*x**2)/(25*a**3*sqrt(1/a)), Ne(a, 0)), (0, True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int x^4 \operatorname{PolyLog}(2, ax^2) \, dx$$
$$= \frac{75\,a^2x^5\operatorname{Li}_2(ax^2) + 30\,a^2x^5\log(-ax^2+1) - 12\,a^2x^5 - 20\,ax^3 - 60\,x}{375\,a^2} - \frac{2\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{25\,a^{\frac{5}{2}}}$$

input
```
integrate(x^4*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
1/375*(75*a^2*x^5*dilog(a*x^2) + 30*a^2*x^5*log(-a*x^2 + 1) - 12*a^2*x^5 -
 20*a*x^3 - 60*x)/a^2 - 2/25*log((a*x - sqrt(a))/(a*x + sqrt(a)))/a^(5/2)
```

3.26.8 Giac [F]

$$\int x^4 \operatorname{PolyLog}(2, ax^2) \, dx = \int x^4\operatorname{Li}_2(ax^2) \, dx$$

input
```
integrate(x^4*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*dilog(a*x^2), x)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int x^4 \operatorname{PolyLog}(2, ax^2) \, dx = \frac{x^5 \operatorname{polylog}(2, a\,x^2)}{5} - \frac{4\,x}{25\,a^2} + \frac{2\,x^5\,\ln(1-a\,x^2)}{25} - \frac{4\,x^5}{125} - \frac{4\,x^3}{75\,a} - \frac{\operatorname{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,4\mathrm{i}}{25\,a^{5/2}}$$

input
```
int(x^4*polylog(2, a*x^2),x)
```

output
```
(x^5*polylog(2, a*x^2))/5 - (atan(a^(1/2)*x*1i)*4i)/(25*a^(5/2)) - (4*x)/(
25*a^2) + (2*x^5*log(1 - a*x^2))/25 - (4*x^5)/125 - (4*x^3)/(75*a)
```

3.27 $\int x^2 \text{PolyLog}(2, ax^2) \, dx$

3.27.1 Optimal result

Integrand size = 11, antiderivative size = 63

$$\int x^2 \text{PolyLog}(2, ax^2) \, dx = -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4\text{arctanh}(\sqrt{a}x)}{9a^{3/2}} + \frac{2}{9}x^3 \log(1 - ax^2) + \frac{1}{3}x^3 \text{PolyLog}(2, ax^2)$$

output
```
-4/9*x/a-4/27*x^3+4/9*arctanh(x*a^(1/2))/a^(3/2)+2/9*x^3*ln(-a*x^2+1)+1/3*
x^3*polylog(2,a*x^2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int x^2 \text{PolyLog}(2, ax^2) \, dx = \frac{1}{27}\left(-\frac{12x}{a} - 4x^3 + \frac{12\text{arctanh}(\sqrt{a}x)}{a^{3/2}} + 6x^3 \log(1 - ax^2) + 9x^3 \text{PolyLog}(2, ax^2)\right)$$

input
```
Integrate[x^2*PolyLog[2, a*x^2],x]
```

output
```
((-12*x)/a - 4*x^3 + (12*ArcTanh[Sqrt[a]*x])/a^(3/2) + 6*x^3*Log[1 - a*x^2
] + 9*x^3*PolyLog[2, a*x^2])/27
```

3.27.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 25, 2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{PolyLog}\left(2,ax^2\right) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)-\frac{2}{3}\int -x^2 \log \left(1-ax^2\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{3}\int x^2 \log \left(1-ax^2\right)dx+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2}{3}\left(\frac{2}{3}a\int \frac{x^4}{1-ax^2}dx+\frac{1}{3}x^3 \log \left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{2}{3}\left(\frac{2}{3}a\int \left(-\frac{x^2}{a}+\frac{1}{a^2\left(1-ax^2\right)}-\frac{1}{a^2}\right)dx+\frac{1}{3}x^3 \log \left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}\left(\frac{2}{3}a\left(\frac{\text{arctanh}\left(\sqrt{a}x\right)}{a^{5/2}}-\frac{x}{a^2}-\frac{x^3}{3a}\right)+\frac{1}{3}x^3 \log \left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)$$

input
```
Int[x^2*PolyLog[2, a*x^2],x]
```

output
```
(2*((2*a*(-(x/a^2) - x^3/(3*a) + ArcTanh[Sqrt[a]*x]/a^(5/2)))/3 + (x^3*Log
[1 - a*x^2])/3))/3 + (x^3*PolyLog[2, a*x^2])/3
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^{3} \operatorname{polylog}\left(2,a \,x^{2}\right)}{3}+\frac{2 x^{3} \ln \left(-a \,x^{2}+1\right)}{9}+\frac{4 a \left(-\frac{\frac{1}{3} a \,x^{3}+x}{a^{2}}+\frac{\operatorname{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{5}{2}}}\right)}{9}$	55
parts	$\frac{x^{3} \operatorname{polylog}\left(2,a \,x^{2}\right)}{3}+\frac{2 x^{3} \ln \left(-a \,x^{2}+1\right)}{9}+\frac{4 a \left(-\frac{\frac{1}{3} a \,x^{3}+x}{a^{2}}+\frac{\operatorname{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{5}{2}}}\right)}{9}$	55
meijerg	$\frac{-\frac{2 x \left(-a \right)^{\frac{5}{2}} \left(20 a \,x^{2}+60\right)}{135 a^{2}}-\frac{4 x \left(-a \right)^{\frac{5}{2}} \left(\ln \left(1-\sqrt{a \,x^{2}}\right)-\ln \left(1+\sqrt{a \,x^{2}}\right)\right)}{9 a^{2} \sqrt{a \,x^{2}}}+\frac{4 x^{3} \left(-a \right)^{\frac{5}{2}} \ln \left(-a \,x^{2}+1\right)}{9 a}+\frac{2 x^{3} \left(-a \right)^{\frac{5}{2}} \operatorname{polylog}\left(2,a \,x^{2}\right)}{3 a}}{2 a \sqrt{-a}}$	116

input
```
int(x^2*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*polylog(2,a*x^2)+2/9*x^3*ln(-a*x^2+1)+4/9*a*(-1/a^2*(1/3*a*x^3+x)+
arctanh(x*a^(1/2))/a^(5/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.27

$$\int x^2 \operatorname{PolyLog}(2, ax^2) \, dx$$
$$= \left[\frac{9\,a^2x^3\mathrm{Li}_2(ax^2) + 6\,a^2x^3\log\left(-ax^2+1\right) - 4\,a^2x^3 - 12\,ax + 6\,\sqrt{a}\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{27\,a^2}, \frac{9\,a^2x^3\mathrm{Li}_2(ax^2) + 6\,a\right.$$

input
```
integrate(x^2*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
[1/27*(9*a^2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12
*a*x + 6*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^2, 1/27*(9*
a^2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12*a*x - 12
*sqrt(-a)*arctan(sqrt(-a)*x))/a^2]
```

3.27.6 Sympy [A] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int x^2 \operatorname{PolyLog}(2, ax^2) \, dx$$
$$= \begin{cases} -\frac{2x^3 \operatorname{Li}_1(ax^2)}{9} + \frac{x^3 \operatorname{Li}_2(ax^2)}{3} - \frac{4x^3}{27} - \frac{4x}{9a} - \frac{4\log\left(x - \sqrt{\frac{1}{a}}\right)}{9a^2\sqrt{\frac{1}{a}}} - \frac{2\operatorname{Li}_1(ax^2)}{9a^2\sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*polylog(2,a*x**2),x)
```

output
```
Piecewise((-2*x**3*polylog(1, a*x**2)/9 + x**3*polylog(2, a*x**2)/3 - 4*x*
*3/27 - 4*x/(9*a) - 4*log(x - sqrt(1/a))/(9*a**2*sqrt(1/a)) - 2*polylog(1,
 a*x**2)/(9*a**2*sqrt(1/a)), Ne(a, 0)), (0, True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int x^2 \operatorname{PolyLog}(2, ax^2)\, dx = \frac{9\, ax^3 \operatorname{Li}_2(ax^2) + 6\, ax^3 \log(-ax^2 + 1) - 4\, ax^3 - 12\, x}{27\, a} - \frac{2\, \log\left(\frac{ax - \sqrt{a}}{ax + \sqrt{a}}\right)}{9\, a^{\frac{3}{2}}}$$

input
```
integrate(x^2*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
1/27*(9*a*x^3*dilog(a*x^2) + 6*a*x^3*log(-a*x^2 + 1) - 4*a*x^3 - 12*x)/a -
 2/9*log((a*x - sqrt(a))/(a*x + sqrt(a)))/a^(3/2)
```

3.27.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(2, ax^2)\, dx = \int x^2 \operatorname{Li}_2(ax^2)\, dx$$

input
```
integrate(x^2*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*dilog(a*x^2), x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{PolyLog}(2, ax^2)\, dx = \frac{x^3 \operatorname{polylog}(2, a\, x^2)}{3} - \frac{4\, x}{9\, a} + \frac{2\, x^3\, \ln(1 - a\, x^2)}{9} - \frac{4\, x^3}{27} - \frac{\operatorname{atan}\left(\sqrt{a}\, x\, 1\mathrm{i}\right)\, 4\mathrm{i}}{9\, a^{3/2}}$$

input
```
int(x^2*polylog(2, a*x^2),x)
```

output
```
(x^3*polylog(2, a*x^2))/3 - (atan(a^(1/2)*x*1i)*4i)/(9*a^(3/2)) - (4*x)/(9
*a) + (2*x^3*log(1 - a*x^2))/9 - (4*x^3)/27
```

3.28 $\int \text{PolyLog}(2, ax^2)\, dx$

3.28.1 Optimal result

Integrand size = 7, antiderivative size = 40

$$\int \text{PolyLog}(2, ax^2)\, dx = -4x + \frac{4\text{arctanh}(\sqrt{a}x)}{\sqrt{a}} + 2x\log(1 - ax^2) + x\,\text{PolyLog}(2, ax^2)$$

output
```
-4*x+2*x*ln(-a*x^2+1)+x*polylog(2,a*x^2)+4*arctanh(x*a^(1/2))/a^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \text{PolyLog}(2, ax^2)\, dx = \frac{4\text{arctanh}(\sqrt{a}x)}{\sqrt{a}} + 2x(-2 + \log(1 - ax^2)) + x\,\text{PolyLog}(2, ax^2)$$

input
```
Integrate[PolyLog[2, a*x^2],x]
```

output
```
(4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*(-2 + Log[1 - a*x^2]) + x*PolyLog[2,
a*x^2]
```

3.28.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {7140, 25, 2898, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx$$

$$\Big\downarrow \text{ 7140}$$

$$x\,\text{PolyLog}\left(2,ax^2\right)-2\int -\log\left(1-ax^2\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$2\int \log\left(1-ax^2\right)dx+x\,\text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 2898}$$

$$2\left(2a\int \frac{x^2}{1-ax^2}dx+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 262}$$

$$2\left(2a\left(\frac{\int \frac{1}{1-ax^2}dx}{a}-\frac{x}{a}\right)+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(2a\left(\frac{\text{arctanh}\left(\sqrt{a}x\right)}{a^{3/2}}-\frac{x}{a}\right)+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)$$

input
```
Int[PolyLog[2, a*x^2],x]
```

output
```
2*(2*a*(-(x/a) + ArcTanh[Sqrt[a]*x]/a^(3/2)) + x*Log[1 - a*x^2]) + x*PolyL
og[2, a*x^2]
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

method	result	size
default	$x\,\mathrm{polylog}\left(2, a\,x^{2}\right)+2 x \ln\left(-a\,x^{2}+1\right)+4 a\left(-\frac{x}{a}+\frac{\mathrm{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{3}{2}}}\right)$	43
parts	$x\,\mathrm{polylog}\left(2, a\,x^{2}\right)+2 x \ln\left(-a\,x^{2}+1\right)+4 a\left(-\frac{x}{a}+\frac{\mathrm{arctanh}\left(x \sqrt{a}\right)}{a^{\frac{3}{2}}}\right)$	43
meijerg	$-\frac{-\frac{8 x \left(-a\right)^{\frac{3}{2}}}{a}-\frac{4 x \left(-a\right)^{\frac{3}{2}} \left(\ln\left(1-\sqrt{a\,x^{2}}\right)-\ln\left(1+\sqrt{a\,x^{2}}\right)\right)}{a \sqrt{a\,x^{2}}}+\frac{4 x \left(-a\right)^{\frac{3}{2}} \ln\left(-a\,x^{2}+1\right)}{a}+\frac{2 x \left(-a\right)^{\frac{3}{2}} \mathrm{polylog}\left(2,a\,x^{2}\right)}{a}}{2 \sqrt{-a}}$	101

input
```
int(polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*polylog(2,a*x^2)+2*x*ln(-a*x^2+1)+4*a*(-x/a+arctanh(x*a^(1/2))/a^(3/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.68

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx$$
$$= \left[\frac{ax\text{Li}_2(ax^2)+2\,ax\log\left(-ax^2+1\right)-4\,ax+2\,\sqrt{a}\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{a}, \frac{ax\text{Li}_2(ax^2)+2\,ax\log\left(-ax^2+1\right)-}{a}\right.$$

input
```
integrate(polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
[(a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1) - 4*a*x + 2*sqrt(a)*log((a*x^2
+ 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, (a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 +
 1) - 4*a*x - 4*sqrt(-a)*arctan(sqrt(-a)*x))/a]
```

3.28.6 Sympy [A] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx$$
$$= \begin{cases} -2x\,\text{Li}_1\left(ax^2\right)+x\,\text{Li}_2\left(ax^2\right)-4x-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{a\sqrt{\frac{1}{a}}}-\frac{2\,\text{Li}_1\left(ax^2\right)}{a\sqrt{\frac{1}{a}}} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(polylog(2,a*x**2),x)
```

output
```
Piecewise((-2*x*polylog(1, a*x**2) + x*polylog(2, a*x**2) - 4*x - 4*log(x
- sqrt(1/a))/(a*sqrt(1/a)) - 2*polylog(1, a*x**2)/(a*sqrt(1/a)), Ne(a, 0))
, (0, True))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx = x\text{Li}_2\left(ax^2\right) + 2\,x\log\left(-ax^2+1\right) - 4\,x - \frac{2\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
x*dilog(a*x^2) + 2*x*log(-a*x^2 + 1) - 4*x - 2*log((a*x - sqrt(a))/(a*x +
sqrt(a)))/sqrt(a)
```

3.28.8 Giac [F]

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx = \int \text{Li}_2\left(ax^2\right)\,dx$$

input
```
integrate(polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \text{PolyLog}\left(2,ax^2\right)\,dx = 2\,x\,\ln\left(1-a\,x^2\right) - 4\,x + x\,\text{polylog}\left(2,a\,x^2\right) - \frac{\text{atan}\left(\sqrt{a}\,x\,1\text{i}\right)\,4\text{i}}{\sqrt{a}}$$

input
```
int(polylog(2, a*x^2),x)
```

output
```
2*x*log(1 - a*x^2) - (atan(a^(1/2)*x*1i)*4i)/a^(1/2) - 4*x + x*polylog(2,
a*x^2)
```

3.29 $\int \frac{\text{PolyLog}(2,ax^2)}{x^2}\,dx$

3.29.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^2}\,dx = 4\sqrt{a}\text{arctanh}\left(\sqrt{a}x\right) + \frac{2\log(1-ax^2)}{x} - \frac{\text{PolyLog}(2,ax^2)}{x}$$

output
```
2*ln(-a*x^2+1)/x-polylog(2,a*x^2)/x+4*arctanh(x*a^(1/2))*a^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^2}\,dx = \frac{4\sqrt{a}x\text{arctanh}\left(\sqrt{a}x\right) + 2\log(1-ax^2) - \text{PolyLog}(2,ax^2)}{x}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^2,x]
```

output
```
(4*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 2*Log[1 - a*x^2] - PolyLog[2, a*x^2])/x
```

3.29.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2905, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^2} \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$2 \int -\frac{\log(1-ax^2)}{x^2} dx - \frac{\text{PolyLog}(2,ax^2)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-2 \int \frac{\log(1-ax^2)}{x^2} dx - \frac{\text{PolyLog}(2,ax^2)}{x}$$

$$\Big\downarrow \text{ 2905}$$

$$-2 \left(-2a \int \frac{1}{1-ax^2} dx - \frac{\log(1-ax^2)}{x}\right) - \frac{\text{PolyLog}(2,ax^2)}{x}$$

$$\Big\downarrow \text{ 219}$$

$$-2 \left(-2\sqrt{a}\text{arctanh}(\sqrt{a}x) - \frac{\log(1-ax^2)}{x}\right) - \frac{\text{PolyLog}(2,ax^2)}{x}$$

input
```
Int[PolyLog[2, a*x^2]/x^2,x]
```

output
```
-2*(-2*Sqrt[a]*ArcTanh[Sqrt[a]*x] - Log[1 - a*x^2]/x) - PolyLog[2, a*x^2]/
x
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2\ln\left(-a\,x^{2}+1\right)}{x}-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{x}+4\operatorname{arctanh}\left(x\sqrt{a}\right)\sqrt{a}$	39
parts	$\frac{2\ln\left(-a\,x^{2}+1\right)}{x}-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{x}+4\operatorname{arctanh}\left(x\sqrt{a}\right)\sqrt{a}$	39
meijerg	$\frac{a\left(-\frac{4x\sqrt{-a}\left(\ln\left(1-\sqrt{a\,x^{2}}\right)-\ln\left(1+\sqrt{a\,x^{2}}\right)\right)}{\sqrt{a\,x^{2}}}+\frac{4\sqrt{-a}\,\ln\left(-a\,x^{2}+1\right)}{xa}-\frac{2\sqrt{-a}\,\operatorname{polylog}\left(2,a\,x^{2}\right)}{xa}\right)}{2\sqrt{-a}}$	92

input
```
int(polylog(2,a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*ln(-a*x^2+1)/x-polylog(2,a*x^2)/x+4*arctanh(x*a^(1/2))*a^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.24

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^2} \, dx = \left[\frac{2\sqrt{a}x \log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right) - \text{Li}_2(ax^2) + 2\log(-ax^2+1)}{x}, -\frac{4\sqrt{-a}x \arctan\left(\sqrt{-a}x\right) + \text{Li}_2(ax^2) - 2\log(-ax^2+1)}{x} \right]$$

input
```
integrate(polylog(2,a*x^2)/x^2,x, algorithm="fricas")
```

output
```
[(2*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - dilog(a*x^2) +
2*log(-a*x^2 + 1))/x, -(4*sqrt(-a)*x*arctan(sqrt(-a)*x) + dilog(a*x^2) - 2
*log(-a*x^2 + 1))/x]
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(34) = 68$.

Time = 15.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.38

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^2} \, dx = \begin{cases} 0 \\ -\frac{\pi^2}{6x} \\ -\frac{4ax^3\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}} - \frac{2ax^3\sqrt{\frac{1}{a}}\,\text{Li}_1(ax^2)}{x^3-\frac{x}{a}} - \frac{2x^2\,\text{Li}_1(ax^2)}{x^3-\frac{x}{a}} - \frac{x^2\,\text{Li}_2(ax^2)}{x^3-\frac{x}{a}} + \frac{4x\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}} + \frac{2x\sqrt{\frac{1}{a}}\,\text{Li}_1(ax^2)}{x^3-\frac{x}{a}} + \frac{2\,\text{Li}_1(a}{ax^3-} \end{cases}$$

input
```
integrate(polylog(2,a*x**2)/x**2,x)
```

output
```
Piecewise((0, Eq(a, 0)), (-pi**2/(6*x), Eq(a, x**(-2))), (-4*a*x**3*sqrt(1
/a)*log(x - sqrt(1/a))/(x**3 - x/a) - 2*a*x**3*sqrt(1/a)*polylog(1, a*x**2
)/(x**3 - x/a) - 2*x**2*polylog(1, a*x**2)/(x**3 - x/a) - x**2*polylog(2,
a*x**2)/(x**3 - x/a) + 4*x*sqrt(1/a)*log(x - sqrt(1/a))/(x**3 - x/a) + 2*x
*sqrt(1/a)*polylog(1, a*x**2)/(x**3 - x/a) + 2*polylog(1, a*x**2)/(a*x**3
- x) + polylog(2, a*x**2)/(a*x**3 - x), True))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^2} dx = -2\sqrt{a}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{\text{Li}_2(ax^2) - 2\log(-ax^2+1)}{x}$$

input
```
integrate(polylog(2,a*x^2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(a)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - (dilog(a*x^2) - 2*log(-a
*x^2 + 1))/x
```

3.29.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^2} dx = \int \frac{\text{Li}_2(ax^2)}{x^2} dx$$

input
```
integrate(polylog(2,a*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^2, x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^2} dx = 4\sqrt{a}\,\text{atanh}\left(\sqrt{a}\,x\right) - \frac{\text{polylog}(2, a\,x^2)}{x} + \frac{2\ln(1-a\,x^2)}{x}$$

input
```
int(polylog(2, a*x^2)/x^2,x)
```

output
```
4*a^(1/2)*atanh(a^(1/2)*x) - polylog(2, a*x^2)/x + (2*log(1 - a*x^2))/x
```

3.30 $\int \frac{\text{PolyLog}(2,ax^2)}{x^4} dx$

3.30.1 Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^4} dx = -\frac{4a}{9x} + \frac{4}{9}a^{3/2}\text{arctanh}(\sqrt{a}x) + \frac{2\log(1-ax^2)}{9x^3} - \frac{\text{PolyLog}(2,ax^2)}{3x^3}$$

output
```
-4/9*a/x+4/9*a^(3/2)*arctanh(x*a^(1/2))+2/9*ln(-a*x^2+1)/x^3-1/3*polylog(2
,a*x^2)/x^3
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^4} dx = -\frac{4ax^2 \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},ax^2\right) - 2\log(1-ax^2) + 3\,\text{PolyLog}(2,ax^2)}{9x^3}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^4,x]
```

output
```
-1/9*(4*a*x^2*Hypergeometric2F1[-1/2, 1, 1/2, a*x^2] - 2*Log[1 - a*x^2] +
3*PolyLog[2, a*x^2])/x^3
```

3.30.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 25, 2905, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^4} dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{3} \int -\frac{\log(1 - ax^2)}{x^4} dx - \frac{\text{PolyLog}(2, ax^2)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3} \int \frac{\log(1 - ax^2)}{x^4} dx - \frac{\text{PolyLog}(2, ax^2)}{3x^3}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{2}{3}\left(-\frac{2}{3}a \int \frac{1}{x^2(1 - ax^2)} dx - \frac{\log(1 - ax^2)}{3x^3}\right) - \frac{\text{PolyLog}(2, ax^2)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}\left(-\frac{2}{3}a\left(a \int \frac{1}{1 - ax^2} dx - \frac{1}{x}\right) - \frac{\log(1 - ax^2)}{3x^3}\right) - \frac{\text{PolyLog}(2, ax^2)}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{3}\left(-\frac{2}{3}a\left(\sqrt{a}\text{arctanh}(\sqrt{a}x) - \frac{1}{x}\right) - \frac{\log(1 - ax^2)}{3x^3}\right) - \frac{\text{PolyLog}(2, ax^2)}{3x^3}$$

input
```
Int[PolyLog[2, a*x^2]/x^4,x]
```

output
```
(-2*((-2*a*(-x^(-1) + Sqrt[a]*ArcTanh[Sqrt[a]*x]))/3 - Log[1 - a*x^2]/(3*x
^3)))/3 - PolyLog[2, a*x^2]/(3*x^3)
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{3x^{3}}+\frac{2\ln\left(-a\,x^{2}+1\right)}{9x^{3}}+\frac{4a\left(\operatorname{arctanh}\left(x\sqrt{a}\right)\sqrt{a}-\frac{1}{x}\right)}{9}$	47
parts	$-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{3x^{3}}+\frac{2\ln\left(-a\,x^{2}+1\right)}{9x^{3}}+\frac{4a\left(\operatorname{arctanh}\left(x\sqrt{a}\right)\sqrt{a}-\frac{1}{x}\right)}{9}$	47
meijerg	$-\frac{a^{2}\left(-\frac{8}{9x\sqrt{-a}}-\frac{4xa\left(\ln\left(1-\sqrt{a\,x^{2}}\right)-\ln\left(1+\sqrt{a\,x^{2}}\right)\right)}{9\sqrt{-a}\,\sqrt{a\,x^{2}}}+\frac{4\ln\left(-a\,x^{2}+1\right)}{9x^{3}\sqrt{-a}\,a}-\frac{2\operatorname{polylog}\left(2,a\,x^{2}\right)}{3x^{3}\sqrt{-a}\,a}\right)}{2\sqrt{-a}}$	105

input
```
int(polylog(2,a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*polylog(2,a*x^2)/x^3+2/9*ln(-a*x^2+1)/x^3+4/9*a*(arctanh(x*a^(1/2))*a
^(1/2)-1/x)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.04

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^4}\,dx = \left[\frac{2\,a^{\frac{3}{2}}x^3\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)-4\,ax^2-3\,\text{Li}_2(ax^2)+2\,\log\left(-ax^2+1\right)}{9\,x^3},\right.$$
$$\left.-\frac{4\,\sqrt{-a}ax^3\arctan\left(\sqrt{-a}x\right)+4\,ax^2+3\,\text{Li}_2(ax^2)-2\,\log\left(-ax^2+1\right)}{9\,x^3}\right]$$

input
```
integrate(polylog(2,a*x^2)/x^4,x, algorithm="fricas")
```

output
```
[1/9*(2*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 4*a*x^2 -
 3*dilog(a*x^2) + 2*log(-a*x^2 + 1))/x^3, -1/9*(4*sqrt(-a)*a*x^3*arctan(sq
rt(-a)*x) + 4*a*x^2 + 3*dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^3]
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(49) = 98.

Time = 72.40 (sec) , antiderivative size = 275, normalized size of antiderivative = 4.91

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^4}\,dx = \begin{cases} 0 \\ -\frac{\pi^2}{18x^3} \\ -\frac{4a^2x^5\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{9x^5-\frac{9x^3}{a}}-\frac{2a^2x^5\sqrt{\frac{1}{a}}\,\text{Li}_1\left(ax^2\right)}{9x^5-\frac{9x^3}{a}}-\frac{4ax^4}{9x^5-\frac{9x^3}{a}}+\frac{4ax^3\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{9x^5-\frac{9x^3}{a}}+\frac{2ax^3\sqrt{\frac{1}{a}}\,\text{Li}_1\left(ax^2\right)}{9x^5-\frac{9x^3}{a}}-\frac{2x^2\,\text{Li}_1\left(ax^2\right)}{9x^5-\frac{9x^3}{a}}-\frac{3x}{9} \end{cases}$$

input
```
integrate(polylog(2,a*x**2)/x**4,x)
```

output
```
Piecewise((0, Eq(a, 0)), (-pi**2/(18*x**3), Eq(a, x**(-2))), (-4*a**2*x**5
*sqrt(1/a)*log(x - sqrt(1/a))/(9*x**5 - 9*x**3/a) - 2*a**2*x**5*sqrt(1/a)*
polylog(1, a*x**2)/(9*x**5 - 9*x**3/a) - 4*a*x**4/(9*x**5 - 9*x**3/a) + 4*
a*x**3*sqrt(1/a)*log(x - sqrt(1/a))/(9*x**5 - 9*x**3/a) + 2*a*x**3*sqrt(1/
a)*polylog(1, a*x**2)/(9*x**5 - 9*x**3/a) - 2*x**2*polylog(1, a*x**2)/(9*x
**5 - 9*x**3/a) - 3*x**2*polylog(2, a*x**2)/(9*x**5 - 9*x**3/a) + 4*x**2/(
9*x**5 - 9*x**3/a) + 2*polylog(1, a*x**2)/(9*a*x**5 - 9*x**3) + 3*polylog(
2, a*x**2)/(9*a*x**5 - 9*x**3), True))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{x^4}\, dx = -\frac{2}{9}\, a^{\frac{3}{2}} \log\left(\frac{ax - \sqrt{a}}{ax + \sqrt{a}}\right) - \frac{4\, ax^2 + 3\, \text{Li}_2(ax^2) - 2\, \log\left(-ax^2 + 1\right)}{9\, x^3}$$

input
```
integrate(polylog(2,a*x^2)/x^4,x, algorithm="maxima")
```

output
```
-2/9*a^(3/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/9*(4*a*x^2 + 3*dilog
(a*x^2) - 2*log(-a*x^2 + 1))/x^3
```

3.30.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{x^4}\, dx = \int \frac{\text{Li}_2(ax^2)}{x^4}\, dx$$

input
```
integrate(polylog(2,a*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^4, x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^4} \, dx = \frac{2 \ln(1 - a x^2)}{9 x^3} - \frac{4 a}{9 x} - \frac{\text{polylog}(2, a x^2)}{3 x^3} - \frac{a^{3/2} \operatorname{atan}\left(\sqrt{a}\, x \, 1\text{i}\right) \, 4\text{i}}{9}$$

input
```
int(polylog(2, a*x^2)/x^4,x)
```

output
```
(2*log(1 - a*x^2))/(9*x^3) - polylog(2, a*x^2)/(3*x^3) - (4*a)/(9*x) - (a^
(3/2)*atan(a^(1/2)*x*1i)*4i)/9
```

3.31 $\int \frac{\text{PolyLog}(2,ax^2)}{x^6} dx$

3.31.1 Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^6} dx = -\frac{4a}{75x^3} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2}\text{arctanh}\left(\sqrt{a}x\right) + \frac{2\log(1-ax^2)}{25x^5} - \frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

output
```
-4/75*a/x^3-4/25*a^2/x+4/25*a^(5/2)*arctanh(x*a^(1/2))+2/25*ln(-a*x^2+1)/x
^5-1/5*polylog(2,a*x^2)/x^5
```

3.31.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^6} dx = -\frac{4ax^2 \text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},ax^2\right) - 6\log(1-ax^2) + 15\,\text{PolyLog}(2,ax^2)}{75x^5}$$

input
```
Integrate[PolyLog[2, a*x^2]/x^6,x]
```

output
```
-1/75*(4*a*x^2*Hypergeometric2F1[-3/2, 1, -1/2, a*x^2] - 6*Log[1 - a*x^2]
+ 15*PolyLog[2, a*x^2])/x^5
```

3.31.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 25, 2905, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^6}\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{5}\int -\frac{\log(1-ax^2)}{x^6}dx-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{5}\int \frac{\log(1-ax^2)}{x^6}dx-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{2}{5}\left(-\frac{2}{5}a\int \frac{1}{x^4(1-ax^2)}dx-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{5}\left(-\frac{2}{5}a\left(a\int \frac{1}{x^2(1-ax^2)}dx-\frac{1}{3x^3}\right)-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{5}\left(-\frac{2}{5}a\left(a\left(a\int \frac{1}{1-ax^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{5}\left(-\frac{2}{5}a\left(a\left(\sqrt{a}\text{arctanh}(\sqrt{a}x)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

input
```
Int[PolyLog[2, a*x^2]/x^6,x]
```

output
```
(-2*((-2*a*(-1/3*1/x^3 + a*(-x^(-1) + Sqrt[a]*ArcTanh[Sqrt[a]*x])))/5 - Lo
g[1 - a*x^2]/(5*x^5)))/5 - PolyLog[2, a*x^2]/(5*x^5)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.31.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{5 x^{5}}+\frac{2 \ln \left(-a\,x^{2}+1\right)}{25 x^{5}}+\frac{4 a \left(a^{\frac{3}{2}} \operatorname{arctanh}\left(x \sqrt{a}\right)-\frac{1}{3 x^{3}}-\frac{a}{x}\right)}{25}$	53
parts	$-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{5 x^{5}}+\frac{2 \ln \left(-a\,x^{2}+1\right)}{25 x^{5}}+\frac{4 a \left(a^{\frac{3}{2}} \operatorname{arctanh}\left(x \sqrt{a}\right)-\frac{1}{3 x^{3}}-\frac{a}{x}\right)}{25}$	53
meijerg	$\frac{a^{3} \left(-\frac{8}{75 x^{3} \left(-a\right)^{\frac{3}{2}}}-\frac{8 a}{25 x \left(-a\right)^{\frac{3}{2}}}-\frac{4 x\,a^{2} \left(\ln \left(1-\sqrt{a\,x^{2}}\right)-\ln \left(1+\sqrt{a\,x^{2}}\right)\right)}{25 \left(-a\right)^{\frac{3}{2}} \sqrt{a\,x^{2}}}+\frac{4 \ln \left(-a\,x^{2}+1\right)}{25 x^{5} \left(-a\right)^{\frac{3}{2}} a}-\frac{2 \operatorname{polylog}\left(2,a\,x^{2}\right)}{5 x^{5} \left(-a\right)^{\frac{3}{2}} a}\right)}{2 \sqrt{-a}}$	118

input
```
int(polylog(2,a*x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*polylog(2,a*x^2)/x^5+2/25*ln(-a*x^2+1)/x^5+4/25*a*(a^(3/2)*arctanh(x*
a^(1/2))-1/3/x^3-a/x)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^6} dx$$
$$= \left[\frac{6 a^{\frac{5}{2}} x^5 \log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right) - 12 a^2 x^4 - 4 ax^2 - 15 \text{Li}_2(ax^2) + 6 \log(-ax^2+1)}{75 x^5}, \right.$$
$$\left. -\frac{12 \sqrt{-a} a^2 x^5 \arctan\left(\sqrt{-a}x\right) + 12 a^2 x^4 + 4 ax^2 + 15 \text{Li}_2(ax^2) - 6 \log(-ax^2+1)}{75 x^5} \right]$$

input
```
integrate(polylog(2,a*x^2)/x^6,x, algorithm="fricas")
```

output
```
[1/75*(6*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 12*a^2*x
^4 - 4*a*x^2 - 15*dilog(a*x^2) + 6*log(-a*x^2 + 1))/x^5, -1/75*(12*sqrt(-a
)*a^2*x^5*arctan(sqrt(-a)*x) + 12*a^2*x^4 + 4*a*x^2 + 15*dilog(a*x^2) - 6*
log(-a*x^2 + 1))/x^5]
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^2)}{x^6} dx = \text{Timed out}$$

input
```
integrate(polylog(2,a*x**2)/x**6,x)
```

output
```
Timed out
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x^6}\,dx = -\frac{2}{25}\,a^{\frac{5}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{12\,a^2x^4+4\,ax^2+15\,\text{Li}_2(ax^2)-6\,\log\left(-ax^2+1\right)}{75\,x^5}$$

input
```
integrate(polylog(2,a*x^2)/x^6,x, algorithm="maxima")
```

output
```
-2/25*a^(5/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/75*(12*a^2*x^4 + 4*
a*x^2 + 15*dilog(a*x^2) - 6*log(-a*x^2 + 1))/x^5
```

3.31.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x^6}\,dx = \int \frac{\text{Li}_2(ax^2)}{x^6}\,dx$$

input
```
integrate(polylog(2,a*x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/x^6, x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x^6}\,dx = \frac{2\,\ln\left(1-a\,x^2\right)}{25\,x^5} - \frac{4\,a^2\,x^2+\frac{4\,a}{3}}{25\,x^3} - \frac{\text{polylog}(2,a\,x^2)}{5\,x^5} - \frac{a^{5/2}\,\text{atan}\left(\sqrt{a}\,x\,1\text{i}\right)\,4\text{i}}{25}$$

input
```
int(polylog(2, a*x^2)/x^6,x)
```

output
```
(2*log(1 - a*x^2))/(25*x^5) - polylog(2, a*x^2)/(5*x^5) - ((4*a)/3 + 4*a^2
*x^2)/(25*x^3) - (a^(5/2)*atan(a^(1/2)*x*1i)*4i)/25
```

3.32 $\int x^5 \operatorname{PolyLog}(3, ax^2)\, dx$

3.32.1 Optimal result

Integrand size = 11, antiderivative size = 88

$$\int x^5 \operatorname{PolyLog}(3, ax^2)\, dx = \frac{x^2}{54a^2} + \frac{x^4}{108a} + \frac{x^6}{162} + \frac{\log(1 - ax^2)}{54a^3} - \frac{1}{54}x^6 \log(1 - ax^2) - \frac{1}{18}x^6 \operatorname{PolyLog}(2, ax^2) + \frac{1}{6}x^6 \operatorname{PolyLog}(3, ax^2)$$

output
```
1/54*x^2/a^2+1/108*x^4/a+1/162*x^6+1/54*ln(-a*x^2+1)/a^3-1/54*x^6*ln(-a*x^
2+1)-1/18*x^6*polylog(2,a*x^2)+1/6*x^6*polylog(3,a*x^2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int x^5 \operatorname{PolyLog}(3, ax^2)\, dx = \frac{6ax^2 + 3a^2x^4 + 2a^3x^6 + 6\log(1 - ax^2) - 6a^3x^6\log(1 - ax^2) - 18a^3x^6 \operatorname{PolyLog}(2, ax^2) + 54a^3x^6 \operatorname{PolyLo}}{324a^3}$$

input
```
Integrate[x^5*PolyLog[3, a*x^2],x]
```

output
```
(6*a*x^2 + 3*a^2*x^4 + 2*a^3*x^6 + 6*Log[1 - a*x^2] - 6*a^3*x^6*Log[1 - a*
x^2] - 18*a^3*x^6*PolyLog[2, a*x^2] + 54*a^3*x^6*PolyLog[3, a*x^2])/(324*a
^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 7145, 25, 2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \operatorname{PolyLog}\left(3,ax^2\right) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)-\frac{1}{3}\int x^5 \operatorname{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{3}\left(\frac{1}{3}\int -x^5 \log\left(1-ax^2\right)dx-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int x^5 \log\left(1-ax^2\right)dx-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3}\left(-\frac{1}{6}\int x^4 \log\left(1-ax^2\right)dx^2-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(-\frac{1}{3}a\int \frac{x^6}{1-ax^2}dx^2-\frac{1}{3}x^6 \log\left(1-ax^2\right)\right)-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(-\frac{1}{3}a\int \left(-\frac{x^4}{a}-\frac{x^2}{a^2}-\frac{1}{a^3\left(ax^2-1\right)}-\frac{1}{a^3}\right)dx^2-\frac{1}{3}x^6 \log\left(1-ax^2\right)\right)-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(-\frac{1}{3}a\left(-\frac{\log\left(1-ax^2\right)}{a^4}-\frac{x^2}{a^3}-\frac{x^4}{2a^2}-\frac{x^6}{3a}\right)-\frac{1}{3}x^6 \log\left(1-ax^2\right)\right)-\frac{1}{6}x^6 \operatorname{PolyLog}\left(2,ax^2\right)\right)+\frac{1}{6}x^6 \operatorname{PolyLog}\left(3,ax^2\right)$$

input
```
Int[x^5*PolyLog[3, a*x^2],x]
```

output
```
((-1/3*(x^6*Log[1 - a*x^2]) - (a*(-(x^2/a^3) - x^4/(2*a^2) - x^6/(3*a) - L
og[1 - a*x^2]/a^4))/3)/6 - (x^6*PolyLog[2, a*x^2])/6)/3 + (x^6*PolyLog[3,
a*x^2])/6
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{\frac{a\,x^{2}\left(4x^{4}a^{2}+6a\,x^{2}+12\right)}{324}+\frac{\left(-4a^{3}x^{6}+4\right)\ln\left(-a\,x^{2}+1\right)}{108}-\frac{a^{3}x^{6}\operatorname{polylog}\left(2,a\,x^{2}\right)}{9}+\frac{a^{3}x^{6}\operatorname{polylog}\left(3,a\,x^{2}\right)}{3}}{2a^{3}}$	80

input
```
int(x^5*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(1/324*a*x^2*(4*a^2*x^4+6*a*x^2+12)+1/108*(-4*a^3*x^6+4)*ln(-a*x^2
+1)-1/9*a^3*x^6*polylog(2,a*x^2)+1/3*a^3*x^6*polylog(3,a*x^2))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^5 \operatorname{PolyLog}(3,ax^2)\,dx = -\frac{18\,a^3x^6\operatorname{Li}_2(ax^2) - 54\,a^3x^6\operatorname{polylog}(3,ax^2) - 2\,a^3x^6 - 3\,a^2x^4 - 6\,ax^2 + 6\,(a^3x^6-1)\log(-ax^2+1)}{324\,a^3}$$

input
```
integrate(x^5*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
-1/324*(18*a^3*x^6*dilog(a*x^2) - 54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6
 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*log(-a*x^2 + 1))/a^3
```

3.32.6 Sympy [F]

$$\int x^5 \operatorname{PolyLog}(3,ax^2)\,dx = \int x^5\operatorname{Li}_3(ax^2)\,dx$$

input
```
integrate(x**5*polylog(3,a*x**2),x)
```

output
```
Integral(x**5*polylog(3, a*x**2), x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^5 \operatorname{PolyLog}(3, ax^2) \, dx = -\frac{18 a^3 x^6 \operatorname{Li}_2(ax^2) - 54 a^3 x^6 \operatorname{Li}_3(ax^2) - 2 a^3 x^6 - 3 a^2 x^4 - 6 ax^2 + 6 (a^3 x^6 - 1) \log(-ax^2 + 1)}{324 a^3}$$

input
```
integrate(x^5*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-1/324*(18*a^3*x^6*dilog(a*x^2) - 54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6
 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*log(-a*x^2 + 1))/a^3
```

3.32.8 Giac [F]

$$\int x^5 \operatorname{PolyLog}(3, ax^2) \, dx = \int x^5 \operatorname{Li}_3(ax^2) \, dx$$

input
```
integrate(x^5*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^5*polylog(3, a*x^2), x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int x^5 \operatorname{PolyLog}(3, ax^2) \, dx = \frac{x^6 \operatorname{polylog}(3, a x^2)}{6} - \frac{x^6 \operatorname{polylog}(2, a x^2)}{18} + \frac{\ln(a x^2 - 1)}{54 a^3} - \frac{x^6 \ln(1 - a x^2)}{54} + \frac{x^6}{162} + \frac{x^2}{54 a^2} + \frac{x^4}{108 a}$$

input
```
int(x^5*polylog(3, a*x^2),x)
```

output
```
(x^6*polylog(3, a*x^2))/6 - (x^6*polylog(2, a*x^2))/18 + log(a*x^2 - 1)/(5
4*a^3) - (x^6*log(1 - a*x^2))/54 + x^6/162 + x^2/(54*a^2) + x^4/(108*a)
```

3.33 $\int x^3 \text{PolyLog}(3, ax^2)\, dx$

3.33.1 Optimal result

Integrand size = 11, antiderivative size = 78

$$\int x^3 \text{PolyLog}(3, ax^2)\, dx = \frac{x^2}{16a} + \frac{x^4}{32} + \frac{\log(1-ax^2)}{16a^2} - \frac{1}{16}x^4 \log(1-ax^2) - \frac{1}{8}x^4 \text{PolyLog}(2, ax^2) + \frac{1}{4}x^4 \text{PolyLog}(3, ax^2)$$

output
```
1/16*x^2/a+1/32*x^4+1/16*ln(-a*x^2+1)/a^2-1/16*x^4*ln(-a*x^2+1)-1/8*x^4*po
lylog(2,a*x^2)+1/4*x^4*polylog(3,a*x^2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int x^3 \text{PolyLog}(3, ax^2)\, dx = \frac{2ax^2 + a^2x^4 + 2\log(1-ax^2) - 2a^2x^4\log(1-ax^2) - 4a^2x^4 \text{PolyLog}(2, ax^2) + 8a^2x^4 \text{PolyLog}(3, ax^2)}{32a^2}$$

input
```
Integrate[x^3*PolyLog[3, a*x^2],x]
```

output
```
(2*a*x^2 + a^2*x^4 + 2*Log[1 - a*x^2] - 2*a^2*x^4*Log[1 - a*x^2] - 4*a^2*x
^4*PolyLog[2, a*x^2] + 8*a^2*x^4*PolyLog[3, a*x^2])/(32*a^2)
```

3.33.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 7145, 25, 2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{PolyLog}(3,ax^2)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)-\frac{1}{2}\int x^3 \operatorname{PolyLog}(2,ax^2)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -x^3\log(1-ax^2)\,dx-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int x^3\log(1-ax^2)\,dx-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int x^2\log(1-ax^2)\,dx^2-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{x^4}{1-ax^2}dx^2-\frac{1}{2}x^4\log(1-ax^2)\right)-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}a\int \left(-\frac{x^2}{a}-\frac{1}{a^2(ax^2-1)}-\frac{1}{a^2}\right)dx^2-\frac{1}{2}x^4\log(1-ax^2)\right)-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(-\frac{\log(1-ax^2)}{a^3}-\frac{x^2}{a^2}-\frac{x^4}{2a}\right)-\frac{1}{2}x^4\log(1-ax^2)\right)-\frac{1}{4}x^4 \operatorname{PolyLog}(2,ax^2)\right)+\frac{1}{4}x^4 \operatorname{PolyLog}(3,ax^2)$$

input
```
Int[x^3*PolyLog[3, a*x^2],x]
```

output
```
((-1/2*(x^4*Log[1 - a*x^2]) - (a*(-(x^2/a^2) - x^4/(2*a) - Log[1 - a*x^2]/
a^3))/2)/4 - (x^4*PolyLog[2, a*x^2])/4)/2 + (x^4*PolyLog[3, a*x^2])/4
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

method	result	size
meijerg	$-\frac{-\frac{a\,x^{2}\left(3a\,x^{2}+6\right)}{48}-\frac{\left(-3x^{4}a^{2}+3\right)\ln\left(-a\,x^{2}+1\right)}{24}+\frac{a^{2}x^{4}\operatorname{polylog}\left(2,a\,x^{2}\right)}{4}-\frac{a^{2}x^{4}\operatorname{polylog}\left(3,a\,x^{2}\right)}{2}}{2a^{2}}$	72

input
```
int(x^3*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(-1/48*a*x^2*(3*a*x^2+6)-1/24*(-3*a^2*x^4+3)*ln(-a*x^2+1)+1/4*a^2
*x^4*polylog(2,a*x^2)-1/2*a^2*x^4*polylog(3,a*x^2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{PolyLog}\left(3, ax^2\right)\, dx = -\frac{4\,a^2x^4\mathrm{Li}_2(ax^2) - 8\,a^2x^4\mathrm{polylog}(3, ax^2) - a^2x^4 - 2\,ax^2 + 2\,(a^2x^4 - 1)\log\left(-ax^2 + 1\right)}{32\,a^2}$$

input
```
integrate(x^3*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
-1/32*(4*a^2*x^4*dilog(a*x^2) - 8*a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*
a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2 + 1))/a^2
```

3.33.6 Sympy [F]

$$\int x^3 \operatorname{PolyLog}\left(3, ax^2\right)\, dx = \int x^3 \operatorname{Li}_3\left(ax^2\right)\, dx$$

input
```
integrate(x**3*polylog(3,a*x**2),x)
```

output
```
Integral(x**3*polylog(3, a*x**2), x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int x^3 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = -\frac{4\, a^2x^4\mathrm{Li}_2(ax^2) - 8\, a^2x^4\mathrm{Li}_3(ax^2) - a^2x^4 - 2\, ax^2 + 2\,(a^2x^4 - 1)\log\left(-ax^2 + 1\right)}{32\, a^2}$$

input
```
integrate(x^3*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-1/32*(4*a^2*x^4*dilog(a*x^2) - 8*a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*
a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2 + 1))/a^2
```

3.33.8 Giac [F]

$$\int x^3 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \int x^3\mathrm{Li}_3(ax^2)\, dx$$

input
```
integrate(x^3*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^3*polylog(3, a*x^2), x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int x^3 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \frac{x^4\,\mathrm{polylog}(3, a\,x^2)}{4} - \frac{x^4\,\mathrm{polylog}(2, a\,x^2)}{8} + \frac{\ln\left(a\,x^2 - 1\right)}{16\,a^2} - \frac{x^4\,\ln\left(1 - a\,x^2\right)}{16} + \frac{x^4}{32} + \frac{x^2}{16\,a}$$

input
```
int(x^3*polylog(3, a*x^2),x)
```

output
```
(x^4*polylog(3, a*x^2))/4 - (x^4*polylog(2, a*x^2))/8 + log(a*x^2 - 1)/(16
*a^2) - (x^4*log(1 - a*x^2))/16 + x^4/32 + x^2/(16*a)
```

3.34 $\int x \,\text{PolyLog}\left(3, ax^2\right) \, dx$

3.34.1 Optimal result

Integrand size = 9, antiderivative size = 60

$$\int x \,\text{PolyLog}\left(3, ax^2\right) \, dx = \frac{x^2}{2} + \frac{(1 - ax^2) \log(1 - ax^2)}{2a} - \frac{1}{2}x^2 \,\text{PolyLog}\left(2, ax^2\right) + \frac{1}{2}x^2 \,\text{PolyLog}\left(3, ax^2\right)$$

output
```
1/2*x^2+1/2*(-a*x^2+1)*ln(-a*x^2+1)/a-1/2*x^2*polylog(2,a*x^2)+1/2*x^2*pol
ylog(3,a*x^2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x \,\text{PolyLog}\left(3, ax^2\right) \, dx = \frac{1}{2}x^2 \left(1 - \log\left(1 - ax^2\right) + \frac{\log(1 - ax^2)}{ax^2} - \text{PolyLog}\left(2, ax^2\right) + \text{PolyLog}\left(3, ax^2\right)\right)$$

input
```
Integrate[x*PolyLog[3, a*x^2],x]
```

output
```
(x^2*(1 - Log[1 - a*x^2] + Log[1 - a*x^2]/(a*x^2) - PolyLog[2, a*x^2] + Po
lyLog[3, a*x^2]))/2
```

3.34.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 7145, 25, 2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(3, ax^2)\, dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2) - \int x \operatorname{PolyLog}(2, ax^2)\, dx$$

$$\Big\downarrow \ 7145$$

$$\int -x \log(1 - ax^2)\, dx - \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2)$$

$$\Big\downarrow \ 25$$

$$-\int x \log(1 - ax^2)\, dx - \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2)$$

$$\Big\downarrow \ 2904$$

$$-\frac{1}{2}\int \log(1 - ax^2)\, dx^2 - \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2)$$

$$\Big\downarrow \ 2836$$

$$\frac{\int \log(1 - ax^2)\, d(1 - ax^2)}{2a} - \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2)$$

$$\Big\downarrow \ 2732$$

$$-\frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^2) + \frac{ax^2 + (1 - ax^2)\log(1 - ax^2) - 1}{2a}$$

input
```
Int[x*PolyLog[3, a*x^2],x]
```

output
```
(-1 + a*x^2 + (1 - a*x^2)*Log[1 - a*x^2])/(2*a) - (x^2*PolyLog[2, a*x^2])/
2 + (x^2*PolyLog[3, a*x^2])/2
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
meijerg	$\frac{a\,x^2+\frac{\left(-2a\,x^2+2\right)\ln\left(-a\,x^2+1\right)}{2}-a\,x^2\operatorname{polylog}\left(2,a\,x^2\right)+x^2a\operatorname{polylog}\left(3,a\,x^2\right)}{2a}$	56

input
```
int(x*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(a*x^2+1/2*(-2*a*x^2+2)*ln(-a*x^2+1)-a*x^2*polylog(2,a*x^2)+x^2*a*po
lylog(3,a*x^2))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int x \operatorname{PolyLog}\left(3, ax^2\right) dx$$
$$= -\frac{ax^2 \mathrm{Li}_2(ax^2) - ax^2 \mathrm{polylog}(3, ax^2) - ax^2 + (ax^2 - 1) \log\left(-ax^2 + 1\right)}{2a}$$

input
```
integrate(x*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
-1/2*(a*x^2*dilog(a*x^2) - a*x^2*polylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*l
og(-a*x^2 + 1))/a
```

3.34.6 Sympy [F]

$$\int x \operatorname{PolyLog}\left(3, ax^2\right) dx = \int x \operatorname{Li}_3\left(ax^2\right) dx$$

input
```
integrate(x*polylog(3,a*x**2),x)
```

output
```
Integral(x*polylog(3, a*x**2), x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int x \operatorname{PolyLog}\left(3, ax^2\right) dx = -\frac{ax^2 \mathrm{Li}_2(ax^2) - ax^2 \mathrm{Li}_3(ax^2) - ax^2 + (ax^2 - 1) \log\left(-ax^2 + 1\right)}{2a}$$

input
```
integrate(x*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-1/2*(a*x^2*dilog(a*x^2) - a*x^2*polylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*l
og(-a*x^2 + 1))/a
```

3.34.8 Giac [F]

$$\int x\,\mathrm{PolyLog}\left(3,ax^2\right)\,dx=\int x\mathrm{Li}_3(ax^2)\,dx$$

input
```
integrate(x*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(x*polylog(3, a*x^2), x)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x\,\mathrm{PolyLog}\left(3,ax^2\right)\,dx=\frac{x^2\,\mathrm{polylog}(3,a\,x^2)}{2}-\frac{x^2\,\mathrm{polylog}(2,a\,x^2)}{2}+\frac{\ln\left(a\,x^2-1\right)}{2\,a}-\frac{x^2\,\ln\left(1-a\,x^2\right)}{2}+\frac{x^2}{2}$$

input
```
int(x*polylog(3, a*x^2),x)
```

output
```
(x^2*polylog(3, a*x^2))/2 - (x^2*polylog(2, a*x^2))/2 + log(a*x^2 - 1)/(2*
a) - (x^2*log(1 - a*x^2))/2 + x^2/2
```

3.35 $\int \frac{\text{PolyLog}(3,ax^2)}{x}\,dx$

3.35.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(3,ax^2)}{x}\,dx = \frac{\text{PolyLog}(4,ax^2)}{2}$$

output
```
1/2*polylog(4,a*x^2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(3,ax^2)}{x}\,dx = \frac{\text{PolyLog}(4,ax^2)}{2}$$

input
```
Integrate[PolyLog[3, a*x^2]/x,x]
```

output
```
PolyLog[4, a*x^2]/2
```

3.35.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3, ax^2)}{x} \, dx$$

$$\Big\downarrow \; 7143$$

$$\frac{\text{PolyLog}(4, ax^2)}{2}$$

input
```
Int[PolyLog[3, a*x^2]/x,x]
```

output
```
PolyLog[4, a*x^2]/2
```

3.35.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.35.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{\operatorname{polylog}(4, a\,x^2)}{2}$	10

input
```
int(polylog(3,a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*polylog(4,a*x^2)
```

3.35.5 Fricas [F]

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x}\,dx=\int \frac{\text{Li}_3(ax^2)}{x}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x^2)/x, x)
```

3.35.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x}\,dx=\int \frac{\text{Li}_3\left(ax^2\right)}{x}\,dx$$

input
```
integrate(polylog(3,a*x**2)/x,x)
```

output
```
Integral(polylog(3, a*x**2)/x, x)
```

3.35.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x}\,dx=\int \frac{\text{Li}_3(ax^2)}{x}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(polylog(3, a*x^2)/x, x)
```

3.35.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x}\,dx = \int \frac{\text{Li}_3(ax^2)}{x}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x, x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x}\,dx = \frac{\text{polylog}(4,a\,x^2)}{2}$$

input
```
int(polylog(3, a*x^2)/x,x)
```

output
```
polylog(4, a*x^2)/2
```

3.36 $\int \frac{\text{PolyLog}(3,ax^2)}{x^3}\,dx$

3.36.1 Optimal result

Integrand size = 11, antiderivative size = 63

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^3}\,dx = a\log(x) - \frac{1}{2}a\log\left(1-ax^2\right) + \frac{\log(1-ax^2)}{2x^2} - \frac{\text{PolyLog}(2,ax^2)}{2x^2} - \frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

output
```
a*ln(x)-1/2*a*ln(-a*x^2+1)+1/2*ln(-a*x^2+1)/x^2-1/2*polylog(2,a*x^2)/x^2-1
/2*polylog(3,a*x^2)/x^2
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^3}\,dx = -\frac{-ax^2\log(-ax^2) - \log(1-ax^2) + ax^2\log(1-ax^2) + \text{PolyLog}(2,ax^2) + \text{PolyLog}(3,ax^2)}{2x^2}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^3,x]
```

output
```
-1/2*(-(a*x^2*Log[-(a*x^2)]) - Log[1 - a*x^2] + a*x^2*Log[1 - a*x^2] + Pol
yLog[2, a*x^2] + PolyLog[3, a*x^2])/x^2
```

3.36.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {7145, 7145, 25, 2904, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^3}dx$$

$$\Big\downarrow \text{7145}$$

$$\int \frac{\text{PolyLog}(2,ax^2)}{x^3}dx-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{7145}$$

$$\int -\frac{\log(1-ax^2)}{x^3}dx-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\log(1-ax^2)}{x^3}dx-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{2}\int \frac{\log(1-ax^2)}{x^4}dx^2-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{1-ax^2}dx^2+\int \frac{1}{x^2}dx^2\right)+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{1-ax^2}dx^2+\log(x^2)\right)+\frac{\log(1-ax^2)}{x^2}\right)-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}+\frac{1}{2}\left(a(\log(x^2)-\log(1-ax^2))+\frac{\log(1-ax^2)}{x^2}\right)$$

input
```
Int[PolyLog[3, a*x^2]/x^3,x]
```

output
```
(a*(Log[x^2] - Log[1 - a*x^2]) + Log[1 - a*x^2]/x^2)/2 - PolyLog[2, a*x^2]
/(2*x^2) - PolyLog[3, a*x^2]/(2*x^2)
```

3.36.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

method	result	size
meijerg	$\frac{a\left(\frac{\left(-8a\,x^2+8\right)\ln\left(-a\,x^2+1\right)}{8a\,x^2}-\frac{\operatorname{polylog}\left(2,a\,x^2\right)}{a\,x^2}-\frac{\operatorname{polylog}\left(3,a\,x^2\right)}{a\,x^2}+2\ln(x)+\ln(-a)\right)}{2}$	68

input
```
int(polylog(3,a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(1/8/a/x^2*(-8*a*x^2+8)*ln(-a*x^2+1)-1/a/x^2*polylog(2,a*x^2)-1/a/x^
2*polylog(3,a*x^2)+2*ln(x)+ln(-a))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int\frac{\text{PolyLog}\left(3,ax^2\right)}{x^3}\,dx$$
$$=-\frac{ax^2\log\left(ax^2-1\right)-2\,ax^2\log\left(x\right)+\text{Li}_2(ax^2)-\log\left(-ax^2+1\right)+\text{polylog}(3,ax^2)}{2\,x^2}$$

input
```
integrate(polylog(3,a*x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + dilog(a*x^2) - log(-a*x^2 +
1) + polylog(3, a*x^2))/x^2
```

3.36.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^3} \, dx = \int \frac{\text{Li}_3(ax^2)}{x^3} \, dx$$

input
```
integrate(polylog(3,a*x**2)/x**3,x)
```

output
```
Integral(polylog(3, a*x**2)/x**3, x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^3} \, dx = a \log(x) - \frac{(ax^2 - 1) \log(-ax^2 + 1) + \text{Li}_2(ax^2) + \text{Li}_3(ax^2)}{2 x^2}$$

input
```
integrate(polylog(3,a*x^2)/x^3,x, algorithm="maxima")
```

output
```
a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2) + polylog(3, a*
x^2))/x^2
```

3.36.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^3} \, dx = \int \frac{\text{Li}_3(ax^2)}{x^3} \, dx$$

input
```
integrate(polylog(3,a*x^2)/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^3, x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^3} \, dx = -\frac{\text{polylog}(2, a\,x^2) - \ln(1 - a\,x^2) + \text{polylog}(3, a\,x^2) - 3\,a\,x^2\,\ln(x) + a\,x^2\,\ln(x\,(a\,x^2 - 1))}{2\,x^2}$$

input
```
int(polylog(3, a*x^2)/x^3,x)
```

output
```
-(polylog(2, a*x^2) - log(1 - a*x^2) + polylog(3, a*x^2) - 3*a*x^2*log(x)
+ a*x^2*log(x*(a*x^2 - 1)))/(2*x^2)
```

3.37 $\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx$

3.37.1 Optimal result

Integrand size = 11, antiderivative size = 78

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx = -\frac{a}{16x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{16}a^2\log(1-ax^2)+\frac{\log(1-ax^2)}{16x^4}-\frac{\text{PolyLog}(2,ax^2)}{8x^4}-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

output
```
-1/16*a/x^2+1/8*a^2*ln(x)-1/16*a^2*ln(-a*x^2+1)+1/16*ln(-a*x^2+1)/x^4-1/8*
polylog(2,a*x^2)/x^4-1/4*polylog(3,a*x^2)/x^4
```

3.37.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx = \frac{G_{5,5}^{2,4}\left(-ax^2\middle|\begin{matrix}1,1,1,1,3\\1,2,0,0,0\end{matrix}\right)}{2x^4}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^5,x]
```

output
```
MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x^2)]/(2*x^4)
```

3.37.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 7145, 25, 2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\int \frac{\text{PolyLog}(2,ax^2)}{x^5}dx-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{\log(1-ax^2)}{x^5}dx-\frac{\text{PolyLog}(2,ax^2)}{4x^4}\right)-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{\log(1-ax^2)}{x^5}dx-\frac{\text{PolyLog}(2,ax^2)}{4x^4}\right)-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{\log(1-ax^2)}{x^6}dx^2-\frac{\text{PolyLog}(2,ax^2)}{4x^4}\right)-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}a\int \frac{1}{x^4(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{2x^4}\right)-\frac{\text{PolyLog}(2,ax^2)}{4x^4}\right)-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}a\int \left(-\frac{a^2}{ax^2-1}+\frac{a}{x^2}+\frac{1}{x^4}\right)dx^2+\frac{\log(1-ax^2)}{2x^4}\right)-\frac{\text{PolyLog}(2,ax^2)}{4x^4}\right)-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}a\left(a\log\left(x^2\right)-a\log\left(1-ax^2\right)-\frac{1}{x^2}\right)+\frac{\log\left(1-ax^2\right)}{2x^4}\right)-\frac{\text{PolyLog}\left(2,ax^2\right)}{4x^4}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{4x^4}$$

input
```
Int[PolyLog[3, a*x^2]/x^5,x]
```

output
```
((Log[1 - a*x^2]/(2*x^4) + (a*(-x^(-2) + a*Log[x^2] - a*Log[1 - a*x^2]))/2
)/4 - PolyLog[2, a*x^2]/(4*x^4))/2 - PolyLog[3, a*x^2]/(4*x^4)
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

method	result	size
meijerg	$-\frac{a^{2}\left(-\frac{81 a\,x^{2}+378}{432 a\,x^{2}}-\frac{\left(-27 x^{4} a^{2}+27\right) \ln\left(-a\,x^{2}+1\right)}{216 a^{2} x^{4}}+\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{4 a^{2} x^{4}}+\frac{\operatorname{polylog}\left(3,a\,x^{2}\right)}{2 a^{2} x^{4}}+\frac{3}{16}-\frac{\ln\left(x\right)}{4}-\frac{\ln\left(-a\right)}{8}+\frac{1}{a\,x^{2}}\right)}{2}$	98

input
```
int(polylog(3,a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(-1/432/a/x^2*(81*a*x^2+378)-1/216/a^2/x^4*(-27*a^2*x^4+27)*ln(-a
*x^2+1)+1/4/a^2/x^4*polylog(2,a*x^2)+1/2/a^2/x^4*polylog(3,a*x^2)+3/16-1/4
*ln(x)-1/8*ln(-a)+1/a/x^2)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x^5}\,dx = -\frac{a^2x^4\log\left(ax^2-1\right)-2\,a^2x^4\log\left(x\right)+ax^2+2\,\text{Li}_2(ax^2)-\log\left(-ax^2+1\right)+4\,\text{polylog}(3,ax^2)}{16\,x^4}$$

input
```
integrate(polylog(3,a*x^2)/x^5,x, algorithm="fricas")
```

output
```
-1/16*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*dilog(a*x^2)
- log(-a*x^2 + 1) + 4*polylog(3, a*x^2))/x^4
```

3.37.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^5}\,dx$$

input
```
integrate(polylog(3,a*x**2)/x**5,x)
```

output
```
Integral(polylog(3, a*x**2)/x**5, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx = \frac{1}{8}a^2\log(x) - \frac{ax^2 + (a^2x^4-1)\log(-ax^2+1) + 2\,\text{Li}_2(ax^2) + 4\,\text{Li}_3(ax^2)}{16\,x^4}$$

input
```
integrate(polylog(3,a*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/8*a^2*log(x) - 1/16*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x
^2) + 4*polylog(3, a*x^2))/x^4
```

3.37.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^5}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^5, x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^5} \, dx = \frac{a^2 \ln(x)}{8} - \frac{\text{polylog}(2, a\,x^2)}{8\,x^4} - \frac{\text{polylog}(3, a\,x^2)}{4\,x^4} - \frac{a^2 \ln(a\,x^2 - 1)}{16} - \frac{a}{16\,x^2} + \frac{\ln(1 - a\,x^2)}{16\,x^4}$$

input
```
int(polylog(3, a*x^2)/x^5,x)
```

output
```
(a^2*log(x))/8 - polylog(2, a*x^2)/(8*x^4) - polylog(3, a*x^2)/(4*x^4) - (
a^2*log(a*x^2 - 1))/16 - a/(16*x^2) + log(1 - a*x^2)/(16*x^4)
```

3.38 $\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx$

3.38.1 Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = -\frac{a}{108x^4}-\frac{a^2}{54x^2}+\frac{1}{27}a^3\log(x)-\frac{1}{54}a^3\log(1-ax^2)+\frac{\log(1-ax^2)}{54x^6}-\frac{\text{PolyLog}(2,ax^2)}{18x^6}-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

output
```
-1/108*a/x^4-1/54*a^2/x^2+1/27*a^3*ln(x)-1/54*a^3*ln(-a*x^2+1)+1/54*ln(-a*
x^2+1)/x^6-1/18*polylog(2,a*x^2)/x^6-1/6*polylog(3,a*x^2)/x^6
```

3.38.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.34

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = \frac{G_{5,5}^{2,4}\left(-ax^2\middle|\begin{array}{c}1,1,1,1,4\\1,3,0,0,0\end{array}\right)}{2x^6}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^7,x]
```

output
```
MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x^2)]/(2*x^6)
```

3.38.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 7145, 25, 2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\int \frac{\text{PolyLog}(2,ax^2)}{x^7}dx-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}\left(\frac{1}{3}\int -\frac{\log(1-ax^2)}{x^7}dx-\frac{\text{PolyLog}(2,ax^2)}{6x^6}\right)-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-\frac{1}{3}\int \frac{\log(1-ax^2)}{x^7}dx-\frac{\text{PolyLog}(2,ax^2)}{6x^6}\right)-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\left(-\frac{1}{6}\int \frac{\log(1-ax^2)}{x^8}dx^2-\frac{\text{PolyLog}(2,ax^2)}{6x^6}\right)-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(\frac{1}{3}a\int \frac{1}{x^6(1-ax^2)}dx^2+\frac{\log(1-ax^2)}{3x^6}\right)-\frac{\text{PolyLog}(2,ax^2)}{6x^6}\right)-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(\frac{1}{3}a\int \left(-\frac{a^3}{ax^2-1}+\frac{a^2}{x^2}+\frac{a}{x^4}+\frac{1}{x^6}\right)dx^2+\frac{\log(1-ax^2)}{3x^6}\right)-\frac{\text{PolyLog}(2,ax^2)}{6x^6}\right)-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{6}\left(\frac{1}{3}a\left(a^2\log\left(x^2\right)-a^2\log\left(1-ax^2\right)-\frac{a}{x^2}-\frac{1}{2x^4}\right)+\frac{\log\left(1-ax^2\right)}{3x^6}\right)-\frac{\text{PolyLog}\left(2,ax^2\right)}{6x^6}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{6x^6}$$

input
```
Int[PolyLog[3, a*x^2]/x^7,x]
```

output
```
((Log[1 - a*x^2]/(3*x^6) + (a*(-1/2*1/x^4 - a/x^2 + a^2*Log[x^2] - a^2*Log
[1 - a*x^2]))/3)/6 - PolyLog[2, a*x^2]/(6*x^6))/3 - PolyLog[3, a*x^2]/(6*x
^6)
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

method	result	si
meijerg	$\frac{a^{3}\left(\frac{64x^{4}a^{2}+152a\,x^{2}+832}{1728a^{2}x^{4}}+\frac{\left(-64a^{3}x^{6}+64\right)\ln\left(-a\,x^{2}+1\right)}{1728a^{3}x^{6}}-\frac{\operatorname{polylog}\left(2,a\,x^{2}\right)}{9a^{3}x^{6}}-\frac{\operatorname{polylog}\left(3,a\,x^{2}\right)}{3a^{3}x^{6}}-\frac{1}{27}+\frac{2\ln\left(x\right)}{27}+\frac{\ln\left(-a\right)}{27}-\frac{1}{2a^{2}x^{4}}-\frac{1}{8a\,x^{2}}\right)}{2}$	11

input
```
int(polylog(3,a*x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*(1/1728/a^2/x^4*(64*a^2*x^4+152*a*x^2+832)+1/1728/a^3/x^6*(-64*a^3
*x^6+64)*ln(-a*x^2+1)-1/9/a^3/x^6*polylog(2,a*x^2)-1/3/a^3/x^6*polylog(3,a
*x^2)-1/27+2/27*ln(x)+1/27*ln(-a)-1/2/a^2/x^4-1/8/a/x^2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x^7}\,dx = -\frac{2\,a^3x^6\log\left(ax^2-1\right)-4\,a^3x^6\log\left(x\right)+2\,a^2x^4+ax^2+6\,\mathrm{Li}_2(ax^2)-2\,\log\left(-ax^2+1\right)+18\,\mathrm{polylog}(3,ax}{108\,x^6}$$

input
```
integrate(polylog(3,a*x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/108*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 +
6*dilog(a*x^2) - 2*log(-a*x^2 + 1) + 18*polylog(3, a*x^2))/x^6
```

3.38.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^7}\,dx$$

input
```
integrate(polylog(3,a*x**2)/x**7,x)
```

output
```
Integral(polylog(3, a*x**2)/x**7, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = \frac{1}{27}\,a^3\log(x) - \frac{2\,a^2x^4 + ax^2 + 2\,(a^3x^6 - 1)\log(-ax^2 + 1) + 6\,\text{Li}_2(ax^2) + 18\,\text{Li}_3(ax^2)}{108\,x^6}$$

input
```
integrate(polylog(3,a*x^2)/x^7,x, algorithm="maxima")
```

output
```
1/27*a^3*log(x) - 1/108*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 +
1) + 6*dilog(a*x^2) + 18*polylog(3, a*x^2))/x^6
```

3.38.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^7}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^7, x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx = \frac{a^3\,\ln(x)}{27} - \frac{\text{polylog}(2,a\,x^2)}{18\,x^6} - \frac{\text{polylog}(3,a\,x^2)}{6\,x^6} - \frac{a^3\,\ln(a\,x^2-1)}{54} - \frac{a}{108\,x^4} + \frac{\ln(1-a\,x^2)}{54\,x^6} - \frac{a^2}{54\,x^2}$$

input
```
int(polylog(3, a*x^2)/x^7,x)
```

output
```
(a^3*log(x))/27 - polylog(2, a*x^2)/(18*x^6) - polylog(3, a*x^2)/(6*x^6) -
 (a^3*log(a*x^2 - 1))/54 - a/(108*x^4) + log(1 - a*x^2)/(54*x^6) - a^2/(54
*x^2)
```

3.39 $\int x^4 \operatorname{PolyLog}(3, ax^2) \, dx$

3.39.1 Optimal result

Integrand size = 11, antiderivative size = 87

$$\int x^4 \operatorname{PolyLog}(3, ax^2) \, dx = \frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{8\operatorname{arctanh}(\sqrt{a}x)}{125a^{5/2}} - \frac{4}{125}x^5 \log(1 - ax^2) - \frac{2}{25}x^5 \operatorname{PolyLog}(2, ax^2) + \frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2)$$

output
```
8/125*x/a^2+8/375*x^3/a+8/625*x^5-8/125*arctanh(x*a^(1/2))/a^(5/2)-4/125*x
^5*ln(-a*x^2+1)-2/25*x^5*polylog(2,a*x^2)+1/5*x^5*polylog(3,a*x^2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int x^4 \operatorname{PolyLog}(3, ax^2) \, dx = \frac{\frac{120x}{a^2} + \frac{40x^3}{a} + 24x^5 - \frac{120\operatorname{arctanh}(\sqrt{a}x)}{a^{5/2}} - 60x^5 \log(1 - ax^2) - 150x^5 \operatorname{PolyLog}(2, ax^2) + 375x^5 \operatorname{PolyLog}(3,}{1875}$$

input
```
Integrate[x^4*PolyLog[3, a*x^2],x]
```

output
```
((120*x)/a^2 + (40*x^3)/a + 24*x^5 - (120*ArcTanh[Sqrt[a]*x])/a^(5/2) - 60
*x^5*Log[1 - a*x^2] - 150*x^5*PolyLog[2, a*x^2] + 375*x^5*PolyLog[3, a*x^2
])/1875
```

3.39.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 7145, 25, 2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \operatorname{PolyLog}(3, ax^2)\, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\int x^4 \operatorname{PolyLog}(2, ax^2)\, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\left(\frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2) - \frac{2}{5}\int -x^4 \log(1-ax^2)\, dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\left(\frac{2}{5}\int x^4 \log(1-ax^2)\, dx + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2)\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\left(\frac{2}{5}\left(\frac{2}{5}a\int \frac{x^6}{1-ax^2}dx + \frac{1}{5}x^5 \log(1-ax^2)\right) + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2)\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\left(\frac{2}{5}\left(\frac{2}{5}a\int \left(-\frac{x^4}{a} - \frac{x^2}{a^2} + \frac{1}{a^3(1-ax^2)} - \frac{1}{a^3}\right)dx + \frac{1}{5}x^5 \log(1-ax^2)\right) + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5 \operatorname{PolyLog}(3, ax^2) - \frac{2}{5}\left(\frac{2}{5}\left(\frac{2}{5}a\left(\frac{\operatorname{arctanh}(\sqrt{a}x)}{a^{7/2}} - \frac{x}{a^3} - \frac{x^3}{3a^2} - \frac{x^5}{5a}\right) + \frac{1}{5}x^5 \log(1-ax^2)\right) + \frac{1}{5}x^5 \operatorname{PolyLog}(2, ax^2)\right)$$

input
```
Int[x^4*PolyLog[3, a*x^2],x]
```

output
```
(-2*((2*((2*a*(-(x/a^3) - x^3/(3*a^2) - x^5/(5*a) + ArcTanh[Sqrt[a]*x]/a^(
7/2)))/5 + (x^5*Log[1 - a*x^2])/5))/5 + (x^5*PolyLog[2, a*x^2])/5))/5 + (x
^5*PolyLog[3, a*x^2])/5
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(69) = 138$.

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

method	result
meijerg	$-\frac{\frac{2x(-a)^{\frac{7}{2}}\left(168x^4a^2+280a\,x^2+840\right)}{13125a^3}+\frac{8x(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{125a^3\sqrt{a\,x^2}}-\frac{8x^5(-a)^{\frac{7}{2}}\ln\left(-a\,x^2+1\right)}{125a}-\frac{4x^5(-a)^{\frac{7}{2}}\operatorname{polylog}\left(2,a\,x^2\right)}{25a}+\frac{2x^5(-}{}}{2a^2\sqrt{-a}}$

input
```
int(x^4*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/(-a)^(1/2)*(2/13125*x*(-a)^(7/2)*(168*a^2*x^4+280*a*x^2+840)/a^3+
8/125*x*(-a)^(7/2)/a^3/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/
2)))-8/125*x^5*(-a)^(7/2)*ln(-a*x^2+1)/a-4/25*x^5*(-a)^(7/2)/a*polylog(2,a
*x^2)+2/5*x^5*(-a)^(7/2)/a*polylog(3,a*x^2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.17

$$\int x^4 \operatorname{PolyLog}\left(3, ax^2\right) \, dx$$
$$= \left[-\frac{150\, a^3 x^5 \mathrm{Li}_2(ax^2) + 60\, a^3 x^5 \log\left(-ax^2+1\right) - 375\, a^3 x^5 \mathrm{polylog}(3, ax^2) - 24\, a^3 x^5 - 40\, a^2 x^3 - 120\, ax -}{1875\, a^3} \right.$$
$$- \frac{150\, a^3 x^5 \mathrm{Li}_2(ax^2) + 60\, a^3 x^5 \log\left(-ax^2+1\right) - 375\, a^3 x^5 \mathrm{polylog}(3, ax^2) - 24\, a^3 x^5 - 40\, a^2 x^3 - 120\, ax - 1}{1875\, a^3}$$

input
```
integrate(x^4*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
[-1/1875*(150*a^3*x^5*dilog(a*x^2) + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*
x^5*polylog(3, a*x^2) - 24*a^3*x^5 - 40*a^2*x^3 - 120*a*x - 60*sqrt(a)*log
((a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^3, -1/1875*(150*a^3*x^5*dilog(a
*x^2) + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*x^5*polylog(3, a*x^2) - 24*a^
3*x^5 - 40*a^2*x^3 - 120*a*x - 120*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

3.39.6 Sympy [F]

$$\int x^4 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \int x^4 \operatorname{Li}_3\left(ax^2\right) \, dx$$

input
```
integrate(x**4*polylog(3,a*x**2),x)
```

output
```
Integral(x**4*polylog(3, a*x**2), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int x^4 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = -\frac{150 \, a^2 x^5 \operatorname{Li}_2(ax^2) + 60 \, a^2 x^5 \log\left(-ax^2 + 1\right) - 375 \, a^2 x^5 \operatorname{Li}_3(ax^2) - 24 \, a^2 x^5 - 40 \, a x^3 - 120 \, x}{1875 \, a^2} + \frac{4 \, \log\left(\frac{ax - \sqrt{a}}{ax + \sqrt{a}}\right)}{125 \, a^{\frac{5}{2}}}$$

input
```
integrate(x^4*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-1/1875*(150*a^2*x^5*dilog(a*x^2) + 60*a^2*x^5*log(-a*x^2 + 1) - 375*a^2*x
^5*polylog(3, a*x^2) - 24*a^2*x^5 - 40*a*x^3 - 120*x)/a^2 + 4/125*log((a*x
 - sqrt(a))/(a*x + sqrt(a)))/a^(5/2)
```

3.39.8 Giac [F]

$$\int x^4 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \int x^4 \operatorname{Li}_3(ax^2) \, dx$$

input
```
integrate(x^4*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^4*polylog(3, a*x^2), x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int x^4 \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \frac{x^5 \operatorname{polylog}(3, a\,x^2)}{5} - \frac{2\,x^5 \operatorname{polylog}(2, a\,x^2)}{25} + \frac{8\,x}{125\,a^2} - \frac{4\,x^5 \ln\left(1 - a\,x^2\right)}{125} + \frac{8\,x^5}{625} + \frac{8\,x^3}{375\,a} + \frac{\operatorname{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,8\mathrm{i}}{125\,a^{5/2}}$$

input
```
int(x^4*polylog(3, a*x^2),x)
```

output
```
(atan(a^(1/2)*x*1i)*8i)/(125*a^(5/2)) - (2*x^5*polylog(2, a*x^2))/25 + (x^
5*polylog(3, a*x^2))/5 + (8*x)/(125*a^2) - (4*x^5*log(1 - a*x^2))/125 + (8
*x^5)/625 + (8*x^3)/(375*a)
```

3.39. $\int x^4 \,\text{PolyLog}\left(3, a x^2\right) \, dx$

3.40 $\int x^2 \operatorname{PolyLog}(3, ax^2)\, dx$

3.40.1 Optimal result

Integrand size = 11, antiderivative size = 77

$$\int x^2 \operatorname{PolyLog}(3, ax^2)\, dx = \frac{8x}{27a} + \frac{8x^3}{81} - \frac{8\operatorname{arctanh}(\sqrt{a}x)}{27a^{3/2}} - \frac{4}{27}x^3 \log(1 - ax^2) - \frac{2}{9}x^3 \operatorname{PolyLog}(2, ax^2) + \frac{1}{3}x^3 \operatorname{PolyLog}(3, ax^2)$$

output
```
8/27*x/a+8/81*x^3-8/27*arctanh(x*a^(1/2))/a^(3/2)-4/27*x^3*ln(-a*x^2+1)-2/
9*x^3*polylog(2,a*x^2)+1/3*x^3*polylog(3,a*x^2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int x^2 \operatorname{PolyLog}(3, ax^2)\, dx = \frac{1}{81}\left(\frac{24x}{a} + 8x^3 - \frac{24\operatorname{arctanh}(\sqrt{a}x)}{a^{3/2}} - 12x^3 \log(1 - ax^2) - 18x^3 \operatorname{PolyLog}(2, ax^2) + 27x^3 \operatorname{PolyLog}(3, ax^2)\right)$$

input
```
Integrate[x^2*PolyLog[3, a*x^2],x]
```

output
```
((24*x)/a + 8*x^3 - (24*ArcTanh[Sqrt[a]*x])/a^(3/2) - 12*x^3*Log[1 - a*x^2
] - 18*x^3*PolyLog[2, a*x^2] + 27*x^3*PolyLog[3, a*x^2])/81
```

3.40.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 7145, 25, 2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{PolyLog}\left(3,ax^2\right) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)-\frac{2}{3}\int x^2 \text{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)-\frac{2}{3}\left(\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)-\frac{2}{3}\int -x^2\log\left(1-ax^2\right)dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)-\frac{2}{3}\left(\frac{2}{3}\int x^2\log\left(1-ax^2\right)dx+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}a\int \frac{x^4}{1-ax^2}dx+\frac{1}{3}x^3\log\left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)- \frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}a\int \left(-\frac{x^2}{a}+\frac{1}{a^2\left(1-ax^2\right)}-\frac{1}{a^2}\right)dx+\frac{1}{3}x^3\log\left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3 \text{PolyLog}\left(3,ax^2\right)- \frac{2}{3}\left(\frac{2}{3}\left(\frac{2}{3}a\left(\frac{\text{arctanh}\left(\sqrt{a}x\right)}{a^{5/2}}-\frac{x}{a^2}-\frac{x^3}{3a}\right)+\frac{1}{3}x^3\log\left(1-ax^2\right)\right)+\frac{1}{3}x^3 \text{PolyLog}\left(2,ax^2\right)\right)$$

input
```
Int[x^2*PolyLog[3, a*x^2],x]
```

output
```
(-2*((2*((2*a*(-(x/a^2) - x^3/(3*a) + ArcTanh[Sqrt[a]*x]/a^(5/2)))/3 + (x^
3*Log[1 - a*x^2])/3))/3 + (x^3*PolyLog[2, a*x^2])/3))/3 + (x^3*PolyLog[3,
a*x^2])/3
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(61) = 122$.

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

method	result
meijerg	$\frac{\frac{2x(-a)^{\frac{5}{2}}\left(40a\,x^{2}+120\right)}{405a^{2}}+\frac{8x(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{a\,x^{2}}\right)-\ln\left(1+\sqrt{a\,x^{2}}\right)\right)}{27a^{2}\sqrt{a\,x^{2}}}-\frac{8x^{3}(-a)^{\frac{5}{2}}\ln\left(-a\,x^{2}+1\right)}{27a}-\frac{4x^{3}(-a)^{\frac{5}{2}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{9a}+\frac{2x^{3}(-a)^{\frac{5}{2}}\operatorname{polylog}(}{3a}}{2a\sqrt{-a}}$

input
```
int(x^2*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(-a)^(1/2)*(2/405*x*(-a)^(5/2)*(40*a*x^2+120)/a^2+8/27*x*(-a)^(5/2)/
a^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8/27*x^3*(-a)^
(5/2)*ln(-a*x^2+1)/a-4/9*x^3*(-a)^(5/2)/a*polylog(2,a*x^2)+2/3*x^3*(-a)^(5
/2)/a*polylog(3,a*x^2))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.25

$$\int x^2 \operatorname{PolyLog}\left(3, ax^2\right) dx$$
$$= \left[-\frac{18\,a^2x^3\mathrm{Li}_2(ax^2) + 12\,a^2x^3\log\left(-ax^2+1\right) - 27\,a^2x^3\mathrm{polylog}(3,ax^2) - 8\,a^2x^3 - 24\,ax - 12\,\sqrt{a}\log\left(\frac{ax^2-}{a}\right.}{81\,a^2}\right.$$
$$-\frac{18\,a^2x^3\mathrm{Li}_2(ax^2) + 12\,a^2x^3\log\left(-ax^2+1\right) - 27\,a^2x^3\mathrm{polylog}(3,ax^2) - 8\,a^2x^3 - 24\,ax - 24\,\sqrt{-a}\arctan}{81\,a^2}$$

input
```
integrate(x^2*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
[-1/81*(18*a^2*x^3*dilog(a*x^2) + 12*a^2*x^3*log(-a*x^2 + 1) - 27*a^2*x^3*
polylog(3, a*x^2) - 8*a^2*x^3 - 24*a*x - 12*sqrt(a)*log((a*x^2 - 2*sqrt(a)
*x + 1)/(a*x^2 - 1)))/a^2, -1/81*(18*a^2*x^3*dilog(a*x^2) + 12*a^2*x^3*log
(-a*x^2 + 1) - 27*a^2*x^3*polylog(3, a*x^2) - 8*a^2*x^3 - 24*a*x - 24*sqrt
(-a)*arctan(sqrt(-a)*x))/a^2]
```

3.40.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}\left(3, ax^2\right) dx = \int x^2 \operatorname{Li}_3\left(ax^2\right) dx$$

input
```
integrate(x**2*polylog(3,a*x**2),x)
```

output
```
Integral(x**2*polylog(3, a*x**2), x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int x^2 \operatorname{PolyLog}(3, ax^2) \, dx$$
$$= -\frac{18\,ax^3\mathrm{Li}_2(ax^2) + 12\,ax^3\log(-ax^2+1) - 27\,ax^3\mathrm{Li}_3(ax^2) - 8\,ax^3 - 24\,x}{81\,a}$$
$$+ \frac{4\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{27\,a^{\frac{3}{2}}}$$

input
```
integrate(x^2*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-1/81*(18*a*x^3*dilog(a*x^2) + 12*a*x^3*log(-a*x^2 + 1) - 27*a*x^3*polylog
(3, a*x^2) - 8*a*x^3 - 24*x)/a + 4/27*log((a*x - sqrt(a))/(a*x + sqrt(a)))
/a^(3/2)
```

3.40.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(3, ax^2) \, dx = \int x^2\mathrm{Li}_3(ax^2)\,dx$$

input
```
integrate(x^2*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*polylog(3, a*x^2), x)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int x^2 \operatorname{PolyLog}(3, ax^2) \, dx = \frac{x^3\,\mathrm{polylog}(3, a\,x^2)}{3} - \frac{2\,x^3\,\mathrm{polylog}(2, a\,x^2)}{9} + \frac{8\,x}{27\,a}$$
$$- \frac{4\,x^3\,\ln(1-a\,x^2)}{27} + \frac{8\,x^3}{81} + \frac{\mathrm{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,8\mathrm{i}}{27\,a^{3/2}}$$

input
```
int(x^2*polylog(3, a*x^2),x)
```

output
```
(atan(a^(1/2)*x*1i)*8i)/(27*a^(3/2)) - (2*x^3*polylog(2, a*x^2))/9 + (x^3*
polylog(3, a*x^2))/3 + (8*x)/(27*a) - (4*x^3*log(1 - a*x^2))/27 + (8*x^3)/
81
```

3.40. $\int x^2 \operatorname{PolyLog}(3, ax^2)\, dx$

3.41 $\int \text{PolyLog}(3, ax^2)\, dx$

3.41.1 Optimal result

Integrand size = 7, antiderivative size = 50

$$\int \text{PolyLog}(3, ax^2)\, dx = 8x - \frac{8\text{arctanh}(\sqrt{a}x)}{\sqrt{a}} - 4x\log(1 - ax^2) - 2x\,\text{PolyLog}(2, ax^2) + x\,\text{PolyLog}(3, ax^2)$$

output
```
8*x-4*x*ln(-a*x^2+1)-2*x*polylog(2,a*x^2)+x*polylog(3,a*x^2)-8*arctanh(x*a
^(1/2))/a^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}(3, ax^2)\, dx = 8x - \frac{8\text{arctanh}(\sqrt{a}x)}{\sqrt{a}} - 4x\log(1 - ax^2) - 2x\,\text{PolyLog}(2, ax^2) + x\,\text{PolyLog}(3, ax^2)$$

input
```
Integrate[PolyLog[3, a*x^2],x]
```

output
```
8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2,
 a*x^2] + x*PolyLog[3, a*x^2]
```

3.41.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {7140, 7140, 25, 2898, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(3,ax^2\right)dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\int \text{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\left(x\,\text{PolyLog}\left(2,ax^2\right)-2\int -\log\left(1-ax^2\right)dx\right)$$

$$\Big\downarrow \text{25}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\left(2\int \log\left(1-ax^2\right)dx+x\,\text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{2898}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\left(2\left(2a\int \frac{x^2}{1-ax^2}dx+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{262}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\left(2\left(2a\left(\frac{\int \frac{1}{1-ax^2}dx}{a}-\frac{x}{a}\right)+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)\right)$$

$$\Big\downarrow \text{219}$$

$$x\,\text{PolyLog}\left(3,ax^2\right)-2\left(2\left(2a\left(\frac{\text{arctanh}\left(\sqrt{a}x\right)}{a^{3/2}}-\frac{x}{a}\right)+x\log\left(1-ax^2\right)\right)+x\,\text{PolyLog}\left(2,ax^2\right)\right)$$

input
```
Int[PolyLog[3, a*x^2],x]
```

output
```
-2*(2*(2*a*(-(x/a) + ArcTanh[Sqrt[a]*x]/a^(3/2)) + x*Log[1 - a*x^2]) + x*P
olyLog[2, a*x^2]) + x*PolyLog[3, a*x^2]
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(46) = 92.

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

method	result	si
meijerg	$-\frac{\frac{16x(-a)^{\frac{3}{2}}}{a}+\frac{8x(-a)^{\frac{3}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{a\sqrt{a\,x^2}}-\frac{8x(-a)^{\frac{3}{2}}\ln\left(-a\,x^2+1\right)}{a}-\frac{4x(-a)^{\frac{3}{2}}\operatorname{polylog}\left(2,a\,x^2\right)}{a}+\frac{2x(-a)^{\frac{3}{2}}\operatorname{polylog}\left(3,a\,x^2\right)}{a}}{2\sqrt{-a}}$	11

input
```
int(polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a)^(1/2)*(16*x*(-a)^(3/2)/a+8*x*(-a)^(3/2)/a/(a*x^2)^(1/2)*(ln(1-(a
*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8*x*(-a)^(3/2)*ln(-a*x^2+1)/a-4*x*(-a)^(
3/2)/a*polylog(2,a*x^2)+2*x*(-a)^(3/2)/a*polylog(3,a*x^2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.66

$$\int \text{PolyLog}\left(3,ax^2\right)\,dx$$
$$= \left[-\frac{2\,ax\text{Li}_2(ax^2)+4\,ax\log\left(-ax^2+1\right)-ax\text{polylog}(3,ax^2)-8\,ax-4\,\sqrt{a}\log\left(\frac{ax^2-2\,\sqrt{a}x+1}{ax^2-1}\right)}{a},\right.$$
$$\left.-\frac{2\,ax\text{Li}_2(ax^2)+4\,ax\log\left(-ax^2+1\right)-ax\text{polylog}(3,ax^2)-8\,ax-8\,\sqrt{-a}\arctan\left(\sqrt{-a}x\right)}{a}\right]$$

input
```
integrate(polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
[-(2*a*x*dilog(a*x^2) + 4*a*x*log(-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*
a*x - 4*sqrt(a)*log((a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, -(2*a*x*dil
og(a*x^2) + 4*a*x*log(-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*a*x - 8*sqrt
(-a)*arctan(sqrt(-a)*x))/a]
```

3.41.6 Sympy [F]

$$\int \text{PolyLog}\left(3,ax^2\right)\,dx = \int \text{Li}_3\left(ax^2\right)\,dx$$

input
```
integrate(polylog(3,a*x**2),x)
```

output
```
Integral(polylog(3, a*x**2), x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \text{PolyLog}\left(3,ax^2\right)\,dx = -2\,x\text{Li}_2\left(ax^2\right) - 4\,x\log\left(-ax^2+1\right) + x\text{Li}_3\left(ax^2\right) + 8\,x + \frac{4\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-2*x*dilog(a*x^2) - 4*x*log(-a*x^2 + 1) + x*polylog(3, a*x^2) + 8*x + 4*lo
g((a*x - sqrt(a))/(a*x + sqrt(a)))/sqrt(a)
```

3.41.8 Giac [F]

$$\int \text{PolyLog}\left(3,ax^2\right)\,dx = \int \text{Li}_3\left(ax^2\right)dx$$

input
```
integrate(polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2), x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \text{PolyLog}\left(3,ax^2\right)\,dx = 8\,x - 4\,x\,\ln\left(1-a\,x^2\right) - 2\,x\,\text{polylog}\left(2,a\,x^2\right) + x\,\text{polylog}\left(3,a\,x^2\right) + \frac{\text{atan}\left(\sqrt{a}\,x\,1\text{i}\right)\,8\text{i}}{\sqrt{a}}$$

input
```
int(polylog(3, a*x^2),x)
```

output
```
8*x + (atan(a^(1/2)*x*1i)*8i)/a^(1/2) - 4*x*log(1 - a*x^2) - 2*x*polylog(2
, a*x^2) + x*polylog(3, a*x^2)
```

3.42 $\int \frac{\text{PolyLog}(3,ax^2)}{x^2}\,dx$

3.42.1 Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^2}\,dx = 8\sqrt{a}\text{arctanh}\left(\sqrt{a}x\right) + \frac{4\log(1-ax^2)}{x} - \frac{2\,\text{PolyLog}(2,ax^2)}{x} - \frac{\text{PolyLog}(3,ax^2)}{x}$$

output
```
4*ln(-a*x^2+1)/x-2*polylog(2,a*x^2)/x-polylog(3,a*x^2)/x+8*arctanh(x*a^(1/
2))*a^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^2}\,dx = \frac{8\sqrt{a}x\text{arctanh}\left(\sqrt{a}x\right) + 4\log(1-ax^2) - 2\,\text{PolyLog}(2,ax^2) - \text{PolyLog}(3,ax^2)}{x}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^2,x]
```

output
```
(8*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 4*Log[1 - a*x^2] - 2*PolyLog[2, a*x^2] -
 PolyLog[3, a*x^2])/x
```

3.42.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 7145, 25, 2905, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x^2}dx$$

$$\Big\downarrow \text{7145}$$

$$2\int \frac{\text{PolyLog}\left(2,ax^2\right)}{x^2}dx-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}$$

$$\Big\downarrow \text{7145}$$

$$2\left(2\int -\frac{\log\left(1-ax^2\right)}{x^2}dx-\frac{\text{PolyLog}\left(2,ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}$$

$$\Big\downarrow \text{25}$$

$$2\left(-2\int \frac{\log\left(1-ax^2\right)}{x^2}dx-\frac{\text{PolyLog}\left(2,ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}$$

$$\Big\downarrow \text{2905}$$

$$2\left(-2\left(-2a\int \frac{1}{1-ax^2}dx-\frac{\log\left(1-ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(2,ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}$$

$$\Big\downarrow \text{219}$$

$$2\left(-2\left(-2\sqrt{a}\text{arctanh}\left(\sqrt{a}x\right)-\frac{\log\left(1-ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(2,ax^2\right)}{x}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}$$

input
```
Int[PolyLog[3, a*x^2]/x^2,x]
```

output
```
2*(-2*(-2*Sqrt[a]*ArcTanh[Sqrt[a]*x] - Log[1 - a*x^2]/x) - PolyLog[2, a*x^
2]/x) - PolyLog[3, a*x^2]/x
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(50) = 100$.

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.07

method	result	size
meijerg	$\frac{a\left(-\frac{8x\sqrt{-a}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{\sqrt{a\,x^2}}+\frac{8\sqrt{-a}\,\ln\left(-a\,x^2+1\right)}{xa}-\frac{4\sqrt{-a}\,\operatorname{polylog}\left(2,a\,x^2\right)}{xa}-\frac{2\sqrt{-a}\,\operatorname{polylog}\left(3,a\,x^2\right)}{xa}\right)}{2\sqrt{-a}}$	112

input
```
int(polylog(3,a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/(-a)^(1/2)*(-8*x*(-a)^(1/2)/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+
(a*x^2)^(1/2)))+8/x*(-a)^(1/2)*ln(-a*x^2+1)/a-4/x*(-a)^(1/2)/a*polylog(2,a
*x^2)-2/x*(-a)^(1/2)/a*polylog(3,a*x^2))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.07

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^2}\,dx = \left[\frac{4\sqrt{a}x\log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right) - 2\,\text{Li}_2(ax^2) + 4\log(-ax^2+1) - \text{polylog}(3, ax^2)}{x}, -\frac{8\sqrt{-a}x\arctan\left(\sqrt{-a}x\right) + 2\,\text{Li}_2(ax^2) - 4\log(-ax^2+1) + \text{polylog}(3, ax^2)}{x}\right]$$

input
```
integrate(polylog(3,a*x^2)/x^2,x, algorithm="fricas")
```

output
```
[(4*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 2*dilog(a*x^2)
+ 4*log(-a*x^2 + 1) - polylog(3, a*x^2))/x, -(8*sqrt(-a)*x*arctan(sqrt(-a)
*x) + 2*dilog(a*x^2) - 4*log(-a*x^2 + 1) + polylog(3, a*x^2))/x]
```

3.42.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^2}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^2}\,dx$$

input
```
integrate(polylog(3,a*x**2)/x**2,x)
```

output
```
Integral(polylog(3, a*x**2)/x**2, x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^2}\,dx = -4\sqrt{a}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{2\,\text{Li}_2(ax^2) - 4\log(-ax^2+1) + \text{Li}_3(ax^2)}{x}$$

input
```
integrate(polylog(3,a*x^2)/x^2,x, algorithm="maxima")
```

output
```
-4*sqrt(a)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - (2*dilog(a*x^2) - 4*log(
-a*x^2 + 1) + polylog(3, a*x^2))/x
```

3.42.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x^2}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^2}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^2, x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{x^2}\,dx = \frac{4\,\ln\left(1-a\,x^2\right)}{x} - \frac{\text{polylog}(3,a\,x^2)}{x} - \frac{2\,\text{polylog}(2,a\,x^2)}{x} - \sqrt{a}\,\text{atan}\left(\sqrt{a}\,x\,1\text{i}\right)\,8\text{i}$$

input
```
int(polylog(3, a*x^2)/x^2,x)
```

output
```
(4*log(1 - a*x^2))/x - (2*polylog(2, a*x^2))/x - polylog(3, a*x^2)/x - a^(
1/2)*atan(a^(1/2)*x*1i)*8i
```

3.43 $\int \frac{\text{PolyLog}(3,ax^2)}{x^4} dx$

3.43.1 Optimal result

Integrand size = 11, antiderivative size = 70

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4} dx = -\frac{8a}{27x} + \frac{8}{27}a^{3/2}\text{arctanh}\left(\sqrt{a}x\right) + \frac{4\log(1-ax^2)}{27x^3} - \frac{2\,\text{PolyLog}(2,ax^2)}{9x^3} - \frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

output
```
-8/27*a/x+8/27*a^(3/2)*arctanh(x*a^(1/2))+4/27*ln(-a*x^2+1)/x^3-2/9*polylo
g(2,a*x^2)/x^3-1/3*polylog(3,a*x^2)/x^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4} dx = -\frac{8ax^2 - 8a^{3/2}x^3\text{arctanh}\left(\sqrt{a}x\right) - 4\log(1-ax^2) + 6\,\text{PolyLog}(2,ax^2) + 9\,\text{PolyLog}(3,ax^2)}{27x^3}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^4,x]
```

output
```
-1/27*(8*a*x^2 - 8*a^(3/2)*x^3*ArcTanh[Sqrt[a]*x] - 4*Log[1 - a*x^2] + 6*P
olyLog[2, a*x^2] + 9*PolyLog[3, a*x^2])/x^3
```

3.43.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {7145, 7145, 25, 2905, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{3}\int \frac{\text{PolyLog}(2,ax^2)}{x^4}dx-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{3}\left(\frac{2}{3}\int -\frac{\log(1-ax^2)}{x^4}dx-\frac{\text{PolyLog}(2,ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(-\frac{2}{3}\int \frac{\log(1-ax^2)}{x^4}dx-\frac{\text{PolyLog}(2,ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2}{3}a\int \frac{1}{x^2(1-ax^2)}dx-\frac{\log(1-ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2}{3}a\left(a\int \frac{1}{1-ax^2}dx-\frac{1}{x}\right)-\frac{\log(1-ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2}{3}a\left(\sqrt{a}\text{arctanh}(\sqrt{a}x)-\frac{1}{x}\right)-\frac{\log(1-ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^2)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

input
```
Int[PolyLog[3, a*x^2]/x^4,x]
```

output
```
(2*((-2*((-2*a*(-x^(-1) + Sqrt[a]*ArcTanh[Sqrt[a]*x]))/3 - Log[1 - a*x^2]/
(3*x^3)))/3 - PolyLog[2, a*x^2]/(3*x^3)))/3 - PolyLog[3, a*x^2]/(3*x^3)
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(56) = 112$.

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.79

method	result	size
meijerg	$-\frac{a^{2}\left(-\frac{16}{27 x \sqrt{-a}}-\frac{8 x a \left(\ln\left(1-\sqrt{a\,x^{2}}\right)-\ln\left(1+\sqrt{a\,x^{2}}\right)\right)}{27 \sqrt{-a}\,\sqrt{a\,x^{2}}}+\frac{8 \ln\left(-a\,x^{2}+1\right)}{27 x^{3} \sqrt{-a}\,a}-\frac{4\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{9 x^{3} \sqrt{-a}\,a}-\frac{2\,\mathrm{polylog}\left(3,a\,x^{2}\right)}{3 x^{3} \sqrt{-a}\,a}\right)}{2 \sqrt{-a}}$	125

input
```
int(polylog(3,a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/(-a)^(1/2)*(-16/27/x/(-a)^(1/2)-8/27*x/(-a)^(1/2)*a/(a*x^2)^(1/2)
*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/27/x^3/(-a)^(1/2)*ln(-a*x^2+1
)/a-4/9/x^3/(-a)^(1/2)/a*polylog(2,a*x^2)-2/3/x^3/(-a)^(1/2)/a*polylog(3,a
*x^2))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.89

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^4} dx = \left[\frac{4a^{\frac{3}{2}}x^3 \log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right) - 8ax^2 - 6\,\text{Li}_2(ax^2) + 4\log(-ax^2+1) - 9\,\text{polylog}(3, ax^2)}{27x^3}, -\frac{8\sqrt{-a}ax^3 \arctan\left(\sqrt{-a}x\right) + 8ax^2 + 6\,\text{Li}_2(ax^2) - 4\log(-ax^2+1) + 9\,\text{polylog}(3, ax^2)}{27x^3} \right]$$

input
```
integrate(polylog(3,a*x^2)/x^4,x, algorithm="fricas")
```

output
```
[1/27*(4*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 8*a*x^2
- 6*dilog(a*x^2) + 4*log(-a*x^2 + 1) - 9*polylog(3, a*x^2))/x^3, -1/27*(8*
sqrt(-a)*a*x^3*arctan(sqrt(-a)*x) + 8*a*x^2 + 6*dilog(a*x^2) - 4*log(-a*x^
2 + 1) + 9*polylog(3, a*x^2))/x^3]
```

3.43.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^4} dx = \int \frac{\text{Li}_3(ax^2)}{x^4} dx$$

input
```
integrate(polylog(3,a*x**2)/x**4,x)
```

output
```
Integral(polylog(3, a*x**2)/x**4, x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4}\,dx = -\frac{4}{27}a^{\frac{3}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{8\,ax^2+6\,\text{Li}_2(ax^2)-4\,\log(-ax^2+1)+9\,\text{Li}_3(ax^2)}{27\,x^3}$$

input
```
integrate(polylog(3,a*x^2)/x^4,x, algorithm="maxima")
```

output
```
-4/27*a^(3/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/27*(8*a*x^2 + 6*dil
og(a*x^2) - 4*log(-a*x^2 + 1) + 9*polylog(3, a*x^2))/x^3
```

3.43.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4}\,dx = \int \frac{\text{Li}_3(ax^2)}{x^4}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^4, x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^4}\,dx = \frac{4\,\ln(1-a\,x^2)}{27\,x^3} - \frac{\text{polylog}(3,a\,x^2)}{3\,x^3} - \frac{8\,a}{27\,x} - \frac{2\,\text{polylog}(2,a\,x^2)}{9\,x^3} - \frac{a^{3/2}\,\text{atan}\left(\sqrt{a}\,x\,1\text{i}\right)\,8\text{i}}{27}$$

input
```
int(polylog(3, a*x^2)/x^4,x)
```

output
```
(4*log(1 - a*x^2))/(27*x^3) - (2*polylog(2, a*x^2))/(9*x^3) - polylog(3, a
*x^2)/(3*x^3) - (8*a)/(27*x) - (a^(3/2)*atan(a^(1/2)*x*1i)*8i)/27
```

3.44 $\int \frac{\text{PolyLog}(3,ax^2)}{x^6}\,dx$

3.44.1 Optimal result

Integrand size = 11, antiderivative size = 80

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^6}\,dx = -\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\text{arctanh}\left(\sqrt{a}x\right)+\frac{4\log(1-ax^2)}{125x^5}-\frac{2\,\text{PolyLog}(2,ax^2)}{25x^5}-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

output
```
-8/375*a/x^3-8/125*a^2/x+8/125*a^(5/2)*arctanh(x*a^(1/2))+4/125*ln(-a*x^2+
1)/x^5-2/25*polylog(2,a*x^2)/x^5-1/5*polylog(3,a*x^2)/x^5
```

3.44.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^6}\,dx = -\frac{8ax^2+24a^2x^4-24a^{5/2}x^5\text{arctanh}\left(\sqrt{a}x\right)-12\log(1-ax^2)+30\,\text{PolyLog}(2,ax^2)+75\,\text{PolyLog}(3,ax^2)}{375x^5}$$

input
```
Integrate[PolyLog[3, a*x^2]/x^6,x]
```

output
```
-1/375*(8*a*x^2 + 24*a^2*x^4 - 24*a^(5/2)*x^5*ArcTanh[Sqrt[a]*x] - 12*Log[
1 - a*x^2] + 30*PolyLog[2, a*x^2] + 75*PolyLog[3, a*x^2])/x^5
```

3.44.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {7145, 7145, 25, 2905, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{x^6}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2}{5}\int \frac{\text{PolyLog}(2,ax^2)}{x^6}dx-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2}{5}\left(\frac{2}{5}\int -\frac{\log(1-ax^2)}{x^6}dx-\frac{\text{PolyLog}(2,ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{5}\left(-\frac{2}{5}\int \frac{\log(1-ax^2)}{x^6}dx-\frac{\text{PolyLog}(2,ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2}{5}a\int \frac{1}{x^4(1-ax^2)}dx-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2}{5}a\left(a\int \frac{1}{x^2(1-ax^2)}dx-\frac{1}{3x^3}\right)-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2}{5}a\left(a\left(a\int \frac{1}{1-ax^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\log(1-ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(2,ax^2)}{5x^5}\right)-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2}{5}a\left(a\left(\sqrt{a}\text{arctanh}(\sqrt{a}x)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{\log\left(1-ax^2\right)}{5x^5}\right)-\frac{\text{PolyLog}\left(2,ax^2\right)}{5x^5}\right)-\frac{\text{PolyLog}\left(3,ax^2\right)}{5x^5}$$

input
```
Int[PolyLog[3, a*x^2]/x^6,x]
```

output
```
(2*((-2*((-2*a*(-1/3*1/x^3 + a*(-x^(-1) + Sqrt[a]*ArcTanh[Sqrt[a]*x])))/5
- Log[1 - a*x^2]/(5*x^5)))/5 - PolyLog[2, a*x^2]/(5*x^5)))/5 - PolyLog[3,
a*x^2]/(5*x^5)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(64) = 128.

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.72

method	result	size
meijerg	$\frac{a^{3}\left(-\frac{16}{375 x^{3} \left(-a \right)^{\frac{3}{2}}}-\frac{16 a}{125 x \left(-a \right)^{\frac{3}{2}}}-\frac{8 x \,a^{2} \left(\ln \left(1-\sqrt{a \,x^{2}}\right)-\ln \left(1+\sqrt{a \,x^{2}}\right)\right)}{125 \left(-a \right)^{\frac{3}{2}} \sqrt{a \,x^{2}}}+\frac{8 \ln \left(-a \,x^{2}+1\right)}{125 x^{5} \left(-a \right)^{\frac{3}{2}} a}-\frac{4 \,\mathrm{polylog}\left(2, a \,x^{2}\right)}{25 x^{5} \left(-a \right)^{\frac{3}{2}} a}-\frac{2 \,\mathrm{polylog}\left(3, a \,x^{2}\right)}{5 x^{5} \left(-a \right)^{\frac{3}{2}} a}\right)}{2 \sqrt{-a}}$	138

input

```
int(polylog(3,a*x^2)/x^6,x,method=_RETURNVERBOSE)
```

output

```
1/2*a^3/(-a)^(1/2)*(-16/375/x^3/(-a)^(3/2)-16/125/x/(-a)^(3/2)*a-8/125*x/(
-a)^(3/2)*a^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/12
5/x^5/(-a)^(3/2)*ln(-a*x^2+1)/a-4/25/x^5/(-a)^(3/2)/a*polylog(2,a*x^2)-2/5
/x^5/(-a)^(3/2)/a*polylog(3,a*x^2))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.88

$$\int \frac{\mathrm{PolyLog}\left(3, ax^2\right)}{x^6}\, dx$$
$$= \left[\frac{12\, a^{\frac{5}{2}} x^5 \log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right) - 24\, a^2x^4 - 8\, ax^2 - 30\, \mathrm{Li}_2(ax^2) + 12\, \log\left(-ax^2+1\right) - 75\, \mathrm{polylog}(3, ax^2)}{375\, x^5},\right.$$
$$\left. -\frac{24\, \sqrt{-a}a^2x^5 \arctan\left(\sqrt{-a}x\right) + 24\, a^2x^4 + 8\, ax^2 + 30\, \mathrm{Li}_2(ax^2) - 12\, \log\left(-ax^2+1\right) + 75\, \mathrm{polylog}(3, ax^2)}{375\, x^5}\right]$$

input

```
integrate(polylog(3,a*x^2)/x^6,x, algorithm="fricas")
```

output

```
[1/375*(12*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 24*a^2
*x^4 - 8*a*x^2 - 30*dilog(a*x^2) + 12*log(-a*x^2 + 1) - 75*polylog(3, a*x^
2))/x^5, -1/375*(24*sqrt(-a)*a^2*x^5*arctan(sqrt(-a)*x) + 24*a^2*x^4 + 8*a
*x^2 + 30*dilog(a*x^2) - 12*log(-a*x^2 + 1) + 75*polylog(3, a*x^2))/x^5]
```

3.44.6 Sympy [F]

$$\int \frac{\mathrm{PolyLog}\left(3,ax^2\right)}{x^6}\,dx=\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x^6}\,dx$$

input
```
integrate(polylog(3,a*x**2)/x**6,x)
```

output
```
Integral(polylog(3, a*x**2)/x**6, x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{\mathrm{PolyLog}\left(3,ax^2\right)}{x^6}\,dx=-\frac{4}{125}\,a^{\frac{5}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)-\frac{24\,a^2x^4+8\,ax^2+30\,\mathrm{Li}_2(ax^2)-12\,\log\left(-ax^2+1\right)+75\,\mathrm{Li}_3(ax^2)}{375\,x^5}$$

input
```
integrate(polylog(3,a*x^2)/x^6,x, algorithm="maxima")
```

output
```
-4/125*a^(5/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/375*(24*a^2*x^4 +
8*a*x^2 + 30*dilog(a*x^2) - 12*log(-a*x^2 + 1) + 75*polylog(3, a*x^2))/x^5
```

3.44.8 Giac [F]

$$\int \frac{\mathrm{PolyLog}\left(3,ax^2\right)}{x^6}\,dx=\int \frac{\mathrm{Li}_3(ax^2)}{x^6}\,dx$$

input
```
integrate(polylog(3,a*x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/x^6, x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\text{PolyLog}(3, ax^2)}{x^6} dx = \frac{4 \ln(1 - a x^2)}{125 x^5} - \frac{\text{polylog}(3, a x^2)}{5 x^5} - \frac{8 a^2 x^2 + \frac{8a}{3}}{125 x^3} - \frac{2 \text{polylog}(2, a x^2)}{25 x^5} - \frac{a^{5/2} \text{atan}\left(\sqrt{a}\, x\, 1\text{i}\right) 8\text{i}}{125}$$

input
```
int(polylog(3, a*x^2)/x^6,x)
```

output
```
(4*log(1 - a*x^2))/(125*x^5) - (2*polylog(2, a*x^2))/(25*x^5) - polylog(3,
 a*x^2)/(5*x^5) - ((8*a)/3 + 8*a^2*x^2)/(125*x^3) - (a^(5/2)*atan(a^(1/2)*
x*1i)*8i)/125
```

3.45 $\int x^2 \text{PolyLog}(2, ax^q)\, dx$

3.45.1 Optimal result

Integrand size = 11, antiderivative size = 71

$$\int x^2 \text{PolyLog}(2, ax^q)\, dx = \frac{aq^2x^{3+q} \text{Hypergeometric2F1}\left(1, \frac{3+q}{q}, 2 + \frac{3}{q}, ax^q\right)}{9(3+q)} + \frac{1}{9}qx^3 \log(1 - ax^q) + \frac{1}{3}x^3 \text{PolyLog}(2, ax^q)$$

output
```
1/9*a*q^2*x^(3+q)*hypergeom([1, (3+q)/q],[2+3/q],a*x^q)/(3+q)+1/9*q*x^3*ln
(1-a*x^q)+1/3*x^3*polylog(2,a*x^q)
```

3.45.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int x^2 \text{PolyLog}(2, ax^q)\, dx = \frac{qx^3\left(aqx^q \text{Hypergeometric2F1}\left(1, \frac{3+q}{q}, 2 + \frac{3}{q}, ax^q\right) + (3+q)\log(1 - ax^q)\right)}{9(3+q)} + \frac{1}{3}x^3 \text{PolyLog}(2, ax^q)$$

input
```
Integrate[x^2*PolyLog[2, a*x^q],x]
```

output
```
(q*x^3*(a*q*x^q*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q] + (3 + q)*
Log[1 - a*x^q]))/(9*(3 + q)) + (x^3*PolyLog[2, a*x^q])/3
```

3.45.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \text{PolyLog}(2, ax^q) \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}x^3 \text{PolyLog}(2, ax^q) - \frac{1}{3}q \int -x^2 \log(1 - ax^q) dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}q \int x^2 \log(1 - ax^q) dx + \frac{1}{3}x^3 \text{PolyLog}(2, ax^q)$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{3}q \left(\frac{1}{3}aq \int \frac{x^{q+2}}{1 - ax^q} dx + \frac{1}{3}x^3 \log(1 - ax^q)\right) + \frac{1}{3}x^3 \text{PolyLog}(2, ax^q)$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}q \left(\frac{aqx^{q+3} \text{Hypergeometric2F1}\left(1, \frac{q+3}{q}, 2 + \frac{3}{q}, ax^q\right)}{3(q+3)} + \frac{1}{3}x^3 \log(1 - ax^q)\right) + \frac{1}{3}x^3 \text{PolyLog}(2, ax^q)$$

input
```
Int[x^2*PolyLog[2, a*x^q],x]
```

output
```
(q*((a*q*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(3*(3
+ q)) + (x^3*Log[1 - a*x^q])/3))/3 + (x^3*PolyLog[2, a*x^q])/3
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	size
meijerg	$-\frac{(-a)^{-\frac{3}{q}}\left(-\frac{q^{2} x^{3} (-a)^{\frac{3}{q}} \ln(1-a\,x^{q})}{9}-\frac{q\,x^{3} (-a)^{\frac{3}{q}} \left(1+\frac{q}{3}\right) \operatorname{polylog}(2,a\,x^{q})}{3+q}-\frac{q^{2} x^{3+q} a (-a)^{\frac{3}{q}} \operatorname{LerchPhi}\left(a\,x^{q},1,\frac{3+q}{q}\right)}{9}\right)}{q}$	108

input
```
int(x^2*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-3/q)/q*(-1/9*q^2*x^3*(-a)^(3/q)*ln(1-a*x^q)-q/(3+q)*x^3*(-a)^(3/q)
*(1+1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,
(3+q)/q))
```

3.45.5 Fricas [F]

$$\int x^2 \operatorname{PolyLog}(2, ax^q)\, dx = \int x^2 \operatorname{Li}_2(ax^q)\, dx$$

input
```
integrate(x^2*polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral(x^2*dilog(a*x^q), x)
```

3.45.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}(2, ax^q)\, dx = \int x^2 \operatorname{Li}_2(ax^q)\, dx$$

input
```
integrate(x**2*polylog(2,a*x**q),x)
```

output
```
Integral(x**2*polylog(2, a*x**q), x)
```

3.45.7 Maxima [F]

$$\int x^2 \operatorname{PolyLog}(2, ax^q)\, dx = \int x^2 \operatorname{Li}_2(ax^q)\, dx$$

input
```
integrate(x^2*polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
-1/27*q^2*x^3 + 1/9*q*x^3*log(-a*x^q + 1) + 1/3*x^3*dilog(a*x^q) - q^2*int
egrate(1/9*x^2/(a*x^q - 1), x)
```

3.45.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(2, ax^q) \, dx = \int x^2 \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate(x^2*polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate(x^2*dilog(a*x^q), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{PolyLog}(2, ax^q) \, dx = \int x^2 \operatorname{polylog}(2, a\, x^q) \, dx$$

input
```
int(x^2*polylog(2, a*x^q),x)
```

output
```
int(x^2*polylog(2, a*x^q), x)
```

3.46 $\int x \operatorname{PolyLog}(2, ax^q) \, dx$

3.46.1 Optimal result

Integrand size = 9, antiderivative size = 71

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \frac{aq^2x^{2+q} \text{Hypergeometric2F1}\left(1, \frac{2+q}{q}, 2\left(1 + \frac{1}{q}\right), ax^q\right)}{4(2+q)} + \frac{1}{4}qx^2 \log(1 - ax^q) + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q)$$

output
```
1/4*a*q^2*x^(2+q)*hypergeom([1, (2+q)/q],[2+2/q],a*x^q)/(2+q)+1/4*q*x^2*ln
(1-a*x^q)+1/2*x^2*polylog(2,a*x^q)
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \frac{qx^2\left(aqx^q \text{Hypergeometric2F1}\left(1, \frac{2+q}{q}, 2 + \frac{2}{q}, ax^q\right) + (2+q)\log(1 - ax^q)\right)}{4(2+q)} + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q)$$

input
```
Integrate[x*PolyLog[2, a*x^q],x]
```

output
```
(q*x^2*(a*q*x^q*Hypergeometric2F1[1, (2 + q)/q, 2 + 2/q, a*x^q] + (2 + q)*
Log[1 - a*x^q]))/(4*(2 + q)) + (x^2*PolyLog[2, a*x^q])/2
```

3.46.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(2, ax^q)\, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q) - \frac{1}{2}q \int -x \log(1 - ax^q)\, dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}q \int x \log(1 - ax^q)\, dx + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q)$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}q\left(\frac{1}{2}aq \int \frac{x^{q+1}}{1 - ax^q}dx + \frac{1}{2}x^2 \log(1 - ax^q)\right) + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q)$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}q\left(\frac{aqx^{q+2} \operatorname{Hypergeometric2F1}\left(1, \frac{q+2}{q}, 2\left(1 + \frac{1}{q}\right), ax^q\right)}{2(q+2)} + \frac{1}{2}x^2 \log(1 - ax^q)\right) + \frac{1}{2}x^2 \operatorname{PolyLog}(2, ax^q)$$

input
```
Int[x*PolyLog[2, a*x^q],x]
```

output
```
(q*((a*q*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])
/(2*(2 + q)) + (x^2*Log[1 - a*x^q])/2))/2 + (x^2*PolyLog[2, a*x^q])/2
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.46.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	size
meijerg	$-\frac{(-a)^{-\frac{2}{q}}\left(-\frac{q^{2}x^{2}(-a)^{\frac{2}{q}}\ln(1-a\,x^{q})}{4}-\frac{q\,x^{2}(-a)^{\frac{2}{q}}\left(1+\frac{q}{2}\right)\operatorname{polylog}(2,a\,x^{q})}{2+q}-\frac{q^{2}x^{2+q}a(-a)^{\frac{2}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{2+q}{q}\right)}{4}\right)}{q}$	108

input
```
int(x*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-2/q)/q*(-1/4*q^2*x^2*(-a)^(2/q)*ln(1-a*x^q)-q/(2+q)*x^2*(-a)^(2/q)
*(1+1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,
(2+q)/q))
```

3.46.5 Fricas [F]

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \int x\operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate(x*polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral(x*dilog(a*x^q), x)
```

3.46.6 Sympy [F]

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \int x \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate(x*polylog(2,a*x**q),x)
```

output
```
Integral(x*polylog(2, a*x**q), x)
```

3.46.7 Maxima [F]

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \int x\operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate(x*polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
-1/8*q^2*x^2 + 1/4*q*x^2*log(-a*x^q + 1) + 1/2*x^2*dilog(a*x^q) - q^2*inte
grate(1/4*x/(a*x^q - 1), x)
```

3.46.8 Giac [F]

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \int x \mathrm{Li}_2(ax^q) \, dx$$

input
```
integrate(x*polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate(x*dilog(a*x^q), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{PolyLog}(2, ax^q) \, dx = \int x \operatorname{polylog}(2, a\, x^q) \, dx$$

input
```
int(x*polylog(2, a*x^q),x)
```

output
```
int(x*polylog(2, a*x^q), x)
```

3.47 $\int \text{PolyLog}(2, ax^q)\, dx$

3.47.1 Optimal result

Integrand size = 7, antiderivative size = 54

$$\int \text{PolyLog}(2, ax^q)\, dx = \frac{aq^2x^{1+q}\,\text{Hypergeometric2F1}\left(1, 1+\frac{1}{q}, 2+\frac{1}{q}, ax^q\right)}{1+q} + qx\log(1-ax^q) + x\,\text{PolyLog}(2, ax^q)$$

output
```
a*q^2*x^(1+q)*hypergeom([1, 1+1/q],[2+1/q],a*x^q)/(1+q)+q*x*ln(1-a*x^q)+x*
polylog(2,a*x^q)
```

3.47.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \text{PolyLog}(2, ax^q)\, dx = qx\left(\frac{aqx^q\,\text{Hypergeometric2F1}\left(1, 1+\frac{1}{q}, 2+\frac{1}{q}, ax^q\right)}{1+q} + \log(1-ax^q)\right) + x\,\text{PolyLog}(2, ax^q)$$

input
```
Integrate[PolyLog[2, a*x^q],x]
```

output
```
q*x*((a*q*x^q*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 + q)
 + Log[1 - a*x^q]) + x*PolyLog[2, a*x^q]
```

3.47.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {7140, 25, 2898, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,ax^q)\,dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}(2,ax^q)-q\int -\log(1-ax^q)dx$$

$$\Big\downarrow \text{25}$$

$$q\int \log(1-ax^q)dx+x\,\text{PolyLog}(2,ax^q)$$

$$\Big\downarrow \text{2898}$$

$$q\left(aq\int \frac{x^q}{1-ax^q}dx+x\log(1-ax^q)\right)+x\,\text{PolyLog}(2,ax^q)$$

$$\Big\downarrow \text{888}$$

$$q\left(\frac{aqx^{q+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{q},2+\frac{1}{q},ax^q\right)}{q+1}+x\log(1-ax^q)\right)+x\,\text{PolyLog}(2,ax^q)$$

input
```
Int[PolyLog[2, a*x^q],x]
```

output
```
q*((a*q*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1
+ q) + x*Log[1 - a*x^q]) + x*PolyLog[2, a*x^q]
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.47.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

method	result	size
meijerg	$-\frac{(-a)^{-\frac{1}{q}}\left(-q^{2}x(-a)^{\frac{1}{q}}\ln(1-a\,x^{q})-qx(-a)^{\frac{1}{q}}\,\text{polylog}(2,a\,x^{q})-q^{2}x^{1+q}a(-a)^{\frac{1}{q}}\,\text{LerchPhi}\left(a\,x^{q},1,\frac{1+q}{q}\right)\right)}{q}$	88

input
```
int(polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-1/q*(-a)^(-1/q)*(-q^2*x*(-a)^(1/q)*ln(1-a*x^q)-q*x*(-a)^(1/q)*polylog(2,a
*x^q)-q^2*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a*x^q,1,(1+q)/q))
```

3.47.5 Fricas [F]

$$\int \text{PolyLog}(2, ax^q)\ dx = \int \text{Li}_2(ax^q)\ dx$$

input
```
integrate(polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral(dilog(a*x^q), x)
```

3.47.6 Sympy [F]

$$\int \text{PolyLog}(2, ax^q)\ dx = \int \text{Li}_2(ax^q)\ dx$$

input
```
integrate(polylog(2,a*x**q),x)
```

output
```
Integral(polylog(2, a*x**q), x)
```

3.47.7 Maxima [F]

$$\int \text{PolyLog}(2, ax^q)\ dx = \int \text{Li}_2(ax^q)\ dx$$

input
```
integrate(polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
-q^2*x - q^2*integrate(1/(a*x^q - 1), x) + q*x*log(-a*x^q + 1) + x*dilog(a
*x^q)
```

3.47.8 Giac [F]

$$\int \text{PolyLog}\left(2,ax^q\right)\,dx=\int \text{Li}_2(ax^q)\,dx$$

input
```
integrate(polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \text{PolyLog}\left(2,ax^q\right)\,dx=\int \text{polylog}(2,a\,x^q)\,dx$$

input
```
int(polylog(2, a*x^q),x)
```

output
```
int(polylog(2, a*x^q), x)
```

3.48 $\int \frac{\text{PolyLog}(2,ax^q)}{x}\,dx$

3.48.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(2,ax^q)}{x}\,dx=\frac{\text{PolyLog}(3,ax^q)}{q}$$

output
```
polylog(3,a*x^q)/q
```

3.48.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,ax^q)}{x}\,dx=\frac{\text{PolyLog}(3,ax^q)}{q}$$

input
```
Integrate[PolyLog[2, a*x^q]/x,x]
```

output
```
PolyLog[3, a*x^q]/q
```

3.48.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} dx$$
$$\Big\downarrow \text{ 7143}$$
$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

input
```
Int[PolyLog[2, a*x^q]/x,x]
```

output
```
PolyLog[3, a*x^q]/q
```

3.48.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.48.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\text{polylog}(3,a\,x^{q})}{q}$	12
default	$\frac{\text{polylog}(3,a\,x^{q})}{q}$	12
meijerg	$\frac{\text{polylog}(3,a\,x^{q})}{q}$	12

input
```
int(polylog(2,a*x^q)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(3,a*x^q)/q
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} \, dx = \frac{\text{polylog}(3, ax^q)}{q}$$

input
```
integrate(polylog(2,a*x^q)/x,x, algorithm="fricas")
```

output
```
polylog(3, a*x^q)/q
```

3.48.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} \, dx = \int \frac{\text{Li}_2(ax^q)}{x} \, dx$$

input
```
integrate(polylog(2,a*x**q)/x,x)
```

output
```
Integral(polylog(2, a*x**q)/x, x)
```

3.48.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} \, dx = \int \frac{\text{Li}_2(ax^q)}{x} \, dx$$

input
```
integrate(polylog(2,a*x^q)/x,x, algorithm="maxima")
```

output
```
-1/6*q^2*log(x)^3 + 1/2*q*log(-a*x^q + 1)*log(x)^2 - q^2*integrate(1/2*log
(x)^2/(a*x*x^q - x), x) + dilog(a*x^q)*log(x)
```

3.48.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} \, dx = \int \frac{\text{Li}_2(ax^q)}{x} \, dx$$

input
```
integrate(polylog(2,a*x^q)/x,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/x, x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{x} \, dx = \int \frac{\text{polylog}(2, a\,x^q)}{x} \, dx$$

input
```
int(polylog(2, a*x^q)/x,x)
```

output
```
int(polylog(2, a*x^q)/x, x)
```

3.49 $\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx$

3.49.1 Optimal result

Integrand size = 11, antiderivative size = 69

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx = -\frac{aq^2x^{-1+q}\,\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q} + \frac{q\log(1-ax^q)}{x} - \frac{\text{PolyLog}(2,ax^q)}{x}$$

output
```
-a*q^2*x^(-1+q)*hypergeom([1, (-1+q)/q],[2-1/q],a*x^q)/(1-q)+q*ln(1-a*x^q)
/x-polylog(2,a*x^q)/x
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx = \frac{q\left(\frac{aqx^q\,\text{Hypergeometric2F1}\left(1,\frac{-1+q}{q},2-\frac{1}{q},ax^q\right)}{-1+q}+\log(1-ax^q)\right)}{x} - \frac{\text{PolyLog}(2,ax^q)}{x}$$

input
```
Integrate[PolyLog[2, a*x^q]/x^2,x]
```

output
```
(q*((a*q*x^q*Hypergeometric2F1[1, (-1 + q)/q, 2 - q^(-1), a*x^q])/(-1 + q)
 + Log[1 - a*x^q]))/x - PolyLog[2, a*x^q]/x
```

3.49.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx$$

$$\Big\downarrow \text{7145}$$

$$q\int -\frac{\log(1-ax^q)}{x^2}dx-\frac{\text{PolyLog}(2,ax^q)}{x}$$

$$\Big\downarrow \text{25}$$

$$-q\int \frac{\log(1-ax^q)}{x^2}dx-\frac{\text{PolyLog}(2,ax^q)}{x}$$

$$\Big\downarrow \text{2905}$$

$$-q\left(-aq\int \frac{x^{q-2}}{1-ax^q}dx-\frac{\log(1-ax^q)}{x}\right)-\frac{\text{PolyLog}(2,ax^q)}{x}$$

$$\Big\downarrow \text{888}$$

$$-q\left(\frac{aqx^{q-1}\,\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q}-\frac{\log(1-ax^q)}{x}\right)-\frac{\text{PolyLog}(2,ax^q)}{x}$$

input
```
Int[PolyLog[2, a*x^q]/x^2,x]
```

output
```
-(q*((a*q*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q]
)/(1 - q) - Log[1 - a*x^q]/x)) - PolyLog[2, a*x^q]/x
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

method	result	size
meijerg	$-\frac{(-a)^{\frac{1}{q}}\left(-\frac{q^{2}(-a)^{-\frac{1}{q}}\ln(1-a\,x^{q})}{x}-\frac{q(-a)^{-\frac{1}{q}}(1-q)\,\mathrm{polylog}(2,a\,x^{q})}{(-1+q)x}-q^{2}x^{-1+q}a(-a)^{-\frac{1}{q}}\,\mathrm{LerchPhi}\left(a\,x^{q},1,\frac{-1+q}{q}\right)\right)}{q}$	106

input
```
int(polylog(2,a*x^q)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(1/q)/q*(-q^2/x*(-a)^(-1/q)*ln(1-a*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)
*polylog(2,a*x^q)-q^2*x^(-1+q)*a*(-a)^(-1/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

3.49.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^2} dx = \int \frac{\text{Li}_2(ax^q)}{x^2} dx$$

input
```
integrate(polylog(2,a*x^q)/x^2,x, algorithm="fricas")
```

output
```
integral(dilog(a*x^q)/x^2, x)
```

3.49.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^2} dx = \int \frac{\text{Li}_2(ax^q)}{x^2} dx$$

input
```
integrate(polylog(2,a*x**q)/x**2,x)
```

output
```
Integral(polylog(2, a*x**q)/x**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^2} dx = \int \frac{\text{Li}_2(ax^q)}{x^2} dx$$

input
```
integrate(polylog(2,a*x^q)/x^2,x, algorithm="maxima")
```

output
```
-q^2*integrate(1/(a*x^2*x^q - x^2), x) + (q^2 + q*log(-a*x^q + 1) - dilog(
a*x^q))/x
```

3.49.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^2} \, dx = \int \frac{\text{Li}_2(ax^q)}{x^2} \, dx$$

input
```
integrate(polylog(2,a*x^q)/x^2,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/x^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^2} \, dx = \int \frac{\text{polylog}(2, a\,x^q)}{x^2} \, dx$$

input
```
int(polylog(2, a*x^q)/x^2,x)
```

output
```
int(polylog(2, a*x^q)/x^2, x)
```

3.50 $\int \frac{\text{PolyLog}(2,ax^q)}{x^3}\,dx$

3.50.1 Optimal result

Integrand size = 11, antiderivative size = 78

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^3}\,dx = -\frac{aq^2x^{-2+q}\,\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{4(2-q)} + \frac{q\log(1-ax^q)}{4x^2} - \frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

output
```
-1/4*a*q^2*x^(-2+q)*hypergeom([1, (-2+q)/q],[2-2/q],a*x^q)/(2-q)+1/4*q*ln(
1-a*x^q)/x^2-1/2*polylog(2,a*x^q)/x^2
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^3}\,dx = \frac{q\left(\frac{aqx^q\,\text{Hypergeometric2F1}\left(1,\frac{-2+q}{q},2-\frac{2}{q},ax^q\right)}{-2+q}+\log(1-ax^q)\right)-2\,\text{PolyLog}(2,ax^q)}{4x^2}$$

input
```
Integrate[PolyLog[2, a*x^q]/x^3,x]
```

output
```
(q*((a*q*x^q*Hypergeometric2F1[1, (-2 + q)/q, 2 - 2/q, a*x^q])/(-2 + q) +
Log[1 - a*x^q]) - 2*PolyLog[2, a*x^q])/(4*x^2)
```

3.50.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^3}\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}q\int -\frac{\log(1-ax^q)}{x^3}dx-\frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}q\int \frac{\log(1-ax^q)}{x^3}dx-\frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{1}{2}q\left(-\frac{1}{2}aq\int \frac{x^{q-3}}{1-ax^q}dx-\frac{\log(1-ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{1}{2}q\left(\frac{aqx^{q-2}\,\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{2(2-q)}-\frac{\log(1-ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

input
```
Int[PolyLog[2, a*x^q]/x^3,x]
```

output
```
-1/2*(q*((a*q*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1))
, a*x^q])/(2*(2 - q)) - Log[1 - a*x^q]/(2*x^2))) - PolyLog[2, a*x^q]/(2*x^
2)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

method	result	size
meijerg	$-\frac{(-a)^{\frac{2}{q}}\left(-\frac{q^{2}(-a)^{-\frac{2}{q}}\ln\left(1-a\,x^{q}\right)}{4x^{2}}-\frac{q(-a)^{-\frac{2}{q}}\left(1-\frac{q}{2}\right)\operatorname{polylog}\left(2,a\,x^{q}\right)}{(-2+q)x^{2}}-\frac{q^{2}x^{-2+q}a(-a)^{-\frac{2}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{-2+q}{q}\right)}{4}\right)}{q}$	108

input
```
int(polylog(2,a*x^q)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(2/q)/q*(-1/4*q^2/x^2*(-a)^(-2/q)*ln(1-a*x^q)-q/(-2+q)/x^2*(-a)^(-2/
q)*(1-1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^
q,1,(-2+q)/q))
```

3.50.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^3} dx = \int \frac{\text{Li}_2(ax^q)}{x^3} dx$$

input
```
integrate(polylog(2,a*x^q)/x^3,x, algorithm="fricas")
```

output
```
integral(dilog(a*x^q)/x^3, x)
```

3.50.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^3} dx = \int \frac{\text{Li}_2(ax^q)}{x^3} dx$$

input
```
integrate(polylog(2,a*x**q)/x**3,x)
```

output
```
Integral(polylog(2, a*x**q)/x**3, x)
```

3.50.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^3} dx = \int \frac{\text{Li}_2(ax^q)}{x^3} dx$$

input
```
integrate(polylog(2,a*x^q)/x^3,x, algorithm="maxima")
```

output
```
-q^2*integrate(1/4/(a*x^3*x^q - x^3), x) + 1/8*(q^2 + 2*q*log(-a*x^q + 1)
- 4*dilog(a*x^q))/x^2
```

3.50.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^3}\, dx = \int \frac{\text{Li}_2(ax^q)}{x^3}\, dx$$

input
```
integrate(polylog(2,a*x^q)/x^3,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/x^3, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^3}\, dx = \int \frac{\text{polylog}(2, a\, x^q)}{x^3}\, dx$$

input
```
int(polylog(2, a*x^q)/x^3,x)
```

output
```
int(polylog(2, a*x^q)/x^3, x)
```

3.51 $\int \frac{\text{PolyLog}(2,ax^q)}{x^4}\,dx$

3.51.1 Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^4}\,dx = -\frac{aq^2x^{-3+q}\,\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{9(3-q)} + \frac{q\log(1-ax^q)}{9x^3} - \frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

output
```
-1/9*a*q^2*x^(-3+q)*hypergeom([1, (-3+q)/q],[2-3/q],a*x^q)/(3-q)+1/9*q*ln(
1-a*x^q)/x^3-1/3*polylog(2,a*x^q)/x^3
```

3.51.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^4}\,dx = \frac{q\left(\frac{aqx^q\,\text{Hypergeometric2F1}\left(1,\frac{-3+q}{q},2-\frac{3}{q},ax^q\right)}{-3+q}+\log(1-ax^q)\right)-3\,\text{PolyLog}(2,ax^q)}{9x^3}$$

input
```
Integrate[PolyLog[2, a*x^q]/x^4,x]
```

output
```
(q*((a*q*x^q*Hypergeometric2F1[1, (-3 + q)/q, 2 - 3/q, a*x^q])/(-3 + q) +
Log[1 - a*x^q]) - 3*PolyLog[2, a*x^q])/(9*x^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{x^4}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{3}q\int -\frac{\log(1-ax^q)}{x^4}dx-\frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}q\int \frac{\log(1-ax^q)}{x^4}dx-\frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{1}{3}q\left(-\frac{1}{3}aq\int \frac{x^{q-4}}{1-ax^q}dx-\frac{\log(1-ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

$$\Big\downarrow \text{888}$$

$$-\frac{1}{3}q\left(\frac{aqx^{q-3}\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{3(3-q)}-\frac{\log(1-ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

input
```
Int[PolyLog[2, a*x^q]/x^4,x]
```

output
```
-1/3*(q*((a*q*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q
])/(3*(3 - q)) - Log[1 - a*x^q]/(3*x^3))) - PolyLog[2, a*x^q]/(3*x^3)
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 3.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result	size
meijerg	$-\frac{(-a)^{\frac{3}{q}} \left(-\frac{q^{2} (-a)^{-\frac{3}{q}} \ln \left(1-a\,x^{q}\right)}{9 x^{3}}-\frac{q (-a)^{-\frac{3}{q}} \left(1-\frac{q}{3}\right) \operatorname{polylog}\left(2, a\,x^{q}\right)}{(-3+q) x^{3}}-\frac{q^{2} x^{-3+q} a (-a)^{-\frac{3}{q}} \operatorname{LerchPhi}\left(a\,x^{q}, 1, \frac{-3+q}{q}\right)}{9}\right)}{q}$	108

input
```
int(polylog(2,a*x^q)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(3/q)/q*(-1/9*q^2/x^3*(-a)^(-3/q)*ln(1-a*x^q)-q/(-3+q)/x^3*(-a)^(-3/
q)*(1-1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^
q,1,(-3+q)/q))
```

3.51.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_2(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(2,a*x^q)/x^4,x, algorithm="fricas")
```

output
```
integral(dilog(a*x^q)/x^4, x)
```

3.51.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_2(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(2,a*x**q)/x**4,x)
```

output
```
Integral(polylog(2, a*x**q)/x**4, x)
```

3.51.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_2(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(2,a*x^q)/x^4,x, algorithm="maxima")
```

output
```
-q^2*integrate(1/9/(a*x^4*x^q - x^4), x) + 1/27*(q^2 + 3*q*log(-a*x^q + 1)
 - 9*dilog(a*x^q))/x^3
```

3.51.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^4} dx = \int \frac{\text{Li}_2(ax^q)}{x^4} dx$$

input
```
integrate(polylog(2,a*x^q)/x^4,x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/x^4, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{x^4} dx = \int \frac{\text{polylog}(2, a\, x^q)}{x^4} dx$$

input
```
int(polylog(2, a*x^q)/x^4,x)
```

output
```
int(polylog(2, a*x^q)/x^4, x)
```

3.52 $\int x^2 \operatorname{PolyLog}(3, ax^q)\, dx$

3.52.1 Optimal result

Integrand size = 11, antiderivative size = 88

$$\int x^2 \operatorname{PolyLog}(3, ax^q)\, dx = -\frac{aq^3x^{3+q}\,\text{Hypergeometric2F1}\left(1, \frac{3+q}{q}, 2+\frac{3}{q}, ax^q\right)}{27(3+q)} - \frac{1}{27}q^2x^3\log(1-ax^q) - \frac{1}{9}qx^3\operatorname{PolyLog}(2, ax^q) + \frac{1}{3}x^3\operatorname{PolyLog}(3, ax^q)$$

output
```
-1/27*a*q^3*x^(3+q)*hypergeom([1, (3+q)/q],[2+3/q],a*x^q)/(3+q)-1/27*q^2*x
^3*ln(1-a*x^q)-1/9*q*x^3*polylog(2,a*x^q)+1/3*x^3*polylog(3,a*x^q)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int x^2 \operatorname{PolyLog}(3, ax^q)\, dx = -\frac{x^3 G_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,\frac{-3+q}{q} \\ 1,0,0,0,-\frac{3}{q} \end{array}\right)}{q}$$

input
```
Integrate[x^2*PolyLog[3, a*x^q],x]
```

output
```
-((x^3*MeijerG[{{1, 1, 1, 1, (-3 + q)/q}, {}}, {{1}, {0, 0, 0, -3/q}}, -(a
*x^q)])/q)
```

3.52.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(3,ax^q)\,dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax^q)-\frac{1}{3}q\int x^2 \operatorname{PolyLog}(2,ax^q)dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax^q)-\frac{1}{3}q\left(\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax^q)-\frac{1}{3}q\int -x^2\log(1-ax^q)dx\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax^q)-\frac{1}{3}q\left(\frac{1}{3}q\int x^2\log(1-ax^q)dx+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \ 2905$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax^q)-\frac{1}{3}q\left(\frac{1}{3}q\left(\frac{1}{3}aq\int \frac{x^{q+2}}{1-ax^q}dx+\frac{1}{3}x^3\log(1-ax^q)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \ 888$$

$$\frac{1}{3}x^3 \operatorname{PolyLog}(3,ax^q)-$$
$$\frac{1}{3}q\left(\frac{1}{3}q\left(\frac{aqx^{q+3}\operatorname{Hypergeometric2F1}\left(1,\frac{q+3}{q},2+\frac{3}{q},ax^q\right)}{3(q+3)}+\frac{1}{3}x^3\log(1-ax^q)\right)+\frac{1}{3}x^3 \operatorname{PolyLog}(2,ax^q)\right)$$

input
```
Int[x^2*PolyLog[3, a*x^q],x]
```

output
```
-1/3*(q*((q*((a*q*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q
])/(3*(3 + q)) + (x^3*Log[1 - a*x^q])/3))/3 + (x^3*PolyLog[2, a*x^q])/3))
+ (x^3*PolyLog[3, a*x^q])/3
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.52.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

method	result
meijerg	$-\frac{(-a)^{-\frac{3}{q}}\left(\frac{q^{3}x^{3}(-a)^{\frac{3}{q}}\ln(1-a\,x^{q})}{27}+\frac{q^{2}x^{3}(-a)^{\frac{3}{q}}\operatorname{polylog}(2,a\,x^{q})}{9}-\frac{q\,x^{3}(-a)^{\frac{3}{q}}\left(1+\frac{q}{3}\right)\operatorname{polylog}(3,a\,x^{q})}{3+q}+\frac{q^{3}x^{3+q}a(-a)^{\frac{3}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{3+q}{q}\right)}{27}\right.}{q}$

input
```
int(x^2*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-3/q)/q*(1/27*q^3*x^3*(-a)^(3/q)*ln(1-a*x^q)+1/9*q^2*x^3*(-a)^(3/q)
*polylog(2,a*x^q)-q/(3+q)*x^3*(-a)^(3/q)*(1+1/3*q)*polylog(3,a*x^q)+1/27*q
^3*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,(3+q)/q))
```

3.52.5 Fricas [F]

$$\int x^2 \operatorname{PolyLog}(3, ax^q) \, dx = \int x^2 \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate(x^2*polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral(x^2*polylog(3, a*x^q), x)
```

3.52.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}(3, ax^q) \, dx = \int x^2 \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate(x**2*polylog(3,a*x**q),x)
```

output
```
Integral(x**2*polylog(3, a*x**q), x)
```

3.52.7 Maxima [F]

$$\int x^2 \operatorname{PolyLog}(3, ax^q) \, dx = \int x^2 \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate(x^2*polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
1/81*q^3*x^3 - 1/27*q^2*x^3*log(-a*x^q + 1) - 1/9*q*x^3*dilog(a*x^q) + q^3
*integrate(1/27*x^2/(a*x^q - 1), x) + 1/3*x^3*polylog(3, a*x^q)
```

3.52.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(3, ax^q) \, dx = \int x^2 \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate(x^2*polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate(x^2*polylog(3, a*x^q), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{PolyLog}(3, ax^q) \, dx = \int x^2 \operatorname{polylog}(3, a\, x^q) \, dx$$

input
```
int(x^2*polylog(3, a*x^q),x)
```

output
```
int(x^2*polylog(3, a*x^q), x)
```

3.53 $\int x \operatorname{PolyLog}(3, ax^q) \, dx$

3.53.1 Optimal result

Integrand size = 9, antiderivative size = 88

$$\int x \operatorname{PolyLog}(3, ax^q) \, dx = -\frac{aq^3x^{2+q} \operatorname{Hypergeometric2F1}\left(1, \frac{2+q}{q}, 2\left(1+\frac{1}{q}\right), ax^q\right)}{8(2+q)} - \frac{1}{8}q^2x^2 \log(1-ax^q) - \frac{1}{4}qx^2 \operatorname{PolyLog}(2, ax^q) + \frac{1}{2}x^2 \operatorname{PolyLog}(3, ax^q)$$

output
```
-1/8*a*q^3*x^(2+q)*hypergeom([1, (2+q)/q],[2+2/q],a*x^q)/(2+q)-1/8*q^2*x^2
*ln(1-a*x^q)-1/4*q*x^2*polylog(2,a*x^q)+1/2*x^2*polylog(3,a*x^q)
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int x \operatorname{PolyLog}(3, ax^q) \, dx = -\frac{x^2 G_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,\frac{-2+q}{q} \\ 1,0,0,0,-\frac{2}{q} \end{array}\right)}{q}$$

input
```
Integrate[x*PolyLog[3, a*x^q],x]
```

output
```
-((x^2*MeijerG[{{1, 1, 1, 1, (-2 + q)/q}, {}}, {{1}, {0, 0, 0, -2/q}}, -(a
*x^q)])/q)
```

3.53.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7145, 7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(3,ax^q)\, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax^q)-\frac{1}{2}q\int x \operatorname{PolyLog}(2,ax^q)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax^q)-\frac{1}{2}q\left(\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax^q)-\frac{1}{2}q\int -x\log(1-ax^q)dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax^q)-\frac{1}{2}q\left(\frac{1}{2}q\int x\log(1-ax^q)dx+\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax^q)-\frac{1}{2}q\left(\frac{1}{2}q\left(\frac{1}{2}aq\int \frac{x^{q+1}}{1-ax^q}dx+\frac{1}{2}x^2\log(1-ax^q)\right)+\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2 \operatorname{PolyLog}(3,ax^q)-$$
$$\frac{1}{2}q\left(\frac{1}{2}q\left(\frac{aqx^{q+2}\,\text{Hypergeometric2F1}\left(1,\frac{q+2}{q},2\left(1+\frac{1}{q}\right),ax^q\right)}{2(q+2)}+\frac{1}{2}x^2\log(1-ax^q)\right)+\frac{1}{2}x^2 \operatorname{PolyLog}(2,ax^q)\right)$$

input
```
Int[x*PolyLog[3, a*x^q],x]
```

output
```
-1/2*(q*((q*((a*q*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1))
, a*x^q])/(2*(2 + q)) + (x^2*Log[1 - a*x^q])/2))/2 + (x^2*PolyLog[2, a*x^q
])/2)) + (x^2*PolyLog[3, a*x^q])/2
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.53.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

method	result
meijerg	$-\frac{(-a)^{-\frac{2}{q}} \left(\frac{q^3 x^2 (-a)^{\frac{2}{q}} \ln(1-a x^q)}{8}+\frac{q^2 x^2 (-a)^{\frac{2}{q}} \operatorname{polylog}(2,a x^q)}{4}-\frac{q x^2 (-a)^{\frac{2}{q}} \left(1+\frac{q}{2}\right) \operatorname{polylog}(3,a x^q)}{2+q}+\frac{q^3 x^{2+q} a (-a)^{\frac{2}{q}} \operatorname{LerchPhi}\left(a x^q,1,\frac{2+q}{q}\right)}{8}\right.}{q}$

input
```
int(x*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(-2/q)/q*(1/8*q^3*x^2*(-a)^(2/q)*ln(1-a*x^q)+1/4*q^2*x^2*(-a)^(2/q)*
polylog(2,a*x^q)-q/(2+q)*x^2*(-a)^(2/q)*(1+1/2*q)*polylog(3,a*x^q)+1/8*q^3
*x^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,(2+q)/q))
```

3.53.5 Fricas [F]

$$\int x\,\text{PolyLog}\left(3,ax^q\right)\,dx=\int x\text{Li}_3(ax^q)\,dx$$

input
```
integrate(x*polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral(x*polylog(3, a*x^q), x)
```

3.53.6 Sympy [F]

$$\int x\,\text{PolyLog}\left(3,ax^q\right)\,dx=\int x\,\text{Li}_3\left(ax^q\right)\,dx$$

input
```
integrate(x*polylog(3,a*x**q),x)
```

output
```
Integral(x*polylog(3, a*x**q), x)
```

3.53.7 Maxima [F]

$$\int x\,\text{PolyLog}\left(3,ax^q\right)\,dx=\int x\text{Li}_3(ax^q)\,dx$$

input
```
integrate(x*polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
1/16*q^3*x^2 - 1/8*q^2*x^2*log(-a*x^q + 1) - 1/4*q*x^2*dilog(a*x^q) + q^3*
integrate(1/8*x/(a*x^q - 1), x) + 1/2*x^2*polylog(3, a*x^q)
```

3.53.8 Giac [F]

$$\int x \operatorname{PolyLog}(3, ax^q) \, dx = \int x \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate(x*polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate(x*polylog(3, a*x^q), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{PolyLog}(3, ax^q) \, dx = \int x \operatorname{polylog}(3, a\, x^q) \, dx$$

input
```
int(x*polylog(3, a*x^q),x)
```

output
```
int(x*polylog(3, a*x^q), x)
```

3.54 $\int \text{PolyLog}(3, ax^q)\, dx$

3.54.1 Optimal result

Integrand size = 7, antiderivative size = 69

$$\int \text{PolyLog}(3, ax^q)\, dx = -\frac{aq^3x^{1+q}\,\text{Hypergeometric2F1}\left(1, 1+\frac{1}{q}, 2+\frac{1}{q}, ax^q\right)}{1+q} - q^2x\log(1-ax^q) - qx\,\text{PolyLog}(2, ax^q) + x\,\text{PolyLog}(3, ax^q)$$

output
```
-a*q^3*x^(1+q)*hypergeom([1, 1+1/q],[2+1/q],a*x^q)/(1+q)-q^2*x*ln(1-a*x^q)
-q*x*polylog(2,a*x^q)+x*polylog(3,a*x^q)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \text{PolyLog}(3, ax^q)\, dx = -\frac{xG_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,\frac{-1+q}{q} \\ 1,0,0,0,-\frac{1}{q} \end{array}\right)}{q}$$

input
```
Integrate[PolyLog[3, a*x^q],x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1, (-1 + q)/q}, {}}, {{1}, {0, 0, 0, -q^(-1)}}, -(
a*x^q)])/q)
```

3.54.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {7140, 7140, 25, 2898, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(3,ax^q)\,dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}(3,ax^q)-q\int \text{PolyLog}(2,ax^q)\,dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}(3,ax^q)-q\left(x\,\text{PolyLog}(2,ax^q)-q\int -\log(1-ax^q)\,dx\right)$$

$$\Big\downarrow \text{25}$$

$$x\,\text{PolyLog}(3,ax^q)-q\left(q\int \log(1-ax^q)\,dx+x\,\text{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \text{2898}$$

$$x\,\text{PolyLog}(3,ax^q)-q\left(q\left(aq\int \frac{x^q}{1-ax^q}dx+x\log(1-ax^q)\right)+x\,\text{PolyLog}(2,ax^q)\right)$$

$$\Big\downarrow \text{888}$$

$$x\,\text{PolyLog}(3,ax^q)-q\left(q\left(\frac{aqx^{q+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{q},2+\frac{1}{q},ax^q\right)}{q+1}+x\log(1-ax^q)\right)+x\,\text{PolyLog}(2,ax^q)\right)$$

input
```
Int[PolyLog[3, a*x^q],x]
```

output
```
-(q*(q*((a*q*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q]
)/(1 + q) + x*Log[1 - a*x^q]) + x*PolyLog[2, a*x^q])) + x*PolyLog[3, a*x^q
]
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

3.54.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.52

method	result
meijerg	$-\frac{(-a)^{-\frac{1}{q}}\left(q^3 x(-a)^{\frac{1}{q}}\ln(1-a\,x^q)+q^2 x(-a)^{\frac{1}{q}}\operatorname{polylog}(2,a\,x^q)-qx(-a)^{\frac{1}{q}}\operatorname{polylog}(3,a\,x^q)+q^3 x^{1+q} a(-a)^{\frac{1}{q}}\operatorname{LerchPhi}\left(a\,x^q,1,\frac{1+q}{q}\right)\right)}{q}$

input
```
int(polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-1/q*(-a)^(-1/q)*(q^3*x*(-a)^(1/q)*ln(1-a*x^q)+q^2*x*(-a)^(1/q)*polylog(2,
a*x^q)-q*x*(-a)^(1/q)*polylog(3,a*x^q)+q^3*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a
*x^q,1,(1+q)/q))
```

3.54.5 Fricas [F]

$$\int \text{PolyLog}\left(3, ax^q\right) dx = \int \text{Li}_3(ax^q)\, dx$$

input
```
integrate(polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x^q), x)
```

3.54.6 Sympy [F]

$$\int \text{PolyLog}\left(3, ax^q\right) dx = \int \text{Li}_3\left(ax^q\right) dx$$

input
```
integrate(polylog(3,a*x**q),x)
```

output
```
Integral(polylog(3, a*x**q), x)
```

3.54.7 Maxima [F]

$$\int \text{PolyLog}\left(3, ax^q\right) dx = \int \text{Li}_3(ax^q)\, dx$$

input
```
integrate(polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
q^3*x + q^3*integrate(1/(a*x^q - 1), x) - q^2*x*log(-a*x^q + 1) - q*x*dilo
g(a*x^q) + x*polylog(3, a*x^q)
```

3.54.8 Giac [F]

$$\int \text{PolyLog}\left(3, ax^q\right) \, dx = \int \text{Li}_3(ax^q) \, dx$$

input
```
integrate(polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \text{PolyLog}\left(3, ax^q\right) \, dx = \int \text{polylog}(3, a\,x^q) \, dx$$

input
```
int(polylog(3, a*x^q),x)
```

output
```
int(polylog(3, a*x^q), x)
```

3.55 $\int \frac{\text{PolyLog}(3,ax^q)}{x} dx$

3.55.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(3,ax^q)}{x} \, dx = \frac{\text{PolyLog}(4,ax^q)}{q}$$

output
```
polylog(4,a*x^q)/q
```

3.55.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(3,ax^q)}{x} \, dx = \frac{\text{PolyLog}(4,ax^q)}{q}$$

input
```
Integrate[PolyLog[3, a*x^q]/x,x]
```

output
```
PolyLog[4, a*x^q]/q
```

3.55.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{x} \, dx$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\text{PolyLog}(4,ax^q)}{q}$$

input
```
Int[PolyLog[3, a*x^q]/x,x]
```

output
```
PolyLog[4, a*x^q]/q
```

3.55.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.55.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\text{polylog}(4,a\,x^q)}{q}$	12
default	$\frac{\text{polylog}(4,a\,x^q)}{q}$	12
meijerg	$\frac{\text{polylog}(4,a\,x^q)}{q}$	12

input
```
int(polylog(3,a*x^q)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(4,a*x^q)/q
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(3, ax^q)}{x} dx = \frac{\text{polylog}(4, ax^q)}{q}$$

input
```
integrate(polylog(3,a*x^q)/x,x, algorithm="fricas")
```

output
```
polylog(4, a*x^q)/q
```

3.55.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x} dx = \int \frac{\text{Li}_3(ax^q)}{x} dx$$

input
```
integrate(polylog(3,a*x**q)/x,x)
```

output
```
Integral(polylog(3, a*x**q)/x, x)
```

3.55.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x} dx = \int \frac{\text{Li}_3(ax^q)}{x} dx$$

input
```
integrate(polylog(3,a*x^q)/x,x, algorithm="maxima")
```

output
```
1/24*q^3*log(x)^4 - 1/6*q^2*log(-a*x^q + 1)*log(x)^3 + q^3*integrate(1/6*l
og(x)^3/(a*x*x^q - x), x) - 1/2*q*dilog(a*x^q)*log(x)^2 + log(x)*polylog(3
, a*x^q)
```

3.55.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x}\,dx = \int \frac{\text{Li}_3(ax^q)}{x}\,dx$$

input
```
integrate(polylog(3,a*x^q)/x,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/x, x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^q)}{x}\,dx = \int \frac{\text{polylog}(3, a\,x^q)}{x}\,dx$$

input
```
int(polylog(3, a*x^q)/x,x)
```

output
```
int(polylog(3, a*x^q)/x, x)
```

3.56 $\int \frac{\text{PolyLog}(3,ax^q)}{x^2}\,dx$

3.56.1 Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^2}\,dx = -\frac{aq^3x^{-1+q}\,\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q} + \frac{q^2\log(1-ax^q)}{x} - \frac{q\,\text{PolyLog}(2,ax^q)}{x} - \frac{\text{PolyLog}(3,ax^q)}{x}$$

output
```
-a*q^3*x^(-1+q)*hypergeom([1, (-1+q)/q],[2-1/q],a*x^q)/(1-q)+q^2*ln(1-a*x^
q)/x-q*polylog(2,a*x^q)/x-polylog(3,a*x^q)/x
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^2}\,dx = -\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{1}{q}\\1,0,0,0,\frac{1}{q}\end{array}\right)}{qx}$$

input
```
Integrate[PolyLog[3, a*x^q]/x^2,x]
```

output
```
-(MeijerG[{{1, 1, 1, 1, 1 + q^(-1)}, {}}, {{1}, {0, 0, 0, q^(-1)}}, -(a*x^
q)]/(q*x))
```

3.56.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^2}dx$$

$$\Big\downarrow \text{ 7145}$$

$$q\int \frac{\text{PolyLog}(2,ax^q)}{x^2}dx-\frac{\text{PolyLog}(3,ax^q)}{x}$$

$$\Big\downarrow \text{ 7145}$$

$$q\left(q\int -\frac{\log(1-ax^q)}{x^2}dx-\frac{\text{PolyLog}(2,ax^q)}{x}\right)-\frac{\text{PolyLog}(3,ax^q)}{x}$$

$$\Big\downarrow \text{ 25}$$

$$q\left(-q\int \frac{\log(1-ax^q)}{x^2}dx-\frac{\text{PolyLog}(2,ax^q)}{x}\right)-\frac{\text{PolyLog}(3,ax^q)}{x}$$

$$\Big\downarrow \text{ 2905}$$

$$q\left(-q\left(-aq\int \frac{x^{q-2}}{1-ax^q}dx-\frac{\log(1-ax^q)}{x}\right)-\frac{\text{PolyLog}(2,ax^q)}{x}\right)-\frac{\text{PolyLog}(3,ax^q)}{x}$$

$$\Big\downarrow \text{ 888}$$

$$q\left(-q\left(\frac{aqx^{q-1}\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q}-\frac{\log(1-ax^q)}{x}\right)-\frac{\text{PolyLog}(2,ax^q)}{x}\right)-\frac{\text{PolyLog}(3,ax^q)}{x}$$

input
```
Int[PolyLog[3, a*x^q]/x^2,x]
```

output
```
q*(-(q*((a*q*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x
^q])/(1 - q) - Log[1 - a*x^q]/x)) - PolyLog[2, a*x^q]/x) - PolyLog[3, a*x^
q]/x
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

method	result
meijerg	$-\frac{(-a)^{\frac{1}{q}}\left(-\frac{q^{3}(-a)^{-\frac{1}{q}}\ln(1-a\,x^{q})}{x}+\frac{q^{2}(-a)^{-\frac{1}{q}}\operatorname{polylog}(2,a\,x^{q})}{x}-\frac{q(-a)^{-\frac{1}{q}}(1-q)\operatorname{polylog}(3,a\,x^{q})}{(-1+q)x}-q^{3}x^{-1+q}a(-a)^{-\frac{1}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{-1}{q}\right.}{q}$

input
```
int(polylog(3,a*x^q)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(1/q)/q*(-q^3/x*(-a)^(-1/q)*ln(1-a*x^q)+q^2/x*(-a)^(-1/q)*polylog(2,
a*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)*polylog(3,a*x^q)-q^3*x^(-1+q)*a*(-a)^(
-1/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

3.56.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^2} dx = \int \frac{\text{Li}_3(ax^q)}{x^2} dx$$

input
```
integrate(polylog(3,a*x^q)/x^2,x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x^q)/x^2, x)
```

3.56.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^2} dx = \int \frac{\text{Li}_3(ax^q)}{x^2} dx$$

input
```
integrate(polylog(3,a*x**q)/x**2,x)
```

output
```
Integral(polylog(3, a*x**q)/x**2, x)
```

3.56.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^2} dx = \int \frac{\text{Li}_3(ax^q)}{x^2} dx$$

input
```
integrate(polylog(3,a*x^q)/x^2,x, algorithm="maxima")
```

output
```
-q^3*integrate(1/(a*x^2*x^q - x^2), x) + (q^3 + q^2*log(-a*x^q + 1) - q*di
log(a*x^q) - polylog(3, a*x^q))/x
```

3.56.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^2}\,dx = \int \frac{\text{Li}_3(ax^q)}{x^2}\,dx$$

input
```
integrate(polylog(3,a*x^q)/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/x^2, x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^2}\,dx = \int \frac{\text{polylog}(3, a\,x^q)}{x^2}\,dx$$

input
```
int(polylog(3, a*x^q)/x^2,x)
```

output
```
int(polylog(3, a*x^q)/x^2, x)
```

3.57 $\int \frac{\text{PolyLog}(3,ax^q)}{x^3}\,dx$

3.57.1 Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^3}\,dx = -\frac{aq^3x^{-2+q}\,\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{8(2-q)} + \frac{q^2\log(1-ax^q)}{8x^2} - \frac{q\,\text{PolyLog}(2,ax^q)}{4x^2} - \frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

output
```
-1/8*a*q^3*x^(-2+q)*hypergeom([1, (-2+q)/q],[2-2/q],a*x^q)/(2-q)+1/8*q^2*l
n(1-a*x^q)/x^2-1/4*q*polylog(2,a*x^q)/x^2-1/2*polylog(3,a*x^q)/x^2
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.43

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^3}\,dx = -\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{2+q}{q}\\1,0,0,0,\frac{2}{q}\end{array}\right)}{qx^2}$$

input
```
Integrate[PolyLog[3, a*x^q]/x^3,x]
```

output
```
-(MeijerG[{{1, 1, 1, 1, (2 + q)/q}, {}}, {{1}, {0, 0, 0, 2/q}}, -(a*x^q)]/
(q*x^2))
```

3.57.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^3}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}q\int \frac{\text{PolyLog}(2,ax^q)}{x^3}dx-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

$$\Big\downarrow \text{7145}$$

$$\frac{1}{2}q\left(\frac{1}{2}q\int -\frac{\log(1-ax^q)}{x^3}dx-\frac{\text{PolyLog}(2,ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}q\left(-\frac{1}{2}q\int \frac{\log(1-ax^q)}{x^3}dx-\frac{\text{PolyLog}(2,ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}q\left(-\frac{1}{2}q\left(-\frac{1}{2}aq\int \frac{x^{q-3}}{1-ax^q}dx-\frac{\log(1-ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}q\left(-\frac{1}{2}q\left(\frac{aqx^{q-2}\,\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{2(2-q)}-\frac{\log(1-ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(2,ax^q)}{2x^2}\right)-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

input
```
Int[PolyLog[3, a*x^q]/x^3,x]
```

output
```
(q*(-1/2*(q*((a*q*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(
-1)), a*x^q])/(2*(2 - q)) - Log[1 - a*x^q]/(2*x^2))) - PolyLog[2, a*x^q]/(
2*x^2)))/2 - PolyLog[3, a*x^q]/(2*x^2)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.57.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

method	result
meijerg	$-\frac{(-a)^{\frac{2}{q}}\left(-\frac{q^{3}(-a)^{-\frac{2}{q}}\ln(1-a\,x^{q})}{8x^{2}}+\frac{q^{2}(-a)^{-\frac{2}{q}}\operatorname{polylog}(2,a\,x^{q})}{4x^{2}}-\frac{q(-a)^{-\frac{2}{q}}\left(1-\frac{q}{2}\right)\operatorname{polylog}(3,a\,x^{q})}{(-2+q)x^{2}}-\frac{q^{3}x^{-2+q}a(-a)^{-\frac{2}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{-2+}{q}\right.}{8}\right.}{q}$

input
```
int(polylog(3,a*x^q)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(2/q)/q*(-1/8*q^3/x^2*(-a)^(-2/q)*ln(1-a*x^q)+1/4*q^2/x^2*(-a)^(-2/q
)*polylog(2,a*x^q)-q/(-2+q)/x^2*(-a)^(-2/q)*(1-1/2*q)*polylog(3,a*x^q)-1/8
*q^3*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^q,1,(-2+q)/q))
```

3.57.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^3} dx = \int \frac{\text{Li}_3(ax^q)}{x^3} dx$$

input
```
integrate(polylog(3,a*x^q)/x^3,x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x^q)/x^3, x)
```

3.57.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^3} dx = \int \frac{\text{Li}_3(ax^q)}{x^3} dx$$

input
```
integrate(polylog(3,a*x**q)/x**3,x)
```

output
```
Integral(polylog(3, a*x**q)/x**3, x)
```

3.57.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^3} dx = \int \frac{\text{Li}_3(ax^q)}{x^3} dx$$

input
```
integrate(polylog(3,a*x^q)/x^3,x, algorithm="maxima")
```

output
```
-q^3*integrate(1/8/(a*x^3*x^q - x^3), x) + 1/16*(q^3 + 2*q^2*log(-a*x^q +
1) - 4*q*dilog(a*x^q) - 8*polylog(3, a*x^q))/x^2
```

3.57.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^3} \, dx = \int \frac{\text{Li}_3(ax^q)}{x^3} \, dx$$

input
```
integrate(polylog(3,a*x^q)/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/x^3, x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^3} \, dx = \int \frac{\text{polylog}(3, a\,x^q)}{x^3} \, dx$$

input
```
int(polylog(3, a*x^q)/x^3,x)
```

output
```
int(polylog(3, a*x^q)/x^3, x)
```

3.58 $\int \frac{\text{PolyLog}(3,ax^q)}{x^4}\,dx$

3.58.1 Optimal result

Integrand size = 11, antiderivative size = 93

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^4}\,dx = -\frac{aq^3x^{-3+q}\,\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{27(3-q)} + \frac{q^2\log(1-ax^q)}{27x^3} - \frac{q\,\text{PolyLog}(2,ax^q)}{9x^3} - \frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

output
```
-1/27*a*q^3*x^(-3+q)*hypergeom([1, (-3+q)/q],[2-3/q],a*x^q)/(3-q)+1/27*q^2
*ln(1-a*x^q)/x^3-1/9*q*polylog(2,a*x^q)/x^3-1/3*polylog(3,a*x^q)/x^3
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^4}\,dx = -\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{3+q}{q}\\1,0,0,0,\frac{3}{q}\end{array}\right)}{qx^3}$$

input
```
Integrate[PolyLog[3, a*x^q]/x^4,x]
```

output
```
-(MeijerG[{{1, 1, 1, 1, (3 + q)/q}, {}}, {{1}, {0, 0, 0, 3/q}}, -(a*x^q)]/
(q*x^3))
```

3.58.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {7145, 7145, 25, 2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{x^4}dx$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{3}q\int \frac{\text{PolyLog}(2,ax^q)}{x^4}dx-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

$$\Big\downarrow \ 7145$$

$$\frac{1}{3}q\left(\frac{1}{3}q\int -\frac{\log(1-ax^q)}{x^4}dx-\frac{\text{PolyLog}(2,ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}q\left(-\frac{1}{3}q\int \frac{\log(1-ax^q)}{x^4}dx-\frac{\text{PolyLog}(2,ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

$$\Big\downarrow \ 2905$$

$$\frac{1}{3}q\left(-\frac{1}{3}q\left(-\frac{1}{3}aq\int \frac{x^{q-4}}{1-ax^q}dx-\frac{\log(1-ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

$$\Big\downarrow \ 888$$

$$\frac{1}{3}q\left(-\frac{1}{3}q\left(\frac{aqx^{q-3}\,\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{3(3-q)}-\frac{\log(1-ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(2,ax^q)}{3x^3}\right)-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

input
```
Int[PolyLog[3, a*x^q]/x^4,x]
```

output
```
(q*(-1/3*(q*((a*q*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a
*x^q])/(3*(3 - q)) - Log[1 - a*x^q]/(3*x^3))) - PolyLog[2, a*x^q]/(3*x^3))
)/3 - PolyLog[3, a*x^q]/(3*x^3)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42

method	result
meijerg	$-\frac{(-a)^{\frac{3}{q}}\left(-\frac{q^{3}(-a)^{-\frac{3}{q}}\ln(1-a\,x^{q})}{27x^{3}}+\frac{q^{2}(-a)^{-\frac{3}{q}}\operatorname{polylog}(2,a\,x^{q})}{9x^{3}}-\frac{q(-a)^{-\frac{3}{q}}\left(1-\frac{q}{3}\right)\operatorname{polylog}(3,a\,x^{q})}{(-3+q)x^{3}}-\frac{q^{3}x^{-3+q}a(-a)^{-\frac{3}{q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{-3+q}{q}\right)}{27}\right)}{q}$

input
```
int(polylog(3,a*x^q)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-(-a)^(3/q)/q*(-1/27*q^3/x^3*(-a)^(-3/q)*ln(1-a*x^q)+1/9*q^2/x^3*(-a)^(-3/
q)*polylog(2,a*x^q)-q/(-3+q)/x^3*(-a)^(-3/q)*(1-1/3*q)*polylog(3,a*x^q)-1/
27*q^3*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^q,1,(-3+q)/q))
```

3.58.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_3(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(3,a*x^q)/x^4,x, algorithm="fricas")
```

output
```
integral(polylog(3, a*x^q)/x^4, x)
```

3.58.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_3(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(3,a*x**q)/x**4,x)
```

output
```
Integral(polylog(3, a*x**q)/x**4, x)
```

3.58.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_3(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(3,a*x^q)/x^4,x, algorithm="maxima")
```

output
```
-q^3*integrate(1/27/(a*x^4*x^q - x^4), x) + 1/81*(q^3 + 3*q^2*log(-a*x^q +
 1) - 9*q*dilog(a*x^q) - 27*polylog(3, a*x^q))/x^3
```

3.58.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^4} \, dx = \int \frac{\text{Li}_3(ax^q)}{x^4} \, dx$$

input
```
integrate(polylog(3,a*x^q)/x^4,x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/x^4, x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^q)}{x^4} \, dx = \int \frac{\text{polylog}(3, a\,x^q)}{x^4} \, dx$$

input
```
int(polylog(3, a*x^q)/x^4,x)
```

output
```
int(polylog(3, a*x^q)/x^4, x)
```

3.59 $\int (dx)^{3/2} \text{PolyLog}(2, ax)\, dx$

3.59.1 Optimal result

Integrand size = 13, antiderivative size = 117

$$\int (dx)^{3/2} \text{PolyLog}(2,ax)\, dx = -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{8d^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\,\text{PolyLog}(2,ax)}{5d}$$

output
```
-8/75*(d*x)^(3/2)/a-8/125*(d*x)^(5/2)/d+8/25*d^(3/2)*arctanh(a^(1/2)*(d*x)
^(1/2)/d^(1/2))/a^(5/2)+4/25*(d*x)^(5/2)*ln(-a*x+1)/d+2/5*(d*x)^(5/2)*poly
log(2,a*x)/d-8/25*d*(d*x)^(1/2)/a^2
```

3.59.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.77

$$\int (dx)^{3/2} \text{PolyLog}(2,ax)\, dx = \frac{2(dx)^{3/2}\left(\frac{4\text{arctanh}(\sqrt{a}\sqrt{x})}{5a^{5/2}} + \frac{2}{75}\sqrt{x}\left(-\frac{2(15+5ax+3a^2x^2)}{a^2} + 15x^2\log(1-ax)\right) + x\right.}{5x^{3/2}}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

output
```
(2*(d*x)^(3/2)*((4*ArcTanh[Sqrt[a]*Sqrt[x]])/(5*a^(5/2)) + (2*Sqrt[x]*((-2
*(15 + 5*a*x + 3*a^2*x^2))/a^2 + 15*x^2*Log[1 - a*x]))/75 + x^(5/2)*PolyLo
g[2, a*x]))/(5*x^(3/2))
```

3.59.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {7145, 25, 2842, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \operatorname{PolyLog}(2,ax) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}(2,ax)}{5d} - \frac{2}{5} \int -(dx)^{3/2} \log(1-ax)dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2}{5} \int (dx)^{3/2} \log(1-ax)dx + \frac{2(dx)^{5/2} \operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{2}{5} \left(\frac{2a \int \frac{(dx)^{5/2}}{1-ax}dx}{5d} + \frac{2(dx)^{5/2} \log(1-ax)}{5d} \right) + \frac{2(dx)^{5/2} \operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2}{5} \left(\frac{2a \left(\frac{d \int \frac{(dx)^{3/2}}{1-ax}dx}{a} - \frac{2(dx)^{5/2}}{5a} \right)}{5d} + \frac{2(dx)^{5/2} \log(1-ax)}{5d} \right) + \frac{2(dx)^{5/2} \operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2}{5} \left(\frac{2a \left(\frac{d \left(\frac{d \int \frac{\sqrt{dx}}{1-ax}dx}{a} - \frac{2(dx)^{3/2}}{3a} \right)}{a} - \frac{2(dx)^{5/2}}{5a} \right)}{5d} + \frac{2(dx)^{5/2} \log(1-ax)}{5d} \right) + \frac{2(dx)^{5/2} \operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \; 73$$

$$\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{2\int\frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}$$

$$\Big\downarrow \; 219$$

3.59. $\int (dx)^{3/2}\operatorname{PolyLog}(2,ax)\,dx$

$$\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}$$

input
```
Int[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

output
```
(2*((2*a*((-2*(d*x)^(5/2))/(5*a) + (d*((-2*(d*x)^(3/2))/(3*a) + (d*((-2*Sq
rt[d*x])/a + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/a^(3/2)))/a)
)/a))/(5*d) + (2*(d*x)^(5/2)*Log[1 - a*x])/(5*d)))/5 + (2*(d*x)^(5/2)*Poly
Log[2, a*x])/(5*d)
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,ax)}{5}+\frac{4(dx)^{\frac{5}{2}}\ln\left(\frac{-adx+d}{d}\right)}{25}+\frac{8a\left(-\frac{\frac{(dx)^{\frac{5}{2}}a^{2}}{5}+\frac{d(dx)^{\frac{3}{2}}a}{3}+d^{2}\sqrt{dx}}{a^{3}}+\frac{d^{3}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{3}\sqrt{ad}}\right)}{25}}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,ax)}{5}+\frac{4(dx)^{\frac{5}{2}}\ln\left(\frac{-adx+d}{d}\right)}{25}+\frac{8a\left(-\frac{\frac{(dx)^{\frac{5}{2}}a^{2}}{5}+\frac{d(dx)^{\frac{3}{2}}a}{3}+d^{2}\sqrt{dx}}{a^{3}}+\frac{d^{3}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{3}\sqrt{ad}}\right)}{25}}{d}$
meijerg	$\frac{(dx)^{\frac{3}{2}}\left(-\frac{2\sqrt{x}\,(-a)^{\frac{7}{2}}\left(84a^{2}x^{2}+140ax+420\right)}{2625a^{3}}-\frac{4\sqrt{x}\,(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{25a^{3}\sqrt{ax}}+\frac{4x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\ln(-ax+1)}{25a}+\frac{2x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\,\mathrm{polylog}}{5a}\right.}{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}a}$

input
```
int((d*x)^(3/2)*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/5*(d*x)^(5/2)*polylog(2,a*x)+2/25*(d*x)^(5/2)*ln((-a*d*x+d)/d)+4/25
*a*(-1/a^3*(1/5*(d*x)^(5/2)*a^2+1/3*d*(d*x)^(3/2)*a+d^2*(d*x)^(1/2))+d^3/a
^3/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int (dx)^{3/2}\,\text{PolyLog}(2,ax)\,dx = \left[\frac{2\left(30\,d\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)+(75\,a^2dx^2\text{Li}_2(ax)+30\,a^2dx^2\log\left(-ax\right.\right.}{375\,a^2}\right.$$

$$-\frac{2\left(60\,d\sqrt{-\frac{d}{a}}\arctan\left(\frac{\sqrt{dx}a\sqrt{-\frac{d}{a}}}{d}\right)-(75\,a^2dx^2\text{Li}_2(ax)+30\,a^2dx^2\log\left(-ax+1\right)-12\,a^2dx^2-20\,adx-60\right.}{375\,a^2}$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="fricas")
```

output
```
[2/375*(30*d*sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)
) + (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) - 12*a^2*d*x^2 -
 20*a*d*x - 60*d)*sqrt(d*x))/a^2, -2/375*(60*d*sqrt(-d/a)*arctan(sqrt(d*x)
*a*sqrt(-d/a)/d) - (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) -
 12*a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2]
```

3.59.6 Sympy [F]

$$\int (dx)^{3/2}\,\text{PolyLog}(2,ax)\,dx = \int (dx)^{\frac{3}{2}}\,\text{Li}_2\left(ax\right)\,dx$$

input
```
integrate((d*x)**(3/2)*polylog(2,a*x),x)
```

output
```
Integral((d*x)**(3/2)*polylog(2, a*x), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int (dx)^{3/2}\,\text{PolyLog}(2,ax)\,dx = -\frac{2\left(\frac{30\,d^3\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}a^2} - \frac{75\,(dx)^{\frac{5}{2}}a^2\text{Li}_2(ax)+30\,(dx)^{\frac{5}{2}}a^2\log(-adx+d)-6\,(5\,a^2\log(d)+2\,a^2)(dx)^{\frac{5}{2}}-20\,(dx)^{\frac{3}{2}}ad-60\,\sqrt{dx}d^2}{a^2}\right)}{375\,d}$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="maxima")
```

output
```
-2/375*(30*d^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(s
qrt(a*d)*a^2) - (75*(d*x)^(5/2)*a^2*dilog(a*x) + 30*(d*x)^(5/2)*a^2*log(-a
*d*x + d) - 6*(5*a^2*log(d) + 2*a^2)*(d*x)^(5/2) - 20*(d*x)^(3/2)*a*d - 60
*sqrt(d*x)*d^2)/a^2)/d
```

3.59.8 Giac [F]

$$\int (dx)^{3/2}\,\text{PolyLog}(2,ax)\,dx = \int (dx)^{\frac{3}{2}}\,\text{Li}_2(ax)\,dx$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*dilog(a*x), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\,\text{PolyLog}(2,ax)\,dx = \int (d\,x)^{3/2}\,\text{polylog}(2,a\,x)\,dx$$

input
```
int((d*x)^(3/2)*polylog(2, a*x),x)
```

output
```
int((d*x)^(3/2)*polylog(2, a*x), x)
```

3.60 $\int \sqrt{dx}\, \text{PolyLog}(2, ax)\, dx$

3.60.1 Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \sqrt{dx}\, \text{PolyLog}(2, ax)\, dx = -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{8\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}} + \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\,\text{PolyLog}(2, ax)}{3d}$$

output
```
-8/27*(d*x)^(3/2)/d+4/9*(d*x)^(3/2)*ln(-a*x+1)/d+2/3*(d*x)^(3/2)*polylog(2
,a*x)/d+8/9*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/2)-8/9*(d*x)
^(1/2)/a
```

3.60.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \sqrt{dx}\, \text{PolyLog}(2, ax)\, dx = \frac{2\sqrt{dx}\left(\frac{12\text{arctanh}(\sqrt{a}\sqrt{x})}{a^{3/2}} + \frac{2\sqrt{x}(-6-2ax+3ax\log(1-ax))}{a} + 9x^{3/2}\,\text{PolyLog}(2, ax)\right)}{27\sqrt{x}}$$

input
```
Integrate[Sqrt[d*x]*PolyLog[2, a*x],x]
```

output
```
(2*Sqrt[d*x]*((12*ArcTanh[Sqrt[a]*Sqrt[x]])/a^(3/2) + (2*Sqrt[x]*(-6 - 2*a
*x + 3*a*x*Log[1 - a*x]))/a + 9*x^(3/2)*PolyLog[2, a*x]))/(27*Sqrt[x])
```

3.60.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7145, 25, 2842, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,\text{PolyLog}(2,ax)\,dx$$

$$\Big\downarrow \; 7145$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}-\frac{2}{3}\int -\sqrt{dx}\log(1-ax)dx$$

$$\Big\downarrow \; 25$$

$$\frac{2}{3}\int \sqrt{dx}\log(1-ax)dx+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}$$

$$\Big\downarrow \; 2842$$

$$\frac{2}{3}\left(\frac{2a\int \frac{(dx)^{3/2}}{1-ax}dx}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}$$

$$\Big\downarrow \; 60$$

$$\frac{2}{3}\left(\frac{2a\left(\frac{d\int \frac{\sqrt{dx}}{1-ax}dx}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}$$

$$\Big\downarrow \; 60$$

$$\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{d\int \frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}$$

$$\Big\downarrow \; 73$$

$$\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{2\int \frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}$$

input
```
Int[Sqrt[d*x]*PolyLog[2, a*x],x]
```

output
```
(2*((2*a*((-2*(d*x)^(3/2))/(3*a) + (d*((-2*Sqrt[d*x])/a + (2*Sqrt[d]*ArcTa
nh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/a^(3/2)))/a))/(3*d) + (2*(d*x)^(3/2)*Log[
1 - a*x])/(3*d)))/3 + (2*(d*x)^(3/2)*PolyLog[2, a*x])/(3*d)
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{3}+\frac{4(dx)^{\frac{3}{2}}\ln\left(\frac{-adx+d}{d}\right)}{9}+\frac{8a\left(-\frac{\frac{a(dx)^{\frac{3}{2}}}{3}+d\sqrt{dx}}{a^{2}}+\frac{d^{2}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{2}\sqrt{ad}}\right)}{9}}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{3}+\frac{4(dx)^{\frac{3}{2}}\ln\left(\frac{-adx+d}{d}\right)}{9}+\frac{8a\left(-\frac{\frac{a(dx)^{\frac{3}{2}}}{3}+d\sqrt{dx}}{a^{2}}+\frac{d^{2}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{2}\sqrt{ad}}\right)}{9}}{d}$
meijerg	$\frac{\sqrt{dx}\left(-\frac{2\sqrt{x}\,(-a)^{\frac{5}{2}}(20ax+60)}{135a^{2}}-\frac{4\sqrt{x}\,(-a)^{\frac{5}{2}}\left(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax})\right)}{9a^{2}\sqrt{ax}}+\frac{4x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\ln(-ax+1)}{9a}+\frac{2x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\operatorname{polylog}(2,ax)}{3a}\right)}{\sqrt{x}\,\sqrt{-a}\,a}$

input
```
int((d*x)^(1/2)*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*(d*x)^(3/2)*polylog(2,a*x)+2/9*(d*x)^(3/2)*ln((-a*d*x+d)/d)+4/9*a
*(-1/a^2*(1/3*a*(d*x)^(3/2)+d*(d*x)^(1/2))+d^2/a^2/(a*d)^(1/2)*arctanh(a*(
d*x)^(1/2)/(a*d)^(1/2))))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \sqrt{dx}\,\text{PolyLog}(2,ax)\,dx$$
$$= \left[\frac{2\left((9\,ax\text{Li}_2(ax)+6\,ax\log\left(-ax+1\right)-4\,ax-12)\sqrt{dx}+6\,\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)\right)}{27\,a},\ \frac{2\left((9\,ax\text{Li}_2(ax\right.}{}\right.$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="fricas")
```

output
```
[2/27*((9*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 6
*sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)))/a, 2/27*(
(9*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) - 12*sqrt(
-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

3.60.6 Sympy [F]

$$\int \sqrt{dx}\,\text{PolyLog}(2,ax)\,dx = \int \sqrt{dx}\,\text{Li}_2\left(ax\right)\,dx$$

input
```
integrate((d*x)**(1/2)*polylog(2,a*x),x)
```

output
```
Integral(sqrt(d*x)*polylog(2, a*x), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \sqrt{dx}\,\mathrm{PolyLog}(2,ax)\,dx = -\frac{2\left(\frac{6\,d^2\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}a} - \frac{9\,(dx)^{\frac{3}{2}}\,a\mathrm{Li}_2(ax)+6\,(dx)^{\frac{3}{2}}\,a\log(-adx+d)-2\,(dx)^{\frac{3}{2}}\,(3\,a\log(d)+2\,a)-12\,\sqrt{dx}d}{a}\right)}{27\,d}$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="maxima")
```

output
```
-2/27*(6*d^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqr
t(a*d)*a) - (9*(d*x)^(3/2)*a*dilog(a*x) + 6*(d*x)^(3/2)*a*log(-a*d*x + d)
- 2*(d*x)^(3/2)*(3*a*log(d) + 2*a) - 12*sqrt(d*x)*d)/a)/d
```

3.60.8 Giac [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}(2,ax)\,dx = \int \sqrt{dx}\mathrm{Li}_2(ax)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*dilog(a*x), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\mathrm{PolyLog}(2,ax)\,dx = \int \sqrt{d\,x}\,\mathrm{polylog}(2,a\,x)\,dx$$

input
```
int((d*x)^(1/2)*polylog(2, a*x),x)
```

output
```
int((d*x)^(1/2)*polylog(2, a*x), x)
```

3.61 $\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx$

3.61.1 Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = -\frac{8\sqrt{dx}}{d} + \frac{8\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} + \frac{4\sqrt{dx}\log(1-ax)}{d} + \frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

output
```
8*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(1/2)/d^(1/2)-8*(d*x)^(1/2)/d+4*l
n(-a*x+1)*(d*x)^(1/2)/d+2*polylog(2,a*x)*(d*x)^(1/2)/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = \frac{8\sqrt{x}\text{arctanh}\left(\sqrt{a}\sqrt{x}\right) + 4\sqrt{a}x(-2+\log(1-ax)) + 2\sqrt{a}x\,\text{PolyLog}(2,ax)}{\sqrt{a}\sqrt{dx}}$$

input
```
Integrate[PolyLog[2, a*x]/Sqrt[d*x],x]
```

output
```
(8*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Sqrt[a]*x*(-2 + Log[1 - a*x]) + 2*
Sqrt[a]*x*PolyLog[2, a*x])/(Sqrt[a]*Sqrt[d*x])
```

3.61.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {7145, 25, 2842, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}-2\int -\frac{\log(1-ax)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{ 25}$$

$$2\int \frac{\log(1-ax)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{2a\int \frac{\sqrt{dx}}{1-ax}dx}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

$$\Big\downarrow \text{ 60}$$

$$2\left(\frac{2a\left(\frac{d\int \frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

$$\Big\downarrow \text{ 73}$$

$$2\left(\frac{2a\left(\frac{2\int \frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{2a\left(\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

input
```
Int[PolyLog[2, a*x]/Sqrt[d*x],x]
```

output
```
2*((2*a*((-2*Sqrt[d*x])/a + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]
])/a^(3/2)))/d + (2*Sqrt[d*x]*Log[1 - a*x])/d) + (2*Sqrt[d*x]*PolyLog[2, a
*x])/d
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,\operatorname{polylog}(2,ax)+4\sqrt{dx}\,\ln\left(\frac{-adx+d}{d}\right)+8a\left(-\frac{\sqrt{dx}}{a}+\frac{d\,\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a\sqrt{ad}}\right)}{d}$	74
default	$\frac{2\sqrt{dx}\,\operatorname{polylog}(2,ax)+4\sqrt{dx}\,\ln\left(\frac{-adx+d}{d}\right)+8a\left(-\frac{\sqrt{dx}}{a}+\frac{d\,\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a\sqrt{ad}}\right)}{d}$	74
meijerg	$\frac{\sqrt{x}\,\sqrt{-a}\left(-\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}}{a}-\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\left(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax})\right)}{a\sqrt{ax}}+\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\ln(-ax+1)}{a}+\frac{2\sqrt{x}\,(-a)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{a}\right)}{\sqrt{dx}\,a}$	109

input
```
int(polylog(2,a*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*polylog(2,a*x)+2*(d*x)^(1/2)*ln((-a*d*x+d)/d)+4*a*(-(d*x)
^(1/2)/a+d/a/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = \left[\frac{2\left(\sqrt{dx}(a\text{Li}_2(ax)+2\,a\log\left(-ax+1\right)-4\,a)+2\,\sqrt{ad}\log\left(\frac{adx+2\,\sqrt{ad}\sqrt{dx}+d}{ax-1}\right)\right)}{ad}, \frac{2\left(\sqrt{dx}(a\text{Li}_2(ax)+2\,a\log\right.}{}\right.$$

input
```
integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
[2*(sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x + 1) - 4*a) + 2*sqrt(a*d)*log((
a*d*x + 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)))/(a*d), 2*(sqrt(d*x)*(a*dilo
g(a*x) + 2*a*log(-a*x + 1) - 4*a) - 4*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*
x)/(a*d*x)))/(a*d)]
```

3.61.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{Li}_2\left(ax\right)}{\sqrt{dx}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(2, a*x)/sqrt(d*x), x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = -\frac{2\left(2\sqrt{dx}(\log(d)+2)-\sqrt{dx}\text{Li}_2(ax)-2\sqrt{dx}\log(-adx+d)+\frac{2d\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}}\right)}{d}$$

input
```
integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*(2*sqrt(d*x)*(log(d) + 2) - sqrt(d*x)*dilog(a*x) - 2*sqrt(d*x)*log(-a*d
*x + d) + 2*d*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqr
t(a*d))/d
```

3.61.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/sqrt(d*x), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{polylog}(2,a\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(polylog(2, a*x)/(d*x)^(1/2),x)
```

output
```
int(polylog(2, a*x)/(d*x)^(1/2), x)
```

3.62 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx$

3.62.1 Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx=\frac{8\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{4\log(1-ax)}{d\sqrt{dx}}-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

output
```
8*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*a^(1/2)/d^(3/2)+4*ln(-a*x+1)/d/(d*x
)^(1/2)-2*polylog(2,a*x)/d/(d*x)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx=\frac{2x\left(4\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\sqrt{a}\sqrt{x}\right)+2\log(1-ax)-\text{PolyLog}(2,ax)\right)}{(dx)^{3/2}}$$

input
```
Integrate[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

output
```
(2*x*(4*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 2*Log[1 - a*x] - PolyLo
g[2, a*x]))/(d*x)^(3/2)
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {7145, 25, 2842, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$2\int -\frac{\log(1-ax)}{(dx)^{3/2}}dx - \frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{\log(1-ax)}{(dx)^{3/2}}dx - \frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 2842}$$

$$-2\left(-\frac{2a\int \frac{1}{\sqrt{dx}(1-ax)}dx}{d} - \frac{2\log(1-ax)}{d\sqrt{dx}}\right) - \frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 73}$$

$$-2\left(-\frac{4a\int \frac{1}{1-ax}d\sqrt{dx}}{d^2} - \frac{2\log(1-ax)}{d\sqrt{dx}}\right) - \frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 219}$$

$$-2\left(-\frac{4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\log(1-ax)}{d\sqrt{dx}}\right) - \frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

output
```
-2*((-4*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*Log[1 -
a*x])/(d*Sqrt[d*x])) - (2*PolyLog[2, a*x])/(d*Sqrt[d*x])
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{\sqrt{dx}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{\sqrt{dx}}+\frac{8a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{\sqrt{ad}}}{d}$	59
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{\sqrt{dx}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{\sqrt{dx}}+\frac{8a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{\sqrt{ad}}}{d}$	59
meijerg	$\frac{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}\left(-\frac{4\sqrt{x}\,\sqrt{-a}\,(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{\sqrt{ax}}+\frac{4\sqrt{-a}\,\ln(-ax+1)}{\sqrt{x}\,a}-\frac{2\sqrt{-a}\,\mathrm{polylog}(2,ax)}{\sqrt{x}\,a}\right)}{(dx)^{\frac{3}{2}}a}$	93

input
```
int(polylog(2,a*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-polylog(2,a*x)/(d*x)^(1/2)+2/(d*x)^(1/2)*ln((-a*d*x+d)/d)+4*a/(a*d)^
(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2)))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.94

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx = \left[\frac{2\left(2\,dx\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\sqrt{dx}(\text{Li}_2(ax)-2\log(-ax+1))\right)}{d^2x},\right.$$
$$\left.-\frac{2\left(4\,dx\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+\sqrt{dx}(\text{Li}_2(ax)-2\log(-ax+1))\right)}{d^2x}\right]$$

input
```
integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
[2*(2*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - sqr
t(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/(d^2*x), -2*(4*d*x*sqrt(-a/d)*arcta
n(sqrt(d*x)*sqrt(-a/d)/(a*x)) + sqrt(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/
(d^2*x)]
```

3.62.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(2, a*x)/(d*x)**(3/2), x)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{2\,a\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}}+\frac{\text{Li}_2(ax)-2\,\log(-adx+d)+2\,\log(d)}{\sqrt{dx}}\right)}{d}$$

input
```
integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*(2*a*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt(a*d)
 + (dilog(a*x) - 2*log(-a*d*x + d) + 2*log(d))/sqrt(d*x))/d
```

3.62.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/(d*x)^(3/2), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx = \int \frac{\text{polylog}(2,a\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int(polylog(2, a*x)/(d*x)^(3/2),x)
```

output
```
int(polylog(2, a*x)/(d*x)^(3/2), x)
```

3.63 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx$

3.63.1 Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx = -\frac{8a}{9d^2\sqrt{dx}}+\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{4\log(1-ax)}{9d(dx)^{3/2}}-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

output
```
8/9*a^(3/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+4/9*ln(-a*x+1)/d/
(d*x)^(3/2)-2/3*polylog(2,a*x)/d/(d*x)^(3/2)-8/9*a/d^2/(d*x)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx = -\frac{2x\left(4ax-4a^{3/2}x^{3/2}\text{arctanh}\left(\sqrt{a}\sqrt{x}\right)-2\log(1-ax)+3\,\text{PolyLog}(2,ax)\right)}{9(dx)^{5/2}}$$

input
```
Integrate[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

output
```
(-2*x*(4*a*x - 4*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 2*Log[1 - a*x]
 + 3*PolyLog[2, a*x]))/(9*(d*x)^(5/2))
```

3.63.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {7145, 25, 2842, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2}{3}\int -\frac{\log(1-ax)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2}{3}\int \frac{\log(1-ax)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 2842}$$

$$-\frac{2}{3}\left(-\frac{2a\int \frac{1}{(dx)^{3/2}(1-ax)}dx}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2}{3}\left(-\frac{2a\left(\frac{a\int \frac{1}{\sqrt{dx}(1-ax)}dx}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2}{3}\left(-\frac{2a\left(\frac{2a\int \frac{1}{1-ax}d\sqrt{dx}}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2}{3}\left(-\frac{2a\left(\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

output
```
(-2*((-2*a*(-2/(d*Sqrt[d*x]) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt
[d]])/d^(3/2)))/(3*d) - (2*Log[1 - a*x])/(3*d*(d*x)^(3/2))))/3 - (2*PolyLo
g[2, a*x])/(3*d*(d*x)^(3/2))
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{3(dx)^{\frac{3}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{a\,\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d\sqrt{ad}}-\frac{1}{d\sqrt{dx}}\right)}{9}}{d}$	75
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{3(dx)^{\frac{3}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{a\,\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d\sqrt{ad}}-\frac{1}{d\sqrt{dx}}\right)}{9}}{d}$	75
meijerg	$\frac{x^{\frac{5}{2}}(-a)^{\frac{5}{2}}\left(-\frac{8}{9\sqrt{x}\,\sqrt{-a}}-\frac{4\sqrt{x}\,a(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{9\sqrt{-a}\,\sqrt{ax}}+\frac{4\ln(-ax+1)}{9x^{\frac{3}{2}}\sqrt{-a}\,a}-\frac{2\,\mathrm{polylog}(2,ax)}{3x^{\frac{3}{2}}\sqrt{-a}\,a}\right)}{(dx)^{\frac{5}{2}}a}$	104

input
```
int(polylog(2,a*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3*polylog(2,a*x)/(d*x)^(3/2)+2/9/(d*x)^(3/2)*ln((-a*d*x+d)/d)+4/9*
a*(a/d/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))-1/d/(d*x)^(1/2)))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int \frac{\mathrm{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx = \left[\frac{2\left(2\,adx^2\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log(-ax+1))\sqrt{dx}\right)}{9\,d^3x^2},\right.$$
$$\left.-\frac{2\left(4\,adx^2\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log(-ax+1))\sqrt{dx}\right)}{9\,d^3x^2}\right]$$

input
```
integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
[2/9*(2*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1))
 - (4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2), -2/9*(4*
a*d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + (4*a*x + 3*dilog(a
*x) - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2)]
```

3.63.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_2\left(ax\right)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)**(5/2),x)
```

output
```
Integral(polylog(2, a*x)/(d*x)**(5/2), x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2, ax)}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{2\,a^2\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}d} + \frac{4\,adx+3\,d\text{Li}_2(ax)-2\,d\log(-adx+d)+2\,d\log(d)}{(dx)^{\frac{3}{2}}d}\right)}{9\,d}$$

input
```
integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/9*(2*a^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt
(a*d)*d) + (4*a*d*x + 3*d*dilog(a*x) - 2*d*log(-a*d*x + d) + 2*d*log(d))/(
(d*x)^(3/2)*d))/d
```

3.63.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_2(ax)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/(d*x)^(5/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{polylog}(2,a\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(polylog(2, a*x)/(d*x)^(5/2),x)
```

output
```
int(polylog(2, a*x)/(d*x)^(5/2), x)
```

3.64 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx$

3.64.1 Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx = -\frac{8a}{75d^2(dx)^{3/2}}-\frac{8a^2}{25d^3\sqrt{dx}}+\frac{8a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{4\log(1-ax)}{25d(dx)^{5/2}}-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

output
```
-8/75*a/d^2/(d*x)^(3/2)+8/25*a^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/
d^(7/2)+4/25*ln(-a*x+1)/d/(d*x)^(5/2)-2/5*polylog(2,a*x)/d/(d*x)^(5/2)-8/2
5*a^2/d^3/(d*x)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx = -\frac{2x\left(4ax+12a^2x^2-12a^{5/2}x^{5/2}\text{arctanh}\left(\sqrt{a}\sqrt{x}\right)-6\log(1-ax)+15\,\text{PolyLog}(2,ax)\right)}{75(dx)^{7/2}}$$

input
```
Integrate[PolyLog[2, a*x]/(d*x)^(7/2),x]
```

output
```
(-2*x*(4*a*x + 12*a^2*x^2 - 12*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] -
6*Log[1 - a*x] + 15*PolyLog[2, a*x]))/(75*(d*x)^(7/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7145, 25, 2842, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2}{5}\int -\frac{\log(1-ax)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2}{5}\int \frac{\log(1-ax)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 2842}$$

$$-\frac{2}{5}\left(-\frac{2a\int \frac{1}{(dx)^{5/2}(1-ax)}dx}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2}{5}\left(-\frac{2a\left(\frac{a\int \frac{1}{(dx)^{3/2}(1-ax)}dx}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{a\int \frac{1}{\sqrt{dx}(1-ax)}dx}{d}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{2a\int \frac{1}{1-ax}d\sqrt{dx}}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

input
```
Int[PolyLog[2, a*x]/(d*x)^(7/2),x]
```

output
```
(-2*((-2*a*(-2/(3*d*(d*x)^(3/2)) + (a*(-2/(d*Sqrt[d*x]) + (2*Sqrt[a]*ArcTa
nh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2)))/d))/(5*d) - (2*Log[1 - a*x])/(5
*d*(d*x)^(5/2))))/5 - (2*PolyLog[2, a*x])/(5*d*(d*x)^(5/2))
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{5(dx)^{\frac{5}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{25(dx)^{\frac{5}{2}}}+\frac{8a\left(\frac{a^{2}\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d^{2}\sqrt{ad}}-\frac{1}{3d(dx)^{\frac{3}{2}}}-\frac{a}{d^{2}\sqrt{dx}}\right)}{25}}{d}$	88
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{5(dx)^{\frac{5}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{25(dx)^{\frac{5}{2}}}+\frac{8a\left(\frac{a^{2}\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d^{2}\sqrt{ad}}-\frac{1}{3d(dx)^{\frac{3}{2}}}-\frac{a}{d^{2}\sqrt{dx}}\right)}{25}}{d}$	88
meijerg	$\frac{x^{\frac{7}{2}}(-a)^{\frac{7}{2}}\left(-\frac{8}{75x^{\frac{3}{2}}(-a)^{\frac{3}{2}}}-\frac{8a}{25\sqrt{x}\,(-a)^{\frac{3}{2}}}-\frac{4\sqrt{x}\,a^{2}\left(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax})\right)}{25(-a)^{\frac{3}{2}}\sqrt{ax}}+\frac{4\ln(-ax+1)}{25x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{2\,\mathrm{polylog}(2,ax)}{5x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}\right)}{(dx)^{\frac{7}{2}}a}$	117

input
```
int(polylog(2,a*x)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/5*polylog(2,a*x)/(d*x)^(5/2)+2/25/(d*x)^(5/2)*ln((-a*d*x+d)/d)+4/2
5*a*(a^2/d^2/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))-1/3/d/(d*x)^(3
/2)-a/d^2/(d*x)^(1/2)))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx = \left[\frac{2\left(6\,a^2dx^3\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-(12\,a^2x^2+4\,ax+15\,\text{Li}_2(ax)-6\,\log(-ax\,-\right.}{75\,d^4x^3}\right.$$

$$\left. -\frac{2\left(12\,a^2dx^3\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+(12\,a^2x^2+4\,ax+15\,\text{Li}_2(ax)-6\,\log(-ax+1))\sqrt{dx}\right)}{75\,d^4x^3}\right]$$

input
```
integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
[2/75*(6*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x -
1)) - (12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d
^4*x^3), -2/75*(12*a^2*d*x^3*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x))
 + (12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d^4*
x^3)]
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(polylog(2,a*x)/(d*x)**(7/2),x)
```

output
```
Timed out
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{\text{PolyLog}(2, ax)}{(dx)^{7/2}} dx = -\frac{2\left(\frac{6a^3 \log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}d^2} + \frac{12a^2d^2x^2+4ad^2x+15d^2\text{Li}_2(ax)-6d^2\log(-adx+d)+6d^2\log(d)}{(dx)^{\frac{5}{2}}d^2}\right)}{75d}$$

input
```
integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/75*(6*a^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqr
t(a*d)*d^2) + (12*a^2*d^2*x^2 + 4*a*d^2*x + 15*d^2*dilog(a*x) - 6*d^2*log(
-a*d*x + d) + 6*d^2*log(d))/((d*x)^(5/2)*d^2))/d
```

3.64.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax)}{(dx)^{7/2}} dx = \int \frac{\text{Li}_2(ax)}{(dx)^{\frac{7}{2}}} dx$$

input
```
integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x)/(d*x)^(7/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax)}{(dx)^{7/2}} dx = \int \frac{\text{polylog}(2, a\,x)}{(d\,x)^{7/2}} dx$$

input
```
int(polylog(2, a*x)/(d*x)^(7/2),x)
```

output
```
int(polylog(2, a*x)/(d*x)^(7/2), x)
```

3.65 $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$

3.65.1 Optimal result

Integrand size = 13, antiderivative size = 153

$$\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx = \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{16d^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{49d} + \frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}$$

output
```
16/1029*d*(d*x)^(3/2)/a^2+16/1715*(d*x)^(5/2)/a+16/2401*(d*x)^(7/2)/d-16/3
43*d^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(7/2)-8/343*(d*x)^(7/2)*
ln(-a*x+1)/d-4/49*(d*x)^(7/2)*polylog(2,a*x)/d+2/7*(d*x)^(7/2)*polylog(3,a
*x)/d+16/343*d^2*(d*x)^(1/2)/a^3
```

3.65.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx = \frac{2(dx)^{5/2}\Big(\frac{8(105+35ax+21a^2x^2+15a^3x^3)}{a^3} - \frac{840\text{arctanh}(\sqrt{a}\sqrt{x})}{a^{7/2}\sqrt{x}} - 420x^3\log(1-ax) - }{36015x^2}$$

input
```
Integrate[(d*x)^(5/2)*PolyLog[3, a*x],x]
```

output
```
(2*(d*x)^(5/2)*((8*(105 + 35*a*x + 21*a^2*x^2 + 15*a^3*x^3))/a^3 - (840*Ar
cTanh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*Sqrt[x]) - 420*x^3*Log[1 - a*x] - 1470*x^
3*PolyLog[2, a*x] + 5145*x^3*PolyLog[3, a*x]))/(36015*x^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {7145, 7145, 25, 2842, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\int (dx)^{5/2}\operatorname{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}-\frac{2}{7}\int -(dx)^{5/2}\log(1-ax)dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2}{7}\int (dx)^{5/2}\log(1-ax)dx+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}\right)$$

$$\Big\downarrow \text{2842}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\int \frac{(dx)^{7/2}}{1-ax}dx}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\int\frac{(dx)^{5/2}}{1-ax}dx}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}\right)$$

$$\Big\downarrow\ 60$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\left(\frac{d\int\frac{(dx)^{3/2}}{1-ax}dx}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}\right)$$

$$\Big\downarrow\ 60$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx}}{1-ax}dx}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax)}{7d}\right)$$

$$\Big\downarrow\ 60$$

3.65. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d} - \frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{Poly}}{7d}$$

$\downarrow$ 73

3.65. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-$$

$$\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{2\int\frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+\frac{2(dx)^{7/2}\,\mathrm{PolyL}}{7d}\right.$$

$$\Big\downarrow \; 219$$

3.65. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax)}{7d}-$$

$$\frac{2}{7}\left(\frac{2}{7}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{a}-\frac{2(dx)^{7/2}}{7a}\right)}{7d}+\frac{2(dx)^{7/2}\log(1-ax)}{7d}\right)+2(dx)^{7}\right.$$

input
```
Int[(d*x)^(5/2)*PolyLog[3, a*x],x]
```

3.65. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax)\,dx$

output
```
(-2*((2*((2*a*((-2*(d*x)^(7/2))/(7*a) + (d*((-2*(d*x)^(5/2))/(5*a) + (d*((
-2*(d*x)^(3/2))/(3*a) + (d*((-2*Sqrt[d*x])/a + (2*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d*x])/Sqrt[d]])/a^(3/2)))/a))/a))/a))/(7*d) + (2*(d*x)^(7/2)*Log[1 -
 a*x])/(7*d)))/7 + (2*(d*x)^(7/2)*PolyLog[2, a*x])/(7*d)))/7 + (2*(d*x)^(7
/2)*PolyLog[3, a*x])/(7*d)
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
meijerg	$\frac{(dx)^{\frac{5}{2}}\left(\frac{2\sqrt{x}\,(-a)^{\frac{9}{2}}\left(360a^3x^3+504a^2x^2+840ax+2520\right)}{108045a^4}+\frac{8\sqrt{x}\,(-a)^{\frac{9}{2}}\left(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax})\right)}{343a^4\sqrt{ax}}-\frac{8x^{\frac{7}{2}}(-a)^{\frac{9}{2}}\ln(-ax+1)}{343a}-\frac{4x^{\frac{7}{2}}(-a)^{\frac{9}{2}}\operatorname{polylog}(2,a}{49a}\right.}{x^{\frac{5}{2}}(-a)^{\frac{5}{2}}a}$

input
```
int((d*x)^(5/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^(5/2)/x^(5/2)/(-a)^(5/2)/a*(2/108045*x^(1/2)*(-a)^(9/2)*(360*a^3*x^3
+504*a^2*x^2+840*a*x+2520)/a^4+8/343*x^(1/2)*(-a)^(9/2)/a^4/(a*x)^(1/2)*(l
n(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8/343*x^(7/2)*(-a)^(9/2)/a*ln(-a*x+1)-
4/49*x^(7/2)*(-a)^(9/2)/a*polylog(2,a*x)+2/7*x^(7/2)*(-a)^(9/2)/a*polylog(
3,a*x))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.82

$$\int (dx)^{5/2}\,\text{PolyLog}(3,ax)\,dx = \left[\frac{2\left(5145\,\sqrt{dx}a^3d^2x^3\text{polylog}(3,ax)+420\,d^2\sqrt{\frac{d}{a}}\log\left(\frac{adx-2\,\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)-2\,(\right.}{}\right.$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="fricas")
```

output
```
[2/36015*(5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) + 420*d^2*sqrt(d/a)*l
og((a*d*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) - 2*(735*a^3*d^2*x^3*d
ilog(a*x) + 210*a^3*d^2*x^3*log(-a*x + 1) - 60*a^3*d^2*x^3 - 84*a^2*d^2*x^
2 - 140*a*d^2*x - 420*d^2)*sqrt(d*x))/a^3, 2/36015*(5145*sqrt(d*x)*a^3*d^2
*x^3*polylog(3, a*x) + 840*d^2*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d)
 - 2*(735*a^3*d^2*x^3*dilog(a*x) + 210*a^3*d^2*x^3*log(-a*x + 1) - 60*a^3*
d^2*x^3 - 84*a^2*d^2*x^2 - 140*a*d^2*x - 420*d^2)*sqrt(d*x))/a^3]
```

3.65.6 Sympy [F]

$$\int (dx)^{5/2}\,\text{PolyLog}(3,ax)\,dx = \int (dx)^{\frac{5}{2}}\,\text{Li}_3\,(ax)\;dx$$

input
```
integrate((d*x)**(5/2)*polylog(3,a*x),x)
```

output
```
Integral((d*x)**(5/2)*polylog(3, a*x), x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.02

$$\int (dx)^{5/2}\,\text{PolyLog}(3,ax)\,dx = \frac{2\left(\frac{420\,d^4\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}a^3} - \frac{1470\,(dx)^{\frac{7}{2}}a^3\text{Li}_2(ax)+420\,(dx)^{\frac{7}{2}}a^3\log(-adx+d)-5145\,(dx)^{\frac{7}{2}}a^3\text{Li}_3(ax)}{a}\right)}{36015\,d}$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="maxima")
```

output
```
2/36015*(420*d^4*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/
(sqrt(a*d)*a^3) - (1470*(d*x)^(7/2)*a^3*dilog(a*x) + 420*(d*x)^(7/2)*a^3*l
og(-a*d*x + d) - 5145*(d*x)^(7/2)*a^3*polylog(3, a*x) - 168*(d*x)^(5/2)*a^
2*d - 60*(7*a^3*log(d) + 2*a^3)*(d*x)^(7/2) - 280*(d*x)^(3/2)*a*d^2 - 840*
sqrt(d*x)*d^3)/a^3)/d
```

3.65.8 Giac [F]

$$\int (dx)^{5/2} \operatorname{PolyLog}(3, ax) \, dx = \int (dx)^{\frac{5}{2}} \operatorname{Li}_3(ax) \, dx$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*polylog(3, a*x), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \operatorname{PolyLog}(3, ax) \, dx = \int (d\,x)^{5/2} \operatorname{polylog}(3, a\,x) \, dx$$

input
```
int((d*x)^(5/2)*polylog(3, a*x),x)
```

output
```
int((d*x)^(5/2)*polylog(3, a*x), x)
```

3.66 $\int (dx)^{3/2} \operatorname{PolyLog}(3, ax) \, dx$

3.66.1 Optimal result

Integrand size = 13, antiderivative size = 136

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax) \, dx = \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{16d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\operatorname{PolyLog}(2, ax)}{25d} + \frac{2(dx)^{5/2}\operatorname{PolyLog}(3, ax)}{5d}$$

output
```
16/375*(d*x)^(3/2)/a+16/625*(d*x)^(5/2)/d-16/125*d^(3/2)*arctanh(a^(1/2)*(
d*x)^(1/2)/d^(1/2))/a^(5/2)-8/125*(d*x)^(5/2)*ln(-a*x+1)/d-4/25*(d*x)^(5/2
)*polylog(2,a*x)/d+2/5*(d*x)^(5/2)*polylog(3,a*x)/d+16/125*d*(d*x)^(1/2)/a
^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax) \, dx = \frac{2d\sqrt{dx}\left(4\left(\frac{30}{a^2} + \frac{10x}{a} + 6x^2 - \frac{30\operatorname{arctanh}(\sqrt{a}\sqrt{x})}{a^{5/2}\sqrt{x}} - 15x^2\log(1-ax)\right) - 150x^2 \operatorname{P}}{1875}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

output
```
(2*d*Sqrt[d*x]*(4*(30/a^2 + (10*x)/a + 6*x^2 - (30*ArcTanh[Sqrt[a]*Sqrt[x]
])/(a^(5/2)*Sqrt[x]) - 15*x^2*Log[1 - a*x]) - 150*x^2*PolyLog[2, a*x] + 37
5*x^2*PolyLog[3, a*x]))/1875
```

3.66.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {7145, 7145, 25, 2842, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \operatorname{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \; 7145$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\int (dx)^{3/2}\operatorname{PolyLog}(2,ax)dx$$

$$\Big\downarrow \; 7145$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}-\frac{2}{5}\int -(dx)^{3/2}\log(1-ax)dx\right)$$

$$\Big\downarrow \; 25$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2}{5}\int (dx)^{3/2}\log(1-ax)dx+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)$$

$$\Big\downarrow \; 2842$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\int \frac{(dx)^{5/2}}{1-ax}dx}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\left(\frac{d\int\frac{(dx)^{3/2}}{1-ax}dx}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\int\frac{\sqrt{dx}}{1-ax}dx}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)$$

$$\Big\downarrow \text{73}$$

3.66. $\int (dx)^{3/2}\operatorname{PolyLog}(3,ax)\,dx$

$$\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{2\int \frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-$$

$$\Big\downarrow \; 219$$

$$\frac{2}{5}\left(\frac{2}{5}\left(\frac{2a\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{a}-\frac{2(dx)^{5/2}}{5a}\right)}{5d}+\frac{2(dx)^{5/2}\log(1-ax)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax)}{5d}\right)\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax)}{5d}-$$

input
```
Int[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

output
```
(-2*((2*((2*a*((-2*(d*x)^(5/2))/(5*a) + (d*((-2*(d*x)^(3/2))/(3*a) + (d*((
-2*Sqrt[d*x])/a + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/a^(3/2)
))/a))/a))/(5*d) + (2*(d*x)^(5/2)*Log[1 - a*x])/(5*d)))/5 + (2*(d*x)^(5/2)
*PolyLog[2, a*x])/(5*d)))/5 + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

3.66. $\int (dx)^{3/2}\operatorname{PolyLog}(3,ax)\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.66.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

method	result
meijerg	$\frac{(dx)^{\frac{3}{2}}\left(\frac{2\sqrt{x}\,(-a)^{\frac{7}{2}}\left(168a^2x^2+280ax+840\right)}{13125a^3}+\frac{8\sqrt{x}\,(-a)^{\frac{7}{2}}\left(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax})\right)}{125a^3\sqrt{ax}}-\frac{8x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\ln(-ax+1)}{125a}-\frac{4x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\operatorname{polylog}(2,ax)}{25a}+\frac{2x^{\frac{5}{2}}(\right.}{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}a}$

input
```
int((d*x)^(3/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^(3/2)/x^(3/2)/(-a)^(3/2)/a*(2/13125*x^(1/2)*(-a)^(7/2)*(168*a^2*x^2+
280*a*x+840)/a^3+8/125*x^(1/2)*(-a)^(7/2)/a^3/(a*x)^(1/2)*(ln(1-(a*x)^(1/2
))-ln(1+(a*x)^(1/2)))-8/125*x^(5/2)*(-a)^(7/2)/a*ln(-a*x+1)-4/25*x^(5/2)*(
-a)^(7/2)/a*polylog(2,a*x)+2/5*x^(5/2)*(-a)^(7/2)/a*polylog(3,a*x))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.68

$$\int (dx)^{3/2}\,\mathrm{PolyLog}(3,ax)\,dx = \left[\frac{2\left(375\sqrt{dx}a^2dx^2\mathrm{polylog}(3,ax)+60\,d\sqrt{\frac{d}{a}}\log\left(\frac{adx-2\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)-2\,(75\,a^2\right.}{1875}\right.$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="fricas")
```

output
```
[2/1875*(375*sqrt(d*x)*a^2*d*x^2*polylog(3, a*x) + 60*d*sqrt(d/a)*log((a*d
*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) - 2*(75*a^2*d*x^2*dilog(a*x)
+ 30*a^2*d*x^2*log(-a*x + 1) - 12*a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/
a^2, 2/1875*(375*sqrt(d*x)*a^2*d*x^2*polylog(3, a*x) + 120*d*sqrt(-d/a)*ar
ctan(sqrt(d*x)*a*sqrt(-d/a)/d) - 2*(75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2
*log(-a*x + 1) - 12*a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2]
```

3.66.6 Sympy [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax)\, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax)\ dx$$

input
```
integrate((d*x)**(3/2)*polylog(3,a*x),x)
```

output
```
Integral((d*x)**(3/2)*polylog(3, a*x), x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax)\, dx = \frac{2\left(\frac{60\, d^3 \log\left(\frac{\sqrt{dx}a - \sqrt{ad}}{\sqrt{dx}a + \sqrt{ad}}\right)}{\sqrt{ad}a^2} - \frac{150\, (dx)^{\frac{5}{2}} a^2 \operatorname{Li}_2(ax) + 60\, (dx)^{\frac{5}{2}} a^2 \log(-adx + d) - 375\, (dx)^{\frac{5}{2}} a^2 \operatorname{Li}_3(ax) - 12}{a^2}\right.}{1875\, d}$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="maxima")
```

output
```
2/1875*(60*d^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(s
qrt(a*d)*a^2) - (150*(d*x)^(5/2)*a^2*dilog(a*x) + 60*(d*x)^(5/2)*a^2*log(-
a*d*x + d) - 375*(d*x)^(5/2)*a^2*polylog(3, a*x) - 12*(5*a^2*log(d) + 2*a^
2)*(d*x)^(5/2) - 40*(d*x)^(3/2)*a*d - 120*sqrt(d*x)*d^2)/a^2)/d
```

3.66.8 Giac [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax)\, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax)\, dx$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*polylog(3, a*x), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax) \, dx = \int (d\,x)^{3/2} \operatorname{polylog}(3, a\,x) \, dx$$

input
```
int((d*x)^(3/2)*polylog(3, a*x),x)
```

output
```
int((d*x)^(3/2)*polylog(3, a*x), x)
```

3.67 $\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx$

3.67.1 Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx = \frac{16\sqrt{dx}}{27a}+\frac{16(dx)^{3/2}}{81d}-\frac{16\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}} - \frac{8(dx)^{3/2}\log(1-ax)}{27d}-\frac{4(dx)^{3/2}\,\text{PolyLog}(2,ax)}{9d} + \frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax)}{3d}$$

output
```
16/81*(d*x)^(3/2)/d-8/27*(d*x)^(3/2)*ln(-a*x+1)/d-4/9*(d*x)^(3/2)*polylog(
2,a*x)/d+2/3*(d*x)^(3/2)*polylog(3,a*x)/d-16/27*arctanh(a^(1/2)*(d*x)^(1/2
)/d^(1/2))*d^(1/2)/a^(3/2)+16/27*(d*x)^(1/2)/a
```

3.67.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx = \frac{2}{81}\sqrt{dx}\left(4\left(\frac{6}{a}+2x-\frac{6\text{arctanh}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}}-3x\log(1-ax)\right) - 18x\,\text{PolyLog}(2,ax)+27x\,\text{PolyLog}(3,ax)\right)$$

input
```
Integrate[Sqrt[d*x]*PolyLog[3, a*x],x]
```

output
```
(2*Sqrt[d*x]*(4*(6/a + 2*x - (6*ArcTanh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[x]
) - 3*x*Log[1 - a*x]) - 18*x*PolyLog[2, a*x] + 27*x*PolyLog[3, a*x]))/81
```

3.67.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {7145, 7145, 25, 2842, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax)}{3d}-\frac{2}{3}\int \sqrt{dx}\,\text{PolyLog}(2,ax)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}-\frac{2}{3}\int -\sqrt{dx}\log(1-ax)dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\int \sqrt{dx}\log(1-ax)dx+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}\right)$$

$$\Big\downarrow \text{2842}$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2a\int \frac{(dx)^{3/2}}{1-ax}dx}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax)}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2a\left(\frac{d\int\frac{\sqrt{dx}}{1-ax}dx}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{d\int\frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{2\int\frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax)}{3d}-\frac{2}{3}\left(\frac{2}{3}\left(\frac{2a\left(\frac{d\left(\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{a}-\frac{2(dx)^{3/2}}{3a}\right)}{3d}+\frac{2(dx)^{3/2}\log(1-ax)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax)}{3d}\right)$$

input

```
Int[Sqrt[d*x]*PolyLog[3, a*x],x]
```

3.67. $\int\sqrt{dx}\operatorname{PolyLog}(3,ax)\,dx$

output
```
(-2*((2*((2*a*((-2*(d*x)^(3/2))/(3*a) + (d*((-2*Sqrt[d*x])/a + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/a^(3/2)))/a))/(3*d) + (2*(d*x)^(3/2)
*Log[1 - a*x])/(3*d)))/3 + (2*(d*x)^(3/2)*PolyLog[2, a*x])/(3*d)))/3 + (2*
(d*x)^(3/2)*PolyLog[3, a*x])/(3*d)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

method	result
meijerg	$\frac{\sqrt{dx}\left(\frac{2\sqrt{x}\,(-a)^{\frac{5}{2}}(40ax+120)}{405a^{2}}+\frac{8\sqrt{x}\,(-a)^{\frac{5}{2}}(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{27a^{2}\sqrt{ax}}-\frac{8x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\ln(-ax+1)}{27a}-\frac{4x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\,\mathrm{polylog}(2,ax)}{9a}+\frac{2x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\,\mathrm{polylog}(}{3a}\right.}{\sqrt{x}\,\sqrt{-a}\,a}$

input
```
int((d*x)^(1/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^(1/2)/x^(1/2)/(-a)^(1/2)/a*(2/405*x^(1/2)*(-a)^(5/2)*(40*a*x+120)/a^
2+8/27*x^(1/2)*(-a)^(5/2)/a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1
/2)))-8/27*x^(3/2)*(-a)^(5/2)/a*ln(-a*x+1)-4/9*x^(3/2)*(-a)^(5/2)/a*polylo
g(2,a*x)+2/3*x^(3/2)*(-a)^(5/2)/a*polylog(3,a*x))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.43

$$\int \sqrt{dx}\,\mathrm{PolyLog}(3,ax)\,dx$$

$$= \left[\frac{2\left(27\sqrt{dx}ax\mathrm{polylog}(3,ax) - 2\left(9\,ax\mathrm{Li}_2(ax) + 6\,ax\log\left(-ax+1\right) - 4\,ax - 12\right)\sqrt{dx} + 12\sqrt{\frac{d}{a}}\log\left(\frac{adx-}{}\right.\right.}{81\,a}\right.$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="fricas")
```

output
```
[2/81*(27*sqrt(d*x)*a*x*polylog(3, a*x) - 2*(9*a*x*dilog(a*x) + 6*a*x*log(
-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 12*sqrt(d/a)*log((a*d*x - 2*sqrt(d*x)*
a*sqrt(d/a) + d)/(a*x - 1)))/a, 2/81*(27*sqrt(d*x)*a*x*polylog(3, a*x) - 2
*(9*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 24*sqrt
(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

3.67.6 Sympy [F]

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx = \int \sqrt{dx}\,\text{Li}_3\,(ax)\,\,dx$$

input
```
integrate((d*x)**(1/2)*polylog(3,a*x),x)
```

output
```
Integral(sqrt(d*x)*polylog(3, a*x), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx$$
$$= \frac{2\left(\frac{12\,d^2\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}a} - \frac{18\,(dx)^{\frac{3}{2}}\,a\text{Li}_2(ax)+12\,(dx)^{\frac{3}{2}}\,a\log(-adx+d)-27\,(dx)^{\frac{3}{2}}\,a\text{Li}_3(ax)-4\,(dx)^{\frac{3}{2}}\,(3\,a\log(d)+2\,a)-24\,\sqrt{dx}d}{a}\right)}{81\,d}$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="maxima")
```

output
```
2/81*(12*d^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqr
t(a*d)*a) - (18*(d*x)^(3/2)*a*dilog(a*x) + 12*(d*x)^(3/2)*a*log(-a*d*x + d
) - 27*(d*x)^(3/2)*a*polylog(3, a*x) - 4*(d*x)^(3/2)*(3*a*log(d) + 2*a) -
24*sqrt(d*x)*d)/a)/d
```

3.67.8 Giac [F]

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx = \int \sqrt{dx}\text{Li}_3(ax)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*polylog(3, a*x), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax)\,dx = \int \sqrt{d\,x}\,\text{polylog}(3,a\,x)\,dx$$

input
```
int((d*x)^(1/2)*polylog(3, a*x),x)
```

output
```
int((d*x)^(1/2)*polylog(3, a*x), x)
```

3.68 $\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx$

3.68.1 Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \frac{16\sqrt{dx}}{d} - \frac{16\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} - \frac{8\sqrt{dx}\log(1-ax)}{d} - \frac{4\sqrt{dx}\,\text{PolyLog}(2,ax)}{d} + \frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}$$

output
```
-16*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(1/2)/d^(1/2)+16*(d*x)^(1/2)/d-
8*ln(-a*x+1)*(d*x)^(1/2)/d-4*polylog(2,a*x)*(d*x)^(1/2)/d+2*polylog(3,a*x)
*(d*x)^(1/2)/d
```

3.68.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \frac{2x\left(8-\frac{8\text{arctanh}(\sqrt{a}\sqrt{x})}{\sqrt{a}\sqrt{x}}-4\log(1-ax)-2\,\text{PolyLog}(2,ax)+\text{PolyLog}(3,ax)\right)}{\sqrt{dx}}$$

input
```
Integrate[PolyLog[3, a*x]/Sqrt[d*x],x]
```

output
```
(2*x*(8 - (8*ArcTanh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[x]) - 4*Log[1 - a*x]
- 2*PolyLog[2, a*x] + PolyLog[3, a*x]))/Sqrt[d*x]
```

3.68.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {7145, 7145, 25, 2842, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-2\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-2\left(\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}-2\int -\frac{\log(1-ax)}{\sqrt{dx}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-2\left(2\int \frac{\log(1-ax)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}\right)$$

$$\Big\downarrow \text{2842}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-2\left(2\left(\frac{2a\int \frac{\sqrt{dx}}{1-ax}dx}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-2\left(2\left(\frac{2a\left(\frac{d\int \frac{1}{\sqrt{dx}(1-ax)}dx}{a}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{dx}\,\mathrm{PolyLog}(3,ax)}{d}-2\left(2\left(\frac{2a\left(\frac{2\int\frac{1}{1-ax}d\sqrt{dx}}{a}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\mathrm{PolyLog}(2,ax)}{d}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{dx}\,\mathrm{PolyLog}(3,ax)}{d}-2\left(2\left(\frac{2a\left(\frac{2\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{a^{3/2}}-\frac{2\sqrt{dx}}{a}\right)}{d}+\frac{2\sqrt{dx}\log(1-ax)}{d}\right)+\frac{2\sqrt{dx}\,\mathrm{PolyLog}(2,ax)}{d}\right)$$

input
```
Int[PolyLog[3, a*x]/Sqrt[d*x],x]
```

output
```
-2*(2*((2*a*((-2*Sqrt[d*x])/a + (2*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqr
t[d]])/a^(3/2)))/d + (2*Sqrt[d*x]*Log[1 - a*x])/d) + (2*Sqrt[d*x]*PolyLog[
2, a*x])/d) + (2*Sqrt[d*x]*PolyLog[3, a*x])/d
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

method	result
meijerg	$\frac{\sqrt{x}\,\sqrt{-a}\left(\frac{16\sqrt{x}\,(-a)^{\frac{3}{2}}}{a}+\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{a\sqrt{ax}}-\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}\ln(-ax+1)}{a}-\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{a}+\frac{2\sqrt{x}\,(-a)^{\frac{3}{2}}\operatorname{polylog}(3,ax)}{a}\right)}{\sqrt{dx}\,a}$

input
```
int(polylog(3,a*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x)^(1/2)*x^(1/2)*(-a)^(1/2)/a*(16*x^(1/2)*(-a)^(3/2)/a+8*x^(1/2)*(-a)
^(3/2)/a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8*x^(1/2)*(-a)^
(3/2)/a*ln(-a*x+1)-4*x^(1/2)*(-a)^(3/2)/a*polylog(2,a*x)+2*x^(1/2)*(-a)^(3
/2)/a*polylog(3,a*x))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.66

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \left[\frac{2\left(\sqrt{dx}a\text{polylog}(3,ax) - 2\sqrt{dx}(a\text{Li}_2(ax) + 2a\log(-ax+1) - 4a) + 4\sqrt{ad}\log\left(\frac{adx - 2\sqrt{ad}\sqrt{dx} + d}{ax-1}\right)\right)}{ad}, \frac{2}{}\right.$$

input
```
integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
[2*(sqrt(d*x)*a*polylog(3, a*x) - 2*sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x
 + 1) - 4*a) + 4*sqrt(a*d)*log((a*d*x - 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x -
1)))/(a*d), 2*(sqrt(d*x)*a*polylog(3, a*x) - 2*sqrt(d*x)*(a*dilog(a*x) + 2
*a*log(-a*x + 1) - 4*a) + 8*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*x)/(a*d*x)
))/(a*d)]
```

3.68.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(3, a*x)/sqrt(d*x), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \frac{2\left(4\sqrt{dx}(\log(d)+2) - 2\sqrt{dx}\text{Li}_2(ax) - 4\sqrt{dx}\log(-adx+d) + \frac{4d\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}} + \sqrt{dx}\text{Li}_3(ax)\right)}{d}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2*(4*sqrt(d*x)*(log(d) + 2) - 2*sqrt(d*x)*dilog(a*x) - 4*sqrt(d*x)*log(-a*
d*x + d) + 4*d*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sq
rt(a*d) + sqrt(d*x)*polylog(3, a*x))/d
```

3.68.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/sqrt(d*x), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx = \int \frac{\text{polylog}(3,a\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(polylog(3, a*x)/(d*x)^(1/2),x)
```

output
```
int(polylog(3, a*x)/(d*x)^(1/2), x)
```

3.69 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx$

3.69.1 Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = \frac{16\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\,\text{PolyLog}(2,ax)}{d\sqrt{dx}} - \frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

output
```
16*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*a^(1/2)/d^(3/2)+8*ln(-a*x+1)/d/(d*
x)^(1/2)-4*polylog(2,a*x)/d/(d*x)^(1/2)-2*polylog(3,a*x)/d/(d*x)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = \frac{2x\left(8\sqrt{a}\sqrt{x}\,\text{arctanh}\left(\sqrt{a}\sqrt{x}\right) + 4\log(1-ax) - 2\,\text{PolyLog}(2,ax) - \text{PolyLog}(3,ax)\right)}{(dx)^{3/2}}$$

input
```
Integrate[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

output
```
(2*x*(8*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Log[1 - a*x] - 2*Poly
Log[2, a*x] - PolyLog[3, a*x]))/(d*x)^(3/2)
```

3.69.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {7145, 7145, 25, 2842, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$2\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{7145}$$

$$2\left(2\int -\frac{\log(1-ax)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{25}$$

$$2\left(-2\int \frac{\log(1-ax)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{2842}$$

$$2\left(-2\left(-\frac{2a\int \frac{1}{\sqrt{dx}(1-ax)}dx}{d}-\frac{2\log(1-ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{73}$$

$$2\left(-2\left(-\frac{4a\int \frac{1}{1-ax}d\sqrt{dx}}{d^2}-\frac{2\log(1-ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{219}$$

$$2\left(-2\left(-\frac{4\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\log(1-ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

output
```
2*(-2*((-4*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*Log[
1 - a*x])/(d*Sqrt[d*x])) - (2*PolyLog[2, a*x])/(d*Sqrt[d*x])) - (2*PolyLog
[3, a*x])/(d*Sqrt[d*x])
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result	size
meijerg	$\frac{x^{\frac{3}{2}} (-a)^{\frac{3}{2}} \left(-\frac{8\sqrt{x}\,\sqrt{-a}\,(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{\sqrt{ax}}+\frac{8\sqrt{-a}\,\ln(-ax+1)}{\sqrt{x}\,a}-\frac{4\sqrt{-a}\,\mathrm{polylog}(2,ax)}{\sqrt{x}\,a}-\frac{2\sqrt{-a}\,\mathrm{polylog}(3,ax)}{\sqrt{x}\,a}\right)}{(dx)^{\frac{3}{2}} a}$	111

input
```
int(polylog(3,a*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x)^(3/2)*x^(3/2)*(-a)^(3/2)/a*(-8*x^(1/2)*(-a)^(1/2)/(a*x)^(1/2)*(ln(
1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/x^(1/2)*(-a)^(1/2)/a*ln(-a*x+1)-4/x^(1
/2)*(-a)^(1/2)/a*polylog(2,a*x)-2/x^(1/2)*(-a)^(1/2)/a*polylog(3,a*x))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

$$\int \frac{\mathrm{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = \left[\frac{2\left(4\,dx\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\sqrt{dx}(\mathrm{Li}_2(ax)-2\log(-ax+1))-\sqrt{dx}\mathrm{polyl}}{d^2x}\right.$$

$$\left.-\frac{2\left(8\,dx\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+2\sqrt{dx}(\mathrm{Li}_2(ax)-2\log(-ax+1))+\sqrt{dx}\mathrm{polylog}(3,ax)\right)}{d^2x}\right]$$

input
```
integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
[2*(4*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - 2*s
qrt(d*x)*(dilog(a*x) - 2*log(-a*x + 1)) - sqrt(d*x)*polylog(3, a*x))/(d^2*
x), -2*(8*d*x*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + 2*sqrt(d*x)*
(dilog(a*x) - 2*log(-a*x + 1)) + sqrt(d*x)*polylog(3, a*x))/(d^2*x)]
```

3.69.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3\left(ax\right)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(3, a*x)/(d*x)**(3/2), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{4\,a\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}}+\frac{2\,\text{Li}_2(ax)-4\,\log(-adx+d)+4\,\log(d)+\text{Li}_3(ax)}{\sqrt{dx}}\right)}{d}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*(4*a*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt(a*d)
 + (2*dilog(a*x) - 4*log(-a*d*x + d) + 4*log(d) + polylog(3, a*x))/sqrt(d*
x))/d
```

3.69.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/(d*x)^(3/2), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{PolyLog}(3, ax)}{(dx)^{3/2}} \, dx = \int \frac{\operatorname{polylog}(3, a\,x)}{(d\,x)^{3/2}} \, dx$$

input
```
int(polylog(3, a*x)/(d*x)^(3/2),x)
```

output
```
int(polylog(3, a*x)/(d*x)^(3/2), x)
```

3.70 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx$

3.70.1 Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = -\frac{16a}{27d^2\sqrt{dx}}+\frac{16a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\,\text{PolyLog}(2,ax)}{9d(dx)^{3/2}}-\frac{2\,\text{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

output
```
16/27*a^(3/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+8/27*ln(-a*x+1)
/d/(d*x)^(3/2)-4/9*polylog(2,a*x)/d/(d*x)^(3/2)-2/3*polylog(3,a*x)/d/(d*x)
^(3/2)-16/27*a/d^2/(d*x)^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = -\frac{2x\left(8ax-8a^{3/2}x^{3/2}\text{arctanh}\left(\sqrt{a}\sqrt{x}\right)-4\log(1-ax)+6\,\text{PolyLog}(2,ax)+9\,\text{PolyLog}(3,ax)\right)}{27(dx)^{5/2}}$$

input
```
Integrate[PolyLog[3, a*x]/(d*x)^(5/2),x]
```

output
```
(-2*x*(8*a*x - 8*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 4*Log[1 - a*x]
 + 6*PolyLog[2, a*x] + 9*PolyLog[3, a*x]))/(27*(d*x)^(5/2))
```

3.70.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7145, 7145, 25, 2842, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(3,ax)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{3}\int \frac{\operatorname{PolyLog}(2,ax)}{(dx)^{5/2}}dx-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{3}\left(\frac{2}{3}\int -\frac{\log(1-ax)}{(dx)^{5/2}}dx-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(-\frac{2}{3}\int \frac{\log(1-ax)}{(dx)^{5/2}}dx-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{2842}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2a\int \frac{1}{(dx)^{3/2}(1-ax)}dx}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2a\left(\frac{a\int \frac{1}{\sqrt{dx}(1-ax)}dx}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2a\left(\frac{2a\int \frac{1}{1-ax}d\sqrt{dx}}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\left(-\frac{2}{3}\left(-\frac{2a\left(\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2\log(1-ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(2,ax)}{3d(dx)^{3/2}}\right)-\frac{2\operatorname{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[3, a*x]/(d*x)^(5/2),x]
```

output
```
(2*((-2*((-2*a*(-2/(d*Sqrt[d*x]) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/
Sqrt[d]])/d^(3/2)))/(3*d) - (2*Log[1 - a*x])/(3*d*(d*x)^(3/2))))/3 - (2*Po
lyLog[2, a*x])/(3*d*(d*x)^(3/2))))/3 - (2*PolyLog[3, a*x])/(3*d*(d*x)^(3/2
))
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

method	result	size
meijerg	$\frac{x^{\frac{5}{2}} (-a)^{\frac{5}{2}} \left(-\frac{16}{27 \sqrt{x}\, \sqrt{-a}}-\frac{8 \sqrt{x}\, a \left(\ln \left(1-\sqrt{a x}\right)-\ln \left(1+\sqrt{a x}\right)\right)}{27 \sqrt{-a}\, \sqrt{a x}}+\frac{8 \ln \left(-a x+1\right)}{27 x^{\frac{3}{2}} \sqrt{-a}\, a}-\frac{4 \,\operatorname{polylog}\left(2, a x\right)}{9 x^{\frac{3}{2}} \sqrt{-a}\, a}-\frac{2 \,\operatorname{polylog}\left(3, a x\right)}{3 x^{\frac{3}{2}} \sqrt{-a}\, a}\right)}{\left(d x\right)^{\frac{5}{2}} a}$	122

input
```
int(polylog(3,a*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x)^(5/2)*x^(5/2)*(-a)^(5/2)/a*(-16/27/x^(1/2)/(-a)^(1/2)-8/27*x^(1/2)
/(-a)^(1/2)*a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/27/x^(3/
2)/(-a)^(1/2)/a*ln(-a*x+1)-4/9/x^(3/2)/(-a)^(1/2)/a*polylog(2,a*x)-2/3/x^(
3/2)/(-a)^(1/2)/a*polylog(3,a*x))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = \left[\frac{2\left(4\,adx^2\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right) - 2\left(4\,ax+3\,\text{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}\right.}{27\,d^3x^2}\right.$$

$$-\frac{2\left(8\,adx^2\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+2\left(4\,ax+3\,\text{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}+9\,\sqrt{dx}\text{polylog}(3,ax)\right)}{27\,d^3x^2}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
[2/27*(4*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)
) - 2*(4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x) - 9*sqrt(d*x)*pol
ylog(3, a*x))/(d^3*x^2), -2/27*(8*a*d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt
(-a/d)/(a*x)) + 2*(4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x) + 9*s
qrt(d*x)*polylog(3, a*x))/(d^3*x^2)]
```

3.70.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3\left(ax\right)}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)**(5/2),x)
```

output
```
Integral(polylog(3, a*x)/(d*x)**(5/2), x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{4\,a^2\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}d}+\frac{8\,adx+6\,d\text{Li}_2(ax)-4\,d\log(-adx+d)+4\,d\log(d)+9\,d\text{Li}_3(ax)}{(dx)^{\frac{3}{2}}d}\right)}{27\,d}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/27*(4*a^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqr
t(a*d)*d) + (8*a*d*x + 6*d*dilog(a*x) - 4*d*log(-a*d*x + d) + 4*d*log(d) +
 9*d*polylog(3, a*x))/((d*x)^(3/2)*d))/d
```

3.70.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/(d*x)^(5/2), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx = \int \frac{\text{polylog}(3,a\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(polylog(3, a*x)/(d*x)^(5/2),x)
```

output
```
int(polylog(3, a*x)/(d*x)^(5/2), x)
```

3.71 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx$

3.71.1 Optimal result

Integrand size = 13, antiderivative size = 125

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\,\text{PolyLog}(2,ax)}{25d(dx)^{5/2}} - \frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

output
```
-16/375*a/d^2/(d*x)^(3/2)+16/125*a^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/
2))/d^(7/2)+8/125*ln(-a*x+1)/d/(d*x)^(5/2)-4/25*polylog(2,a*x)/d/(d*x)^(5/
2)-2/5*polylog(3,a*x)/d/(d*x)^(5/2)-16/125*a^2/d^3/(d*x)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = -\frac{2x\left(8ax + 24a^2x^2 - 24a^{5/2}x^{5/2}\text{arctanh}\left(\sqrt{a}\sqrt{x}\right) - 12\log(1-ax) + 30\,\text{PolyLog}(2,ax) + 75\,\text{PolyLog}(3,ax}{375(dx)^{7/2}}$$

input
```
Integrate[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

output
```
(-2*x*(8*a*x + 24*a^2*x^2 - 24*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] -
12*Log[1 - a*x] + 30*PolyLog[2, a*x] + 75*PolyLog[3, a*x]))/(375*(d*x)^(7/
2))
```

3.71.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {7145, 7145, 25, 2842, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{5}\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{7145}$$

$$\frac{2}{5}\left(\frac{2}{5}\int -\frac{\log(1-ax)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}\left(-\frac{2}{5}\int \frac{\log(1-ax)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{2842}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2a\int \frac{1}{(dx)^{5/2}(1-ax)}dx}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2a\left(\frac{a\int \frac{1}{(dx)^{3/2}(1-ax)}dx}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{a\int\frac{1}{\sqrt{dx}(1-ax)}dx}{d}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{2a\int\frac{1}{1-ax}d\sqrt{dx}}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2}{5}\left(-\frac{2}{5}\left(-\frac{2a\left(\frac{a\left(\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2}{d\sqrt{dx}}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2\log(1-ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

input
```
Int[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

output
```
(2*((-2*((-2*a*(-2/(3*d*(d*x)^(3/2)) + (a*(-2/(d*Sqrt[d*x]) + (2*Sqrt[a]*A
rcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2)))/d))/(5*d) - (2*Log[1 - a*x]
)/(5*d*(d*x)^(5/2))))/5 - (2*PolyLog[2, a*x])/(5*d*(d*x)^(5/2))))/5 - (2*P
olyLog[3, a*x])/(5*d*(d*x)^(5/2))
```

3.71. $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

method	result	size
meijerg	$\frac{x^{\frac{7}{2}}(-a)^{\frac{7}{2}}\left(-\frac{16}{375x^{\frac{3}{2}}(-a)^{\frac{3}{2}}}-\frac{16a}{125\sqrt{x}\,(-a)^{\frac{3}{2}}}-\frac{8\sqrt{x}\,a^{2}(\ln(1-\sqrt{ax})-\ln(1+\sqrt{ax}))}{125(-a)^{\frac{3}{2}}\sqrt{ax}}+\frac{8\ln(-ax+1)}{125x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{4\,\mathrm{polylog}(2,ax)}{25x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{2\,\mathrm{polylog}(3,ax)}{5x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}\right)}{(dx)^{\frac{7}{2}}a}$	135

input
```
int(polylog(3,a*x)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x)^(7/2)*x^(7/2)*(-a)^(7/2)/a*(-16/375/x^(3/2)/(-a)^(3/2)-16/125/x^(1
/2)/(-a)^(3/2)*a-8/125*x^(1/2)/(-a)^(3/2)*a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2
))-ln(1+(a*x)^(1/2)))+8/125/x^(5/2)/(-a)^(3/2)/a*ln(-a*x+1)-4/25/x^(5/2)/(
-a)^(3/2)/a*polylog(2,a*x)-2/5/x^(5/2)/(-a)^(3/2)/a*polylog(3,a*x))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.56

$$\int \frac{\mathrm{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = \left[\frac{2\left(12\,a^{2}dx^{3}\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\left(12\,a^{2}x^{2}+4\,ax+15\,\mathrm{Li}_{2}(ax)-6\,\log\left(-a\right.\right.\right.}{375\,d^{4}x^{3}}\right.$$

$$-\frac{2\left(24\,a^{2}dx^{3}\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+2\left(12\,a^{2}x^{2}+4\,ax+15\,\mathrm{Li}_{2}(ax)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}+75\,\sqrt{dx}\mathrm{p}\right.}{375\,d^{4}x^{3}}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
[2/375*(12*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x
- 1)) - 2*(12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x)
 - 75*sqrt(d*x)*polylog(3, a*x))/(d^4*x^3), -2/375*(24*a^2*d*x^3*sqrt(-a/d
)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + 2*(12*a^2*x^2 + 4*a*x + 15*dilog(a*
x) - 6*log(-a*x + 1))*sqrt(d*x) + 75*sqrt(d*x)*polylog(3, a*x))/(d^4*x^3)]
```

3.71.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = \int \frac{\text{Li}_3(ax)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)**(7/2),x)
```

output
```
Integral(polylog(3, a*x)/(d*x)**(7/2), x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = -\frac{2\left(\frac{12\,a^3\log\left(\frac{\sqrt{dx}a-\sqrt{ad}}{\sqrt{dx}a+\sqrt{ad}}\right)}{\sqrt{ad}d^2}+\frac{24\,a^2d^2x^2+8\,ad^2x+30\,d^2\text{Li}_2(ax)-12\,d^2\log(-adx+d)+12\,d^2\log(d)+75\,d^2\text{Li}_3(ax)}{(dx)^{\frac{5}{2}}d^2}\right)}{375\,d}$$

input
```
integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/375*(12*a^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(s
qrt(a*d)*d^2) + (24*a^2*d^2*x^2 + 8*a*d^2*x + 30*d^2*dilog(a*x) - 12*d^2*l
og(-a*d*x + d) + 12*d^2*log(d) + 75*d^2*polylog(3, a*x))/((d*x)^(5/2)*d^2)
)/d
```

3.71.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx = \int \frac{\text{Li}_3(ax)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x)/(d*x)^(7/2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax)}{(dx)^{7/2}} \, dx = \int \frac{\text{polylog}(3, a\,x)}{(d\,x)^{7/2}} \, dx$$

input
```
int(polylog(3, a*x)/(d*x)^(7/2),x)
```

output
```
int(polylog(3, a*x)/(d*x)^(7/2), x)
```

3.72 $\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^2) \, dx$

3.72.1 Optimal result

Integrand size = 15, antiderivative size = 140

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^2) \, dx = -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{16d^{3/2} \arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2} \operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \operatorname{PolyLog}(2, ax^2)}{5d}$$

output
```
-32/125*(d*x)^(5/2)/d+16/25*d^(3/2)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^
(5/4)+16/25*d^(3/2)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(5/4)+8/25*(d*x
)^(5/2)*ln(-a*x^2+1)/d+2/5*(d*x)^(5/2)*polylog(2,a*x^2)/d-32/25*d*(d*x)^(1
/2)/a
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^2) \, dx = \frac{2(dx)^{3/2} \left(\frac{40 \arctan\left(\sqrt[4]{a}\sqrt{x}\right) + 40 \operatorname{arctanh}\left(\sqrt[4]{a}\sqrt{x}\right) + 4\sqrt[4]{a}\sqrt{x}\left(-20 - 4ax^2 + 5ax^2 \log(1 - ax^2\right.}{a^{5/4}}\right.}{125x^{3/2}}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
```

output
```
(2*(d*x)^(3/2)*((40*ArcTan[a^(1/4)*Sqrt[x]] + 40*ArcTanh[a^(1/4)*Sqrt[x]]
+ 4*a^(1/4)*Sqrt[x]*(-20 - 4*a*x^2 + 5*a*x^2*Log[1 - a*x^2]))/a^(5/4) + 25
*x^(5/2)*PolyLog[2, a*x^2]))/(125*x^(3/2))
```

3.72.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7145, 25, 2905, 8, 262, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \operatorname{PolyLog}\left(2,ax^2\right) \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}-\frac{4}{5} \int -(dx)^{3/2} \log \left(1-ax^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{4}{5} \int (dx)^{3/2} \log \left(1-ax^2\right)dx+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$$\Big\downarrow \text{2905}$$

$$\frac{4}{5} \left(\frac{4a \int \frac{x(dx)^{5/2}}{1-ax^2}dx}{5d}+\frac{2(dx)^{5/2} \log \left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$$\Big\downarrow \text{8}$$

$$\frac{4}{5} \left(\frac{4a \int \frac{(dx)^{7/2}}{1-ax^2}dx}{5d^2}+\frac{2(dx)^{5/2} \log \left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{5} \left(\frac{4a \left(\frac{d^2 \int \frac{(dx)^{3/2}}{1-ax^2}dx}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2} \log \left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}dx}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$\Big\downarrow$ 266

$$\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\int \frac{1}{1-ax^2}d\sqrt{dx}}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$\Big\downarrow$ 756

$$\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$\Big\downarrow$ 218

3.72. $\int (dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)\,dx$

$$\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

$$\Big\downarrow \; 221$$

3.72. $\int (dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)\,dx$

$$\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(2,ax^2\right)}{5d}$$

input
```
Int[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
```

output
```
(4*((4*a*((-2*d*(d*x)^(5/2))/(5*a) + (d^2*((-2*d*Sqrt[d*x])/a + (2*d*((Sqr
t[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4)) + (Sqrt[d]*ArcTanh[(
a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4))))/a))/a))/(5*d^2) + (2*(d*x)^(5/2
)*Log[1 - a*x^2])/(5*d)))/5 + (2*(d*x)^(5/2)*PolyLog[2, a*x^2])/(5*d)
```

3.72. $\int (dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)\,dx$

3.72.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

method	result
meijerg	$-\frac{(dx)^{\frac{3}{2}}\left(-\frac{4\sqrt{x}\,(-a)^{\frac{9}{4}}\left(144a\,x^{2}+720\right)}{1125a^{2}}-\frac{16\sqrt{x}\,(-a)^{\frac{9}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{25a^{2}\left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{16x^{\frac{5}{2}}(-a)^{\frac{9}{4}}\cdots}{2\cdots}\right.}{2x^{\frac{3}{2}}(-a)^{\frac{5}{4}}}$
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{5}+\frac{8(dx)^{\frac{5}{2}}\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25}+\frac{32a\left(-\frac{\frac{a(dx)^{\frac{5}{2}}}{5}+d^{2}\sqrt{dx}}{a^{2}}+\frac{d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4a^{2}}\cdots\right.}{25}\cdots}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{5}+\frac{8(dx)^{\frac{5}{2}}\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25}+\frac{32a\left(-\frac{\frac{a(dx)^{\frac{5}{2}}}{5}+d^{2}\sqrt{dx}}{a^{2}}+\frac{d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4a^{2}}\cdots\right.}{25}\cdots}{d}$

input
```
int((d*x)^(3/2)*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(3/2)/x^(3/2)/(-a)^(5/4)*(-4/1125*x^(1/2)*(-a)^(9/4)*(144*a*x^2
+720)/a^2-16/25*x^(1/2)*(-a)^(9/4)/a^2/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-
ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))+16/25*x^(5/2)*(-a)^(9/4)*ln(-
a*x^2+1)/a+4/5*x^(5/2)*(-a)^(9/4)/a*polylog(2,a*x^2))
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.34

$$\int (dx)^{3/2}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \frac{2\left(20\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}d+8\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)+20i\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}d+8i\,a\right.\right.}{}$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
2/125*(20*a*(d^6/a^5)^(1/4)*log(8*sqrt(d*x)*d + 8*a*(d^6/a^5)^(1/4)) + 20*
I*a*(d^6/a^5)^(1/4)*log(8*sqrt(d*x)*d + 8*I*a*(d^6/a^5)^(1/4)) - 20*I*a*(d
^6/a^5)^(1/4)*log(8*sqrt(d*x)*d - 8*I*a*(d^6/a^5)^(1/4)) - 20*a*(d^6/a^5)^
(1/4)*log(8*sqrt(d*x)*d - 8*a*(d^6/a^5)^(1/4)) + (25*a*d*x^2*dilog(a*x^2)
+ 20*a*d*x^2*log(-a*x^2 + 1) - 16*a*d*x^2 - 80*d)*sqrt(d*x))/a
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^{3/2}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(3/2)*polylog(2,a*x**2),x)
```

output
```
Timed out
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int (dx)^{3/2}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \frac{2\left(\frac{25\,(dx)^{\frac{5}{2}}a\text{Li}_2\left(ax^2\right)+20\,(dx)^{\frac{5}{2}}a\log\left(-ad^2x^2+d^2\right)-8\,(dx)^{\frac{5}{2}}(5\,a\log(d)+2\,a)-80\,\sqrt{dx}d^2}{a}+\frac{20\left(\frac{2}{-}\right.}{}\right.}{125\,d}$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
2/125*((25*(d*x)^(5/2)*a*dilog(a*x^2) + 20*(d*x)^(5/2)*a*log(-a*d^2*x^2 +
d^2) - 8*(d*x)^(5/2)*(5*a*log(d) + 2*a) - 80*sqrt(d*x)*d^2)/a + 20*(2*d^3*
arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/sqrt(sqrt(a)*d) - d^3*log((sqrt(
d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(sqrt(a)*d)))/sqr
t(sqrt(a)*d))/a)/d
```

3.72.8 Giac [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}\left(2, ax^2\right) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_2\left(ax^2\right) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*dilog(a*x^2), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \operatorname{PolyLog}\left(2, ax^2\right) \, dx = \int \operatorname{polylog}\left(2, a\,x^2\right) \, (d\,x)^{3/2} \, dx$$

input
```
int(polylog(2, a*x^2)*(d*x)^(3/2),x)
```

output
```
int(polylog(2, a*x^2)*(d*x)^(3/2), x)
```

3.73 $\int \sqrt{dx}\, \text{PolyLog}\left(2,ax^2\right)\, dx$

3.73.1 Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \sqrt{dx}\, \text{PolyLog}\left(2,ax^2\right)\, dx = -\frac{32(dx)^{3/2}}{27d} - \frac{16\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{16\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\,\text{PolyLog}\left(2,ax^2\right)}{3d}$$

output
```
-32/27*(d*x)^(3/2)/d+8/9*(d*x)^(3/2)*ln(-a*x^2+1)/d+2/3*(d*x)^(3/2)*polylo
g(2,a*x^2)/d-16/9*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)+16/9
*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)
```

3.73.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \sqrt{dx}\, \text{PolyLog}\left(2,ax^2\right)\, dx = \frac{2\sqrt{dx}\left(\frac{4\left(-6\arctan\left(\sqrt[4]{a}\sqrt{x}\right)+6\text{arctanh}\left(\sqrt[4]{a}\sqrt{x}\right)+a^{3/4}x^{3/2}\left(-4+3\log\left(1-ax^2\right)\right)\right)}{a^{3/4}} + 9x^{3/2}\,\text{PolyLog}\left(2,ax^2\right)\right)}{27\sqrt{x}}$$

input
```
Integrate[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

output
```
(2*Sqrt[d*x]*((4*(-6*ArcTan[a^(1/4)*Sqrt[x]] + 6*ArcTanh[a^(1/4)*Sqrt[x]]
+ a^(3/4)*x^(3/2)*(-4 + 3*Log[1 - a*x^2])))/a^(3/4) + 9*x^(3/2)*PolyLog[2,
 a*x^2]))/(27*Sqrt[x])
```

3.73.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7145, 25, 2905, 8, 262, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\, \text{PolyLog}\left(2, ax^2\right) \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2} \text{PolyLog}\left(2, ax^2\right)}{3d} - \frac{4}{3} \int -\sqrt{dx} \log\left(1 - ax^2\right) dx$$

$$\Big\downarrow \text{25}$$

$$\frac{4}{3} \int \sqrt{dx} \log\left(1 - ax^2\right) dx + \frac{2(dx)^{3/2} \text{PolyLog}\left(2, ax^2\right)}{3d}$$

$$\Big\downarrow \text{2905}$$

$$\frac{4}{3} \left(\frac{4a \int \frac{x(dx)^{3/2}}{1-ax^2} dx}{3d} + \frac{2(dx)^{3/2} \log\left(1 - ax^2\right)}{3d} \right) + \frac{2(dx)^{3/2} \text{PolyLog}\left(2, ax^2\right)}{3d}$$

$$\Big\downarrow \text{8}$$

$$\frac{4}{3} \left(\frac{4a \int \frac{(dx)^{5/2}}{1-ax^2} dx}{3d^2} + \frac{2(dx)^{3/2} \log\left(1 - ax^2\right)}{3d} \right) + \frac{2(dx)^{3/2} \text{PolyLog}\left(2, ax^2\right)}{3d}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{3} \left(\frac{4a \left(\frac{d^2 \int \frac{\sqrt{dx}}{1-ax^2} dx}{a} - \frac{2d(dx)^{3/2}}{3a} \right)}{3d^2} + \frac{2(dx)^{3/2} \log\left(1 - ax^2\right)}{3d} \right) + \frac{2(dx)^{3/2} \text{PolyLog}\left(2, ax^2\right)}{3d}$$

$$\Big\downarrow \ 266$$

$$\frac{4}{3}\left(\frac{4a\left(\frac{2d\int \frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\int \frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)}{3d}$$

$$\Big\downarrow \ 827$$

$$\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)}{3d}$$

$$\Big\downarrow \ 218$$

$$\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)}{3d}$$

$$\Big\downarrow \ 221$$

3.73. $\int \sqrt{dx}\operatorname{PolyLog}\left(2,ax^2\right)\,dx$

$$\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}\left(2,ax^2\right)}{3d}$$

input
```
Int[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

output
```
(4*((4*a*((-2*d*(d*x)^(3/2))/(3*a) + (2*d^3*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x
])/Sqrt[d]]/(a^(3/4)*Sqrt[d]) + ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^
(3/4)*Sqrt[d])))/a))/(3*d^2) + (2*(d*x)^(3/2)*Log[1 - a*x^2])/(3*d)))/3 +
(2*(d*x)^(3/2)*PolyLog[2, a*x^2])/(3*d)
```

3.73.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.73. $\int \sqrt{dx}\,\text{PolyLog}\left(2, ax^2\right)\,dx$

3.73.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
meijerg	$-\frac{\sqrt{dx}\left(-\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}}{27a}-\frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{9a\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\ln\left(-a\,x^{2}+1\right)}{9a}+\right.}{2\sqrt{x}\,(-a)^{\frac{3}{4}}}$
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{3}+\frac{8(dx)^{\frac{3}{2}}\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{9}+\frac{32a\left(-\frac{(dx)^{\frac{3}{2}}}{3a}-\frac{d^{2}\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{9}}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{3}+\frac{8(dx)^{\frac{3}{2}}\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{9}+\frac{32a\left(-\frac{(dx)^{\frac{3}{2}}}{3a}-\frac{d^{2}\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{9}}{d}$

input
```
int((d*x)^(1/2)*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(1/2)/x^(1/2)/(-a)^(3/4)*(-64/27*x^(3/2)*(-a)^(7/4)/a-16/9*x^(3
/2)*(-a)^(7/4)/a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*
arctan((a*x^2)^(1/4)))+16/9*x^(3/2)*(-a)^(7/4)*ln(-a*x^2+1)/a+4/3*x^(3/2)*
(-a)^(7/4)/a*polylog(2,a*x^2))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \frac{2}{27}\sqrt{dx}\left(9\,x\text{Li}_2\left(ax^2\right)+12\,x\log\left(-ax^2+1\right)-16\,x\right) + \frac{8}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(512\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}d\right) - \frac{8}{9}i\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(512i\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}d\right) + \frac{8}{9}i\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-512i\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}d\right) - \frac{8}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-512\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}d\right)$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
2/27*sqrt(d*x)*(9*x*dilog(a*x^2) + 12*x*log(-a*x^2 + 1) - 16*x) + 8/9*(d^2
/a^3)^(1/4)*log(512*a^2*(d^2/a^3)^(3/4) + 512*sqrt(d*x)*d) - 8/9*I*(d^2/a^
3)^(1/4)*log(512*I*a^2*(d^2/a^3)^(3/4) + 512*sqrt(d*x)*d) + 8/9*I*(d^2/a^3
)^(1/4)*log(-512*I*a^2*(d^2/a^3)^(3/4) + 512*sqrt(d*x)*d) - 8/9*(d^2/a^3)^
(1/4)*log(-512*a^2*(d^2/a^3)^(3/4) + 512*sqrt(d*x)*d)
```

3.73.6 Sympy [F]

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \int \sqrt{dx}\,\text{Li}_2\left(ax^2\right)\,dx$$

input
```
integrate((d*x)**(1/2)*polylog(2,a*x**2),x)
```

output
```
Integral(sqrt(d*x)*polylog(2, a*x**2), x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)\,dx =$$

$$-\frac{2\left(12\,d^2\left(\frac{2\,\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)+8\,(dx)^{\frac{3}{2}}\,(3\,\log\left(d\right)+2)-9\,(dx)^{\frac{3}{2}}\,\text{Li}_2(ax^2)-12\,(dx)^{\frac{3}{2}}\,\text{lo}}{27\,d}$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
-2/27*(12*d^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d
)*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a)
+ sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + 8*(d*x)^(3/2)*(3*log(d) +
 2) - 9*(d*x)^(3/2)*dilog(a*x^2) - 12*(d*x)^(3/2)*log(-a*d^2*x^2 + d^2))/d
```

3.73.8 Giac [F]

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \int \sqrt{dx}\text{Li}_2\left(ax^2\right)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*dilog(a*x^2), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)\,dx = \int \text{polylog}\left(2,a\,x^2\right)\,\sqrt{d\,x}\,dx$$

input
```
int(polylog(2, a*x^2)*(d*x)^(1/2),x)
```

output
```
int(polylog(2, a*x^2)*(d*x)^(1/2), x)
```

3.74 $\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx$

3.74.1 Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx=-\frac{32\sqrt{dx}}{d}+\frac{16\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{16\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{8\sqrt{dx}\log(1-ax^2)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

output
```
16*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)+16*arctanh(a^(1/4)*
(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)-32*(d*x)^(1/2)/d+8*ln(-a*x^2+1)*(d*x)
^(1/2)/d+2*polylog(2,a*x^2)*(d*x)^(1/2)/d
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx=\frac{5x\,\text{Gamma}\left(\frac{5}{4}\right)\left(-16+16\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+4\log(1-ax^2)+\text{PolyLog}(2,ax^2)\right)}{2\sqrt{dx}\,\text{Gamma}\left(\frac{9}{4}\right)}$$

input
```
Integrate[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

output
```
(5*x*Gamma[5/4]*(-16 + 16*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1
- a*x^2] + PolyLog[2, a*x^2]))/(2*Sqrt[d*x]*Gamma[9/4])
```

3.74.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7145, 25, 2905, 8, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}-4\int -\frac{\log(1-ax^2)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{25}$$

$$4\int \frac{\log(1-ax^2)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

$$\Big\downarrow \text{2905}$$

$$4\left(\frac{4a\int \frac{x\sqrt{dx}}{1-ax^2}dx}{d}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

$$\Big\downarrow \text{8}$$

$$4\left(\frac{4a\int \frac{(dx)^{3/2}}{1-ax^2}dx}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

$$\Big\downarrow \text{262}$$

$$4\left(\frac{4a\left(\frac{d^2\int \frac{1}{\sqrt{dx}(1-ax^2)}dx}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

$$\Big\downarrow \ 266$$

$$4\left(\frac{4a\left(\frac{2d\int\frac{1}{1-ax^2}d\sqrt{dx}}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}$$

$$\Big\downarrow \ 756$$

$$4\left(\frac{4a\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}$$

$$\Big\downarrow \ 218$$

$$4\left(\frac{4a\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}$$

$$\Big\downarrow \ 221$$

3.74. $\int\frac{\text{PolyLog}\left(2,ax^2\right)}{\sqrt{dx}}\,dx$

$$4\left(\frac{4a\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}$$

input
```
Int[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

output
```
4*((4*a*((-2*d*Sqrt[d*x])/a + (2*d*((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(2*a^(1/4)) + (Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^
(1/4))))/a))/d^2 + (2*Sqrt[d*x]*Log[1 - a*x^2])/d) + (2*Sqrt[d*x]*PolyLog[
2, a*x^2])/d
```

3.74.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.74. $\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx$

3.74.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

method	result
meijerg	$-\frac{\sqrt{x}\left(-\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}}{a}-\frac{16\sqrt{x}\,(-a)^{\frac{5}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{a\left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{16\sqrt{x}\,(-a)^{\frac{5}{4}}\ln\left(-a\,x^{2}+1\right)}{a}+}{2\sqrt{dx}\,(-a)^{\frac{1}{4}}}$
derivativedivides	$\frac{2\sqrt{dx}\,\operatorname{polylog}\left(2,a\,x^{2}\right)+8\sqrt{dx}\,\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)+32a\left(-\frac{\sqrt{dx}}{a}+\frac{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a}\right)}{d}$
default	$\frac{2\sqrt{dx}\,\operatorname{polylog}\left(2,a\,x^{2}\right)+8\sqrt{dx}\,\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)+32a\left(-\frac{\sqrt{dx}}{a}+\frac{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a}\right)}{d}$

input
```
int(polylog(2,a*x^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(1/2)*x^(1/2)/(-a)^(1/4)*(-64*x^(1/2)*(-a)^(5/4)/a-16*x^(1/2)*(
-a)^(5/4)/a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arcta
n((a*x^2)^(1/4)))+16*x^(1/2)*(-a)^(5/4)*ln(-a*x^2+1)/a+4*x^(1/2)*(-a)^(5/4
)/a*polylog(2,a*x^2))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx = \frac{2\left(4\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-4i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(-i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{\right.}\right.}{d}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2*(4*d*(1/(a*d^2))^(1/4)*log(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 4*I*d*(1/(
a*d^2))^(1/4)*log(I*d*(1/(a*d^2))^(1/4) + sqrt(d*x)) - 4*I*d*(1/(a*d^2))^(
1/4)*log(-I*d*(1/(a*d^2))^(1/4) + sqrt(d*x)) - 4*d*(1/(a*d^2))^(1/4)*log(-
d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + sqrt(d*x)*(dilog(a*x^2) + 4*log(-a*x^2
+ 1) - 16))/d
```

3.74.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{\sqrt{dx}}\,dx=\int \frac{\text{Li}_2\left(ax^2\right)}{\sqrt{dx}}\,dx$$

input
```
integrate(polylog(2,a*x**2)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(2, a*x**2)/sqrt(d*x), x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{\sqrt{dx}}\,dx = -\frac{2\left(8\sqrt{dx}(\log(d)+2)-\frac{8\,d\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}}-\sqrt{dx}\text{Li}_2(ax^2)-4\sqrt{dx}\log\left(-ad^2x^2+d^2\right)+\frac{4\,d\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}\cdots}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}\cdots}}\right.}{\sqrt{\sqrt{a}d}}\cdots\right.}{d}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*(8*sqrt(d*x)*(log(d) + 2) - 8*d*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d
))/sqrt(sqrt(a)*d) - sqrt(d*x)*dilog(a*x^2) - 4*sqrt(d*x)*log(-a*d^2*x^2 +
 d^2) + 4*d*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
 sqrt(sqrt(a)*d)))/sqrt(sqrt(a)*d))/d
```

3.74.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^2)}{\sqrt{dx}} \, dx = \int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}} \, dx$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/sqrt(d*x), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^2)}{\sqrt{dx}} \, dx = \int \frac{\text{polylog}(2, a\,x^2)}{\sqrt{d\,x}} \, dx$$

input
```
int(polylog(2, a*x^2)/(d*x)^(1/2),x)
```

output
```
int(polylog(2, a*x^2)/(d*x)^(1/2), x)
```

3.75 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}\,dx$

3.75.1 Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}\,dx = -\frac{16\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a}\,\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

output
```
-16*a^(1/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+16*a^(1/4)*arctanh
(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+8*ln(-a*x^2+1)/d/(d*x)^(1/2)-2*polyl
og(2,a*x^2)/d/(d*x)^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}\,dx = \frac{x\,\text{Gamma}\left(\frac{3}{4}\right)\left(16ax^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+12\log(1-ax^2)-3\,\text{Pol}}{2(dx)^{3/2}\,\text{Gamma}\left(\frac{7}{4}\right)}$$

input
```
Integrate[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

output
```
(x*Gamma[3/4]*(16*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 12*Log[1 -
 a*x^2] - 3*PolyLog[2, a*x^2]))/(2*(d*x)^(3/2)*Gamma[7/4])
```

3.75.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {7145, 25, 2905, 8, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$4\int -\frac{\log(1-ax^2)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\log(1-ax^2)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{2905}$$

$$-4\left(-\frac{4a\int \frac{x}{\sqrt{dx}(1-ax^2)}dx}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{8}$$

$$-4\left(-\frac{4a\int \frac{\sqrt{dx}}{1-ax^2}dx}{d^2}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{266}$$

$$-4\left(-\frac{8a\int \frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{d^3}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{27}$$

$$-4\left(-\frac{8a\int \frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{827}$$

$$-4\left(-\frac{8a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{d}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{218}$$

$$-4\left(-\frac{8a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{221}$$

$$-4\left(-\frac{8a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

output
```
-4*((-8*a*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(a^(3/4)*Sqrt[d]) + Ar
cTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^(3/4)*Sqrt[d])))/d - (2*Log[1 - a*
x^2])/(d*Sqrt[d*x])) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])
```

3.75.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.75. $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}\,dx$

3.75.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

method	result
meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{4}}\left(-\frac{16x^{\frac{3}{2}}(-a)^{\frac{3}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{16(-a)^{\frac{3}{4}}\ln\left(-a\,x^{2}+1\right)}{\sqrt{x}\,a}-\frac{4(-a)^{\frac{3}{4}}\operatorname{poly}}{\sqrt{}}\right.}{2(dx)^{\frac{3}{2}}}$
derivativedivides	$\frac{-\frac{2\operatorname{polylog}\left(2,a\,x^{2}\right)}{\sqrt{dx}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{\sqrt{dx}}-\frac{8\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}}{d}$
default	$\frac{-\frac{2\operatorname{polylog}\left(2,a\,x^{2}\right)}{\sqrt{dx}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{\sqrt{dx}}-\frac{8\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}}{d}$

input
```
int(polylog(2,a*x^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(3/2)*x^(3/2)*(-a)^(1/4)*(-16*x^(3/2)*(-a)^(3/4)/(a*x^2)^(3/4)*
(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))+16/x^(1/
2)*(-a)^(3/4)*ln(-a*x^2+1)/a-4/x^(1/2)*(-a)^(3/4)/a*polylog(2,a*x^2))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{PolyLog}(2, ax^2)}{(dx)^{3/2}}\,dx = \frac{2\left(4\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(512\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+512\,\sqrt{dx}a\right)-4i\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(512i\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+5\right.\right.}{}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*(4*d^2*x*(a/d^6)^(1/4)*log(512*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - 4*
I*d^2*x*(a/d^6)^(1/4)*log(512*I*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) + 4*I
*d^2*x*(a/d^6)^(1/4)*log(-512*I*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - 4*d
^2*x*(a/d^6)^(1/4)*log(-512*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - sqrt(d*
x)*(dilog(a*x^2) - 4*log(-a*x^2 + 1)))/(d^2*x)
```

3.75.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x**2)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(2, a*x**2)/(d*x)**(3/2), x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{3/2}}\,dx = -\frac{2\left(4\,a\left(\frac{2\,\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)+\frac{\text{Li}_2\left(ax^2\right)-4\,\log\left(-ad^2x^2+d^2\right)+8\,\log\left(d\right)}{\sqrt{dx}}\right)}{d}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*(4*a*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*sqrt
(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt
(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + (dilog(a*x^2) - 4*log(-a*d^2*x^
2 + d^2) + 8*log(d))/sqrt(d*x))/d
```

3.75.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{(dx)^{3/2}}\, dx = \int \frac{\text{Li}_2(ax^2)}{(dx)^{\frac{3}{2}}}\, dx$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/(d*x)^(3/2), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2, ax^2\right)}{(dx)^{3/2}}\, dx = \int \frac{\text{polylog}(2, a\, x^2)}{(d\, x)^{3/2}}\, dx$$

input
```
int(polylog(2, a*x^2)/(d*x)^(3/2),x)
```

output
```
int(polylog(2, a*x^2)/(d*x)^(3/2), x)
```

3.76 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{5/2}}\,dx$

3.76.1 Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{5/2}}\,dx = \frac{16a^{3/4}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\,\text{PolyLog}(2,ax^2)}{3d(dx)^{3/2}}$$

output
```
16/9*a^(3/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+16/9*a^(3/4)*arct
anh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+8/9*ln(-a*x^2+1)/d/(d*x)^(3/2)-2/
3*polylog(2,a*x^2)/d/(d*x)^(3/2)
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{5/2}}\,dx = \frac{x\,\text{Gamma}\left(\frac{1}{4}\right)\left(16ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+4\log(1-ax^2)-3\,\text{Poly}\right.}{18(dx)^{5/2}\,\text{Gamma}\left(\frac{5}{4}\right)}$$

input
```
Integrate[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

output
```
(x*Gamma[1/4]*(16*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 -
a*x^2] - 3*PolyLog[2, a*x^2]))/(18*(d*x)^(5/2)*Gamma[5/4])
```

3.76.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {7145, 25, 2905, 8, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{4}{3}\int -\frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4}{3}\int \frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 2905}$$

$$-\frac{4}{3}\left(-\frac{4a\int \frac{x}{(dx)^{3/2}(1-ax^2)}dx}{3d}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 8}$$

$$-\frac{4}{3}\left(-\frac{4a\int \frac{1}{\sqrt{dx}(1-ax^2)}dx}{3d^2}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{4}{3}\left(-\frac{8a\int \frac{1}{1-ax^2}d\sqrt{dx}}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 756}$$

$$-\frac{4}{3}\left(-\frac{8a\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{4}{3}\left(-\frac{8a\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{4}{3}\left(-\frac{8a\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

output
```
(-4*((-8*a*((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4)) + (S
qrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4))))/(3*d^3) - (2*Lo
g[1 - a*x^2])/(3*d*(d*x)^(3/2))))/3 - (2*PolyLog[2, a*x^2])/(3*d*(d*x)^(3/
2))
```

3.76.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.76.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

method	result
meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{16\sqrt{x}\,(-a)^{\frac{1}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{9\left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{16(-a)^{\frac{1}{4}}\ln\left(-a\,x^{2}+1\right)}{9x^{\frac{3}{2}}a}-\frac{4(-a)^{\frac{1}{4}}\,\mathrm{poly}}{3x}\right.}{2(dx)^{\frac{5}{2}}}$
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{9d^{2}}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{9d^{2}}}{d}$

input
```
int(polylog(2,a*x^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(5/2)*x^(5/2)*(-a)^(3/4)*(-16/9*x^(1/2)*(-a)^(1/4)/(a*x^2)^(1/4
)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))+16/9/x
^(3/2)*(-a)^(1/4)*ln(-a*x^2+1)/a-4/3/x^(3/2)*(-a)^(1/4)/a*polylog(2,a*x^2)
)
```

3.76.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.80

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}}\, dx = \frac{2\left(4\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(8\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}} + 8\sqrt{dx}a\right) + 4i\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(8i\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}} + \right.\right.}{}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(4*d^3*x^2*(a^3/d^10)^(1/4)*log(8*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x)*a
) + 4*I*d^3*x^2*(a^3/d^10)^(1/4)*log(8*I*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x
)*a) - 4*I*d^3*x^2*(a^3/d^10)^(1/4)*log(-8*I*d^3*(a^3/d^10)^(1/4) + 8*sqrt
(d*x)*a) - 4*d^3*x^2*(a^3/d^10)^(1/4)*log(-8*d^3*(a^3/d^10)^(1/4) + 8*sqrt
(d*x)*a) - sqrt(d*x)*(3*dilog(a*x^2) - 4*log(-a*x^2 + 1)))/(d^3*x^2)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(polylog(2,a*x**2)/(d*x)**(5/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}} dx = \frac{2 \left(\frac{8 a \arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d} - \frac{4 a \log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d} - \frac{3 \text{Li}_2(ax^2) - 4 \log(-ad^2x^2+d^2) + 8 \log(d)}{(dx)^{\frac{3}{2}}} \right)}{9 d}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/9*(8*a*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*d) - 4
*a*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(sqr
t(a)*d)))/(sqrt(sqrt(a)*d)*d) - (3*dilog(a*x^2) - 4*log(-a*d^2*x^2 + d^2)
+ 8*log(d))/(d*x)^(3/2))/d
```

3.76.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}} dx = \int \frac{\text{Li}_2(ax^2)}{(dx)^{\frac{5}{2}}} dx$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/(d*x)^(5/2), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}} dx = \int \frac{\text{polylog}(2, a\, x^2)}{(d\, x)^{5/2}} dx$$

input
```
int(polylog(2, a*x^2)/(d*x)^(5/2),x)
```

output
```
int(polylog(2, a*x^2)/(d*x)^(5/2), x)
```

3.77 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx$

3.77.1 Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx = -\frac{32a}{25d^3\sqrt{dx}} - \frac{16a^{5/4}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

output
```
-16/25*a^(5/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+16/25*a^(5/4)*a
rctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+8/25*ln(-a*x^2+1)/d/(d*x)^(5/2
)-2/5*polylog(2,a*x^2)/d/(d*x)^(5/2)-32/25*a/d^3/(d*x)^(1/2)
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx = -\frac{x\,\text{Gamma}\left(-\frac{1}{4}\right)\left(-48ax^2+16a^2x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+12\log(1-ax^2)-15\,\text{PolyLog}(2,a\right.}{150(dx)^{7/2}\,\text{Gamma}\left(\frac{3}{4}\right)}$$

input
```
Integrate[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```

output
```
-1/150*(x*Gamma[-1/4]*(-48*a*x^2 + 16*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/
4, a*x^2] + 12*Log[1 - a*x^2] - 15*PolyLog[2, a*x^2]))/((d*x)^(7/2)*Gamma[
3/4])
```

3.77.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 25, 2905, 8, 264, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{4}{5}\int -\frac{\log(1-ax^2)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4}{5}\int \frac{\log(1-ax^2)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{2905}$$

$$-\frac{4}{5}\left(-\frac{4a\int \frac{x}{(dx)^{5/2}(1-ax^2)}dx}{5d}-\frac{2\log(1-ax^2)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{8}$$

$$-\frac{4}{5}\left(-\frac{4a\int \frac{1}{(dx)^{3/2}(1-ax^2)}dx}{5d^2}-\frac{2\log(1-ax^2)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{a\int \frac{\sqrt{dx}}{1-ax^2}dx}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log(1-ax^2)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 266$$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\int \frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{d^3}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\mathrm{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\int \frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\mathrm{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 827$$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\mathrm{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\mathrm{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \ 221$$

3.77. $\int \frac{\mathrm{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx$

$$-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\operatorname{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}$$

input
```
Int[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```

output
```
(-4*((-4*a*(-2/(d*Sqrt[d*x]) + (2*a*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[
d]]/(a^(3/4)*Sqrt[d]) + ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^(3/4)*Sq
rt[d])))/d))/(5*d^2) - (2*Log[1 - a*x^2])/(5*d*(d*x)^(5/2))))/5 - (2*PolyL
og[2, a*x^2])/(5*d*(d*x)^(5/2))
```

3.77.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.77. $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx$

3.77.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

method	result
meijerg	$-\frac{x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{64}{25\sqrt{x}\,(-a)^{\frac{1}{4}}}-\frac{16x^{\frac{3}{2}}a\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{25(-a)^{\frac{1}{4}}\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{16\ln\left(-a\,x^{2}+1\right)}{25x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}-\frac{4\,\mathrm{polylog}(}{5x^{\frac{5}{2}}(-}\right.}{2(dx)^{\frac{7}{2}}}$
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25(dx)^{\frac{5}{2}}}+\frac{32a\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{4d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}-\frac{1}{d^{2}\sqrt{dx}}\right)}{25}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25(dx)^{\frac{5}{2}}}+\frac{32a\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{4d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}-\frac{1}{d^{2}\sqrt{dx}}\right)}{25}}{d}$

input
```
int(polylog(2,a*x^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(7/2)*x^(7/2)*(-a)^(5/4)*(-64/25/x^(1/2)/(-a)^(1/4)-16/25*x^(3/
2)/(-a)^(1/4)*a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*a
rctan((a*x^2)^(1/4)))+16/25/x^(5/2)/(-a)^(1/4)*ln(-a*x^2+1)/a-4/5/x^(5/2)/
(-a)^(1/4)/a*polylog(2,a*x^2))
```

3.77.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.70

$$\int \frac{\mathrm{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx = \frac{2\left(4\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(512\,d^{11}\left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}}+512\sqrt{dx}a^4\right)-4i\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(512i\,d^{11}\right.\right.}{}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/25*(4*d^4*x^3*(a^5/d^14)^(1/4)*log(512*d^11*(a^5/d^14)^(3/4) + 512*sqrt(
d*x)*a^4) - 4*I*d^4*x^3*(a^5/d^14)^(1/4)*log(512*I*d^11*(a^5/d^14)^(3/4) +
 512*sqrt(d*x)*a^4) + 4*I*d^4*x^3*(a^5/d^14)^(1/4)*log(-512*I*d^11*(a^5/d^
14)^(3/4) + 512*sqrt(d*x)*a^4) - 4*d^4*x^3*(a^5/d^14)^(1/4)*log(-512*d^11*
(a^5/d^14)^(3/4) + 512*sqrt(d*x)*a^4) - (16*a*x^2 + 5*dilog(a*x^2) - 4*log
(-a*x^2 + 1))*sqrt(d*x))/(d^4*x^3)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate(polylog(2,a*x**2)/(d*x)**(7/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{7/2}}\, dx = -\frac{2\left(\frac{4\,a^2\left(\frac{2\,\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)}{d^2}+\frac{16\,ad^2x^2+5\,d^2\text{Li}_2\left(ax^2\right)-4\,d^2\log\left(-ad^2x^2+d^2\right)+8\,d^2\log\left(d\right)}{(dx)^{\frac{5}{2}}\,d^2}\right)}{25\,d}$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/25*(4*a^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)
*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
 sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/d^2 + (16*a*d^2*x^2 + 5*d^2*
dilog(a*x^2) - 4*d^2*log(-a*d^2*x^2 + d^2) + 8*d^2*log(d))/((d*x)^(5/2)*d^
2))/d
```

3.77.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{7/2}}\,dx = \int \frac{\text{Li}_2(ax^2)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^2)/(d*x)^(7/2), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{7/2}}\,dx = \int \frac{\text{polylog}(2,a\,x^2)}{(d\,x)^{7/2}}\,dx$$

input
```
int(polylog(2, a*x^2)/(d*x)^(7/2),x)
```

output
```
int(polylog(2, a*x^2)/(d*x)^(7/2), x)
```

3.78 $\int (dx)^{5/2} \operatorname{PolyLog}(3, ax^2) \, dx$

3.78.1 Optimal result

Integrand size = 15, antiderivative size = 161

$$\int (dx)^{5/2} \operatorname{PolyLog}(3, ax^2) \, dx = \frac{128 d (dx)^{3/2}}{1029 a} + \frac{128 (dx)^{7/2}}{2401 d} + \frac{64 d^{5/2} \arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343 a^{7/4}} - \frac{64 d^{5/2} \operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343 a^{7/4}} - \frac{32 (dx)^{7/2} \log(1 - ax^2)}{343 d} - \frac{8 (dx)^{7/2} \operatorname{PolyLog}(2, ax^2)}{49 d} + \frac{2 (dx)^{7/2} \operatorname{PolyLog}(3, ax^2)}{7 d}$$

output
```
128/1029*d*(d*x)^(3/2)/a+128/2401*(d*x)^(7/2)/d+64/343*d^(5/2)*arctan(a^(1
/4)*(d*x)^(1/2)/d^(1/2))/a^(7/4)-64/343*d^(5/2)*arctanh(a^(1/4)*(d*x)^(1/2
)/d^(1/2))/a^(7/4)-32/343*(d*x)^(7/2)*ln(-a*x^2+1)/d-8/49*(d*x)^(7/2)*poly
log(2,a*x^2)/d+2/7*(d*x)^(7/2)*polylog(3,a*x^2)/d
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int (dx)^{5/2} \operatorname{PolyLog}(3, ax^2) \, dx =$$

$$-\frac{11 d (dx)^{3/2} \operatorname{Gamma}\left(\frac{11}{4}\right) \left(-448 - 192 ax^2 + 448 \operatorname{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) + 336 ax^2 \log(1 - ax^2) \cdots\right.}{14406 a \operatorname{Gamma}\left(\frac{15}{4}\right)}$$

input
```
Integrate[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```

output
```
(-11*d*(d*x)^(3/2)*Gamma[11/4]*(-448 - 192*a*x^2 + 448*Hypergeometric2F1[3
/4, 1, 7/4, a*x^2] + 336*a*x^2*Log[1 - a*x^2] + 588*a*x^2*PolyLog[2, a*x^2
] - 1029*a*x^2*PolyLog[3, a*x^2]))/(14406*a*Gamma[15/4])
```

3.78.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {7145, 7145, 25, 2905, 8, 262, 262, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} \operatorname{PolyLog}\left(3,ax^2\right) \, dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(3,ax^2\right)}{7d}-\frac{4}{7} \int (dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(3,ax^2\right)}{7d}-\frac{4}{7} \left(\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(2,ax^2\right)}{7d}-\frac{4}{7} \int -(dx)^{5/2} \log \left(1-ax^2\right)dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(3,ax^2\right)}{7d}-\frac{4}{7} \left(\frac{4}{7} \int (dx)^{5/2} \log \left(1-ax^2\right)dx+\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(2,ax^2\right)}{7d}\right)$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(3,ax^2\right)}{7d}- \frac{4}{7} \left(\frac{4}{7} \left(\frac{4a \int \frac{x(dx)^{7/2}}{1-ax^2}dx}{7d}+\frac{2(dx)^{7/2} \log \left(1-ax^2\right)}{7d}\right)+\frac{2(dx)^{7/2} \operatorname{PolyLog}\left(2,ax^2\right)}{7d}\right)$$

$$\Big\downarrow \text{ 8}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\int\frac{(dx)^{9/2}}{1-ax^2}dx}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax^2)}{7d}\right)$$

$\downarrow$ 262

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\int\frac{(dx)^{5/2}}{1-ax^2}dx}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax^2)}{7d}\right)$$

$\downarrow$ 262

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{d^2\int\frac{\sqrt{dx}}{1-ax^2}dx}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax^2)}{7d}\right)$$

$\downarrow$ 266

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\int\frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax^2)}{7d}\right)$$

$\downarrow$ 27

3.78. $\int(dx)^{5/2}\operatorname{PolyLog}(3,ax^2)\,dx$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-$$

$$\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{2d^3\int\frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{PolyLog}(2,ax^2)}{7d}\right.$$

$$\Big\downarrow \text{827}$$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d}-$$

$$\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{2d^3\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(dx)^{7/2}\operatorname{Pol}}{}\right.$$

$$\Big\downarrow \text{218}$$

3.78. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax^2)\,dx$

$$\frac{2(dx)^{7/2}\,\text{PolyLog}\left(3,ax^2\right)}{7d}-$$

$$\frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{2d^3\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log\left(1-ax^2\right)}{7d}\right)+\frac{2(dx)^{7/2}\,\text{I}}{}$$

$\downarrow$ 221

3.78. $\int (dx)^{5/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx$

$$\frac{2(dx)^{7/2}\operatorname{PolyLog}(3,ax^2)}{7d} - \frac{4}{7}\left(\frac{4}{7}\left(\frac{4a\left(\frac{d^2\left(\frac{2d^3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{a}-\frac{2d(dx)^{7/2}}{7a}\right)}{7d^2}+\frac{2(dx)^{7/2}\log(1-ax^2)}{7d}\right)+\frac{2(d\ldots\right)$$

input
```
Int[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```

output
```
(-4*((4*((4*a*((-2*d*(d*x)^(7/2))/(7*a) + (d^2*((-2*d*(d*x)^(3/2))/(3*a) +
 (2*d^3*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(a^(3/4)*Sqrt[d]) + ArcT
anh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^(3/4)*Sqrt[d])))/a))/a))/(7*d^2) + (
2*(d*x)^(7/2)*Log[1 - a*x^2])/(7*d)))/7 + (2*(d*x)^(7/2)*PolyLog[2, a*x^2]
)/(7*d)))/7 + (2*(d*x)^(7/2)*PolyLog[3, a*x^2])/(7*d)
```

3.78. $\int (dx)^{5/2}\operatorname{PolyLog}(3,ax^2)\,dx$

3.78.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.78.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

method	result
meijerg	$-\frac{(dx)^{\frac{5}{2}}\left(\frac{4x^{\frac{3}{2}}(-a)^{\frac{11}{4}}\left(2112a\,x^{2}+4928\right)}{79233a^{2}}+\frac{64x^{\frac{3}{2}}(-a)^{\frac{11}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{343a^{2}\left(a\,x^{2}\right)^{\frac{3}{4}}}-\frac{64x^{\frac{7}{2}}(-a)^{\frac{11}{4}}\ln\left(-a\,x^{2}+\right.}{343a}\right.}{2x^{\frac{5}{2}}(-a)^{\frac{7}{4}}}$

input
```
int((d*x)^(5/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(5/2)/x^(5/2)/(-a)^(7/4)*(4/79233*x^(3/2)*(-a)^(11/4)*(2112*a*x
^2+4928)/a^2+64/343*x^(3/2)*(-a)^(11/4)/a^2/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1
/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))-64/343*x^(7/2)*(-a)^(11/
4)*ln(-a*x^2+1)/a-16/49*x^(7/2)*(-a)^(11/4)/a*polylog(2,a*x^2)+4/7*x^(7/2)
*(-a)^(11/4)/a*polylog(3,a*x^2))
```

3.78.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.46

$$\int (dx)^{5/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx = \frac{2\left(1029\sqrt{dx}ad^2x^3\text{polylog}(3,ax^2)-336\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\log\left(32768\sqrt{dx}d^7+327\right.\right.}{}$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
2/7203*(1029*sqrt(d*x)*a*d^2*x^3*polylog(3, a*x^2) - 336*(d^10/a^7)^(1/4)*
a*log(32768*sqrt(d*x)*d^7 + 32768*(d^10/a^7)^(3/4)*a^5) + 336*I*(d^10/a^7)
^(1/4)*a*log(32768*sqrt(d*x)*d^7 + 32768*I*(d^10/a^7)^(3/4)*a^5) - 336*I*(
d^10/a^7)^(1/4)*a*log(32768*sqrt(d*x)*d^7 - 32768*I*(d^10/a^7)^(3/4)*a^5)
+ 336*(d^10/a^7)^(1/4)*a*log(32768*sqrt(d*x)*d^7 - 32768*(d^10/a^7)^(3/4)*
a^5) - 4*(147*a*d^2*x^3*dilog(a*x^2) + 84*a*d^2*x^3*log(-a*x^2 + 1) - 48*a
*d^2*x^3 - 112*d^2*x)*sqrt(d*x))/a
```

3.78.6 Sympy [F]

$$\int (dx)^{5/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx=\int (dx)^{\frac{5}{2}}\,\text{Li}_3\left(ax^2\right)\,dx$$

input
```
integrate((d*x)**(5/2)*polylog(3,a*x**2),x)
```

output
```
Integral((d*x)**(5/2)*polylog(3, a*x**2), x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int (dx)^{5/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx=\frac{2\left(\frac{336\,d^4\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)}{a}-\frac{588\,(dx)^{\frac{7}{2}}\,a\text{Li}_2\left(ax^2\right)+336\,(dx)^{\frac{7}{2}}\,a\log\left(-a\right.}{}\right.}{7203\,d}$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
2/7203*(336*d^4*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)
*d)*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a
) + sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/a - (588*(d*x)^(7/2)*a*di
log(a*x^2) + 336*(d*x)^(7/2)*a*log(-a*d^2*x^2 + d^2) - 1029*(d*x)^(7/2)*a*
polylog(3, a*x^2) - 96*(d*x)^(7/2)*(7*a*log(d) + 2*a) - 448*(d*x)^(3/2)*d^
2)/a)/d
```

3.78.8 Giac [F]

$$\int (dx)^{5/2} \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \int (dx)^{\frac{5}{2}} \operatorname{Li}_3(ax^2) \, dx$$

input
```
integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*polylog(3, a*x^2), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \operatorname{PolyLog}\left(3, ax^2\right) \, dx = \int \operatorname{polylog}\left(3, a\,x^2\right) \, (d\,x)^{5/2} \, dx$$

input
```
int(polylog(3, a*x^2)*(d*x)^(5/2),x)
```

output
```
int(polylog(3, a*x^2)*(d*x)^(5/2), x)
```

3.79 $\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^2)\, dx$

3.79.1 Optimal result

Integrand size = 15, antiderivative size = 161

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^2)\, dx = \frac{128 d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{64d^{3/2}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\operatorname{PolyLog}(2, ax^2)}{25d} + \frac{2(dx)^{5/2}\operatorname{PolyLog}(3, ax^2)}{5d}$$

output
```
128/625*(d*x)^(5/2)/d-64/125*d^(3/2)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a
^(5/4)-64/125*d^(3/2)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(5/4)-32/125*
(d*x)^(5/2)*ln(-a*x^2+1)/d-8/25*(d*x)^(5/2)*polylog(2,a*x^2)/d+2/5*(d*x)^(
5/2)*polylog(3,a*x^2)/d+128/125*d*(d*x)^(1/2)/a
```

3.79.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^2)\, dx =$$
$$-\frac{9d\sqrt{dx}\operatorname{Gamma}\left(\frac{9}{4}\right)\left(-320 - 64ax^2 + 320\operatorname{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) + 80ax^2\log(1-ax^2) + 100a\right.}{1250a\operatorname{Gamma}\left(\frac{13}{4}\right)}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```

output
```
(-9*d*Sqrt[d*x]*Gamma[9/4]*(-320 - 64*a*x^2 + 320*Hypergeometric2F1[1/4, 1
, 5/4, a*x^2] + 80*a*x^2*Log[1 - a*x^2] + 100*a*x^2*PolyLog[2, a*x^2] - 12
5*a*x^2*PolyLog[3, a*x^2]))/(1250*a*Gamma[13/4])
```

3.79.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {7145, 7145, 25, 2905, 8, 262, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \operatorname{PolyLog}\left(3,ax^2\right) \, dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(3,ax^2\right)}{5d}-\frac{4}{5} \int (dx)^{3/2} \operatorname{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(3,ax^2\right)}{5d}-\frac{4}{5} \left(\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}-\frac{4}{5} \int -(dx)^{3/2} \log\left(1-ax^2\right)dx\right)$$

$$\Big\downarrow \ 25$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(3,ax^2\right)}{5d}-\frac{4}{5} \left(\frac{4}{5} \int (dx)^{3/2} \log\left(1-ax^2\right)dx+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}\right)$$

$$\Big\downarrow \ 2905$$

$$\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(3,ax^2\right)}{5d}- \frac{4}{5} \left(\frac{4}{5} \left(\frac{4a\int \frac{x(dx)^{5/2}}{1-ax^2}dx}{5d}+\frac{2(dx)^{5/2} \log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2} \operatorname{PolyLog}\left(2,ax^2\right)}{5d}\right)$$

$$\Big\downarrow \ 8$$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax^2)}{5d}-\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\int\frac{(dx)^{7/2}}{1-ax^2}dx}{5d^2}+\frac{2(dx)^{5/2}\log(1-ax^2)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax^2)}{5d}\right)$$

$\downarrow$ 262

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax^2)}{5d}-\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\int\frac{(dx)^{3/2}}{1-ax^2}dx}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log(1-ax^2)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax^2)}{5d}\right)$$

$\downarrow$ 262

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax^2)}{5d}-\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{d^2\int\frac{1}{\sqrt{dx}(1-ax^2)}dx}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log(1-ax^2)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax^2)}{5d}\right)$$

$\downarrow$ 266

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}(3,ax^2)}{5d}-\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\int\frac{1}{1-ax^2}d\sqrt{dx}}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log(1-ax^2)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{PolyLog}(2,ax^2)}{5d}\right)$$

$\downarrow$ 756

3.79. $\int (dx)^{3/2}\operatorname{PolyLog}(3,ax^2)\,dx$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(3,ax^2\right)}{5d}-$$

$$\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\operatorname{Po}\ldots}{\ldots}\right.$$

$\downarrow$ 218

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(3,ax^2\right)}{5d}-$$

$$\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+\frac{2(dx)^{5/2}\ldots}{\ldots}\right.$$

$\downarrow$ 221

3.79. $\int(dx)^{3/2}\operatorname{PolyLog}\left(3,ax^2\right)\,dx$

$$\frac{2(dx)^{5/2}\operatorname{PolyLog}\left(3,ax^2\right)}{5d}-$$

$$\frac{4}{5}\left(\frac{4}{5}\left(\frac{4a\left(\frac{d^2\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{a}-\frac{2d(dx)^{5/2}}{5a}\right)}{5d^2}+\frac{2(dx)^{5/2}\log\left(1-ax^2\right)}{5d}\right)+2(\right.$$

input
```
Int[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```

output
```
(-4*((4*((4*a*((-2*d*(d*x)^(5/2))/(5*a) + (d^2*((-2*d*Sqrt[d*x])/a + (2*d*
((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4)) + (Sqrt[d]*ArcT
anh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4))))/a))/a))/(5*d^2) + (2*(d*x)
^(5/2)*Log[1 - a*x^2])/(5*d)))/5 + (2*(d*x)^(5/2)*PolyLog[2, a*x^2])/(5*d)
))/5 + (2*(d*x)^(5/2)*PolyLog[3, a*x^2])/(5*d)
```

3.79. $\int (dx)^{3/2}\operatorname{PolyLog}\left(3,ax^2\right)\,dx$

3.79.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.79.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

method	result
meijerg	$-\frac{(dx)^{\frac{3}{2}}\left(\frac{4\sqrt{x}\,(-a)^{\frac{9}{4}}\left(576a\,x^{2}+2880\right)}{5625a^{2}}+\frac{64\sqrt{x}\,(-a)^{\frac{9}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{125a^{2}\left(a\,x^{2}\right)^{\frac{1}{4}}}-\frac{64x^{\frac{5}{2}}(-a)^{\frac{9}{4}}\ln\left(-a\,x^{2}+1\right)}{125a}-$

input
```
int((d*x)^(3/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(3/2)/x^(3/2)/(-a)^(5/4)*(4/5625*x^(1/2)*(-a)^(9/4)*(576*a*x^2+
2880)/a^2+64/125*x^(1/2)*(-a)^(9/4)/a^2/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))
-ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))-64/125*x^(5/2)*(-a)^(9/4)*ln
(-a*x^2+1)/a-16/25*x^(5/2)*(-a)^(9/4)/a*polylog(2,a*x^2)+4/5*x^(5/2)*(-a)^
(9/4)/a*polylog(3,a*x^2))
```

3.79.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29

$$\int (dx)^{3/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx = \frac{2\left(125\sqrt{dx}adx^2\text{polylog}(3,ax^2)-80\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(32\sqrt{dx}d+32\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)\right.}{}$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
2/625*(125*sqrt(d*x)*a*d*x^2*polylog(3, a*x^2) - 80*a*(d^6/a^5)^(1/4)*log(
32*sqrt(d*x)*d + 32*a*(d^6/a^5)^(1/4)) - 80*I*a*(d^6/a^5)^(1/4)*log(32*sqr
t(d*x)*d + 32*I*a*(d^6/a^5)^(1/4)) + 80*I*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*
x)*d - 32*I*a*(d^6/a^5)^(1/4)) + 80*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*x)*d -
 32*a*(d^6/a^5)^(1/4)) - 4*(25*a*d*x^2*dilog(a*x^2) + 20*a*d*x^2*log(-a*x^
2 + 1) - 16*a*d*x^2 - 80*d)*sqrt(d*x))/a
```

3.79.6 Sympy [F]

$$\int (dx)^{3/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx=\int (dx)^{\frac{3}{2}}\,\text{Li}_3\left(ax^2\right)\,dx$$

input
```
integrate((d*x)**(3/2)*polylog(3,a*x**2),x)
```

output
```
Integral((d*x)**(3/2)*polylog(3, a*x**2), x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int (dx)^{3/2}\,\text{PolyLog}\left(3,ax^2\right)\,dx=$$

$$-\frac{2\left(\frac{100\,(dx)^{\frac{5}{2}}a\text{Li}_2(ax^2)+80\,(dx)^{\frac{5}{2}}a\log(-ad^2x^2+d^2)-125\,(dx)^{\frac{5}{2}}a\text{Li}_3(ax^2)-32\,(dx)^{\frac{5}{2}}(5\,a\log(d)+2\,a)-320\,\sqrt{dx}d^2}{a}+\frac{80\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}}\right.}{}\right.}{625\,d}$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
-2/625*((100*(d*x)^(5/2)*a*dilog(a*x^2) + 80*(d*x)^(5/2)*a*log(-a*d^2*x^2
+ d^2) - 125*(d*x)^(5/2)*a*polylog(3, a*x^2) - 32*(d*x)^(5/2)*(5*a*log(d)
+ 2*a) - 320*sqrt(d*x)*d^2)/a + 80*(2*d^3*arctan(sqrt(d*x)*sqrt(a)/sqrt(sq
rt(a)*d))/sqrt(sqrt(a)*d) - d^3*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/
(sqrt(d*x)*sqrt(a) + sqrt(sqrt(a)*d)))/sqrt(sqrt(a)*d))/a)/d
```

3.79.8 Giac [F]

$$\int (dx)^{3/2} \text{PolyLog}\left(3, ax^2\right) dx = \int (dx)^{\frac{3}{2}} \text{Li}_3(ax^2)\, dx$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*polylog(3, a*x^2), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \text{PolyLog}\left(3, ax^2\right) dx = \int \text{polylog}\left(3, a\,x^2\right) (d\,x)^{3/2}\, dx$$

input
```
int(polylog(3, a*x^2)*(d*x)^(3/2),x)
```

output
```
int(polylog(3, a*x^2)*(d*x)^(3/2), x)
```

3.80 $\int \sqrt{dx}\,\text{PolyLog}(3,ax^2)\,dx$

3.80.1 Optimal result

Integrand size = 15, antiderivative size = 146

$$\begin{aligned}\int \sqrt{dx}\,\text{PolyLog}(3,ax^2)\,dx &= \frac{128(dx)^{3/2}}{81d}+\frac{64\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}\\ &\quad-\frac{64\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{32(dx)^{3/2}\log(1-ax^2)}{27d}\\ &\quad-\frac{8(dx)^{3/2}\,\text{PolyLog}(2,ax^2)}{9d}+\frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax^2)}{3d}\end{aligned}$$

output
```
128/81*(d*x)^(3/2)/d-32/27*(d*x)^(3/2)*ln(-a*x^2+1)/d-8/9*(d*x)^(3/2)*poly
log(2,a*x^2)/d+2/3*(d*x)^(3/2)*polylog(3,a*x^2)/d+64/27*arctan(a^(1/4)*(d*
x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)-64/27*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2
))*d^(1/2)/a^(3/4)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.47

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax^2)\,dx =$$
$$-\frac{7x\sqrt{dx}\,\text{Gamma}\left(\frac{7}{4}\right)\left(-64+64\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+48\log(1-ax^2)+36\,\text{PolyLog}(2,a\ldots}{162\,\text{Gamma}\left(\frac{11}{4}\right)}$$

input
```
Integrate[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

output
```
(-7*x*Sqrt[d*x]*Gamma[7/4]*(-64 + 64*Hypergeometric2F1[3/4, 1, 7/4, a*x^2]
 + 48*Log[1 - a*x^2] + 36*PolyLog[2, a*x^2] - 27*PolyLog[3, a*x^2]))/(162*
Gamma[11/4])
```

3.80.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {7145, 7145, 25, 2905, 8, 262, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}-\frac{4}{3}\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}-\frac{4}{3}\left(\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{3d}-\frac{4}{3}\int -\sqrt{dx}\log\left(1-ax^2\right)dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}-\frac{4}{3}\left(\frac{4}{3}\int \sqrt{dx}\log\left(1-ax^2\right)dx+\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{3d}\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\int \frac{x(dx)^{3/2}}{1-ax^2}dx}{3d}+\frac{2(dx)^{3/2}\log\left(1-ax^2\right)}{3d}\right)+\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{3d}\right)$$

$$\Big\downarrow \text{8}$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\int\frac{(dx)^{5/2}}{1-ax^2}dx}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$$\Big\downarrow \ 262$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{d^2\int\frac{\sqrt{dx}}{1-ax^2}dx}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$$\Big\downarrow \ 266$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{2d\int\frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\int\frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$$\Big\downarrow \ 827$$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$$\Big\downarrow \ 218$$

3.80. $\int\sqrt{dx}\operatorname{PolyLog}(3,ax^2)\,dx$

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

$\Big\downarrow$ 221

$$\frac{2(dx)^{3/2}\operatorname{PolyLog}(3,ax^2)}{3d}-\frac{4}{3}\left(\frac{4}{3}\left(\frac{4a\left(\frac{2d^3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{a}-\frac{2d(dx)^{3/2}}{3a}\right)}{3d^2}+\frac{2(dx)^{3/2}\log(1-ax^2)}{3d}\right)+\frac{2(dx)^{3/2}\operatorname{PolyLog}(2,ax^2)}{3d}\right)$$

input
```
Int[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

output
```
(-4*((4*((4*a*((-2*d*(d*x)^(3/2))/(3*a) + (2*d^3*(-1/2*ArcTan[(a^(1/4)*Sqr
t[d*x])/Sqrt[d]]/(a^(3/4)*Sqrt[d]) + ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/
(2*a^(3/4)*Sqrt[d])))/a))/(3*d^2) + (2*(d*x)^(3/2)*Log[1 - a*x^2])/(3*d)))
/3 + (2*(d*x)^(3/2)*PolyLog[2, a*x^2])/(3*d)))/3 + (2*(d*x)^(3/2)*PolyLog[
3, a*x^2])/(3*d)
```

3.80. $\int \sqrt{dx}\operatorname{PolyLog}(3,ax^2)\,dx$

3.80.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
meijerg	$-\frac{\sqrt{dx}\left(\frac{256x^{\frac{3}{2}}(-a)^{\frac{7}{4}}}{81a}+\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{27a\left(a\,x^{2}\right)^{\frac{3}{4}}}-\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\ln\left(-a\,x^{2}+1\right)}{27a}-\frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\,\mathrm{po}}{9a}\right.}{2\sqrt{x}\,(-a)^{\frac{3}{4}}}$

input
```
int((d*x)^(1/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^(1/2)/x^(1/2)/(-a)^(3/4)*(256/81*x^(3/2)*(-a)^(7/4)/a+64/27*x^(
3/2)*(-a)^(7/4)/a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2
*arctan((a*x^2)^(1/4)))-64/27*x^(3/2)*(-a)^(7/4)*ln(-a*x^2+1)/a-16/9*x^(3/
2)*(-a)^(7/4)/a*polylog(2,a*x^2)+4/3*x^(3/2)*(-a)^(7/4)/a*polylog(3,a*x^2)
)
```

3.80.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \sqrt{dx}\,\mathrm{PolyLog}\left(3,ax^2\right)\,dx &= \frac{2}{3}\sqrt{dx}x\mathrm{polylog}\left(3,ax^2\right)\\ &\quad-\frac{8}{81}\sqrt{dx}\left(9\,x\mathrm{Li}_2\left(ax^2\right)+12\,x\log\left(-ax^2+1\right)-16\,x\right)\\ &\quad-\frac{32}{27}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(32768\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}d\right)\\ &\quad+\frac{32}{27}i\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(32768i\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}d\right)\\ &\quad-\frac{32}{27}i\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-32768i\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}d\right)\\ &\quad+\frac{32}{27}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-32768\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}d\right)\end{aligned}$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*x)*x*polylog(3, a*x^2) - 8/81*sqrt(d*x)*(9*x*dilog(a*x^2) + 12*
x*log(-a*x^2 + 1) - 16*x) - 32/27*(d^2/a^3)^(1/4)*log(32768*a^2*(d^2/a^3)^
(3/4) + 32768*sqrt(d*x)*d) + 32/27*I*(d^2/a^3)^(1/4)*log(32768*I*a^2*(d^2/
a^3)^(3/4) + 32768*sqrt(d*x)*d) - 32/27*I*(d^2/a^3)^(1/4)*log(-32768*I*a^2
*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d) + 32/27*(d^2/a^3)^(1/4)*log(-32768*a
^2*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d)
```

3.80.6 Sympy [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}\left(3,ax^2\right)\,dx = \int \sqrt{dx}\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

input
```
integrate((d*x)**(1/2)*polylog(3,a*x**2),x)
```

output
```
Integral(sqrt(d*x)*polylog(3, a*x**2), x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05

$$\int \sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)\,dx$$
$$=\frac{2\left(48\,d^2\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)+32\,(dx)^{\frac{3}{2}}\,(3\log(d)+2)-36\,(dx)^{\frac{3}{2}}\,\text{Li}_2(ax^2)-48\,(dx)^{\frac{3}{2}}\,\text{lo}}{81\,d}$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
2/81*(48*d^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)
*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
 sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + 32*(d*x)^(3/2)*(3*log(d) +
 2) - 36*(d*x)^(3/2)*dilog(a*x^2) - 48*(d*x)^(3/2)*log(-a*d^2*x^2 + d^2) +
 27*(d*x)^(3/2)*polylog(3, a*x^2))/d
```

3.80.8 Giac [F]

$$\int \sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)\,dx=\int \sqrt{dx}\text{Li}_3(ax^2)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*polylog(3, a*x^2), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)\,dx=\int \text{polylog}\left(3,a\,x^2\right)\,\sqrt{d\,x}\,dx$$

input
```
int(polylog(3, a*x^2)*(d*x)^(1/2),x)
```

output
```
int(polylog(3, a*x^2)*(d*x)^(1/2), x)
```

3.81 $\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$

3.81.1 Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx = \frac{128\sqrt{dx}}{d} - \frac{64\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{64\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{32\sqrt{dx}\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d} + \frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}$$

output
```
-64*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)-64*arctanh(a^(1/4)
*(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)+128*(d*x)^(1/2)/d-32*ln(-a*x^2+1)*(d
*x)^(1/2)/d-8*polylog(2,a*x^2)*(d*x)^(1/2)/d+2*polylog(3,a*x^2)*(d*x)^(1/2
)/d
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx = -\frac{5x\,\text{Gamma}\left(\frac{5}{4}\right)\left(-64+64\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+16\log\left(1-ax^2\right)+4\,\text{PolyLog}\left(2,ax^2\right)-\right.}{2\sqrt{dx}\,\text{Gamma}\left(\frac{9}{4}\right)}$$

input
```
Integrate[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

output
```
(-5*x*Gamma[5/4]*(-64 + 64*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[
1 - a*x^2] + 4*PolyLog[2, a*x^2] - PolyLog[3, a*x^2]))/(2*Sqrt[d*x]*Gamma[
9/4])
```

3.81.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7145, 7145, 25, 2905, 8, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}-4\int -\frac{\log(1-ax^2)}{\sqrt{dx}}dx\right)$$

$$\Big\downarrow \ 25$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\int \frac{\log(1-ax^2)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\left(\frac{4a\int\frac{x\sqrt{dx}}{1-ax^2}dx}{d}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{8}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\left(\frac{4a\int\frac{(dx)^{3/2}}{1-ax^2}dx}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\left(\frac{4a\left(\frac{d^2\int\frac{1}{\sqrt{dx}(1-ax^2)}dx}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\left(\frac{4a\left(\frac{2d\int\frac{1}{1-ax^2}d\sqrt{dx}}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^2)}{d}-4\left(4\left(\frac{4a\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log(1-ax^2)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}\right)$$

$$\Big\downarrow \text{218}$$

3.81. $\int\frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$

$$\frac{2\sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)}{d}-4\left(4\left(\frac{4a\left(\frac{2d\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}\right)$$

$\downarrow$ 221

$$\frac{2\sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)}{d}-4\left(4\left(\frac{4a\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{2d\sqrt{dx}}{a}\right)}{d^2}+\frac{2\sqrt{dx}\log\left(1-ax^2\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,a\right.}{d}$$

input
```
Int[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

output
```
-4*(4*((4*a*((-2*d*Sqrt[d*x])/a + (2*d*((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x]
)/Sqrt[d]])/(2*a^(1/4)) + (Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(
2*a^(1/4))))/a))/d^2 + (2*Sqrt[d*x]*Log[1 - a*x^2])/d) + (2*Sqrt[d*x]*Poly
Log[2, a*x^2])/d) + (2*Sqrt[d*x]*PolyLog[3, a*x^2])/d
```

3.81. $\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$

3.81.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.81.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.10

method	result
meijerg	$-\frac{\sqrt{x}\left(\frac{256\sqrt{x}\,(-a)^{\frac{5}{4}}}{a}+\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{a\left(a\,x^{2}\right)^{\frac{1}{4}}}-\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}\ln\left(-a\,x^{2}+1\right)}{a}-\frac{16\sqrt{x}\,(-a)^{\frac{5}{4}}\,\mathrm{po}}{a}}{2\sqrt{dx}\,(-a)^{\frac{1}{4}}}$

input
```
int(polylog(3,a*x^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(1/2)*x^(1/2)/(-a)^(1/4)*(256*x^(1/2)*(-a)^(5/4)/a+64*x^(1/2)*(
-a)^(5/4)/a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arcta
n((a*x^2)^(1/4)))-64*x^(1/2)*(-a)^(5/4)*ln(-a*x^2+1)/a-16*x^(1/2)*(-a)^(5/
4)/a*polylog(2,a*x^2)+4*x^(1/2)*(-a)^(5/4)/a*polylog(3,a*x^2))
```

3.81.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

$$\int \frac{\text{PolyLog}(3, ax^2)}{\sqrt{dx}}\,dx = -\frac{2\left(16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+16i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-16i\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(-i\,d\left(\frac{1}{ad^2}\right)\right.\right.}{}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(16*d*(1/(a*d^2))^(1/4)*log(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 16*I*d*(
1/(a*d^2))^(1/4)*log(I*d*(1/(a*d^2))^(1/4) + sqrt(d*x)) - 16*I*d*(1/(a*d^2
))^(1/4)*log(-I*d*(1/(a*d^2))^(1/4) + sqrt(d*x)) - 16*d*(1/(a*d^2))^(1/4)*
log(-d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 4*sqrt(d*x)*(dilog(a*x^2) + 4*log(
-a*x^2 + 1) - 16) - sqrt(d*x)*polylog(3, a*x^2))/d
```

3.81.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_3(ax^2)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(3,a*x**2)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(3, a*x**2)/sqrt(d*x), x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$\int \frac{\text{PolyLog}(3, ax^2)}{\sqrt{dx}}\, dx = \frac{2\left(32\sqrt{dx}(\log(d)+2) - \frac{32\, d \arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}} - 4\sqrt{dx}\text{Li}_2(ax^2) - 16\sqrt{dx}\log(-ad^2x^2+d^2) + \frac{16\, d \log\left(\frac{\sqrt{dx}\sqrt{a}-}{\sqrt{dx}\sqrt{a}+}\right)}{\sqrt{\sqrt{a}d}}\right.}{d}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2*(32*sqrt(d*x)*(log(d) + 2) - 32*d*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*
d))/sqrt(sqrt(a)*d) - 4*sqrt(d*x)*dilog(a*x^2) - 16*sqrt(d*x)*log(-a*d^2*x
^2 + d^2) + 16*d*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt
(a) + sqrt(sqrt(a)*d)))/sqrt(sqrt(a)*d) + sqrt(d*x)*polylog(3, a*x^2))/d
```

3.81.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(3, ax^2\right)}{\sqrt{dx}} \, dx = \int \frac{\text{Li}_3(ax^2)}{\sqrt{dx}} \, dx$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/sqrt(d*x), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3, ax^2\right)}{\sqrt{dx}} \, dx = \int \frac{\text{polylog}(3, a\,x^2)}{\sqrt{d\,x}} \, dx$$

input
```
int(polylog(3, a*x^2)/(d*x)^(1/2),x)
```

output
```
int(polylog(3, a*x^2)/(d*x)^(1/2), x)
```

3.82 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx$

3.82.1 Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx = -\frac{64\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{64\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{32\log(1-ax^2)}{d\sqrt{dx}}-\frac{8\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}-\frac{2\,\text{PolyLog}(3,ax^2)}{d\sqrt{dx}}$$

output
```
-64*a^(1/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+64*a^(1/4)*arctanh
(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+32*ln(-a*x^2+1)/d/(d*x)^(1/2)-8*poly
log(2,a*x^2)/d/(d*x)^(1/2)-2*polylog(3,a*x^2)/d/(d*x)^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.58

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx = \frac{x\,\text{Gamma}\left(\frac{3}{4}\right)\left(64ax^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+48\log(1-ax^2)-12\,\text{Po}\right.}{2(dx)^{3/2}\,\text{Gamma}\left(\frac{7}{4}\right)}$$

input
```
Integrate[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```

output
```
(x*Gamma[3/4]*(64*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 48*Log[1 -
 a*x^2] - 12*PolyLog[2, a*x^2] - 3*PolyLog[3, a*x^2]))/(2*(d*x)^(3/2)*Gamm
a[7/4])
```

3.82.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {7145, 7145, 25, 2905, 8, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$4\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{7145}$$

$$4\left(4\int -\frac{\log\left(1-ax^2\right)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{25}$$

$$4\left(-4\int \frac{\log\left(1-ax^2\right)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{2905}$$

$$4\left(-4\left(-\frac{4a\int \frac{x}{\sqrt{dx}(1-ax^2)}dx}{d}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{8}$$

$$4\left(-4\left(-\frac{4a\int \frac{\sqrt{dx}}{1-ax^2}dx}{d^2}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{266}$$

$$4\left(-4\left(-\frac{8a\int \frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{d^3}-\frac{2\log\left(1-ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{27}$$

$$4\left(-4\left(-\frac{8a\int\frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{827}$$

$$4\left(-4\left(-\frac{8a\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int\frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{218}$$

$$4\left(-4\left(-\frac{8a\left(\frac{\int\frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{221}$$

$$4\left(-4\left(-\frac{8a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2\log(1-ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```

output
```
4*(-4*((-8*a*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(a^(3/4)*Sqrt[d]) +
 ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^(3/4)*Sqrt[d])))/d - (2*Log[1 -
 a*x^2])/(d*Sqrt[d*x])) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])) - (2*PolyLo
g[3, a*x^2])/(d*Sqrt[d*x])
```

3.82. $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx$

3.82.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.82.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

method	result
meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{4}}\left(-\frac{64x^{\frac{3}{2}}(-a)^{\frac{3}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{\left(a\,x^2\right)^{\frac{3}{4}}}+\frac{64(-a)^{\frac{3}{4}}\ln\left(-a\,x^2+1\right)}{\sqrt{x}\,a}-\frac{16(-a)^{\frac{3}{4}}\operatorname{polylog}\left(2,a\,x^2\right)}{\sqrt{x}\,a}-\right.}{2(dx)^{\frac{3}{2}}}$

input
```
int(polylog(3,a*x^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(3/2)*x^(3/2)*(-a)^(1/4)*(-64*x^(3/2)*(-a)^(3/4)/(a*x^2)^(3/4)*
(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))+64/x^(1/
2)*(-a)^(3/4)*ln(-a*x^2+1)/a-16/x^(1/2)*(-a)^(3/4)/a*polylog(2,a*x^2)-4/x^
(1/2)*(-a)^(3/4)/a*polylog(3,a*x^2))
```

3.82.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.54

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{3/2}}\,dx = \frac{2\left(16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(32768\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}a\right)-16i\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(32768i\,d^5\right.\right.}{}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*(16*d^2*x*(a/d^6)^(1/4)*log(32768*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x)*a)
 - 16*I*d^2*x*(a/d^6)^(1/4)*log(32768*I*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x
)*a) + 16*I*d^2*x*(a/d^6)^(1/4)*log(-32768*I*d^5*(a/d^6)^(3/4) + 32768*sqr
t(d*x)*a) - 16*d^2*x*(a/d^6)^(1/4)*log(-32768*d^5*(a/d^6)^(3/4) + 32768*sq
rt(d*x)*a) - 4*sqrt(d*x)*(dilog(a*x^2) - 4*log(-a*x^2 + 1)) - sqrt(d*x)*po
lylog(3, a*x^2))/(d^2*x)
```

3.82.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x**2)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(3, a*x**2)/(d*x)**(3/2), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{3/2}}\,dx = -\frac{2\left(16\,a\left(\frac{2\,\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)+\frac{4\,\text{Li}_2(ax^2)-16\,\log(-ad^2x^2+d^2)+32\,\log(d)+\text{Li}_3(ax^2)}{\sqrt{dx}}\right)}{d}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2*(16*a*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*sqr
t(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqr
t(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + (4*dilog(a*x^2) - 16*log(-a*d^
2*x^2 + d^2) + 32*log(d) + polylog(3, a*x^2))/sqrt(d*x))/d
```

3.82.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{3/2}}\,dx=\int \frac{\mathrm{polylog}(3,a\,x^2)}{{\left(d\,x\right)}^{3/2}}\,dx$$

input
```
int(polylog(3, a*x^2)/(d*x)^(3/2),x)
```

output
```
int(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

3.83 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{5/2}} \, dx$

3.83.1 Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{5/2}} \, dx = \frac{64a^{3/4}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\,\text{PolyLog}(2,ax^2)}{9d(dx)^{3/2}} - \frac{2\,\text{PolyLog}(3,ax^2)}{3d(dx)^{3/2}}$$

output
```
64/27*a^(3/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+64/27*a^(3/4)*ar
ctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+32/27*ln(-a*x^2+1)/d/(d*x)^(3/2
)-8/9*polylog(2,a*x^2)/d/(d*x)^(3/2)-2/3*polylog(3,a*x^2)/d/(d*x)^(3/2)
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{5/2}} \, dx = \frac{x\,\text{Gamma}\left(\frac{1}{4}\right)\left(64ax^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right) + 16\log(1-ax^2) - 12\,\text{Po}\right.}{54(dx)^{5/2}\,\text{Gamma}\left(\frac{5}{4}\right)}$$

input
```
Integrate[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

output
```
(x*Gamma[1/4]*(64*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1 -
 a*x^2] - 12*PolyLog[2, a*x^2] - 9*PolyLog[3, a*x^2]))/(54*(d*x)^(5/2)*Gam
ma[5/4])
```

3.83.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7145, 7145, 25, 2905, 8, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{4}{3}\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{4}{3}\left(\frac{4}{3}\int -\frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4}{3}\left(-\frac{4}{3}\int \frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{4a\int \frac{x}{(dx)^{3/2}(1-ax^2)}dx}{3d}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{4a\int \frac{1}{\sqrt{dx}(1-ax^2)}dx}{3d^2}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{8a\int \frac{1}{1-ax^2}d\sqrt{dx}}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 756$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{8a\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{8a\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{4}{3}\left(-\frac{4}{3}\left(-\frac{8a\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{3d^3}-\frac{2\log\left(1-ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

output
```
(4*((-4*((-8*a*((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4))
+ (Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(2*a^(1/4))))/(3*d^3) - (
2*Log[1 - a*x^2])/(3*d*(d*x)^(3/2))))/3 - (2*PolyLog[2, a*x^2])/(3*d*(d*x)
^(3/2))))/3 - (2*PolyLog[3, a*x^2])/(3*d*(d*x)^(3/2))
```

3.83. $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{5/2}}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

method	result
meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{64\sqrt{x}\,(-a)^{\frac{1}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{27\left(a\,x^2\right)^{\frac{1}{4}}}+\frac{64(-a)^{\frac{1}{4}}\ln\left(-a\,x^2+1\right)}{27x^{\frac{3}{2}}a}-\frac{16(-a)^{\frac{1}{4}}\operatorname{polylog}\left(2,a\,x^2\right)}{9x^{\frac{3}{2}}a}\right.}{2(dx)^{\frac{5}{2}}}$

input
```
int(polylog(3,a*x^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(5/2)*x^(5/2)*(-a)^(3/4)*(-64/27*x^(1/2)*(-a)^(1/4)/(a*x^2)^(1/
4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))+64/27
/x^(3/2)*(-a)^(1/4)*ln(-a*x^2+1)/a-16/9/x^(3/2)*(-a)^(1/4)/a*polylog(2,a*x
^2)-4/3/x^(3/2)*(-a)^(1/4)/a*polylog(3,a*x^2))
```

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{PolyLog}\left(3,ax^2\right)}{(dx)^{5/2}}\,dx = \frac{2\left(16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(32\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+32\,\sqrt{dx}a\right)+16i\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(32i\,d^3\left(\frac{a^3}{d^{10}}\right.\right.\right.}{}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(16*d^3*x^2*(a^3/d^10)^(1/4)*log(32*d^3*(a^3/d^10)^(1/4) + 32*sqrt(d*
x)*a) + 16*I*d^3*x^2*(a^3/d^10)^(1/4)*log(32*I*d^3*(a^3/d^10)^(1/4) + 32*s
qrt(d*x)*a) - 16*I*d^3*x^2*(a^3/d^10)^(1/4)*log(-32*I*d^3*(a^3/d^10)^(1/4)
 + 32*sqrt(d*x)*a) - 16*d^3*x^2*(a^3/d^10)^(1/4)*log(-32*d^3*(a^3/d^10)^(1
/4) + 32*sqrt(d*x)*a) - 4*sqrt(d*x)*(3*dilog(a*x^2) - 4*log(-a*x^2 + 1)) -
 9*sqrt(d*x)*polylog(3, a*x^2))/(d^3*x^2)
```

3.83.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x**2)/(d*x)**(5/2),x)
```

output
```
Integral(polylog(3, a*x**2)/(d*x)**(5/2), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{5/2}}\,dx = \frac{2\left(\frac{32\,a\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d} - \frac{16\,a\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d} - \frac{12\,\text{Li}_2(ax^2)-16\,\log(-ad^2x^2+d^2)+32\,\log(d)+9\,\text{Li}}{(dx)^{\frac{3}{2}}}}{27\,d}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/27*(32*a*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*d) -
 16*a*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(
sqrt(a)*d)))/(sqrt(sqrt(a)*d)*d) - (12*dilog(a*x^2) - 16*log(-a*d^2*x^2 +
d^2) + 32*log(d) + 9*polylog(3, a*x^2))/(d*x)^(3/2))/d
```

3.83.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{5/2}}\,dx=\int \frac{\mathrm{polylog}(3,a\,x^2)}{{(d\,x)}^{5/2}}\,dx$$

input
```
int(polylog(3, a*x^2)/(d*x)^(5/2),x)
```

output
```
int(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

3.84 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx$

3.84.1 Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = -\frac{128a}{125d^3\sqrt{dx}} - \frac{64a^{5/4}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\,\text{PolyLog}(2,ax^2)}{25d(dx)^{5/2}} - \frac{2\,\text{PolyLog}(3,ax^2)}{5d(dx)^{5/2}}$$

output
```
-64/125*a^(5/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+64/125*a^(5/4)
*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+32/125*ln(-a*x^2+1)/d/(d*x)^
(5/2)-8/25*polylog(2,a*x^2)/d/(d*x)^(5/2)-2/5*polylog(3,a*x^2)/d/(d*x)^(5/
2)-128/125*a/d^3/(d*x)^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = -\frac{x\,\text{Gamma}\left(-\frac{1}{4}\right)\left(-192ax^2+64a^2x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+48\log(1-ax^2)-60\,\text{PolyLog}(2,\dots\right.}{750(dx)^{7/2}\,\text{Gamma}\left(\frac{3}{4}\right)}$$

input
```
Integrate[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

output
```
-1/750*(x*Gamma[-1/4]*(-192*a*x^2 + 64*a^2*x^4*Hypergeometric2F1[3/4, 1, 7
/4, a*x^2] + 48*Log[1 - a*x^2] - 60*PolyLog[2, a*x^2] - 75*PolyLog[3, a*x^
2]))/((d*x)^(7/2)*Gamma[3/4])
```

3.84.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {7145, 7145, 25, 2905, 8, 264, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{4}{5}\int \frac{\text{PolyLog}\left(2,ax^2\right)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{7145}$$

$$\frac{4}{5}\left(\frac{4}{5}\int -\frac{\log\left(1-ax^2\right)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4}{5}\left(-\frac{4}{5}\int \frac{\log\left(1-ax^2\right)}{(dx)^{7/2}}dx-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{2905}$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\int \frac{x}{(dx)^{5/2}(1-ax^2)}dx}{5d}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{8}$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\int \frac{1}{(dx)^{3/2}(1-ax^2)}dx}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{a\int \frac{\sqrt{dx}}{1-ax^2}dx}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\int \frac{d^3x}{d^2-ad^2x^2}d\sqrt{dx}}{d^3}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\int \frac{dx}{d^2-ad^2x^2}d\sqrt{dx}}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 827$$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}}{2\sqrt{a}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 218$$

3.84. $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx$

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}}{2\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

$\downarrow$ 221

$$\frac{4}{5}\left(-\frac{4}{5}\left(-\frac{4a\left(\frac{2a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2a^{3/4}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\log\left(1-ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}$$

input
```
Int[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

output
```
(4*((-4*((-4*a*(-2/(d*Sqrt[d*x]) + (2*a*(-1/2*ArcTan[(a^(1/4)*Sqrt[d*x])/S
qrt[d]]/(a^(3/4)*Sqrt[d]) + ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*a^(3/4
)*Sqrt[d])))/d))/(5*d^2) - (2*Log[1 - a*x^2])/(5*d*(d*x)^(5/2))))/5 - (2*P
olyLog[2, a*x^2])/(5*d*(d*x)^(5/2))))/5 - (2*PolyLog[3, a*x^2])/(5*d*(d*x)
^(5/2))
```

3.84. $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx$

3.84.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

method	result
meijerg	$-\frac{x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{256}{125\sqrt{x}\,(-a)^{\frac{1}{4}}}-\frac{64x^{\frac{3}{2}}a\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{125(-a)^{\frac{1}{4}}\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{64\ln\left(-a\,x^{2}+1\right)}{125x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}-\frac{16\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{25x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}-4\right.}{2(dx)^{\frac{7}{2}}}$

input
```
int(polylog(3,a*x^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(7/2)*x^(7/2)*(-a)^(5/4)*(-256/125/x^(1/2)/(-a)^(1/4)-64/125*x^
(3/2)/(-a)^(1/4)*a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+
2*arctan((a*x^2)^(1/4)))+64/125/x^(5/2)/(-a)^(1/4)*ln(-a*x^2+1)/a-16/25/x^
(5/2)/(-a)^(1/4)/a*polylog(2,a*x^2)-4/5/x^(5/2)/(-a)^(1/4)/a*polylog(3,a*x
^2))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.55

$$\int \frac{\mathrm{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = \frac{2\left(16\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(32768\,d^{11}\left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}a^4\right)-16i\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(\right.\right.}{}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/125*(16*d^4*x^3*(a^5/d^14)^(1/4)*log(32768*d^11*(a^5/d^14)^(3/4) + 32768
*sqrt(d*x)*a^4) - 16*I*d^4*x^3*(a^5/d^14)^(1/4)*log(32768*I*d^11*(a^5/d^14
)^(3/4) + 32768*sqrt(d*x)*a^4) + 16*I*d^4*x^3*(a^5/d^14)^(1/4)*log(-32768*
I*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x)*a^4) - 16*d^4*x^3*(a^5/d^14)^(1/
4)*log(-32768*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x)*a^4) - 4*(16*a*x^2 +
 5*dilog(a*x^2) - 4*log(-a*x^2 + 1))*sqrt(d*x) - 25*sqrt(d*x)*polylog(3, a
*x^2))/(d^4*x^3)
```

3.84.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x**2)/(d*x)**(7/2),x)
```

output
```
Integral(polylog(3, a*x**2)/(d*x)**(7/2), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = -\frac{2\left(\frac{16\,a^2\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}\sqrt{a}}\right)}{d^2}+\frac{64\,ad^2x^2+20\,d^2\text{Li}_2(ax^2)-16\,d^2\log(-ad^2x^2+d^2)+32\,d^2\log(d)+25\,d^2\text{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}d^2}\right)}{125\,d}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/125*(16*a^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*
d)*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a)
 + sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/d^2 + (64*a*d^2*x^2 + 20*d
^2*dilog(a*x^2) - 16*d^2*log(-a*d^2*x^2 + d^2) + 32*d^2*log(d) + 25*d^2*po
lylog(3, a*x^2))/((d*x)^(5/2)*d^2))/d
```

3.84.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = \int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/(d*x)^(7/2), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx = \int \frac{\text{polylog}(3,a\,x^2)}{(d\,x)^{7/2}}\,dx$$

input
```
int(polylog(3, a*x^2)/(d*x)^(7/2),x)
```

output
```
int(polylog(3, a*x^2)/(d*x)^(7/2), x)
```

3.85 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}\,dx$

3.85.1 Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}\,dx = -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{64a^{7/4}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\,\text{PolyLog}(2,ax^2)}{49d(dx)^{7/2}} - \frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

output
```
-128/1029*a/d^3/(d*x)^(3/2)+64/343*a^(7/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1
/2))/d^(9/2)+64/343*a^(7/4)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(9/2)+3
2/343*ln(-a*x^2+1)/d/(d*x)^(7/2)-8/49*polylog(2,a*x^2)/d/(d*x)^(7/2)-2/7*p
olylog(3,a*x^2)/d/(d*x)^(7/2)
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}\,dx = -\frac{\sqrt{dx}\,\text{Gamma}\left(-\frac{3}{4}\right)\left(-64ax^2+192a^2x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+48\log(1-ax^2)-84\,\text{PolyLog}\right.}{686d^5x^4\,\text{Gamma}\left(\frac{1}{4}\right)}$$

input
```
Integrate[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```

output
```
-1/686*(Sqrt[d*x]*Gamma[-3/4]*(-64*a*x^2 + 192*a^2*x^4*Hypergeometric2F1[1
/4, 1, 5/4, a*x^2] + 48*Log[1 - a*x^2] - 84*PolyLog[2, a*x^2] - 147*PolyLo
g[3, a*x^2]))/(d^5*x^4*Gamma[1/4])
```

3.85.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7145, 7145, 25, 2905, 8, 264, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{4}{7}\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{9/2}}dx-\frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{7145}$$

$$\frac{4}{7}\left(\frac{4}{7}\int -\frac{\log(1-ax^2)}{(dx)^{9/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4}{7}\left(-\frac{4}{7}\int \frac{\log(1-ax^2)}{(dx)^{9/2}}dx-\frac{2\,\text{PolyLog}(2,ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{2905}$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\int \frac{x}{(dx)^{7/2}(1-ax^2)}dx}{7d}-\frac{2\log(1-ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{8}$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\int \frac{1}{(dx)^{5/2}(1-ax^2)}dx}{7d^2}-\frac{2\log(1-ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^2)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^2)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\left(\frac{a\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}dx}{d^2}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\log\left(1-ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\left(\frac{2a\int \frac{1}{1-ax^2}d\sqrt{dx}}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\log\left(1-ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 756$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\left(\frac{2a\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{\sqrt{a}xd+d}d\sqrt{dx}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\log\left(1-ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\left(\frac{2a\left(\frac{1}{2}d\int \frac{1}{d-\sqrt{a}dx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\log\left(1-ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 221$$

3.85. $\int \frac{\text{PolyLog}\left(3,ax^2\right)}{(dx)^{9/2}}\,dx$

$$\frac{4}{7}\left(-\frac{4}{7}\left(-\frac{4a\left(\frac{2a\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{a}}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\log\left(1-ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^2\right)}{7d(dx)^{7/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}$$

input
```
Int[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```

output
```
(4*((-4*((-4*a*(-2/(3*d*(d*x)^(3/2)) + (2*a*((Sqrt[d]*ArcTan[(a^(1/4)*Sqrt
[d*x])/Sqrt[d]])/(2*a^(1/4)) + (Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(2*a^(1/4))))/d^3))/(7*d^2) - (2*Log[1 - a*x^2])/(7*d*(d*x)^(7/2))))/7
 - (2*PolyLog[2, a*x^2])/(7*d*(d*x)^(7/2))))/7 - (2*PolyLog[3, a*x^2])/(7*
d*(d*x)^(7/2))
```

3.85.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

method	result
meijerg	$-\frac{x^{\frac{9}{2}} (-a)^{\frac{7}{4}} \left(-\frac{256}{1029 x^{\frac{3}{2}} (-a)^{\frac{3}{4}}}-\frac{64 \sqrt{x}\, a \left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2 \arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{343 (-a)^{\frac{3}{4}} \left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{64 \ln\left(-a\,x^{2}+1\right)}{343 x^{\frac{7}{2}} (-a)^{\frac{3}{4}} a}-\frac{16 \operatorname{polylog}\left(2,a\,x^{2}\right)}{49 x^{\frac{7}{2}} (-a)^{\frac{3}{4}} a}-\right.}{2 (d x)^{\frac{9}{2}}}$

input
```
int(polylog(3,a*x^2)/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x)^(9/2)*x^(9/2)*(-a)^(7/4)*(-256/1029/x^(3/2)/(-a)^(3/4)-64/343*x
^(1/2)/(-a)^(3/4)*a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))
-2*arctan((a*x^2)^(1/4)))+64/343/x^(7/2)/(-a)^(3/4)*ln(-a*x^2+1)/a-16/49/x
^(7/2)/(-a)^(3/4)/a*polylog(2,a*x^2)-4/7/x^(7/2)/(-a)^(3/4)/a*polylog(3,a*
x^2))
```

3.85.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.55

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{9/2}} \, dx = \frac{2 \left(48 \, d^5 x^4 \left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}} \log\left(32 \, d^5 \left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}} + 32 \sqrt{dx} a^2\right) + 48i \, d^5 x^4 \left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}} \log\left(32i \, d^5 \left(\frac{a}{d}\right.\right.\right.}{}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="fricas")
```

output
```
2/1029*(48*d^5*x^4*(a^7/d^18)^(1/4)*log(32*d^5*(a^7/d^18)^(1/4) + 32*sqrt(
d*x)*a^2) + 48*I*d^5*x^4*(a^7/d^18)^(1/4)*log(32*I*d^5*(a^7/d^18)^(1/4) +
32*sqrt(d*x)*a^2) - 48*I*d^5*x^4*(a^7/d^18)^(1/4)*log(-32*I*d^5*(a^7/d^18)
^(1/4) + 32*sqrt(d*x)*a^2) - 48*d^5*x^4*(a^7/d^18)^(1/4)*log(-32*d^5*(a^7/
d^18)^(1/4) + 32*sqrt(d*x)*a^2) - 4*(16*a*x^2 + 21*dilog(a*x^2) - 12*log(-
a*x^2 + 1))*sqrt(d*x) - 147*sqrt(d*x)*polylog(3, a*x^2))/(d^5*x^4)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{9/2}} \, dx = \text{Timed out}$$

input
```
integrate(polylog(3,a*x**2)/(d*x)**(9/2),x)
```

output
```
Timed out
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

$$\int \frac{\text{PolyLog}\left(3, ax^2\right)}{(dx)^{9/2}}\,dx = \frac{2\left(\frac{48\left(\frac{2\,a^2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d} - \frac{a^2\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}d}}\right)}{\sqrt{\sqrt{a}d}d}\right)}{d^2} - \frac{64\,ad^2x^2+84\,d^2\text{Li}_2\left(ax^2\right)-48\,d^2\log\left(-ad^2x^2+d^2\right)\cdots}{(dx)^{\frac{7}{2}}d^2}\cdots\right.}{1029\,d}$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="maxima")
```

output
```
2/1029*(48*(2*a^2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*
d)*d) - a^2*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
 sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*d))/d^2 - (64*a*d^2*x^2 + 84*d^2*dilog
(a*x^2) - 48*d^2*log(-a*d^2*x^2 + d^2) + 96*d^2*log(d) + 147*d^2*polylog(3
, a*x^2))/((d*x)^(7/2)*d^2))/d
```

3.85.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(3, ax^2\right)}{(dx)^{9/2}}\,dx = \int \frac{\text{Li}_3\left(ax^2\right)}{(dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^2)/(d*x)^(9/2), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3, ax^2\right)}{(dx)^{9/2}}\,dx = \int \frac{\text{polylog}\left(3, a\,x^2\right)}{(d\,x)^{9/2}}\,dx$$

input
```
int(polylog(3, a*x^2)/(d*x)^(9/2),x)
```

output
```
int(polylog(3, a*x^2)/(d*x)^(9/2), x)
```

3.86 $\int (dx)^{3/2}\,\text{PolyLog}\left(2, ax^q\right)\,dx$

3.86.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int (dx)^{3/2}\,\text{PolyLog}\left(2, ax^q\right)\,dx = \frac{8adq^2x^{2+q}\sqrt{dx}\,\text{Hypergeometric2F1}\left(1, \frac{\frac{5}{2}+q}{q}, \frac{1}{2}\left(4+\frac{5}{q}\right), ax^q\right)}{25(5+2q)} + \frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d} + \frac{2(dx)^{5/2}\,\text{PolyLog}\left(2, ax^q\right)}{5d}$$

output
```
4/25*q*(d*x)^(5/2)*ln(1-a*x^q)/d+2/5*(d*x)^(5/2)*polylog(2,a*x^q)/d+8/25*a
*d*q^2*x^(2+q)*hypergeom([1, (5/2+q)/q],[2+5/2/q],a*x^q)*(d*x)^(1/2)/(5+2*
q)
```

3.86.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int (dx)^{3/2}\,\text{PolyLog}\left(2, ax^q\right)\,dx = \frac{2x(dx)^{3/2}\left(4aq^2x^q\,\text{Hypergeometric2F1}\left(1, \frac{\frac{5}{2}+q}{q}, 2+\frac{5}{2q}, ax^q\right) + (5+2q)\,(2q\right.}{25(5+2q)}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

output
```
(2*x*(d*x)^(3/2)*(4*a*q^2*x^q*Hypergeometric2F1[1, (5/2 + q)/q, 2 + 5/(2*q
), a*x^q] + (5 + 2*q)*(2*q*Log[1 - a*x^q] + 5*PolyLog[2, a*x^q])))/(25*(5
+ 2*q))
```

3.86.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \text{PolyLog}\left(2, ax^q\right) dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{5/2} \text{PolyLog}\left(2, ax^q\right)}{5d} - \frac{2}{5} q \int -(dx)^{3/2} \log\left(1 - ax^q\right) dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5} q \int (dx)^{3/2} \log\left(1 - ax^q\right) dx + \frac{2(dx)^{5/2} \text{PolyLog}\left(2, ax^q\right)}{5d}$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{5} q \left(\frac{2aq \int \frac{x^{q-1}(dx)^{5/2}}{1-ax^q} dx}{5d} + \frac{2(dx)^{5/2} \log\left(1 - ax^q\right)}{5d}\right) + \frac{2(dx)^{5/2} \text{PolyLog}\left(2, ax^q\right)}{5d}$$

$$\Big\downarrow \text{30}$$

$$\frac{2}{5} q \left(\frac{2adq\sqrt{dx} \int \frac{x^{q+\frac{3}{2}}}{1-ax^q} dx}{5\sqrt{x}} + \frac{2(dx)^{5/2} \log\left(1 - ax^q\right)}{5d}\right) + \frac{2(dx)^{5/2} \text{PolyLog}\left(2, ax^q\right)}{5d}$$

$$\Big\downarrow \text{888}$$

$$\frac{2}{5} q \left(\frac{4adq\sqrt{dx}x^{q+2} \text{Hypergeometric2F1}\left(1, \frac{q+\frac{5}{2}}{q}, \frac{1}{2}\left(4 + \frac{5}{q}\right), ax^q\right)}{5(2q+5)} + \frac{2(dx)^{5/2} \log\left(1 - ax^q\right)}{5d}\right) + \frac{2(dx)^{5/2} \text{PolyLog}\left(2, ax^q\right)}{5d}$$

input
```
Int[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

output
```
(2*q*((4*a*d*q*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 +
5/q)/2, a*x^q])/(5*(5 + 2*q)) + (2*(d*x)^(5/2)*Log[1 - a*x^q])/(5*d)))/5 +
 (2*(d*x)^(5/2)*PolyLog[2, a*x^q])/(5*d)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.86.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

method	result	si
meijerg	$-\frac{(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(-\frac{4q^{2}x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\ln(1-a\,x^{q})}{25}-\frac{2q\,x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\left(1+\frac{2q}{5}\right)\operatorname{polylog}(2,a\,x^{q})}{5+2q}-\frac{4q^{2}x^{\frac{5}{2}+q}a(-a)^{\frac{5}{2q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{5+2q}{2q}\right)}{25}\right)}{x^{\frac{3}{2}}q}$	1

input
```
int((d*x)^(3/2)*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(-4/25*q^2*x^(5/2)*(-a)^(5/2/q)*ln(1-
a*x^q)-2*q/(5+2*q)*x^(5/2)*(-a)^(5/2/q)*(1+2/5*q)*polylog(2,a*x^q)-4/25*q^
2*x^(5/2+q)*a*(-a)^(5/2/q)*LerchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

3.86.5 Fricas [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x*dilog(a*x^q), x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^q) \, dx = \text{Timed out}$$

input
```
integrate((d*x)**(3/2)*polylog(2,a*x**q),x)
```

output
```
Timed out
```

3.86.7 Maxima [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
8*d^(3/2)*q^3*integrate(1/25*x^(3/2)/((2*a^2*q - 5*a^2)*x^(2*q) - 2*(2*a*q
 - 5*a)*x^q + 2*q - 5), x) + 2/125*(25*((2*a*d^(3/2)*q - 5*a*d^(3/2))*x*x^
q - (2*d^(3/2)*q - 5*d^(3/2))*x)*x^(3/2)*dilog(a*x^q) + 10*((2*a*d^(3/2)*q
^2 - 5*a*d^(3/2)*q)*x*x^q - (2*d^(3/2)*q^2 - 5*d^(3/2)*q)*x)*x^(3/2)*log(-
a*x^q + 1) + 4*(2*d^(3/2)*q^3*x - (2*a*d^(3/2)*q^3 - 5*a*d^(3/2)*q^2)*x*x^
q)*x^(3/2))/((2*a*q - 5*a)*x^q - 2*q + 5)
```

3.86.8 Giac [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*dilog(a*x^q), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \operatorname{PolyLog}(2, ax^q) \, dx = \int (d\,x)^{3/2} \operatorname{polylog}(2, a\,x^q) \, dx$$

input
```
int((d*x)^(3/2)*polylog(2, a*x^q),x)
```

output
```
int((d*x)^(3/2)*polylog(2, a*x^q), x)
```

3.87 $\int \sqrt{dx}\,\text{PolyLog}\left(2, ax^q\right)\,dx$

3.87.1 Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \sqrt{dx}\,\text{PolyLog}\left(2, ax^q\right)\,dx = \frac{8aq^2x^{1+q}\sqrt{dx}\,\text{Hypergeometric2F1}\left(1, \frac{\frac{3}{2}+q}{q}, \frac{1}{2}\left(4+\frac{3}{q}\right), ax^q\right)}{9(3+2q)} + \frac{4q(dx)^{3/2}\log\left(1-ax^q\right)}{9d} + \frac{2(dx)^{3/2}\,\text{PolyLog}\left(2, ax^q\right)}{3d}$$

output
```
4/9*q*(d*x)^(3/2)*ln(1-a*x^q)/d+2/3*(d*x)^(3/2)*polylog(2,a*x^q)/d+8/9*a*q
^2*x^(1+q)*hypergeom([1, (3/2+q)/q],[2+3/2/q],a*x^q)*(d*x)^(1/2)/(3+2*q)
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \sqrt{dx}\,\text{PolyLog}\left(2, ax^q\right)\,dx = \frac{2x\sqrt{dx}\left(4aq^2x^q\,\text{Hypergeometric2F1}\left(1, \frac{\frac{3}{2}+q}{q}, 2+\frac{3}{2q}, ax^q\right) + (3+2q)\left(2q\log\left(1-ax^q\right)+3\,\text{PolyLog}(2, ax^q \right.\right.}{9(3+2q)}$$

input
```
Integrate[Sqrt[d*x]*PolyLog[2, a*x^q],x]
```

output
```
(2*x*Sqrt[d*x]*(4*a*q^2*x^q*Hypergeometric2F1[1, (3/2 + q)/q, 2 + 3/(2*q),
 a*x^q] + (3 + 2*q)*(2*q*Log[1 - a*x^q] + 3*PolyLog[2, a*x^q])))/(9*(3 + 2
*q))
```

3.87.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,\text{PolyLog}(2,ax^q)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{3d}-\frac{2}{3}q\int -\sqrt{dx}\log(1-ax^q)\,dx$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}q\int \sqrt{dx}\log(1-ax^q)\,dx+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{3d}$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{3}q\left(\frac{2aq\int \frac{x^{q-1}(dx)^{3/2}}{1-ax^q}dx}{3d}+\frac{2(dx)^{3/2}\log(1-ax^q)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{3d}$$

$$\Big\downarrow \text{30}$$

$$\frac{2}{3}q\left(\frac{2aq\sqrt{dx}\int \frac{x^{q+\frac{1}{2}}}{1-ax^q}dx}{3\sqrt{x}}+\frac{2(dx)^{3/2}\log(1-ax^q)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{3d}$$

$$\Big\downarrow \text{888}$$

$$\frac{2}{3}q\left(\frac{4aq\sqrt{dx}x^{q+1}\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{3}{2}}{q},\frac{1}{2}\left(4+\frac{3}{q}\right),ax^q\right)}{3(2q+3)}+\frac{2(dx)^{3/2}\log(1-ax^q)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{3d}$$

input
```
Int[Sqrt[d*x]*PolyLog[2, a*x^q],x]
```

output
```
(2*q*((4*a*q*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/
q)/2, a*x^q])/(3*(3 + 2*q)) + (2*(d*x)^(3/2)*Log[1 - a*x^q])/(3*d)))/3 + (
2*(d*x)^(3/2)*PolyLog[2, a*x^q])/(3*d)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.87.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

method	result	siz
meijerg	$-\frac{\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(-\frac{4q^{2}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\ln(1-a\,x^{q})}{9}-\frac{2q\,x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\left(1+\frac{2q}{3}\right)\operatorname{polylog}(2,a\,x^{q})}{3+2q}-\frac{4q^{2}x^{\frac{3}{2}+q}a(-a)^{\frac{3}{2q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{3+2q}{2q}\right)}{9}\right)}{\sqrt{x}\,q}$	12

input
```
int((d*x)^(1/2)*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(-4/9*q^2*x^(3/2)*(-a)^(3/2/q)*ln(1-a
*x^q)-2*q/(3+2*q)*x^(3/2)*(-a)^(3/2/q)*(1+2/3*q)*polylog(2,a*x^q)-4/9*q^2*
x^(3/2+q)*a*(-a)^(3/2/q)*LerchPhi(a*x^q,1,1/2*(3+2*q)/q))
```

3.87.5 Fricas [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}\left(2,ax^q\right)\,dx=\int \sqrt{dx}\mathrm{Li}_2(ax^q)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*dilog(a*x^q), x)
```

3.87.6 Sympy [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}\left(2,ax^q\right)\,dx=\int \sqrt{dx}\,\mathrm{Li}_2\left(ax^q\right)\,dx$$

input
```
integrate((d*x)**(1/2)*polylog(2,a*x**q),x)
```

output
```
Integral(sqrt(d*x)*polylog(2, a*x**q), x)
```

3.87.7 Maxima [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}\left(2,ax^q\right)\,dx=\int \sqrt{dx}\mathrm{Li}_2(ax^q)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
8*sqrt(d)*q^3*integrate(1/9*sqrt(x)/((2*a^2*q - 3*a^2)*x^(2*q) - 2*(2*a*q
- 3*a)*x^q + 2*q - 3), x) + 2/27*(9*((2*a*sqrt(d)*q - 3*a*sqrt(d))*x*x^q -
 (2*sqrt(d)*q - 3*sqrt(d))*x)*sqrt(x)*dilog(a*x^q) + 6*((2*a*sqrt(d)*q^2 -
 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - 3*sqrt(d)*q)*x)*sqrt(x)*log(-a*x^
q + 1) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - 3*a*sqrt(d)*q^2)*x*x^q)*s
qrt(x))/((2*a*q - 3*a)*x^q - 2*q + 3)
```

3.87.8 Giac [F]

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^q\right)\,dx = \int \sqrt{dx}\text{Li}_2(ax^q)\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*dilog(a*x^q), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^q\right)\,dx = \int \sqrt{d\,x}\,\text{polylog}(2,a\,x^q)\,dx$$

input
```
int((d*x)^(1/2)*polylog(2, a*x^q),x)
```

output
```
int((d*x)^(1/2)*polylog(2, a*x^q), x)
```

3.88 $\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}\,dx$

3.88.1 Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}\,dx = \frac{8aq^2x^q\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{\frac{1}{2}+q}{q},\frac{1}{2}\left(4+\frac{1}{q}\right),ax^q\right)}{d(1+2q)} + \frac{4q\sqrt{dx}\log(1-ax^q)}{d} + \frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}$$

output
```
8*a*q^2*x^q*hypergeom([1, (1/2+q)/q],[2+1/2/q],a*x^q)*(d*x)^(1/2)/d/(1+2*q
)+4*q*ln(1-a*x^q)*(d*x)^(1/2)/d+2*polylog(2,a*x^q)*(d*x)^(1/2)/d
```

3.88.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}\,dx = -\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1-\frac{1}{2q}\\1,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

input
```
Integrate[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, -1/2*1/q}}, -(a*x^
q)])/(q*Sqrt[d*x]))
```

3.88.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}-2q\int -\frac{\log(1-ax^q)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{25}$$

$$2q\int \frac{\log(1-ax^q)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}$$

$$\Big\downarrow \text{2905}$$

$$2q\left(\frac{2aq\int \frac{x^{q-1}\sqrt{dx}}{1-ax^q}dx}{d}+\frac{2\sqrt{dx}\log(1-ax^q)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}$$

$$\Big\downarrow \text{30}$$

$$2q\left(\frac{2aq\sqrt{dx}\int \frac{x^{q-\frac{1}{2}}}{1-ax^q}dx}{d\sqrt{x}}+\frac{2\sqrt{dx}\log(1-ax^q)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}$$

$$\Big\downarrow \text{888}$$

$$2q\left(\frac{4aq\sqrt{dx}x^q\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{1}{2}}{q},\frac{1}{2}\left(4+\frac{1}{q}\right),ax^q\right)}{d(2q+1)}+\frac{2\sqrt{dx}\log(1-ax^q)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}$$

input
```
Int[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

output
```
2*q*((4*a*q*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2
, a*x^q])/(d*(1 + 2*q)) + (2*Sqrt[d*x]*Log[1 - a*x^q])/d) + (2*Sqrt[d*x]*P
olyLog[2, a*x^q])/d
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.88.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

method	result	size
meijerg	$-\frac{\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(-4q^{2}\sqrt{x}\,(-a)^{\frac{1}{2q}}\ln(1-a\,x^{q})-2q\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(2,a\,x^{q})-4q^{2}x^{\frac{1}{2}+q}a(-a)^{\frac{1}{2q}}\operatorname{LerchPhi}\left(a\,x^{q},1,\frac{1+2q}{2q}\right)\right)}{\sqrt{dx}\,q}$	109

input
```
int(polylog(2,a*x^q)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(-4*q^2*x^(1/2)*(-a)^(1/2/q)*ln(1-a
*x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-4*q^2*x^(1/2+q)*a*(-a)^(1/
2/q)*LerchPhi(a*x^q,1,1/2*(1+2*q)/q))
```

3.88.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*dilog(a*x^q)/(d*x), x)
```

3.88.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(2,a*x**q)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(2, a*x**q)/sqrt(d*x), x)
```

3.88.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
8*q^3*integrate(1/(((2*a^2*sqrt(d)*q - a^2*sqrt(d))*x^(2*q) - 2*(2*a*sqrt(
d)*q - a*sqrt(d))*x^q + 2*sqrt(d)*q - sqrt(d))*sqrt(x)), x) - 2*(((2*a*sqr
t(d)*q - a*sqrt(d))*x*x^q - (2*sqrt(d)*q - sqrt(d))*x)*dilog(a*x^q)/sqrt(x
) + 2*((2*a*sqrt(d)*q^2 - a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - sqrt(d)*q)
*x)*log(-a*x^q + 1)/sqrt(x) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - a*sq
rt(d)*q^2)*x*x^q)/sqrt(x))/(2*d*q - (2*a*d*q - a*d)*x^q - d)
```

3.88.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}} \, dx = \int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}} \, dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/sqrt(d*x), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}} \, dx = \int \frac{\text{polylog}(2, a\,x^q)}{\sqrt{d\,x}} \, dx$$

input
```
int(polylog(2, a*x^q)/(d*x)^(1/2),x)
```

output
```
int(polylog(2, a*x^q)/(d*x)^(1/2), x)
```

3.89 $\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}} \, dx$

3.89.1 Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}} \, dx = -\frac{8aq^2x^q \text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right),\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{4q\log(1-ax^q)}{d\sqrt{dx}} - \frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

output
```
-8*a*q^2*x^q*hypergeom([1, 1-1/2/q],[2-1/2/q],a*x^q)/d/(1-2*q)/(d*x)^(1/2)
+4*q*ln(1-a*x^q)/d/(d*x)^(1/2)-2*polylog(2,a*x^q)/d/(d*x)^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.49

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}} \, dx = -\frac{xG_{4,4}^{1,4}\left(-ax^q\middle| \begin{array}{c} 1,1,1,1+\frac{1}{2q} \\ 1,0,0,\frac{1}{2q} \end{array}\right)}{q(dx)^{3/2}}$$

input
```
Integrate[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 1/(2*q)}}, -(a*x^q
)])/(q*(d*x)^(3/2)))
```

3.89.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 7145}$$

$$2q\int -\frac{\log(1-ax^q)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 25}$$

$$-2q\int \frac{\log(1-ax^q)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 2905}$$

$$-2q\left(-\frac{2aq\int \frac{x^{q-1}}{\sqrt{dx}(1-ax^q)}dx}{d}-\frac{2\log(1-ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 30}$$

$$-2q\left(-\frac{2aq\sqrt{x}\int \frac{x^{q-\frac{3}{2}}}{1-ax^q}dx}{d\sqrt{dx}}-\frac{2\log(1-ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 888}$$

$$-2q\left(\frac{4aqx^q\,\text{Hypergeometric2F1}\left(1,-\frac{\frac{1}{2}-q}{q},\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{2\log(1-ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

output
```
-2*q*((4*a*q*x^q*Hypergeometric2F1[1, -((1/2 - q)/q), (4 - q^(-1))/2, a*x^
q])/(d*(1 - 2*q)*Sqrt[d*x]) - (2*Log[1 - a*x^q])/(d*Sqrt[d*x])) - (2*PolyL
og[2, a*x^q])/(d*Sqrt[d*x])
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.89.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

method	result	size
meijerg	$-\frac{x^{\frac{3}{2}} (-a)^{\frac{1}{2q}} \left(-\frac{4 q^{2} (-a)^{-\frac{1}{2q}} \ln\left(1-a\,x^{q}\right)}{\sqrt{x}}-\frac{2 q (-a)^{-\frac{1}{2q}} \left(1-2q\right) \operatorname{polylog}\left(2,a\,x^{q}\right)}{\left(2q-1\right)\sqrt{x}}-4 q^{2} x^{q-\frac{1}{2}} a (-a)^{-\frac{1}{2q}} \operatorname{LerchPhi}\left(a\,x^{q},1,\frac{2q-1}{2q}\right)\right)}{\left(d x\right)^{\frac{3}{2}} q}$	121

input
```
int(polylog(2,a*x^q)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-4*q^2/x^(1/2)*(-a)^(-1/2/q)*ln(1-a
*x^q)-2*q/(2*q-1)/x^(1/2)*(-a)^(-1/2/q)*(1-2*q)*polylog(2,a*x^q)-4*q^2*x^(
q-1/2)*a*(-a)^(-1/2/q)*LerchPhi(a*x^q,1,1/2*(2*q-1)/q))
```

3.89.5 Fricas [F]

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{3/2}}\,dx=\int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*dilog(a*x^q)/(d^2*x^2), x)
```

3.89.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{3/2}}\,dx=\int \frac{\text{Li}_2\left(ax^q\right)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x**q)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(2, a*x**q)/(d*x)**(3/2), x)
```

3.89.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{3/2}}\,dx=\int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
8*q^3*integrate(1/((2*d^(3/2)*q + (2*a^2*d^(3/2)*q + a^2*d^(3/2))*x^(2*q)
- 2*(2*a*d^(3/2)*q + a*d^(3/2))*x^q + d^(3/2))*x^(3/2)), x) + 2*(((2*a*sqr
t(d)*q + a*sqrt(d))*x*x^q - (2*sqrt(d)*q + sqrt(d))*x)*dilog(a*x^q)/x^(3/2
) - 2*((2*a*sqrt(d)*q^2 + a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 + sqrt(d)*q)
*x)*log(-a*x^q + 1)/x^(3/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 + a*sq
rt(d)*q^2)*x*x^q)/x^(3/2))/(2*d^2*q + d^2 - (2*a*d^2*q + a*d^2)*x^q)
```

3.89.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, ax^q)}{(dx)^{3/2}} \, dx = \int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/(d*x)^(3/2), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, ax^q)}{(dx)^{3/2}} \, dx = \int \frac{\text{polylog}(2, a\,x^q)}{(d\,x)^{3/2}} \, dx$$

input
```
int(polylog(2, a*x^q)/(d*x)^(3/2),x)
```

output
```
int(polylog(2, a*x^q)/(d*x)^(3/2), x)
```

3.90 $\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}}\,dx$

3.90.1 Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}}\,dx = -\frac{8aq^2x^{-1+q}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right),\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{9d^2(3-2q)\sqrt{dx}} + \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

output
```
4/9*q*ln(1-a*x^q)/d/(d*x)^(3/2)-2/3*polylog(2,a*x^q)/d/(d*x)^(3/2)-8/9*a*q
^2*x^(-1+q)*hypergeom([1, 1-3/2/q],[2-3/2/q],a*x^q)/d^2/(3-2*q)/(d*x)^(1/2
)
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}}\,dx = -\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1+\frac{3}{2q}\\1,0,0,\frac{3}{2q}\end{array}\right)}{q(dx)^{5/2}}$$

input
```
Integrate[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 3/(2*q)}}, -(a*x^q
)])/(q*(d*x)^(5/2)))
```

3.90.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2}{3}q\int -\frac{\log(1-ax^q)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{2}{3}q\int \frac{\log(1-ax^q)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 2905$$

$$-\frac{2}{3}q\left(-\frac{2aq\int \frac{x^{q-1}}{(dx)^{3/2}(1-ax^q)}dx}{3d}-\frac{2\log(1-ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 30$$

$$-\frac{2}{3}q\left(-\frac{2aq\sqrt{x}\int \frac{x^{q-\frac{5}{2}}}{1-ax^q}dx}{3d^2\sqrt{dx}}-\frac{2\log(1-ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 888$$

$$-\frac{2}{3}q\left(\frac{4aqx^{q-1}\,\text{Hypergeometric2F1}\left(1,-\frac{\frac{3}{2}-q}{q},\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{3d^2(3-2q)\sqrt{dx}}-\frac{2\log(1-ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

output
```
(-2*q*((4*a*q*x^(-1 + q)*Hypergeometric2F1[1, -((3/2 - q)/q), (4 - 3/q)/2,
 a*x^q])/(3*d^2*(3 - 2*q)*Sqrt[d*x]) - (2*Log[1 - a*x^q])/(3*d*(d*x)^(3/2)
)))/3 - (2*PolyLog[2, a*x^q])/(3*d*(d*x)^(3/2))
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.90.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result	size
meijerg	$-\frac{x^{\frac{5}{2}} (-a)^{\frac{3}{2q}} \left(-\frac{4 q^{2} (-a)^{-\frac{3}{2q}} \ln\left(1-a\,x^{q}\right)}{9 x^{\frac{3}{2}}}-\frac{2 q (-a)^{-\frac{3}{2q}} \left(1-\frac{2 q}{3}\right) \operatorname{polylog}\left(2, a\,x^{q}\right)}{(-3+2 q) x^{\frac{3}{2}}}-\frac{4 q^{2} x^{q-\frac{3}{2}} a (-a)^{-\frac{3}{2q}} \operatorname{LerchPhi}\left(a\,x^{q}, 1, \frac{-3+2 q}{2 q}\right)}{9}\right)}{(d x)^{\frac{5}{2}} q}$	121

input
```
int(polylog(2,a*x^q)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*ln(1
-a*x^q)-2*q/(-3+2*q)/x^(3/2)*(-a)^(-3/2/q)*(1-2/3*q)*polylog(2,a*x^q)-4/9*
q^2*x^(q-3/2)*a*(-a)^(-3/2/q)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

3.90.5 Fricas [F]

$$\int \frac{\operatorname{PolyLog}(2, ax^q)}{(dx)^{5/2}}\,dx = \int \frac{\operatorname{Li}_2(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*dilog(a*x^q)/(d^3*x^3), x)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{PolyLog}(2, ax^q)}{(dx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(polylog(2,a*x**q)/(d*x)**(5/2),x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
8*q^3*integrate(1/9/((2*d^(5/2)*q + 3*d^(5/2) + (2*a^2*d^(5/2)*q + 3*a^2*d
^(5/2))*x^(2*q) - 2*(2*a*d^(5/2)*q + 3*a*d^(5/2))*x^q)*x^(5/2)), x) + 2/27
*(9*((2*a*sqrt(d)*q + 3*a*sqrt(d))*x*x^q - (2*sqrt(d)*q + 3*sqrt(d))*x)*di
log(a*x^q)/x^(5/2) - 6*((2*a*sqrt(d)*q^2 + 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(
d)*q^2 + 3*sqrt(d)*q)*x)*log(-a*x^q + 1)/x^(5/2) + 4*(2*sqrt(d)*q^3*x - (2
*a*sqrt(d)*q^3 + 3*a*sqrt(d)*q^2)*x*x^q)/x^(5/2))/(2*d^3*q + 3*d^3 - (2*a*
d^3*q + 3*a*d^3)*x^q)
```

3.90.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(dilog(a*x^q)/(d*x)^(5/2), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,ax^q\right)}{(dx)^{5/2}}\,dx = \int \frac{\text{polylog}(2,a\,x^q)}{(d\,x)^{5/2}}\,dx$$

input
```
int(polylog(2, a*x^q)/(d*x)^(5/2),x)
```

output
```
int(polylog(2, a*x^q)/(d*x)^(5/2), x)
```

3.91 $\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx$

3.91.1 Optimal result

Integrand size = 15, antiderivative size = 125

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = -\frac{16adq^3x^{2+q}\sqrt{dx}\,\text{Hypergeometric2F1}\left(1, \frac{\frac{5}{2}+q}{q}, \frac{1}{2}\left(4+\frac{5}{q}\right), ax^q\right)}{125(5+2q)} - \frac{8q^2(dx)^{5/2}\log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2}\operatorname{PolyLog}(2, ax^q)}{25d} + \frac{2(dx)^{5/2}\operatorname{PolyLog}(3, ax^q)}{5d}$$

output
```
-8/125*q^2*(d*x)^(5/2)*ln(1-a*x^q)/d-4/25*q*(d*x)^(5/2)*polylog(2,a*x^q)/d
+2/5*(d*x)^(5/2)*polylog(3,a*x^q)/d-16/125*a*d*q^3*x^(2+q)*hypergeom([1, (
5/2+q)/q],[2+5/2/q],a*x^q)*(d*x)^(1/2)/(5+2*q)
```

3.91.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = -\frac{x(dx)^{3/2} G_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,1-\frac{5}{2q} \\ 1,0,0,0,-\frac{5}{2q} \end{array}\right)}{q}$$

input
```
Integrate[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

output
```
-((x*(d*x)^(3/2)*MeijerG[{{1, 1, 1, 1, 1 - 5/(2*q)}, {}}, {{1}, {0, 0, 0,
-5/(2*q)}}, -(a*x^q)])/q)
```

3.91.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \text{PolyLog}(3,ax^q)\, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d} - \frac{2}{5}q\int (dx)^{3/2}\text{PolyLog}(2,ax^q)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d} - \frac{2}{5}q\left(\frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d} - \frac{2}{5}q\int -(dx)^{3/2}\log(1-ax^q)\,dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d} - \frac{2}{5}q\left(\frac{2}{5}q\int (dx)^{3/2}\log(1-ax^q)\,dx + \frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d} - \frac{2}{5}q\left(\frac{2}{5}q\left(\frac{2aq\int \frac{x^{q-1}(dx)^{5/2}}{1-ax^q}dx}{5d} + \frac{2(dx)^{5/2}\log(1-ax^q)}{5d}\right) + \frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}\right)$$

$$\Big\downarrow \text{30}$$

$$\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d} - \frac{2}{5}q\left(\frac{2}{5}q\left(\frac{2adq\sqrt{dx}\int \frac{x^{q+\frac{3}{2}}}{1-ax^q}dx}{5\sqrt{x}} + \frac{2(dx)^{5/2}\log(1-ax^q)}{5d}\right) + \frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}\right)$$

$$\Big\downarrow 888$$

$$\frac{2(dx)^{5/2}\,\text{PolyLog}\left(3,ax^q\right)}{5d} - \frac{2}{5}q\left(\frac{2}{5}q\left(\frac{4adq\sqrt{dx}x^{q+2}\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{5}{2}}{q},\frac{1}{2}\left(4+\frac{5}{q}\right),ax^q\right)}{5(2q+5)}+\frac{2(dx)^{5/2}\log\left(1-ax^q\right)}{5d}\right)+\frac{2(dx)^{5/2}\,\text{Poly}}{5d}\right.$$

input
```
Int[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

output
```
(-2*q*((2*q*((4*a*d*q*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q
, (4 + 5/q)/2, a*x^q]))/(5*(5 + 2*q)) + (2*(d*x)^(5/2)*Log[1 - a*x^q])/(5*d
)))/5 + (2*(d*x)^(5/2)*PolyLog[2, a*x^q])/(5*d)))/5 + (2*(d*x)^(5/2)*PolyL
og[3, a*x^q])/(5*d)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.91.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

method	result
meijerg	$-\frac{(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(\frac{8q^3x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\ln(1-ax^q)}{125}+\frac{4q^2x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\operatorname{polylog}(2,ax^q)}{25}-\frac{2qx^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\left(1+\frac{2q}{5}\right)\operatorname{polylog}(3,ax^q)}{5+2q}+\frac{8q^3x^{\frac{5}{2}+q}a(-a)^{\frac{5}{2q}}\,\mathrm{L}}{1}\right.}{x^{\frac{3}{2}}q}$

input
```
int((d*x)^(3/2)*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(8/125*q^3*x^(5/2)*(-a)^(5/2/q)*ln(1-
a*x^q)+4/25*q^2*x^(5/2)*(-a)^(5/2/q)*polylog(2,a*x^q)-2*q/(5+2*q)*x^(5/2)*
(-a)^(5/2/q)*(1+2/5*q)*polylog(3,a*x^q)+8/125*q^3*x^(5/2+q)*a*(-a)^(5/2/q)
*LerchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

3.91.5 Fricas [F]

$$\int (dx)^{3/2}\,\mathrm{PolyLog}\left(3,ax^q\right)\,dx=\int (dx)^{\frac{3}{2}}\,\mathrm{Li}_3(ax^q)\,dx$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x*polylog(3, a*x^q), x)
```

3.91.6 Sympy [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate((d*x)**(3/2)*polylog(3,a*x**q),x)
```

output
```
Integral((d*x)**(3/2)*polylog(3, a*x**q), x)
```

3.91.7 Maxima [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
-16*d^(3/2)*q^4*integrate(1/125*x^(3/2)/(a^2*(2*q - 5)*x^(2*q) - 2*a*(2*q
- 5)*x^q + 2*q - 5), x) - 2/625*(50*((2*q^2 - 5*q)*a*d^(3/2)*x*x^q - (2*q^
2 - 5*q)*d^(3/2)*x)*x^(3/2)*dilog(a*x^q) + 20*((2*q^3 - 5*q^2)*a*d^(3/2)*x
*x^q - (2*q^3 - 5*q^2)*d^(3/2)*x)*x^(3/2)*log(-a*x^q + 1) - 125*(a*d^(3/2)
*(2*q - 5)*x*x^q - d^(3/2)*(2*q - 5)*x)*x^(3/2)*polylog(3, a*x^q) + 8*(2*d
^(3/2)*q^4*x - (2*q^4 - 5*q^3)*a*d^(3/2)*x*x^q)*x^(3/2))/(a*(2*q - 5)*x^q
- 2*q + 5)
```

3.91.8 Giac [F]

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = \int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*polylog(3, a*x^q), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \operatorname{PolyLog}(3, ax^q) \, dx = \int (d\,x)^{3/2} \operatorname{polylog}(3, a\,x^q) \, dx$$

input
```
int((d*x)^(3/2)*polylog(3, a*x^q),x)
```

output
```
int((d*x)^(3/2)*polylog(3, a*x^q), x)
```

3.92 $\int \sqrt{dx}\,\text{PolyLog}(3,ax^q)\,dx$

3.92.1 Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax^q)\,dx = -\frac{16aq^3x^{1+q}\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{\frac{3}{2}+q}{q},\frac{1}{2}\left(4+\frac{3}{q}\right),ax^q\right)}{27(3+2q)} - \frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\,\text{PolyLog}(2,ax^q)}{9d} + \frac{2(dx)^{3/2}\,\text{PolyLog}(3,ax^q)}{3d}$$

output
```
-8/27*q^2*(d*x)^(3/2)*ln(1-a*x^q)/d-4/9*q*(d*x)^(3/2)*polylog(2,a*x^q)/d+2
/3*(d*x)^(3/2)*polylog(3,a*x^q)/d-16/27*a*q^3*x^(1+q)*hypergeom([1, (3/2+q
)/q],[2+3/2/q],a*x^q)*(d*x)^(1/2)/(3+2*q)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax^q)\,dx = -\frac{x\sqrt{dx}\,G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{3}{2q}\\1,0,0,0,-\frac{3}{2q}\end{array}\right)}{q}$$

input
```
Integrate[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

output
```
-((x*Sqrt[d*x]*MeijerG[{{1, 1, 1, 1, 1 - 3/(2*q)}, {}}, {{1}, {0, 0, 0, -3
/(2*q)}}, -(a*x^q)])/q)
```

3.92.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\,\text{PolyLog}(3,ax^q)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}(3,ax^q)}{3d}-\frac{2}{3}q\int \sqrt{dx}\,\text{PolyLog}(2,ax^q)dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}(3,ax^q)}{3d}-\frac{2}{3}q\left(\frac{2(dx)^{3/2}\text{PolyLog}(2,ax^q)}{3d}-\frac{2}{3}q\int -\sqrt{dx}\log(1-ax^q)dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}(3,ax^q)}{3d}-\frac{2}{3}q\left(\frac{2}{3}q\int \sqrt{dx}\log(1-ax^q)dx+\frac{2(dx)^{3/2}\text{PolyLog}(2,ax^q)}{3d}\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}(3,ax^q)}{3d}-\frac{2}{3}q\left(\frac{2}{3}q\left(\frac{2aq\int \frac{x^{q-1}(dx)^{3/2}}{1-ax^q}dx}{3d}+\frac{2(dx)^{3/2}\log(1-ax^q)}{3d}\right)+\frac{2(dx)^{3/2}\text{PolyLog}(2,ax^q)}{3d}\right)$$

$$\Big\downarrow \text{30}$$

$$\frac{2(dx)^{3/2}\text{PolyLog}(3,ax^q)}{3d}-\frac{2}{3}q\left(\frac{2}{3}q\left(\frac{2aq\sqrt{dx}\int \frac{x^{q+\frac{1}{2}}}{1-ax^q}dx}{3\sqrt{x}}+\frac{2(dx)^{3/2}\log(1-ax^q)}{3d}\right)+\frac{2(dx)^{3/2}\text{PolyLog}(2,ax^q)}{3d}\right)$$

$$\Big\downarrow 888$$

$$\frac{2(dx)^{3/2}\,\text{PolyLog}\left(3,ax^q\right)}{3d}-\frac{2}{3}q\left(\frac{2}{3}q\left(\frac{4aq\sqrt{dx}x^{q+1}\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{3}{2}}{q},\frac{1}{2}\left(4+\frac{3}{q}\right),ax^q\right)}{3(2q+3)}+\frac{2(dx)^{3/2}\log\left(1-ax^q\right)}{3d}\right)+\frac{2(dx)^{3/2}\,\text{PolyL}}{3d}\right.$$

input
```
Int[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

output
```
(-2*q*((2*q*((4*a*q*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q,
(4 + 3/q)/2, a*x^q])/(3*(3 + 2*q)) + (2*(d*x)^(3/2)*Log[1 - a*x^q])/(3*d))
)/3 + (2*(d*x)^(3/2)*PolyLog[2, a*x^q])/(3*d)))/3 + (2*(d*x)^(3/2)*PolyLog
[3, a*x^q])/(3*d)
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.92.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

method	result
meijerg	$-\frac{\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(\frac{8q^{3}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\ln(1-a\,x^{q})}{27}+\frac{4q^{2}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\operatorname{polylog}(2,a\,x^{q})}{9}-\frac{2q\,x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\left(1+\frac{2q}{3}\right)\operatorname{polylog}(3,a\,x^{q})}{3+2q}+\frac{8q^{3}x^{\frac{3}{2}+q}a(-a)^{\frac{3}{2q}}\,\mathrm{Ler}}{27}\right.}{\sqrt{x}\,q}$

input
```
int((d*x)^(1/2)*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(8/27*q^3*x^(3/2)*(-a)^(3/2/q)*ln(1-a
*x^q)+4/9*q^2*x^(3/2)*(-a)^(3/2/q)*polylog(2,a*x^q)-2*q/(3+2*q)*x^(3/2)*(-
a)^(3/2/q)*(1+2/3*q)*polylog(3,a*x^q)+8/27*q^3*x^(3/2+q)*a*(-a)^(3/2/q)*Le
rchPhi(a*x^q,1,1/2*(3+2*q)/q))
```

3.92.5 Fricas [F]

$$\int \sqrt{dx}\,\mathrm{PolyLog}\left(3,ax^{q}\right)\,dx=\int \sqrt{dx}\mathrm{Li}_{3}(ax^{q})\,dx$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*polylog(3, a*x^q), x)
```

3.92.6 Sympy [F]

$$\int \sqrt{dx}\, \text{PolyLog}\left(3, ax^q\right)\, dx = \int \sqrt{dx}\, \text{Li}_3\left(ax^q\right)\, dx$$

input
```
integrate((d*x)**(1/2)*polylog(3,a*x**q),x)
```

output
```
Integral(sqrt(d*x)*polylog(3, a*x**q), x)
```

3.92.7 Maxima [F]

$$\int \sqrt{dx}\, \text{PolyLog}\left(3, ax^q\right)\, dx = \int \sqrt{dx}\text{Li}_3(ax^q)\, dx$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
-16*sqrt(d)*q^4*integrate(1/27*sqrt(x)/(a^2*(2*q - 3)*x^(2*q) - 2*a*(2*q -
 3)*x^q + 2*q - 3), x) - 2/81*(18*((2*q^2 - 3*q)*a*sqrt(d)*x*x^q - (2*q^2
- 3*q)*sqrt(d)*x)*sqrt(x)*dilog(a*x^q) + 12*((2*q^3 - 3*q^2)*a*sqrt(d)*x*x
^q - (2*q^3 - 3*q^2)*sqrt(d)*x)*sqrt(x)*log(-a*x^q + 1) - 27*(a*sqrt(d)*(2
*q - 3)*x*x^q - sqrt(d)*(2*q - 3)*x)*sqrt(x)*polylog(3, a*x^q) + 8*(2*sqrt
(d)*q^4*x - (2*q^4 - 3*q^3)*a*sqrt(d)*x*x^q)*sqrt(x))/(a*(2*q - 3)*x^q - 2
*q + 3)
```

3.92.8 Giac [F]

$$\int \sqrt{dx}\, \text{PolyLog}\left(3, ax^q\right)\, dx = \int \sqrt{dx}\text{Li}_3(ax^q)\, dx$$

input
```
integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*polylog(3, a*x^q), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\,\text{PolyLog}\,(3,ax^q)\;dx = \int \sqrt{d\,x}\,\text{polylog}(3,a\,x^q)\;dx$$

input
```
int((d*x)^(1/2)*polylog(3, a*x^q),x)
```

output
```
int((d*x)^(1/2)*polylog(3, a*x^q), x)
```

3.93 $\int \frac{\text{PolyLog}(3,ax^q)}{\sqrt{dx}}\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \frac{\text{PolyLog}(3,ax^q)}{\sqrt{dx}}\,dx = -\frac{16aq^3x^q\sqrt{dx}\,\text{Hypergeometric2F1}\left(1,\frac{\frac{1}{2}+q}{q},\frac{1}{2}\left(4+\frac{1}{q}\right),ax^q\right)}{d(1+2q)} - \frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d} + \frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}$$

output
```
-16*a*q^3*x^q*hypergeom([1, (1/2+q)/q],[2+1/2/q],a*x^q)*(d*x)^(1/2)/d/(1+2
*q)-8*q^2*ln(1-a*x^q)*(d*x)^(1/2)/d-4*q*polylog(2,a*x^q)*(d*x)^(1/2)/d+2*p
olylog(3,a*x^q)*(d*x)^(1/2)/d
```

3.93.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.43

$$\int \frac{\text{PolyLog}(3,ax^q)}{\sqrt{dx}}\,dx = -\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{1}{2q}\\1,0,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

input
```
Integrate[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, 0, -1/2*1/q}},
-(a*x^q)])/(q*Sqrt[d*x]))
```

3.93.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}-2q\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}-2q\left(\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}-2q\int -\frac{\log(1-ax^q)}{\sqrt{dx}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}-2q\left(2q\int \frac{\log(1-ax^q)}{\sqrt{dx}}dx+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}\right)$$

$$\Big\downarrow \text{2905}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}-2q\left(2q\left(\frac{2aq\int \frac{x^{q-1}\sqrt{dx}}{1-ax^q}dx}{d}+\frac{2\sqrt{dx}\log(1-ax^q)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}\right)$$

$$\Big\downarrow \text{30}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax^q)}{d}-2q\left(2q\left(\frac{2aq\sqrt{dx}\int \frac{x^{q-\frac{1}{2}}}{1-ax^q}dx}{d\sqrt{x}}+\frac{2\sqrt{dx}\log(1-ax^q)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}\right)$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{dx}\,\text{PolyLog}\left(3,ax^q\right)}{d}-$$
$$2q\left(2q\left(\frac{4aq\sqrt{dx}x^q\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{1}{2}}{q},\frac{1}{2}\left(4+\frac{1}{q}\right),ax^q\right)}{d(2q+1)}+\frac{2\sqrt{dx}\log\left(1-ax^q\right)}{d}\right)+\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax\right.}{d}\right.$$

input
```
Int[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```

output
```
-2*q*(2*q*((4*a*q*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(
-1))/2, a*x^q])/(d*(1 + 2*q)) + (2*Sqrt[d*x]*Log[1 - a*x^q])/d) + (2*Sqrt[
d*x]*PolyLog[2, a*x^q])/d) + (2*Sqrt[d*x]*PolyLog[3, a*x^q])/d
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

method	result
meijerg	$-\frac{\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(8q^{3}\sqrt{x}\,(-a)^{\frac{1}{2q}}\ln(1-a\,x^{q})+4q^{2}\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(2,a\,x^{q})-2q\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(3,a\,x^{q})+8q^{3}x^{\frac{1}{2}+q}a(-a)^{\frac{1}{2q}}\,\mathrm{Lerc}\right.}{\sqrt{dx}\,q}$

input
```
int(polylog(3,a*x^q)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(8*q^3*x^(1/2)*(-a)^(1/2/q)*ln(1-a*
x^q)+4*q^2*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*
polylog(3,a*x^q)+8*q^3*x^(1/2+q)*a*(-a)^(1/2/q)*LerchPhi(a*x^q,1,1/2*(1+2*
q)/q))
```

3.93.5 Fricas [F]

$$\int \frac{\mathrm{PolyLog}\left(3, ax^{q}\right)}{\sqrt{dx}}\, dx = \int \frac{\mathrm{Li}_{3}(ax^{q})}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*polylog(3, a*x^q)/(d*x), x)
```

3.93.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(3,a*x**q)/(d*x)**(1/2),x)
```

output
```
Integral(polylog(3, a*x**q)/sqrt(d*x), x)
```

3.93.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-16*q^4*integrate(1/((a^2*sqrt(d)*(2*q - 1)*x^(2*q) - 2*a*sqrt(d)*(2*q - 1
)*x^q + sqrt(d)*(2*q - 1))*sqrt(x)), x) - 2*(2*((2*q^2 - q)*a*x*x^q - (2*q
^2 - q)*x)*dilog(a*x^q)/sqrt(x) + 4*((2*q^3 - q^2)*a*x*x^q - (2*q^3 - q^2)
*x)*log(-a*x^q + 1)/sqrt(x) - (a*(2*q - 1)*x*x^q - (2*q - 1)*x)*polylog(3,
 a*x^q)/sqrt(x) + 8*(2*q^4*x - (2*q^4 - q^3)*a*x*x^q)/sqrt(x))/(a*sqrt(d)*
(2*q - 1)*x^q - sqrt(d)*(2*q - 1))
```

3.93.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{\sqrt{dx}}\, dx = \int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}}\, dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/sqrt(d*x), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3,ax^q\right)}{\sqrt{dx}}\,dx=\int \frac{\text{polylog}(3,a\,x^q)}{\sqrt{d\,x}} \,dx$$

input
```
int(polylog(3, a*x^q)/(d*x)^(1/2),x)
```

output
```
int(polylog(3, a*x^q)/(d*x)^(1/2), x)
```

3.94 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}}\,dx$

3.94.1 Optimal result

Integrand size = 15, antiderivative size = 119

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}}\,dx = -\frac{16aq^3x^q\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right),\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{8q^2\log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}} - \frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

output
```
-16*a*q^3*x^q*hypergeom([1, 1-1/2/q],[2-1/2/q],a*x^q)/d/(1-2*q)/(d*x)^(1/2
)+8*q^2*ln(1-a*x^q)/d/(d*x)^(1/2)-4*q*polylog(2,a*x^q)/d/(d*x)^(1/2)-2*pol
ylog(3,a*x^q)/d/(d*x)^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}}\,dx = -\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{1}{2q}\\1,0,0,0,\frac{1}{2q}\end{array}\right)}{q(dx)^{3/2}}$$

input
```
Integrate[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 0, 1/(2*q)}}, -
(a*x^q)])/(q*(d*x)^(3/2)))
```

3.94.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \; 7145$$

$$2q\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 7145$$

$$2q\left(2q\int -\frac{\log(1-ax^q)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 25$$

$$2q\left(-2q\int \frac{\log(1-ax^q)}{(dx)^{3/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 2905$$

$$2q\left(-2q\left(-\frac{2aq\int \frac{x^{q-1}}{\sqrt{dx}(1-ax^q)}dx}{d}-\frac{2\log(1-ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 30$$

$$2q\left(-2q\left(-\frac{2aq\sqrt{x}\int \frac{x^{q-\frac{3}{2}}}{1-ax^q}dx}{d\sqrt{dx}}-\frac{2\log(1-ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 888$$

$$2q\left(-2q\left(\frac{4aqx^q\,\text{Hypergeometric2F1}\left(1,-\frac{\frac{1}{2}-q}{q},\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{2\log\left(1-ax^q\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^q\right)}{d\sqrt{dx}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^q\right)}{d\sqrt{dx}}$$

input
```
Int[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```

output
```
2*q*(-2*q*((4*a*q*x^q*Hypergeometric2F1[1, -((1/2 - q)/q), (4 - q^(-1))/2,
 a*x^q])/(d*(1 - 2*q)*Sqrt[d*x]) - (2*Log[1 - a*x^q])/(d*Sqrt[d*x])) - (2*
PolyLog[2, a*x^q])/(d*Sqrt[d*x])) - (2*PolyLog[3, a*x^q])/(d*Sqrt[d*x])
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.94.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

method	result
meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-\frac{8q^{3}(-a)^{-\frac{1}{2q}}\ln(1-a\,x^{q})}{\sqrt{x}}+\frac{4q^{2}(-a)^{-\frac{1}{2q}}\operatorname{polylog}(2,a\,x^{q})}{\sqrt{x}}-\frac{2q(-a)^{-\frac{1}{2q}}(1-2q)\operatorname{polylog}(3,a\,x^{q})}{(2q-1)\sqrt{x}}-8q^{3}x^{q-\frac{1}{2}}a(-a)^{-\frac{1}{2q}}\operatorname{LerchPh}\right.}{(dx)^{\frac{3}{2}}q}$

input
```
int(polylog(3,a*x^q)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-8*q^3/x^(1/2)*(-a)^(-1/2/q)*ln(1-a
*x^q)+4*q^2/x^(1/2)*(-a)^(-1/2/q)*polylog(2,a*x^q)-2*q/(2*q-1)/x^(1/2)*(-a
)^(-1/2/q)*(1-2*q)*polylog(3,a*x^q)-8*q^3*x^(q-1/2)*a*(-a)^(-1/2/q)*LerchP
hi(a*x^q,1,1/2*(2*q-1)/q))
```

3.94.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*polylog(3, a*x^q)/(d^2*x^2), x)
```

3.94.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}}\,dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x**q)/(d*x)**(3/2),x)
```

output
```
Integral(polylog(3, a*x**q)/(d*x)**(3/2), x)
```

3.94.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}}\, dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\, dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
16*q^4*integrate(1/((a^2*d^(3/2)*(2*q + 1)*x^(2*q) - 2*a*d^(3/2)*(2*q + 1)
*x^q + d^(3/2)*(2*q + 1))*x^(3/2)), x) - 2*(2*((2*q^2 + q)*a*x*x^q - (2*q^
2 + q)*x)*dilog(a*x^q)/x^(3/2) - 4*((2*q^3 + q^2)*a*x*x^q - (2*q^3 + q^2)*
x)*log(-a*x^q + 1)/x^(3/2) + (a*(2*q + 1)*x*x^q - (2*q + 1)*x)*polylog(3,
a*x^q)/x^(3/2) + 8*(2*q^4*x - (2*q^4 + q^3)*a*x*x^q)/x^(3/2))/(a*d^(3/2)*(
2*q + 1)*x^q - d^(3/2)*(2*q + 1))
```

3.94.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}}\, dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\, dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/(d*x)^(3/2), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}}\, dx = \int \frac{\text{polylog}(3, a\, x^q)}{(d\, x)^{3/2}}\, dx$$

input
```
int(polylog(3, a*x^q)/(d*x)^(3/2),x)
```

output
```
int(polylog(3, a*x^q)/(d*x)^(3/2), x)
```

3.95 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}}\,dx$

3.95.1 Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}}\,dx = -\frac{16aq^3x^{-1+q}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right),\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{27d^2(3-2q)\sqrt{dx}} + \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\,\text{PolyLog}(2,ax^q)}{9d(dx)^{3/2}} - \frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

output
```
8/27*q^2*ln(1-a*x^q)/d/(d*x)^(3/2)-4/9*q*polylog(2,a*x^q)/d/(d*x)^(3/2)-2/
3*polylog(3,a*x^q)/d/(d*x)^(3/2)-16/27*a*q^3*x^(-1+q)*hypergeom([1, 1-3/2/
q],[2-3/2/q],a*x^q)/d^2/(3-2*q)/(d*x)^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.39

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}}\,dx = -\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{3}{2q}\\1,0,0,0,\frac{3}{2q}\end{array}\right)}{q(dx)^{5/2}}$$

input
```
Integrate[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

output
```
-((x*MeijerG[{{1, 1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 0, 3/(2*q)}}, -
(a*x^q)])/(q*(d*x)^(5/2)))
```

3.95.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \ 7145$$

$$\frac{2}{3}q\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 7145$$

$$\frac{2}{3}q\left(\frac{2}{3}q\int -\frac{\log(1-ax^q)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2}{3}q\left(-\frac{2}{3}q\int \frac{\log(1-ax^q)}{(dx)^{5/2}}dx-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 2905$$

$$\frac{2}{3}q\left(-\frac{2}{3}q\left(-\frac{2aq\int \frac{x^{q-1}}{(dx)^{3/2}(1-ax^q)}dx}{3d}-\frac{2\log(1-ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 30$$

$$\frac{2}{3}q\left(-\frac{2}{3}q\left(-\frac{2aq\sqrt{x}\int \frac{x^{q-\frac{5}{2}}}{1-ax^q}dx}{3d^2\sqrt{dx}}-\frac{2\log(1-ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 888$$

$$\frac{2}{3}q\left(-\frac{2}{3}q\left(\frac{4aqx^{q-1}\,\text{Hypergeometric2F1}\left(1,-\frac{\frac{3}{2}-q}{q},\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{3d^2(3-2q)\sqrt{dx}}-\frac{2\log\left(1-ax^q\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(2,ax^q\right)}{3d(dx)^{3/2}}\right)-\frac{2\,\text{PolyLog}\left(3,ax^q\right)}{3d(dx)^{3/2}}$$

input
```
Int[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

output
```
(2*q*((-2*q*((4*a*q*x^(-1 + q)*Hypergeometric2F1[1, -((3/2 - q)/q), (4 - 3
/q)/2, a*x^q])/(3*d^2*(3 - 2*q)*Sqrt[d*x]) - (2*Log[1 - a*x^q])/(3*d*(d*x)
^(3/2))))/3 - (2*PolyLog[2, a*x^q])/(3*d*(d*x)^(3/2))))/3 - (2*PolyLog[3,
a*x^q])/(3*d*(d*x)^(3/2))
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

method	result
meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{8q^{3}(-a)^{-\frac{3}{2q}}\ln(1-a\,x^{q})}{27x^{\frac{3}{2}}}+\frac{4q^{2}(-a)^{-\frac{3}{2q}}\operatorname{polylog}(2,a\,x^{q})}{9x^{\frac{3}{2}}}-\frac{2q(-a)^{-\frac{3}{2q}}\left(1-\frac{2q}{3}\right)\operatorname{polylog}(3,a\,x^{q})}{(-3+2q)x^{\frac{3}{2}}}-\frac{8q^{3}x^{q-\frac{3}{2}}a(-a)^{-\frac{3}{2q}}\operatorname{LerchPh}}{27}\right.}{(dx)^{\frac{5}{2}}q}$

input
```
int(polylog(3,a*x^q)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-8/27*q^3/x^(3/2)*(-a)^(-3/2/q)*ln(
1-a*x^q)+4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*polylog(2,a*x^q)-2*q/(-3+2*q)/x^(3/
2)*(-a)^(-3/2/q)*(1-2/3*q)*polylog(3,a*x^q)-8/27*q^3*x^(q-3/2)*a*(-a)^(-3/
2/q)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

3.95.5 Fricas [F]

$$\int \frac{\operatorname{PolyLog}(3,ax^q)}{(dx)^{5/2}}\,dx=\int \frac{\operatorname{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*polylog(3, a*x^q)/(d^3*x^3), x)
```

3.95.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x**q)/(d*x)**(5/2),x)
```

output
```
Integral(polylog(3, a*x**q)/(d*x)**(5/2), x)
```

3.95.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
16*q^4*integrate(1/27/((a^2*d^(5/2)*(2*q + 3)*x^(2*q) - 2*a*d^(5/2)*(2*q +
 3)*x^q + d^(5/2)*(2*q + 3))*x^(5/2)), x) - 2/81*(18*((2*q^2 + 3*q)*a*x*x^
q - (2*q^2 + 3*q)*x)*dilog(a*x^q)/x^(5/2) - 12*((2*q^3 + 3*q^2)*a*x*x^q -
(2*q^3 + 3*q^2)*x)*log(-a*x^q + 1)/x^(5/2) + 27*(a*(2*q + 3)*x*x^q - (2*q
+ 3)*x)*polylog(3, a*x^q)/x^(5/2) + 8*(2*q^4*x - (2*q^4 + 3*q^3)*a*x*x^q)/
x^(5/2))/(a*d^(5/2)*(2*q + 3)*x^q - d^(5/2)*(2*q + 3))
```

3.95.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{5/2}}\,dx = \int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(polylog(3, a*x^q)/(d*x)^(5/2), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\,(3,ax^q)}{(dx)^{5/2}}\,dx = \int \frac{\text{polylog}(3,a\,x^q)}{(d\,x)^{5/2}}\,dx$$

input
```
int(polylog(3, a*x^q)/(d*x)^(5/2),x)
```

output
```
int(polylog(3, a*x^q)/(d*x)^(5/2), x)
```

3.96 $\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$

3.96.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = -x \text{PolyLog}\left(\frac{1}{2}, ax\right) + x \text{PolyLog}\left(\frac{3}{2}, ax\right) + \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

output
```
-x*polylog(1/2,a*x)+x*polylog(3/2,a*x)+Unintegrable(polylog(-1/2,a*x),x)
```

3.96.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$$

input
```
Integrate[PolyLog[3/2, a*x],x]
```

output
```
Integrate[PolyLog[3/2, a*x], x]
```

3.96.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7140, 7140, 7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(\frac{3}{2},ax\right)\,dx$$
$$\Big\downarrow \; 7140$$
$$x\,\text{PolyLog}\left(\frac{3}{2},ax\right)-\int \text{PolyLog}\left(\frac{1}{2},ax\right)dx$$
$$\Big\downarrow \; 7140$$
$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx-x\,\text{PolyLog}\left(\frac{1}{2},ax\right)+x\,\text{PolyLog}\left(\frac{3}{2},ax\right)$$
$$\Big\downarrow \; 7142$$
$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx-x\,\text{PolyLog}\left(\frac{1}{2},ax\right)+x\,\text{PolyLog}\left(\frac{3}{2},ax\right)$$

input
```
Int[PolyLog[3/2, a*x],x]
```

output
```
$Aborted
```

3.96.3.1 Defintions of rubi rules used

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.96.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \text{polylog}\left(\frac{3}{2}, ax\right) dx$$

input
```
int(polylog(3/2,a*x),x)
```

output
```
int(polylog(3/2,a*x),x)
```

3.96.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{Li}_{\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(3/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(3/2, a*x), x)
```

3.96.6 Sympy [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{Li}_{\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(3/2,a*x),x)
```

output
```
Integral(polylog(3/2, a*x), x)
```

3.96.7 Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{Li}_{\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(3/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(3/2, a*x), x)
```

3.96.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{Li}_{\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(3/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(3/2, a*x), x)
```

3.96.9 Mupad [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx = \int \text{polylog}\left(\frac{3}{2}, a\, x\right) dx$$

input
```
int(polylog(3/2, a*x),x)
```

output
```
int(polylog(3/2, a*x), x)
```

3.97 $\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$

3.97.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx = x\,\text{PolyLog}\left(\frac{1}{2}, ax\right) - \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

output
```
x*polylog(1/2,a*x)-Unintegrable(polylog(-1/2,a*x),x)
```

3.97.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx = \int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$$

input
```
Integrate[PolyLog[1/2, a*x],x]
```

output
```
Integrate[PolyLog[1/2, a*x], x]
```

3.97.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7140, 7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(\frac{1}{2},ax\right)dx$$

$$\Big\downarrow \text{7140}$$

$$x\,\text{PolyLog}\left(\frac{1}{2},ax\right)-\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$

$$\Big\downarrow \text{7142}$$

$$x\,\text{PolyLog}\left(\frac{1}{2},ax\right)-\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$

input
```
Int[PolyLog[1/2, a*x],x]
```

output
```
$Aborted
```

3.97.3.1 Defintions of rubi rules used

rule 7140
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Simp[p*q   Int[PolyLog[n - 1, a*(b*x^p)^q], x], x]
/; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.97.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \mathrm{polylog}\left(\frac{1}{2}, ax\right) dx$$

input
```
int(polylog(1/2,a*x),x)
```

output
```
int(polylog(1/2,a*x),x)
```

3.97.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \mathrm{PolyLog}\left(\frac{1}{2}, ax\right) dx = \int \mathrm{Li}_{\frac{1}{2}}(ax)\, dx$$

input
```
integrate(polylog(1/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(1/2, a*x), x)
```

3.97.6 Sympy [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \mathrm{PolyLog}\left(\frac{1}{2}, ax\right) dx = \int \mathrm{Li}_{\frac{1}{2}}(ax)\, dx$$

input
```
integrate(polylog(1/2,a*x),x)
```

output
```
Integral(polylog(1/2, a*x), x)
```

3.97.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) \, dx = \int \text{Li}_{\frac{1}{2}}(ax) \, dx$$

input
```
integrate(polylog(1/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(1/2, a*x), x)
```

3.97.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) \, dx = \int \text{Li}_{\frac{1}{2}}(ax) \, dx$$

input
```
integrate(polylog(1/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(1/2, a*x), x)
```

3.97.9 Mupad [N/A]

Not integrable

Time = 5.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) \, dx = \int \text{polylog}\left(\frac{1}{2}, a\,x\right) \, dx$$

input
```
int(polylog(1/2, a*x),x)
```

output
```
int(polylog(1/2, a*x), x)
```

3.98 $\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx$

3.98.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\text{Int}\left(\text{PolyLog}\left(-\frac{1}{2},ax\right),x\right)$$

output
```
Unintegrable(polylog(-1/2,a*x),x)
```

3.98.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx$$

input
```
Integrate[PolyLog[-1/2, a*x],x]
```

output
```
Integrate[PolyLog[-1/2, a*x], x]
```

3.98.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$
$$\big\downarrow \text{ 7142}$$
$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$

input
```
Int[PolyLog[-1/2, a*x],x]
```

output
```
$Aborted
```

3.98.3.1 Defintions of rubi rules used

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.98.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \text{polylog}\left(-\frac{1}{2},ax\right)dx$$

input
```
int(polylog(-1/2,a*x),x)
```

output
```
int(polylog(-1/2,a*x),x)
```

3.98.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)\,dx$$

input
```
integrate(polylog(-1/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(-1/2, a*x), x)
```

3.98.6 Sympy [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{Li}_{-\frac{1}{2}}\left(ax\right)\,dx$$

input
```
integrate(polylog(-1/2,a*x),x)
```

output
```
Integral(polylog(-1/2, a*x), x)
```

3.98.7 Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)\,dx$$

input
```
integrate(polylog(-1/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(-1/2, a*x), x)
```

3.98.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)\,dx$$

input
```
integrate(polylog(-1/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(-1/2, a*x), x)
```

3.98.9 Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)\,dx=\int \text{polylog}\left(-\frac{1}{2},a\,x\right)\,dx$$

input
```
int(polylog(-1/2, a*x),x)
```

output
```
int(polylog(-1/2, a*x), x)
```

3.99 $\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$

3.99.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx = x\,\text{PolyLog}\left(-\frac{1}{2}, ax\right) - \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

output
```
x*polylog(-1/2,a*x)-Unintegrable(polylog(-1/2,a*x),x)
```

3.99.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$$

input
```
Integrate[PolyLog[-3/2, a*x],x]
```

output
```
Integrate[PolyLog[-3/2, a*x], x]
```

3.99.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7141, 7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(-\frac{3}{2},ax\right)\,dx$$

$$\Big\downarrow \text{7141}$$

$$x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)-\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$

$$\Big\downarrow \text{7142}$$

$$x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)-\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx$$

input
```
Int[PolyLog[-3/2, a*x],x]
```

output
```
$Aborted
```

3.99.3.1 Defintions of rubi rules used

rule 7141
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*(PolyL
og[n + 1, a*(b*x^p)^q]/(p*q)), x] - Simp[1/(p*q)   Int[PolyLog[n + 1, a*(b*
x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && LtQ[n, -1]
```

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.99.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \operatorname{polylog}\left(-\frac{3}{2}, ax\right) dx$$

input
```
int(polylog(-3/2,a*x),x)
```

output
```
int(polylog(-3/2,a*x),x)
```

3.99.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \operatorname{Li}_{-\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(-3/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(-3/2, a*x), x)
```

3.99.6 Sympy [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \operatorname{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \operatorname{Li}_{-\frac{3}{2}}(ax)\, dx$$

input
```
integrate(polylog(-3/2,a*x),x)
```

output
```
Integral(polylog(-3/2, a*x), x)
```

3.99.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \text{Li}_{-\frac{3}{2}}(ax) \, dx$$

input
```
integrate(polylog(-3/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(-3/2, a*x), x)
```

3.99.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \text{Li}_{-\frac{3}{2}}(ax) \, dx$$

input
```
integrate(polylog(-3/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(-3/2, a*x), x)
```

3.99.9 Mupad [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx = \int \text{polylog}\left(-\frac{3}{2}, a\,x\right) dx$$

input
```
int(polylog(-3/2, a*x),x)
```

output
```
int(polylog(-3/2, a*x), x)
```

3.100 $\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$

3.100.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx = x \text{PolyLog}\left(-\frac{3}{2}, ax\right) - x \text{PolyLog}\left(-\frac{1}{2}, ax\right) + \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

output
```
x*polylog(-3/2,a*x)-x*polylog(-1/2,a*x)+Unintegrable(polylog(-1/2,a*x),x)
```

3.100.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

input
```
Integrate[PolyLog[-5/2, a*x],x]
```

output
```
Integrate[PolyLog[-5/2, a*x], x]
```

3.100.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7141, 7141, 7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(-\frac{5}{2},ax\right)dx$$

$$\Big\downarrow \text{7141}$$

$$x\,\text{PolyLog}\left(-\frac{3}{2},ax\right)-\int \text{PolyLog}\left(-\frac{3}{2},ax\right)dx$$

$$\Big\downarrow \text{7141}$$

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx+x\,\text{PolyLog}\left(-\frac{3}{2},ax\right)-x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)$$

$$\Big\downarrow \text{7142}$$

$$\int \text{PolyLog}\left(-\frac{1}{2},ax\right)dx+x\,\text{PolyLog}\left(-\frac{3}{2},ax\right)-x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)$$

input
```
Int[PolyLog[-5/2, a*x],x]
```

output
```
$Aborted
```

3.100.3.1 Defintions of rubi rules used

rule 7141
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*(PolyL
og[n + 1, a*(b*x^p)^q]/(p*q)), x] - Simp[1/(p*q)   Int[PolyLog[n + 1, a*(b*
x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && LtQ[n, -1]
```

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.100.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \operatorname{polylog}\left(-\frac{5}{2}, ax\right) dx$$

input
```
int(polylog(-5/2,a*x),x)
```

output
```
int(polylog(-5/2,a*x),x)
```

3.100.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \operatorname{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \operatorname{Li}_{-\frac{5}{2}}(ax) \, dx$$

input
```
integrate(polylog(-5/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(-5/2, a*x), x)
```

3.100.6 Sympy [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \operatorname{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \operatorname{Li}_{-\frac{5}{2}}(ax) \, dx$$

input
```
integrate(polylog(-5/2,a*x),x)
```

output
```
Integral(polylog(-5/2, a*x), x)
```

3.100.7 Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \text{Li}_{-\frac{5}{2}}(ax)\, dx$$

input
```
integrate(polylog(-5/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(-5/2, a*x), x)
```

3.100.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \text{Li}_{-\frac{5}{2}}(ax)\, dx$$

input
```
integrate(polylog(-5/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(-5/2, a*x), x)
```

3.100.9 Mupad [N/A]

Not integrable

Time = 5.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx = \int \text{polylog}\left(-\frac{5}{2}, a\,x\right) dx$$

input
```
int(polylog(-5/2, a*x),x)
```

output
```
int(polylog(-5/2, a*x), x)
```

3.101 $\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right) \, dx$

3.101.1 Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right) \, dx = x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)$$

output
```
x*polylog(-1/2,a*x)
```

3.101.2 Mathematica [F]

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right) \, dx = \int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right) \, dx$$

input
```
Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]
```

output
```
Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x], x]
```

3.101.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\,\text{PolyLog}\left(-\frac{1}{2},ax\right)$$

input
```
Int[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]
```

output
```
x*PolyLog[-1/2, a*x]
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.101.4 Maple [F]

$$\int \left(\text{polylog}\left(-\frac{3}{2},ax\right)+\text{polylog}\left(-\frac{1}{2},ax\right)\right)dx$$

input
```
int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

output
```
int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

3.101.5 Fricas [F]

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)+\text{Li}_{-\frac{3}{2}}(ax)\,dx$$

input
```
integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

3.101.6 Sympy [F]

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx=\int \left(\text{Li}_{-\frac{3}{2}}\left(ax\right)+\text{Li}_{-\frac{1}{2}}\left(ax\right)\right)\,dx$$

input
```
integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

output
```
Integral(polylog(-3/2, a*x) + polylog(-1/2, a*x), x)
```

3.101.7 Maxima [F]

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)+\text{Li}_{-\frac{3}{2}}(ax)\,dx$$

input
```
integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

3.101.8 Giac [F]

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx=\int \text{Li}_{-\frac{1}{2}}(ax)+\text{Li}_{-\frac{3}{2}}(ax)\,dx$$

input
```
integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx=x\,\text{polylog}\left(-\frac{1}{2},a\,x\right)$$

input
```
int(polylog(-1/2, a*x) + polylog(-3/2, a*x),x)
```

output
```
x*polylog(-1/2, a*x)
```

3.102 $\int (dx)^m \operatorname{PolyLog}(2,ax)\,dx$

3.102.1 Optimal result

Integrand size = 11, antiderivative size = 78

$$\int (dx)^m \operatorname{PolyLog}(2,ax)\,dx = \frac{a(dx)^{2+m}\operatorname{Hypergeometric2F1}(1,2+m,3+m,ax)}{d^2(1+m)^2(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\operatorname{PolyLog}(2,ax)}{d(1+m)}$$

output
```
a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^2/(2+m)+(d*x)^(1+m)*
ln(-a*x+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)
```

3.102.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int (dx)^m \operatorname{PolyLog}(2,ax)\,dx = \frac{x(dx)^m(ax\operatorname{Hypergeometric2F1}(1,2+m,3+m,ax)+(2+m)(\log(1-ax)+(1+m)\operatorname{PolyLog}(2,ax)))}{(1+m)^2(2+m)}$$

input
```
Integrate[(d*x)^m*PolyLog[2, a*x],x]
```

output
```
(x*(d*x)^m*(a*x*Hypergeometric2F1[1, 2 + m, 3 + m, a*x] + (2 + m)*(Log[1 -
 a*x] + (1 + m)*PolyLog[2, a*x])))/((1 + m)^2*(2 + m))
```

3.102.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {7145, 25, 2842, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,ax)(dx)^m\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int -(dx)^m\log(1-ax)dx}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (dx)^m\log(1-ax)dx}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{2842}$$

$$\frac{\frac{a\int \frac{(dx)^{m+1}}{1-ax}dx}{d(m+1)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{a(dx)^{m+2}\text{Hypergeometric2F1}(1,m+2,m+3,ax)}{d^2(m+1)(m+2)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}$$

input
```
Int[(d*x)^m*PolyLog[2, a*x],x]
```

output
```
((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)))/(1 + m) + ((d*x)^(1 + m
)*PolyLog[2, a*x])/(d*(1 + m))
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.85

method	result
meijerg	$\frac{(dx)^{m} x^{-m} (-a)^{-m} \left(\frac{x^{m} (-a)^{m} \left(-a\, m^{2} x-2 a m x-m^{2}-3 m-2\right)}{(2+m)(1+m)^{3} m}-\frac{x^{1+m} a (-a)^{m} (-m-2) \ln(-a x+1)}{(2+m)(1+m)^{2}}+\frac{x^{1+m} a (-a)^{m} \operatorname{polylog}(2,a x)}{1+m}+\frac{x^{m} (-a)^{m}}{}\right.}{a}$

input
```
int((d*x)^m*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2
)/(1+m)^3/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*ln(-a*x+1)+x^(1+m)*a*(
-a)^m/(1+m)*polylog(2,a*x)+x^m*(-a)^m/(1+m)^2*LerchPhi(a*x,1,m))
```

3.102.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax) \, dx = \int (dx)^m \operatorname{Li}_2(ax) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x),x, algorithm="fricas")
```

output
```
integral((d*x)^m*dilog(a*x), x)
```

3.102.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax) \, dx = \int (dx)^m \operatorname{Li}_2(ax) \, dx$$

input
```
integrate((d*x)**m*polylog(2,a*x),x)
```

output
```
Integral((d*x)**m*polylog(2, a*x), x)
```

3.102.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax) \, dx = \int (dx)^m \operatorname{Li}_2(ax) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x),x, algorithm="maxima")
```

output
```
-a*d^m*integrate(-x*x^m/(m^2 - (a*m^2 + 2*a*m + a)*x + 2*m + 1), x) + ((d^
m*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*log(-a*x + 1))/(m^2 + 2*m + 1)
```

3.102.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax)\, dx = \int (dx)^m \operatorname{Li}_2(ax)\, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*dilog(a*x), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(2, ax)\, dx = \int (d\,x)^m \operatorname{polylog}(2, a\,x)\, dx$$

input
```
int((d*x)^m*polylog(2, a*x),x)
```

output
```
int((d*x)^m*polylog(2, a*x), x)
```

3.103 $\int (dx)^m \operatorname{PolyLog}(3, ax)\, dx$

3.103.1 Optimal result

Integrand size = 11, antiderivative size = 102

$$\int (dx)^m \operatorname{PolyLog}(3, ax)\, dx = -\frac{a(dx)^{2+m} \operatorname{Hypergeometric2F1}(1, 2+m, 3+m, ax)}{d^2(1+m)^3(2+m)} - \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\operatorname{PolyLog}(2, ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\operatorname{PolyLog}(3, ax)}{d(1+m)}$$

output
```
-a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^3/(2+m)-(d*x)^(1+m)
*ln(-a*x+1)/d/(1+m)^3-(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)^2+(d*x)^(1+m)*pol
ylog(3,a*x)/d/(1+m)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int (dx)^m \operatorname{PolyLog}(3, ax)\, dx = -\frac{x(dx)^m \operatorname{Gamma}(2+m) \left(a(1+m)x \operatorname{Gamma}(1+m)\, {}_2\tilde{F}_1(1, 2+m; 3+m; ax) + \log(1-ax) + (1+m)\right.}{(1+m)^4 \operatorname{Gamma}(1+m}$$

input
```
Integrate[(d*x)^m*PolyLog[3, a*x],x]
```

output
```
-((x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegul
arized[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x]
- PolyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x]))/((1 + m)^4
*Gamma[1 + m]))
```

3.103.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {7145, 7145, 25, 2842, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(3,ax)(dx)^m\,dx$$

$$\Big\downarrow \ 7145$$

$$\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int (dx)^m\,\text{PolyLog}(2,ax)dx}{m+1}$$

$$\Big\downarrow \ 7145$$

$$\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int -(dx)^m\log(1-ax)dx}{m+1}}{m+1}$$

$$\Big\downarrow \ 25$$

$$\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\int (dx)^m\log(1-ax)dx}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}$$

$$\Big\downarrow \ 2842$$

$$\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\frac{a\int \frac{(dx)^{m+1}}{1-ax}dx}{d(m+1)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}$$

$$\Big\downarrow \ 74$$

$$\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\frac{a(dx)^{m+2}\,\text{Hypergeometric2F1}(1,m+2,m+3,ax)}{d^2(m+1)(m+2)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[3, a*x],x]
```

output
```
-((((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)
*(2 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)))/(1 + m) + ((d*x)^(1
+ m)*PolyLog[2, a*x])/(d*(1 + m)))/(1 + m)) + ((d*x)^(1 + m)*PolyLog[3, a*
x])/(d*(1 + m))
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

method	result
meijerg	$\frac{(dx)^m x^{-m} (-a)^{-m} \left(\frac{x^m (-a)^m \left(a\, m^2 x+2amx+m^2+3m+2\right)}{(2+m)(1+m)^4 m}-\frac{x^{1+m} a(-a)^m \ln(-ax+1)}{(1+m)^3}+\frac{x^{1+m} a(-a)^m (-m-2)\operatorname{polylog}(2,ax)}{(2+m)(1+m)^2}+\frac{x^{1+m} a(-a)}{}\right.}{a}$

input
```
int((d*x)^m*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(a*m^2*x+2*a*m*x+m^2+3*m+2)
/(1+m)^4/m-x^(1+m)*a*(-a)^m/(1+m)^3*ln(-a*x+1)+1/(2+m)*x^(1+m)*a*(-a)^m*(-
m-2)/(1+m)^2*polylog(2,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(3,a*x)+1/(2+m)*
x^m*(-a)^m*(-m-2)/(1+m)^3*LerchPhi(a*x,1,m))
```

3.103.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax) \, dx = \int (dx)^m \operatorname{Li}_3(ax) \, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(3, a*x), x)
```

3.103.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax) \, dx = \int (dx)^m \operatorname{Li}_3(ax) \, dx$$

input
```
integrate((d*x)**m*polylog(3,a*x),x)
```

output
```
Integral((d*x)**m*polylog(3, a*x), x)
```

3.103.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax) \, dx = \int (dx)^m \operatorname{Li}_3(ax) \, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x),x, algorithm="maxima")
```

output
```
a*d^m*integrate(-x*x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x + 3*m^2 + 3*m +
1), x) - (d^m*(m + 1)*x*x^m*dilog(a*x) - (m^2 + 2*m + 1)*d^m*x*x^m*polylog
(3, a*x) + d^m*x*x^m*log(-a*x + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

3.103.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax)\, dx = \int (dx)^m \operatorname{Li}_3(ax)\, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(3, a*x), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(3, ax)\, dx = \int (d\,x)^m \operatorname{polylog}(3, a\,x)\, dx$$

input
```
int((d*x)^m*polylog(3, a*x),x)
```

output
```
int((d*x)^m*polylog(3, a*x), x)
```

3.104 $\int (dx)^m \text{PolyLog}(4, ax)\, dx$

3.104.1 Optimal result

Integrand size = 11, antiderivative size = 121

$$\int (dx)^m \text{PolyLog}(4, ax)\, dx = \frac{a(dx)^{2+m}\,\text{Hypergeometric2F1}(1, 2+m, 3+m, ax)}{d^2(1+m)^4(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m}\,\text{PolyLog}(2, ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\,\text{PolyLog}(3, ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\,\text{PolyLog}(4, ax)}{d(1+m)}$$

output
```
a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^4/(2+m)+(d*x)^(1+m)*
ln(-a*x+1)/d/(1+m)^4+(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)^3-(d*x)^(1+m)*poly
log(3,a*x)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a*x)/d/(1+m)
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int (dx)^m \text{PolyLog}(4, ax)\, dx$$

$$= \frac{x(dx)^m\,\text{Gamma}(2+m)\,\left(a(1+m)x\,\text{Gamma}(1+m)\,{}_2\tilde{F}_1(1, 2+m; 3+m; ax) + \log(1-ax) + (1+m)\,\text{P}\right.}{}$$

input
```
Integrate[(d*x)^m*PolyLog[4, a*x],x]
```

output
```
(x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegular
ized[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x] -
PolyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x] + PolyLog[4, a
*x] + 3*m*PolyLog[4, a*x] + 3*m^2*PolyLog[4, a*x] + m^3*PolyLog[4, a*x]))/
((1 + m)^5*Gamma[1 + m])
```

3.104.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {7145, 7145, 7145, 25, 2842, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(4,ax)(dx)^m\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int (dx)^m\,\text{PolyLog}(3,ax)dx}{m+1}$$

$$\Big\downarrow \text{7145}$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int (dx)^m\,\text{PolyLog}(2,ax)dx}{m+1}}{m+1}$$

$$\Big\downarrow \text{7145}$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}-\frac{\int -(dx)^m\log(1-ax)dx}{m+1}}{m+1}}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\int (dx)^m\log(1-ax)dx}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}}{m+1}$$

$$\Big\downarrow \text{2842}$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\frac{a\int\frac{(dx)^{m+1}}{1-ax}dx}{d(m+1)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}}{m+1}$$

$$\Big\downarrow \; 74$$

$$\frac{\text{PolyLog}(4,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\text{PolyLog}(3,ax)(dx)^{m+1}}{d(m+1)}-\frac{\frac{\frac{a(dx)^{m+2}\,\text{Hypergeometric2F1}(1,m+2,m+3,ax)}{d^2(m+1)(m+2)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)}}{m+1}+\frac{\text{PolyLog}(2,ax)(dx)^{m+1}}{d(m+1)}}{m+1}}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[4, a*x],x]
```

output
```
-((-((((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 +
 m)*(2 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)))/(1 + m) + ((d*x)^
(1 + m)*PolyLog[2, a*x])/(d*(1 + m)))/(1 + m)) + ((d*x)^(1 + m)*PolyLog[3,
 a*x])/(d*(1 + m)))/(1 + m)) + ((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.64

method	result
meijerg	$\frac{(dx)^m x^{-m}(-a)^{-m}\left(\frac{x^m(-a)^m\left(-a\,m^2x-2amx-m^2-3m-2\right)}{(2+m)(1+m)^5m}-\frac{x^{1+m}a(-a)^m(-m-2)\ln(-ax+1)}{(2+m)(1+m)^4}+\frac{x^{1+m}a(-a)^m\,\mathrm{polylog}(2,ax)}{(1+m)^3}+\frac{x^{1+m}a(-}{}\right.}{a}$

input
```
int((d*x)^m*polylog(4,a*x),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2
)/(1+m)^5/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^4*ln(-a*x+1)+x^(1+m)*a*(
-a)^m/(1+m)^3*polylog(2,a*x)+1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*polyl
og(3,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(4,a*x)+x^m*(-a)^m/(1+m)^4*LerchPh
i(a*x,1,m))
```

3.104.5 Fricas [F]

$$\int (dx)^m \,\mathrm{PolyLog}(4,ax)\,dx = \int (dx)^m \,\mathrm{Li}_4(ax)\,dx$$

input
```
integrate((d*x)^m*polylog(4,a*x),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(4, a*x), x)
```

3.104.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(4,ax)\,dx = \int (dx)^m \operatorname{Li}_4(ax)\,dx$$

input
```
integrate((d*x)**m*polylog(4,a*x),x)
```

output
```
Integral((d*x)**m*polylog(4, a*x), x)
```

3.104.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(4,ax)\,dx = \int (dx)^m \operatorname{Li}_4(ax)\,dx$$

input
```
integrate((d*x)^m*polylog(4,a*x),x, algorithm="maxima")
```

output
```
-a*d^m*integrate(-x*x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2
+ 4*a*m + a)*x + 4*m + 1), x) + ((d^m*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^
m*log(-a*x + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*polylog(4, a
*x) - (d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x))/(m^4 + 4*m^3 + 6*m^
2 + 4*m + 1)
```

3.104.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(4,ax)\,dx = \int (dx)^m \operatorname{Li}_4(ax)\,dx$$

input
```
integrate((d*x)^m*polylog(4,a*x),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(4, a*x), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(4, ax)\, dx = \int (d\,x)^m \operatorname{polylog}(4, a\,x)\, dx$$

input
```
int((d*x)^m*polylog(4, a*x),x)
```

output
```
int((d*x)^m*polylog(4, a*x), x)
```

3.105 $\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx$

3.105.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx = \frac{4a(dx)^{3+m} \operatorname{Hypergeometric2F1}\left(1, \frac{3+m}{2}, \frac{5+m}{2}, ax^2\right)}{d^3(1+m)^2(3+m)} + \frac{2(dx)^{1+m} \log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{PolyLog}(2, ax^2)}{d(1+m)}$$

output
```
4*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^2/(3
+m)+2*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^2)/d/(1
+m)
```

3.105.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx = \frac{x(dx)^m \left(4ax^2 \operatorname{Hypergeometric2F1}\left(1, \frac{3+m}{2}, \frac{5+m}{2}, ax^2\right) + (3+m)\left(2\log(1-ax^2) + (1+m)\operatorname{PolyLog}(2, a\right.\right.}{(1+m)^2(3+m)}$$

input
```
Integrate[(d*x)^m*PolyLog[2, a*x^2],x]
```

output
```
(x*(d*x)^m*(4*a*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2] + (3
 + m)*(2*Log[1 - a*x^2] + (1 + m)*PolyLog[2, a*x^2])))/((1 + m)^2*(3 + m))
```

3.105.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {7145, 25, 2905, 8, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(2,ax^2\right)(dx)^m\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\int -(dx)^m\log\left(1-ax^2\right)dx}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int (dx)^m\log\left(1-ax^2\right)dx}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{2905}$$

$$\frac{2\left(\frac{2a\int \frac{x(dx)^{m+1}}{1-ax^2}dx}{d(m+1)}+\frac{\log(1-ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{8}$$

$$\frac{2\left(\frac{2a\int \frac{(dx)^{m+2}}{1-ax^2}dx}{d^2(m+1)}+\frac{\log(1-ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\left(\frac{2a(dx)^{m+3}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)(m+3)}+\frac{\log(1-ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}$$

input
```
Int[(d*x)^m*PolyLog[2, a*x^2],x]
```

output
```
(2*((2*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/
(d^3*(1 + m)*(3 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m))))/(1 +
m) + ((d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))
```

3.105.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.105.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left(\frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} (-12-4m)}{(3+m)(1+m)^3 a}-\frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} (-6-2m) \ln\left(-a\,x^2+1\right)}{(3+m)(1+m)^2 a}+\frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} \operatorname{polylog}\left(2,a\,x^2\right)}{(1+m)a}+\right.}{2}$

input
```
int((d*x)^m*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-
12-4*m)/(1+m)^3/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2*ln(-a*
x^2+1)/a+2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(2,a*x^2)+2/(3+m)*x^(1+
m)*(-a)^(3/2+1/2*m)*(6+2*m)/(1+m)^2/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

3.105.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx = \int (dx)^m \operatorname{Li}_2(ax^2)\, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="fricas")
```

output
```
integral((d*x)^m*dilog(a*x^2), x)
```

3.105.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx = \int (dx)^m \operatorname{Li}_2(ax^2)\, dx$$

input
```
integrate((d*x)**m*polylog(2,a*x**2),x)
```

output
```
Integral((d*x)**m*polylog(2, a*x**2), x)
```

3.105.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^2)\, dx = \int (dx)^m \operatorname{Li}_2(ax^2)\, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="maxima")
```

output
```
-4*a*d^m*integrate(x^2*x^m/((a*m^2 + 2*a*m + a)*x^2 - m^2 - 2*m - 1), x) +
 ((d^m*m + d^m)*x*x^m*dilog(a*x^2) + 2*d^m*x*x^m*log(-a*x^2 + 1))/(m^2 + 2
*m + 1)
```

3.105.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}\left(2, ax^2\right) \, dx = \int (dx)^m \operatorname{Li}_2\left(ax^2\right) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*dilog(a*x^2), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}\left(2, ax^2\right) \, dx = \int \operatorname{polylog}\left(2, a\,x^2\right) \, (d\,x)^m \, dx$$

input
```
int(polylog(2, a*x^2)*(d*x)^m,x)
```

output
```
int(polylog(2, a*x^2)*(d*x)^m, x)
```

3.106 $\int (dx)^m \text{PolyLog}(3, ax^2) \, dx$

3.106.1 Optimal result

Integrand size = 13, antiderivative size = 118

$$\int (dx)^m \text{PolyLog}(3, ax^2) \, dx = -\frac{8a(dx)^{3+m} \text{Hypergeometric2F1}\left(1, \frac{3+m}{2}, \frac{5+m}{2}, ax^2\right)}{d^3(1+m)^3(3+m)} - \frac{4(dx)^{1+m} \log(1-ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{PolyLog}(2, ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}(3, ax^2)}{d(1+m)}$$

output
```
-8*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^3/(
3+m)-4*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^3-2*(d*x)^(1+m)*polylog(2,a*x^2)/d
/(1+m)^2+(d*x)^(1+m)*polylog(3,a*x^2)/d/(1+m)
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int (dx)^m \text{PolyLog}(3, ax^2) \, dx = -\frac{2x(dx)^m \text{Gamma}\left(\frac{3+m}{2}\right) \left(2a(1+m)x^2 \text{Gamma}\left(\frac{1+m}{2}\right) {}_2\tilde{F}_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; ax^2\right) + 4\log(1-ax^2) + 2(1+\right.}{(1+m)^4 \text{Gamma}\left(\frac{1}{\right.}}$$

input
```
Integrate[(d*x)^m*PolyLog[3, a*x^2],x]
```

output
```
(-2*x*(d*x)^m*Gamma[(3 + m)/2]*(2*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeom
etricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 4*Log[1 - a*x^2]
 + 2*(1 + m)*PolyLog[2, a*x^2] - PolyLog[3, a*x^2] - 2*m*PolyLog[3, a*x^2]
 - m^2*PolyLog[3, a*x^2]))/((1 + m)^4*Gamma[(1 + m)/2])
```

3.106.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {7145, 7145, 25, 2905, 8, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(3,ax^2\right)(dx)^m\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\int (dx)^m\text{PolyLog}\left(2,ax^2\right)dx}{m+1}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}(2,ax^2)(dx)^{m+1}}{d(m+1)}-\frac{2\int -(dx)^m\log(1-ax^2)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\int (dx)^m\log(1-ax^2)dx}{m+1}+\frac{\text{PolyLog}(2,ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a\int\frac{x(dx)^{m+1}}{1-ax^2}dx}{d(m+1)}+\frac{\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{\text{PolyLog}(3,ax^2)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a\int\frac{(dx)^{m+2}}{1-ax^2}dx}{d^2(m+1)}+\frac{\log(1-ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$\downarrow$ 278

$$\frac{\text{PolyLog}(3,ax^2)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)(m+3)}+\frac{\log(1-ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^2)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[3, a*x^2],x]
```

output
```
(-2*((2*((2*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x
^2])/(d^3*(1 + m)*(3 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m))))/
(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))))/(1 + m) + ((d*x)
^(1 + m)*PolyLog[3, a*x^2])/(d*(1 + m))
```

3.106.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.106.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.85

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(24+8m)}{(3+m)(1+m)^4 a}-\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(12+4m)\ln\left(-a\,x^2+1\right)}{(3+m)(1+m)^3 a}+\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-6-2m)\,\mathrm{polylog}\left(2,a\right.}{(3+m)(1+m)^2 a}\right.}{2}$

input
```
int((d*x)^m*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(2
4+8*m)/(1+m)^4/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3*ln(-a*x
^2+1)/a+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*polylog(2,a*x^
2)+2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(3,a*x^2)+2/(3+m)*x^(1+m)*(-a
)^(3/2+1/2*m)*(-12-4*m)/(1+m)^3/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

3.106.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}\left(3,ax^2\right)\,dx=\int (dx)^m \operatorname{Li}_3(ax^2)\,dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(3, a*x^2), x)
```

3.106.6 Sympy [F]

$$\int (dx)^m \text{PolyLog}\left(3, ax^2\right)\,dx = \int (dx)^m \text{Li}_3\left(ax^2\right)\,dx$$

input
```
integrate((d*x)**m*polylog(3,a*x**2),x)
```

output
```
Integral((d*x)**m*polylog(3, a*x**2), x)
```

3.106.7 Maxima [F]

$$\int (dx)^m \text{PolyLog}\left(3, ax^2\right)\,dx = \int (dx)^m \text{Li}_3(ax^2)\,dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="maxima")
```

output
```
8*a*d^m*integrate(x^2*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^2 - m^3 - 3*m^2 - 3
*m - 1), x) - (2*d^m*(m + 1)*x*x^m*dilog(a*x^2) - (m^2 + 2*m + 1)*d^m*x*x^
m*polylog(3, a*x^2) + 4*d^m*x*x^m*log(-a*x^2 + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

3.106.8 Giac [F]

$$\int (dx)^m \text{PolyLog}\left(3, ax^2\right)\,dx = \int (dx)^m \text{Li}_3(ax^2)\,dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(3, a*x^2), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^2\right)\,dx = \int \operatorname{polylog}\left(3, a\,x^2\right)\,(d\,x)^m\,dx$$

input
```
int(polylog(3, a*x^2)*(d*x)^m,x)
```

output
```
int(polylog(3, a*x^2)*(d*x)^m, x)
```

3.107 $\int (dx)^m \operatorname{PolyLog}(4, ax^2) \, dx$

3.107.1 Optimal result

Integrand size = 13, antiderivative size = 142

$$\int (dx)^m \operatorname{PolyLog}(4, ax^2) \, dx = \frac{16a(dx)^{3+m} \operatorname{Hypergeometric2F1}\left(1, \frac{3+m}{2}, \frac{5+m}{2}, ax^2\right)}{d^3(1+m)^4(3+m)} + \frac{8(dx)^{1+m} \log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m} \operatorname{PolyLog}(2, ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \operatorname{PolyLog}(3, ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{PolyLog}(4, ax^2)}{d(1+m)}$$

output
```
16*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^4/(
3+m)+8*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^4+4*(d*x)^(1+m)*polylog(2,a*x^2)/d
/(1+m)^3-2*(d*x)^(1+m)*polylog(3,a*x^2)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a*
x^2)/d/(1+m)
```

3.107.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.17

$$\int (dx)^m \operatorname{PolyLog}(4, ax^2) \, dx$$
$$= \frac{2x(dx)^m \operatorname{Gamma}\left(\frac{3+m}{2}\right)\left(4a(1+m)x^2 \operatorname{Gamma}\left(\frac{1+m}{2}\right) {}_2\tilde{F}_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; ax^2\right) + 8\log(1-ax^2) + 4(1+m\right.}{}$$

input
```
Integrate[(d*x)^m*PolyLog[4, a*x^2],x]
```

output
```
(2*x*(d*x)^m*Gamma[(3 + m)/2]*(4*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeome
tricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 8*Log[1 - a*x^2]
+ 4*(1 + m)*PolyLog[2, a*x^2] - 2*PolyLog[3, a*x^2] - 4*m*PolyLog[3, a*x^2
] - 2*m^2*PolyLog[3, a*x^2] + PolyLog[4, a*x^2] + 3*m*PolyLog[4, a*x^2] +
3*m^2*PolyLog[4, a*x^2] + m^3*PolyLog[4, a*x^2]))/((1 + m)^5*Gamma[(1 + m)
/2])
```

3.107.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {7145, 7145, 7145, 25, 2905, 8, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(4,ax^2\right)(dx)^m\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\int (dx)^m\text{PolyLog}\left(3,ax^2\right)dx}{m+1}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}(3,ax^2)(dx)^{m+1}}{d(m+1)}-\frac{2\int (dx)^m\text{PolyLog}(2,ax^2)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}(3,ax^2)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\int -(dx)^m\log\left(1-ax^2\right)dx}{m+1}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\int(dx)^m\log\left(1-ax^2\right)dx}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 2905

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a\int\frac{x(dx)^{m+1}}{1-ax^2}dx}{d(m+1)}+\frac{\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 8

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a\int\frac{(dx)^{m+2}}{1-ax^2}dx}{d^2(m+1)}+\frac{\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 278

$$\frac{\text{PolyLog}\left(4,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{\text{PolyLog}\left(3,ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{2\left(\frac{2\left(\frac{2a(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)(m+3)}+\frac{\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^2\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

3.107. $\int(dx)^m\,\text{PolyLog}\left(4,ax^2\right)\,dx$

input
```
Int[(d*x)^m*PolyLog[4, a*x^2],x]
```

output
```
(-2*((-2*((2*((2*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2
, a*x^2])/(d^3*(1 + m)*(3 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m
))))/(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))))/(1 + m) + (
(d*x)^(1 + m)*PolyLog[3, a*x^2])/(d*(1 + m))))/(1 + m) + ((d*x)^(1 + m)*Po
lyLog[4, a*x^2])/(d*(1 + m))
```

3.107.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.68 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.82

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left(\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-48-16m)}{(3+m)(1+m)^5 a} - \frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-24-8m)\ln\left(-a\,x^2+1\right)}{(3+m)(1+m)^4 a} + \frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(12+4m)\,\mathrm{polylog}}{(3+m)(1+m)^3 a}\right.}{}$

input
```
int((d*x)^m*polylog(4,a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-
48-16*m)/(1+m)^5/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-24-8*m)/(1+m)^4*ln(-
a*x^2+1)/a+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3/a*polylog(2,a
*x^2)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*polylog(3,a*x^2)
+2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(4,a*x^2)+2/(3+m)*x^(1+m)*(-a)^
(3/2+1/2*m)*(24+8*m)/(1+m)^4/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

3.107.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^2\right)\, dx = \int (dx)^m \operatorname{Li}_4(ax^2)\, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(4, a*x^2), x)
```

3.107.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^2\right)\, dx = \int (dx)^m \operatorname{Li}_4\left(ax^2\right)\, dx$$

input
```
integrate((d*x)**m*polylog(4,a*x**2),x)
```

output
```
Integral((d*x)**m*polylog(4, a*x**2), x)
```

3.107.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^2\right) \, dx = \int (dx)^m \operatorname{Li}_4(ax^2) \, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="maxima")
```

output
```
-16*a*d^m*integrate(-x^2*x^m/(m^4 + 4*m^3 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4
*a*m + a)*x^2 + 6*m^2 + 4*m + 1), x) + (4*(d^m*m + d^m)*x*x^m*dilog(a*x^2)
 + 8*d^m*x*x^m*log(-a*x^2 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x
^m*polylog(4, a*x^2) - 2*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^2)
)/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

3.107.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^2\right) \, dx = \int (dx)^m \operatorname{Li}_4(ax^2) \, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(4, a*x^2), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^2\right) \, dx = \int \operatorname{polylog}\left(4, a\,x^2\right) \, (d\,x)^m \, dx$$

input
```
int(polylog(4, a*x^2)*(d*x)^m,x)
```

output
```
int(polylog(4, a*x^2)*(d*x)^m, x)
```

3.108 $\int (dx)^m \operatorname{PolyLog}(2, ax^3)\, dx$

3.108.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int (dx)^m \operatorname{PolyLog}(2, ax^3)\, dx = \frac{9a(dx)^{4+m} \operatorname{Hypergeometric2F1}\left(1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right)}{d^4(1+m)^2(4+m)} + \frac{3(dx)^{1+m} \log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{PolyLog}(2, ax^3)}{d(1+m)}$$

output
```
9*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^2/(4
+m)+3*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^3)/d/(1
+m)
```

3.108.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int (dx)^m \operatorname{PolyLog}(2, ax^3)\, dx = \frac{x(dx)^m \left(9ax^3 \operatorname{Hypergeometric2F1}\left(1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right) + (4+m)\left(3\log(1-ax^3) + (1+m)\operatorname{PolyLog}(2, ax^3)\right)\right)}{(1+m)^2(4+m)}$$

input
```
Integrate[(d*x)^m*PolyLog[2, a*x^3],x]
```

output
```
(x*(d*x)^m*(9*a*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3] + (4
 + m)*(3*Log[1 - a*x^3] + (1 + m)*PolyLog[2, a*x^3])))/((1 + m)^2*(4 + m))
```

3.108.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {7145, 25, 2905, 8, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(2,ax^3\right)(dx)^m\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int -(dx)^m\log\left(1-ax^3\right)dx}{m+1}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3\int (dx)^m\log\left(1-ax^3\right)dx}{m+1}+\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{3\left(\frac{3a\int \frac{x^2(dx)^{m+1}}{1-ax^3}dx}{d(m+1)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{3\left(\frac{3a\int \frac{(dx)^{m+3}}{1-ax^3}dx}{d^3(m+1)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{3\left(\frac{3a(dx)^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)(m+4)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}$$

input
```
Int[(d*x)^m*PolyLog[2, a*x^3],x]
```

output
```
(3*((3*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/
(d^4*(1 + m)*(4 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m))))/(1 +
m) + ((d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))
```

3.108.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.108.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 3.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

method	result
meijerg	$-\frac{(dx)^{m} x^{-m} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left(\frac{3 x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (-36-9 m)}{(4+m)(1+m)^{3} a}-\frac{3 x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (-12-3 m) \ln\left(-a\, x^{3}+1\right)}{(4+m)(1+m)^{2} a}+\frac{3 x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} \operatorname{polylog}\left(2,a\, x^{3}\right)}{(1+m) a}+\right.}{3}$

input
```
int((d*x)^m*polylog(2,a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-
36-9*m)/(1+m)^3/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*ln(-a
*x^3+1)/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(2,a*x^3)+3/(4+m)*x^(1
+m)*(-a)^(4/3+1/3*m)*(12+3*m)/(1+m)^2/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

3.108.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}\left(2,ax^3\right)\,dx = \int (dx)^m \operatorname{Li}_2\left(ax^3\right)\,dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="fricas")
```

output
```
integral((d*x)^m*dilog(a*x^3), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}\left(2,ax^3\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*polylog(2,a*x**3),x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}\left(2,ax^3\right)\,dx = \int (dx)^m \operatorname{Li}_2\left(ax^3\right)\,dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="maxima")
```

output
```
-9*a*d^m*integrate(x^3*x^m/((a*m^2 + 2*a*m + a)*x^3 - m^2 - 2*m - 1), x) +
 ((d^m*m + d^m)*x*x^m*dilog(a*x^3) + 3*d^m*x*x^m*log(-a*x^3 + 1))/(m^2 + 2
*m + 1)
```

3.108.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^3) \, dx = \int (dx)^m \operatorname{Li}_2(ax^3) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="giac")
```

output
```
integrate((d*x)^m*dilog(a*x^3), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(2, ax^3) \, dx = \int \operatorname{polylog}(2, a\,x^3) \, (d\,x)^m \, dx$$

input
```
int(polylog(2, a*x^3)*(d*x)^m,x)
```

output
```
int(polylog(2, a*x^3)*(d*x)^m, x)
```

3.109 $\int (dx)^m \operatorname{PolyLog}(3, ax^3)\, dx$

3.109.1 Optimal result

Integrand size = 13, antiderivative size = 118

$$\int (dx)^m \operatorname{PolyLog}(3, ax^3)\, dx = -\frac{27a(dx)^{4+m} \operatorname{Hypergeometric2F1}\left(1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right)}{d^4(1+m)^3(4+m)} - \frac{9(dx)^{1+m} \log(1-ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \operatorname{PolyLog}(2, ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{PolyLog}(3, ax^3)}{d(1+m)}$$

output
```
-27*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^3/
(4+m)-9*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^3-3*(d*x)^(1+m)*polylog(2,a*x^3)/
d/(1+m)^2+(d*x)^(1+m)*polylog(3,a*x^3)/d/(1+m)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07

$$\int (dx)^m \operatorname{PolyLog}(3, ax^3)\, dx =$$

$$-\frac{3x(dx)^m \operatorname{Gamma}\left(\frac{4+m}{3}\right) \left(3a(1+m)x^3 \operatorname{Gamma}\left(\frac{1+m}{3}\right) {}_2\tilde{F}_1\left(1, \frac{4+m}{3}; \frac{7+m}{3}; ax^3\right) + 9\log(1-ax^3) + 3(1+\right.}{(1+m)^4 \operatorname{Gamma}\left(\frac{1}{\right.}}$$

input
```
Integrate[(d*x)^m*PolyLog[3, a*x^3],x]
```

output
```
(-3*x*(d*x)^m*Gamma[(4 + m)/3]*(3*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeom
etricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 9*Log[1 - a*x^3]
 + 3*(1 + m)*PolyLog[2, a*x^3] - PolyLog[3, a*x^3] - 2*m*PolyLog[3, a*x^3]
 - m^2*PolyLog[3, a*x^3]))/((1 + m)^4*Gamma[(1 + m)/3])
```

3.109.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {7145, 7145, 25, 2905, 8, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{PolyLog}\left(3,ax^3\right)(dx)^m\,dx$$

$$\Big\downarrow \ 7145$$

$$\frac{\operatorname{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int (dx)^m \operatorname{PolyLog}\left(2,ax^3\right)dx}{m+1}$$

$$\Big\downarrow \ 7145$$

$$\frac{\operatorname{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\operatorname{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int -(dx)^m\log\left(1-ax^3\right)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \ 25$$

$$\frac{\operatorname{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\int (dx)^m\log\left(1-ax^3\right)dx}{m+1}+\frac{\operatorname{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \ 2905$$

$$\frac{\operatorname{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a\int \frac{x^2(dx)^{m+1}}{1-ax^3}dx}{d(m+1)}+\frac{\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\operatorname{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \ 8$$

$$\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a\int\frac{(dx)^{m+3}}{1-ax^3}dx}{d^3(m+1)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \text{888}$$

$$\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)(m+4)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[3, a*x^3],x]
```

output
```
(-3*((3*((3*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x
^3])/(d^4*(1 + m)*(4 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m))))/
(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))))/(1 + m) + ((d*x)
^(1 + m)*PolyLog[3, a*x^3])/(d*(1 + m))
```

3.109.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.85

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left(\frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (108+27m)}{(4+m)(1+m)^4 a}-\frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (36+9m) \ln\left(-a\,x^3+1\right)}{(4+m)(1+m)^3 a}+\frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (-12-3m) \operatorname{polylog}(\right.}{(4+m)(1+m)^2 a}}{3}$

input
```
int((d*x)^m*polylog(3,a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(1
08+27*m)/(1+m)^4/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*ln(
-a*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(2,a*x
^3)/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(3,a*x^3)+3/(4+m)*x^(1+m)*
(-a)^(4/3+1/3*m)*(-36-9*m)/(1+m)^3/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

3.109.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^3\right)\, dx = \int (dx)^m \operatorname{Li}_3(ax^3)\, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(3, a*x^3), x)
```

3.109.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^3\right) \, dx = \int (dx)^m \operatorname{Li}_3\left(ax^3\right) \, dx$$

input
```
integrate((d*x)**m*polylog(3,a*x**3),x)
```

output
```
Integral((d*x)**m*polylog(3, a*x**3), x)
```

3.109.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^3\right) \, dx = \int (dx)^m \operatorname{Li}_3(ax^3) \, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="maxima")
```

output
```
27*a*d^m*integrate(x^3*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^3 - m^3 - 3*m^2 -
3*m - 1), x) - (3*d^m*(m + 1)*x*x^m*dilog(a*x^3) - (m^2 + 2*m + 1)*d^m*x*x
^m*polylog(3, a*x^3) + 9*d^m*x*x^m*log(-a*x^3 + 1))/(m^3 + 3*m^2 + 3*m + 1
)
```

3.109.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^3\right) \, dx = \int (dx)^m \operatorname{Li}_3(ax^3) \, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(3, a*x^3), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}\left(3, ax^3\right) \, dx = \int \operatorname{polylog}\left(3, a\, x^3\right) \, (d\, x)^m \, dx$$

input
```
int(polylog(3, a*x^3)*(d*x)^m,x)
```

output
```
int(polylog(3, a*x^3)*(d*x)^m, x)
```

3.110 $\int (dx)^m \text{PolyLog}(4, ax^3) \, dx$

3.110.1 Optimal result

Integrand size = 13, antiderivative size = 142

$$\int (dx)^m \text{PolyLog}(4, ax^3) \, dx = \frac{81a(dx)^{4+m} \text{Hypergeometric2F1}\left(1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right)}{d^4(1+m)^4(4+m)} + \frac{27(dx)^{1+m} \log(1-ax^3)}{d(1+m)^4} + \frac{9(dx)^{1+m} \text{PolyLog}(2, ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{PolyLog}(3, ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}(4, ax^3)}{d(1+m)}$$

output
```
81*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^4/(
4+m)+27*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^4+9*(d*x)^(1+m)*polylog(2,a*x^3)/
d/(1+m)^3-3*(d*x)^(1+m)*polylog(3,a*x^3)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a
*x^3)/d/(1+m)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.17

$$\int (dx)^m \text{PolyLog}(4, ax^3) \, dx$$

$$= \frac{3x(dx)^m \text{Gamma}\left(\frac{4+m}{3}\right) \left(9a(1+m)x^3 \text{Gamma}\left(\frac{1+m}{3}\right) {}_2\tilde{F}_1\left(1, \frac{4+m}{3}; \frac{7+m}{3}; ax^3\right) + 27\log(1-ax^3) + 9(1+m\right.}{}$$

input
```
Integrate[(d*x)^m*PolyLog[4, a*x^3],x]
```

output
```
(3*x*(d*x)^m*Gamma[(4 + m)/3]*(9*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeome
tricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 27*Log[1 - a*x^3]
 + 9*(1 + m)*PolyLog[2, a*x^3] - 3*PolyLog[3, a*x^3] - 6*m*PolyLog[3, a*x^
3] - 3*m^2*PolyLog[3, a*x^3] + PolyLog[4, a*x^3] + 3*m*PolyLog[4, a*x^3] +
 3*m^2*PolyLog[4, a*x^3] + m^3*PolyLog[4, a*x^3]))/((1 + m)^5*Gamma[(1 + m
)/3])
```

3.110.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7145, 7145, 7145, 25, 2905, 8, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(4,ax^3\right)(dx)^m\,dx$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int (dx)^m\,\text{PolyLog}\left(3,ax^3\right)dx}{m+1}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int (dx)^m\,\text{PolyLog}\left(2,ax^3\right)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{ 7145}$$

$$\frac{\text{PolyLog}\left(4,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}\left(3,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}\left(2,ax^3\right)(dx)^{m+1}}{d(m+1)}-\frac{3\int -(dx)^m\log\left(1-ax^3\right)dx}{m+1}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\text{PolyLog}(4,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\int(dx)^m\log(1-ax^3)dx}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 2905

$$\frac{\text{PolyLog}(4,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a\int\frac{x^2(dx)^{m+1}}{1-ax^3}dx}{d(m+1)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 8

$$\frac{\text{PolyLog}(4,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a\int\frac{(dx)^{m+3}}{1-ax^3}dx}{d^3(m+1)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$\downarrow$ 888

$$\frac{\text{PolyLog}(4,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{\text{PolyLog}(3,ax^3)(dx)^{m+1}}{d(m+1)}-\frac{3\left(\frac{3\left(\frac{3a(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)(m+4)}+\frac{\log(1-ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}+\frac{\text{PolyLog}(2,ax^3)(dx)^{m+1}}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

3.110. $\int(dx)^m\,\text{PolyLog}(4,ax^3)\,dx$

input
```
Int[(d*x)^m*PolyLog[4, a*x^3],x]
```

output
```
(-3*((-3*((3*((3*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3
, a*x^3])/(d^4*(1 + m)*(4 + m)) + ((d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m
))))/(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))))/(1 + m) + (
(d*x)^(1 + m)*PolyLog[3, a*x^3])/(d*(1 + m))))/(1 + m) + ((d*x)^(1 + m)*Po
lyLog[4, a*x^3])/(d*(1 + m))
```

3.110.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.63 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.82

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left(\frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (-324-81m)}{(4+m)(1+m)^5 a} - \frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (-108-27m) \ln\left(-a\,x^3+1\right)}{(4+m)(1+m)^4 a} + \frac{3x^{1+m} (-a)^{\frac{4}{3}+\frac{m}{3}} (36+9m)\,\mathrm{poly}}{(4+m)(1+m)^3 a}\right.}{}$

input
```
int((d*x)^m*polylog(4,a*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-
324-81*m)/(1+m)^5/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-108-27*m)/(1+m)^4/a
*ln(-a*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*polylog(
2,a*x^3)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(3,a*x^
3)/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(4,a*x^3)+3/(4+m)*x^(1+m)*(
-a)^(4/3+1/3*m)*(108+27*m)/(1+m)^4/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

3.110.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^3\right)\, dx = \int (dx)^m \operatorname{Li}_4(ax^3)\, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(4, a*x^3), x)
```

3.110.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}\left(4, ax^3\right)\, dx = \int (dx)^m \operatorname{Li}_4\left(ax^3\right)\, dx$$

input
```
integrate((d*x)**m*polylog(4,a*x**3),x)
```

output
```
Integral((d*x)**m*polylog(4, a*x**3), x)
```

3.110.7 Maxima [F]

$$\int (dx)^m \,\text{PolyLog}\left(4, ax^3\right) \,dx = \int (dx)^m \,\text{Li}_4(ax^3) \,dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="maxima")
```

output
```
-81*a*d^m*integrate(-x^3*x^m/(m^4 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*a*m + a
)*x^3 + 4*m^3 + 6*m^2 + 4*m + 1), x) + (9*(d^m*m + d^m)*x*x^m*dilog(a*x^3)
 + 27*d^m*x*x^m*log(-a*x^3 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*
x^m*polylog(4, a*x^3) - 3*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^3
))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

3.110.8 Giac [F]

$$\int (dx)^m \,\text{PolyLog}\left(4, ax^3\right) \,dx = \int (dx)^m \,\text{Li}_4(ax^3) \,dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(4, a*x^3), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \,\text{PolyLog}\left(4, ax^3\right) \,dx = \int \text{polylog}\left(4, a\,x^3\right) \,(d\,x)^m \,dx$$

input
```
int(polylog(4, a*x^3)*(d*x)^m,x)
```

output
```
int(polylog(4, a*x^3)*(d*x)^m, x)
```

3.111 $\int (dx)^m \operatorname{PolyLog}(2, ax^q)\, dx$

3.111.1 Optimal result

Integrand size = 13, antiderivative size = 101

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q)\, dx = \frac{aq^2x^{1+q}(dx)^m \operatorname{Hypergeometric2F1}\left(1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, ax^q\right)}{(1+m)^2(1+m+q)} + \frac{q(dx)^{1+m}\log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\operatorname{PolyLog}(2, ax^q)}{d(1+m)}$$

output
```
a*q^2*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)^
2/(1+m+q)+q*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^q)
/d/(1+m)
```

3.111.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q)\, dx$$
$$= \frac{x(dx)^m \left(aq^2x^q \operatorname{Hypergeometric2F1}\left(1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, ax^q\right) + (1+m+q)\left(q\log(1-ax^q) + (1+m)\operatorname{Po}\right.\right.}{(1+m)^2(1+m+q)}$$

input
```
Integrate[(d*x)^m*PolyLog[2, a*x^q],x]
```

output
```
(x*(d*x)^m*(a*q^2*x^q*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/q,
 a*x^q] + (1 + m + q)*(q*Log[1 - a*x^q] + (1 + m)*PolyLog[2, a*x^q])))/((1
 + m)^2*(1 + m + q))
```

3.111.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q) \, dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1} \operatorname{PolyLog}(2, ax^q)}{d(m+1)} - \frac{q \int -(dx)^m \log(1 - ax^q) \, dx}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{q \int (dx)^m \log(1 - ax^q) \, dx}{m+1} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ax^q)}{d(m+1)}$$

$$\Big\downarrow \text{2905}$$

$$\frac{q \left(\frac{aq \int \frac{x^{q-1}(dx)^{m+1}}{1-ax^q} dx}{d(m+1)} + \frac{(dx)^{m+1} \log(1-ax^q)}{d(m+1)} \right)}{m+1} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ax^q)}{d(m+1)}$$

$$\Big\downarrow \text{30}$$

$$\frac{q \left(\frac{aqx^{-m}(dx)^m \int \frac{x^{m+q}}{1-ax^q} dx}{m+1} + \frac{(dx)^{m+1} \log(1-ax^q)}{d(m+1)} \right)}{m+1} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ax^q)}{d(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{q \left(\frac{aqx^{q+1}(dx)^m \operatorname{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right)}{(m+1)(m+q+1)} + \frac{(dx)^{m+1} \log(1-ax^q)}{d(m+1)} \right)}{m+1} + \frac{(dx)^{m+1} \operatorname{PolyLog}(2, ax^q)}{d(m+1)}$$

input
```
Int[(d*x)^m*PolyLog[2, a*x^q],x]
```

output
```
(q*((a*q*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*
q)/q, a*x^q])/((1 + m)*(1 + m + q)) + ((d*x)^(1 + m)*Log[1 - a*x^q])/(d*(1
 + m))))/(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m))
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 2.92 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{m}{q}-\frac{1}{q}}\left(-\frac{q^2 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\ln(1-a x^q)}{(1+m)^2}-\frac{q x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,a x^q)}{1+m}-\frac{q^2 x^{1+m+q}a(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{LerchPhi}\left(a x^q,1\right.}{(1+m)^2}\right.}{q}$

input
```
int((d*x)^m*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*ln(
1-a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*polylog(2,a*x^q)-q^2*x^(1+m+q)*a*(
-a)^(m/q+1/q)/(1+m)^2*LerchPhi(a*x^q,1,(1+m+q)/q))
```

3.111.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q)\, dx = \int (dx)^m \operatorname{Li}_2(ax^q)\, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="fricas")
```

output
```
integral((d*x)^m*dilog(a*x^q), x)
```

3.111.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q)\, dx = \int (dx)^m \operatorname{Li}_2(ax^q)\, dx$$

input
```
integrate((d*x)**m*polylog(2,a*x**q),x)
```

output
```
Integral((d*x)**m*polylog(2, a*x**q), x)
```

3.111.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q) \, dx = \int (dx)^m \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="maxima")
```

output
```
-d^m*q^2*integrate(-x^m/(m^2 - (a*m^2 + 2*a*m + a)*x^q + 2*m + 1), x) - (d
^m*q^2*x*x^m - (d^m*m + d^m)*q*x*x^m*log(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m
+ d^m)*x*x^m*dilog(a*x^q))/(m^3 + 3*m^2 + 3*m + 1)
```

3.111.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q) \, dx = \int (dx)^m \operatorname{Li}_2(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="giac")
```

output
```
integrate((d*x)^m*dilog(a*x^q), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(2, ax^q) \, dx = \int (d\,x)^m \operatorname{polylog}(2, a\,x^q) \, dx$$

input
```
int((d*x)^m*polylog(2, a*x^q),x)
```

output
```
int((d*x)^m*polylog(2, a*x^q), x)
```

3.112 $\int (dx)^m \operatorname{PolyLog}(3, ax^q)\, dx$

3.112.1 Optimal result

Integrand size = 13, antiderivative size = 130

$$\int (dx)^m \operatorname{PolyLog}(3,ax^q)\, dx = -\frac{aq^3x^{1+q}(dx)^m \operatorname{Hypergeometric2F1}\left(1,\frac{1+m+q}{q},\frac{1+m+2q}{q},ax^q\right)}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\operatorname{PolyLog}(2,ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\operatorname{PolyLog}(3,ax^q)}{d(1+m)}$$

output
```
-a*q^3*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)
^3/(1+m+q)-q^2*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*polylog(2,a
*x^q)/d/(1+m)^2+(d*x)^(1+m)*polylog(3,a*x^q)/d/(1+m)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int (dx)^m \operatorname{PolyLog}(3,ax^q)\, dx = -\frac{x(dx)^m G_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,1-\frac{1+m}{q} \\ 1,0,0,0,-\frac{1+m}{q} \end{array}\right)}{q}$$

input
```
Integrate[(d*x)^m*PolyLog[3, a*x^q],x]
```

output
```
-((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0, -(
(1 + m)/q)}}, -(a*x^q)])/q)
```

3.112.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \operatorname{PolyLog}(3,ax^q)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\int (dx)^m \operatorname{PolyLog}(2,ax^q)dx}{m+1}$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}-\frac{q\int -(dx)^m\log(1-ax^q)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\int (dx)^m\log(1-ax^q)dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \text{2905}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aq\int \frac{x^{q-1}(dx)^{m+1}}{1-ax^q}dx}{d(m+1)}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \text{30}$$

$$\frac{(dx)^{m+1}\,\text{PolyLog}\left(3,ax^q\right)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aqx^{-m}(dx)^m\int\frac{x^{m+q}}{1-ax^q}dx}{m+1}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\,\text{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}$$

$$\Big\downarrow \; 888$$

$$\frac{(dx)^{m+1}\,\text{PolyLog}\left(3,ax^q\right)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aqx^{q+1}(dx)^m\,\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ax^q\right)}{(m+1)(m+q+1)}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\,\text{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[3, a*x^q],x]
```

output
```
-((q*((q*((a*q*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 +
m + 2*q)/q, a*x^q])/((1 + m)*(1 + m + q)) + ((d*x)^(1 + m)*Log[1 - a*x^q])
/(d*(1 + m))))/(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m))))/(
1 + m)) + ((d*x)^(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m))
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

method	result
meijerg	$-\frac{(dx)^m x^{-m} (-a)^{-\frac{m}{q}-\frac{1}{q}} \left(\frac{q^3 x^{1+m} (-a)^{\frac{m}{q}+\frac{1}{q}} \ln(1-a\,x^q)}{(1+m)^3} + \frac{q^2 x^{1+m} (-a)^{\frac{m}{q}+\frac{1}{q}} \operatorname{polylog}(2,a\,x^q)}{(1+m)^2} - \frac{q\,x^{1+m} (-a)^{\frac{m}{q}+\frac{1}{q}} \operatorname{polylog}(3,a\,x^q)}{1+m} + \cdots \right.}{q}$

input
```
int((d*x)^m*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*ln(1
-a*x^q)+q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*polylog(2,a*x^q)-q*x^(1+m)*(-a)
^(m/q+1/q)/(1+m)*polylog(3,a*x^q)+q^3*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^3*L
erchPhi(a*x^q,1,(1+m+q)/q))
```

3.112.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax^q) \, dx = \int (dx)^m \operatorname{Li}_3(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(3, a*x^q), x)
```

3.112.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax^q)\,dx = \int (dx)^m \operatorname{Li}_3(ax^q)\,dx$$

input
```
integrate((d*x)**m*polylog(3,a*x**q),x)
```

output
```
Integral((d*x)**m*polylog(3, a*x**q), x)
```

3.112.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax^q)\,dx = \int (dx)^m \operatorname{Li}_3(ax^q)\,dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="maxima")
```

output
```
d^m*q^3*integrate(-x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x^q + 3*m^2 + 3*m
+ 1), x) + (d^m*q^3*x*x^m - (m^2*q + 2*m*q + q)*d^m*x*x^m*dilog(a*x^q) - (
m*q^2 + q^2)*d^m*x*x^m*log(-a*x^q + 1) + (m^3 + 3*m^2 + 3*m + 1)*d^m*x*x^m
*polylog(3, a*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

3.112.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(3, ax^q)\,dx = \int (dx)^m \operatorname{Li}_3(ax^q)\,dx$$

input
```
integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(3, a*x^q), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(3, ax^q) \, dx = \int (d\,x)^m \operatorname{polylog}(3, a\,x^q) \, dx$$

input
```
int((d*x)^m*polylog(3, a*x^q),x)
```

output
```
int((d*x)^m*polylog(3, a*x^q), x)
```

3.113 $\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx$

3.113.1 Optimal result

Integrand size = 13, antiderivative size = 154

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \frac{aq^4x^{1+q}(dx)^m \operatorname{Hypergeometric2F1}\left(1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, ax^q\right)}{(1+m)^4(1+m+q)} + \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\operatorname{PolyLog}(2, ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\operatorname{PolyLog}(3, ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\operatorname{PolyLog}(4, ax^q)}{d(1+m)}$$

output
```
a*q^4*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)^
4/(1+m+q)+q^3*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^4+q^2*(d*x)^(1+m)*polylog(2,
a*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*polylog(3,a*x^q)/d/(1+m)^2+(d*x)^(1+m)*poly
log(4,a*x^q)/d/(1+m)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = -\frac{x(dx)^m G_{6,6}^{1,6}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,1,1-\frac{1+m}{q} \\ 1,0,0,0,0,-\frac{1+m}{q} \end{array}\right)}{q}$$

input
```
Integrate[(d*x)^m*PolyLog[4, a*x^q],x]
```

output
```
-((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0,
 0, -((1 + m)/q)}}, -(a*x^q)])/q)
```

3.113.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {7145, 7145, 7145, 25, 2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \operatorname{PolyLog}(4,ax^q)\,dx$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\int (dx)^m\operatorname{PolyLog}(3,ax^q)dx}{m+1}$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\int (dx)^m\operatorname{PolyLog}(2,ax^q)dx}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{7145}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}-\frac{q\int -(dx)^m\log(1-ax^q)dx}{m+1}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\int (dx)^m\log(1-ax^q)dx}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \text{2905}$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aq\int\frac{x^{q-1}(dx)^{m+1}}{1-ax^q}dx}{d(m+1)}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \; 30$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aqx^{-m}(dx)^m\int\frac{x^{m+q}}{1-ax^q}dx}{m+1}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

$$\Big\downarrow \; 888$$

$$\frac{(dx)^{m+1}\operatorname{PolyLog}(4,ax^q)}{d(m+1)}-\frac{q\left(\frac{(dx)^{m+1}\operatorname{PolyLog}(3,ax^q)}{d(m+1)}-\frac{q\left(\frac{q\left(\frac{aqx^{q+1}(dx)^m\operatorname{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ax^q\right)}{(m+1)(m+q+1)}+\frac{(dx)^{m+1}\log(1-ax^q)}{d(m+1)}\right)}{m+1}+\frac{(dx)^{m+1}\operatorname{PolyLog}(2,ax^q)}{d(m+1)}\right)}{m+1}\right)}{m+1}$$

input
```
Int[(d*x)^m*PolyLog[4, a*x^q],x]
```

output
```
-((q*(-((q*((q*((a*q*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q,
 (1 + m + 2*q)/q, a*x^q])/((1 + m)*(1 + m + q)) + ((d*x)^(1 + m)*Log[1 - a
*x^q])/(d*(1 + m))))/(1 + m) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m
))))/(1 + m)) + ((d*x)^(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m))))/(1 + m)) +
 ((d*x)^(1 + m)*PolyLog[4, a*x^q])/(d*(1 + m))
```

3.113. $\int (dx)^m \operatorname{PolyLog}(4,ax^q)\, dx$

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

rule 7145
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Simp[p
*(q/(m + 1))   Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
 b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.41

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{m}{q}-\frac{1}{q}}\left(-\frac{q^4 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\ln(1-a\,x^q)}{(1+m)^4}-\frac{q^3 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,a\,x^q)}{(1+m)^3}+\frac{q^2 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(3,a\,x^q)}{(1+m)^2}-q \right.}{q}$

input
```
int((d*x)^m*polylog(4,a*x^q),x,method=_RETURNVERBOSE)
```

output
```
-(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^4*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^4*ln(
1-a*x^q)-q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*polylog(2,a*x^q)+q^2*x^(1+m)*(
-a)^(m/q+1/q)/(1+m)^2*polylog(3,a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*poly
log(4,a*x^q)-q^4*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^4*LerchPhi(a*x^q,1,(1+m+
q)/q))
```

3.113.5 Fricas [F]

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \int (dx)^m \operatorname{Li}_4(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="fricas")
```

output
```
integral((d*x)^m*polylog(4, a*x^q), x)
```

3.113.6 Sympy [F]

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \int (dx)^m \operatorname{Li}_4(ax^q) \, dx$$

input
```
integrate((d*x)**m*polylog(4,a*x**q),x)
```

output
```
Integral((d*x)**m*polylog(4, a*x**q), x)
```

3.113.7 Maxima [F]

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \int (dx)^m \operatorname{Li}_4(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="maxima")
```

output
```
-d^m*q^4*integrate(-x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2
+ 4*a*m + a)*x^q + 4*m + 1), x) - (d^m*q^4*x*x^m - (d^m*m + d^m)*q^3*x*x^m
*log(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m + d^m)*q^2*x*x^m*dilog(a*x^q) + (d^m
*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*q*x*x^m*polylog(3, a*x^q) - (d^m*m^4 + 4
*d^m*m^3 + 6*d^m*m^2 + 4*d^m*m + d^m)*x*x^m*polylog(4, a*x^q))/(m^5 + 5*m^
4 + 10*m^3 + 10*m^2 + 5*m + 1)
```

3.113.8 Giac [F]

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \int (dx)^m \operatorname{Li}_4(ax^q) \, dx$$

input
```
integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="giac")
```

output
```
integrate((d*x)^m*polylog(4, a*x^q), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \operatorname{PolyLog}(4, ax^q) \, dx = \int (d\,x)^m \operatorname{polylog}(4, a\,x^q) \, dx$$

input
```
int((d*x)^m*polylog(4, a*x^q),x)
```

output
```
int((d*x)^m*polylog(4, a*x^q), x)
```

3.114 $\int x \operatorname{PolyLog}(n, ax) \, dx$

3.114.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int x \operatorname{PolyLog}(n, ax) \, dx = \operatorname{Int}(x \operatorname{PolyLog}(n, ax), x)$$

output
```
Unintegrable(x*polylog(n,a*x),x)
```

3.114.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int x \operatorname{PolyLog}(n, ax) \, dx = \int x \operatorname{PolyLog}(n, ax) \, dx$$

input
```
Integrate[x*PolyLog[n, a*x],x]
```

output
```
Integrate[x*PolyLog[n, a*x], x]
```

3.114.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(n, ax) \, dx$$

$$\Big\downarrow \text{7147}$$

$$\int x \operatorname{PolyLog}(n, ax) dx$$

input
```
Int[x*PolyLog[n, a*x],x]
```

output
```
$Aborted
```

3.114.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.114.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \operatorname{polylog}(n, ax) \, dx$$

input
```
int(x*polylog(n,a*x),x)
```

output
```
int(x*polylog(n,a*x),x)
```

3.114.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int x \operatorname{PolyLog}(n, ax)\, dx = \int x \operatorname{Li}_n(ax)\, dx$$

input
```
integrate(x*polylog(n,a*x),x, algorithm="fricas")
```

output
```
integral(x*polylog(n, a*x), x)
```

3.114.6 Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \operatorname{PolyLog}(n, ax)\, dx = \int x \operatorname{Li}_n(ax)\, dx$$

input
```
integrate(x*polylog(n,a*x),x)
```

output
```
Integral(x*polylog(n, a*x), x)
```

3.114.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int x \operatorname{PolyLog}(n, ax)\, dx = \int x \operatorname{Li}_n(ax)\, dx$$

input
```
integrate(x*polylog(n,a*x),x, algorithm="maxima")
```

output
```
integrate(x*polylog(n, a*x), x)
```

3.114.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int x \operatorname{PolyLog}(n, ax)\, dx = \int x\operatorname{Li}_n(ax)\, dx$$

input
```
integrate(x*polylog(n,a*x),x, algorithm="giac")
```

output
```
integrate(x*polylog(n, a*x), x)
```

3.114.9 Mupad [N/A]

Not integrable

Time = 5.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int x \operatorname{PolyLog}(n, ax)\, dx = \int x \operatorname{polylog}(n, a\, x)\, dx$$

input
```
int(x*polylog(n, a*x),x)
```

output
```
int(x*polylog(n, a*x), x)
```

3.115 $\int \text{PolyLog}(n, ax) \, dx$

3.115.1 Optimal result

Integrand size = 5, antiderivative size = 5

$$\int \text{PolyLog}(n, ax) \, dx = \text{Int}(\text{PolyLog}(n, ax), x)$$

output
```
Unintegrable(polylog(n,a*x),x)
```

3.115.2 Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \text{PolyLog}(n, ax) \, dx = \int \text{PolyLog}(n, ax) \, dx$$

input
```
Integrate[PolyLog[n, a*x],x]
```

output
```
Integrate[PolyLog[n, a*x], x]
```

3.115.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(n, ax)\, dx$$

$$\Big\downarrow \; 7142$$

$$\int \text{PolyLog}(n, ax) dx$$

input
```
Int[PolyLog[n, a*x],x]
```

output
```
$Aborted
```

3.115.3.1 Defintions of rubi rules used

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.115.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \text{polylog}\left(n, ax\right) dx$$

input
```
int(polylog(n,a*x),x)
```

output
```
int(polylog(n,a*x),x)
```

3.115.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \mathrm{PolyLog}(n, ax)\,dx = \int \mathrm{Li}_n(ax)\,dx$$

input
```
integrate(polylog(n,a*x),x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x), x)
```

3.115.6 Sympy [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \mathrm{PolyLog}(n, ax)\,dx = \int \mathrm{Li}_n\,(ax)\;dx$$

input
```
integrate(polylog(n,a*x),x)
```

output
```
Integral(polylog(n, a*x), x)
```

3.115.7 Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \mathrm{PolyLog}(n, ax)\,dx = \int \mathrm{Li}_n(ax)\,dx$$

input
```
integrate(polylog(n,a*x),x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x), x)
```

3.115.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \text{PolyLog}(n, ax)\, dx = \int \text{Li}_n(ax)\, dx$$

input
```
integrate(polylog(n,a*x),x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x), x)
```

3.115.9 Mupad [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \text{PolyLog}(n, ax)\, dx = \int \text{polylog}(n, a\,x)\ dx$$

input
```
int(polylog(n, a*x),x)
```

output
```
int(polylog(n, a*x), x)
```

3.116 $\int \frac{\text{PolyLog}(n,ax)}{x}\,dx$

3.116.1 Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \frac{\text{PolyLog}(n,ax)}{x}\,dx = \text{PolyLog}(1+n,ax)$$

output
```
polylog(1+n,a*x)
```

3.116.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(n,ax)}{x}\,dx = \text{PolyLog}(1+n,ax)$$

input
```
Integrate[PolyLog[n, a*x]/x,x]
```

output
```
PolyLog[1 + n, a*x]
```

3.116.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n, ax)}{x} \, dx$$
$$\Big\downarrow \ 7143$$
$$\text{PolyLog}(n+1, ax)$$

input
```
Int[PolyLog[n, a*x]/x,x]
```

output
```
PolyLog[1 + n, a*x]
```

3.116.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.116.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\text{polylog}(1+n, ax)$	8
default	$\text{polylog}(1+n, ax)$	8

input
```
int(polylog(n,a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(1+n,a*x)
```

3.116.5 Fricas [F]

$$\int \frac{\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\text{Li}_n(ax)}{x}\,dx$$

input
```
integrate(polylog(n,a*x)/x,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x)/x, x)
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\text{PolyLog}(n,ax)}{x}\,dx = \text{Li}_{n+1}(ax)$$

input
```
integrate(polylog(n,a*x)/x,x)
```

output
```
polylog(n + 1, a*x)
```

3.116.7 Maxima [F]

$$\int \frac{\text{PolyLog}(n,ax)}{x}\,dx = \int \frac{\text{Li}_n(ax)}{x}\,dx$$

input
```
integrate(polylog(n,a*x)/x,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x)/x, x)
```

3.116.8 Giac [F]

$$\int \frac{\text{PolyLog}(n, ax)}{x} \, dx = \int \frac{\text{Li}_n(ax)}{x} \, dx$$

input
```
integrate(polylog(n,a*x)/x,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x)/x, x)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(n, ax)}{x} \, dx = \text{polylog}(n + 1, a\, x)$$

input
```
int(polylog(n, a*x)/x,x)
```

output
```
polylog(n + 1, a*x)
```

3.117 $\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx$

3.117.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx = \text{Int}\left(\frac{\text{PolyLog}(n,ax)}{x^2},x\right)$$

output
```
Unintegrable(polylog(n,a*x)/x^2,x)
```

3.117.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx = \int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx$$

input
```
Integrate[PolyLog[n, a*x]/x^2,x]
```

output
```
Integrate[PolyLog[n, a*x]/x^2, x]
```

3.117.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx$$
$$\Big\downarrow \ 7147$$
$$\int \frac{\text{PolyLog}(n,ax)}{x^2}dx$$

input
```
Int[PolyLog[n, a*x]/x^2,x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.117.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\text{polylog}\,(n,ax)}{x^2}dx$$

input
```
int(polylog(n,a*x)/x^2,x)
```

output
```
int(polylog(n,a*x)/x^2,x)
```

3.117.5 Fricas [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} \, dx = \int \frac{\text{Li}_n(ax)}{x^2} \, dx$$

input
```
integrate(polylog(n,a*x)/x^2,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x)/x^2, x)
```

3.117.6 Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} \, dx = \int \frac{\text{Li}_n(ax)}{x^2} \, dx$$

input
```
integrate(polylog(n,a*x)/x**2,x)
```

output
```
Integral(polylog(n, a*x)/x**2, x)
```

3.117.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} \, dx = \int \frac{\text{Li}_n(ax)}{x^2} \, dx$$

input
```
integrate(polylog(n,a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x)/x^2, x)
```

3.117.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} \, dx = \int \frac{\text{Li}_n(ax)}{x^2} \, dx$$

input
```
integrate(polylog(n,a*x)/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x)/x^2, x)
```

3.117.9 Mupad [N/A]

Not integrable

Time = 5.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} \, dx = \int \frac{\text{polylog}(n, a\,x)}{x^2} \, dx$$

input
```
int(polylog(n, a*x)/x^2,x)
```

output
```
int(polylog(n, a*x)/x^2, x)
```

3.118 $\int \frac{\text{PolyLog}(n,ax)}{x^3} dx$

3.118.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{\text{PolyLog}(n,ax)}{x^3} dx = \text{Int}\left(\frac{\text{PolyLog}(n,ax)}{x^3},x\right)$$

output
```
Unintegrable(polylog(n,a*x)/x^3,x)
```

3.118.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^3} dx = \int \frac{\text{PolyLog}(n,ax)}{x^3} dx$$

input
```
Integrate[PolyLog[n, a*x]/x^3,x]
```

output
```
Integrate[PolyLog[n, a*x]/x^3, x]
```

3.118.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n,ax)}{x^3} \, dx$$

$$\Big\downarrow \; 7147$$

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}dx$$

input
```
Int[PolyLog[n, a*x]/x^3,x]
```

output
```
$Aborted
```

3.118.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.118.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\text{polylog}\left(n,ax\right)}{x^3}dx$$

input
```
int(polylog(n,a*x)/x^3,x)
```

output
```
int(polylog(n,a*x)/x^3,x)
```

3.118.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx = \int \frac{\text{Li}_n(ax)}{x^3}\,dx$$

input
```
integrate(polylog(n,a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x)/x^3, x)
```

3.118.6 Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx = \int \frac{\text{Li}_n\left(ax\right)}{x^3}\,dx$$

input
```
integrate(polylog(n,a*x)/x**3,x)
```

output
```
Integral(polylog(n, a*x)/x**3, x)
```

3.118.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx = \int \frac{\text{Li}_n(ax)}{x^3}\,dx$$

input
```
integrate(polylog(n,a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x)/x^3, x)
```

3.118.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx = \int \frac{\text{Li}_n(ax)}{x^3}\,dx$$

input
```
integrate(polylog(n,a*x)/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x)/x^3, x)
```

3.118.9 Mupad [N/A]

Not integrable

Time = 5.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx = \int \frac{\text{polylog}(n,a\,x)}{x^3}\,dx$$

input
```
int(polylog(n, a*x)/x^3,x)
```

output
```
int(polylog(n, a*x)/x^3, x)
```

3.119 $\int x \,\text{PolyLog}(n, ax^q) \, dx$

3.119.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int x \,\text{PolyLog}(n, ax^q) \, dx = \text{Int}(x \,\text{PolyLog}(n, ax^q), x)$$

output
```
Unintegrable(x*polylog(n,a*x^q),x)
```

3.119.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int x \,\text{PolyLog}(n, ax^q) \, dx = \int x \,\text{PolyLog}(n, ax^q) \, dx$$

input
```
Integrate[x*PolyLog[n, a*x^q],x]
```

output
```
Integrate[x*PolyLog[n, a*x^q], x]
```

3.119.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx$$

$$\Big\downarrow \ 7147$$

$$\int x \operatorname{PolyLog}(n, ax^q) dx$$

input
```
Int[x*PolyLog[n, a*x^q],x]
```

output
```
$Aborted
```

3.119.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.119.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \operatorname{polylog}(n, a\, x^q) dx$$

input
```
int(x*polylog(n,a*x^q),x)
```

output
```
int(x*polylog(n,a*x^q),x)
```

3.119.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx = \int x\mathrm{Li}_n(ax^q) \, dx$$

input
```
integrate(x*polylog(n,a*x^q),x, algorithm="fricas")
```

output
```
integral(x*polylog(n, a*x^q), x)
```

3.119.6 Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx = \int x \operatorname{Li}_n(ax^q) \, dx$$

input
```
integrate(x*polylog(n,a*x**q),x)
```

output
```
Integral(x*polylog(n, a*x**q), x)
```

3.119.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx = \int x\mathrm{Li}_n(ax^q) \, dx$$

input
```
integrate(x*polylog(n,a*x^q),x, algorithm="maxima")
```

output
```
integrate(x*polylog(n, a*x^q), x)
```

3.119.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx = \int x \operatorname{Li}_n(ax^q) \, dx$$

input
```
integrate(x*polylog(n,a*x^q),x, algorithm="giac")
```

output
```
integrate(x*polylog(n, a*x^q), x)
```

3.119.9 Mupad [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int x \operatorname{PolyLog}(n, ax^q) \, dx = \int x \operatorname{polylog}(n, a\,x^q) \, dx$$

input
```
int(x*polylog(n, a*x^q),x)
```

output
```
int(x*polylog(n, a*x^q), x)
```

3.120 $\int \text{PolyLog}(n, ax^q)\,dx$

3.120.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \text{PolyLog}(n, ax^q)\,dx = \text{Int}(\text{PolyLog}(n, ax^q), x)$$

output
```
Unintegrable(polylog(n,a*x^q),x)
```

3.120.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}(n, ax^q)\,dx = \int \text{PolyLog}(n, ax^q)\,dx$$

input
```
Integrate[PolyLog[n, a*x^q],x]
```

output
```
Integrate[PolyLog[n, a*x^q], x]
```

3.120.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(n, ax^q) \, dx$$

$$\Big\downarrow \text{7142}$$

$$\int \text{PolyLog}(n, ax^q)dx$$

input
```
Int[PolyLog[n, a*x^q],x]
```

output
```
$Aborted
```

3.120.3.1 Defintions of rubi rules used

rule 7142
```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Unintegrable[
PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, n, p, q}, x]
```

3.120.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{polylog}(n, a\,x^q)\,dx$$

input
```
int(polylog(n,a*x^q),x)
```

output
```
int(polylog(n,a*x^q),x)
```

3.120.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(n, ax^q\right) \, dx = \int \text{Li}_n(ax^q) \, dx$$

input
```
integrate(polylog(n,a*x^q),x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x^q), x)
```

3.120.6 Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(n, ax^q\right) \, dx = \int \text{Li}_n\left(ax^q\right) \, dx$$

input
```
integrate(polylog(n,a*x**q),x)
```

output
```
Integral(polylog(n, a*x**q), x)
```

3.120.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(n, ax^q\right) \, dx = \int \text{Li}_n(ax^q) \, dx$$

input
```
integrate(polylog(n,a*x^q),x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x^q), x)
```

3.120.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(n, ax^q\right)\, dx = \int \text{Li}_n(ax^q)\, dx$$

input
```
integrate(polylog(n,a*x^q),x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x^q), x)
```

3.120.9 Mupad [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \text{PolyLog}\left(n, ax^q\right)\, dx = \int \text{polylog}(n, a\, x^q)\, dx$$

input
```
int(polylog(n, a*x^q),x)
```

output
```
int(polylog(n, a*x^q), x)
```

3.121 $\int \frac{\text{PolyLog}(n,ax^q)}{x} \, dx$

3.121.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{\text{PolyLog}(n,ax^q)}{x} \, dx = \frac{\text{PolyLog}(1+n,ax^q)}{q}$$

output
```
polylog(1+n,a*x^q)/q
```

3.121.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(n,ax^q)}{x} \, dx = \frac{\text{PolyLog}(1+n,ax^q)}{q}$$

input
```
Integrate[PolyLog[n, a*x^q]/x,x]
```

output
```
PolyLog[1 + n, a*x^q]/q
```

3.121.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x} \, dx$$

$$\Big\downarrow \ 7143$$

$$\frac{\text{PolyLog}(n+1, ax^q)}{q}$$

input
```
Int[PolyLog[n, a*x^q]/x,x]
```

output
```
PolyLog[1 + n, a*x^q]/q
```

3.121.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.121.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\text{polylog}(1+n, a\, x^{q})}{q}$	14
default	$\frac{\text{polylog}(1+n, a\, x^{q})}{q}$	14

input
```
int(polylog(n,a*x^q)/x,x,method=_RETURNVERBOSE)
```

output
```
polylog(1+n,a*x^q)/q
```

3.121.5 Fricas [F]

$$\int \frac{\text{PolyLog}\,(n, ax^q)}{x}\,dx = \int \frac{\text{Li}_n(ax^q)}{x}\,dx$$

input
```
integrate(polylog(n,a*x^q)/x,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x^q)/x, x)
```

3.121.6 Sympy [F]

$$\int \frac{\text{PolyLog}\,(n, ax^q)}{x}\,dx = \int \frac{\text{Li}_n\,(ax^q)}{x}\,dx$$

input
```
integrate(polylog(n,a*x**q)/x,x)
```

output
```
Integral(polylog(n, a*x**q)/x, x)
```

3.121.7 Maxima [F]

$$\int \frac{\text{PolyLog}\,(n, ax^q)}{x}\,dx = \int \frac{\text{Li}_n(ax^q)}{x}\,dx$$

input
```
integrate(polylog(n,a*x^q)/x,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x^q)/x, x)
```

3.121.8 Giac [F]

$$\int \frac{\text{PolyLog}(n, ax^q)}{x}\, dx = \int \frac{\text{Li}_n(ax^q)}{x}\, dx$$

input
```
integrate(polylog(n,a*x^q)/x,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x^q)/x, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x}\, dx = \int \frac{\text{polylog}(n, a\, x^q)}{x}\, dx$$

input
```
int(polylog(n, a*x^q)/x,x)
```

output
```
int(polylog(n, a*x^q)/x, x)
```

3.122 $\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx$

3.122.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx=\text{Int}\left(\frac{\text{PolyLog}(n,ax^q)}{x^2},x\right)$$

output
```
Unintegrable(polylog(n,a*x^q)/x^2,x)
```

3.122.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx=\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx$$

input
```
Integrate[PolyLog[n, a*x^q]/x^2,x]
```

output
```
Integrate[PolyLog[n, a*x^q]/x^2, x]
```

3.122.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} \, dx$$
$$\Big\downarrow \; 7147$$
$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} dx$$

input
```
Int[PolyLog[n, a*x^q]/x^2,x]
```

output
```
$Aborted
```

3.122.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.122.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{polylog}(n, a\,x^q)}{x^2} dx$$

input
```
int(polylog(n,a*x^q)/x^2,x)
```

output
```
int(polylog(n,a*x^q)/x^2,x)
```

3.122.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx = \int \frac{\text{Li}_n(ax^q)}{x^2}\,dx$$

input
```
integrate(polylog(n,a*x^q)/x^2,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x^q)/x^2, x)
```

3.122.6 Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx = \int \frac{\text{Li}_n\left(ax^q\right)}{x^2}\,dx$$

input
```
integrate(polylog(n,a*x**q)/x**2,x)
```

output
```
Integral(polylog(n, a*x**q)/x**2, x)
```

3.122.7 Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx = \int \frac{\text{Li}_n(ax^q)}{x^2}\,dx$$

input
```
integrate(polylog(n,a*x^q)/x^2,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x^q)/x^2, x)
```

3.122.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^2} \, dx$$

input
```
integrate(polylog(n,a*x^q)/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x^q)/x^2, x)
```

3.122.9 Mupad [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} \, dx = \int \frac{\text{polylog}(n, a\, x^q)}{x^2} \, dx$$

input
```
int(polylog(n, a*x^q)/x^2,x)
```

output
```
int(polylog(n, a*x^q)/x^2, x)
```

3.123 $\int \frac{\text{PolyLog}(n,ax^q)}{x^3}\,dx$

3.123.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^3}\,dx = \text{Int}\left(\frac{\text{PolyLog}(n,ax^q)}{x^3},x\right)$$

output
```
Unintegrable(polylog(n,a*x^q)/x^3,x)
```

3.123.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^3}\,dx = \int \frac{\text{PolyLog}(n,ax^q)}{x^3}\,dx$$

input
```
Integrate[PolyLog[n, a*x^q]/x^3,x]
```

output
```
Integrate[PolyLog[n, a*x^q]/x^3, x]
```

3.123.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7147}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} dx$$
$$\Big\downarrow \; 7147$$
$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} dx$$

input
```
Int[PolyLog[n, a*x^q]/x^3,x]
```

output
```
$Aborted
```

3.123.3.1 Defintions of rubi rules used

rule 7147
```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Unintegrable[(d*x)^m*PolyLog[n, a*(b*x^p)^q], x] /; FreeQ[{a, b, d, m
, n, p, q}, x]
```

3.123.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\text{polylog}(n, a\, x^q)}{x^3} dx$$

input
```
int(polylog(n,a*x^q)/x^3,x)
```

output
```
int(polylog(n,a*x^q)/x^3,x)
```

3.123.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

input
```
integrate(polylog(n,a*x^q)/x^3,x, algorithm="fricas")
```

output
```
integral(polylog(n, a*x^q)/x^3, x)
```

3.123.6 Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

input
```
integrate(polylog(n,a*x**q)/x**3,x)
```

output
```
Integral(polylog(n, a*x**q)/x**3, x)
```

3.123.7 Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

input
```
integrate(polylog(n,a*x^q)/x^3,x, algorithm="maxima")
```

output
```
integrate(polylog(n, a*x^q)/x^3, x)
```

3.123.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

input
```
integrate(polylog(n,a*x^q)/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(n, a*x^q)/x^3, x)
```

3.123.9 Mupad [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} \, dx = \int \frac{\text{polylog}(n, a\, x^q)}{x^3} \, dx$$

input
```
int(polylog(n, a*x^q)/x^3,x)
```

output
```
int(polylog(n, a*x^q)/x^3, x)
```

3.124 $\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx$

3.124.1 Optimal result

Integrand size = 13, antiderivative size = 260

$$\begin{aligned}\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx = &-\frac{a^2x}{3b^2}+\frac{a(1-ac)x}{6b^2c}-\frac{(1-ac)^2x}{9b^2c^2}+\frac{ax^2}{12b}\\ &-\frac{(1-ac)x^2}{18bc}-\frac{x^3}{27}+\frac{a(1-ac)^2\log(1-ac-bcx)}{6b^3c^2}\\ &-\frac{(1-ac)^3\log(1-ac-bcx)}{9b^3c^3}\\ &-\frac{ax^2\log(1-ac-bcx)}{6b}+\frac{1}{9}x^3\log(1-ac-bcx)\\ &-\frac{a^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c}\\ &+\frac{a^3\operatorname{PolyLog}(2,c(a+bx))}{3b^3}+\frac{1}{3}x^3\operatorname{PolyLog}(2,c(a+bx))\end{aligned}$$

output
```
-1/3*a^2*x/b^2+1/6*a*(-a*c+1)*x/b^2/c-1/9*(-a*c+1)^2*x/b^2/c^2+1/12*a*x^2/
b-1/18*(-a*c+1)*x^2/b/c-1/27*x^3+1/6*a*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^3/c^2
-1/9*(-a*c+1)^3*ln(-b*c*x-a*c+1)/b^3/c^3-1/6*a*x^2*ln(-b*c*x-a*c+1)/b+1/9*
x^3*ln(-b*c*x-a*c+1)-1/3*a^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/c+1/3*a^3
*polylog(2,c*(b*x+a))/b^3+1/3*x^3*polylog(2,c*(b*x+a))
```

3.124.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.55

$$\int x^2 \operatorname{PolyLog}(2, c(a + bx))\,dx = \frac{-bcx(12 + 66a^2c^2 + 6bcx + 4b^2c^2x^2 - 3ac(14 + 5bcx)) + 6(-2 + 11a^3c^3 + 2b^3c^3x^3 + 6a^2c^2(-3 + bcx) + \cdots}{108b^3c^3}$$

input
```
Integrate[x^2*PolyLog[2, c*(a + b*x)],x]
```

output
```
(-(b*c*x*(12 + 66*a^2*c^2 + 6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x)
)) + 6*(-2 + 11*a^3*c^3 + 2*b^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c
- 3*b^2*c^3*x^2))*Log[1 - a*c - b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2,
 c*(a + b*x)])/(108*b^3*c^3)
```

3.124.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7152, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(2, c(a + bx))\,dx$$

$$\Big\downarrow \text{7152}$$

$$\frac{1}{3}b\int \frac{x^3 \log(-ac - bxc + 1)}{a + bx}dx + \frac{1}{3}x^3 \operatorname{PolyLog}(2, c(a + bx))$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{3}b\int \left(-\frac{\log(-ac - bxc + 1)a^3}{b^3(a + bx)} + \frac{\log(-ac - bxc + 1)a^2}{b^3} - \frac{x\log(-ac - bxc + 1)a}{b^2} + \frac{x^2\log(-ac - bxc + 1)}{b}\right)dx + \frac{1}{3}x^3 \operatorname{PolyLog}(2, c(a + bx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}b\left(\frac{a^3 \operatorname{PolyLog}(2,c(a+bx))}{b^4}-\frac{a^2(-ac-bcx+1)\log(-ac-bcx+1)}{b^4c}-\frac{a^2x}{b^3}-\frac{(1-ac)^3\log(-ac-bcx+1)}{3b^4c^3}+\right.$$
$$\frac{1}{3}x^3 \operatorname{PolyLog}(2,c(a+bx))$$

input
```
Int[x^2*PolyLog[2, c*(a + b*x)],x]
```

output
```
(x^3*PolyLog[2, c*(a + b*x)])/3 + (b*(-((a^2*x)/b^3) + (a*(1 - a*c)*x)/(2*
b^3*c) - ((1 - a*c)^2*x)/(3*b^3*c^2) + (a*x^2)/(4*b^2) - ((1 - a*c)*x^2)/(
6*b^2*c) - x^3/(9*b) + (a*(1 - a*c)^2*Log[1 - a*c - b*c*x])/(2*b^4*c^2) -
((1 - a*c)^3*Log[1 - a*c - b*c*x])/(3*b^4*c^3) - (a*x^2*Log[1 - a*c - b*c*
x])/(2*b^2) + (x^3*Log[1 - a*c - b*c*x])/(3*b) - (a^2*(1 - a*c - b*c*x)*Lo
g[1 - a*c - b*c*x])/(b^4*c) + (a^3*PolyLog[2, c*(a + b*x)])/b^4))/3
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.124.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{36x^3\operatorname{polylog}(2,c(bx+a))b^3c^3+12x^3\ln(1-c(bx+a))b^3c^3-4x^3b^3c^3-18x^2\ln(1-c(bx+a))a\,b^2c^3-12+15x^2a\,b^2c^3+36x\ln(}{}$
parts	$\frac{x^3\operatorname{polylog}(2,c(bx+a))}{3}+\frac{-3a^2c^3((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)-3a\,c^2\left(\frac{(-bcx-ac+1)^2\ln(-bcx-ac+1)}{2}-\right.}{}$
derivativedivides	$\frac{-\frac{\operatorname{polylog}(2,bcx+ac)a^3c^3}{3}+\operatorname{polylog}(2,bcx+ac)a^2c^2(bcx+ac)-\operatorname{polylog}(2,bcx+ac)ac(bcx+ac)^2+\frac{\operatorname{polylog}(2,bcx+ac)(bcx+ac)^3}{3}}{}$
default	$\frac{-\frac{\operatorname{polylog}(2,bcx+ac)a^3c^3}{3}+\operatorname{polylog}(2,bcx+ac)a^2c^2(bcx+ac)-\operatorname{polylog}(2,bcx+ac)ac(bcx+ac)^2+\frac{\operatorname{polylog}(2,bcx+ac)(bcx+ac)^3}{3}}{}$

input
```
int(x^2*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/108*(36*x^3*polylog(2,c*(b*x+a))*b^3*c^3+12*x^3*ln(1-c*(b*x+a))*b^3*c^3-
4*x^3*b^3*c^3-18*x^2*ln(1-c*(b*x+a))*a*b^2*c^3-12+15*x^2*a*b^2*c^3+36*x*ln
(1-c*(b*x+a))*a^2*b*c^3-66*x*a^2*b*c^3+36*polylog(2,c*(b*x+a))*a^3*c^3+66*
ln(1-c*(b*x+a))*a^3*c^3-6*x^2*b^2*c^2+117*a^3*c^3+42*x*a*b*c^2-108*ln(1-c*
(b*x+a))*a^2*c^2-129*a^2*c^2-12*b*c*x+54*ln(1-c*(b*x+a))*a*c+60*a*c-12*ln(
1-c*(b*x+a)))/b^3/c^3
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.63

$$\int x^2\operatorname{PolyLog}(2,c(a+bx))\,dx =$$
$$-\frac{4\,b^3c^3x^3-3\,(5\,ab^2c^3-2\,b^2c^2)x^2+6\,(11\,a^2bc^3-7\,abc^2+2\,bc)x-36\,(b^3c^3x^3+a^3c^3)\operatorname{Li}_2(bcx+ac)-6\,(2}{108\,b^3c^3}$$

input
```
integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-1/108*(4*b^3*c^3*x^3 - 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 + 6*(11*a^2*b*c^3
- 7*a*b*c^2 + 2*b*c)*x - 36*(b^3*c^3*x^3 + a^3*c^3)*dilog(b*c*x + a*c) - 6
*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^
2 + 9*a*c - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.90

$$\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx$$
$$= \begin{cases} 0 \\ \frac{x^3 \operatorname{Li}_2(ac)}{3} \\ 0 \\ -\frac{11a^3 \operatorname{Li}_1(ac+bcx)}{18b^3} + \frac{a^3 \operatorname{Li}_2(ac+bcx)}{3b^3} - \frac{a^2 x \operatorname{Li}_1(ac+bcx)}{3b^2} - \frac{11a^2x}{18b^2} + \frac{a^2 \operatorname{Li}_1(ac+bcx)}{b^3c} + \frac{ax^2 \operatorname{Li}_1(ac+bcx)}{6b} + \frac{5ax^2}{36b} + \frac{7ax}{18b^2c} - \frac{a \operatorname{Li}}{} \end{cases}$$

input
```
integrate(x**2*polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x**3*polylog(2, a*c)/3, Eq(b, 0)), (0
, Eq(c, 0)), (-11*a**3*polylog(1, a*c + b*c*x)/(18*b**3) + a**3*polylog(2,
 a*c + b*c*x)/(3*b**3) - a**2*x*polylog(1, a*c + b*c*x)/(3*b**2) - 11*a**2
*x/(18*b**2) + a**2*polylog(1, a*c + b*c*x)/(b**3*c) + a*x**2*polylog(1, a
*c + b*c*x)/(6*b) + 5*a*x**2/(36*b) + 7*a*x/(18*b**2*c) - a*polylog(1, a*c
 + b*c*x)/(2*b**3*c**2) - x**3*polylog(1, a*c + b*c*x)/9 + x**3*polylog(2,
 a*c + b*c*x)/3 - x**3/27 - x**2/(18*b*c) - x/(9*b**2*c**2) + polylog(1, a
*c + b*c*x)/(9*b**3*c**3), True))
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

$$\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx$$
$$= -\frac{(\log(bcx+ac)\log(-bcx-ac+1) + \operatorname{Li}_2(-bcx-ac+1))a^3}{3\,b^3}$$
$$+ \frac{36\,b^3c^3x^3\operatorname{Li}_2(bcx+ac) - 4\,b^3c^3x^3 + 3\,(5\,ab^2c^3 - 2\,b^2c^2)x^2 - 6\,(11\,a^2bc^3 - 7\,abc^2 + 2\,bc)x + 6\,(2\,b^3c^3x^3 -}{108\,b^3c^3}$$

input
```
integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^
3/b^3 + 1/108*(36*b^3*c^3*x^3*dilog(b*c*x + a*c) - 4*b^3*c^3*x^3 + 3*(5*a*
b^2*c^3 - 2*b^2*c^2)*x^2 - 6*(11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*x + 6*(2*b
^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9
*a*c - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

3.124.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx = \int x^2 \operatorname{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*dilog((b*x + a)*c), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{PolyLog}(2, c(a+bx))\,dx = \int x^2 \operatorname{polylog}(2, c\,(a+b\,x))\,dx$$

input
```
int(x^2*polylog(2, c*(a + b*x)),x)
```

output
```
int(x^2*polylog(2, c*(a + b*x)), x)
```

3.125 $\int x \operatorname{PolyLog}(2, c(a+bx))\, dx$

3.125.1 Optimal result

Integrand size = 11, antiderivative size = 152

$$\int x \operatorname{PolyLog}(2, c(a+bx))\, dx = \frac{ax}{2b} - \frac{(1-ac)x}{4bc} - \frac{x^2}{8} - \frac{(1-ac)^2 \log(1-ac-bcx)}{4b^2c^2} + \frac{1}{4}x^2 \log(1-ac-bcx) + \frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} - \frac{a^2 \operatorname{PolyLog}(2, c(a+bx))}{2b^2} + \frac{1}{2}x^2 \operatorname{PolyLog}(2, c(a+bx))$$

output
```
1/2*a*x/b-1/4*(-a*c+1)*x/b/c-1/8*x^2-1/4*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^2/c
^2+1/4*x^2*ln(-b*c*x-a*c+1)+1/2*a*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c-1/
2*a^2*polylog(2,c*(b*x+a))/b^2+1/2*x^2*polylog(2,c*(b*x+a))
```

3.125.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

$$\int x \operatorname{PolyLog}(2, c(a+bx))\, dx = \frac{-bcx(2-6ac+bcx) + (-2-6a^2c^2+2b^2c^2x^2-4ac(-2+bcx))\log(1-ac-bcx) - 4c^2(a^2-b^2x^2)\operatorname{Poly}}{8b^2c^2}$$

input
```
Integrate[x*PolyLog[2, c*(a + b*x)],x]
```

output
```
(-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-
2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a +
 b*x)])/(8*b^2*c^2)
```

3.125.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {7152, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(2, c(a+bx)) \, dx$$

$$\Big\downarrow \; 7152$$

$$\frac{1}{2} b \int \frac{x^2 \log(-ac - bxc + 1)}{a + bx} dx + \frac{1}{2} x^2 \operatorname{PolyLog}(2, c(a+bx))$$

$$\Big\downarrow \; 2863$$

$$\frac{1}{2} b \int \left(\frac{\log(-ac - bxc + 1) a^2}{b^2 (a + bx)} - \frac{\log(-ac - bxc + 1) a}{b^2} + \frac{x \log(-ac - bxc + 1)}{b} \right) dx + \frac{1}{2} x^2 \operatorname{PolyLog}(2, c(a+bx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} b \left(-\frac{a^2 \operatorname{PolyLog}(2, c(a+bx))}{b^3} - \frac{(1-ac)^2 \log(-ac - bcx + 1)}{2 b^3 c^2} + \frac{a(-ac - bcx + 1) \log(-ac - bcx + 1)}{b^3 c} - \frac{x(1-}{2b^2c} \right. \frac{1}{2} x^2 \operatorname{PolyLog}(2, c(a+bx))$$

input
```
Int[x*PolyLog[2, c*(a + b*x)],x]
```

output
```
(x^2*PolyLog[2, c*(a + b*x)])/2 + (b*((a*x)/b^2 - ((1 - a*c)*x)/(2*b^2*c)
- x^2/(4*b) - ((1 - a*c)^2*Log[1 - a*c - b*c*x])/(2*b^3*c^2) + (x^2*Log[1
- a*c - b*c*x])/(2*b) + (a*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b^3*c)
 - (a^2*PolyLog[2, c*(a + b*x)])/b^3))/2
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.125.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08

method	result
parts	$\frac{x^2 \operatorname{polylog}(2,c(bx+a))}{2} - \frac{-2a\,c^2((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)-c\left(\frac{(-bcx-ac+1)^2\ln(-bcx-ac+1)}{2} - \frac{(-bcx \ldots}{}\right.}{2b^2c^3}$
derivativedivides	$\frac{-\operatorname{polylog}(2,bcx+ac)ac(bcx+ac)+\frac{\operatorname{polylog}(2,bcx+ac)(bcx+ac)^2}{2}+ac((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)+\frac{(-bcx- \ldots}{}}{b^2c^2}$
default	$\frac{-\operatorname{polylog}(2,bcx+ac)ac(bcx+ac)+\frac{\operatorname{polylog}(2,bcx+ac)(bcx+ac)^2}{2}+ac((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)+\frac{(-bcx- \ldots}{}}{b^2c^2}$
parallelrisch	$\frac{4x^2\operatorname{polylog}(2,c(bx+a))b^2c^2+2x^2\ln(1-c(bx+a))b^2c^2-2-x^2b^2c^2-4x\ln(1-c(bx+a))ab\,c^2+6xab\,c^2-4\operatorname{polylog}(2,c(bx+ \ldots}{8b^2c^2}$

input
```
int(x*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*polylog(2,c*(b*x+a))-1/2/b^2/c^3*(-2*a*c^2*((-b*c*x-a*c+1)*ln(-b*c
*x-a*c+1)-1+b*c*x+a*c)-c*(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/4*(-b*c*
x-a*c+1)^2)+c*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)+a^2*c^3*dilog(
-b*c*x-a*c+1))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.72

$$\int x\,\mathrm{PolyLog}(2,c(a+bx))\,dx = -\frac{b^2c^2x^2-2\,(3\,abc^2-bc)x-4\,(b^2c^2x^2-a^2c^2)\mathrm{Li}_2(bcx+ac)-2\,(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1)\log}{8\,b^2c^2}$$

input
```
integrate(x*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-1/8*(b^2*c^2*x^2 - 2*(3*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - a^2*c^2)*dilo
g(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log
(-b*c*x - a*c + 1))/(b^2*c^2)
```

3.125.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x\,\mathrm{PolyLog}(2,c(a+bx))\,dx = \begin{cases} 0 \\ \frac{x^2\,\mathrm{Li}_2(ac)}{2} \\ 0 \\ \frac{3a^2\,\mathrm{Li}_1(ac+bcx)}{4b^2}-\frac{a^2\,\mathrm{Li}_2(ac+bcx)}{2b^2}+\frac{ax\,\mathrm{Li}_1(ac+bcx)}{2b}+\frac{3ax}{4b}-\frac{a\,\mathrm{Li}_1(ac+bcx)}{b^2c}-\frac{x^2\,\mathrm{Li}_1(ac+bcx)}{4}+\frac{x^2\,\mathrm{Li}_2(ac+bcx)}{2}-\frac{x^2}{8}-\frac{x}{4bc}- \end{cases}$$

input
```
integrate(x*polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x**2*polylog(2, a*c)/2, Eq(b, 0)), (0
, Eq(c, 0)), (3*a**2*polylog(1, a*c + b*c*x)/(4*b**2) - a**2*polylog(2, a*
c + b*c*x)/(2*b**2) + a*x*polylog(1, a*c + b*c*x)/(2*b) + 3*a*x/(4*b) - a*
polylog(1, a*c + b*c*x)/(b**2*c) - x**2*polylog(1, a*c + b*c*x)/4 + x**2*p
olylog(2, a*c + b*c*x)/2 - x**2/8 - x/(4*b*c) + polylog(1, a*c + b*c*x)/(4
*b**2*c**2), True))
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int x \operatorname{PolyLog}(2,c(a+bx))\,dx = \frac{(\log(bcx+ac)\log(-bcx-ac+1)+\operatorname{Li}_2(-bcx-ac+1))a^2}{2\,b^2} + \frac{4\,b^2c^2x^2\operatorname{Li}_2(bcx+ac)-b^2c^2x^2+2\,(3\,abc^2-bc)x+2\,(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1)\log(-bcx-}{8\,b^2c^2}$$

input
```
integrate(x*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^2
/b^2 + 1/8*(4*b^2*c^2*x^2*dilog(b*c*x + a*c) - b^2*c^2*x^2 + 2*(3*a*b*c^2
- b*c)*x + 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log(-b*c*
x - a*c + 1))/(b^2*c^2)
```

3.125.8 Giac [F]

$$\int x \operatorname{PolyLog}(2,c(a+bx))\,dx = \int x\operatorname{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*dilog((b*x + a)*c), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{PolyLog}(2,c(a+bx))\,dx = \int x\,\mathrm{polylog}(2,c\,(a+b\,x))\,dx$$

input
```
int(x*polylog(2, c*(a + b*x)),x)
```

output
```
int(x*polylog(2, c*(a + b*x)), x)
```

3.126 $\int \text{PolyLog}(2, c(a + bx)) \, dx$

3.126.1 Optimal result

Integrand size = 9, antiderivative size = 60

$$\int \text{PolyLog}(2, c(a + bx)) \, dx = -x - \frac{(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{a \text{PolyLog}(2, c(a + bx))}{b} + x \text{PolyLog}(2, c(a + bx))$$

output
```
-x-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+a*polylog(2,c*(b*x+a))/b+x*polylog(
2,c*(b*x+a))
```

3.126.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \text{PolyLog}(2, c(a + bx)) \, dx = \frac{-c(a + bx) + (-1 + c(a + bx)) \log(1 - c(a + bx)) + c(a + bx) \text{PolyLog}(2, c(a + bx))}{bc}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)],x]
```

output
```
(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Po
lyLog[2, c*(a + b*x)])/(b*c)
```

3.126.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {7149, 25, 2868, 2840, 2838, 2894, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,c(a+bx))\,dx$$

$$\Big\downarrow \text{7149}$$

$$-\int -\log(1-c(a+bx))dx+a\int -\frac{\log(1-c(a+bx))}{a+bx}dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\int \log(1-c(a+bx))dx-a\int \frac{\log(1-c(a+bx))}{a+bx}dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2868}$$

$$-a\int \frac{\log(-ac-bxc+1)}{a+bx}dx+\int \log(1-c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2840}$$

$$\int \log(1-c(a+bx))dx-\frac{a\int \frac{\log(1-c(a+bx))}{a+bx}d(a+bx)}{b}+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$\int \log(1-c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2894}$$

$$\int \log(-ac-bxc+1)dx+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{\int \log(-ac-bxc+1)d(-ac-bxc+1)}{bc}+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2732}$$

$$x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}-\frac{(-ac-bcx+1)\log(-ac-bcx+1)+ac+bcx-1}{bc}$$

input
```
Int[PolyLog[2, c*(a + b*x)],x]
```

output
```
-((-1 + a*c + b*c*x + (1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c)) + (a*
PolyLog[2, c*(a + b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2868
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

rule 7149
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Simp[p   Int[PolyLog[n - 1, c*(a + b*x)^p], x
], x] + Simp[a*p   Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /;
FreeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

3.126.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{(bcx+ac)\operatorname{polylog}(2,bcx+ac)-(-bcx-ac+1)\ln(-bcx-ac+1)+1-bcx-ac}{bc}$
default	$\frac{(bcx+ac)\operatorname{polylog}(2,bcx+ac)-(-bcx-ac+1)\ln(-bcx-ac+1)+1-bcx-ac}{bc}$
parts	$x\operatorname{polylog}(2,c(bx+a))+\frac{-c((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)+a\,c^{2}\operatorname{dilog}(-bcx-ac+1)}{b\,c^{2}}$
parallelrisch	$\frac{x\operatorname{polylog}(2,c(bx+a))a^{2}b\,c^{2}+x\ln(1-c(bx+a))a^{2}b\,c^{2}-x\,a^{2}b\,c^{2}+\operatorname{polylog}(2,c(bx+a))a^{3}c^{2}+\ln(1-c(bx+a))a^{3}c^{2}-x\operatorname{polylog}}{abc(ac-1)}$

input
```
int(polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/c*((b*c*x+a*c)*polylog(2,b*c*x+a*c)-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)+1-
b*c*x-a*c)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \operatorname{PolyLog}(2,c(a+bx))\,dx = -\frac{bcx-(bcx+ac)\operatorname{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x
- a*c + 1))/(b*c)
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \text{PolyLog}(2,c(a+bx))\,dx$$
$$=\begin{cases} 0 & \text{for } b=0 \wedge c=0 \\ x\,\text{Li}_2\left(ac\right) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ -\frac{a\,\text{Li}_1(ac+bcx)}{b}+\frac{a\,\text{Li}_2(ac+bcx)}{b}-x\,\text{Li}_1\left(ac+bcx\right)+x\,\text{Li}_2\left(ac+bcx\right)-x+\frac{\text{Li}_1(ac+bcx)}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x*polylog(2, a*c), Eq(b, 0)), (0, Eq(
c, 0)), (-a*polylog(1, a*c + b*c*x)/b + a*polylog(2, a*c + b*c*x)/b - x*po
lylog(1, a*c + b*c*x) + x*polylog(2, a*c + b*c*x) - x + polylog(1, a*c + b
*c*x)/(b*c), True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \text{PolyLog}(2,c(a+bx))\,dx$$
$$=-\frac{(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\text{Li}_2(-bcx-ac+1))a}{b}$$
$$+\frac{bcx\text{Li}_2(bcx+ac)-bcx+(bcx+ac-1)\log\left(-bcx-ac+1\right)}{bc}$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b +
(b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1
))/(b*c)
```

3.126.8 Giac [F]

$$\int \text{PolyLog}(2, c(a + bx))\,dx = \int \text{Li}_2((bx + a)c)\ dx$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c), x)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \text{PolyLog}(2, c(a + bx))\,dx = \frac{\text{polylog}(2, c\,(a + b\,x))\ (a + b\,x)}{b} - x - \frac{\ln\left(1 - c\,(a + b\,x)\right)}{b\,c} + \frac{\ln\left(1 - c\,(a + b\,x)\right)\ (a + b\,x)}{b}$$

input
```
int(polylog(2, c*(a + b*x)),x)
```

output
```
(polylog(2, c*(a + b*x))*(a + b*x))/b - x - log(1 - c*(a + b*x))/(b*c) + (
log(1 - c*(a + b*x))*(a + b*x))/b
```

3.127 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x}\,dx$

3.127.1 Optimal result

Integrand size = 13, antiderivative size = 401

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x}\,dx = \log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx)) + \frac{1}{2}\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right) - \log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right) + \frac{1}{2}\left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\left(\log(x) + \log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)^2 + \left(\log(1-c(a+bx)) - \log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) + \log(x)\,\text{PolyLog}(2,c(a+bx)) + \log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{PolyLog}\left(2, -\frac{bx}{a(1-c(a+bx))}\right) - \log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{PolyLog}\left(2,-\frac{bcx}{1-c(a+bx)}\right) + \left(\log(x)+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\text{PolyLog}(2,1 - c(a+bx)) - \text{PolyLog}\left(3,-\frac{bx}{a}\right)+\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right) - \text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)-\text{PolyLog}(3,1-c(a+bx))$$

output
```
ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))+1/2*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a
)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2+1/2*(
ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2+(ln(1-c*(b*x
+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)+ln(x)*polylog(2,c*(b*x+a)
)+ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))-ln(-a*(1-c*(b*x
+a))/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))+(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))
*polylog(2,1-c*(b*x+a))-polylog(3,-b*x/a)+polylog(3,-b*x/a/(1-c*(b*x+a)))-
polylog(3,-b*c*x/(1-c*(b*x+a)))-polylog(3,1-c*(b*x+a))
```

3.127.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.05

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x}\,dx$$
$$= \log(x)\log\left(1+\frac{bx}{a}\right)\log(1-ac-bcx) + \frac{1}{2}\left(-\log(c(a+bx))+\log\left(1+\frac{bx}{a}\right)\right)\log(1-ac-bcx)(-2\log(x)+\log(1-ac-bcx)) + \left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\log(1-ac-bcx)\log\left(\frac{a(-1+ac+bcx)}{bx}\right) + \frac{1}{2}\left(\log\left(\frac{1-ac}{bcx}\right)-\log\left(\frac{(1-ac)(a+bx)}{bx}\right) + \log\left(1+\frac{bx}{a}\right)\right)\log^2\left(\frac{a(-1+ac+bcx)}{bx}\right) + \left(\log(1-ac-bcx)-\log\left(\frac{a(-1+ac+bcx)}{bx}\right)\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right) + \left(\log(x)+\log\left(\frac{a(-1+ac+bcx)}{bx}\right)\right)\text{PolyLog}(2,1-ac-bcx) + \log\left(\frac{a(-1+ac+bcx)}{bx}\right)\left(-\text{PolyLog}\left(2,\frac{a(-1+ac+bcx)}{bx}\right) + \text{PolyLog}\left(2,\frac{-1+ac+bcx}{bcx}\right)\right) + \log(x)\text{PolyLog}(2,ac+bcx)-\text{PolyLog}\left(3,-\frac{bx}{a}\right)-\text{PolyLog}(3,1-ac-bcx) + \text{PolyLog}\left(3,\frac{a(-1+ac+bcx)}{bx}\right)-\text{PolyLog}\left(3,\frac{-1+ac+bcx}{bcx}\right)$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/x,x]
```

output
```
Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1
 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 +
(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*
c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/
(b*x)] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1
 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)
] + (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*
x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))
/(b*x)] + PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + Log[x]*PolyLog[2, a*c
+ b*c*x] - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[
3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]
```

3.127.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7151, 2890, 2885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{ 7151}$$

$$b\int \frac{\log(x)\log(-ac-bxc+1)}{a+bx}dx+\log(x)\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{ 2890}$$

$$\int \frac{\log\left(\frac{a+bx}{b}-\frac{a}{b}\right)\log(1-c(a+bx))}{a+bx}d(a+bx)+\log(x)\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{ 2885}$$

$$\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)-\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)-\text{PolyLog}(3,1-c(a+bx))+\text{PolyLog}\left(2,-\frac{bx}{a(1-c(a+bx))}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)-\text{PolyLog}\left(2,-\frac{bcx}{1-c(a+bx)}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\,\text{PolyLog}(2,c(a+bx))+\text{PolyLog}\left(2,1-\frac{a+bx}{a}\right)\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)+\text{PolyLog}(2,1-c(a+bx))\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log\left(\frac{a+bx}{b}-\frac{a}{b}\right)\right)+\frac{1}{2}\left(\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)+\log\left(\frac{a+bx}{a}\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)-\frac{1}{2}\left(\log\left(\frac{a+bx}{a}\right)-\log(c(a+bx))\right)\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log\left(\frac{a+bx}{b}-\frac{a}{b}\right)\right)^2+\log\left(\frac{a+bx}{a}\right)\log\left(\frac{a+bx}{b}-\frac{a}{b}\right)\log(1-c(a+bx))-\text{PolyLog}\left(3,1-\frac{a+bx}{a}\right)$$

input
```
Int[PolyLog[2, c*(a + b*x)]/x,x]
```

output
```
Log[(a + b*x)/a]*Log[-(a/b) + (a + b*x)/b]*Log[1 - c*(a + b*x)] + ((Log[(a
 + b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/
(a*(1 - c*(a + b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/2 - ((Log[(
a + b*x)/a] - Log[c*(a + b*x)])*(Log[-(a/b) + (a + b*x)/b] + Log[-((a*(1 -
 c*(a + b*x)))/(b*x))])^2)/2 + Log[x]*PolyLog[2, c*(a + b*x)] + (Log[1 - c
*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - (a + b*x
)/a] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a
 + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1
- c*(a + b*x)))] + (Log[-(a/b) + (a + b*x)/b] + Log[-((a*(1 - c*(a + b*x))
)/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, 1 - (a + b*x)/a] + Pol
yLog[3, -((b*x)/(a*(1 - c*(a + b*x))))] - PolyLog[3, -((b*c*x)/(1 - c*(a +
 b*x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

3.127. $\int \frac{\text{PolyLog}(2,c(a+bx))}{x}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 2885
```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

rule 2890
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Simp[1/l   Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f +
g*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
 c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

rule 7151
```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Simp[b/e   Int[Log[d
 + e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e},
x] && NeQ[c*(b*d - a*e) + e, 0]
```

3.127.4 Maple [F]

$$\int \frac{\text{polylog}\,(2, c(bx + a))}{x} dx$$

input
```
int(polylog(2,c*(b*x+a))/x,x)
```

output
```
int(polylog(2,c*(b*x+a))/x,x)
```

3.127.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x} \, dx = \int \frac{\text{Li}_2((bx + a)c)}{x} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/x, x)
```

3.127.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x} \, dx = \int \frac{\text{Li}_2(ac + bcx)}{x} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x,x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/x, x)
```

3.127.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x} \, dx = \int \frac{\text{Li}_2((bx + a)c)}{x} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(dilog((b*x + a)*c)/x, x)
```

3.127.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x} \, dx = \int \frac{\text{Li}_2((bx+a)c)}{x} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/x, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x} \, dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{x} \, dx$$

input
```
int(polylog(2, c*(a + b*x))/x,x)
```

output
```
int(polylog(2, c*(a + b*x))/x, x)
```

3.128 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$

3.128.1 Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2}\,dx = -\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{b\,\text{PolyLog}(2,c(a+bx))}{a} - \frac{\text{PolyLog}(2,c(a+bx))}{x} - \frac{b\,\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a}$$

output

```
-b*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a-b*polylog(2,c*(b*x+a))/a-polylog(
2,c*(b*x+a))/x-b*polylog(2,1-b*c*x/(-a*c+1))/a
```

3.128.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2}\,dx = -\frac{(a+bx)\,\text{PolyLog}(2,c(a+bx)) + bx\left(\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx) + \text{PolyLog}\left(2,\frac{-1+ac+bcx}{-1+ac}\right)\right)}{ax}$$

input

```
Integrate[PolyLog[2, c*(a + b*x)]/x^2,x]
```

output

```
-(((a + b*x)*PolyLog[2, c*(a + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 -
 a*c - b*c*x] + PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/(a*x))
```

3.128.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {7152, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2}dx$$

$$\Big\downarrow \text{7152}$$

$$-b\int \frac{\log(-ac-bxc+1)}{x(a+bx)}dx-\frac{\text{PolyLog}(2,c(a+bx))}{x}$$

$$\Big\downarrow \text{2863}$$

$$-b\int \left(\frac{\log(-ac-bxc+1)}{ax}-\frac{b\log(-ac-bxc+1)}{a(a+bx)}\right)dx-\frac{\text{PolyLog}(2,c(a+bx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}(2,c(a+bx))}{x}-b\left(\frac{\text{PolyLog}(2,c(a+bx))}{a}+\frac{\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a}+\frac{\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a}\right)$$

input
```
Int[PolyLog[2, c*(a + b*x)]/x^2,x]
```

output
```
-(PolyLog[2, c*(a + b*x)]/x) - b*((Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*
c*x])/a + PolyLog[2, c*(a + b*x)]/a + PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a)
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.128.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{\operatorname{polylog}(2,c(bx+a))}{x}-b\left(\frac{\operatorname{dilog}(-bcx-ac+1)}{a}+\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{bcx}{ac-1}\right)}{a}\right)$	80
derivativedivides	$bc\left(-\frac{\operatorname{polylog}(2,bcx+ac)}{bcx}-\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{bcx}{ac-1}\right)}{ac}-\frac{\operatorname{dilog}(-bcx-ac+1)}{ac}\right)$	94
default	$bc\left(-\frac{\operatorname{polylog}(2,bcx+ac)}{bcx}-\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{bcx}{ac-1}\right)}{ac}-\frac{\operatorname{dilog}(-bcx-ac+1)}{ac}\right)$	94

input
```
int(polylog(2,c*(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-polylog(2,c*(b*x+a))/x-b*(1/a*dilog(-b*c*x-a*c+1)+1/a*(dilog(-b*c*x/(a*c-
1))+ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1))))
```

3.128.5 Fricas [F]

$$\int \frac{\operatorname{PolyLog}(2,c(a+bx))}{x^2}\,dx=\int \frac{\operatorname{Li}_2((bx+a)c)}{x^2}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/x^2, x)
```

3.128.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x^2}\, dx = \int \frac{\text{Li}_2\,(ac + bcx)}{x^2}\, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x**2,x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/x**2, x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x^2}\, dx = \frac{(\log\,(bcx + ac)\log\,(-bcx - ac + 1) + \text{Li}_2(-bcx - ac + 1))b}{a} - \frac{\left(\log\,(-bcx - ac + 1)\log\left(-\frac{bcx+ac-1}{ac-1} + 1\right) + \text{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b}{a} - \frac{\text{Li}_2(bcx + ac)}{x}$$

input
```
integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/a - (
log(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x
 + a*c - 1)/(a*c - 1)))*b/a - dilog(b*c*x + a*c)/x
```

3.128.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{x^2}\, dx = \int \frac{\text{Li}_2((bx + a)c)}{x^2}\, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/x^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x^2}\, dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{x^2}\, dx$$

input
```
int(polylog(2, c*(a + b*x))/x^2,x)
```

output
```
int(polylog(2, c*(a + b*x))/x^2, x)
```

3.129 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3} \, dx$

3.129.1 Optimal result

Integrand size = 13, antiderivative size = 173

$$\begin{aligned}\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx = {} & \frac{b^2c\log(x)}{2a(1-ac)} - \frac{b^2c\log(1-ac-bcx)}{2a(1-ac)} + \frac{b\log(1-ac-bcx)}{2ax} \\ & + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2\,\text{PolyLog}(2,c(a+bx))}{2a^2} \\ & - \frac{\text{PolyLog}(2,c(a+bx))}{2x^2} + \frac{b^2\,\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}\end{aligned}$$

output
```
1/2*b^2*c*ln(x)/a/(-a*c+1)-1/2*b^2*c*ln(-b*c*x-a*c+1)/a/(-a*c+1)+1/2*b*ln(
-b*c*x-a*c+1)/a/x+1/2*b^2*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^2+1/2*b^2*
polylog(2,c*(b*x+a))/a^2-1/2*polylog(2,c*(b*x+a))/x^2+1/2*b^2*polylog(2,1-
b*c*x/(-a*c+1))/a^2
```

3.129.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx$$
$$= \frac{-((-1+ac)\left(a^2-b^2x^2\right)\text{PolyLog}(2,c(a+bx))) + bx\left(-abcx\log(x) + \left(a(-1+ac+bcx) + b(-1+ac)x\,\text{l}\right.\right.}{2a^2(-1+ac)x^2}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/x^3,x]
```

output
```
(-((-1 + a*c)*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*Lo
g[x]) + (a*(-1 + a*c + b*c*x) + b*(-1 + a*c)*x*Log[(b*c*x)/(1 - a*c)])*Log
[1 - a*c - b*c*x] + b*(-1 + a*c)*x*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c
)]))/(2*a^2*(-1 + a*c)*x^2)
```

3.129.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7152, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3} \, dx$$

$$\Big\downarrow \text{7152}$$

$$-\frac{1}{2}b\int \frac{\log(-ac-bxc+1)}{x^2(a+bx)}dx-\frac{\text{PolyLog}(2,c(a+bx))}{2x^2}$$

$$\Big\downarrow \text{2863}$$

$$-\frac{1}{2}b\int \left(\frac{\log(-ac-bxc+1)b^2}{a^2(a+bx)}-\frac{\log(-ac-bxc+1)b}{a^2x}+\frac{\log(-ac-bxc+1)}{ax^2}\right)dx-\frac{\text{PolyLog}(2,c(a+bx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}b\left(-\frac{b\,\text{PolyLog}(2,c(a+bx))}{a^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a^2}-\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a^2}-\frac{bc\log(x)}{a(1-ac)}+\frac{bc\,\text{l}}{\ldots}\right.$$
$$\frac{\text{PolyLog}(2,c(a+bx))}{2x^2}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/x^3,x]
```

output
```
-1/2*PolyLog[2, c*(a + b*x)]/x^2 - (b*(-((b*c*Log[x])/(a*(1 - a*c))) + (b*
c*Log[1 - a*c - b*c*x])/(a*(1 - a*c)) - Log[1 - a*c - b*c*x]/(a*x) - (b*Lo
g[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a^2 - (b*PolyLog[2, c*(a + b*x)
])/a^2 - (b*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a^2))/2
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.129.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

method	result
parts	$-\frac{\operatorname{polylog}(2,c(bx+a))}{2x^2}+\frac{b^2c\left(\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{bcx}{ac-1}\right)}{a^2c}+\frac{\operatorname{dilog}(-bcx-ac+1)}{a^2c}+\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+ \cdots}{(ac- \cdots}}{a} \cdots\right.}{2}$
derivativedivides	$b^2c^2\left(-\frac{\operatorname{polylog}(2,bcx+ac)}{2b^2c^2x^2}+\frac{\operatorname{dilog}(-bcx-ac+1)}{2a^2c^2}+\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(ac-1)bcx}}{2ac}+\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\cdots}{\cdots}\right.$
default	$b^2c^2\left(-\frac{\operatorname{polylog}(2,bcx+ac)}{2b^2c^2x^2}+\frac{\operatorname{dilog}(-bcx-ac+1)}{2a^2c^2}+\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(ac-1)bcx}}{2ac}+\frac{\operatorname{dilog}\left(-\frac{bcx}{ac-1}\right)+\cdots}{\cdots}\right.$

input
```
int(polylog(2,c*(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*polylog(2,c*(b*x+a))/x^2+1/2*b^2*c*(1/a^2/c*(dilog(-b*c*x/(a*c-1))+ln
(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1)))+1/a^2/c*dilog(-b*c*x-a*c+1)+1/a*(-1/(a*
c-1)*ln(-b*c*x)-ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c-1)/b/c/x))
```

3.129.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx = \int \frac{\text{Li}_2((bx+a)c)}{x^3}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/x^3, x)
```

3.129.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx = \int \frac{\text{Li}_2\left(ac+bcx\right)}{x^3}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x**3,x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/x**3, x)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx \\
&= -\frac{b^2c\log(x)}{2\,(a^2c-a)} - \frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))b^2}{2\,a^2} \\
&\quad + \frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right)+\text{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b^2}{2\,a^2} \\
&\quad - \frac{(a^2c-a)\text{Li}_2(bcx+ac)-(b^2cx^2+(abc-b)x)\log(-bcx-ac+1)}{2\,(a^2c-a)x^2}
\end{aligned}$$

input
```
integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*c*log(x)/(a^2*c - a) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1
) + dilog(-b*c*x - a*c + 1))*b^2/a^2 + 1/2*(log(-b*c*x - a*c + 1)*log(-(b*
c*x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x + a*c - 1)/(a*c - 1)))*b^2/a^
2 - 1/2*((a^2*c - a)*dilog(b*c*x + a*c) - (b^2*c*x^2 + (a*b*c - b)*x)*log(
-b*c*x - a*c + 1))/((a^2*c - a)*x^2)
```

3.129.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx=\int \frac{\text{Li}_2((bx+a)c)}{x^3}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/x^3, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx=\int \frac{\text{polylog}(2,c\,(a+b\,x))}{x^3}\,dx$$

input
```
int(polylog(2, c*(a + b*x))/x^3,x)
```

output
```
int(polylog(2, c*(a + b*x))/x^3, x)
```

3.130 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx$

3.130.1 Optimal result

Integrand size = 13, antiderivative size = 276

$$\begin{aligned}\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx =& -\frac{b^2c}{6a(1-ac)x}+\frac{b^3c^2\log(x)}{6a(1-ac)^2}-\frac{b^3c\log(x)}{3a^2(1-ac)} \\ &-\frac{b^3c^2\log(1-ac-bcx)}{6a(1-ac)^2}+\frac{b^3c\log(1-ac-bcx)}{3a^2(1-ac)} \\ &+\frac{b\log(1-ac-bcx)}{6ax^2}-\frac{b^2\log(1-ac-bcx)}{3a^2x} \\ &-\frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3}-\frac{b^3\,\text{PolyLog}(2,c(a+bx))}{3a^3} \\ &-\frac{\text{PolyLog}(2,c(a+bx))}{3x^3}-\frac{b^3\,\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{3a^3}\end{aligned}$$

output
```
-1/6*b^2*c/a/(-a*c+1)/x+1/6*b^3*c^2*ln(x)/a/(-a*c+1)^2-1/3*b^3*c*ln(x)/a^2
/(-a*c+1)-1/6*b^3*c^2*ln(-b*c*x-a*c+1)/a/(-a*c+1)^2+1/3*b^3*c*ln(-b*c*x-a*
c+1)/a^2/(-a*c+1)+1/6*b*ln(-b*c*x-a*c+1)/a/x^2-1/3*b^2*ln(-b*c*x-a*c+1)/a^
2/x-1/3*b^3*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^3-1/3*b^3*polylog(2,c*(b
*x+a))/a^3-1/3*polylog(2,c*(b*x+a))/x^3-1/3*b^3*polylog(2,1-b*c*x/(-a*c+1)
)/a^3
```

3.130.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.80

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx = -\frac{b\left(\frac{2ab^2c\log(x)}{1-ac}+\frac{2ab^2c\log(1-ac-bcx)}{-1+ac}-\frac{a^2\log(1-ac-bcx)}{x^2}+\frac{2ab\log(1-ac-bcx)}{x}+2b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)-\cdots\right)}{6a^3} - \frac{\text{PolyLog}(2,ac+bcx)}{3x^3}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/x^4,x]
```

output
```
-1/6*(b*((2*a*b^2*c*Log[x])/(1 - a*c) + (2*a*b^2*c*Log[1 - a*c - b*c*x])/(
-1 + a*c) - (a^2*Log[1 - a*c - b*c*x])/x^2 + (2*a*b*Log[1 - a*c - b*c*x])/
x + 2*b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x] - (a^2*b*c*(-1 + a*c
 + b*c*x*Log[x] - b*c*x*Log[1 - a*c - b*c*x]))/((-1 + a*c)^2*x) + 2*b^2*Po
lyLog[2, c*(a + b*x)] + 2*b^2*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/
a^3 - PolyLog[2, a*c + b*c*x]/(3*x^3)
```

3.130.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {7152, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx$$

$$\Big\downarrow \ 7152$$

$$-\frac{1}{3}b\int \frac{\log(-ac-bxc+1)}{x^3(a+bx)}dx - \frac{\text{PolyLog}(2,c(a+bx))}{3x^3}$$

$$\Big\downarrow \ 2863$$

$$-\frac{1}{3}b\int \left(-\frac{\log(-ac-bxc+1)b^3}{a^3(a+bx)}+\frac{\log(-ac-bxc+1)b^2}{a^3x}-\frac{\log(-ac-bxc+1)b}{a^2x^2}+\frac{\log(-ac-bxc+1)}{ax^3}\right)dx - \frac{\text{PolyLog}(2,c(a+bx))}{3x^3}$$

$\downarrow$ 2009

$$-\frac{1}{3}b\left(\frac{b^2\operatorname{PolyLog}(2,c(a+bx))}{a^3}+\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a^3}+\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a^3}+\frac{b^2c\log(x)}{a^2(1-ac)}-\right.$$
$$\frac{\operatorname{PolyLog}(2,c(a+bx))}{3x^3}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/x^4,x]
```

output
```
-1/3*PolyLog[2, c*(a + b*x)]/x^3 - (b*((b*c)/(2*a*(1 - a*c)*x) - (b^2*c^2*
Log[x])/(2*a*(1 - a*c)^2) + (b^2*c*Log[x])/(a^2*(1 - a*c)) + (b^2*c^2*Log[
1 - a*c - b*c*x])/(2*a*(1 - a*c)^2) - (b^2*c*Log[1 - a*c - b*c*x])/(a^2*(1
 - a*c)) - Log[1 - a*c - b*c*x]/(2*a*x^2) + (b*Log[1 - a*c - b*c*x])/(a^2*
x) + (b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a^3 + (b^2*PolyLog[
2, c*(a + b*x)])/a^3 + (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a^3))/3
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.130.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.88

method	result
parts	$-\frac{\text{polylog}(2,c(bx+a))}{3x^3}-\frac{b^3c^2\left(\frac{\text{dilog}(-bcx-ac+1)}{a^3c^2}+\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(ac-1)bcx}}{a^2c}+\frac{-\frac{\frac{ac-1}{bcx}+\ln(-bcx)}{2(ac-1)^2}+\frac{\ln(-bc\cdots}{\cdots}}{\cdots}\cdots\right.}{3}$
derivativedivides	$b^3c^3\left(-\frac{\text{polylog}(2,bcx+ac)}{3b^3c^3x^3}-\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(ac-1)bcx}}{3a^2c^2}-\frac{\text{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{b\cdots}{ac\cdots}\right.}{3a^3c^3}\right.$
default	$b^3c^3\left(-\frac{\text{polylog}(2,bcx+ac)}{3b^3c^3x^3}-\frac{-\frac{\ln(-bcx)}{ac-1}-\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(ac-1)bcx}}{3a^2c^2}-\frac{\text{dilog}\left(-\frac{bcx}{ac-1}\right)+\ln(-bcx-ac+1)\ln\left(-\frac{b\cdots}{ac\cdots}\right.}{3a^3c^3}\right.$

input
```
int(polylog(2,c*(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*polylog(2,c*(b*x+a))/x^3-1/3*b^3*c^2*(1/a^3/c^2*dilog(-b*c*x-a*c+1)+1
/a^2/c*(-1/(a*c-1)*ln(-b*c*x)-ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c-1)/b/c/
x)+1/a*(-1/2/(a*c-1)^2*((a*c-1)/b/c/x+ln(-b*c*x))+1/2*ln(-b*c*x-a*c+1)*(-b
*c*x+a*c-1)*(-b*c*x-a*c+1)/b^2/c^2/x^2/(a*c-1)^2)+1/a^3/c^2*(dilog(-b*c*x/
(a*c-1))+ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1))))
```

3.130.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx=\int \frac{\text{Li}_2((bx+a)c)}{x^4}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/x^4, x)
```

3.130.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x^4} \, dx = \int \frac{\text{Li}_2(ac+bcx)}{x^4} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x**4,x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/x**4, x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x^4} \, dx = \frac{(\log(bcx+ac)\log(-bcx-ac+1) + \text{Li}_2(-bcx-ac+1))b^3}{3a^3} - \frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right) + \text{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b^3}{3a^3} + \frac{(3ab^3c^2-2b^3c)\log(x)}{6(a^4c^2-2a^3c+a^2)} + \frac{(a^2b^2c^2-ab^2c)x^2 - 2(a^4c^2-2a^3c+a^2)\text{Li}_2(bcx+ac) - ((3ab^3c^2-2b^3c)x^3 + 2(a^2b^2c^2-2ab^2c+b^2)}{6(a^4c^2-2a^3c+a^2)x^3}$$

input
```
integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="maxima")
```

output
```
1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^3
/a^3 - 1/3*(log(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) +
dilog((b*c*x + a*c - 1)/(a*c - 1)))*b^3/a^3 + 1/6*(3*a*b^3*c^2 - 2*b^3*c)*
log(x)/(a^4*c^2 - 2*a^3*c + a^2) + 1/6*((a^2*b^2*c^2 - a*b^2*c)*x^2 - 2*(a
^4*c^2 - 2*a^3*c + a^2)*dilog(b*c*x + a*c) - ((3*a*b^3*c^2 - 2*b^3*c)*x^3
+ 2*(a^2*b^2*c^2 - 2*a*b^2*c + b^2)*x^2 - (a^3*b*c^2 - 2*a^2*b*c + a*b)*x)
*log(-b*c*x - a*c + 1))/((a^4*c^2 - 2*a^3*c + a^2)*x^3)
```

3.130.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x^4} \, dx = \int \frac{\text{Li}_2((bx+a)c)}{x^4} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/x^4, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{x^4} \, dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{x^4} \, dx$$

input
```
int(polylog(2, c*(a + b*x))/x^4,x)
```

output
```
int(polylog(2, c*(a + b*x))/x^4, x)
```

3.131 $\int x^2 \operatorname{PolyLog}(3, c(a+bx))\, dx$

3.131.1 Optimal result

Integrand size = 13, antiderivative size = 347

$$\begin{aligned}\int x^2 \operatorname{PolyLog}(3,c(a+bx))\,dx &= \frac{11a^2x}{18b^2}-\frac{5a(1-ac)x}{36b^2c}+\frac{(1-ac)^2x}{27b^2c^2}-\frac{5ax^2}{72b} \\ &+\frac{(1-ac)x^2}{54bc}+\frac{x^3}{81}-\frac{5a(1-ac)^2\log(1-ac-bcx)}{36b^3c^2} \\ &+\frac{(1-ac)^3\log(1-ac-bcx)}{27b^3c^3} \\ &+\frac{5ax^2\log(1-ac-bcx)}{36b}-\frac{1}{27}x^3\log(1-ac-bcx) \\ &+\frac{11a^2(1-ac-bcx)\log(1-ac-bcx)}{18b^3c} \\ &-\frac{11a^3\operatorname{PolyLog}(2,c(a+bx))}{18b^3} \\ &-\frac{a^2x\operatorname{PolyLog}(2,c(a+bx))}{3b^2}+\frac{ax^2\operatorname{PolyLog}(2,c(a+bx))}{6b} \\ &-\frac{1}{9}x^3\operatorname{PolyLog}(2,c(a+bx))+\frac{2a^3\operatorname{PolyLog}(3,c(a+bx))}{3b^3} \\ &-\frac{(a^3-b^3x^3)\operatorname{PolyLog}(3,c(a+bx))}{3b^3}\end{aligned}$$

output
```
11/18*a^2*x/b^2-5/36*a*(-a*c+1)*x/b^2/c+1/27*(-a*c+1)^2*x/b^2/c^2-5/72*a*x
^2/b+1/54*(-a*c+1)*x^2/b/c+1/81*x^3-5/36*a*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^3
/c^2+1/27*(-a*c+1)^3*ln(-b*c*x-a*c+1)/b^3/c^3+5/36*a*x^2*ln(-b*c*x-a*c+1)/
b-1/27*x^3*ln(-b*c*x-a*c+1)+11/18*a^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/
c-11/18*a^3*polylog(2,c*(b*x+a))/b^3-1/3*a^2*x*polylog(2,c*(b*x+a))/b^2+1/
6*a*x^2*polylog(2,c*(b*x+a))/b-1/9*x^3*polylog(2,c*(b*x+a))+2/3*a^3*polylo
g(3,c*(b*x+a))/b^3-1/3*(-b^3*x^3+a^3)*polylog(3,c*(b*x+a))/b^3
```

3.131.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.85

$$\int x^2 \operatorname{PolyLog}(3, c(a+bx))\, dx$$
$$= \frac{24ac - 150a^2c^2 + 575a^3c^3 + 24bcx - 138abc^2x + 510a^2bc^3x + 12b^2c^2x^2 - 57ab^2c^3x^2 + 8b^3c^3x^3 + 24\log(1}{}$$

input
```
Integrate[x^2*PolyLog[3, c*(a + b*x)],x]
```

output
```
(24*a*c - 150*a^2*c^2 + 575*a^3*c^3 + 24*b*c*x - 138*a*b*c^2*x + 510*a^2*b
*c^3*x + 12*b^2*c^2*x^2 - 57*a*b^2*c^3*x^2 + 8*b^3*c^3*x^3 + 24*Log[1 - a*
c - b*c*x] - 162*a*c*Log[1 - a*c - b*c*x] + 648*a^2*c^2*Log[1 - a*c - b*c*
x] - 510*a^3*c^3*Log[1 - a*c - b*c*x] - 396*a^2*b*c^3*x*Log[1 - a*c - b*c*
x] + 90*a*b^2*c^3*x^2*Log[1 - a*c - b*c*x] - 24*b^3*c^3*x^3*Log[1 - a*c -
b*c*x] - 36*c^3*(11*a^3 + 6*a^2*b*x - 3*a*b^2*x^2 + 2*b^3*x^3)*PolyLog[2,
c*(a + b*x)] + 216*c^3*(a^3 + b^3*x^3)*PolyLog[3, c*(a + b*x)])/(648*b^3*c
^3)
```

3.131.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(3, c(a+bx))\, dx$$

$\Big\downarrow$ 7153

$$\frac{\int \left(\frac{2\,\text{PolyLog}(2,c(a+bx))a^3}{a+bx} - \text{PolyLog}(2,c(a+bx))a^2 + bx\,\text{PolyLog}(2,c(a+bx))a - b^2x^2\,\text{PolyLog}(2,c(a+bx))\right)dx}{3b^2} - \frac{\left(a^3-b^3x^3\right)\text{PolyLog}(3,c(a+bx))}{3b^3}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{11a^3\,\text{PolyLog}(2,c(a+bx))}{6b} + \frac{2a^3\,\text{PolyLog}(3,c(a+bx))}{b} - a^2x\,\text{PolyLog}(2,c(a+bx)) + \frac{11a^2(-ac-bcx+1)\log(-ac-bcx+1)}{6bc} + \frac{11a^2x}{6}}{} - \frac{\left(a^3-b^3x^3\right)\text{PolyLog}(3,c(a+bx))}{3b^3}$$

input
```
Int[x^2*PolyLog[3, c*(a + b*x)],x]
```

output
```
-1/3*((a^3 - b^3*x^3)*PolyLog[3, c*(a + b*x)])/b^3 + ((11*a^2*x)/6 - (5*a*
(1 - a*c)*x)/(12*c) + ((1 - a*c)^2*x)/(9*c^2) - (5*a*b*x^2)/24 + (b*(1 - a
*c)*x^2)/(18*c) + (b^2*x^3)/27 - (5*a*(1 - a*c)^2*Log[1 - a*c - b*c*x])/(1
2*b*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(9*b*c^3) + (5*a*b*x^2*Log[1
 - a*c - b*c*x])/12 - (b^2*x^3*Log[1 - a*c - b*c*x])/9 + (11*a^2*(1 - a*c
- b*c*x)*Log[1 - a*c - b*c*x])/(6*b*c) - (11*a^3*PolyLog[2, c*(a + b*x)])/
(6*b) - a^2*x*PolyLog[2, c*(a + b*x)] + (a*b*x^2*PolyLog[2, c*(a + b*x)])/
2 - (b^2*x^3*PolyLog[2, c*(a + b*x)])/3 + (2*a^3*PolyLog[3, c*(a + b*x)])/
b)/(3*b^2)
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7153
```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Simp[p/((m + 1)*b^m)   Int[ExpandIntegrand[PolyLog[n -
 1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
 /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

3.131.4 Maple [F]

$$\int x^2 \operatorname{polylog}(3, c(bx + a))\, dx$$

input
```
int(x^2*polylog(3,c*(b*x+a)),x)
```

output
```
int(x^2*polylog(3,c*(b*x+a)),x)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.63

$$\int x^2 \operatorname{PolyLog}(3, c(a + bx))\, dx = \frac{8 b^3c^3x^3 - 3(19 ab^2c^3 - 4 b^2c^2)x^2 + 6(85 a^2bc^3 - 23 abc^2 + 4 bc)x - 36(2 b^3c^3x^3 - 3 ab^2c^3x^2 + 6 a^2bc^3x +}{}$$

input
```
integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

output
```
1/648*(8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(85*a^2*b*c^3
- 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^
3*x + 11*a^3*c^3)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2
 + 66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*
c + 1) + 216*(b^3*c^3*x^3 + a^3*c^3)*polylog(3, b*c*x + a*c))/(b^3*c^3)
```

3.131.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}(3, c(a + bx))\, dx = \int x^2 \operatorname{Li}_3(ac + bcx)\, dx$$

input
```
integrate(x**2*polylog(3,c*(b*x+a)),x)
```

output
```
Integral(x**2*polylog(3, a*c + b*c*x), x)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.76

$$\int x^2 \operatorname{PolyLog}(3, c(a+bx))\,dx$$
$$= \frac{11\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\operatorname{Li}_2(-bcx-ac+1)\right)a^3}{18\,b^3} + \frac{a^3\operatorname{Li}_3(bcx+ac)}{3\,b^3}$$
$$+ \frac{216\,b^3c^3x^3\operatorname{Li}_3(bcx+ac)+8\,b^3c^3x^3-3\left(19\,ab^2c^3-4\,b^2c^2\right)x^2+6\left(85\,a^2bc^3-23\,abc^2+4\,bc\right)x-36\left(2\,b^3c\right.}{}$$

input
```
integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

output
```
11/18*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a
^3/b^3 + 1/3*a^3*polylog(3, b*c*x + a*c)/b^3 + 1/648*(216*b^3*c^3*x^3*poly
log(3, b*c*x + a*c) + 8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6
*(85*a^2*b*c^3 - 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x
^2 + 6*a^2*b*c^3*x)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x
^2 + 66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x -
a*c + 1))/(b^3*c^3)
```

3.131.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}(3, c(a+bx))\,dx = \int x^2\operatorname{Li}_3((bx+a)c)\,dx$$

input
```
integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*polylog(3, (b*x + a)*c), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{PolyLog}(3, c(a+bx)) \, dx = \int x^2 \operatorname{polylog}(3, c\,(a+b\,x)) \, dx$$

input
```
int(x^2*polylog(3, c*(a + b*x)),x)
```

output
```
int(x^2*polylog(3, c*(a + b*x)), x)
```

3.132 $\int x \operatorname{PolyLog}(3, c(a+bx)) \, dx$

3.132.1 Optimal result

Integrand size = 11, antiderivative size = 198

$$\begin{aligned}\int x \operatorname{PolyLog}(3, c(a+bx)) \, dx = & -\frac{3ax}{4b} + \frac{(1-ac)x}{8bc} + \frac{x^2}{16} + \frac{(1-ac)^2 \log(1-ac-bcx)}{8b^2c^2} \\ & - \frac{1}{8}x^2 \log(1-ac-bcx) - \frac{3a(1-ac-bcx)\log(1-ac-bcx)}{4b^2c} \\ & + \frac{3a^2 \operatorname{PolyLog}(2, c(a+bx))}{4b^2} + \frac{ax \operatorname{PolyLog}(2, c(a+bx))}{2b} \\ & - \frac{1}{4}x^2 \operatorname{PolyLog}(2, c(a+bx)) \\ & - \frac{(a^2-b^2x^2) \operatorname{PolyLog}(3, c(a+bx))}{2b^2}\end{aligned}$$

output
```
-3/4*a*x/b+1/8*(-a*c+1)*x/b/c+1/16*x^2+1/8*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^2
/c^2-1/8*x^2*ln(-b*c*x-a*c+1)-3/4*a*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c+
3/4*a^2*polylog(2,c*(b*x+a))/b^2+1/2*a*x*polylog(2,c*(b*x+a))/b-1/4*x^2*po
lylog(2,c*(b*x+a))-1/2*(-b^2*x^2+a^2)*polylog(3,c*(b*x+a))/b^2
```

3.132.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00

$$\int x \operatorname{PolyLog}(3, c(a+bx))\,dx$$
$$= \frac{2ac - 15a^2c^2 + 2bcx - 14abc^2x + b^2c^2x^2 + 2\log(1-ac-bcx) - 16ac\log(1-ac-bcx) + 14a^2c^2\log(1-$$

input
```
Integrate[x*PolyLog[3, c*(a + b*x)],x]
```

output
```
(2*a*c - 15*a^2*c^2 + 2*b*c*x - 14*a*b*c^2*x + b^2*c^2*x^2 + 2*Log[1 - a*c
 - b*c*x] - 16*a*c*Log[1 - a*c - b*c*x] + 14*a^2*c^2*Log[1 - a*c - b*c*x]
+ 12*a*b*c^2*x*Log[1 - a*c - b*c*x] - 2*b^2*c^2*x^2*Log[1 - a*c - b*c*x] +
 4*c^2*(3*a^2 + 2*a*b*x - b^2*x^2)*PolyLog[2, c*(a + b*x)] - 8*c^2*(a^2 -
b^2*x^2)*PolyLog[3, c*(a + b*x)])/(16*b^2*c^2)
```

3.132.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(3, c(a+bx))\,dx$$
$$\Big\downarrow \text{ 7153}$$
$$\frac{\int (a \operatorname{PolyLog}(2, c(a+bx)) - bx \operatorname{PolyLog}(2, c(a+bx)))dx}{2b} - \frac{(a^2-b^2x^2)\operatorname{PolyLog}(3, c(a+bx))}{2b^2}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\frac{3a^2 \operatorname{PolyLog}(2,c(a+bx))}{2b} + \frac{(1-ac)^2\log(-ac-bcx+1)}{4bc^2} - \frac{1}{2}bx^2 \operatorname{PolyLog}(2, c(a+bx)) + ax \operatorname{PolyLog}(2, c(a+bx)) - \frac{1}{4}bx^2\log(-}{2b}$$
$$\frac{(a^2-b^2x^2)\operatorname{PolyLog}(3, c(a+bx))}{2b^2}$$

input
```
Int[x*PolyLog[3, c*(a + b*x)],x]
```

output
```
((-3*a*x)/2 + ((1 - a*c)*x)/(4*c) + (b*x^2)/8 + ((1 - a*c)^2*Log[1 - a*c -
 b*c*x])/(4*b*c^2) - (b*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(2*b*c) + (3*a^2*PolyLog[2, c*(a + b*x)])/(2*b) +
 a*x*PolyLog[2, c*(a + b*x)] - (b*x^2*PolyLog[2, c*(a + b*x)])/2)/(2*b) -
((a^2 - b^2*x^2)*PolyLog[3, c*(a + b*x)])/(2*b^2)
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7153
```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Simp[p/((m + 1)*b^m)   Int[ExpandIntegrand[PolyLog[n -
 1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
 /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

3.132.4 Maple [F]

$$\int x \operatorname{polylog}(3, c(bx+a))\, dx$$

input
```
int(x*polylog(3,c*(b*x+a)),x)
```

output
```
int(x*polylog(3,c*(b*x+a)),x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int x \operatorname{PolyLog}(3, c(a+bx))\, dx = \frac{b^2c^2x^2 - 2\,(7\,abc^2 - bc)x - 4\,(b^2c^2x^2 - 2\,abc^2x - 3\,a^2c^2)\operatorname{Li}_2(bcx + ac) - 2\,(b^2c^2x^2 - 6\,abc^2x - 7\,a^2c^2 + 8\,c}{16\,b^2c^2}$$

input
```
integrate(x*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*b*c^2*x -
 3*a^2*c^2)*dilog(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2
+ 8*a*c - 1)*log(-b*c*x - a*c + 1) + 8*(b^2*c^2*x^2 - a^2*c^2)*polylog(3,
b*c*x + a*c))/(b^2*c^2)
```

3.132.6 Sympy [F]

$$\int x \operatorname{PolyLog}(3, c(a+bx))\, dx = \int x \operatorname{Li}_3(ac+bcx)\, dx$$

input
```
integrate(x*polylog(3,c*(b*x+a)),x)
```

output
```
Integral(x*polylog(3, a*c + b*c*x), x)
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97

$$\int x \operatorname{PolyLog}(3, c(a+bx))\, dx$$
$$= -\frac{3\left(\log(bcx+ac)\log(-bcx-ac+1)+\operatorname{Li}_2(-bcx-ac+1)\right)a^2}{4\,b^2} - \frac{a^2\operatorname{Li}_3(bcx+ac)}{2\,b^2}$$
$$+ \frac{8\,b^2c^2x^2\operatorname{Li}_3(bcx+ac)+b^2c^2x^2-2\,(7\,abc^2-bc)x-4\,(b^2c^2x^2-2\,abc^2x)\operatorname{Li}_2(bcx+ac)-2\,(b^2c^2x^2-6\,a}{16\,b^2c^2}$$

input
```
integrate(x*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-3/4*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^
2/b^2 - 1/2*a^2*polylog(3, b*c*x + a*c)/b^2 + 1/16*(8*b^2*c^2*x^2*polylog(
3, b*c*x + a*c) + b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2
*a*b*c^2*x)*dilog(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2
+ 8*a*c - 1)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

3.132.8 Giac [F]

$$\int x \operatorname{PolyLog}(3, c(a+bx)) \, dx = \int x\operatorname{Li}_3((bx+a)c) \, dx$$

input
```
integrate(x*polylog(3,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*polylog(3, (b*x + a)*c), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int x \operatorname{PolyLog}(3, c(a+bx)) \, dx = \int x \operatorname{polylog}(3, c\,(a+b\,x)) \, dx$$

input
```
int(x*polylog(3, c*(a + b*x)),x)
```

output
```
int(x*polylog(3, c*(a + b*x)), x)
```

3.133 $\int \text{PolyLog}(3,c(a+bx))\,dx$

3.133.1 Optimal result

Integrand size = 9, antiderivative size = 84

$$\int \text{PolyLog}(3,c(a+bx))\,dx = x+\frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} - \frac{a\,\text{PolyLog}(2,c(a+bx))}{b} - x\,\text{PolyLog}(2,c(a+bx)) + \frac{a\,\text{PolyLog}(3,c(a+bx))}{b} + x\,\text{PolyLog}(3,c(a+bx))$$

output
```
x+(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c-a*polylog(2,c*(b*x+a))/b-x*polylog(2
,c*(b*x+a))+a*polylog(3,c*(b*x+a))/b+x*polylog(3,c*(b*x+a))
```

3.133.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int \text{PolyLog}(3,c(a+bx))\,dx = \frac{(a+bx)\left(1-\log(1-c(a+bx))+\frac{\log(1-c(a+bx))}{c(a+bx)}-\text{PolyLog}(2,c(a+bx))+\text{PolyLog}(3,c(a+bx))\right)}{b}$$

input
```
Integrate[PolyLog[3, c*(a + b*x)],x]
```

output
```
((a + b*x)*(1 - Log[1 - c*(a + b*x)] + Log[1 - c*(a + b*x)]/(c*(a + b*x))
- PolyLog[2, c*(a + b*x)] + PolyLog[3, c*(a + b*x)]))/b
```

3.133.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {7149, 7143, 7149, 25, 2868, 2840, 2838, 2894, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(3,c(a+bx))\,dx$$

$$\Big\downarrow \text{7149}$$

$$-\int \text{PolyLog}(2,c(a+bx))dx+a\int \frac{\text{PolyLog}(2,c(a+bx))}{a+bx}dx+x\,\text{PolyLog}(3,c(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$-\int \text{PolyLog}(2,c(a+bx))dx+x\,\text{PolyLog}(3,c(a+bx))+\frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{7149}$$

$$\int -\log(1-c(a+bx))dx-a\int -\frac{\log(1-c(a+bx))}{a+bx}dx-x\,\text{PolyLog}(2,c(a+bx))+x\,\text{PolyLog}(3,c(a+bx))+\frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$-\int \log(1-c(a+bx))dx+a\int \frac{\log(1-c(a+bx))}{a+bx}dx-x\,\text{PolyLog}(2,c(a+bx))+x\,\text{PolyLog}(3,c(a+bx))+\frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2868}$$

$$a\int \frac{\log(-ac-bxc+1)}{a+bx}dx-\int \log(1-c(a+bx))dx-x\,\text{PolyLog}(2,c(a+bx))+x\,\text{PolyLog}(3,c(a+bx))+\frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2840}$$

$$-\int \log(1-c(a+bx))dx+\frac{a\int \frac{\log(1-c(a+bx))}{a+bx}d(a+bx)}{b}-x\,\text{PolyLog}(2,c(a+bx))+x\,\text{PolyLog}(3,c(a+bx))+\frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\int \log(1-c(a+bx))dx - x\,\text{PolyLog}(2,c(a+bx)) - \frac{a\,\text{PolyLog}(2,c(a+bx))}{b} + x\,\text{PolyLog}(3,c(a+bx)) + \frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2894}$$

$$-\int \log(-ac-bxc+1)dx - x\,\text{PolyLog}(2,c(a+bx)) - \frac{a\,\text{PolyLog}(2,c(a+bx))}{b} + x\,\text{PolyLog}(3,c(a+bx)) + \frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log(-ac-bxc+1)d(-ac-bxc+1)}{bc} - x\,\text{PolyLog}(2,c(a+bx)) - \frac{a\,\text{PolyLog}(2,c(a+bx))}{b} + x\,\text{PolyLog}(3,c(a+bx)) + \frac{a\,\text{PolyLog}(3,c(a+bx))}{b}$$

$$\Big\downarrow \text{2732}$$

$$-x\,\text{PolyLog}(2,c(a+bx)) - \frac{a\,\text{PolyLog}(2,c(a+bx))}{b} + x\,\text{PolyLog}(3,c(a+bx)) + \frac{a\,\text{PolyLog}(3,c(a+bx))}{b} + \frac{(-ac-bcx+1)\log(-ac-bcx+1)+ac+bcx-1}{bc}$$

input
```
Int[PolyLog[3, c*(a + b*x)],x]
```

output
```
(-1 + a*c + b*c*x + (1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) - (a*Pol
yLog[2, c*(a + b*x)])/b - x*PolyLog[2, c*(a + b*x)] + (a*PolyLog[3, c*(a +
 b*x)])/b + x*PolyLog[3, c*(a + b*x)]
```

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2868
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7149
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Simp[p   Int[PolyLog[n - 1, c*(a + b*x)^p], x
], x] + Simp[a*p   Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /;
FreeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

3.133.4 Maple [F]

$$\int \operatorname{polylog}(3, c(bx + a))\, dx$$

input
```
int(polylog(3,c*(b*x+a)),x)
```

output
```
int(polylog(3,c*(b*x+a)),x)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \text{PolyLog}(3, c(a+bx))\, dx = \frac{bcx - (bcx+ac)\text{Li}_2(bcx+ac) - (bcx+ac-1)\log(-bcx-ac+1) + (bcx+ac)\text{polylog}(3, bcx+ac)}{bc}$$

input
```
integrate(polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

output
```
(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
 a*c + 1) + (b*c*x + a*c)*polylog(3, b*c*x + a*c))/(b*c)
```

3.133.6 Sympy [F]

$$\int \text{PolyLog}(3, c(a+bx))\, dx = \int \text{Li}_3\left(c(a+bx)\right)\, dx$$

input
```
integrate(polylog(3,c*(b*x+a)),x)
```

output
```
Integral(polylog(3, c*(a + b*x)), x)
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \text{PolyLog}(3, c(a+bx))\, dx = \frac{(\log(bcx+ac)\log(-bcx-ac+1) + \text{Li}_2(-bcx-ac+1))a}{b} + \frac{a\text{Li}_3(bcx+ac)}{b} - \frac{bcx\text{Li}_2(bcx+ac) - bcx\text{Li}_3(bcx+ac) - bcx + (bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

input
```
integrate(polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

output
```
(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + a
*polylog(3, b*c*x + a*c)/b - (b*c*x*dilog(b*c*x + a*c) - b*c*x*polylog(3,
b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))/(b*c)
```

3.133.8 Giac [F]

$$\int \text{PolyLog}(3, c(a+bx))\,dx = \int \text{Li}_3((bx+a)c)\,dx$$

input
```
integrate(polylog(3,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(polylog(3, (b*x + a)*c), x)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \text{PolyLog}(3, c(a+bx))\,dx = x - \frac{\text{polylog}(2, c\,(a+b\,x))\,\,(a+b\,x)}{b} + \frac{\text{polylog}(3, c\,(a+b\,x))\,\,(a+b\,x)}{b} + \frac{\ln\left(c\,(a+b\,x)-1\right)}{b\,c} - \frac{\ln\left(1-c\,(a+b\,x)\right)\,\,(a+b\,x)}{b}$$

input
```
int(polylog(3, c*(a + b*x)),x)
```

output
```
x - (polylog(2, c*(a + b*x))*(a + b*x))/b + (polylog(3, c*(a + b*x))*(a +
b*x))/b + log(c*(a + b*x) - 1)/(b*c) - (log(1 - c*(a + b*x))*(a + b*x))/b
```

3.134 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x} dx$

3.134.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x} dx = \text{Int}\left(\frac{\text{PolyLog}(3,ac+bcx)}{x},x\right)$$

output
```
int(polylog(3,b*c*x+a*c)/x,x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x} dx = \int \frac{\text{PolyLog}(3,c(a+bx))}{x} dx$$

input
```
Integrate[PolyLog[3, c*(a + b*x)]/x,x]
```

output
```
Integrate[PolyLog[3, c*(a + b*x)]/x, x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x} \, dx$$
$$\Big\downarrow \text{7292}$$
$$\int \frac{\text{PolyLog}(3,ac+bxc)}{x}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\text{PolyLog}(3,ac+bxc)}{x}dx$$

input
```
Int[PolyLog[3, c*(a + b*x)]/x,x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.134.4 Maple [F]

$$\int \frac{\text{polylog}(3, c(bx+a))}{x} dx$$

input
```
int(polylog(3,c*(b*x+a))/x,x)
```

output
```
int(polylog(3,c*(b*x+a))/x,x)
```

3.134.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x} dx = \int \frac{\text{Li}_3((bx+a)c)}{x} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(polylog(3, b*c*x + a*c)/x, x)
```

3.134.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x} dx = \int \frac{\text{Li}_3(ac+bcx)}{x} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x,x)
```

output
```
Integral(polylog(3, a*c + b*c*x)/x, x)
```

3.134.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x} \, dx = \int \frac{\text{Li}_3((bx+a)c)}{x} \, dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(polylog(3, (b*x + a)*c)/x, x)
```

3.134.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x} \, dx = \int \frac{\text{Li}_3((bx+a)c)}{x} \, dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(polylog(3, (b*x + a)*c)/x, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x} \, dx = \int \frac{\text{polylog}(3, c\,(a+b\,x))}{x} \, dx$$

input
```
int(polylog(3, c*(a + b*x))/x,x)
```

output
```
int(polylog(3, c*(a + b*x))/x, x)
```

3.135 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x^2} \, dx$

3.135.1 Optimal result

Integrand size = 13, antiderivative size = 486

$$\int \frac{\operatorname{PolyLog}(3,c(a+bx))}{x^2}\,dx = \frac{b\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{a} + \frac{b\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)}{2a} + \frac{b\left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\left(\log(x)+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)^2}{2a} + \frac{b\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a}\right)}{a} + \frac{b\log(x)\operatorname{PolyLog}(2,c(a+bx))}{a} + \frac{b\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\operatorname{PolyLog}\left(2,-\frac{bx}{a(1-c(a+bx))}\right)}{a} - \frac{b\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\operatorname{PolyLog}\left(2,-\frac{bcx}{1-c(a+bx)}\right)}{a} + \frac{b\left(\log(x)+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\operatorname{PolyLog}(2,1-c(a+bx))}{a} - \frac{b\operatorname{PolyLog}\left(3,-\frac{bx}{a}\right)}{a} - \frac{2b\operatorname{PolyLog}(3,c(a+bx))}{a} + \frac{\left(b-\frac{a}{x}\right)\operatorname{PolyLog}(3,c(a+bx))}{a} + \frac{b\operatorname{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{a} - \frac{b\operatorname{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{a} - \frac{b\operatorname{PolyLog}(3,1-c(a+bx))}{a}$$

output
```
b*ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))/a+1/2*b*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*
(b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2
/a+1/2*b*(ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/a+
b*(ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/a+b*ln(x)*p
olylog(2,c*(b*x+a))/a+b*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*
x+a)))/a-b*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/a+b*(l
n(x)+ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/a-b*polylog(3,-b*x/a
)/a-2*b*polylog(3,c*(b*x+a))/a+(b-a/x)*polylog(3,c*(b*x+a))/a+b*polylog(3,
-b*x/a/(1-c*(b*x+a)))/a-b*polylog(3,-b*c*x/(1-c*(b*x+a)))/a-b*polylog(3,1-
c*(b*x+a))/a
```

3.135.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.98

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} \, dx = -\frac{\text{PolyLog}(3, c(a+bx))}{x}$$
$$+\frac{b\Big(\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-ac-bcx)+\frac{1}{2}\left(-\log(c(a+bx))+\log\left(1+\frac{bx}{a}\right)\right)\log(1-ac-bcx)(-2\log(\cdots}{\cdots}$$

input
```
Integrate[PolyLog[3, c*(a + b*x)]/x^2,x]
```

output
```
-(PolyLog[3, c*(a + b*x)]/x) + (b*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b
*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*L
og[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*L
og[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b
*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a])*Log[(a*(-1 +
 a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*
c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] - Log[a + b*x])*PolyLog[2,
c*(a + b*x)] + Log[a + b*x]*PolyLog[2, c*(a + b*x)] + (Log[x] + Log[(a*(-1
 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c +
b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1
 + a*c + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, c*(a + b*x
)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)
] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]))/a
```

3.135.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} \, dx$$
$$\Big\downarrow \; 7153$$
$$\frac{\left(b-\frac{a}{x}\right)\text{PolyLog}(3, c(a+bx))}{a} - b^2 \int \left(\frac{2\,\text{PolyLog}(2, c(a+bx))}{a(a+bx)} - \frac{\text{PolyLog}(2, c(a+bx))}{abx}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(b-\frac{a}{x}\right)\operatorname{PolyLog}(3,c(a+bx))}{a}-$$

$$b^2\left(\frac{2\operatorname{PolyLog}(3,c(a+bx))}{ab}-\frac{\operatorname{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{ab}+\frac{\operatorname{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{ab}+\frac{\operatorname{PolyLog}(3,1-c(a+bx\right.$$

input
```
Int[PolyLog[3, c*(a + b*x)]/x^2,x]
```

output
```
((b - a/x)*PolyLog[3, c*(a + b*x)])/a - b^2*(-((Log[x]*Log[1 + (b*x)/a]*Lo
g[1 - c*(a + b*x)])/(a*b)) - ((Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a
+ b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(1 -
 c*(a + b*x)))/(b*x))]^2)/(2*a*b) - ((Log[c*(a + b*x)] - Log[1 + (b*x)/a])
*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*a*b) - ((Log[1 - c*(
a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)])/(
a*b) - (Log[x]*PolyLog[2, c*(a + b*x)])/(a*b) - (Log[-((a*(1 - c*(a + b*x)
))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(a*b) + (Log[-((a*(
1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))])/(a*b)
- ((Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*
x)])/(a*b) + PolyLog[3, -((b*x)/a)]/(a*b) + (2*PolyLog[3, c*(a + b*x)])/(a
*b) - PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))]/(a*b) + PolyLog[3, -((b*c
*x)/(1 - c*(a + b*x)))]/(a*b) + PolyLog[3, 1 - c*(a + b*x)]/(a*b))
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7153
```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Simp[p/((m + 1)*b^m)   Int[ExpandIntegrand[PolyLog[n -
 1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
 /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

3.135.4 Maple [F]

$$\int \frac{\text{polylog}\left(3, c(bx+a)\right)}{x^2}dx$$

input
```
int(polylog(3,c*(b*x+a))/x^2,x)
```

output
```
int(polylog(3,c*(b*x+a))/x^2,x)
```

3.135.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} dx = \int \frac{\text{Li}_3((bx+a)c)}{x^2} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
integral(polylog(3, b*c*x + a*c)/x^2, x)
```

3.135.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} dx = \int \frac{\text{Li}_3\left(ac+bcx\right)}{x^2} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x**2,x)
```

output
```
Integral(polylog(3, a*c + b*c*x)/x**2, x)
```

3.135.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} \, dx = \int \frac{\text{Li}_3((bx+a)c)}{x^2} \, dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
b*integrate(dilog(b*c*x + a*c)/(b*x^2 + a*x), x) - polylog(3, b*c*x + a*c)
/x
```

3.135.8 Giac [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} \, dx = \int \frac{\text{Li}_3((bx+a)c)}{x^2} \, dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="giac")
```

output
```
integrate(polylog(3, (b*x + a)*c)/x^2, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} \, dx = \int \frac{\text{polylog}(3, c\,(a+b\,x))}{x^2} \, dx$$

input
```
int(polylog(3, c*(a + b*x))/x^2,x)
```

output
```
int(polylog(3, c*(a + b*x))/x^2, x)
```

3.136 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx$

3.136.1 Optimal result

Integrand size = 13, antiderivative size = 629

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx$$
$$= -\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2}-\frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2}$$
$$-\frac{b^2\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)}{4a^2}$$
$$-\frac{b^2\left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\left(\log(x)+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)^2}{4a^2}$$
$$-\frac{b^2\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{2a^2}$$
$$-\frac{b^2\,\text{PolyLog}(2,c(a+bx))}{2a^2}-\frac{b\,\text{PolyLog}(2,c(a+bx))}{2ax}-\frac{b^2\log(x)\,\text{PolyLog}(2,c(a+bx))}{2a^2}$$
$$-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{b^2\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{PolyLog}\left(2,-\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}$$
$$+\frac{b^2\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{PolyLog}\left(2,-\frac{bcx}{1-c(a+bx)}\right)}{2a^2}$$
$$-\frac{b^2\left(\log(x)+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)\text{PolyLog}(2,1-c(a+bx))}{2a^2}+\frac{b^2\,\text{PolyLog}\left(3,-\frac{bx}{a}\right)}{2a^2}$$
$$+\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{PolyLog}(3,c(a+bx))}{2a^2}-\frac{b^2\,\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}$$
$$+\frac{b^2\,\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{2a^2}+\frac{b^2\,\text{PolyLog}(3,1-c(a+bx))}{2a^2}$$

output
```
-1/2*b^2*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^2-1/2*b^2*ln(x)*ln(1+b*x/a)
*ln(1-c*(b*x+a))/a^2-1/4*b^2*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a)))-ln((-
a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2/a^2-1/4*b^2*(l
n(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/a^2-1/2*b^2*(
ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/a^2-1/2*b^2*po
lylog(2,c*(b*x+a))/a^2-1/2*b*polylog(2,c*(b*x+a))/a/x-1/2*b^2*ln(x)*polylo
g(2,c*(b*x+a))/a^2-1/2*b^2*polylog(2,1-b*c*x/(-a*c+1))/a^2-1/2*b^2*ln(-a*(
1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))/a^2+1/2*b^2*ln(-a*(1-c*(
b*x+a))/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/a^2-1/2*b^2*(ln(x)+ln(-a*(1-c
*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/a^2+1/2*b^2*polylog(3,-b*x/a)/a^2+1
/2*(b^2-a^2/x^2)*polylog(3,c*(b*x+a))/a^2-1/2*b^2*polylog(3,-b*x/a/(1-c*(b
*x+a)))/a^2+1/2*b^2*polylog(3,-b*c*x/(1-c*(b*x+a)))/a^2+1/2*b^2*polylog(3,
1-c*(b*x+a))/a^2
```

3.136.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.91

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx$$
$$= \frac{-\text{PolyLog}(3,c(a+bx)) + \frac{bx\left(-((a+bx\log(x)-bx\log(a+bx))\,\text{PolyLog}(2,c(a+bx)))+bx\left(\log(c(a+bx))\log(1-ac-bcx)-\log(x)\log\left(1+\right.\right.\right.}{}}{}$$

input
```
Integrate[PolyLog[3, c*(a + b*x)]/x^3,x]
```

output
```
(-PolyLog[3, c*(a + b*x)] + (b*x*(-((a + b*x*Log[x] - b*x*Log[a + b*x])*Po
lyLog[2, c*(a + b*x)]) + b*x*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x] - Log[
x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((Log[c*(a + b*x)] - Log[1 + (b
*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 - (Log[
c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b
*c*x))/(b*x)] - ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)
] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 - Log[x]*(Log
[1 - a*c - b*c*x] - Log[1 + (b*c*x)/(-1 + a*c)]) - (Log[1 - a*c - b*c*x] -
 Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + PolyLog[2, (b
*c*x)/(1 - a*c)] - Log[a + b*x]*PolyLog[2, c*(a + b*x)] + PolyLog[2, 1 - a
*c - b*c*x] - (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 -
a*c - b*c*x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(PolyLog[2, (a*(-1 + a*c
+ b*c*x))/(b*x)] - PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + PolyLog[3, -(
(b*x)/a)] + PolyLog[3, c*(a + b*x)] + PolyLog[3, 1 - a*c - b*c*x] - PolyLo
g[3, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)
])))/a^2)/(2*x^2)
```

3.136.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx$$

$$\Big\downarrow \text{7153}$$

$$\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{PolyLog}(3,c(a+bx))}{2a^2}-\frac{1}{2}b^3\int\left(\frac{\text{PolyLog}(2,c(a+bx))}{a^2bx}-\frac{\text{PolyLog}(2,c(a+bx))}{ab^2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{PolyLog}(3,c(a+bx))}{2a^2}-$$

$$\frac{1}{2}b^3\left(\frac{\text{PolyLog}(2,c(a+bx))}{a^2b}+\frac{\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a^2b}+\frac{\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{a^2b}-\frac{\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{a^2b}\right.$$

input
```
Int[PolyLog[3, c*(a + b*x)]/x^3,x]
```

output
```
((b^2 - a^2/x^2)*PolyLog[3, c*(a + b*x)])/(2*a^2) - (b^3*((Log[(b*c*x)/(1
- a*c)]*Log[1 - a*c - b*c*x])/(a^2*b) + (Log[x]*Log[1 + (b*x)/a]*Log[1 - c
*(a + b*x)])/(a^2*b) + ((Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x)
)] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(1 - c*(a
+ b*x)))/(b*x))]^2)/(2*a^2*b) + ((Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(Lo
g[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*a^2*b) + ((Log[1 - c*(a
+ b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)])/(a^
2*b) + PolyLog[2, c*(a + b*x)]/(a^2*b) + PolyLog[2, c*(a + b*x)]/(a*b^2*x)
 + (Log[x]*PolyLog[2, c*(a + b*x)])/(a^2*b) + PolyLog[2, 1 - (b*c*x)/(1 -
a*c)]/(a^2*b) + (Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a
*(1 - c*(a + b*x))))])/(a^2*b) - (Log[-((a*(1 - c*(a + b*x)))/(b*x))]*Poly
Log[2, -((b*c*x)/(1 - c*(a + b*x)))])/(a^2*b) + ((Log[x] + Log[-((a*(1 - c
*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(a^2*b) - PolyLog[3, -(
(b*x)/a)]/(a^2*b) + PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))]/(a^2*b) - P
olyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/(a^2*b) - PolyLog[3, 1 - c*(a + b*
x)]/(a^2*b)))/2
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7153
```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Simp[p/((m + 1)*b^m)   Int[ExpandIntegrand[PolyLog[n -
 1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
 /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

3.136.4 Maple [F]

$$\int \frac{\text{polylog}(3, c(bx + a))}{x^3} dx$$

input
```
int(polylog(3,c*(b*x+a))/x^3,x)
```

output
```
int(polylog(3,c*(b*x+a))/x^3,x)
```

3.136.5 Fricas [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^3} dx = \int \frac{\text{Li}_3((bx+a)c)}{x^3} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
integral(polylog(3, b*c*x + a*c)/x^3, x)
```

3.136.6 Sympy [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^3} dx = \int \frac{\text{Li}_3\left(ac+bcx\right)}{x^3} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x**3,x)
```

output
```
Integral(polylog(3, a*c + b*c*x)/x**3, x)
```

3.136.7 Maxima [F]

$$\int \frac{\text{PolyLog}(3, c(a+bx))}{x^3} dx = \int \frac{\text{Li}_3((bx+a)c)}{x^3} dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
b*integrate(1/2*dilog(b*c*x + a*c)/(b*x^3 + a*x^2), x) - 1/2*polylog(3, b*
c*x + a*c)/x^2
```

3.136.8 Giac [F]

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx=\int \frac{\text{Li}_3((bx+a)c)}{x^3}\,dx$$

input
```
integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="giac")
```

output
```
integrate(polylog(3, (b*x + a)*c)/x^3, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx=\int \frac{\text{polylog}(3,c\,(a+b\,x))}{x^3}\,dx$$

input
```
int(polylog(3, c*(a + b*x))/x^3,x)
```

output
```
int(polylog(3, c*(a + b*x))/x^3, x)
```

3.137 $\int (d+ex)^3 \operatorname{PolyLog}(2, c(a+bx)) \, dx$

3.137.1 Optimal result

Integrand size = 17, antiderivative size = 605

$$\begin{aligned}\int (d+ex)^3\operatorname{PolyLog}(2,c(a+bx))\,dx = &-\frac{(bd-ae)^3x}{4b^3}-\frac{(bd-ae)^2(bcd+e-ace)x}{8b^3c}\\
&-\frac{(bd-ae)(bcd+e-ace)^2x}{12b^3c^2}\\
&-\frac{(bcd+e-ace)^3x}{16b^3c^3}-\frac{(bd-ae)^2(d+ex)^2}{16b^2e}\\
&-\frac{(bd-ae)(bcd+e-ace)(d+ex)^2}{24b^2ce}\\
&-\frac{(bcd+e-ace)^2(d+ex)^2}{32b^2c^2e}-\frac{(bd-ae)(d+ex)^3}{36be}\\
&-\frac{(bcd+e-ace)(d+ex)^3}{48bce}-\frac{(d+ex)^4}{64e}\\
&-\frac{(bd-ae)^2(bcd+e-ace)^2\log(1-ac-bcx)}{8b^4c^2e}\\
&-\frac{(bd-ae)(bcd+e-ace)^3\log(1-ac-bcx)}{12b^4c^3e}\\
&-\frac{(bcd+e-ace)^4\log(1-ac-bcx)}{16b^4c^4e}\\
&-\frac{(bd-ae)^3(1-ac-bcx)\log(1-ac-bcx)}{4b^4c}\\
&+\frac{(bd-ae)^2(d+ex)^2\log(1-ac-bcx)}{8b^2e}\\
&+\frac{(bd-ae)(d+ex)^3\log(1-ac-bcx)}{12be}\\
&+\frac{(d+ex)^4\log(1-ac-bcx)}{16e}\\
&-\frac{(bd-ae)^4\operatorname{PolyLog}(2,c(a+bx))}{4b^4e}\\
&+\frac{(d+ex)^4\operatorname{PolyLog}(2,c(a+bx))}{4e}\end{aligned}$$

output
```
-1/4*(-a*e+b*d)^3*x/b^3-1/8*(-a*e+b*d)^2*(-a*c*e+b*c*d+e)*x/b^3/c-1/12*(-a
*e+b*d)*(-a*c*e+b*c*d+e)^2*x/b^3/c^2-1/16*(-a*c*e+b*c*d+e)^3*x/b^3/c^3-1/1
6*(-a*e+b*d)^2*(e*x+d)^2/b^2/e-1/24*(-a*e+b*d)*(-a*c*e+b*c*d+e)*(e*x+d)^2/
b^2/c/e-1/32*(-a*c*e+b*c*d+e)^2*(e*x+d)^2/b^2/c^2/e-1/36*(-a*e+b*d)*(e*x+d
)^3/b/e-1/48*(-a*c*e+b*c*d+e)*(e*x+d)^3/b/c/e-1/64*(e*x+d)^4/e-1/8*(-a*e+b
*d)^2*(-a*c*e+b*c*d+e)^2*ln(-b*c*x-a*c+1)/b^4/c^2/e-1/12*(-a*e+b*d)*(-a*c*
e+b*c*d+e)^3*ln(-b*c*x-a*c+1)/b^4/c^3/e-1/16*(-a*c*e+b*c*d+e)^4*ln(-b*c*x-
a*c+1)/b^4/c^4/e-1/4*(-a*e+b*d)^3*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^4/c+1/
8*(-a*e+b*d)^2*(e*x+d)^2*ln(-b*c*x-a*c+1)/b^2/e+1/12*(-a*e+b*d)*(e*x+d)^3*
ln(-b*c*x-a*c+1)/b/e+1/16*(e*x+d)^4*ln(-b*c*x-a*c+1)/e-1/4*(-a*e+b*d)^4*po
lylog(2,c*(b*x+a))/b^4/e+1/4*(e*x+d)^4*polylog(2,c*(b*x+a))/e
```

3.137.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.80

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx$$
$$= \frac{12e(-1+ac+bcx)\left(\left(3-13ac+23a^2c^2-25a^3c^3\right)e^2+bce\left(8\left(2-7ac+11a^2c^2\right)d+\left(3-10ac+13a^2c^2\right)\right.\right.}{}$$

input
```
Integrate[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```

output
```
(12*e*(-1 + a*c + b*c*x)*((3 - 13*a*c + 23*a^2*c^2 - 25*a^3*c^3)*e^2 + b*c
*e*(8*(2 - 7*a*c + 11*a^2*c^2)*d + (3 - 10*a*c + 13*a^2*c^2)*e*x) + b^3*c^
3*x*(36*d^2 + 16*d*e*x + 3*e^2*x^2) + b^2*c^2*(-36*(-1 + 3*a*c)*d^2 - 8*(-
2 + 5*a*c)*d*e*x + (3 - 7*a*c)*e^2*x^2))*Log[1 - a*c - b*c*x] + b*c*(300*a
^3*c^3*e^3*x - 6*a^2*c^2*e^2*x*(46*e + b*c*(176*d + 13*e*x)) + 4*a*c*(39*e
^3*x + 3*b*c*e^2*x*(56*d + 5*e*x) + b^2*c^2*(-144*d^3 + 324*d^2*e*x + 60*d
*e^2*x^2 + 7*e^3*x^3)) - x*(36*e^3 + 6*b*c*e^2*(32*d + 3*e*x) + 12*b^2*c^2
*e*(36*d^2 + 8*d*e*x + e^2*x^2) + b^3*c^3*(576*d^3 + 216*d^2*e*x + 64*d*e^
2*x^2 + 9*e^3*x^3)) + 576*b^2*c^2*d^3*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*
x)]) - 144*c^4*(-4*a*b^3*d^3 + 6*a^2*b^2*d^2*e - 4*a^3*b*d*e^2 + a^4*e^3 -
 b^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*PolyLog[2, c*(a + b*x)
])/(576*b^4*c^4)
```

3.137.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx$$

$$\Big\downarrow \; 7152$$

$$\frac{b\int \frac{(d+ex)^4\log(-ac-bxc+1)}{a+bx}dx}{4e}+\frac{(d+ex)^4\operatorname{PolyLog}(2,c(a+bx))}{4e}$$

$$\Big\downarrow \; 2865$$

$$\frac{b\int \left(\frac{\log(-ac-bxc+1)(bd-ae)^4}{b^4(a+bx)}+\frac{e\log(-ac-bxc+1)(bd-ae)^3}{b^4}+\frac{e(d+ex)\log(-ac-bxc+1)(bd-ae)^2}{b^3}+\frac{e(d+ex)^2\log(-ac-bxc+1)(bd-ae)}{b^2}\right.}{4e}$$

$$\frac{(d+ex)^4\operatorname{PolyLog}(2,c(a+bx))}{4e}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(-\frac{(-ace+bcd+e)^4\log(-ac-bcx+1)}{4b^5c^4}-\frac{(bd-ae)(-ace+bcd+e)^3\log(-ac-bcx+1)}{3b^5c^3}-\frac{(bd-ae)^2(-ace+bcd+e)^2\log(-ac-bcx+1)}{2b^5c^2}-\frac{(bd-ae)}{}\right.}{}$$

$$\frac{(d+ex)^4\operatorname{PolyLog}(2,c(a+bx))}{4e}$$

input
```
Int[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```

output
```
((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e) + (b*(-((e*(b*d - a*e)^3*x)/b^
4) - (e*(b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(2*b^4*c) - (e*(b*d - a*e)*(b
*c*d + e - a*c*e)^2*x)/(3*b^4*c^2) - (e*(b*c*d + e - a*c*e)^3*x)/(4*b^4*c^
3) - ((b*d - a*e)^2*(d + e*x)^2)/(4*b^3) - ((b*d - a*e)*(b*c*d + e - a*c*e
)*(d + e*x)^2)/(6*b^3*c) - ((b*c*d + e - a*c*e)^2*(d + e*x)^2)/(8*b^3*c^2)
 - ((b*d - a*e)*(d + e*x)^3)/(9*b^2) - ((b*c*d + e - a*c*e)*(d + e*x)^3)/(
12*b^2*c) - (d + e*x)^4/(16*b) - ((b*d - a*e)^2*(b*c*d + e - a*c*e)^2*Log[
1 - a*c - b*c*x])/(2*b^5*c^2) - ((b*d - a*e)*(b*c*d + e - a*c*e)^3*Log[1 -
 a*c - b*c*x])/(3*b^5*c^3) - ((b*c*d + e - a*c*e)^4*Log[1 - a*c - b*c*x])/
(4*b^5*c^4) - (e*(b*d - a*e)^3*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b^
5*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c - b*c*x])/(2*b^3) + ((b*d -
a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(3*b^2) + ((d + e*x)^4*Log[1 - a*c
- b*c*x])/(4*b) - ((b*d - a*e)^4*PolyLog[2, c*(a + b*x)])/b^5))/(4*e)
```

3.137.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.137.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 531, normalized size of antiderivative = 0.88

method	result
parts	$\frac{\operatorname{polylog}(2,c(bx+a))e^3x^4}{4}+\operatorname{polylog}(2,c(bx+a))\,e^2d\,x^3+\frac{3\operatorname{polylog}(2,c(bx+a))e\,d^2x^2}{2}+\operatorname{polylog}(2,c$
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((e*x+d)^3*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*polylog(2,c*(b*x+a))*e^3*x^4+polylog(2,c*(b*x+a))*e^2*d*x^3+3/2*polylo
g(2,c*(b*x+a))*e*d^2*x^2+polylog(2,c*(b*x+a))*d^3*x+1/4*polylog(2,c*(b*x+a
))/e*d^4-1/4/e/c*(-e^4/b^4/c^3*(1/4*(-b*c*x-a*c+1)^4*ln(-b*c*x-a*c+1)-1/16
*(-b*c*x-a*c+1)^4)-e^3*(4*a*c*e-4*b*c*d-3*e)/b^4/c^3*(1/3*(-b*c*x-a*c+1)^3
*ln(-b*c*x-a*c+1)-1/9*(-b*c*x-a*c+1)^3)-e^2*(6*a^2*c^2*e^2-12*a*b*c^2*d*e+
6*b^2*c^2*d^2-8*a*c*e^2+8*b*c*d*e+3*e^2)/b^4/c^3*(1/2*(-b*c*x-a*c+1)^2*ln(
-b*c*x-a*c+1)-1/4*(-b*c*x-a*c+1)^2)-e*(4*a^3*c^3*e^3-12*a^2*b*c^3*d*e^2+12
*a*b^2*c^3*d^2*e-4*b^3*c^3*d^3-6*a^2*c^2*e^3+12*a*b*c^2*d*e^2-6*b^2*c^2*d^
2*e+4*a*c*e^3-4*b*c*d*e^2-e^3)/b^4/c^3*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+
b*c*x+a*c)+c*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e+b^4*d^
4)/b^4*dilog(-b*c*x-a*c+1))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.08

$$\int (d+ex)^3\operatorname{PolyLog}(2,c(a+bx))\,dx =$$
$$-\frac{9\,b^4c^4e^3x^4+4\,(16\,b^4c^4de^2-(7\,ab^3c^4-3\,b^3c^3)e^3)x^3+6\,(36\,b^4c^4d^2e-8\,(5\,ab^3c^4-2\,b^3c^3)de^2+(13\,a^2b^2c}{}$$

input
```
integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-1/576*(9*b^4*c^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3
)*e^3)*x^3 + 6*(36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13
*a^2*b^2*c^4 - 10*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 3
6*(3*a*b^3*c^4 - b^3*c^3)*d^2*e + 8*(11*a^2*b^2*c^4 - 7*a*b^2*c^3 + 2*b^2*
c^2)*d*e^2 - (25*a^3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*e^3)*x - 1
44*(b^4*c^4*e^3*x^4 + 4*b^4*c^4*d*e^2*x^3 + 6*b^4*c^4*d^2*e*x^2 + 4*b^4*c^
4*d^3*x + 4*a*b^3*c^4*d^3 - 6*a^2*b^2*c^4*d^2*e + 4*a^3*b*c^4*d*e^2 - a^4*
c^4*e^3)*dilog(b*c*x + a*c) - 12*(3*b^4*c^4*e^3*x^4 + 48*(a*b^3*c^4 - b^3*
c^3)*d^3 - 36*(3*a^2*b^2*c^4 - 4*a*b^2*c^3 + b^2*c^2)*d^2*e + 8*(11*a^3*b*
c^4 - 18*a^2*b*c^3 + 9*a*b*c^2 - 2*b*c)*d*e^2 - (25*a^4*c^4 - 48*a^3*c^3 +
 36*a^2*c^2 - 16*a*c + 3)*e^3 + 4*(4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 +
6*(6*b^4*c^4*d^2*e - 4*a*b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3)*x^2 + 12*(4*b^4*
c^4*d^3 - 6*a*b^3*c^4*d^2*e + 4*a^2*b^2*c^4*d*e^2 - a^3*b*c^4*e^3)*x)*log(
-b*c*x - a*c + 1))/(b^4*c^4)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 28.61 (sec) , antiderivative size = 1028, normalized size of antiderivative = 1.70

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x*
*3 + e**3*x**4/4)*polylog(2, a*c), Eq(b, 0)), (0, Eq(c, 0)), (25*a**4*e**3
*polylog(1, a*c + b*c*x)/(48*b**4) - a**4*e**3*polylog(2, a*c + b*c*x)/(4*
b**4) - 11*a**3*d*e**2*polylog(1, a*c + b*c*x)/(6*b**3) + a**3*d*e**2*poly
log(2, a*c + b*c*x)/b**3 + a**3*e**3*x*polylog(1, a*c + b*c*x)/(4*b**3) +
25*a**3*e**3*x/(48*b**3) - a**3*e**3*polylog(1, a*c + b*c*x)/(b**4*c) + 9*
a**2*d**2*e*polylog(1, a*c + b*c*x)/(4*b**2) - 3*a**2*d**2*e*polylog(2, a*
c + b*c*x)/(2*b**2) - a**2*d*e**2*x*polylog(1, a*c + b*c*x)/b**2 - 11*a**2
*d*e**2*x/(6*b**2) - a**2*e**3*x**2*polylog(1, a*c + b*c*x)/(8*b**2) - 13*
a**2*e**3*x**2/(96*b**2) + 3*a**2*d*e**2*polylog(1, a*c + b*c*x)/(b**3*c)
- 23*a**2*e**3*x/(48*b**3*c) + 3*a**2*e**3*polylog(1, a*c + b*c*x)/(4*b**4
*c**2) - a*d**3*polylog(1, a*c + b*c*x)/b + a*d**3*polylog(2, a*c + b*c*x)
/b + 3*a*d**2*e*x*polylog(1, a*c + b*c*x)/(2*b) + 9*a*d**2*e*x/(4*b) + a*d
*e**2*x**2*polylog(1, a*c + b*c*x)/(2*b) + 5*a*d*e**2*x**2/(12*b) + a*e**3
*x**3*polylog(1, a*c + b*c*x)/(12*b) + 7*a*e**3*x**3/(144*b) - 3*a*d**2*e*
polylog(1, a*c + b*c*x)/(b**2*c) + 7*a*d*e**2*x/(6*b**2*c) + 5*a*e**3*x**2
/(48*b**2*c) - 3*a*d*e**2*polylog(1, a*c + b*c*x)/(2*b**3*c**2) + 13*a*e**
3*x/(48*b**3*c**2) - a*e**3*polylog(1, a*c + b*c*x)/(3*b**4*c**3) - d**3*x
*polylog(1, a*c + b*c*x) + d**3*x*polylog(2, a*c + b*c*x) - d**3*x - 3*d**
2*e*x**2*polylog(1, a*c + b*c*x)/4 + 3*d**2*e*x**2*polylog(2, a*c + b*c...
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.13

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx =$$

$$-\frac{(4\,ab^3d^3 - 6\,a^2b^2d^2e + 4\,a^3bde^2 - a^4e^3)(\log(bcx+ac)\log(-bcx-ac+1) + \mathrm{Li}_2(-bcx-ac+1))}{4\,b^4}$$

$$-\frac{9\,b^4c^4e^3x^4 + 4\,(16\,b^4c^4de^2 - (7\,ab^3c^4 - 3\,b^3c^3)e^3)x^3 + 6\,(36\,b^4c^4d^2e - 8\,(5\,ab^3c^4 - 2\,b^3c^3)de^2 + (13\,a^2b^2c^{\cdots}}{}$$

input
```
integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e^2 - a^4*e^3)*(log(b*c*x
+ a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))/b^4 - 1/576*(9*b^4
*c^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3)*e^3)*x^3 +
6*(36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13*a^2*b^2*c^4
- 10*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 36*(3*a*b^3*c^
4 - b^3*c^3)*d^2*e + 8*(11*a^2*b^2*c^4 - 7*a*b^2*c^3 + 2*b^2*c^2)*d*e^2 -
(25*a^3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*e^3)*x - 144*(b^4*c^4*e
^3*x^4 + 4*b^4*c^4*d*e^2*x^3 + 6*b^4*c^4*d^2*e*x^2 + 4*b^4*c^4*d^3*x)*dilo
g(b*c*x + a*c) - 12*(3*b^4*c^4*e^3*x^4 + 48*(a*b^3*c^4 - b^3*c^3)*d^3 - 36
*(3*a^2*b^2*c^4 - 4*a*b^2*c^3 + b^2*c^2)*d^2*e + 8*(11*a^3*b*c^4 - 18*a^2*
b*c^3 + 9*a*b*c^2 - 2*b*c)*d*e^2 - (25*a^4*c^4 - 48*a^3*c^3 + 36*a^2*c^2 -
 16*a*c + 3)*e^3 + 4*(4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 + 6*(6*b^4*c^4*
d^2*e - 4*a*b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3)*x^2 + 12*(4*b^4*c^4*d^3 - 6*a
*b^3*c^4*d^2*e + 4*a^2*b^2*c^4*d*e^2 - a^3*b*c^4*e^3)*x)*log(-b*c*x - a*c
+ 1))/(b^4*c^4)
```

3.137.8 Giac [F]

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx = \int (ex+d)^3 \operatorname{Li}_2((bx+a)c)\,dx$$

input
```
integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*dilog((b*x + a)*c), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \operatorname{PolyLog}(2,c(a+bx))\,dx = \int \operatorname{polylog}(2,c\,(a+b\,x))\,(d+e\,x)^3\,dx$$

input
```
int(polylog(2, c*(a + b*x))*(d + e*x)^3,x)
```

output
```
int(polylog(2, c*(a + b*x))*(d + e*x)^3, x)
```

3.138 $\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx$

3.138.1 Optimal result

Integrand size = 17, antiderivative size = 385

$$\begin{aligned}\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx =& -\frac{(bd-ae)^2x}{3b^2}-\frac{(bd-ae)(bcd+e-ace)x}{6b^2c}\\
&-\frac{(bcd+e-ace)^2x}{9b^2c^2}-\frac{(bd-ae)(d+ex)^2}{12be}\\
&-\frac{(bcd+e-ace)(d+ex)^2}{18bce}-\frac{(d+ex)^3}{27e}\\
&-\frac{(bd-ae)(bcd+e-ace)^2\log(1-ac-bcx)}{6b^3c^2e}\\
&-\frac{(bcd+e-ace)^3\log(1-ac-bcx)}{9b^3c^3e}\\
&-\frac{(bd-ae)^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c}\\
&+\frac{(bd-ae)(d+ex)^2\log(1-ac-bcx)}{6be}\\
&+\frac{(d+ex)^3\log(1-ac-bcx)}{9e}\\
&-\frac{(bd-ae)^3\operatorname{PolyLog}(2,c(a+bx))}{3b^3e}\\
&+\frac{(d+ex)^3\operatorname{PolyLog}(2,c(a+bx))}{3e}\end{aligned}$$

output
```
-1/3*(-a*e+b*d)^2*x/b^2-1/6*(-a*e+b*d)*(-a*c*e+b*c*d+e)*x/b^2/c-1/9*(-a*c*
e+b*c*d+e)^2*x/b^2/c^2-1/12*(-a*e+b*d)*(e*x+d)^2/b/e-1/18*(-a*c*e+b*c*d+e)
*(e*x+d)^2/b/c/e-1/27*(e*x+d)^3/e-1/6*(-a*e+b*d)*(-a*c*e+b*c*d+e)^2*ln(-b*
c*x-a*c+1)/b^3/c^2/e-1/9*(-a*c*e+b*c*d+e)^3*ln(-b*c*x-a*c+1)/b^3/c^3/e-1/3
*(-a*e+b*d)^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/c+1/6*(-a*e+b*d)*(e*x+d)
^2*ln(-b*c*x-a*c+1)/b/e+1/9*(e*x+d)^3*ln(-b*c*x-a*c+1)/e-1/3*(-a*e+b*d)^3*
polylog(2,c*(b*x+a))/b^3/e+1/3*(e*x+d)^3*polylog(2,c*(b*x+a))/e
```

3.138.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.71

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx$$
$$= \frac{6e(-1+ac+bcx)\left(\left(2-7ac+11a^2c^2\right)e+b^2c^2x(9d+2ex)+bc((9-27ac)d+(2-5ac)ex)\right)\log(1-ac}{}$$

input
```
Integrate[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

output
```
(6*e*(-1 + a*c + b*c*x)*((2 - 7*a*c + 11*a^2*c^2)*e + b^2*c^2*x*(9*d + 2*e
*x) + b*c*((9 - 27*a*c)*d + (2 - 5*a*c)*e*x))*Log[1 - a*c - b*c*x] + b*c*(
-66*a^2*c^2*e^2*x - x*(12*e^2 + 6*b*c*e*(9*d + e*x) + b^2*c^2*(108*d^2 + 2
7*d*e*x + 4*e^2*x^2)) + 3*a*c*(14*e^2*x + b*c*(-36*d^2 + 54*d*e*x + 5*e^2*
x^2)) + 108*b*c*d^2*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*x)]) + 36*c^3*(3*a
*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2 + b^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*Poly
Log[2, c*(a + b*x)])/(108*b^3*c^3)
```

3.138.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx$$
$$\Big\downarrow \text{ 7152}$$

$$\frac{b\int \frac{(d+ex)^3\log(-ac-bxc+1)}{a+bx}dx}{3e}+\frac{(d+ex)^3\operatorname{PolyLog}(2,c(a+bx))}{3e}$$

$$\Big\downarrow \text{2865}$$

$$\frac{b\int \left(\frac{\log(-ac-bxc+1)(bd-ae)^3}{b^3(a+bx)}+\frac{e\log(-ac-bxc+1)(bd-ae)^2}{b^3}+\frac{e(d+ex)\log(-ac-bxc+1)(bd-ae)}{b^2}+\frac{e(d+ex)^2\log(-ac-bxc+1)}{b}\right)dx}{3e}+\frac{(d+ex)^3\operatorname{PolyLog}(2,c(a+bx))}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{(-ace+bcd+e)^3\log(-ac-bcx+1)}{3b^4c^3}-\frac{(bd-ae)(-ace+bcd+e)^2\log(-ac-bcx+1)}{2b^4c^2}-\frac{(bd-ae)^3\operatorname{PolyLog}(2,c(a+bx))}{b^4}-\frac{e(-ac-bcx+1)(bd-a}{b^4}\right.}{}$$

$$\frac{(d+ex)^3\operatorname{PolyLog}(2,c(a+bx))}{3e}$$

input
```
Int[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

output
```
((d + e*x)^3*PolyLog[2, c*(a + b*x)])/(3*e) + (b*(-((e*(b*d - a*e)^2*x)/b^
3) - (e*(b*d - a*e)*(b*c*d + e - a*c*e)*x)/(2*b^3*c) - (e*(b*c*d + e - a*c
*e)^2*x)/(3*b^3*c^2) - ((b*d - a*e)*(d + e*x)^2)/(4*b^2) - ((b*c*d + e - a
*c*e)*(d + e*x)^2)/(6*b^2*c) - (d + e*x)^3/(9*b) - ((b*d - a*e)*(b*c*d + e
 - a*c*e)^2*Log[1 - a*c - b*c*x])/(2*b^4*c^2) - ((b*c*d + e - a*c*e)^3*Log
[1 - a*c - b*c*x])/(3*b^4*c^3) - (e*(b*d - a*e)^2*(1 - a*c - b*c*x)*Log[1
- a*c - b*c*x])/(b^4*c) + ((b*d - a*e)*(d + e*x)^2*Log[1 - a*c - b*c*x])/(
2*b^2) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(3*b) - ((b*d - a*e)^3*PolyLog
[2, c*(a + b*x)])/b^4))/(3*e)
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.138.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.50

method	result
parts	$\frac{\operatorname{polylog}(2,c(bx+a))e^2x^3}{3}+\operatorname{polylog}(2,c(bx+a))\,ed\,x^2+\operatorname{polylog}(2,c(bx+a))\,d^2x+\frac{\operatorname{polylog}(2,\ldots}{3}$
parallelrisch	$\frac{-12e^2-108\operatorname{polylog}(2,c(bx+a))a^2b\,c^3de-162\ln(1-c(bx+a))a^2b\,c^3de+216\ln(1-c(bx+a))ab\,c^2de+36x^3\operatorname{polylog}(2,c(bx\ldots}{}$
derivativedivides	$\frac{-\frac{c\,e^2\operatorname{polylog}(2,bcx+ac)a^3}{3b^2}+\frac{ce\operatorname{polylog}(2,bcx+ac)a^2d}{b}-c\operatorname{polylog}(2,bcx+ac)a\,d^2+\frac{bc\operatorname{polylog}(2,bcx+ac)d^3}{3e}+\frac{e^2\operatorname{polylog}(2,bcx+ac)\ldots}{b^2}}{}$
default	$\frac{-\frac{c\,e^2\operatorname{polylog}(2,bcx+ac)a^3}{3b^2}+\frac{ce\operatorname{polylog}(2,bcx+ac)a^2d}{b}-c\operatorname{polylog}(2,bcx+ac)a\,d^2+\frac{bc\operatorname{polylog}(2,bcx+ac)d^3}{3e}+\frac{e^2\operatorname{polylog}(2,bcx+ac)\ldots}{b^2}}{}$

input
```
int((e*x+d)^2*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/3*polylog(2,c*(b*x+a))*e^2*x^3+polylog(2,c*(b*x+a))*e*d*x^2+polylog(2,c*
(b*x+a))*d^2*x+1/3*polylog(2,c*(b*x+a))/e*d^3-1/3/e/c*(e/b^3/c^2*(3*a^2*c^
2*e^2*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)-6*a*b*c^2*d*e*((-b*c*x
-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)+3*b^2*c^2*d^2*((-b*c*x-a*c+1)*ln(-b*
c*x-a*c+1)-1+b*c*x+a*c)+3*a*c*e^2*(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1
/4*(-b*c*x-a*c+1)^2)-3*b*c*d*e*(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/4*
(-b*c*x-a*c+1)^2)-3*a*c*e^2*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)+
3*b*c*d*e*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)+e^2*(1/3*(-b*c*x-a
*c+1)^3*ln(-b*c*x-a*c+1)-1/9*(-b*c*x-a*c+1)^3)-2*e^2*(1/2*(-b*c*x-a*c+1)^2
*ln(-b*c*x-a*c+1)-1/4*(-b*c*x-a*c+1)^2)+e^2*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+
1)-1+b*c*x+a*c))-c*(a^3*e^3-3*a^2*b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)/b^3*dilog
(-b*c*x-a*c+1))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.97

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx = -\frac{4\,b^3c^3e^2x^3+3\,(9\,b^3c^3de-(5\,ab^2c^3-2\,b^2c^2)e^2)x^2+6\,(18\,b^3c^3d^2-9\,(3\,ab^2c^3-b^2c^2)de+(11\,a^2bc^3-7\,a$$

input
```
integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-1/108*(4*b^3*c^3*e^2*x^3 + 3*(9*b^3*c^3*d*e - (5*a*b^2*c^3 - 2*b^2*c^2)*e
^2)*x^2 + 6*(18*b^3*c^3*d^2 - 9*(3*a*b^2*c^3 - b^2*c^2)*d*e + (11*a^2*b*c^
3 - 7*a*b*c^2 + 2*b*c)*e^2)*x - 36*(b^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 +
3*b^3*c^3*d^2*x + 3*a*b^2*c^3*d^2 - 3*a^2*b*c^3*d*e + a^3*c^3*e^2)*dilog(b
*c*x + a*c) - 6*(2*b^3*c^3*e^2*x^3 + 18*(a*b^2*c^3 - b^2*c^2)*d^2 - 9*(3*a
^2*b*c^3 - 4*a*b*c^2 + b*c)*d*e + (11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^
2 + 3*(3*b^3*c^3*d*e - a*b^2*c^3*e^2)*x^2 + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3
*d*e + a^2*b*c^3*e^2)*x)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.46

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx = \begin{cases} 0 \\ \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\operatorname{Li}_2(ac) \\ 0 \\ -\frac{11a^3e^2\operatorname{Li}_1(ac+bcx)}{18b^3}+\frac{a^3e^2\operatorname{Li}_2(ac+bcx)}{3b^3}+\frac{3a^2de\operatorname{Li}_1(ac+bcx)}{2b^2}-\frac{a^2de\operatorname{Li}_2(ac+bcx)}{b^2}-\frac{a^2e^2x\operatorname{Li}_1(ac+bcx)}{3b^2}-\frac{11a^2e^2x}{18b^2}+\frac{a^2e^2\operatorname{Li}_1(\cdots}{b^3\cdots} \end{cases}$$

input
```
integrate((e*x+d)**2*polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*pol
ylog(2, a*c), Eq(b, 0)), (0, Eq(c, 0)), (-11*a**3*e**2*polylog(1, a*c + b*
c*x)/(18*b**3) + a**3*e**2*polylog(2, a*c + b*c*x)/(3*b**3) + 3*a**2*d*e*p
olylog(1, a*c + b*c*x)/(2*b**2) - a**2*d*e*polylog(2, a*c + b*c*x)/b**2 -
a**2*e**2*x*polylog(1, a*c + b*c*x)/(3*b**2) - 11*a**2*e**2*x/(18*b**2) +
a**2*e**2*polylog(1, a*c + b*c*x)/(b**3*c) - a*d**2*polylog(1, a*c + b*c*x
)/b + a*d**2*polylog(2, a*c + b*c*x)/b + a*d*e*x*polylog(1, a*c + b*c*x)/b
 + 3*a*d*e*x/(2*b) + a*e**2*x**2*polylog(1, a*c + b*c*x)/(6*b) + 5*a*e**2*
x**2/(36*b) - 2*a*d*e*polylog(1, a*c + b*c*x)/(b**2*c) + 7*a*e**2*x/(18*b*
*2*c) - a*e**2*polylog(1, a*c + b*c*x)/(2*b**3*c**2) - d**2*x*polylog(1, a
*c + b*c*x) + d**2*x*polylog(2, a*c + b*c*x) - d**2*x - d*e*x**2*polylog(1
, a*c + b*c*x)/2 + d*e*x**2*polylog(2, a*c + b*c*x) - d*e*x**2/4 - e**2*x*
*3*polylog(1, a*c + b*c*x)/9 + e**2*x**3*polylog(2, a*c + b*c*x)/3 - e**2*
x**3/27 + d**2*polylog(1, a*c + b*c*x)/(b*c) - d*e*x/(2*b*c) - e**2*x**2/(
18*b*c) + d*e*polylog(1, a*c + b*c*x)/(2*b**2*c**2) - e**2*x/(9*b**2*c**2)
 + e**2*polylog(1, a*c + b*c*x)/(9*b**3*c**3), True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.05

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx$$
$$= -\frac{(3\,ab^2d^2 - 3\,a^2bde + a^3e^2)(\log(bcx+ac)\log(-bcx-ac+1) + \operatorname{Li}_2(-bcx-ac+1))}{3\,b^3}$$
$$- \frac{4\,b^3c^3e^2x^3 + 3\,(9\,b^3c^3de - (5\,ab^2c^3 - 2\,b^2c^2)e^2)x^2 + 6\,(18\,b^3c^3d^2 - 9\,(3\,ab^2c^3 - b^2c^2)de + (11\,a^2bc^3 - 7\,a}{}$$

input
```
integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*(log(b*c*x + a*c)*log(-b*c*x -
a*c + 1) + dilog(-b*c*x - a*c + 1))/b^3 - 1/108*(4*b^3*c^3*e^2*x^3 + 3*(9*
b^3*c^3*d*e - (5*a*b^2*c^3 - 2*b^2*c^2)*e^2)*x^2 + 6*(18*b^3*c^3*d^2 - 9*(
3*a*b^2*c^3 - b^2*c^2)*d*e + (11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*e^2)*x - 3
6*(b^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 + 3*b^3*c^3*d^2*x)*dilog(b*c*x + a*
c) - 6*(2*b^3*c^3*e^2*x^3 + 18*(a*b^2*c^3 - b^2*c^2)*d^2 - 9*(3*a^2*b*c^3
- 4*a*b*c^2 + b*c)*d*e + (11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^2 + 3*(3*
b^3*c^3*d*e - a*b^2*c^3*e^2)*x^2 + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3*d*e + a^
2*b*c^3*e^2)*x)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

3.138.8 Giac [F]

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx=\int (ex+d)^2{\rm Li}_2((bx+a)c)\,dx$$

input
```
integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*dilog((b*x + a)*c), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \operatorname{PolyLog}(2,c(a+bx))\,dx=\int \mathrm{polylog}(2,c\,(a+b\,x))\,\left(d+e\,x\right)^2\,dx$$

input
```
int(polylog(2, c*(a + b*x))*(d + e*x)^2,x)
```

output
```
int(polylog(2, c*(a + b*x))*(d + e*x)^2, x)
```

3.139 $\int (d+ex) \operatorname{PolyLog}(2,c(a+bx))\,dx$

3.139.1 Optimal result

Integrand size = 15, antiderivative size = 210

$$\int (d+ex) \operatorname{PolyLog}(2,c(a+bx))\,dx = -\frac{(bd-ae)x}{2b} - \frac{(bcd+e-ace)x}{4bc} - \frac{(d+ex)^2}{8e} - \frac{(bcd+e-ace)^2\log(1-ac-bcx)}{4b^2c^2e} - \frac{(bd-ae)(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} + \frac{(d+ex)^2\log(1-ac-bcx)}{4e} - \frac{(bd-ae)^2\operatorname{PolyLog}(2,c(a+bx))}{2b^2e} + \frac{(d+ex)^2\operatorname{PolyLog}(2,c(a+bx))}{2e}$$

output
```
-1/2*(-a*e+b*d)*x/b-1/4*(-a*c*e+b*c*d+e)*x/b/c-1/8*(e*x+d)^2/e-1/4*(-a*c*e
+b*c*d+e)^2*ln(-b*c*x-a*c+1)/b^2/c^2/e-1/2*(-a*e+b*d)*(-b*c*x-a*c+1)*ln(-b
*c*x-a*c+1)/b^2/c+1/4*(e*x+d)^2*ln(-b*c*x-a*c+1)/e-1/2*(-a*e+b*d)^2*polylo
g(2,c*(b*x+a))/b^2/e+1/2*(e*x+d)^2*polylog(2,c*(b*x+a))/e
```

3.139.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.77

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx = \frac{e(-bcx(2-6ac+bcx)+(-2-6a^2c^2+2b^2c^2x^2-4ac(-2+bcx))\log(1-ac-bcx)-4a^2c^2\,\text{PolyLog}(2,}{8b^2c^2}$$
$$+\frac{d(-c(a+bx)+(-1+c(a+bx))\log(1-c(a+bx))+c(a+bx)\,\text{PolyLog}(2,c(a+bx)))}{bc}$$
$$+\frac{1}{2}ex^2\,\text{PolyLog}(2,ac+bcx)$$

input
```
Integrate[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

output
```
(e*(-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c
*(-2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, c*(a + b*x)]))/
(8*b^2*c^2) + (d*(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)]
 + c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (e*x^2*PolyLog[2, a*c + b
*c*x])/2
```

3.139.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$\Big\downarrow \; 7152$$
$$\frac{b\int \frac{(d+ex)^2\log(-ac-bxc+1)}{a+bx}dx}{2e}+\frac{(d+ex)^2\,\text{PolyLog}(2,c(a+bx))}{2e}$$
$$\Big\downarrow \; 2865$$
$$\frac{b\int \left(\frac{\log(-ac-bxc+1)(bd-ae)^2}{b^2(a+bx)}+\frac{e\log(-ac-bxc+1)(bd-ae)}{b^2}+\frac{e(d+ex)\log(-ac-bxc+1)}{b}\right)dx}{2e}+\frac{(d+ex)^2\,\text{PolyLog}(2,c(a+bx))}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{(-ace+bcd+e)^2\log(-ac-bcx+1)}{2b^3c^2}-\frac{(bd-ae)^2\operatorname{PolyLog}(2,c(a+bx))}{b^3}-\frac{e(-ac-bcx+1)(bd-ae)\log(-ac-bcx+1)}{b^3c}-\frac{ex(-ace+bcd+e)}{2b^2c}-\ldots\right.}{2e}$$

$$\frac{(d+ex)^2\operatorname{PolyLog}(2,c(a+bx))}{2e}$$

input
```
Int[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

output
```
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e) + (b*(-((e*(b*d - a*e)*x)/b^2)
 - (e*(b*c*d + e - a*c*e)*x)/(2*b^2*c) - (d + e*x)^2/(4*b) - ((b*c*d + e -
 a*c*e)^2*Log[1 - a*c - b*c*x])/(2*b^3*c^2) - (e*(b*d - a*e)*(1 - a*c - b*
c*x)*Log[1 - a*c - b*c*x])/(b^3*c) + ((d + e*x)^2*Log[1 - a*c - b*c*x])/(2
*b) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/b^3))/(2*e)
```

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.139.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.08

method	result
parts	$\frac{\text{polylog}(2,c(bx+a))e\,x^2}{2} + \text{polylog}\left(2, c(bx+a)\right) dx - \frac{-\frac{2ace((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)-2dbc((-}{}}{}$
derivativedivides	$\frac{-\frac{\text{polylog}(2,bcx+ac)ae(bcx+ac)}{b}+\text{polylog}(2,bcx+ac)d(bcx+ac)+\frac{\text{polylog}(2,bcx+ac)e(bcx+ac)^2}{2bc}-\frac{-2ace((-bcx-ac+1)\ln(-bcx-}{}}{}$
default	$\frac{-\frac{\text{polylog}(2,bcx+ac)ae(bcx+ac)}{b}+\text{polylog}(2,bcx+ac)d(bcx+ac)+\frac{\text{polylog}(2,bcx+ac)e(bcx+ac)^2}{2bc}-\frac{-2ace((-bcx-ac+1)\ln(-bcx-}{}}{}$
parallelrisch	$\frac{4x^2\,\text{polylog}(2,c(bx+a))b^2c^2e+2x^2\ln(1-c(bx+a))b^2c^2e-x^2b^2c^2e+8x\,\text{polylog}(2,c(bx+a))b^2c^2d-4x\ln(1-c(bx+a))ab\,c^2}{}$

input
```
int((e*x+d)*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*polylog(2,c*(b*x+a))*e*x^2+polylog(2,c*(b*x+a))*d*x-1/2/c*(-1/b^2/c*(2
*a*c*e*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)-2*d*b*c*((-b*c*x-a*c+
1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c)+e*(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-
1/4*(-b*c*x-a*c+1)^2)-e*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+b*c*x+a*c))+a*c
*(a*e-2*b*d)/b^2*dilog(-b*c*x-a*c+1))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.84

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx =$$
$$-\frac{b^2c^2ex^2+2\left(4\,b^2c^2d-(3\,abc^2-bc)e\right)x-4\left(b^2c^2ex^2+2\,b^2c^2dx+2\,abc^2d-a^2c^2e\right)\text{Li}_2(bcx+ac)-2\,(b^2c}{8\,b^2c^2}$$

input
```
integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b*c)*e)*x - 4*(b^2*c^2
*e*x^2 + 2*b^2*c^2*d*x + 2*a*b*c^2*d - a^2*c^2*e)*dilog(b*c*x + a*c) - 2*(
b^2*c^2*e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2
*c^2*d - a*b*c^2*e)*x)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.20

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$=\begin{cases} 0 \\ \left(dx+\frac{ex^2}{2}\right)\text{Li}_2\,(ac) \\ 0 \\ \frac{3a^2e\,\text{Li}_1(ac+bcx)}{4b^2}-\frac{a^2e\,\text{Li}_2(ac+bcx)}{2b^2}-\frac{ad\,\text{Li}_1(ac+bcx)}{b}+\frac{ad\,\text{Li}_2(ac+bcx)}{b}+\frac{aex\,\text{Li}_1(ac+bcx)}{2b}+\frac{3aex}{4b}-\frac{ae\,\text{Li}_1(ac+bcx)}{b^2c}-dx\,\text{Li}_1 \end{cases}$$

input
```
integrate((e*x+d)*polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), ((d*x + e*x**2/2)*polylog(2, a*c), Eq(
b, 0)), (0, Eq(c, 0)), (3*a**2*e*polylog(1, a*c + b*c*x)/(4*b**2) - a**2*e
*polylog(2, a*c + b*c*x)/(2*b**2) - a*d*polylog(1, a*c + b*c*x)/b + a*d*po
lylog(2, a*c + b*c*x)/b + a*e*x*polylog(1, a*c + b*c*x)/(2*b) + 3*a*e*x/(4
*b) - a*e*polylog(1, a*c + b*c*x)/(b**2*c) - d*x*polylog(1, a*c + b*c*x) +
 d*x*polylog(2, a*c + b*c*x) - d*x - e*x**2*polylog(1, a*c + b*c*x)/4 + e*
x**2*polylog(2, a*c + b*c*x)/2 - e*x**2/8 + d*polylog(1, a*c + b*c*x)/(b*c
) - e*x/(4*b*c) + e*polylog(1, a*c + b*c*x)/(4*b**2*c**2), True))
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$=-\frac{(2\,abd-a^2e)(\log\,(bcx+ac)\log\,(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))}{2\,b^2}$$
$$-\frac{b^2c^2ex^2+2\,(4\,b^2c^2d-(3\,abc^2-bc)e)x-4\,(b^2c^2ex^2+2\,b^2c^2dx)\text{Li}_2(bcx+ac)-2\,(b^2c^2ex^2+4\,(abc^2-b}{8\,b^2c^2}$$

input
```
integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*d - a^2*e)*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*
c*x - a*c + 1))/b^2 - 1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b
*c)*e)*x - 4*(b^2*c^2*e*x^2 + 2*b^2*c^2*d*x)*dilog(b*c*x + a*c) - 2*(b^2*c
^2*e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2*c^2*
d - a*b*c^2*e)*x)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

3.139.8 Giac [F]

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx = \int (ex+d)\text{Li}_2((bx+a)c)\,dx$$

input
```
integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((e*x + d)*dilog((b*x + a)*c), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\,\text{PolyLog}(2,c(a+bx))\,dx = \int \text{polylog}(2,c\,(a+b\,x))\,(d+e\,x)\,dx$$

input
```
int(polylog(2, c*(a + b*x))*(d + e*x),x)
```

output
```
int(polylog(2, c*(a + b*x))*(d + e*x), x)
```

3.140 $\int \text{PolyLog}(2, c(a + bx)) \, dx$

3.140.1 Optimal result

Integrand size = 9, antiderivative size = 60

$$\int \text{PolyLog}(2, c(a + bx)) \, dx = -x - \frac{(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{a \, \text{PolyLog}(2, c(a + bx))}{b} + x \, \text{PolyLog}(2, c(a + bx))$$

output
```
-x-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+a*polylog(2,c*(b*x+a))/b+x*polylog(
2,c*(b*x+a))
```

3.140.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \text{PolyLog}(2, c(a + bx)) \, dx = \frac{-c(a + bx) + (-1 + c(a + bx)) \log(1 - c(a + bx)) + c(a + bx) \, \text{PolyLog}(2, c(a + bx))}{bc}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)],x]
```

output
```
(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Po
lyLog[2, c*(a + b*x)])/(b*c)
```

3.140.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {7149, 25, 2868, 2840, 2838, 2894, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,c(a+bx))\,dx$$

$$\Big\downarrow \text{7149}$$

$$-\int -\log(1-c(a+bx))dx+a\int -\frac{\log(1-c(a+bx))}{a+bx}dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{25}$$

$$\int \log(1-c(a+bx))dx-a\int \frac{\log(1-c(a+bx))}{a+bx}dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2868}$$

$$-a\int \frac{\log(-ac-bxc+1)}{a+bx}dx+\int \log(1-c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2840}$$

$$\int \log(1-c(a+bx))dx-\frac{a\int \frac{\log(1-c(a+bx))}{a+bx}d(a+bx)}{b}+x\,\text{PolyLog}(2,c(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$\int \log(1-c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2894}$$

$$\int \log(-ac-bxc+1)dx+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{\int \log(-ac-bxc+1)d(-ac-bxc+1)}{bc}+x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}$$

$$\Big\downarrow \text{2732}$$

$$x\,\text{PolyLog}(2,c(a+bx))+\frac{a\,\text{PolyLog}(2,c(a+bx))}{b}-\frac{(-ac-bcx+1)\log(-ac-bcx+1)+ac+bcx-1}{bc}$$

input
```
Int[PolyLog[2, c*(a + b*x)],x]
```

output
```
-((-1 + a*c + b*c*x + (1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c)) + (a*
PolyLog[2, c*(a + b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2868
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

rule 7149
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Simp[p   Int[PolyLog[n - 1, c*(a + b*x)^p], x
], x] + Simp[a*p   Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /;
FreeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

3.140.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{(bcx+ac)\,\mathrm{polylog}(2,bcx+ac)-(-bcx-ac+1)\ln(-bcx-ac+1)+1-bcx-ac}{bc}$
default	$\frac{(bcx+ac)\,\mathrm{polylog}(2,bcx+ac)-(-bcx-ac+1)\ln(-bcx-ac+1)+1-bcx-ac}{bc}$
parts	$x\,\mathrm{polylog}\left(2,c(bx+a)\right)+\frac{-c((-bcx-ac+1)\ln(-bcx-ac+1)-1+bcx+ac)+a\,c^2\,\mathrm{dilog}(-bcx-ac+1)}{b\,c^2}$
parallelrisch	$\frac{x\,\mathrm{polylog}(2,c(bx+a))a^2b\,c^2+x\ln(1-c(bx+a))a^2b\,c^2-x\,a^2b\,c^2+\mathrm{polylog}(2,c(bx+a))a^3c^2+\ln(1-c(bx+a))a^3c^2-x\,\mathrm{polylog}}{abc(ac-1)}$

input
```
int(polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/c*((b*c*x+a*c)*polylog(2,b*c*x+a*c)-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)+1-
b*c*x-a*c)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \mathrm{PolyLog}(2,c(a+bx))\,dx = -\frac{bcx-(bcx+ac)\mathrm{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

output
```
-(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x
- a*c + 1))/(b*c)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \text{PolyLog}(2, c(a+bx))\,dx = \begin{cases} 0 & \text{for } b = 0 \wedge c = 0 \\ x\,\text{Li}_2\left(ac\right) & \text{for } b = 0 \\ 0 & \text{for } c = 0 \\ -\frac{a\,\text{Li}_1(ac+bcx)}{b} + \frac{a\,\text{Li}_2(ac+bcx)}{b} - x\,\text{Li}_1\left(ac+bcx\right) + x\,\text{Li}_2\left(ac+bcx\right) - x + \frac{\text{Li}_1(ac+bcx)}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(polylog(2,c*(b*x+a)),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x*polylog(2, a*c), Eq(b, 0)), (0, Eq(
c, 0)), (-a*polylog(1, a*c + b*c*x)/b + a*polylog(2, a*c + b*c*x)/b - x*po
lylog(1, a*c + b*c*x) + x*polylog(2, a*c + b*c*x) - x + polylog(1, a*c + b
*c*x)/(b*c), True))
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \text{PolyLog}(2, c(a+bx))\,dx = -\frac{(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right) + \text{Li}_2(-bcx-ac+1))a}{b} + \frac{bcx\text{Li}_2(bcx+ac) - bcx + (bcx+ac-1)\log\left(-bcx-ac+1\right)}{bc}$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

output
```
-(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b +
(b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1
))/(b*c)
```

3.140.8 Giac [F]

$$\int \text{PolyLog}(2, c(a+bx))\,dx = \int \text{Li}_2((bx+a)c)\,dx$$

input
```
integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c), x)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \text{PolyLog}(2, c(a+bx))\,dx = \frac{\text{polylog}(2, c\,(a+b\,x))\,(a+b\,x)}{b} - x - \frac{\ln(1-c\,(a+b\,x))}{b\,c} + \frac{\ln(1-c\,(a+b\,x))\,(a+b\,x)}{b}$$

input
```
int(polylog(2, c*(a + b*x)),x)
```

output
```
(polylog(2, c*(a + b*x))*(a + b*x))/b - x - log(1 - c*(a + b*x))/(b*c) + (
log(1 - c*(a + b*x))*(a + b*x))/b
```

3.141 $\int \frac{\text{PolyLog}(2,c(a+bx))}{d+ex}\,dx$

3.141.1 Optimal result

Integrand size = 17, antiderivative size = 591

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{d+ex}\,dx$$
$$= \frac{\left(\log(c(a+bx))+\log\left(\frac{bcd+e-ace}{bc(d+ex)}\right)-\log\left(\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2e}$$
$$+\frac{\log(c(a+bx))\log(d+ex)\log(1-c(a+bx))}{e}$$
$$-\frac{\left(\log(c(a+bx))-\log\left(-\frac{e(a+bx)}{bd-ae}\right)\right)\left(\log\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)+\log(1-c(a+bx))\right)^2}{2e}$$
$$+\frac{\log(d+ex)\,\text{PolyLog}(2,c(a+bx))}{e}$$
$$+\frac{\left(\log\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)+\log(1-c(a+bx))\right)\text{PolyLog}\left(2,\frac{b(d+ex)}{bd-ae}\right)}{e}$$
$$+\frac{\left(\log(d+ex)-\log\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)\right)\text{PolyLog}(2,1-c(a+bx))}{e}$$
$$-\frac{\log\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)\text{PolyLog}\left(2,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}$$
$$+\frac{\log\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)\text{PolyLog}\left(2,\frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right)}{e}$$
$$-\frac{\text{PolyLog}\left(3,\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{\text{PolyLog}(3,1-c(a+bx))}{e}$$
$$-\frac{\text{PolyLog}\left(3,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\text{PolyLog}\left(3,\frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right)}{e}$$

output
```
1/2*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b
*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/e+ln(c*(b*x+a))
*ln(e*x+d)*ln(1-c*(b*x+a))/e-1/2*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))
*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/e+ln(e*x+d)*po
lylog(2,c*(b*x+a))/e+(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a
)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/e+(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(
1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/e-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+
a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/e+ln(b*(e*x+d)/(-a*e+b*d)/(1-
c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e-polylog(3,b*(e
*x+d)/(-a*e+b*d))/e-polylog(3,1-c*(b*x+a))/e-polylog(3,-e*(1-c*(b*x+a))/b/
c/(e*x+d))/e+polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e
```

3.141.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.05

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{d+ex}\,dx$$
$$= \frac{\log(c(a+bx))\log(1-ac-bcx)\log(d+ex)+\frac{1}{2}\left(\log(c(a+bx))-\log\left(\frac{e(a+bx)}{-bd+ae}\right)\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)\left(-2\log(1\right.}{}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```

output
```
(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] -
 Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log
[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)
] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[
-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/
((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e -
 a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*
c*e)/(e - a*c*e - b*c*e*x)]))/2 + Log[d + e*x]*PolyLog[2, c*(a + b*x)] + (
Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*Pol
yLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b
*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + L
og[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d
+ e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*
(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d +
e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] +
PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])/e
```

3.141.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7151, 2890, 2885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{d+ex}\,dx$$
$$\Big\downarrow \; 7151$$

$$\frac{b\int \frac{\log(-ac-bxc+1)\log(d+ex)}{a+bx}dx}{e}+\frac{\log(d+ex)\operatorname{PolyLog}(2,c(a+bx))}{e}$$

$$\Big\downarrow \text{2890}$$

$$\frac{\int \frac{\log(1-c(a+bx))\log\left(d-\frac{ae}{b}+\frac{e(a+bx)}{b}\right)}{a+bx}d(a+bx)}{e}+\frac{\log(d+ex)\operatorname{PolyLog}(2,c(a+bx))}{e}$$

$$\Big\downarrow \text{2885}$$

$$\frac{-\operatorname{PolyLog}\left(3,-\frac{e(1-c(a+bx))}{c\left(b\left(d-\frac{ae}{b}\right)+e(a+bx)\right)}\right)+\operatorname{PolyLog}\left(3,\frac{b\left(d-\frac{ae}{b}\right)(1-c(a+bx))}{b\left(d-\frac{ae}{b}\right)+e(a+bx)}\right)-\operatorname{PolyLog}\left(2,-\frac{e(1-c(a+bx))}{c\left(b\left(d-\frac{ae}{b}\right)+e(a+bx)\right)}\right)\log\left(\frac{b}{(1}\right.}{}$$

$$\frac{\log(d+ex)\operatorname{PolyLog}(2,c(a+bx))}{e}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```

output
```
(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + (((Log[c*(a + b*x)] + Log[(b*c*
d + e - a*c*e)/(c*(b*(d - (a*e)/b) + e*(a + b*x)))] - Log[((b*c*d + e - a*
c*e)*(a + b*x))/(b*(d - (a*e)/b) + e*(a + b*x))])*Log[(b*(d - (a*e)/b) + e
*(a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/2 - ((Log[c*(a + b*x)] - L
og[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - c*(a + b*x)] + Log[(b*(d - (a*e
)/b) + e*(a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))])^2)/2 + Log[c*(a + b*
x)]*Log[1 - c*(a + b*x)]*Log[d - (a*e)/b + (e*(a + b*x))/b] - (Log[(b*(d -
 (a*e)/b) + e*(a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))] - Log[d - (a*e)/
b + (e*(a + b*x))/b])*PolyLog[2, 1 - c*(a + b*x)] - Log[(b*(d - (a*e)/b) +
 e*(a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a +
 b*x)))/(c*(b*(d - (a*e)/b) + e*(a + b*x))))] + Log[(b*(d - (a*e)/b) + e*(
a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, (b*(d - (a*e)/b)*(1
- c*(a + b*x)))/(b*(d - (a*e)/b) + e*(a + b*x))] + (Log[1 - c*(a + b*x)] +
 Log[(b*(d - (a*e)/b) + e*(a + b*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*Pol
yLog[2, 1 + (e*(a + b*x))/(b*d - a*e)] - PolyLog[3, 1 - c*(a + b*x)] - Pol
yLog[3, -((e*(1 - c*(a + b*x)))/(c*(b*(d - (a*e)/b) + e*(a + b*x))))] + Po
lyLog[3, (b*(d - (a*e)/b)*(1 - c*(a + b*x)))/(b*(d - (a*e)/b) + e*(a + b*x
))] - PolyLog[3, 1 + (e*(a + b*x))/(b*d - a*e)])/e
```

3.141. $\int \frac{\operatorname{PolyLog}(2,c(a+bx))}{d+ex}\,dx$

3.141.3.1 Defintions of rubi rules used

rule 2885
```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

rule 2890
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Simp[1/l   Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f +
g*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
 c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

rule 7151
```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Simp[b/e   Int[Log[d
 + e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e},
x] && NeQ[c*(b*d - a*e) + e, 0]
```

3.141.4 Maple [F]

$$\int \frac{\text{polylog}\,(2, c(bx + a))}{ex + d} dx$$

input
```
int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

output
```
int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

3.141.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{d + ex} dx = \int \frac{\text{Li}_2((bx + a)c)}{ex + d} dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/(e*x + d), x)
```

3.141.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{d + ex} dx = \int \frac{\text{Li}_2(ac + bcx)}{d + ex} dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d),x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/(d + e*x), x)
```

3.141.7 Maxima [F]

$$\int \frac{\text{PolyLog}(2, c(a + bx))}{d + ex} dx = \int \frac{\text{Li}_2((bx + a)c)}{ex + d} dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(dilog((b*x + a)*c)/(e*x + d), x)
```

3.141.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{d+ex}\,dx = \int \frac{\text{Li}_2((bx+a)c)}{ex+d}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/(e*x + d), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{d+ex}\,dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{d+e\,x}\,dx$$

input
```
int(polylog(2, c*(a + b*x))/(d + e*x),x)
```

output
```
int(polylog(2, c*(a + b*x))/(d + e*x), x)
```

3.142 $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx$

3.142.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx = \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)}+\frac{b\,\text{PolyLog}(2,c(a+bx))}{e(bd-ae)} - \frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}+\frac{b\,\text{PolyLog}\left(2,\frac{e(1-ac-bcx)}{bcd+e-ace}\right)}{e(bd-ae)}$$

output
```
b*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)+b*polylog
(2,c*(b*x+a))/e/(-a*e+b*d)-polylog(2,c*(b*x+a))/e/(e*x+d)+b*polylog(2,e*(-
b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)
```

3.142.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx = \frac{-\frac{\text{PolyLog}(2,c(a+bx))}{d+ex}+\frac{b\left(\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)+\text{PolyLog}(2,c(a+bx))+\text{PolyLog}\left(2,\frac{e(-1+ac+bcx)}{-bcd-e+ace}\right)\right)}{bd-ae}}{e}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

output
```
(-(PolyLog[2, c*(a + b*x)]/(d + e*x)) + (b*(Log[1 - a*c - b*c*x]*Log[(b*c*
(d + e*x))/(b*c*d + e - a*c*e)] + PolyLog[2, c*(a + b*x)] + PolyLog[2, (e*
(-1 + a*c + b*c*x))/(-(b*c*d) - e + a*c*e)]))/(b*d - a*e))/e
```

3.142.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{7152}$$

$$-\frac{b\int \frac{\log(-ac-bxc+1)}{(a+bx)(d+ex)}dx}{e}-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}$$

$$\Big\downarrow \text{2865}$$

$$-\frac{b\int \left(\frac{b\log(-ac-bxc+1)}{(bd-ae)(a+bx)}-\frac{e\log(-ac-bxc+1)}{(bd-ae)(d+ex)}\right)dx}{e}-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}-\frac{b\left(-\frac{\text{PolyLog}(2,c(a+bx))}{bd-ae}-\frac{\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{bcd-ace+e}\right)}{bd-ae}-\frac{\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{bd-ae}\right)}{e}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

output
```
-(PolyLog[2, c*(a + b*x)]/(e*(d + e*x))) - (b*(-((Log[1 - a*c - b*c*x]*Log
[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(b*d - a*e)) - PolyLog[2, c*(a + b*
x)]/(b*d - a*e) - PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(b
*d - a*e)))/e
```

3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.142.4 Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.30

method	result
parts	$-\frac{\operatorname{polylog}(2,c(bx+a))}{e(ex+d)}+\frac{-\frac{bc\,\operatorname{dilog}(-bcx-ac+1)}{ae-db}-\frac{bce\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(\cdots}{aec\cdots}\right)}{e}\cdots\right)}{ae-db}}{ec}\cdots$
derivativedivides	$\frac{\frac{b^2c^2\operatorname{polylog}(2,bcx+ac)}{(aec-bcd-e(bcx+ac))e}+\frac{b^2c^2\left(-\frac{e\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{c(ae-db)}-\cdots\right)}{e}}{bc}\cdots$
default	$\frac{\frac{b^2c^2\operatorname{polylog}(2,bcx+ac)}{(aec-bcd-e(bcx+ac))e}+\frac{b^2c^2\left(-\frac{e\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{c(ae-db)}-\cdots\right)}{e}}{bc}\cdots$

input
```
int(polylog(2,c*(b*x+a))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-polylog(2,c*(b*x+a))/e/(e*x+d)+1/e/c*(-b*c/(a*e-b*d)*dilog(-b*c*x-a*c+1)-
b*c*e/(a*e-b*d)*(dilog((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e
+ln(-b*c*x-a*c+1)*ln((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e))
```

3.142.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx=\int \frac{\text{Li}_2((bx+a)c)}{(ex+d)^2}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.142.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx=\int \frac{\text{Li}_2(ac+bcx)}{(d+ex)^2}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)**2,x)
```

output
```
Integral(polylog(2, a*c + b*c*x)/(d + e*x)**2, x)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx = -\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))b}{bde-ae^2} + \frac{\left(\log(-bcx-ac+1)\log\left(\frac{bcex+(ac-1)e}{bcd-(ac-1)e}+1\right)+\text{Li}_2\left(-\frac{bcex+(ac-1)e}{bcd-(ac-1)e}\right)\right)b}{bde-ae^2} - \frac{\text{Li}_2(bcx+ac)}{e^2x+de}$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/(b*d
*e - a*e^2) + (log(-b*c*x - a*c + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d -
(a*c - 1)*e) + 1) + dilog(-(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))
*b/(b*d*e - a*e^2) - dilog(b*c*x + a*c)/(e^2*x + d*e)
```

3.142.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx = \int \frac{\text{Li}_2((bx+a)c)}{(ex+d)^2}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/(e*x + d)^2, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx = \int \frac{\text{polylog}(2,c\,(a+b\,x))}{(d+e\,x)^2}\,dx$$

input
```
int(polylog(2, c*(a + b*x))/(d + e*x)^2,x)
```

output
```
int(polylog(2, c*(a + b*x))/(d + e*x)^2, x)
```

3.143 $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx$

3.143.1 Optimal result

Integrand size = 17, antiderivative size = 278

$$\begin{aligned}\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx &= \frac{b^2c\log(1-ac-bcx)}{2e(bd-ae)(bcd+e-ace)} - \frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} \\ &\quad - \frac{b^2c\log(d+ex)}{2e(bd-ae)(bcd+e-ace)} \\ &\quad + \frac{b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd-ae)^2} \\ &\quad + \frac{b^2\,\text{PolyLog}(2,c(a+bx))}{2e(bd-ae)^2} - \frac{\text{PolyLog}(2,c(a+bx))}{2e(d+ex)^2} \\ &\quad + \frac{b^2\,\text{PolyLog}\left(2,\frac{e(1-ac-bcx)}{bcd+e-ace}\right)}{2e(bd-ae)^2}\end{aligned}$$

output
```
1/2*b^2*c*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)-1/2*b*ln(-b*c*x-a
*c+1)/e/(-a*e+b*d)/(e*x+d)-1/2*b^2*c*ln(e*x+d)/e/(-a*e+b*d)/(-a*c*e+b*c*d+
e)+1/2*b^2*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^
2+1/2*b^2*polylog(2,c*(b*x+a))/e/(-a*e+b*d)^2-1/2*polylog(2,c*(b*x+a))/e/(
e*x+d)^2+1/2*b^2*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)
^2
```

3.143.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.68

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^3}\,dx$$
$$= \frac{-\frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2} + \frac{b\left(-\frac{(bd-ae)\log(1-ac-bcx)}{d+ex} + \frac{bc(bd-ae)(\log(1-ac-bcx)-\log(d+ex))}{bcd+e-ace} + b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right) + b\,\text{PolyLog}(2,c(a\cdots\right.}{(bd-ae)^2}}{2e}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

output
```
(-(PolyLog[2, c*(a + b*x)]/(d + e*x)^2) + (b*(-(((b*d - a*e)*Log[1 - a*c -
 b*c*x])/(d + e*x)) + (b*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x
]))/(b*c*d + e - a*c*e) + b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*
d + e - a*c*e)] + b*PolyLog[2, c*(a + b*x)] + b*PolyLog[2, (e*(-1 + a*c +
b*c*x))/(-(b*c*d) + (-1 + a*c)*e)]))/(b*d - a*e)^2)/(2*e)
```

3.143.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^3}\,dx$$

$$\Big\downarrow \ 7152$$

$$-\frac{b\int \frac{\log(-ac-bxc+1)}{(a+bx)(d+ex)^2}dx}{2e} - \frac{\text{PolyLog}(2, c(a+bx))}{2e(d+ex)^2}$$

$$\Big\downarrow \ 2865$$

$$-\frac{b\int \left(\frac{\log(-ac-bxc+1)b^2}{(bd-ae)^2(a+bx)} - \frac{e\log(-ac-bxc+1)b}{(bd-ae)^2(d+ex)} - \frac{e\log(-ac-bxc+1)}{(bd-ae)(d+ex)^2}\right)dx}{2e} - \frac{\text{PolyLog}(2, c(a+bx))}{2e(d+ex)^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\text{PolyLog}(2,c(a+bx))}{2e(d+ex)^2}-\frac{b\left(-\frac{b\,\text{PolyLog}(2,c(a+bx))}{(bd-ae)^2}-\frac{b\,\text{PolyLog}\left(2,\frac{e(-ac-bxc+1)}{bcd-ace+e}\right)}{(bd-ae)^2}-\frac{b\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{(bd-ae)^2}-\frac{bc\log(-ac-bcx+1)}{(bd-ae)(-ace+bcd+e)}+\frac{\log(-ac-bc}{(d+ex)(bd}\right.}{2e}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

output
```
-1/2*PolyLog[2, c*(a + b*x)]/(e*(d + e*x)^2) - (b*(-((b*c*Log[1 - a*c - b*
c*x])/((b*d - a*e)*(b*c*d + e - a*c*e))) + Log[1 - a*c - b*c*x]/((b*d - a*
e)*(d + e*x)) + (b*c*Log[d + e*x])/((b*d - a*e)*(b*c*d + e - a*c*e)) - (b*
Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(b*d - a*e)
^2 - (b*PolyLog[2, c*(a + b*x)])/(b*d - a*e)^2 - (b*PolyLog[2, (e*(1 - a*c
 - b*c*x))/(b*c*d + e - a*c*e)])/(b*d - a*e)^2))/(2*e)
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.143.4 Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.13

method	result
parts	$-\frac{\operatorname{polylog}(2,c(bx+a))}{2e(ex+d)^2}+\frac{\frac{b^2ce\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{(ae-db)^2}+\frac{b^2}{}}{}$
derivativedivides	$\frac{-\frac{b^3c^3\operatorname{polylog}(2,bcx+ac)}{2(aec-bcd-e(bcx+ac))^2e}-\frac{b^3c^3\left(-\frac{e\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{c^2(ae-db)^2}\right.}{}}{bc}$
default	$\frac{-\frac{b^3c^3\operatorname{polylog}(2,bcx+ac)}{2(aec-bcd-e(bcx+ac))^2e}-\frac{b^3c^3\left(-\frac{e\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{c^2(ae-db)^2}\right.}{}}{bc}$

input
```
int(polylog(2,c*(b*x+a))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*polylog(2,c*(b*x+a))/e/(e*x+d)^2+1/2/e/c*(b^2*c*e/(a*e-b*d)^2*(dilog(
(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e+ln(-b*c*x-a*c+1)*ln((a
*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e)+b^2*c^2*e/(a*e-b*d)*(-1
/(a*c*e-b*c*d-e)*ln(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/e+ln(-b*c*x-a*c+1)*(-b
*c*x-a*c+1)/(a*c*e-b*c*d-e)/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e))+c/(a*e-b*d)^
2*b^2*dilog(-b*c*x-a*c+1))
```

3.143.5 Fricas [F]

$$\int \frac{\operatorname{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx=\int \frac{\operatorname{Li}_2((bx+a)c)}{(ex+d)^3}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)**3,x)
```

output
```
Timed out
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.36

$$\begin{aligned}
&\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx \\
&= \frac{b^2c\log(bcx+ac-1)}{2\left(b^2cd^2e-(2\,abc-b)de^2+(a^2c-a)e^3\right)} - \frac{b^2c\log(ex+d)}{2\left(b^2cd^2e-(2\,abc-b)de^2+(a^2c-a)e^3\right)} \\
&\quad - \frac{\left(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1)\right)b^2}{2\left(b^2d^2e-2\,abde^2+a^2e^3\right)} \\
&\quad + \frac{\left(\log(-bcx-ac+1)\log\left(\frac{bcex+(ac-1)e}{bcd-(ac-1)e}+1\right)+\text{Li}_2\left(-\frac{bcex+(ac-1)e}{bcd-(ac-1)e}\right)\right)b^2}{2\left(b^2d^2e-2\,abde^2+a^2e^3\right)} \\
&\quad - \frac{(bd-ae)\text{Li}_2(bcx+ac)+(bex+bd)\log(-bcx-ac+1)}{2\left(bd^3e-ad^2e^2+(bde^3-ae^4)x^2+2\left(bd^2e^2-ade^3\right)x\right)}
\end{aligned}$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*b^2*c*log(b*c*x + a*c - 1)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (a^2*c
 - a)*e^3) - 1/2*b^2*c*log(e*x + d)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (
a^2*c - a)*e^3) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c
*x - a*c + 1))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/2*(log(-b*c*x -
 a*c + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + dilog(-
(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b^2/(b^2*d^2*e - 2*a*b*d*e
^2 + a^2*e^3) - 1/2*((b*d - a*e)*dilog(b*c*x + a*c) + (b*e*x + b*d)*log(-b
*c*x - a*c + 1))/(b*d^3*e - a*d^2*e^2 + (b*d*e^3 - a*e^4)*x^2 + 2*(b*d^2*e
^2 - a*d*e^3)*x)
```

3.143.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^3} \, dx = \int \frac{\text{Li}_2((bx+a)c)}{(ex+d)^3} \, dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/(e*x + d)^3, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^3} \, dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{(d+e\,x)^3} \, dx$$

input
```
int(polylog(2, c*(a + b*x))/(d + e*x)^3,x)
```

output
```
int(polylog(2, c*(a + b*x))/(d + e*x)^3, x)
```

3.144 $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4} dx$

3.144.1 Optimal result

Integrand size = 17, antiderivative size = 448

$$\begin{aligned}\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4} dx &= \frac{b^2c}{6e(bd-ae)(bcd+e-ace)(d+ex)} \\ &+ \frac{b^3c^2\log(1-ac-bcx)}{6e(bd-ae)(bcd+e-ace)^2} + \frac{b^3c\log(1-ac-bcx)}{3e(bd-ae)^2(bcd+e-ace)} \\ &- \frac{b\log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2\log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} \\ &- \frac{b^3c^2\log(d+ex)}{6e(bd-ae)(bcd+e-ace)^2} - \frac{b^3c\log(d+ex)}{3e(bd-ae)^2(bcd+e-ace)} \\ &+ \frac{b^3\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{3e(bd-ae)^3} \\ &+ \frac{b^3\,\text{PolyLog}(2,c(a+bx))}{3e(bd-ae)^3} - \frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3} \\ &+ \frac{b^3\,\text{PolyLog}\left(2,\frac{e(1-ac-bcx)}{bcd+e-ace}\right)}{3e(bd-ae)^3}\end{aligned}$$

output
```
1/6*b^2*c/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)/(e*x+d)+1/6*b^3*c^2*ln(-b*c*x-a*c+
1)/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)^2+1/3*b^3*c*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)
^2/(-a*c*e+b*c*d+e)-1/6*b*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)/(e*x+d)^2-1/3*b^2*
ln(-b*c*x-a*c+1)/e/(-a*e+b*d)^2/(e*x+d)-1/6*b^3*c^2*ln(e*x+d)/e/(-a*e+b*d)
/(-a*c*e+b*c*d+e)^2-1/3*b^3*c*ln(e*x+d)/e/(-a*e+b*d)^2/(-a*c*e+b*c*d+e)+1/
3*b^3*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^3+1/3
*b^3*polylog(2,c*(b*x+a))/e/(-a*e+b*d)^3-1/3*polylog(2,c*(b*x+a))/e/(e*x+d
)^3+1/3*b^3*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^3
```

3.144.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.70

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx$$
$$= \frac{-\frac{2\,\text{PolyLog}(2,c(a+bx))}{(d+ex)^3}+\frac{b\left(-\frac{(bd-ae)^2\log(1-ac-bcx)}{(d+ex)^2}-\frac{2b(bd-ae)\log(1-ac-bcx)}{d+ex}+\frac{2b^2c(bd-ae)(\log(1-ac-bcx)-\log(d+ex))}{bcd+e-ace}+\frac{bc(bd-ae)^2(bcd+e-ac\cdots}{\cdots}\right.}{\cdots}}{6\cdots}$$

input
```
Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```

output
```
((-2*PolyLog[2, c*(a + b*x)])/(d + e*x)^3 + (b*(-(((b*d - a*e)^2*Log[1 - a
*c - b*c*x])/(d + e*x)^2) - (2*b*(b*d - a*e)*Log[1 - a*c - b*c*x])/(d + e*
x) + (2*b^2*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x]))/(b*c*d +
e - a*c*e) + (b*c*(b*d - a*e)^2*(b*c*d + e - a*c*e + b*c*(d + e*x)*Log[1 -
 a*c - b*c*x] - b*c*(d + e*x)*Log[d + e*x]))/((b*c*d + e - a*c*e)^2*(d + e
*x)) + 2*b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]
 + 2*b^2*PolyLog[2, c*(a + b*x)] + 2*b^2*PolyLog[2, (e*(-1 + a*c + b*c*x))
/(-(b*c*d) + (-1 + a*c)*e)]))/(b*d - a*e)^3)/(6*e)
```

3.144.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7152, 2865, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx$$

$$\Big\downarrow \; 7152$$

$$-\frac{b\int \frac{\log(-ac-bxc+1)}{(a+bx)(d+ex)^3}dx}{3e}-\frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3}$$

$$\Big\downarrow \; 2865$$

$$-\frac{b\int \left(\frac{\log(-ac-bxc+1)b^3}{(bd-ae)^3(a+bx)}-\frac{e\log(-ac-bxc+1)b^2}{(bd-ae)^3(d+ex)}-\frac{e\log(-ac-bxc+1)b}{(bd-ae)^2(d+ex)^2}-\frac{e\log(-ac-bxc+1)}{(bd-ae)(d+ex)^3}\right)dx}{3e}-\frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{b\left(-\frac{b^2c^2\log(-ac-bcx+1)}{2(bd-ae)(-ace+bcd+e)^2}+\frac{b^2c^2\log(d+ex)}{2(bd-ae)(-ace+bcd+e)^2}-\frac{b^2\,\text{PolyLog}(2,c(a+bx))}{(bd-ae)^3}-\frac{b^2\,\text{PolyLog}\left(2,\frac{e(-ac-bxc+1)}{bcd-ace+e}\right)}{(bd-ae)^3}-\frac{b^2c\log(-ac-b}{(bd-ae)^2(-ace-}\right.}{}$$

$$\frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3}$$

input
```
Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```

output
```
-1/3*PolyLog[2, c*(a + b*x)]/(e*(d + e*x)^3) - (b*(-1/2*(b*c)/((b*d - a*e)
*(b*c*d + e - a*c*e)*(d + e*x)) - (b^2*c^2*Log[1 - a*c - b*c*x])/(2*(b*d -
 a*e)*(b*c*d + e - a*c*e)^2) - (b^2*c*Log[1 - a*c - b*c*x])/((b*d - a*e)^2
*(b*c*d + e - a*c*e)) + Log[1 - a*c - b*c*x]/(2*(b*d - a*e)*(d + e*x)^2) +
 (b*Log[1 - a*c - b*c*x])/((b*d - a*e)^2*(d + e*x)) + (b^2*c^2*Log[d + e*x
])/(2*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^2*c*Log[d + e*x])/((b*d - a*
e)^2*(b*c*d + e - a*c*e)) - (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/
(b*c*d + e - a*c*e)])/(b*d - a*e)^3 - (b^2*PolyLog[2, c*(a + b*x)])/(b*d -
 a*e)^3 - (b^2*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(b*d
 - a*e)^3))/(3*e)
```

3.144. $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx$

3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2865
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

rule 7152
```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Simp[b/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x)
), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.144.4 Maple [A] (verified)

Time = 24.59 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{\operatorname{polylog}(2,c(bx+a))}{3e(ex+d)^{3}}+\frac{-\frac{b^{3}c\operatorname{dilog}(-bcx-ac+1)}{(ae-db)^{3}}-\frac{b^{3}ce\left(\frac{\operatorname{dilog}\left(\frac{aec-bcd+e(-bcx-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-bcx-ac+1)\ln\left(\frac{aec-bcd+}{a}\right.}{e}\right.}{(ae-db)^{3}}}{}$
derivativedivides	$\frac{\frac{b^{4}c^{4}\operatorname{polylog}(2,bcx+ac)}{3(aec-bcd-e(bcx+ac))^{3}e}+\frac{b^{4}c^{4}\left(-\frac{e\left(-\frac{\ln(aec-bcd+e(-bcx-ac+1)-e)}{(aec-bcd-e)e}+\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(aec-bcd-e)(aec-bcd+e(-bcx-ac+1)-e)}\right)}{c^{2}(ae-db)^{2}}-\frac{\operatorname{dilog}(-b}{c^{3}(ae}\right.}{}}{}$
default	$\frac{\frac{b^{4}c^{4}\operatorname{polylog}(2,bcx+ac)}{3(aec-bcd-e(bcx+ac))^{3}e}+\frac{b^{4}c^{4}\left(-\frac{e\left(-\frac{\ln(aec-bcd+e(-bcx-ac+1)-e)}{(aec-bcd-e)e}+\frac{\ln(-bcx-ac+1)(-bcx-ac+1)}{(aec-bcd-e)(aec-bcd+e(-bcx-ac+1)-e)}\right)}{c^{2}(ae-db)^{2}}-\frac{\operatorname{dilog}(-b}{c^{3}(ae}\right.}{}}{}$

input
```
int(polylog(2,c*(b*x+a))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*polylog(2,c*(b*x+a))/e/(e*x+d)^3+1/3/e/c*(-b^3*c/(a*e-b*d)^3*dilog(-b
*c*x-a*c+1)-b^3*c*e/(a*e-b*d)^3*(dilog((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a
*c*e-b*c*d-e))/e+ln(-b*c*x-a*c+1)*ln((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c
*e-b*c*d-e))/e)-b^3*c^3*e/(a*e-b*d)*(-1/2/(a*c*e-b*c*d-e)^2*(-(a*c*e-b*c*d
-e)/e/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)+1/e*ln(a*e*c-b*c*d+e*(-b*c*x-a*c+1)
-e))+1/2*ln(-b*c*x-a*c+1)*(2*a*e*c-2*b*c*d+e*(-b*c*x-a*c+1)-2*e)*(-b*c*x-a
*c+1)/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)^2/(a*c*e-b*c*d-e)^2)-b^3*c^2*e/(a*e
-b*d)^2*(-1/(a*c*e-b*c*d-e)*ln(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/e+ln(-b*c*x
-a*c+1)*(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)))
```

3.144.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx=\int \frac{\text{Li}_2((bx+a)c)}{(ex+d)^4}\,dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(dilog(b*c*x + a*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3
*e*x + d^4), x)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx=\text{Timed out}$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)**4,x)
```

output
```
Timed out
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. 2(422) = 844.

Time = 0.24 (sec) , antiderivative size = 1428, normalized size of antiderivative = 3.19

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^4} dx = \text{Too large to display}$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^
3/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) + 1/3*(log(-b*c*
x - a*c + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + dilo
g(-(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b^3/(b^3*d^3*e - 3*a*b^
2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) - 1/6*(3*b^4*c^2*d - (3*a*b^3*c^2 - 2
*b^3*c)*e)*log(e*x + d)/(b^4*c^2*d^4*e - 2*(2*a*b^3*c^2 - b^3*c)*d^3*e^2 +
 (6*a^2*b^2*c^2 - 6*a*b^2*c + b^2)*d^2*e^3 - 2*(2*a^3*b*c^2 - 3*a^2*b*c +
a*b)*d*e^4 + (a^4*c^2 - 2*a^3*c + a^2)*e^5) + 1/6*(b^4*c^2*d^4 - (2*a*b^3*
c^2 - b^3*c)*d^3*e + (a^2*b^2*c^2 - a*b^2*c)*d^2*e^2 + (b^4*c^2*d^2*e^2 -
(2*a*b^3*c^2 - b^3*c)*d*e^3 + (a^2*b^2*c^2 - a*b^2*c)*e^4)*x^2 + 2*(b^4*c^
2*d^3*e - (2*a*b^3*c^2 - b^3*c)*d^2*e^2 + (a^2*b^2*c^2 - a*b^2*c)*d*e^3)*x
 - 2*(b^4*c^2*d^4 - 2*(2*a*b^3*c^2 - b^3*c)*d^3*e + (6*a^2*b^2*c^2 - 6*a*b
^2*c + b^2)*d^2*e^2 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d*e^3 + (a^4*c^2 -
 2*a^3*c + a^2)*e^4)*dilog(b*c*x + a*c) + (4*(a*b^3*c^2 - b^3*c)*d^3*e - (
5*a^2*b^2*c^2 - 8*a*b^2*c + 3*b^2)*d^2*e^2 + (a^3*b*c^2 - 2*a^2*b*c + a*b)
*d*e^3 + (3*b^4*c^2*d*e^3 - (3*a*b^3*c^2 - 2*b^3*c)*e^4)*x^3 + (7*b^4*c^2*
d^2*e^2 - (5*a*b^3*c^2 - 2*b^3*c)*d*e^3 - 2*(a^2*b^2*c^2 - 2*a*b^2*c + b^2
)*e^4)*x^2 + (4*b^4*c^2*d^3*e + 2*(a*b^3*c^2 - 2*b^3*c)*d^2*e^2 - (7*a^2*b
^2*c^2 - 12*a*b^2*c + 5*b^2)*d*e^3 + (a^3*b*c^2 - 2*a^2*b*c + a*b)*e^4)*x)
*log(-b*c*x - a*c + 1))/(b^4*c^2*d^7*e - 2*(2*a*b^3*c^2 - b^3*c)*d^6*e^...
```

3.144.8 Giac [F]

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^4} dx = \int \frac{\text{Li}_2((bx+a)c)}{(ex+d)^4} dx$$

input
```
integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(dilog((b*x + a)*c)/(e*x + d)^4, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^4} \, dx = \int \frac{\text{polylog}(2, c\,(a+b\,x))}{(d+e\,x)^4} \, dx$$

input
```
int(polylog(2, c*(a + b*x))/(d + e*x)^4,x)
```

output
```
int(polylog(2, c*(a + b*x))/(d + e*x)^4, x)
```

3.145 $\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx$

3.145.1 Optimal result

Integrand size = 9, antiderivative size = 46

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x)$$

output
```
ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,
1-x)
```

3.145.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x)$$

input
```
Integrate[PolyLog[2, x]/(-1 + x),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x]
```

3.145.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7150, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,x)}{x-1} \, dx$$

$$\Big\downarrow \text{7150}$$

$$\int \frac{\log^2(1-x)}{x}dx+\text{PolyLog}(2,x)\log(1-x)$$

$$\Big\downarrow \text{2843}$$

$$2\int \frac{\log(1-x)\log(x)}{1-x}dx+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{2881}$$

$$-2\int \frac{\log(1-x)\log(x)}{1-x}d(1-x)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{2821}$$

$$-2\left(\int \frac{\text{PolyLog}(2,1-x)}{1-x}d(1-x)-\text{PolyLog}(2,1-x)\log(1-x)\right)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{7143}$$

$$\text{PolyLog}(2,x)\log(1-x)-2(\text{PolyLog}(3,1-x)-\text{PolyLog}(2,1-x)\log(1-x))+\log(x)\log^2(1-x)$$

input
```
Int[PolyLog[2, x]/(-1 + x),x]
```

output
```
Log[1 - x]^2*Log[x] + Log[1 - x]*PolyLog[2, x] - 2*(-(Log[1 - x]*PolyLog[2
, 1 - x]) + PolyLog[3, 1 - x])
```

3.145.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7150
```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Simp[b/e   I
nt[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c*(b*d - a*e) + e, 0]
```

3.145.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
default	$\ln(x-1)\,\text{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\,\text{polylog}(2,1-x)-2\,\text{polylog}(3,1-x$
parts	$\ln(x-1)\,\text{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\,\text{polylog}(2,1-x)-2\,\text{polylog}(3,1-x$

input
```
int(polylog(2,x)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)*polylog(2,x)+ln(x-1)^2*ln(x)+2*ln(x-1)*polylog(2,1-x)-2*polylog(3,
1-x)+(ln(1-x)-ln(x-1))*(dilog(x)+ln(x-1)*ln(x))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \log(x)\log(-x+1)^2 + (\text{Li}_2(x) + 2\,\text{Li}_2(-x+1))\log(-x+1) - 2\,\text{polylog}(3,-x+1)$$

input
```
integrate(polylog(2,x)/(-1+x),x, algorithm="fricas")
```

output
```
log(x)*log(-x + 1)^2 + (dilog(x) + 2*dilog(-x + 1))*log(-x + 1) - 2*polylo
g(3, -x + 1)
```

3.145.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \int \frac{\text{Li}_2(x)}{x-1}\,dx$$

input
```
integrate(polylog(2,x)/(-1+x),x)
```

output
```
Integral(polylog(2, x)/(x - 1), x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\,\text{Li}_2(-x+1)\log(-x+1) - 2\,\text{Li}_3(-x+1)$$

input
```
integrate(polylog(2,x)/(-1+x),x, algorithm="maxima")
```

output
```
log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1)
- 2*polylog(3, -x + 1)
```

3.145.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \int \frac{\text{Li}_2(x)}{x-1}\,dx$$

input
```
integrate(polylog(2,x)/(-1+x),x, algorithm="giac")
```

output
```
integrate(dilog(x)/(x - 1), x)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx = \ln(1-x)^2\ln(x) - 2\,\text{polylog}(3,1-x) + 2\ln(1-x)\,\text{polylog}(2,1-x) + \ln(1-x)\,\text{polylog}(2,x)$$

input
```
int(polylog(2, x)/(x - 1),x)
```

output
```
log(1 - x)^2*log(x) - 2*polylog(3, 1 - x) + 2*log(1 - x)*polylog(2, 1 - x)
 + log(1 - x)*polylog(2, x)
```

3.146 $\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx$

3.146.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x)$$

output
```
ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,
1-x)
```

3.146.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x)$$

input
```
Integrate[-(PolyLog[2, x]/(1 - x)),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x]
```

3.146.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {25, 7150, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\text{PolyLog}(2,x)}{1-x}dx$$

$$\Big\downarrow \text{7150}$$

$$\int \frac{\log^2(1-x)}{x}dx+\text{PolyLog}(2,x)\log(1-x)$$

$$\Big\downarrow \text{2843}$$

$$2\int \frac{\log(1-x)\log(x)}{1-x}dx+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{2881}$$

$$-2\int \frac{\log(1-x)\log(x)}{1-x}d(1-x)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{2821}$$

$$-2\left(\int \frac{\text{PolyLog}(2,1-x)}{1-x}d(1-x)-\text{PolyLog}(2,1-x)\log(1-x)\right)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

$$\Big\downarrow \text{7143}$$

$$\text{PolyLog}(2,x)\log(1-x)-2(\text{PolyLog}(3,1-x)-\text{PolyLog}(2,1-x)\log(1-x))+\log(x)\log^2(1-x)$$

input
```
Int[-(PolyLog[2, x]/(1 - x)),x]
```

output
```
Log[1 - x]^2*Log[x] + Log[1 - x]*PolyLog[2, x] - 2*(-(Log[1 - x]*PolyLog[2
, 1 - x]) + PolyLog[3, 1 - x])
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7150
```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Simp[b/e   I
nt[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c*(b*d - a*e) + e, 0]
```

3.146.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
default	$\ln(x-1)\,\mathrm{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\,\mathrm{polylog}(2,1-x)-2\,\mathrm{polylog}(3,1-x$
parts	$\ln(x-1)\,\mathrm{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\,\mathrm{polylog}(2,1-x)-2\,\mathrm{polylog}(3,1-x$

input
```
int(-polylog(2,x)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)*polylog(2,x)+ln(x-1)^2*ln(x)+2*ln(x-1)*polylog(2,1-x)-2*polylog(3,
1-x)+(ln(1-x)-ln(x-1))*(dilog(x)+ln(x-1)*ln(x))
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int -\frac{\mathrm{PolyLog}(2,x)}{1-x}\,dx = \log(x)\log(-x+1)^2+(\mathrm{Li}_2(x)+2\,\mathrm{Li}_2(-x+1))\log(-x+1) - 2\,\mathrm{polylog}(3,-x+1)$$

input
```
integrate(-polylog(2,x)/(1-x),x, algorithm="fricas")
```

output
```
log(x)*log(-x + 1)^2 + (dilog(x) + 2*dilog(-x + 1))*log(-x + 1) - 2*polylo
g(3, -x + 1)
```

3.146.6 Sympy [F]

$$\int -\frac{\mathrm{PolyLog}(2,x)}{1-x}\,dx = \int \frac{\mathrm{Li}_2(x)}{x-1}\,dx$$

input
```
integrate(-polylog(2,x)/(1-x),x)
```

output
```
Integral(polylog(2, x)/(x - 1), x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx = \log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\,\text{Li}_2(-x+1)\log(-x+1) - 2\,\text{Li}_3(-x+1)$$

input
```
integrate(-polylog(2,x)/(1-x),x, algorithm="maxima")
```

output
```
log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1)
- 2*polylog(3, -x + 1)
```

3.146.8 Giac [F]

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx = \int \frac{\text{Li}_2(x)}{x-1}\,dx$$

input
```
integrate(-polylog(2,x)/(1-x),x, algorithm="giac")
```

output
```
integrate(dilog(x)/(x - 1), x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx = \ln(1-x)^2\ \ln(x) - 2\,\text{polylog}(3,1-x) + 2\ \ln(1-x)\ \text{polylog}(2,1-x) + \ln(1-x)\ \text{polylog}(2,x)$$

input
```
int(polylog(2, x)/(x - 1),x)
```

output
```
log(1 - x)^2*log(x) - 2*polylog(3, 1 - x) + 2*log(1 - x)*polylog(2, 1 - x)
 + log(1 - x)*polylog(2, x)
```

3.147 $\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx$

3.147.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x)$$

output
```
ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,
1-x)-polylog(3,x)
```

3.147.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x)$$

input
```
Integrate[PolyLog[2, x]/((-1 + x)*x),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

3.147.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,x)}{(x-1)x}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\text{PolyLog}(2,x)}{x-1}-\frac{\text{PolyLog}(2,x)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2\,\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\,\text{PolyLog}(2,1-x)\log(1-x)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

input
```
Int[PolyLog[2, x]/((-1 + x)*x),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.147.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

method	result
default	$\ln(x-1)\operatorname{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\operatorname{polylog}(2,1-x)-2\operatorname{polylog}(3,1-x$
parts	$\ln(x-1)\operatorname{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\operatorname{polylog}(2,1-x)-2\operatorname{polylog}(3,1-x$

input
```
int(polylog(2,x)/(x-1)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)*polylog(2,x)+ln(x-1)^2*ln(x)+2*ln(x-1)*polylog(2,1-x)-2*polylog(3,
1-x)+(ln(1-x)-ln(x-1))*(dilog(x)+ln(x-1)*ln(x))-polylog(3,x)
```

3.147.5 Fricas [F]

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx=\int \frac{\text{Li}_2(x)}{(x-1)x}\,dx$$

input
```
integrate(polylog(2,x)/(-1+x)/x,x, algorithm="fricas")
```

output
```
integral(dilog(x)/(x^2 - x), x)
```

3.147.6 Sympy [F]

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx=\int \frac{\text{Li}_2\left(x\right)}{x\left(x-1\right)}\,dx$$

input
```
integrate(polylog(2,x)/(-1+x)/x,x)
```

output
```
Integral(polylog(2, x)/(x*(x - 1)), x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx = \log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\,\text{Li}_2(-x+1)\log(-x+1) - \text{Li}_3(x) - 2\,\text{Li}_3(-x+1)$$

input
```
integrate(polylog(2,x)/(-1+x)/x,x, algorithm="maxima")
```

output
```
log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1)
- polylog(3, x) - 2*polylog(3, -x + 1)
```

3.147.8 Giac [F]

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx = \int \frac{\text{Li}_2(x)}{(x-1)x}\,dx$$

input
```
integrate(polylog(2,x)/(-1+x)/x,x, algorithm="giac")
```

output
```
integrate(dilog(x)/((x - 1)*x), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx = \int \frac{\text{polylog}(2,x)}{x\,(x-1)}\,dx$$

input
```
int(polylog(2, x)/(x*(x - 1)),x)
```

output
```
int(polylog(2, x)/(x*(x - 1)), x)
```

3.148 $\int -\frac{\text{PolyLog}(2,x)}{(1-x)x} dx$

3.148.1 Optimal result

Integrand size = 15, antiderivative size = 51

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x} dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x)$$

output
```
ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,
1-x)-polylog(3,x)
```

3.148.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x} dx = \log^2(1-x)\log(x) + 2\log(1-x)\,\text{PolyLog}(2,1-x) + \log(1-x)\,\text{PolyLog}(2,x) - 2\,\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x)$$

input
```
Integrate[-(PolyLog[2, x]/((1 - x)*x)),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

3.148.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\text{PolyLog}(2,x)}{(1-x)x}dx$$

$$\Big\downarrow \ 7293$$

$$-\int \left(\frac{\text{PolyLog}(2,x)}{x}-\frac{\text{PolyLog}(2,x)}{x-1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-2\,\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\,\text{PolyLog}(2,1-x)\log(1-x)+\text{PolyLog}(2,x)\log(1-x)+\log(x)\log^2(1-x)$$

input
```
Int[-(PolyLog[2, x]/((1 - x)*x)),x]
```

output
```
Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[
2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.148.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

method	result
default	$\ln(x-1)\operatorname{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\operatorname{polylog}(2,1-x)-2\operatorname{polylog}(3,1-x$
parts	$\ln(x-1)\operatorname{polylog}(2,x)+\ln(x-1)^2\ln(x)+2\ln(x-1)\operatorname{polylog}(2,1-x)-2\operatorname{polylog}(3,1-x$

input
```
int(-polylog(2,x)/(1-x)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)*polylog(2,x)+ln(x-1)^2*ln(x)+2*ln(x-1)*polylog(2,1-x)-2*polylog(3,
1-x)+(ln(1-x)-ln(x-1))*(dilog(x)+ln(x-1)*ln(x))-polylog(3,x)
```

3.148.5 Fricas [F]

$$\int -\frac{\operatorname{PolyLog}(2,x)}{(1-x)x}\,dx=\int \frac{\operatorname{Li}_2(x)}{(x-1)x}\,dx$$

input
```
integrate(-polylog(2,x)/(1-x)/x,x, algorithm="fricas")
```

output
```
integral(dilog(x)/(x^2 - x), x)
```

3.148.6 Sympy [F]

$$\int -\frac{\operatorname{PolyLog}(2,x)}{(1-x)x}\,dx=\int \frac{\operatorname{Li}_2(x)}{x(x-1)}\,dx$$

input
```
integrate(-polylog(2,x)/(1-x)/x,x)
```

output
```
Integral(polylog(2, x)/(x*(x - 1)), x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx = \log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\,\text{Li}_2(-x+1)\log(-x+1) - \text{Li}_3(x) - 2\,\text{Li}_3(-x+1)$$

input
```
integrate(-polylog(2,x)/(1-x)/x,x, algorithm="maxima")
```

output
```
log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1)
- polylog(3, x) - 2*polylog(3, -x + 1)
```

3.148.8 Giac [F]

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx = \int \frac{\text{Li}_2(x)}{(x-1)x}\,dx$$

input
```
integrate(-polylog(2,x)/(1-x)/x,x, algorithm="giac")
```

output
```
integrate(dilog(x)/((x - 1)*x), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx = \int \frac{\text{polylog}(2,x)}{x\,(x-1)}\,dx$$

input
```
int(polylog(2, x)/(x*(x - 1)),x)
```

output
```
int(polylog(2, x)/(x*(x - 1)), x)
```

3.149 $\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

3.149.1 Optimal result

Integrand size = 34, antiderivative size = 35

$$\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(1+n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

output
```
polylog(1+n,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

3.149.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(1+n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

input
```
Integrate[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

3.149.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{7164}$$

$$\frac{\text{PolyLog}\left(n+1,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

input
```
Int[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

3.149.3.1 Defintions of rubi rules used

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.149.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\text{polylog}\left(1+n,e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(ad-bc)n}$	37
default	$-\frac{\text{polylog}\left(1+n,e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(ad-bc)n}$	37

input
```
int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
-1/(a*d-b*c)/n*polylog(1+n,e*((b*x+a)/(d*x+c))^n)
```

3.149.5 Fricas [F]

$$\int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

output
```
integral(polylog(n, e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c + (b*c + a*d
)*x), x)
```

3.149.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_n\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate(polylog(n,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

output
```
Integral(polylog(n, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*
x)), x)
```

3.149.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

output
```
integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.149.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

output
```
integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{polylog}\left(n, e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(polylog(n, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int(polylog(n, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.150 $\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

3.150.1 Optimal result

Integrand size = 34, antiderivative size = 33

$$\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

output
```
polylog(4,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

3.150.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

input
```
Integrate[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[4, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

3.150.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \ 7164$$

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

input
```
Int[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[4, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

3.150.3.1 Defintions of rubi rules used

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.150.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\operatorname{polylog}\left(4,e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(ad-bc)n}$	35
default	$-\frac{\operatorname{polylog}\left(4,e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(ad-bc)n}$	35

input
```
int(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
-1/(a*d-b*c)/n*polylog(4,e*((b*x+a)/(d*x+c))^n)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{polylog}\left(4, e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bc-ad)n}$$

input
```
integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

output
```
polylog(4, e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)
```

3.150.6 Sympy [F]

$$\int \frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_3\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate(polylog(3,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

output
```
Integral(polylog(3, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*
x)), x)
```

3.150.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\text{Li}_3\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

output
```
-1/6*(3*(n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x + c)
^2)*dilog(e*e^(n*log(b*x + a) - n*log(d*x + c))) + (n^2*log(b*x + a)^3 - 3
*n^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^2 - n^2
*log(d*x + c)^3)*log(-(b*x + a)^n*e + (d*x + c)^n) - (n^2*log(b*x + a)^3 -
 3*n^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^2 - n
^2*log(d*x + c)^3)*log((d*x + c)^n) - 6*(log(b*x + a) - log(d*x + c))*poly
log(3, e*e^(n*log(b*x + a) - n*log(d*x + c))))/(b*c - a*d) + integrate(1/6
*(e*n^3*log(b*x + a)^3 - 3*e*n^3*log(b*x + a)^2*log(d*x + c) + 3*e*n^3*log
(b*x + a)*log(d*x + c)^2 - e*n^3*log(d*x + c)^3)*(b*x + a)^n/((b*d*e*x^2 +
 a*c*e + (b*c*e + a*d*e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*
(d*x + c)^n), x)
```

3.150.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_3\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

output
```
integrate(polylog(3, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{polylog}\left(3,e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(polylog(3, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int(polylog(3, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.151 $\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

3.151.1 Optimal result

Integrand size = 34, antiderivative size = 33

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

output
```
polylog(3,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

3.151.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

input
```
Integrate[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[3, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

3.151.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \; 7164$$

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

input
```
Int[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

3.151.3.1 Defintions of rubi rules used

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.151.4 Maple [A] (verified)

Time = 28.95 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\text{polylog}\left(3,e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(ad-bc)n}$	35
default	$-\frac{\text{polylog}\left(3,e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(ad-bc)n}$	35

input
```
int(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

output
```
-1/(a*d-b*c)/n*polylog(3,e*((b*x+a)/(d*x+c))^n)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{polylog}\left(3,e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bc-ad)n}$$

input
```
integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

output
```
polylog(3, e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(polylog(2,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

output
```
1/2*(2*(log(b*x + a) - log(d*x + c))*dilog(e*e^(n*log(b*x + a) - n*log(d*x
 + c))) + (n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x +
c)^2)*log(-(b*x + a)^n*e + (d*x + c)^n) - (n*log(b*x + a)^2 - 2*n*log(b*x
+ a)*log(d*x + c) + n*log(d*x + c)^2)*log((d*x + c)^n))/(b*c - a*d) + inte
grate(-1/2*(e*n^2*log(b*x + a)^2 - 2*e*n^2*log(b*x + a)*log(d*x + c) + e*n
^2*log(d*x + c)^2)*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c + a*d)*e*x)*(b*x
 + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

3.151.8 Giac [F]

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="g
iac")
```

output
```
integrate(dilog(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\text{polylog}\left(2,e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(polylog(2, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int(polylog(2, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.152 $\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

3.152.1 Optimal result

Integrand size = 37, antiderivative size = 33

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

output
```
polylog(2,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

3.152.2 Mathematica [F]

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

input
```
Integrate[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

output
```
-Integrate[Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)), x]
```

3.152.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {25, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\log \left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\log \left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \; 2998$$

$$\frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

input
```
Int[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

output
```
PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

3.152.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{\operatorname{dilog}\left(1-e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(ad-bc)n}$	37
default	$-\frac{\operatorname{dilog}\left(1-e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(ad-bc)n}$	37

input
```
int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)/n*dilog(1-e*((b*x+a)/(d*x+c))^n)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(a+bx)(c+dx)}\,dx = \frac{\operatorname{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bc-ad)n}$$

input
```
integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fric
as")
```

output
```
dilog(e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(-ln(1-e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int -\frac{\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxi
ma")
```

output
```
-((log(b*x + a) - log(d*x + c))*log(-(b*x + a)^n*e + (d*x + c)^n) - (log(b
*x + a) - log(d*x + c))*log((d*x + c)^n))/(b*c - a*d) + integrate((e*n*log
(b*x + a) - e*n*log(d*x + c))*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c*e + a
*d*e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

3.152.8 Giac [F]

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int -\frac{\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="giac
")
```

output
```
integrate(-log(-e*((b*x + a)/(d*x + c))^n + 1)/((b*x + a)*(d*x + c)), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int -\frac{\ln\left(1-e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(-log(1 - e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int(-log(1 - e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.153 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

3.153.1 Optimal result

Integrand size = 53, antiderivative size = 36

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

output
```
-ln(1-e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

3.153.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{e\log\left((bc-ad)en\left(-1+e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bcen-aden}$$

input
```
Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*
x)/(c + d*x))^n)),x]
```

output
```
-((e*Log[(b*c - a*d)*e*n*(-1 + e*((a + b*x)/(c + d*x))^n)])/(b*c*e*n - a*d
*e*n))
```

3.153.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {27, 7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{7235}$$

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

input
```
Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c
+ d*x))^n)),x]
```

output
```
-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n))
```

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

3.153.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\ln\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{n(ad-bc)}$	35
default	$\frac{\ln\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{n(ad-bc)}$	35
parallelrisch	$\frac{\ln\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{n(ad-bc)}$	35
norman	$\frac{\ln\left(-1+e\,{\mathrm e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)}{n(ad-bc)}$	37
risch	$\frac{\ln(bx+a)}{ad-bc}-\frac{\ln(-dx-c)}{ad-bc}-\frac{\ln\left(\frac{bx+a}{dx+c}\right)}{ad-bc}+\frac{\ln\left(\left(\frac{bx+a}{dx+c}\right)^{n}-\frac{1}{e}\right)}{n(ad-bc)}$	102

input
```
int(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,me
thod=_RETURNVERBOSE)
```

output
```
1/n/(a*d-b*c)*ln(-1+e*((b*x+a)/(d*x+c))^n)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n-1\right)}{(bc-ad)n}$$

input
```
integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c
),x, algorithm="fricas")
```

output
```
-log(e*((b*x + a)/(d*x + c))^n - 1)/((b*c - a*d)*n)
```

3.153.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 102.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(aden\left(\frac{a+bx}{c+dx}\right)^n - adn - bcen\left(\frac{a+bx}{c+dx}\right)^n + bcn\right)}{n\,(ad-bc)}$$

input
```
integrate(-e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x
+c),x)
```

output
```
log(a*d*e*n*((a + b*x)/(c + d*x))**n - a*d*n - b*c*e*n*((a + b*x)/(c + d*x
))**n + b*c*n)/(n*(a*d - b*c))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -e\left(\frac{\log\left(-(bx+a)^n e + (dx+c)^n\right)}{bcen - aden} - \frac{\log(dx+c)}{bce - ade}\right)$$

input
```
integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c
),x, algorithm="maxima")
```

output
```
-e*(log(-(b*x + a)^n*e + (d*x + c)^n)/(b*c*e*n - a*d*e*n) - log(d*x + c)/(
b*c*e - a*d*e))
```

3.153.8 Giac [F]

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int -\frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n - 1\right)}\,dx$$

input
```
integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c
),x, algorithm="giac")
```

output
```
integrate(-e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d
*x + c))^n - 1)), x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n-1\right)}{a\,d\,n-b\,c\,n}$$

input
```
int(-(e*((a + b*x)/(c + d*x))^n)/((e*((a + b*x)/(c + d*x))^n - 1)*(a + b*x
)*(c + d*x)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n - 1)/(a*d*n - b*c*n)
```

3.154 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx$

3.154.1 Optimal result

Integrand size = 53, antiderivative size = 36

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx=\frac{1}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

output
```
1/(-a*d+b*c)/n/(1-e*((b*x+a)/(d*x+c))^n)
```

3.154.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx=-\frac{1}{(bc-ad)n\left(-1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

input
```
Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*
x)/(c + d*x))^n)^2),x]
```

output
```
-(1/((b*c - a*d)*n*(-1 + e*((a + b*x)/(c + d*x))^n)))
```

3.154.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {27, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}dx$$

$$\Big\downarrow \text{ 7237}$$

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

input
```
Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c
+ d*x))^n)^2),x]
```

output
```
1/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n))
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.154.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{1}{\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)n(ad-bc)}$	36
default	$\frac{1}{\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)n(ad-bc)}$	36
risch	$\frac{1}{\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)n(ad-bc)}$	36
parallelrisch	$\frac{1}{\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)n(ad-bc)}$	36
norman	$\frac{e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}}{n(ad-bc)\left(-1+e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)}$	56

input
```
int(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/(-1+e*((b*x+a)/(d*x+c))^n)/n/(a*d-b*c)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^{n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^{n}\right)^{2}}\,dx=-\frac{1}{(bc-ad)en\left(\frac{bx+a}{dx+c}\right)^{n}-(bc-ad)n}$$

input
```
integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+
c),x, algorithm="fricas")
```

output
```
-1/((b*c - a*d)*e*n*((b*x + a)/(d*x + c))^n - (b*c - a*d)*n)
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)**2/(b*x+a)/(d
*x+c),x)
```

output
```
Timed out
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx = -\frac{(bx+a)^n e}{(bcen-aden)(bx+a)^n-(bcn-adn)(dx+c)^n}$$

input
```
integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+
c),x, algorithm="maxima")
```

output
```
-(b*x + a)^n*e/((b*c*e*n - a*d*e*n)*(b*x + a)^n - (b*c*n - a*d*n)*(d*x + c
)^n)
```

3.154.8 Giac [F]

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx = \int \frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n-1\right)^2}\,dx$$

input
```
integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+
c),x, algorithm="giac")
```

output
```
integrate(e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*
x + c))^n - 1)^2), x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx = \frac{1}{n\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n-1\right)\,\left(a\,d-b\,c\right)}$$

input
```
int((e*((a + b*x)/(c + d*x))^n)/((e*((a + b*x)/(c + d*x))^n - 1)^2*(a + b*
x)*(c + d*x)),x)
```

output
```
1/(n*(e*((a + b*x)/(c + d*x))^n - 1)*(a*d - b*c))
```

3.155 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$

3.155.1 Optimal result

Integrand size = 76, antiderivative size = 52

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx=\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

output
```
e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(-a*d+b*c)/n
```

3.155.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx=\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(-1+e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

input
```
Integrate[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((
a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

output
```
(e*((a + b*x)/(c + d*x))^n)/((b*c - a*d)*n*(-1 + e*((a + b*x)/(c + d*x))^n
)^2)
```

3.155.3 Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {7292, 27, 7243, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^2\left(\frac{a+bx}{c+dx}\right)^{2n}+e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}dx$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n\left(e\left(\frac{a+bx}{c+dx}\right)^n+1\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n\left(e\left(\frac{a+bx}{c+dx}\right)^n+1\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}dx$$

$$\Big\downarrow \text{7243}$$

$$\frac{\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+1}{\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}d\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\Big\downarrow \text{38}$$

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

input
```
Int[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((a + b*
x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

output
```
(e*((a + b*x)/(c + d*x))^n)/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n)
^2)
```

3.155. $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$

3.155.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 7243
```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((a_.) + (b_.)*(y_))^(m_.), x_Symbol] :
> With[{q = DerivativeDivides[y, u, x]}, Simp[q   Subst[Int[(a + b*x)^m*(c
+ d*x)^n, x], x, y], x] /;  !FalseQ[q]] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[v, y]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.155.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result	size
risch	$-\frac{e\left(\frac{bx+a}{dx+c}\right)^{n}}{n(ad-bc)\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}$	53
parallelrisch	$-\frac{e\left(\frac{bx+a}{dx+c}\right)^{n}}{n(ad-bc)\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}$	53
norman	$-\frac{e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}}{n(ad-bc)\left(-1+e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)^{2}}$	57
derivativedivides	$\frac{e\left(-\frac{1}{e\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}-\frac{1}{e\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\right)}{(ad-bc)n}$	69
default	$\frac{e\left(-\frac{1}{e\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}-\frac{1}{e\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\right)}{(ad-bc)n}$	69

3.155. $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^{n}+e^{2}\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^{n}\right)^{3}}\,dx$

input
```
int(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c
))^n)^3/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-e/n/(a*d-b*c)*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx = \frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bc-ad)e^2n\left(\frac{bx+a}{dx+c}\right)^{2n}-2(bc-ad)en\left(\frac{bx+a}{dx+c}\right)^n+(bc-ad)n}$$

input
```
integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/
(d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
e*((b*x + a)/(d*x + c))^n/((b*c - a*d)*e^2*n*((b*x + a)/(d*x + c))^(2*n) -
 2*(b*c - a*d)*e*n*((b*x + a)/(d*x + c))^n + (b*c - a*d)*n)
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(-(1+e*((b*x+a)/(d*x+c))**n)*e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a
)/(d*x+c))**n)**3/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(51) = 102$.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.06

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n + e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$$
$$= \frac{1}{2}\left(\frac{(bx+a)^{2n}e}{(bce^2n-ade^2n)(bx+a)^{2n}+(bcn-adn)(dx+c)^{2n}-2\,(bcen-aden)e^{(n\log(bx+a)+n\log(dx+c))}} - \frac{}{(bce^2}\right.$$

input
```
integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/
(d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*((b*x + a)^(2*n)*e/((b*c*e^2*n - a*d*e^2*n)*(b*x + a)^(2*n) + (b*c*n -
 a*d*n)*(d*x + c)^(2*n) - 2*(b*c*e*n - a*d*e*n)*e^(n*log(b*x + a) + n*log(
d*x + c))) - ((b*x + a)^(2*n)*e - 2*e^(n*log(b*x + a) + n*log(d*x + c)))/(
(b*c*e^2*n - a*d*e^2*n)*(b*x + a)^(2*n) + (b*c*n - a*d*n)*(d*x + c)^(2*n)
- 2*(b*c*e*n - a*d*e*n)*e^(n*log(b*x + a) + n*log(d*x + c))))*e
```

3.155.8 Giac [F]

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n + e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx = \int -\frac{\left(e\left(\frac{bx+a}{dx+c}\right)^n+1\right)e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n-1\right)^3}\,dx$$

input
```
integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/
(d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(-(e*((b*x + a)/(d*x + c))^n + 1)*e*((b*x + a)/(d*x + c))^n/((b*x
 + a)*(d*x + c)*(e*((b*x + a)/(d*x + c))^n - 1)^3), x)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$$
$$=-\frac{e\left(\frac{a+b\,x}{c+d\,x}\right)^n}{n\,\left(a\,d-b\,c\right)\,\left(e^2\left(\frac{a}{c+d\,x}+\frac{b\,x}{c+d\,x}\right)^{2\,n}-2\,e\left(\frac{a+b\,x}{c+d\,x}\right)^n+1\right)}$$

input
```
int(-(e*(e*((a + b*x)/(c + d*x))^n + 1)*((a + b*x)/(c + d*x))^n)/((e*((a +
 b*x)/(c + d*x))^n - 1)^3*(a + b*x)*(c + d*x)),x)
```

output
```
-(e*((a + b*x)/(c + d*x))^n)/(n*(a*d - b*c)*(e^2*(a/(c + d*x) + (b*x)/(c +
 d*x))^(2*n) - 2*e*((a + b*x)/(c + d*x))^n + 1))
```

3.156 $\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

3.156.1 Optimal result

Integrand size = 19, antiderivative size = 135

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x^3 \operatorname{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \operatorname{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \operatorname{PolyLog}\left(3+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{6 \operatorname{PolyLog}\left(4+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)}$$

output
```
x^3*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-3*x^2*polylog(2+n,d*(F^(c
*(b*x+a)))^p)/b^2/c^2/p^2/ln(F)^2+6*x*polylog(3+n,d*(F^(c*(b*x+a)))^p)/b^3
/c^3/p^3/ln(F)^3-6*polylog(4+n,d*(F^(c*(b*x+a)))^p)/b^4/c^4/p^4/ln(F)^4
```

3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int x^3 \operatorname{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\frac{x^3 \operatorname{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2 \operatorname{PolyLog}\left(2+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x \operatorname{PolyLog}\left(3+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\frac{6 \operatorname{PolyLog}\left(4+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)}$$

input
```
Integrate[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLo
g[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 +
 n, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(
F^(c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```

3.156.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {7163, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{x^3 \operatorname{PolyLog}\left(n+1,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3\int x^2 \operatorname{PolyLog}\left(n+1,d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{x^3 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\int x \operatorname{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}\right)}{bcp\log(F)}$$

$$\Big\downarrow \text{7163}$$

$$\frac{x^3 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int \operatorname{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}\right)}{bcp\log(F)}\right)}{bcp\log(F)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{x^3 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int F^{-c(a+bx)} \operatorname{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)dF^{c(a+bx)}}{b^2c^2p\log^2(F)}\right)}{bcp\log(F)}\right)}{bcp\log(F)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{x^3 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3\left(\frac{x^2 \operatorname{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\operatorname{PolyLog}\left(n+4,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}\right)}{bcp\log(F)}\right)}{bcp\log(F)}$$

input
```
Int[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*((x^2*Poly
Log[2 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*((x*PolyLog[3 + n,
d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[4 + n, d*(F^(c*(a + b*x))
)^p]/(b^2*c^2*p^2*Log[F]^2)))/(b*c*p*Log[F])))/(b*c*p*Log[F])
```

3.156. $\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) \, dx$

3.156.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.156.4 Maple [F]

$$\int x^3 \operatorname{polylog}\left(n, d\left(F^{c(bx+a)}\right)^p\right) dx$$

input
```
int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

output
```
int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

3.156.5 Fricas [F]

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^3 \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

output
```
integral(x^3*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

3.156.6 Sympy [F]

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^3 \operatorname{Li}_n\left(d\left(F^{ac+bcx}\right)^p\right) dx$$

input
```
integrate(x**3*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

output
```
Integral(x**3*polylog(n, d*(F**(a*c + b*c*x))**p), x)
```

3.156.7 Maxima [F]

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^3 \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

output
```
integrate(x^3*polylog(n, F^((b*x + a)*c*p)*d), x)
```

3.156.8 Giac [F]

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^3 \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

output
```
integrate(x^3*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^3 \operatorname{polylog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$$

input
```
int(x^3*polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

output
```
int(x^3*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.157 $\int x^2 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

3.157.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int x^2 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x^2 \text{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)} - \frac{2x \text{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2 \log^2(F)} + \frac{2 \text{PolyLog}\left(3+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3 \log^3(F)}$$

output
```
x^2*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-2*x*polylog(2+n,d*(F^(c*(
b*x+a)))^p)/b^2/c^2/p^2/ln(F)^2+2*polylog(3+n,d*(F^(c*(b*x+a)))^p)/b^3/c^3
/p^3/ln(F)^3
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int x^2 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x^2 \text{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)} - \frac{2x \text{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2 \log^2(F)} + \frac{2 \text{PolyLog}\left(3+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3 \log^3(F)}$$

input
```
Integrate[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[
2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n,
d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```

3.157.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$$

$$\Big\downarrow \text{7163}$$

$$\frac{x^2 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\int x \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}$$

$$\Big\downarrow \text{7163}$$

$$\frac{x^2 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int \operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}\right)}{bcp\log(F)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{x^2 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int F^{-c(a+bx)} \operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)dF^{c(a+bx)}}{b^2c^2p\log^2(F)}\right)}{bcp\log(F)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{x^2 \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\left(\frac{x \operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\operatorname{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}\right)}{bcp\log(F)}$$

input
```
Int[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*((x*PolyLo
g[2 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[3 + n, d*(F^(c*(
a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)))/(b*c*p*Log[F])
```

3.157.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.157.4 Maple [F]

$$\int x^2 \operatorname{polylog}\left(n, d\left(F^{c(bx+a)}\right)^p\right) dx$$

input
```
int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

output
```
int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

3.157.5 Fricas [F]

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^2 \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

output
```
integral(x^2*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

3.157.6 Sympy [F]

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^2 \operatorname{Li}_n\left(d\left(F^{ac+bcx}\right)^p\right) dx$$

input
```
integrate(x**2*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

output
```
Integral(x**2*polylog(n, d*(F**(a*c + b*c*x))**p), x)
```

3.157.7 Maxima [F]

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^2 \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

output
```
integrate(x^2*polylog(n, F^((b*x + a)*c*p)*d), x)
```

3.157.8 Giac [F]

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^2 \mathrm{Li}_n(\left(F^{(bx+a)c}\right)^p d)\, dx$$

input
```
integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

output
```
integrate(x^2*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x^2 \operatorname{polylog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$$

input
```
int(x^2*polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

output
```
int(x^2*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.158 $\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

3.158.1 Optimal result

Integrand size = 17, antiderivative size = 65

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x \operatorname{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\operatorname{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

output
```
x*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-polylog(2+n,d*(F^(c*(b*x+a)
))^p)/b^2/c^2/p^2/ln(F)^2
```

3.158.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x \operatorname{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\operatorname{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

input
```
Integrate[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n,
d*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

3.158.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$$
$$\Big\downarrow \text{7163}$$
$$\frac{x \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right) dx}{bcp\log(F)}$$
$$\Big\downarrow \text{2720}$$
$$\frac{x \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int F^{-c(a+bx)} \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right) dF^{c(a+bx)}}{b^2c^2p\log^2(F)}$$
$$\Big\downarrow \text{7143}$$
$$\frac{x \operatorname{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\operatorname{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

input
```
Int[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
(x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n,
d*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

3.158.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.158.4 Maple [F]

$$\int x \operatorname{polylog}\left(n, d\left(F^{c(bx+a)}\right)^{p}\right) dx$$

input
```
int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

output
```
int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

3.158.5 Fricas [F]

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^{p}\right) \, dx = \int x \operatorname{Li}_{n}(\left(F^{(bx+a)c}\right)^{p} d) \, dx$$

input
```
integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

output
```
integral(x*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

3.158.6 Sympy [F]

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x \operatorname{Li}_n\left(d\left(F^{ac+bcx}\right)^p\right) dx$$

input
```
integrate(x*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

output
```
Integral(x*polylog(n, d*(F**(a*c + b*c*x))**p), x)
```

3.158.7 Maxima [F]

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

output
```
integrate(x*polylog(n, F^((b*x + a)*c*p)*d), x)
```

3.158.8 Giac [F]

$$\int x \operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int x \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

output
```
integrate(x*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int x\,\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\int x\,\text{polylog}\left(n,d\,\left(F^{c\,(a+b\,x)}\right)^p\right)\,dx$$

input
```
int(x*polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

output
```
int(x*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.159 $\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx$

3.159.1 Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\frac{\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

output
```
polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)
```

3.159.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\frac{\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

input
```
Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

3.159.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int F^{-c(a+bx)}\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\text{PolyLog}\left(n+1,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

input
```
Int[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

output
```
PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

3.159.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.159.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\text{polylog}\left(1+n,d\left(F^{c(bx+a)}\right)^{p}\right)}{bcp\ln(F)}$	32
default	$\frac{\text{polylog}\left(1+n,d\left(F^{c(bx+a)}\right)^{p}\right)}{bcp\ln(F)}$	32

input
```
int(polylog(n,d*(F^(c*(b*x+a)))^p),x,method=_RETURNVERBOSE)
```

output
```
polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)
```

3.159.5 Fricas [F]

$$\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\int \text{Li}_n(\left(F^{(bx+a)c}\right)^p d)\,dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

output
```
integral(polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

3.159.6 Sympy [F]

$$\int \text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx=\int \text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx$$

input
```
integrate(polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

output
```
Integral(polylog(n, d*(F**(c*(a + b*x)))**p), x)
```

3.159.7 Maxima [F]

$$\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int \text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

output
```
integrate(polylog(n, F^((b*x + a)*c*p)*d), x)
```

3.159.8 Giac [F]

$$\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int \text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

output
```
integrate(polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx = \int \text{polylog}\left(n, d\left(F^{c\,(a+b\,x)}\right)^p\right) dx$$

input
```
int(polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

output
```
int(polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.160 $\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx$

3.160.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx=\text{Int}\left(\frac{\text{PolyLog}\left(n,d\left(F^{ac+bcx}\right)^p\right)}{x},x\right)$$

output
```
CannotIntegrate(polylog(n,d*(F^(b*c*x+a*c))^p)/x,x)
```

3.160.2 Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx=\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx$$

input
```
Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
```

output
```
Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x, x]
```

3.160.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}dx$$
$$\Big\downarrow \text{7292}$$
$$\int \frac{\text{PolyLog}\left(n,d\left(F^{ac+bxc}\right)^p\right)}{x}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\text{PolyLog}\left(n,d\left(F^{ac+bxc}\right)^p\right)}{x}dx$$

input
```
Int[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
```

output
```
$Aborted
```

3.160.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.160.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{polylog}\left(n, d\left(F^{c(bx+a)}\right)^{p}\right)}{x}dx$$

input
```
int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

output
```
int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

3.160.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^{p}\right)}{x} dx = \int \frac{\operatorname{Li}_{n}(\left(F^{(bx+a)c}\right)^{p}d)}{x} dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="fricas")
```

output
```
integral(polylog(n, (F^(b*c*x + a*c))^p*d)/x, x)
```

3.160.6 Sympy [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^{p}\right)}{x} dx = \int \frac{\operatorname{Li}_{n}\left(d\left(F^{ac+bcx}\right)^{p}\right)}{x} dx$$

input
```
integrate(polylog(n,d*(F**(c*(b*x+a)))**p)/x,x)
```

output
```
Integral(polylog(n, d*(F**(a*c + b*c*x))**p)/x, x)
```

3.160.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx = \int \frac{\text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right)}{x}\,dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="maxima")
```

output
```
integrate(polylog(n, F^((b*x + a)*c*p)*d)/x, x)
```

3.160.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx = \int \frac{\text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right)}{x}\,dx$$

input
```
integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="giac")
```

output
```
integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)
```

3.160.9 Mupad [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx = \int \frac{\text{polylog}\left(n, d\left(F^{c\,(a+b\,x)}\right)^p\right)}{x}\,dx$$

input
```
int(polylog(n, d*(F^(c*(a + b*x)))^p)/x,x)
```

output
```
int(polylog(n, d*(F^(c*(a + b*x)))^p)/x, x)
```

3.161 $\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx$

3.161.1 Optimal result

Integrand size = 16, antiderivative size = 300

$$\begin{aligned}
\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx &= \frac{355x}{576c^3}+\frac{139x^2}{1152c^2}+\frac{67x^3}{1728c}+\frac{3x^4}{256}+\frac{139\log(1-cx)}{576c^4} \\
&\quad -\frac{x^2\log(1-cx)}{8c^2}-\frac{5x^3\log(1-cx)}{72c} \\
&\quad -\frac{3}{64}x^4\log(1-cx)+\frac{3(1-cx)\log(1-cx)}{8c^4} \\
&\quad -\frac{\log^2(1-cx)}{16c^4}+\frac{1}{16}x^4\log^2(1-cx) \\
&\quad -\frac{\log(cx)\log^2(1-cx)}{4c^4}-\frac{x\operatorname{PolyLog}(2,cx)}{4c^3} \\
&\quad -\frac{x^2\operatorname{PolyLog}(2,cx)}{8c^2}-\frac{x^3\operatorname{PolyLog}(2,cx)}{12c} \\
&\quad -\frac{1}{16}x^4\operatorname{PolyLog}(2,cx)-\frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{4c^4} \\
&\quad +\frac{1}{4}x^4\log(1-cx)\operatorname{PolyLog}(2,cx) \\
&\quad -\frac{\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{2c^4} \\
&\quad +\frac{\operatorname{PolyLog}(3,1-cx)}{2c^4}
\end{aligned}$$

output
```
355/576*x/c^3+139/1152*x^2/c^2+67/1728*x^3/c+3/256*x^4+139/576*ln(-c*x+1)/
c^4-1/8*x^2*ln(-c*x+1)/c^2-5/72*x^3*ln(-c*x+1)/c-3/64*x^4*ln(-c*x+1)+3/8*(
-c*x+1)*ln(-c*x+1)/c^4-1/16*ln(-c*x+1)^2/c^4+1/16*x^4*ln(-c*x+1)^2-1/4*ln(
c*x)*ln(-c*x+1)^2/c^4-1/4*x*polylog(2,c*x)/c^3-1/8*x^2*polylog(2,c*x)/c^2-
1/12*x^3*polylog(2,c*x)/c-1/16*x^4*polylog(2,c*x)-1/4*ln(-c*x+1)*polylog(2
,c*x)/c^4+1/4*x^4*ln(-c*x+1)*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,-c*x+
1)/c^4+1/2*polylog(3,-c*x+1)/c^4
```

3.161.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.74

$$\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx) \, dx$$
$$= \frac{4260cx + 834c^2x^2 + 268c^3x^3 + 81c^4x^4 + 4260 \log(1-cx) - 2592cx \log(1-cx) - 864c^2x^2 \log(1-cx) - \cdots}{}$$

input
```
Integrate[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(4260*c*x + 834*c^2*x^2 + 268*c^3*x^3 + 81*c^4*x^4 + 4260*Log[1 - c*x] - 2
592*c*x*Log[1 - c*x] - 864*c^2*x^2*Log[1 - c*x] - 480*c^3*x^3*Log[1 - c*x]
 - 324*c^4*x^4*Log[1 - c*x] - 432*Log[1 - c*x]^2 + 432*c^4*x^4*Log[1 - c*x
]^2 - 1728*Log[c*x]*Log[1 - c*x]^2 + 144*(-(c*x*(12 + 6*c*x + 4*c^2*x^2 +
3*c^3*x^3)) + 12*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog[2, c*x] - 3456*Log[1
 - c*x]*PolyLog[2, 1 - c*x] + 3456*PolyLog[3, 1 - c*x])/(6912*c^4)
```

3.161.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7157, 2009, 2845, 2857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \operatorname{PolyLog}(2,cx) \log(1-cx) \, dx$$

$$\Big\downarrow \; 7157$$

$$\frac{1}{4}c\int\left(-\frac{\text{PolyLog}(2,cx)x^3}{c}-\frac{\text{PolyLog}(2,cx)x^2}{c^2}-\frac{\text{PolyLog}(2,cx)x}{c^3}+\frac{\text{PolyLog}(2,cx)}{c^4(1-cx)}-\frac{\text{PolyLog}(2,cx)}{c^4}\right)dx+$$
$$\frac{1}{4}\int x^3\log^2(1-cx)dx+\frac{1}{4}x^4\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\int x^3\log^2(1-cx)dx+$$
$$\frac{1}{4}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^5}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^5}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^5}-\frac{\log(cx)\log^2(1-cx)}{c^5}\right.$$
$$\frac{1}{4}x^4\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{ 2845}$$

$$\frac{1}{4}\left(\frac{1}{2}c\int\frac{x^4\log(1-cx)}{1-cx}dx+\frac{1}{4}x^4\log^2(1-cx)\right)+$$
$$\frac{1}{4}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^5}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^5}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^5}-\frac{\log(cx)\log^2(1-cx)}{c^5}\right.$$
$$\frac{1}{4}x^4\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{ 2857}$$

$$\frac{1}{4}\left(\frac{1}{2}c\int\left(-\frac{\log(1-cx)x^3}{c}-\frac{\log(1-cx)x^2}{c^2}-\frac{\log(1-cx)x}{c^3}-\frac{\log(1-cx)}{c^4(cx-1)}-\frac{\log(1-cx)}{c^4}\right)dx+\frac{1}{4}x^4\log^2(1-cx)\right.$$
$$\frac{1}{4}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^5}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^5}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^5}-\frac{\log(cx)\log^2(1-cx)}{c^5}\right.$$
$$\frac{1}{4}x^4\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^5}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^5}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^5}-\frac{\log(cx)\log^2(1-cx)}{c^5}\right.$$
$$\frac{1}{4}\left(\frac{1}{2}c\left(-\frac{\log^2(1-cx)}{2c^5}+\frac{(1-cx)\log(1-cx)}{c^5}+\frac{13\log(1-cx)}{12c^5}+\frac{25x}{12c^4}+\frac{13x^2}{24c^3}-\frac{x^2\log(1-cx)}{2c^3}+\frac{7x^3}{36c^2}-\frac{x^3\log(\right.\right.$$
$$\frac{1}{4}x^4\,\text{PolyLog}(2,cx)\log(1-cx)$$

input
```
Int[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```

3.161. $\int x^3\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

output
```
((x^4*Log[1 - c*x]^2)/4 + (c*((25*x)/(12*c^4) + (13*x^2)/(24*c^3) + (7*x^3
)/(36*c^2) + x^4/(16*c) + (13*Log[1 - c*x])/(12*c^5) - (x^2*Log[1 - c*x])/
(2*c^3) - (x^3*Log[1 - c*x])/(3*c^2) - (x^4*Log[1 - c*x])/(4*c) + ((1 - c*
x)*Log[1 - c*x])/c^5 - Log[1 - c*x]^2/(2*c^5)))/2)/4 + (x^4*Log[1 - c*x]*P
olyLog[2, c*x])/4 + (c*((205*x)/(144*c^4) + (61*x^2)/(288*c^3) + (25*x^3)/
(432*c^2) + x^4/(64*c) + (61*Log[1 - c*x])/(144*c^5) - (x^2*Log[1 - c*x])/
(4*c^3) - (x^3*Log[1 - c*x])/(9*c^2) - (x^4*Log[1 - c*x])/(16*c) + ((1 - c
*x)*Log[1 - c*x])/c^5 - (Log[c*x]*Log[1 - c*x]^2)/c^5 - (x*PolyLog[2, c*x]
)/c^4 - (x^2*PolyLog[2, c*x])/(2*c^3) - (x^3*PolyLog[2, c*x])/(3*c^2) - (x
^4*PolyLog[2, c*x])/(4*c) - (Log[1 - c*x]*PolyLog[2, c*x])/c^5 - (2*Log[1
- c*x]*PolyLog[2, 1 - c*x])/c^5 + (2*PolyLog[3, 1 - c*x])/c^5))/4
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.161.4 Maple [F]

$$\int x^{3} \ln \left(-c x +1\right) \operatorname{polylog}\left(2, c x \right)d x$$

input
```
int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

3.161.5 Fricas [F]

$$\int x^3 \log(1-cx) \text{PolyLog}(2,cx) \, dx = \int x^{3} {\rm Li}_2\left(c x\right) \log \left(-c x + 1\right) \,d x$$

input
```
integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(x^3*dilog(c*x)*log(-c*x + 1), x)
```

3.161.6 Sympy [F]

$$\int x^3 \log(1-cx) \text{PolyLog}(2,cx) \, dx = \int x^{3} \log{\left(- c x + 1 \right)} \operatorname{Li}_{2}\left(c x\right)\, dx$$

input
```
integrate(x**3*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral(x**3*log(-c*x + 1)*polylog(2, c*x), x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.25

$$\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx$$
$$= \frac{9\,c^4\left(\frac{3\,c^3x^4+4\,c^2x^3+6\,cx^2+12\,x}{c^4}+\frac{12\,\log(cx-1)}{c^5}\right)+24\,c^3\left(\frac{2\,c^2x^3+3\,cx^2+6\,x}{c^3}+\frac{6\,\log(cx-1)}{c^4}\right)+108\,c^2\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right.}{}$$
$$+\frac{\left(48\,c^4x^4\mathrm{Li}_2(cx)-3\,c^4x^4-4\,c^3x^3-6\,c^2x^2-12\,cx+12\,(c^4x^4-1)\log(-cx+1)\right)\log(-cx+1)}{192\,c^4}$$

input
```
integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
1/6912*(9*c^4*((3*c^3*x^4 + 4*c^2*x^3 + 6*c*x^2 + 12*x)/c^4 + 12*log(c*x -
 1)/c^5) + 24*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) +
 108*c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 432*c*(x/c + log(c*x -
 1)/c^2) + 2*(27*c^4*x^4 + 92*c^3*x^3 + 300*c^2*x^2 + 1680*c*x - 72*(3*c^4
*x^4 + 4*c^3*x^3 + 6*c^2*x^2 + 12*c*x + 12*log(-c*x + 1))*dilog(c*x) - 12*
(9*c^4*x^4 + 14*c^3*x^3 + 27*c^2*x^2 + 90*c*x - 140)*log(-c*x + 1))/c - 17
28*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))/c)/c^3 + 1/192*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*c^3*x
^3 - 6*c^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*log(-c*x + 1)/c^
4
```

3.161.8 Giac [F]

$$\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx = \int x^3 \mathrm{Li}_2(cx) \log(-cx+1)\,dx$$

input
```
integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate(x^3*dilog(c*x)*log(-c*x + 1), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \log(1-cx) \operatorname{PolyLog}(2,cx) \, dx = \int x^3 \ln(1-c\,x) \operatorname{polylog}(2,c\,x) \, dx$$

input
```
int(x^3*log(1 - c*x)*polylog(2, c*x),x)
```

output
```
int(x^3*log(1 - c*x)*polylog(2, c*x), x)
```

3.162 $\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx$

3.162.1 Optimal result

Integrand size = 16, antiderivative size = 258

$$\begin{aligned}\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx =& \frac{31x}{36c^2}+\frac{11x^2}{72c}+\frac{x^3}{27}+\frac{11\log(1-cx)}{36c^3} \\ &-\frac{7x^2\log(1-cx)}{36c}-\frac{1}{9}x^3\log(1-cx) \\ &+\frac{5(1-cx)\log(1-cx)}{9c^3}-\frac{\log^2(1-cx)}{9c^3} \\ &+\frac{1}{9}x^3\log^2(1-cx)-\frac{\log(cx)\log^2(1-cx)}{3c^3} \\ &-\frac{x\operatorname{PolyLog}(2,cx)}{3c^2}-\frac{x^2\operatorname{PolyLog}(2,cx)}{6c} \\ &-\frac{1}{9}x^3\operatorname{PolyLog}(2,cx)-\frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{3c^3} \\ &+\frac{1}{3}x^3\log(1-cx)\operatorname{PolyLog}(2,cx) \\ &-\frac{2\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{3c^3} \\ &+\frac{2\operatorname{PolyLog}(3,1-cx)}{3c^3}\end{aligned}$$

output
```
31/36*x/c^2+11/72*x^2/c+1/27*x^3+11/36*ln(-c*x+1)/c^3-7/36*x^2*ln(-c*x+1)/
c-1/9*x^3*ln(-c*x+1)+5/9*(-c*x+1)*ln(-c*x+1)/c^3-1/9*ln(-c*x+1)^2/c^3+1/9*
x^3*ln(-c*x+1)^2-1/3*ln(c*x)*ln(-c*x+1)^2/c^3-1/3*x*polylog(2,c*x)/c^2-1/6
*x^2*polylog(2,c*x)/c-1/9*x^3*polylog(2,c*x)-1/3*ln(-c*x+1)*polylog(2,c*x)
/c^3+1/3*x^3*ln(-c*x+1)*polylog(2,c*x)-2/3*ln(-c*x+1)*polylog(2,-c*x+1)/c^
3+2/3*polylog(3,-c*x+1)/c^3
```

3.162.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx$$
$$= \frac{186cx + 33c^2x^2 + 8c^3x^3 + 186\log(1-cx) - 120cx\log(1-cx) - 42c^2x^2\log(1-cx) - 24c^3x^3\log(1-cx)}{}$$

input
```
Integrate[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(186*c*x + 33*c^2*x^2 + 8*c^3*x^3 + 186*Log[1 - c*x] - 120*c*x*Log[1 - c*x
] - 42*c^2*x^2*Log[1 - c*x] - 24*c^3*x^3*Log[1 - c*x] - 24*Log[1 - c*x]^2
+ 24*c^3*x^3*Log[1 - c*x]^2 - 72*Log[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(6 +
3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] - 144
*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 144*PolyLog[3, 1 - c*x])/(216*c^3)
```

3.162.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7157, 2009, 2845, 2857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(2,cx)\log(1-cx)\,dx$$

$$\Big\downarrow \text{7157}$$

$$\frac{1}{3}c\int \left(-\frac{\operatorname{PolyLog}(2,cx)x^2}{c} - \frac{\operatorname{PolyLog}(2,cx)x}{c^2} + \frac{\operatorname{PolyLog}(2,cx)}{c^3(1-cx)} - \frac{\operatorname{PolyLog}(2,cx)}{c^3}\right)dx + \frac{1}{3}\int x^2\log^2(1-cx)dx + \frac{1}{3}x^3\operatorname{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\int x^2\log^2(1-cx)dx + \frac{1}{3}c\left(\frac{2\operatorname{PolyLog}(3,1-cx)}{c^4} - \frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{c^4} - \frac{2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{c^4} - \frac{\log(cx)\log^2(1-cx)}{c^4}\right. \frac{1}{3}x^3\operatorname{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{3}\left(\frac{2}{3}c\int \frac{x^3\log(1-cx)}{1-cx}dx+\frac{1}{3}x^3\log^2(1-cx)\right)+$$
$$\frac{1}{3}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2857}$$

$$\frac{1}{3}\left(\frac{2}{3}c\int\left(-\frac{\log(1-cx)x^2}{c}-\frac{\log(1-cx)x}{c^2}-\frac{\log(1-cx)}{c^3(cx-1)}-\frac{\log(1-cx)}{c^3}\right)dx+\frac{1}{3}x^3\log^2(1-cx)\right)+$$
$$\frac{1}{3}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}\left(\frac{2}{3}c\left(-\frac{\log^2(1-cx)}{2c^4}+\frac{(1-cx)\log(1-cx)}{c^4}+\frac{5\log(1-cx)}{6c^4}+\frac{11x}{6c^3}+\frac{5x^2}{12c^2}-\frac{x^2\log(1-cx)}{2c^2}+\frac{x^3}{9c}-\frac{x^3\log(1-\right.\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)\log(1-cx)$$

input
```
Int[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
((x^3*Log[1 - c*x]^2)/3 + (2*c*((11*x)/(6*c^3) + (5*x^2)/(12*c^2) + x^3/(9
*c) + (5*Log[1 - c*x])/(6*c^4) - (x^2*Log[1 - c*x])/(2*c^2) - (x^3*Log[1 -
 c*x])/(3*c) + ((1 - c*x)*Log[1 - c*x])/c^4 - Log[1 - c*x]^2/(2*c^4)))/3)/
3 + (x^3*Log[1 - c*x]*PolyLog[2, c*x])/3 + (c*((49*x)/(36*c^3) + (13*x^2)/
(72*c^2) + x^3/(27*c) + (13*Log[1 - c*x])/(36*c^4) - (x^2*Log[1 - c*x])/(4
*c^2) - (x^3*Log[1 - c*x])/(9*c) + ((1 - c*x)*Log[1 - c*x])/c^4 - (Log[c*x
]*Log[1 - c*x]^2)/c^4 - (x*PolyLog[2, c*x])/c^3 - (x^2*PolyLog[2, c*x])/(2
*c^2) - (x^3*PolyLog[2, c*x])/(3*c) - (Log[1 - c*x]*PolyLog[2, c*x])/c^4 -
 (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^4 + (2*PolyLog[3, 1 - c*x])/c^4))/
3
```

3.162. $\int x^2\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.162.4 Maple [F]

$$\int x^2 \ln(-cx+1) \operatorname{polylog}(2, cx) \, dx$$

input
```
int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

3.162.5 Fricas [F]

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\, dx = \int x^2 \operatorname{Li}_2(cx) \log(-cx+1)\, dx$$

input
```
integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(x^2*dilog(c*x)*log(-c*x + 1), x)
```

3.162.6 Sympy [F]

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\, dx = \int x^2 \log(-cx+1) \operatorname{Li}_2(cx)\, dx$$

input
```
integrate(x**2*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral(x**2*log(-c*x + 1)*polylog(2, c*x), x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.15

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\, dx$$
$$= \frac{4\,c^3\left(\frac{2\,c^2x^3+3\,cx^2+6\,x}{c^3}+\frac{6\,\log(cx-1)}{c^4}\right)+18\,c^2\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right)+72\,c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{16\,c^3x^3+69\,c^2x^2+426\,cx-}{}}{} + \frac{(18\,c^3x^3\operatorname{Li}_2(cx)-2\,c^3x^3-3\,c^2x^2-6\,cx+6\,(c^3x^3-1)\log(-cx+1))\log(-cx+1)}{54\,c^3}$$

input
```
integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
1/648*(4*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) + 18*c
^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 72*c*(x/c + log(c*x - 1)/c^2
) + (16*c^3*x^3 + 69*c^2*x^2 + 426*c*x - 36*(2*c^3*x^3 + 3*c^2*x^2 + 6*c*x
 + 6*log(-c*x + 1))*dilog(c*x) - 6*(8*c^3*x^3 + 15*c^2*x^2 + 48*c*x - 71)*
log(-c*x + 1))/c - 216*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-
c*x + 1) - 2*polylog(3, -c*x + 1))/c)/c^2 + 1/54*(18*c^3*x^3*dilog(c*x) -
2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*log(-c*x +
1)/c^3
```

3.162.8 Giac [F]

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx = \int x^2 \operatorname{Li}_2(cx) \log(-cx+1)\,dx$$

input
```
integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate(x^2*dilog(c*x)*log(-c*x + 1), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \log(1-cx) \operatorname{PolyLog}(2,cx)\,dx = \int x^2 \ln(1-c\,x)\ \operatorname{polylog}(2,c\,x)\,dx$$

input
```
int(x^2*log(1 - c*x)*polylog(2, c*x),x)
```

output
```
int(x^2*log(1 - c*x)*polylog(2, c*x), x)
```

3.163 $\int x \log(1-cx) \text{PolyLog}(2,cx)\,dx$

3.163.1 Optimal result

Integrand size = 14, antiderivative size = 262

$$\begin{aligned}\int x\log(1-cx)\,\text{PolyLog}(2,cx)\,dx &= \frac{13x}{8c}+\frac{x^2}{16}+\frac{(1-cx)^2}{8c^2}+\frac{\log(1-cx)}{8c^2}-\frac{1}{8}x^2\log(1-cx)\\
&\quad+\frac{3(1-cx)\log(1-cx)}{2c^2}-\frac{(1-cx)^2\log(1-cx)}{4c^2}\\
&\quad-\frac{(1-cx)\log^2(1-cx)}{2c^2}+\frac{(1-cx)^2\log^2(1-cx)}{4c^2}\\
&\quad-\frac{\log(cx)\log^2(1-cx)}{2c^2}-\frac{x\,\text{PolyLog}(2,cx)}{2c}\\
&\quad-\frac{1}{4}x^2\,\text{PolyLog}(2,cx)-\frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{2c^2}\\
&\quad+\frac{1}{2}x^2\log(1-cx)\,\text{PolyLog}(2,cx)\\
&\quad-\frac{\log(1-cx)\,\text{PolyLog}(2,1-cx)}{c^2}+\frac{\text{PolyLog}(3,1-cx)}{c^2}\end{aligned}$$

output
```
13/8*x/c+1/16*x^2+1/8*(-c*x+1)^2/c^2+1/8*ln(-c*x+1)/c^2-1/8*x^2*ln(-c*x+1)
+3/2*(-c*x+1)*ln(-c*x+1)/c^2-1/4*(-c*x+1)^2*ln(-c*x+1)/c^2-1/2*(-c*x+1)*ln
(-c*x+1)^2/c^2+1/4*(-c*x+1)^2*ln(-c*x+1)^2/c^2-1/2*ln(c*x)*ln(-c*x+1)^2/c^
2-1/2*x*polylog(2,c*x)/c-1/4*x^2*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,c
*x)/c^2+1/2*x^2*ln(-c*x+1)*polylog(2,c*x)-ln(-c*x+1)*polylog(2,-c*x+1)/c^2
+polylog(3,-c*x+1)/c^2
```

3.163.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.61

$$\int x\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$=\frac{-14+22cx+3c^2x^2+22\log(1-cx)-16cx\log(1-cx)-6c^2x^2\log(1-cx)-4\log^2(1-cx)+4c^2x^2\log}{}$$

input
```
Integrate[x*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*Log[1 - c*x] - 6*c^2*
x^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 - c*x]^2 - 8*Log[c*x
]*Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2)*Log[1 - c*x])*Poly
Log[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*PolyLog[3, 1 - c*x]
)/(16*c^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7157, 2009, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\text{PolyLog}(2,cx)\log(1-cx)\,dx$$
$$\Big\downarrow \text{7157}$$
$$\frac{1}{2}c\int\left(-\frac{x\,\text{PolyLog}(2,cx)}{c}+\frac{\text{PolyLog}(2,cx)}{c^2(1-cx)}-\frac{\text{PolyLog}(2,cx)}{c^2}\right)dx+\frac{1}{2}\int x\log^2(1-cx)dx+\frac{1}{2}x^2\,\text{PolyLog}(2,cx)\log(1-cx)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\int x\log^2(1-cx)dx+\frac{1}{2}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-cx}{c^3}\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2848}$$

$$\frac{1}{2}\int\left(\frac{\log^2(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}\right)dx+$$
$$\frac{1}{2}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-cx)}{c^3}\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(1-cx)^2}{4c^2}+\frac{(1-cx)^2\log^2(1-cx)}{2c^2}-\frac{(1-cx)\log^2(1-cx)}{c^2}-\frac{(1-cx)^2\log(1-cx)}{2c^2}+\frac{2(1-cx)\log(1-cx)}{c^2}+\right.$$
$$\frac{1}{2}c\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-cx)}{c^3}\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(2,cx)\log(1-cx)$$

input
```
Int[x*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
((2*x)/c + (1 - c*x)^2/(4*c^2) + (2*(1 - c*x)*Log[1 - c*x])/c^2 - ((1 - c*
x)^2*Log[1 - c*x])/(2*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/c^2 + ((1 - c*x)^2
*Log[1 - c*x]^2)/(2*c^2))/2 + (x^2*Log[1 - c*x]*PolyLog[2, c*x])/2 + (c*((
5*x)/(4*c^2) + x^2/(8*c) + Log[1 - c*x]/(4*c^3) - (x^2*Log[1 - c*x])/(4*c)
 + ((1 - c*x)*Log[1 - c*x])/c^3 - (Log[c*x]*Log[1 - c*x]^2)/c^3 - (x*PolyL
og[2, c*x])/c^2 - (x^2*PolyLog[2, c*x])/(2*c) - (Log[1 - c*x]*PolyLog[2, c
*x])/c^3 - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^3 + (2*PolyLog[3, 1 - c*
x])/c^3))/2
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.163.4 Maple [F]

$$\int x \ln(-cx+1) \operatorname{polylog}(2, cx)\, dx$$

input
```
int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

3.163.5 Fricas [F]

$$\int x \log(1-cx) \operatorname{PolyLog}(2, cx)\, dx = \int x \operatorname{Li}_2(cx) \log(-cx+1)\, dx$$

input
```
integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(x*dilog(c*x)*log(-c*x + 1), x)
```

3.163.6 Sympy [F]

$$\int x \log(1-cx) \operatorname{PolyLog}(2, cx)\, dx = \int x \log(-cx+1) \operatorname{Li}_2(cx)\, dx$$

input
```
integrate(x*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral(x*log(-c*x + 1)*polylog(2, c*x), x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

$$\int x\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$=\frac{c^2\left(\frac{cx^2+2x}{c^2}+\frac{2\log(cx-1)}{c^3}\right)+4c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{2\left(c^2x^2+8cx-2\left(c^2x^2+2cx+2\log(-cx+1)\right)\text{Li}_2(cx)-2\left(c^2x^2+3cx-4\right)\log(-cx\right.}{c}}{16c}$$
$$+\frac{\left(4c^2x^2\text{Li}_2(cx)-c^2x^2-2cx+2\left(c^2x^2-1\right)\log\left(-cx+1\right)\right)\log\left(-cx+1\right)}{8c^2}$$

input
```
integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
1/16*(c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 4*c*(x/c + log(c*x -
1)/c^2) + 2*(c^2*x^2 + 8*c*x - 2*(c^2*x^2 + 2*c*x + 2*log(-c*x + 1))*dilog
(c*x) - 2*(c^2*x^2 + 3*c*x - 4)*log(-c*x + 1))/c - 8*(log(c*x)*log(-c*x +
1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1))/c)/c + 1/
8*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))
*log(-c*x + 1)/c^2
```

3.163.8 Giac [F]

$$\int x\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int x\text{Li}_2(cx)\log\left(-cx+1\right)\,dx$$

input
```
integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate(x*dilog(c*x)*log(-c*x + 1), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int x\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int x\,\ln\left(1-c\,x\right)\,\text{polylog}(2,c\,x)\,dx$$

input
```
int(x*log(1 - c*x)*polylog(2, c*x),x)
```

output
```
int(x*log(1 - c*x)*polylog(2, c*x), x)
```

3.164 $\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

3.164.1 Optimal result

Integrand size = 13, antiderivative size = 132

$$\begin{aligned}\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx = 3x+\frac{3(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}\\ -\frac{\log(cx)\log^2(1-cx)}{c}-x\,\text{PolyLog}(2,cx)\\ -\frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{c}\\ +x\log(1-cx)\,\text{PolyLog}(2,cx)\\ -\frac{2\log(1-cx)\,\text{PolyLog}(2,1-cx)}{c}\\ +\frac{2\,\text{PolyLog}(3,1-cx)}{c}\end{aligned}$$

output
```
3*x+3*(-c*x+1)*ln(-c*x+1)/c-(-c*x+1)*ln(-c*x+1)^2/c-ln(c*x)*ln(-c*x+1)^2/c
-x*polylog(2,c*x)-ln(-c*x+1)*polylog(2,c*x)/c+x*ln(-c*x+1)*polylog(2,c*x)-
2*ln(-c*x+1)*polylog(2,-c*x+1)/c+2*polylog(3,-c*x+1)/c
```

3.164.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$=\frac{-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)+(-}{c}$$

input
```
Integrate[Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*L
og[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x
])*PolyLog[2, c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 -
 c*x])/c
```

3.164.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {7154, 25, 2836, 2733, 2732, 7239, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,cx)\log(1-cx)\,dx$$
$$\Big\downarrow \text{ 7154}$$
$$c\int -\left(\left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)\right)dx+\int \log^2(1-cx)dx+x\,\text{PolyLog}(2,cx)\log(1-cx)$$
$$\Big\downarrow \text{ 25}$$
$$-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx+\int \log^2(1-cx)dx+x\,\text{PolyLog}(2,cx)\log(1-cx)$$
$$\Big\downarrow \text{ 2836}$$
$$-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx-\frac{\int \log^2(1-cx)d(1-cx)}{c}+x\,\text{PolyLog}(2,cx)\log(1-cx)$$
$$\Big\downarrow \text{ 2733}$$

$$-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right) \text{PolyLog}(2,cx)dx-\frac{(1-cx)\log^2(1-cx)-2\int \log(1-cx)d(1-cx)}{c}+x\,\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2732}$$

$$-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right) \text{PolyLog}(2,cx)dx+x\,\text{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-cx)\log(1-cx)-1)}{c}$$

$$\Big\downarrow \text{7239}$$

$$-c\int \frac{x\,\text{PolyLog}(2,cx)}{cx-1}dx+x\,\text{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-cx)\log(1-cx)-1)}{c}$$

$$\Big\downarrow \text{7293}$$

$$-c\int \left(\frac{\text{PolyLog}(2,cx)}{c}+\frac{\text{PolyLog}(2,cx)}{c(cx-1)}\right)dx+x\,\text{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-cx)\log(1-cx)-1)}{c}$$

$$\Big\downarrow \text{2009}$$

$$-c\Bigg(-\frac{2\,\text{PolyLog}(3,1-cx)}{c^2}+\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^2}+\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^2}+\frac{\log(cx)\log^2(1-c}{c^2}$$
$$x\,\text{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-cx)\log(1-cx)-1)}{c}$$

input
```
Int[Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
-(((1 - c*x)*Log[1 - c*x]^2 - 2*(-1 + c*x + (1 - c*x)*Log[1 - c*x]))/c) +
x*Log[1 - c*x]*PolyLog[2, c*x] - c*(-(x/c) - ((1 - c*x)*Log[1 - c*x])/c^2
+ (Log[c*x]*Log[1 - c*x]^2)/c^2 + (x*PolyLog[2, c*x])/c + (Log[1 - c*x]*Po
lyLog[2, c*x])/c^2 + (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 - (2*PolyLog
[3, 1 - c*x])/c^2)
```

3.164. $\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 7154
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Simp[b   Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a
*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Simp[e*h*n   Int[Poly
Log[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b
, c, d, e, f, g, h, n}, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.164.4 Maple [F]

$$\int \ln \left(-c x+1\right) \operatorname{polylog}\left(2, c x\right)d x$$

input
```
int(ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(ln(-c*x+1)*polylog(2,c*x),x)
```

3.164.5 Fricas [F]

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int \text{Li}_2(cx)\log\left(-cx+1\right)\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(dilog(c*x)*log(-c*x + 1), x)
```

3.164.6 Sympy [F]

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int \log\left(-cx+1\right)\text{Li}_2\left(cx\right)\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x), x)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$= c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{(cx\text{Li}_2(cx)-cx+(cx-1)\log(-cx+1))\log(-cx+1)}{c}$$
$$-\frac{\log(cx)\log(-cx+1)^2+2\,\text{Li}_2(-cx+1)\log(-cx+1)-2\,\text{Li}_3(-cx+1)}{c}$$
$$+\frac{2\,cx-(cx+\log(-cx+1))\text{Li}_2(cx)-2\,(cx-1)\log(-cx+1)}{c}$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
c*(x/c + log(c*x - 1)/c^2) + (c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x +
1))*log(-c*x + 1)/c - (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c
*x + 1) - 2*polylog(3, -c*x + 1))/c + (2*c*x - (c*x + log(-c*x + 1))*dilog
(c*x) - 2*(c*x - 1)*log(-c*x + 1))/c
```

3.164.8 Giac [F]

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int \text{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int \ln(1-c\,x)\,\text{polylog}(2,c\,x)\,dx$$

input
```
int(log(1 - c*x)*polylog(2, c*x),x)
```

output
```
int(log(1 - c*x)*polylog(2, c*x), x)
```

3.165 $\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx$

3.165.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx = -\frac{1}{2}\,\text{PolyLog}(2,cx)^2$$

output
```
-1/2*polylog(2,c*x)^2
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx = -\frac{1}{2}\,\text{PolyLog}(2,cx)^2$$

input
```
Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

output
```
-1/2*PolyLog[2, c*x]^2
```

3.165.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {7155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}\,dx$$

$$\Big\downarrow \; 7155$$

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

input
```
Int[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

output
```
-1/2*PolyLog[2, c*x]^2
```

3.165.3.1 Defintions of rubi rules used

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{\text{polylog}(2,cx)^2}{2}$	10
default	$-\frac{\text{polylog}(2,cx)^2}{2}$	10
parallelrisch	$-\frac{\text{polylog}(2,cx)^2}{2}$	10

input
```
int(ln(-c*x+1)*polylog(2,c*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*polylog(2,c*x)^2
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx=-\frac{1}{2}\,\text{Li}_2(cx)^2$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
```

output
```
-1/2*dilog(c*x)^2
```

3.165.6 Sympy [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx=\int \frac{\log(-cx+1)\,\text{Li}_2\,(cx)}{x}\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x)/x,x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x)/x, x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx=-\frac{1}{2}\,\text{Li}_2(cx)^2$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
```

output
```
-1/2*dilog(c*x)^2
```

3.165.8 Giac [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x}\,dx = \int \frac{\mathrm{Li}_2(cx)\log(-cx+1)}{x}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1)/x, x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x}\,dx = -\frac{\mathrm{polylog}(2,c\,x)^2}{2}$$

input
```
int((log(1 - c*x)*polylog(2, c*x))/x,x)
```

output
```
-polylog(2, c*x)^2/2
```

3.166 $\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx$

3.166.1 Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx = \frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx) - 2c\,\text{PolyLog}(2,cx)+c\log(1-cx)\,\text{PolyLog}(2,cx) - \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x} + 2c\log(1-cx)\,\text{PolyLog}(2,1-cx) - c\,\text{PolyLog}(3,cx)-2c\,\text{PolyLog}(3,1-cx)$$

output
```
(-c*x+1)*ln(-c*x+1)^2/x+c*ln(c*x)*ln(-c*x+1)^2-2*c*polylog(2,c*x)+c*ln(-c*
x+1)*polylog(2,c*x)-ln(-c*x+1)*polylog(2,c*x)/x+2*c*ln(-c*x+1)*polylog(2,-
c*x+1)-c*polylog(3,c*x)-2*c*polylog(3,-c*x+1)
```

3.166.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx = 2c\log(cx)\log(1-cx)-c\log^2(1-cx) + \frac{\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx) + \frac{(-1+cx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x} + 2c(1+\log(1-cx))\,\text{PolyLog}(2,1-cx) - c\,\text{PolyLog}(3,cx)-2c\,\text{PolyLog}(3,1-cx)$$

input
```
Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

output
```
2*c*Log[c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*
x]*Log[1 - c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 +
 Log[1 - c*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1
- c*x]
```

3.166.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7157, 2009, 2844, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)\log(1-cx)}{x^2}dx$$

$$\Big\downarrow \text{7157}$$

$$-c\int \left(\frac{\text{PolyLog}(2,cx)}{x}+\frac{c\,\text{PolyLog}(2,cx)}{1-cx}\right)dx-\int \frac{\log^2(1-cx)}{x^2}dx-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\int \frac{\log^2(1-cx)}{x^2}dx-$$
$$c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(cx$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}$$

$$\Big\downarrow \text{2844}$$

$$2c\int \frac{\log(1-cx)}{x}dx-$$
$$c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(cx$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}+\frac{(1-cx)\log^2(1-cx)}{x}$$

$$\Big\downarrow \text{2838}$$

$$-2c\,\text{PolyLog}(2,cx)-$$
$$c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(cx$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}+\frac{(1-cx)\log^2(1-cx)}{x}$$

input
```
Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

output
```
((1 - c*x)*Log[1 - c*x]^2)/x - 2*c*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog
[2, c*x])/x - c*(-(Log[c*x]*Log[1 - c*x]^2) - Log[1 - c*x]*PolyLog[2, c*x]
 - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + PolyLog[3, c*x] + 2*PolyLog[3, 1 -
 c*x])
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.166.4 Maple [F]

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^2}dx$$

input
```
int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

output
```
int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

3.166.5 Fricas [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

output
```
integral(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.166.6 Sympy [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx = \left(\log(cx)\log(-cx+1)^2 + 2\,\text{Li}_2(-cx+1)\log(-cx+1) - 2\,\text{Li}_3(-cx+1)\right)c + 2\left(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1)\right)c - c\text{Li}_3(cx) + \frac{(cx-1)\text{Li}_2(cx)\log(-cx+1) - (cx-1)\log(-cx+1)^2}{x}$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

output
```
(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3,
 -c*x + 1))*c + 2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c - c*polylog
(3, c*x) + ((c*x - 1)*dilog(c*x)*log(-c*x + 1) - (c*x - 1)*log(-c*x + 1)^2
)/x
```

3.166.8 Giac [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{\ln(1-c\,x)\,\text{polylog}(2,c\,x)}{x^2}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x))/x^2,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x))/x^2, x)
```

3.167 $\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx$

3.167.1 Optimal result

Integrand size = 16, antiderivative size = 191

$$\begin{aligned}\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx &= -c^2\log(x)+c^2\log(1-cx)-\frac{c\log(1-cx)}{x}\\ &\quad -\frac{1}{4}c^2\log^2(1-cx)+\frac{\log^2(1-cx)}{4x^2}\\ &\quad +\frac{1}{2}c^2\log(cx)\log^2(1-cx)-\frac{1}{2}c^2\,\text{PolyLog}(2,cx)\\ &\quad +\frac{c\,\text{PolyLog}(2,cx)}{2x}+\frac{1}{2}c^2\log(1-cx)\,\text{PolyLog}(2,cx)\\ &\quad -\frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{2x^2}\\ &\quad +c^2\log(1-cx)\,\text{PolyLog}(2,1-cx)\\ &\quad -\frac{1}{2}c^2\,\text{PolyLog}(3,cx)-c^2\,\text{PolyLog}(3,1-cx)\end{aligned}$$

output
```
-c^2*ln(x)+c^2*ln(-c*x+1)-c*ln(-c*x+1)/x-1/4*c^2*ln(-c*x+1)^2+1/4*ln(-c*x+
1)^2/x^2+1/2*c^2*ln(c*x)*ln(-c*x+1)^2-1/2*c^2*polylog(2,c*x)+1/2*c*polylog
(2,c*x)/x+1/2*c^2*ln(-c*x+1)*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,c*x)/
x^2+c^2*ln(-c*x+1)*polylog(2,-c*x+1)-1/2*c^2*polylog(3,c*x)-c^2*polylog(3,
-c*x+1)
```

3.167.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.97

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \frac{1}{4}\Bigg(-2c^2\log(x)-2c^2\log(cx)+4c^2\log(1-cx) - \frac{4c\log(1-cx)}{x}+2c^2\log(cx)\log(1-cx) - c^2\log^2(1-cx)+\frac{\log^2(1-cx)}{x^2}+2c^2\log(cx)\log^2(1-cx) + \frac{2(cx+(-1+c^2x^2)\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2} + 2c^2(1+2\log(1-cx))\,\text{PolyLog}(2,1-cx) - 2c^2\,\text{PolyLog}(3,cx)-4c^2\,\text{PolyLog}(3,1-cx)\Bigg)$$

input
```
Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

output
```
(-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c*x] - (4*c*Log[1 - c*x])/
x + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]^2 + Log[1 - c*x]^2/x^2
+ 2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c^2*x^2)*Log[1 - c*x])*P
olyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 2*c
^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x])/4
```

3.167.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7157, 2009, 2845, 2857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)\log(1-cx)}{x^3}\,dx$$

$$\Big\downarrow \text{7157}$$

$$-\frac{1}{2}c\int \left(\frac{\text{PolyLog}(2,cx)c^2}{1-cx}+\frac{\text{PolyLog}(2,cx)c}{x}+\frac{\text{PolyLog}(2,cx)}{x^2}\right)dx-\frac{1}{2}\int \frac{\log^2(1-cx)}{x^3}dx - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\int \frac{\log^2(1-cx)}{x^3}dx - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{2x^2} - \frac{1}{2}c\left(-\frac{\text{PolyLog}(2,cx)}{x} + c\,\text{PolyLog}(3,cx) + 2c\,\text{PolyLog}(3,1-cx) - c\,\text{PolyLog}(2,cx)\log(1-cx) - 2c\,\text{PolyLog}(2,\right.$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(c\int \frac{\log(1-cx)}{x^2(1-cx)}dx + \frac{\log^2(1-cx)}{2x^2}\right) - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{2x^2} - \frac{1}{2}c\left(-\frac{\text{PolyLog}(2,cx)}{x} + c\,\text{PolyLog}(3,cx) + 2c\,\text{PolyLog}(3,1-cx) - c\,\text{PolyLog}(2,cx)\log(1-cx) - 2c\,\text{PolyLog}(2,\right.$$

$$\Big\downarrow \text{2857}$$

$$\frac{1}{2}\left(c\int \left(-\frac{\log(1-cx)c^2}{cx-1} + \frac{\log(1-cx)c}{x} + \frac{\log(1-cx)}{x^2}\right)dx + \frac{\log^2(1-cx)}{2x^2}\right) - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{2x^2} - \frac{1}{2}c\left(-\frac{\text{PolyLog}(2,cx)}{x} + c\,\text{PolyLog}(3,cx) + 2c\,\text{PolyLog}(3,1-cx) - c\,\text{PolyLog}(2,cx)\log(1-cx) - 2c\,\text{PolyLog}(2,\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(c\left(-c\,\text{PolyLog}(2,cx) - \frac{1}{2}c\log^2(1-cx) + c\log(1-cx) - \frac{\log(1-cx)}{x} - c\log(x)\right) + \frac{\log^2(1-cx)}{2x^2}\right) - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{2x^2} - \frac{1}{2}c\left(-\frac{\text{PolyLog}(2,cx)}{x} + c\,\text{PolyLog}(3,cx) + 2c\,\text{PolyLog}(3,1-cx) - c\,\text{PolyLog}(2,cx)\log(1-cx) - 2c\,\text{PolyLog}(2,\right.$$

input
```
Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

output
```
-1/2*(Log[1 - c*x]*PolyLog[2, c*x])/x^2 + (Log[1 - c*x]^2/(2*x^2) + c*(-(c
*Log[x]) + c*Log[1 - c*x] - Log[1 - c*x]/x - (c*Log[1 - c*x]^2)/2 - c*Poly
Log[2, c*x]))/2 - (c*(c*Log[x] - c*Log[1 - c*x] + Log[1 - c*x]/x - c*Log[c
*x]*Log[1 - c*x]^2 - PolyLog[2, c*x]/x - c*Log[1 - c*x]*PolyLog[2, c*x] -
2*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] + c*PolyLog[3, c*x] + 2*c*PolyLog[3,
1 - c*x]))/2
```

3.167. $\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx$

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.167.4 Maple [F]

$$\int \frac{\ln \left(-cx+1\right) \operatorname{polylog}\left(2,cx\right)}{x^{3}}dx$$

input
```
int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

output
```
int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

3.167.5 Fricas [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^3}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

output
```
integral(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.167.6 Sympy [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{\log(-cx+1)\,\text{Li}_2(cx)}{x^3}\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x)/x**3, x)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx \\
&= \frac{1}{2}\left(\log(cx)\log(-cx+1)^2 + 2\,\text{Li}_2(-cx+1)\log(-cx+1) - 2\,\text{Li}_3(-cx+1)\right)c^2 \\
&\quad + \frac{1}{2}\left(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1)\right)c^2 - c^2\log(x) - \frac{1}{2}c^2\text{Li}_3(cx) \\
&\quad - \frac{(c^2x^2-1)\log(-cx+1)^2 - 2\,(cx + (c^2x^2-1)\log(-cx+1))\text{Li}_2(cx) - 4\,(c^2x^2-cx)\log(-cx+1)}{4\,x^2}
\end{aligned}$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*c^2 + 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^2 -
 c^2*log(x) - 1/2*c^2*polylog(3, c*x) - 1/4*((c^2*x^2 - 1)*log(-c*x + 1)^2
 - 2*(c*x + (c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x) - 4*(c^2*x^2 - c*x)*lo
g(-c*x + 1))/x^2
```

3.167.8 Giac [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^3}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{\ln(1-c\,x)\,\text{polylog}(2,c\,x)}{x^3}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x))/x^3,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x))/x^3, x)
```

3.168 $\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^4}\,dx$

3.168.1 Optimal result

Integrand size = 16, antiderivative size = 245

$$\begin{aligned}\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^4}\,dx &= \frac{7c^2}{36x} - \frac{3}{4}c^3\log(x) + \frac{3}{4}c^3\log(1-cx) - \frac{7c\log(1-cx)}{36x^2} \\ &\quad - \frac{5c^2\log(1-cx)}{9x} - \frac{1}{9}c^3\log^2(1-cx) \\ &\quad + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3\log(cx)\log^2(1-cx) \\ &\quad - \frac{2}{9}c^3\operatorname{PolyLog}(2,cx) + \frac{c\operatorname{PolyLog}(2,cx)}{6x^2} \\ &\quad + \frac{c^2\operatorname{PolyLog}(2,cx)}{3x} + \frac{1}{3}c^3\log(1-cx)\operatorname{PolyLog}(2,cx) \\ &\quad - \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{3x^3} \\ &\quad + \frac{2}{3}c^3\log(1-cx)\operatorname{PolyLog}(2,1-cx) \\ &\quad - \frac{1}{3}c^3\operatorname{PolyLog}(3,cx) - \frac{2}{3}c^3\operatorname{PolyLog}(3,1-cx)\end{aligned}$$

output
```
7/36*c^2/x-3/4*c^3*ln(x)+3/4*c^3*ln(-c*x+1)-7/36*c*ln(-c*x+1)/x^2-5/9*c^2*
ln(-c*x+1)/x-1/9*c^3*ln(-c*x+1)^2+1/9*ln(-c*x+1)^2/x^3+1/3*c^3*ln(c*x)*ln(
-c*x+1)^2-2/9*c^3*polylog(2,c*x)+1/6*c*polylog(2,c*x)/x^2+1/3*c^2*polylog(
2,c*x)/x+1/3*c^3*ln(-c*x+1)*polylog(2,c*x)-1/3*ln(-c*x+1)*polylog(2,c*x)/x
^3+2/3*c^3*ln(-c*x+1)*polylog(2,-c*x+1)-1/3*c^3*polylog(3,c*x)-2/3*c^3*pol
ylog(3,-c*x+1)
```

3.168.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx$$
$$= \frac{7c^2x^2 - 4c^3x^3 - 15c^3x^3\log(x) - 12c^3x^3\log(cx) - 7cx\log(1-cx) - 20c^2x^2\log(1-cx) + 27c^3x^3\log(1-\ldots}{\ldots}$$

input
```
Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

output
```
(7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[x] - 12*c^3*x^3*Log[c*x] - 7*c*x*L
og[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] + 27*c^3*x^3*Log[1 - c*x] + 8*c^3*x^
3*Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]^2 - 4*c^3*x^3*Log[1 - c*x]^2 + 12
*c^3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*x*(1 + 2*c*x) + 2*(-1 + c^3*x^3)*L
og[1 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3*(1 + 3*Log[1 - c*x])*PolyLog[2, 1
 - c*x] - 12*c^3*x^3*PolyLog[3, c*x] - 24*c^3*x^3*PolyLog[3, 1 - c*x])/(36
*x^3)
```

3.168.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7157, 2009, 2845, 2857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)\log(1-cx)}{x^4}\,dx$$

$$\Big\downarrow \text{7157}$$

$$-\frac{1}{3}c\int \left(\frac{\text{PolyLog}(2,cx)c^3}{1-cx}+\frac{\text{PolyLog}(2,cx)c^2}{x}+\frac{\text{PolyLog}(2,cx)c}{x^2}+\frac{\text{PolyLog}(2,cx)}{x^3}\right)dx - \frac{1}{3}\int \frac{\log^2(1-cx)}{x^4}dx - \frac{\text{PolyLog}(2,cx)\log(1-cx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\int \frac{\log^2(1-cx)}{x^4}dx -$$
$$\frac{1}{3}c\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{3x^3}$$

$\big\downarrow$ 2845

$$\frac{1}{3}\left(\frac{2}{3}c\int \frac{\log(1-cx)}{x^3(1-cx)}dx+\frac{\log^2(1-cx)}{3x^3}\right)-$$
$$\frac{1}{3}c\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{3x^3}$$

$\big\downarrow$ 2857

$$\frac{1}{3}\left(\frac{2}{3}c\int \left(-\frac{\log(1-cx)c^3}{cx-1}+\frac{\log(1-cx)c^2}{x}+\frac{\log(1-cx)c}{x^2}+\frac{\log(1-cx)}{x^3}\right)dx+\frac{\log^2(1-cx)}{3x^3}\right)-$$
$$\frac{1}{3}c\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{3x^3}$$

$\big\downarrow$ 2009

$$-\frac{1}{3}c\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{1}{3}\Big(\frac{2}{3}c\Big(-c^2\operatorname{PolyLog}(2,cx)-\frac{1}{2}c^2\log^2(1-cx)-\frac{3}{2}c^2\log(x)+\frac{3}{2}c^2\log(1-cx)-\frac{\log(1-cx)}{2x^2}+\frac{c}{2x}-\frac{c\log(1-cx)}{x}$$
$$\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{3x^3}$$

input
```
Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

output
```
-1/3*(Log[1 - c*x]*PolyLog[2, c*x])/x^3 + (Log[1 - c*x]^2/(3*x^3) + (2*c*(
c/(2*x) - (3*c^2*Log[x])/2 + (3*c^2*Log[1 - c*x])/2 - Log[1 - c*x]/(2*x^2)
 - (c*Log[1 - c*x])/x - (c^2*Log[1 - c*x]^2)/2 - c^2*PolyLog[2, c*x]))/3)/
3 - (c*(-1/4*c/x + (5*c^2*Log[x])/4 - (5*c^2*Log[1 - c*x])/4 + Log[1 - c*x
]/(4*x^2) + (c*Log[1 - c*x])/x - c^2*Log[c*x]*Log[1 - c*x]^2 - PolyLog[2,
c*x]/(2*x^2) - (c*PolyLog[2, c*x])/x - c^2*Log[1 - c*x]*PolyLog[2, c*x] -
2*c^2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + c^2*PolyLog[3, c*x] + 2*c^2*PolyL
og[3, 1 - c*x]))/3
```

3.168. $\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^4}\,dx$

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.168.4 Maple [F]

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2, cx)}{x^4} dx$$

input
```
int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

output
```
int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

3.168.5 Fricas [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^4}\,dx=\int \frac{\mathrm{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

output
```
integral(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.168.6 Sympy [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^4}\,dx=\int \frac{\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x)/x**4, x)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\begin{aligned}&\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^4}\,dx\\&=\frac{1}{3}\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)c^3\\&\quad+\frac{2}{9}\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)c^3-\frac{3}{4}\,c^3\log(x)-\frac{1}{3}\,c^3\mathrm{Li}_3(cx)\\&\quad+\frac{7\,c^2x^2-4\,(c^3x^3-1)\log(-cx+1)^2+6\left(2\,c^2x^2+cx+2\,(c^3x^3-1)\log(-cx+1)\right)\mathrm{Li}_2(cx)+(27\,c^3x^3-}{36\,x^3}\end{aligned}$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

output
```
1/3*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*c^3 + 2/9*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^3 -
 3/4*c^3*log(x) - 1/3*c^3*polylog(3, c*x) + 1/36*(7*c^2*x^2 - 4*(c^3*x^3 -
 1)*log(-c*x + 1)^2 + 6*(2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1))*
dilog(c*x) + (27*c^3*x^3 - 20*c^2*x^2 - 7*c*x)*log(-c*x + 1))/x^3
```

3.168.8 Giac [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{\ln(1-c\,x)\,\,\text{polylog}(2,c\,x)}{x^4}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x))/x^4,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x))/x^4, x)
```

3.169 $\int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^5}\,dx$

3.169.1 Optimal result

Integrand size = 16, antiderivative size = 287

$$\begin{aligned}\int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^5}\,dx =& \frac{5c^2}{144x^2}+\frac{7c^3}{36x}-\frac{41}{72}c^4\log(x)+\frac{41}{72}c^4\log(1-cx)\\ &-\frac{5c\log(1-cx)}{72x^3}-\frac{c^2\log(1-cx)}{8x^2}-\frac{3c^3\log(1-cx)}{8x}\\ &-\frac{1}{16}c^4\log^2(1-cx)+\frac{\log^2(1-cx)}{16x^4}\\ &+\frac{1}{4}c^4\log(cx)\log^2(1-cx)-\frac{1}{8}c^4\text{PolyLog}(2,cx)\\ &+\frac{c\,\text{PolyLog}(2,cx)}{12x^3}+\frac{c^2\,\text{PolyLog}(2,cx)}{8x^2}\\ &+\frac{c^3\,\text{PolyLog}(2,cx)}{4x}+\frac{1}{4}c^4\log(1-cx)\text{PolyLog}(2,cx)\\ &-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{4x^4}\\ &+\frac{1}{2}c^4\log(1-cx)\text{PolyLog}(2,1-cx)\\ &-\frac{1}{4}c^4\text{PolyLog}(3,cx)-\frac{1}{2}c^4\text{PolyLog}(3,1-cx)\end{aligned}$$

output
```
5/144*c^2/x^2+7/36*c^3/x-41/72*c^4*ln(x)+41/72*c^4*ln(-c*x+1)-5/72*c*ln(-c
*x+1)/x^3-1/8*c^2*ln(-c*x+1)/x^2-3/8*c^3*ln(-c*x+1)/x-1/16*c^4*ln(-c*x+1)^
2+1/16*ln(-c*x+1)^2/x^4+1/4*c^4*ln(c*x)*ln(-c*x+1)^2-1/8*c^4*polylog(2,c*x
)+1/12*c*polylog(2,c*x)/x^3+1/8*c^2*polylog(2,c*x)/x^2+1/4*c^3*polylog(2,c
*x)/x+1/4*c^4*ln(-c*x+1)*polylog(2,c*x)-1/4*ln(-c*x+1)*polylog(2,c*x)/x^4+
1/2*c^4*ln(-c*x+1)*polylog(2,-c*x+1)-1/4*c^4*polylog(3,c*x)-1/2*c^4*polylo
g(3,-c*x+1)
```

3.169.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.97

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx$$
$$= \frac{5c^2x^2 + 28c^3x^3 - 18c^4x^4 - 49c^4x^4\log(x) - 33c^4x^4\log(cx) - 10cx\log(1-cx) - 18c^2x^2\log(1-cx) - 54c}{}$$

input
```
Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

output
```
(5*c^2*x^2 + 28*c^3*x^3 - 18*c^4*x^4 - 49*c^4*x^4*Log[x] - 33*c^4*x^4*Log[
c*x] - 10*c*x*Log[1 - c*x] - 18*c^2*x^2*Log[1 - c*x] - 54*c^3*x^3*Log[1 -
c*x] + 82*c^4*x^4*Log[1 - c*x] + 18*c^4*x^4*Log[c*x]*Log[1 - c*x] + 9*Log[
1 - c*x]^2 - 9*c^4*x^4*Log[1 - c*x]^2 + 36*c^4*x^4*Log[c*x]*Log[1 - c*x]^2
 + 6*(c*x*(2 + 3*c*x + 6*c^2*x^2) + 6*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog
[2, c*x] + 18*c^4*x^4*(1 + 4*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 36*c^4*x^
4*PolyLog[3, c*x] - 72*c^4*x^4*PolyLog[3, 1 - c*x])/(144*x^4)
```

3.169.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {7157, 2009, 2845, 2857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)\log(1-cx)}{x^5}\,dx$$
$$\Big\downarrow \; 7157$$

$$-\frac{1}{4}c\int\left(\frac{\text{PolyLog}(2,cx)c^4}{1-cx}+\frac{\text{PolyLog}(2,cx)c^3}{x}+\frac{\text{PolyLog}(2,cx)c^2}{x^2}+\frac{\text{PolyLog}(2,cx)c}{x^3}+\frac{\text{PolyLog}(2,cx)}{x^4}\right)dx-\frac{1}{4}\int\frac{\log^2(1-cx)}{x^5}dx-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\int\frac{\log^2(1-cx)}{x^5}dx-\frac{1}{4}c\Big(c^3\text{PolyLog}(3,cx)+2c^3\text{PolyLog}(3,1-cx)-c^3\text{PolyLog}(2,cx)\log(1-cx)-2c^3\text{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{4x^4}$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{4}\left(\frac{1}{2}c\int\frac{\log(1-cx)}{x^4(1-cx)}dx+\frac{\log^2(1-cx)}{4x^4}\right)-\frac{1}{4}c\Big(c^3\text{PolyLog}(3,cx)+2c^3\text{PolyLog}(3,1-cx)-c^3\text{PolyLog}(2,cx)\log(1-cx)-2c^3\text{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{4x^4}$$

$$\Big\downarrow \text{2857}$$

$$\frac{1}{4}\left(\frac{1}{2}c\int\left(-\frac{\log(1-cx)c^4}{cx-1}+\frac{\log(1-cx)c^3}{x}+\frac{\log(1-cx)c^2}{x^2}+\frac{\log(1-cx)c}{x^3}+\frac{\log(1-cx)}{x^4}\right)dx+\frac{\log^2(1-cx)}{4x^4}\right)-$$
$$\frac{1}{4}c\Big(c^3\text{PolyLog}(3,cx)+2c^3\text{PolyLog}(3,1-cx)-c^3\text{PolyLog}(2,cx)\log(1-cx)-2c^3\text{PolyLog}(2,1-cx)\log(1-c$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}c\Big(c^3\text{PolyLog}(3,cx)+2c^3\text{PolyLog}(3,1-cx)-c^3\text{PolyLog}(2,cx)\log(1-cx)-2c^3\text{PolyLog}(2,1-cx)\log(1-$$
$$\frac{1}{4}\Big(\frac{1}{2}c\Big(-c^3\text{PolyLog}(2,cx)-\frac{1}{2}c^3\log^2(1-cx)-\frac{11}{6}c^3\log(x)+\frac{11}{6}c^3\log(1-cx)+\frac{5c^2}{6x}-\frac{c^2\log(1-cx)}{x}-\frac{\log(1-}{3x}$$
$$\frac{\text{PolyLog}(2,cx)\log(1-cx)}{4x^4}$$

input
```
Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

3.169. $\int\frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx$

output
```
-1/4*(Log[1 - c*x]*PolyLog[2, c*x])/x^4 + (Log[1 - c*x]^2/(4*x^4) + (c*(c/
(6*x^2) + (5*c^2)/(6*x) - (11*c^3*Log[x])/6 + (11*c^3*Log[1 - c*x])/6 - Lo
g[1 - c*x]/(3*x^3) - (c*Log[1 - c*x])/(2*x^2) - (c^2*Log[1 - c*x])/x - (c^
3*Log[1 - c*x]^2)/2 - c^3*PolyLog[2, c*x]))/2)/4 - (c*(-1/18*c/x^2 - (13*c
^2)/(36*x) + (49*c^3*Log[x])/36 - (49*c^3*Log[1 - c*x])/36 + Log[1 - c*x]/
(9*x^3) + (c*Log[1 - c*x])/(4*x^2) + (c^2*Log[1 - c*x])/x - c^3*Log[c*x]*L
og[1 - c*x]^2 - PolyLog[2, c*x]/(3*x^3) - (c*PolyLog[2, c*x])/(2*x^2) - (c
^2*PolyLog[2, c*x])/x - c^3*Log[1 - c*x]*PolyLog[2, c*x] - 2*c^3*Log[1 - c
*x]*PolyLog[2, 1 - c*x] + c^3*PolyLog[3, c*x] + 2*c^3*PolyLog[3, 1 - c*x])
)/4
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.169.4 Maple [F]

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^5}dx$$

input
```
int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

output
```
int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

3.169.5 Fricas [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{\mathrm{Li}_2(cx)\log(-cx+1)}{x^5}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

output
```
integral(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.169.6 Sympy [F]

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^5}\,dx$$

input
```
integrate(ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

output
```
Integral(log(-c*x + 1)*polylog(2, c*x)/x**5, x)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.75

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx$$
$$= \frac{1}{4}\left(\log(cx)\log(-cx+1)^2 + 2\,\text{Li}_2(-cx+1)\log(-cx+1) - 2\,\text{Li}_3(-cx+1)\right)c^4$$
$$+ \frac{1}{8}\left(\log(cx)\log(-cx+1) + \text{Li}_2(-cx+1)\right)c^4 - \frac{41}{72}\,c^4\log(x) - \frac{1}{4}\,c^4\text{Li}_3(cx)$$
$$+ \frac{28\,c^3x^3 + 5\,c^2x^2 - 9\left(c^4x^4-1\right)\log(-cx+1)^2 + 6\left(6\,c^3x^3 + 3\,c^2x^2 + 2\,cx + 6\left(c^4x^4-1\right)\log(-cx+1)\right)\text{I}}{144\,x^4}$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

output
```
1/4*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*c^4 + 1/8*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^4 -
 41/72*c^4*log(x) - 1/4*c^4*polylog(3, c*x) + 1/144*(28*c^3*x^3 + 5*c^2*x^
2 - 9*(c^4*x^4 - 1)*log(-c*x + 1)^2 + 6*(6*c^3*x^3 + 3*c^2*x^2 + 2*c*x + 6
*(c^4*x^4 - 1)*log(-c*x + 1))*dilog(c*x) + 2*(41*c^4*x^4 - 27*c^3*x^3 - 9*
c^2*x^2 - 5*c*x)*log(-c*x + 1))/x^4
```

3.169.8 Giac [F]

$$\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^5}\,dx$$

input
```
integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

output
```
integrate(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{\ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)}{x^5}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x))/x^5,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x))/x^5, x)
```

3.170 $\int x^2(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx$

3.170.1 Optimal result

Integrand size = 20, antiderivative size = 423

$$\begin{aligned}
&\int x^2(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx\\
&=\frac{121hx}{108c^2}+\frac{13hx^2}{216c}+\frac{hx^3}{81}+\frac{h(1-cx)^2}{6c^3}-\frac{2h(1-cx)^3}{81c^3}+\frac{13h\log(1-cx)}{108c^3}\\
&\quad-\frac{hx^2\log(1-cx)}{12c}-\frac{1}{27}hx^3\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{3c^3}\\
&\quad+\frac{h\log^2(1-cx)}{9c^3}-\frac{h\log(cx)\log^2(1-cx)}{3c^3}+\frac{1}{9}x^3\log(1-cx)(g+h\log(1-cx))\\
&\quad+\frac{(1-cx)(g+2h\log(1-cx))}{3c^3}-\frac{(1-cx)^2(g+2h\log(1-cx))}{6c^3}\\
&\quad+\frac{(1-cx)^3(g+2h\log(1-cx))}{27c^3}-\frac{\log(1-cx)(g+2h\log(1-cx))}{9c^3}\\
&\quad-\frac{hx\operatorname{PolyLog}(2,cx)}{3c^2}-\frac{hx^2\operatorname{PolyLog}(2,cx)}{6c}-\frac{1}{9}hx^3\operatorname{PolyLog}(2,cx)\\
&\quad-\frac{h\log(1-cx)\operatorname{PolyLog}(2,cx)}{3c^3}+\frac{1}{3}x^3(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\\
&\quad-\frac{2h\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{3c^3}+\frac{2h\operatorname{PolyLog}(3,1-cx)}{3c^3}
\end{aligned}$$

output
```
121/108*h*x/c^2+13/216*h*x^2/c+1/81*h*x^3+1/6*h*(-c*x+1)^2/c^3-2/81*h*(-c*
x+1)^3/c^3+13/108*h*ln(-c*x+1)/c^3-1/12*h*x^2*ln(-c*x+1)/c-1/27*h*x^3*ln(-
c*x+1)+1/3*h*(-c*x+1)*ln(-c*x+1)/c^3+1/9*h*ln(-c*x+1)^2/c^3-1/3*h*ln(c*x)*
ln(-c*x+1)^2/c^3+1/9*x^3*ln(-c*x+1)*(g+h*ln(-c*x+1))+1/3*(-c*x+1)*(g+2*h*l
n(-c*x+1))/c^3-1/6*(-c*x+1)^2*(g+2*h*ln(-c*x+1))/c^3+1/27*(-c*x+1)^3*(g+2*
h*ln(-c*x+1))/c^3-1/9*ln(-c*x+1)*(g+2*h*ln(-c*x+1))/c^3-1/3*h*x*polylog(2,
c*x)/c^2-1/6*h*x^2*polylog(2,c*x)/c-1/9*h*x^3*polylog(2,c*x)-1/3*h*ln(-c*x
+1)*polylog(2,c*x)/c^3+1/3*x^3*(g+h*ln(-c*x+1))*polylog(2,c*x)-2/3*h*ln(-c
*x+1)*polylog(2,-c*x+1)/c^3+2/3*h*polylog(3,-c*x+1)/c^3
```

3.170.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.60

$$\int x^2(g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx$$
$$=\frac{g(-cx(6+3cx+2c^2x^2)+6(-1+c^3x^3)\log(1-cx)+18c^3x^3\,\text{PolyLog}(2,cx))}{54c^3}$$
$$+\frac{h\big(186cx+33c^2x^2+8c^3x^3+186\log(1-cx)-120cx\log(1-cx)-42c^2x^2\log(1-cx)-24c^3x^3\log(1-}{}$$

input
```
Integrate[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
(g*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x] + 18*c^
3*x^3*PolyLog[2, c*x]))/(54*c^3) + (h*(186*c*x + 33*c^2*x^2 + 8*c^3*x^3 +
186*Log[1 - c*x] - 120*c*x*Log[1 - c*x] - 42*c^2*x^2*Log[1 - c*x] - 24*c^3
*x^3*Log[1 - c*x] - 24*Log[1 - c*x]^2 + 24*c^3*x^3*Log[1 - c*x]^2 - 72*Log
[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^
3)*Log[1 - c*x])*PolyLog[2, c*x] - 144*Log[1 - c*x]*PolyLog[2, 1 - c*x] +
144*PolyLog[3, 1 - c*x]))/(216*c^3)
```

3.170.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {7157, 2009, 2883, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)\,dx$$

$$\Big\downarrow \; 7157$$

$$\frac{1}{3}ch\int \left(-\frac{\operatorname{PolyLog}(2,cx)x^2}{c}-\frac{\operatorname{PolyLog}(2,cx)x}{c^2}+\frac{\operatorname{PolyLog}(2,cx)}{c^3(1-cx)}-\frac{\operatorname{PolyLog}(2,cx)}{c^3}\right)dx+\frac{1}{3}\int x^2\log(1-cx)(g+h\log(1-cx))dx+\frac{1}{3}x^3\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\int x^2\log(1-cx)(g+h\log(1-cx))dx+\frac{1}{3}ch\left(\frac{2\operatorname{PolyLog}(3,1-cx)}{c^4}-\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-c\ldots}{c^4}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \; 2883$$

$$\frac{1}{3}\left(\frac{1}{3}c\int\frac{x^3(g+2h\log(1-cx))}{1-cx}dx+\frac{1}{3}x^3\log(1-cx)(h\log(1-cx)+g)\right)+\frac{1}{3}ch\left(\frac{2\operatorname{PolyLog}(3,1-cx)}{c^4}-\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-c\ldots}{c^4}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \; 2858$$

$$\frac{1}{3}\left(\frac{1}{3}x^3\log(1-cx)(h\log(1-cx)+g)-\frac{1}{3}\int\frac{x^3(g+2h\log(1-cx))}{1-cx}d(1-cx)\right)+\frac{1}{3}ch\left(\frac{2\operatorname{PolyLog}(3,1-cx)}{c^4}-\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-c\ldots}{c^4}\right.$$
$$\frac{1}{3}x^3\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{3}x^3\log(1-cx)(h\log(1-cx)+g)-\frac{\int\frac{c^3x^3(g+2h\log(1-cx))}{1-cx}d(1-cx)}{3c^3}\right)+$$
$$\frac{1}{3}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$\downarrow$ 2772

$$\frac{1}{3}\left(\frac{1}{3}x^3\log(1-cx)(h\log(1-cx)+g)-\frac{-2h\int\left(-\frac{1}{3}(1-cx)^2+\frac{3}{2}(1-cx)-3+\frac{\log(1-cx)}{1-cx}\right)d(1-cx)-\frac{1}{3}(1-cx)}{}\right.$$
$$\frac{1}{3}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{3}x^3\log(1-cx)(h\log(1-cx)+g)-\frac{-\frac{1}{3}(1-cx)^3(2h\log(1-cx)+g)+\frac{3}{2}(1-cx)^2(2h\log(1-cx)+g)-3(1}{}\right.$$
$$\frac{1}{3}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^4}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^4}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^4}-\frac{\log(cx)\log^2(1-cx)}{c^4}\right.$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

input
```
Int[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
((x^3*Log[1 - c*x]*(g + h*Log[1 - c*x]))/3 - (-3*(1 - c*x)*(g + 2*h*Log[1
- c*x]) + (3*(1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/2 - ((1 - c*x)^3*(g + 2*h
*Log[1 - c*x]))/3 + Log[1 - c*x]*(g + 2*h*Log[1 - c*x]) - 2*h*(-3*(1 - c*x
) + (3*(1 - c*x)^2)/4 - (1 - c*x)^3/9 + Log[1 - c*x]^2/2))/(3*c^3))/3 + (x
^3*(g + h*Log[1 - c*x])*PolyLog[2, c*x])/3 + (c*h*((49*x)/(36*c^3) + (13*x
^2)/(72*c^2) + x^3/(27*c) + (13*Log[1 - c*x])/(36*c^4) - (x^2*Log[1 - c*x]
)/(4*c^2) - (x^3*Log[1 - c*x])/(9*c) + ((1 - c*x)*Log[1 - c*x])/c^4 - (Log
[c*x]*Log[1 - c*x]^2)/c^4 - (x*PolyLog[2, c*x])/c^3 - (x^2*PolyLog[2, c*x]
)/(2*c^2) - (x^3*PolyLog[2, c*x])/(3*c) - (Log[1 - c*x]*PolyLog[2, c*x])/c
^4 - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^4 + (2*PolyLog[3, 1 - c*x])/c^
4))/3
```

3.170. $\int x^2(g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx$

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.170.4 Maple [F]

$$\int x^2(g+h\ln(-cx+1))\operatorname{polylog}(2,cx)\,dx$$

input
```
int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

3.170.5 Fricas [F]

$$\int x^2(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx=\int(h\log(-cx+1)+g)x^2\operatorname{Li}_2(cx)\,dx$$

input
```
integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(h*x^2*dilog(c*x)*log(-c*x + 1) + g*x^2*dilog(c*x), x)
```

3.170.6 Sympy [F]

$$\int x^2(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx=\int x^2(g+h\log(-cx+1))\operatorname{Li}_2(cx)\,dx$$

input
```
integrate(x**2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
Integral(x**2*(g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

3.170.7 Maxima [F]

$$\int x^2(g+h\log(1-cx))\,\mathrm{PolyLog}(2,cx)\,dx=\int (h\log{(-cx+1)}+g)x^2\mathrm{Li}_2(cx)\,dx$$

input
```
integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-1/18*h*((2*c^3*x^3 + 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 - 1)*log(-c*x + 1))*d
ilog(c*x)/c^3 - integrate((6*(c^3*x^3 - 1)*log(-c*x + 1)^2 - (2*c^3*x^3 +
3*c^2*x^2 + 6*c*x)*log(-c*x + 1))/x, x)/c^3) + 1/54*(18*c^3*x^3*dilog(c*x)
 - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*g/c^3
```

3.170.8 Giac [F]

$$\int x^2(g+h\log(1-cx))\,\mathrm{PolyLog}(2,cx)\,dx=\int (h\log{(-cx+1)}+g)x^2\mathrm{Li}_2(cx)\,dx$$

input
```
integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*x^2*dilog(c*x), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int x^2(g+h\log(1-cx))\,\mathrm{PolyLog}(2,cx)\,dx=\int x^2\,(g+h\,\ln{(1-c\,x)})\,\mathrm{polylog}(2,c\,x)\,dx$$

input
```
int(x^2*(g + h*log(1 - c*x))*polylog(2, c*x),x)
```

output
```
int(x^2*(g + h*log(1 - c*x))*polylog(2, c*x), x)
```

3.171 $\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx$

3.171.1 Optimal result

Integrand size = 18, antiderivative size = 330

$$
\begin{aligned}
\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx &= \frac{13hx}{8c}+\frac{hx^2}{16}+\frac{h(1-cx)^2}{8c^2}+\frac{h\log(1-cx)}{8c^2} \\
&-\frac{1}{8}hx^2\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{2c^2} \\
&+\frac{h\log^2(1-cx)}{4c^2}-\frac{h\log(cx)\log^2(1-cx)}{2c^2} \\
&+\frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx)) \\
&+\frac{(1-cx)(g+2h\log(1-cx))}{2c^2} \\
&-\frac{(1-cx)^2(g+2h\log(1-cx))}{8c^2} \\
&-\frac{\log(1-cx)(g+2h\log(1-cx))}{4c^2} \\
&-\frac{hx\operatorname{PolyLog}(2,cx)}{2c}-\frac{1}{4}hx^2\operatorname{PolyLog}(2,cx) \\
&-\frac{h\log(1-cx)\operatorname{PolyLog}(2,cx)}{2c^2} \\
&+\frac{1}{2}x^2(g+h\log(1-cx))\operatorname{PolyLog}(2,cx) \\
&-\frac{h\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{c^2} \\
&+\frac{h\operatorname{PolyLog}(3,1-cx)}{c^2}
\end{aligned}
$$

output
```
13/8*h*x/c+1/16*h*x^2+1/8*h*(-c*x+1)^2/c^2+1/8*h*ln(-c*x+1)/c^2-1/8*h*x^2*
ln(-c*x+1)+1/2*h*(-c*x+1)*ln(-c*x+1)/c^2+1/4*h*ln(-c*x+1)^2/c^2-1/2*h*ln(c
*x)*ln(-c*x+1)^2/c^2+1/4*x^2*ln(-c*x+1)*(g+h*ln(-c*x+1))+1/2*(-c*x+1)*(g+2
*h*ln(-c*x+1))/c^2-1/8*(-c*x+1)^2*(g+2*h*ln(-c*x+1))/c^2-1/4*ln(-c*x+1)*(g
+2*h*ln(-c*x+1))/c^2-1/2*h*x*polylog(2,c*x)/c-1/4*h*x^2*polylog(2,c*x)-1/2
*h*ln(-c*x+1)*polylog(2,c*x)/c^2+1/2*x^2*(g+h*ln(-c*x+1))*polylog(2,c*x)-h
*ln(-c*x+1)*polylog(2,-c*x+1)/c^2+h*polylog(3,-c*x+1)/c^2
```

3.171.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx$$
$$=\frac{g(-cx(2+cx)+2(-1+c^2x^2)\log(1-cx)+4c^2x^2\operatorname{PolyLog}(2,cx))}{8c^2}$$
$$+\frac{h(-14+22cx+3c^2x^2+22\log(1-cx)-16cx\log(1-cx)-6c^2x^2\log(1-cx)-4\log^2(1-cx)+4c^2x}{}$$

input
```
Integrate[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
(g*(-(c*x*(2 + c*x)) + 2*(-1 + c^2*x^2)*Log[1 - c*x] + 4*c^2*x^2*PolyLog[2
, c*x]))/(8*c^2) + (h*(-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x
*Log[1 - c*x] - 6*c^2*x^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[
1 - c*x]^2 - 8*Log[c*x]*Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x
^2)*Log[1 - c*x])*PolyLog[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] +
16*PolyLog[3, 1 - c*x]))/(16*c^2)
```

3.171.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {7157, 2009, 2883, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)\,dx$$

$$\Big\downarrow \text{7157}$$

$$\frac{1}{2}ch\int\left(-\frac{x\,\text{PolyLog}(2,cx)}{c}+\frac{\text{PolyLog}(2,cx)}{c^2(1-cx)}-\frac{\text{PolyLog}(2,cx)}{c^2}\right)dx+\frac{1}{2}\int x\log(1-cx)(g+ h\log(1-cx))dx+\frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\int x\log(1-cx)(g+h\log(1-cx))dx+ \frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-c}{c^3}\right. \frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{2883}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int\frac{x^2(g+2h\log(1-cx))}{1-cx}dx+\frac{1}{2}x^2\log(1-cx)(h\log(1-cx)+g)\right)+ \frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-c}{c^3}\right. \frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\log(1-cx)(h\log(1-cx)+g)-\frac{1}{2}\int\frac{x^2(g+2h\log(1-cx))}{1-cx}d(1-cx)\right)+ \frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-c}{c^3}\right. \frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\log(1-cx)(h\log(1-cx)+g)-\frac{\int\frac{c^2x^2(g+2h\log(1-cx))}{1-cx}d(1-cx)}{2c^2}\right)+ \frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-c}{c^3}\right. \frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{2772}$$

3.171. $\int x(g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx$

$$\frac{1}{2}\left(\frac{1}{2}x^2\log(1-cx)(h\log(1-cx)+g)-\frac{-2h\int\left(\frac{1}{2}(1-cx)-2+\frac{\log(1-cx)}{1-cx}\right)d(1-cx)+\frac{1}{2}(1-cx)^2(2h\log(1-cx)}{2}\right.$$
$$\frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-cx)}{c^3}\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{2}x^2\log(1-cx)(h\log(1-cx)+g)-\frac{\frac{1}{2}(1-cx)^2(2h\log(1-cx)+g)-2(1-cx)(2h\log(1-cx)+g)+\log(1-}{2}\right.$$
$$\frac{1}{2}ch\left(\frac{2\,\text{PolyLog}(3,1-cx)}{c^3}-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^3}-\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^3}-\frac{\log(cx)\log^2(1-cx)}{c^3}\right.$$
$$\frac{1}{2}x^2\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

input
```
Int[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
((x^2*Log[1 - c*x]*(g + h*Log[1 - c*x]))/2 - (-2*(1 - c*x)*(g + 2*h*Log[1
- c*x]) + ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/2 + Log[1 - c*x]*(g + 2*h*L
og[1 - c*x]) - 2*h*(-2*(1 - c*x) + (1 - c*x)^2/4 + Log[1 - c*x]^2/2))/(2*c
^2))/2 + (x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x])/2 + (c*h*((5*x)/(4*c^2
) + x^2/(8*c) + Log[1 - c*x]/(4*c^3) - (x^2*Log[1 - c*x])/(4*c) + ((1 - c*
x)*Log[1 - c*x])/c^3 - (Log[c*x]*Log[1 - c*x]^2)/c^3 - (x*PolyLog[2, c*x])
/c^2 - (x^2*PolyLog[2, c*x])/(2*c) - (Log[1 - c*x]*PolyLog[2, c*x])/c^3 -
(2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^3 + (2*PolyLog[3, 1 - c*x])/c^3))/2
```

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.171.4 Maple [F]

$$\int x(g+h\ln(-cx+1))\operatorname{polylog}(2,cx)\,dx$$

input
```
int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

3.171.5 Fricas [F]

$$\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx=\int (h\log(-cx+1)+g)x\mathrm{Li}_2(cx)\,dx$$

input
```
integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(h*x*dilog(c*x)*log(-c*x + 1) + g*x*dilog(c*x), x)
```

3.171.6 Sympy [F]

$$\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx=\int x(g+h\log(-cx+1))\,\mathrm{Li}_2(cx)\,dx$$

input
```
integrate(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
Integral(x*(g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

3.171.7 Maxima [F]

$$\int x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx=\int (h\log(-cx+1)+g)x\mathrm{Li}_2(cx)\,dx$$

input
```
integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-1/4*h*((c^2*x^2 + 2*c*x - 2*(c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x)/c^2 -
 integrate((2*(c^2*x^2 - 1)*log(-c*x + 1)^2 - (c^2*x^2 + 2*c*x)*log(-c*x +
 1))/x, x)/c^2) + 1/8*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2
 - 1)*log(-c*x + 1))*g/c^2
```

3.171.8 Giac [F]

$$\int x(g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx=\int (h\log(-cx+1)+g)x\text{Li}_2(cx)\,dx$$

input
```
integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*x*dilog(c*x), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int x(g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx=\int x\,(g+h\,\ln(1-c\,x))\,\text{polylog}(2,c\,x)\,dx$$

input
```
int(x*(g + h*log(1 - c*x))*polylog(2, c*x),x)
```

output
```
int(x*(g + h*log(1 - c*x))*polylog(2, c*x), x)
```

3.172 $\int (g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx$

3.172.1 Optimal result

Integrand size = 17, antiderivative size = 167

$$\begin{aligned}\int (g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\,dx = &-gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}\\&+\frac{3h(1-cx)\log(1-cx)}{c}\\&-\frac{h(1-cx)\log^2(1-cx)}{c}\\&-\frac{h\log(cx)\log^2(1-cx)}{c}-hx\operatorname{PolyLog}(2,cx)\\&-\frac{h\log(1-cx)\operatorname{PolyLog}(2,cx)}{c}\\&+x(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)\\&-\frac{2h\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{c}\\&+\frac{2h\operatorname{PolyLog}(3,1-cx)}{c}\end{aligned}$$

output
```
-g*x+3*h*x-g*(-c*x+1)*ln(-c*x+1)/c+3*h*(-c*x+1)*ln(-c*x+1)/c-h*(-c*x+1)*ln
(-c*x+1)^2/c-h*ln(c*x)*ln(-c*x+1)^2/c-h*x*polylog(2,c*x)-h*ln(-c*x+1)*poly
log(2,c*x)/c+x*(g+h*ln(-c*x+1))*polylog(2,c*x)-2*h*ln(-c*x+1)*polylog(2,-c
*x+1)/c+2*h*polylog(3,-c*x+1)/c
```

3.172.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int (g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx = g\left(-x+\left(-\frac{1}{c}+x\right)\log(1-cx)+x\,\text{PolyLog}(2,cx)\right) + \frac{h\left(-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)\right.}{}$$

input
```
Integrate[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
g*(-x + (-c^(-1) + x)*Log[1 - c*x] + x*PolyLog[2, c*x]) + (h*(-2 + 3*c*x +
 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*Log[1 - c*x]^2
 - Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2,
 c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

3.172.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {7154, 25, 2811, 2807, 2009, 7239, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,cx)(h\log(1-cx)+g)\,dx$$

$$\Big\downarrow \text{ 7154}$$

$$\int \log(1-cx)(g+h\log(1-cx))dx + ch\int -\left(\left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)\right)dx + x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{ 25}$$

$$\int \log(1-cx)(g+h\log(1-cx))dx - ch\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx + x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow \text{ 2811}$$

$$-\frac{\int \log(1-cx)(g+h\log(1-cx))d(1-cx)}{c} - ch\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx + x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow 2807$$

$$-\frac{\int \left(h\log^2(1-cx)+g\log(1-cx)\right)d(1-cx)}{c}-ch\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx+x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)$$

$$\Big\downarrow 2009$$

$$-ch\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\text{PolyLog}(2,cx)dx+x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)-\frac{-g(1-cx)+g(1-cx)\log(1-cx)+2h(1-cx)+h(1-cx)\log^2(1-cx)-2h(1-cx)\log(1-cx)}{c}$$

$$\Big\downarrow 7239$$

$$-ch\int \frac{x\,\text{PolyLog}(2,cx)}{cx-1}dx+x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)-\frac{-g(1-cx)+g(1-cx)\log(1-cx)+2h(1-cx)+h(1-cx)\log^2(1-cx)-2h(1-cx)\log(1-cx)}{c}$$

$$\Big\downarrow 7293$$

$$-ch\int \left(\frac{\text{PolyLog}(2,cx)}{c}+\frac{\text{PolyLog}(2,cx)}{c(cx-1)}\right)dx+x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)-\frac{-g(1-cx)+g(1-cx)\log(1-cx)+2h(1-cx)+h(1-cx)\log^2(1-cx)-2h(1-cx)\log(1-cx)}{c}$$

$$\Big\downarrow 2009$$

$$-ch\left(-\frac{2\,\text{PolyLog}(3,1-cx)}{c^2}+\frac{\text{PolyLog}(2,cx)\log(1-cx)}{c^2}+\frac{2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{c^2}+\frac{\log(cx)\log^2(1-\right.$$
$$x\,\text{PolyLog}(2,cx)(h\log(1-cx)+g)-\frac{-g(1-cx)+g(1-cx)\log(1-cx)+2h(1-cx)+h(1-cx)\log^2(1-cx)-2h(1-cx)\log(1-cx)}{c}$$

input
```
Int[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

output
```
-((-(g*(1 - c*x)) + 2*h*(1 - c*x) + g*(1 - c*x)*Log[1 - c*x] - 2*h*(1 - c*
x)*Log[1 - c*x] + h*(1 - c*x)*Log[1 - c*x]^2)/c) + x*(g + h*Log[1 - c*x])*
PolyLog[2, c*x] - c*h*(-(x/c) - ((1 - c*x)*Log[1 - c*x])/c^2 + (Log[c*x]*L
og[1 - c*x]^2)/c^2 + (x*PolyLog[2, c*x])/c + (Log[1 - c*x]*PolyLog[2, c*x]
)/c^2 + (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 - (2*PolyLog[3, 1 - c*x])
/c^2)
```

3.172. $\int (g+h\log(1-cx))\,\text{PolyLog}(2,cx)\,dx$

3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2807
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

rule 2811
```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.), x_Symbol]
:> Simp[1/Coeff[v, x, 1]   Subst[Int[(a + b*Log[x])^p*(c + d*Log[x])^q, x],
 x, v], x] /; FreeQ[{a, b, c, d, p, q}, x] && LinearQ[v, x] && NeQ[Coeff[v,
 x, 0], 0]
```

rule 7154
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Simp[b   Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a
*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Simp[e*h*n   Int[Poly
Log[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b
, c, d, e, f, g, h, n}, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.172.4 Maple [F]

$$\int (g + h\ln(-cx+1))\,\mathrm{polylog}(2, cx)\,dx$$

input
```
int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

3.172.5 Fricas [F]

$$\int (g + h\log(1-cx))\,\mathrm{PolyLog}(2, cx)\,dx = \int (h\log(-cx+1) + g)\mathrm{Li}_2(cx)\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral(h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x), x)
```

3.172.6 Sympy [F]

$$\int (g + h\log(1-cx))\,\mathrm{PolyLog}(2, cx)\,dx = \int (g + h\log(-cx+1))\,\mathrm{Li}_2(cx)\,dx$$

input
```
integrate((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

output
```
Integral((g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

3.172.7 Maxima [F]

$$\int (g + h \log(1 - cx)) \operatorname{PolyLog}(2, cx) \, dx = \int (h \log(-cx + 1) + g) \operatorname{Li}_2(cx) \, dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-h*((c*x - (c*x - 1)*log(-c*x + 1))*dilog(c*x)/c - integrate(-(c*x*log(-c*
x + 1) - (c*x - 1)*log(-c*x + 1)^2)/x, x)/c) + (c*x*dilog(c*x) - c*x + (c*
x - 1)*log(-c*x + 1))*g/c
```

3.172.8 Giac [F]

$$\int (g + h \log(1 - cx)) \operatorname{PolyLog}(2, cx) \, dx = \int (h \log(-cx + 1) + g) \operatorname{Li}_2(cx) \, dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int (g + h \log(1 - cx)) \operatorname{PolyLog}(2, cx) \, dx = \int (g + h \ln(1 - c x)) \operatorname{polylog}(2, c x) \, dx$$

input
```
int((g + h*log(1 - c*x))*polylog(2, c*x),x)
```

output
```
int((g + h*log(1 - c*x))*polylog(2, c*x), x)
```

3.173 $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx$

3.173.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=-\frac{1}{2}h\,\text{PolyLog}(2,cx)^2+g\,\text{PolyLog}(3,cx)$$

output
```
-1/2*h*polylog(2,c*x)^2+g*polylog(3,c*x)
```

3.173.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=-\frac{1}{2}h\,\text{PolyLog}(2,cx)^2+g\,\text{PolyLog}(3,cx)$$

input
```
Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
```

output
```
-1/2*(h*PolyLog[2, c*x]^2) + g*PolyLog[3, c*x]
```

3.173.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {7156, 7143, 7155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}\,dx$$

$$\Big\downarrow \text{7156}$$

$$g\int \frac{\text{PolyLog}(2,cx)}{x}dx+h\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}dx$$

$$\Big\downarrow \text{7143}$$

$$h\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x}dx+g\,\text{PolyLog}(3,cx)$$

$$\Big\downarrow \text{7155}$$

$$g\,\text{PolyLog}(3,cx)-\frac{1}{2}h\,\text{PolyLog}(2,cx)^2$$

input
```
Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
```

output
```
-1/2*(h*PolyLog[2, c*x]^2) + g*PolyLog[3, c*x]
```

3.173.3.1 Defintions of rubi rules used

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

rule 7156
```
Int[((Log[1 + (e_.)*(x_)]*(h_.) + (g_))*PolyLog[2, (c_.)*(x_)])/(x_), x_Sym
bol] :> Simp[g   Int[PolyLog[2, c*x]/x, x], x] + Simp[h   Int[(Log[1 + e*x]
*PolyLog[2, c*x])/x, x], x] /; FreeQ[{c, e, g, h}, x] && EqQ[c + e, 0]
```

3.173.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{h\,\text{polylog}(2,cx)^2}{2}+g\,\text{polylog}\left(3,cx\right)$	19
parts	$-\frac{h\,\text{polylog}(2,cx)^2}{2}+g\,\text{polylog}\left(3,cx\right)$	19

input
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*h*polylog(2,c*x)^2+g*polylog(3,c*x)
```

3.173.5 Fricas [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=\int \frac{(h\log\left(-cx+1\right)+g)\text{Li}_2(cx)}{x}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="fricas")
```

output
```
integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x, x)
```

3.173.6 Sympy [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=\int \frac{(g+h\log\left(-cx+1\right))\,\text{Li}_2\left(cx\right)}{x}\,dx$$

input
```
integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)
```

output
```
Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x, x)
```

3.173.7 Maxima [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=\int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="maxima")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

3.173.8 Giac [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=\int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}\,dx=g\,\text{polylog}(3,c\,x)-\frac{h\,\text{polylog}(2,c\,x)^2}{2}$$

input
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x,x)
```

output
```
g*polylog(3, c*x) - (h*polylog(2, c*x)^2)/2
```

3.174 $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx$

3.174.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$\begin{aligned}
&\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx \\
&= ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-cx))}{x} \\
&\quad + c(g+2h\log(1-cx))\log\left(1-\frac{1}{1-cx}\right)+ch\log(1-cx)\,\text{PolyLog}(2,cx) \\
&\quad - \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x}-2ch\,\text{PolyLog}\left(2,\frac{1}{1-cx}\right) \\
&\quad + 2ch\log(1-cx)\,\text{PolyLog}(2,1-cx)-ch\,\text{PolyLog}(3,cx)-2ch\,\text{PolyLog}(3,1-cx)
\end{aligned}$$

output
```
c*h*ln(c*x)*ln(-c*x+1)^2+ln(-c*x+1)*(g+h*ln(-c*x+1))/x+c*(g+2*h*ln(-c*x+1)
)*ln(1-1/(-c*x+1))+c*h*ln(-c*x+1)*polylog(2,c*x)-(g+h*ln(-c*x+1))*polylog(
2,c*x)/x-2*c*h*polylog(2,1/(-c*x+1))+2*c*h*ln(-c*x+1)*polylog(2,-c*x+1)-c*
h*polylog(3,c*x)-2*c*h*polylog(3,-c*x+1)
```

3.174.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx$$
$$=\frac{g(cx\log(x)+(1-cx)\log(1-cx)-\text{PolyLog}(2,cx))}{x}+h\left(2c\log(cx)\log(1-cx)-c\log^2(1-cx)+\frac{\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)+\frac{(-1+cx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x}+2c(1+\log(1-cx))\,\text{PolyLog}(2,1-cx)-c\,\text{PolyLog}(3,cx)-2c\,\text{PolyLog}(3,1-cx)\right)$$

input
```
Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

output
```
(g*(c*x*Log[x] + (1 - c*x)*Log[1 - c*x] - PolyLog[2, c*x]))/x + h*(2*c*Log
[c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x]*Log[
1 - c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + Log[1
- c*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c*x])
```

3.174.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {7157, 2009, 2883, 2858, 27, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x^2}\,dx$$
$$\Big\downarrow \text{7157}$$
$$-\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^2}dx-ch\int \left(\frac{\text{PolyLog}(2,cx)}{x}+\frac{c\,\text{PolyLog}(2,cx)}{1-cx}\right)dx-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}$$
$$\Big\downarrow \text{2009}$$

$$-\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^2}dx-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

$\downarrow$ 2883

$$c\int \frac{g+2h\log(1-cx)}{x(1-cx)}dx-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}+\frac{\log(1-cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

$\downarrow$ 2858

$$-\int \frac{g+2h\log(1-cx)}{x(1-cx)}d(1-cx)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}+$$
$$\frac{\log(1-cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

$\downarrow$ 27

$$-c\int \frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}+$$
$$\frac{\log(1-cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

$\downarrow$ 2779

$$-c\left(2h\int \frac{\log\left(1-\frac{1}{1-cx}\right)}{1-cx}d(1-cx)-\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)\right)-$$
$$\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}+\frac{\log(1-cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

$\downarrow$ 2838

$$-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}-$$
$$c\left(2h\,\text{PolyLog}\left(2,\frac{1}{1-cx}\right)-\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)\right)+$$
$$\frac{\log(1-cx)(h\log(1-cx)+g)}{x}-$$
$$ch\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-cx)\log(1-cx)-\log(c$$

input
```
Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

3.174. $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx$

output
```
(Log[1 - c*x]*(g + h*Log[1 - c*x]))/x - ((g + h*Log[1 - c*x])*PolyLog[2, c
*x])/x - c*(-((g + 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)]) + 2*h*PolyLo
g[2, (1 - c*x)^(-1)]) - c*h*(-(Log[c*x]*Log[1 - c*x]^2) - Log[1 - c*x]*Pol
yLog[2, c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + PolyLog[3, c*x] + 2*Po
lyLog[3, 1 - c*x])
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

3.174. $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx$

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.174.4 Maple [F]

$$\int \frac{(g + h\ln(-cx+1))\,\text{polylog}(2, cx)}{x^2}dx$$

input
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
```

output
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
```

3.174.5 Fricas [F]

$$\int \frac{(g + h\log(1-cx))\,\text{PolyLog}(2, cx)}{x^2}\,dx = \int \frac{(h\log(-cx+1) + g)\text{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

output
```
integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^2, x)
```

3.174.6 Sympy [F]

$$\int \frac{(g + h\log(1-cx))\,\text{PolyLog}(2, cx)}{x^2}\,dx = \int \frac{(g + h\log(-cx+1))\,\text{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**2,x)
```

output
```
Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**2, x)
```

3.174.7 Maxima [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

output
```
(c*log(x) - ((c*x - 1)*log(-c*x + 1) + dilog(c*x))/x)*g + h*integrate(dilo
g(c*x)*log(-c*x + 1)/x^2, x)
```

3.174.8 Giac [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^2, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(g+h\,\ln(1-c\,x))\,\text{polylog}(2,c\,x)}{x^2}\,dx$$

input
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^2,x)
```

output
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^2, x)
```

3.175 $\int \frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^3}\,dx$

3.175.1 Optimal result

Integrand size = 20, antiderivative size = 266

$$\begin{aligned}\int \frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^3}\,dx &= -c^2h\log(x)+\frac{1}{2}c^2h\log(1-cx)-\frac{ch\log(1-cx)}{2x}+\frac{1}{2}c^2h\log(cx)\log^2(1-cx)\\ &\quad+\frac{\log(1-cx)(g+h\log(1-cx))}{4x^2}-\frac{c(1-cx)(g+2h\log(1-cx))}{4x}\\ &\quad+\frac{1}{4}c^2(g+2h\log(1-cx))\log\left(1-\frac{1}{1-cx}\right)+\frac{ch\operatorname{PolyLog}(2,cx)}{2x}\\ &\quad+\frac{1}{2}c^2h\log(1-cx)\operatorname{PolyLog}(2,cx)-\frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{2x^2}\\ &\quad-\frac{1}{2}c^2h\operatorname{PolyLog}\left(2,\frac{1}{1-cx}\right)+c^2h\log(1-cx)\operatorname{PolyLog}(2,1-cx)\\ &\quad-\frac{1}{2}c^2h\operatorname{PolyLog}(3,cx)-c^2h\operatorname{PolyLog}(3,1-cx)\end{aligned}$$

output
```
-c^2*h*ln(x)+1/2*c^2*h*ln(-c*x+1)-1/2*c*h*ln(-c*x+1)/x+1/2*c^2*h*ln(c*x)*l
n(-c*x+1)^2+1/4*ln(-c*x+1)*(g+h*ln(-c*x+1))/x^2-1/4*c*(-c*x+1)*(g+2*h*ln(-
c*x+1))/x+1/4*c^2*(g+2*h*ln(-c*x+1))*ln(1-1/(-c*x+1))+1/2*c*h*polylog(2,c*
x)/x+1/2*c^2*h*ln(-c*x+1)*polylog(2,c*x)-1/2*(g+h*ln(-c*x+1))*polylog(2,c*
x)/x^2-1/2*c^2*h*polylog(2,1/(-c*x+1))+c^2*h*ln(-c*x+1)*polylog(2,-c*x+1)-
1/2*c^2*h*polylog(3,c*x)-c^2*h*polylog(3,-c*x+1)
```

3.175.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.89

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx = \frac{g(-cx+c^2x^2\log(x)+\log(1-cx)-c^2x^2\log(1-cx)-2\,\text{PolyLog}(2,cx))}{4x^2} + \frac{1}{4}h\Bigg(-2c^2\log(x)-2c^2\log(cx)+4c^2\log(1-cx)-\frac{4c\log(1-cx)}{x} + 2c^2\log(cx)\log(1-cx)-c^2\log^2(1-cx)+\frac{\log^2(1-cx)}{x^2}+2c^2\log(cx)\log^2(1-cx) + \frac{2(cx+(-1+c^2x^2)\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2} + 2c^2(1+2\log(1-cx))\,\text{PolyLog}(2,1-cx)-2c^2\,\text{PolyLog}(3,cx)-4c^2\,\text{PolyLog}(3,1-cx)\Bigg)$$

input
```
Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

output
```
(g*(-(c*x) + c^2*x^2*Log[x] + Log[1 - c*x] - c^2*x^2*Log[1 - c*x] - 2*Poly
Log[2, c*x]))/(4*x^2) + (h*(-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 -
 c*x] - (4*c*Log[1 - c*x])/x + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c
*x]^2 + Log[1 - c*x]^2/x^2 + 2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1
 + c^2*x^2)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x]
)*PolyLog[2, 1 - c*x] - 2*c^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x])
)/4
```

3.175.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {7157, 2009, 2883, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x^3}\,dx$$

$$\Big\downarrow \text{ 7157}$$

$$-\frac{1}{2}ch\int\left(\frac{\text{PolyLog}(2,cx)c^2}{1-cx}+\frac{\text{PolyLog}(2,cx)c}{x}+\frac{\text{PolyLog}(2,cx)}{x^2}\right)dx-\frac{1}{2}\int\frac{\log(1-cx)(g+h\log(1-cx))}{x^3}dx-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\int\frac{\log(1-cx)(g+h\log(1-cx))}{x^3}dx-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \text{2883}$$

$$\frac{1}{2}\left(\frac{1}{2}c\int\frac{g+2h\log(1-cx)}{x^2(1-cx)}dx+\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}\right)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}\int\frac{g+2h\log(1-cx)}{x^2(1-cx)}d(1-cx)\right)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\int\frac{g+2h\log(1-cx)}{c^2x^2(1-cx)}d(1-cx)\right)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\left(\int\frac{g+2h\log(1-cx)}{c^2x^2}d(1-cx)+\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)\right)\right)-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \text{2751}$$

3.175. $\int\frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}dx$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\left(\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)-2h\int\frac{1}{cx}d(1-cx)+\frac{(1-cx)(2h\log(\right.\right.$$
$$\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-$$
$$\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\left(\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)+\frac{(1-cx)(2h\log(1-cx)+g)}{cx}+2h\log\right.\right.$$
$$\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-$$
$$\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \; 2779$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\left(2h\int\frac{\log\left(1-\frac{1}{1-cx}\right)}{1-cx}d(1-cx)+\frac{(1-cx)(2h\log(1-cx)+g)}{cx}-\log\left(\right.\right.\right.$$
$$\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-$$
$$\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{2}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{2x^2}-\frac{1}{2}c^2\left(\frac{(1-cx)(2h\log(1-cx)+g)}{cx}-\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)\right.\right.$$
$$\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{2x^2}-$$
$$\frac{1}{2}ch\left(-\frac{\text{PolyLog}(2,cx)}{x}+c\,\text{PolyLog}(3,cx)+2c\,\text{PolyLog}(3,1-cx)-c\,\text{PolyLog}(2,cx)\log(1-cx)-2c\,\text{PolyLog}(2\right.$$

input
```
Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

3.175. $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx$

output
```
-1/2*((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + ((Log[1 - c*x]*(g + h*Lo
g[1 - c*x]))/(2*x^2) - (c^2*(2*h*Log[c*x] + ((1 - c*x)*(g + 2*h*Log[1 - c*
x]))/(c*x) - (g + 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)] + 2*h*PolyLog[
2, (1 - c*x)^(-1)]))/2)/2 - (c*h*(c*Log[x] - c*Log[1 - c*x] + Log[1 - c*x]
/x - c*Log[c*x]*Log[1 - c*x]^2 - PolyLog[2, c*x]/x - c*Log[1 - c*x]*PolyLo
g[2, c*x] - 2*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] + c*PolyLog[3, c*x] + 2*c
*PolyLog[3, 1 - c*x]))/2
```

3.175.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.175.4 Maple [F]

$$\int \frac{(g + h\ln(-cx + 1))\,\mathrm{polylog}(2, cx)}{x^3}dx$$

input
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
```

output
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
```

3.175.5 Fricas [F]

$$\int \frac{(g + h\log(1 - cx))\,\mathrm{PolyLog}(2, cx)}{x^3}\,dx = \int \frac{(h\log(-cx + 1) + g)\mathrm{Li}_2(cx)}{x^3}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

output
```
integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^3, x)
```

3.175.6 Sympy [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx=\int \frac{(g+h\log{(-cx+1)})\,\text{Li}_2\,(cx)}{x^3}\,dx$$

input
```
integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**3,x)
```

output
```
Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**3, x)
```

3.175.7 Maxima [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx=\int \frac{(h\log{(-cx+1)}+g)\text{Li}_2(cx)}{x^3}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

output
```
1/4*(c^2*log(x) - (c*x + (c^2*x^2 - 1)*log(-c*x + 1) + 2*dilog(c*x))/x^2)*
g + h*integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.175.8 Giac [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx=\int \frac{(h\log{(-cx+1)}+g)\text{Li}_2(cx)}{x^3}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^3, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{(g+h\,\ln(1-c\,x))\,\operatorname{polylog}(2,c\,x)}{x^3}\,dx$$

input
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^3,x)
```

output
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^3, x)
```

3.176 $\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx$

3.176.1 Optimal result

Integrand size = 20, antiderivative size = 340

$$\begin{aligned}
&\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx \\
&= \frac{7c^2h}{36x} - \frac{3}{4}c^3h\log(x) + \frac{19}{36}c^3h\log(1-cx) - \frac{ch\log(1-cx)}{12x^2} - \frac{c^2h\log(1-cx)}{3x} \\
&\quad + \frac{1}{3}c^3h\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx))}{9x^3} - \frac{c(g+2h\log(1-cx))}{18x^2} \\
&\quad - \frac{c^2(1-cx)(g+2h\log(1-cx))}{9x} + \frac{1}{9}c^3(g+2h\log(1-cx))\log\left(1-\frac{1}{1-cx}\right) \\
&\quad + \frac{ch\,\text{PolyLog}(2,cx)}{6x^2} + \frac{c^2h\,\text{PolyLog}(2,cx)}{3x} + \frac{1}{3}c^3h\log(1-cx)\,\text{PolyLog}(2,cx) \\
&\quad - \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{3x^3} - \frac{2}{9}c^3h\,\text{PolyLog}\left(2,\frac{1}{1-cx}\right) \\
&\quad + \frac{2}{3}c^3h\log(1-cx)\,\text{PolyLog}(2,1-cx) - \frac{1}{3}c^3h\,\text{PolyLog}(3,cx) - \frac{2}{3}c^3h\,\text{PolyLog}(3,1-cx)
\end{aligned}$$

output
```
7/36*c^2*h/x-3/4*c^3*h*ln(x)+19/36*c^3*h*ln(-c*x+1)-1/12*c*h*ln(-c*x+1)/x^
2-1/3*c^2*h*ln(-c*x+1)/x+1/3*c^3*h*ln(c*x)*ln(-c*x+1)^2+1/9*ln(-c*x+1)*(g+
h*ln(-c*x+1))/x^3-1/18*c*(g+2*h*ln(-c*x+1))/x^2-1/9*c^2*(-c*x+1)*(g+2*h*ln
(-c*x+1))/x+1/9*c^3*(g+2*h*ln(-c*x+1))*ln(1-1/(-c*x+1))+1/6*c*h*polylog(2,
c*x)/x^2+1/3*c^2*h*polylog(2,c*x)/x+1/3*c^3*h*ln(-c*x+1)*polylog(2,c*x)-1/
3*(g+h*ln(-c*x+1))*polylog(2,c*x)/x^3-2/9*c^3*h*polylog(2,1/(-c*x+1))+2/3*
c^3*h*ln(-c*x+1)*polylog(2,-c*x+1)-1/3*c^3*h*polylog(3,c*x)-2/3*c^3*h*poly
log(3,-c*x+1)
```

3.176.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.89

$$\int \frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^4}\,dx$$
$$= -\frac{g(cx(1+2cx)-2c^3x^3\log(x)+2(-1+c^3x^3)\log(1-cx)+6\operatorname{PolyLog}(2,cx))}{18x^3}$$
$$+\frac{h\left(7c^2x^2-4c^3x^3-15c^3x^3\log(x)-12c^3x^3\log(cx)-7cx\log(1-cx)-20c^2x^2\log(1-cx)+27c^3x^3\log(\right.}{}$$

input
```
Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

output
```
-1/18*(g*(c*x*(1 + 2*c*x) - 2*c^3*x^3*Log[x] + 2*(-1 + c^3*x^3)*Log[1 - c*
x] + 6*PolyLog[2, c*x]))/x^3 + (h*(7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[
x] - 12*c^3*x^3*Log[c*x] - 7*c*x*Log[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] +
27*c^3*x^3*Log[1 - c*x] + 8*c^3*x^3*Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]
^2 - 4*c^3*x^3*Log[1 - c*x]^2 + 12*c^3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*
x*(1 + 2*c*x) + 2*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3
*(1 + 3*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 12*c^3*x^3*PolyLog[3, c*x] - 2
4*c^3*x^3*PolyLog[3, 1 - c*x]))/(36*x^3)
```

3.176.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {7157, 2009, 2883, 2858, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{x^4}\,dx$$

$$\Big\downarrow \text{ 7157}$$

$$-\frac{1}{3}ch\int \left(\frac{\operatorname{PolyLog}(2,cx)c^3}{1-cx}+\frac{\operatorname{PolyLog}(2,cx)c^2}{x}+\frac{\operatorname{PolyLog}(2,cx)c}{x^2}+\frac{\operatorname{PolyLog}(2,cx)}{x^3}\right)dx-\frac{1}{3}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^4}dx-\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^4}dx-$$
$$\frac{1}{3}ch\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow 2883$$

$$\frac{1}{3}\left(\frac{1}{3}c\int \frac{g+2h\log(1-cx)}{x^3(1-cx)}dx+\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}\right)-$$
$$\frac{1}{3}ch\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow 2858$$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}\int \frac{g+2h\log(1-cx)}{x^3(1-cx)}d(1-cx)\right)-$$
$$\frac{1}{3}ch\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\int \frac{g+2h\log(1-cx)}{c^3x^3(1-cx)}d(1-cx)\right)-$$
$$\frac{1}{3}ch\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow 2789$$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int \frac{g+2h\log(1-cx)}{c^3x^3}d(1-cx)+\int \frac{g+2h\log(1-cx)}{c^2x^2(1-cx)}d(1-cx)\right)\right)-$$
$$\frac{1}{3}ch\Big(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow 2756$$

3.176. $\int \frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^4}\,dx$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{c^2x^2(1-cx)}d(1-cx)-h\int\frac{1}{c^2x^2(1-cx)}d(1-cx)+\frac{2h\log(\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$\downarrow$ 54

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{c^2x^2(1-cx)}d(1-cx)-h\int\left(\frac{1}{1-cx}+\frac{1}{cx}+\frac{1}{c^2x^2}\right)d(1-cx\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{c^2x^2(1-cx)}d(1-cx)+\frac{2h\log(1-cx)+g}{2c^2x^2}-h\left(\frac{1}{cx}-\log(cx)\right.\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$\downarrow$ 2789

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{c^2x^2}d(1-cx)+\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)+\frac{2h}{}\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$\downarrow$ 2751

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)-2h\int\frac{1}{cx}d(1-cx)+\frac{2h\log(1-cx)+}{2c^2x^2}\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$\downarrow$ 16

3.176. $\int\frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^4}\,dx$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\int\frac{g+2h\log(1-cx)}{cx(1-cx)}d(1-cx)+\frac{2h\log(1-cx)+g}{2c^2x^2}+\frac{(1-cx)(2h\log\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow \text{2779}$$

$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(2h\int\frac{\log\left(1-\frac{1}{1-cx}\right)}{1-cx}d(1-cx)+\frac{2h\log(1-cx)+g}{2c^2x^2}+\frac{(1-cx)(2h\log\right.\right.$$
$$\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{3}ch\left(c^2\operatorname{PolyLog}(3,cx)+2c^2\operatorname{PolyLog}(3,1-cx)-c^2\operatorname{PolyLog}(2,cx)\log(1-cx)-2c^2\operatorname{PolyLog}(2,1-cx)\log(1\right.$$
$$\frac{1}{3}\left(\frac{\log(1-cx)(h\log(1-cx)+g)}{3x^3}-\frac{1}{3}c^3\left(\frac{2h\log(1-cx)+g}{2c^2x^2}+\frac{(1-cx)(2h\log(1-cx)+g)}{cx}-\log\left(1-\frac{1}{1-cx}\right)\right.\right.$$
$$\frac{\operatorname{PolyLog}(2,cx)(h\log(1-cx)+g)}{3x^3}$$

input
```
Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

output
```
-1/3*((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3 + ((Log[1 - c*x]*(g + h*Lo
g[1 - c*x]))/(3*x^3) - (c^3*(2*h*Log[c*x] - h*(1/(c*x) - Log[c*x] + Log[1
- c*x]) + (g + 2*h*Log[1 - c*x])/(2*c^2*x^2) + ((1 - c*x)*(g + 2*h*Log[1 -
 c*x]))/(c*x) - (g + 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)] + 2*h*PolyL
og[2, (1 - c*x)^(-1)]))/3)/3 - (c*h*(-1/4*c/x + (5*c^2*Log[x])/4 - (5*c^2*
Log[1 - c*x])/4 + Log[1 - c*x]/(4*x^2) + (c*Log[1 - c*x])/x - c^2*Log[c*x]
*Log[1 - c*x]^2 - PolyLog[2, c*x]/(2*x^2) - (c*PolyLog[2, c*x])/x - c^2*Lo
g[1 - c*x]*PolyLog[2, c*x] - 2*c^2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + c^2*
PolyLog[3, c*x] + 2*c^2*PolyLog[3, 1 - c*x]))/3
```

3.176. $\int \frac{(g+h\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^4}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

3.176. $\int \frac{(g+h\log(1-cx))\,\mathrm{PolyLog}(2,cx)}{x^4}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2883
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Simp[
e*(n/(m + 1))   Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
 e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.176.4 Maple [F]

$$\int \frac{(g + h\ln(-cx+1))\,\mathrm{polylog}(2, cx)}{x^4} dx$$

input
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

output
```
int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

3.176.5 Fricas [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

output
```
integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^4, x)
```

3.176.6 Sympy [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(g+h\log(-cx+1))\,\text{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**4,x)
```

output
```
Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**4, x)
```

3.176.7 Maxima [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

output
```
1/18*(2*c^3*log(x) - (2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1) + 6*
dilog(c*x))/x^3)*g + h*integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.176.8 Giac [F]

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="giac")
```

output
```
integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^4, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(1-cx))\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(g+h\,\ln(1-c\,x))\,\,\text{polylog}(2,c\,x)}{x^4}\,dx$$

input
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^4,x)
```

output
```
int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^4, x)
```

3.177 $\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx$

3.177.1 Optimal result

Integrand size = 27, antiderivative size = 2995

$$\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx = \text{Too large to display}$$

output
```
-1/3*a^2*g*x/b^2+1/3*a^3*g*polylog(2,c*(b*x+a))/b^3-1/9*h*n*x^3*polylog(2,
c*(b*x+a))-2/27*h*n*x^3*ln(-b*c*x-a*c+1)+1/12*a*x^2*(g+h*ln(f*(e*x+d)^n))/
b-1/27*x^3*(g+h*ln(f*(e*x+d)^n))+1/3*a*d*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+
1)/b^2/c/e+2/27*(-a*c+1)^3*h*n*ln(-b*c*x-a*c+1)/b^3/c^3+5/36*a*h*n*x^2*ln(
-b*c*x-a*c+1)/b+5/36*d*h*n*x^2*ln(-b*c*x-a*c+1)/e+1/9*d^3*h*n*ln(-b*c*x-a*
c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e^3-1/3*a^2*h*(e*x+d)*ln(f*(e*x+d)^n
)/b^2/e-1/3*a^2*(-a*c+1)*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+h*ln(f*(
e*x+d)^n))/b^3/c+1/6*a*(-a*c+1)^2*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g
+h*ln(f*(e*x+d)^n))/b^3/c^2-1/6*a^3*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)
/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b
*d)/(1-c*(b*x+a)))^2/b^3+1/6*d^3*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/
c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)
/(1-c*(b*x+a)))^2/e^3+1/6*a^3*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d))
)*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/b^3-1/6*d^3*h
*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c
*(b*x+a)))+ln(1-c*(b*x+a)))^2/e^3+1/3*d^3*h*n*ln(e*x+d)*polylog(2,c*(b*x+a
))/e^3-1/3*a^3*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a))
)*polylog(2,b*(e*x+d)/(-a*e+b*d))/b^3+1/3*d^3*h*n*(ln(b*(e*x+d)/(-a*e+b*d)
/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/e^3-1/3*a
^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c...
```

3.177.2 Mathematica [A] (verified)

Time = 9.50 (sec) , antiderivative size = 2610, normalized size of antiderivative = 0.87

$$\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx=\text{Result too large to show}$$

input
```
Integrate[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

output
```
((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(12 + 66*a^2*c^2 +
 6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x))) + 6*(-2 + 11*a^3*c^3 + 2
*b^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c - 3*b^2*c^3*x^2))*Log[1 - a
*c - b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2, c*(a + b*x)]))/(108*b^3*c^
3) + (h*n*(36*b^3*c^3*(e*x*(-6*d^2 + 3*d*e*x - 2*e^2*x^2) + 6*(d^3 + e^3*x
^3)*Log[d + e*x])*PolyLog[2, c*(a + b*x)] - 216*b^2*c^2*d^2*e*(1 - a*c - b
*c*x + (-1 + a*c + b*c*x - a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - a*
c*PolyLog[2, 1 - a*c - b*c*x]) - 27*b*c*d*e^2*(c*(-4*a^2*c + a*(4 - 6*b*c*
x) + b*x*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2*c^2*x^2 + 4*a*c*(-2 + b*c*x
) - 4*a^2*c^2*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2
, 1 - a*c - b*c*x]) - 2*e^3*(-(c*(36*a^3*c^2 - 3*a*b*c*x*(14 + 5*b*c*x) +
6*a^2*c*(-6 + 11*b*c*x) + 2*b*x*(6 + 3*b*c*x + 2*b^2*c^2*x^2))) - 6*(2 - 1
1*a^3*c^3 - 2*b^3*c^3*x^3 - 6*a^2*c^2*(-3 + b*c*x) + 3*a*c*(-3 + b^2*c^2*x
^2) + 6*a^3*c^3*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 36*a^3*c^3*PolyLo
g[2, 1 - a*c - b*c*x]) + 216*b^3*c^3*d^3*(Log[c*(a + b*x)]*Log[1 - a*c - b
*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)
])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e
*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a
*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1
+ a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x...
```

3.177.3 Rubi [A] (verified)

Time = 5.42 (sec) , antiderivative size = 3373, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7157, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\,\text{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)\,dx$$

$\big\downarrow$ 7157

$$\frac{1}{3}b\int\left(-\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)a^3}{b^3(a+bx)}+\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)a^2}{b^3}-\frac{x\log$$
$$\frac{1}{3}ehn\int\left(-\frac{\text{PolyLog}(2,c(a+bx))d^3}{e^3(d+ex)}+\frac{\text{PolyLog}(2,c(a+bx))d^2}{e^3}-\frac{x\,\text{PolyLog}(2,c(a+bx))d}{e^2}+\frac{x^2\,\text{PolyLog}(2,c(a}{e}$$
$$\frac{1}{3}x^3\,\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))x^3-$$
$$\frac{1}{3}ehn\left(\frac{\text{PolyLog}(2,c(a+bx))a^3}{3b^3e}-\frac{xa^2}{3b^2e}-\frac{(-ac-bxc+1)\log(-ac-bxc+1)a^2}{3b^3ce}+\frac{d\,\text{PolyLog}(2,c(a+bx))a^2}{2b^2e^2}+\right.$$
$$\frac{1}{3}b\left(-\frac{hn\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)a^3}{2b^4}+\frac{hn\left(\log(c(a\right.}{}\right.$$

input
```
Int[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

output
```
(x^3*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/3 - (e*h*n*(-((d^
2*x)/e^3) - (a*d*x)/(2*b*e^2) + ((1 - a*c)*d*x)/(4*b*c*e^2) - (a^2*x)/(3*b
^2*e) + (a*(1 - a*c)*x)/(6*b^2*c*e) - ((1 - a*c)^2*x)/(9*b^2*c^2*e) + (d*x
^2)/(8*e^2) + (a*x^2)/(12*b*e) - ((1 - a*c)*x^2)/(18*b*c*e) - x^3/(27*e) +
 ((1 - a*c)^2*d*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e^2) + (a*(1 - a*c)^2*Log
[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(9*b
^3*c^3*e) - (d*x^2*Log[1 - a*c - b*c*x])/(4*e^2) - (a*x^2*Log[1 - a*c - b*
c*x])/(6*b*e) + (x^3*Log[1 - a*c - b*c*x])/(9*e) - (d^2*(1 - a*c - b*c*x)*
Log[1 - a*c - b*c*x])/(b*c*e^3) - (a*d*(1 - a*c - b*c*x)*Log[1 - a*c - b*c
*x])/(2*b^2*c*e^2) - (a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c
*e) - (d^3*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] -
Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*
d - a*e)*(1 - c*(a + b*x)))]^2)/(2*e^4) - (d^3*Log[c*(a + b*x)]*Log[d + e*
x]*Log[1 - c*(a + b*x)])/e^4 + (d^3*(Log[c*(a + b*x)] - Log[-((e*(a + b*x)
)/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log
[1 - c*(a + b*x)])^2)/(2*e^4) + (a*d^2*PolyLog[2, c*(a + b*x)])/(b*e^3) +
(a^2*d*PolyLog[2, c*(a + b*x)])/(2*b^2*e^2) + (a^3*PolyLog[2, c*(a + b*x)]
)/(3*b^3*e) + (d^2*x*PolyLog[2, c*(a + b*x)])/e^3 - (d*x^2*PolyLog[2, c*(a
 + b*x)])/(2*e^2) + (x^3*PolyLog[2, c*(a + b*x)])/(3*e) - (d^3*Log[d + e*x
]*PolyLog[2, c*(a + b*x)])/e^4 - (d^3*(Log[(b*(d + e*x))/((b*d - a*e)*(...
```

3.177. $\int x^2\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))\,dx$

3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.177.4 Maple [F]

$$\int x^2(g+h\ln(f(ex+d)^n))\operatorname{polylog}(2,c(bx+a))\,dx$$

input
```
int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

output
```
int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

3.177.5 Fricas [F]

$$\int x^2(g+h\log(f(d+ex)^n))\operatorname{PolyLog}(2,c(a+bx))\,dx = \int (h\log((ex+d)^nf)+g)x^2\operatorname{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fr
icas")
```

output
```
integral(h*x^2*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*x^2*dilog(b*c*x +
 a*c), x)
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx=\text{Timed out}$$

input
```
integrate(x**2*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

output
```
Timed out
```

3.177.7 Maxima [F]

$$\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$=\int(h\log((ex+d)^nf)+g)x^2\text{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="ma
xima")
```

output
```
1/18*(6*e^3*h*x^3*log((e*x + d)^n) + 3*d*e^2*h*n*x^2 - 6*d^2*e*h*n*x + 6*d
^3*h*n*log(e*x + d) - 2*(e^3*h*n - 3*e^3*h*log(f) - 3*e^3*g)*x^3)*dilog(b*
c*x + a*c)/e^3 + integrate(1/18*(6*b*e^3*h*x^3*log(-b*c*x - a*c + 1)*log((
e*x + d)^n) + (3*b*d*e^2*h*n*x^2 - 6*b*d^2*e*h*n*x + 6*b*d^3*h*n*log(e*x +
 d) - 2*(b*e^3*h*n - 3*b*e^3*h*log(f) - 3*b*e^3*g)*x^3)*log(-b*c*x - a*c +
 1))/(b*e^3*x + a*e^3), x)
```

3.177.8 Giac [F]

$$\int x^2(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$=\int(h\log((ex+d)^nf)+g)x^2\text{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="gi
ac")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*x^2*dilog((b*x + a)*c), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int x^2(g+h\log(f(d+ex)^n))\operatorname{PolyLog}(2,c(a+bx))\,dx$$
$$=\int x^2\operatorname{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)\,dx$$

input
```
int(x^2*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
```

output
```
int(x^2*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.178 $\int x(g+h\log(f(d+ex)^n))\text{PolyLog}(2,c(a+bx))\,dx$

3.178.1 Optimal result

Integrand size = 25, antiderivative size = 2252

$$\int x(g+h\log(f(d+ex)^n))\text{PolyLog}(2,c(a+bx))\,dx=\text{Too large to display}$$

output
```
-1/4*h*n*x^2*ln(-b*c*x-a*c+1)+1/2*a*g*x/b-1/2*a^2*g*polylog(2,c*(b*x+a))/b
^2-1/4*h*n*x^2*polylog(2,c*(b*x+a))-1/4*d^2*h*n*polylog(2,e*(-b*c*x-a*c+1)
/(-a*c*e+b*c*d+e))/e^2-1/2*a^2*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/b^2+1/2
*d^2*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/e^2-1/2*a^2*h*n*polylog(3,1-c*(b*
x+a))/b^2+1/2*d^2*h*n*polylog(3,1-c*(b*x+a))/e^2-1/2*a^2*h*n*polylog(3,-e*
(1-c*(b*x+a))/b/c/(e*x+d))/b^2+1/2*d^2*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/
(e*x+d))/e^2+1/2*a^2*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/b^2
-1/2*d^2*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e^2-1/8*x^2*(g+
h*ln(f*(e*x+d)^n))-7/8*d*h*n*x/e+1/4*a^2*h*n*polylog(2,c*(b*x+a))/b^2+1/2*
x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))+3/16*h*n*x^2+1/4*x^2*ln(-b*
c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))+1/8*d^2*h*n*ln(e*x+d)/e^2-3/4*a*h*n*(-b*c
*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c-1/4*(-a*c+1)*h*(e*x+d)*ln(f*(e*x+d)^n)/b/
c/e+1/2*a^2*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/b^2-1/2*d^2*h*n*ln
(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/e^2+1/2*a*d*h*n*polylog(2,c*(b*x+a))
/b/e-1/2*a*d*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/b/e+1/2*a*(-
a*c+1)*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b^2/c+1/4*d^2*h*n*(ln(c
*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)
))+ln(1-c*(b*x+a)))^2/e^2-1/2*d^2*h*n*ln(e*x+d)*polylog(2,c*(b*x+a))/e^2+1
/2*a^2*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylo
g(2,b*(e*x+d)/(-a*e+b*d))/b^2-1/4*(-a*c+1)^2*h*n*polylog(2,b*c*(e*x+d)/...
```

3.178.2 Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 1996, normalized size of antiderivative = 0.89

$$\int x(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))\,dx=\text{Too large to display}$$

input
```
Integrate[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

output
```
((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(2 - 6*a*c + b*c*x
)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2 + b*c*x))*Log[1 - a*c - b
*c*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]))/(8*b^2*c^2) + (h*n
*(4*b^2*c^2*(e*x*(2*d - e*x) - 2*(d^2 - e^2*x^2)*Log[d + e*x])*PolyLog[2,
c*(a + b*x)] + 8*b*c*d*e*(1 - a*c - b*c*x + (-1 + a*c + b*c*x - a*c*Log[c*
(a + b*x)])*Log[1 - a*c - b*c*x] - a*c*PolyLog[2, 1 - a*c - b*c*x]) + e^2*
(c*(-4*a^2*c + a*(4 - 6*b*c*x) + b*x*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2
*c^2*x^2 + 4*a*c*(-2 + b*c*x) - 4*a^2*c^2*Log[c*(a + b*x)])*Log[1 - a*c -
b*c*x] - 4*a^2*c^2*PolyLog[2, 1 - a*c - b*c*x]) - 8*b^2*c^2*d^2*(Log[c*(a
+ b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a
 + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c -
 b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e
*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d +
e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*
e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a
 + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e -
a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(
-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x]
+ Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d
 + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*...
```

3.178.3 Rubi [A] (verified)

Time = 3.77 (sec) , antiderivative size = 2358, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7157, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\,\mathrm{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)\,dx$$

$$\Big\downarrow \; 7157$$

$$\frac{1}{2}b\int\left(\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)a^2}{b^2(a+bx)}-\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)a}{b^2}+\frac{x\log(-\right.$$
$$\frac{1}{2}ehn\int\left(\frac{\operatorname{PolyLog}(2,c(a+bx))d^2}{e^2(d+ex)}-\frac{\operatorname{PolyLog}(2,c(a+bx))d}{e^2}+\frac{x\operatorname{PolyLog}(2,c(a+bx))}{e}\right)dx+$$
$$\frac{1}{2}x^2\operatorname{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(g+h\log\left(f(d+ex)^n\right)\right)\operatorname{PolyLog}(2,c(a+bx))x^2-$$
$$\frac{1}{2}ehn\left(-\frac{\operatorname{PolyLog}(2,c(a+bx))a^2}{2b^2e}+\frac{xa}{2be}+\frac{(-ac-bxc+1)\log(-ac-bxc+1)a}{2b^2ce}-\frac{d\operatorname{PolyLog}(2,c(a+bx))a}{be^2}-\frac{x}{8}\right.$$
$$\frac{1}{2}b\left(\frac{hn\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)a^2}{2b^3}-\frac{hn\left(\log(c(a+\right.}{}\right.$$

input
```
Int[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

output
```
(x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (e*h*n*((d*x)
/e^2 + (a*x)/(2*b*e) - ((1 - a*c)*x)/(4*b*c*e) - x^2/(8*e) - ((1 - a*c)^2*
Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) + (x^2*Log[1 - a*c - b*c*x])/(4*e) + (
d*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c*e^2) + (a*(1 - a*c - b*c*x)
*Log[1 - a*c - b*c*x])/(2*b^2*c*e) + (d^2*(Log[c*(a + b*x)] + Log[(b*c*d +
 e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d +
 e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*e^3) + (
d^2*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/e^3 - (d^2*(Log[c*
(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d -
 a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*e^3) - (a*d*PolyLo
g[2, c*(a + b*x)])/(b*e^2) - (a^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) - (d*
x*PolyLog[2, c*(a + b*x)])/e^2 + (x^2*PolyLog[2, c*(a + b*x)])/(2*e) + (d^
2*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e^3 + (d^2*(Log[(b*(d + e*x))/((b*
d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*
x))/(b*d - a*e)])/e^3 + (d^2*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e
)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e^3 - (d^2*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))
/(b*c*(d + e*x)))])/e^3 + (d^2*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a +
b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e^3 - (
d^2*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/e^3 - (d^2*PolyLog[3, 1 - c*...
```

3.178. $\int x(g+h\log\left(f(d+ex)^n\right))\operatorname{PolyLog}(2,c(a+bx))\,dx$

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.178.4 Maple [F]

$$\int x(g + h\ln(f(ex+d)^n))\,\mathrm{polylog}(2, c(bx+a))\,dx$$

input
```
int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

output
```
int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

3.178.5 Fricas [F]

$$\int x(g + h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2, c(a+bx))\,dx$$
$$= \int (h\log((ex+d)^n f) + g)x\mathrm{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fric
as")
```

output
```
integral(h*x*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*x*dilog(b*c*x + a*c
), x)
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int x(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))\,dx=\text{Timed out}$$

input
```
integrate(x*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

output
```
Timed out
```

3.178.7 Maxima [F]

$$\int x(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))\,dx = \int (h\log((ex+d)^n f)+g)x\mathrm{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxi
ma")
```

output
```
1/4*(2*e^2*h*x^2*log((e*x + d)^n) + 2*d*e*h*n*x - 2*d^2*h*n*log(e*x + d) -
 (e^2*h*n - 2*e^2*h*log(f) - 2*e^2*g)*x^2)*dilog(b*c*x + a*c)/e^2 + integr
ate(1/4*(2*b*e^2*h*x^2*log(-b*c*x - a*c + 1)*log((e*x + d)^n) + (2*b*d*e*h
*n*x - 2*b*d^2*h*n*log(e*x + d) - (b*e^2*h*n - 2*b*e^2*h*log(f) - 2*b*e^2*
g)*x^2)*log(-b*c*x - a*c + 1))/(b*e^2*x + a*e^2), x)
```

3.178.8 Giac [F]

$$\int x(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))\,dx = \int (h\log((ex+d)^n f)+g)x\mathrm{Li}_2((bx+a)c)\,dx$$

input
```
integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac
")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*x*dilog((b*x + a)*c), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int x(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx$$
$$=\int x\,\text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,\left(d+e\,x\right)^n\right)\right)\,dx$$

input
```
int(x*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
```

output
```
int(x*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.179 $\int (g + h\log(f(d + ex)^n))\,\text{PolyLog}(2, c(a+bx))\,dx$

3.179.1 Optimal result

Integrand size = 24, antiderivative size = 1653

$$\int (g + h\log(f(d + ex)^n))\,\text{PolyLog}(2, c(a + bx))\,dx = \text{Too large to display}$$

output

```
-h*(e*x+d)*ln(f*(e*x+d)^n)/e+d*h*n*ln(c*(b*x+a))*ln(-b*c*x-a*c+1)*ln(-e*x-
d)/e-a*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/b+3*h*n*x-d*h*n*ln(b*(e
*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1))*polylog(2,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/
e+d*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1))*polylog(2,(-a*e+b*d)*(-b*c
*x-a*c+1)/b/(e*x+d))/e+d*h*n*(ln(-b*c*x-a*c+1)+ln(b*(e*x+d)/(-a*e+b*d)/(-b
*c*x-a*c+1)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/e-a*h*n*(ln(b*(e*x+d)/(-a*e+
b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/b-g*x
+h*x*ln(-b*c*x-a*c+1)*ln(f*(e*x+d)^n)+a*g*polylog(2,c*(b*x+a))/b-h*n*x*pol
ylog(2,b*c*x+a*c)+2*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+1/2*d*h*n*(ln(
c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/
(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1))^2/e-1/2*d*h*n*(ln(c*(b*x
+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(-b*c*x-a*c+1)+ln(b*(e*x+d)/(-a*e+b*d)/
(-b*c*x-a*c+1)))^2/e-1/2*a*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x
+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*
(b*x+a)))^2/b+1/2*a*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e
*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/b-d*h*n*polylog(3,-b*c*
x-a*c+1)/e-d*h*n*polylog(3,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/e-d*h*n*polylog(
3,b*(e*x+d)/(-a*e+b*d))/e+d*h*n*polylog(3,(-a*e+b*d)*(-b*c*x-a*c+1)/b/(e*x
+d))/e+a*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/b+a*h*n*polylog(3,1-c*(b*x+a)
)/b+a*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/b-a*h*n*polylog(3,(-a...
```

3.179.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 1546, normalized size of antiderivative = 0.94

$$\int (g+h\log (f(d+ex)^n)) \operatorname{PolyLog}(2,c(a+bx))\,dx = \text{Too large to display}$$

input
```
Integrate[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

output
```
((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x) + (-1 + a*c + b*c
*x)*Log[1 - a*c - b*c*x] + c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (
h*n*((-(e*x) + (d + e*x)*Log[d + e*x])*PolyLog[2, c*(a + b*x)] + (-e + a*c
*e + 2*b*c*e*x - b*c*d*Log[d + e*x] - b*c*e*x*Log[d + e*x] + Log[1 - a*c -
 b*c*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b*c*x)*Log[d + e*x] + (b*
c*d + e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + e*(-1 + a*c +
 b*c*x + (1 - a*c - b*c*x + a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] + a
*c*PolyLog[2, 1 - a*c - b*c*x]) + (b*c*d + e - a*c*e)*PolyLog[2, (e*(-1 +
a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)] + b*c*d*(Log[c*(a + b*x)]*Log[1 -
 a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d
) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(
b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(
b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a
*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d
- a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x
)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*
x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d +
 e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d -
 a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLo...
```

3.179.3 Rubi [A] (verified)

Time = 4.01 (sec) , antiderivative size = 1697, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {7154, 7239, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{PolyLog}(2,c(a+bx)) (h\log (f(d+ex)^n)+g)\, dx$$

$$\Big\downarrow \text{7154}$$

$$b\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)dx-ehn\int \left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)\text{PolyLog}(2,c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$$\Big\downarrow \text{7239}$$

$$b\int \frac{x\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)}{a+bx}dx-ehn\int \left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)\text{PolyLog}(2,c(a+bx))dx+x\,\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$$\Big\downarrow \text{7292}$$

$$b\int \frac{x\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)}{a+bx}dx-ehn\int \frac{x\,\text{PolyLog}(2,ac+bxc)}{d+ex}dx+x\,\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$$\Big\downarrow \text{7293}$$

$$b\int \left(\frac{hx\log\left(f(d+ex)^n\right)\log(-ac-bxc+1)}{a+bx}+\frac{gx\log(-ac-bxc+1)}{a+bx}\right)dx-ehn\int \left(\frac{\text{PolyLog}(2,ac+bxc)}{e}+\frac{d\,\text{PolyLog}(2,ac+bxc)}{e(-d-ex)}\right)dx+x\,\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)$$

$$\Big\downarrow \text{2009}$$

$$x\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))-ehn\left(-\frac{d\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(-ac-bxc+1)}\right)}{2e^2}+\frac{d\,\text{PolyLog}\left(2,\ldots\right.}{\ldots}\right.$$

$$b\left(-\frac{ahn\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2b^2}+\frac{ahn\,\text{PolyLog}\left(2\ldots\right.}{\ldots}\right.$$

input
```
Int[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

3.179. $\int \left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))\,dx$

output
```
x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)] - e*h*n*(-(x/e) - ((1
 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c*e) - (d*Log[c*(a + b*x)]*Log[1
- a*c - b*c*x]*Log[-d - e*x])/e^2 - (d*(Log[c*(a + b*x)] + Log[(b*c*d + e
- a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*
x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e^2) + (d*(
Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - a*c - b*c*x
] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])^2)/(2*e^2) + (a*Po
lyLog[2, c*(a + b*x)])/(b*e) - (d*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d
 - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e^2 + (x*PolyLog
[2, a*c + b*c*x])/e - (d*Log[-d - e*x]*PolyLog[2, a*c + b*c*x])/e^2 + (d*L
og[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c
 - b*c*x))/(b*c*(d + e*x)))])/e^2 - (d*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
 a*c - b*c*x))]*PolyLog[2, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])
/e^2 - (d*(Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c
- b*c*x))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/e^2 + (d*PolyLog[3, 1 -
 a*c - b*c*x])/e^2 + (d*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))
)])/e^2 - (d*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e^
2 + (d*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/e^2) + b*(-((g*x)/b) + (2*h*
n*x)/b - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b^2*c) + (h*n*(1 - a*
c - b*c*x)*Log[1 - a*c - b*c*x])/(b^2*c) + (d*h*n*Log[1 - a*c - b*c*x]*...
```

3.179.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7154
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Simp[b   Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a
*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Simp[e*h*n   Int[Poly
Log[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b
, c, d, e, f, g, h, n}, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.179.4 Maple [F]

$$\int (g + h \ln (f (ex + d)^n)) \operatorname{polylog}(2, c(bx + a))\, dx$$

input
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

output
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

3.179.5 Fricas [F]

$$\int (g + h \log (f (d + ex)^n)) \operatorname{PolyLog}(2, c(a + bx))\, dx = \int (h \log ((ex + d)^n f) + g)\operatorname{Li}_2((bx + a)c)\, dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fricas
")
```

output
```
integral(h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c), x
)
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int (g + h \log (f (d + ex)^n)) \operatorname{PolyLog}(2, c(a + bx))\, dx = \text{Timed out}$$

input
```
integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

output
```
Timed out
```

3.179.7 Maxima [F]

$$\int (g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx = \int (h\log((ex+d)^n f)+g)\text{Li}_2((bx+a)c)\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxima
")
```

output
```
(d*h*n*log(e*x + d) + e*h*x*log((e*x + d)^n) - (e*h*n - e*h*log(f) - e*g)*
x)*dilog(b*c*x + a*c)/e + integrate((b*e*h*x*log(-b*c*x - a*c + 1)*log((e*
x + d)^n) + (b*d*h*n*log(e*x + d) - (b*e*h*n - b*e*h*log(f) - b*e*g)*x)*lo
g(-b*c*x - a*c + 1))/(b*e*x + a*e), x)
```

3.179.8 Giac [F]

$$\int (g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx = \int (h\log((ex+d)^n f)+g)\text{Li}_2((bx+a)c)\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int (g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))\,dx = \int \text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)\,dx$$

input
```
int(polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
```

output
```
int(polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.180 $\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx$

3.180.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx = \mathrm{Int}\left(\frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x},x\right)$$

output
```
Unintegrable((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

3.180.2 Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx$$

input
```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
```

output
```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

3.180.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7161}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2, c(a+bx)) (h \log (f(d+ex)^n)+g)}{x} \, dx$$

$$\Big\downarrow \text{ 7161}$$

$$\int \frac{\text{PolyLog}(2, c(a+bx)) (h \log (f(d+ex)^n)+g)}{x}dx$$

input
```
Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
```

output
```
$Aborted
```

3.180.3.1 Defintions of rubi rules used

rule 7161
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_.)*(x_)^(m_)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Unintegrable[Px*x^m*(g
 + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m, n}, x] && PolyQ[Px, x]
```

3.180.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(g+h \ln (f(ex+d)^n)) \operatorname{polylog}(2, c(bx+a))}{x}dx$$

input
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

output
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

3.180.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{(h\log((ex+d)^n f)+g)\text{Li}_2((bx+a)c)}{x}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="fric
as")
```

output
```
integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/
x, x)
```

3.180.6 Sympy [N/A]

Not integrable

Time = 126.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(ac+bcx)}{x}\,dx$$

input
```
integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x,x)
```

output
```
Integral((g + h*log(f*(d + e*x)**n))*polylog(2, a*c + b*c*x)/x, x)
```

3.180.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{(h\log((ex+d)^n f)+g)\mathrm{Li}_2((bx+a)c)}{x}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="maxi
ma")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

3.180.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{(h\log((ex+d)^n f)+g)\mathrm{Li}_2((bx+a)c)}{x}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="giac
")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

3.180.9 Mupad [N/A]

Not integrable

Time = 8.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x}\,dx = \int \frac{\text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)}{x}\,dx$$

input
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x,x)
```

output
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x, x)
```

3.181 $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$

3.181.1 Optimal result

Integrand size = 27, antiderivative size = 2498

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^2}\,dx = \text{Too large to display}$$

output
```
-b*h*n*polylog(3,1-c*(b*x+a))/a-b*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+
d))/a-e*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/d+e*h*n*polylog(
3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/d+b*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))
/b/(e*x+d))/a-b*g*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a+b*h*(n*ln(e*x+d)-l
n(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/a+b*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*p
olylog(2,1-b*c*x/(-a*c+1))/a+b*h*n*polylog(3,1-b*c*x/(-a*c+1))/a-b*h*n*pol
ylog(3,d*(-b*c*x-a*c+1)/(-a*c+1)/(e*x+d))/a+b*h*n*polylog(3,-e*(-b*c*x-a*c
+1)/b/c/(e*x+d))/a-b*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/a-e*h*n*polylog(3
,-b*x/a)/d+b*h*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)*(n*ln(e*x+d)-ln(f*(e*x+
d)^n))/a+e*h*n*(ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a
)/d+e*h*n*ln(x)*polylog(2,c*(b*x+a))/d-e*h*n*ln(e*x+d)*polylog(2,c*(b*x+a)
)/d-b*h*n*(ln(e*x+d)-ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))*polylog(2,1-b*
c*x/(-a*c+1))/a-b*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,d*(-
b*c*x-a*c+1)/(-a*c+1)/(e*x+d))/a+b*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1
))*polylog(2,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a+b*h*n*(ln(b*(e*x+d)/(-a*e+b*
d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/a-e*h*n
*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x
+d)/(-a*e+b*d))/d-b*h*n*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*
c+1)))*polylog(2,1+e*x/d)/a+e*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/
a/(1-c*(b*x+a)))/d-e*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*c*x/(1-c...
```

3.181.2 Mathematica [A] (verified)

Time = 7.15 (sec) , antiderivative size = 2247, normalized size of antiderivative = 0.90

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]
```

output
```
-(((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*((a + b*x)*PolyLog[2, c*(
a + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x] + PolyLog[2,
(-1 + a*c + b*c*x)/(-1 + a*c)])))/(a*x)) + (h*n*(a*(e*x*Log[x] - (d + e*x)
*Log[d + e*x])*PolyLog[2, c*(a + b*x)] + x*(a*e*(Log[x]*Log[1 + (b*x)/a]*L
og[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c
- b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1
 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Lo
g[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a]
)*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*
(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] + Log[(a*(-1
+ a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b
*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1
+ a*c + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b
*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c +
b*c*x)/(b*c*x)]) - a*e*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x]
 + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x
))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)])
)/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d +
e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]
+ (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a +...
```

3.181.3 Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 2413, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7157, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)}{x^2}\,dx$$

$$\Big\downarrow \; 7157$$

$$-b\int \left(\frac{\log(-ac-bxc+1)\,(g+h\log(f(d+ex)^n))}{ax}-\frac{b\log(-ac-bxc+1)\,(g+h\log(f(d+ex)^n))}{a(a+bx)}\right)dx+ ehn\int\left(\frac{\text{PolyLog}(2,c(a+bx))}{dx}-\frac{e\,\text{PolyLog}(2,c(a+bx))}{d(d+ex)}\right)dx-\frac{\text{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)}{x}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x}+ ehn\left(-\frac{\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2d}+\frac{\text{PolyLog}\left(2,-\frac{e(1}{}\right.}{}\right.$$

$$b\left(\frac{hn\left(\log\left(\frac{bcx}{1-ac}\right)+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)x}{(1-ac)(d+ex)}\right)\right)\log^2\left(\frac{(1-ac)(d+ex)}{d(-ac-bxc+1)}\right)}{2a}+\frac{hn\,\text{PolyLog}\left(2,\frac{d(-ac-bxc+1)}{(1-ac)(d+ex)}\right)\text{lo}}{a}\right.$$

input
```
Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]
```

output
```
-(((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x) + e*h*n*(-1/2*((
Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d
+ e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))]^2)/d + (Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/d -
 (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/d + ((Log[c*(a + b*x
)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*d) + ((Log[1 + (b*x)/a] +
Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + ((Log[c*(a + b*x
)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
2*d) + ((Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyL
og[2, -((b*x)/a)])/d + (Log[x]*PolyLog[2, c*(a + b*x)])/d - (Log[d + e*x]*
PolyLog[2, c*(a + b*x)])/d - ((Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a +
b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/d +
 (Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*
x))))])/d - (Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 -
 c*(a + b*x)))])/d - ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/d + ((Log[x] + Log[-((a*(1 -
c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/d + (Log[(b*(d + e*x))
/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b...
```

3.181. $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.181.4 Maple [F]

$$\int \frac{(g+h\ln(f(ex+d)^n))\operatorname{polylog}(2,c(bx+a))}{x^2}dx$$

input
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

output
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

3.181.5 Fricas [F]

$$\int \frac{(g+h\log(f(d+ex)^n))\operatorname{PolyLog}(2,c(a+bx))}{x^2}\,dx = \int \frac{(h\log((ex+d)^nf)+g)\operatorname{Li}_2((bx+a)c)}{x^2}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="fr
icas")
```

output
```
integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/
x^2, x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**2,x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^2}\,dx = \int \frac{(h\log((ex + d)^n f) + g)\operatorname{Li}_2((bx + a)c)}{x^2}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="ma
xima")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

3.181.8 Giac [F]

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^2}\,dx = \int \frac{(h\log((ex + d)^n f) + g)\operatorname{Li}_2((bx + a)c)}{x^2}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="gi
ac")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$$
$$= \int \frac{\text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)}{x^2}\,dx$$

input
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^2,x)
```

output
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^2, x)
```

3.182 $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^3}\,dx$

3.182.1 Optimal result

Integrand size = 27, antiderivative size = 3119

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^3}\,dx = \text{Too large to display}$$

output
```
1/4*b^2*h*n*(ln(b*c*x/(-a*c+1))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*
e+b*c*d+e)*x/(-a*c+1)/(e*x+d)))*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))^2/a^
2-1/4*b^2*h*n*(ln(b*c*x/(-a*c+1))-ln(-e*x/d))*(ln(-b*c*x-a*c+1)+ln((-a*c+1
)*(e*x+d)/d/(-b*c*x-a*c+1)))^2/a^2-1/2*b^2*h*ln(b*c*x/(-a*c+1))*ln(-b*c*x-
a*c+1)*(n*ln(e*x+d)-ln(f*(e*x+d)^n))/a^2+1/2*b^2*c*ln(-e*x/d)*(g+h*ln(f*(e
*x+d)^n))/a/(-a*c+1)-1/2*b^2*c*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+h*
ln(f*(e*x+d)^n))/a/(-a*c+1)-1/4*b^2*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)
/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b
*d)/(1-c*(b*x+a)))^2/a^2+1/4*e^2*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/
c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)
/(1-c*(b*x+a)))^2/d^2+1/4*b^2*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d))
)*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/a^2-1/4*e^2*h
*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c
*(b*x+a)))+ln(1-c*(b*x+a)))^2/d^2-1/4*e^2*h*n*(ln(1+b*x/a)+ln((-a*c+1)/(1-
c*(b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)
^2/d^2-1/4*e^2*h*n*(ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/
b/x))^2/d^2-1/2*e^2*h*n*(ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog
(2,-b*x/a)/d^2-1/2*e^2*h*n*ln(x)*polylog(2,c*(b*x+a))/d^2+1/2*e^2*h*n*ln(e
*x+d)*polylog(2,c*(b*x+a))/d^2+1/2*b^2*h*n*(ln(e*x+d)-ln((-a*c+1)*(e*x+d)/
d/(-b*c*x-a*c+1)))*polylog(2,1-b*c*x/(-a*c+1))/a^2+1/2*b^2*h*n*ln((-a*c...
```

3.182.2 Mathematica [A] (verified)

Time = 13.02 (sec) , antiderivative size = 2700, normalized size of antiderivative = 0.87

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{PolyLog}(2,c(a+bx))}{x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

output
```
((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-((-1 + a*c)*(a^2 - b^2*x^
2)*PolyLog[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*Log[x]) + (a*(-1 + a*c + b*c*
x) + b*(-1 + a*c)*x*Log[(b*c*x)/(1 - a*c)])*Log[1 - a*c - b*c*x] + b*(-1 +
 a*c)*x*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)])))/(2*a^2*(-1 + a*c)*x^2
) - (h*n*(((d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*x^2)*Log[d + e*x])*PolyLog
[2, c*(a + b*x)])/x^2 + (b*d*e*(Log[x]*Log[1 - a*c - b*c*x] - Log[c*(a + b
*x)]*Log[1 - a*c - b*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - PolyLog[2
, (b*c*x)/(1 - a*c)] - PolyLog[2, 1 - a*c - b*c*x]))/a + e^2*(Log[x]*Log[1
 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])
*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a +
b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/
(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log
[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c
*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x]
+ Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*
(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + Po
lyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3
, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3,
 (-1 + a*c + b*c*x)/(b*c*x)]) - e^2*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]
*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])...
```

3.182.3 Rubi [A] (verified)

Time = 4.49 (sec) , antiderivative size = 2903, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7157, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\mathrm{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)}{x^3}\,dx$$

$$\Big\downarrow \ 7157$$

$$-\frac{1}{2}b\int\left(\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)b^2}{a^2(a+bx)}-\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)b}{a^2x}+\frac{\log(-a\right.$$

$$\frac{1}{2}ehn\int\left(\frac{\text{PolyLog}(2,c(a+bx))e^2}{d^2(d+ex)}-\frac{\text{PolyLog}(2,c(a+bx))e}{d^2x}+\frac{\text{PolyLog}(2,c(a+bx))}{dx^2}\right)dx-\frac{\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))}{2x^2}+$$

$$\frac{1}{2}ehn\left(\frac{e\left(\log(c(a+bx))+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2d^2}-\frac{e\,\text{PolyLog}\left(2,-\right.}{}\right.$$

$$\frac{1}{2}b\left(-\frac{bhn\left(\log\left(\frac{bcx}{1-ac}\right)+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)x}{(1-ac)(d+ex)}\right)\right)\log^2\left(\frac{(1-ac)(d+ex)}{d(-ac-bxc+1)}\right)}{2a^2}-\frac{bhn\,\text{PolyLog}\left(2,\frac{d(-ac-bxc+}{(1-ac)(d+e}\right.}{a^2}\right.$$

input
```
Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

output
```
-1/2*((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2 + (e*h*n*(-(
(b*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(a*d)) + (e*(Log[c*(a + b*
x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*
(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)
))]^2)/(2*d^2) - (e*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/d^2 + (e
*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/d^2 - (e*(Log[c*(a +
b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)
*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*d^2) - (e*(Log[1 + (b*x
)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1
- c*(a + b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d^2) - (e*(Log
[c*(a + b*x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b
*x))])^2)/(2*d^2) - (e*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))
/(b*x))])*PolyLog[2, -((b*x)/a)])/d^2 - (b*PolyLog[2, c*(a + b*x)])/(a*d)
- PolyLog[2, c*(a + b*x)]/(d*x) - (e*Log[x]*PolyLog[2, c*(a + b*x)])/d^2 +
 (e*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/d^2 - (b*PolyLog[2, 1 - (b*c*x)/
(1 - a*c)])/(a*d) + (e*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]
 + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/d^2 - (e*L
og[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))
))])/d^2 + (e*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1
- c*(a + b*x)))])/d^2 + (e*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*...
```

3.182. $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^3}\,dx$

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.182.4 Maple [F]

$$\int \frac{(g+h\ln (f(ex+d)^{n}))\,\mathrm{polylog}(2,c(bx+a))}{x^{3}}dx$$

input
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
```

output
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
```

3.182.5 Fricas [F]

$$\int \frac{(g+h\log (f(d+ex)^{n}))\,\mathrm{PolyLog}(2,c(a+bx))}{x^{3}}\,dx = \int \frac{(h\log ((ex+d)^{n}f)+g)\mathrm{Li}_2((bx+a)c)}{x^{3}}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="fr
icas")
```

output
```
integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/
x^3, x)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(g + h\log(f(d+ex)^n))\,\text{PolyLog}(2, c(a+bx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**3,x)
```

output
```
Timed out
```

3.182.7 Maxima [F]

$$\int \frac{(g + h\log(f(d+ex)^n))\,\text{PolyLog}(2, c(a+bx))}{x^3}\,dx$$
$$= \int \frac{(h\log((ex+d)^n f) + g)\text{Li}_2((bx+a)c)}{x^3}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="ma
xima")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

3.182.8 Giac [F]

$$\int \frac{(g + h\log(f(d+ex)^n))\,\text{PolyLog}(2, c(a+bx))}{x^3}\,dx$$
$$= \int \frac{(h\log((ex+d)^n f) + g)\text{Li}_2((bx+a)c)}{x^3}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="gi
ac")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^3}\,dx = \int \frac{\text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)}{x^3}\,dx$$

input
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^3,x)
```

output
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^3, x)
```

3.183 $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^4}\,dx$

3.183.1 Optimal result

Integrand size = 27, antiderivative size = 3733

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^4}\,dx = \text{Too large to display}$$

output
```
-1/3*b^3*h*n*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))*poly
log(2,1+e*x/d)/a^3+1/3*e^3*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(
1-c*(b*x+a)))/d^3-1/3*e^3*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*c*x/(1
-c*(b*x+a)))/d^3+1/3*b^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+
a))))*polylog(2,1-c*(b*x+a))/a^3-1/3*e^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e
+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/d^3+1/3*e^3*h*n*(ln(x)+ln(-a*
(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/d^3-1/3*b^3*h*n*ln(b*(e*x+d)/(-
a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a^3+1/3*e^
3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/
c/(e*x+d))/d^3+1/3*b^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(
2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a^3-1/3*e^3*h*n*ln(b*(e*x+d)/(-a*e+b
*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/d^3-1/6*b
^3*h*n*(ln(b*c*x/(-a*c+1))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c
*d+e)*x/(-a*c+1)/(e*x+d)))*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))^2/a^3+1/6
*b^3*h*n*(ln(b*c*x/(-a*c+1))-ln(-e*x/d))*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*
x+d)/d/(-b*c*x-a*c+1)))^2/a^3+1/3*b^3*h*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1
)*(n*ln(e*x+d)-ln(f*(e*x+d)^n))/a^3-1/6*b^2*c*(g+h*ln(f*(e*x+d)^n))/a/(-a*
c+1)/x+1/6*b^3*c^2*ln(-e*x/d)*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)^2-1/3*b^3*c
*ln(-e*x/d)*(g+h*ln(f*(e*x+d)^n))/a^2/(-a*c+1)-1/6*b^3*c^2*ln(e*(-b*c*x-a*
c+1)/(-a*c*e+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)^2+1/3*b^3*c*ln(...
```

3.183.2 Mathematica [A] (verified)

Time = 16.09 (sec) , antiderivative size = 3341, normalized size of antiderivative = 0.89

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

output
```
(g + h*(-(n*Log[d + e*x]) + Log[f*(d + e*x)^n]))*(-1/6*(b*((2*a*b^2*c*Log[
x])/(1 - a*c) + (2*a*b^2*c*Log[1 - a*c - b*c*x])/(-1 + a*c) - (a^2*Log[1 -
 a*c - b*c*x])/x^2 + (2*a*b*Log[1 - a*c - b*c*x])/x + 2*b^2*Log[(b*c*x)/(1
 - a*c)]*Log[1 - a*c - b*c*x] - (a^2*b*c*(-1 + a*c + b*c*x*Log[x] - b*c*x*
Log[1 - a*c - b*c*x]))/((-1 + a*c)^2*x) + 2*b^2*PolyLog[2, c*(a + b*x)] +
2*b^2*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/a^3 - PolyLog[2, a*c + b
*c*x]/(3*x^3)) + h*n*((-1/6*e/(d*x^2) + e^2/(3*d^2*x) + (e^3*Log[x])/(3*d^
3) - (e^3*Log[d + e*x])/(3*d^3) - Log[d + e*x]/(3*x^3))*PolyLog[2, c*(a +
b*x)] - (b*((-2*d*e^2*(Log[x]*Log[1 - a*c - b*c*x] - Log[c*(a + b*x)]*Log[
1 - a*c - b*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - PolyLog[2, (b*c*x)
/(1 - a*c)] - PolyLog[2, 1 - a*c - b*c*x]))/a + (d^2*e*((a*(b*c*x*Log[x] -
 (-1 + a*c + b*c*x)*Log[1 - a*c - b*c*x]))/((-1 + a*c)*x) + b*(Log[x]*(-Lo
g[1 - a*c - b*c*x] + Log[1 + (b*c*x)/(-1 + a*c)]) + PolyLog[2, (b*c*x)/(1
- a*c)]) + b*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x] + PolyLog[2, 1 - a*c -
 b*c*x])))/a^2 - (2*e^3*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((
-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Lo
g[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c
 - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - L
og[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c
*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(...
```

3.183.3 Rubi [A] (verified)

Time = 5.34 (sec) , antiderivative size = 3741, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7157, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,c(a+bx))\,(h\log(f(d+ex)^n)+g)}{x^4}\,dx$$

$$\Big\downarrow \ 7157$$

$$-\frac{1}{3}b\int\left(-\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)b^3}{a^3(a+bx)}+\frac{\log(-ac-bxc+1)\left(g+h\log\left(f(d+ex)^n\right)\right)b^2}{a^3x}-\frac{\log(\right.$$
$$\frac{1}{3}ehn\int\left(-\frac{\text{PolyLog}(2,c(a+bx))e^3}{d^3(d+ex)}+\frac{\text{PolyLog}(2,c(a+bx))e^2}{d^3x}-\frac{\text{PolyLog}(2,c(a+bx))e}{d^2x^2}+\frac{\text{PolyLog}(2,c(a+bx}{dx^3}\right.$$
$$\frac{\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)}{3x^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))}{3x^3}+$$
$$\frac{1}{3}ehn\left(\frac{c\log(x)b^2}{2a(1-ac)d}+\frac{\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bxc+1)b^2}{2a^2d}-\frac{c\log(-ac-bxc+1)b^2}{2a(1-ac)d}+\frac{\text{PolyLog}(2,c(a+bx))b^2}{2a^2d}+\frac{\text{P}}{}\right.$$
$$\frac{1}{3}b\left(\frac{hn\left(\log\left(\frac{bcx}{1-ac}\right)+\log\left(\frac{bcd-ace+e}{bc(d+ex)}\right)-\log\left(\frac{(bcd-ace+e)x}{(1-ac)(d+ex)}\right)\right)\log^2\left(\frac{(1-ac)(d+ex)}{d(-ac-bxc+1)}\right)b^2}{2a^3}-\frac{hn\left(\log\left(\frac{bcx}{1-ac}\right)-\log\left(-\frac{ex}{d}\right.\right.}{}\right.$$

input
```
Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

output
```
-1/3*((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3 + (e*h*n*((b
^2*c*Log[x])/(2*a*(1 - a*c)*d) - (b^2*c*Log[1 - a*c - b*c*x])/(2*a*(1 - a*
c)*d) + (b*Log[1 - a*c - b*c*x])/(2*a*d*x) + (b^2*Log[(b*c*x)/(1 - a*c)]*L
og[1 - a*c - b*c*x])/(2*a^2*d) + (b*e*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c -
 b*c*x])/(a*d^2) - (e^2*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(
d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*d^3) + (e^2*Log[x]*Log[1 +
 (b*x)/a]*Log[1 - c*(a + b*x)])/d^3 - (e^2*Log[c*(a + b*x)]*Log[d + e*x]*L
og[1 - c*(a + b*x)])/d^3 + (e^2*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b
*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 -
 c*(a + b*x)])^2)/(2*d^3) + (e^2*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*
(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(
1 - c*(a + b*x)))/(b*x))]^2)/(2*d^3) + (e^2*(Log[c*(a + b*x)] - Log[1 + (b
*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*d^3) + (e^2*(
Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((
b*x)/a)])/d^3 + (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2*d) + (b*e*PolyLog[2,
c*(a + b*x)])/(a*d^2) - PolyLog[2, c*(a + b*x)]/(2*d*x^2) + (e*PolyLog[2,
c*(a + b*x)])/(d^2*x) + (e^2*Log[x]*PolyLog[2, c*(a + b*x)])/d^3 - (e^2*Lo
g[d + e*x]*PolyLog[2, c*(a + b*x)])/d^3 + (b^2*PolyLog[2, 1 - (b*c*x)/(1 -
 a*c)])/(2*a^2*d) + (b*e*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(a*d^2) - (...
```

3.183. $\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^4}\,dx$

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

3.183.4 Maple [F]

$$\int \frac{(g + h\ln(f(ex + d)^n))\,\mathrm{polylog}(2, c(bx + a))}{x^4} dx$$

input
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
```

output
```
int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
```

3.183.5 Fricas [F]

$$\int \frac{(g + h\log(f(d + ex)^n))\,\mathrm{PolyLog}(2, c(a + bx))}{x^4} dx = \int \frac{(h\log((ex + d)^n f) + g)\mathrm{Li}_2((bx + a)c)}{x^4} dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="fr
icas")
```

output
```
integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/
x^4, x)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**4,x)
```

output
```
Timed out
```

3.183.7 Maxima [F]

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^4}\,dx = \int \frac{(h\log((ex + d)^n f) + g)\operatorname{Li}_2((bx + a)c)}{x^4}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="ma
xima")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

3.183.8 Giac [F]

$$\int \frac{(g + h\log(f(d + ex)^n))\operatorname{PolyLog}(2, c(a + bx))}{x^4}\,dx = \int \frac{(h\log((ex + d)^n f) + g)\operatorname{Li}_2((bx + a)c)}{x^4}\,dx$$

input
```
integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="gi
ac")
```

output
```
integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x^4}\,dx = \int \frac{\text{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,(d+e\,x)^n\right)\right)}{x^4}\,dx$$

input
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^4,x)
```

output
```
int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^4, x)
```

3.184 $\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

3.184.1 Optimal result

Integrand size = 21, antiderivative size = 661

$$\begin{aligned}
&\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx \\
&= \frac{53bx}{192c^3}+\frac{11ax}{27c^2}+\frac{49(3b+4ac)x}{432c^3}+\frac{29bx^2}{384c^2}+\frac{5ax^2}{54c}+\frac{13(3b+4ac)x^2}{864c^2}+\frac{2ax^3}{81}+\frac{17bx^3}{576c} \\
&+\frac{(3b+4ac)x^3}{324c}+\frac{3bx^4}{256}+\frac{29b\log(1-cx)}{192c^4}+\frac{5a\log(1-cx)}{27c^3}+\frac{13(3b+4ac)\log(1-cx)}{432c^4} \\
&-\frac{bx^2\log(1-cx)}{16c^2}-\frac{ax^2\log(1-cx)}{9c}-\frac{(3b+4ac)x^2\log(1-cx)}{48c^2}-\frac{2}{27}ax^3\log(1-cx) \\
&-\frac{bx^3\log(1-cx)}{24c}-\frac{(3b+4ac)x^3\log(1-cx)}{108c}-\frac{3}{64}bx^4\log(1-cx) \\
&+\frac{b(1-cx)\log(1-cx)}{8c^4}+\frac{2a(1-cx)\log(1-cx)}{9c^3}+\frac{(3b+4ac)(1-cx)\log(1-cx)}{12c^4} \\
&-\frac{b\log^2(1-cx)}{16c^4}-\frac{a\log^2(1-cx)}{9c^3}+\frac{1}{9}ax^3\log^2(1-cx)+\frac{1}{16}bx^4\log^2(1-cx) \\
&-\frac{(3b+4ac)\log(cx)\log^2(1-cx)}{12c^4}-\frac{(3b+4ac)x\,\text{PolyLog}(2,cx)}{12c^3} \\
&-\frac{(3b+4ac)x^2\,\text{PolyLog}(2,cx)}{24c^2}-\frac{(3b+4ac)x^3\,\text{PolyLog}(2,cx)}{36c}-\frac{1}{16}bx^4\,\text{PolyLog}(2,cx) \\
&-\frac{(3b+4ac)\log(1-cx)\,\text{PolyLog}(2,cx)}{12c^4}+\frac{1}{12}\left(4ax^3+3bx^4\right)\log(1-cx)\,\text{PolyLog}(2,cx) \\
&-\frac{(3b+4ac)\log(1-cx)\,\text{PolyLog}(2,1-cx)}{6c^4}+\frac{(3b+4ac)\,\text{PolyLog}(3,1-cx)}{6c^4}
\end{aligned}$$

output
```
-1/12*(4*a*c+3*b)*x*polylog(2,c*x)/c^3-1/24*(4*a*c+3*b)*x^2*polylog(2,c*x)
/c^2-1/36*(4*a*c+3*b)*x^3*polylog(2,c*x)/c-1/16*b*x^2*ln(-c*x+1)/c^2-1/9*a
*x^2*ln(-c*x+1)/c-1/48*(4*a*c+3*b)*x^2*ln(-c*x+1)/c^2-1/24*b*x^3*ln(-c*x+1
)/c-1/108*(4*a*c+3*b)*x^3*ln(-c*x+1)/c+1/8*b*(-c*x+1)*ln(-c*x+1)/c^4+2/9*a
*(-c*x+1)*ln(-c*x+1)/c^3+1/12*(4*a*c+3*b)*(-c*x+1)*ln(-c*x+1)/c^4-1/12*(4*
a*c+3*b)*ln(c*x)*ln(-c*x+1)^2/c^4-1/12*(4*a*c+3*b)*ln(-c*x+1)*polylog(2,c*
x)/c^4-1/6*(4*a*c+3*b)*ln(-c*x+1)*polylog(2,-c*x+1)/c^4+2/81*a*x^3+3/256*b
*x^4+53/192*b*x/c^3+11/27*a*x/c^2+49/432*(4*a*c+3*b)*x/c^3+29/384*b*x^2/c^
2+5/54*a*x^2/c+13/864*(4*a*c+3*b)*x^2/c^2+17/576*b*x^3/c+1/324*(4*a*c+3*b)
*x^3/c-1/16*b*x^4*polylog(2,c*x)+1/6*(4*a*c+3*b)*polylog(3,-c*x+1)/c^4+29/
192*b*ln(-c*x+1)/c^4+5/27*a*ln(-c*x+1)/c^3+13/432*(4*a*c+3*b)*ln(-c*x+1)/c
^4-2/27*a*x^3*ln(-c*x+1)-3/64*b*x^4*ln(-c*x+1)-1/16*b*ln(-c*x+1)^2/c^4-1/9
*a*ln(-c*x+1)^2/c^3+1/9*a*x^3*ln(-c*x+1)^2+1/16*b*x^4*ln(-c*x+1)^2+1/12*(3
*b*x^4+4*a*x^3)*ln(-c*x+1)*polylog(2,c*x)
```

3.184.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.64

$$\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$= \frac{4260bcx + 5952ac^2x + 834bc^2x^2 + 1056ac^3x^2 + 268bc^3x^3 + 256ac^4x^3 + 81bc^4x^4 + 4260b\log(1-cx) + 59}{}$$

input
```
Integrate[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(4260*b*c*x + 5952*a*c^2*x + 834*b*c^2*x^2 + 1056*a*c^3*x^2 + 268*b*c^3*x^
3 + 256*a*c^4*x^3 + 81*b*c^4*x^4 + 4260*b*Log[1 - c*x] + 5952*a*c*Log[1 -
c*x] - 2592*b*c*x*Log[1 - c*x] - 3840*a*c^2*x*Log[1 - c*x] - 864*b*c^2*x^2
*Log[1 - c*x] - 1344*a*c^3*x^2*Log[1 - c*x] - 480*b*c^3*x^3*Log[1 - c*x] -
 768*a*c^4*x^3*Log[1 - c*x] - 324*b*c^4*x^4*Log[1 - c*x] - 432*b*Log[1 - c
*x]^2 - 768*a*c*Log[1 - c*x]^2 + 768*a*c^4*x^3*Log[1 - c*x]^2 + 432*b*c^4*
x^4*Log[1 - c*x]^2 - 1728*b*Log[c*x]*Log[1 - c*x]^2 - 2304*a*c*Log[c*x]*Lo
g[1 - c*x]^2 + 48*(-(c*x*(8*a*c*(6 + 3*c*x + 2*c^2*x^2) + 3*b*(12 + 6*c*x
+ 4*c^2*x^2 + 3*c^3*x^3))) + 12*(4*a*c*(-1 + c^3*x^3) + 3*b*(-1 + c^4*x^4)
)*Log[1 - c*x])*PolyLog[2, c*x] - 1152*(3*b + 4*a*c)*Log[1 - c*x]*PolyLog[
2, 1 - c*x] + 3456*b*PolyLog[3, 1 - c*x] + 4608*a*c*PolyLog[3, 1 - c*x])/(
6912*c^4)
```

3.184.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)\,\text{PolyLog}(2,cx)\log(1-cx)\,dx$$

$$\Big\downarrow \text{7158}$$

$$c\int\left(-\frac{b\,\text{PolyLog}(2,cx)x^3}{4c}-\frac{(3b+4ac)\,\text{PolyLog}(2,cx)x^2}{12c^2}-\frac{(3b+4ac)\,\text{PolyLog}(2,cx)x}{12c^3}-\frac{(3b+4ac)\,\text{PolyLog}(2,\right.$$
$$\int\left(\frac{1}{4}b\log^2(1-cx)x^3+\frac{1}{3}a\log^2(1-cx)x^2\right)dx+\frac{1}{12}\left(4ax^3+3bx^4\right)\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{(4ac+3b)\,\text{PolyLog}(3,1-cx)}{6c^5}-\frac{(4ac+3b)\,\text{PolyLog}(2,cx)\log(1-cx)}{12c^5}-\frac{(4ac+3b)\,\text{PolyLog}(2,1-cx)\log(1-\right.$$
$$\frac{1}{12}\left(4ax^3+3bx^4\right)\text{PolyLog}(2,cx)\log(1-cx)-\frac{a\log^2(1-cx)}{9c^3}+\frac{2a(1-cx)\log(1-cx)}{9c^3}+$$
$$\frac{5a\log(1-cx)}{27c^3}+\frac{11ax}{27c^2}+\frac{1}{9}ax^3\log^2(1-cx)-\frac{2}{27}ax^3\log(1-cx)+\frac{5ax^2}{54c}-\frac{ax^2\log(1-cx)}{9c}+$$
$$\frac{2ax^3}{81}-\frac{b\log^2(1-cx)}{16c^4}+\frac{b(1-cx)\log(1-cx)}{8c^4}+\frac{13b\log(1-cx)}{96c^4}+\frac{25bx}{96c^3}+\frac{13bx^2}{192c^2}-$$
$$\frac{bx^2\log(1-cx)}{16c^2}+\frac{1}{16}bx^4\log^2(1-cx)-\frac{1}{32}bx^4\log(1-cx)+\frac{7bx^3}{288c}-\frac{bx^3\log(1-cx)}{24c}+\frac{bx^4}{128}$$

input
```
Int[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(25*b*x)/(96*c^3) + (11*a*x)/(27*c^2) + (13*b*x^2)/(192*c^2) + (5*a*x^2)/(
54*c) + (2*a*x^3)/81 + (7*b*x^3)/(288*c) + (b*x^4)/128 + (13*b*Log[1 - c*x
])/(96*c^4) + (5*a*Log[1 - c*x])/(27*c^3) - (b*x^2*Log[1 - c*x])/(16*c^2)
- (a*x^2*Log[1 - c*x])/(9*c) - (2*a*x^3*Log[1 - c*x])/27 - (b*x^3*Log[1 -
c*x])/(24*c) - (b*x^4*Log[1 - c*x])/32 + (b*(1 - c*x)*Log[1 - c*x])/(8*c^4
) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) - (b*Log[1 - c*x]^2)/(16*c^4) - (
a*Log[1 - c*x]^2)/(9*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]
^2)/16 + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyLog[2, c*x])/12 + c*((b*x)/
(64*c^4) + (49*(3*b + 4*a*c)*x)/(432*c^4) + (b*x^2)/(128*c^3) + (13*(3*b +
 4*a*c)*x^2)/(864*c^3) + (b*x^3)/(192*c^2) + ((3*b + 4*a*c)*x^3)/(324*c^2)
 + (b*x^4)/(256*c) + (b*Log[1 - c*x])/(64*c^5) + (13*(3*b + 4*a*c)*Log[1 -
 c*x])/(432*c^5) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^3) - ((3*b + 4*a
*c)*x^3*Log[1 - c*x])/(108*c^2) - (b*x^4*Log[1 - c*x])/(64*c) + ((3*b + 4*
a*c)*(1 - c*x)*Log[1 - c*x])/(12*c^5) - ((3*b + 4*a*c)*Log[c*x]*Log[1 - c*
x]^2)/(12*c^5) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(12*c^4) - ((3*b + 4*a*
c)*x^2*PolyLog[2, c*x])/(24*c^3) - ((3*b + 4*a*c)*x^3*PolyLog[2, c*x])/(36
*c^2) - (b*x^4*PolyLog[2, c*x])/(16*c) - ((3*b + 4*a*c)*Log[1 - c*x]*PolyL
og[2, c*x])/(12*c^5) - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6
*c^5) + ((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^5))
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

3.184.4 Maple [F]

$$\int x^2(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)\,dx$$

input
```
int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

3.184.5 Fricas [F]

$$\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int (bx+a)x^2\text{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)*dilog(c*x)*log(-c*x + 1), x)
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Timed out
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.63

$$\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx =$$

$$-\frac{1}{6912}c\left(\frac{576\left(\log\left(cx\right)\log\left(-cx+1\right)^2+2\,\text{Li}_2(-cx+1)\log\left(-cx+1\right)-2\,\text{Li}_3(-cx+1)\right)(4\,ac+3\,b)}{c^5}-\frac{8}{}\right.$$

$$+\frac{1}{1728}\left(\frac{32\left(18\,c^3x^3\text{Li}_2(cx)-2\,c^3x^3-3\,c^2x^2-6\,cx+6\left(c^3x^3-1\right)\log\left(-cx+1\right)\right)a}{c^3}+\frac{9\left(48\,c^4x^4\text{Li}_2(cx)-\right.}{}\right.$$

$$+1)$$

input
```
integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-1/6912*c*(576*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1)
 - 2*polylog(3, -c*x + 1))*(4*a*c + 3*b)/c^5 - (81*b*c^4*x^4 + 4*(64*a*c^4
 + 67*b*c^3)*x^3 + 6*(176*a*c^3 + 139*b*c^2)*x^2 + 12*(496*a*c^2 + 355*b*c
)*x - 48*(9*b*c^4*x^4 + 4*(4*a*c^4 + 3*b*c^3)*x^3 + 6*(4*a*c^3 + 3*b*c^2)*
x^2 + 12*(4*a*c^2 + 3*b*c)*x + 12*(4*a*c + 3*b)*log(-c*x + 1))*dilog(c*x)
- 4*(54*b*c^4*x^4 + 4*(32*a*c^4 + 21*b*c^3)*x^3 + 6*(40*a*c^3 + 27*b*c^2)*
x^2 - 1488*a*c + 12*(64*a*c^2 + 45*b*c)*x - 1065*b)*log(-c*x + 1))/c^5) +
1/1728*(32*(18*c^3*x^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3
*x^3 - 1)*log(-c*x + 1))*a/c^3 + 9*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*
c^3*x^3 - 6*c^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*b/c^4)*log(
-c*x + 1)
```

3.184.8 Giac [F]

$$\int x^2(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int(bx+a)x^2\text{Li}_2(cx)\log\left(-cx+1\right)\,dx$$

input
```
integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((b*x + a)*x^2*dilog(c*x)*log(-c*x + 1), x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx=\int x^2\,\ln(1-c\,x)\,\operatorname{polylog}(2,c\,x)\,(a+b\,x)\,dx$$

input
```
int(x^2*log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

output
```
int(x^2*log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.185 $\int x(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx$

3.185.1 Optimal result

Integrand size = 19, antiderivative size = 546

$$\int x(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx = \frac{4bx}{9c^2}+\frac{ax}{c}+\frac{5(2b+3ac)x}{24c^2}+\frac{bx^2}{9c}+\frac{(2b+3ac)x^2}{48c}+\frac{bx^3}{27}+\frac{a(1-cx)^2}{8c^2}+\frac{2b\log(1-cx)}{9c^3}+\frac{(2b+3ac)\log(1-cx)}{24c^3}-\frac{bx^2\log(1-cx)}{9c}-\frac{(2b+3ac)x^2\log(1-cx)}{24c}-\frac{1}{9}bx^3\log(1-cx)+\frac{2b(1-cx)\log(1-cx)}{9c^3}+\frac{a(1-cx)\log(1-cx)}{c^2}+\frac{(2b+3ac)(1-cx)\log(1-cx)}{6c^3}-\frac{a(1-cx)^2\log(1-cx)}{4c^2}-\frac{b\log^2(1-cx)}{9c^3}+\frac{1}{9}bx^3\log^2(1-cx)-\frac{a(1-cx)\log^2(1-cx)}{2c^2}+\frac{a(1-cx)^2\log^2(1-cx)}{4c^2}-\frac{(2b+3ac)\log(cx)\log^2(1-cx)}{6c^3}-\frac{(2b+3ac)x\,\text{PolyLog}(2,cx)}{6c^2}-\frac{(2b+3ac)x^2\,\text{PolyLog}(2,cx)}{12c}-\frac{1}{9}bx^3\,\text{PolyLog}(2,cx)-\frac{(2b+3ac)\log(1-cx)\,\text{PolyLog}(2,cx)}{6c^3}+\frac{1}{6}(3ax^2+2bx^3)\log(1-cx)\,\text{PolyLog}(2,cx)-\frac{(2b+3ac)\log(1-cx)\,\text{PolyLog}(2,1-cx)}{3c^3}+\frac{(2b+3ac)\,\text{PolyLog}(3,1-cx)}{3c^3}$$

output
```
-1/6*(3*a*c+2*b)*x*polylog(2,c*x)/c^2-1/12*(3*a*c+2*b)*x^2*polylog(2,c*x)/
c-1/9*b*x^2*ln(-c*x+1)/c-1/24*(3*a*c+2*b)*x^2*ln(-c*x+1)/c+2/9*b*(-c*x+1)*
ln(-c*x+1)/c^3+1/6*(3*a*c+2*b)*(-c*x+1)*ln(-c*x+1)/c^3-1/4*a*(-c*x+1)^2*ln
(-c*x+1)/c^2-1/2*a*(-c*x+1)*ln(-c*x+1)^2/c^2+1/4*a*(-c*x+1)^2*ln(-c*x+1)^2
/c^2-1/6*(3*a*c+2*b)*ln(c*x)*ln(-c*x+1)^2/c^3-1/6*(3*a*c+2*b)*ln(-c*x+1)*p
olylog(2,c*x)/c^3-1/3*(3*a*c+2*b)*ln(-c*x+1)*polylog(2,-c*x+1)/c^3+1/27*b*
x^3-1/9*b*ln(-c*x+1)^2/c^3+1/9*b*x^3*ln(-c*x+1)^2+1/6*(2*b*x^3+3*a*x^2)*ln
(-c*x+1)*polylog(2,c*x)+a*x/c+a*(-c*x+1)*ln(-c*x+1)/c^2+4/9*b*x/c^2+5/24*(
3*a*c+2*b)*x/c^2+1/9*b*x^2/c+1/48*(3*a*c+2*b)*x^2/c+1/8*a*(-c*x+1)^2/c^2-1
/9*b*x^3*polylog(2,c*x)+1/3*(3*a*c+2*b)*polylog(3,-c*x+1)/c^3+2/9*b*ln(-c*
x+1)/c^3+1/24*(3*a*c+2*b)*ln(-c*x+1)/c^3-1/9*b*x^3*ln(-c*x+1)
```

3.185.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.66

$$\int x(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)\,dx$$
$$= \frac{-378ac+372bcx+594ac^2x+66bc^2x^2+81ac^3x^2+16bc^3x^3+372b\log(1-cx)+594ac\log(1-cx)-24}{}$$

input
```
Integrate[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(-378*a*c + 372*b*c*x + 594*a*c^2*x + 66*b*c^2*x^2 + 81*a*c^3*x^2 + 16*b*c
^3*x^3 + 372*b*Log[1 - c*x] + 594*a*c*Log[1 - c*x] - 240*b*c*x*Log[1 - c*x
] - 432*a*c^2*x*Log[1 - c*x] - 84*b*c^2*x^2*Log[1 - c*x] - 162*a*c^3*x^2*L
og[1 - c*x] - 48*b*c^3*x^3*Log[1 - c*x] - 48*b*Log[1 - c*x]^2 - 108*a*c*Lo
g[1 - c*x]^2 + 108*a*c^3*x^2*Log[1 - c*x]^2 + 48*b*c^3*x^3*Log[1 - c*x]^2
- 144*b*Log[c*x]*Log[1 - c*x]^2 - 216*a*c*Log[c*x]*Log[1 - c*x]^2 + 12*(-(
c*x*(9*a*c*(2 + c*x) + 2*b*(6 + 3*c*x + 2*c^2*x^2))) + 6*(3*a*c*(-1 + c^2*
x^2) + 2*b*(-1 + c^3*x^3))*Log[1 - c*x])*PolyLog[2, c*x] - 144*(2*b + 3*a*
c)*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 288*b*PolyLog[3, 1 - c*x] + 432*a*c*
PolyLog[3, 1 - c*x])/(432*c^3)
```

3.185.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)\operatorname{PolyLog}(2,cx)\log(1-cx)\,dx$$

$$\Big\downarrow \text{7158}$$

$$c\int\left(-\frac{b\operatorname{PolyLog}(2,cx)x^2}{3c}-\frac{(2b+3ac)\operatorname{PolyLog}(2,cx)x}{6c^2}-\frac{(2b+3ac)\operatorname{PolyLog}(2,cx)}{6c^3}+\frac{(2b+3ac)\operatorname{PolyLog}(2,cx}{6c^3(1-cx)}\right.$$
$$\int\left(\frac{1}{3}bx^2\log^2(1-cx)+\frac{1}{2}ax\log^2(1-cx)\right)dx+\frac{1}{6}\left(3ax^2+2bx^3\right)\operatorname{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{(3ac+2b)\operatorname{PolyLog}(3,1-cx)}{3c^4}-\frac{(3ac+2b)\operatorname{PolyLog}(2,cx)\log(1-cx)}{6c^4}-\frac{(3ac+2b)\operatorname{PolyLog}(2,1-cx)\log(1-}{3c^4}\right.$$
$$\frac{1}{6}\left(3ax^2+2bx^3\right)\operatorname{PolyLog}(2,cx)\log(1-cx)+\frac{a(1-cx)^2}{8c^2}+\frac{a(1-cx)^2\log^2(1-cx)}{4c^2}-$$
$$\frac{a(1-cx)\log^2(1-cx)}{2c^2}-\frac{a(1-cx)^2\log(1-cx)}{4c^2}+\frac{a(1-cx)\log(1-cx)}{c^2}+\frac{ax}{c}-\frac{b\log^2(1-cx)}{9c^3}+$$
$$\frac{2b(1-cx)\log(1-cx)}{9c^3}+\frac{5b\log(1-cx)}{27c^3}+\frac{11bx}{27c^2}+\frac{1}{9}bx^3\log^2(1-cx)-\frac{2}{27}bx^3\log(1-cx)+\frac{5bx^2}{54c}-$$
$$\frac{bx^2\log(1-cx)}{9c}+\frac{2bx^3}{81}$$

input
```
Int[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(11*b*x)/(27*c^2) + (a*x)/c + (5*b*x^2)/(54*c) + (2*b*x^3)/81 + (a*(1 - c*
x)^2)/(8*c^2) + (5*b*Log[1 - c*x])/(27*c^3) - (b*x^2*Log[1 - c*x])/(9*c) -
 (2*b*x^3*Log[1 - c*x])/27 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1
- c*x)*Log[1 - c*x])/c^2 - (a*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*Log[1
 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (a*(1 - c*x)*Log[1 - c*x]^
2)/(2*c^2) + (a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) + ((3*a*x^2 + 2*b*x^3)
*Log[1 - c*x]*PolyLog[2, c*x])/6 + c*((b*x)/(27*c^3) + (5*(2*b + 3*a*c)*x)
/(24*c^3) + (b*x^2)/(54*c^2) + ((2*b + 3*a*c)*x^2)/(48*c^2) + (b*x^3)/(81*
c) + (b*Log[1 - c*x])/(27*c^4) + ((2*b + 3*a*c)*Log[1 - c*x])/(24*c^4) - (
(2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c^2) - (b*x^3*Log[1 - c*x])/(27*c) + (
(2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^4) - ((2*b + 3*a*c)*Log[c*x]*Lo
g[1 - c*x]^2)/(6*c^4) - ((2*b + 3*a*c)*x*PolyLog[2, c*x])/(6*c^3) - ((2*b
+ 3*a*c)*x^2*PolyLog[2, c*x])/(12*c^2) - (b*x^3*PolyLog[2, c*x])/(9*c) - (
(2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^4) - ((2*b + 3*a*c)*Log[1
 - c*x]*PolyLog[2, 1 - c*x])/(3*c^4) + ((2*b + 3*a*c)*PolyLog[3, 1 - c*x])
/(3*c^4))
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

3.185.4 Maple [F]

$$\int x(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)\,dx$$

input
```
int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

3.185.5 Fricas [F]

$$\int x(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx=\int (bx+a)x\text{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)*dilog(c*x)*log(-c*x + 1), x)
```

3.185.6 Sympy [F]

$$\int x(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx=\int x(a+bx)\log(-cx+1)\operatorname{Li}_2(cx)\,dx$$

input
```
integrate(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral(x*(a + b*x)*log(-c*x + 1)*polylog(2, c*x), x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.63

$$\int x(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx=$$
$$-\frac{1}{432}c\left(\frac{72\left(\log(cx)\log(-cx+1)^2+2\operatorname{Li}_2(-cx+1)\log(-cx+1)-2\operatorname{Li}_3(-cx+1)\right)(3ac+2b)}{c^4}-\frac{16b\cdots}{\cdots}\right.$$
$$+\frac{1}{216}\left(\frac{27\left(4c^2x^2\operatorname{Li}_2(cx)-c^2x^2-2cx+2\left(c^2x^2-1\right)\log(-cx+1)\right)a}{c^2}+\frac{4\left(18c^3x^3\operatorname{Li}_2(cx)-2c^3x^3-3c\cdots\right.}{\cdots}\right.$$
$$+1)$$

input
```
integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-1/432*c*(72*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) -
 2*polylog(3, -c*x + 1))*(3*a*c + 2*b)/c^4 - (16*b*c^3*x^3 + 3*(27*a*c^3 +
 22*b*c^2)*x^2 + 6*(99*a*c^2 + 62*b*c)*x - 12*(4*b*c^3*x^3 + 3*(3*a*c^3 +
2*b*c^2)*x^2 + 6*(3*a*c^2 + 2*b*c)*x + 6*(3*a*c + 2*b)*log(-c*x + 1))*dilo
g(c*x) - 2*(16*b*c^3*x^3 + 6*(9*a*c^3 + 5*b*c^2)*x^2 - 297*a*c + 6*(27*a*c
^2 + 16*b*c)*x - 186*b)*log(-c*x + 1))/c^4) + 1/216*(27*(4*c^2*x^2*dilog(c
*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*a/c^2 + 4*(18*c^3*x
^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x +
 1))*b/c^3)*log(-c*x + 1)
```

3.185.8 Giac [F]

$$\int x(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int(bx+a)x\text{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((b*x + a)*x*dilog(c*x)*log(-c*x + 1), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx=\int x\,\ln(1-c\,x)\,\text{polylog}(2,c\,x)\,(a+b\,x)\,dx$$

input
```
int(x*log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

output
```
int(x*log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.186 $\int (a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$

3.186.1 Optimal result

Integrand size = 18, antiderivative size = 390

$$\begin{aligned}\int (a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx &= 2ax+\frac{9bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b(1-cx)^2}{8c^2}+\frac{b\log(1-cx)}{8c^2}\\&\quad-\frac{1}{8}bx^2\log(1-cx)+\frac{b(1-cx)\log(1-cx)}{c^2}+\frac{2a(1-cx)\log(1-cx)}{c}\\&\quad+\frac{(b+2ac)(1-cx)\log(1-cx)}{2c^2}-\frac{b(1-cx)^2\log(1-cx)}{4c^2}\\&\quad-\frac{b(1-cx)\log^2(1-cx)}{2c^2}-\frac{a(1-cx)\log^2(1-cx)}{c}+\frac{b(1-cx)^2\log^2(1-cx)}{4c^2}\\&\quad-\frac{(b+2ac)\log(cx)\log^2(1-cx)}{2c^2}-\frac{(b+2ac)x\,\text{PolyLog}(2,cx)}{2c}-\frac{1}{4}bx^2\,\text{PolyLog}(2,cx)\\&\quad-\frac{(b+2ac)\log(1-cx)\,\text{PolyLog}(2,cx)}{2c^2}+\frac{1}{2}(2ax+bx^2)\log(1-cx)\,\text{PolyLog}(2,cx)\\&\quad-\frac{(b+2ac)\log(1-cx)\,\text{PolyLog}(2,1-cx)}{c^2}+\frac{(b+2ac)\,\text{PolyLog}(3,1-cx)}{c^2}\end{aligned}$$

output
```
2*a*x+9/8*b*x/c+1/2*(2*a*c+b)*x/c+1/16*b*x^2+1/8*b*(-c*x+1)^2/c^2+1/8*b*ln
(-c*x+1)/c^2-1/8*b*x^2*ln(-c*x+1)+b*(-c*x+1)*ln(-c*x+1)/c^2+2*a*(-c*x+1)*l
n(-c*x+1)/c+1/2*(2*a*c+b)*(-c*x+1)*ln(-c*x+1)/c^2-1/4*b*(-c*x+1)^2*ln(-c*x
+1)/c^2-1/2*b*(-c*x+1)*ln(-c*x+1)^2/c^2-a*(-c*x+1)*ln(-c*x+1)^2/c+1/4*b*(-
c*x+1)^2*ln(-c*x+1)^2/c^2-1/2*(2*a*c+b)*ln(c*x)*ln(-c*x+1)^2/c^2-1/2*(2*a*
c+b)*x*polylog(2,c*x)/c-1/4*b*x^2*polylog(2,c*x)-1/2*(2*a*c+b)*ln(-c*x+1)*
polylog(2,c*x)/c^2+1/2*(b*x^2+2*a*x)*ln(-c*x+1)*polylog(2,c*x)-(2*a*c+b)*l
n(-c*x+1)*polylog(2,-c*x+1)/c^2+(2*a*c+b)*polylog(3,-c*x+1)/c^2
```

3.186.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.73

$$\int (a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx$$
$$= \frac{-14b-32ac+22bcx+48ac^2x+3bc^2x^2+22b\log(1-cx)+48ac\log(1-cx)-16bcx\log(1-cx)-48a}{}$$

input
```
Integrate[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
(-14*b - 32*a*c + 22*b*c*x + 48*a*c^2*x + 3*b*c^2*x^2 + 22*b*Log[1 - c*x]
+ 48*a*c*Log[1 - c*x] - 16*b*c*x*Log[1 - c*x] - 48*a*c^2*x*Log[1 - c*x] -
6*b*c^2*x^2*Log[1 - c*x] - 4*b*Log[1 - c*x]^2 - 16*a*c*Log[1 - c*x]^2 + 16
*a*c^2*x*Log[1 - c*x]^2 + 4*b*c^2*x^2*Log[1 - c*x]^2 - 8*b*Log[c*x]*Log[1
- c*x]^2 - 16*a*c*Log[c*x]*Log[1 - c*x]^2 + 4*(-(c*x*(2*b + 4*a*c + b*c*x)
) + 2*(-1 + c*x)*(b + 2*a*c + b*c*x)*Log[1 - c*x])*PolyLog[2, c*x] - 16*(b
 + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*b*PolyLog[3, 1 - c*x] + 32
*a*c*PolyLog[3, 1 - c*x])/(16*c^2)
```

3.186.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\,\text{PolyLog}(2,cx)\log(1-cx)\,dx$$

$$\Big\downarrow \text{7158}$$

$$\int \left(a\log^2(1-cx)+\frac{1}{2}bx\log^2(1-cx)+\frac{a^2\log^2(1-cx)}{2bx}\right)dx+ c\int \left(\frac{\text{PolyLog}(2,cx)(b+ac)^2}{2bc^2(1-cx)}-\frac{(b+2ac)\,\text{PolyLog}(2,cx)}{2c^2}-\frac{bx\,\text{PolyLog}(2,cx)}{2c}\right)dx+ \frac{(a+bx)^2\,\text{PolyLog}(2,cx)\log(1-cx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\operatorname{PolyLog}(3,1-cx)}{b}+\frac{a^2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{b}+\frac{a^2\log(cx)\log^2(1-cx)}{2b}+ c\left(\frac{(ac+b)^2\operatorname{PolyLog}(3,1-cx)}{bc^3}-\frac{(ac+b)^2\operatorname{PolyLog}(2,cx)\log(1-cx)}{2bc^3}-\frac{(ac+b)^2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{bc^3}\right. \frac{(a+bx)^2\operatorname{PolyLog}(2,cx)\log(1-cx)}{2b}-\frac{a(1-cx)\log^2(1-cx)}{c}+\frac{2a(1-cx)\log(1-cx)}{c}+2ax+ \frac{b(1-cx)^2}{8c^2}+\frac{b(1-cx)^2\log^2(1-cx)}{4c^2}-\frac{b(1-cx)\log^2(1-cx)}{2c^2}-\frac{b(1-cx)^2\log(1-cx)}{4c^2}+ \frac{b(1-cx)\log(1-cx)}{c^2}+\frac{bx}{c}$$

input
```
Int[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

output
```
2*a*x + (b*x)/c + (b*(1 - c*x)^2)/(8*c^2) + (b*(1 - c*x)*Log[1 - c*x])/c^2
 + (2*a*(1 - c*x)*Log[1 - c*x])/c - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) -
 (b*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (
b*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) + (a^2*Log[c*x]*Log[1 - c*x]^2)/(2*b
) + ((a + b*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*b) + (a^2*Log[1 - c*x]*P
olyLog[2, 1 - c*x])/b - (a^2*PolyLog[3, 1 - c*x])/b + c*((b*x)/(8*c^2) + (
(b + 2*a*c)*x)/(2*c^2) + (b*x^2)/(16*c) + (b*Log[1 - c*x])/(8*c^3) - (b*x^
2*Log[1 - c*x])/(8*c) + ((b + 2*a*c)*(1 - c*x)*Log[1 - c*x])/(2*c^3) - ((b
 + a*c)^2*Log[c*x]*Log[1 - c*x]^2)/(2*b*c^3) - ((b + 2*a*c)*x*PolyLog[2, c
*x])/(2*c^2) - (b*x^2*PolyLog[2, c*x])/(4*c) - ((b + a*c)^2*Log[1 - c*x]*P
olyLog[2, c*x])/(2*b*c^3) - ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])
/(b*c^3) + ((b + a*c)^2*PolyLog[3, 1 - c*x])/(b*c^3))
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

3.186.4 Maple [F]

$$\int (bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)\,dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

3.186.5 Fricas [F]

$$\int (a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)\,dx = \int (bx+a)\mathrm{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

3.186.6 Sympy [F]

$$\int (a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)\,dx = \int (a+bx)\log(-cx+1)\,\mathrm{Li}_2(cx)\,dx$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

output
```
Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x), x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.66

$$\int (a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx =$$
$$-\frac{1}{16}c\left(\frac{8\left(\log(cx)\log(-cx+1)^2+2\,\text{Li}_2(-cx+1)\log(-cx+1)-2\,\text{Li}_3(-cx+1)\right)(2\,ac+b)}{c^3}-\frac{3\,bc^2x^2\ldots}{\ldots}\right.$$
$$+\frac{1}{8}\left(\frac{8\,(cx\text{Li}_2(cx)-cx+(cx-1)\log(-cx+1))a}{c}+\frac{(4\,c^2x^2\text{Li}_2(cx)-c^2x^2-2\,cx+2\,(c^2x^2-1)\log(-c\ldots}{c^2}\right.$$
$$+1)$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

output
```
-1/16*c*(8*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2
*polylog(3, -c*x + 1))*(2*a*c + b)/c^3 - (3*b*c^2*x^2 + 2*(24*a*c^2 + 11*b
*c)*x - 4*(b*c^2*x^2 + 2*(2*a*c^2 + b*c)*x + 2*(2*a*c + b)*log(-c*x + 1))*
dilog(c*x) - 2*(2*b*c^2*x^2 - 24*a*c + 2*(8*a*c^2 + 3*b*c)*x - 11*b)*log(-
c*x + 1))/c^3) + 1/8*(8*(c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1))*a
/c + (4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x +
1))*b/c^2)*log(-c*x + 1)
```

3.186.8 Giac [F]

$$\int (a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)\,dx = \int (bx+a)\text{Li}_2(cx)\log(-cx+1)\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)\,dx = \int \ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)\ (a+b\,x)\ dx$$

input
```
int(log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

output
```
int(log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.187 $\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}\,dx$

3.187.1 Optimal result

Integrand size = 21, antiderivative size = 153

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}\,dx &= 3bx+\frac{3b(1-cx)\log(1-cx)}{c}-\frac{b(1-cx)\log^2(1-cx)}{c}-\frac{b\log(cx)\log^2(1-cx)}{c}\\ &\quad -bx\operatorname{PolyLog}(2,cx)-\frac{b\log(1-cx)\operatorname{PolyLog}(2,cx)}{c}+bx\log(1-cx)\operatorname{PolyLog}(2,cx)\\ &\quad -\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2-\frac{2b\log(1-cx)\operatorname{PolyLog}(2,1-cx)}{c}+\frac{2b\operatorname{PolyLog}(3,1-cx)}{c}\end{aligned}$$

output
```
3*b*x+3*b*(-c*x+1)*ln(-c*x+1)/c-b*(-c*x+1)*ln(-c*x+1)^2/c-b*ln(c*x)*ln(-c*
x+1)^2/c-b*x*polylog(2,c*x)-b*ln(-c*x+1)*polylog(2,c*x)/c+b*x*ln(-c*x+1)*p
olylog(2,c*x)-1/2*a*polylog(2,c*x)^2-2*b*ln(-c*x+1)*polylog(2,-c*x+1)/c+2*
b*polylog(3,-c*x+1)/c
```

3.187.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}\,dx &= \frac{b(-cx+(-1+cx)\log(1-cx))\operatorname{PolyLog}(2,cx)}{c}-\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2\\ &\quad +\frac{b(-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)}{c}\end{aligned}$$

input
```
Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

output
```
(b*(-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2, c*x])/c - (a*PolyLog[2, c
*x]^2)/2 + (b*(-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 -
c*x]^2 + c*x*Log[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 - 2*Log[1 - c*x]*Pol
yLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

3.187.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {7159, 27, 7154, 25, 2836, 2733, 2732, 7155, 7239, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\operatorname{PolyLog}(2,cx)\log(1-cx)}{x}dx$$

$$\Big\downarrow \text{7159}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+\int b\log(1-cx)\operatorname{PolyLog}(2,cx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+b\int \log(1-cx)\operatorname{PolyLog}(2,cx)dx$$

$$\Big\downarrow \text{7154}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+ b\left(c\int -\left(\left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)\right)dx+\int \log^2(1-cx)dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)\right)$$

$$\Big\downarrow \text{25}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+ b\left(-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)dx+\int \log^2(1-cx)dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)\right)$$

$$\Big\downarrow \text{2836}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+$$
$$b\left(-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)dx-\frac{\int \log^2(1-cx)d(1-cx)}{c}+x\operatorname{PolyLog}(2,cx)\log(1-cx)\right)$$

$\Big\downarrow$ 2733

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+$$
$$b\left(-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)dx-\frac{(1-cx)\log^2(1-cx)-2\int \log(1-cx)d(1-cx)}{c}+x\operatorname{PolyLog}(2,c\right.$$

$\Big\downarrow$ 2732

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx+$$
$$b\left(-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-}{c}\right.$$

$\Big\downarrow$ 7155

$$b\left(-c\int \left(\frac{1}{c}-\frac{1}{c(1-cx)}\right)\operatorname{PolyLog}(2,cx)dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-}{c}\right.$$
$$\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2$$

$\Big\downarrow$ 7239

$$b\left(-c\int \frac{x\operatorname{PolyLog}(2,cx)}{cx-1}dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+(1-cx)\log(1-cx)}{c}\right.$$
$$\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2$$

$\Big\downarrow$ 7293

$$b\left(-c\int \left(\frac{\operatorname{PolyLog}(2,cx)}{c}+\frac{\operatorname{PolyLog}(2,cx)}{c(cx-1)}\right)dx+x\operatorname{PolyLog}(2,cx)\log(1-cx)-\frac{(1-cx)\log^2(1-cx)-2(cx+}{c}\right.$$
$$\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2$$

$\Big\downarrow$ 2009

$$b\left(-c\left(-\frac{2\operatorname{PolyLog}(3,1-cx)}{c^2}+\frac{\operatorname{PolyLog}(2,cx)\log(1-cx)}{c^2}+\frac{2\operatorname{PolyLog}(2,1-cx)\log(1-cx)}{c^2}+\frac{\log(cx)\log^2(1-}{c^2}\right.\right.$$
$$\frac{1}{2}a\operatorname{PolyLog}(2,cx)^2$$

3.187. $\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx$

input
```
Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

output
```
-1/2*(a*PolyLog[2, c*x]^2) + b*(-(((1 - c*x)*Log[1 - c*x]^2 - 2*(-1 + c*x
+ (1 - c*x)*Log[1 - c*x]))/c) + x*Log[1 - c*x]*PolyLog[2, c*x] - c*(-(x/c)
 - ((1 - c*x)*Log[1 - c*x])/c^2 + (Log[c*x]*Log[1 - c*x]^2)/c^2 + (x*PolyL
og[2, c*x])/c + (Log[1 - c*x]*PolyLog[2, c*x])/c^2 + (2*Log[1 - c*x]*PolyL
og[2, 1 - c*x])/c^2 - (2*PolyLog[3, 1 - c*x])/c^2))
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 7154
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Simp[b   Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a
*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Simp[e*h*n   Int[Poly
Log[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b
, c, d, e, f, g, h, n}, x]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

rule 7159
```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Simp[Coeff[Px, x, -m - 1]   Int[(g + h*Log[1 + e*x])*(Pol
yLog[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
 + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[P
x, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.187.4 Maple [F]

$$\int \frac{(bx+a)\ln(-cx+1)\operatorname{polylog}(2,cx)}{x}dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

3.187.5 Fricas [F]

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}\,dx = \int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

3.187.6 Sympy [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx = \int \frac{(a+bx)\log\left(-cx+1\right)\text{Li}_2\left(cx\right)}{x}\,dx$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

output
```
Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x, x)
```

3.187.7 Maxima [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx = \int \frac{(bx+a)\text{Li}_2(cx)\log\left(-cx+1\right)}{x}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

3.187.8 Giac [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x}\,dx = \int \frac{(bx+a)\text{Li}_2(cx)\log\left(-cx+1\right)}{x}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x}\,dx = \int \frac{\ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)\ (a+b\,x)}{x}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x, x)
```

3.188 $\int \frac{(a+bx)\log(1-cx)\text{PolyLog}(2,cx)}{x^2}\,dx$

3.188.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\text{PolyLog}(2,cx)}{x^2}\,dx &= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\,\text{PolyLog}(2,cx) \\ &\quad + ac\log(1-cx)\,\text{PolyLog}(2,cx) - \frac{a\log(1-cx)\,\text{PolyLog}(2,cx)}{x} - \frac{1}{2}b\,\text{PolyLog}(2,cx)^2 \\ &\quad + 2ac\log(1-cx)\,\text{PolyLog}(2,1-cx) - ac\,\text{PolyLog}(3,cx) - 2ac\,\text{PolyLog}(3,1-cx)\end{aligned}$$

output
```
a*(-c*x+1)*ln(-c*x+1)^2/x+a*c*ln(c*x)*ln(-c*x+1)^2-2*a*c*polylog(2,c*x)+a*
c*ln(-c*x+1)*polylog(2,c*x)-a*ln(-c*x+1)*polylog(2,c*x)/x-1/2*b*polylog(2,
c*x)^2+2*a*c*ln(-c*x+1)*polylog(2,-c*x+1)-a*c*polylog(3,c*x)-2*a*c*polylog
(3,-c*x+1)
```

3.188.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^2}\,dx = 2ac\log(cx)\log(1-cx) - ac\log^2(1-cx) + \frac{a\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) + \frac{a(-1+cx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x} - \frac{1}{2}b\operatorname{PolyLog}(2,cx)^2 + 2ac(1+\log(1-cx))\operatorname{PolyLog}(2,1-cx) - ac\operatorname{PolyLog}(3,cx) - 2ac\operatorname{PolyLog}(3,1-cx)$$

input
```
Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

output
```
2*a*c*Log[c*x]*Log[1 - c*x] - a*c*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x +
a*c*Log[c*x]*Log[1 - c*x]^2 + (a*(-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/
x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*(1 + Log[1 - c*x])*PolyLog[2, 1 - c*x]
 - a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

3.188.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7159, 27, 7155, 7157, 2009, 2844, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\operatorname{PolyLog}(2,cx)\log(1-cx)}{x^2}\,dx$$

$$\Big\downarrow \text{7159}$$

$$\int \frac{a\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^2}dx + b\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^2}dx + b\int \frac{\log(1-cx)\operatorname{PolyLog}(2,cx)}{x}dx$$

$$\Big\downarrow \text{7155}$$

$$a\int \frac{\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}dx-\frac{1}{2}b\,\text{PolyLog}(2,cx)^2$$

$$\Big\downarrow \text{7157}$$

$$a\left(-c\int \left(\frac{\text{PolyLog}(2,cx)}{x}+\frac{c\,\text{PolyLog}(2,cx)}{1-cx}\right)dx-\int \frac{\log^2(1-cx)}{x^2}dx-\frac{\text{PolyLog}(2,cx)\log(1-cx)}{x}\right)-\frac{1}{2}b\,\text{PolyLog}(2,cx)^2$$

$$\Big\downarrow \text{2009}$$

$$a\Bigg(-\int \frac{\log^2(1-cx)}{x^2}dx-c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1$$
$$\frac{1}{2}b\,\text{PolyLog}(2,cx)^2$$

$$\Big\downarrow \text{2844}$$

$$a\Bigg(2c\int \frac{\log(1-cx)}{x}dx-c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-$$
$$\frac{1}{2}b\,\text{PolyLog}(2,cx)^2$$

$$\Big\downarrow \text{2838}$$

$$a\Big(-2c\,\text{PolyLog}(2,cx)-c\big(\text{PolyLog}(3,cx)+2\,\text{PolyLog}(3,1-cx)-\text{PolyLog}(2,cx)\log(1-cx)-2\,\text{PolyLog}(2,1-$$
$$\frac{1}{2}b\,\text{PolyLog}(2,cx)^2$$

input
```
Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

output
```
-1/2*(b*PolyLog[2, c*x]^2) + a*(((1 - c*x)*Log[1 - c*x]^2)/x - 2*c*PolyLog
[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x - c*(-(Log[c*x]*Log[1 - c*x]^2
) - Log[1 - c*x]*PolyLog[2, c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + Po
lyLog[3, c*x] + 2*PolyLog[3, 1 - c*x]))
```

3.188. $\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^2}\,dx$

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

rule 7157
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Simp[b/(m + 1)   Int
[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)
/(a + b*x), x], x], x] - Simp[e*h*(n/(m + 1))   Int[ExpandIntegrand[PolyLog
[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

rule 7159
```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Simp[Coeff[Px, x, -m - 1]   Int[(g + h*Log[1 + e*x])*(Pol
yLog[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
 + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[P
x, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

3.188. $\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx$

3.188.4 Maple [F]

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^2}dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

3.188.5 Fricas [F]

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.188.6 Sympy [F]

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(a+bx)\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^2}\,dx$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

output
```
Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

3.188.7 Maxima [F]

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.188.8 Giac [F]

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^2}\,dx = \int \frac{\ln(1-c\,x)\,\mathrm{polylog}(2,c\,x)\,(a+b\,x)}{x^2}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^2,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^2, x)
```

3.189 $\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx$

3.189.1 Optimal result

Integrand size = 21, antiderivative size = 331

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx &= -ac^2\log(x)+ac^2\log(1-cx)-\frac{ac\log(1-cx)}{x}-\frac{1}{4}ac^2\log^2(1-cx)+\frac{a\log^2(1-cx)}{4x^2}\\
&\quad+\frac{b(1-cx)\log^2(1-cx)}{x}-\frac{b^2\log(cx)\log^2(1-cx)}{2a}+\frac{(b+ac)^2\log(cx)\log^2(1-cx)}{2a}\\
&\quad-2bc\,\text{PolyLog}(2,cx)-\frac{1}{2}ac^2\,\text{PolyLog}(2,cx)+\frac{ac\,\text{PolyLog}(2,cx)}{2x}\\
&\quad+\frac{(b+ac)^2\log(1-cx)\,\text{PolyLog}(2,cx)}{2a}-\frac{(a+bx)^2\log(1-cx)\,\text{PolyLog}(2,cx)}{2ax^2}\\
&\quad-\frac{b^2\log(1-cx)\,\text{PolyLog}(2,1-cx)}{a}+\frac{(b+ac)^2\log(1-cx)\,\text{PolyLog}(2,1-cx)}{a}\\
&\quad-\frac{1}{2}c(2b+ac)\,\text{PolyLog}(3,cx)+\frac{b^2\,\text{PolyLog}(3,1-cx)}{a}-\frac{(b+ac)^2\,\text{PolyLog}(3,1-cx)}{a}\end{aligned}$$

output
```
-a*c^2*ln(x)+a*c^2*ln(-c*x+1)-a*c*ln(-c*x+1)/x-1/4*a*c^2*ln(-c*x+1)^2+1/4*
a*ln(-c*x+1)^2/x^2+b*(-c*x+1)*ln(-c*x+1)^2/x-1/2*b^2*ln(c*x)*ln(-c*x+1)^2/
a+1/2*(a*c+b)^2*ln(c*x)*ln(-c*x+1)^2/a-2*b*c*polylog(2,c*x)-1/2*a*c^2*poly
log(2,c*x)+1/2*a*c*polylog(2,c*x)/x+1/2*(a*c+b)^2*ln(-c*x+1)*polylog(2,c*x
)/a-1/2*(b*x+a)^2*ln(-c*x+1)*polylog(2,c*x)/a/x^2-b^2*ln(-c*x+1)*polylog(2
,-c*x+1)/a+(a*c+b)^2*ln(-c*x+1)*polylog(2,-c*x+1)/a-1/2*c*(a*c+2*b)*polylo
g(3,c*x)+b^2*polylog(3,-c*x+1)/a-(a*c+b)^2*polylog(3,-c*x+1)/a
```

3.189.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^3}\,dx = \frac{1}{4}\left(-4ac^2\log(cx)+4ac^2\log(1-cx)-\frac{4ac\log(1-cx)}{x}+8bc\log(cx)\log(1-cx) + 2ac^2\log(cx)\log(1-cx)-4bc\log^2(1-cx)-ac^2\log^2(1-cx)+\frac{a\log^2(1-cx)}{x^2} + \frac{4b\log^2(1-cx)}{x}+4bc\log(cx)\log^2(1-cx)+2ac^2\log(cx)\log^2(1-cx) + \frac{2(acx+(-1+cx)(a+2bx+acx)\log(1-cx))\operatorname{PolyLog}(2,cx)}{x^2} + 2c(4b+ac+2(2b+ac)\log(1-cx))\operatorname{PolyLog}(2,1-cx)-4bc\operatorname{PolyLog}(3,cx) - 2ac^2\operatorname{PolyLog}(3,cx)-8bc\operatorname{PolyLog}(3,1-cx)-4ac^2\operatorname{PolyLog}(3,1-cx)\right)$$

input
```
Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

output
```
(-4*a*c^2*Log[c*x] + 4*a*c^2*Log[1 - c*x] - (4*a*c*Log[1 - c*x])/x + 8*b*c
*Log[c*x]*Log[1 - c*x] + 2*a*c^2*Log[c*x]*Log[1 - c*x] - 4*b*c*Log[1 - c*x
]^2 - a*c^2*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x^2 + (4*b*Log[1 - c*x]^2)
/x + 4*b*c*Log[c*x]*Log[1 - c*x]^2 + 2*a*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*
(a*c*x + (-1 + c*x)*(a + 2*b*x + a*c*x)*Log[1 - c*x])*PolyLog[2, c*x])/x^2
 + 2*c*(4*b + a*c + 2*(2*b + a*c)*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 4*b*
c*PolyLog[3, c*x] - 2*a*c^2*PolyLog[3, c*x] - 8*b*c*PolyLog[3, 1 - c*x] -
4*a*c^2*PolyLog[3, 1 - c*x])/4
```

3.189.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\operatorname{PolyLog}(2,cx)\log(1-cx)}{x^3}\,dx$$

$$\Big\downarrow \text{7160}$$

$$\int \left(-\frac{b^2\log^2(1-cx)}{2ax}-\frac{b\log^2(1-cx)}{x^2}-\frac{a\log^2(1-cx)}{2x^3}\right)dx+c\int \left(-\frac{\text{PolyLog}(2,cx)(b+ac)^2}{2a(1-cx)}-\frac{(2b+ac)\,\text{PolyLog}(2,cx)}{2x}-\frac{a\,\text{PolyLog}(2,cx)}{2x^2}\right)dx-\frac{(a+bx)^2\,\text{PolyLog}(2,cx)\log(1-cx)}{2ax^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\,\text{PolyLog}(3,1-cx)}{a}-\frac{b^2\,\text{PolyLog}(2,1-cx)\log(1-cx)}{a}-\frac{b^2\log(cx)\log^2(1-cx)}{2a}-\frac{(a+bx)^2\,\text{PolyLog}(2,cx)\log(1-cx)}{2ax^2}+c\left(-\frac{(ac+b)^2\,\text{PolyLog}(3,1-cx)}{ac}-\frac{1}{2}(ac+2b)\,\text{PolyLog}(3,cx)+\frac{(ac+b)^2\,\text{PolyLog}(2,cx)\log(1-cx)}{2ac}+\frac{(ac+b)^2}{}\right.$$
$$\frac{1}{2}ac^2\,\text{PolyLog}(2,cx)-\frac{1}{4}ac^2\log^2(1-cx)-\frac{1}{2}ac^2\log(x)+\frac{1}{2}ac^2\log(1-cx)+\frac{a\log^2(1-cx)}{4x^2}-\frac{ac\log(1-cx)}{2x}-2bc\,\text{PolyLog}(2,cx)+\frac{b(1-cx)\log^2(1-cx)}{x}$$

input
```
Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

output
```
-1/2*(a*c^2*Log[x]) + (a*c^2*Log[1 - c*x])/2 - (a*c*Log[1 - c*x])/(2*x) -
(a*c^2*Log[1 - c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1
 - c*x]^2)/x - (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) - 2*b*c*PolyLog[2, c*x]
 - (a*c^2*PolyLog[2, c*x])/2 - ((a + b*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/
(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a + (b^2*PolyLog[3, 1 -
 c*x])/a + c*(-1/2*(a*c*Log[x]) + (a*c*Log[1 - c*x])/2 - (a*Log[1 - c*x])/
(2*x) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2)/(2*a*c) + (a*PolyLog[2, c*x]
)/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*c) + ((b + a*c)^
2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(a*c) - ((2*b + a*c)*PolyLog[3, c*x])/
2 - ((b + a*c)^2*PolyLog[3, 1 - c*x])/(a*c))
```

3.189. $\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx$

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.189.4 Maple [F]

$$\int \frac{(bx+a)\ln(-cx+1)\operatorname{polylog}(2,cx)}{x^3}dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

3.189.5 Fricas [F]

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{(bx+a)\operatorname{Li}_2(cx)\log(-cx+1)}{x^3}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.189.6 Sympy [F]

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^3}\,dx=\int \frac{(a+bx)\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^3}\,dx$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

output
```
Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**3, x)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^3}\,dx=-ac^2\log(x)$$
$$+\frac{1}{2}\left(ac^2+2\,bc\right)\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)$$
$$+\frac{1}{2}\left(ac^2+4\,bc\right)\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)-\frac{1}{2}\left(ac^2+2\,bc\right)\mathrm{Li}_3(cx)$$
$$-\frac{((ac^2+4\,bc)x^2-4\,bx-a)\log(-cx+1)^2-2\,(acx+((ac^2+2\,bc)x^2-2\,bx-a)\log(-cx+1))\mathrm{Li}_2(cx)}{4\,x^2}$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

output
```
-a*c^2*log(x) + 1/2*(a*c^2 + 2*b*c)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c
*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/2*(a*c^2 + 4*b*c)*(log
(c*x)*log(-c*x + 1) + dilog(-c*x + 1)) - 1/2*(a*c^2 + 2*b*c)*polylog(3, c*
x) - 1/4*(((a*c^2 + 4*b*c)*x^2 - 4*b*x - a)*log(-c*x + 1)^2 - 2*(a*c*x + (
(a*c^2 + 2*b*c)*x^2 - 2*b*x - a)*log(-c*x + 1))*dilog(c*x) - 4*(a*c^2*x^2
- a*c*x)*log(-c*x + 1))/x^2
```

3.189.8 Giac [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^3}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^3}\,dx = \int \frac{\ln(1-c\,x)\;\text{polylog}(2,c\,x)\;(a+b\,x)}{x^3}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^3,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^3, x)
```

3.190 $\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx$

3.190.1 Optimal result

Integrand size = 21, antiderivative size = 460

$$\begin{aligned}
&\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx \\
&= \frac{7ac^2}{36x} - \frac{1}{2}bc^2\log(x) - \frac{5}{12}ac^3\log(x) - \frac{1}{6}c^2(3b+2ac)\log(x) + \frac{1}{2}bc^2\log(1-cx) \\
&+ \frac{5}{12}ac^3\log(1-cx) + \frac{1}{6}c^2(3b+2ac)\log(1-cx) - \frac{7ac\log(1-cx)}{36x^2} - \frac{bc\log(1-cx)}{2x} \\
&- \frac{2ac^2\log(1-cx)}{9x} - \frac{c(3b+2ac)\log(1-cx)}{6x} - \frac{1}{4}bc^2\log^2(1-cx) - \frac{1}{9}ac^3\log^2(1-cx) \\
&+ \frac{a\log^2(1-cx)}{9x^3} + \frac{b\log^2(1-cx)}{4x^2} + \frac{1}{6}c^2(3b+2ac)\log(cx)\log^2(1-cx) \\
&- \frac{1}{2}bc^2\,\text{PolyLog}(2,cx) - \frac{2}{9}ac^3\,\text{PolyLog}(2,cx) + \frac{ac\,\text{PolyLog}(2,cx)}{6x^2} \\
&+ \frac{c(3b+2ac)\,\text{PolyLog}(2,cx)}{6x} + \frac{1}{6}c^2(3b+2ac)\log(1-cx)\,\text{PolyLog}(2,cx) \\
&- \frac{1}{6}\left(\frac{2a}{x^3} + \frac{3b}{x^2}\right)\log(1-cx)\,\text{PolyLog}(2,cx) + \frac{1}{3}c^2(3b+2ac)\log(1-cx)\,\text{PolyLog}(2,1-cx) \\
&- \frac{1}{6}c^2(3b+2ac)\,\text{PolyLog}(3,cx) - \frac{1}{3}c^2(3b+2ac)\,\text{PolyLog}(3,1-cx)
\end{aligned}$$

output
```
1/6*a*c*polylog(2,c*x)/x^2+1/6*c*(2*a*c+3*b)*polylog(2,c*x)/x-7/36*a*c*ln(
-c*x+1)/x^2-1/2*b*c*ln(-c*x+1)/x-2/9*a*c^2*ln(-c*x+1)/x-1/6*c*(2*a*c+3*b)*
ln(-c*x+1)/x+1/6*c^2*(2*a*c+3*b)*ln(c*x)*ln(-c*x+1)^2+1/6*c^2*(2*a*c+3*b)*
ln(-c*x+1)*polylog(2,c*x)+1/3*c^2*(2*a*c+3*b)*ln(-c*x+1)*polylog(2,-c*x+1)
+7/36*a*c^2/x-1/2*b*c^2*polylog(2,c*x)-2/9*a*c^3*polylog(2,c*x)-1/6*c^2*(2
*a*c+3*b)*polylog(3,c*x)-1/3*c^2*(2*a*c+3*b)*polylog(3,-c*x+1)-1/2*b*c^2*l
n(x)-5/12*a*c^3*ln(x)-1/6*c^2*(2*a*c+3*b)*ln(x)+1/2*b*c^2*ln(-c*x+1)+5/12*
a*c^3*ln(-c*x+1)+1/6*c^2*(2*a*c+3*b)*ln(-c*x+1)-1/4*b*c^2*ln(-c*x+1)^2-1/9
*a*c^3*ln(-c*x+1)^2+1/9*a*ln(-c*x+1)^2/x^3+1/4*b*ln(-c*x+1)^2/x^2-1/6*(2*a
/x^3+3*b/x^2)*ln(-c*x+1)*polylog(2,c*x)
```

3.190.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx$$
$$= \frac{1}{36}\left(-7ac^3+\frac{7ac^2}{x}-36bc^2\log(cx)-27ac^3\log(cx)+36bc^2\log(1-cx)+27ac^3\log(1-cx)\right.$$
$$-\frac{7ac\log(1-cx)}{x^2}-\frac{36bc\log(1-cx)}{x}-\frac{20ac^2\log(1-cx)}{x}+18bc^2\log(cx)\log(1-cx)$$
$$+8ac^3\log(cx)\log(1-cx)-9bc^2\log^2(1-cx)-4ac^3\log^2(1-cx)+\frac{4a\log^2(1-cx)}{x^3}$$
$$+\frac{9b\log^2(1-cx)}{x^2}+18bc^2\log(cx)\log^2(1-cx)+12ac^3\log(cx)\log^2(1-cx)$$
$$+\frac{6(cx(a+3bx+2acx)+(-2a-3bx+3bc^2x^3+2ac^3x^3)\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}$$
$$+2c^2(9b+4ac+6(3b+2ac)\log(1-cx))\,\text{PolyLog}(2,1-cx)-18bc^2\,\text{PolyLog}(3,cx)$$
$$\left.-12ac^3\,\text{PolyLog}(3,cx)-36bc^2\,\text{PolyLog}(3,1-cx)-24ac^3\,\text{PolyLog}(3,1-cx)\right)$$

input
```
Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

output
```
(-7*a*c^3 + (7*a*c^2)/x - 36*b*c^2*Log[c*x] - 27*a*c^3*Log[c*x] + 36*b*c^2
*Log[1 - c*x] + 27*a*c^3*Log[1 - c*x] - (7*a*c*Log[1 - c*x])/x^2 - (36*b*c
*Log[1 - c*x])/x - (20*a*c^2*Log[1 - c*x])/x + 18*b*c^2*Log[c*x]*Log[1 - c
*x] + 8*a*c^3*Log[c*x]*Log[1 - c*x] - 9*b*c^2*Log[1 - c*x]^2 - 4*a*c^3*Log
[1 - c*x]^2 + (4*a*Log[1 - c*x]^2)/x^3 + (9*b*Log[1 - c*x]^2)/x^2 + 18*b*c
^2*Log[c*x]*Log[1 - c*x]^2 + 12*a*c^3*Log[c*x]*Log[1 - c*x]^2 + (6*(c*x*(a
 + 3*b*x + 2*a*c*x) + (-2*a - 3*b*x + 3*b*c^2*x^3 + 2*a*c^3*x^3)*Log[1 - c
*x])*PolyLog[2, c*x])/x^3 + 2*c^2*(9*b + 4*a*c + 6*(3*b + 2*a*c)*Log[1 - c
*x])*PolyLog[2, 1 - c*x] - 18*b*c^2*PolyLog[3, c*x] - 12*a*c^3*PolyLog[3,
c*x] - 36*b*c^2*PolyLog[3, 1 - c*x] - 24*a*c^3*PolyLog[3, 1 - c*x])/36
```

3.190.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\operatorname{PolyLog}(2,cx)\log(1-cx)}{x^4}\,dx$$

$$\Big\downarrow \; 7160$$

$$c\int \left(-\frac{(3b+2ac)\operatorname{PolyLog}(2,cx)c^2}{6(1-cx)}-\frac{(3b+2ac)\operatorname{PolyLog}(2,cx)c}{6x}-\frac{(3b+2ac)\operatorname{PolyLog}(2,cx)}{6x^2}-\frac{a\operatorname{PolyLog}(2,cx}{3x^3}\right.$$
$$\int\left(-\frac{b\log^2(1-cx)}{2x^3}-\frac{a\log^2(1-cx)}{3x^4}\right)dx-\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\operatorname{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \; 2009$$

$$c\left(-\frac{1}{6}c(2ac+3b)\operatorname{PolyLog}(3,cx)-\frac{1}{3}c(2ac+3b)\operatorname{PolyLog}(3,1-cx)+\frac{(2ac+3b)\operatorname{PolyLog}(2,cx)}{6x}+\frac{1}{6}c(2ac+3b)\right.$$
$$\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\operatorname{PolyLog}(2,cx)\log(1-cx)-\frac{2}{9}ac^3\operatorname{PolyLog}(2,cx)-\frac{1}{9}ac^3\log^2(1-cx)-\frac{1}{3}ac^3\log(x)+$$
$$\frac{1}{3}ac^3\log(1-cx)+\frac{ac^2}{9x}-\frac{2ac^2\log(1-cx)}{9x}+\frac{a\log^2(1-cx)}{9x^3}-\frac{ac\log(1-cx)}{9x^2}-\frac{1}{2}bc^2\operatorname{PolyLog}(2,cx)-$$
$$\frac{1}{4}bc^2\log^2(1-cx)-\frac{1}{2}bc^2\log(x)+\frac{1}{2}bc^2\log(1-cx)+\frac{b\log^2(1-cx)}{4x^2}-\frac{bc\log(1-cx)}{2x}$$

input
```
Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

output
```
(a*c^2)/(9*x) - (b*c^2*Log[x])/2 - (a*c^3*Log[x])/3 + (b*c^2*Log[1 - c*x])
/2 + (a*c^3*Log[1 - c*x])/3 - (a*c*Log[1 - c*x])/(9*x^2) - (b*c*Log[1 - c*
x])/(2*x) - (2*a*c^2*Log[1 - c*x])/(9*x) - (b*c^2*Log[1 - c*x]^2)/4 - (a*c
^3*Log[1 - c*x]^2)/9 + (a*Log[1 - c*x]^2)/(9*x^3) + (b*Log[1 - c*x]^2)/(4*
x^2) - (b*c^2*PolyLog[2, c*x])/2 - (2*a*c^3*PolyLog[2, c*x])/9 - (((2*a)/x
^3 + (3*b)/x^2)*Log[1 - c*x]*PolyLog[2, c*x])/6 + c*((a*c)/(12*x) - (a*c^2
*Log[x])/12 - (c*(3*b + 2*a*c)*Log[x])/6 + (a*c^2*Log[1 - c*x])/12 + (c*(3
*b + 2*a*c)*Log[1 - c*x])/6 - (a*Log[1 - c*x])/(12*x^2) - ((3*b + 2*a*c)*L
og[1 - c*x])/(6*x) + (c*(3*b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/6 + (a*Poly
Log[2, c*x])/(6*x^2) + ((3*b + 2*a*c)*PolyLog[2, c*x])/(6*x) + (c*(3*b + 2
*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/6 + (c*(3*b + 2*a*c)*Log[1 - c*x]*Poly
Log[2, 1 - c*x])/3 - (c*(3*b + 2*a*c)*PolyLog[3, c*x])/6 - (c*(3*b + 2*a*c
)*PolyLog[3, 1 - c*x])/3)
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.190.4 Maple [F]

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^4}dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

3.190.5 Fricas [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.190.6 Sympy [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx = \int \frac{(a+bx)\log(-cx+1)\,\text{Li}_2(cx)}{x^4}\,dx$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

output
```
Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**4, x)
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.62

$$\begin{aligned}
&\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx \\
&= \frac{1}{6}\left(2\,ac^3+3\,bc^2\right)\left(\log(cx)\log(-cx+1)^2+2\,\text{Li}_2(-cx+1)\log(-cx+1)-2\,\text{Li}_3(-cx+1)\right) \\
&\quad+\frac{1}{18}\left(4\,ac^3+9\,bc^2\right)\left(\log(cx)\log(-cx+1)+\text{Li}_2(-cx+1)\right) \\
&\quad-\frac{1}{4}\left(3\,ac^3+4\,bc^2\right)\log(x)-\frac{1}{6}\left(2\,ac^3+3\,bc^2\right)\text{Li}_3(cx) \\
&\quad+\frac{7\,ac^2x^2-((4\,ac^3+9\,bc^2)x^3-9\,bx-4\,a)\log(-cx+1)^2+6\,(acx+(2\,ac^2+3\,bc)x^2+((2\,ac^3+3\,bc^2)x^3 \cdots}{36 \cdots}
\end{aligned}$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(2*a*c^3 + 3*b*c^2)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(
-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/18*(4*a*c^3 + 9*b*c^2)*(log(c*x)*l
og(-c*x + 1) + dilog(-c*x + 1)) - 1/4*(3*a*c^3 + 4*b*c^2)*log(x) - 1/6*(2*
a*c^3 + 3*b*c^2)*polylog(3, c*x) + 1/36*(7*a*c^2*x^2 - ((4*a*c^3 + 9*b*c^2
)*x^3 - 9*b*x - 4*a)*log(-c*x + 1)^2 + 6*(a*c*x + (2*a*c^2 + 3*b*c)*x^2 +
((2*a*c^3 + 3*b*c^2)*x^3 - 3*b*x - 2*a)*log(-c*x + 1))*dilog(c*x) + (9*(3*
a*c^3 + 4*b*c^2)*x^3 - 7*a*c*x - 4*(5*a*c^2 + 9*b*c)*x^2)*log(-c*x + 1))/x
^3
```

3.190.8 Giac [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx=\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^4}\,dx=\int \frac{\ln(1-c\,x)\ \text{polylog}(2,c\,x)\ (a+b\,x)}{x^4}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^4,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^4, x)
```

3.191 $\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx$

3.191.1 Optimal result

Integrand size = 21, antiderivative size = 584

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^5}\,dx =& \frac{5ac^2}{144x^2}+\frac{bc^2}{9x}+\frac{19ac^3}{144x}+\frac{c^2(4b+3ac)}{48x}-\frac{1}{3}bc^3\log(x)\\
&-\frac{37}{144}ac^4\log(x)-\frac{5}{48}c^3(4b+3ac)\log(x)\\
&+\frac{1}{3}bc^3\log(1-cx)+\frac{37}{144}ac^4\log(1-cx)\\
&+\frac{5}{48}c^3(4b+3ac)\log(1-cx)-\frac{5ac\log(1-cx)}{72x^3}\\
&-\frac{bc\log(1-cx)}{9x^2}-\frac{ac^2\log(1-cx)}{16x^2}\\
&-\frac{c(4b+3ac)\log(1-cx)}{48x^2}-\frac{2bc^2\log(1-cx)}{9x}\\
&-\frac{ac^3\log(1-cx)}{8x}-\frac{c^2(4b+3ac)\log(1-cx)}{12x}\\
&-\frac{1}{9}bc^3\log^2(1-cx)-\frac{1}{16}ac^4\log^2(1-cx)\\
&+\frac{a\log^2(1-cx)}{16x^4}+\frac{b\log^2(1-cx)}{9x^3}\\
&+\frac{1}{12}c^3(4b+3ac)\log(cx)\log^2(1-cx)\\
&-\frac{2}{9}bc^3\,\mathrm{PolyLog}(2,cx)\\
&-\frac{1}{8}ac^4\,\mathrm{PolyLog}(2,cx)+\frac{ac\,\mathrm{PolyLog}(2,cx)}{12x^3}\\
&+\frac{c(4b+3ac)\,\mathrm{PolyLog}(2,cx)}{24x^2}\\
&+\frac{c^2(4b+3ac)\,\mathrm{PolyLog}(2,cx)}{12x}\\
&+\frac{1}{12}c^3(4b+3ac)\log(1-cx)\,\mathrm{PolyLog}(2,cx)\\
&-\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\log(1-cx)\,\mathrm{PolyLog}(2,cx)\\
&+\frac{1}{6}c^3(4b+3ac)\log(1-cx)\,\mathrm{PolyLog}(2,1-cx)\\
&-\frac{1}{12}c^3(4b+3ac)\,\mathrm{PolyLog}(3,cx)\\
&-\frac{1}{6}c^3(4b+3ac)\,\mathrm{PolyLog}(3,1-cx)\end{aligned}$$

output
```
1/12*a*c*polylog(2,c*x)/x^3+1/24*c*(3*a*c+4*b)*polylog(2,c*x)/x^2+1/12*c^2
*(3*a*c+4*b)*polylog(2,c*x)/x-5/72*a*c*ln(-c*x+1)/x^3-1/9*b*c*ln(-c*x+1)/x
^2-1/16*a*c^2*ln(-c*x+1)/x^2-1/48*c*(3*a*c+4*b)*ln(-c*x+1)/x^2-2/9*b*c^2*l
n(-c*x+1)/x-1/8*a*c^3*ln(-c*x+1)/x-1/12*c^2*(3*a*c+4*b)*ln(-c*x+1)/x+1/12*
c^3*(3*a*c+4*b)*ln(c*x)*ln(-c*x+1)^2+1/12*c^3*(3*a*c+4*b)*ln(-c*x+1)*polyl
og(2,c*x)+1/6*c^3*(3*a*c+4*b)*ln(-c*x+1)*polylog(2,-c*x+1)+1/16*a*ln(-c*x+
1)^2/x^4+1/9*b*ln(-c*x+1)^2/x^3-1/12*(3*a/x^4+4*b/x^3)*ln(-c*x+1)*polylog(
2,c*x)+5/144*a*c^2/x^2+1/9*b*c^2/x+19/144*a*c^3/x+1/48*c^2*(3*a*c+4*b)/x-2
/9*b*c^3*polylog(2,c*x)-1/8*a*c^4*polylog(2,c*x)-1/12*c^3*(3*a*c+4*b)*poly
log(3,c*x)-1/6*c^3*(3*a*c+4*b)*polylog(3,-c*x+1)-1/3*b*c^3*ln(x)-37/144*a*
c^4*ln(x)-5/48*c^3*(3*a*c+4*b)*ln(x)+1/3*b*c^3*ln(-c*x+1)+37/144*a*c^4*ln(
-c*x+1)+5/48*c^3*(3*a*c+4*b)*ln(-c*x+1)-1/9*b*c^3*ln(-c*x+1)^2-1/16*a*c^4*
ln(-c*x+1)^2
```

3.191.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)\log(1-cx)\,\mathrm{PolyLog}(2,cx)}{x^5}\,dx =$$

$$-\frac{-5ac^2x^2-28bc^2x^3-28ac^3x^3+28bc^3x^4+33ac^4x^4+108bc^3x^4\log(cx)+82ac^4x^4\log(cx)+10acx\log($$

input
```
Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

output
```
-1/144*(-5*a*c^2*x^2 - 28*b*c^2*x^3 - 28*a*c^3*x^3 + 28*b*c^3*x^4 + 33*a*c
^4*x^4 + 108*b*c^3*x^4*Log[c*x] + 82*a*c^4*x^4*Log[c*x] + 10*a*c*x*Log[1 -
 c*x] + 28*b*c*x^2*Log[1 - c*x] + 18*a*c^2*x^2*Log[1 - c*x] + 80*b*c^2*x^3
*Log[1 - c*x] + 54*a*c^3*x^3*Log[1 - c*x] - 108*b*c^3*x^4*Log[1 - c*x] - 8
2*a*c^4*x^4*Log[1 - c*x] - 32*b*c^3*x^4*Log[c*x]*Log[1 - c*x] - 18*a*c^4*x
^4*Log[c*x]*Log[1 - c*x] - 9*a*Log[1 - c*x]^2 - 16*b*x*Log[1 - c*x]^2 + 16
*b*c^3*x^4*Log[1 - c*x]^2 + 9*a*c^4*x^4*Log[1 - c*x]^2 - 48*b*c^3*x^4*Log[
c*x]*Log[1 - c*x]^2 - 36*a*c^4*x^4*Log[c*x]*Log[1 - c*x]^2 - 6*(c*x*(4*b*x
*(1 + 2*c*x) + a*(2 + 3*c*x + 6*c^2*x^2)) + (8*b*x*(-1 + c^3*x^3) + 6*a*(-
1 + c^4*x^4))*Log[1 - c*x])*PolyLog[2, c*x] - 2*c^3*x^4*(16*b + 9*a*c + 12
*(4*b + 3*a*c)*Log[1 - c*x])*PolyLog[2, 1 - c*x] + 48*b*c^3*x^4*PolyLog[3,
 c*x] + 36*a*c^4*x^4*PolyLog[3, c*x] + 96*b*c^3*x^4*PolyLog[3, 1 - c*x] +
72*a*c^4*x^4*PolyLog[3, 1 - c*x])/x^4
```

3.191.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\,\text{PolyLog}(2,cx)\log(1-cx)}{x^5}\,dx$$

$$\Big\downarrow \; 7160$$

$$c\int \left(-\frac{(4b+3ac)\,\text{PolyLog}(2,cx)c^3}{12(1-cx)}-\frac{(4b+3ac)\,\text{PolyLog}(2,cx)c^2}{12x}-\frac{(4b+3ac)\,\text{PolyLog}(2,cx)c}{12x^2}-\frac{(4b+3ac)\,\text{Po}}{12x}\right.$$
$$\int \left(-\frac{b\log^2(1-cx)}{3x^4}-\frac{a\log^2(1-cx)}{4x^5}\right)dx-\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\text{PolyLog}(2,cx)\log(1-cx)$$

$$\Big\downarrow \; 2009$$

$$c\left(-\frac{1}{12}c^2(3ac+4b)\,\text{PolyLog}(3,cx)-\frac{1}{6}c^2(3ac+4b)\,\text{PolyLog}(3,1-cx)+\frac{1}{12}c^2(3ac+4b)\,\text{PolyLog}(2,cx)\log(1-c\right.$$
$$\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\text{PolyLog}(2,cx)\log(1-cx)-\frac{1}{8}ac^4\,\text{PolyLog}(2,cx)-\frac{1}{16}ac^4\log^2(1-cx)-$$
$$\frac{11}{48}ac^4\log(x)+\frac{11}{48}ac^4\log(1-cx)+\frac{5ac^3}{48x}-\frac{ac^3\log(1-cx)}{8x}+\frac{ac^2}{48x^2}-\frac{ac^2\log(1-cx)}{16x^2}+$$
$$\frac{a\log^2(1-cx)}{16x^4}-\frac{ac\log(1-cx)}{24x^3}-\frac{2}{9}bc^3\,\text{PolyLog}(2,cx)-\frac{1}{9}bc^3\log^2(1-cx)-\frac{1}{3}bc^3\log(x)+$$
$$\frac{1}{3}bc^3\log(1-cx)+\frac{bc^2}{9x}-\frac{2bc^2\log(1-cx)}{9x}+\frac{b\log^2(1-cx)}{9x^3}-\frac{bc\log(1-cx)}{9x^2}$$

input
```
Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

output
```
(a*c^2)/(48*x^2) + (b*c^2)/(9*x) + (5*a*c^3)/(48*x) - (b*c^3*Log[x])/3 - (
11*a*c^4*Log[x])/48 + (b*c^3*Log[1 - c*x])/3 + (11*a*c^4*Log[1 - c*x])/48
- (a*c*Log[1 - c*x])/(24*x^3) - (b*c*Log[1 - c*x])/(9*x^2) - (a*c^2*Log[1
- c*x])/(16*x^2) - (2*b*c^2*Log[1 - c*x])/(9*x) - (a*c^3*Log[1 - c*x])/(8*
x) - (b*c^3*Log[1 - c*x]^2)/9 - (a*c^4*Log[1 - c*x]^2)/16 + (a*Log[1 - c*x
]^2)/(16*x^4) + (b*Log[1 - c*x]^2)/(9*x^3) - (2*b*c^3*PolyLog[2, c*x])/9 -
 (a*c^4*PolyLog[2, c*x])/8 - (((3*a)/x^4 + (4*b)/x^3)*Log[1 - c*x]*PolyLog
[2, c*x])/12 + c*((a*c)/(72*x^2) + (a*c^2)/(36*x) + (c*(4*b + 3*a*c))/(48*
x) - (a*c^3*Log[x])/36 - (5*c^2*(4*b + 3*a*c)*Log[x])/48 + (a*c^3*Log[1 -
c*x])/36 + (5*c^2*(4*b + 3*a*c)*Log[1 - c*x])/48 - (a*Log[1 - c*x])/(36*x^
3) - ((4*b + 3*a*c)*Log[1 - c*x])/(48*x^2) - (c*(4*b + 3*a*c)*Log[1 - c*x]
)/(12*x) + (c^2*(4*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/12 + (a*PolyLog[2,
c*x])/(12*x^3) + ((4*b + 3*a*c)*PolyLog[2, c*x])/(24*x^2) + (c*(4*b + 3*a*
c)*PolyLog[2, c*x])/(12*x) + (c^2*(4*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*
x])/12 + (c^2*(4*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/6 - (c^2*(4*
b + 3*a*c)*PolyLog[3, c*x])/12 - (c^2*(4*b + 3*a*c)*PolyLog[3, 1 - c*x])/6
)
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.191.4 Maple [F]

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^5}dx$$

input
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

output
```
int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

3.191.5 Fricas [F]

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^5}\,dx=\int \frac{(bx+a)\operatorname{Li}_2(cx)\log(-cx+1)}{x^5}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

output
```
integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

output
```
Timed out
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.58

$$\begin{aligned}&\int \frac{(a+bx)\log(1-cx)\operatorname{PolyLog}(2,cx)}{x^5}\,dx\\&=\frac{1}{12}\left(3\,ac^4+4\,bc^3\right)\left(\log(cx)\log(-cx+1)^2+2\operatorname{Li}_2(-cx+1)\log(-cx+1)-2\operatorname{Li}_3(-cx+1)\right)\\&\quad+\frac{1}{72}\left(9\,ac^4+16\,bc^3\right)\left(\log(cx)\log(-cx+1)+\operatorname{Li}_2(-cx+1)\right)\\&\quad-\frac{1}{72}\left(41\,ac^4+54\,bc^3\right)\log(x)-\frac{1}{12}\left(3\,ac^4+4\,bc^3\right)\operatorname{Li}_3(cx)\\&\quad+\frac{5\,ac^2x^2+28\,(ac^3+bc^2)x^3-((9\,ac^4+16\,bc^3)x^4-16\,bx-9\,a)\log(-cx+1)^2+6\,(2\,(3\,ac^3+4\,bc^2)x^3+\cdots}{\cdots}\end{aligned}$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

output
```
1/12*(3*a*c^4 + 4*b*c^3)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log
(-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/72*(9*a*c^4 + 16*b*c^3)*(log(c*x)
*log(-c*x + 1) + dilog(-c*x + 1)) - 1/72*(41*a*c^4 + 54*b*c^3)*log(x) - 1/
12*(3*a*c^4 + 4*b*c^3)*polylog(3, c*x) + 1/144*(5*a*c^2*x^2 + 28*(a*c^3 +
b*c^2)*x^3 - ((9*a*c^4 + 16*b*c^3)*x^4 - 16*b*x - 9*a)*log(-c*x + 1)^2 + 6
*(2*(3*a*c^3 + 4*b*c^2)*x^3 + 2*a*c*x + (3*a*c^2 + 4*b*c)*x^2 + 2*((3*a*c^
4 + 4*b*c^3)*x^4 - 4*b*x - 3*a)*log(-c*x + 1))*dilog(c*x) + 2*((41*a*c^4 +
 54*b*c^3)*x^4 - (27*a*c^3 + 40*b*c^2)*x^3 - 5*a*c*x - (9*a*c^2 + 14*b*c)*
x^2)*log(-c*x + 1))/x^4
```

3.191.8 Giac [F]

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^5}\,dx$$

input
```
integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\log(1-cx)\,\text{PolyLog}(2,cx)}{x^5}\,dx = \int \frac{\ln(1-c\,x)\;\text{polylog}(2,c\,x)\;(a+b\,x)}{x^5}\,dx$$

input
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^5,x)
```

output
```
int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^5, x)
```

3.192 $\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx$

3.192.1 Optimal result

Integrand size = 24, antiderivative size = 900

$$\int x(a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)\,dx$$
$$\begin{aligned}
&= \frac{53cx}{192d^3}+\frac{11bx}{27d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5(3c+4bd+6ad^2)\,x}{48d^3}\\
&+\frac{29cx^2}{384d^2}+\frac{5bx^2}{54d}+\frac{(3c+4bd)x^2}{216d^2}+\frac{(3c+4bd+6ad^2)\,x^2}{96d^2}+\frac{2bx^3}{81}\\
&+\frac{17cx^3}{576d}+\frac{(3c+4bd)x^3}{324d}+\frac{3cx^4}{256}+\frac{a(1-dx)^2}{8d^2}+\frac{29c\log(1-dx)}{192d^4}\\
&+\frac{5b\log(1-dx)}{27d^3}+\frac{(3c+4bd)\log(1-dx)}{108d^4}+\frac{(3c+4bd+6ad^2)\log(1-dx)}{48d^4}\\
&-\frac{cx^2\log(1-dx)}{16d^2}-\frac{bx^2\log(1-dx)}{9d}-\frac{(3c+4bd+6ad^2)\,x^2\log(1-dx)}{48d^2}\\
&-\frac{2}{27}bx^3\log(1-dx)-\frac{cx^3\log(1-dx)}{24d}-\frac{(3c+4bd)x^3\log(1-dx)}{108d}\\
&-\frac{3}{64}cx^4\log(1-dx)+\frac{c(1-dx)\log(1-dx)}{8d^4}+\frac{2b(1-dx)\log(1-dx)}{9d^3}\\
&+\frac{a(1-dx)\log(1-dx)}{d^2}+\frac{(3c+4bd+6ad^2)\,(1-dx)\log(1-dx)}{12d^4}\\
&-\frac{a(1-dx)^2\log(1-dx)}{4d^2}-\frac{c\log^2(1-dx)}{16d^4}-\frac{b\log^2(1-dx)}{9d^3}+\frac{1}{9}bx^3\log^2(1-dx)\\
&+\frac{1}{16}cx^4\log^2(1-dx)-\frac{a(1-dx)\log^2(1-dx)}{2d^2}+\frac{a(1-dx)^2\log^2(1-dx)}{4d^2}\\
&-\frac{(3c+4bd+6ad^2)\log(dx)\log^2(1-dx)}{12d^4}-\frac{(3c+4bd+6ad^2)\,x\,\mathrm{PolyLog}(2,dx)}{12d^3}\\
&-\frac{(3c+4bd+6ad^2)\,x^2\,\mathrm{PolyLog}(2,dx)}{24d^2}-\frac{(3c+4bd)x^3\,\mathrm{PolyLog}(2,dx)}{36d}\\
&-\frac{1}{16}cx^4\,\mathrm{PolyLog}(2,dx)-\frac{(3c+4bd+6ad^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)}{12d^4}\\
&+\frac{1}{12}(6ax^2+4bx^3+3cx^4)\log(1-dx)\,\mathrm{PolyLog}(2,dx)\\
&-\frac{(3c+4bd+6ad^2)\log(1-dx)\,\mathrm{PolyLog}(2,1-dx)}{6d^4}\\
&+\frac{(3c+4bd+6ad^2)\,\mathrm{PolyLog}(3,1-dx)}{6d^4}
\end{aligned}$$

output
```
-1/12*(6*a*d^2+4*b*d+3*c)*x*polylog(2,d*x)/d^3-1/24*(6*a*d^2+4*b*d+3*c)*x^
2*polylog(2,d*x)/d^2-1/36*(4*b*d+3*c)*x^3*polylog(2,d*x)/d-1/16*c*x^2*ln(-
d*x+1)/d^2-1/9*b*x^2*ln(-d*x+1)/d-1/48*(6*a*d^2+4*b*d+3*c)*x^2*ln(-d*x+1)/
d^2-1/24*c*x^3*ln(-d*x+1)/d-1/108*(4*b*d+3*c)*x^3*ln(-d*x+1)/d+1/8*c*(-d*x
+1)*ln(-d*x+1)/d^4+2/9*b*(-d*x+1)*ln(-d*x+1)/d^3+1/12*(6*a*d^2+4*b*d+3*c)*
(-d*x+1)*ln(-d*x+1)/d^4-1/4*a*(-d*x+1)^2*ln(-d*x+1)/d^2-1/2*a*(-d*x+1)*ln(
-d*x+1)^2/d^2+1/4*a*(-d*x+1)^2*ln(-d*x+1)^2/d^2-1/12*(6*a*d^2+4*b*d+3*c)*l
n(d*x)*ln(-d*x+1)^2/d^4-1/12*(6*a*d^2+4*b*d+3*c)*ln(-d*x+1)*polylog(2,d*x)
/d^4-1/6*(6*a*d^2+4*b*d+3*c)*ln(-d*x+1)*polylog(2,-d*x+1)/d^4+2/81*b*x^3+3
/256*c*x^4+a*x/d+a*(-d*x+1)*ln(-d*x+1)/d^2+29/192*c*ln(-d*x+1)/d^4+5/27*b*
ln(-d*x+1)/d^3+1/108*(4*b*d+3*c)*ln(-d*x+1)/d^4+1/48*(6*a*d^2+4*b*d+3*c)*l
n(-d*x+1)/d^4-2/27*b*x^3*ln(-d*x+1)-3/64*c*x^4*ln(-d*x+1)-1/16*c*ln(-d*x+1
)^2/d^4-1/9*b*ln(-d*x+1)^2/d^3+1/9*b*x^3*ln(-d*x+1)^2+1/16*c*x^4*ln(-d*x+1
)^2+1/12*(3*c*x^4+4*b*x^3+6*a*x^2)*ln(-d*x+1)*polylog(2,d*x)+53/192*c*x/d^
3+11/27*b*x/d^2+1/108*(4*b*d+3*c)*x/d^3+5/48*(6*a*d^2+4*b*d+3*c)*x/d^3+29/
384*c*x^2/d^2+5/54*b*x^2/d+1/216*(4*b*d+3*c)*x^2/d^2+1/96*(6*a*d^2+4*b*d+3
*c)*x^2/d^2+17/576*c*x^3/d+1/324*(4*b*d+3*c)*x^3/d+1/8*a*(-d*x+1)^2/d^2-1/
16*c*x^4*polylog(2,d*x)+1/6*(6*a*d^2+4*b*d+3*c)*polylog(3,-d*x+1)/d^4
```

3.192.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.65

$$\int x\left(a+bx+cx^2\right)\log(1-dx)\operatorname{PolyLog}(2,dx)\,dx$$
$$=\frac{\frac{355cdx}{4}+124bd^2x+198ad^3x+\frac{139}{8}cd^2x^2+22bd^3x^2+27ad^4x^2+\frac{67}{12}cd^3x^3+\frac{16}{3}bd^4x^3+\frac{27}{16}cd^4x^4+\frac{355}{4}c\log(1}{}$$

input
```
Integrate[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

output
```
((355*c*d*x)/4 + 124*b*d^2*x + 198*a*d^3*x + (139*c*d^2*x^2)/8 + 22*b*d^3*
x^2 + 27*a*d^4*x^2 + (67*c*d^3*x^3)/12 + (16*b*d^4*x^3)/3 + (27*c*d^4*x^4)
/16 + (355*c*Log[1 - d*x])/4 + 124*b*d*Log[1 - d*x] + 198*a*d^2*Log[1 - d*
x] - 54*c*d*x*Log[1 - d*x] - 80*b*d^2*x*Log[1 - d*x] - 144*a*d^3*x*Log[1 -
 d*x] - 18*c*d^2*x^2*Log[1 - d*x] - 28*b*d^3*x^2*Log[1 - d*x] - 54*a*d^4*x
^2*Log[1 - d*x] - 10*c*d^3*x^3*Log[1 - d*x] - 16*b*d^4*x^3*Log[1 - d*x] -
(27*c*d^4*x^4*Log[1 - d*x])/4 - 9*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2
 - 36*a*d^2*Log[1 - d*x]^2 + 36*a*d^4*x^2*Log[1 - d*x]^2 + 16*b*d^4*x^3*Lo
g[1 - d*x]^2 + 9*c*d^4*x^4*Log[1 - d*x]^2 - 36*c*Log[d*x]*Log[1 - d*x]^2 -
 48*b*d*Log[d*x]*Log[1 - d*x]^2 - 72*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d*
x*(3*c*(12 + 6*d*x + 4*d^2*x^2 + 3*d^3*x^3) + 4*d*(9*a*d*(2 + d*x) + 2*b*(
6 + 3*d*x + 2*d^2*x^2)))) + 12*(-4*b*d - 6*a*d^2 + 6*a*d^4*x^2 + 4*b*d^4*x
^3 + 3*c*(-1 + d^4*x^4))*Log[1 - d*x])*PolyLog[2, d*x] - 24*(3*c + 4*b*d +
 6*a*d^2)*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 24*(3*c + 4*b*d + 6*a*d^2)*Po
lyLog[3, 1 - d*x])/(144*d^4)
```

3.192.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{7158, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \operatorname{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{7158}$$

$$d\int\left(-\frac{c\operatorname{PolyLog}(2,dx)x^3}{4d}-\frac{(3c+4bd)\operatorname{PolyLog}(2,dx)x^2}{12d^2}-\frac{(6ad^2+4bd+3c)\operatorname{PolyLog}(2,dx)x}{12d^3}-\frac{(6ad^2+4bd\right.$$

$$\int\left(\frac{1}{4}c\log^2(1-dx)x^3+\frac{1}{3}b\log^2(1-dx)x^2+\frac{1}{2}a\log^2(1-dx)x\right)dx+\frac{1}{12}\operatorname{PolyLog}(2,dx)\log(1-dx)\left(6ax^2+4bx^3+3cx^4\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}c\log^2(1-dx)x^4+\frac{cx^4}{128}-\frac{1}{32}c\log(1-dx)x^4+\frac{1}{9}b\log^2(1-dx)x^3+\frac{2bx^3}{81}-\frac{2}{27}b\log(1-dx)x^3-\frac{c\log(1-dx)x^3}{24d}+\frac{7cx^3}{288d}-\frac{b\log(1-dx)x^2}{9d}-\frac{c\log(1-dx)x^2}{16d^2}+\frac{5bx^2}{54d}+\frac{13cx^2}{192d^2}+\frac{ax}{d}+\frac{11bx}{27d^2}+\frac{25cx}{96d^3}+\frac{a(1-dx)^2}{8d^2}+\frac{a(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{a(1-dx)\log^2(1-dx)}{2d^2}-\frac{b\log^2(1-dx)}{9d^3}-\frac{c\log^2(1-dx)}{16d^4}-\frac{a(1-dx)^2\log(1-dx)}{4d^2}+\frac{a(1-dx)\log(1-dx)}{d^2}+\frac{2b(1-dx)\log(1-dx)}{9d^3}+\frac{c(1-dx)\log(1-dx)}{8d^4}+\frac{5b\log(1-dx)}{27d^3}+\frac{13c\log(1-dx)}{96d^4}+\frac{1}{12}(3cx^4+4bx^3+6ax^2)\log(1-dx)\operatorname{PolyLog}(2,dx)+d\left(-\frac{c\log(1-dx)x^4}{64d}-\frac{c\operatorname{PolyLog}(2,dx)x^4}{16d}+\frac{cx^4}{256d}+\frac{(3c+4bd)x^3}{324d^2}-\frac{(3c+4bd)\log(1-dx)x^3}{108d^2}-\frac{(3c+4bd)\operatorname{Poly}}{36d}\right.$$

input
```
Int[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

output
```
(25*c*x)/(96*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + (13*c*x^2)/(192*d^2) + (
5*b*x^2)/(54*d) + (2*b*x^3)/81 + (7*c*x^3)/(288*d) + (c*x^4)/128 + (a*(1 -
 d*x)^2)/(8*d^2) + (13*c*Log[1 - d*x])/(96*d^4) + (5*b*Log[1 - d*x])/(27*d
^3) - (c*x^2*Log[1 - d*x])/(16*d^2) - (b*x^2*Log[1 - d*x])/(9*d) - (2*b*x^
3*Log[1 - d*x])/27 - (c*x^3*Log[1 - d*x])/(24*d) - (c*x^4*Log[1 - d*x])/32
 + (c*(1 - d*x)*Log[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*Log[1 - d*x])/(9*d^
3) + (a*(1 - d*x)*Log[1 - d*x])/d^2 - (a*(1 - d*x)^2*Log[1 - d*x])/(4*d^2)
 - (c*Log[1 - d*x]^2)/(16*d^4) - (b*Log[1 - d*x]^2)/(9*d^3) + (b*x^3*Log[1
 - d*x]^2)/9 + (c*x^4*Log[1 - d*x]^2)/16 - (a*(1 - d*x)*Log[1 - d*x]^2)/(2
*d^2) + (a*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) + ((6*a*x^2 + 4*b*x^3 + 3*c
*x^4)*Log[1 - d*x]*PolyLog[2, d*x])/12 + d*((c*x)/(64*d^4) + ((3*c + 4*b*d
)*x)/(108*d^4) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^4) + (c*x^2)/(128*d^3
) + ((3*c + 4*b*d)*x^2)/(216*d^3) + ((3*c + 4*b*d + 6*a*d^2)*x^2)/(96*d^3)
 + (c*x^3)/(192*d^2) + ((3*c + 4*b*d)*x^3)/(324*d^2) + (c*x^4)/(256*d) + (
c*Log[1 - d*x])/(64*d^5) + ((3*c + 4*b*d)*Log[1 - d*x])/(108*d^5) + ((3*c
+ 4*b*d + 6*a*d^2)*Log[1 - d*x])/(48*d^5) - ((3*c + 4*b*d + 6*a*d^2)*x^2*L
og[1 - d*x])/(48*d^3) - ((3*c + 4*b*d)*x^3*Log[1 - d*x])/(108*d^2) - (c*x^
4*Log[1 - d*x])/(64*d) + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*Log[1 - d*x])/
(12*d^5) - ((3*c + 4*b*d + 6*a*d^2)*Log[d*x]*Log[1 - d*x]^2)/(12*d^5) - ((
3*c + 4*b*d + 6*a*d^2)*x*PolyLog[2, d*x])/(12*d^4) - ((3*c + 4*b*d + 6*...
```

3.192. $\int x(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)\,dx$

3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

3.192.4 Maple [F]

$$\int x\left(c\,x^{2}+bx+a\right)\ln\left(-dx+1\right)\operatorname{polylog}\left(2,dx\right)dx$$

input
```
int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

output
```
int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

3.192.5 Fricas [F]

$$\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx=\int\left(cx^2+bx+a\right)x\text{Li}_2(dx)\log\left(-dx+1\right)\,dx$$

input
```
integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas"
)
```

output
```
integral((c*x^3 + b*x^2 + a*x)*dilog(d*x)*log(-d*x + 1), x)
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

output
```
Timed out
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.58

$$\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx =$$
$$-\frac{1}{6912}d\left(\frac{576\,(6\,ad^2+4\,bd+3\,c)\left(\log(dx)\log(-dx+1)^2+2\,\text{Li}_2(-dx+1)\log(-dx+1)-2\,\text{Li}_3(-dx+\right.}{d^5}\right.$$
$$+\frac{1}{1728}\left(\frac{216\,(4\,d^2x^2\text{Li}_2(dx)-d^2x^2-2\,dx+2\,(d^2x^2-1)\log(-dx+1))a}{d^2}+\frac{32\,(18\,d^3x^3\text{Li}_2(dx)-2\,d^3x^3\right.$$
$$+1)$$

input
```
integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima"
)
```

output
```
-1/6912*d*(576*(6*a*d^2 + 4*b*d + 3*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog
(-d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^5 - (81*c*d^4*x^4 + 4
*(64*b*d^4 + 67*c*d^3)*x^3 + 6*(216*a*d^4 + 176*b*d^3 + 139*c*d^2)*x^2 + 1
2*(792*a*d^3 + 496*b*d^2 + 355*c*d)*x - 48*(9*c*d^4*x^4 + 4*(4*b*d^4 + 3*c
*d^3)*x^3 + 6*(6*a*d^4 + 4*b*d^3 + 3*c*d^2)*x^2 + 12*(6*a*d^3 + 4*b*d^2 +
3*c*d)*x + 12*(6*a*d^2 + 4*b*d + 3*c)*log(-d*x + 1))*dilog(d*x) - 4*(54*c*
d^4*x^4 + 4*(32*b*d^4 + 21*c*d^3)*x^3 - 2376*a*d^2 + 6*(72*a*d^4 + 40*b*d^
3 + 27*c*d^2)*x^2 - 1488*b*d + 12*(108*a*d^3 + 64*b*d^2 + 45*c*d)*x - 1065
*c)*log(-d*x + 1))/d^5) + 1/1728*(216*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*
d*x + 2*(d^2*x^2 - 1)*log(-d*x + 1))*a/d^2 + 32*(18*d^3*x^3*dilog(d*x) - 2
*d^3*x^3 - 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*b/d^3 + 9*(4
8*d^4*x^4*dilog(d*x) - 3*d^4*x^4 - 4*d^3*x^3 - 6*d^2*x^2 - 12*d*x + 12*(d^
4*x^4 - 1)*log(-d*x + 1))*c/d^4)*log(-d*x + 1)
```

3.192.8 Giac [F]

$$\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx = \int (cx^2+bx+a)x\text{Li}_2(dx)\log(-dx+1)\,dx$$

input
```
integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*x*dilog(d*x)*log(-d*x + 1), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx = \int x\,\ln(1-d\,x)\,\text{polylog}(2,d\,x)\,\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(x*log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2),x)
```

output
```
int(x*log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2), x)
```

3.193 $\int (a+bx+cx^2) \log(1-dx) \operatorname{PolyLog}(2,dx)\,dx$

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 645

$$
\begin{aligned}
&\int (a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)\,dx \\
&= 2ax+\frac{4cx}{9d^2}+\frac{bx}{d}+\frac{(2c+3bd)x}{24d^2}+\frac{(2c+3d(b+2ad))x}{6d^2}+\frac{cx^2}{9d}+\frac{(2c+3bd)x^2}{48d} \\
&+\frac{cx^3}{27}+\frac{b(1-dx)^2}{8d^2}+\frac{2c\log(1-dx)}{9d^3}+\frac{(2c+3bd)\log(1-dx)}{24d^3} \\
&-\frac{cx^2\log(1-dx)}{9d}-\frac{(2c+3bd)x^2\log(1-dx)}{24d}-\frac{1}{9}cx^3\log(1-dx) \\
&+\frac{2c(1-dx)\log(1-dx)}{9d^3}+\frac{b(1-dx)\log(1-dx)}{d^2}+\frac{2a(1-dx)\log(1-dx)}{d} \\
&+\frac{(2c+3d(b+2ad))(1-dx)\log(1-dx)}{6d^3}-\frac{b(1-dx)^2\log(1-dx)}{4d^2}-\frac{c\log^2(1-dx)}{9d^3} \\
&+\frac{1}{9}cx^3\log^2(1-dx)-\frac{b(1-dx)\log^2(1-dx)}{2d^2}-\frac{a(1-dx)\log^2(1-dx)}{d} \\
&+\frac{b(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{(2c+3d(b+2ad))\log(dx)\log^2(1-dx)}{6d^3} \\
&-\frac{(2c+3d(b+2ad))x\operatorname{PolyLog}(2,dx)}{6d^2}-\frac{(2c+3bd)x^2\operatorname{PolyLog}(2,dx)}{12d} \\
&-\frac{1}{9}cx^3\operatorname{PolyLog}(2,dx)-\frac{(2c+3d(b+2ad))\log(1-dx)\operatorname{PolyLog}(2,dx)}{6d^3} \\
&+\frac{1}{6}(6ax+3bx^2+2cx^3)\log(1-dx)\operatorname{PolyLog}(2,dx) \\
&-\frac{(2c+3d(b+2ad))\log(1-dx)\operatorname{PolyLog}(2,1-dx)}{3d^3} \\
&+\frac{(2c+3d(b+2ad))\operatorname{PolyLog}(3,1-dx)}{3d^3}
\end{aligned}
$$

output
```
-1/12*(3*b*d+2*c)*x^2*polylog(2,d*x)/d-1/6*(2*c+3*d*(2*a*d+b))*x*polylog(2
,d*x)/d^2-1/6*(2*c+3*d*(2*a*d+b))*ln(-d*x+1)*polylog(2,d*x)/d^3-1/3*(2*c+3
*d*(2*a*d+b))*ln(-d*x+1)*polylog(2,-d*x+1)/d^3-1/9*c*x^2*ln(-d*x+1)/d+1/27
*c*x^3+2*a*x-1/24*(3*b*d+2*c)*x^2*ln(-d*x+1)/d+2/9*c*(-d*x+1)*ln(-d*x+1)/d
^3+2*a*(-d*x+1)*ln(-d*x+1)/d+1/6*(2*c+3*d*(2*a*d+b))*(-d*x+1)*ln(-d*x+1)/d
^3-1/4*b*(-d*x+1)^2*ln(-d*x+1)/d^2-1/2*b*(-d*x+1)*ln(-d*x+1)^2/d^2+1/4*b*(
-d*x+1)^2*ln(-d*x+1)^2/d^2-1/6*(2*c+3*d*(2*a*d+b))*ln(d*x)*ln(-d*x+1)^2/d^
3+b*x/d+b*(-d*x+1)*ln(-d*x+1)/d^2-a*(-d*x+1)*ln(-d*x+1)^2/d+2/9*c*ln(-d*x+
1)/d^3+1/24*(3*b*d+2*c)*ln(-d*x+1)/d^3-1/9*c*x^3*ln(-d*x+1)-1/9*c*ln(-d*x+
1)^2/d^3+1/9*c*x^3*ln(-d*x+1)^2+1/6*(2*c*x^3+3*b*x^2+6*a*x)*ln(-d*x+1)*pol
ylog(2,d*x)+4/9*c*x/d^2+1/24*(3*b*d+2*c)*x/d^2+1/6*(2*c+3*d*(2*a*d+b))*x/d
^2+1/9*c*x^2/d+1/48*(3*b*d+2*c)*x^2/d+1/8*b*(-d*x+1)^2/d^2-1/9*c*x^3*polyl
og(2,d*x)+1/3*(2*c+3*d*(2*a*d+b))*polylog(3,-d*x+1)/d^3
```

3.193.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.73

$$\int (a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx = \frac{31cdx+\frac{99}{2}bd^2x+108ad^3x+\frac{11}{2}cd^2x^2+\frac{27}{4}bd^3x^2+\frac{4}{3}cd^3x^3+31c\log(1-dx)+\frac{99}{2}bd\log(1-dx)+108ad^2}{}$$

input
```
Integrate[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

output
```
(31*c*d*x + (99*b*d^2*x)/2 + 108*a*d^3*x + (11*c*d^2*x^2)/2 + (27*b*d^3*x^
2)/4 + (4*c*d^3*x^3)/3 + 31*c*Log[1 - d*x] + (99*b*d*Log[1 - d*x])/2 + 108
*a*d^2*Log[1 - d*x] - 20*c*d*x*Log[1 - d*x] - 36*b*d^2*x*Log[1 - d*x] - 10
8*a*d^3*x*Log[1 - d*x] - 7*c*d^2*x^2*Log[1 - d*x] - (27*b*d^3*x^2*Log[1 -
d*x])/2 - 4*c*d^3*x^3*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 9*b*d*Log[1 - d*
x]^2 - 36*a*d^2*Log[1 - d*x]^2 + 36*a*d^3*x*Log[1 - d*x]^2 + 9*b*d^3*x^2*L
og[1 - d*x]^2 + 4*c*d^3*x^3*Log[1 - d*x]^2 - 12*c*Log[d*x]*Log[1 - d*x]^2
- 18*b*d*Log[d*x]*Log[1 - d*x]^2 - 36*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d
*x*(9*d*(2*b + 4*a*d + b*d*x) + 2*c*(6 + 3*d*x + 2*d^2*x^2))) + 6*(-1 + d*
x)*(3*d*(b + 2*a*d + b*d*x) + 2*c*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyL
og[2, d*x] - 12*(2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x] +
 12*(2*c + 3*d*(b + 2*a*d))*PolyLog[3, 1 - d*x])/(36*d^3)
```

3.193.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \ 7158$$

$$d\int \left(-\frac{c\,\text{PolyLog}(2,dx)x^2}{3d}-\frac{(2c+3bd)\,\text{PolyLog}(2,dx)x}{6d^2}-\frac{(2c+3d(b+2ad))\,\text{PolyLog}(2,dx)}{6d^3}+\frac{(2c+3d(b+2a}{6d^3(}\right.$$
$$\int\left(\frac{1}{3}cx^2\log^2(1-dx)+a\log^2(1-dx)+\frac{1}{2}bx\log^2(1-dx)\right)dx+\frac{1}{6}\text{PolyLog}(2,dx)\log(1-dx)\left(6ax+3bx^2+2cx^3\right)$$

$$\Big\downarrow \ 2009$$

$$d\left(\frac{\text{PolyLog}(3,1-dx)(3d(2ad+b)+2c)}{3d^4}-\frac{\text{PolyLog}(2,dx)\log(1-dx)(3d(2ad+b)+2c)}{6d^4}-\frac{\text{PolyLog}(2,1-dx)\,l}{}\right.$$
$$\frac{1}{6}\text{PolyLog}(2,dx)\log(1-dx)\left(6ax+3bx^2+2cx^3\right)-\frac{a(1-dx)\log^2(1-dx)}{d}+$$
$$\frac{2a(1-dx)\log(1-dx)}{d}+2ax+\frac{b(1-dx)^2}{8d^2}+\frac{b(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{b(1-dx)\log^2(1-dx)}{2d^2}-$$
$$\frac{b(1-dx)^2\log(1-dx)}{4d^2}+\frac{b(1-dx)\log(1-dx)}{d^2}+\frac{bx}{d}-\frac{c\log^2(1-dx)}{9d^3}+\frac{2c(1-dx)\log(1-dx)}{9d^3}+$$
$$\frac{5c\log(1-dx)}{27d^3}+\frac{11cx}{27d^2}+\frac{1}{9}cx^3\log^2(1-dx)-\frac{2}{27}cx^3\log(1-dx)+\frac{5cx^2}{54d}-\frac{cx^2\log(1-dx)}{9d}+\frac{2cx^3}{81}$$

input
```
Int[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

output
```
2*a*x + (11*c*x)/(27*d^2) + (b*x)/d + (5*c*x^2)/(54*d) + (2*c*x^3)/81 + (b
*(1 - d*x)^2)/(8*d^2) + (5*c*Log[1 - d*x])/(27*d^3) - (c*x^2*Log[1 - d*x])
/(9*d) - (2*c*x^3*Log[1 - d*x])/27 + (2*c*(1 - d*x)*Log[1 - d*x])/(9*d^3)
+ (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x])/d - (b*(1
- d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1
 - d*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 -
 d*x]^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) + ((6*a*x + 3*b*x^2 +
2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 + d*((c*x)/(27*d^3) + ((2*c + 3*b
*d)*x)/(24*d^3) + ((2*c + 3*d*(b + 2*a*d))*x)/(6*d^3) + (c*x^2)/(54*d^2) +
 ((2*c + 3*b*d)*x^2)/(48*d^2) + (c*x^3)/(81*d) + (c*Log[1 - d*x])/(27*d^4)
 + ((2*c + 3*b*d)*Log[1 - d*x])/(24*d^4) - ((2*c + 3*b*d)*x^2*Log[1 - d*x]
)/(24*d^2) - (c*x^3*Log[1 - d*x])/(27*d) + ((2*c + 3*d*(b + 2*a*d))*(1 - d
*x)*Log[1 - d*x])/(6*d^4) - ((2*c + 3*d*(b + 2*a*d))*Log[d*x]*Log[1 - d*x]
^2)/(6*d^4) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x])/(6*d^3) - ((2*c
+ 3*b*d)*x^2*PolyLog[2, d*x])/(12*d^2) - (c*x^3*PolyLog[2, d*x])/(9*d) - (
(2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^4) - ((2*c + 3*
d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^4) + ((2*c + 3*d*(b
+ 2*a*d))*PolyLog[3, 1 - d*x])/(3*d^4))
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

3.193.4 Maple [F]

$$\int \left(c\,x^{2}+bx+a\right)\ln \left(-dx+1\right)\operatorname{polylog}\left(2,dx\right)dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

3.193.5 Fricas [F]

$$\int \left(a+bx+cx^2\right)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx=\int \left(cx^{2}+bx+a\right)\text{Li}_2(dx)\log\left(-dx+1\right)\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

3.193.6 Sympy [F]

$$\int \left(a+bx+cx^2\right)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx=\int \left(a+bx+cx^{2}\right)\log\left(-dx+1\right)\text{Li}_{2}\left(dx\right)\,dx$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

output
```
Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x), x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.64

$$\int (a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx =$$
$$-\frac{1}{432}d\left(\frac{72\,(6\,ad^2+3\,bd+2\,c)\big(\log(dx)\log(-dx+1)^2+2\,\text{Li}_2(-dx+1)\log(-dx+1)-2\,\text{Li}_3(-dx+1\right.$$
$$+\frac{1}{216}\left(\frac{216\,(dx\text{Li}_2(dx)-dx+(dx-1)\log(-dx+1))a}{d}+\frac{27\,(4\,d^2x^2\text{Li}_2(dx)-d^2x^2-2\,dx+2\,(d^2x^2-\right.$$
$$+1)$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")
```

output
```
-1/432*d*(72*(6*a*d^2 + 3*b*d + 2*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-
d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^4 - (16*c*d^3*x^3 + 3*(
27*b*d^3 + 22*c*d^2)*x^2 + 6*(216*a*d^3 + 99*b*d^2 + 62*c*d)*x - 12*(4*c*d
^3*x^3 + 3*(3*b*d^3 + 2*c*d^2)*x^2 + 6*(6*a*d^3 + 3*b*d^2 + 2*c*d)*x + 6*(
6*a*d^2 + 3*b*d + 2*c)*log(-d*x + 1))*dilog(d*x) - 2*(16*c*d^3*x^3 - 648*a
*d^2 + 6*(9*b*d^3 + 5*c*d^2)*x^2 - 297*b*d + 6*(72*a*d^3 + 27*b*d^2 + 16*c
*d)*x - 186*c)*log(-d*x + 1))/d^4) + 1/216*(216*(d*x*dilog(d*x) - d*x + (d
*x - 1)*log(-d*x + 1))*a/d + 27*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x +
2*(d^2*x^2 - 1)*log(-d*x + 1))*b/d^2 + 4*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^
3 - 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*c/d^3)*log(-d*x + 1
)
```

3.193.8 Giac [F]

$$\int (a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)\,dx = \int (cx^2+bx+a)\text{Li}_2(dx)\log(-dx+1)\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)\,dx$$
$$=\int \ln (1-d\,x)\ \mathrm{polylog}(2,d\,x)\ \left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2),x)
```

output
```
int(log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2), x)
```

3.194 $\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x}\,dx$

3.194.1 Optimal result

Integrand size = 26, antiderivative size = 402

$$\begin{aligned}
&\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x}\,dx \\
&= 2bx+\frac{9cx}{8d}+\frac{(c+2bd)x}{2d}+\frac{cx^2}{16}+\frac{c(1-dx)^2}{8d^2}+\frac{c\log(1-dx)}{8d^2}-\frac{1}{8}cx^2\log(1-dx) \\
&+\frac{c(1-dx)\log(1-dx)}{d^2}+\frac{2b(1-dx)\log(1-dx)}{d}+\frac{(c+2bd)(1-dx)\log(1-dx)}{2d^2} \\
&-\frac{c(1-dx)^2\log(1-dx)}{4d^2}-\frac{c(1-dx)\log^2(1-dx)}{2d^2}-\frac{b(1-dx)\log^2(1-dx)}{d} \\
&+\frac{c(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{(c+2bd)\log(dx)\log^2(1-dx)}{2d^2}-\frac{(c+2bd)x\operatorname{PolyLog}(2,dx)}{2d} \\
&-\frac{1}{4}cx^2\operatorname{PolyLog}(2,dx)-\frac{(c+2bd)\log(1-dx)\operatorname{PolyLog}(2,dx)}{2d^2} \\
&+\frac{1}{2}\left(2bx+cx^2\right)\log(1-dx)\operatorname{PolyLog}(2,dx)-\frac{1}{2}a\operatorname{PolyLog}(2,dx)^2 \\
&-\frac{(c+2bd)\log(1-dx)\operatorname{PolyLog}(2,1-dx)}{d^2}+\frac{(c+2bd)\operatorname{PolyLog}(3,1-dx)}{d^2}
\end{aligned}$$

output
```
2*b*x+9/8*c*x/d+1/2*(2*b*d+c)*x/d+1/16*c*x^2+1/8*c*(-d*x+1)^2/d^2+1/8*c*ln
(-d*x+1)/d^2-1/8*c*x^2*ln(-d*x+1)+c*(-d*x+1)*ln(-d*x+1)/d^2+2*b*(-d*x+1)*l
n(-d*x+1)/d+1/2*(2*b*d+c)*(-d*x+1)*ln(-d*x+1)/d^2-1/4*c*(-d*x+1)^2*ln(-d*x
+1)/d^2-1/2*c*(-d*x+1)*ln(-d*x+1)^2/d^2-b*(-d*x+1)*ln(-d*x+1)^2/d+1/4*c*(-
d*x+1)^2*ln(-d*x+1)^2/d^2-1/2*(2*b*d+c)*ln(d*x)*ln(-d*x+1)^2/d^2-1/2*(2*b*
d+c)*x*polylog(2,d*x)/d-1/4*c*x^2*polylog(2,d*x)-1/2*(2*b*d+c)*ln(-d*x+1)*
polylog(2,d*x)/d^2+1/2*(c*x^2+2*b*x)*ln(-d*x+1)*polylog(2,d*x)-1/2*a*polyl
og(2,d*x)^2-(2*b*d+c)*ln(-d*x+1)*polylog(2,-d*x+1)/d^2+(2*b*d+c)*polylog(3
,-d*x+1)/d^2
```

3.194.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x}\,dx$$
$$= \frac{-14c-32bd+22cdx+48bd^2x+3cd^2x^2+22c\log(1-dx)+48bd\log(1-dx)-16cdx\log(1-dx)-48}{}$$

input
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

output
```
(-14*c - 32*b*d + 22*c*d*x + 48*b*d^2*x + 3*c*d^2*x^2 + 22*c*Log[1 - d*x]
+ 48*b*d*Log[1 - d*x] - 16*c*d*x*Log[1 - d*x] - 48*b*d^2*x*Log[1 - d*x] -
6*c*d^2*x^2*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2 + 16
*b*d^2*x*Log[1 - d*x]^2 + 4*c*d^2*x^2*Log[1 - d*x]^2 - 8*c*Log[d*x]*Log[1
- d*x]^2 - 16*b*d*Log[d*x]*Log[1 - d*x]^2 + 4*(-(d*x*(2*c + 4*b*d + c*d*x)
) + 2*(-1 + d*x)*(c + 2*b*d + c*d*x)*Log[1 - d*x])*PolyLog[2, d*x] - 8*a*d
^2*PolyLog[2, d*x]^2 - 16*(c + 2*b*d)*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 1
6*c*PolyLog[3, 1 - d*x] + 32*b*d*PolyLog[3, 1 - d*x])/(16*d^2)
```

3.194.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {7159, 9, 7155, 7158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)}{x}dx$$

$$\Big\downarrow \ 7159$$

$$a\int \frac{\log(1-dx)\,\text{PolyLog}(2,dx)}{x}dx+\int \frac{\left(cx^2+bx\right)\log(1-dx)\,\text{PolyLog}(2,dx)}{x}dx$$

$$\Big\downarrow \ 9$$

$$a\int \frac{\log(1-dx)\,\text{PolyLog}(2,dx)}{x}dx+\int (b+cx)\log(1-dx)\,\text{PolyLog}(2,dx)dx$$

$$\Big\downarrow \ 7155$$

$$\int (b+cx)\log(1-dx)\,\text{PolyLog}(2,dx)dx-\frac{1}{2}a\,\text{PolyLog}(2,dx)^2$$

$$\Big\downarrow \ 7158$$

$$\int \left(b\log^2(1-dx)+\frac{1}{2}cx\log^2(1-dx)+\frac{b^2\log^2(1-dx)}{2cx}\right)dx+ d\int \left(\frac{\text{PolyLog}(2,dx)(c+bd)^2}{2cd^2(1-dx)}-\frac{(c+2bd)\,\text{PolyLog}(2,dx)}{2d^2}-\frac{cx\,\text{PolyLog}(2,dx)}{2d}\right)dx- \frac{1}{2}a\,\text{PolyLog}(2,dx)^2+\frac{(b+cx)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2c}$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{2}a\,\text{PolyLog}(2,dx)^2-\frac{b^2\,\text{PolyLog}(3,1-dx)}{c}+\frac{b^2\,\text{PolyLog}(2,1-dx)\log(1-dx)}{c}+ \frac{b^2\log(dx)\log^2(1-dx)}{2c}+ d\left(\frac{(bd+c)^2\,\text{PolyLog}(3,1-dx)}{cd^3}-\frac{(bd+c)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2cd^3}-\frac{(bd+c)^2\,\text{PolyLog}(2,1-dx)\log(1-dx}{cd^3}\right. \frac{(b+cx)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2c}-\frac{b(1-dx)\log^2(1-dx)}{d}+\frac{2b(1-dx)\log(1-dx)}{d}+2bx+ \frac{c(1-dx)^2}{8d^2}+\frac{c(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{c(1-dx)\log^2(1-dx)}{2d^2}-\frac{c(1-dx)^2\log(1-dx)}{4d^2}+ \frac{c(1-dx)\log(1-dx)}{d^2}+\frac{cx}{d}$$

input
```
Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

3.194. $\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x}dx$

output
```
2*b*x + (c*x)/d + (c*(1 - d*x)^2)/(8*d^2) + (c*(1 - d*x)*Log[1 - d*x])/d^2
 + (2*b*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) -
 (c*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (
c*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) + (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*c
) + ((b + c*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*c) - (a*PolyLog[2, d*x]^
2)/2 + (b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/c - (b^2*PolyLog[3, 1 - d*x]
)/c + d*((c*x)/(8*d^2) + ((c + 2*b*d)*x)/(2*d^2) + (c*x^2)/(16*d) + (c*Log
[1 - d*x])/(8*d^3) - (c*x^2*Log[1 - d*x])/(8*d) + ((c + 2*b*d)*(1 - d*x)*L
og[1 - d*x])/(2*d^3) - ((c + b*d)^2*Log[d*x]*Log[1 - d*x]^2)/(2*c*d^3) - (
(c + 2*b*d)*x*PolyLog[2, d*x])/(2*d^2) - (c*x^2*PolyLog[2, d*x])/(4*d) - (
(c + b*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*c*d^3) - ((c + b*d)^2*Log[1 -
 d*x]*PolyLog[2, 1 - d*x])/(c*d^3) + ((c + b*d)^2*PolyLog[3, 1 - d*x])/(c*
d^3))
```

3.194.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

rule 7158
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Simp[b   Int[Exp
andIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x),
x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d
 + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px,
 x]
```

rule 7159
```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Simp[Coeff[Px, x, -m - 1]   Int[(g + h*Log[1 + e*x])*(Pol
yLog[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
 + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[P
x, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

3.194.4 Maple [F]

$$\int \frac{(c\,x^2 + bx + a)\ln(-dx + 1)\,\mathrm{polylog}(2, dx)}{x}dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

3.194.5 Fricas [F]

$$\int \frac{(a + bx + cx^2)\log(1 - dx)\,\mathrm{PolyLog}(2, dx)}{x}\,dx = \int \frac{(cx^2 + bx + a)\mathrm{Li}_2(dx)\log(-dx + 1)}{x}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="fricas"
)
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

3.194.6 Sympy [F]

$$\int \frac{(a + bx + cx^2)\log(1 - dx)\,\mathrm{PolyLog}(2, dx)}{x}\,dx$$
$$= \int \frac{(a + bx + cx^2)\log(-dx + 1)\,\mathrm{Li}_2(dx)}{x}\,dx$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

output
```
Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x, x)
```

3.194.7 Maxima [F]

$$\int \frac{(a + bx + cx^2) \log(1 - dx) \operatorname{PolyLog}(2, dx)}{x} \, dx = \int \frac{(cx^2 + bx + a)\operatorname{Li}_2(dx) \log(-dx + 1)}{x} \, dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="maxima"
)
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

3.194.8 Giac [F]

$$\int \frac{(a + bx + cx^2) \log(1 - dx) \operatorname{PolyLog}(2, dx)}{x} \, dx = \int \frac{(cx^2 + bx + a)\operatorname{Li}_2(dx) \log(-dx + 1)}{x} \, dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2) \log(1 - dx) \operatorname{PolyLog}(2, dx)}{x} \, dx = \int \frac{\ln(1 - d\,x)\ \operatorname{polylog}(2, d\,x)\ (c\,x^2 + b\,x + a)}{x} \, dx$$

input
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x,x)
```

output
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x, x)
```

3.195 $\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx$

3.195.1 Optimal result

Integrand size = 26, antiderivative size = 218

$$\begin{aligned}
&\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx\\
&= 3cx+\frac{3c(1-dx)\log(1-dx)}{d}-\frac{c(1-dx)\log^2(1-dx)}{d}+\frac{a(1-dx)\log^2(1-dx)}{x}\\
&\quad+\left(a-\frac{c}{d^2}\right)d\log(dx)\log^2(1-dx)-2ad\operatorname{PolyLog}(2,dx)-cx\operatorname{PolyLog}(2,dx)\\
&\quad+\left(a-\frac{c}{d^2}\right)d\log(1-dx)\operatorname{PolyLog}(2,dx)-\left(\frac{a}{x}-cx\right)\log(1-dx)\operatorname{PolyLog}(2,dx)\\
&\quad-\frac{1}{2}b\operatorname{PolyLog}(2,dx)^2+2\left(a-\frac{c}{d^2}\right)d\log(1-dx)\operatorname{PolyLog}(2,1-dx)\\
&\quad-ad\operatorname{PolyLog}(3,dx)-2\left(a-\frac{c}{d^2}\right)d\operatorname{PolyLog}(3,1-dx)
\end{aligned}$$

output
```
3*c*x+3*c*(-d*x+1)*ln(-d*x+1)/d-c*(-d*x+1)*ln(-d*x+1)^2/d+a*(-d*x+1)*ln(-d
*x+1)^2/x+(a-c/d^2)*d*ln(d*x)*ln(-d*x+1)^2-2*a*d*polylog(2,d*x)-c*x*polylo
g(2,d*x)+(a-c/d^2)*d*ln(-d*x+1)*polylog(2,d*x)-(a/x-c*x)*ln(-d*x+1)*polylo
g(2,d*x)-1/2*b*polylog(2,d*x)^2+2*(a-c/d^2)*d*ln(-d*x+1)*polylog(2,-d*x+1)
-a*d*polylog(3,d*x)-2*(a-c/d^2)*d*polylog(3,-d*x+1)
```

3.195.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx$$
$$= \frac{2(-cdx^2+(ad+cx)(-1+dx)\log(1-dx))\operatorname{PolyLog}(2,dx)-bdx\operatorname{PolyLog}(2,dx)^2+2(-2cx+3cdx^2+}{}$$

input
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

output
```
(2*(-(c*d*x^2) + (a*d + c*x)*(-1 + d*x)*Log[1 - d*x])*PolyLog[2, d*x] - b*
d*x*PolyLog[2, d*x]^2 + 2*(-2*c*x + 3*c*d*x^2 + 3*c*x*Log[1 - d*x] - 3*c*d
*x^2*Log[1 - d*x] + 2*a*d^2*x*Log[d*x]*Log[1 - d*x] + a*d*Log[1 - d*x]^2 -
 c*x*Log[1 - d*x]^2 - a*d^2*x*Log[1 - d*x]^2 + c*d*x^2*Log[1 - d*x]^2 - c*
x*Log[d*x]*Log[1 - d*x]^2 + a*d^2*x*Log[d*x]*Log[1 - d*x]^2 + 2*x*(a*d^2 +
 (-c + a*d^2)*Log[1 - d*x])*PolyLog[2, 1 - d*x] - a*d^2*x*PolyLog[3, d*x]
+ 2*c*x*PolyLog[3, 1 - d*x] - 2*a*d^2*x*PolyLog[3, 1 - d*x]))/(2*d*x)
```

3.195.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7159, 7155, 7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)}{x^2}\,dx$$
$$\Big\downarrow \text{7159}$$
$$\int \frac{\left(cx^2+a\right)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}dx+b\int \frac{\log(1-dx)\operatorname{PolyLog}(2,dx)}{x}dx$$
$$\Big\downarrow \text{7155}$$
$$\int \frac{\left(cx^2+a\right)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}dx-\frac{1}{2}b\operatorname{PolyLog}(2,dx)^2$$
$$\Big\downarrow \text{7160}$$

$$d\int \left(-\frac{c\,\text{PolyLog}(2,dx)}{d}-\frac{a\,\text{PolyLog}(2,dx)}{x}+\frac{(c-ad^2)\,\text{PolyLog}(2,dx)}{d(1-dx)}\right)dx+\int \left(c\log^2(1-dx)-\frac{a\log^2(1-dx)}{x^2}\right)dx-\left(\frac{a}{x}-cx\right)\text{PolyLog}(2,dx)\log(1-dx)-\frac{1}{2}b\,\text{PolyLog}(2,dx)^2$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(\frac{2(c-ad^2)\,\text{PolyLog}(3,1-dx)}{d^2}-\frac{(c-ad^2)\,\text{PolyLog}(2,dx)\log(1-dx)}{d^2}-\frac{2(c-ad^2)\,\text{PolyLog}(2,1-dx)\log(1\right.$$
$$\left(\frac{a}{x}-cx\right)\text{PolyLog}(2,dx)\log(1-dx)-2ad\,\text{PolyLog}(2,dx)+\frac{a(1-dx)\log^2(1-dx)}{x}-\frac{1}{2}b\,\text{PolyLog}(2,dx)^2-\frac{c(1-dx)\log^2(1-dx)}{d}+\frac{2c(1-dx)\log(1-dx)}{d}+2cx$$

input
```
Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

output
```
2*c*x + (2*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d +
(a*(1 - d*x)*Log[1 - d*x]^2)/x - 2*a*d*PolyLog[2, d*x] - (a/x - c*x)*Log[1
 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^2)/2 + d*((c*x)/d + (c*(1 - d
*x)*Log[1 - d*x])/d^2 - ((c - a*d^2)*Log[d*x]*Log[1 - d*x]^2)/d^2 - (c*x*P
olyLog[2, d*x])/d - ((c - a*d^2)*Log[1 - d*x]*PolyLog[2, d*x])/d^2 - (2*(c
 - a*d^2)*Log[1 - d*x]*PolyLog[2, 1 - d*x])/d^2 - a*PolyLog[3, d*x] + (2*(
c - a*d^2)*PolyLog[3, 1 - d*x])/d^2)
```

3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

rule 7159
```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Simp[Coeff[Px, x, -m - 1]   Int[(g + h*Log[1 + e*x])*(Pol
yLog[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
 + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[P
x, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.195.4 Maple [F]

$$\int \frac{(c\,x^{2}+bx+a)\ln\left(-dx+1\right)\operatorname{polylog}\left(2,dx\right)}{x^{2}}dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

3.195.5 Fricas [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^2}\,dx=\int \frac{(cx^2+bx+a)\text{Li}_2(dx)\log\left(-dx+1\right)}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="frica
s")
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

3.195.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^2}\,dx =\int \frac{(a+bx+cx^2)\log\left(-dx+1\right)\text{Li}_2\left(dx\right)}{x^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**2,x)
```

output
```
Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x**2, x)
```

3.195.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx = \int \frac{(cx^2+bx+a)\mathrm{Li}_2(dx)\log(-dx+1)}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="maxim
a")
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

3.195.8 Giac [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx = \int \frac{(cx^2+bx+a)\mathrm{Li}_2(dx)\log(-dx+1)}{x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="giac"
)
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^2}\,dx = \int \frac{\ln(1-d\,x)\ \operatorname{polylog}(2,d\,x)\ (c\,x^2+b\,x+a)}{x^2}\,dx$$

input
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^2,x)
```

output
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^2, x)
```

3.196 $\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^3}\,dx$

3.196.1 Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}
&\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^3}\,dx \\
&= -ad^2\log(x)+ad^2\log(1-dx)-\frac{ad\log(1-dx)}{x}-\frac{1}{4}ad^2\log^2(1-dx) \\
&\quad+\frac{a\log^2(1-dx)}{4x^2}+\frac{b(1-dx)\log^2(1-dx)}{x}-\frac{b^2\log(dx)\log^2(1-dx)}{2a} \\
&\quad+\frac{(b+ad)^2\log(dx)\log^2(1-dx)}{2a}-2bd\operatorname{PolyLog}(2,dx)-\frac{1}{2}ad^2\operatorname{PolyLog}(2,dx) \\
&\quad+\frac{ad\operatorname{PolyLog}(2,dx)}{2x}+\frac{(b+ad)^2\log(1-dx)\operatorname{PolyLog}(2,dx)}{2a} \\
&\quad-\frac{(a+bx)^2\log(1-dx)\operatorname{PolyLog}(2,dx)}{2ax^2}-\frac{1}{2}c\operatorname{PolyLog}(2,dx)^2 \\
&\quad-\frac{b^2\log(1-dx)\operatorname{PolyLog}(2,1-dx)}{a}+\frac{(b+ad)^2\log(1-dx)\operatorname{PolyLog}(2,1-dx)}{a} \\
&\quad-\frac{1}{2}d(2b+ad)\operatorname{PolyLog}(3,dx)+\frac{b^2\operatorname{PolyLog}(3,1-dx)}{a}-\frac{(b+ad)^2\operatorname{PolyLog}(3,1-dx)}{a}
\end{aligned}$$

output
```
-a*d^2*ln(x)+a*d^2*ln(-d*x+1)-a*d*ln(-d*x+1)/x-1/4*a*d^2*ln(-d*x+1)^2+1/4*
a*ln(-d*x+1)^2/x^2+b*(-d*x+1)*ln(-d*x+1)^2/x-1/2*b^2*ln(d*x)*ln(-d*x+1)^2/
a+1/2*(a*d+b)^2*ln(d*x)*ln(-d*x+1)^2/a-2*b*d*polylog(2,d*x)-1/2*a*d^2*poly
log(2,d*x)+1/2*a*d*polylog(2,d*x)/x+1/2*(a*d+b)^2*ln(-d*x+1)*polylog(2,d*x
)/a-1/2*(b*x+a)^2*ln(-d*x+1)*polylog(2,d*x)/a/x^2-1/2*c*polylog(2,d*x)^2-b
^2*ln(-d*x+1)*polylog(2,-d*x+1)/a+(a*d+b)^2*ln(-d*x+1)*polylog(2,-d*x+1)/a
-1/2*d*(a*d+2*b)*polylog(3,d*x)+b^2*polylog(3,-d*x+1)/a-(a*d+b)^2*polylog(
3,-d*x+1)/a
```

3.196.2 Mathematica [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}\,dx$$

input
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```

output
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3, x]
```

3.196.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7159, 7155, 7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)}{x^3}\,dx$$

$$\Big\downarrow \text{7159}$$

$$\int \frac{(a+bx)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}dx + c\int \frac{\log(1-dx)\,\text{PolyLog}(2,dx)}{x}dx$$

$$\Big\downarrow \text{7155}$$

$$\int \frac{(a+bx)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}dx - \frac{1}{2}c\,\text{PolyLog}(2,dx)^2$$

$$\Big\downarrow \text{7160}$$

$$\int \left(-\frac{b^2 \log^2(1-dx)}{2ax} - \frac{b \log^2(1-dx)}{x^2} - \frac{a \log^2(1-dx)}{2x^3}\right) dx + d\int \left(-\frac{\text{PolyLog}(2,dx)(b+ad)^2}{2a(1-dx)} - \frac{(2b+ad)\,\text{PolyLog}(2,dx)}{2x} - \frac{a\,\text{PolyLog}(2,dx)}{2x^2}\right) dx - \frac{(a+bx)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2ax^2} - \frac{1}{2}c\,\text{PolyLog}(2,dx)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\,\text{PolyLog}(3,1-dx)}{a} - \frac{b^2\,\text{PolyLog}(2,1-dx)\log(1-dx)}{a} - \frac{b^2 \log(dx)\log^2(1-dx)}{2a} - \frac{(a+bx)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2ax^2} + d\left(-\frac{(ad+b)^2\,\text{PolyLog}(3,1-dx)}{ad} - \frac{1}{2}(ad+2b)\,\text{PolyLog}(3,dx) + \frac{(ad+b)^2\,\text{PolyLog}(2,dx)\log(1-dx)}{2ad} + \frac{(ad+b\right. \frac{1}{2}ad^2\,\text{PolyLog}(2,dx) - \frac{1}{4}ad^2\log^2(1-dx) - \frac{1}{2}ad^2\log(x) + \frac{1}{2}ad^2\log(1-dx) + \frac{a\log^2(1-dx)}{4x^2} - \frac{ad\log(1-dx)}{2x} - 2bd\,\text{PolyLog}(2,dx) + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{1}{2}c\,\text{PolyLog}(2,dx)^2$$

input
```
Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```

output
```
-1/2*(a*d^2*Log[x]) + (a*d^2*Log[1 - d*x])/2 - (a*d*Log[1 - d*x])/(2*x) -
(a*d^2*Log[1 - d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1
 - d*x]^2)/x - (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) - 2*b*d*PolyLog[2, d*x]
 - (a*d^2*PolyLog[2, d*x])/2 - ((a + b*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/
(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x
])/a + (b^2*PolyLog[3, 1 - d*x])/a + d*(-1/2*(a*d*Log[x]) + (a*d*Log[1 - d
*x])/2 - (a*Log[1 - d*x])/(2*x) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2)/(2
*a*d) + (a*PolyLog[2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d
*x])/(2*a*d) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(a*d) - ((2*
b + a*d)*PolyLog[3, d*x])/2 - ((b + a*d)^2*PolyLog[3, 1 - d*x])/(a*d))
```

3.196.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7155
```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

3.196. $\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}\,dx$

rule 7159
```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Simp[Coeff[Px, x, -m - 1]   Int[(g + h*Log[1 + e*x])*(Pol
yLog[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
 + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[P
x, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.196.4 Maple [F]

$$\int \frac{(c\,x^2+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)}{x^3}dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

3.196.5 Fricas [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{(cx^2+bx+a)\mathrm{Li}_2(dx)\log(-dx+1)}{x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="frica
s")
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

3.196.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{(a+bx+cx^2)\log(-dx+1)\operatorname{Li}_2(dx)}{x^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**3,x)
```

output
```
Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x**3, x)
```

3.196.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{(cx^2+bx+a)\operatorname{Li}_2(dx)\log(-dx+1)}{x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="maxim
a")
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

3.196.8 Giac [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{(cx^2+bx+a)\operatorname{Li}_2(dx)\log(-dx+1)}{x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="giac"
)
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^3}\,dx = \int \frac{\ln(1-d\,x)\,\text{polylog}(2,d\,x)\,\left(c\,x^2+b\,x+a\right)}{x^3}\,dx$$

input
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^3,x)
```

output
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^3, x)
```

3.197 $\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^4}\,dx$

3.197.1 Optimal result

Integrand size = 26, antiderivative size = 515

$$\begin{aligned}
&\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^4}\,dx \\
&= \frac{7ad^2}{36x} - \frac{1}{2}bd^2\log(x) - \frac{5}{12}ad^3\log(x) - \frac{1}{6}d^2(3b+2ad)\log(x) + \frac{1}{2}bd^2\log(1-dx) \\
&\quad + \frac{5}{12}ad^3\log(1-dx) + \frac{1}{6}d^2(3b+2ad)\log(1-dx) - \frac{7ad\log(1-dx)}{36x^2} \\
&\quad - \frac{bd\log(1-dx)}{2x} - \frac{2ad^2\log(1-dx)}{9x} - \frac{d(3b+2ad)\log(1-dx)}{6x} - \frac{1}{4}bd^2\log^2(1-dx) \\
&\quad - \frac{1}{9}ad^3\log^2(1-dx) + \frac{a\log^2(1-dx)}{9x^3} + \frac{b\log^2(1-dx)}{4x^2} + \frac{c(1-dx)\log^2(1-dx)}{x} \\
&\quad + \frac{1}{6}d(6c+d(3b+2ad))\log(dx)\log^2(1-dx) - 2cd\operatorname{PolyLog}(2,dx) \\
&\quad - \frac{1}{2}bd^2\operatorname{PolyLog}(2,dx) - \frac{2}{9}ad^3\operatorname{PolyLog}(2,dx) + \frac{ad\operatorname{PolyLog}(2,dx)}{6x^2} \\
&\quad + \frac{d(3b+2ad)\operatorname{PolyLog}(2,dx)}{6x} + \frac{1}{6}d(6c+d(3b+2ad))\log(1-dx)\operatorname{PolyLog}(2,dx) \\
&\quad - \frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)\log(1-dx)\operatorname{PolyLog}(2,dx) \\
&\quad + \frac{1}{3}d(6c+d(3b+2ad))\log(1-dx)\operatorname{PolyLog}(2,1-dx) \\
&\quad - \frac{1}{6}d(6c+d(3b+2ad))\operatorname{PolyLog}(3,dx) - \frac{1}{3}d(6c+d(3b+2ad))\operatorname{PolyLog}(3,1-dx)
\end{aligned}$$

output
```
-1/6*d^2*(2*a*d+3*b)*ln(x)+1/2*b*d^2*ln(-d*x+1)+5/12*a*d^3*ln(-d*x+1)+1/6*
d^2*(2*a*d+3*b)*ln(-d*x+1)-1/4*b*d^2*ln(-d*x+1)^2-1/9*a*d^3*ln(-d*x+1)^2+1
/9*a*ln(-d*x+1)^2/x^3+1/4*b*ln(-d*x+1)^2/x^2-1/6*(2*a/x^3+3*b/x^2+6*c/x)*l
n(-d*x+1)*polylog(2,d*x)+7/36*a*d^2/x-2*c*d*polylog(2,d*x)-1/2*b*d^2*polyl
og(2,d*x)-2/9*a*d^3*polylog(2,d*x)-1/6*d*(6*c+d*(2*a*d+3*b))*polylog(3,d*x
)-1/3*d*(6*c+d*(2*a*d+3*b))*polylog(3,-d*x+1)+1/6*a*d*polylog(2,d*x)/x^2+1
/6*d*(2*a*d+3*b)*polylog(2,d*x)/x-7/36*a*d*ln(-d*x+1)/x^2-1/2*b*d*ln(-d*x+
1)/x-2/9*a*d^2*ln(-d*x+1)/x-1/6*d*(2*a*d+3*b)*ln(-d*x+1)/x+1/6*d*(6*c+d*(2
*a*d+3*b))*ln(d*x)*ln(-d*x+1)^2+1/6*d*(6*c+d*(2*a*d+3*b))*ln(-d*x+1)*polyl
og(2,d*x)+1/3*d*(6*c+d*(2*a*d+3*b))*ln(-d*x+1)*polylog(2,-d*x+1)-5/12*a*d^
3*ln(x)-1/2*b*d^2*ln(x)+c*(-d*x+1)*ln(-d*x+1)^2/x
```

3.197.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^4}\,dx \\
&= \frac{1}{36}\Bigg(-7ad^3+\frac{7ad^2}{x}-36bd^2\log(dx)-27ad^3\log(dx)+36bd^2\log(1-dx)+27ad^3\log(1-dx) \\
&\quad -\frac{7ad\log(1-dx)}{x^2}-\frac{36bd\log(1-dx)}{x}-\frac{20ad^2\log(1-dx)}{x}+72cd\log(dx)\log(1-dx) \\
&\quad +18bd^2\log(dx)\log(1-dx)+8ad^3\log(dx)\log(1-dx)-36cd\log^2(1-dx)-9bd^2\log^2(1-dx) \\
&\quad -4ad^3\log^2(1-dx)+\frac{4a\log^2(1-dx)}{x^3}+\frac{9b\log^2(1-dx)}{x^2}+\frac{36c\log^2(1-dx)}{x} \\
&\quad +36cd\log(dx)\log^2(1-dx)+18bd^2\log(dx)\log^2(1-dx)+12ad^3\log(dx)\log^2(1-dx) \\
&\quad +\frac{6(dx(a+3bx+2adx)+(-1+dx)\left(3x(b+2cx+bdx)+2a(1+dx+d^2x^2)\right)\log(1-dx))\operatorname{PolyLog}(2,}{x^3} \\
&\quad +2d\left(36c+9bd+4ad^2+6\left(6c+3bd+2ad^2\right)\log(1-dx)\right)\operatorname{PolyLog}(2,1-dx) \\
&\quad -36cd\operatorname{PolyLog}(3,dx)-18bd^2\operatorname{PolyLog}(3,dx)-12ad^3\operatorname{PolyLog}(3,dx) \\
&\quad -72cd\operatorname{PolyLog}(3,1-dx)-36bd^2\operatorname{PolyLog}(3,1-dx)-24ad^3\operatorname{PolyLog}(3,1-dx)\Bigg)
\end{aligned}
$$

input
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

output
```
(-7*a*d^3 + (7*a*d^2)/x - 36*b*d^2*Log[d*x] - 27*a*d^3*Log[d*x] + 36*b*d^2
*Log[1 - d*x] + 27*a*d^3*Log[1 - d*x] - (7*a*d*Log[1 - d*x])/x^2 - (36*b*d
*Log[1 - d*x])/x - (20*a*d^2*Log[1 - d*x])/x + 72*c*d*Log[d*x]*Log[1 - d*x
] + 18*b*d^2*Log[d*x]*Log[1 - d*x] + 8*a*d^3*Log[d*x]*Log[1 - d*x] - 36*c*
d*Log[1 - d*x]^2 - 9*b*d^2*Log[1 - d*x]^2 - 4*a*d^3*Log[1 - d*x]^2 + (4*a*
Log[1 - d*x]^2)/x^3 + (9*b*Log[1 - d*x]^2)/x^2 + (36*c*Log[1 - d*x]^2)/x +
 36*c*d*Log[d*x]*Log[1 - d*x]^2 + 18*b*d^2*Log[d*x]*Log[1 - d*x]^2 + 12*a*
d^3*Log[d*x]*Log[1 - d*x]^2 + (6*(d*x*(a + 3*b*x + 2*a*d*x) + (-1 + d*x)*(
3*x*(b + 2*c*x + b*d*x) + 2*a*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2
, d*x])/x^3 + 2*d*(36*c + 9*b*d + 4*a*d^2 + 6*(6*c + 3*b*d + 2*a*d^2)*Log[
1 - d*x])*PolyLog[2, 1 - d*x] - 36*c*d*PolyLog[3, d*x] - 18*b*d^2*PolyLog[
3, d*x] - 12*a*d^3*PolyLog[3, d*x] - 72*c*d*PolyLog[3, 1 - d*x] - 36*b*d^2
*PolyLog[3, 1 - d*x] - 24*a*d^3*PolyLog[3, 1 - d*x])/36
```

3.197.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)}{x^4}dx$$

$$\Big\downarrow \text{7160}$$

$$d\int \left(-\frac{(6c+d(3b+2ad))\,\text{PolyLog}(2,dx)}{6x}-\frac{d(6c+d(3b+2ad))\,\text{PolyLog}(2,dx)}{6(1-dx)}-\frac{(3b+2ad)\,\text{PolyLog}(2,dx)}{6x^2}-\right.$$
$$\int \left(-\frac{c\log^2(1-dx)}{x^2}-\frac{b\log^2(1-dx)}{2x^3}-\frac{a\log^2(1-dx)}{3x^4}\right)dx-\frac{1}{6}\text{PolyLog}(2,dx)\log(1-$$
$$dx)\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$d\left(-\frac{1}{6}\operatorname{PolyLog}(3,dx)(d(2ad+3b)+6c)-\frac{1}{3}\operatorname{PolyLog}(3,1-dx)(d(2ad+3b)+6c)+\frac{1}{6}\operatorname{PolyLog}(2,dx)\log(1-dx\right.$$

$$\frac{1}{6}\operatorname{PolyLog}(2,dx)\log(1-dx)\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)-\frac{2}{9}ad^3\operatorname{PolyLog}(2,dx)-\frac{1}{9}ad^3\log^2(1-dx)-$$

$$\frac{1}{3}ad^3\log(x)+\frac{1}{3}ad^3\log(1-dx)+\frac{ad^2}{9x}-\frac{2ad^2\log(1-dx)}{9x}+\frac{a\log^2(1-dx)}{9x^3}-\frac{ad\log(1-dx)}{9x^2}-$$

$$\frac{1}{2}bd^2\operatorname{PolyLog}(2,dx)-\frac{1}{4}bd^2\log^2(1-dx)-\frac{1}{2}bd^2\log(x)+\frac{1}{2}bd^2\log(1-dx)+\frac{b\log^2(1-dx)}{4x^2}-$$

$$\frac{bd\log(1-dx)}{2x}-2cd\operatorname{PolyLog}(2,dx)+\frac{c(1-dx)\log^2(1-dx)}{x}$$

input
```
Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

output
```
(a*d^2)/(9*x) - (b*d^2*Log[x])/2 - (a*d^3*Log[x])/3 + (b*d^2*Log[1 - d*x])
/2 + (a*d^3*Log[1 - d*x])/3 - (a*d*Log[1 - d*x])/(9*x^2) - (b*d*Log[1 - d*
x])/(2*x) - (2*a*d^2*Log[1 - d*x])/(9*x) - (b*d^2*Log[1 - d*x]^2)/4 - (a*d
^3*Log[1 - d*x]^2)/9 + (a*Log[1 - d*x]^2)/(9*x^3) + (b*Log[1 - d*x]^2)/(4*
x^2) + (c*(1 - d*x)*Log[1 - d*x]^2)/x - 2*c*d*PolyLog[2, d*x] - (b*d^2*Pol
yLog[2, d*x])/2 - (2*a*d^3*PolyLog[2, d*x])/9 - (((2*a)/x^3 + (3*b)/x^2 +
(6*c)/x)*Log[1 - d*x]*PolyLog[2, d*x])/6 + d*((a*d)/(12*x) - (a*d^2*Log[x]
)/12 - (d*(3*b + 2*a*d)*Log[x])/6 + (a*d^2*Log[1 - d*x])/12 + (d*(3*b + 2*
a*d)*Log[1 - d*x])/6 - (a*Log[1 - d*x])/(12*x^2) - ((3*b + 2*a*d)*Log[1 -
d*x])/(6*x) + ((6*c + d*(3*b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/6 + (a*Pol
yLog[2, d*x])/(6*x^2) + ((3*b + 2*a*d)*PolyLog[2, d*x])/(6*x) + ((6*c + d*
(3*b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/6 + ((6*c + d*(3*b + 2*a*d))*
Log[1 - d*x]*PolyLog[2, 1 - d*x])/3 - ((6*c + d*(3*b + 2*a*d))*PolyLog[3,
d*x])/6 - ((6*c + d*(3*b + 2*a*d))*PolyLog[3, 1 - d*x])/3)
```

3.197.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.197. $\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^4}\,dx$

3.197.4 Maple [F]

$$\int \frac{(c\,x^2+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)}{x^4}dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

3.197.5 Fricas [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)}{x^4}\,dx = \int \frac{(cx^2+bx+a)\mathrm{Li}_2(dx)\log(-dx+1)}{x^4}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="frica
s")
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

3.197.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)}{x^4}\,dx = \int \frac{(a+bx+cx^2)\log(-dx+1)\,\mathrm{Li}_2(dx)}{x^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**4,x)
```

output
```
Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x**4, x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(2\,ad^3+3\,bd^2+6\,cd\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^2+2\,\text{Li}_2(-dx+1)\log\left(-dx+1\right)-2\,\text{Li}_3(-dx+1)\right)$$
$$+\frac{1}{18}\left(4\,ad^3+9\,bd^2+36\,cd\right)\left(\log\left(dx\right)\log\left(-dx+1\right)+\text{Li}_2(-dx+1)\right)$$
$$-\frac{1}{4}\left(3\,ad^3+4\,bd^2\right)\log\left(x\right)-\frac{1}{6}\left(2\,ad^3+3\,bd^2+6\,cd\right)\text{Li}_3(dx)$$
$$+\frac{7\,ad^2x^2-((4\,ad^3+9\,bd^2+36\,cd)x^3-36\,cx^2-9\,bx-4\,a)\log\left(-dx+1\right)^2+6\left(adx+(2\,ad^2+3\,bd)x^2\right.}{}$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="maxim
a")
```

output
```
1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x +
 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/18*(4*a*d^3 + 9*b*d^2 + 36
*c*d)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/4*(3*a*d^3 + 4*b*d^2)
*log(x) - 1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*polylog(3, d*x) + 1/36*(7*a*d^2*
x^2 - ((4*a*d^3 + 9*b*d^2 + 36*c*d)*x^3 - 36*c*x^2 - 9*b*x - 4*a)*log(-d*x
 + 1)^2 + 6*(a*d*x + (2*a*d^2 + 3*b*d)*x^2 + ((2*a*d^3 + 3*b*d^2 + 6*c*d)*
x^3 - 6*c*x^2 - 3*b*x - 2*a)*log(-d*x + 1))*dilog(d*x) + (9*(3*a*d^3 + 4*b
*d^2)*x^3 - 7*a*d*x - 4*(5*a*d^2 + 9*b*d)*x^2)*log(-d*x + 1))/x^3
```

3.197.8 Giac [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^4}\,dx = \int \frac{(cx^2+bx+a)\text{Li}_2(dx)\log\left(-dx+1\right)}{x^4}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="giac"
)
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^4}\,dx = \int \frac{\ln(1-d\,x)\;\mathrm{polylog}(2,d\,x)\;(c\,x^2+b\,x+a)}{x^4}\,dx$$

input
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^4,x)
```

output
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^4, x)
```

3.198 $\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx$

3.198.1 Optimal result

Integrand size = 26, antiderivative size = 767

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^5}\,dx$$
$$= \frac{5ad^2}{144x^2}+\frac{bd^2}{9x}+\frac{19ad^3}{144x}+\frac{d^2(4b+3ad)}{48x}-\frac{1}{2}cd^2\log(x)-\frac{1}{3}bd^3\log(x)-\frac{37}{144}ad^4\log(x)$$
$$-\frac{1}{48}d^3(4b+3ad)\log(x)-\frac{1}{12}d^2(6c+d(4b+3ad))\log(x)+\frac{1}{2}cd^2\log(1-dx)$$
$$+\frac{1}{3}bd^3\log(1-dx)+\frac{37}{144}ad^4\log(1-dx)+\frac{1}{48}d^3(4b+3ad)\log(1-dx)$$
$$+\frac{1}{12}d^2(6c+d(4b+3ad))\log(1-dx)-\frac{5ad\log(1-dx)}{72x^3}-\frac{bd\log(1-dx)}{9x^2}$$
$$-\frac{ad^2\log(1-dx)}{16x^2}-\frac{d(4b+3ad)\log(1-dx)}{48x^2}-\frac{cd\log(1-dx)}{2x}-\frac{2bd^2\log(1-dx)}{9x}$$
$$-\frac{ad^3\log(1-dx)}{8x}-\frac{d(6c+d(4b+3ad))\log(1-dx)}{12x}-\frac{1}{4}cd^2\log^2(1-dx)$$
$$-\frac{1}{9}bd^3\log^2(1-dx)-\frac{1}{16}ad^4\log^2(1-dx)+\frac{a\log^2(1-dx)}{16x^4}+\frac{b\log^2(1-dx)}{9x^3}$$
$$+\frac{c\log^2(1-dx)}{4x^2}+\frac{1}{12}d^2(6c+d(4b+3ad))\log(dx)\log^2(1-dx)-\frac{1}{2}cd^2\operatorname{PolyLog}(2,dx)$$
$$-\frac{2}{9}bd^3\operatorname{PolyLog}(2,dx)-\frac{1}{8}ad^4\operatorname{PolyLog}(2,dx)+\frac{ad\operatorname{PolyLog}(2,dx)}{12x^3}$$
$$+\frac{d(4b+3ad)\operatorname{PolyLog}(2,dx)}{24x^2}+\frac{d(6c+d(4b+3ad))\operatorname{PolyLog}(2,dx)}{12x}$$
$$+\frac{1}{12}d^2(6c+d(4b+3ad))\log(1-dx)\operatorname{PolyLog}(2,dx)$$
$$-\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)\log(1-dx)\operatorname{PolyLog}(2,dx)$$
$$+\frac{1}{6}d^2(6c+d(4b+3ad))\log(1-dx)\operatorname{PolyLog}(2,1-dx)$$
$$-\frac{1}{12}d^2(6c+d(4b+3ad))\operatorname{PolyLog}(3,dx)-\frac{1}{6}d^2(6c+d(4b+3ad))\operatorname{PolyLog}(3,1-dx)$$

output
```
-1/8*a*d^4*polylog(2,d*x)-1/12*d^2*(6*c+d*(3*a*d+4*b))*polylog(3,d*x)-1/6*
d^2*(6*c+d*(3*a*d+4*b))*polylog(3,-d*x+1)-1/2*c*d^2*ln(x)-1/3*b*d^3*ln(x)-
37/144*a*d^4*ln(x)-1/48*d^3*(3*a*d+4*b)*ln(x)-1/12*d^2*(6*c+d*(3*a*d+4*b))
*ln(x)+1/2*c*d^2*ln(-d*x+1)+1/3*b*d^3*ln(-d*x+1)+37/144*a*d^4*ln(-d*x+1)+1
/48*d^3*(3*a*d+4*b)*ln(-d*x+1)+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)-1/4
*c*d^2*ln(-d*x+1)^2-1/9*b*d^3*ln(-d*x+1)^2-1/16*a*d^4*ln(-d*x+1)^2+1/16*a*
ln(-d*x+1)^2/x^4+1/9*b*ln(-d*x+1)^2/x^3+1/4*c*ln(-d*x+1)^2/x^2-1/12*(3*a/x
^4+4*b/x^3+6*c/x^2)*ln(-d*x+1)*polylog(2,d*x)+5/144*a*d^2/x^2+1/9*b*d^2/x+
19/144*a*d^3/x+1/48*d^2*(3*a*d+4*b)/x-1/2*c*d^2*polylog(2,d*x)-2/9*b*d^3*p
olylog(2,d*x)+1/12*a*d*polylog(2,d*x)/x^3+1/24*d*(3*a*d+4*b)*polylog(2,d*x
)/x^2+1/12*d*(6*c+d*(3*a*d+4*b))*polylog(2,d*x)/x-5/72*a*d*ln(-d*x+1)/x^3-
1/9*b*d*ln(-d*x+1)/x^2-1/16*a*d^2*ln(-d*x+1)/x^2-1/48*d*(3*a*d+4*b)*ln(-d*
x+1)/x^2-1/2*c*d*ln(-d*x+1)/x-2/9*b*d^2*ln(-d*x+1)/x-1/8*a*d^3*ln(-d*x+1)/
x-1/12*d*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)/x+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln(
d*x)*ln(-d*x+1)^2+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)*polylog(2,d*x)+1
/6*d^2*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)*polylog(2,-d*x+1)
```

3.198.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx+cx^2)\log(1-dx)\operatorname{PolyLog}(2,dx)}{x^5}\,dx = \frac{1}{144}\Bigg(-28bd^3-33ad^4+\frac{5ad^2}{x^2}+\frac{28bd^2}{x}$$
$$+\frac{28ad^3}{x}-144cd^2\log(dx)-108bd^3\log(dx)-82ad^4\log(dx)+144cd^2\log(1-dx)$$
$$+108bd^3\log(1-dx)+82ad^4\log(1-dx)-\frac{10ad\log(1-dx)}{x^3}-\frac{28bd\log(1-dx)}{x^2}$$
$$-\frac{18ad^2\log(1-dx)}{x^2}-\frac{144cd\log(1-dx)}{x}-\frac{80bd^2\log(1-dx)}{x}-\frac{54ad^3\log(1-dx)}{x}$$
$$+72cd^2\log(dx)\log(1-dx)+32bd^3\log(dx)\log(1-dx)+18ad^4\log(dx)\log(1-dx)$$
$$-36cd^2\log^2(1-dx)-16bd^3\log^2(1-dx)-9ad^4\log^2(1-dx)+\frac{9a\log^2(1-dx)}{x^4}$$
$$+\frac{16b\log^2(1-dx)}{x^3}+\frac{36c\log^2(1-dx)}{x^2}+72cd^2\log(dx)\log^2(1-dx)$$
$$+48bd^3\log(dx)\log^2(1-dx)+36ad^4\log(dx)\log^2(1-dx)$$
$$+\frac{6(dx(4x(b+3cx+2bdx)+a(2+3dx+6d^2x^2))+2(-4bx-6cx^2+6cd^2x^4+4bd^3x^4+3a(-1+d^4x^4}{x^4}$$
$$+2d^2\big(36c+16bd+9ad^2+12\big(6c+4bd+3ad^2\big)\log(1-dx)\big)\operatorname{PolyLog}(2,1-dx)$$
$$-72cd^2\operatorname{PolyLog}(3,dx)-48bd^3\operatorname{PolyLog}(3,dx)-36ad^4\operatorname{PolyLog}(3,dx)$$
$$-144cd^2\operatorname{PolyLog}(3,1-dx)-96bd^3\operatorname{PolyLog}(3,1-dx)-72ad^4\operatorname{PolyLog}(3,1-dx)\Bigg)$$

input
```
Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

output
```
(-28*b*d^3 - 33*a*d^4 + (5*a*d^2)/x^2 + (28*b*d^2)/x + (28*a*d^3)/x - 144*
c*d^2*Log[d*x] - 108*b*d^3*Log[d*x] - 82*a*d^4*Log[d*x] + 144*c*d^2*Log[1
- d*x] + 108*b*d^3*Log[1 - d*x] + 82*a*d^4*Log[1 - d*x] - (10*a*d*Log[1 -
d*x])/x^3 - (28*b*d*Log[1 - d*x])/x^2 - (18*a*d^2*Log[1 - d*x])/x^2 - (144
*c*d*Log[1 - d*x])/x - (80*b*d^2*Log[1 - d*x])/x - (54*a*d^3*Log[1 - d*x])
/x + 72*c*d^2*Log[d*x]*Log[1 - d*x] + 32*b*d^3*Log[d*x]*Log[1 - d*x] + 18*
a*d^4*Log[d*x]*Log[1 - d*x] - 36*c*d^2*Log[1 - d*x]^2 - 16*b*d^3*Log[1 - d
*x]^2 - 9*a*d^4*Log[1 - d*x]^2 + (9*a*Log[1 - d*x]^2)/x^4 + (16*b*Log[1 -
d*x]^2)/x^3 + (36*c*Log[1 - d*x]^2)/x^2 + 72*c*d^2*Log[d*x]*Log[1 - d*x]^2
 + 48*b*d^3*Log[d*x]*Log[1 - d*x]^2 + 36*a*d^4*Log[d*x]*Log[1 - d*x]^2 + (
6*(d*x*(4*x*(b + 3*c*x + 2*b*d*x) + a*(2 + 3*d*x + 6*d^2*x^2)) + 2*(-4*b*x
 - 6*c*x^2 + 6*c*d^2*x^4 + 4*b*d^3*x^4 + 3*a*(-1 + d^4*x^4))*Log[1 - d*x])
*PolyLog[2, d*x])/x^4 + 2*d^2*(36*c + 16*b*d + 9*a*d^2 + 12*(6*c + 4*b*d +
 3*a*d^2)*Log[1 - d*x])*PolyLog[2, 1 - d*x] - 72*c*d^2*PolyLog[3, d*x] - 4
8*b*d^3*PolyLog[3, d*x] - 36*a*d^4*PolyLog[3, d*x] - 144*c*d^2*PolyLog[3,
1 - d*x] - 96*b*d^3*PolyLog[3, 1 - d*x] - 72*a*d^4*PolyLog[3, 1 - d*x])/14
4
```

3.198.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{PolyLog}(2,dx)\log(1-dx)\left(a+bx+cx^2\right)}{x^5}\,dx$$

$$\Big\downarrow \; 7160$$

$$d\int \left(-\frac{(6c+d(4b+3ad))\,\text{PolyLog}(2,dx)d^2}{12(1-dx)}-\frac{(6c+d(4b+3ad))\,\text{PolyLog}(2,dx)d}{12x}-\frac{(6c+d(4b+3ad))\,\text{PolyLog}}{12x^2}\right.$$
$$\int \left(-\frac{c\log^2(1-dx)}{2x^3}-\frac{b\log^2(1-dx)}{3x^4}-\frac{a\log^2(1-dx)}{4x^5}\right)dx-\frac{1}{12}\text{PolyLog}(2,dx)\log(1-dx)\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{16}a\log^2(1-dx)d^4-\frac{11}{48}a\log(x)d^4+\frac{11}{48}a\log(1-dx)d^4-\frac{1}{8}a\,\text{PolyLog}(2,dx)d^4-\frac{1}{9}b\log^2(1-dx)d^3-\frac{1}{3}b\log(x)d^3+\frac{1}{3}b\log(1-dx)d^3-\frac{a\log(1-dx)d^3}{8x}-\frac{2}{9}b\,\text{PolyLog}(2,dx)d^3+\frac{5ad^3}{48x}-\frac{1}{4}c\log^2(1-dx)d^2-\frac{1}{2}c\log(x)d^2+\frac{1}{2}c\log(1-dx)d^2-\frac{2b\log(1-dx)d^2}{9x}-\frac{a\log(1-dx)d^2}{16x^2}-\frac{1}{2}c\,\text{PolyLog}(2,dx)d^2+\frac{bd^2}{9x}+\frac{ad^2}{48x^2}-\frac{c\log(1-dx)d}{2x}-\frac{b\log(1-dx)d}{9x^2}-\frac{a\log(1-dx)d}{24x^3}+\left(-\frac{1}{36}a\log(x)d^3+\frac{1}{36}a\log(1-dx)d^3-\frac{1}{48}(4b+3ad)\log(x)d^2+\frac{1}{48}(4b+3ad)\log(1-dx)d^2+\frac{ad^2}{36x}+\frac{1}{12}(6c+d(\right.$$
$$\frac{c\log^2(1-dx)}{4x^2}+\frac{b\log^2(1-dx)}{9x^3}+\frac{a\log^2(1-dx)}{16x^4}-\frac{1}{12}\left(\frac{3a}{x^4}+\frac{6c}{x^2}+\frac{4b}{x^3}\right)\log(1-dx)\,\text{PolyLog}(2,dx)$$

input
```
Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

output
```
(a*d^2)/(48*x^2) + (b*d^2)/(9*x) + (5*a*d^3)/(48*x) - (c*d^2*Log[x])/2 - (
b*d^3*Log[x])/3 - (11*a*d^4*Log[x])/48 + (c*d^2*Log[1 - d*x])/2 + (b*d^3*L
og[1 - d*x])/3 + (11*a*d^4*Log[1 - d*x])/48 - (a*d*Log[1 - d*x])/(24*x^3)
- (b*d*Log[1 - d*x])/(9*x^2) - (a*d^2*Log[1 - d*x])/(16*x^2) - (c*d*Log[1
- d*x])/(2*x) - (2*b*d^2*Log[1 - d*x])/(9*x) - (a*d^3*Log[1 - d*x])/(8*x)
- (c*d^2*Log[1 - d*x]^2)/4 - (b*d^3*Log[1 - d*x]^2)/9 - (a*d^4*Log[1 - d*x
]^2)/16 + (a*Log[1 - d*x]^2)/(16*x^4) + (b*Log[1 - d*x]^2)/(9*x^3) + (c*Lo
g[1 - d*x]^2)/(4*x^2) - (c*d^2*PolyLog[2, d*x])/2 - (2*b*d^3*PolyLog[2, d*
x])/9 - (a*d^4*PolyLog[2, d*x])/8 - (((3*a)/x^4 + (4*b)/x^3 + (6*c)/x^2)*L
og[1 - d*x]*PolyLog[2, d*x])/12 + d*((a*d)/(72*x^2) + (a*d^2)/(36*x) + (d*
(4*b + 3*a*d))/(48*x) - (a*d^3*Log[x])/36 - (d^2*(4*b + 3*a*d)*Log[x])/48
- (d*(6*c + d*(4*b + 3*a*d))*Log[x])/12 + (a*d^3*Log[1 - d*x])/36 + (d^2*(
4*b + 3*a*d)*Log[1 - d*x])/48 + (d*(6*c + d*(4*b + 3*a*d))*Log[1 - d*x])/1
2 - (a*Log[1 - d*x])/(36*x^3) - ((4*b + 3*a*d)*Log[1 - d*x])/(48*x^2) - ((
6*c + d*(4*b + 3*a*d))*Log[1 - d*x])/(12*x) + (d*(6*c + d*(4*b + 3*a*d))*L
og[d*x]*Log[1 - d*x]^2)/12 + (a*PolyLog[2, d*x])/(12*x^3) + ((4*b + 3*a*d)
*PolyLog[2, d*x])/(24*x^2) + ((6*c + d*(4*b + 3*a*d))*PolyLog[2, d*x])/(12
*x) + (d*(6*c + d*(4*b + 3*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/12 + (d*(6*
c + d*(4*b + 3*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/6 - (d*(6*c + d*(4*
b + 3*a*d))*PolyLog[3, d*x])/12 - (d*(6*c + d*(4*b + 3*a*d))*PolyLog[3,...
```

3.198. $\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7160
```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (S
imp[b   Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]
, u/(a + b*x), x], x], x] - Simp[e*h*n   Int[ExpandIntegrand[PolyLog[2, c*(
a + b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n},
x] && PolyQ[Px, x] && IntegerQ[m]
```

3.198.4 Maple [F]

$$\int \frac{(c\,x^2+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)}{x^5}dx$$

input
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

output
```
int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

3.198.5 Fricas [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\mathrm{PolyLog}(2,dx)}{x^5}\,dx = \int \frac{(cx^2+bx+a)\mathrm{Li}_2(dx)\log(-dx+1)}{x^5}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="frica
s")
```

output
```
integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**5,x)
```

output
```
Timed out
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx$$
$$= \frac{1}{12}\left(3\,ad^4+4\,bd^3+6\,cd^2\right)\left(\log(dx)\log(-dx+1)^2+2\,\text{Li}_2(-dx+1)\log(-dx+1)-2\,\text{Li}_3(-dx+1)\right)$$
$$+\frac{1}{72}\left(9\,ad^4+16\,bd^3+36\,cd^2\right)\left(\log(dx)\log(-dx+1)+\text{Li}_2(-dx+1)\right)$$
$$-\frac{1}{72}\left(41\,ad^4+54\,bd^3+72\,cd^2\right)\log(x)-\frac{1}{12}\left(3\,ad^4+4\,bd^3+6\,cd^2\right)\text{Li}_3(dx)$$
$$+\frac{5\,ad^2x^2+28\,(ad^3+bd^2)x^3-((9\,ad^4+16\,bd^3+36\,cd^2)x^4-36\,cx^2-16\,bx-9\,a)\log(-dx+1)^2+6\,(2$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="maxim
a")
```

output
```
1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*
x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/72*(9*a*d^4 + 16*b*d^3
+ 36*c*d^2)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/72*(41*a*d^4 +
54*b*d^3 + 72*c*d^2)*log(x) - 1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*polylog(3
, d*x) + 1/144*(5*a*d^2*x^2 + 28*(a*d^3 + b*d^2)*x^3 - ((9*a*d^4 + 16*b*d^
3 + 36*c*d^2)*x^4 - 36*c*x^2 - 16*b*x - 9*a)*log(-d*x + 1)^2 + 6*(2*(3*a*d
^3 + 4*b*d^2 + 6*c*d)*x^3 + 2*a*d*x + (3*a*d^2 + 4*b*d)*x^2 + 2*((3*a*d^4
+ 4*b*d^3 + 6*c*d^2)*x^4 - 6*c*x^2 - 4*b*x - 3*a)*log(-d*x + 1))*dilog(d*x
) + 2*((41*a*d^4 + 54*b*d^3 + 72*c*d^2)*x^4 - (27*a*d^3 + 40*b*d^2 + 72*c*
d)*x^3 - 5*a*d*x - (9*a*d^2 + 14*b*d)*x^2)*log(-d*x + 1))/x^4
```

3.198.8 Giac [F]

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx = \int \frac{(cx^2+bx+a)\text{Li}_2(dx)\log(-dx+1)}{x^5}\,dx$$

input
```
integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="giac"
)
```

output
```
integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\,\text{PolyLog}(2,dx)}{x^5}\,dx = \int \frac{\ln(1-d\,x)\ \text{polylog}(2,d\,x)\ (c\,x^2+b\,x+a)}{x^5}\,dx$$

input
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^5,x)
```

output
```
int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^5, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
            return 1
        elif isinstance(expn,list):
            return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
        elif  is_sqrt(expn):
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        elif isinstance(expn,Pow):    #type(expn,'`^`')
            if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
                return expnType(expn.args[0])    #ExpnType(op(1,expn))
            elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
                if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                    return 1
                else:
                    return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
            else:
                return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
        elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
            m1 = expnType(expn.args[0])
            m2 = expnType(list(expn.args[1:]))
            return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
        elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
            return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
        elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
        elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif is_appell_function(expn.func):
            m1 = max(map(expnType, list(expn.args)))
            return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif isinstance(expn,RootSum):
            m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            return max(7,m1)
        elif str(expn).find("Integral") != -1:
            m1 = max(map(expnType, list(expn.args)))
            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        else:
            return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```