Computer Algebra Independent Integration Tests

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359-MIT

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**321**]. This is test number [359].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.38 (319)	0.62 (2)
Maple	95.95 (308)	4.05 (13)
Fricas	95.95 (308)	4.05 (13)
Rubi	94.70 (304)	5.30 (17)
Maxima	92.52 (297)	7.48 (24)
Giac	92.21 (296)	7.79 (25)
Mupad	90.03 (289)	9.97 (32)
Reduce	81.93 (263)	18.07 (58)
Sympy	81.93 (263)	18.07 (58)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

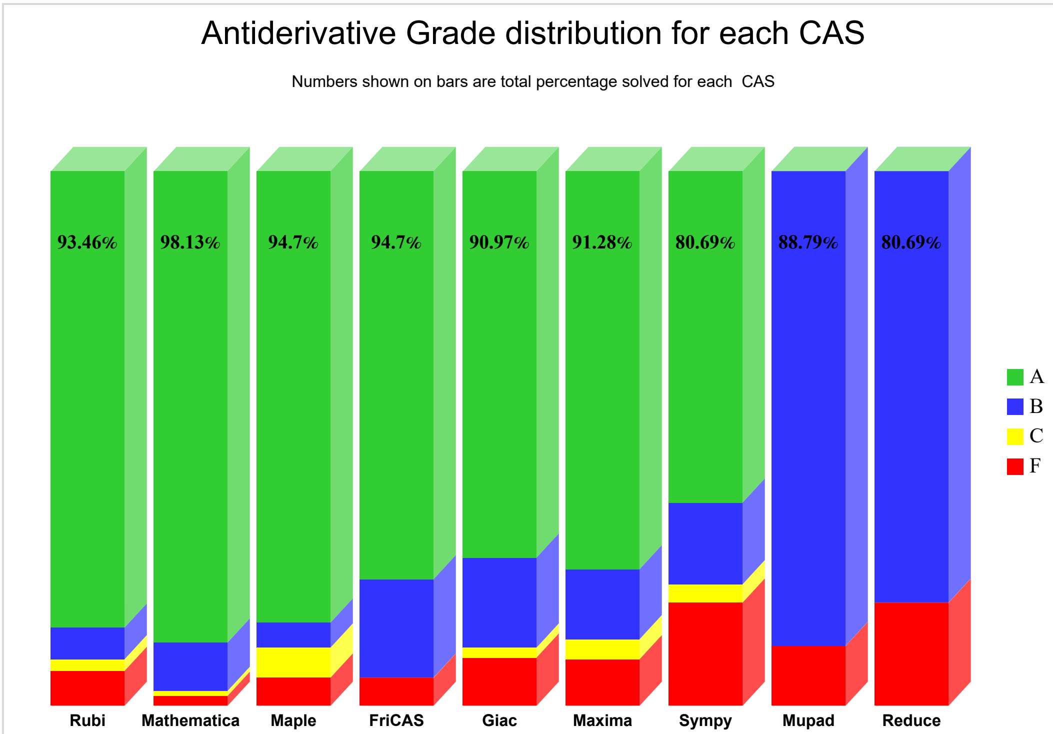
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

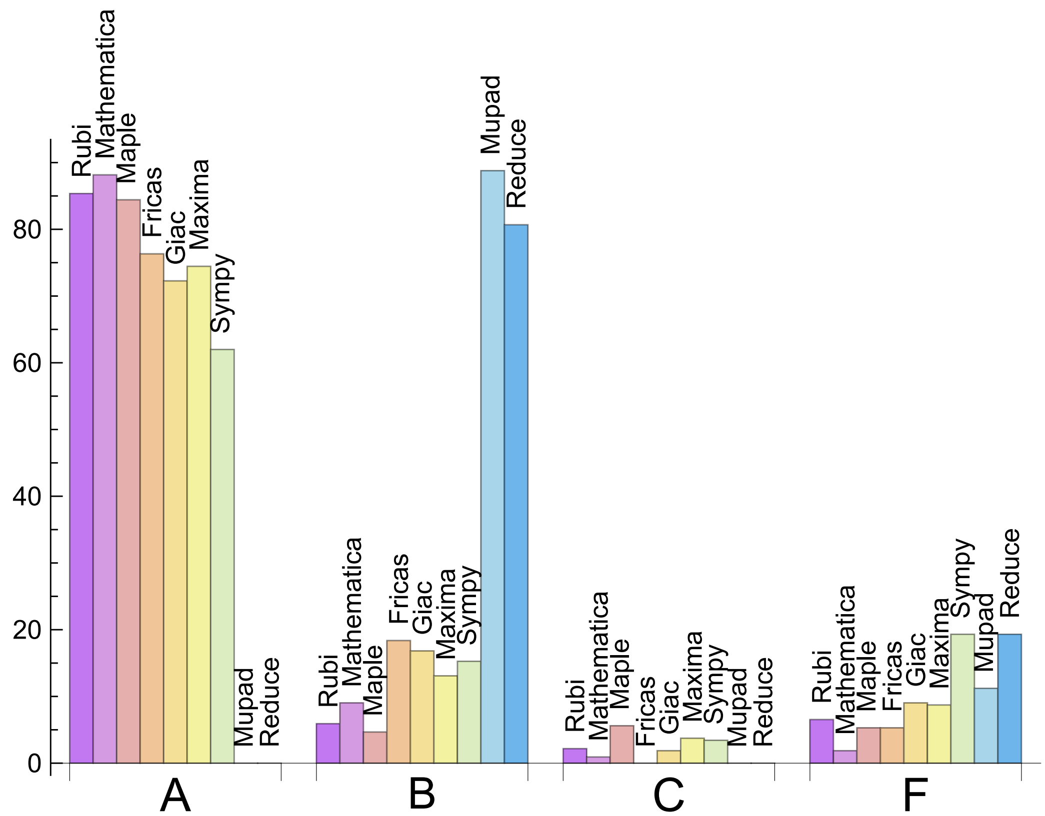
System	% A grade	% B grade	% C grade	% F grade
Mathematica	88.162	9.034	0.935	1.869
Rubi	85.358	5.919	2.181	6.542
Maple	84.424	4.673	5.607	5.296
Fricas	76.324	18.380	0.000	5.296
Maxima	74.455	13.084	3.738	8.723
Giac	72.274	16.822	1.869	9.034
Sympy	61.994	15.265	3.427	19.315
Mupad	0.000	88.785	0.000	11.215
Reduce	0.000	80.685	0.000	19.315

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	2	100.00	0.00	0.00
Fricas	13	46.15	23.08	30.77
Maple	13	84.62	15.38	0.00
Rubi	17	100.00	0.00	0.00
Maxima	24	91.67	4.17	4.17
Giac	25	92.00	4.00	4.00
Mupad	32	0.00	100.00	0.00
Reduce	58	100.00	0.00	0.00
Sympy	58	86.21	13.79	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Fricas	0.12
Giac	0.14
Mathematica	0.15
Mupad	0.17
Rubi	0.26
Reduce	0.32
Sympy	1.11
Maple	2.06

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	20.45	1.23	13.00	0.86
Fricas	25.36	1.61	17.00	1.00
Rubi	28.70	1.81	18.00	1.00
Maple	88.08	4.18	14.00	0.92
Mathematica	97.53	4.49	18.00	1.00
Maxima	106.75	5.74	15.00	0.97
Reduce	109.25	5.67	16.00	1.00
Giac	111.94	6.79	16.00	1.00
Sympy	133.32	6.18	15.00	0.92

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

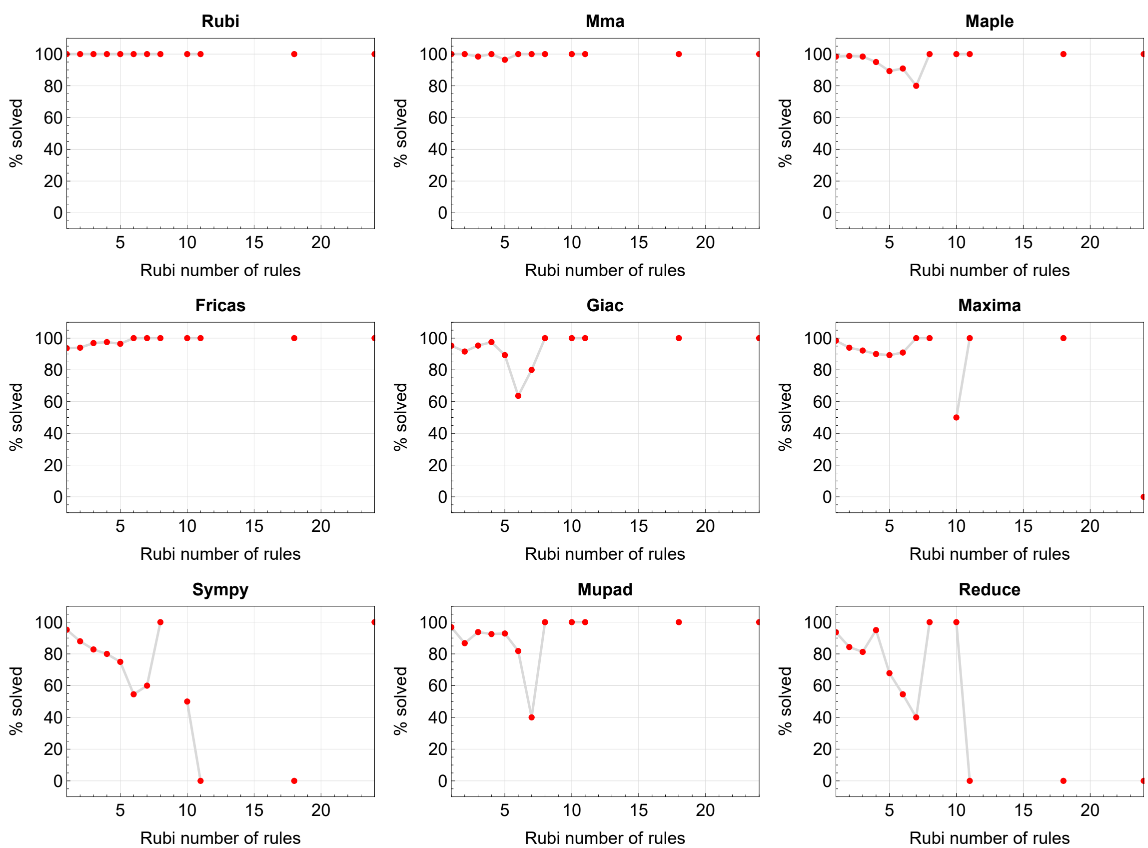


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

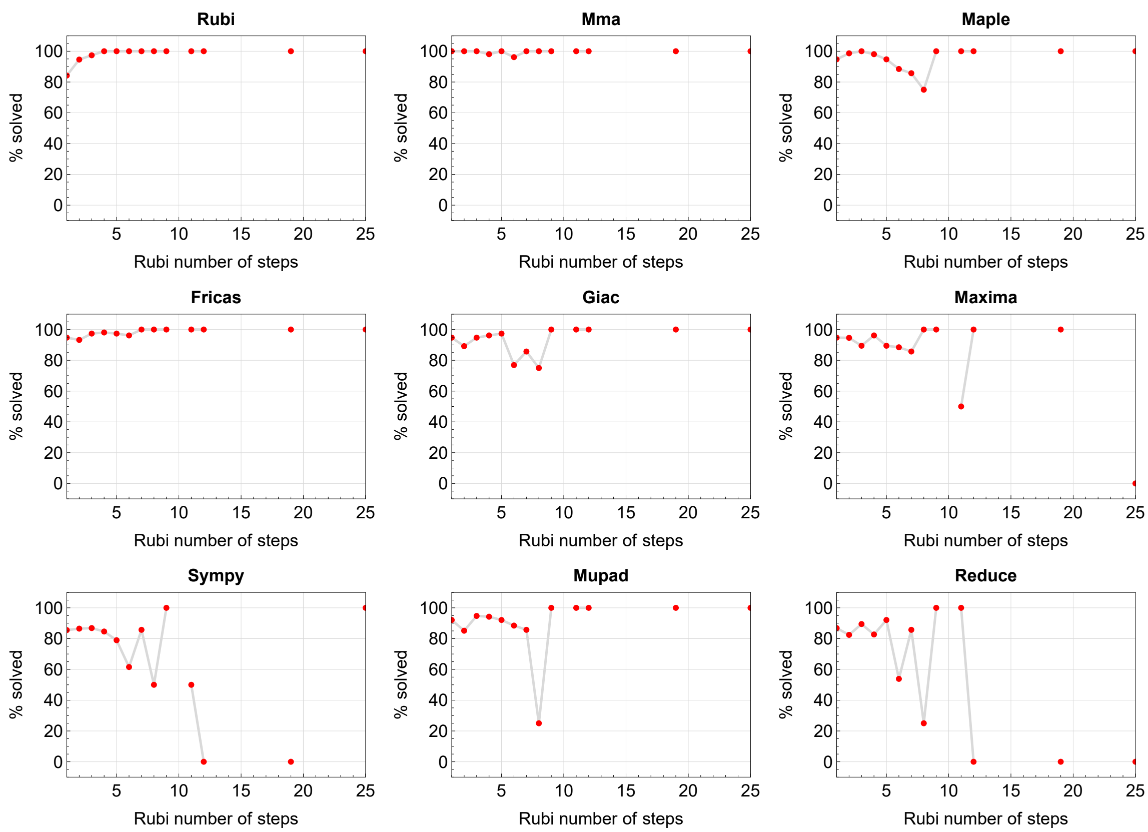


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

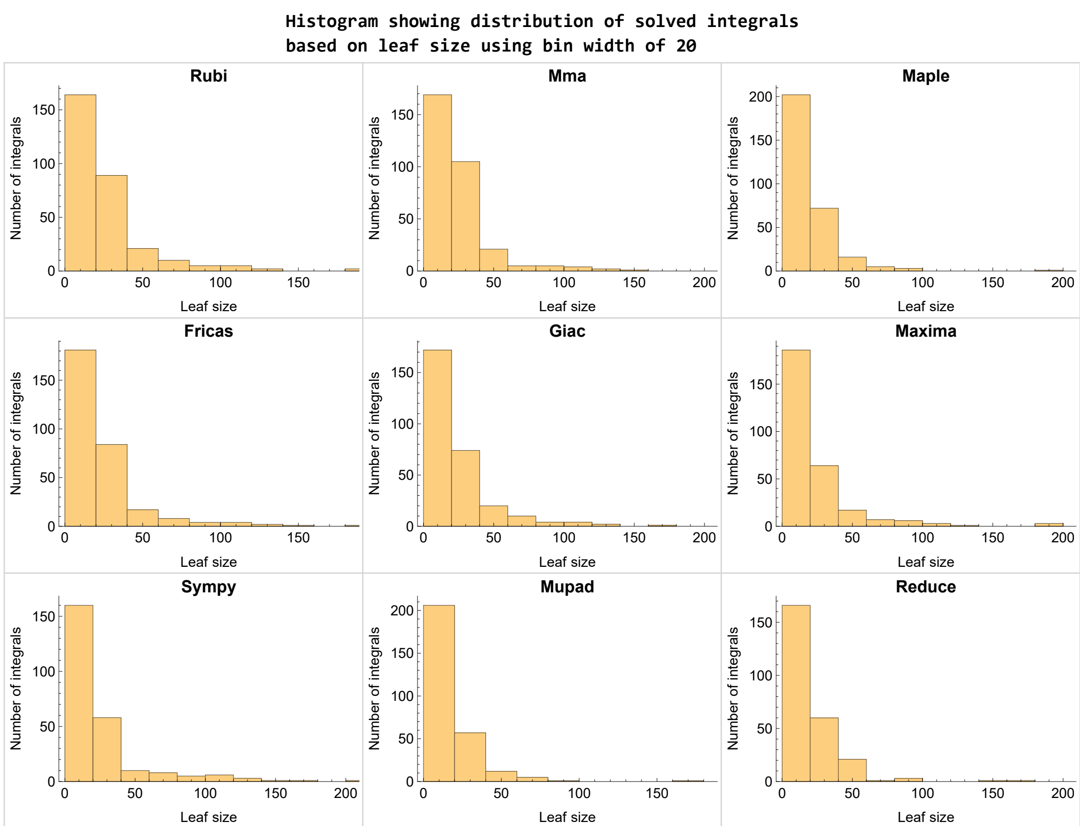


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

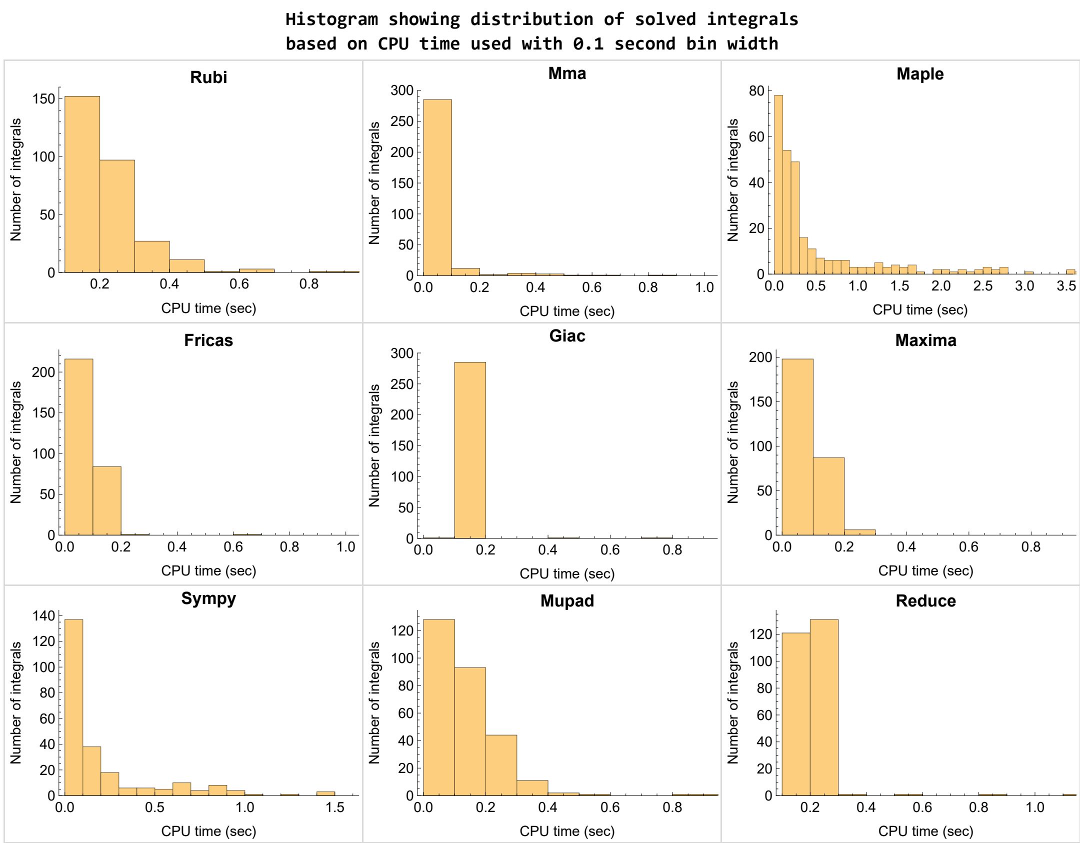


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

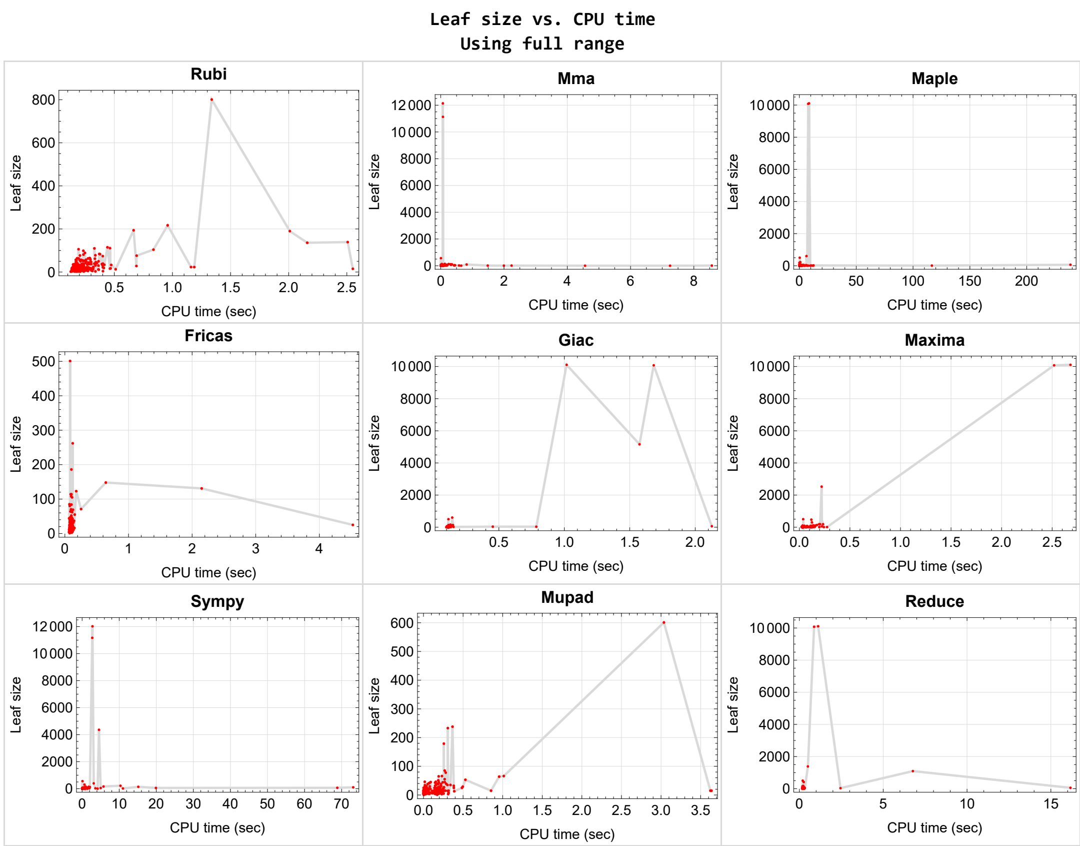


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{194, 225, 226, 236\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{7, 115, 219, 315\}$

Mathematica $\{278\}$

Maple $\{77, 146, 150, 185, 233, 264, 276\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

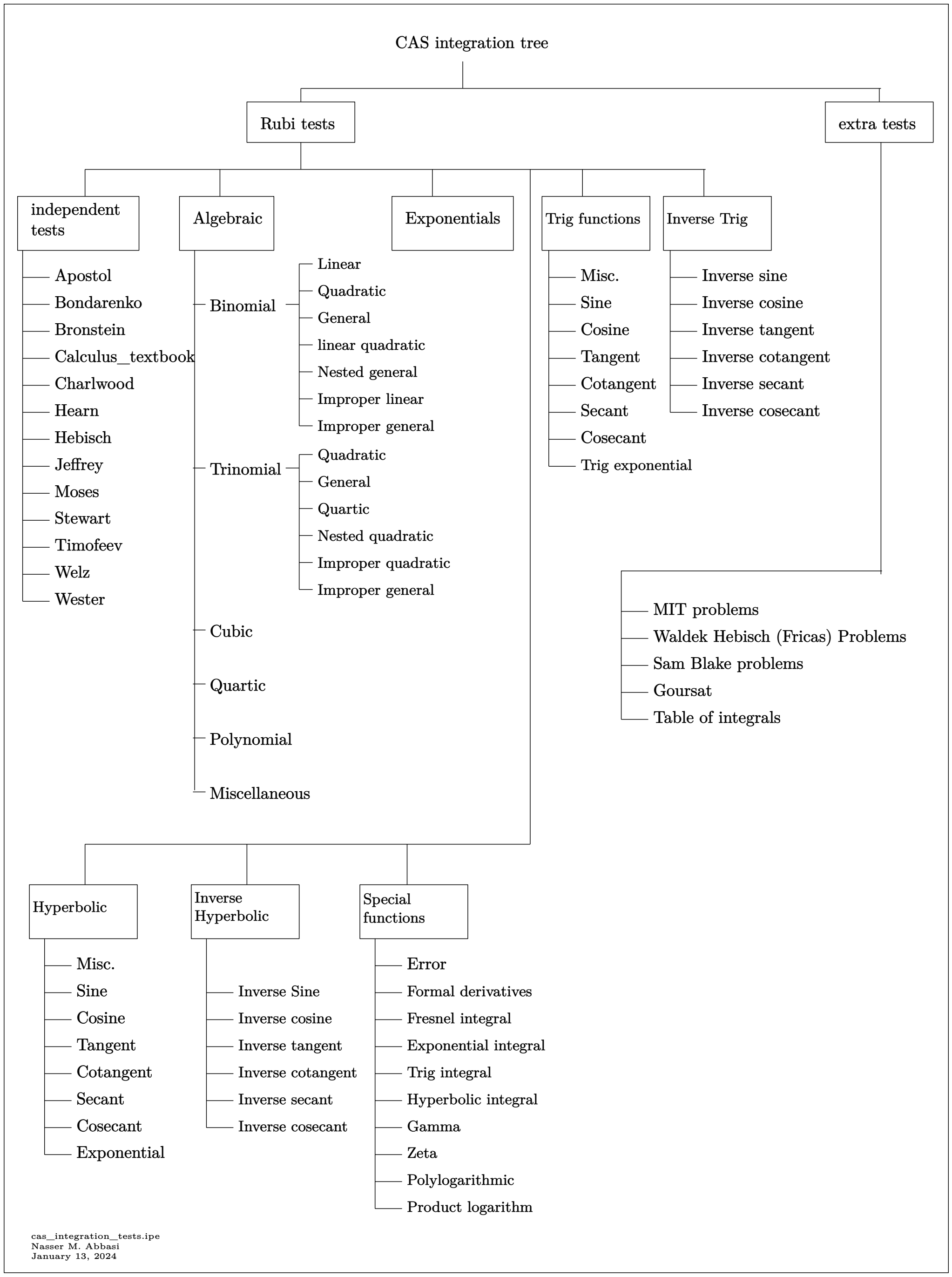
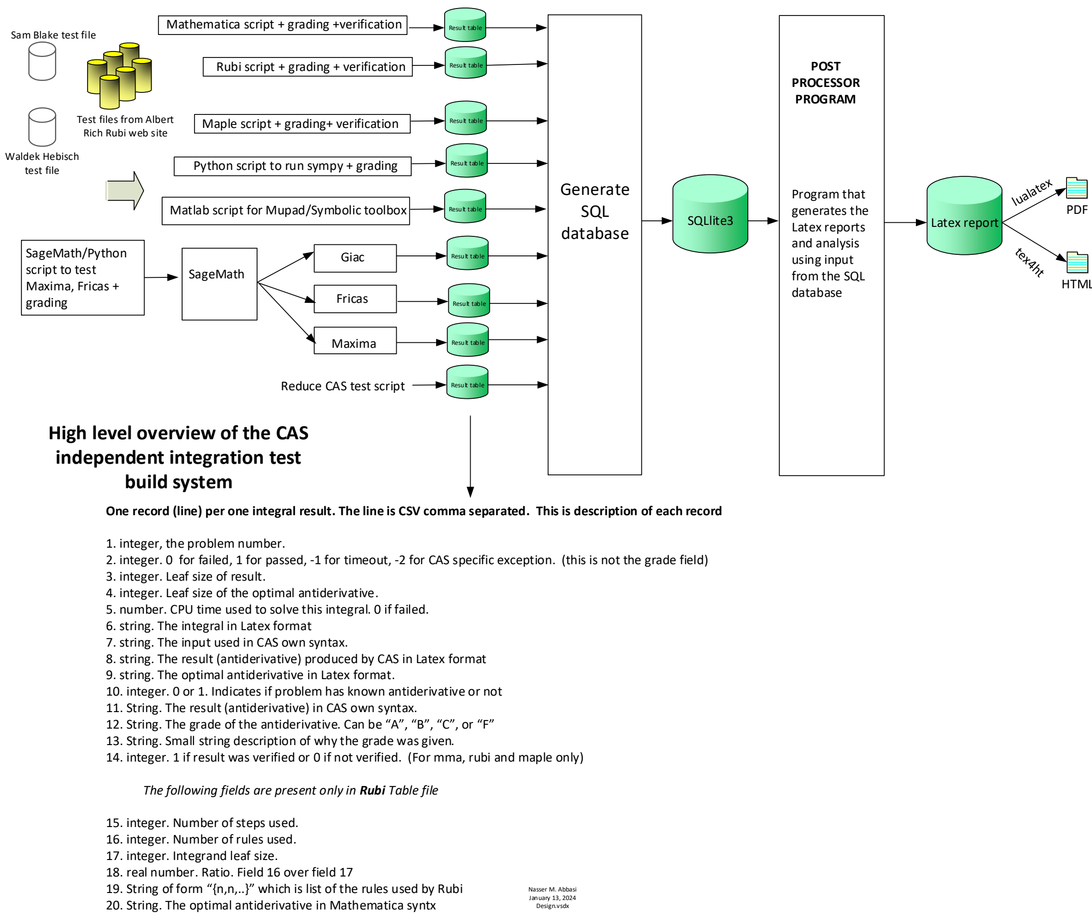


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 227, 228, 229, 230, 231, 232, 233, 235, 237, 238, 239, 240, 241, 247, 248, 249, 251, 252, 253, 255, 256, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 273, 274, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 295, 296, 297, 299, 300, 301, 302, 303, 304, 306, 308, 309, 310, 311, 312, 314, 315, 316, 318, 321 }

B grade { 41, 46, 52, 84, 85, 118, 119, 187, 196, 207, 242, 244, 254, 266, 272, 276, 287, 305, 319 }

C grade { 20, 234, 246, 257, 258, 288, 294 }

F normal fail { 77, 140, 147, 180, 211, 243, 245, 250, 265, 275, 277, 293, 298, 307, 313, 317, 320 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 62, 63, 65, 66, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 98, 99, 100, 101, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 235, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321 }

B grade { 20, 43, 55, 57, 64, 67, 71, 78, 89, 94, 97, 105, 129, 168, 170, 183, 193, 196, 197, 207, 213, 220, 227, 238, 247, 258, 290, 310, 315 }

C grade { 102, 147, 209 }

F normal fail { 266, 294 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 195, 197, 198, 199, 200, 201, 202, 203, 204, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 251, 252, 254, 255, 256, 260, 262, 264, 267, 269, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 295, 296, 297, 298, 299, 300, 301, 303, 305, 306, 308, 310, 311, 314, 316, 321 }

B grade { 46, 52, 64, 71, 81, 105, 135, 196, 211, 227, 265, 304, 309, 312, 319 }

C grade { 19, 20, 116, 132, 205, 207, 244, 253, 257, 258, 261, 263, 268, 276, 278, 293, 313, 317 }

F normal fail { 173, 250, 259, 266, 270, 294, 302, 307, 315, 318, 320 }

F(-1) timedout fail { 188, 291 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 44, 45, 47, 48, 49, 50, 51, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 179, 181, 183, 184, 185, 187, 188, 189, 191, 192, 195, 198, 201, 202, 203, 204, 206, 208, 209, 210, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 229, 230, 231, 232, 233, 235, 237, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 252, 254, 256, 258, 260, 261, 262, 263, 264, 265, 267, 269, 271, 272, 273, 274, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 288, 289, 293, 295, 296, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 311, 314, 315, 316, 317, 319, 320, 321 }

B grade { 7, 20, 27, 30, 36, 43, 52, 53, 55, 64, 71, 78, 80, 89, 94, 121, 129, 135, 137, 140, 149, 156, 159, 168, 169, 170, 178, 180, 182, 186, 190, 193, 196, 197, 200, 207, 211, 213, 224, 228, 234, 238, 239, 243, 247, 253, 255, 257, 275, 280, 290, 292, 294, 297, 304, 310, 312, 313, 318 }

C grade { }

F normal fail { 11, 205, 266, 268, 270, 287 }

F(-1) timedout fail { 46, 199, 291 }

F(-2) exception fail { 105, 227, 259, 302 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 24, 25, 28, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 148, 149, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 185, 186, 189, 190, 192, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 233, 234, 237, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 260, 262, 263, 264, 265, 267, 269, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 291, 292, 295, 296, 298, 300, 304, 306, 308, 310, 311, 312, 314, 316, 317, 318, 319, 321 }

B grade { 7, 19, 26, 27, 29, 30, 32, 43, 64, 71, 74, 78, 89, 90, 93, 94, 105, 113, 128, 145, 156, 180, 184, 187, 193, 196, 197, 207, 208, 227, 238, 239, 244, 254, 255, 258, 261, 290, 293, 297, 301, 303 }

C grade { 23, 119, 147, 150, 167, 173, 246, 256, 277, 289, 302, 309 }

F normal fail { 20, 42, 52, 58, 59, 116, 170, 188, 191, 235, 240, 257, 259, 266, 268, 270, 294, 299, 307, 313, 315, 320 }

F(-1) timedout fail { 305 }

F(-2) exception fail { 46 }

Giac

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 31, 33, 34, 35, 37, 39, 40, 41, 42, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 118, 120, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 171, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 195, 198, 199, 200, 202, 203, 204, 206, 210, 212, 214, 215, 216, 218, 220, 221, 222, 223, 224, 229, 230, 231, 233, 234, 235, 237, 241, 242, 243, 245, 248, 249, 251, 253, 254, 256, 260, 261, 262, 263, 264, 267, 269, 272, 273, 274, 276, 277, 279, 280, 281, 282, 283, 284, 286, 288, 289, 291, 292, 295, 296, 299, 300, 301, 303, 305, 306, 307, 308, 310, 311, 312, 314, 316, 317, 318, 319, 320, 321 }

B grade { 4, 16, 19, 20, 29, 30, 36, 43, 46, 55, 58, 64, 71, 77, 78, 89, 94, 105, 113, 117, 121, 129, 137, 145, 168, 170, 180, 181, 187, 193, 196, 197, 207, 208, 209, 213, 217, 227, 228, 232, 238, 239, 244, 247, 255, 257, 258, 265, 285, 290, 293, 298, 304, 313 }

C grade { 119, 150, 246, 287, 302, 309 }

F normal fail { 11, 32, 38, 59, 74, 116, 128, 172, 201, 205, 211, 240, 252, 259, 266, 268, 270, 271, 275, 278, 294, 297, 315 }

F(-1) timedout fail { 250 }

F(-2) exception fail { 219 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 227, 228, 229, 230, 231, 232, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 260, 261, 263, 264, 267, 269, 272, 273, 274, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 316, 317, 319, 321 }

C grade { }

F normal fail { }

F(-1) timedout fail { 11, 42, 52, 63, 81, 93, 115, 116, 172, 191, 205, 223, 233, 250, 258, 259, 262, 265, 266, 268, 270, 271, 275, 278, 291, 294, 297, 307, 309, 315, 318, 320 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 9, 13, 15, 17, 19, 22, 23, 25, 26, 27, 28, 33, 34, 35, 40, 41, 44, 45, 47, 48, 50, 54, 55, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 72, 73, 75, 76, 77, 79, 80, 82, 83, 84, 85, 86, 88, 90, 91, 92, 95, 96, 97, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 114, 116, 117, 120, 122, 123, 124, 125, 126, 127, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 164, 165, 166, 167, 168, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 190, 192, 195, 198, 199, 200, 201, 202, 203, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 229, 230, 231, 232, 237, 240, 242, 245, 247, 248, 249, 251, 253, 254, 255, 256, 261, 262, 263, 264, 265, 269, 273, 274, 279, 281, 282, 283, 285, 286, 287, 289, 293, 295, 296, 298, 300, 303, 305, 306, 308, 309, 310, 311, 312, 314, 316, 321 }

B grade { 1, 2, 14, 16, 18, 29, 31, 36, 38, 39, 43, 51, 53, 64, 70, 78, 87, 89, 93, 94, 103, 105, 118, 119, 121, 137, 142, 159, 189, 193, 197, 204, 211, 227, 228, 234, 238, 244, 260, 268, 272, 280, 284, 288, 292, 294, 302, 304, 315 }

C grade { 3, 8, 24, 98, 112, 161, 223, 241, 252, 267, 278 }

F normal fail { 10, 11, 12, 20, 21, 30, 32, 37, 42, 49, 52, 58, 59, 71, 74, 81, 113, 115, 128, 147, 163, 169, 170, 172, 173, 184, 191, 233, 235, 239, 243, 246, 250, 257, 258, 259, 266, 270, 271, 275, 276, 277, 290, 297, 299, 307, 313, 317, 318, 320 }

F(-1) timedout fail { 46, 132, 188, 196, 207, 291, 301, 319 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 206, 208, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 227, 228, 229, 230, 231, 232, 234, 235, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 253, 254, 255, 260, 261, 263, 264, 265, 267, 269, 272, 273, 274, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 298, 299, 300, 302, 303, 304, 306, 308, 310, 311, 312, 314, 315, 316, 317, 321 }

C grade { }

F normal fail { 10, 11, 20, 21, 32, 37, 38, 47, 59, 61, 65, 68, 74, 81, 115, 128, 132, 143, 164, 170, 172, 173, 184, 187, 188, 201, 205, 207, 209, 214, 224, 233, 243, 250, 252, 256, 257, 258, 259, 262, 266, 268, 270, 271, 275, 276, 277, 278, 294, 297, 301, 305, 307, 309, 313, 318, 319, 320 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	112	13	89	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.48	0.52	3.56	0.56
time (sec)	N/A	0.209	0.066	4.773	0.031	0.104	1.073	0.111	0.193	0.182

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	19	31	24	23	13	65	13	42	14
N.S.	1	1.12	1.82	1.41	1.35	0.76	3.82	0.76	2.47	0.82
time (sec)	N/A	0.253	0.066	3.680	0.032	0.090	0.647	0.113	0.205	0.142

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	32	21	18	22	22	144	22	22	19
N.S.	1	1.52	1.00	0.86	1.05	1.05	6.86	1.05	1.05	0.90
time (sec)	N/A	0.155	0.015	0.596	0.040	0.079	0.976	0.105	0.196	0.024

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	22	21	23	20	60	31	31
N.S.	1	0.96	1.00	0.81	0.78	0.85	0.74	2.22	1.15	1.15
time (sec)	N/A	0.181	0.008	0.320	0.027	0.117	0.051	0.121	0.220	0.223

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	18	20	21	37	21	18	24
N.S.	1	1.00	0.96	0.67	0.74	0.78	1.37	0.78	0.67	0.89
time (sec)	N/A	0.169	0.007	0.787	0.037	0.127	0.289	0.112	0.216	0.156

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	15	15	10	11	11	8	11	12	11
N.S.	1	1.25	1.25	0.83	0.92	0.92	0.67	0.92	1.00	0.92
time (sec)	N/A	0.157	0.012	0.112	0.027	0.098	0.037	0.111	0.190	0.031

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	33	50	69	46	46	151	30
N.S.	1	1.00	1.03	1.10	1.67	2.30	1.53	1.53	5.03	1.00
time (sec)	N/A	0.214	0.016	1.026	0.031	0.128	0.078	0.125	0.182	0.213

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	24	10	11	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.71	0.71	0.79	0.71
time (sec)	N/A	0.151	0.015	1.131	0.110	0.089	0.605	0.114	0.199	0.230

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	17	13	12	15	11	10	13	28	11
N.S.	1	1.31	1.00	0.92	1.15	0.85	0.77	1.00	2.15	0.85
time (sec)	N/A	0.146	0.003	0.274	0.032	0.085	0.056	0.117	0.202	0.039

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	110	40	49	80	70	0	80	5	65
N.S.	1	1.55	0.56	0.69	1.13	0.99	0.00	1.13	0.07	0.92
time (sec)	N/A	0.329	0.027	0.454	0.111	0.098	0.000	0.117	0.184	0.192

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	70	56	0	0	0	14	0
N.S.	1	1.00	1.00	0.79	0.63	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.248	0.009	0.763	0.149	0.000	0.000	0.000	0.192	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	63	51	36	35	35	0	36	34	35
N.S.	1	1.19	0.96	0.68	0.66	0.66	0.00	0.68	0.64	0.66
time (sec)	N/A	0.193	0.018	0.274	0.032	0.099	0.000	0.120	0.177	0.158

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.202	0.011	0.171	0.067	0.105	0.062	0.113	0.201	0.231

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	50	29	36	28	31	66	28	32	25
N.S.	1	1.72	1.00	1.24	0.97	1.07	2.28	0.97	1.10	0.86
time (sec)	N/A	0.170	0.037	0.262	0.031	0.131	68.827	0.115	0.182	0.383

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	20	16
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	1.05	0.84
time (sec)	N/A	0.136	0.005	0.460	0.107	0.094	0.049	0.111	0.188	0.135

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	37	26	25	25	26	29	25	13
N.S.	1	1.00	1.76	1.24	1.19	1.19	1.24	1.38	1.19	0.62
time (sec)	N/A	0.146	0.004	0.079	0.025	0.093	0.097	0.117	0.184	0.149

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73	0.73
time (sec)	N/A	0.153	0.006	0.079	0.041	0.099	0.074	0.110	0.204	0.161

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	14	15	11	15	16	32	14
N.S.	1	1.00	1.00	1.75	1.88	1.38	1.88	2.00	4.00	1.75
time (sec)	N/A	0.196	0.082	0.464	0.032	0.110	0.096	0.111	0.178	0.062

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	26	65	10	8	79	10	10
N.S.	1	1.00	0.91	2.36	5.91	0.91	0.73	7.18	0.91	0.91
time (sec)	N/A	0.366	0.102	1.419	0.134	0.100	0.084	0.121	0.202	0.055

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	111	93	57	0	105	0	81	9	233
N.S.	1	3.96	3.32	2.04	0.00	3.75	0.00	2.89	0.32	8.32
time (sec)	N/A	0.463	0.819	1.681	0.000	0.113	0.000	0.114	0.197	0.307

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	20	18	37	0	37	26	21
N.S.	1	1.00	0.88	0.77	0.69	1.42	0.00	1.42	1.00	0.81
time (sec)	N/A	0.172	0.063	0.115	0.127	0.118	0.000	0.119	0.183	0.087

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	11	11	10	11	12	11
N.S.	1	1.00	0.64	0.55	0.50	0.50	0.45	0.50	0.55	0.50
time (sec)	N/A	0.184	0.015	0.122	0.029	0.128	0.040	0.110	0.192	0.021

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	12	6	3	6	6	6
N.S.	1	1.00	1.00	1.17	2.00	1.00	0.50	1.00	1.00	1.00
time (sec)	N/A	0.147	0.015	0.088	0.066	0.088	0.036	0.114	0.190	0.213

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	61	38	52	124	42	35	41
N.S.	1	1.00	1.00	1.15	0.72	0.98	2.34	0.79	0.66	0.77
time (sec)	N/A	0.175	0.073	0.513	0.113	0.101	2.098	0.106	0.208	0.068

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	0.81
time (sec)	N/A	0.175	0.000	0.080	0.023	0.085	0.029	0.112	0.229	0.013

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	21	6	5	6	6	6
N.S.	1	1.00	1.00	1.17	3.50	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.146	0.004	0.128	0.037	0.116	0.040	0.111	0.206	0.134

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	16	18	20	4	4	14
N.S.	1	1.00	1.00	1.25	4.00	4.50	5.00	1.00	1.00	3.50
time (sec)	N/A	0.138	0.078	0.633	0.111	0.121	0.657	0.121	0.223	0.107

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88	0.88
time (sec)	N/A	0.158	0.004	0.244	0.034	0.080	0.030	0.111	0.218	0.018

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	24	301	29	162	39	38	39
N.S.	1	1.00	1.82	1.41	17.71	1.71	9.53	2.29	2.24	2.29
time (sec)	N/A	0.278	0.051	0.808	0.125	0.100	5.775	0.114	0.189	0.286

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	10	20	0	10	16	4
N.S.	1	1.00	1.00	1.25	2.50	5.00	0.00	2.50	4.00	1.00
time (sec)	N/A	0.175	0.002	0.928	0.031	0.091	0.000	0.108	0.184	0.007

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	13	20	29	13	32	13
N.S.	1	1.00	1.59	0.82	0.76	1.18	1.71	0.76	1.88	0.76
time (sec)	N/A	0.207	0.024	1.501	0.039	0.105	0.033	0.109	0.180	0.159

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	10	15
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	0.77	1.15
time (sec)	N/A	0.318	0.108	2.641	0.195	0.131	0.000	0.000	0.190	0.276

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	30	23	24	25	36	22	26	22
N.S.	1	1.29	0.88	0.68	0.71	0.74	1.06	0.65	0.76	0.65
time (sec)	N/A	0.278	0.007	2.787	0.029	0.122	0.030	0.131	0.185	0.025

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	20	16
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	1.05	0.84
time (sec)	N/A	0.151	0.004	0.257	0.112	0.087	0.052	0.109	0.193	0.018

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	11	10	13	15	13	10	13
N.S.	1	1.00	1.00	0.65	0.59	0.76	0.88	0.76	0.59	0.76
time (sec)	N/A	0.158	0.005	0.251	0.036	0.094	0.122	0.105	0.215	0.158

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	7	6	17	15	25	17	11
N.S.	1	1.00	1.00	2.33	2.00	5.67	5.00	8.33	5.67	3.67
time (sec)	N/A	0.158	0.002	0.231	0.040	0.099	0.076	0.114	0.213	0.141

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	11	11	8	7	21	0	20	17	16
N.S.	1	0.46	0.46	0.33	0.29	0.88	0.00	0.83	0.71	0.67
time (sec)	N/A	0.192	0.055	2.232	0.122	0.098	0.000	0.114	0.210	0.245

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	22	0	13	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	1.57	0.00	0.93	0.86
time (sec)	N/A	0.176	0.123	0.310	0.048	0.075	0.397	0.000	0.186	0.200

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	30	66	23	49	27
N.S.	1	1.00	1.00	0.77	0.74	0.97	2.13	0.74	1.58	0.87
time (sec)	N/A	0.338	0.062	5.294	0.044	0.096	0.870	0.115	0.198	0.259

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.235	0.006	0.100	0.067	0.085	0.138	0.113	0.181	0.187

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	19	8	12	11	11	10	11	12	11
N.S.	1	2.38	1.00	1.50	1.38	1.38	1.25	1.38	1.50	1.38
time (sec)	N/A	0.247	0.011	0.109	0.033	0.109	0.048	0.108	0.183	0.028

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	43	0	33	0	16	31	0
N.S.	1	1.00	1.17	1.05	0.00	0.80	0.00	0.39	0.76	0.00
time (sec)	N/A	0.350	0.184	3.842	0.000	0.134	0.000	0.114	0.197	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	25	7	15	15	12	16	19	6
N.S.	1	1.00	4.17	1.17	2.50	2.50	2.00	2.67	3.17	1.00
time (sec)	N/A	0.229	0.003	0.253	0.039	0.076	0.050	0.109	0.195	0.150

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.338	0.002	0.160	0.061	0.093	0.071	0.107	0.176	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	47	27	36	37	26	25	42	26
N.S.	1	1.28	1.62	0.93	1.24	1.28	0.90	0.86	1.45	0.90
time (sec)	N/A	0.274	0.048	1.573	0.113	0.082	0.262	0.126	0.174	0.027

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	801	12	602	0	0	0	601	1095	601
N.S.	1	66.75	1.00	50.17	0.00	0.00	0.00	50.08	91.25	50.08
time (sec)	N/A	1.338	0.005	6.243	0.000	0.000	0.000	0.144	6.769	3.036

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	7	7	7	7	11	7
N.S.	1	1.00	1.00	1.10	0.70	0.70	0.70	0.70	1.10	0.70
time (sec)	N/A	0.229	0.015	0.455	0.064	0.087	0.791	0.109	0.184	0.030

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	22	32	46	29	23	56	21
N.S.	1	1.00	0.86	0.79	1.14	1.64	1.04	0.82	2.00	0.75
time (sec)	N/A	0.212	0.011	0.059	0.033	0.081	0.052	0.112	0.190	0.028

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	57	64	39	43	32	0	29	26	43
N.S.	1	1.50	1.68	1.03	1.13	0.84	0.00	0.76	0.68	1.13
time (sec)	N/A	0.169	0.063	0.434	0.123	0.084	0.000	0.129	0.180	0.146

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	18	18	15	15	14	15	15	12	14
N.S.	1	0.90	0.90	0.75	0.75	0.70	0.75	0.75	0.60	0.70
time (sec)	N/A	0.161	0.009	0.275	0.028	0.094	0.051	0.112	0.188	0.033

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	11	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.52	0.43
time (sec)	N/A	0.151	0.004	0.375	0.033	0.110	1.717	0.108	0.182	0.134

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	51	21	41	0	37	0	19	23	0
N.S.	1	2.55	1.05	2.05	0.00	1.85	0.00	0.95	1.15	0.00
time (sec)	N/A	0.233	0.070	0.399	0.000	0.099	0.000	0.114	0.191	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	22	20	19	186	138	16	54	53
N.S.	1	1.07	0.81	0.74	0.70	6.89	5.11	0.59	2.00	1.96
time (sec)	N/A	0.167	0.018	0.221	0.040	0.102	15.155	0.113	0.179	0.529

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	16	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.70	0.74
time (sec)	N/A	0.144	0.039	1.341	0.121	0.099	0.095	0.119	0.178	0.019

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	17	23	10	11	21	10	29	9	11
N.S.	1	1.70	2.30	1.00	1.10	2.10	1.00	2.90	0.90	1.10
time (sec)	N/A	0.157	0.054	0.951	0.116	0.079	0.300	0.114	0.179	0.156

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	7	8	7	7	8	8
N.S.	1	1.00	1.00	1.12	0.88	1.00	0.88	0.88	1.00	1.00
time (sec)	N/A	0.142	0.002	0.135	0.035	0.094	0.041	0.110	0.184	0.012

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	14	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.40	1.00
time (sec)	N/A	0.172	0.012	0.263	0.025	0.097	0.208	0.112	0.183	0.137

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	70	53	0	64	0	117	73	64
N.S.	1	1.09	1.32	1.00	0.00	1.21	0.00	2.21	1.38	1.21
time (sec)	N/A	0.295	0.134	0.087	0.000	0.087	0.000	0.142	0.199	0.955

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	18	0	22	0	0	59	179
N.S.	1	1.00	1.00	0.39	0.00	0.48	0.00	0.00	1.28	3.89
time (sec)	N/A	0.341	0.154	3.023	0.000	0.112	0.000	0.000	0.227	0.257

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	29	32	44	27	28	50	32
N.S.	1	1.00	0.95	0.78	0.86	1.19	0.73	0.76	1.35	0.86
time (sec)	N/A	0.218	0.008	0.080	0.036	0.067	0.049	0.109	0.187	0.146

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	79	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	15.80	1.00
time (sec)	N/A	0.175	0.022	4.858	0.095	0.091	4.179	0.109	0.176	0.226

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	19	17	14	17	13	10	16	17	13
N.S.	1	1.12	1.00	0.82	1.00	0.76	0.59	0.94	1.00	0.76
time (sec)	N/A	0.154	0.004	0.286	0.023	0.076	0.056	0.108	0.188	0.039

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	12	15	14	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.71	0.88	0.82	0.00
time (sec)	N/A	0.191	0.007	0.812	0.110	0.098	0.082	0.123	0.183	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	567	501	501	501	561	501	500	12
N.S.	1	1.00	24.65	21.78	21.78	21.78	24.39	21.78	21.74	0.52
time (sec)	N/A	0.172	0.006	0.302	0.038	0.082	0.099	0.115	0.200	0.182

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	12	12	11	9	11
N.S.	1	1.00	1.00	0.92	0.85	0.92	0.92	0.85	0.69	0.85
time (sec)	N/A	0.204	0.007	1.222	0.033	0.096	0.460	0.116	0.181	0.191

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	18	16	13	12	12	15	12	12	12
N.S.	1	1.12	1.00	0.81	0.75	0.75	0.94	0.75	0.75	0.75
time (sec)	N/A	0.156	0.022	0.212	0.120	0.082	0.445	0.114	0.172	0.147

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	54	22	24	28	26	36	18	21
N.S.	1	1.00	2.16	0.88	0.96	1.12	1.04	1.44	0.72	0.84
time (sec)	N/A	0.148	0.021	0.500	0.108	0.088	0.083	0.113	0.215	0.075

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	9	9	10	9	16	9
N.S.	1	1.00	1.00	0.83	0.75	0.75	0.83	0.75	1.33	0.75
time (sec)	N/A	0.159	0.021	0.238	0.111	0.089	0.237	0.111	0.233	0.064

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	13	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	1.08	0.67
time (sec)	N/A	0.145	0.004	0.447	0.112	0.086	0.050	0.111	0.192	0.163

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	13	13	9	8	15	22	8	13	13
N.S.	1	1.62	1.62	1.12	1.00	1.88	2.75	1.00	1.62	1.62
time (sec)	N/A	0.187	0.021	0.809	0.042	0.094	0.344	0.119	0.200	0.217

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	39	27	26	80	0	49	56	30
N.S.	1	1.00	3.55	2.45	2.36	7.27	0.00	4.45	5.09	2.73
time (sec)	N/A	0.212	0.004	0.253	0.082	0.077	0.000	0.118	0.197	0.038

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	20	17	19	19	41	19	18	19
N.S.	1	1.15	0.74	0.63	0.70	0.70	1.52	0.70	0.67	0.70
time (sec)	N/A	0.164	0.014	1.461	0.025	0.077	0.104	0.113	0.176	0.146

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	16	11	9	10	11	11	10
N.S.	1	1.00	1.00	1.60	1.10	0.90	1.00	1.10	1.10	1.00
time (sec)	N/A	0.194	0.028	0.054	0.055	0.085	0.685	0.114	0.171	0.192

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	10	15
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	0.77	1.15
time (sec)	N/A	0.303	0.013	2.646	0.234	0.097	0.000	0.000	0.203	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	14	5	7	14	13	16
N.S.	1	1.00	1.00	1.00	1.00	0.36	0.50	1.00	0.93	1.14
time (sec)	N/A	0.147	0.004	0.097	0.041	0.084	0.036	0.113	0.187	0.164

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	3	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	1.00	0.67
time (sec)	N/A	0.134	0.000	0.115	0.033	0.081	0.032	0.111	0.194	0.005

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	B	B	B
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	7	0	7	8	7	7	5	20	7	7
N.S.	1	0.00	1.00	1.14	1.00	1.00	0.71	2.86	1.00	1.00
time (sec)	N/A	0.000	0.646	1.672	0.109	0.084	0.474	0.126	0.178	0.228

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	8	19	19	15	19	16	19
N.S.	1	1.00	2.09	0.73	1.73	1.73	1.36	1.73	1.45	1.73
time (sec)	N/A	0.144	0.000	0.076	0.026	0.077	0.020	0.112	0.185	0.018

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	23	22	41	27	23	21	25
N.S.	1	1.00	1.23	0.66	0.63	1.17	0.77	0.66	0.60	0.71
time (sec)	N/A	0.146	0.074	2.365	0.109	0.080	0.141	0.114	0.176	0.138

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	20	19	45	15	19	45	45
N.S.	1	1.00	1.07	0.74	0.70	1.67	0.56	0.70	1.67	1.67
time (sec)	N/A	0.155	0.027	1.958	0.036	0.091	0.052	0.109	0.183	0.091

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	13	5	0	6	12	0
N.S.	1	1.00	1.00	2.73	0.87	0.33	0.00	0.40	0.80	0.00
time (sec)	N/A	0.291	0.012	2.701	0.028	0.104	0.000	0.122	0.216	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	7	7	5	7	6	6
N.S.	1	1.00	1.00	0.78	0.78	0.78	0.56	0.78	0.67	0.67
time (sec)	N/A	0.154	0.000	0.086	0.032	0.071	0.026	0.115	0.217	0.009

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75	0.75
time (sec)	N/A	0.144	0.000	0.082	0.032	0.093	0.044	0.119	0.189	0.009

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	19	6	10	9	9	8	10	6	7
N.S.	1	3.17	1.00	1.67	1.50	1.50	1.33	1.67	1.00	1.17
time (sec)	N/A	0.165	0.011	0.109	0.026	0.089	0.043	0.112	0.181	0.049

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	29	14	11	10	19	14	10	18	18
N.S.	1	2.07	1.00	0.79	0.71	1.36	1.00	0.71	1.29	1.29
time (sec)	N/A	0.235	0.021	1.414	0.036	0.091	0.034	0.116	0.204	0.146

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	7	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.70	0.80
time (sec)	N/A	0.198	0.022	0.735	0.039	0.101	0.126	0.108	0.206	0.185

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00	1.00
time (sec)	N/A	0.155	0.005	0.220	0.126	0.091	0.028	0.113	0.199	0.018

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	57	31	26	21	21	48	21	21	28
N.S.	1	1.90	1.03	0.87	0.70	0.70	1.60	0.70	0.70	0.93
time (sec)	N/A	0.256	0.009	0.108	0.050	0.083	0.160	0.111	0.204	0.013

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	17	21	19	19	9	18
N.S.	1	1.00	2.38	1.50	2.12	2.62	2.38	2.38	1.12	2.25
time (sec)	N/A	0.178	0.013	1.441	0.037	0.110	0.064	0.106	0.186	0.234

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	21	6	5	6	6	6
N.S.	1	1.00	1.00	1.17	3.50	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.142	0.001	0.091	0.024	0.076	0.052	0.111	0.181	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	16	18	15	14	14	12	14	14	14
N.S.	1	0.89	1.00	0.83	0.78	0.78	0.67	0.78	0.78	0.78
time (sec)	N/A	0.150	0.003	0.285	0.034	0.084	0.035	0.115	0.178	0.017

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	5	4	4	3	4	4	4
N.S.	1	1.00	1.00	0.62	0.50	0.50	0.38	0.50	0.50	0.50
time (sec)	N/A	0.139	0.003	1.661	0.109	0.078	0.054	0.114	0.205	0.030

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	17	89	28	105	26	51	0
N.S.	1	1.00	1.73	1.13	5.93	1.87	7.00	1.73	3.40	0.00
time (sec)	N/A	0.219	0.010	1.270	0.032	0.107	0.614	0.113	0.203	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	7	17	17	15	18	21	6
N.S.	1	1.00	2.88	0.88	2.12	2.12	1.88	2.25	2.62	0.75
time (sec)	N/A	0.143	0.003	0.276	0.039	0.074	0.048	0.104	0.191	0.173

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	23	22	41	27	23	21	25
N.S.	1	1.00	1.23	0.66	0.63	1.17	0.77	0.66	0.60	0.71
time (sec)	N/A	0.146	0.001	0.481	0.116	0.084	0.162	0.118	0.188	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	14	14	14	14	13
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.82	0.82	0.82	0.76
time (sec)	N/A	0.200	0.019	4.954	0.029	0.110	0.060	0.107	0.202	0.388

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	14	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	1.27	0.91
time (sec)	N/A	0.175	0.014	0.293	0.036	0.084	0.213	0.111	0.191	0.013

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	14	24	39	18	17	18
N.S.	1	1.00	1.00	0.82	0.64	1.09	1.77	0.82	0.77	0.82
time (sec)	N/A	0.153	0.019	0.980	0.112	0.084	0.615	0.109	0.180	0.048

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	10	8	8
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.00	0.80	0.80
time (sec)	N/A	0.145	0.000	0.047	0.032	0.080	0.043	0.111	0.192	0.009

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	29	26	21	21	34	21	25	21
N.S.	1	1.06	0.83	0.74	0.60	0.60	0.97	0.60	0.71	0.60
time (sec)	N/A	0.301	0.018	0.397	0.026	0.105	0.168	0.115	0.191	0.206

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	9	9	7	9	12	11
N.S.	1	1.00	1.00	1.00	0.90	0.90	0.70	0.90	1.20	1.10
time (sec)	N/A	0.154	0.045	0.139	0.037	0.099	0.046	0.113	0.185	0.038

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	37	11	19	14	14	31	14	13	23
N.S.	1	1.09	0.32	0.56	0.41	0.41	0.91	0.41	0.38	0.68
time (sec)	N/A	0.226	0.003	0.109	0.042	0.088	0.082	0.109	0.172	0.154

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	11	21	13
N.S.	1	1.00	1.00	0.80	0.73	0.87	1.73	0.73	1.40	0.87
time (sec)	N/A	0.180	0.005	1.799	0.027	0.109	0.150	0.109	0.178	0.023

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.140	0.017	0.645	0.039	0.116	0.027	0.110	0.182	0.167

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12138	10076	10076	0	12024	10076	10075	15
N.S.	1	1.00	527.74	438.09	438.09	0.00	522.78	438.09	438.04	0.65
time (sec)	N/A	1.188	0.066	7.771	2.520	0.000	2.789	1.683	0.879	3.623

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	22	12	22	13	14
N.S.	1	1.00	1.00	0.94	0.88	1.38	0.75	1.38	0.81	0.88
time (sec)	N/A	0.271	0.002	0.124	0.034	0.083	0.061	0.108	0.171	0.163

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	16	12	11	11	10	12	11	11
N.S.	1	1.00	1.07	0.80	0.73	0.73	0.67	0.80	0.73	0.73
time (sec)	N/A	0.234	0.002	0.267	0.025	0.096	0.041	0.107	0.195	0.014

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	20	16	16	15	13	15	15	15	14
N.S.	1	0.95	0.76	0.76	0.71	0.62	0.71	0.71	0.71	0.67
time (sec)	N/A	0.277	0.004	0.200	0.105	0.084	0.088	0.109	0.203	0.011

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	13	8
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.68	0.42
time (sec)	N/A	0.240	0.003	0.672	0.031	0.077	0.063	0.107	0.173	0.052

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	28	16	18	17	17	19	27	16	15
N.S.	1	1.47	0.84	0.95	0.89	0.89	1.00	1.42	0.84	0.79
time (sec)	N/A	0.690	0.031	0.494	0.198	0.110	0.740	0.118	0.178	0.257

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	25	24	23	23	22	23	22	22
N.S.	1	1.04	1.00	0.96	0.92	0.92	0.88	0.92	0.88	0.88
time (sec)	N/A	0.234	0.002	0.354	0.106	0.092	0.105	0.119	0.213	0.018

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	22	17	13	22	68	18	18	26
N.S.	1	1.27	1.00	0.77	0.59	1.00	3.09	0.82	0.82	1.18
time (sec)	N/A	0.230	0.003	0.803	0.138	0.101	0.884	0.112	0.186	0.182

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	55	15
N.S.	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	2.89	0.79
time (sec)	N/A	0.239	0.007	0.787	0.131	0.102	0.000	0.136	0.180	0.019

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	24	19	19	18	15	22	19	18
N.S.	1	1.16	0.96	0.76	0.76	0.72	0.60	0.88	0.76	0.72
time (sec)	N/A	0.189	0.004	0.057	0.036	0.082	0.060	0.116	0.189	0.040

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	6	11	17	16	11	0	12	39	0
N.S.	1	0.55	1.00	1.55	1.45	1.00	0.00	1.09	3.55	0.00
time (sec)	N/A	0.216	0.050	0.711	0.055	0.081	0.000	0.113	0.215	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	41	37	57	0	26	42	0	23	0
N.S.	1	1.11	1.00	1.54	0.00	0.70	1.14	0.00	0.62	0.00
time (sec)	N/A	0.182	0.034	0.096	0.000	0.085	0.156	0.000	0.189	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	30	20	16	19	23	32	40	14	32
N.S.	1	1.50	1.00	0.80	0.95	1.15	1.60	2.00	0.70	1.60
time (sec)	N/A	0.210	0.016	0.274	0.119	0.117	0.168	0.111	0.182	0.218

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	29	8	7	6	5	29	6	14	6
N.S.	1	5.80	1.60	1.40	1.20	1.00	5.80	1.20	2.80	1.20
time (sec)	N/A	0.183	0.019	2.544	0.027	0.090	0.027	0.111	0.178	0.161

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	C	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	37	16	13	19	12	31	22	13	14
N.S.	1	2.31	1.00	0.81	1.19	0.75	1.94	1.38	0.81	0.88
time (sec)	N/A	0.160	0.016	0.280	0.031	0.087	0.633	0.114	0.194	0.020

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	5	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.62	0.75
time (sec)	N/A	0.193	0.004	0.275	0.036	0.099	0.121	0.108	0.200	0.020

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	7	6	17	15	25	17	11
N.S.	1	1.00	1.00	2.33	2.00	5.67	5.00	8.33	5.67	3.67
time (sec)	N/A	0.164	0.000	0.040	0.030	0.087	0.078	0.115	0.201	0.001

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	3	3	4	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	0.75	0.75	1.00	0.75
time (sec)	N/A	0.177	0.006	0.446	0.036	0.090	0.138	0.107	0.213	0.158

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61	0.61
time (sec)	N/A	0.181	0.003	0.102	0.029	0.095	0.051	0.105	0.217	0.020

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	34	22	19	18	18	20	18	16	18
N.S.	1	1.55	1.00	0.86	0.82	0.82	0.91	0.82	0.73	0.82
time (sec)	N/A	0.194	0.013	0.292	0.139	0.074	0.095	0.116	0.190	0.034

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00	1.00
time (sec)	N/A	0.147	0.001	0.135	0.036	0.108	0.045	0.111	0.181	0.031

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	28	21	20	27	22	21	38	22
N.S.	1	1.27	1.27	0.95	0.91	1.23	1.00	0.95	1.73	1.00
time (sec)	N/A	0.163	0.008	0.389	0.043	0.088	0.057	0.116	0.232	0.166

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	16	15	17	16	15	19	16	18	18
N.S.	1	1.07	1.00	1.13	1.07	1.00	1.27	1.07	1.20	1.20
time (sec)	N/A	0.266	0.008	0.089	0.028	0.085	0.102	0.108	0.235	0.194

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	10	15
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	0.77	1.15
time (sec)	N/A	0.512	0.012	2.233	0.205	0.098	0.000	0.000	0.200	0.002

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	16	5	4	14	3	25	13	4
N.S.	1	1.00	2.67	0.83	0.67	2.33	0.50	4.17	2.17	0.67
time (sec)	N/A	0.201	0.012	0.296	0.115	0.087	0.082	0.113	0.188	0.145

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	11	7	7	11	7
N.S.	1	1.00	1.00	1.14	1.00	1.57	1.00	1.00	1.57	1.00
time (sec)	N/A	0.303	0.003	0.123	0.030	0.087	0.040	0.111	0.196	0.157

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	31	26	25	25	27	25	29	25
N.S.	1	0.97	0.89	0.74	0.71	0.71	0.77	0.71	0.83	0.71
time (sec)	N/A	0.414	0.012	3.516	0.037	0.112	3.576	0.111	0.173	0.210

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	37	32	40	39	47	0	48	13	44
N.S.	1	0.52	0.45	0.56	0.55	0.66	0.00	0.68	0.18	0.62
time (sec)	N/A	0.393	0.047	2.040	0.118	0.103	0.000	0.111	0.188	0.242

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	15	13	10	9	9	8	10	10	9
N.S.	1	1.25	1.08	0.83	0.75	0.75	0.67	0.83	0.83	0.75
time (sec)	N/A	0.301	0.017	0.133	0.032	0.099	0.038	0.115	0.186	0.031

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	12	19	12	12	12
N.S.	1	1.00	1.14	0.93	0.86	0.86	1.36	0.86	0.86	0.86
time (sec)	N/A	0.290	0.005	0.151	0.117	0.089	0.028	0.101	0.206	0.020

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	17	7
N.S.	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	3.40	1.40
time (sec)	N/A	0.312	0.012	1.905	0.029	0.088	0.033	0.115	0.178	0.243

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.134	0.002	2.735	0.031	0.091	0.036	0.108	0.173	0.020

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	12	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	4.00	1.00
time (sec)	N/A	0.176	0.001	0.266	0.041	0.085	0.052	0.115	0.185	0.016

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	11	11	11	8	11	10	10
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.53	0.73	0.67	0.67
time (sec)	N/A	0.149	0.000	0.075	0.024	0.067	0.021	0.109	0.182	0.017

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	9	9	8	9	7	7
N.S.	1	1.00	0.86	0.71	0.64	0.64	0.57	0.64	0.50	0.50
time (sec)	N/A	0.146	0.000	0.076	0.027	0.070	0.042	0.110	0.185	0.016

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	9	8	20	14	12	15	8
N.S.	1	0.00	1.00	0.82	0.73	1.82	1.27	1.09	1.36	0.73
time (sec)	N/A	0.000	0.026	0.684	0.136	0.085	0.123	0.117	0.191	0.207

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.186	0.004	0.124	0.023	0.097	0.065	0.110	0.197	0.233

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	20	36	16	21	10
N.S.	1	1.00	1.00	0.79	1.14	1.43	2.57	1.14	1.50	0.71
time (sec)	N/A	0.211	0.007	0.310	0.128	0.104	0.222	0.109	0.196	0.171

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	39	34	33	25	37	49	8	25
N.S.	1	1.11	0.87	0.76	0.73	0.56	0.82	1.09	0.18	0.56
time (sec)	N/A	0.209	0.029	0.273	0.118	4.526	0.186	0.125	0.185	0.485

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75	0.75
time (sec)	N/A	0.145	0.000	0.057	0.042	0.089	0.039	0.112	0.189	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	21	5	5	21	6	5
N.S.	1	1.00	1.00	1.00	3.50	0.83	0.83	3.50	1.00	0.83
time (sec)	N/A	0.152	0.005	1.332	0.026	0.093	0.095	0.112	0.216	0.163

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	13	13	12	13	16	13
N.S.	1	1.00	0.62	0.54	0.50	0.50	0.46	0.50	0.62	0.50
time (sec)	N/A	0.192	0.020	0.123	0.044	0.085	0.043	0.107	0.212	0.166

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	C	A	F	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	27	10	473	12	0	12	10	9
N.S.	1	0.00	2.70	1.00	47.30	1.20	0.00	1.20	1.00	0.90
time (sec)	N/A	0.000	0.419	116.559	0.119	0.095	0.000	0.118	0.206	0.041

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	50	35	34	32	51	34	33	34
N.S.	1	1.11	0.89	0.62	0.61	0.57	0.91	0.61	0.59	0.61
time (sec)	N/A	0.333	0.028	0.081	0.024	0.082	5.003	0.112	0.176	0.018

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	21	8	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.33	0.89	0.78	0.78	0.78
time (sec)	N/A	0.211	1.996	0.576	0.039	0.098	0.310	0.116	0.185	0.183

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	27	10	19	29	13	14
N.S.	1	1.00	1.00	0.93	1.80	0.67	1.27	1.93	0.87	0.93
time (sec)	N/A	0.230	0.034	2.404	0.071	0.089	1.499	0.109	0.203	0.034

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	20	17	22	19	37	19	18	20
N.S.	1	1.15	0.74	0.63	0.81	0.70	1.37	0.70	0.67	0.74
time (sec)	N/A	0.161	0.004	0.270	0.124	0.084	0.162	0.112	0.186	0.018

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	26	18	29	20
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.45	1.00
time (sec)	N/A	0.165	0.007	0.065	0.118	0.080	0.048	0.114	0.184	0.168

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	25	20	19	19	34	38	23	30
N.S.	1	0.93	0.60	0.48	0.45	0.45	0.81	0.90	0.55	0.71
time (sec)	N/A	0.230	0.011	0.129	0.048	0.090	0.467	0.113	0.182	0.033

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	13	5	8	18	16
N.S.	1	1.00	1.00	1.14	1.00	1.86	0.71	1.14	2.57	2.29
time (sec)	N/A	0.145	0.004	0.274	0.038	0.100	0.024	0.115	0.173	0.064

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	19	20	19	25
N.S.	1	1.00	1.00	0.74	0.70	0.70	0.70	0.74	0.70	0.93
time (sec)	N/A	0.181	0.007	0.283	0.131	0.109	0.072	0.118	0.169	0.031

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	30	20	16	30	39	36	26	25	42
N.S.	1	1.50	1.00	0.80	1.50	1.95	1.80	1.30	1.25	2.10
time (sec)	N/A	0.184	0.015	0.246	0.117	0.090	0.176	0.117	0.169	0.076

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	0.69
time (sec)	N/A	0.138	0.009	1.223	0.032	0.100	0.074	0.117	0.197	0.022

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	13	9	8	13	10	9
N.S.	1	1.00	1.00	0.85	1.00	0.69	0.62	1.00	0.77	0.69
time (sec)	N/A	0.155	0.002	0.056	0.033	0.090	0.043	0.111	0.184	0.022

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	5	4	3	8	7	5	6	5
N.S.	1	1.00	1.67	1.33	1.00	2.67	2.33	1.67	2.00	1.67
time (sec)	N/A	0.156	0.001	0.789	0.035	0.084	0.184	0.111	0.193	0.012

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	11	11	10	11	12	11
N.S.	1	1.00	0.64	0.55	0.50	0.50	0.45	0.50	0.55	0.50
time (sec)	N/A	0.187	0.001	0.119	0.032	0.136	0.042	0.112	0.197	0.001

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	7	14	17	8	11	8
N.S.	1	1.00	1.00	0.90	0.70	1.40	1.70	0.80	1.10	0.80
time (sec)	N/A	0.143	0.012	0.523	0.111	0.112	0.569	0.109	0.201	0.057

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	17	13	12	15	11	10	15	11	11
N.S.	1	1.31	1.00	0.92	1.15	0.85	0.77	1.15	0.85	0.85
time (sec)	N/A	0.135	0.003	0.245	0.029	0.084	0.044	0.108	0.180	0.153

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	14	22	0	22	13	17
N.S.	1	1.00	1.00	0.86	0.67	1.05	0.00	1.05	0.62	0.81
time (sec)	N/A	0.164	0.003	0.142	0.043	0.105	0.000	0.111	0.190	0.175

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	23	22	21	32	21	19	22
N.S.	1	1.00	1.00	0.68	0.65	0.62	0.94	0.62	0.56	0.65
time (sec)	N/A	0.179	0.037	0.055	0.135	0.109	0.283	0.107	0.207	0.153

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	13	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.59	0.73
time (sec)	N/A	0.232	0.019	0.365	0.039	0.093	0.101	0.110	0.193	0.189

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	22	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	1.16	0.89
time (sec)	N/A	0.152	0.007	0.256	0.111	0.083	0.049	0.108	0.182	0.021

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	11	9	8	9	10	9
N.S.	1	1.00	1.00	1.00	1.10	0.90	0.80	0.90	1.00	0.90
time (sec)	N/A	0.162	0.022	0.191	0.069	0.082	0.044	0.107	0.175	0.034

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	18	5	25	15	6
N.S.	1	1.00	5.00	0.88	0.75	2.25	0.62	3.12	1.88	0.75
time (sec)	N/A	0.153	0.030	1.355	0.119	0.091	0.532	0.119	0.186	0.006

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	12	23	0	10	17	12
N.S.	1	1.00	1.00	1.58	1.00	1.92	0.00	0.83	1.42	1.00
time (sec)	N/A	0.160	0.011	0.244	0.119	0.096	0.000	0.111	0.180	0.157

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	22	7	16
N.S.	1	1.00	3.33	1.42	0.00	2.00	0.00	1.83	0.58	1.33
time (sec)	N/A	0.167	0.010	1.095	0.000	0.103	0.000	0.114	0.187	0.018

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	15	14	15	14	12	14
N.S.	1	1.00	1.00	0.83	0.83	0.78	0.83	0.78	0.67	0.78
time (sec)	N/A	0.152	0.010	0.263	0.039	0.078	0.054	0.109	0.180	0.032

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	38	35	26	43	0	0	18	0
N.S.	1	1.00	0.73	0.67	0.50	0.83	0.00	0.00	0.35	0.00
time (sec)	N/A	0.241	0.043	3.549	0.127	0.120	0.000	0.000	0.202	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	0	137	33	0	33	54	45
N.S.	1	1.00	0.95	0.00	3.51	0.85	0.00	0.85	1.38	1.15
time (sec)	N/A	0.402	0.454	0.000	0.174	0.087	0.000	0.115	0.192	0.268

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	20	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.74	0.70
time (sec)	N/A	0.171	0.032	0.885	0.026	0.083	0.128	0.107	0.176	0.017

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.263	0.025	7.878	0.060	0.095	0.773	0.113	0.181	0.318

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	6	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	1.00	0.83
time (sec)	N/A	0.165	0.010	0.123	0.037	0.068	0.041	0.112	0.204	0.016

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.75
time (sec)	N/A	0.163	0.021	0.612	0.034	0.106	0.074	0.111	0.183	0.151

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	12	8	7	10	16	10	6	6	6
N.S.	1	1.50	1.00	0.88	1.25	2.00	1.25	0.75	0.75	0.75
time (sec)	N/A	0.188	0.001	0.451	0.028	0.150	0.254	0.117	0.182	0.174

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	14	14	17	14	416	14
N.S.	1	1.00	1.00	0.79	0.74	0.74	0.89	0.74	21.89	0.74
time (sec)	N/A	0.144	0.006	0.254	0.120	0.089	0.057	0.120	0.260	0.022

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	6	52	34	5	45	5	5
N.S.	1	0.00	1.00	1.20	10.40	6.80	1.00	9.00	1.00	1.00
time (sec)	N/A	0.000	0.011	6.880	0.044	0.087	0.160	0.113	0.186	0.033

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	7	77	7	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	1.00	11.00	1.00	0.86
time (sec)	N/A	0.200	0.070	1.290	0.034	0.111	0.070	0.116	0.190	0.161

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	44	5	4	4	4
N.S.	1	1.00	1.00	1.25	1.00	11.00	1.25	1.00	1.00	1.00
time (sec)	N/A	0.243	7.248	9.565	0.041	0.094	0.873	0.107	0.200	0.075

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	14	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.40	1.00
time (sec)	N/A	0.168	0.004	0.194	0.033	0.084	0.197	0.117	0.203	0.001

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	42	6	0	6	86	6
N.S.	1	1.00	1.00	0.83	7.00	1.00	0.00	1.00	14.33	1.00
time (sec)	N/A	0.196	0.005	2.173	0.043	0.121	0.000	0.112	0.180	0.019

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	6	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	1.00	0.83
time (sec)	N/A	0.148	0.005	0.184	0.059	0.118	0.099	0.112	0.181	0.171

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	12	16	3	12	6	6
N.S.	1	1.00	1.33	1.17	2.00	2.67	0.50	2.00	1.00	1.00
time (sec)	N/A	0.168	0.005	0.185	0.030	0.096	0.078	0.116	0.179	0.160

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	16	7	8	36	9	8	21	43	26
N.S.	1	2.29	1.00	1.14	5.14	1.29	1.14	3.00	6.14	3.71
time (sec)	N/A	0.289	0.006	1.174	0.043	0.111	1.404	0.118	0.195	0.195

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	0	0	6	0	6	11	6
N.S.	1	1.00	1.00	0.00	0.00	0.60	0.00	0.60	1.10	0.60
time (sec)	N/A	0.168	0.024	180.000	0.000	0.094	0.000	0.112	200.032	0.110

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	219	3	46	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	73.00	1.00	15.33	1.00
time (sec)	N/A	0.193	0.008	0.238	0.126	0.110	10.339	0.117	0.258	0.172

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	21	9
N.S.	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	1.91	0.82
time (sec)	N/A	0.134	0.002	0.112	0.033	0.082	0.398	0.123	0.269	0.022

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	7	0	29	0	15	6	0
N.S.	1	1.00	1.00	0.35	0.00	1.45	0.00	0.75	0.30	0.00
time (sec)	N/A	0.154	0.066	0.070	0.000	0.085	0.000	0.132	0.282	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	12	8	11	11	7	12	12	12
N.S.	1	1.00	1.71	1.14	1.57	1.57	1.00	1.71	1.71	1.71
time (sec)	N/A	0.245	0.055	0.109	0.078	0.086	0.060	0.117	0.257	0.211

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	17	19	14	16	24	3
N.S.	1	1.00	3.00	1.33	5.67	6.33	4.67	5.33	8.00	1.00
time (sec)	N/A	0.169	0.007	0.100	0.029	0.106	0.058	0.120	0.275	0.028

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	5	3	5	5	5	5	5	5
N.S.	1	1.00	1.67	1.00	1.67	1.67	1.67	1.67	1.67	1.67
time (sec)	N/A	0.367	0.027	0.029	0.343	0.094	1.068	0.111	0.250	0.161

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.205	0.040	0.088	0.073	0.077	0.058	0.109	0.250	0.157

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	15	15	12	11	27	0	11	17	5
N.S.	1	2.50	2.50	2.00	1.83	4.50	0.00	1.83	2.83	0.83
time (sec)	N/A	0.204	0.016	0.035	0.035	0.088	0.000	0.118	0.252	0.171

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	13	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	6.50	1.00
time (sec)	N/A	0.207	0.002	0.082	0.046	0.071	0.047	0.110	0.276	0.176

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	13	12	13	24	14	10	14	12	13
N.S.	1	1.08	1.00	1.08	2.00	1.17	0.83	1.17	1.00	1.08
time (sec)	N/A	0.249	0.005	0.072	0.049	0.094	0.116	0.111	0.246	0.166

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	0	60	29	47	29
N.S.	1	1.00	1.00	0.77	0.74	0.00	1.54	0.74	1.21	0.74
time (sec)	N/A	0.329	0.042	2.408	0.030	0.000	0.262	0.115	0.242	0.496

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4	4
N.S.	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00	1.00
time (sec)	N/A	0.402	4.561	9.915	0.026	0.108	0.892	0.116	0.251	0.069

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	29	17	22	0	41	17
N.S.	1	1.00	1.00	0.75	1.21	0.71	0.92	0.00	1.71	0.71
time (sec)	N/A	0.319	0.006	0.050	0.077	0.091	0.426	0.000	0.448	0.172

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	13	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.59	0.73
time (sec)	N/A	0.401	0.004	0.028	0.046	0.099	0.102	0.108	0.251	0.001

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	12	10	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.88	0.75	0.62	0.75
time (sec)	N/A	0.194	0.002	0.099	0.112	0.082	0.083	0.115	0.254	0.184

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	89	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	2.97	0.73
time (sec)	N/A	0.364	0.024	1.273	0.031	0.096	0.816	0.117	0.270	0.247

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	34	34	185	30	0	29	0	11	0
N.S.	1	1.48	1.48	8.04	1.30	0.00	1.26	0.00	0.48	0.00
time (sec)	N/A	0.269	0.005	0.040	0.107	0.000	0.409	0.000	0.261	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	4	4	3	4	3	4
N.S.	1	1.00	1.00	1.67	1.33	1.33	1.00	1.33	1.00	1.33
time (sec)	N/A	0.207	0.000	0.010	0.030	0.072	0.015	0.098	0.230	0.001

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	C	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	139	131	16	2527	123	0	134	33	15
N.S.	1	12.64	11.91	1.45	229.73	11.18	0.00	12.18	3.00	1.36
time (sec)	N/A	2.507	0.248	1.040	0.220	0.178	0.000	0.149	0.247	0.853

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	11	9	34	7	8	33	8	8
N.S.	1	1.00	1.38	1.12	4.25	0.88	1.00	4.12	1.00	1.00
time (sec)	N/A	0.155	0.009	0.298	0.051	0.084	0.178	0.108	0.259	0.032

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	33	21	22	44	11	46
N.S.	1	1.00	1.21	1.12	1.38	0.88	0.92	1.83	0.46	1.92
time (sec)	N/A	0.215	0.065	0.164	0.154	0.101	0.533	0.121	0.233	0.155

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	0.67	0.67
time (sec)	N/A	0.182	0.010	0.091	0.114	0.080	0.069	0.109	0.222	0.081

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	A	B	B	F	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	12	5	11	7	0	5	5
N.S.	1	0.00	1.00	2.40	1.00	2.20	1.40	0.00	1.00	1.00
time (sec)	N/A	0.000	0.013	0.034	0.088	0.080	0.069	0.000	0.263	0.196

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	19	11	12	13	11	8	13	18	11
N.S.	1	1.73	1.00	1.09	1.18	1.00	0.73	1.18	1.64	1.00
time (sec)	N/A	0.173	0.004	0.084	0.037	0.096	0.041	0.117	0.250	0.035

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	9	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	4.50	1.00
time (sec)	N/A	0.125	0.001	0.076	0.125	0.097	0.066	0.111	0.249	0.016

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	18	9	10	18	9	10	10	14	9
N.S.	1	2.00	1.00	1.11	2.00	1.00	1.11	1.11	1.56	1.00
time (sec)	N/A	0.293	0.009	0.040	0.031	0.098	0.050	0.105	0.250	0.168

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	13	10	10	9	9	7	9	10	9
N.S.	1	1.30	1.00	1.00	0.90	0.90	0.70	0.90	1.00	0.90
time (sec)	N/A	0.256	0.012	0.014	0.037	0.099	0.032	0.106	0.269	0.028

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	13	13
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.65	0.65
time (sec)	N/A	0.250	0.008	0.030	0.029	0.084	0.130	0.110	0.237	0.204

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	22	20	15	107	33	20
N.S.	1	1.00	1.00	0.88	0.88	0.80	0.60	4.28	1.32	0.80
time (sec)	N/A	0.239	0.005	0.069	0.047	0.083	0.063	0.118	0.235	0.053

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	0.75
time (sec)	N/A	0.262	0.005	0.212	0.113	0.074	0.052	0.117	0.253	0.031

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	33	20	13	5	12	17	0	5	12
N.S.	1	1.65	1.00	0.65	0.25	0.60	0.85	0.00	0.25	0.60
time (sec)	N/A	0.473	0.005	0.096	0.197	0.093	0.529	0.000	0.252	0.227

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	16	25	20	8	19	10	19	10	16
N.S.	1	1.33	2.08	1.67	0.67	1.58	0.83	1.58	0.83	1.33
time (sec)	N/A	0.463	0.025	6.605	0.038	0.087	0.950	0.113	0.239	0.247

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	16	17	16	16	15	16	16	16
N.S.	1	1.06	1.00	1.06	1.00	1.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.263	0.002	0.053	0.112	0.093	0.065	0.112	0.248	0.026

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	13	13	12	13	13	9
N.S.	1	1.00	1.00	0.67	0.87	0.87	0.80	0.87	0.87	0.60
time (sec)	N/A	0.238	0.002	0.149	0.039	0.070	0.033	0.113	0.258	0.155

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	23	23	22	19	37	40	21	0
N.S.	1	1.00	0.82	0.82	0.79	0.68	1.32	1.43	0.75	0.00
time (sec)	N/A	0.182	0.006	0.057	0.124	0.111	0.803	0.124	0.247	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	12	13	21	13	47	17	13	10	24
N.S.	1	0.92	1.00	1.62	1.00	3.62	1.31	1.00	0.77	1.85
time (sec)	N/A	0.311	2.238	12.444	0.029	0.101	0.695	0.111	0.238	0.253

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	9	9	8	11	9	11
N.S.	1	1.00	1.22	1.00	1.00	1.00	0.89	1.22	1.00	1.22
time (sec)	N/A	0.181	1.172	0.052	0.258	0.091	0.524	0.103	0.278	0.195

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	1038	14	14	14	13	14
N.S.	1	1.00	1.14	0.86	74.14	1.00	1.00	1.00	0.93	1.00
time (sec)	N/A	0.199	6.459	0.106	0.583	0.110	0.742	0.120	0.255	0.197

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	11128	10106	10106	0	11171	10106	10105	15
N.S.	1	1.00	483.83	439.39	439.39	0.00	485.70	439.39	439.35	0.65
time (sec)	N/A	1.160	0.070	8.499	2.681	0.000	2.731	1.018	1.120	3.637

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	21	19	10	9	19	29	19	27	15
N.S.	1	1.91	1.73	0.91	0.82	1.73	2.64	1.73	2.45	1.36
time (sec)	N/A	0.237	0.012	3.940	0.041	0.109	0.277	0.119	0.278	0.197

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00	1.00
time (sec)	N/A	0.274	0.014	0.033	0.077	0.083	0.080	0.110	0.268	0.228

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	16	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.70	0.74
time (sec)	N/A	0.143	0.004	0.191	0.128	0.096	0.101	0.116	0.238	0.001

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	22	21	21	20	35	12	21
N.S.	1	1.00	1.00	0.79	0.75	0.75	0.71	1.25	0.43	0.75
time (sec)	N/A	0.216	0.027	0.032	0.027	0.095	0.558	0.454	0.242	0.202

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	72	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	14.40	1.00	1.00
time (sec)	N/A	0.185	0.054	0.182	0.035	0.096	0.065	0.113	0.226	0.053

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	12	9	5	0	5	17	0
N.S.	1	1.00	1.00	0.21	0.16	0.09	0.00	0.09	0.30	0.00
time (sec)	N/A	0.214	0.005	0.050	0.274	0.077	0.000	0.115	0.265	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	29	22	11	10	29	42	10	11	10
N.S.	1	2.07	1.57	0.79	0.71	2.07	3.00	0.71	0.79	0.71
time (sec)	N/A	0.220	0.005	0.096	0.038	0.101	0.217	0.121	0.243	0.174

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	55	34	30	0	33	0	33	45	51
N.S.	1	1.22	0.76	0.67	0.00	0.73	0.00	0.73	1.00	1.13
time (sec)	N/A	0.203	0.209	0.036	0.000	0.076	0.000	0.119	0.256	0.187

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	7	5	7	7	7	7	7	7
N.S.	1	1.00	1.40	1.00	1.40	1.40	1.40	1.40	1.40	1.40
time (sec)	N/A	0.170	1.089	0.064	0.315	0.091	0.523	0.110	0.265	0.194

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	18	20	17	16	16	15	16	16	16
N.S.	1	0.90	1.00	0.85	0.80	0.80	0.75	0.80	0.80	0.80
time (sec)	N/A	0.249	0.003	0.165	0.028	0.095	0.120	0.110	0.256	0.171

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	7	17	17	15	18	21	6
N.S.	1	1.00	2.88	0.88	2.12	2.12	1.88	2.25	2.62	0.75
time (sec)	N/A	0.141	0.003	0.080	0.028	0.080	0.040	0.113	0.237	0.001

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	10	20	0	10	16	2
N.S.	1	1.00	1.00	1.50	5.00	10.00	0.00	5.00	8.00	1.00
time (sec)	N/A	0.177	0.001	0.102	0.035	0.098	0.000	0.114	0.255	0.006

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	0	7	7	0	11	7
N.S.	1	1.00	1.00	0.89	0.00	0.78	0.78	0.00	1.22	0.78
time (sec)	N/A	0.181	0.009	0.031	0.000	0.085	0.057	0.000	0.258	0.167

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	26	24	23	21	306	30	18	10
N.S.	1	1.06	0.74	0.69	0.66	0.60	8.74	0.86	0.51	0.29
time (sec)	N/A	0.169	0.015	0.067	0.038	0.095	0.629	0.111	0.241	0.211

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	194	20	21	20	20	19	21	20	20
N.S.	1	8.08	0.83	0.88	0.83	0.83	0.79	0.88	0.83	0.83
time (sec)	N/A	0.666	0.006	0.019	0.122	0.084	0.056	0.110	0.224	0.029

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	F	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	6	5	65	0	5	17	5
N.S.	1	0.00	1.00	1.20	1.00	13.00	0.00	1.00	3.40	1.00
time (sec)	N/A	0.000	0.081	11.888	0.243	0.103	0.000	0.114	0.274	0.254

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	33	12	40	79	14	80	55	28	14
N.S.	1	2.75	1.00	3.33	6.58	1.17	6.67	4.58	2.33	1.17
time (sec)	N/A	0.238	0.021	5.988	0.035	0.085	0.149	0.123	0.230	0.305

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	18	12	16	11	10	11	10	11
N.S.	1	0.00	1.80	1.20	1.60	1.10	1.00	1.10	1.00	1.10
time (sec)	N/A	0.000	0.008	0.108	0.032	0.084	0.284	0.112	0.256	0.197

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	C	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	42	3	4	14	3	0	40	55	3
N.S.	1	14.00	1.00	1.33	4.67	1.00	0.00	13.33	18.33	1.00
time (sec)	N/A	0.252	0.000	0.618	0.109	0.077	0.000	0.126	16.167	0.311

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	18	5	25	15	6
N.S.	1	1.00	5.00	0.88	0.75	2.25	0.62	3.12	1.88	0.75
time (sec)	N/A	0.144	0.008	0.214	0.123	0.083	0.231	0.115	0.248	0.145

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	16	15	14	13	31	63	46	37	26
N.S.	1	0.47	0.44	0.41	0.38	0.91	1.85	1.35	1.09	0.76
time (sec)	N/A	0.182	0.062	0.095	0.123	0.121	0.287	0.109	0.243	0.182

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	19	14	20	22	18
N.S.	1	1.00	1.00	0.89	1.00	1.06	0.78	1.11	1.22	1.00
time (sec)	N/A	0.158	0.005	0.069	0.038	0.094	0.078	0.108	0.259	0.160

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	56	0	50	70	0	0	12	0
N.S.	1	0.00	1.00	0.00	0.89	1.25	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.021	0.000	0.224	0.097	0.000	0.000	0.252	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	17
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.85
time (sec)	N/A	0.204	0.002	0.020	0.024	0.076	0.018	0.110	0.263	0.016

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	13	12	32	11	65	0	40	11
N.S.	1	1.00	0.62	0.57	1.52	0.52	3.10	0.00	1.90	0.52
time (sec)	N/A	0.204	8.568	0.124	0.118	0.100	1.276	0.000	0.242	0.056

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	25	33	31	25	39	29	18	38	38
N.S.	1	1.47	1.94	1.82	1.47	2.29	1.71	1.06	2.24	2.24
time (sec)	N/A	0.213	0.033	0.298	0.026	0.081	0.027	0.118	0.230	0.221

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	31	8	7	61	10	5	6	6	6
N.S.	1	3.88	1.00	0.88	7.62	1.25	0.62	0.75	0.75	0.75
time (sec)	N/A	0.399	0.030	0.106	0.037	0.113	0.156	0.119	0.257	0.151

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	23	14	39	42	19	37	46	14
N.S.	1	1.00	1.35	0.82	2.29	2.47	1.12	2.18	2.71	0.82
time (sec)	N/A	0.202	0.009	10.596	0.034	0.076	0.169	0.109	0.254	0.171

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	30	22	23	31	22	24	51	11	22
N.S.	1	0.91	0.67	0.70	0.94	0.67	0.73	1.55	0.33	0.67
time (sec)	N/A	0.215	0.013	0.089	0.148	0.111	0.081	0.113	0.257	0.080

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	217	6	44	0	15	0	35	29	35
N.S.	1	36.17	1.00	7.33	0.00	2.50	0.00	5.83	4.83	5.83
time (sec)	N/A	0.959	0.055	0.515	0.000	0.113	0.000	0.110	0.258	0.341

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	C	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	81	21	28	45	16	0	5161	49	0
N.S.	1	9.00	2.33	3.11	5.00	1.78	0.00	573.44	5.44	0.00
time (sec)	N/A	0.235	0.577	1.606	0.126	0.138	0.000	1.574	0.239	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	56	0	0	0	0	0	27	0
N.S.	1	1.16	0.84	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.337	0.038	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	66	62	394	62	175	78
N.S.	1	1.00	1.00	0.75	0.79	0.74	4.69	0.74	2.08	0.93
time (sec)	N/A	0.371	0.101	238.451	0.071	0.114	3.124	0.111	0.231	0.280

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	58	16	19	23	16	32
N.S.	1	1.00	1.00	1.05	2.90	0.80	0.95	1.15	0.80	1.60
time (sec)	N/A	0.319	0.074	0.263	0.125	0.117	0.063	0.119	0.239	0.382

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	89	53	62	83	119	50	36	0
N.S.	1	1.00	1.41	0.84	0.98	1.32	1.89	0.79	0.57	0.00
time (sec)	N/A	0.274	0.131	0.276	0.121	0.088	0.981	0.115	0.273	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	14	17	20	17	16	14
N.S.	1	1.00	1.00	1.24	0.67	0.81	0.95	0.81	0.76	0.67
time (sec)	N/A	0.279	0.012	0.159	0.031	0.113	0.350	0.109	0.252	0.182

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	41	40	103	39	36	39	41	39
N.S.	1	1.00	0.54	0.53	1.36	0.51	0.47	0.51	0.54	0.51
time (sec)	N/A	0.692	0.432	0.061	0.032	0.079	0.047	0.107	0.246	0.177

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	A	A	A	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	38	80	34	34	34	163	33	0
N.S.	1	0.00	1.00	2.11	0.89	0.89	0.89	4.29	0.87	0.00
time (sec)	N/A	0.000	0.012	0.361	0.112	0.091	0.115	0.130	0.259	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	115	0	0	0	0	0	0	510	0
N.S.	1	2.21	0.00	0.00	0.00	0.00	0.00	0.00	9.81	0.00
time (sec)	N/A	0.441	0.000	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	33	29	46	29	138	61	28	46
N.S.	1	1.00	0.52	0.45	0.72	0.45	2.16	0.95	0.44	0.72
time (sec)	N/A	0.180	0.018	0.118	0.033	0.096	1.774	0.103	0.261	0.016

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	77	70	40	0	0	56	0	8	0
N.S.	1	1.79	1.63	0.93	0.00	0.00	1.30	0.00	0.19	0.00
time (sec)	N/A	0.205	0.025	0.152	0.000	0.000	0.786	0.000	0.257	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	58	31	30	48	34	30	27	31
N.S.	1	1.12	1.41	0.76	0.73	1.17	0.83	0.73	0.66	0.76
time (sec)	N/A	0.170	0.079	0.159	0.127	0.091	0.103	0.112	0.280	0.025

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	142	0	0	0	0	0	213	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	2.15	0.00
time (sec)	N/A	0.234	0.128	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	29	36	0	0	33	0
N.S.	1	1.00	1.00	0.86	0.81	1.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.268	0.133	0.051	0.078	0.100	0.000	0.000	0.233	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	104	47	42	41	45	85	41	56	55
N.S.	1	2.36	1.07	0.95	0.93	1.02	1.93	0.93	1.27	1.25
time (sec)	N/A	0.837	0.031	0.230	0.041	0.111	0.968	0.112	0.226	0.267

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	64	33	30	58	29	68	42	28	30
N.S.	1	1.14	0.59	0.54	1.04	0.52	1.21	0.75	0.50	0.54
time (sec)	N/A	0.291	0.029	0.153	0.033	0.087	0.808	0.113	0.227	0.079

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	25	23	28	37	26	23	42	28
N.S.	1	1.16	0.81	0.74	0.90	1.19	0.84	0.74	1.35	0.90
time (sec)	N/A	0.150	0.010	0.096	0.118	0.092	0.059	0.110	0.264	0.026

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	132	96	115	262	0	0	8	0
N.S.	1	0.00	1.22	0.89	1.06	2.43	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.036	0.538	0.163	0.123	0.000	0.000	0.236	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	A	A	F	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	83	29	17	8	8	0	32	104	8
N.S.	1	2.59	0.91	0.53	0.25	0.25	0.00	1.00	3.25	0.25
time (sec)	N/A	0.377	0.013	2.548	0.058	0.107	0.000	0.786	0.248	0.112

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	C	A	F	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	21	22	27	13	0	13	17	19
N.S.	1	0.00	1.00	1.05	1.29	0.62	0.00	0.62	0.81	0.90
time (sec)	N/A	0.000	0.038	0.221	0.084	0.089	0.000	0.109	0.265	0.016

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	114	13	117	114	31	0	48	0
N.S.	1	1.04	1.12	0.13	1.15	1.12	0.30	0.00	0.47	0.00
time (sec)	N/A	0.192	0.302	0.132	0.142	0.091	1.950	0.000	0.294	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	4	4	3	4	3	4
N.S.	1	1.00	1.00	1.67	1.33	1.33	1.00	1.33	1.00	1.33
time (sec)	N/A	0.129	0.000	0.014	0.029	0.065	0.021	0.111	0.240	0.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	5	4	3	8	7	5	6	5
N.S.	1	1.00	1.67	1.33	1.00	2.67	2.33	1.67	2.00	1.67
time (sec)	N/A	0.166	0.000	0.115	0.034	0.089	0.243	0.118	0.263	0.001

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	7	7	7	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	1.00	1.00	1.00	0.86
time (sec)	N/A	0.246	0.018	0.034	0.034	0.101	0.082	0.115	0.247	0.180

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	20	22	25	22
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.67	0.73	0.83	0.73
time (sec)	N/A	0.236	0.000	0.076	0.038	0.074	0.022	0.114	0.261	0.011

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	24	26	26	27	23	25	26
N.S.	1	1.00	1.00	0.67	0.72	0.72	0.75	0.64	0.69	0.72
time (sec)	N/A	0.234	0.002	0.027	0.030	0.075	0.018	0.111	0.279	0.012

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	20	20	17	16	16	15	16	16	8
N.S.	1	2.00	2.00	1.70	1.60	1.60	1.50	1.60	1.60	0.80
time (sec)	N/A	0.174	0.012	2.089	0.053	0.098	0.086	0.110	0.254	0.173

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	24	14	24	17	28	42	14
N.S.	1	1.00	1.00	2.00	1.17	2.00	1.42	2.33	3.50	1.17
time (sec)	N/A	0.237	0.166	1.125	0.128	0.100	0.042	0.117	0.268	0.202

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	8	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	1.00	0.88
time (sec)	N/A	0.258	0.004	0.059	0.041	0.096	0.075	0.114	0.278	0.216

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	F	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	3	1	2	1	0	0	26	1	1
N.S.	1	3.00	1.00	2.00	1.00	0.00	0.00	26.00	1.00	1.00
time (sec)	N/A	0.235	0.000	0.103	0.028	0.000	0.023	0.114	0.272	0.003

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	50	1	2	1	1	58	1	1	1
N.S.	1	50.00	1.00	2.00	1.00	1.00	58.00	1.00	1.00	1.00
time (sec)	N/A	0.364	0.001	0.222	0.027	0.075	0.033	0.109	0.255	0.173

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	81	8	8	8	8	9
N.S.	1	1.00	1.00	1.00	9.00	0.89	0.89	0.89	0.89	1.00
time (sec)	N/A	0.259	0.021	0.311	0.077	0.087	0.186	0.116	0.261	0.193

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	100	63	79	114	0	73	45	238
N.S.	1	1.00	2.04	1.29	1.61	2.33	0.00	1.49	0.92	4.86
time (sec)	N/A	0.258	0.050	0.338	0.109	0.103	0.000	0.123	0.253	0.366

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	15	13	0	11	0	0	15	1385	0
N.S.	1	1.15	1.00	0.00	0.85	0.00	0.00	1.15	106.54	0.00
time (sec)	N/A	2.552	1.485	180.000	0.029	0.000	0.000	0.106	0.508	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	17	17	14	13	131	34	13	27	13
N.S.	1	1.89	1.89	1.56	1.44	14.56	3.78	1.44	3.00	1.44
time (sec)	N/A	0.160	0.064	2.500	0.034	2.150	0.197	0.135	0.255	0.209

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	B	A	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	21	30	91	16	17	119	17	22
N.S.	1	0.00	1.11	1.58	4.79	0.84	0.89	6.26	0.89	1.16
time (sec)	N/A	0.000	0.047	0.223	0.050	0.131	0.093	0.124	0.252	0.193

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	F	F	B	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	74	0	0	0	30	56	0	18	0
N.S.	1	8.22	0.00	0.00	0.00	3.33	6.22	0.00	2.00	0.00
time (sec)	N/A	0.401	0.000	0.000	0.000	0.104	19.907	0.000	0.247	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	24	24	26	24	23	24
N.S.	1	1.00	1.00	0.75	0.75	0.75	0.81	0.75	0.72	0.75
time (sec)	N/A	0.154	0.000	0.022	0.033	0.073	0.018	0.117	0.231	0.014

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	14	16	14	10
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.14	1.00	0.71
time (sec)	N/A	0.157	0.005	0.040	0.047	0.095	0.043	0.111	0.249	0.024

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	47	34	28	80	65	0	0	6	0
N.S.	1	1.21	0.87	0.72	2.05	1.67	0.00	0.00	0.15	0.00
time (sec)	N/A	0.301	0.011	0.038	0.045	0.101	0.000	0.000	0.227	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	24	33	28	32	27	72	33	28
N.S.	1	0.00	1.00	1.38	1.17	1.33	1.12	3.00	1.38	1.17
time (sec)	N/A	0.000	0.028	0.093	0.088	0.088	0.053	0.110	2.453	0.194

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	23	20	19	0	34	0	22	18	17
N.S.	1	0.88	0.77	0.73	0.00	1.31	0.00	0.85	0.69	0.65
time (sec)	N/A	0.199	0.008	0.184	0.000	0.089	0.000	0.117	0.244	0.029

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	42	33	32	35	83	32	35	38
N.S.	1	1.11	0.95	0.75	0.73	0.80	1.89	0.73	0.80	0.86
time (sec)	N/A	0.297	0.011	0.493	0.040	0.116	0.212	0.111	0.243	0.185

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	17	11	12	54	17	0	18	19	11
N.S.	1	1.55	1.00	1.09	4.91	1.55	0.00	1.64	1.73	1.00
time (sec)	N/A	0.468	0.010	0.112	0.117	0.099	0.000	0.117	0.277	0.185

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	C	F(-2)	B	C	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	41	0	83	41	7	9
N.S.	1	1.00	1.00	0.00	3.15	0.00	6.38	3.15	0.54	0.69
time (sec)	N/A	0.283	0.012	0.000	0.126	0.000	0.312	0.115	0.250	0.222

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	94	22	15	12	38	35
N.S.	1	1.00	1.00	1.67	7.83	1.83	1.25	1.00	3.17	2.92
time (sec)	N/A	0.363	0.013	0.372	0.126	0.094	11.015	0.125	0.328	0.302

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	86	19	85	82	85	88	85
N.S.	1	1.00	1.00	4.53	1.00	4.47	4.32	4.47	4.63	4.47
time (sec)	N/A	0.410	0.037	0.122	0.027	0.072	0.632	0.133	0.291	0.268

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F(-1)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	190	86	67	0	71	100	66	9	66
N.S.	1	2.21	1.00	0.78	0.00	0.83	1.16	0.77	0.10	0.77
time (sec)	N/A	2.009	0.019	0.381	0.000	0.254	73.120	0.120	0.243	0.233

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	19	22	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85	0.85
time (sec)	N/A	0.175	0.019	0.059	0.115	0.089	0.041	0.110	0.242	0.021

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	A	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	25	0	0	12	0	12	8	0
N.S.	1	0.00	1.09	0.00	0.00	0.52	0.00	0.52	0.35	0.00
time (sec)	N/A	0.000	0.042	0.000	0.000	0.095	0.000	0.110	0.280	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.83
time (sec)	N/A	0.139	0.037	0.059	0.024	0.075	0.264	0.113	0.236	0.048

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	34	46	15	27	42	13	0
N.S.	1	1.00	0.96	1.48	2.00	0.65	1.17	1.83	0.57	0.00
time (sec)	N/A	0.185	0.026	0.339	0.132	0.100	1.494	0.128	0.245	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	24	8	12	17	7	8	7	7
N.S.	1	1.00	2.67	0.89	1.33	1.89	0.78	0.89	0.78	0.78
time (sec)	N/A	0.183	0.016	0.107	0.031	0.092	0.123	0.121	0.250	0.041

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	23	34	24	20	22	22	18
N.S.	1	1.00	0.91	1.05	1.55	1.09	0.91	1.00	1.00	0.82
time (sec)	N/A	0.263	0.010	0.039	0.032	0.108	0.059	0.115	0.249	0.021

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	39	34	40	17	31	16	38
N.S.	1	1.00	1.00	1.77	1.55	1.82	0.77	1.41	0.73	1.73
time (sec)	N/A	0.154	0.026	0.086	0.024	0.093	0.827	0.114	0.261	0.052

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	F	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	21	52	0	84	0	136	14	15
N.S.	1	0.00	1.00	2.48	0.00	4.00	0.00	6.48	0.67	0.71
time (sec)	N/A	0.000	0.057	1.516	0.000	0.103	0.000	0.115	0.227	0.317

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	15	21	17	17
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.88	1.24	1.00	1.00
time (sec)	N/A	0.305	0.044	0.095	0.079	0.093	0.072	0.109	0.249	0.221

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	37	107	0	0	37	4362	0	34	0
N.S.	1	0.82	2.38	0.00	0.00	0.82	96.93	0.00	0.76	0.00
time (sec)	N/A	0.291	0.350	0.000	0.000	0.144	4.546	0.000	0.266	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	45	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.83	0.81
time (sec)	N/A	0.228	0.003	0.049	0.030	0.071	0.021	0.109	0.282	0.195

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	A	A	F	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	18	34	23	23	0	23	34	18
N.S.	1	0.00	1.00	1.89	1.28	1.28	0.00	1.28	1.89	1.00
time (sec)	N/A	0.000	0.009	0.298	0.132	0.079	0.000	0.111	0.267	0.201

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	65	77	0	56	108	0	57	24	0
N.S.	1	1.07	1.26	0.00	0.92	1.77	0.00	0.93	0.39	0.00
time (sec)	N/A	0.288	0.189	0.000	0.123	0.099	0.000	0.127	0.222	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	136	64	228	52	55	0	61	49	66
N.S.	1	2.12	1.00	3.56	0.81	0.86	0.00	0.95	0.77	1.03
time (sec)	N/A	2.159	0.323	0.806	0.039	0.155	0.000	2.127	200.042	1.014

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	A	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	105	0	0	148	0	83	75	0
N.S.	1	0.00	1.00	0.00	0.00	1.41	0.00	0.79	0.71	0.00
time (sec)	N/A	0.000	0.343	0.000	0.000	0.643	0.000	0.123	0.286	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	25	18	17	17	15	19	25	23
N.S.	1	1.16	1.00	0.72	0.68	0.68	0.60	0.76	1.00	0.92
time (sec)	N/A	0.189	0.004	0.051	0.041	0.081	0.089	0.153	0.274	0.053

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [305] had the largest ratio of [3.42857000000000012]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	11	0.273
2	A	7	6	1.12	9	0.667
3	A	2	2	1.52	13	0.154
4	A	4	4	0.96	8	0.500
5	A	2	2	1.00	5	0.400
6	A	5	4	1.25	9	0.444
7	A	6	5	1.00	9	0.556
8	A	4	3	1.00	13	0.231
9	A	5	4	1.31	11	0.364
10	A	12	11	1.55	6	1.833
11	A	2	2	1.00	12	0.167
12	A	6	5	1.19	21	0.238
13	A	2	2	1.00	8	0.250
14	A	4	3	1.72	17	0.176
15	A	2	2	1.00	7	0.286
16	A	2	2	1.00	12	0.167
17	A	1	1	1.00	7	0.143
18	A	2	2	1.00	13	0.154
19	A	3	2	1.00	17	0.118
20	C	5	4	3.96	7	0.571
21	A	4	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	9	0.222
23	A	1	1	1.00	11	0.091
24	A	6	5	1.00	17	0.294
25	A	2	2	1.00	19	0.105
26	A	1	1	1.00	9	0.111
27	A	4	3	1.00	14	0.214
28	A	2	2	1.00	11	0.182
29	A	3	3	1.00	27	0.111
30	A	5	4	1.00	4	1.000
31	A	5	4	1.00	9	0.444
32	A	6	6	1.00	11	0.545
33	A	7	7	1.29	4	1.750
34	A	3	3	1.00	12	0.250
35	A	1	1	1.00	3	0.333
36	A	2	2	1.00	2	1.000
37	A	4	3	0.46	15	0.200
38	A	1	1	1.00	13	0.077
39	A	3	3	1.00	11	0.273
40	A	1	1	1.00	8	0.125
41	B	5	4	2.38	7	0.571
42	A	5	4	1.00	15	0.267
43	A	4	3	1.00	12	0.250
44	A	2	2	1.00	8	0.250
45	A	4	3	1.28	13	0.231
46	B	3	3	66.75	9	0.333
47	A	4	3	1.00	9	0.333
48	A	4	4	1.00	34	0.118
49	A	4	3	1.50	15	0.200
50	A	5	4	0.90	9	0.444
51	A	1	1	1.00	8	0.125
52	B	5	4	2.55	21	0.190
53	A	4	3	1.07	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	11	0.182
55	A	3	2	1.70	14	0.143
56	A	1	1	1.00	4	0.250
57	A	2	2	1.00	6	0.333
58	A	11	10	1.09	23	0.435
59	A	6	5	1.00	21	0.238
60	A	4	4	1.00	34	0.118
61	A	1	1	1.00	12	0.083
62	A	7	6	1.12	9	0.667
63	A	2	2	1.00	15	0.133
64	A	2	2	1.00	9	0.222
65	A	4	3	1.00	7	0.429
66	A	5	4	1.12	15	0.267
67	A	3	2	1.00	11	0.182
68	A	4	3	1.00	9	0.333
69	A	3	2	1.00	9	0.222
70	A	3	3	1.62	11	0.273
71	A	4	4	1.00	6	0.667
72	A	4	3	1.15	13	0.231
73	A	2	2	1.00	10	0.200
74	A	6	6	1.00	11	0.545
75	A	1	1	1.00	11	0.091
76	A	1	1	1.00	3	0.333
77	F	0	0	N/A	0.000	N/A
78	A	2	2	1.00	13	0.154
79	A	2	2	1.00	11	0.182
80	A	1	1	1.00	15	0.067
81	A	6	6	1.00	13	0.462
82	A	2	2	1.00	27	0.074
83	A	1	1	1.00	2	0.500
84	B	5	4	3.17	11	0.364
85	B	5	5	2.07	9	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	1.00	15	0.267
87	A	3	3	1.00	4	0.750
88	A	6	5	1.90	7	0.714
89	A	6	5	1.00	5	1.000
90	A	1	1	1.00	9	0.111
91	A	4	3	0.89	11	0.273
92	A	3	2	1.00	10	0.200
93	A	8	7	1.00	6	1.167
94	A	3	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182
96	A	6	5	1.00	9	0.556
97	A	2	2	1.00	8	0.250
98	A	4	3	1.00	13	0.231
99	A	1	1	1.00	4	0.250
100	A	9	8	1.06	6	1.333
101	A	1	1	1.00	16	0.062
102	A	5	4	1.09	13	0.308
103	A	2	2	1.00	9	0.222
104	A	1	1	1.00	6	0.167
105	A	2	2	1.00	9	0.222
106	A	2	2	1.00	2	1.000
107	A	2	2	1.00	9	0.222
108	A	3	3	0.95	4	0.750
109	A	2	2	1.00	10	0.200
110	A	2	2	1.47	18	0.111
111	A	3	3	1.04	4	0.750
112	A	5	4	1.27	13	0.308
113	A	5	5	1.00	8	0.625
114	A	3	3	1.16	19	0.158
115	A	4	3	0.55	12	0.250
116	A	4	3	1.11	15	0.200
117	A	2	2	1.50	6	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	B	1	1	5.80	11	0.091
119	B	2	2	2.31	11	0.182
120	A	4	3	1.00	12	0.250
121	A	2	2	1.00	2	1.000
122	A	3	2	1.00	7	0.286
123	A	2	2	1.00	6	0.333
124	A	7	6	1.55	12	0.500
125	A	1	1	1.00	3	0.333
126	A	2	2	1.27	12	0.167
127	A	4	3	1.07	9	0.333
128	A	6	6	1.00	11	0.545
129	A	1	1	1.00	9	0.111
130	A	5	4	1.00	14	0.286
131	A	6	5	0.97	10	0.500
132	A	6	5	0.52	11	0.455
133	A	4	3	1.25	15	0.200
134	A	5	5	1.00	4	1.250
135	A	5	4	1.00	7	0.571
136	A	1	1	1.00	14	0.071
137	A	3	3	1.00	2	1.500
138	A	1	1	1.00	7	0.143
139	A	1	1	1.00	6	0.167
140	F	0	0	N/A	0.000	N/A
141	A	3	2	1.00	11	0.182
142	A	5	5	1.00	8	0.625
143	A	6	5	1.11	10	0.500
144	A	1	1	1.00	2	0.500
145	A	1	1	1.00	11	0.091
146	A	2	2	1.00	11	0.182
147	F	0	0	N/A	0.000	N/A
148	A	4	3	1.11	25	0.120
149	A	5	4	1.00	9	0.444

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	1	1	1.00	21	0.048
151	A	4	3	1.15	13	0.231
152	A	4	3	1.00	12	0.250
153	A	5	4	0.93	11	0.364
154	A	1	1	1.00	6	0.167
155	A	2	2	1.00	15	0.133
156	A	2	2	1.50	6	0.333
157	A	1	1	1.00	13	0.077
158	A	1	1	1.00	6	0.167
159	A	2	2	1.00	2	1.000
160	A	2	2	1.00	9	0.222
161	A	4	3	1.00	13	0.231
162	A	5	4	1.31	11	0.364
163	A	2	2	1.00	2	1.000
164	A	2	2	1.00	12	0.167
165	A	6	5	1.00	6	0.833
166	A	3	3	1.00	7	0.429
167	A	1	1	1.00	12	0.083
168	A	4	3	1.00	11	0.273
169	A	5	4	1.00	8	0.500
170	A	2	2	1.00	8	0.250
171	A	4	3	1.00	9	0.333
172	A	2	2	1.00	18	0.111
173	A	6	5	1.00	25	0.200
174	A	1	1	1.00	10	0.100
175	A	4	3	1.00	13	0.231
176	A	3	2	1.00	11	0.182
177	A	2	2	1.00	4	0.500
178	A	4	3	1.50	7	0.429
179	A	3	2	1.00	11	0.182
180	F	0	0	N/A	0.000	N/A
181	A	1	1	1.00	15	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	4	1.00	10	0.400
183	A	2	2	1.00	6	0.333
184	A	4	3	1.00	13	0.231
185	A	1	1	1.00	10	0.100
186	A	4	4	1.00	4	1.000
187	B	6	5	2.29	25	0.200
188	A	5	4	1.00	13	0.308
189	A	4	3	1.00	13	0.231
190	A	1	1	1.00	9	0.111
191	A	3	2	1.00	15	0.133
192	A	2	2	1.00	14	0.143
193	A	4	3	1.00	5	0.600
194	N/A	4	0	1.00	3	0.000
195	A	1	1	1.00	13	0.077
196	B	1	1	2.50	17	0.059
197	A	1	1	1.00	9	0.111
198	A	2	2	1.08	4	0.500
199	A	1	1	1.00	21	0.048
200	A	5	4	1.00	10	0.400
201	A	3	2	1.00	13	0.154
202	A	6	5	1.00	6	0.833
203	A	4	3	1.00	11	0.273
204	A	3	3	1.00	11	0.273
205	A	1	1	1.48	9	0.111
206	A	1	1	1.00	1	1.000
207	B	3	3	12.64	21	0.143
208	A	1	1	1.00	20	0.050
209	A	5	5	1.00	11	0.455
210	A	2	2	1.00	9	0.222
211	F	0	0	N/A	0.000	N/A
212	A	5	4	1.73	18	0.222
213	A	1	1	1.00	9	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	2.00	12	0.167
215	A	5	4	1.30	7	0.571
216	A	3	2	1.00	9	0.222
217	A	2	2	1.00	12	0.167
218	A	4	3	1.00	11	0.273
219	A	5	4	1.65	15	0.267
220	A	4	3	1.33	21	0.143
221	A	3	3	1.06	6	0.500
222	A	1	1	1.00	18	0.056
223	A	2	2	1.00	6	0.333
224	A	8	7	0.92	8	0.875
225	N/A	2	0	1.00	9	0.000
226	N/A	2	0	1.00	14	0.000
227	A	2	2	1.00	9	0.222
228	A	7	6	1.91	15	0.400
229	A	4	4	1.00	12	0.333
230	A	2	2	1.00	11	0.182
231	A	4	3	1.00	11	0.273
232	A	1	1	1.00	11	0.091
233	A	2	2	1.00	9	0.222
234	C	4	4	2.07	12	0.333
235	A	3	3	1.22	19	0.158
236	N/A	1	0	1.00	5	0.000
237	A	6	5	0.90	8	0.625
238	A	3	2	1.00	11	0.182
239	A	4	3	1.00	4	0.750
240	A	1	1	1.00	17	0.059
241	A	4	3	1.06	13	0.231
242	B	6	5	8.08	23	0.217
243	F	0	0	N/A	0.000	N/A
244	B	1	1	2.75	19	0.053
245	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	C	1	1	14.00	37	0.027
247	A	3	2	1.00	11	0.182
248	A	4	3	0.47	8	0.375
249	A	4	4	1.00	8	0.500
250	F	0	0	N/A	0.000	N/A
251	A	2	2	1.00	14	0.143
252	A	2	2	1.00	22	0.091
253	A	5	4	1.47	9	0.444
254	B	11	10	3.88	11	0.909
255	A	6	5	1.00	9	0.556
256	A	4	3	0.91	9	0.333
257	C	3	3	36.17	16	0.188
258	C	2	2	9.00	19	0.105
259	A	6	5	1.16	12	0.417
260	A	3	3	1.00	13	0.231
261	A	2	2	1.00	21	0.095
262	A	1	1	1.00	34	0.029
263	A	1	1	1.00	7	0.143
264	A	3	3	1.00	20	0.150
265	F	0	0	N/A	0.000	N/A
266	B	4	3	2.21	22	0.136
267	A	2	2	1.00	13	0.154
268	A	2	2	1.79	6	0.333
269	A	2	2	1.12	19	0.105
270	A	2	2	1.00	12	0.167
271	A	2	2	1.00	16	0.125
272	B	4	4	2.36	11	0.364
273	A	5	4	1.14	29	0.138
274	A	3	3	1.16	7	0.429
275	F	0	0	N/A	0.000	N/A
276	B	6	5	2.59	39	0.128
277	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	3	3	1.04	15	0.200
279	A	1	1	1.00	1	1.000
280	A	2	2	1.00	2	1.000
281	A	5	4	1.00	19	0.211
282	A	2	2	1.00	29	0.069
283	A	3	3	1.00	17	0.176
284	A	1	1	2.00	13	0.077
285	A	1	1	1.00	25	0.040
286	A	1	1	1.00	13	0.077
287	B	1	1	3.00	43	0.023
288	C	1	1	50.00	19	0.053
289	A	1	1	1.00	11	0.091
290	A	1	1	1.00	31	0.032
291	A	5	4	1.15	15	0.267
292	A	1	1	1.89	17	0.059
293	F	0	0	N/A	0.000	N/A
294	C	6	5	8.22	16	0.312
295	A	1	1	1.00	20	0.050
296	A	2	2	1.00	17	0.118
297	A	8	7	1.21	4	1.750
298	F	0	0	N/A	0.000	N/A
299	A	2	2	0.88	13	0.154
300	A	5	5	1.11	6	0.833
301	A	7	6	1.55	17	0.353
302	A	1	1	1.00	25	0.040
303	A	5	5	1.00	8	0.625
304	A	1	1	1.00	40	0.025
305	B	25	24	2.21	7	3.429
306	A	6	5	1.00	18	0.278
307	F	0	0	N/A	0.000	N/A
308	A	1	1	1.00	16	0.062
309	A	2	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	4	3	1.00	8	0.375
311	A	6	6	1.00	4	1.500
312	A	5	4	1.00	9	0.444
313	F	0	0	N/A	0.000	N/A
314	A	2	2	1.00	25	0.080
315	A	7	6	0.82	17	0.353
316	A	1	1	1.00	17	0.059
317	F	0	0	N/A	0.000	N/A
318	A	8	7	1.07	14	0.500
319	B	19	18	2.12	47	0.383
320	F	0	0	N/A	0.000	N/A
321	A	4	3	1.16	16	0.188

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

input
```
Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\sin(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sin(2x)\sin(3x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

input
```
Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
parallelrisch	$\frac{1}{16}+\frac{\cos(6x)}{24}-\frac{\cos(4x)}{16}-\frac{\cos(2x)}{8}$	21
orering	$-\frac{5\cos(x)\sin(2x)\sin(3x)}{48}+\frac{\sin(x)\cos(2x)\sin(3x)}{48}-\frac{11\sin(x)\sin(2x)\cos(3x)}{48}-\frac{7\cos(x)\cos(2x)\cos(3x)}{48}$	50

input
```
int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sin(x)\sin(2x)\sin(3x)\,dx=\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(19) = 38$.

Time = 1.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.48

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{\sin(x)\sin(2x)\cos(3x)}{3} + \frac{\sin(x)\sin(3x)\cos(2x)}{8} - \frac{\cos(x)\cos(2x)\cos(3x)}{24}$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

output
```
x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*c
os(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - sin(x)*sin(2*x)*cos(3*x)
/3 + sin(x)*sin(3*x)*cos(2*x)/8 - cos(x)*cos(2*x)*cos(3*x)/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-4/3*sin(x)^6 + 3/2*sin(x)^4
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{\sin(x)^4\left(8\sin(x)^2-9\right)}{6}$$

input
```
int(sin(2*x)*sin(3*x)*sin(x),x)
```

output
```
-(sin(x)^4*(8*sin(x)^2 - 9))/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx = &-\frac{\cos(3x)\cos(2x)\cos(x)}{24} + \frac{\cos(3x)\cos(2x)\sin(x)\,x}{4}\\ &+ \frac{\cos(3x)\cos(x)\sin(2x)\,x}{4}\\ &- \frac{\cos(3x)\sin(2x)\sin(x)}{3} - \frac{\cos(2x)\cos(x)\sin(3x)\,x}{4}\\ &+ \frac{\cos(2x)\sin(3x)\sin(x)}{8} + \frac{\sin(3x)\sin(2x)\sin(x)\,x}{4}\end{aligned}$$

input
```
int(sin(x)*sin(2*x)*sin(3*x),x)
```

output
```
( - cos(3*x)*cos(2*x)*cos(x) + 6*cos(3*x)*cos(2*x)*sin(x)*x + 6*cos(3*x)*c
os(x)*sin(2*x)*x - 8*cos(3*x)*sin(2*x)*sin(x) - 6*cos(2*x)*cos(x)*sin(3*x)
*x + 3*cos(2*x)*sin(3*x)*sin(x) + 6*sin(3*x)*sin(2*x)*sin(x)*x)/24
```

3.2 $\int \cos(x)\sin^3(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cos(x)\sin^3(2x)\,dx = -\frac{8}{5}\cos^5(x) + \frac{8\cos^7(x)}{7}$$

output
```
-8/5*cos(x)^5+8/7*cos(x)^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \cos(x)\sin^3(2x)\,dx = -\frac{3\cos(x)}{8} - \frac{1}{8}\cos(3x) + \frac{1}{40}\cos(5x) + \frac{1}{56}\cos(7x)$$

input
```
Integrate[Cos[x]*Sin[2*x]^3,x]
```

output
```
(-3*Cos[x])/8 - Cos[3*x]/8 + Cos[5*x]/40 + Cos[7*x]/56
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4775, 3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)^3\cos(x)dx$$

$$\Big\downarrow \text{4775}$$

$$8\int \cos^4(x)\sin^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(x)^4\sin(x)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-8\int \cos^4(x)\left(1-\cos^2(x)\right)d\cos(x)$$

$$\Big\downarrow \text{244}$$

$$-8\int \left(\cos^4(x)-\cos^6(x)\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-8\left(\frac{\cos^5(x)}{5}-\frac{\cos^7(x)}{7}\right)$$

input
```
Int[Cos[x]*Sin[2*x]^3,x]
```

output
```
-8*(Cos[x]^5/5 - Cos[x]^7/7)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(5x)}{40}+\frac{\cos(7x)}{56}$	24
risch	$-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(5x)}{40}+\frac{\cos(7x)}{56}$	24
parallelrisch	$-\frac{8}{21}-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(7x)}{56}+\frac{\cos(5x)}{40}$	25
orering	$-\frac{9\sin(x)\sin(2x)^3}{35}-\frac{22\cos(x)\sin(2x)^2\cos(2x)}{35}-\frac{8\cos(2x)^2\sin(x)\sin(2x)}{35}-\frac{16\cos(x)\cos(2x)^3}{35}$	50

input
```
int(cos(x)*sin(2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*cos(x)-1/8*cos(3*x)+1/40*cos(5*x)+1/56*cos(7*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin^3(2x)\,dx = \frac{8}{7}\cos{(x)}^7 - \frac{8}{5}\cos{(x)}^5$$

input
```
integrate(cos(x)*sin(2*x)^3,x, algorithm="fricas")
```

output
```
8/7*cos(x)^7 - 8/5*cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(15) = 30.

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.82

$$\int \cos(x)\sin^3(2x)\,dx = -\frac{9\sin{(x)}\sin^3{(2x)}}{35} - \frac{8\sin{(x)}\sin{(2x)}\cos^2{(2x)}}{35} - \frac{22\sin^2{(2x)}\cos{(x)}\cos{(2x)}}{35} - \frac{16\cos{(x)}\cos^3{(2x)}}{35}$$

input
```
integrate(cos(x)*sin(2*x)**3,x)
```

output
```
-9*sin(x)*sin(2*x)**3/35 - 8*sin(x)*sin(2*x)*cos(2*x)**2/35 - 22*sin(2*x)*
*2*cos(x)*cos(2*x)/35 - 16*cos(x)*cos(2*x)**3/35
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \cos(x)\sin^3(2x)\,dx = \frac{1}{56}\cos(7x) + \frac{1}{40}\cos(5x) - \frac{1}{8}\cos(3x) - \frac{3}{8}\cos(x)$$

input
```
integrate(cos(x)*sin(2*x)^3,x, algorithm="maxima")
```

output
```
1/56*cos(7*x) + 1/40*cos(5*x) - 1/8*cos(3*x) - 3/8*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin^3(2x)\,dx = \frac{8}{7}\cos(x)^7 - \frac{8}{5}\cos(x)^5$$

input
```
integrate(cos(x)*sin(2*x)^3,x, algorithm="giac")
```

output
```
8/7*cos(x)^7 - 8/5*cos(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos(x)\sin^3(2x)\,dx = \frac{8\cos(x)^5\,\left(5\cos(x)^2 - 7\right)}{35}$$

input
```
int(sin(2*x)^3*cos(x),x)
```

output
```
(8*cos(x)^5*(5*cos(x)^2 - 7))/35
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int \cos(x)\sin^3(2x)\,dx = -\frac{6\cos(2x)\cos(x)\sin(2x)^2}{35} - \frac{16\cos(2x)\cos(x)}{35} - \frac{\sin(2x)^3\sin(x)}{35} - \frac{8\sin(2x)\sin(x)}{35} + \frac{8}{21}$$

input
```
int(cos(x)*sin(2*x)^3,x)
```

output
```
( - 18*cos(2*x)*cos(x)*sin(2*x)**2 - 48*cos(2*x)*cos(x) - 3*sin(2*x)**3*si
n(x) - 24*sin(2*x)*sin(x) + 40)/105
```

3.3 $\int \sqrt[3]{-1+x}(1+x)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx=\frac{3}{70}(-1+x)^{4/3}\left(37+26x+7x^2\right)$$

output
```
3/70*(-1+x)^(4/3)*(7*x^2+26*x+37)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx=\frac{3}{70}(-1+x)^{4/3}\left(37+26x+7x^2\right)$$

input
```
Integrate[(-1 + x)^(1/3)*(1 + x)^2,x]
```

output
```
(3*(-1 + x)^(4/3)*(37 + 26*x + 7*x^2))/70
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{x-1}(x+1)^2\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left((x-1)^{7/3}+4(x-1)^{4/3}+4\sqrt[3]{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{10}(x-1)^{10/3}+\frac{12}{7}(x-1)^{7/3}+3(x-1)^{4/3}$$

input
```
Int[(-1 + x)^(1/3)*(1 + x)^2,x]
```

output
```
3*(-1 + x)^(4/3) + (12*(-1 + x)^(7/3))/7 + (3*(-1 + x)^(10/3))/10
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{3(x-1)^{\frac{4}{3}}(7x^2+26x+37)}{70}$
orering	$\frac{3(x-1)^{\frac{4}{3}}(7x^2+26x+37)}{70}$
trager	$\left(\frac{3}{10}x^3+\frac{57}{70}x^2+\frac{33}{70}x-\frac{111}{70}\right)(x-1)^{\frac{1}{3}}$
derivativedivides	$\frac{3(x-1)^{\frac{10}{3}}}{10}+\frac{12(x-1)^{\frac{7}{3}}}{7}+3(x-1)^{\frac{4}{3}}$
default	$\frac{3(x-1)^{\frac{10}{3}}}{10}+\frac{12(x-1)^{\frac{7}{3}}}{7}+3(x-1)^{\frac{4}{3}}$
risch	$\frac{3(x-1)^{\frac{1}{3}}(7x^3+19x^2+11x-37)}{70}$
meijerg	$\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x^3\operatorname{hypergeom}\left(\left[-\frac{1}{3},3\right],[4],x\right)}{3(-\operatorname{signum}(x-1))^{\frac{1}{3}}}+\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x^2\operatorname{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],x\right)}{(-\operatorname{signum}(x-1))^{\frac{1}{3}}}+\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x\operatorname{hy}}{(-\operatorname{signu}}$

input
```
int((x-1)^(1/3)*(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
3/70*(x-1)^(4/3)*(7*x^2+26*x+37)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx=\frac{3}{70}\left(7x^3+19x^2+11x-37\right)(x-1)^{\frac{1}{3}}$$

input
```
integrate((-1+x)^(1/3)*(1+x)^2,x, algorithm="fricas")
```

output
```
3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(x - 1)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.86

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx = \begin{cases} \frac{3\sqrt[3]{x-1}(x+1)^3}{10} - \frac{3\sqrt[3]{x-1}(x+1)^2}{35} - \frac{9\sqrt[3]{x-1}(x+1)}{35} - \frac{54\sqrt[3]{x-1}}{35} & \text{for } |x+1|>2 \\ -\frac{3\sqrt[3]{1-x}(x+1)^3e^{-\frac{2i\pi}{3}}}{10} + \frac{3\sqrt[3]{1-x}(x+1)^2e^{-\frac{2i\pi}{3}}}{35} + \frac{9\sqrt[3]{1-x}(x+1)e^{-\frac{2i\pi}{3}}}{35} + \frac{54\sqrt[3]{1-x}e^{-\frac{2i\pi}{3}}}{35} & \text{otherwise} \end{cases}$$

input
```
integrate((-1+x)**(1/3)*(1+x)**2,x)
```

output
```
Piecewise((3*(x - 1)**(1/3)*(x + 1)**3/10 - 3*(x - 1)**(1/3)*(x + 1)**2/35
 - 9*(x - 1)**(1/3)*(x + 1)/35 - 54*(x - 1)**(1/3)/35, Abs(x + 1) > 2), (-
3*(1 - x)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(1 - x)**(1/3)*(x + 1)**
2*exp(-2*I*pi/3)/35 + 9*(1 - x)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35 + 54*(1 -
 x)**(1/3)*exp(-2*I*pi/3)/35, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx = \frac{3}{10}\,(x-1)^{\frac{10}{3}} + \frac{12}{7}\,(x-1)^{\frac{7}{3}} + 3\,(x-1)^{\frac{4}{3}}$$

input
```
integrate((-1+x)^(1/3)*(1+x)^2,x, algorithm="maxima")
```

output
```
3/10*(x - 1)^(10/3) + 12/7*(x - 1)^(7/3) + 3*(x - 1)^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx = \frac{3}{10}\,(x-1)^{\frac{10}{3}} + \frac{12}{7}\,(x-1)^{\frac{7}{3}} + 3\,(x-1)^{\frac{4}{3}}$$

input
```
integrate((-1+x)^(1/3)*(1+x)^2,x, algorithm="giac")
```

output
```
3/10*(x - 1)^(10/3) + 12/7*(x - 1)^(7/3) + 3*(x - 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx = \frac{3\,(x-1)^{4/3}\,\left(40\,x+7\,(x-1)^2+30\right)}{70}$$

input
```
int((x - 1)^(1/3)*(x + 1)^2,x)
```

output
```
(3*(x - 1)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sqrt[3]{-1+x}(1+x)^2\,dx = \frac{3(x-1)^{\frac{1}{3}}\,(7x^3+19x^2+11x-37)}{70}$$

input
```
int((-1+x)^(1/3)*(1+x)^2,x)
```

output
```
(3*(x - 1)**(1/3)*(7*x**3 + 19*x**2 + 11*x - 37))/70
```

3.4 $\int x \log \left(1+\frac{1}{x}\right) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int x \log \left(1+\frac{1}{x}\right) \, dx=\frac{x}{2}+\frac{1}{2} x^2 \log \left(1+\frac{1}{x}\right)-\frac{1}{2} \log (1+x)$$

output
```
1/2*x+1/2*x^2*ln(1+1/x)-1/2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x \log \left(1+\frac{1}{x}\right) \, dx=\frac{x}{2}+\frac{1}{2} x^2 \log \left(1+\frac{1}{x}\right)-\frac{1}{2} \log (1+x)$$

input
```
Integrate[x*Log[1 + x^(-1)],x]
```

output
```
x/2 + (x^2*Log[1 + x^(-1)])/2 - Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2905, 772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(\frac{1}{x}+1\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2} \int \frac{1}{1+\frac{1}{x}}dx+\frac{1}{2} x^2 \log \left(\frac{1}{x}+1\right)$$

$$\Big\downarrow \text{772}$$

$$\frac{1}{2} \int \frac{x}{x+1}dx+\frac{1}{2} x^2 \log \left(\frac{1}{x}+1\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(1+\frac{1}{-x-1}\right)dx+\frac{1}{2} x^2 \log \left(\frac{1}{x}+1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} x^2 \log \left(\frac{1}{x}+1\right)+\frac{1}{2} (x-\log (x+1))$$

input
```
Int[x*Log[1 + x^(-1)],x]
```

output
```
(x^2*Log[1 + x^(-1)])/2 + (x - Log[1 + x])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{2}+\frac{x^{2} \ln \left(\frac{1}{x}+1\right)}{2}-\frac{\ln \left(x+1\right)}{2}$	22
parts	$\frac{x}{2}+\frac{x^{2} \ln \left(\frac{1}{x}+1\right)}{2}-\frac{\ln \left(x+1\right)}{2}$	22
derivativedivides	$\frac{x}{2}+\frac{\ln \left(\frac{1}{x}\right)}{2}-\frac{\ln \left(\frac{1}{x}+1\right) \left(\frac{1}{x}+1\right) \left(\frac{1}{x}-1\right) x^{2}}{2}$	32
default	$\frac{x}{2}+\frac{\ln \left(\frac{1}{x}\right)}{2}-\frac{\ln \left(\frac{1}{x}+1\right) \left(\frac{1}{x}+1\right) \left(\frac{1}{x}-1\right) x^{2}}{2}$	32
parallelrisch	$\frac{x^{2} \ln \left(\frac{x+1}{x}\right)}{2}-\frac{1}{2}-\frac{\ln \left(x \right)}{2}+\frac{x}{2}-\frac{\ln \left(\frac{x+1}{x}\right)}{2}$	33

input
```
int(x*ln(1/x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*x^2*ln(1/x+1)-1/2*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x \log \left(1+\frac{1}{x}\right) dx = \frac{1}{2} x^2 \log \left(\frac{x+1}{x}\right) + \frac{1}{2} x - \frac{1}{2} \log (x+1)$$

input
```
integrate(x*log(1+1/x),x, algorithm="fricas")
```

output
```
1/2*x^2*log((x + 1)/x) + 1/2*x - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x \log \left(1+\frac{1}{x}\right) dx = \frac{x^2 \log \left(1+\frac{1}{x}\right)}{2} + \frac{x}{2} - \frac{\log (x+1)}{2}$$

input
```
integrate(x*ln(1+1/x),x)
```

output
```
x**2*log(1 + 1/x)/2 + x/2 - log(x + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int x \log \left(1+\frac{1}{x}\right) dx = \frac{1}{2} x^2 \log \left(\frac{1}{x}+1\right) + \frac{1}{2} x - \frac{1}{2} \log (x+1)$$

input
```
integrate(x*log(1+1/x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(1/x + 1) + 1/2*x - 1/2*log(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int x\log\left(1+\frac{1}{x}\right)\,dx = \frac{1}{2\left(\frac{x+1}{x}-1\right)} + \frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x}-1\right)^2} - \frac{1}{2}\log\left(\frac{|x+1|}{|x|}\right) + \frac{1}{2}\log\left(\left|\frac{x+1}{x}-1\right|\right)$$

input
```
integrate(x*log(1+1/x),x, algorithm="giac")
```

output
```
1/2/((x + 1)/x - 1) + 1/2*log((x + 1)/x)/((x + 1)/x - 1)^2 - 1/2*log(abs(x
 + 1)/abs(x)) + 1/2*log(abs((x + 1)/x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x\log\left(1+\frac{1}{x}\right)\,dx = \frac{x}{2} - \frac{\ln\left(x\,(x+1)\right)}{4} - \frac{\ln\left(\frac{1}{x}+1\right)}{4} + \frac{x^2\,\ln\left(\frac{1}{x}+1\right)}{2}$$

input
```
int(x*log(1/x + 1),x)
```

output
```
x/2 - log(x*(x + 1))/4 - log(1/x + 1)/4 + (x^2*log(1/x + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x\log\left(1+\frac{1}{x}\right)\,dx = \frac{\log\left(\frac{x+1}{x}\right)x^2}{2} - \frac{\log\left(\frac{x+1}{x}\right)}{2} - \frac{\log(x)}{2} + \frac{x}{2}$$

input
```
int(x*log(1+1/x),x)
```

output
```
(log((x + 1)/x)*x**2 - log((x + 1)/x) - log(x) + x)/2
```

3.5 $\int \sin^2(\log(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 27

$$\int \sin^2(\log(x))\,dx=\frac{2x}{5}-\frac{2}{5}x\cos(\log(x))\sin(\log(x))+\frac{1}{5}x\sin^2(\log(x))$$

output
```
2/5*x-2/5*x*cos(ln(x))*sin(ln(x))+1/5*x*sin(ln(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin^2(\log(x))\,dx=\frac{x}{2}-\frac{1}{10}x\cos(2\log(x))-\frac{1}{5}x\sin(2\log(x))$$

input
```
Integrate[Sin[Log[x]]^2,x]
```

output
```
x/2 - (x*Cos[2*Log[x]])/10 - (x*Sin[2*Log[x]])/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4980, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(\log(x))\,dx$$

$$\Big\downarrow \text{4980}$$

$$\frac{2\int 1dx}{5}+\frac{1}{5}x\sin^2(\log(x))-\frac{2}{5}x\sin(\log(x))\cos(\log(x))$$

$$\Big\downarrow \text{24}$$

$$\frac{2x}{5}+\frac{1}{5}x\sin^2(\log(x))-\frac{2}{5}x\sin(\log(x))\cos(\log(x))$$

input
```
Int[Sin[Log[x]]^2,x]
```

output
```
(2*x)/5 - (2*x*Cos[Log[x]]*Sin[Log[x]])/5 + (x*Sin[Log[x]]^2)/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 4980
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (-Simp[b*d*n*
p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n
^2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int
[Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
 IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{x(-5+\cos(2\ln(x))+2\sin(2\ln(x)))}{10}$	18
default	$\frac{(\sin(\ln(x))-2\cos(\ln(x)))x\sin(\ln(x))}{5}+\frac{2x}{5}$	20
risch	$\frac{x}{2}+\left(-\frac{1}{20}+\frac{i}{10}\right)x\,x^{2i}+\left(-\frac{1}{20}-\frac{i}{10}\right)x\,x^{-2i}$	27
norman	$\frac{\frac{2x}{5}-\frac{4x\tan\left(\frac{\ln(x)}{2}\right)}{5}+\frac{8x\tan\left(\frac{\ln(x)}{2}\right)^2}{5}+\frac{4x\tan\left(\frac{\ln(x)}{2}\right)^3}{5}+\frac{2x\tan\left(\frac{\ln(x)}{2}\right)^4}{5}}{\left(1+\tan\left(\frac{\ln(x)}{2}\right)^2\right)^2}$	55

input
```
int(sin(ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/10*x*(-5+cos(2*ln(x))+2*sin(2*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sin^2(\log(x))\,dx=-\frac{1}{5}\,x\cos\left(\log\left(x\right)\right)^2-\frac{2}{5}\,x\cos\left(\log\left(x\right)\right)\sin\left(\log\left(x\right)\right)+\frac{3}{5}\,x$$

input
```
integrate(sin(log(x))^2,x, algorithm="fricas")
```

output
```
-1/5*x*cos(log(x))^2 - 2/5*x*cos(log(x))*sin(log(x)) + 3/5*x
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sin^2(\log(x))\,dx = \frac{3x\sin^2(\log(x))}{5} - \frac{2x\sin(\log(x))\cos(\log(x))}{5} + \frac{2x\cos^2(\log(x))}{5}$$

input
```
integrate(sin(ln(x))**2,x)
```

output
```
3*x*sin(log(x))**2/5 - 2*x*sin(log(x))*cos(log(x))/5 + 2*x*cos(log(x))**2/
5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sin^2(\log(x))\,dx = -\frac{1}{10}\,x\cos(2\,\log(x)) - \frac{1}{5}\,x\sin(2\,\log(x)) + \frac{1}{2}\,x$$

input
```
integrate(sin(log(x))^2,x, algorithm="maxima")
```

output
```
-1/10*x*cos(2*log(x)) - 1/5*x*sin(2*log(x)) + 1/2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sin^2(\log(x))\,dx = -\frac{1}{5}\,x\cos(\log(x))^2 - \frac{2}{5}\,x\cos(\log(x))\sin(\log(x)) + \frac{3}{5}\,x$$

input
```
integrate(sin(log(x))^2,x, algorithm="giac")
```

output
```
-1/5*x*cos(log(x))^2 - 2/5*x*cos(log(x))*sin(log(x)) + 3/5*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sin^2(\log(x))\,dx = \frac{x}{2} + \frac{x\left(2\sin\left(\ln\left(x\right)\right)^2 - 1\right)}{10} - \frac{x\,\sin\left(2\,\ln\left(x\right)\right)}{5}$$

input
```
int(sin(log(x))^2,x)
```

output
```
x/2 + (x*(2*sin(log(x))^2 - 1))/10 - (x*sin(2*log(x)))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sin^2(\log(x))\,dx = \frac{x\left(-2\cos\left(\log(x)\right)\sin\left(\log(x)\right) + \sin\left(\log(x)\right)^2 + 2\right)}{5}$$

input
```
int(sin(log(x))^2,x)
```

output
```
(x*( - 2*cos(log(x))*sin(log(x)) + sin(log(x))**2 + 2))/5
```

3.6 $\int \frac{1}{1+3e^x} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{1+3e^x} dx = x - \log(1+3e^x)$$

output
```
x-ln(1+3*exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+3e^x} dx = \log(e^x) - \log(1+3e^x)$$

input
```
Integrate[(1 + 3*E^x)^(-1),x]
```

output
```
Log[E^x] - Log[1 + 3*E^x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3e^x+1}\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^{-x}}{3e^x+1}de^x$$

$$\Big\downarrow \text{ 47}$$

$$\int e^{-x}de^x-3\int \frac{1}{1+3e^x}de^x$$

$$\Big\downarrow \text{ 14}$$

$$\log\left(e^x\right)-3\int \frac{1}{1+3e^x}de^x$$

$$\Big\downarrow \text{ 16}$$

$$\log\left(e^x\right)-\log\left(3e^x+1\right)$$

input
```
Int[(1 + 3*E^x)^(-1),x]
```

output
```
Log[E^x] - Log[1 + 3*E^x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
risch	$x-\ln\left(\frac{1}{3}+{\mathrm e}^{x}\right)$	10
parallelrisch	$x-\ln\left(\frac{1}{3}+{\mathrm e}^{x}\right)$	10
norman	$x-\ln\left(1+3\,{\mathrm e}^{x}\right)$	12
derivativedivides	$-\ln\left(1+3\,{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	14
default	$-\ln\left(1+3\,{\mathrm e}^{x}\right)+\ln\left({\mathrm e}^{x}\right)$	14

input
```
int(1/(1+3*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x-ln(1/3+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+3e^x}\,dx = x - \log\left(3\,e^x + 1\right)$$

input
```
integrate(1/(1+3*exp(x)),x, algorithm="fricas")
```

output
```
x - log(3*e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+3e^x}\,dx = x - \log\left(e^x + \frac{1}{3}\right)$$

input
```
integrate(1/(1+3*exp(x)),x)
```

output
```
x - log(exp(x) + 1/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+3e^x}\,dx = x - \log\left(3\,e^x + 1\right)$$

input
```
integrate(1/(1+3*exp(x)),x, algorithm="maxima")
```

output
```
x - log(3*e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+3e^x}\,dx = x-\log\left(3\,e^x+1\right)$$

input
```
integrate(1/(1+3*exp(x)),x, algorithm="giac")
```

output
```
x - log(3*e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+3e^x}\,dx = x-\ln\left(3\,\mathrm{e}^x+1\right)$$

input
```
int(1/(3*exp(x) + 1),x)
```

output
```
x - log(3*exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+3e^x}\,dx = -\log(3e^x+1)+x$$

input
```
int(1/(1+3*exp(x)),x)
```

output
```
- log(3*e**x + 1) + x
```

3.7 $\int \csc^3(x)\sec^5(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 30

$$\int \csc^3(x)\sec^5(x)\,dx = -\frac{1}{2}\cot^2(x) + 3\log(\tan(x)) + \frac{3\tan^2(x)}{2} + \frac{\tan^4(x)}{4}$$

output
```
-1/2*cot(x)^2+3*ln(tan(x))+3/2*tan(x)^2+1/4*tan(x)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \csc^3(x)\sec^5(x)\,dx = -\frac{1}{2}\csc^2(x) - 3\log(\cos(x)) + 3\log(\sin(x)) + \sec^2(x) + \frac{\sec^4(x)}{4}$$

input
```
Integrate[Csc[x]^3*Sec[x]^5,x]
```

output
```
-1/2*Csc[x]^2 - 3*Log[Cos[x]] + 3*Log[Sin[x]] + Sec[x]^2 + Sec[x]^4/4
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(x)^3\sec(x)^5dx$$

$$\Big\downarrow \text{ 3100}$$

$$\int \left(\tan^2(x)+1\right)^3\cot^3(x)d\tan(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \cot^2(x)\left(\tan^2(x)+1\right)^3d\tan^2(x)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\cot^2(x)+3\cot(x)+\tan^2(x)+3\right)d\tan^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\tan^4(x)}{2}+3\tan^2(x)-\cot(x)+3\log\left(\tan^2(x)\right)\right)$$

input
```
Int[Csc[x]^3*Sec[x]^5,x]
```

output
```
(-Cot[x] + 3*Log[Tan[x]^2] + 3*Tan[x]^2 + Tan[x]^4/2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result
default	$\frac{1}{4\sin(x)^{2}\cos(x)^{4}}+\frac{3}{4\sin(x)^{2}\cos(x)^{2}}-\frac{3}{2\sin(x)^{2}}+3\ln(\tan(x))$
norman	$\frac{-\frac{1}{8}-10\tan(\frac{x}{2})^{6}+\frac{57\tan(\frac{x}{2})^{4}}{8}+\frac{57\tan(\frac{x}{2})^{8}}{8}-\frac{\tan(\frac{x}{2})^{12}}{8}}{\tan(\frac{x}{2})^{2}\left(\tan(\frac{x}{2})^{2}-1\right)^{4}}+3\ln\left(\tan\left(\frac{x}{2}\right)\right)-3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-3\ln\left(1+\mathrm{t}\right.$
risch	$\frac{6\,\mathrm{e}^{10ix}+12\,\mathrm{e}^{8ix}-4\,\mathrm{e}^{6ix}+12\,\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{4}(\mathrm{e}^{2ix}-1)^{2}}+3\ln(\mathrm{e}^{2ix}-1)-3\ln(\mathrm{e}^{2ix}+1)$
parallelrisch	$\frac{(-384\cos(2x)-96\cos(4x)-288)\ln(1+\tan(\frac{x}{2}))+(-384\cos(2x)-96\cos(4x)-288)\ln(\tan(\frac{x}{2})-1)+(384\cos(2x)+96\cos(4x)-}{32\cos(4x)+128\,\mathrm{co}}$

input
```
int(csc(x)^3*sec(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4/sin(x)^2/cos(x)^4+3/4/sin(x)^2/cos(x)^2-3/2/sin(x)^2+3*ln(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.30

$$\int \csc^3(x)\sec^5(x)\,dx = \frac{6\cos(x)^4 - 3\cos(x)^2 - 6\left(\cos(x)^6 - \cos(x)^4\right)\log\left(\cos(x)^2\right) + 6\left(\cos(x)^6 - \cos(x)^4\right)\log\left(-\frac{1}{4}\cos(x)^2\right.}{4\left(\cos(x)^6 - \cos(x)^4\right)}$$

input
```
integrate(csc(x)^3*sec(x)^5,x, algorithm="fricas")
```

output
```
1/4*(6*cos(x)^4 - 3*cos(x)^2 - 6*(cos(x)^6 - cos(x)^4)*log(cos(x)^2) + 6*(
cos(x)^6 - cos(x)^4)*log(-1/4*cos(x)^2 + 1/4) - 1)/(cos(x)^6 - cos(x)^4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \csc^3(x)\sec^5(x)\,dx = \frac{3\log\left(\cos^2(x) - 1\right)}{2} - 3\log(\cos(x)) - \frac{-6\cos^4(x) + 3\cos^2(x) + 1}{4\cos^6(x) - 4\cos^4(x)}$$

input
```
integrate(csc(x)**3*sec(x)**5,x)
```

output
```
3*log(cos(x)**2 - 1)/2 - 3*log(cos(x)) - (-6*cos(x)**4 + 3*cos(x)**2 + 1)/
(4*cos(x)**6 - 4*cos(x)**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \csc^3(x)\sec^5(x)\,dx = -\frac{6\,\sin(x)^4 - 9\,\sin(x)^2 + 2}{4\,(\sin(x)^6 - 2\,\sin(x)^4 + \sin(x)^2)} - \frac{3}{2}\,\log\left(\sin(x)^2 - 1\right) + \frac{3}{2}\,\log\left(\sin(x)^2\right)$$

input
```
integrate(csc(x)^3*sec(x)^5,x, algorithm="maxima")
```

output
```
-1/4*(6*sin(x)^4 - 9*sin(x)^2 + 2)/(sin(x)^6 - 2*sin(x)^4 + sin(x)^2) - 3/
2*log(sin(x)^2 - 1) + 3/2*log(sin(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \csc^3(x)\sec^5(x)\,dx = -\frac{6\,\sin(x)^4 - 9\,\sin(x)^2 + 2}{4\,\left(\sin(x)^2 - 1\right)^2\sin(x)^2} - \frac{3}{2}\,\log\left(-\sin(x)^2 + 1\right) + 3\,\log\left(|\sin(x)|\right)$$

input
```
integrate(csc(x)^3*sec(x)^5,x, algorithm="giac")
```

output
```
-1/4*(6*sin(x)^4 - 9*sin(x)^2 + 2)/((sin(x)^2 - 1)^2*sin(x)^2) - 3/2*log(-
sin(x)^2 + 1) + 3*log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \csc^3(x)\sec^5(x)\,dx = 3\,\ln\left(\tan(x)\right) + \frac{\frac{3}{4\cos(x)^2} + \frac{1}{4\cos(x)^4}}{\sin\left(x\right)^2} - \frac{3}{2\sin\left(x\right)^2}$$

input
```
int(1/(cos(x)^5*sin(x)^3),x)
```

output
```
3*log(tan(x)) + (3/(4*cos(x)^2) + 1/(4*cos(x)^4))/sin(x)^2 - 3/(2*sin(x)^2
)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.03

$$\int \csc^3(x)\sec^5(x)\,dx$$
$$= \frac{-12\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^6 + 24\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^4 - 12\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^2 - 12\log\left(\tan\right.}{}$$

input
```
int(csc(x)^3*sec(x)^5,x)
```

output
```
( - 12*log(tan(x/2) - 1)*sin(x)**6 + 24*log(tan(x/2) - 1)*sin(x)**4 - 12*l
og(tan(x/2) - 1)*sin(x)**2 - 12*log(tan(x/2) + 1)*sin(x)**6 + 24*log(tan(x
/2) + 1)*sin(x)**4 - 12*log(tan(x/2) + 1)*sin(x)**2 + 12*log(tan(x/2))*sin
(x)**6 - 24*log(tan(x/2))*sin(x)**4 + 12*log(tan(x/2))*sin(x)**2 - 6*sin(x
)**6 + 6*sin(x)**4 + 3*sin(x)**2 - 2)/(4*sin(x)**2*(sin(x)**4 - 2*sin(x)**
2 + 1))
```

3.8 $\int \frac{1}{x\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

output
```
1/2*arctan((x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 + x^4]),x]
```

output
```
ArcTan[Sqrt[-1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4-1}}\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{1}{x^4\sqrt{x^4-1}}dx^4$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{2}\int \frac{1}{x^8+1}d\sqrt{x^4-1}$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
Int[1/(x*Sqrt[-1 + x^4]),x]
```

output
```
ArcTan[Sqrt[-1 + x^4]]/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
elliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
pseudoelliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^4-1}}{x^2}\right)}{2}$	28
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(\left(-2\ln(2)+4\ln(x)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^4+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^4-1)}}$	61

input
```
int(1/x/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(x**4-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(x**(-2))/2, 1/Abs(x**4) > 1), (-asin(x**(-2))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{\operatorname{atan}\left(\sqrt{x^4-1}\right)}{2}$$

input
```
int(1/(x*(x^4 - 1)^(1/2)),x)
```

output
```
atan((x^4 - 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = atan\left(\sqrt{x^4-1}+x^2\right)$$

input
```
int(1/x/(x^4-1)^(1/2),x)
```

output
```
atan(sqrt(x**4 - 1) + x**2)
```

3.9 $\int \frac{1}{x(1+x^5)} dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{x(1+x^5)} dx = \log(x) - \frac{1}{5}\log(1+x^5)$$

output
```
ln(x)-1/5*ln(x^5+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x^5)} dx = \log(x) - \frac{1}{5}\log(1+x^5)$$

input
```
Integrate[1/(x*(1 + x^5)),x]
```

output
```
Log[x] - Log[1 + x^5]/5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^5+1)} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{5} \int \frac{1}{x^5 (x^5+1)}dx^5$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{5} \left(\int \frac{1}{x^5}dx^5-\int \frac{1}{x^5+1}dx^5\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{5} \left(\log \left(x^5\right)-\int \frac{1}{x^5+1}dx^5\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{5} \left(\log \left(x^5\right)-\log \left(x^5+1\right)\right)$$

input
```
Int[1/(x*(1 + x^5)),x]
```

output
```
(Log[x^5] - Log[1 + x^5])/5
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
meijerg	$\ln(x)-\frac{\ln(x^5+1)}{5}$	12
risch	$\ln(x)-\frac{\ln(x^5+1)}{5}$	12
default	$\ln(x)-\frac{\ln(x+1)}{5}-\frac{\ln(x^4-x^3+x^2-x+1)}{5}$	29
norman	$\ln(x)-\frac{\ln(x+1)}{5}-\frac{\ln(x^4-x^3+x^2-x+1)}{5}$	29
parallelrisch	$\ln(x)-\frac{\ln(x+1)}{5}-\frac{\ln(x^4-x^3+x^2-x+1)}{5}$	29

input
```
int(1/x/(x^5+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/5*ln(x^5+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^5)}\,dx = -\frac{1}{5}\log\left(x^5+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^5+1),x, algorithm="fricas")
```

output
```
-1/5*log(x^5 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x(1+x^5)}\,dx = \log\left(x\right)-\frac{\log\left(x^5+1\right)}{5}$$

input
```
integrate(1/x/(x**5+1),x)
```

output
```
log(x) - log(x**5 + 1)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(1+x^5)}\,dx = -\frac{1}{5}\log\left(x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

input
```
integrate(1/x/(x^5+1),x, algorithm="maxima")
```

output
```
-1/5*log(x^5 + 1) + 1/5*log(x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x^5)}\,dx = -\frac{1}{5}\log\left(\left|x^5+1\right|\right)+\log\left(|x|\right)$$

input
```
integrate(1/x/(x^5+1),x, algorithm="giac")
```

output
```
-1/5*log(abs(x^5 + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^5)}\,dx = \ln(x)-\frac{\ln(x^5+1)}{5}$$

input
```
int(1/(x*(x^5 + 1)),x)
```

output
```
log(x) - log(x^5 + 1)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.15

$$\int \frac{1}{x(1+x^5)}\,dx = -\frac{\log(x^4-x^3+x^2-x+1)}{5}-\frac{\log(x+1)}{5}+\log(x)$$

input
```
int(1/x/(x^5+1),x)
```

output
```
( - log(x**4 - x**3 + x**2 - x + 1) - log(x + 1) + 5*log(x))/5
```

3.10 $\int \sqrt{\tan(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 71

$$\int \sqrt{\tan(x)}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(x)}}{1+\tan(x)}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(-1+tan(x)^(1/2)*2^(1/2))*2^(1/2)+1/2*arctan(1+tan(x)^(1/2)*2^(1
/2))*2^(1/2)-1/2*arctanh(2^(1/2)*tan(x)^(1/2)/(1+tan(x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \sqrt{\tan(x)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{-\tan^2(x)}\right)\right)\sqrt[4]{-\tan(x)}}{\sqrt[4]{\tan(x)}}$$

input
```
Integrate[Sqrt[Tan[x]],x]
```

output
```
((ArcTan[(-Tan[x]^2)^(1/4)] - ArcTanh[(-Tan[x]^2)^(1/4)])*(-Tan[x])^(1/4))
/Tan[x]^(1/4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.55, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.833$, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)}dx$$

$$\Big\downarrow \text{3957}$$

$$\int \frac{\sqrt{\tan(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{\tan(x)+1}{\tan^2(x)+1}d\sqrt{\tan(x)}-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}+\frac{1}{2}\int \frac{1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)-\frac{1}{2}\int \frac{1-\tan(x}{\tan^2(x)+}\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(x)-1}d\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(x)-1}d\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$\downarrow$ 217

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\sqrt{\tan(x)}\right)$$

$\downarrow$ 1479

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}\right.\right.$$

$\downarrow$ 25

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\right.\right.$$

$\downarrow$ 27

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(x)}}{\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1}d\sqrt{\tan(x)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x\right.}}{\sqrt{2}}\right.\right.$$

$\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\mathrm{t}\right.}{}\right.\right.$$

input

```
Int[Sqrt[Tan[x]],x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[
Tan[x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
- Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result	size
lookup	$\frac{\sqrt{\tan(x)}\,\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\sqrt{\tan(x)}\,\cos(x)+\sin(x)\right)}{2}$	49
default	$\frac{\sqrt{\tan(x)}\,\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\sqrt{\tan(x)}\,\cos(x)+\sin(x)\right)}{2}$	49
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(x)-\sqrt{2}\,\sqrt{\tan(x)}+1}{\tan(x)+\sqrt{2}\,\sqrt{\tan(x)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(x)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(x)}\right)\right)}{4}$	62

input
```
int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x)
)-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\tan(x)}-1\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*sqrt(tan(x)) + 1) + 1/2*sqrt(2)*arctan(sqrt(2)*
sqrt(tan(x)) - 1) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) + 1
/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1)
```

Sympy [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan(x)}\,dx$$

input
```
integrate(tan(x)**(1/2),x)
```

output
```
Integral(sqrt(tan(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \sqrt{\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

input
```
integrate(tan(x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*s
qrt(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \sqrt{\tan(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{4} + \frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)+\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{2}$$

input
```
int(tan(x)^(1/2),x)
```

output
```
(2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*ta
n(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1
/2)*tan(x)^(1/2) + 1)))/2
```

Reduce [F]

$$\int \sqrt{\tan(x)}\,dx = \int \sqrt{\tan(x)}dx$$

input
```
int(tan(x)^(1/2),x)
```

output
```
int(sqrt(tan(x)),x)
```

3.11 $\int \frac{\log(1+x)}{1+x^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int \frac{\log(1+x)}{1+x^2} dx = -\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x) + \frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x) - \frac{1}{2}i\,\mathrm{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(1+x)\right) + \frac{1}{2}i\,\mathrm{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(1+x)\right)$$

output
```
-1/2*I*ln((1/2-1/2*I)*(I-x))*ln(1+x)+1/2*I*ln((-1/2-1/2*I)*(I+x))*ln(1+x)-
1/2*I*polylog(2,(1/2-1/2*I)*(1+x))+1/2*I*polylog(2,(1/2+1/2*I)*(1+x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{1+x^2}\,dx = -\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x) + \frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x) - \frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(1+x)\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(1+x)\right)$$

input
```
Integrate[Log[1 + x]/(1 + x^2),x]
```

output
```
(-1/2*I)*Log[(1/2 - I/2)*(I - x)]*Log[1 + x] + (I/2)*Log[(-1/2 - I/2)*(I +
 x)]*Log[1 + x] - (I/2)*PolyLog[2, (1/2 - I/2)*(1 + x)] + (I/2)*PolyLog[2,
 (1/2 + I/2)*(1 + x)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x+1)}{x^2+1}\,dx$$

$$\Big\downarrow \text{2856}$$

$$\int \left(\frac{i\log(x+1)}{2(-x+i)}+\frac{i\log(x+1)}{2(x+i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(x+1)\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(x+1)\right)-\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-x+i)\right)\log(x+1)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(x+i)\right)\log(x+1)$$

input
```
Int[Log[1 + x]/(1 + x^2),x]
```

output
```
(-1/2*I)*Log[(1/2 - I/2)*(I - x)]*Log[1 + x] + (I/2)*Log[(-1/2 - I/2)*(I +
 x)]*Log[1 + x] - (I/2)*PolyLog[2, (1/2 - I/2)*(1 + x)] + (I/2)*PolyLog[2,
 (1/2 + I/2)*(1 + x)]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
default	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
risch	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
parts	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$

input
```
int(ln(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*ln(x+1)*ln(1/2-1/2*x+1/2*I*(x+1))+1/2*I*ln(x+1)*ln(1/2-1/2*x-1/2*I*
(x+1))-1/2*I*dilog(1/2-1/2*x+1/2*I*(x+1))+1/2*I*dilog(1/2-1/2*x-1/2*I*(x+1
))
```

Fricas [F]

$$\int \frac{\log(1+x)}{1+x^2}\,dx = \int \frac{\log(x+1)}{x^2+1}\,dx$$

input
```
integrate(log(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(log(x + 1)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log(1+x)}{1+x^2}\,dx = \int \frac{\log(x+1)}{x^2+1}\,dx$$

input
```
integrate(ln(1+x)/(x**2+1),x)
```

output
```
Integral(log(x + 1)/(x**2 + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\begin{aligned}\int \frac{\log(1+x)}{1+x^2}\,dx &= \frac{1}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2},\frac{1}{2}x+\frac{1}{2}\right)\log\left(x^2+1\right)\\ &\quad -\frac{1}{2}\arctan(x)\log\left(\frac{1}{2}x^2+x+\frac{1}{2}\right)+\arctan(x)\log(x+1)\\ &\quad +\frac{1}{2}i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i-\frac{1}{2}\right)x+\frac{1}{2}i+\frac{1}{2}\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)x-\frac{1}{2}i+\frac{1}{2}\right)\end{aligned}$$

input
```
integrate(log(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan2(1/2*x + 1/2, 1/2*x + 1/2)*log(x^2 + 1) - 1/2*arctan(x)*log(1/2
*x^2 + x + 1/2) + arctan(x)*log(x + 1) + 1/2*I*dilog((1/2*I - 1/2)*x + 1/2
*I + 1/2) - 1/2*I*dilog(-(1/2*I + 1/2)*x - 1/2*I + 1/2)
```

Giac [F]

$$\int \frac{\log(1+x)}{1+x^2}\,dx = \int \frac{\log(x+1)}{x^2+1}\,dx$$

input
```
integrate(log(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(log(x + 1)/(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1+x)}{1+x^2}\,dx = \int \frac{\ln(x+1)}{x^2+1}\,dx$$

input
```
int(log(x + 1)/(x^2 + 1),x)
```

output
```
int(log(x + 1)/(x^2 + 1), x)
```

Reduce [F]

$$\int \frac{\log(1+x)}{1+x^2}\,dx = \int \frac{\log(x+1)}{x^2+1}dx$$

input
```
int(log(1+x)/(x^2+1),x)
```

output
```
int(log(x + 1)/(x**2 + 1),x)
```

3.12 $\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}} dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}} dx = 6\sqrt[6]{x}+3\sqrt[3]{x}+2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+x+6\log\left(1-\sqrt[6]{x}\right)$$

output
```
6*x^(1/6)+3*x^(1/3)+2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+x+6*ln(1-x^(1/6))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}} dx = 6\sqrt[6]{x}+3\sqrt[3]{x}+2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+x+6\log\left(-1+\sqrt[6]{x}\right)$$

input
```
Integrate[Sqrt[x]/(-x^(1/3) + Sqrt[x]),x]
```

output
```
6*x^(1/6) + 3*x^(1/3) + 2*Sqrt[x] + (3*x^(2/3))/2 + (6*x^(5/6))/5 + x + 6*
Log[-1 + x^(1/6)]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {10, 25, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int -\frac{\sqrt[6]{x}}{1-\sqrt[6]{x}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sqrt[6]{x}}{1-\sqrt[6]{x}}dx$$

$$\Big\downarrow \text{ 798}$$

$$-6\int \frac{x}{1-\sqrt[6]{x}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 49}$$

$$-6\int \left(-x^{5/6}-x^{2/3}-\sqrt{x}-\sqrt[3]{x}-\sqrt[6]{x}+\frac{1}{1-\sqrt[6]{x}}-1\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-6\left(-\frac{x^{5/6}}{5}-\frac{x^{2/3}}{4}-\frac{x}{6}-\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}-\sqrt[6]{x}-\log\left(1-\sqrt[6]{x}\right)\right)$$

input
```
Int[Sqrt[x]/(-x^(1/3) + Sqrt[x]),x]
```

output
```
-6*(-x^(1/6) - x^(1/3)/2 - Sqrt[x]/3 - x^(2/3)/4 - x^(5/6)/5 - x/6 - Log[1
 - x^(1/6)])
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$x+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{2}{3}}}{2}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\ln\left(x^{\frac{1}{6}}-1\right)$	36
default	$x+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{2}{3}}}{2}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\ln\left(x^{\frac{1}{6}}-1\right)$	36
meijerg	$\frac{x^{\frac{1}{6}}\left(70x^{\frac{5}{6}}+84x^{\frac{2}{3}}+105\sqrt{x}+140x^{\frac{1}{3}}+210x^{\frac{1}{6}}+420\right)}{70}+6\ln\left(1-x^{\frac{1}{6}}\right)$	44

input
```
int(x^(1/2)/(-x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+6/5*x^(5/6)+3/2*x^(2/3)+2*x^(1/2)+3*x^(1/3)+6*x^(1/6)+6*ln(x^(1/6)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = x + \frac{6}{5}\,x^{\frac{5}{6}} + \frac{3}{2}\,x^{\frac{2}{3}} + 2\,\sqrt{x} + 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} + 6\,\log\left(x^{\frac{1}{6}} - 1\right)$$

input
```
integrate(x^(1/2)/(-x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

output
```
x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(
x^(1/6) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx$$

input
```
integrate(x**(1/2)/(-x**(1/3)+x**(1/2)),x)
```

output
```
Integral(sqrt(x)/(-x**(1/3) + sqrt(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = x + \frac{6}{5}\,x^{\frac{5}{6}} + \frac{3}{2}\,x^{\frac{2}{3}} + 2\,\sqrt{x} + 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} + 6\,\log\left(x^{\frac{1}{6}} - 1\right)$$

input
```
integrate(x^(1/2)/(-x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(
x^(1/6) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = x + \frac{6}{5}\,x^{\frac{5}{6}} + \frac{3}{2}\,x^{\frac{2}{3}} + 2\,\sqrt{x} + 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} + 6\,\log\left(\left|x^{\frac{1}{6}} - 1\right|\right)$$

input
```
integrate(x^(1/2)/(-x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(
abs(x^(1/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = x + 6\,\ln\left(x^{1/6} - 1\right) + 2\,\sqrt{x} + 3\,x^{1/3} + \frac{3\,x^{2/3}}{2} + 6\,x^{1/6} + \frac{6\,x^{5/6}}{5}$$

input
```
int(x^(1/2)/(x^(1/2) - x^(1/3)),x)
```

output
```
x + 6*log(x^(1/6) - 1) + 2*x^(1/2) + 3*x^(1/3) + (3*x^(2/3))/2 + 6*x^(1/6)
 + (6*x^(5/6))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx = \frac{6x^{\frac{5}{6}}}{5}+6x^{\frac{1}{6}}+\frac{3x^{\frac{2}{3}}}{2}+3x^{\frac{1}{3}}+2\sqrt{x}+6\log\left(x^{\frac{1}{6}}-1\right)+x$$

input
```
int(x^(1/2)/(-x^(1/3)+x^(1/2)),x)
```

output
```
(12*x**(5/6) + 60*x**(1/6) + 15*x**(2/3) + 30*x**(1/3) + 20*sqrt(x) + 60*l
og(x**(1/6) - 1) + 10*x)/10
```

3.13 $\int x^x(1+\log(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 3

$$\int x^x(1+\log(x))\,dx = x^x$$

output
```
x^x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
Integrate[x^x*(1 + Log[x]),x]
```

output
```
x^x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^x(\log(x)+1)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int (x^x+x^x\log(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$x^x$$

input
```
Int[x^x*(1 + Log[x]),x]
```

output
```
x^x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	x^{x}	4
default	x^{x}	4
risch	x^{x}	4
parallelrisch	x^{x}	4
norman	$\mathrm{e}^{\ln(x) x}$	6

input
```
int(x^x*(1+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x^x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^{x}(1+\log(x))\,dx = x^{x}$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="fricas")
```

output
```
x^x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int x^{x}(1+\log(x))\,dx = x^{x}$$

input
```
integrate(x**x*(1+ln(x)),x)
```

output
```
x**x
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx=x^x$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="maxima")
```

output
```
x^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx=x^x$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="giac")
```

output
```
x^x
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx=x^x$$

input
```
int(x^x*(log(x) + 1),x)
```

output
```
x^x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
int(x^x*(1+log(x)),x)
```

output
```
x**x
```

3.14 $\int x^{13/2}\sqrt{1+x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx=\frac{4}{525}\left(1+x^{5/2}\right)^{3/2}\left(8-12x^{5/2}+15x^5\right)$$

output
```
4/525*(1+x^(5/2))^(3/2)*(8-12*x^(5/2)+15*x^5)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx=\frac{4}{525}\left(1+x^{5/2}\right)^{3/2}\left(8-12x^{5/2}+15x^5\right)$$

input
```
Integrate[x^(13/2)*Sqrt[1 + x^(5/2)],x]
```

output
```
(4*(1 + x^(5/2))^(3/2)*(8 - 12*x^(5/2) + 15*x^5))/525
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13/2}\sqrt{x^{5/2}+1}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{5}\int x^5\sqrt{x^{5/2}+1}dx^{5/2}$$

$$\Big\downarrow \text{53}$$

$$\frac{2}{5}\int \left(\left(x^{5/2}+1\right)^{5/2}-2\left(x^{5/2}+1\right)^{3/2}+\sqrt{x^{5/2}+1}\right)dx^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}\left(\frac{2}{7}\left(x^{5/2}+1\right)^{7/2}-\frac{4}{5}\left(x^{5/2}+1\right)^{5/2}+\frac{2}{3}\left(x^{5/2}+1\right)^{3/2}\right)$$

input
```
Int[x^(13/2)*Sqrt[1 + x^(5/2)],x]
```

output
```
(2*((2*(1 + x^(5/2))^(3/2))/3 - (4*(1 + x^(5/2))^(5/2))/5 + (2*(1 + x^(5/2
))^(7/2))/7))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result	size
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(1+x^{\frac{5}{2}}\right)^{\frac{3}{2}}\left(8-12x^{\frac{5}{2}}+15x^{5}\right)}{105}}{5\sqrt{\pi}}$	36

input
```
int(x^(13/2)*(1+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/Pi^(1/2)*(32/105*Pi^(1/2)-4/105*Pi^(1/2)*(1+x^(5/2))^(3/2)*(8-12*x^(5
/2)+15*x^5))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = \frac{4}{525}\left(3\,x^{5}+\left(15\,x^{7}-4\,x^{2}\right)\sqrt{x}+8\right)\sqrt{x^{\frac{5}{2}}+1}$$

input
```
integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="fricas")
```

output
```
4/525*(3*x^5 + (15*x^7 - 4*x^2)*sqrt(x) + 8)*sqrt(x^(5/2) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 68.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = \frac{4x^{\frac{15}{2}}\sqrt{x^{\frac{5}{2}}+1}}{35} - \frac{16x^{\frac{5}{2}}\sqrt{x^{\frac{5}{2}}+1}}{525} + \frac{4x^5\sqrt{x^{\frac{5}{2}}+1}}{175} + \frac{32\sqrt{x^{\frac{5}{2}}+1}}{525}$$

input
```
integrate(x**(13/2)*(1+x**(5/2))**(1/2),x)
```

output
```
4*x**(15/2)*sqrt(x**(5/2) + 1)/35 - 16*x**(5/2)*sqrt(x**(5/2) + 1)/525 + 4
*x**5*sqrt(x**(5/2) + 1)/175 + 32*sqrt(x**(5/2) + 1)/525
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = \frac{4}{35}\left(x^{\frac{5}{2}}+1\right)^{\frac{7}{2}} - \frac{8}{25}\left(x^{\frac{5}{2}}+1\right)^{\frac{5}{2}} + \frac{4}{15}\left(x^{\frac{5}{2}}+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="maxima")
```

output
```
4/35*(x^(5/2) + 1)^(7/2) - 8/25*(x^(5/2) + 1)^(5/2) + 4/15*(x^(5/2) + 1)^(
3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = \frac{4}{35}\left(x^{\frac{5}{2}}+1\right)^{\frac{7}{2}} - \frac{8}{25}\left(x^{\frac{5}{2}}+1\right)^{\frac{5}{2}} + \frac{4}{15}\left(x^{\frac{5}{2}}+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="giac")
```

output
```
4/35*(x^(5/2) + 1)^(7/2) - 8/25*(x^(5/2) + 1)^(5/2) + 4/15*(x^(5/2) + 1)^(
3/2)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = -\frac{4\left(x^{5/2}+1\right)^{3/2}\left(42\,x^{5/2}-15\left(x^{5/2}+1\right)^2+7\right)}{525}$$

input
```
int(x^(13/2)*(x^(5/2) + 1)^(1/2),x)
```

output
```
-(4*(x^(5/2) + 1)^(3/2)*(42*x^(5/2) - 15*(x^(5/2) + 1)^2 + 7))/525
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int x^{13/2}\sqrt{1+x^{5/2}}\,dx = \frac{4\sqrt{\sqrt{x}\,x^2+1}\left(15\sqrt{x}\,x^7-4\sqrt{x}\,x^2+3x^5+8\right)}{525}$$

input
```
int(x^(13/2)*(1+x^(5/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*x**2 + 1)*(15*sqrt(x)*x**7 - 4*sqrt(x)*x**2 + 3*x**5 + 8))
/525
```

3.15 $\int \frac{1}{(1+x^2)^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \frac{1}{(1+x^2)^2} dx = \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
x/(2*x^2+2)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+x^2)^2} dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x)\right)$$

input
```
Integrate[(1 + x^2)^(-2),x]
```

output
```
(x/(1 + x^2) + ArcTan[x])/2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^2} \, dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}$$

input
```
Int[(1 + x^2)^(-2),x]
```

output
```
x/(2*(1 + x^2)) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	17
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(i+x)x^2+i\ln(x-i)-i\ln(i+x)-2x}{4(x^2+1)}$	52

input
```
int(1/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

input
```
integrate(1/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**2+1)**2,x)
```

output
```
x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(1/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(1/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{\mathrm{atan}(x)}{2} + \frac{x}{2\,(x^2+1)}$$

input
```
int(1/(x^2 + 1)^2,x)
```

output
```
atan(x)/2 + x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1+x^2)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) + x}{2x^2 + 2}$$

input
```
int(1/(x^2+1)^2,x)
```

output
```
(atan(x)*x**2 + atan(x) + x)/(2*(x**2 + 1))
```

3.16 $\int \frac{1}{36-13x^2+x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{36-13x^2+x^4} \, dx = -\frac{1}{15}\operatorname{arctanh}\left(\frac{x}{3}\right)+\frac{1}{10}\operatorname{arctanh}\left(\frac{x}{2}\right)$$

output
```
-1/15*arctanh(1/3*x)+1/10*arctanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{36-13x^2+x^4} \, dx = -\frac{1}{20}\log(2-x)+\frac{1}{30}\log(3-x)+\frac{1}{20}\log(2+x)-\frac{1}{30}\log(3+x)$$

input
```
Integrate[(36 - 13*x^2 + x^4)^(-1),x]
```

output
```
-1/20*Log[2 - x] + Log[3 - x]/30 + Log[2 + x]/20 - Log[3 + x]/30
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-13x^2+36}dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{5}\int \frac{1}{x^2-9}dx-\frac{1}{5}\int \frac{1}{x^2-4}dx$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{10}\text{arctanh}\left(\frac{x}{2}\right)-\frac{1}{15}\text{arctanh}\left(\frac{x}{3}\right)$$

input
```
Int[(36 - 13*x^2 + x^4)^(-1),x]
```

output
```
-1/15*ArcTanh[x/3] + ArcTanh[x/2]/10
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\ln(3+x)}{30}+\frac{\ln(-3+x)}{30}-\frac{\ln(x-2)}{20}+\frac{\ln(x+2)}{20}$	26
norman	$-\frac{\ln(3+x)}{30}+\frac{\ln(-3+x)}{30}-\frac{\ln(x-2)}{20}+\frac{\ln(x+2)}{20}$	26
risch	$-\frac{\ln(3+x)}{30}+\frac{\ln(-3+x)}{30}-\frac{\ln(x-2)}{20}+\frac{\ln(x+2)}{20}$	26
parallelrisch	$-\frac{\ln(3+x)}{30}+\frac{\ln(-3+x)}{30}-\frac{\ln(x-2)}{20}+\frac{\ln(x+2)}{20}$	26

input
```
int(1/(x^4-13*x^2+36),x,method=_RETURNVERBOSE)
```

output
```
-1/30*ln(3+x)+1/30*ln(-3+x)-1/20*ln(x-2)+1/20*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{36-13x^2+x^4}\,dx = -\frac{1}{30}\log(x+3)+\frac{1}{20}\log(x+2)-\frac{1}{20}\log(x-2)+\frac{1}{30}\log(x-3)$$

input
```
integrate(1/(x^4-13*x^2+36),x, algorithm="fricas")
```

output
```
-1/30*log(x + 3) + 1/20*log(x + 2) - 1/20*log(x - 2) + 1/30*log(x - 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{36-13x^2+x^4}\,dx = \frac{\log(x-3)}{30}-\frac{\log(x-2)}{20}+\frac{\log(x+2)}{20}-\frac{\log(x+3)}{30}$$

input
```
integrate(1/(x**4-13*x**2+36),x)
```

output
```
log(x - 3)/30 - log(x - 2)/20 + log(x + 2)/20 - log(x + 3)/30
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{36-13x^2+x^4}\,dx = -\frac{1}{30}\log(x+3)+\frac{1}{20}\log(x+2)-\frac{1}{20}\log(x-2)+\frac{1}{30}\log(x-3)$$

input
```
integrate(1/(x^4-13*x^2+36),x, algorithm="maxima")
```

output
```
-1/30*log(x + 3) + 1/20*log(x + 2) - 1/20*log(x - 2) + 1/30*log(x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{1}{36-13x^2+x^4}\,dx = -\frac{1}{30}\log(|x+3|)+\frac{1}{20}\log(|x+2|) - \frac{1}{20}\log(|x-2|)+\frac{1}{30}\log(|x-3|)$$

input
```
integrate(1/(x^4-13*x^2+36),x, algorithm="giac")
```

output
```
-1/30*log(abs(x + 3)) + 1/20*log(abs(x + 2)) - 1/20*log(abs(x - 2)) + 1/30
*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{36-13x^2+x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{10}-\frac{\operatorname{atanh}\left(\frac{x}{3}\right)}{15}$$

input
```
int(1/(x^4 - 13*x^2 + 36),x)
```

output
```
atanh(x/2)/10 - atanh(x/3)/15
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{36-13x^2+x^4}\,dx=\frac{\log(x-3)}{30}-\frac{\log(x-2)}{20}-\frac{\log(x+3)}{30}+\frac{\log(x+2)}{20}$$

input
```
int(1/(x^4-13*x^2+36),x)
```

output
```
(2*log(x - 3) - 3*log(x - 2) - 2*log(x + 3) + 3*log(x + 2))/60
```

3.17 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x) \log(\log(x))$$

output
```
-ln(x)+ln(x)*ln(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x) \log(\log(x))$$

input
```
Integrate[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x))}{x} \, dx$$

$$\Big\downarrow \text{ 3001}$$

$$\log(x)\log(\log(x)) - \log(x)$$

input
```
Int[Log[Log[x]]/x,x]
```

output
```
-Log[x] + Log[x]*Log[Log[x]]
```

Defintions of rubi rules used

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\ln(x) + \ln(x)\ln(\ln(x))$	12
default	$-\ln(x) + \ln(x)\ln(\ln(x))$	12
norman	$-\ln(x) + \ln(x)\ln(\ln(x))$	12
risch	$-\ln(x) + \ln(x)\ln(\ln(x))$	12

input
```
int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x)*ln(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="fricas")
```

output
```
log(x)*log(log(x)) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(ln(ln(x))/x,x)
```

output
```
log(x)*log(log(x)) - log(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="maxima")
```

output
```
log(x)*log(log(x)) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \log(\log(x)) - \log(x)$$

input
```
integrate(log(log(x))/x,x, algorithm="giac")
```

output
```
log(x)*log(log(x)) - log(x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x} \, dx = \ln(x) \, (\ln(\ln(x)) - 1)$$

input
```
int(log(log(x))/x,x)
```

output
```
log(x)*(log(log(x)) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\log(\log(x))}{x} \, dx = \log(x) \left(\log(\log(x)) - 1\right)$$

input
```
int(log(log(x))/x,x)
```

output
```
log(x)*(log(log(x)) - 1)
```

3.18 $\int \frac{1+\cot(x)}{1-\cot(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = \log(\cos(x)-\sin(x))$$

output
```
ln(cos(x)-sin(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = \log(\cos(x)-\sin(x))$$

input
```
Integrate[(1 + Cot[x])/(1 - Cot[x]),x]
```

output
```
Log[Cos[x] - Sin[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)+1}{1-\cot(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\tan\left(x+\frac{\pi}{2}\right)}{\tan\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{4013}$$

$$\log(\cos(x)-\sin(x))$$

input
```
Int[(1 + Cot[x])/(1 - Cot[x]),x]
```

output
```
Log[Cos[x] - Sin[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

method	result	size
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec(x)^2}}\right)+\ln(\tan(x)-1)$	14
risch	$-ix+\ln(e^{2ix}-i)$	15
derivativedivides	$-\frac{\ln(1+\cot(x)^2)}{2}+\ln(-1+\cot(x))$	16
default	$-\frac{\ln(1+\cot(x)^2)}{2}+\ln(-1+\cot(x))$	16
norman	$-\frac{\ln(1+\tan(x)^2)}{2}+\ln(\tan(x)-1)$	16

input
```
int((1+cot(x))/(1-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1/(sec(x)^2)^(1/2))+ln(tan(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = \frac{1}{2}\log(-\sin(2x)+1)$$

input
```
integrate((1+cot(x))/(1-cot(x)),x, algorithm="fricas")
```

output
```
1/2*log(-sin(2*x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = \log\left(\tan\left(x\right)-1\right) - \frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

input
```
integrate((1+cot(x))/(1-cot(x)),x)
```

output
```
log(tan(x) - 1) - log(tan(x)**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = -\frac{1}{2}\log\left(\tan\left(x\right)^{2}+1\right) + \log\left(\tan\left(x\right)-1\right)$$

input
```
integrate((1+cot(x))/(1-cot(x)),x, algorithm="maxima")
```

output
```
-1/2*log(tan(x)^2 + 1) + log(tan(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = -\frac{1}{2}\log\left(\tan\left(x\right)^{2}+1\right) + \log\left(\left|\tan\left(x\right)-1\right|\right)$$

input
```
integrate((1+cot(x))/(1-cot(x)),x, algorithm="giac")
```

output
```
-1/2*log(tan(x)^2 + 1) + log(abs(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x\,2\mathrm{i}} - \mathrm{i}\right)$$

input
```
int(-(cot(x) + 1)/(cot(x) - 1),x)
```

output
```
log(exp(x*2i) - 1i) - x*1i
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 4.00

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,dx = -\log\left(\tan\left(\frac{x}{2}\right)^2+1\right) + \log\left(-\sqrt{2}+\tan\left(\frac{x}{2}\right)+1\right) + \log\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int((1+cot(x))/(1-cot(x)),x)
```

output
```
 - log(tan(x/2)**2 + 1) + log( - sqrt(2) + tan(x/2) + 1) + log(sqrt(2) + t
an(x/2) + 1)
```

3.19 $\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))} dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))} dx = -\log\left(1+\frac{\cos(x)}{x}\right)$$

output
```
-ln(1+cos(x)/x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))} dx = \log(x)-\log(x+\cos(x))$$

input
```
Integrate[(Cos[x] + x*Sin[x])/(x*(x + Cos[x])),x]
```

output
```
Log[x] - Log[x + Cos[x]]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7263, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(x)+\cos(x)}{x(x+\cos(x))}dx$$

$$\Big\downarrow \text{ 7263}$$

$$-\int \frac{1}{\frac{\cos(x)}{x}+1}d\frac{\cos(x)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$-\log\left(\frac{\cos(x)}{x}+1\right)$$

input
```
Int[(Cos[x] + x*Sin[x])/(x*(x + Cos[x])),x]
```

output
```
-Log[1 + Cos[x]/x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 7263
```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Simp[(-c)*q   Subst[
Int[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{
a, b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Inte
gerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

method	result	size
risch	$ix+\ln(x)-\ln\left(2\,\mathrm{e}^{ix}x+\mathrm{e}^{2ix}+1\right)$	26
parallelrisch	$\ln(x)-\ln\left(2+\sec\left(\frac{x}{2}\right)^{2}(x-1)\right)+\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)$	26
norman	$-\ln\left(x\tan\left(\frac{x}{2}\right)^{2}-\tan\left(\frac{x}{2}\right)^{2}+x+1\right)+\ln(x)+\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$	35

input
```
int((cos(x)+x*sin(x))/x/(x+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x+ln(x)-ln(2*exp(I*x)*x+exp(2*I*x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int\frac{\cos(x)+x\sin(x)}{x(x+\cos(x))}\,dx=-\log\left(x+\cos\left(x\right)\right)+\log\left(x\right)$$

input
```
integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="fricas")
```

output
```
-log(x + cos(x)) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int\frac{\cos(x)+x\sin(x)}{x(x+\cos(x))}\,dx=\log\left(x\right)-\log\left(x+\cos\left(x\right)\right)$$

input
```
integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x)
```

output
```
log(x) - log(x + cos(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(11) = 22.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 5.91

$$\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))}\,dx = -\frac{1}{2}\log\left(4\,x^2\cos\left(x\right)^2+4\,x^2\sin\left(x\right)^2+4\,x\sin\left(2\,x\right)\sin\left(x\right)+2\left(2\,x\cos\left(x\right)+1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^2+4\,x\cos\left(x\right)+\sin\left(2\,x\right)^2+1\right)+\log\left(x\right)$$

input
```
integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="maxima")
```

output
```
-1/2*log(4*x^2*cos(x)^2 + 4*x^2*sin(x)^2 + 4*x*sin(2*x)*sin(x) + 2*(2*x*co
s(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*x*cos(x) + sin(2*x)^2 + 1) + log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 7.18

$$\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))}\,dx = -\frac{1}{2}\log\left(\frac{4\left(x^2\tan\left(\frac{1}{2}\,x\right)^4-2\,x\tan\left(\frac{1}{2}\,x\right)^4+2\,x^2\tan\left(\frac{1}{2}\,x\right)^2+\tan\left(\frac{1}{2}\,x\right)^4+x^2-2\,\tan\left(\frac{1}{2}\,x\right)^2+2\,x+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)+\log\left(|x|\right)$$

input
```
integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="giac")
```

output
```
-1/2*log(4*(x^2*tan(1/2*x)^4 - 2*x*tan(1/2*x)^4 + 2*x^2*tan(1/2*x)^2 + tan
(1/2*x)^4 + x^2 - 2*tan(1/2*x)^2 + 2*x + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2
 + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x) + x\sin(x)}{x(x + \cos(x))}\, dx = \ln(x) - \ln(x + \cos(x))$$

input
```
int((cos(x) + x*sin(x))/(x*(x + cos(x))),x)
```

output
```
log(x) - log(x + cos(x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x) + x\sin(x)}{x(x + \cos(x))}\, dx = -\log(\cos(x) + x) + \log(x)$$

input
```
int((cos(x)+x*sin(x))/x/(x+cos(x)),x)
```

output
```
- log(cos(x) + x) + log(x)
```

3.20 $\int \frac{1}{\sec(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \frac{1}{\sec(x)+\sin(x)}\,dx = \arctan(\cos(x)+\sin(x)) - \frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
arctan(cos(x)+sin(x))-1/3*arctanh(1/3*(cos(x)-sin(x))*3^(1/2))*3^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

Time = 0.82 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\begin{aligned}
&\int \frac{1}{\sec(x)+\sin(x)}\,dx \\
&= \arctan\left(1-\left(-1+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)+\arctan\left(1+\left(1+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right) \\
&\quad+\frac{-\log\left(\sec^2\left(\frac{x}{2}\right)\left(\sqrt{3}+\cos(x)-\sin(x)\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{3}-\cos(x)+\sin(x)\right)\right)}{2\sqrt{3}}
\end{aligned}$$

input
```
Integrate[(Sec[x] + Sin[x])^(-1),x]
```

output
```
ArcTan[1 - (-1 + Sqrt[3])*Tan[x/2]] + ArcTan[1 + (1 + Sqrt[3])*Tan[x/2]] +
 (-Log[Sec[x/2]^2*(Sqrt[3] + Cos[x] - Sin[x])] + Log[-(Sec[x/2]^2*(Sqrt[3]
 - Cos[x] + Sin[x]))])/(2*Sqrt[3])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4902, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\sec(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(x)+\sec(x)}dx$$

$$\Big\downarrow \; 4902$$

$$2\int \frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-2\tan^3\left(\frac{x}{2}\right)+2\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 2492$$

$$2\int \left(\frac{3i-\sqrt{3}}{6\left(-i\tan^2\left(\frac{x}{2}\right)+\left(i+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+i\right)}+\frac{3i+\sqrt{3}}{6\left(-i\tan^2\left(\frac{x}{2}\right)+\left(i-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)+i\right)}\right)d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{6}\left(3+i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)-\sqrt{3}+i}{\sqrt{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{6}\left(3-i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)+\sqrt{3}+i}{\sqrt{2\left(1-i\sqrt{3}\right)}}\right)\right)$$

input
```
Int[(Sec[x] + Sin[x])^(-1),x]
```

output
```
2*(((3 + I*Sqrt[3])*ArcTan[(I - Sqrt[3] - (2*I)*Tan[x/2])/Sqrt[2*(1 + I*Sq
rt[3])]])/6 - ((3 - I*Sqrt[3])*ArcTan[(I + Sqrt[3] - (2*I)*Tan[x/2])/Sqrt[
2*(1 - I*Sqrt[3])]])/6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

method	result
default	$\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-2\_Z^{3}+2\_Z^{2}+2\_Z+1\right)}\frac{\left(-\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{2\_R^{3}-3\_R^{2}+2\_R+1}$
risch	$\frac{i\ln\left(e^{ix}-\frac{1}{2}+\frac{i}{2}+\frac{i\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)}{2}+\frac{\ln\left(e^{ix}-\frac{1}{2}+\frac{i}{2}+\frac{i\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{i\ln\left(e^{ix}-\frac{1}{2}+\frac{i}{2}-\frac{i\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)}{2}-\frac{\ln\left(e^{ix}-\frac{1}{2}+\frac{i}{2}-\frac{i\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\sqrt{3}}{6}-$

input
```
int(1/(sec(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
sum((-_R^2+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z
^3+2*_Z^2+2*_Z+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.75

$$\begin{aligned}\int \frac{1}{\sec(x)+\sin(x)}\,dx =& -\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}+\cos(x)\right)\sin(x)-\sqrt{3}\cos(x)-2\right)\\&+\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}-\cos(x)\right)\sin(x)-\sqrt{3}\cos(x)+2\right)\\&-\frac{1}{2}\arctan\left(-\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)+2}{\cos(x)+\sin(x)}\right)\\&+\frac{1}{2}\arctan\left(-\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)-2}{\cos(x)+\sin(x)}\right)\end{aligned}$$

input
```
integrate(1/(sec(x)+sin(x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*log((sqrt(3) + cos(x))*sin(x) - sqrt(3)*cos(x) - 2) + 1/12*s
qrt(3)*log((sqrt(3) - cos(x))*sin(x) - sqrt(3)*cos(x) + 2) - 1/2*arctan(-(
sqrt(3)*cos(x) - sqrt(3)*sin(x) + 2)/(cos(x) + sin(x))) + 1/2*arctan(-(sqr
t(3)*cos(x) - sqrt(3)*sin(x) - 2)/(cos(x) + sin(x)))
```

Sympy [F]

$$\int \frac{1}{\sec(x) + \sin(x)}\, dx = \int \frac{1}{\sin(x) + \sec(x)}\, dx$$

input
```
integrate(1/(sec(x)+sin(x)),x)
```

output
```
Integral(1/(sin(x) + sec(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sec(x) + \sin(x)}\, dx = \int \frac{1}{\sec(x) + \sin(x)}\, dx$$

input
```
integrate(1/(sec(x)+sin(x)),x, algorithm="maxima")
```

output
```
integrate(1/(sec(x) + sin(x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sec(x)+\sin(x)}\,dx = \frac{1}{2}\,\pi + \frac{1}{6}\,\sqrt{3}\log\left(\left(\sqrt{3}+\tan\left(\frac{1}{2}\,x\right)-1\right)^2+\tan\left(\frac{1}{2}\,x\right)^2\right) - \frac{1}{6}\,\sqrt{3}\log\left(\left(\sqrt{3}-\tan\left(\frac{1}{2}\,x\right)+1\right)^2+\tan\left(\frac{1}{2}\,x\right)^2\right) + \arctan\left(\left(\sqrt{3}+1\right)\tan\left(\frac{1}{2}\,x\right)+1\right) + \arctan\left(-\left(\sqrt{3}-1\right)\tan\left(\frac{1}{2}\,x\right)+1\right)$$

input

```
integrate(1/(sec(x)+sin(x)),x, algorithm="giac")
```

output

```
1/2*pi + 1/6*sqrt(3)*log((sqrt(3) + tan(1/2*x) - 1)^2 + tan(1/2*x)^2) - 1/
6*sqrt(3)*log((sqrt(3) - tan(1/2*x) + 1)^2 + tan(1/2*x)^2) + arctan((sqrt(
3) + 1)*tan(1/2*x) + 1) + arctan(-(sqrt(3) - 1)*tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 8.32

$$\int \frac{1}{\sec(x)+\sin(x)}\,dx = -\text{atan}\left(\frac{96\tan\left(\frac{x}{2}\right)}{64\tan\left(\frac{x}{2}\right)+64-\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} + \frac{\sqrt{3}\,32\text{i}}{64\tan\left(\frac{x}{2}\right)+64-\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} + \frac{32}{64\tan\left(\frac{x}{2}\right)+64-\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} + \frac{\sqrt{3}\tan\left(\frac{x}{2}\right)\,32\text{i}}{64\tan\left(\frac{x}{2}\right)+64-\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}}\right)\left(-1+\frac{\sqrt{3}\,1\text{i}}{3}\right) - \text{atan}\left(-\frac{96\tan\left(\frac{x}{2}\right)}{64\tan\left(\frac{x}{2}\right)+64+\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} + \frac{\sqrt{3}\,32\text{i}}{64\tan\left(\frac{x}{2}\right)+64+\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} - \frac{32}{64\tan\left(\frac{x}{2}\right)+64+\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}} + \frac{\sqrt{3}\tan\left(\frac{x}{2}\right)\,32\text{i}}{64\tan\left(\frac{x}{2}\right)+64+\sqrt{3}\tan\left(\frac{x}{2}\right)\,64\text{i}}\right)\left(1+\frac{\sqrt{3}\,1\text{i}}{3}\right)$$

input
```
int(1/(sin(x) + 1/cos(x)),x)
```

output
```
- atan((96*tan(x/2))/(64*tan(x/2) - 3^(1/2)*tan(x/2)*64i + 64) + (3^(1/2)*
32i)/(64*tan(x/2) - 3^(1/2)*tan(x/2)*64i + 64) + 32/(64*tan(x/2) - 3^(1/2)
*tan(x/2)*64i + 64) + (3^(1/2)*tan(x/2)*32i)/(64*tan(x/2) - 3^(1/2)*tan(x/
2)*64i + 64))*((3^(1/2)*1i)/3 - 1) - atan((3^(1/2)*32i)/(64*tan(x/2) + 3^(
1/2)*tan(x/2)*64i + 64) - (96*tan(x/2))/(64*tan(x/2) + 3^(1/2)*tan(x/2)*64
i + 64) - 32/(64*tan(x/2) + 3^(1/2)*tan(x/2)*64i + 64) + (3^(1/2)*tan(x/2)
*32i)/(64*tan(x/2) + 3^(1/2)*tan(x/2)*64i + 64))*((3^(1/2)*1i)/3 + 1)
```

Reduce [F]

$$\int \frac{1}{\sec(x)+\sin(x)}\,dx=\int \frac{1}{\sec{(x)}+\sin{(x)}}dx$$

input
```
int(1/(sec(x)+sin(x)),x)
```

output
```
int(1/(sec(x) + sin(x)),x)
```

3.21 $\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = -\operatorname{arctanh}\left(\frac{2+e^x}{2\sqrt{1+e^x+e^{2x}}}\right)$$

output
```
-arctanh(1/2*(2+exp(x))/(1+exp(x)+exp(2*x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = 2\operatorname{arctanh}\left(e^x-\sqrt{1+e^x+e^{2x}}\right)$$

input
```
Integrate[1/Sqrt[1 + E^x + E^(2*x)],x]
```

output
```
2*ArcTanh[E^x - Sqrt[1 + E^x + E^(2*x)]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2720, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e^x+e^{2x}+1}}dx$$
$$\Big\downarrow \ 2720$$
$$\int \frac{e^{-x}}{\sqrt{e^x+e^{2x}+1}}de^x$$
$$\Big\downarrow \ 1154$$
$$-2\int \frac{1}{4-e^{2x}}d\frac{2+e^x}{\sqrt{1+e^x+e^{2x}}}$$
$$\Big\downarrow \ 219$$
$$-\text{arctanh}\left(\frac{e^x+2}{2\sqrt{e^x+e^{2x}+1}}\right)$$

input
```
Int[1/Sqrt[1 + E^x + E^(2*x)],x]
```

output
```
-ArcTanh[(2 + E^x)/(2*Sqrt[1 + E^x + E^(2*x)])]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

method	result	size
default	$-\operatorname{arctanh}\left(\frac{2+{\mathrm e}^{x}}{2 \sqrt{1+{\mathrm e}^{x}+{\mathrm e}^{2 x}}}\right)$	20

input
```
int(1/(1+exp(x)+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/2*(2+exp(x))/(1+exp(x)+exp(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = -\log\left(\sqrt{e^{(2\,x)}+e^x+1}-e^x+1\right)+\log\left(\sqrt{e^{(2\,x)}+e^x+1}-e^x-1\right)$$

input
```
integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(e^(2*x) + e^x + 1) - e^x + 1) + log(sqrt(e^(2*x) + e^x + 1) - e^
x - 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = \int \frac{1}{\sqrt{e^{2x}+e^x+1}}\,dx$$

input
```
integrate(1/(1+exp(x)+exp(2*x))**(1/2),x)
```

output
```
Integral(1/sqrt(exp(2*x) + exp(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = -\operatorname{arsinh}\left(\frac{2}{3}\sqrt{3}e^{(-x)} + \frac{1}{3}\sqrt{3}\right)$$

input
```
integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(2/3*sqrt(3)*e^(-x) + 1/3*sqrt(3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = -\log\left(\sqrt{e^{(2x)}+e^x+1}-e^x+1\right)+\log\left(-\sqrt{e^{(2x)}+e^x+1}+e^x+1\right)$$

input
```
integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(e^(2*x) + e^x + 1) - e^x + 1) + log(-sqrt(e^(2*x) + e^x + 1) + e
^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = x-\ln\left(\frac{\mathrm{e}^x}{2}+\sqrt{\mathrm{e}^{2\,x}+\mathrm{e}^x+1}+1\right)$$

input
```
int(1/(exp(2*x) + exp(x) + 1)^(1/2),x)
```

output
```
x - log(exp(x)/2 + (exp(2*x) + exp(x) + 1)^(1/2) + 1)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx = \int \frac{\sqrt{e^{2x}+e^x+1}}{e^{2x}+e^x+1}dx$$

input
```
int(1/(1+exp(x)+exp(2*x))^(1/2),x)
```

output
```
int(sqrt(e**(2*x) + e**x + 1)/(e**(2*x) + e**x + 1),x)
```

3.22 $\int e^{x^2} x^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int e^{x^2} x^3 \, dx = -\frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2$$

output
```
-1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} e^{x^2} \left(-1 + x^2\right)$$

input
```
Integrate[E^x^2*x^3,x]
```

output
```
(E^x^2*(-1 + x^2))/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x^3 \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2} e^{x^2} x^2 - \int e^{x^2} x dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{2} e^{x^2} x^2 - \frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x^3,x]
```

output
```
-1/2*E^x^2 + (E^x^2*x^2)/2
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(x^2-1)e^{x^2}}{2}$	12
orering	$\frac{(x^2-1)e^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)e^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)e^{x^2}}{4}$	16
derivativedivides	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
default	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
norman	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parts	$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\,\mathrm{erfi}(x)}{3}-\frac{2\left(-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

input
```
int(exp(x^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2-1)*exp(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx=\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int e^{x^2} x^3 \, dx = \frac{(x^2 - 1) e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x**3,x)
```

output
```
(x**2 - 1)*exp(x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} (x^2 - 1) e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} (x^2 - 1) e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx = \frac{\mathrm{e}^{x^2}\,(x^2-1)}{2}$$

input
```
int(x^3*exp(x^2),x)
```

output
```
(exp(x^2)*(x^2 - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int e^{x^2}x^3\,dx = \frac{e^{x^2}(x^2-1)}{2}$$

input
```
int(exp(x^2)*x^3,x)
```

output
```
(e**(x**2)*(x**2 - 1))/2
```

3.23 $\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) \, dx=\frac{x}{\log(x)}$$

output
```
x/ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) \, dx=\frac{x}{\log(x)}$$

input
```
Integrate[-Log[x]^(-2) + Log[x]^(-1),x]
```

output
```
x/Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\log(x)}-\frac{1}{\log^2(x)}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x}{\log(x)}$$

input
```
Int[-Log[x]^(-2) + Log[x]^(-1),x]
```

output
```
x/Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{x}{\ln(x)}$	7
norman	$\frac{x}{\ln(x)}$	7
risch	$\frac{x}{\ln(x)}$	7
parallelrisch	$\frac{x}{\ln(x)}$	7
parts	$\frac{x}{\ln(x)}$	7

input
```
int(-1/ln(x)^2+1/ln(x),x,method=_RETURNVERBOSE)
```

output
```
x/ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) dx = \frac{x}{\log(x)}$$

input
```
integrate(-1/log(x)^2+1/log(x),x, algorithm="fricas")
```

output
```
x/log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) dx = \frac{x}{\log(x)}$$

input
```
integrate(-1/ln(x)**2+1/ln(x),x)
```

output
```
x/log(x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right)\,dx=\mathrm{Ei}(\log{(x)})-\Gamma(-1,-\log{(x)})$$

input
```
integrate(-1/log(x)^2+1/log(x),x, algorithm="maxima")
```

output
```
Ei(log(x)) - gamma(-1, -log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right)\,dx=\frac{x}{\log{(x)}}$$

input
```
integrate(-1/log(x)^2+1/log(x),x, algorithm="giac")
```

output
```
x/log(x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right)\,dx=\frac{x}{\ln{(x)}}$$

input
```
int(1/log(x) - 1/log(x)^2,x)
```

output
```
x/log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) dx = \frac{x}{\log(x)}$$

input
```
int(-1/log(x)^2+1/log(x),x)
```

output
```
x/log(x)
```

3.24 $\int \sqrt{2-x}\sqrt{-1+x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx=\frac{1}{4}\sqrt{2-x}\sqrt{-1+x}-\frac{1}{2}(2-x)^{3/2}\sqrt{-1+x}+\frac{1}{4}\arcsin\left(\sqrt{-1+x}\right)$$

output
```
1/4*(2-x)^(1/2)*(-1+x)^(1/2)-1/2*(2-x)^(3/2)*(-1+x)^(1/2)+1/4*arcsin((-1+x
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx=\frac{1}{4}\sqrt{-2+3x-x^2}\left(-3+2x-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{\frac{-2+x}{-1+x}}}\right)}{\sqrt{-2+x}\sqrt{-1+x}}\right)$$

input
```
Integrate[Sqrt[2 - x]*Sqrt[-1 + x],x]
```

output
```
(Sqrt[-2 + 3*x - x^2]*(-3 + 2*x - ArcTanh[1/Sqrt[(-2 + x)/(-1 + x)]]/(Sqrt
[-2 + x]*Sqrt[-1 + x])))/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-x}\sqrt{x-1}\,dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\int \frac{\sqrt{2-x}}{\sqrt{x-1}}dx-\frac{1}{2}(2-x)^{3/2}\sqrt{x-1}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-x}\sqrt{x-1}}dx+\sqrt{2-x}\sqrt{x-1}\right)-\frac{1}{2}(2-x)^{3/2}\sqrt{x-1}$$

$$\Big\downarrow \text{62}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{-x^2+3x-2}}dx+\sqrt{2-x}\sqrt{x-1}\right)-\frac{1}{2}(2-x)^{3/2}\sqrt{x-1}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\left(\sqrt{2-x}\sqrt{x-1}-\frac{1}{2}\int \frac{1}{\sqrt{1-(3-2x)^2}}d(3-2x)\right)-\frac{1}{2}(2-x)^{3/2}\sqrt{x-1}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}\left(\sqrt{2-x}\sqrt{x-1}-\frac{1}{2}\arcsin(3-2x)\right)-\frac{1}{2}(2-x)^{3/2}\sqrt{x-1}$$

input
```
Int[Sqrt[2 - x]*Sqrt[-1 + x],x]
```

output
```
-1/2*((2 - x)^(3/2)*Sqrt[-1 + x]) + (Sqrt[2 - x]*Sqrt[-1 + x] - ArcSin[3 -
 2*x]/2)/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{2-x}\,(x-1)^{\frac{3}{2}}}{2}-\frac{\sqrt{2-x}\,\sqrt{x-1}}{4}+\frac{\sqrt{(x-1)(2-x)}\,\arcsin(-3+2x)}{8\sqrt{x-1}\,\sqrt{2-x}}$	61
risch	$-\frac{(-3+2x)(x-2)\sqrt{x-1}\,\sqrt{(x-1)(2-x)}}{4\sqrt{-(x-2)(x-1)}\,\sqrt{2-x}}+\frac{\sqrt{(x-1)(2-x)}\,\arcsin(-3+2x)}{8\sqrt{x-1}\,\sqrt{2-x}}$	76

input
```
int((2-x)^(1/2)*(x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2-x)^(1/2)*(x-1)^(3/2)-1/4*(2-x)^(1/2)*(x-1)^(1/2)+1/8*((x-1)*(2-x))^
(1/2)/(x-1)^(1/2)/(2-x)^(1/2)*arcsin(-3+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = \frac{1}{4}\,(2\,x-3)\sqrt{x-1}\sqrt{-x+2} - \frac{1}{8}\,\arctan\left(\frac{(2\,x-3)\sqrt{x-1}\sqrt{-x+2}}{2\,(x^2-3\,x+2)}\right)$$

input
```
integrate((2-x)^(1/2)*(-1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*x - 3)*sqrt(x - 1)*sqrt(-x + 2) - 1/8*arctan(1/2*(2*x - 3)*sqrt(x -
 1)*sqrt(-x + 2)/(x^2 - 3*x + 2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = \begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{x-1}\right)}{4} + \frac{i(x-1)^{\frac{5}{2}}}{2\sqrt{x-2}} - \frac{3i(x-1)^{\frac{3}{2}}}{4\sqrt{x-2}} + \frac{i\sqrt{x-1}}{4\sqrt{x-2}} & \text{for } |x-1| > 1 \\ \frac{\operatorname{asin}\left(\sqrt{x-1}\right)}{4} - \frac{(x-1)^{\frac{5}{2}}}{2\sqrt{2-x}} + \frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{2-x}} - \frac{\sqrt{x-1}}{4\sqrt{2-x}} & \text{otherwise} \end{cases}$$

input
```
integrate((2-x)**(1/2)*(-1+x)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(x - 1))/4 + I*(x - 1)**(5/2)/(2*sqrt(x - 2)) - 3*
I*(x - 1)**(3/2)/(4*sqrt(x - 2)) + I*sqrt(x - 1)/(4*sqrt(x - 2)), Abs(x -
1) > 1), (asin(sqrt(x - 1))/4 - (x - 1)**(5/2)/(2*sqrt(2 - x)) + 3*(x - 1)
**(3/2)/(4*sqrt(2 - x)) - sqrt(x - 1)/(4*sqrt(2 - x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = \frac{1}{2}\sqrt{-x^2+3x-2}x - \frac{3}{4}\sqrt{-x^2+3x-2} + \frac{1}{8}\arcsin(2x-3)$$

input
```
integrate((2-x)^(1/2)*(-1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 3*x - 2)*x - 3/4*sqrt(-x^2 + 3*x - 2) + 1/8*arcsin(2*x - 3
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = \frac{1}{4}(2x+1)\sqrt{x-1}\sqrt{-x+2} - \sqrt{x-1}\sqrt{-x+2} + \frac{1}{4}\arcsin\left(\sqrt{x-1}\right)$$

input
```
integrate((2-x)^(1/2)*(-1+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*x + 1)*sqrt(x - 1)*sqrt(-x + 2) - sqrt(x - 1)*sqrt(-x + 2) + 1/4*ar
csin(sqrt(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = \left(\frac{x}{2}-\frac{3}{4}\right)\sqrt{x-1}\,\sqrt{2-x} - \frac{\ln\left(x-\frac{3}{2}-\sqrt{x-1}\,\sqrt{2-x}\,1\mathrm{i}\right)\,1\mathrm{i}}{8}$$

input
```
int((x - 1)^(1/2)*(2 - x)^(1/2),x)
```

output
```
(x/2 - 3/4)*(x - 1)^(1/2)*(2 - x)^(1/2) - (log(x - (x - 1)^(1/2)*(2 - x)^(
1/2)*1i - 3/2)*1i)/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \sqrt{2-x}\sqrt{-1+x}\,dx = -\frac{asin\left(\sqrt{-x+2}\right)}{4} + \frac{\sqrt{x-1}\,\sqrt{-x+2}\,x}{2} - \frac{3\sqrt{x-1}\,\sqrt{-x+2}}{4}$$

input
```
int((2-x)^(1/2)*(-1+x)^(1/2),x)
```

output
```
( - asin(sqrt( - x + 2)) + 2*sqrt(x - 1)*sqrt( - x + 2)*x - 3*sqrt(x - 1)*
sqrt( - x + 2))/4
```

3.25 $\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx = x-\frac{x^2}{2}+\frac{x^3}{3}$$

output
```
x-1/2*x^2+1/3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx = x-\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
Integrate[(-1 + x^6)/(-1 - x + x^3 + x^4),x]
```

output
```
x - x^2/2 + x^3/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6-1}{x^4+x^3-x-1} \, dx$$
$$\Big\downarrow \text{ 2019}$$
$$\int \left(x^2-x+1\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x^3}{3}-\frac{x^2}{2}+x$$

input
```
Int[(-1 + x^6)/(-1 - x + x^3 + x^4),x]
```

output
```
x - x^2/2 + x^3/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
norman	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
risch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parallelrisch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parts	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
gosper	$\frac{x\left(2x^{2}-3x+6\right)}{6}$	14
orering	$\frac{x\left(2x^{2}-3x+6\right)\left(x^{6}-1\right)}{6\left(x^{2}-x+1\right)\left(x^{4}+x^{3}-x-1\right)}$	42

input
```
int((x^6-1)/(x^4+x^3-x-1),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2+1/3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^6}{-1-x+x^3+x^4}\,dx=\frac{1}{3}\,x^{3}-\frac{1}{2}\,x^{2}+x$$

input
```
integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="fricas")
```

output
```
1/3*x^3 - 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx = \frac{x^3}{3} - \frac{x^2}{2} + x$$

input
```
integrate((x**6-1)/(x**4+x**3-x-1),x)
```

output
```
x**3/3 - x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="maxima")
```

output
```
1/3*x^3 - 1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="giac")
```

output
```
1/3*x^3 - 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^6}{-1-x+x^3+x^4}\,dx=\frac{x\,(2\,x^2-3\,x+6)}{6}$$

input
```
int(-(x^6 - 1)/(x - x^3 - x^4 + 1),x)
```

output
```
(x*(2*x^2 - 3*x + 6))/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^6}{-1-x+x^3+x^4}\,dx=\frac{x(2x^2-3x+6)}{6}$$

input
```
int((x^6-1)/(x^4+x^3-x-1),x)
```

output
```
(x*(2*x**2 - 3*x + 6))/6
```

3.26 $\int \left(2 \log(x) + \log^2(x)\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \left(2 \log(x) + \log^2(x)\right) \, dx = x \log^2(x)$$

output
```
x*ln(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2 \log(x) + \log^2(x)\right) \, dx = x \log^2(x)$$

input
```
Integrate[2*Log[x] + Log[x]^2,x]
```

output
```
x*Log[x]^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\log^2(x)+2\log(x)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$x\log^2(x)$$

input
```
Int[2*Log[x] + Log[x]^2,x]
```

output
```
x*Log[x]^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x\ln\left(x\right)^{2}$	7
norman	$x\ln\left(x\right)^{2}$	7
risch	$x\ln\left(x\right)^{2}$	7
parallelrisch	$x\ln\left(x\right)^{2}$	7
parts	$x\ln\left(x\right)^{2}$	7
orering	$\left(2\ln\left(x\right)+\ln\left(x\right)^{2}\right)x-2\ln\left(x\right)x$	18

input
```
int(2*ln(x)+ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x) + \log^2(x)\right) \, dx = x \log{(x)}^2$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="fricas")
```

output
```
x*log(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \left(2\log(x) + \log^2(x)\right) \, dx = x \log{(x)}^2$$

input
```
integrate(2*ln(x)+ln(x)**2,x)
```

output
```
x*log(x)**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(6) = 12.

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.50

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = \left(\log\left(x\right)^2-2\,\log\left(x\right)+2\right)x+2\,x\log\left(x\right)-2\,x$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="maxima")
```

output
```
(log(x)^2 - 2*log(x) + 2)*x + 2*x*log(x) - 2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\log\left(x\right)^2$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="giac")
```

output
```
x*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\ln\left(x\right)^2$$

input
```
int(2*log(x) + log(x)^2,x)
```

output
```
x*log(x)^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2 \log(x) + \log^2(x)\right)\, dx = \log(x)^2 x$$

input
```
int(2*log(x)+log(x)^2,x)
```

output
```
log(x)**2*x
```

3.27 $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 4

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = \arcsin\left(x^2\right)$$

output
```
arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = \arcsin\left(x^2\right)$$

input
```
Integrate[(2*x)/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {27, 807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \; 27$$

$$2\int \frac{x}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \; 807$$

$$\int \frac{1}{\sqrt{1-x^4}}dx^2$$

$$\Big\downarrow \; 223$$

$$\arcsin\left(x^2\right)$$

input
```
Int[(2*x)/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\arcsin(x^2)$	5
meijerg	$\arcsin(x^2)$	5
elliptic	$\arcsin(x^2)$	5
pseudoelliptic	$\arcsin(x^2)$	5
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^4+1}+x^2\right)$	29

input
```
int(2*x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(4) = 8$.

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 4.50

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = -2\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input
```
integrate(2*x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = 2\left(\begin{cases} -\frac{i\operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{\operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2*x/(-x**4+1)**(1/2),x)
```

output
```
2*Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(4) = 8.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(2*x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-arctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = \arcsin\left(x^2\right)$$

input
```
integrate(2*x/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
arcsin(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = \operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)$$

input
```
int((2*x)/(1 - x^4)^(1/2),x)
```

output
```
atan(x^2/(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2x}{\sqrt{1-x^4}}\,dx = asin(x^2)$$

input
```
int(2*x/(-x^4+1)^(1/2),x)
```

output
```
asin(x**2)
```

3.28 $\int \frac{1+x^2}{1+x} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{1+x} dx = -x + \frac{x^2}{2} + 2\log(1+x)$$

output
```
-x+1/2*x^2+2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{1+x} dx = \frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

input
```
Integrate[(1 + x^2)/(1 + x),x]
```

output
```
(-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x+1} \, dx$$

$$\Big\downarrow \text{ 476}$$

$$\int \left(x+\frac{2}{x+1}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}-x+2\log(x+1)$$

input
```
Int[(1 + x^2)/(1 + x),x]
```

output
```
-x + x^2/2 + 2*Log[1 + x]
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\frac{x^{2}}{2}+2\ln\left(x+1\right)$	16
norman	$-x+\frac{x^{2}}{2}+2\ln\left(x+1\right)$	16
meijerg	$-\frac{x(-3x+6)}{6}+2\ln\left(x+1\right)$	16
risch	$-x+\frac{x^{2}}{2}+2\ln\left(x+1\right)$	16
parallelrisch	$-x+\frac{x^{2}}{2}+2\ln\left(x+1\right)$	16

input
```
int((x^2+1)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+2*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx=\frac{1}{2}\,x^{2}-x+2\,\log\left(x+1\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{1+x}\,dx = \frac{x^2}{2} - x + 2\log{(x+1)}$$

input
```
integrate((x**2+1)/(1+x),x)
```

output
```
x**2/2 - x + 2*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(x+1\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(|x+1|\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="giac")
```

output
```
1/2*x^2 - x + 2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\,\ln(x+1) - x + \frac{x^2}{2}$$

input
```
int((x^2 + 1)/(x + 1),x)
```

output
```
2*log(x + 1) - x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\log(x+1) + \frac{x^2}{2} - x$$

input
```
int((x^2+1)/(1+x),x)
```

output
```
(4*log(x + 1) + x**2 - 2*x)/2
```

3.29 $\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 17

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx=-x-\cos(x)+\frac{\cos(x)}{1+\sin(x)}$$

output
```
-x-cos(x)+cos(x)/(1+sin(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx=-x-\cos(x)-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(-2 - 2*Sin[x] + Sin[x]^2 + Sin[x]^3)/(1 + 2*Sin[x] + Sin[x]^2),
x]
```

output
```
-x - Cos[x] - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)+\sin^2(x)-2\sin(x)-2}{\sin^2(x)+2\sin(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^3+\sin(x)^2-2\sin(x)-2}{\sin(x)^2+2\sin(x)+1}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\sin(x)+\frac{1}{-\sin(x)-1}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-x-\cos(x)+\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[(-2 - 2*Sin[x] + Sin[x]^2 + Sin[x]^3)/(1 + 2*Sin[x] + Sin[x]^2),x]
```

output
```
-x - Cos[x] + Cos[x]/(1 + Sin[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result
parallelrisch	$\frac{1-\cos(2x)-2x\cos(x)-2\sin(x)}{2\cos(x)}$
default	$\frac{2}{1+\tan(\frac{x}{2})}-\frac{2}{1+\tan(\frac{x}{2})^2}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$
risch	$-x-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{2}{i+e^{ix}}$
norman	$\frac{-2\tan(\frac{x}{2})-2\tan(\frac{x}{2})^2-2\tan(\frac{x}{2})^3-2\tan(\frac{x}{2})^4+2\tan(\frac{x}{2})^8+2\tan(\frac{x}{2})^7+2\tan(\frac{x}{2})^6+2\tan(\frac{x}{2})^5-x-3x\tan(\frac{x}{2})-6x\tan(\frac{x}{2})^2-}{\left(1+\tan(\frac{x}{2})^2\right)^3(1+\tan(\frac{x}{2}))^3}$

input
```
int((-2-2*sin(x)+sin(x)^2+sin(x)^3)/(1+2*sin(x)+sin(x)^2),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(1-cos(2*x)-2*x*cos(x)-2*sin(x))/cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx = -\frac{x\cos(x)+\cos(x)^2+(x+\cos(x)+1)\sin(x)+x-1}{\cos(x)+\sin(x)+1}$$

input
```
integrate((-2-2*sin(x)+sin(x)^2+sin(x)^3)/(1+2*sin(x)+sin(x)^2),x, algorit
hm="fricas")
```

output
```
-(x*cos(x) + cos(x)^2 + (x + cos(x) + 1)*sin(x) + x - 1)/(cos(x) + sin(x)
+ 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(12) = 24$.

Time = 5.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 9.53

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx = -\frac{x\tan^3\left(\frac{x}{2}\right)}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1} - \frac{x\tan^2\left(\frac{x}{2}\right)}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1} - \frac{x\tan\left(\frac{x}{2}\right)}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1} - \frac{x}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1} + \frac{2\tan^2\left(\frac{x}{2}\right)}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1} - \frac{2\tan\left(\frac{x}{2}\right)}{\tan^3\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}$$

input
```
integrate((-2-2*sin(x)+sin(x)**2+sin(x)**3)/(1+2*sin(x)+sin(x)**2),x)
```

output
```
-x*tan(x/2)**3/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - x*tan(x/2)**2/
(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - x*tan(x/2)/(tan(x/2)**3 + tan
(x/2)**2 + tan(x/2) + 1) - x/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) +
2*tan(x/2)**2/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - 2*tan(x/2)/(tan
(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 17.71

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx$$
$$= -\frac{4\left(\frac{12\sin(x)}{\cos(x)+1}+\frac{11\sin(x)^2}{(\cos(x)+1)^2}+\frac{9\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{4\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^5}{(\cos(x)+1)^5}+1\right)}$$
$$+\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+4\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}$$
$$+\frac{4\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}$$
$$+\frac{4\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate((-2-2*sin(x)+sin(x)^2+sin(x)^3)/(1+2*sin(x)+sin(x)^2),x, algorit
hm="maxima")
```

output
```
-4/3*(12*sin(x)/(cos(x) + 1) + 11*sin(x)^2/(cos(x) + 1)^2 + 9*sin(x)^3/(co
s(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)/(cos(x) + 1) + 4*si
n(x)^2/(cos(x) + 1)^2 + 4*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1
)^4 + sin(x)^5/(cos(x) + 1)^5 + 1) + 2/3*(9*sin(x)/(cos(x) + 1) + 3*sin(x)
^2/(cos(x) + 1)^2 + 4)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2
+ sin(x)^3/(cos(x) + 1)^3 + 1) + 4/3*(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(
cos(x) + 1)^2 + 2)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + si
n(x)^3/(cos(x) + 1)^3 + 1) + 4/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(co
s(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) - 2*a
rctan(sin(x)/(cos(x) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx = -x+\frac{2\left(\tan\left(\frac{1}{2}x\right)^2-\tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate((-2-2*sin(x)+sin(x)^2+sin(x)^3)/(1+2*sin(x)+sin(x)^2),x, algorit
hm="giac")
```

output
```
-x + 2*(tan(1/2*x)^2 - tan(1/2*x))/(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*
x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx = -x-\frac{2\tan\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)^2}{\left(\tan\left(\frac{x}{2}\right)^2+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

input
```
int(-(2*sin(x) - sin(x)^2 - sin(x)^3 + 2)/(2*sin(x) + sin(x)^2 + 1),x)
```

output
```
- x - (2*tan(x/2) - 2*tan(x/2)^2)/((tan(x/2)^2 + 1)*(tan(x/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx = \frac{-\cos(x)\tan\left(\frac{x}{2}\right)-\cos(x)-\tan\left(\frac{x}{2}\right)x-2\tan\left(\frac{x}{2}\right)-x}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int((-2-2*sin(x)+sin(x)^2+sin(x)^3)/(1+2*sin(x)+sin(x)^2),x)
```

output
```
( - cos(x)*tan(x/2) - cos(x) - tan(x/2)*x - 2*tan(x/2) - x)/(tan(x/2) + 1)
```

3.30 $\int \operatorname{csch}^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

output
```
-coth(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

input
```
Integrate[Csch[x]^2,x]
```

output
```
-Coth[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -\csc(ix)^2dx$$
$$\Big\downarrow \ 25$$
$$-\int \csc(ix)^2dx$$
$$\Big\downarrow \ 4254$$
$$-i\int 1d(-i\coth(x))$$
$$\Big\downarrow \ 24$$
$$-\coth(x)$$

input
```
Int[Csch[x]^2,x]
```

output
```
-Coth[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\coth\left(x\right)$	5
risch	$-\frac{2}{{\mathrm e}^{2 x}-1}$	11
parallelrisch	$-\frac{\coth\left(\frac{x}{2}\right)}{2}-\frac{\tanh\left(\frac{x}{2}\right)}{2}$	14

input
```
int(csch(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-coth(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(4) = 8.

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \operatorname{csch}^2(x)\,dx = -\frac{2}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

input
```
integrate(csch(x)^2,x, algorithm="fricas")
```

output
```
-2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \operatorname{csch}^2(x)\,dx = \int \operatorname{csch}^2(x)\,dx$$

input
```
integrate(csch(x)**2,x)
```

output
```
Integral(csch(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(4) = 8.

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^2(x)\,dx = \frac{2}{e^{(-2x)} - 1}$$

input
```
integrate(csch(x)^2,x, algorithm="maxima")
```

output
```
2/(e^(-2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(4) = 8$.

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^2(x)\,dx = -\frac{2}{e^{(2x)}-1}$$

input
```
integrate(csch(x)^2,x, algorithm="giac")
```

output
```
-2/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

input
```
int(1/sinh(x)^2,x)
```

output
```
-coth(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \operatorname{csch}^2(x)\,dx = -\frac{2e^{2x}}{e^{2x}-1}$$

input
```
int(csch(x)^2,x)
```

output
```
( - 2*e**(2*x))/(e**(2*x) - 1)
```

3.31 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^4(x)\tan^2(x)\,dx=\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

output
```
1/3*tan(x)^3+1/5*tan(x)^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sec^4(x)\tan^2(x)\,dx=-\frac{2\tan(x)}{15}-\frac{1}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)$$

input
```
Integrate[Sec[x]^4*Tan[x]^2,x]
```

output
```
(-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sec^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2\sec(x)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^2(x)\left(\tan^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^4(x)+\tan^2(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

input
```
Int[Sec[x]^4*Tan[x]^2,x]
```

output
```
Tan[x]^3/3 + Tan[x]^5/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\tan(x)^3}{3}+\frac{\tan(x)^5}{5}$	14
default	$\frac{\tan(x)^3}{3}+\frac{\tan(x)^5}{5}$	14
risch	$-\frac{4i(15\,\mathrm{e}^{6ix}-5\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1)}{15(\mathrm{e}^{2ix}+1)^5}$	36

input
```
int(sec(x)^4*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(x)^3+1/5*tan(x)^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{\left(2\,\cos\left(x\right)^4+\cos\left(x\right)^2-3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^5}$$

input
```
integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

output
```
-1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \sec^4(x)\tan^2(x)\,dx = -\frac{2\sin\left(x\right)}{15\cos\left(x\right)} - \frac{\sin\left(x\right)}{15\cos^3\left(x\right)} + \frac{\sin\left(x\right)}{5\cos^5\left(x\right)}$$

input
```
integrate(sec(x)**4*tan(x)**2,x)
```

output
```
-2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\,\tan\left(x\right)^5 + \frac{1}{3}\,\tan\left(x\right)^3$$

input
```
integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

output
```
1/5*tan(x)^5 + 1/3*tan(x)^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{1}{5}\tan\left(x\right)^5 + \frac{1}{3}\tan\left(x\right)^3$$

input
```
integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

output
```
1/5*tan(x)^5 + 1/3*tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{\tan(x)^5}{5} + \frac{\tan(x)^3}{3}$$

input
```
int(tan(x)^2/cos(x)^4,x)
```

output
```
tan(x)^3/3 + tan(x)^5/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \sec^4(x)\tan^2(x)\,dx = \frac{\sin\left(x\right)^3\left(-2\sin\left(x\right)^2+5\right)}{15\cos\left(x\right)\left(\sin\left(x\right)^4-2\sin\left(x\right)^2+1\right)}$$

input
```
int(sec(x)^4*tan(x)^2,x)
```

output
```
(sin(x)**3*( - 2*sin(x)**2 + 5))/(15*cos(x)*(sin(x)**4 - 2*sin(x)**2 + 1))
```

3.32 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

input
```
Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\csc(x)-\sin(x)}dx$$

$$\Big\downarrow \ 4897$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \ 4900$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \ 3069$$

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

input
```
Int[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$2\sqrt{\cos\left(x\right)\cot\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-ix}}{\mathrm{e}^{2ix}-1}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}$	51

input
```
int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \int \sqrt{-\sin(x) + \csc(x)}\, dx$$

input
```
integrate((csc(x)-sin(x))**(1/2),x)
```

output
```
Integral(sqrt(-sin(x) + csc(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 14.46

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \frac{\left(\left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan(\sin(x), \cos(x) - 1)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\right.\right.}{}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
(((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(
x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin
(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) -
((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(
1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)^(1/4))
```

Giac [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\int \sqrt{\csc(x)-\sin(x)}\,dx$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\frac{2\left|\cos(x)\right|}{\cos(x)\sqrt{\frac{1}{\sin(x)}}}$$

input
```
int((1/sin(x) - sin(x))^(1/2),x)
```

output
```
(2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Reduce [F]

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \int \sqrt{\csc\left(x\right) - \sin\left(x\right)}dx$$

input
```
int((csc(x)-sin(x))^(1/2),x)
```

output
```
int(sqrt(csc(x) - sin(x)),x)
```

3.33 $\int \cos^6(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 34

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)$$

output
```
5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

input
```
Integrate[Cos[x]^6,x]
```

output
```
(5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)$$

$$\Big\downarrow \ 24$$

$$\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)$$

input
```
Int[Cos[x]^6,x]
```

output
```
(Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/
2))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\tan\left(\frac{x}{2}\right)^3}{24}+\frac{15\tan\left(\frac{x}{2}\right)^5}{4}-\frac{15\tan\left(\frac{x}{2}\right)^7}{4}+\frac{5\tan\left(\frac{x}{2}\right)^9}{24}-\frac{11\tan\left(\frac{x}{2}\right)^{11}}{8}+\frac{15x\tan\left(\frac{x}{2}\right)^2}{8}+\frac{75x\tan\left(\frac{x}{2}\right)^4}{16}+\frac{25x\tan\left(\frac{x}{2}\right)^6}{4}+\frac{75x\tan\left(\frac{x}{2}\right)^8}{16}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^6}$
orering	$x\cos\left(x\right)^6+\frac{11\cos(x)^5\sin(x)}{16}+\frac{49x\left(30\cos(x)^4\sin(x)^2-6\cos(x)^6\right)}{144}+\frac{5\cos(x)^3\sin(x)^3}{6}+\frac{7x\left(360\cos(x)^2\sin(x)^4-\right.}{}$

input
```
int(cos(x)^6,x,method=_RETURNVERBOSE)
```

output
```
5/16*x+1/192*sin(6*x)+3/64*sin(4*x)+15/64*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \cos^6(x)\,dx = \frac{1}{48}\left(8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{5}{16}\,x$$

input
```
integrate(cos(x)^6,x, algorithm="fricas")
```

output
```
1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \cos^6(x)\,dx = \frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

input
```
integrate(cos(x)**6,x)
```

output
```
5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cos^6(x)\,dx = -\frac{1}{48}\sin(2x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)^6,x, algorithm="maxima")
```

output
```
-1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5}{16}\,x + \frac{1}{192}\,\sin\left(6\,x\right) + \frac{3}{64}\,\sin\left(4\,x\right) + \frac{15}{64}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cos^6(x)\,dx = \frac{5\,x}{16} + \frac{15\,\sin\left(2\,x\right)}{64} + \frac{3\,\sin\left(4\,x\right)}{64} + \frac{\sin\left(6\,x\right)}{192}$$

input
```
int(cos(x)^6,x)
```

output
```
(5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \cos^6(x)\,dx = \frac{\cos\left(x\right)\sin\left(x\right)^5}{6} - \frac{13\cos\left(x\right)\sin\left(x\right)^3}{24} + \frac{11\cos\left(x\right)\sin\left(x\right)}{16} + \frac{5x}{16}$$

input
```
int(cos(x)^6,x)
```

output
```
(8*cos(x)*sin(x)**5 - 26*cos(x)*sin(x)**3 + 33*cos(x)*sin(x) + 15*x)/48
```

3.34 $\int \frac{1}{1+2x^2+x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{1+2x^2+x^4} \, dx = \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
x/(2*x^2+2)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+2x^2+x^4} \, dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x)\right)$$

input
```
Integrate[(1 + 2*x^2 + x^4)^(-1),x]
```

output
```
(x/(1 + x^2) + ArcTan[x])/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1379, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+2x^2+1}dx$$

$$\Big\downarrow \text{1379}$$

$$\int \frac{1}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}$$

input
```
Int[(1 + 2*x^2 + x^4)^(-1),x]
```

output
```
x/(2*(1 + x^2)) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(i+x)x^2+i\ln(x-i)-i\ln(i+x)-2x}{4(x^2+1)}$	52

input
```
int(1/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x^2+x^4}\,dx=\frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**4+2*x**2+1),x)
```

output
```
x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{\operatorname{atan}(x)}{2} + \frac{x}{2\,(x^2+1)}$$

input
```
int(1/(2*x^2 + x^4 + 1),x)
```

output
```
atan(x)/2 + x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{atan(x)\,x^2 + atan(x) + x}{2x^2+2}$$

input
```
int(1/(x^4+2*x^2+1),x)
```

output
```
(atan(x)*x**2 + atan(x) + x)/(2*(x**2 + 1))
```

3.35 $\int \cos(\log(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

output
```
1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

input
```
Integrate[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(x))\,dx$$

$$\Big\downarrow \text{ 4979}$$

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

input
```
Int[Cos[Log[x]],x]
```

output
```
(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

input
```
int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(cos(ln(x))+sin(ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos\left(\log\left(x\right)\right) + \frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

input
```
integrate(cos(log(x)),x, algorithm="fricas")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x))\,dx = \frac{x\sin\left(\log\left(x\right)\right)}{2} + \frac{x\cos\left(\log\left(x\right)\right)}{2}$$

input
```
integrate(cos(ln(x)),x)
```

output
```
x*sin(log(x))/2 + x*cos(log(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x(\cos(\log(x)) + \sin(\log(x)))$$

input
```
integrate(cos(log(x)),x, algorithm="maxima")
```

output
```
1/2*x*(cos(log(x)) + sin(log(x)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

input
```
integrate(cos(log(x)),x, algorithm="giac")
```

output
```
1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x))\,dx = \frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(2^(1/2)*x*sin(pi/4 + log(x)))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x))\,dx = \frac{x(\cos(\log(x)) + \sin(\log(x)))}{2}$$

input
```
int(cos(log(x)),x)
```

output
```
(x*(cos(log(x)) + sin(log(x))))/2
```

3.36 $\int \sec(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

output
```
arctanh(sin(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sec(x)\,dx = \coth^{-1}(\sin(x))$$

input
```
Integrate[Sec[x],x]
```

output
```
ArcCoth[Sin[x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\text{arctanh}(\sin(x))$$

input
```
Int[Sec[x],x]
```

output
```
ArcTanh[Sin[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

method	result	size
lookup	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)$	7
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$-\ln\left(\mathrm{e}^{ix}-i\right)+\ln\left(i+\mathrm{e}^{ix}\right)$	22

input
```
int(sec(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\,dx = \frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x)\,dx = -\frac{\log\left(\sin\left(x\right)-1\right)}{2} + \frac{\log\left(\sin\left(x\right)+1\right)}{2}$$

input
```
integrate(sec(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

$$\int \sec(x)\,dx = \log\left(\sec\left(x\right)+\tan\left(x\right)\right)$$

input
```
integrate(sec(x),x, algorithm="maxima")
```

output
```
log(sec(x) + tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(3) = 6.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 8.33

$$\int \sec(x)\,dx = \frac{1}{4}\,\log\left(\left|\frac{1}{\sin\left(x\right)}+\sin\left(x\right)+2\right|\right) - \frac{1}{4}\,\log\left(\left|\frac{1}{\sin\left(x\right)}+\sin\left(x\right)-2\right|\right)$$

input
```
integrate(sec(x),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \sec(x)\, dx = \ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

input
```
int(1/cos(x),x)
```

output
```
log(1/cos(x)) + log(sin(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\, dx = -\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)$$

input
```
int(sec(x),x)
```

output
```
- log(tan(x/2) - 1) + log(tan(x/2) + 1)
```

3.37 $\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx = \frac{x}{6}-\frac{1}{6}\arctan\left(\frac{\cos(x)\sin(x)}{2+\cos^2(x)}\right)$$

output
```
1/6*x-1/6*arctan(cos(x)*sin(x)/(2+cos(x)^2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx = \frac{1}{6}\arctan\left(\frac{2\tan(x)}{3}\right)$$

input
```
Integrate[(9*Cos[x]^2 + 4*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(2*Tan[x])/3]/6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\sin^2(x)+9\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{4\sin(x)^2+9\cos(x)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{4\tan^2(x)+9}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{6}\arctan\left(\frac{2\tan(x)}{3}\right)$$

input
```
Int[(9*Cos[x]^2 + 4*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(2*Tan[x])/3]/6
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\arctan\left(\frac{2\tan(x)}{3}\right)}{6}$	8
risch	$\frac{i\ln\left(e^{2ix}+5\right)}{12}-\frac{i\ln\left(e^{2ix}+\frac{1}{5}\right)}{12}$	24
parallelrisch	$-\frac{i\left(\ln\left(3\tan\left(\frac{x}{2}\right)^{2}-4i\tan\left(\frac{x}{2}\right)-3\right)-\ln\left(3\tan\left(\frac{x}{2}\right)^{2}+4i\tan\left(\frac{x}{2}\right)-3\right)\right)}{12}$	43

input
```
int(1/(9*cos(x)^2+4*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(2/3*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=-\frac{1}{12}\arctan\left(\frac{13\cos(x)^2-4}{12\cos(x)\sin(x)}\right)$$

input
```
integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/12*arctan(1/12*(13*cos(x)^2 - 4)/(cos(x)*sin(x)))
```

Sympy [F]

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=\int \frac{1}{4\sin^2(x)+9\cos^2(x)}\,dx$$

input
```
integrate(1/(9*cos(x)**2+4*sin(x)**2),x)
```

output
```
Integral(1/(4*sin(x)**2 + 9*cos(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.29

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=\frac{1}{6}\arctan\left(\frac{2}{3}\tan(x)\right)$$

input
```
integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="maxima")
```

output
```
1/6*arctan(2/3*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=\frac{1}{6}x-\frac{1}{6}\arctan\left(\frac{\sin(2x)}{\cos(2x)+5}\right)$$

input
```
integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="giac")
```

output
```
1/6*x - 1/6*arctan(sin(2*x)/(cos(2*x) + 5))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=\frac{x}{6}-\frac{\operatorname{atan}(\tan(x))}{6}+\frac{\operatorname{atan}\left(\frac{2\tan(x)}{3}\right)}{6}$$

input
```
int(1/(9*cos(x)^2 + 4*sin(x)^2),x)
```

output
```
x/6 - atan(tan(x))/6 + atan((2*tan(x))/3)/6
```

Reduce [F]

$$\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx=\int \frac{1}{9\cos(x)^2+4\sin(x)^2}dx$$

input
```
int(1/(9*cos(x)^2+4*sin(x)^2),x)
```

output
```
int(1/(9*cos(x)**2 + 4*sin(x)**2),x)
```

3.38 $\int \frac{1}{x^2(1+x^4)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x^2(1+x^4)^{3/4}}\,dx = -\frac{\sqrt[4]{1+x^4}}{x}$$

output
```
-(x^4+1)^(1/4)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1+x^4)^{3/4}}\,dx = -\frac{\sqrt[4]{1+x^4}}{x}$$

input
```
Integrate[1/(x^2*(1 + x^4)^(3/4)),x]
```

output
```
-((1 + x^4)^(1/4)/x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^4+1)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

input
```
Int[1/(x^2*(1 + x^4)^(3/4)),x]
```

output
```
-((1 + x^4)^(1/4)/x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13
trager	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13
meijerg	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13
risch	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13
pseudoelliptic	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13
orering	$-\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}$	13

input
```
int(1/x^2/(x^4+1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-(x^4+1)^(1/4)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (1+x^4)^{3/4}}\, dx = -\frac{(x^4+1)^{\frac{1}{4}}}{x}$$

input
```
integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="fricas")
```

output
```
-(x^4 + 1)^(1/4)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^2 (1+x^4)^{3/4}} \, dx = \frac{\sqrt[4]{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(x**4+1)**(3/4),x)
```

output
```
(1 + x**(-4))**(1/4)*gamma(-1/4)/(4*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (1+x^4)^{3/4}} \, dx = -\frac{(x^4+1)^{\frac{1}{4}}}{x}$$

input
```
integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="maxima")
```

output
```
-(x^4 + 1)^(1/4)/x
```

Giac [F]

$$\int \frac{1}{x^2 (1+x^4)^{3/4}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (1+x^4)^{3/4}} \, dx = -\frac{(x^4+1)^{1/4}}{x}$$

input
```
int(1/(x^2*(x^4 + 1)^(3/4)),x)
```

output
```
-(x^4 + 1)^(1/4)/x
```

Reduce [F]

$$\int \frac{1}{x^2 (1+x^4)^{3/4}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{4}} x^2} dx$$

input
```
int(1/x^2/(x^4+1)^(3/4),x)
```

output
```
int(1/((x**4 + 1)**(3/4)*x**2),x)
```

3.39 $\int \cos(x)\cos(3x)\cos(5x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{\sin(x)}{4} + \frac{1}{12}\sin(3x) + \frac{1}{28}\sin(7x) + \frac{1}{36}\sin(9x)$$

output
```
1/4*sin(x)+1/12*sin(3*x)+1/28*sin(7*x)+1/36*sin(9*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{\sin(x)}{4} + \frac{1}{12}\sin(3x) + \frac{1}{28}\sin(7x) + \frac{1}{36}\sin(9x)$$

input
```
Integrate[Cos[x]*Cos[3*x]*Cos[5*x],x]
```

output
```
Sin[x]/4 + Sin[3*x]/12 + Sin[7*x]/28 + Sin[9*x]/36
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(3x)\cos(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cos(3x)\cos(5x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{\cos(x)}{4}+\frac{1}{4}\cos(3x)+\frac{1}{4}\cos(7x)+\frac{1}{4}\cos(9x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(x)}{4}+\frac{1}{12}\sin(3x)+\frac{1}{28}\sin(7x)+\frac{1}{36}\sin(9x)$$

input
```
Int[Cos[x]*Cos[3*x]*Cos[5*x],x]
```

output
```
Sin[x]/4 + Sin[3*x]/12 + Sin[7*x]/28 + Sin[9*x]/36
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24
risch	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24
parallelrisch	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24
orering	$-\frac{11\sin(x)\cos(3x)\cos(5x)}{63}-\frac{17\cos(x)\sin(3x)\cos(5x)}{63}+\frac{25\cos(x)\cos(3x)\sin(5x)}{63}-\frac{10\sin(x)\sin(3x)\sin(5x)}{63}$	50

input
```
int(cos(x)*cos(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(x)+1/12*sin(3*x)+1/28*sin(7*x)+1/36*sin(9*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{1}{63}\left(448\cos(x)^8-640\cos(x)^6+240\cos(x)^4+5\cos(x)^2+10\right)\sin(x)$$

input
```
integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="fricas")
```

output
```
1/63*(448*cos(x)^8 - 640*cos(x)^6 + 240*cos(x)^4 + 5*cos(x)^2 + 10)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = -\frac{10\sin(x)\sin(3x)\sin(5x)}{63} - \frac{11\sin(x)\cos(3x)\cos(5x)}{63} - \frac{17\sin(3x)\cos(x)\cos(5x)}{63} + \frac{25\sin(5x)\cos(x)\cos(3x)}{63}$$

input
```
integrate(cos(x)*cos(3*x)*cos(5*x),x)
```

output
```
-10*sin(x)*sin(3*x)*sin(5*x)/63 - 11*sin(x)*cos(3*x)*cos(5*x)/63 - 17*sin(
3*x)*cos(x)*cos(5*x)/63 + 25*sin(5*x)*cos(x)*cos(3*x)/63
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{1}{36}\sin(9x) + \frac{1}{28}\sin(7x) + \frac{1}{12}\sin(3x) + \frac{1}{4}\sin(x)$$

input
```
integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="maxima")
```

output
```
1/36*sin(9*x) + 1/28*sin(7*x) + 1/12*sin(3*x) + 1/4*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{1}{36}\sin(9x) + \frac{1}{28}\sin(7x) + \frac{1}{12}\sin(3x) + \frac{1}{4}\sin(x)$$

input
```
integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="giac")
```

output
```
1/36*sin(9*x) + 1/28*sin(7*x) + 1/12*sin(3*x) + 1/4*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = \frac{64\sin(x)^9}{9} - \frac{128\sin(x)^7}{7} + 16\sin(x)^5 - \frac{17\sin(x)^3}{3} + \sin(x)$$

input
```
int(cos(3*x)*cos(5*x)*cos(x),x)
```

output
```
sin(x) - (17*sin(x)^3)/3 + 16*sin(x)^5 - (128*sin(x)^7)/7 + (64*sin(x)^9)/
9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \cos(x)\cos(3x)\cos(5x)\,dx = -\frac{11\cos(5x)\cos(3x)\sin(x)}{63} - \frac{17\cos(5x)\cos(x)\sin(3x)}{63} + \frac{25\cos(3x)\cos(x)\sin(5x)}{63} - \frac{10\sin(5x)\sin(3x)\sin(x)}{63}$$

input
```
int(cos(x)*cos(3*x)*cos(5*x),x)
```

output
```
( - 11*cos(5*x)*cos(3*x)*sin(x) - 17*cos(5*x)*cos(x)*sin(3*x) + 25*cos(3*x
)*cos(x)*sin(5*x) - 10*sin(5*x)*sin(3*x)*sin(x))/63
```

3.40 $\int \left(\frac{1}{\log(x)}+\log(\log(x))\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 5

$$\int \left(\frac{1}{\log(x)}+\log(\log(x))\right)\,dx = x\log(\log(x))$$

output
```
x*ln(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)}+\log(\log(x))\right)\,dx = x\log(\log(x))$$

input
```
Integrate[Log[x]^(-1) + Log[Log[x]],x]
```

output
```
x*Log[Log[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\log(\log(x)) + \frac{1}{\log(x)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x \log(\log(x))$$

input
```
Int[Log[x]^(-1) + Log[Log[x]],x]
```

output
```
x*Log[Log[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$x \ln (\ln (x))$	6
norman	$x \ln (\ln (x))$	6
risch	$x \ln (\ln (x))$	6
parallelrisch	$x \ln (\ln (x))$	6
parts	$x \ln (\ln (x))$	6

input
```
int(1/ln(x)+ln(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)}+\log(\log(x))\right)\,dx=x\log\left(\log\left(x\right)\right)$$

input
```
integrate(1/log(x)+log(log(x)),x, algorithm="fricas")
```

output
```
x*log(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)}+\log(\log(x))\right)\,dx=x\log\left(\log\left(x\right)\right)$$

input
```
integrate(1/ln(x)+ln(ln(x)),x)
```

output
```
x*log(log(x))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)} + \log(\log(x))\right) dx = x \log(\log(x))$$

input
```
integrate(1/log(x)+log(log(x)),x, algorithm="maxima")
```

output
```
x*log(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)} + \log(\log(x))\right) dx = x \log(\log(x))$$

input
```
integrate(1/log(x)+log(log(x)),x, algorithm="giac")
```

output
```
x*log(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)} + \log(\log(x))\right) dx = x \ln(\ln(x))$$

input
```
int(log(log(x)) + 1/log(x),x)
```

output
```
x*log(log(x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\log(x)} + \log(\log(x))\right) dx = \log(\log(x)) x$$

input
```
int(1/log(x)+log(log(x)),x)
```

output
```
log(log(x))*x
```

3.41 $\int \frac{1}{2+e^x} dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \frac{1}{2+e^x} dx = -\text{arctanh}(1+e^x)$$

output
```
-arctanh(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+e^x} dx = -\text{arctanh}(1+e^x)$$

input
```
Integrate[(2 + E^x)^(-1),x]
```

output
```
-ArcTanh[1 + E^x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^x+2} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^{-x}}{e^x+2}de^x$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\int e^{-x}de^x}{2}-\frac{1}{2}\int \frac{1}{2+e^x}de^x$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\log(e^x)}{2}-\frac{1}{2}\int \frac{1}{2+e^x}de^x$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(e^x)}{2}-\frac{1}{2}\log(e^x+2)$$

input
```
Int[(2 + E^x)^(-1),x]
```

output
```
Log[E^x]/2 - Log[2 + E^x]/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
norman	$\frac{x}{2}-\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	12
risch	$\frac{x}{2}-\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	12
parallelrisch	$\frac{x}{2}-\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	12
derivativedivides	$\frac{\ln\left({\mathrm e}^{x}\right)}{2}-\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	14
default	$\frac{\ln\left({\mathrm e}^{x}\right)}{2}-\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	14

input
```
int(1/(2+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*ln(2+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{1}{2+e^x}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\log\left(e^x+2\right)$$

input
```
integrate(1/(2+exp(x)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*log(e^x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{2+e^x}\,dx = \frac{x}{2} - \frac{\log\left(e^x+2\right)}{2}$$

input
```
integrate(1/(2+exp(x)),x)
```

output
```
x/2 - log(exp(x) + 2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{1}{2+e^x}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\log\left(e^x+2\right)$$

input
```
integrate(1/(2+exp(x)),x, algorithm="maxima")
```

output
```
1/2*x - 1/2*log(e^x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{1}{2+e^x} \, dx = \frac{1}{2} \, x - \frac{1}{2} \, \log \left(e^x + 2 \right)$$

input
```
integrate(1/(2+exp(x)),x, algorithm="giac")
```

output
```
1/2*x - 1/2*log(e^x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{1}{2+e^x} \, dx = \frac{x}{2} - \frac{\ln \left(e^x + 2 \right)}{2}$$

input
```
int(1/(exp(x) + 2),x)
```

output
```
x/2 - log(exp(x) + 2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{1}{2+e^x} \, dx = -\frac{\log(e^x + 2)}{2} + \frac{x}{2}$$

input
```
int(1/(2+exp(x)),x)
```

output
```
( - log(e**x + 2) + x)/2
```

3.42 $\int \sqrt{\frac{x}{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{\frac{x}{1-x^3}}\,dx=\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\arcsin\left(x^{3/2}\right)}{3\sqrt{x}}$$

output

```
2/3*(x/(-x^3+1))^(1/2)*(-x^3+1)^(1/2)*arcsin(x^(3/2))/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{x}{1-x^3}}\,dx=\frac{2\sqrt{-\frac{x}{-1+x^3}}\sqrt{-1+x^3}\log\left(x^{3/2}+\sqrt{-1+x^3}\right)}{3\sqrt{x}}$$

input

```
Integrate[Sqrt[x/(1 - x^3)],x]
```

output

```
(2*Sqrt[-(x/(-1 + x^3))]*Sqrt[-1 + x^3]*Log[x^(3/2) + Sqrt[-1 + x^3]])/(3*
Sqrt[x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {7270, 851, 807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{1-x^3}}dx$$

$$\Big\downarrow \text{7270}$$

$$\frac{\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\int \frac{\sqrt{x}}{\sqrt{1-x^3}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\int \frac{x}{\sqrt{1-x^3}}d\sqrt{x}}{\sqrt{x}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\int \frac{1}{\sqrt{1-x}}dx^{3/2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\arcsin\left(x^{3/2}\right)}{3\sqrt{x}}$$

input
```
Int[Sqrt[x/(1 - x^3)],x]
```

output
```
(2*Sqrt[x/(1 - x^3)]*Sqrt[1 - x^3]*ArcSin[x^(3/2)])/(3*Sqrt[x])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result
default	$\frac{2\sqrt{-\frac{x}{x^3-1}}\,(x^3-1)\operatorname{arctanh}\left(\frac{\sqrt{x^4-x}}{x^2}\right)}{3\sqrt{(x^3-1)x}}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\sqrt{-\frac{x}{x^3-1}}\,x^4+2\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{-\frac{x}{x^3-1}}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{3}$
elliptic	$\frac{2\sqrt{-\frac{x}{x^3-1}}\,\sqrt{(x^3-1)x}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)}}\,(x-1)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x-1)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}}\right.\right.}{x\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(x-1)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\right.}}$

input
```
int((x/(-x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-x/(x^3-1))^(1/2)*(x^3-1)/((x^3-1)*x)^(1/2)*arctanh((x^4-x)^(1/2)/x^2
)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \frac{1}{3}\,\arctan\left(\frac{2\,(x^4-x)\sqrt{-\frac{x}{x^3-1}}}{2\,x^3-1}\right)$$

input
```
integrate((x/(-x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
1/3*arctan(2*(x^4 - x)*sqrt(-x/(x^3 - 1))/(2*x^3 - 1))
```

Sympy [F]

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \int \sqrt{\frac{x}{1-x^3}}\,dx$$

input
```
integrate((x/(-x**3+1))**(1/2),x)
```

output
```
Integral(sqrt(x/(1 - x**3)), x)
```

Maxima [F]

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \int \sqrt{-\frac{x}{x^3-1}}\,dx$$

input
```
integrate((x/(-x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x/(x^3 - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.39

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{\frac{1}{x^3}-1}\right)\operatorname{sgn}\left(x^3-1\right)$$

input
```
integrate((x/(-x^3+1))^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(1/x^3 - 1))*sgn(x^3 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \int \sqrt{-\frac{x}{x^3-1}}\,dx$$

input
```
int((-x/(x^3 - 1))^(1/2),x)
```

output
```
int((-x/(x^3 - 1))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{x}{1-x^3}}\,dx = \frac{i\left(-\log\left(\sqrt{x^3-1}-\sqrt{x}\,x\right)+\log\left(\sqrt{x^3-1}+\sqrt{x}\,x\right)\right)}{3}$$

input
```
int((x/(-x^3+1))^(1/2),x)
```

output
```
(i*( - log(sqrt(x**3 - 1) - sqrt(x)*x) + log(sqrt(x**3 - 1) + sqrt(x)*x)))
/3
```

3.43 $\int \frac{4x}{1-x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 6

$$\int \frac{4x}{1-x^4}\,dx = 2\text{arctanh}\left(x^2\right)$$

output
```
2*arctanh(x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{4x}{1-x^4}\,dx = -4\left(\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(1+x^2\right)\right)$$

input
```
Integrate[(4*x)/(1 - x^4),x]
```

output
```
-4*(Log[1 - x^2]/4 - Log[1 + x^2]/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {27, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x}{1-x^4} \, dx$$
$$\Big\downarrow \; 27$$
$$4\int \frac{x}{1-x^4}dx$$
$$\Big\downarrow \; 807$$
$$2\int \frac{1}{1-x^4}dx^2$$
$$\Big\downarrow \; 219$$
$$2\text{arctanh}\left(x^2\right)$$

input
```
Int[(4*x)/(1 - x^4),x]
```

output
```
2*ArcTanh[x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
meijerg	$2 \operatorname{arctanh}(x^2)$	7
risch	$\ln(x^2+1) - \ln(x^2-1)$	16
default	$-\ln(x-1) - \ln(x+1) + \ln(x^2+1)$	20
norman	$-\ln(x-1) - \ln(x+1) + \ln(x^2+1)$	20
parallelrisch	$-\ln(x-1) - \ln(x+1) + \ln(x^2+1)$	20

input
```
int(4*x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
2*arctanh(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{4x}{1-x^4}\,dx = \log\left(x^2+1\right) - \log\left(x^2-1\right)$$

input
```
integrate(4*x/(-x^4+1),x, algorithm="fricas")
```

output
```
log(x^2 + 1) - log(x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{4x}{1-x^4} \, dx = -\log\left(x^2-1\right) + \log\left(x^2+1\right)$$

input
```
integrate(4*x/(-x**4+1),x)
```

output
```
-log(x**2 - 1) + log(x**2 + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{4x}{1-x^4} \, dx = \log\left(x^2+1\right) - \log\left(x^2-1\right)$$

input
```
integrate(4*x/(-x^4+1),x, algorithm="maxima")
```

output
```
log(x^2 + 1) - log(x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{4x}{1-x^4} \, dx = \log\left(x^2+1\right) - \log\left(\left|x^2-1\right|\right)$$

input
```
integrate(4*x/(-x^4+1),x, algorithm="giac")
```

output
```
log(x^2 + 1) - log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{4x}{1-x^4}\,dx = 2\,\operatorname{atanh}\left(x^2\right)$$

input
```
int(-(4*x)/(x^4 - 1),x)
```

output
```
2*atanh(x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{4x}{1-x^4}\,dx = \log\left(x^2+1\right) - \log(x-1) - \log(x+1)$$

input
```
int(4*x/(-x^4+1),x)
```

output
```
log(x**2 + 1) - log(x - 1) - log(x + 1)
```

3.44 $\int x^x(1+\log(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 3

$$\int x^x(1+\log(x))\,dx = x^x$$

output
```
x^x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
Integrate[x^x*(1 + Log[x]),x]
```

output
```
x^x
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^x(\log(x)+1)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int (x^x + x^x\log(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$x^x$$

input
```
Int[x^x*(1 + Log[x]),x]
```

output
```
x^x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	x^x	4
default	x^x	4
risch	x^x	4
parallelrisch	x^x	4
norman	$e^{\ln(x)x}$	6

input
```
int(x^x*(1+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x^x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="fricas")
```

output
```
x^x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
integrate(x**x*(1+ln(x)),x)
```

output
```
x**x
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="maxima")
```

output
```
x^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
integrate(x^x*(1+log(x)),x, algorithm="giac")
```

output
```
x^x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
int(x^x*(log(x) + 1),x)
```

output
```
x^x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int x^x(1+\log(x))\,dx = x^x$$

input
```
int(x^x*(1+log(x)),x)
```

output
```
x**x
```

3.45 $\int \sqrt{6x-x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \sqrt{6x-x^2}\,dx = \frac{1}{2}\left((-3+x)\sqrt{-((-6+x)x)} - 9\arcsin\left(1-\frac{x}{3}\right)\right)$$

output
```
1/2*(-3+x)*(-(-6+x)*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \sqrt{6x-x^2}\,dx = \frac{1}{2}\sqrt{-((-6+x)x)}\left(-3+x+\frac{18\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-6+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[6*x - x^2],x]
```

output
```
(Sqrt[-((-6 + x)*x)]*(-3 + x + (18*Log[Sqrt[-6 + x] - Sqrt[x]])/(Sqrt[-6 +
 x]*Sqrt[x])))/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{6x-x^2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}dx-\frac{1}{2}(3-x)\sqrt{6x-x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{3}{4}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-2x)^2}}d(6-2x)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{9}{2}\arcsin\left(\frac{1}{6}(6-2x)\right)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

input
```
Int[Sqrt[6*x - x^2],x]
```

output
```
-1/2*((3 - x)*Sqrt[6*x - x^2]) - (9*ArcSin[(6 - 2*x)/6])/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{(-3+x)(-6+x)x}{2\sqrt{-(-6+x)x}}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$
default	$-\frac{(-2x+6)\sqrt{-x^{2}+6x}}{4}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$
pseudoelliptic	$-9\arctan\left(\frac{\sqrt{-(-6+x)x}}{x}\right)+\frac{(-3+x)\sqrt{-(-6+x)x}}{2}$
meijerg	$-\frac{18i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{6}\,(3-x)\sqrt{-\frac{x}{6}+1}}{36}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)}{2}\right)}{\sqrt{\pi}}$
trager	$\left(-\frac{3}{2}+\frac{x}{2}\right)\sqrt{-x^{2}+6x}+\frac{9\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)x+3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{-x^{2}+6x}\right)}{2}$

input
```
int((-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3+x)*(-6+x)*x/(-(-6+x)*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \sqrt{6x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+6\,x}(x-3) - 9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x-6}\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/(x - 6))
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \sqrt{6x-x^2}\,dx = \left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{-x^2+6x} + \frac{9\operatorname{asin}\left(\frac{x}{3}-1\right)}{2}$$

input
```
integrate((-x**2+6*x)**(1/2),x)
```

output
```
(x/2 - 3/2)*sqrt(-x**2 + 6*x) + 9*asin(x/3 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \sqrt{6x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+6\,x}x - \frac{3}{2}\sqrt{-x^2+6\,x} - \frac{9}{2}\arcsin\left(-\frac{1}{3}\,x+1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sqrt{6x-x^2}\,dx=\frac{1}{2}\sqrt{-x^2+6x}(x-3)+\frac{9}{2}\arcsin\left(\frac{1}{3}x-1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \sqrt{6x-x^2}\,dx=\frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2}+\left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{6\,x-x^2}$$

input
```
int((6*x - x^2)^(1/2),x)
```

output
```
(9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \sqrt{6x-x^2}\,dx=\frac{\sqrt{x}\,\sqrt{-x+6}\,x}{2}-\frac{3\sqrt{x}\,\sqrt{-x+6}}{2}-9\log\left(\frac{\sqrt{-x+6}+\sqrt{x}\,i}{\sqrt{6}}\right)i$$

input
```
int((-x^2+6*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt( - x + 6)*x - 3*sqrt(x)*sqrt( - x + 6) - 18*log((sqrt( - x +
 6) + sqrt(x)*i)/sqrt(6))*i)/2
```

3.46 $\int \sin^{99}(x)\sin(101x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \sin^{99}(x)\sin(101x)\,dx = \frac{1}{100}\sin^{100}(x)\sin(100x)$$

output
```
1/100*sin(x)^100*sin(100*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sin^{99}(x)\sin(101x)\,dx = \frac{1}{100}\sin^{100}(x)\sin(100x)$$

input
```
Integrate[Sin[x]^99*Sin[101*x],x]
```

output
```
(Sin[x]^100*Sin[100*x])/100
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 801 vs. $2(12) = 24$.

Time = 1.34 (sec) , antiderivative size = 801, normalized size of antiderivative = 66.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{99}(x) \sin(101x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^{99} \sin(101x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{\cos(2x)}{633825300114114700748351602688} + \frac{99\cos(4x)}{633825300114114700748351602688} - \frac{4851\cos(6x)}{633825300114114700748351$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(2x)}{1267650600228229401496703205376}+\frac{99\sin(4x)}{2535301200456458802993406410752}-$$
$$\frac{1617\sin(6x)}{1267650600228229401496703205376}+\frac{156849\sin(8x)}{5070602400912917605986812821504}-$$
$$\frac{470547\sin(10x)}{792281625142643375935439503360}+\frac{2980131\sin(12x)}{316912650057057350374175801344}-$$
$$\frac{20009451\sin(14x)}{158456325028528675187087900672}+\frac{1860878943\sin(16x)}{1267650600228229401496703205376}-$$
$$\frac{4755579521\sin(18x)}{316912650057057350374175801344}+\frac{432757736411\sin(20x)}{3169126500570573503741758013440}-$$
$$\frac{354074511609\sin(22x)}{316912650057057350374175801344}+\frac{10504210511067\sin(24x)}{1267650600228229401496703205376}-$$
$$\frac{8888178124749\sin(26x)}{158456325028528675187087900672}+\frac{110467356693309\sin(28x)}{316912650057057350374175801344}-$$
$$\frac{1583365445937429\sin(30x)}{792281625142643375935439503360}+\frac{26917212580936293\sin(32x)}{2535301200456458802993406410752}-$$
$$\frac{33250674364686009\sin(34x)}{633825300114114700748351602688}+\frac{306645108029882083\sin(36x)}{1267650600228229401496703205376}-$$
$$\frac{661707864696061337\sin(38x)}{633825300114114700748351602688}+\frac{5359833704038096829 7\sin(40x)}{12676506002282294014967032053760}-$$
$$\frac{25523017638276651 57\sin(42x)}{158456325028528675187087900672}+\frac{18330167212944140673\sin(44x)}{316912650057057350374175801344}-$$
$$\frac{31081587882818325489\sin(46x)}{158456325028528675187087900672}+\frac{79776075565900368755 1\sin(48x)}{1267650600228229401496703205376}-$$
$$\frac{1515745435752107006346 9\sin(50x)}{7922816251426433759354395033600}+\frac{349787408250486232233 9\sin(52x)}{633825300114114700748351602688}-$$
$$\frac{479338300195110762690 9\sin(54x)}{316912650057057350374175801344}+\frac{499881370203472652520 51\sin(56x)}{1267650600228229401496703205376}-$$
$$\frac{155135597649353581816 71\sin(58x)}{158456325028528675187087900672}+\frac{367154247770136810299 547\sin(60x)}{1584563250285286751870879006720}-$$
$$\frac{82905797883579279745059\sin(62x)}{158456325028528675187087900672}+\frac{572050005396697030240907 1\sin(64x)}{5070602400912917605986812821504}-$$
$$\frac{29469242702254089436652 79\sin(66x)}{1267650600228229401496703205376}+\frac{1161434859441778818973962 9\sin(68x)}{2535301200456458802993406410752}-$$
$$\frac{547533576593981443230582 51\sin(70x)}{6338253001141147007483516026880}+\frac{790881832857973195777508 07\sin(72x)}{5070602400912917605986812821504}-$$
$$\frac{213751846718371133993921 1\sin(74x)}{79228162514264337593543950336}+\frac{708756123329335865348264 7\sin(76x)}{158456325028528675187087900672}-$$
$$\frac{563370251877164405789646 3\sin(78x)}{79228162514264337593543950336}+\frac{343655853645070287531684243\sin(80x)}{3169126500570573503741758013440}-$$
$$\frac{2514555026671246006329396 9\sin(82x)}{158456325028528675187087900672}+\frac{706470221779064354159211 51\sin(84x)}{316912650057057350374175801344}-$$
$$\frac{4764566611998340993166775 3\sin(86x)}{158456325028528675187087900672}+\frac{246891178985368578736823811\sin(88x)}{633825300114114700748351602688}-$$
$$\frac{1920264725441755612397518 53\sin(90x)}{396140812571321687967719751680}+\frac{918387477385187466798813 21\sin(92x)}{158456325028528675187087900672}-$$
$$\frac{527584295519150246884424 61\sin(94x)}{79228162514264337593543950336}+\frac{932065588750498769495816811\sin(96x)}{1267650600228229401496703205376}-$$
$$\frac{2472827072195200817029718 07\sin(98x)}{316912650057057350374175801344}+\frac{1261141806819552416685156215 7\sin(100x)}{1584563250285286751870879006720 0}-$$
$$\frac{2472827072195200817029718 07\sin(102x)}{316912650057057350374175801344}+\frac{932065588750498769495816811\sin(104x)}{1267650600228229401496703205376}-$$
$$\frac{527584295519150246884424 61\sin(106x)}{79228162514264337593543950336}+\frac{918387477385187466798813 21\sin(108x)}{158456325028528675187087900672}-$$
$$\frac{1920264725441755612397518 53\sin(110x)}{396140812571321687967719751680}+\frac{246891178985368578736823811\sin(112x)}{633825300114114700748351602688}-$$
$$\frac{4764566611998340993166775 3\sin(114x)}{158456325028528675187087900672}+\frac{706470221779064354159211 51\sin(116x)}{316912650057057350374175801344}-$$

input
```
Int[Sin[x]^99*Sin[101*x],x]
```

output
```
-1/1267650600228229401496703205376*Sin[2*x] + (99*Sin[4*x])/25353012004564
58802993406410752 - (1617*Sin[6*x])/1267650600228229401496703205376 + (156
849*Sin[8*x])/5070602400912917605986812821504 - (470547*Sin[10*x])/7922816
251426433759354395033600 + (2980131*Sin[12*x])/316912650057057350374175801
344 - (20009451*Sin[14*x])/158456325028528675187087900672 + (1860878943*Sin
[16*x])/1267650600228229401496703205376 - (4755579521*Sin[18*x])/316912650
057057350374175801344 + (432757736411*Sin[20*x])/31691265005705735037417580
13440 - (354074511609*Sin[22*x])/316912650057057350374175801344 + (105042
10511067*Sin[24*x])/1267650600228229401496703205376 - (888178124749*Sin[2
6*x])/158456325028528675187087900672 + (110467356693309*Sin[28*x])/3169126
50057057350374175801344 - (1583365445937429*Sin[30*x])/7922816251426433759
35439503360 + (26917212580936293*Sin[32*x])/25353012004564588029934064107
52 - (33250674364686009*Sin[34*x])/633825300114114700748351602688 + (306645
108029882083*Sin[36*x])/1267650600228229401496703205376 - (661707864696061
337*Sin[38*x])/633825300114114700748351602688 + (53598337040380968297*Sin[
40*x])/12676506002282294014967032053760 - (2552301763827665157*Sin[42*x])/
158456325028528675187087900672 + (18330167212944140673*Sin[44*x])/31691265
0057057350374175801344 - (31081587882818325489*Sin[46*x])/1584563250285286
75187087900672 + (79776075565900368755 1*Sin[48*x])/126765060022822940149 67
03205376 - (151574543575210700634 69*Sin[50*x])/79228162514264337593543 9...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(10) = 20$.

Time = 6.24 (sec) , antiderivative size = 602, normalized size of antiderivative = 50.17

Expression too large to display

input
```
int(sin(x)^99*sin(101*x),x)
```

output
```
91838747738518746679881321/158456325028528675187087900672*sin(92*x)-332506
74364686009/633825300114114700748351602688*sin(166*x)-1/126765060022822940
1496703205376*sin(2*x)+306645108029882083/1267650600228229401496703205376*
sin(36*x)+797760755659003687551/1267650600228229401496703205376*sin(48*x)-
31081587882818325489/158456325028528675187087900672*sin(46*x)+790881832857
97319577750807/5070602400912917605986812821504*sin(128*x)+3671542477701368
10299547/1584563250285286751870879006720*sin(60*x)-15157454357521070063469
/7922816251426433759354395033600*sin(50*x)+3497874082504862322339/63382530
0114114700748351602688*sin(148*x)-47645666119983409931667753/1584563250285
28675187087900672*sin(86*x)-1617/1267650600228229401496703205376*sin(6*x)+
432757736411/3169126500570573503741758013440*sin(20*x)+1833016721294414067
3/316912650057057350374175801344*sin(44*x)-82905797883579279745059/1584563
25028528675187087900672*sin(138*x)+99/2535301200456458802993406410752*sin(
4*x)-247282707219520081702971807/316912650057057350374175801344*sin(102*x)
+110467356693309/316912650057057350374175801344*sin(28*x)+9183874773851874
6679881321/158456325028528675187087900672*sin(108*x)-8888178124749/1584563
25028528675187087900672*sin(26*x)+1/126765060022822940149670320537600*sin(
200*x)+53598337040380968297/12676506002282294014967032053760*sin(40*x)+126
11418068195524166851562157/15845632502852867518708790067200*sin(100*x)-255
2301763827665157/158456325028528675187087900672*sin(158*x)-310815878828...
```

Fricas [F(-1)]

Timed out.

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Timed out}$$

input
```
integrate(sin(x)^99*sin(101*x),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**99*sin(101*x),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(x)^99*sin(101*x),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: Memory limit reached. Please j
ump to an outer pointer, quit program and enlarge thememory limits before
executing the program again.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(10) = 20$.

Time = 0.14 (sec) , antiderivative size = 601, normalized size of antiderivative = 50.08

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Too large to display}$$

input
```
integrate(sin(x)^99*sin(101*x),x, algorithm="giac")
```

output
```
1/126765060022822940149670320537600*sin(200*x) - 1/126765060022822940149670
3205376*sin(198*x) + 99/2535301200456458802993406410752*sin(196*x) - 1617
/1267650600228229401496703205376*sin(194*x) + 156849/507060240091291760598
6812821504*sin(192*x) - 470547/792281625142643375935439503360*sin(190*x) +
 2980131/316912650057057350374175801344*sin(188*x) - 20009451/158456325028
528675187087900672*sin(186*x) + 1860878943/1267650600228229401496703205376
*sin(184*x) - 4755579521/316912650057057350374175801344*sin(182*x) + 43275
7736411/3169126500570573503741758013440*sin(180*x) - 354074511609/31691265
0057057350374175801344*sin(178*x) + 10504210511067/126765060022822940149670
3205376*sin(176*x) - 8888178124749/158456325028528675187087900672*sin(174
*x) + 110467356693309/316912650057057350374175801344*sin(172*x) - 15833654
45937429/792281625142643375935439503360*sin(170*x) + 26917212580936293/253
5301200456458802993406410752*sin(168*x) - 33250674364686009/63382530011411
4700748351602688*sin(166*x) + 306645108029882083/1267650600228229401496703
205376*sin(164*x) - 661707864696061337/633825300114114700748351602688*sin(
162*x) + 53598337040380968297/12676506002282294014967032053760*sin(160*x)
- 2552301763827665157/158456325028528675187087900672*sin(158*x) + 18330167
212944140673/316912650057057350374175801344*sin(156*x) - 31081587882818325
489/158456325028528675187087900672*sin(154*x) + 797760755659003687551/1267
650600228229401496703205376*sin(152*x) - 15157454357521070063469/792281...
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 601, normalized size of antiderivative = 50.08

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Too large to display}$$

input
```
int(sin(101*x)*sin(x)^99,x)
```

output

```
(99*sin(4*x))/2535301200456458802993406410752 - sin(2*x)/12676506002282294
01496703205376 - (1617*sin(6*x))/1267650600228229401496703205376 + (156849
*sin(8*x))/5070602400912917605986812821504 - (470547*sin(10*x))/7922816251
426433759354395033600 + (2980131*sin(12*x))/316912650057057350374175801344
- (20009451*sin(14*x))/158456325028528675187087900672 + (1860878943*sin(16
*x))/1267650600228229401496703205376 - (4755579521*sin(18*x))/316912650057
057350374175801344 + (432757736411*sin(20*x))/31691265005705735037417580134
40 - (354074511609*sin(22*x))/316912650057057350374175801344 + (105042105
11067*sin(24*x))/1267650600228229401496703205376 - (8888178124749*sin(26*x
))/158456325028528675187087900672 + (110467356693309*sin(28*x))/3169126500
57057350374175801344 - (1583365445937429*sin(30*x))/79228162514264337593543
9503360 + (26917212580936293*sin(32*x))/2535301200456458802993406410752 -
 (33250674364686009*sin(34*x))/633825300114114700748351602688 + (306645108
029882083*sin(36*x))/1267650600228229401496703205376 - (661707864696061337
*sin(38*x))/633825300114114700748351602688 + (5359833704038096829 7*sin(40*
x))/12676506002282294014967032053760 - (2552301763827665157*sin(42*x))/158
456325028528675187087900672 + (18330167212944140673*sin(44*x))/31691265005
7057350374175801344 - (31081587882818325489*sin(46*x))/1584563250285286751
87087900672 + (79776075565900368755 1*sin(48*x))/12676506002282294014967032
05376 - (151574543575210700634 69*sin(50*x))/7922816251426433759354395 03...
```

Reduce [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 1095, normalized size of antiderivative = 91.25

$$\int \sin^{99}(x)\sin(101x)\,dx = \text{Too large to display}$$

input

```
int(sin(x)^99*sin(101*x),x)
```

output
```
( - 320081776557627923877917559357444*cos(101*x)*sin(x)**99 + 3921001762830
94206750449010212864*cos(101*x)*sin(x)**97 - 310479323260899086161580032557
67168*cos(101*x)*sin(x)**95 + 178645641541226085014362737816371200*cos(101*
x)*sin(x)**93 - 796089640118088741345253950394204l6*cos(101*x)*sin(x)**91
+ 2859637786213660873516504321810759 68*cos(101*x)*sin(x)**89 - 85093780933
22717310555190671892807 68*cos(101*x)*sin(x)**87 + 213935372628496542061740
3783840792576*cos(101*x)*sin(x)**85 - 461201256209983487415708424414953472
0*cos(101*x)*sin(x)**83 + 862344986199216377843766905650597 0688*cos(101*x)
*sin(x)**81 - 14111099774168995273807094819737042944*cos(101*x)*sin(x)**79
 + 20354072286617356946909952780716212224*cos(101*x)*sin(x)**77 - 26029727
058847196864798304998415925248*cos(101*x)*sin(x)**75 + 2965209055348972549
25350149304614912 00*cos(101*x)*sin(x)**73 - 302037573544848831761170617198
65425920*cos(101*x)*sin(x)**71 + 27594241553637843637004006755097640960*co
s(101*x)*sin(x)**69 - 22666698419059657273253291263115919360*cos(101*x)*si
n(x)**67 + 16772446520930690171071963514976337920*cos(101*x)*sin(x)**65 -
11195984840672604478764340215388569600*cos(101*x)*sin(x)**63 + 67486908622
94319921921838407609221120*cos(101*x)*sin(x)**61 - 367562627321387067176l0
01275572879360*cos(101*x)*sin(x)**59 + 180926252976666649004116029416144 89
60*cos(101*x)*sin(x)**57 - 8047890182574135223260010673025843 20*cos(101*x)
*sin(x)**55 + 3233527305498536473631254288269312 00*cos(101*x)*sin(x)**5...
```

3.47 $\int e^{e^{x^2}} x\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int e^{e^{x^2}} x\,dx = \frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

output
```
1/2*Ei(exp(x^2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{e^{x^2}} x\,dx = \frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

input
```
Integrate[E^E^x^2*x,x]
```

output
```
ExpIntegralEi[E^x^2]/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 2720, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{e^{x^2}} x\,dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int e^{e^{x^2}}dx^2$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}\int \frac{e^{e^{x^2}}}{x^2}de^{x^2}$$

$$\Big\downarrow \; 2609$$

$$\frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

input
```
Int[E^E^x^2*x,x]
```

output
```
ExpIntegralEi[E^x^2]/2
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{\operatorname{expIntegral}_1\left(-{\mathrm e}^{x^{2}}\right)}{2}$	11
derivativedivides	$-\frac{\operatorname{expIntegral}_1\left(-{\mathrm e}^{x^{2}}\right)}{2}$	13
default	$-\frac{\operatorname{expIntegral}_1\left(-{\mathrm e}^{x^{2}}\right)}{2}$	13

input
```
int(exp(exp(1)^(x^2))*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*Ei(1,-exp(x^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{e^{x^2}} x\,dx = \frac{1}{2}\,\mathrm{Ei}\left(e^{\left(x^2\right)}\right)$$

input
```
integrate(exp(exp(1)^(x^2))*x,x, algorithm="fricas")
```

output
```
1/2*Ei(e^(x^2))
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{e^{x^2}} x\,dx = \frac{\operatorname{Ei}\left(e^{x^2}\right)}{2}$$

input
```
integrate(exp(exp(1)**(x**2))*x,x)
```

output
```
Ei(exp(x**2))/2
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{e^{x^2}} x\,dx = \frac{1}{2}\operatorname{Ei}\left(e^{(x^2)}\right)$$

input
```
integrate(exp(exp(1)^(x^2))*x,x, algorithm="maxima")
```

output
```
1/2*Ei(e^(x^2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{e^{x^2}} x\,dx = \frac{1}{2}\operatorname{Ei}\left(e^{(x^2)}\right)$$

input
```
integrate(exp(exp(1)^(x^2))*x,x, algorithm="giac")
```

output
```
1/2*Ei(e^(x^2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{e^{x^2}} x\,dx = \frac{\mathrm{ei}\left(\mathrm{e}^{x^2}\right)}{2}$$

input
```
int(x*exp(exp(x^2)),x)
```

output
```
ei(exp(x^2))/2
```

Reduce [F]

$$\int e^{e^{x^2}} x\,dx = \int e^{e^{x^2}} x dx$$

input
```
int(exp(exp(1)^(x^2))*x,x)
```

output
```
int(e**(e**(x**2))*x,x)
```

3.48 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x)$$

output
```
8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{2(4+9x+9x^2)}{3(1+x)^3} + \log(1+x)$$

input
```
Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
(2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2006, 2007, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+3x-1}{x^4+4x^3+6x^2+4x+1}dx$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{(x-1)^3}{x^4+4x^3+6x^2+4x+1}dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{(x-1)^3}{(x+1)^4}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{x+1}-\frac{6}{(x+1)^2}+\frac{12}{(x+1)^3}-\frac{8}{(x+1)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

input
```
Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(x+1)^3}+\ln(x+1)$	22
default	$\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}+\ln(x+1)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(x+1)$	32
parallelrisch	$\frac{3\ln(x+1)x^3+8+9\ln(x+1)x^2+9x\ln(x+1)+18x^2+3\ln(x+1)+18x}{3x^3+9x^2+9x+3}$	59

input
```
int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
(6*x+6*x^2+8/3)/(x+1)^3+ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log(x+1)+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^
2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log(x+1)$$

input
```
integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

output
```
(18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log(x+1)$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x+1)^3} + \log\left(|x+1|\right)$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln\left(x+1\right) + \frac{6\,x^2+6\,x+\frac{8}{3}}{(x+1)^3}$$

input
```
int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

output
```
log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{3\log(x+1)\,x^3+9\log(x+1)\,x^2+9\log(x+1)\,x+3\log(x+1)-6x^3+2}{3x^3+9x^2+9x+3}$$

input
```
int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

output
```
(3*log(x + 1)*x**3 + 9*log(x + 1)*x**2 + 9*log(x + 1)*x + 3*log(x + 1) - 6
*x**3 + 2)/(3*(x**3 + 3*x**2 + 3*x + 1))
```

3.49 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \sqrt{\frac{1-x}{1+x}}(1+x) - 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

output
```
((1-x)/(1+x))^(1/2)*(1+x)-2*arctan(((1-x)/(1+x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
(Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]
/(-1 + x)]))/Sqrt[1 - x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x+1}}dx$$

$$\Big\downarrow \text{ 2051}$$

$$-4\int \frac{1-x}{(x+1)\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{ 252}$$

$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-x}{x+1}+1}d\sqrt{\frac{1-x}{x+1}}-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
-4*(-1/2*Sqrt[(1 - x)/(1 + x)]/(1 + (1 - x)/(1 + x)) + ArcTan[Sqrt[(1 - x)
/(1 + x)]]/2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\sqrt{-x^{2}+1}+\arcsin(x)\right)}{\sqrt{-(x-1)(x+1)}}$
risch	$(x+1)\sqrt{-\frac{x-1}{x+1}}-\frac{\arcsin(x)\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}{x-1}$
trager	$(x+1)\sqrt{-\frac{x-1}{x+1}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\Big(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x-1}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{}$

input
```
int(((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = (x+1)\sqrt{-\frac{x-1}{x+1}} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \int \sqrt{\frac{1-x}{x+1}}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(
x + 1)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\,\pi\mathrm{sgn}(x+1) + \arcsin{(x)}\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int((-(x - 1)/(x + 1))^(1/2),x)
```

output
```
- 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)
/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \sqrt{x+1}\,\sqrt{1-x}$$

input
```
int(((1-x)/(1+x))^(1/2),x)
```

output
```
 - 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)
```

3.50 $\int \frac{1}{-1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{1}{-1+\sqrt{x}} \, dx = 2\sqrt{x} + 2\log\left(1-\sqrt{x}\right)$$

output
```
2*x^(1/2)+2*ln(1-x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{-1+\sqrt{x}} \, dx = 2\sqrt{x} + 2\log\left(-1+\sqrt{x}\right)$$

input
```
Integrate[(-1 + Sqrt[x])^(-1),x]
```

output
```
2*Sqrt[x] + 2*Log[-1 + Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {774, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}-1} \, dx$$
$$\Big\downarrow \; 774$$
$$2 \int -\frac{\sqrt{x}}{1-\sqrt{x}} d\sqrt{x}$$
$$\Big\downarrow \; 25$$
$$-2 \int \frac{\sqrt{x}}{1-\sqrt{x}} d\sqrt{x}$$
$$\Big\downarrow \; 49$$
$$-2 \int \left(\frac{1}{1-\sqrt{x}}-1\right) d\sqrt{x}$$
$$\Big\downarrow \; 2009$$
$$2\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

input
```
Int[(-1 + Sqrt[x])^(-1),x]
```

output
```
2*(Sqrt[x] + Log[1 - Sqrt[x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2\sqrt{x}+2\ln\left(\sqrt{x}-1\right)$	15
meijerg	$2\sqrt{x}+2\ln\left(-\sqrt{x}+1\right)$	17
trager	$2\sqrt{x}+\ln\left(2\sqrt{x}-1-x\right)$	18
default	$\ln\left(x-1\right)+2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)$	25

input
```
int(1/(x^(1/2)-1),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)+2*ln(x^(1/2)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\sqrt{x} + 2\log\left(\sqrt{x} - 1\right)$$

input
```
integrate(1/(-1+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) + 2*log(sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\sqrt{x} + 2\log\left(\sqrt{x} - 1\right)$$

input
```
integrate(1/(-1+x**(1/2)),x)
```

output
```
2*sqrt(x) + 2*log(sqrt(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\sqrt{x} + 2\log\left(\sqrt{x} - 1\right) - 2$$

input
```
integrate(1/(-1+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) + 2*log(sqrt(x) - 1) - 2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\sqrt{x} + 2\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(1/(-1+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) + 2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\ln\left(\sqrt{x}-1\right) + 2\sqrt{x}$$

input
```
int(1/(x^(1/2) - 1),x)
```

output
```
2*log(x^(1/2) - 1) + 2*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{-1+\sqrt{x}}\,dx = 2\sqrt{x} + 2\log\left(\sqrt{x}-1\right)$$

input
```
int(1/(-1+x^(1/2)),x)
```

output
```
2*(sqrt(x) + log(sqrt(x) - 1))
```

3.51 $\int \sqrt[4]{x} \log(x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt[4]{x} \log(x)\, dx = -\frac{16x^{5/4}}{25} + \frac{4}{5}x^{5/4}\log(x)$$

output
```
-16/25*x^(5/4)+4/5*x^(5/4)*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt[4]{x} \log(x)\, dx = \frac{4}{25}x^{5/4}(-4 + 5\log(x))$$

input
```
Integrate[x^(1/4)*Log[x],x]
```

output
```
(4*x^(5/4)*(-4 + 5*Log[x]))/25
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{x}\log(x)\,dx$$

$$\Big\downarrow \; 2741$$

$$\frac{4}{5}x^{5/4}\log(x) - \frac{16x^{5/4}}{25}$$

input
```
Int[x^(1/4)*Log[x],x]
```

output
```
(-16*x^(5/4))/25 + (4*x^(5/4)*Log[x])/5
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14
default	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14
risch	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14
orering	$\frac{24x^{\frac{5}{4}}\ln(x)}{25}-\frac{16x^{2}\left(\frac{\ln(x)}{4x^{\frac{3}{4}}}+\frac{1}{x^{\frac{3}{4}}}\right)}{25}$	25

input
```
int(x^(1/4)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-16/25*x^(5/4)+4/5*x^(5/4)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \sqrt[4]{x}\log(x)\,dx=\frac{4}{25}\,(5\,x\log{(x)}-4\,x)x^{\frac{1}{4}}$$

input
```
integrate(x^(1/4)*log(x),x, algorithm="fricas")
```

output
```
4/25*(5*x*log(x) - 4*x)*x^(1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(19) = 38$.

Time = 1.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \sqrt[4]{x}\log(x)\,dx = \begin{cases} -\frac{4x^{\frac{5}{4}}\log\left(\frac{1}{x}\right)}{5} + \frac{4x^{\frac{5}{4}}\log(x)}{5} - \frac{32x^{\frac{5}{4}}}{25} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \frac{4x^{\frac{5}{4}}\log(x)}{5} - \frac{16x^{\frac{5}{4}}}{25} & \text{for } |x| < 1 \\ -\frac{4x^{\frac{5}{4}}\log\left(\frac{1}{x}\right)}{5} - \frac{16x^{\frac{5}{4}}}{25} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{9}{4}, \frac{9}{4} \\ \frac{5}{4}, \frac{5}{4} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{9}{4}, \frac{9}{4}, 1 & \\ & \frac{5}{4}, \frac{5}{4}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/4)*ln(x),x)
```

output
```
Piecewise((-4*x**(5/4)*log(1/x)/5 + 4*x**(5/4)*log(x)/5 - 32*x**(5/4)/25,
(Abs(x) < 1) & (1/Abs(x) < 1)), (4*x**(5/4)*log(x)/5 - 16*x**(5/4)/25, Abs
(x) < 1), (-4*x**(5/4)*log(1/x)/5 - 16*x**(5/4)/25, 1/Abs(x) < 1), (-meije
rg(((1,), (9/4, 9/4)), ((5/4, 5/4), (0,)), x) + meijerg(((9/4, 9/4, 1), ()
), ((), (5/4, 5/4, 0)), x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[4]{x}\log(x)\,dx = \frac{4}{5}\,x^{\frac{5}{4}}\log(x) - \frac{16}{25}\,x^{\frac{5}{4}}$$

input
```
integrate(x^(1/4)*log(x),x, algorithm="maxima")
```

output
```
4/5*x^(5/4)*log(x) - 16/25*x^(5/4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt[4]{x}\log(x)\,dx = \frac{4}{5}\,x^{\frac{5}{4}}\log\left(x\right) - \frac{16}{25}\,x^{\frac{5}{4}}$$

input
```
integrate(x^(1/4)*log(x),x, algorithm="giac")
```

output
```
4/5*x^(5/4)*log(x) - 16/25*x^(5/4)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \sqrt[4]{x}\log(x)\,dx = \frac{4\,x^{5/4}\,\left(\ln\left(x\right) - \frac{4}{5}\right)}{5}$$

input
```
int(x^(1/4)*log(x),x)
```

output
```
(4*x^(5/4)*(log(x) - 4/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \sqrt[4]{x}\log(x)\,dx = \frac{4x^{\frac{5}{4}}(5\log(x) - 4)}{25}$$

input
```
int(x^(1/4)*log(x),x)
```

output
```
(4*x**(1/4)*x*(5*log(x) - 4))/25
```

3.52 $\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx = -\frac{2(1-\sqrt{x})}{\sqrt{1-x}}$$

output
```
(-2+2*x^(1/2))/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx = \frac{2(-\sqrt{x}+x)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/((1 + Sqrt[x])*Sqrt[x - x^2]),x]
```

output
```
(2*(-Sqrt[x] + x))/Sqrt[-((-1 + x)*x)]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(20) = 40.

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2467, 1388, 946, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x}+1\right)\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{1-x}\sqrt{x}\int \frac{1}{\left(\sqrt{x}+1\right)\sqrt{1-x}\sqrt{x}}dx}{\sqrt{x-x^2}}$$

$$\Big\downarrow \text{ 1388}$$

$$\frac{\sqrt{1-x}\sqrt{x}\int \frac{1}{\sqrt{1-\sqrt{x}}\left(\sqrt{x}+1\right)^{3/2}\sqrt{x}}dx}{\sqrt{x-x^2}}$$

$$\Big\downarrow \text{ 946}$$

$$\frac{2\sqrt{1-x}\sqrt{x}\int \frac{1}{\sqrt{1-\sqrt{x}}\left(\sqrt{x}+1\right)^{3/2}}d\sqrt{x}}{\sqrt{x-x^2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{2\sqrt{1-\sqrt{x}}\sqrt{1-x}\sqrt{x}}{\sqrt{\sqrt{x}+1}\sqrt{x-x^2}}$$

input
```
Int[1/((1 + Sqrt[x])*Sqrt[x - x^2]),x]
```

output
```
(-2*Sqrt[1 - Sqrt[x]]*Sqrt[1 - x]*Sqrt[x])/(Sqrt[1 + Sqrt[x]]*Sqrt[x - x^2
])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{2\sqrt{-(x-1)x}}{\sqrt{x}\,(x-1)}-\frac{2\sqrt{-(x-1)^{2}-x+1}}{x-1}$	41

input
```
int(1/(x^(1/2)+1)/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(x-1)*x)^(1/2)/x^(1/2)/(x-1)-2/(x-1)*(-(x-1)^2-x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}\,dx = -\frac{2\left(\sqrt{-x^2+x}x-\sqrt{-x^2+x}\sqrt{x}\right)}{x^2-x}$$

input
```
integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-x^2 + x)*x - sqrt(-x^2 + x)*sqrt(x))/(x^2 - x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{-x\left(x-1\right)}\left(\sqrt{x}+1\right)}\,dx$$

input
```
integrate(1/(1+x**(1/2))/(-x**2+x)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x*(x - 1))*(sqrt(x) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+x}\left(\sqrt{x}+1\right)}\,dx$$

input
```
integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + x)*(sqrt(x) + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx = \frac{4}{\frac{\sqrt{-x+1}-1}{\sqrt{x}}-1}$$

input
```
integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
4/((sqrt(-x + 1) - 1)/sqrt(x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{x-x^2}\,(\sqrt{x}+1)}\,dx$$

input
```
int(1/((x - x^2)^(1/2)*(x^(1/2) + 1)),x)
```

output
```
int(1/((x - x^2)^(1/2)*(x^(1/2) + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx = \frac{-2\sqrt{1-x}\,i+2\sqrt{x}-2}{\sqrt{1-x}}$$

input
```
int(1/(1+x^(1/2))/(-x^2+x)^(1/2),x)
```

output
```
(2*( - sqrt( - x + 1)*i + sqrt(x) - 1))/sqrt( - x + 1)
```

3.53 $\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10} \sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10} \sqrt{x}}\,dx = \frac{4}{9\left(1+\sqrt[4]{x}\right)^9} - \frac{1}{2\left(1+\sqrt[4]{x}\right)^8}$$

output
```
4/9/(1+x^(1/4))^9-1/2/(1+x^(1/4))^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10} \sqrt{x}}\,dx = \frac{-1-9\sqrt[4]{x}}{18\left(1+\sqrt[4]{x}\right)^9}$$

input
```
Integrate[1/((1 + x^(1/4))^10*Sqrt[x]),x]
```

output
```
(-1 - 9*x^(1/4))/(18*(1 + x^(1/4))^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt[4]{x}+1\right)^{10}\sqrt{x}}dx$$

$$\Big\downarrow \text{798}$$

$$4\int \frac{\sqrt[4]{x}}{\left(\sqrt[4]{x}+1\right)^{10}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{53}$$

$$4\int \left(\frac{1}{\left(\sqrt[4]{x}+1\right)^{9}}-\frac{1}{\left(\sqrt[4]{x}+1\right)^{10}}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{1}{9\left(\sqrt[4]{x}+1\right)^{9}}-\frac{1}{8\left(\sqrt[4]{x}+1\right)^{8}}\right)$$

input
```
Int[1/((1 + x^(1/4))^10*Sqrt[x]),x]
```

output
```
4*(1/(9*(1 + x^(1/4))^9) - 1/(8*(1 + x^(1/4))^8))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{4}{9\left(1+x^{\frac{1}{4}}\right)^{9}}-\frac{1}{2\left(1+x^{\frac{1}{4}}\right)^{8}}$	20
default	$\frac{4}{9\left(1+x^{\frac{1}{4}}\right)^{9}}-\frac{1}{2\left(1+x^{\frac{1}{4}}\right)^{8}}$	20
meijerg	$\frac{\sqrt{x}\left(x^{\frac{7}{4}}+9x^{\frac{3}{2}}+36x^{\frac{5}{4}}+84x+126x^{\frac{3}{4}}+126\sqrt{x}+84x^{\frac{1}{4}}+36\right)}{18\left(1+x^{\frac{1}{4}}\right)^{9}}$	46

input
```
int(1/(1+x^(1/4))^10/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/9/(1+x^(1/4))^9-1/2/(1+x^(1/4))^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.89

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx =$$
$$-\frac{9\,x^7+4209\,x^6+71109\,x^5+227277\,x^4+184587\,x^3+36099\,x^2-16\,(5\,x^6+648\,x^5+6813\,x^4+15288$$

input
```
integrate(1/(1+x^(1/4))^10/x^(1/2),x, algorithm="fricas")
```

output
```
-1/18*(9*x^7 + 4209*x^6 + 71109*x^5 + 227277*x^4 + 184587*x^3 + 36099*x^2
- 16*(5*x^6 + 648*x^5 + 6813*x^4 + 15288*x^3 + 8847*x^2 + 1152*x + 15)*x^(
3/4) + 4*(99*x^6 + 5544*x^5 + 38027*x^4 + 60552*x^3 + 24777*x^2 + 2064*x +
 9)*sqrt(x) - 32*(45*x^6 + 1311*x^5 + 6066*x^4 + 6894*x^3 + 1969*x^2 + 99*
x)*x^(1/4) + 999*x - 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4
 + 84*x^3 - 36*x^2 + 9*x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(22) = 44$.

Time = 15.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.11

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx = -\frac{\sqrt[4]{x}}{162x^{\frac{9}{4}}+1512x^{\frac{7}{4}}+2268x^{\frac{5}{4}}+648x^{\frac{3}{4}}+18\sqrt[4]{x}+18x^{\frac{5}{2}}+2268x^{\frac{3}{2}}+162\sqrt{x}+648x^2+1512x} - \frac{9\sqrt{x}}{162x^{\frac{9}{4}}+1512x^{\frac{7}{4}}+2268x^{\frac{5}{4}}+648x^{\frac{3}{4}}+18\sqrt[4]{x}+18x^{\frac{5}{2}}+2268x^{\frac{3}{2}}+162\sqrt{x}+648x^2+1512x}$$

input
```
integrate(1/(1+x**(1/4))**10/x**(1/2),x)
```

output
```
-x**(1/4)/(162*x**(9/4) + 1512*x**(7/4) + 2268*x**(5/4) + 648*x**(3/4) + 1
8*x**(1/4) + 18*x**(5/2) + 2268*x**(3/2) + 162*sqrt(x) + 648*x**2 + 1512*x
) - 9*sqrt(x)/(162*x**(9/4) + 1512*x**(7/4) + 2268*x**(5/4) + 648*x**(3/4)
 + 18*x**(1/4) + 18*x**(5/2) + 2268*x**(3/2) + 162*sqrt(x) + 648*x**2 + 15
12*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx = -\frac{1}{2\left(x^{\frac{1}{4}}+1\right)^8}+\frac{4}{9\left(x^{\frac{1}{4}}+1\right)^9}$$

input
```
integrate(1/(1+x^(1/4))^10/x^(1/2),x, algorithm="maxima")
```

output
```
-1/2/(x^(1/4) + 1)^8 + 4/9/(x^(1/4) + 1)^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx = -\frac{9\,x^{\frac{1}{4}}+1}{18\left(x^{\frac{1}{4}}+1\right)^{9}}$$

input
```
integrate(1/(1+x^(1/4))^10/x^(1/2),x, algorithm="giac")
```

output
```
-1/18*(9*x^(1/4) + 1)/(x^(1/4) + 1)^9
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx = -\frac{9\,x^{1/4}+1}{2268\,x+\sqrt{x}\,\left(1512\,x+648\right)+x^{3/4}\,\left(648\,x+1512\right)+x^{1/4}\,\left(18\,x^2+2268\,x+162\right)+162\,x^2+18}$$

input
```
int(1/(x^(1/2)*(x^(1/4) + 1)^10),x)
```

output
```
-(9*x^(1/4) + 1)/(2268*x + x^(1/2)*(1512*x + 648) + x^(3/4)*(648*x + 1512)
 + x^(1/4)*(2268*x + 18*x^2 + 162) + 162*x^2 + 18)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx$$
$$= \frac{-9x^{\frac{1}{4}}-1}{648x^{\frac{7}{4}}+1512x^{\frac{3}{4}}+18x^{\frac{9}{4}}+2268x^{\frac{5}{4}}+162x^{\frac{1}{4}}+1512\sqrt{x}\,x+648\sqrt{x}+162x^{2}+2268x+18}$$

input
```
int(1/(1+x^(1/4))^10/x^(1/2),x)
```

output
```
( - 9*x**(1/4) - 1)/(18*(36*x**(3/4)*x + 84*x**(3/4) + x**(1/4)*x**2 + 126
*x**(1/4)*x + 9*x**(1/4) + 84*sqrt(x)*x + 36*sqrt(x) + 9*x**2 + 126*x + 1)
)
```

3.54 $\int \sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\,dx$$
$$\Big\downarrow \text{211}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$
$$\Big\downarrow \text{223}$$
$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

input
```
int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx=\frac{1}{2}\sqrt{-x^2+1}x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 + asin(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \frac{x\sqrt{1-x^2}}{2}$$

input
```
int((1 - x^2)^(1/2),x)
```

output
```
asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \sqrt{1-x^2}\,dx = \frac{asin(x)}{2} + \frac{\sqrt{-x^2+1}\,x}{2}$$

input
```
int((-x^2+1)^(1/2),x)
```

output
```
(asin(x) + sqrt( - x**2 + 1)*x)/2
```

3.55 $\int \frac{1}{\sqrt{1-4x-x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = \arcsin\left(\frac{2+x}{\sqrt{5}}\right)$$

output
```
arcsin(1/5*(2+x)*5^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = 2\arctan\left(\frac{x}{-1+\sqrt{1-4x-x^2}}\right)$$

input
```
Integrate[1/Sqrt[1 - 4*x - x^2],x]
```

output
```
2*ArcTan[x/(-1 + Sqrt[1 - 4*x - x^2])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-4x+1}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{20}(-2x-4)^2}}d(-2x-4)}{2\sqrt{5}}$$

$$\Big\downarrow \text{223}$$

$$-\arcsin\left(\frac{-2x-4}{2\sqrt{5}}\right)$$

input
```
Int[1/Sqrt[1 - 4*x - x^2],x]
```

output
```
-ArcSin[(-4 - 2*x)/(2*Sqrt[5])]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\arcsin\left(\frac{(x+2)\sqrt{5}}{5}\right)$	10
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-4x+1}-2\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	39

input
```
int(1/(-x^2-4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/5*(x+2)*5^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2-4\,x+1}-1}{x}\right)$$

input
```
integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-x^2 - 4*x + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{5}(x+2)}{5}\right)$$

input
```
integrate(1/(-x**2-4*x+1)**(1/2),x)
```

output
```
asin(sqrt(5)*(x + 2)/5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = -\arcsin\left(-\frac{1}{5}\sqrt{5}(x+2)\right)$$

input
```
integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/5*sqrt(5)*(x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(9) = 18.

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-4\,x+1}(x+2)+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}(x+2)\right)$$

input
```
integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 4*x + 1)*(x + 2) + 5/2*arcsin(1/5*sqrt(5)*(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = \operatorname{asin}\left(\frac{\sqrt{20}\,(2\,x+4)}{20}\right)$$

input
```
int(1/(1 - x^2 - 4*x)^(1/2),x)
```

output
```
asin((20^(1/2)*(2*x + 4))/20)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-4x-x^2}}\,dx = asin\left(\frac{x+2}{\sqrt{5}}\right)$$

input
```
int(1/(-x^2-4*x+1)^(1/2),x)
```

output
```
asin((x + 2)/sqrt(5))
```

3.56 $\int \log \left(\frac{1}{x}\right) dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int \log \left(\frac{1}{x}\right) dx = x + x \log \left(\frac{1}{x}\right)$$

output
```
x+x*ln(1/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1}{x}\right) dx = x + x \log \left(\frac{1}{x}\right)$$

input
```
Integrate[Log[x^(-1)],x]
```

output
```
x + x*Log[x^(-1)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{2732}$$

$$x + x\log\left(\frac{1}{x}\right)$$

input
```
Int[Log[x^(-1)],x]
```

output
```
x + x*Log[x^(-1)]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$x+x\ln\left(\frac{1}{x}\right)$	9
default	$x+x\ln\left(\frac{1}{x}\right)$	9
norman	$x+x\ln\left(\frac{1}{x}\right)$	9
risch	$x+x\ln\left(\frac{1}{x}\right)$	9
parallelrisch	$x+x\ln\left(\frac{1}{x}\right)$	9
parts	$x+x\ln\left(\frac{1}{x}\right)$	9
orering	$x+x\ln\left(\frac{1}{x}\right)$	9

input
```
int(ln(1/x),x,method=_RETURNVERBOSE)
```

output
```
x+x*ln(1/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{1}{x}\right)\,dx = x\log\left(\frac{1}{x}\right) + x$$

input
```
integrate(log(1/x),x, algorithm="fricas")
```

output
```
x*log(1/x) + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \log\left(\frac{1}{x}\right) dx = x\log\left(\frac{1}{x}\right) + x$$

input
```
integrate(ln(1/x),x)
```

output
```
x*log(1/x) + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \log\left(\frac{1}{x}\right) dx = -x\log(x) + x$$

input
```
integrate(log(1/x),x, algorithm="maxima")
```

output
```
-x*log(x) + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \log\left(\frac{1}{x}\right) dx = -x\log(x) + x$$

input
```
integrate(log(1/x),x, algorithm="giac")
```

output
```
-x*log(x) + x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{1}{x}\right) dx = x\left(\ln\left(\frac{1}{x}\right) + 1\right)$$

input
```
int(log(1/x),x)
```

output
```
x*(log(1/x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log\left(\frac{1}{x}\right) dx = x(-\log(x) + 1)$$

input
```
int(log(1/x),x)
```

output
```
x*( - log(x) + 1)
```

3.57 $\int \frac{1}{1+\sin(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos(x)}{1+\sin(x)}$$

output
```
-cos(x)/(1+sin(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{1+\sin(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(1 + Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+1} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(x)+1} dx$$

$$\Big\downarrow \ 3127$$

$$-\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[(1 + Sin[x])^(-1),x]
```

output
```
-(Cos[x]/(1 + Sin[x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
parallelrisch	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

input
```
int(1/(1+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(1+tan(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos(x) - \sin(x) + 1}{\cos(x) + \sin(x) + 1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="fricas")
```

output
```
-(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

input
```
integrate(1/(1+sin(x)),x)
```

output
```
-2/(tan(x/2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 1),x)
```

output
```
-2/(tan(x/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{1+\sin(x)}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(1+sin(x)),x)
```

output
```
(2*tan(x/2))/(tan(x/2) + 1)
```

3.58 $\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = -\frac{1}{2}\sqrt{2012-x}\sqrt{x}+\frac{x}{2} + 503\text{arctanh}\left(\frac{\sqrt{2012-x}\sqrt{x}}{1006}\right)+503\log(1006-x)$$

output
```
-1/2*(2012-x)^(1/2)*x^(1/2)+1/2*x+503*arctanh(1/1006*(2012-x)^(1/2)*x^(1/2
))+503*ln(1006-x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = \frac{1}{2}\left(x-\sqrt{-((-2012+x)x)}+2012\log\left(2\sqrt{503}-\sqrt{x}\right) + 2012\log\left(\sqrt{2012-x}+\sqrt{x}\right) - 2012\log\left(-1006+\sqrt{503}\sqrt{x}\right)\right)$$

input
```
Integrate[Sqrt[x]/(Sqrt[2012 - x] + Sqrt[x]),x]
```

output
```
(x - Sqrt[-((-2012 + x)*x)] + 2012*Log[2*Sqrt[503] - Sqrt[x]] + 2012*Log[S
qrt[2012 - x] + Sqrt[x]] - 2012*Log[-1006 + Sqrt[503]*Sqrt[x]])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2035, 2532, 27, 243, 49, 380, 27, 291, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \frac{x}{\sqrt{2012-x}+\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2532}$$

$$2\left(\int \frac{\sqrt{2012-x}x}{2(1006-x)}d\sqrt{x}-\int \frac{x^{3/2}}{2(1006-x)}d\sqrt{x}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\int \frac{\sqrt{2012-x}x}{1006-x}d\sqrt{x}-\frac{1}{2}\int \frac{x^{3/2}}{1006-x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{243}$$

$$2\left(\frac{1}{2}\int \frac{\sqrt{2012-x}x}{1006-x}d\sqrt{x}-\frac{1}{4}\int \frac{x}{1006-x}dx\right)$$

$$\Big\downarrow \text{49}$$

$$2\left(\frac{1}{2}\int \frac{\sqrt{2012-x}x}{1006-x}d\sqrt{x}-\frac{1}{4}\int \left(-1-\frac{1006}{x-1006}\right)dx\right)$$

$$\Big\downarrow \text{380}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{2024072}{(1006-x)\sqrt{2012-x}}d\sqrt{x}-\frac{1}{2}\sqrt{2012-x}\sqrt{x}\right)-\frac{1}{4}\int \left(-1-\frac{1006}{x-1006}\right)dx\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(1012036\int \frac{1}{(1006-x)\sqrt{2012-x}}d\sqrt{x}-\frac{1}{2}\sqrt{2012-x}\sqrt{x}\right)-\frac{1}{4}\int \left(-1-\frac{1006}{x-1006}\right)dx\right)$$

$$\Big\downarrow \text{291}$$

$$2\left(\frac{1}{2}\left(1012036\int \frac{1}{1006-1006x}d\frac{\sqrt{x}}{\sqrt{2012-x}}-\frac{1}{2}\sqrt{2012-x}\sqrt{x}\right)-\frac{1}{4}\int \left(-1-\frac{1006}{x-1006}\right)dx\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(1006\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{2012-x}}\right)-\frac{1}{2}\sqrt{2012-x}\sqrt{x}\right)-\frac{1}{4}\int \left(-1-\frac{1006}{x-1006}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{2}\left(1006\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{2012-x}}\right)-\frac{1}{2}\sqrt{2012-x}\sqrt{x}\right)+\frac{1}{4}(x+1006\log(1006-x))\right)$$

input
```
Int[Sqrt[x]/(Sqrt[2012 - x] + Sqrt[x]),x]
```

output
```
2*((-1/2*(Sqrt[2012 - x]*Sqrt[x]) + 1006*ArcTanh[Sqrt[x]/Sqrt[2012 - x]])/
2 + (x + 1006*Log[1006 - x])/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2532
```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Simp[-d   Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
 + Simp[c   Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)),
x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{2}+503\ln\left(x-1006\right)-\frac{\left(\sqrt{-x(-2012+x)}-1006\,\operatorname{arctanh}\left(\frac{1006}{\sqrt{-x(-2012+x)}}\right)\right)\sqrt{x}\,\sqrt{2012-x}}{2\sqrt{-x(-2012+x)}}$	53

input
```
int(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+503*ln(x-1006)-1/2*((-x*(-2012+x))^(1/2)-1006*arctanh(1006/(-x*(-201
2+x))^(1/2)))*x^(1/2)*(2012-x)^(1/2)/(-x*(-2012+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2012} + 503\,\log\left(x-1006\right) + 503\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2012}}{x}\right) - 503\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2012}}{x}\right)$$

input
```
integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*sqrt(x)*sqrt(-x + 2012) + 503*log(x - 1006) + 503*log((x + sqr
t(x)*sqrt(-x + 2012))/x) - 503*log(-(x - sqrt(x)*sqrt(-x + 2012))/x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2012-x}}\,dx$$

input
```
integrate(x**(1/2)/((2012-x)**(1/2)+x**(1/2)),x)
```

output
```
Integral(sqrt(x)/(sqrt(x) + sqrt(2012 - x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x}+\sqrt{-x+2012}}\,dx$$

input
```
integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(sqrt(x) + sqrt(-x + 2012)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(39) = 78$.

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx \\
&= \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2012} + 503\,\log\left(|x-1006|\right) \\
&\quad + 503\,\log\left(\left|-\frac{2\,\sqrt{503}-\sqrt{-x+2012}}{\sqrt{x}} + \frac{\sqrt{x}}{2\,\sqrt{503}-\sqrt{-x+2012}} + 2\right|\right) \\
&\quad - 503\,\log\left(\left|-\frac{2\,\sqrt{503}-\sqrt{-x+2012}}{\sqrt{x}} + \frac{\sqrt{x}}{2\,\sqrt{503}-\sqrt{-x+2012}} - 2\right|\right)
\end{aligned}$$

input
```
integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x - 1/2*sqrt(x)*sqrt(-x + 2012) + 503*log(abs(x - 1006)) + 503*log(abs
(-(2*sqrt(503) - sqrt(-x + 2012))/sqrt(x) + sqrt(x)/(2*sqrt(503) - sqrt(-x
 + 2012)) + 2)) - 503*log(abs(-(2*sqrt(503) - sqrt(-x + 2012))/sqrt(x) + s
qrt(x)/(2*sqrt(503) - sqrt(-x + 2012)) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = \frac{x}{2} + 1006\,\mathrm{atanh}\left(\frac{2\sqrt{503}\sqrt{x}-\sqrt{x}\sqrt{2012-x}}{x+2\sqrt{503}\sqrt{2012-x}-2012}\right) + 503\,\ln(x-1006) - \frac{\sqrt{x}\sqrt{2012-x}}{2}$$

input
```
int(x^(1/2)/((2012 - x)^(1/2) + x^(1/2)),x)
```

output
```
x/2 + 1006*atanh((2*503^(1/2)*x^(1/2) - x^(1/2)*(2012 - x)^(1/2))/(x + 2*5
03^(1/2)*(2012 - x)^(1/2) - 2012)) + 503*log(x - 1006) - (x^(1/2)*(2012 -
x)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}}\,dx = -\frac{\sqrt{x}\sqrt{-x+2012}}{2} - 1006\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{x}}{2\sqrt{503}}\right)}{2}\right)^2+1\right) + 1006\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{x}}{2\sqrt{503}}\right)}{2}\right)-1\right) + 1006\log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{x}}{2\sqrt{503}}\right)}{2}\right)-1\right) + \frac{x}{2}$$

input
```
int(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x)
```

output
```
( - sqrt(x)*sqrt( - x + 2012) - 2012*log(tan(asin(sqrt(x)/(2*sqrt(503)))/2
)**2 + 1) + 2012*log( - sqrt(2) + tan(asin(sqrt(x)/(2*sqrt(503)))/2) - 1)
+ 2012*log(sqrt(2) + tan(asin(sqrt(x)/(2*sqrt(503)))/2) - 1) + x)/2
```

3.59 $\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}} \, dx = -\frac{2\sqrt{x}\sqrt{1+x+x^2}\arctan\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}$$

output
```
-2*x^(1/2)*(x^2+x+1)^(1/2)*arctan(x^(1/2)/(x^2+x+1)^(1/2))/(x^3+x^2+x)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}} \, dx = -\frac{2\sqrt{x}\sqrt{1+x+x^2}\arctan\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x(1+x+x^2)}}$$

input
```
Integrate[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/Sqrt[x*(1
 + x + x^2)]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2467, 25, 2035, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{x^2+x+1}\int -\frac{1-x}{\sqrt{x}(x+1)\sqrt{x^2+x+1}}dx}{\sqrt{x^3+x^2+x}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{x}\sqrt{x^2+x+1}\int \frac{1-x}{\sqrt{x}(x+1)\sqrt{x^2+x+1}}dx}{\sqrt{x^3+x^2+x}}$$

$$\Big\downarrow \text{2035}$$

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\int \frac{1-x}{(x+1)\sqrt{x^2+x+1}}d\sqrt{x}}{\sqrt{x^3+x^2+x}}$$

$$\Big\downarrow \text{2212}$$

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\int \frac{1}{x+1}d\frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{\sqrt{x^3+x^2+x}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{\sqrt{x^3+x^2+x}}$$

input
```
Int[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/Sqrt[x +
x^2 + x^3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

method	result
default	$2\arctan\left(\frac{\sqrt{(x^2+x+1)x}}{x}\right)$
pseudoelliptic	$2\arctan\left(\frac{\sqrt{(x^2+x+1)x}}{x}\right)$
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2+\operatorname{RootOf}\left(\_Z^2+1\right)-2\sqrt{x^3+x^2+x}}{(x+1)^2}\right)$
elliptic	$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}-\frac{4\left(\frac{1}{2}+i\right.}{}$

input
```
int((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(((x^2+x+1)*x)^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = -\arctan\left(\frac{2\sqrt{x^3+x^2+x}}{x^2+1}\right)$$

input
```
integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(x^3 + x^2 + x)/(x^2 + 1))
```

Sympy [F]

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = \int \frac{x-1}{\sqrt{x\left(x^2+x+1\right)}\left(x+1\right)}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x**3+x**2+x)**(1/2),x)
```

output
```
Integral((x - 1)/(sqrt(x*(x**2 + x + 1))*(x + 1)), x)
```

Maxima [F]

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = \int \frac{x-1}{\sqrt{x^3+x^2+x}(x+1)}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

Giac [F]

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = \int \frac{x-1}{\sqrt{x^3+x^2+x}(x+1)}\,dx$$

input
```
integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.89

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx$$
$$= \frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\sqrt{3}+1i\right)\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-2\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}\,1i}{2};\mathrm{as}\right.\right.}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)x}}$$

input
```
int((x - 1)/((x + 1)*(x + x^2 + x^3)^(1/2)),x)
```

output
```
((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/
2)*(3^(1/2) + 1i)*(ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^
(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - 2*ellipticPi(1/2 - (3^(1/2)*1
i)/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 + 1/2)))*1i)/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{x^2+x+1}}{x^4+2x^3+2x^2+x}dx\right)+\int \frac{\sqrt{x}\,\sqrt{x^2+x+1}}{x^3+2x^2+2x+1}dx$$

input
```
int((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x**2 + x + 1))/(x**4 + 2*x**3 + 2*x**2 + x),x) + int(
(sqrt(x)*sqrt(x**2 + x + 1))/(x**3 + 2*x**2 + 2*x + 1),x)
```

3.60 $\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 37

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = -\frac{8}{(1-x)^2}+\frac{32}{1-x}+7x+\frac{x^2}{2}+24\log(1-x)$$

output
```
-8/(1-x)^2+32/(1-x)+7*x+1/2*x^2+24*ln(1-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = -\frac{8}{(-1+x)^2}-\frac{32}{-1+x}+8(-1+x)+\frac{1}{2}(-1+x)^2+24\log(-1+x)$$

input
```
Integrate[(1 + 4*x + 6*x^2 + 4*x^3 + x^4)/(-1 + 3*x - 3*x^2 + x^3),x]
```

output
```
-8/(-1 + x)^2 - 32/(-1 + x) + 8*(-1 + x) + (-1 + x)^2/2 + 24*Log[-1 + x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2006, 2007, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+4x^3+6x^2+4x+1}{x^3-3x^2+3x-1} \, dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{(x+1)^4}{x^3-3x^2+3x-1}dx$$

$$\Big\downarrow \text{ 2007}$$

$$\int \frac{(x+1)^4}{(x-1)^3}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(x+\frac{24}{x-1}+\frac{32}{(x-1)^2}+\frac{16}{(x-1)^3}+7\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}+7x+\frac{32}{1-x}-\frac{8}{(1-x)^2}+24\log(1-x)$$

input
```
Int[(1 + 4*x + 6*x^2 + 4*x^3 + x^4)/(-1 + 3*x - 3*x^2 + x^3),x]
```

output
```
-8/(1 - x)^2 + 32/(1 - x) + 7*x + x^2/2 + 24*Log[1 - x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
norman	$\frac{-52x+6x^3+\frac{1}{2}x^4+\frac{75}{2}}{(x-1)^2}+24\ln(x-1)$	29
default	$\frac{x^2}{2}+7x-\frac{32}{x-1}-\frac{8}{(x-1)^2}+24\ln(x-1)$	30
risch	$\frac{x^2}{2}+7x+\frac{-32x+24}{x^2-2x+1}+24\ln(x-1)$	32
parallelrisch	$\frac{x^4+48x^2\ln(x-1)+12x^3+75-96\ln(x-1)x+48\ln(x-1)-104x}{2x^2-4x+2}$	48

input
```
int((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x,method=_RETURNVERBOSE)
```

output
```
(-52*x+6*x^3+1/2*x^4+75/2)/(x-1)^2+24*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = \frac{x^4+12\,x^3-27\,x^2+48\,(x^2-2\,x+1)\log(x-1)-50\,x+48}{2\,(x^2-2\,x+1)}$$

input
```
integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
```

output
```
1/2*(x^4 + 12*x^3 - 27*x^2 + 48*(x^2 - 2*x + 1)*log(x - 1) - 50*x + 48)/(x
^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = \frac{x^2}{2}+7x+\frac{24-32x}{x^2-2x+1}+24\log(x-1)$$

input
```
integrate((x**4+4*x**3+6*x**2+4*x+1)/(x**3-3*x**2+3*x-1),x)
```

output
```
x**2/2 + 7*x + (24 - 32*x)/(x**2 - 2*x + 1) + 24*log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = \frac{1}{2}\,x^2+7\,x-\frac{8\,(4\,x-3)}{x^2-2\,x+1}+24\,\log(x-1)$$

input
```
integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 7*x - 8*(4*x - 3)/(x^2 - 2*x + 1) + 24*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = \frac{1}{2}\,x^2+7\,x-\frac{8\,(4\,x-3)}{(x-1)^2}+24\,\log\left(|x-1|\right)$$

input
```
integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
```

output
```
1/2*x^2 + 7*x - 8*(4*x - 3)/(x - 1)^2 + 24*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = 7\,x+24\,\ln\left(x-1\right)-\frac{32\,x-24}{x^2-2\,x+1}+\frac{x^2}{2}$$

input
```
int((4*x + 6*x^2 + 4*x^3 + x^4 + 1)/(3*x - 3*x^2 + x^3 - 1),x)
```

output
```
7*x + 24*log(x - 1) - (32*x - 24)/(x^2 - 2*x + 1) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3}\,dx = \frac{48\log(x-1)\,x^2-96\log(x-1)\,x+48\log(x-1)+x^4+12x^3-52x^2+23}{2x^2-4x+2}$$

input
```
int((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x)
```

output
```
(48*log(x - 1)*x**2 - 96*log(x - 1)*x + 48*log(x - 1) + x**4 + 12*x**3 - 5
2*x**2 + 23)/(2*(x**2 - 2*x + 1))
```

3.61 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

output
```
ln(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx=\log(x)\sin(x)$$

input
```
Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
```

output
```
Log[x]*Sin[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\sin(x)}{x}+\log(x)\cos(x)\right)dx$$
$$\Big\downarrow \; 2009$$
$$\log(x)\sin(x)$$

input
```
Int[Cos[x]*Log[x] + Sin[x]/x,x]
```

output
```
Log[x]*Sin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result
risch	$\ln(x)\sin(x)$
parallelrisch	$\ln(x)\sin(x)$
norman	$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}$
orering	$-\frac{4x(2x^{2}-1)\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)}{4x^{4}-5x^{2}-2}-\frac{2(4x^{4}-3x^{2}-1)\left(-\ln(x)\sin(x)+\frac{2\cos(x)}{x}-\frac{\sin(x)}{x^{2}}\right)}{4x^{4}-5x^{2}-2}-\frac{4x(2x^{2}-1)\left(-\cos(x)\ln(x)-\right.}{4x^{4}-5}$

input
```
int(cos(x)*ln(x)+sin(x)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx = \log\left(x\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
```

output
```
log(x)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right) dx = \log\left(x\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*ln(x)+sin(x)/x,x)
```

output
```
log(x)*sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right)\,dx = \log{(x)}\sin{(x)}$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
```

output
```
log(x)*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right)\,dx = \log{(x)}\sin{(x)}$$

input
```
integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
```

output
```
log(x)*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \left(\cos(x)\log(x) + \frac{\sin(x)}{x}\right)\,dx = \ln{(x)}\,\sin{(x)}$$

input
```
int(cos(x)*log(x) + sin(x)/x,x)
```

output
```
log(x)*sin(x)
```

Reduce [F]

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)dx$$
$$=\frac{-2\left(\int \frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2x+x}dx\right)\tan\left(\frac{x}{2}\right)^2-2\left(\int \frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2x+x}dx\right)+2\log(x)\tan\left(\frac{x}{2}\right)+si(x)\tan\left(\frac{x}{2}\right)^2+si(x)}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cos(x)*log(x)+sin(x)/x,x)
```

output
```
( - 2*int(tan(x/2)/(tan(x/2)**2*x + x),x)*tan(x/2)**2 - 2*int(tan(x/2)/(ta
n(x/2)**2*x + x),x) + 2*log(x)*tan(x/2) + si(x)*tan(x/2)**2 + si(x))/(tan(
x/2)**2 + 1)
```

3.62 $\int \frac{1}{-x+x^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{-x+x^3} \, dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

output
```
-ln(x)+1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3} \, dx = -\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[(-x + x^3)^(-1),x]
```

output
```
-Log[x] + Log[1 - x^2]/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2026, 243, 25, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3-x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x(x^2-1)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int -\frac{1}{x^2(1-x^2)}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^2(1-x^2)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(-\int \frac{1}{x^2}dx^2-\int \frac{1}{1-x^2}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-\int \frac{1}{1-x^2}dx^2-\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(1-x^2\right)-\log\left(x^2\right)\right)$$

input
```
Int[(-x + x^3)^(-1),x]
```

output
```
(-Log[x^2] + Log[1 - x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
risch	$-\ln \left(x \right)+\frac{\ln \left(x^{2}-1\right)}{2}$	14
default	$-\ln \left(x \right)+\frac{\ln \left(x +1\right)}{2}+\frac{\ln \left(x -1\right)}{2}$	18
norman	$-\ln \left(x \right)+\frac{\ln \left(x +1\right)}{2}+\frac{\ln \left(x -1\right)}{2}$	18
parallelrisch	$-\ln \left(x \right)+\frac{\ln \left(x +1\right)}{2}+\frac{\ln \left(x -1\right)}{2}$	18
meijerg	$-\ln \left(x \right)-\frac{i \pi}{2}+\frac{\ln \left(-x^{2}+1\right)}{2}$	20

input
```
int(1/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx = \frac{1}{2}\log\left(x^2-1\right)-\log\left(x\right)$$

input
```
integrate(1/(x^3-x),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{-x+x^3}\,dx = -\log\left(x\right)+\frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(1/(x**3-x),x)
```

output
```
-log(x) + log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3}\,dx = \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

input
```
integrate(1/(x^3-x),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{-x+x^3}\,dx = -\frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(1/(x^3-x),x, algorithm="giac")
```

output
```
-1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{-x+x^3}\,dx = \frac{\ln\left(x^2-1\right)}{2} - \ln(x)$$

input
```
int(-1/(x - x^3),x)
```

output
```
log(x^2 - 1)/2 - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+x^3}\,dx = \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \log(x)$$

input
```
int(1/(x^3-x),x)
```

output
```
(log(x - 1) + log(x + 1) - 2*log(x))/2
```

3.63 $\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx = x - \sqrt{1-x^2} \arcsin(x)$$

output
```
x-(-x^2+1)^(1/2)*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx = x - \sqrt{1-x^2} \arcsin(x)$$

input
```
Integrate[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x - Sqrt[1 - x^2]*ArcSin[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx$$

$$\Big\downarrow \text{ 5182}$$

$$\int 1 dx - \sqrt{1-x^2} \arcsin(x)$$

$$\Big\downarrow \text{ 24}$$

$$x - \sqrt{1-x^2} \arcsin(x)$$

input
```
Int[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

output
```
x - Sqrt[1 - x^2]*ArcSin[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x-\sqrt{-x^{2}+1}\, \arcsin \left(x \right)$	16
orering	$\frac{\left(x^{2}-2\right) \arcsin \left(x \right)}{\sqrt{-x^{2}+1}}-\left(x -1\right) \left(x +1\right) \left(\frac{\arcsin \left(x \right)}{\sqrt{-x^{2}+1}}+\frac{x}{-x^{2}+1}+\frac{x^{2} \arcsin \left(x \right)}{\left(-x^{2}+1\right)^{\frac{3}{2}}}\right)$	66

input
```
int(x*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x-(-x^2+1)^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\, dx = -\sqrt{-x^{2} + 1} \arcsin\left(x\right) + x$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1)*arcsin(x) + x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\, dx = x - \sqrt{1 - x^{2}} \operatorname{asin}{\left(x \right)}$$

input
```
integrate(x*asin(x)/(-x**2+1)**(1/2),x)
```

output
```
x - sqrt(1 - x**2)*asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx = -\sqrt{-x^2+1} \arcsin(x) + x$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)*arcsin(x) + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx = -\sqrt{-x^2+1} \arcsin(x) + x$$

input
```
integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)*arcsin(x) + x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} \, dx = \int \frac{x \operatorname{asin}(x)}{\sqrt{1-x^2}} \, dx$$

input
```
int((x*asin(x))/(1 - x^2)^(1/2),x)
```

output
```
int((x*asin(x))/(1 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}\,asin(x) + x$$

input
```
int(x*asin(x)/(-x^2+1)^(1/2),x)
```

output
```
- sqrt( - x**2 + 1)*asin(x) + x
```

3.64 $\int (1-x)^{99} x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int (1-x)^{99} x \, dx = -\frac{1}{100}(1-x)^{100} + \frac{1}{101}(1-x)^{101}$$

output
```
-1/100*(1-x)^100+1/101*(1-x)^101
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 567 vs. 2(23) = 46.

Time = 0.01 (sec) , antiderivative size = 567, normalized size of antiderivative = 24.65

$$\int (1-x)^{99} x \, dx = \text{Too large to display}$$

input
```
Integrate[(1 - x)^99*x,x]
```

output
```
x^2/2 - 33*x^3 + (4851*x^4)/4 - (156849*x^5)/5 + 627396*x^6 - 10217592*x^7
 + 140066157*x^8 - 1654114616*x^9 + (85600431378*x^10)/5 - 157366449604*x^
11 + 1298273209233*x^12 - 9696194317908*x^13 + 66026466069564*x^14 - (2062
057324941768*x^15)/5 + (4750096337812287*x^16)/2 - 12666923567499432*x^17
+ 62806829355518017*x^18 - 290505891817783026*x^19 + (12572449429225165403
*x^20)/10 - 5104603527655330314*x^21 + 19490304378320352108*x^22 - 7013281
3684308016488*x^23 + 238292173768273828749*x^24 - (19146258135816088501224
*x^25)/25 + 2331916055003241548226*x^26 - 6736646381120475583764*x^27 + 18
488763007525700846649*x^28 - 48264408157576669898532*x^29 + (5998576442441
67183024612*x^30)/5 - 284248449886557530554488*x^31 + (2570079734390957672
096829*x^32)/4 - 1386787891870780679371896*x^33 + (57205000539669703024090
71*x^34)/2 - (28206275157871771317939099*x^35)/5 + (4258594484619855669571
1973*x^36)/4 - 19237666204653402059452899*x^37 + 3330028769928308192747402
4*x^38 - 55246631151825154632274992*x^39 + (439428796464188236515924114*x^
40)/5 - 134109601422466453670901168*x^41 + 196374773511468735732390996*x^4
2 - 276016272695076305811040776*x^43 + 372502480574415750374856978*x^44 -
(2414047083412492769871166152*x^45)/5 + 601126348833940887359223192*x^46 -
 719077854633508484642475024*x^47 + 826548729646668720118931889*x^48 - 913
043842041304917057126672*x^49 + (24233705307512968006891237086*x^50)/25 -
989130828878080326811887228*x^51 + 970109082168886474373197089*x^52 - 9...
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{99} x \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((1-x)^{99}-(1-x)^{100}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{101}(1-x)^{101}-\frac{1}{100}(1-x)^{100}$$

input
```
Int[(1 - x)^99*x,x]
```

output
```
-1/100*(1 - x)^100 + (1 - x)^101/101
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 501, normalized size of antiderivative = 21.78

method	result	size
gosper	Expression too large to display	501
default	Expression too large to display	502
risch	Expression too large to display	502
parallelrisch	Expression too large to display	502
orering	Expression too large to display	513

input
```
int((1-x)^99*x,x,method=_RETURNVERBOSE)
```

output
```
-1/10100*x^2*(100*x^99-9999*x^98+494900*x^97-16165050*x^96+391960800*x^95-
7524830775*x^94+119129952480*x^93-1599564027600*x^92+18592781869200*x^91-1
90037092945700*x^90+1729128713835600*x^89-14145670154903560*x^88+104900475
306026400*x^87-710003828928813300*x^86+4411583725232560800*x^85-2528966019
2321540400*x^84+134332724433331476360*x^83-663667626664673365350*x^82+3059
800945425883628200*x^81-13203492783118238531700*x^80+534659954674417560296
00*x^79-203648157735809402877030*x^78+731164847106703494794400*x^77-247919
4963740288382850800*x^76+7952742286283782215118800*x^75-241721508964678117
32795300*x^74+69714962380376909325764496*x^73-191035745261543332611892200*
x^72+497964017002692209469746400*x^71-1236085967492793928008474900*x^70+29
24823134349146195851039200*x^69-6603091490864731434780756240*x^68+14234925
496610562332226630300*x^67-29326230200904915179092563225*x^66+577770505450
66400054331617100*x^65-108925997889075399236629520550*x^64+196625391061305
336057915852480*x^63-340025750167248881400829989825*x^62+5635848691159747 5
4134709022400*x^61-895722354016828362340647962400*x^60+1365609490550246519
634102631200*x^59-1997897787191193993562250150280*x^58+2805764446902887448
586210864800*x^57-3783394480727510220367051779600*x^56+4899707046811384846
791142017600*x^55-6095468885616544243924694533800*x^54+7285651347867363554
793785087040*x^53-8367877730115588952806888703200*x^52+9236242400221923655
456660172400*x^51-9798101729905753391169290598900*x^50+9990221371668611...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 501, normalized size of antiderivative = 21.78

$$\int (1-x)^{99}x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)^99*x,x, algorithm="fricas")
```

output
```
-1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131
/4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757
*x^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7
0297408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 665013
48729372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 +
 1307276513180023617*x^82 - 5293662917568490696*x^81 + 2016318393423855474
03/10*x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 7874
00226364730912388*x^77 + 2393282266977011062653*x^76 - 1725617880702398745
68724/25*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^7
3 + 122384749256712270099849*x^72 - 289586448945460019391192*x^71 + 326885
7173695411601376612/5*x^70 - 1409398564020847755666003*x^69 + 116143485944
17788189739629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178
066659311/2*x^66 - 97339302505596701018770224/5*x^65 + 1346636634325738144
16170293/4*x^64 - 55800482090690569716307824*x^63 + 8868538158582459033075
7224*x^62 - 135208860450519457389515112*x^61 + 989058310490690095822896114
/5*x^60 - 277798460089394796889723848*x^59 + 374593512943317843600698196*x
^58 - 485119509585285628395162576*x^57 + 603511770853123192467791538*x^56
- 3606758093003645324155339152/5*x^55 + 828502745555998906218503832*x^54 -
 914479445566527094599669324*x^53 + 970109082168886474373197089*x^52 - 989
130828878080326811887228*x^51 + 24233705307512968006891237086/25*x^50 -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 24.39

$$\int (1-x)^{99}x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)**99*x,x)
```

output

```
-x**101/101 + 99*x**100/100 - 49*x**99 + 3201*x**98/2 - 38808*x**97 + 2980
131*x**96/4 - 58975224*x**95/5 + 158372676*x**94 - 1840869492*x**93 + 1881
5553757*x**92 - 171200862756*x**91 + 7002807007378*x**90/5 - 1038618567386
4*x**89 + 70297408804833*x**88 - 436790467844808*x**87 + 2503926751715004*
x**86 - 66501348729372018*x**85/5 + 131419332012806607*x**84/2 - 302950588
656028082*x**83 + 1307276513180023617*x**82 - 5293662917568490696*x**81 +
201631839342385547403*x**80/10 - 72392559119475593544*x**79 + 245464847895
078057708*x**78 - 787400226364730912388*x**77 + 2393282266977011062653*x**
76 - 172561788070239874568724*x**75/25 + 18914430223915181446722*x**74 - 4
9303368020068535591064*x**73 + 122384749256712270099849*x**72 - 2895864489
45460019391192*x**71 + 3268857173695411601376612*x**70/5 - 140939856402084
7755666003*x**69 + 11614348594417788189739629*x**68/4 - 572050005396697030
2409071*x**67 + 21569504532490178066659311*x**66/2 - 973393025055967010187
70224*x**65/5 + 134663663432573814416170293*x**64/4 - 55800482090690569716
307824*x**63 + 88685381585824590330757224*x**62 - 135208860450519457389515
112*x**61 + 989058310490690095822896114*x**60/5 - 277798460089394796889723
848*x**59 + 374593512943317843600698196*x**58 - 48511950958528562839516257
6*x**57 + 603511770853123192467791538*x**56 - 3606758093003645324155339152
*x**55/5 + 828502745555998906218503832*x**54 - 914479445566527094599669324
*x**53 + 970109082168886474373197089*x**52 - 98913082887808032681188722...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 501, normalized size of antiderivative = 21.78

$$\int (1-x)^{99} x \, dx = \text{Too large to display}$$

input

```
integrate((1-x)^99*x,x, algorithm="maxima")
```

output
```
-1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131
/4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757
*x^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7
0297408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 665013
48729372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 +
 1307276513180023617*x^82 - 5293662917568490696*x^81 + 201631839342385547403/10*x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 7874
00226364730912388*x^77 + 2393282266977011062653*x^76 - 172561788070239874568724/25*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^7
3 + 122384749256712270099849*x^72 - 289586448945460019391192*x^71 + 326885
7173695411601376612/5*x^70 - 1409398564020847755666003*x^69 + 116143485944
17788189739629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178
066659311/2*x^66 - 97339302505596701018770224/5*x^65 + 134663663432573814416170293/4*x^64 - 55800482090690569716307824*x^63 + 886853815858245903307
57224*x^62 - 135208860450519457389515112*x^61 + 989058310490690095822896114
/5*x^60 - 277798460089394796889723848*x^59 + 374593512943317843600698196*x
^58 - 485119509585285628395162576*x^57 + 603511770853123192467791538*x^56
- 3606758093003645324155339152/5*x^55 + 828502745555998906218503832*x^54 -
 914479445566527094599669324*x^53 + 970109082168886474373197089*x^52 - 989
130828878080326811887228*x^51 + 24233705307512968006891237086/25*x^50 -...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(15) = 30$.

Time = 0.11 (sec) , antiderivative size = 501, normalized size of antiderivative = 21.78

$$\int (1-x)^{99}x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)^99*x,x, algorithm="giac")
```

output
```
-1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131
/4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757
*x^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7
0297408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 665013
48729372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 +
 1307276513180023617*x^82 - 5293662917568490696*x^81 + 2016318393423855474
03/10*x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 7874
00226364730912388*x^77 + 2393282266977011062653*x^76 - 1725617880702398745
68724/25*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^7
3 + 122384749256712270099849*x^72 - 289586448945460019391192*x^71 + 326885
7173695411601376612/5*x^70 - 1409398564020847755666003*x^69 + 116143485944
17788189739629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178
066659311/2*x^66 - 97339302505596701018770224/5*x^65 + 1346636634325738144
16170293/4*x^64 - 55800482090690569716307824*x^63 + 8868538158582459033075
7224*x^62 - 135208860450519457389515112*x^61 + 989058310490690095822896114
/5*x^60 - 277798460089394796889723848*x^59 + 374593512943317843600698196*x
^58 - 485119509585285628395162576*x^57 + 603511770853123192467791538*x^56
- 3606758093003645324155339152/5*x^55 + 828502745555998906218503832*x^54 -
 914479445566527094599669324*x^53 + 970109082168886474373197089*x^52 - 989
130828878080326811887228*x^51 + 24233705307512968006891237086/25*x^50 -...
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int (1-x)^{99} x \, dx = -\frac{(100\,x+1)\,(x-1)^{100}}{10100}$$

input
```
int(-x*(x - 1)^99,x)
```

output
```
-((100*x + 1)*(x - 1)^100)/10100
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 500, normalized size of antiderivative = 21.74

$$\int (1-x)^{99}x\,dx = \text{Too large to display}$$

input
```
int((1-x)^99*x,x)
```

output
```
(x**2*( - 100*x**99 + 9999*x**98 - 494900*x**97 + 16165050*x**96 - 3919608
00*x**95 + 7524830775*x**94 - 119129952480*x**93 + 1599564027600*x**92 - 1
8592781869200*x**91 + 190037092945700*x**90 - 1729128713835600*x**89 + 141
45670154903560*x**88 - 104900475306026400*x**87 + 710003828928813300*x**86
 - 4411583725232560800*x**85 + 25289660192321540400*x**84 - 13433272443333
1476360*x**83 + 663667626664673365350*x**82 - 3059800945425883628200*x**81
 + 13203492783118238531700*x**80 - 53465995467441756029600*x**79 + 2036481
57735809402877030*x**78 - 731164847106703494794400*x**77 + 247919496374028
8382850800*x**76 - 7952742286283782215118800*x**75 + 241721508964678117327
95300*x**74 - 69714962380376909325764496*x**73 + 1910357452615433326118922
00*x**72 - 497964017002692209469746400*x**71 + 123608596749279392800847490
0*x**70 - 2924823134349146195851039200*x**69 + 660309149086473143478075624
0*x**68 - 14234925496610562332226630300*x**67 + 293262302009049151790925632
225*x**66 - 57777050545066400054331617100*x**65 + 108925997889075399236629
520550*x**64 - 196625391061305336057915852480*x**63 + 34002575016724888140
0829989825*x**62 - 563584869115974754134709022400*x**61 + 8957223540168283
62340647962400*x**60 - 1365609490550246519634102631200*x**59 + 19978977871
91193993562250150280*x**58 - 2805764446902887448586210864800*x**57 + 37833
94480727510220367051779600*x**56 - 4899707046811384846791142017600*x**55 +
 6095468885616544243924694533800*x**54 - 728565134786736355479378508704...
```

3.65 $\int \csc(x)\sin(4x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \csc(x)\sin(4x)\,dx = 4\sin(x) - \frac{8\sin^3(x)}{3}$$

output
```
4*sin(x)-8/3*sin(x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \csc(x)\sin(4x)\,dx = 4\sin(x) - \frac{8\sin^3(x)}{3}$$

input
```
Integrate[Csc[x]*Sin[4*x],x]
```

output
```
4*Sin[x] - (8*Sin[x]^3)/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4878, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(4x)}{\sin(x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \left(4-8\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$4\sin(x)-\frac{8\sin^3(x)}{3}$$

input
```
Int[Csc[x]*Sin[4*x],x]
```

output
```
4*Sin[x] - (8*Sin[x]^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
risch	$2\sin\left(x\right)+\frac{2\sin\left(3x\right)}{3}$	12
default	$\frac{8\left(2+\cos\left(x\right)^{2}\right)\sin\left(x\right)}{3}-4\sin\left(x\right)$	16

input
```
int(csc(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sin(x)+2/3*sin(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \csc(x)\sin(4x)\,dx=\frac{4}{3}\,\left(2\,\cos\left(x\right)^{2}+1\right)\sin\left(x\right)$$

input
```
integrate(csc(x)*sin(4*x),x, algorithm="fricas")
```

output
```
4/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \csc(x)\sin(4x)\,dx = -\frac{8\sin^3(x)}{3} + 4\sin(x)$$

input
```
integrate(csc(x)*sin(4*x),x)
```

output
```
-8*sin(x)**3/3 + 4*sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc(x)\sin(4x)\,dx = -\frac{8}{3}\sin(x)^3 + 4\sin(x)$$

input
```
integrate(csc(x)*sin(4*x),x, algorithm="maxima")
```

output
```
-8/3*sin(x)^3 + 4*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc(x)\sin(4x)\,dx = \frac{2}{3}\sin(3x) + 2\sin(x)$$

input
```
integrate(csc(x)*sin(4*x),x, algorithm="giac")
```

output
```
2/3*sin(3*x) + 2*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \csc(x)\sin(4x)\,dx = 4\,\sin(x) - \frac{8\sin(x)^3}{3}$$

input
```
int(sin(4*x)/sin(x),x)
```

output
```
4*sin(x) - (8*sin(x)^3)/3
```

Reduce [F]

$$\int \csc(x)\sin(4x)\,dx = \int \csc(x)\sin(4x)\,dx$$

input
```
int(csc(x)*sin(4*x),x)
```

output
```
int(csc(x)*sin(4*x),x)
```

3.66 $\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

output
```
6*x^(1/6)-6*arctan(x^(1/6))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

input
```
Integrate[1/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
6*x^(1/6) - 6*ArcTan[x^(1/6)]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {864, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt[3]{x}+1\right)\sqrt{x}}dx$$

$$\Big\downarrow \text{864}$$

$$3\int \frac{\sqrt[6]{x}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{60}$$

$$3\left(2\sqrt[6]{x}-\int \frac{1}{\left(\sqrt[3]{x}+1\right)\sqrt[6]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{73}$$

$$3\left(2\sqrt[6]{x}-2\int \frac{1}{x^{2/3}+1}d\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{216}$$

$$3\left(2\sqrt[6]{x}-2\arctan\left(\sqrt[6]{x}\right)\right)$$

input
```
Int[1/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
3*(2*x^(1/6) - 2*ArcTan[x^(1/6)])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	13
default	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	13
meijerg	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	13

input
```
int(1/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/6)-6*arctan(x^(1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(1/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

output
```
6*x^(1/6) - 6*arctan(x^(1/6))
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x} + 6\,\mathrm{atan}\left(\frac{1}{\sqrt[6]{x}}\right)$$

input
```
integrate(1/(1+x**(1/3))/x**(1/2),x)
```

output
```
6*x**(1/6) + 6*atan(x**(-1/6))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(1/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

output
```
6*x^(1/6) - 6*arctan(x^(1/6))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(1/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

output
```
6*x^(1/6) - 6*arctan(x^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\,x^{1/6} - 6\,\mathrm{atan}\left(x^{1/6}\right)$$

input
```
int(1/(x^(1/2)*(x^(1/3) + 1)),x)
```

output
```
6*x^(1/6) - 6*atan(x^(1/6))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = -6atan\left(x^{\frac{1}{6}}\right) + 6x^{\frac{1}{6}}$$

input
```
int(1/(1+x^(1/3))/x^(1/2),x)
```

output
```
6*( - atan(x**(1/6)) + x**(1/6))
```

3.67 $\int \frac{1}{\sqrt{-1+2x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+2x^2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(2^(1/2)*x/(2*x^2-1)^(1/2))*2^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{-\log\left(\sqrt{2}-\frac{2x}{\sqrt{-1+2x^2}}\right)+\log\left(\sqrt{2}+\frac{2x}{\sqrt{-1+2x^2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[1/Sqrt[-1 + 2*x^2],x]
```

output
```
(-Log[Sqrt[2] - (2*x)/Sqrt[-1 + 2*x^2]] + Log[Sqrt[2] + (2*x)/Sqrt[-1 + 2*
x^2]])/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-1}}\,dx$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-\frac{2x^2}{2x^2-1}}d\frac{x}{\sqrt{2x^2-1}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{2x^2-1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[-1 + 2*x^2],x]
```

output
```
ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + 2*x^2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\ln\left(x\sqrt{2}+\sqrt{2x^{2}-1}\right)\sqrt{2}}{2}$	22
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2x^{2}-1}\,\sqrt{2}}{2x}\right)}{2}$	24
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}-2\right)\sqrt{2x^{2}-1}+2x\right)}{2}$	31
meijerg	$\frac{\sqrt{-\operatorname{signum}(2x^{2}-1)}\,\sqrt{2}\,\arcsin\left(x\sqrt{2}\right)}{2\sqrt{\operatorname{signum}(2x^{2}-1)}}$	34

input
```
int(1/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x*2^(1/2)+(2*x^2-1)^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(2\,\sqrt{2}\sqrt{2\,x^{2}-1}x+4\,x^{2}-1\right)$$

input
```
integrate(1/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(2*sqrt(2)*sqrt(2*x^2 - 1)*x + 4*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{2}\log\left(2x+\sqrt{2}\sqrt{2x^2-1}\right)}{2}$$

input
```
integrate(1/(2*x**2-1)**(1/2),x)
```

output
```
sqrt(2)*log(2*x + sqrt(2)*sqrt(2*x**2 - 1))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{1}{2}\sqrt{2}\log\left(2\sqrt{2}\sqrt{2x^2-1}+4x\right)$$

input
```
integrate(1/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(2*sqrt(2)*sqrt(2*x^2 - 1) + 4*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{1}{2}\sqrt{2x^2-1}x+\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}x+\sqrt{2x^2-1}\right|\right)$$

input
```
integrate(1/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2*x^2 - 1)*x + 1/4*sqrt(2)*log(abs(-sqrt(2)*x + sqrt(2*x^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x+\sqrt{2\,x^2-1}\right)}{2}$$

input
```
int(1/(2*x^2 - 1)^(1/2),x)
```

output
```
(2^(1/2)*log(2^(1/2)*x + (2*x^2 - 1)^(1/2)))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{2}\log\left(\sqrt{2x^2-1}+\sqrt{2}\,x\right)}{2}$$

input
```
int(1/(2*x^2-1)^(1/2),x)
```

output
```
(sqrt(2)*log(sqrt(2*x**2 - 1) + sqrt(2)*x))/2
```

3.68 $\int \frac{1}{\sqrt{-1+e^x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\arctan\left(\sqrt{-1+e^x}\right)$$

output
```
2*arctan((-1+exp(x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\arctan\left(\sqrt{-1+e^x}\right)$$

input
```
Integrate[1/Sqrt[-1 + E^x],x]
```

output
```
2*ArcTan[Sqrt[-1 + E^x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e^x-1}}dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \frac{e^{-x}}{\sqrt{e^x-1}}de^x$$
$$\Big\downarrow \text{ 73}$$
$$2\int \frac{1}{1+e^{2x}}d\sqrt{-1+e^x}$$
$$\Big\downarrow \text{ 216}$$
$$2\arctan\left(\sqrt{e^x-1}\right)$$

input
```
Int[1/Sqrt[-1 + E^x],x]
```

output
```
2*ArcTan[Sqrt[-1 + E^x]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$2 \arctan\left(\sqrt{e^{x}-1}\right)$	10
default	$2 \arctan\left(\sqrt{e^{x}-1}\right)$	10

input
```
int(1/(exp(x)-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((exp(x)-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\,\arctan\left(\sqrt{e^x-1}\right)$$

input
```
integrate(1/(-1+exp(x))^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(e^x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\,\mathrm{atan}\left(\sqrt{e^x-1}\right)$$

input
```
integrate(1/(-1+exp(x))**(1/2),x)
```

output
```
2*atan(sqrt(exp(x) - 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\,\arctan\left(\sqrt{e^x-1}\right)$$

input
```
integrate(1/(-1+exp(x))^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(e^x - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\,\arctan\left(\sqrt{e^x-1}\right)$$

input
```
integrate(1/(-1+exp(x))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = 2\,\mathrm{atan}\left(\sqrt{\mathrm{e}^x-1}\right)$$

input
```
int(1/(exp(x) - 1)^(1/2),x)
```

output
```
2*atan((exp(x) - 1)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+e^x}}\,dx = \int \frac{\sqrt{e^x-1}}{e^x-1}dx$$

input
```
int(1/(-1+exp(x))^(1/2),x)
```

output
```
int(sqrt(e**x - 1)/(e**x - 1),x)
```

3.69 $\int \frac{x}{4+x^4} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{4+x^4} \, dx = \frac{1}{4} \arctan\left(\frac{x^2}{2}\right)$$

output
```
1/4*arctan(1/2*x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{4+x^4} \, dx = \frac{1}{4} \arctan\left(\frac{x^2}{2}\right)$$

input
```
Integrate[x/(4 + x^4),x]
```

output
```
ArcTan[x^2/2]/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+4}\,dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{x^4+4}dx^2$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\arctan\left(\frac{x^2}{2}\right)$$

input
```
Int[x/(4 + x^4),x]
```

output
```
ArcTan[x^2/2]/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
meijerg	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
risch	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
parallelrisch	$-\frac{i\ln(x-1-i)}{8}+\frac{i\ln(x-1+i)}{8}+\frac{i\ln(x+1-i)}{8}-\frac{i\ln(x+1+i)}{8}$	38

input
```
int(x/(x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(1/2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+x^4}\,dx = \frac{1}{4}\arctan\left(\frac{1}{2}\,x^2\right)$$

input
```
integrate(x/(x^4+4),x, algorithm="fricas")
```

output
```
1/4*arctan(1/2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{4+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{2}\right)}{4}$$

input
```
integrate(x/(x**4+4),x)
```

output
```
atan(x**2/2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+x^4}\,dx = \frac{1}{4}\arctan\left(\frac{1}{2}x^2\right)$$

input
```
integrate(x/(x^4+4),x, algorithm="maxima")
```

output
```
1/4*arctan(1/2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+x^4}\,dx = \frac{1}{4}\arctan\left(\frac{1}{2}x^2\right)$$

input
```
integrate(x/(x^4+4),x, algorithm="giac")
```

output
```
1/4*arctan(1/2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{4+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{2}\right)}{4}$$

input
```
int(x/(x^4 + 4),x)
```

output
```
atan(x^2/2)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{4+x^4}\,dx = \frac{atan(x-1)}{4} - \frac{atan(x+1)}{4}$$

input
```
int(x/(x^4+4),x)
```

output
```
(atan(x - 1) - atan(x + 1))/4
```

3.70 $\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx = \frac{2}{-1+\cot(x)}$$

output
```
2/(-1+cot(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx = \frac{2\sin(x)}{\cos(x)-\sin(x)}$$

input
```
Integrate[2/(Cos[x] - Sin[x])^2,x]
```

output
```
(2*Sin[x])/(Cos[x] - Sin[x])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {27, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx$$

$$\Big\downarrow \; 27$$

$$2\int \frac{1}{(\cos(x)-\sin(x))^2}dx$$

$$\Big\downarrow \; 3042$$

$$2\int \frac{1}{(\cos(x)-\sin(x))^2}dx$$

$$\Big\downarrow \; 3554$$

$$\frac{2\sin(x)}{\cos(x)-\sin(x)}$$

input
```
Int[2/(Cos[x] - Sin[x])^2,x]
```

output
```
(2*Sin[x])/(Cos[x] - Sin[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{2}{\tan(x)-1}$	9
risch	$\frac{2}{e^{2ix}-i}$	13
parallelrisch	$\frac{2\sin(x)}{\cos(x)-\sin(x)}$	14
norman	$-\frac{4\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}+2\tan\left(\frac{x}{2}\right)-1}$	23

input
```
int(2/(cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx=\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}$$

input
```
integrate(2/(cos(x)-sin(x))^2,x, algorithm="fricas")
```

output
```
(cos(x) + sin(x))/(cos(x) - sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(5) = 10.

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx = -\frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1}$$

input
```
integrate(2/(cos(x)-sin(x))**2,x)
```

output
```
-4*tan(x/2)/(tan(x/2)**2 + 2*tan(x/2) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx = -\frac{2}{\tan(x)-1}$$

input
```
integrate(2/(cos(x)-sin(x))^2,x, algorithm="maxima")
```

output
```
-2/(tan(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx = -\frac{2}{\tan(x)-1}$$

input
```
integrate(2/(cos(x)-sin(x))^2,x, algorithm="giac")
```

output
```
-2/(tan(x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx=\frac{2\,\sin{(x)}}{\cos{(x)}-\sin{(x)}}$$

input
```
int(2/(cos(x) - sin(x))^2,x)
```

output
```
(2*sin(x))/(cos(x) - sin(x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx=\frac{2\cos{(x)}}{\cos{(x)}-\sin{(x)}}$$

input
```
int(2/(cos(x)-sin(x))^2,x)
```

output
```
(2*cos(x))/(cos(x) - sin(x))
```

3.71 $\int x \coth(x)\mathbf{csch}(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int x\coth(x)\text{csch}(x)\,dx = -\text{arctanh}(\cosh(x)) - x\text{csch}(x)$$

output
```
-arctanh(cosh(x))-x*csch(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.55

$$\int x\coth(x)\text{csch}(x)\,dx = -\frac{1}{2}x\coth\left(\frac{x}{2}\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{1}{2}x\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[x*Coth[x]*Csch[x],x]
```

output
```
-1/2*(x*Coth[x/2]) - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (x*Tanh[x/2])/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5942, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\coth(x)\text{csch}(x)\,dx$$

$$\Big\downarrow \text{5942}$$

$$\int \text{csch}(x)dx - x\text{csch}(x)$$

$$\Big\downarrow \text{3042}$$

$$-x\text{csch}(x) + \int i\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-x\text{csch}(x) + i\int \csc(ix)dx$$

$$\Big\downarrow \text{4257}$$

$$-\text{arctanh}(\cosh(x)) - x\text{csch}(x)$$

input
```
Int[x*Coth[x]*Csch[x],x]
```

output
```
-ArcTanh[Cosh[x]] - x*Csch[x]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 5942
```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
 /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] &&
EqQ[q, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

method	result	size
risch	$-\frac{2\,\mathrm{e}^{x} x}{\mathrm{e}^{2x}-1}-\ln\left(1+\mathrm{e}^{x}\right)+\ln\left(\mathrm{e}^{x}-1\right)$	27

input
```
int(x*coth(x)*csch(x),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*x/(exp(2*x)-1)-ln(1+exp(x))+ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 7.27

$$\int x\coth(x)\text{csch}(x)\,dx =$$
$$-\frac{2\,x\cosh\left(x\right) + \left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - (\cosh(}{\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) +}$$

input
```
integrate(x*coth(x)*csch(x),x, algorithm="fricas")
```

output
```
-(2*x*cosh(x) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x
) + sinh(x) + 1) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cos
h(x) + sinh(x) - 1) + 2*x*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1)
```

Sympy [F]

$$\int x\coth(x)\text{csch}(x)\,dx = \int x\coth\left(x\right)\text{csch}\left(x\right)\,dx$$

input
```
integrate(x*coth(x)*csch(x),x)
```

output
```
Integral(x*coth(x)*csch(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int x \coth(x)\operatorname{csch}(x)\,dx = -\frac{2\,xe^{x}}{e^{(2\,x)}-1} - \log\left(e^{x}+1\right) + \log\left(e^{x}-1\right)$$

input
```
integrate(x*coth(x)*csch(x),x, algorithm="maxima")
```

output
```
-2*x*e^x/(e^(2*x) - 1) - log(e^x + 1) + log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.45

$$\int x \coth(x)\operatorname{csch}(x)\,dx$$
$$= -\frac{2\,xe^{x} + e^{(2\,x)}\log\left(e^{x}+1\right) - e^{(2\,x)}\log\left(e^{x}-1\right) - \log\left(e^{x}+1\right) + \log\left(e^{x}-1\right)}{e^{(2\,x)}-1}$$

input
```
integrate(x*coth(x)*csch(x),x, algorithm="giac")
```

output
```
-(2*x*e^x + e^(2*x)*log(e^x + 1) - e^(2*x)*log(e^x - 1) - log(e^x + 1) + l
og(e^x - 1))/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int x \coth(x)\operatorname{csch}(x)\,dx = \ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)-\frac{2\,x\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x}-1}$$

input
```
int((x*coth(x))/sinh(x),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - (2*x*exp(x))/(exp(2*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int x \coth(x)\operatorname{csch}(x)\,dx = \frac{e^{2x}\log(e^{x}-1)-e^{2x}\log(e^{x}+1)-2e^{x}x-\log(e^{x}-1)+\log(e^{x}+1)}{e^{2x}-1}$$

input
```
int(x*coth(x)*csch(x),x)
```

output
```
(e**(2*x)*log(e**x - 1) - e**(2*x)*log(e**x + 1) - 2*e**x*x - log(e**x - 1
) + log(e**x + 1))/(e**(2*x) - 1)
```

3.72 $\int x^5\sqrt{1+x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^5\sqrt{1+x^3}\,dx = -\frac{2}{9}\left(1+x^3\right)^{3/2}+\frac{2}{15}\left(1+x^3\right)^{5/2}$$

output
```
-2/9*(x^3+1)^(3/2)+2/15*(x^3+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2}{45}\left(1+x^3\right)^{3/2}\left(-2+3x^3\right)$$

input
```
Integrate[x^5*Sqrt[1 + x^3],x]
```

output
```
(2*(1 + x^3)^(3/2)*(-2 + 3*x^3))/45
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{x^3+1}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^3\sqrt{x^3+1}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(x^3+1\right)^{3/2}-\sqrt{x^3+1}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{5}\left(x^3+1\right)^{5/2}-\frac{2}{3}\left(x^3+1\right)^{3/2}\right)$$

input
```
Int[x^5*Sqrt[1 + x^3],x]
```

output
```
((-2*(1 + x^3)^(3/2))/3 + (2*(1 + x^3)^(5/2))/5)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{2\left(x^{3}+1\right)^{\frac{3}{2}}\left(3x^{3}-2\right)}{45}$	17
risch	$\frac{2\left(3x^{6}+x^{3}-2\right)\sqrt{x^{3}+1}}{45}$	20
trager	$\left(\frac{2}{15}x^{6}+\frac{2}{45}x^{3}-\frac{4}{45}\right)\sqrt{x^{3}+1}$	21
gosper	$\frac{2\left(x+1\right)\left(x^{2}-x+1\right)\left(3x^{3}-2\right)\sqrt{x^{3}+1}}{45}$	28
orering	$\frac{2\left(x+1\right)\left(x^{2}-x+1\right)\left(3x^{3}-2\right)\sqrt{x^{3}+1}}{45}$	28
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(x^{3}+1\right)^{\frac{3}{2}}\left(-3x^{3}+2\right)}{15}}{6\sqrt{\pi}}$	31
default	$\frac{2x^{6}\sqrt{x^{3}+1}}{15}+\frac{2x^{3}\sqrt{x^{3}+1}}{45}-\frac{4\sqrt{x^{3}+1}}{45}$	35
elliptic	$\frac{2x^{6}\sqrt{x^{3}+1}}{15}+\frac{2x^{3}\sqrt{x^{3}+1}}{45}-\frac{4\sqrt{x^{3}+1}}{45}$	35

input
```
int(x^5*(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(x^3+1)^(3/2)*(3*x^3-2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2}{45}\left(3\,x^6+x^3-2\right)\sqrt{x^3+1}$$

input
```
integrate(x^5*(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*x^6 + x^3 - 2)*sqrt(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2x^6\sqrt{x^3+1}}{15} + \frac{2x^3\sqrt{x^3+1}}{45} - \frac{4\sqrt{x^3+1}}{45}$$

input
```
integrate(x**5*(x**3+1)**(1/2),x)
```

output
```
2*x**6*sqrt(x**3 + 1)/15 + 2*x**3*sqrt(x**3 + 1)/45 - 4*sqrt(x**3 + 1)/45
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}} - \frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(x^3 + 1)^(5/2) - 2/9*(x^3 + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}} - \frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/15*(x^3 + 1)^(5/2) - 2/9*(x^3 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2\left(x^3+1\right)^{5/2}}{15} - \frac{2\left(x^3+1\right)^{3/2}}{9}$$

input
```
int(x^5*(x^3 + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)^(5/2))/15 - (2*(x^3 + 1)^(3/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^5\sqrt{1+x^3}\,dx = \frac{2\sqrt{x^3+1}\,(3x^6+x^3-2)}{45}$$

input
```
int(x^5*(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*(3*x**6 + x**3 - 2))/45
```

3.73 $\int \frac{-1+x^7}{\log(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{-1+x^7}{\log(x)}\,dx = \text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)$$

output
```
Ei(8*ln(x))-Li(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^7}{\log(x)}\,dx = \text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)$$

input
```
Integrate[(-1 + x^7)/Log[x],x]
```

output
```
ExpIntegralEi[8*Log[x]] - LogIntegral[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7-1}{\log(x)} \, dx$$

$$\Big\downarrow \text{ 2767}$$

$$\int \left(\frac{x^7}{\log(x)}-\frac{1}{\log(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\text{ExpIntegralEi}(8\log(x))-\text{LogIntegral}(x)$$

input
```
Int[(-1 + x^7)/Log[x],x]
```

output
```
ExpIntegralEi[8*Log[x]] - LogIntegral[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

method	result	size
default	$-\operatorname{expIntegral}_1(-8\ln(x)) + \operatorname{expIntegral}_1(-\ln(x))$	16
risch	$-\operatorname{expIntegral}_1(-8\ln(x)) + \operatorname{expIntegral}_1(-\ln(x))$	16

input
```
int((x^7-1)/ln(x),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-8*ln(x))+Ei(1,-ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{-1+x^7}{\log(x)}\,dx = \text{log\_integral}\left(x^8\right) - \text{log\_integral}\,(x)$$

input
```
integrate((x^7-1)/log(x),x, algorithm="fricas")
```

output
```
log_integral(x^8) - log_integral(x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^7}{\log(x)}\,dx = -\operatorname{Ei}(\log(x)) + \operatorname{Ei}(8\log(x))$$

input
```
integrate((x**7-1)/ln(x),x)
```

output
```
-Ei(log(x)) + Ei(8*log(x))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+x^7}{\log(x)}\,dx = \mathrm{Ei}(8\log(x)) - \mathrm{Ei}(\log(x))$$

input
```
integrate((x^7-1)/log(x),x, algorithm="maxima")
```

output
```
Ei(8*log(x)) - Ei(log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+x^7}{\log(x)}\,dx = \mathrm{Ei}(8\log(x)) - \mathrm{Ei}(\log(x))$$

input
```
integrate((x^7-1)/log(x),x, algorithm="giac")
```

output
```
Ei(8*log(x)) - Ei(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^7}{\log(x)}\,dx = \mathrm{ei}(8\ln(x)) - \mathrm{logint}(x)$$

input
```
int((x^7 - 1)/log(x),x)
```

output
```
ei(8*log(x)) - logint(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-1+x^7}{\log(x)}\,dx = -ei(\log(x)) + ei(8\log(x))$$

input
```
int((x^7-1)/log(x),x)
```

output
```
- ei(log(x)) + ei(8*log(x))
```

3.74 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

input
```
Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc(x)-\sin(x)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \sqrt{\cos(x)\cot(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\cos(x)\cot(x)}dx$$
$$\Big\downarrow \text{4900}$$
$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$\Big\downarrow \text{3069}$$
$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

input
```
Int[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$2\sqrt{\cos\left(x\right)\cot\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-ix}}{\mathrm{e}^{2ix}-1}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}$	51

input
```
int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \int \sqrt{-\sin(x) + \csc(x)}\, dx$$

input
```
integrate((csc(x)-sin(x))**(1/2),x)
```

output
```
Integral(sqrt(-sin(x) + csc(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 14.46

$$\int \sqrt{\csc(x) - \sin(x)}\, dx$$
$$= \frac{\left(\left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x), \cos(x) - 1\right)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\right.\right.}{}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
(((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(
x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin
(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) -
((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(
1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)^(1/4))
```

Giac [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\int \sqrt{\csc\left(x\right)-\sin\left(x\right)}\,dx$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\frac{2\left|\cos\left(x\right)\right|}{\cos\left(x\right)\,\sqrt{\frac{1}{\sin(x)}}}$$

input
```
int((1/sin(x) - sin(x))^(1/2),x)
```

output
```
(2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Reduce [F]

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \int \sqrt{\csc(x) - \sin(x)} dx$$

input
```
int((csc(x)-sin(x))^(1/2),x)
```

output
```
int(sqrt(csc(x) - sin(x)),x)
```

3.75 $\int (-2\log(2x)+\log(x^2))\ dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int (-2\log(2x)+\log(x^2))\ dx = -2x\log(2x)+x\log(x^2)$$

output
```
-2*x*ln(2*x)+x*ln(x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (-2\log(2x)+\log(x^2))\ dx = -2x\log(2x)+x\log(x^2)$$

input
```
Integrate[-2*Log[2*x] + Log[x^2],x]
```

output
```
-2*x*Log[2*x] + x*Log[x^2]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\log \left(x^2\right)-2 \log (2 x)\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$x \log \left(x^2\right)-2 x \log (2 x)$$

input
```
Int[-2*Log[2*x] + Log[x^2],x]
```

output
```
-2*x*Log[2*x] + x*Log[x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
orering	$\left(-2 \ln \left(2 x \right)+\ln \left(x^{2}\right)\right) x$	14
default	$-2 x \ln \left(2 x \right)+x \ln \left(x^{2}\right)$	15
norman	$-2 x \ln \left(2 x \right)+x \ln \left(x^{2}\right)$	15
risch	$-2 x \ln \left(2 x \right)+x \ln \left(x^{2}\right)$	15
parallelrisch	$-2 x \ln \left(2 x \right)+x \ln \left(x^{2}\right)$	15
parts	$-2 x \ln \left(2 x \right)+x \ln \left(x^{2}\right)$	15

input
```
int(-2*ln(2*x)+ln(x^2),x,method=_RETURNVERBOSE)
```

output
```
(-2*ln(2*x)+ln(x^2))*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.36

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx=-2\,x\log\left(2\right)$$

input
```
integrate(-2*log(2*x)+log(x^2),x, algorithm="fricas")
```

output
```
-2*x*log(2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx=-2x\log\left(2\right)$$

input
```
integrate(-2*ln(2*x)+ln(x**2),x)
```

output
```
-2*x*log(2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx = x\log\left(x^2\right)-2\,x\log\left(2\,x\right)$$

input
```
integrate(-2*log(2*x)+log(x^2),x, algorithm="maxima")
```

output
```
x*log(x^2) - 2*x*log(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx = x\log\left(x^2\right)-2\,x\log\left(2\,x\right)$$

input
```
integrate(-2*log(2*x)+log(x^2),x, algorithm="giac")
```

output
```
x*log(x^2) - 2*x*log(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx = -x\left(2\,\ln\left(2\,x\right)-\ln\left(x^2\right)\right)$$

input
```
int(log(x^2) - 2*log(2*x),x)
```

output
```
-x*(2*log(2*x) - log(x^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(-2\log(2x)+\log\left(x^2\right)\right)\,dx = x\left(\log\left(x^2\right)-2\log(2x)\right)$$

input
```
int(-2*log(2*x)+log(x^2),x)
```

output
```
x*(log(x**2) - 2*log(2*x))
```

3.76 $\int e^x\,dx$

Optimal result

Integrand size = 3, antiderivative size = 3

$$\int e^x\,dx = e^x$$

output
```
exp(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x\,dx = e^x$$

input
```
Integrate[E^x,x]
```

output
```
E^x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \, dx$$
$$\Big\downarrow \text{2624}$$
$$e^x$$

input
```
Int[E^x,x]
```

output
```
E^x
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

method	result	size
gosper	e^x	3
lookup	e^x	3
derivativedivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
parallelrisch	e^x	3
orering	e^x	3
meijerg	$e^x - 1$	5

input
```
int(exp(x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x \, dx = e^x$$

input
```
integrate(exp(x),x, algorithm="fricas")
```

output
```
e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x)
```

output
```
exp(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x, algorithm="maxima")
```

output
```
e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = e^x$$

input
```
integrate(exp(x),x, algorithm="giac")
```

output
```
e^x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x\,dx = \mathrm{e}^x$$

input
```
int(exp(x),x)
```

output
```
exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x\,dx = e^x$$

input
```
int(exp(x),x)
```

output
```
e**x
```

3.77 $\int \frac{\cos(x)\log(x)-\frac{\sin(x)}{x}}{\log^2(x)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 7

$$\int \frac{\cos(x)\log(x)-\frac{\sin(x)}{x}}{\log^2(x)}\,dx=\frac{\sin(x)}{\log(x)}$$

output
```
sin(x)/ln(x)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\log(x)-\frac{\sin(x)}{x}}{\log^2(x)}\,dx=\frac{\sin(x)}{\log(x)}$$

input
```
Integrate[(Cos[x]*Log[x] - Sin[x]/x)/Log[x]^2,x]
```

output
```
Sin[x]/Log[x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\cos(x)-\frac{\sin(x)}{x}}{\log^2(x)}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{\cos(x)}{\log(x)}-\frac{\sin(x)}{x\log^2(x)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\int \frac{\cos(x)}{\log(x)}dx-\int \frac{\sin(x)}{x\log^2(x)}dx$$

input
```
Int[(Cos[x]*Log[x] - Sin[x]/x)/Log[x]^2,x]
```

output
```
$Aborted
```

Maple [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\sin(x)}{\ln(x)}$	8
parallelrisch	$\frac{\sin(x)}{\ln(x)}$	8
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\ln(x)}$	21

input
```
int((cos(x)*ln(x)-sin(x)/x)/ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)/ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{\sin(x)}{\log(x)}$$

input
```
integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="fricas")
```

output
```
sin(x)/log(x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{\sin(x)}{\log(x)}$$

input
```
integrate((cos(x)*ln(x)-sin(x)/x)/ln(x)**2,x)
```

output
```
sin(x)/log(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{\sin(x)}{\log(x)}$$

input
```
integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="maxima")
```

output
```
sin(x)/log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.86

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{2\tan\left(\frac{1}{2}x\right)}{\log\left(x\right)\tan\left(\frac{1}{2}x\right)^2 + \log\left(x\right)}$$

input
```
integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="giac")
```

output
```
2*tan(1/2*x)/(log(x)*tan(1/2*x)^2 + log(x))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{\sin\left(x\right)}{\ln\left(x\right)}$$

input
```
int((cos(x)*log(x) - sin(x)/x)/log(x)^2,x)
```

output
```
sin(x)/log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx = \frac{\sin\left(x\right)}{\log\left(x\right)}$$

input
```
int((cos(x)*log(x)-sin(x)/x)/log(x)^2,x)
```

output
```
sin(x)/log(x)
```

3.78 $\int (-1+3x-3x^2+x^3) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int (-1+3x-3x^2+x^3) \, dx = \frac{1}{4}(1-x)^4$$

output
```
1/4*(1-x)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int (-1+3x-3x^2+x^3) \, dx = -x+\frac{3x^2}{2}-x^3+\frac{x^4}{4}$$

input
```
Integrate[-1 + 3*x - 3*x^2 + x^3,x]
```

output
```
-x + (3*x^2)/2 - x^3 + x^4/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3-3x^2+3x-1\right)\,dx$$

$$\Big\downarrow \text{2006}$$

$$\int (x-1)^3dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{4}(1-x)^4$$

input
```
Int[-1 + 3*x - 3*x^2 + x^3,x]
```

output
```
(1 - x)^4/4
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(x-1)^4}{4}$	8
gosper	$\frac{x(x^3-4x^2+6x-4)}{4}$	17
norman	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
risch	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
parallelrisch	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
parts	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
orering	$\frac{x(x^3-4x^2+6x-4)(x^3-3x^2+3x-1)}{4(x-1)^3}$	35

input
```
int(x^3-3*x^2+3*x-1,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x-1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \left(-1+3x-3x^2+x^3\right) \, dx = \frac{1}{4}\,x^4 - x^3 + \frac{3}{2}\,x^2 - x$$

input
```
integrate(x^3-3*x^2+3*x-1,x, algorithm="fricas")
```

output
```
1/4*x^4 - x^3 + 3/2*x^2 - x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \left(-1+3x-3x^2+x^3\right)\,dx=\frac{x^4}{4}-x^3+\frac{3x^2}{2}-x$$

input
```
integrate(x**3-3*x**2+3*x-1,x)
```

output
```
x**4/4 - x**3 + 3*x**2/2 - x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \left(-1+3x-3x^2+x^3\right)\,dx=\frac{1}{4}\,x^4-x^3+\frac{3}{2}\,x^2-x$$

input
```
integrate(x^3-3*x^2+3*x-1,x, algorithm="maxima")
```

output
```
1/4*x^4 - x^3 + 3/2*x^2 - x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \left(-1+3x-3x^2+x^3\right)\,dx=\frac{1}{4}\,x^4-x^3+\frac{3}{2}\,x^2-x$$

input
```
integrate(x^3-3*x^2+3*x-1,x, algorithm="giac")
```

output
```
1/4*x^4 - x^3 + 3/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-1+3x-3x^2+x^3)\ dx = \frac{x^4}{4}-x^3+\frac{3\,x^2}{2}-x$$

input
```
int(3*x - 3*x^2 + x^3 - 1,x)
```

output
```
(3*x^2)/2 - x - x^3 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int (-1+3x-3x^2+x^3)\ dx = \frac{x(x^3-4x^2+6x-4)}{4}$$

input
```
int(x^3-3*x^2+3*x-1,x)
```

output
```
(x*(x**3 - 4*x**2 + 6*x - 4))/4
```

3.79 $\int \sqrt{12-3x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)$$

output
```
1/2*3^(1/2)*x*(-x^2+4)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\sqrt{3}\left(x\sqrt{4-x^2}-8\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)\right)$$

input
```
Integrate[Sqrt[12 - 3*x^2],x]
```

output
```
(Sqrt[3]*(x*Sqrt[4 - x^2] - 8*ArcTan[Sqrt[4 - x^2]/(2 + x)]))/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{12-3x^2}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$6\int \frac{1}{\sqrt{12-3x^2}}dx+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

$$\Big\downarrow \text{ 223}$$

$$2\sqrt{3}\arcsin\left(\frac{x}{2}\right)+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

input
```
Int[Sqrt[12 - 3*x^2],x]
```

output
```
(Sqrt[3]*x*Sqrt[4 - x^2])/2 + 2*Sqrt[3]*ArcSin[x/2]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{x\sqrt{-3x^2+12}}{2}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	23
risch	$-\frac{3x(x^2-4)}{2\sqrt{-3x^2+12}}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	28
meijerg	$\frac{i\sqrt{3}\left(-i\sqrt{\pi}\,x\sqrt{-\frac{x^2}{4}+1}-2i\sqrt{\pi}\,\arcsin\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{-3x^2+12}}{2}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{-3x^2+12}\,\sqrt{3}}{3x}\right)$	37
trager	$\frac{x\sqrt{-3x^2+12}}{2}+2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+3\right)\sqrt{-3x^2+12}+3x\right)$	43

input
```
int((-3*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-3*x^2+12)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sqrt{12-3x^2}\,dx = \frac{1}{2}\,\sqrt{-3\,x^2+12}x-2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+12}x}{3\,(x^2-4)}\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-3*x^2 + 12)*x - 2*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 12)*x
/(x^2 - 4))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{12-3x^2}\,dx=\frac{\sqrt{3}x\sqrt{4-x^2}}{2}+2\sqrt{3}\,\mathrm{asin}\left(\frac{x}{2}\right)$$

input
```
integrate((-3*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*x*sqrt(4 - x**2)/2 + 2*sqrt(3)*asin(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\,\sqrt{-3\,x^2+12}x+2\,\sqrt{3}\arcsin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 12)*x + 2*sqrt(3)*arcsin(1/2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\,\sqrt{3}\left(\sqrt{-x^2+4}x+4\,\arcsin\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*(sqrt(-x^2 + 4)*x + 4*arcsin(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sqrt{12-3x^2}\,dx = 2\sqrt{3}\,\mathrm{asin}\left(\frac{x}{2}\right)+\frac{\sqrt{3}\,x\,\sqrt{4-x^2}}{2}$$

input
```
int((12 - 3*x^2)^(1/2),x)
```

output
```
2*3^(1/2)*asin(x/2) + (3^(1/2)*x*(4 - x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sqrt{12-3x^2}\,dx = \frac{\sqrt{3}\,\left(4asin\left(\frac{x}{2}\right)+\sqrt{-x^2+4}\,x\right)}{2}$$

input
```
int((-3*x^2+12)^(1/2),x)
```

output
```
(sqrt(3)*(4*asin(x/2) + sqrt( - x**2 + 4)*x))/2
```

3.80 $\int \left((-3+x)^7 + x - \sin(3-x) \right) dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \left((-3+x)^7 + x - \sin(3-x) \right) dx = \frac{1}{8}(3-x)^8 + \frac{x^2}{2} - \cos(3-x)$$

output
```
1/8*(3-x)^8+1/2*x^2-cos(-3+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left((-3+x)^7 + x - \sin(3-x) \right) dx = \frac{1}{8}(-3+x)^8 + \frac{x^2}{2} - \cos(3)\cos(x) - \sin(3)\sin(x)$$

input
```
Integrate[(-3 + x)^7 + x - Sin[3 - x],x]
```

output
```
(-3 + x)^8/8 + x^2/2 - Cos[3]*Cos[x] - Sin[3]*Sin[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((x-3)^7+x-\sin(3-x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+\frac{1}{8}(3-x)^8-\cos(3-x)$$

input
```
Int[(-3 + x)^7 + x - Sin[3 - x],x]
```

output
```
(3 - x)^8/8 + x^2/2 - Cos[3 - x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
default	$\frac{x^2}{2}+\frac{(-3+x)^8}{8}-\cos(-3+x)$
derivativedivides	$-9+3x+\frac{(-3+x)^2}{2}+\frac{(-3+x)^8}{8}-\cos(-3+x)$
parts	$-2187x+\frac{x^8}{8}+2552x^2-1701x^3+\frac{2835x^4}{4}-189x^5+\frac{63x^6}{2}-3x^7-\cos(-3+x)$
risch	$2552x^2+\frac{x^8}{8}-3x^7+\frac{63x^6}{2}-189x^5+\frac{2835x^4}{4}-1701x^3-2187x+\frac{6561}{8}-\cos(-3+x)$
parallelrisch	$\frac{x^8}{8}-3x^7+\frac{63x^6}{2}-189x^5+\frac{2835x^4}{4}-1701x^3+2552x^2-2187x+1-\cos(-3+x)$
norman	$\frac{-2187x+2552x^2-1701x^3+\frac{2835x^4}{4}-189x^5+\frac{63x^6}{2}-3x^7+\frac{x^8}{8}-2187x\tan\left(-\frac{3}{2}+\frac{x}{2}\right)^2+2552x^2\tan\left(-\frac{3}{2}+\frac{x}{2}\right)^2-1701x^3\tan}{1+\tan(-}$
orering	$\frac{x\left(x^7-24x^6+308x^5-2520x^4+13230x^3-43848x^2+88456x-99144\right)\left((-3+x)^7+x+\sin(-3+x)\right)}{8x^7-168x^6+1848x^5-12600x^4+52920x^3-131544x^2+176912x-99144}-7(-3+x)^6-$

input
```
int((-3+x)^7+x+sin(-3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/8*(-3+x)^8-cos(-3+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx=\frac{1}{8}\,x^8-3\,x^7+\frac{63}{2}\,x^6-189\,x^5+\frac{2835}{4}\,x^4-1701\,x^3+2552\,x^2-2187\,x-\cos(x-3)$$

input
```
integrate((-3+x)^7+x+sin(-3+x),x, algorithm="fricas")
```

output
```
1/8*x^8 - 3*x^7 + 63/2*x^6 - 189*x^5 + 2835/4*x^4 - 1701*x^3 + 2552*x^2 -
2187*x - cos(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx=\frac{x^2}{2}+\frac{(x-3)^8}{8}-\cos{(x-3)}$$

input
```
integrate((-3+x)**7+x+sin(-3+x),x)
```

output
```
x**2/2 + (x - 3)**8/8 - cos(x - 3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx=\frac{1}{8}\,(x-3)^8+\frac{1}{2}\,x^2-\cos{(x-3)}$$

input
```
integrate((-3+x)^7+x+sin(-3+x),x, algorithm="maxima")
```

output
```
1/8*(x - 3)^8 + 1/2*x^2 - cos(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx=\frac{1}{8}\,(x-3)^8+\frac{1}{2}\,x^2-\cos{(x-3)}$$

input
```
integrate((-3+x)^7+x+sin(-3+x),x, algorithm="giac")
```

output
```
1/8*(x - 3)^8 + 1/2*x^2 - cos(x - 3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx = 2552\,x^2-\cos\left(x-3\right)-2187\,x-1701\,x^3+\frac{2835\,x^4}{4}-189\,x^5+\frac{63\,x^6}{2}-3\,x^7+\frac{x^8}{8}$$

input
```
int(x + sin(x - 3) + (x - 3)^7,x)
```

output
```
2552*x^2 - cos(x - 3) - 2187*x - 1701*x^3 + (2835*x^4)/4 - 189*x^5 + (63*x
^6)/2 - 3*x^7 + x^8/8
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \left((-3+x)^7+x-\sin(3-x)\right)\,dx = -\cos\left(x-3\right)+\frac{x^8}{8}-3x^7+\frac{63x^6}{2}-189x^5+\frac{2835x^4}{4}-1701x^3+2552x^2-2187x$$

input
```
int((-3+x)^7+x+sin(-3+x),x)
```

output
```
( - 8*cos(x - 3) + x**8 - 24*x**7 + 252*x**6 - 1512*x**5 + 5670*x**4 - 136
08*x**3 + 20416*x**2 - 17496*x)/8
```

3.81 $\int \sin(x)\sqrt{1+\tan^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = -\cos(x)\log(\cos(x))\sqrt{\sec^2(x)}$$

output
```
-cos(x)*ln(cos(x))*(sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = -\cos(x)\log(\cos(x))\sqrt{\sec^2(x)}$$

input
```
Integrate[Sin[x]*Sqrt[1 + Tan[x]^2],x]
```

output
```
-(Cos[x]*Log[Cos[x]]*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 4140, 3042, 4613, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\tan^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\tan(x)^2+1}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sin(x)\sqrt{\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\sec(x)^2}dx$$

$$\Big\downarrow \text{4613}$$

$$\cos(x)\sqrt{\sec^2(x)}\int \tan(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(x)\sqrt{\sec^2(x)}\int \tan(x)dx$$

$$\Big\downarrow \text{3956}$$

$$-\cos(x)\sqrt{\sec^2(x)}\log(\cos(x))$$

input
```
Int[Sin[x]*Sqrt[1 + Tan[x]^2],x]
```

output
```
-(Cos[x]*Log[Cos[x]]*Sqrt[Sec[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time = 2.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result
default	$-\left(\ln\left(-\cot\left(x\right)+1+\csc\left(x\right)\right)-\ln\left(\frac{2}{1+\cos\left(x\right)}\right)+\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)\right)\sqrt{\sec\left(x\right)^{2}}\,\cos\left(x\right.$
risch	$2i\sqrt{\frac{{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,x\cos\left(x\right)-2\sqrt{\frac{{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\ln\left({\mathrm e}^{2 i x}+1\right)\cos\left(x\right)$

input
```
int(sin(x)*(1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(ln(-cot(x)+1+csc(x))-ln(2/(1+cos(x)))+ln(-cot(x)+csc(x)-1))*(sec(x)^2)^(
1/2)*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.33

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = \log\left(-\cos\left(x\right)\right)$$

input
```
integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-cos(x))
```

Sympy [F]

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = \int \sqrt{\tan^2\left(x\right)+1}\sin\left(x\right)\,dx$$

input
```
integrate(sin(x)*(1+tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tan(x)**2 + 1)*sin(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = -\sqrt{\frac{1}{\cos\left(x\right)^2}}\cos\left(x\right)\log\left(\cos\left(x\right)\right)$$

input
```
integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(cos(x)^(-2))*cos(x)*log(cos(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = -\log(|\cos(x)|)$$

input
```
integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = \int \sin(x)\,\sqrt{\tan(x)^2+1}\,dx$$

input
```
int(sin(x)*(tan(x)^2 + 1)^(1/2),x)
```

output
```
int(sin(x)*(tan(x)^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sin(x)\sqrt{1+\tan^2(x)}\,dx = \int \sqrt{\tan(x)^2+1}\,\sin(x)\,dx$$

input
```
int(sin(x)*(1+tan(x)^2)^(1/2),x)
```

output
```
int(sqrt(tan(x)**2 + 1)*sin(x),x)
```

3.82 $\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 9

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{1}{2}(1+x)^2$$

output
```
1/2*(1+x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=x+\frac{x^2}{2}$$

input
```
Integrate[(-1 + x^2 - x^3 + x^5)/(-1 + x - x^3 + x^4),x]
```

output
```
x + x^2/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5-x^3+x^2-1}{x^4-x^3+x-1} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int (x+1)dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2}(x+1)^2$$

input
```
Int[(-1 + x^2 - x^3 + x^5)/(-1 + x - x^3 + x^4),x]
```

output
```
(1 + x)^2/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{x(x+2)}{2}$	7
default	$\frac{1}{2}x^2+x$	8
norman	$\frac{1}{2}x^2+x$	8
risch	$\frac{1}{2}x^2+x$	8
parallelrisch	$\frac{1}{2}x^2+x$	8
parts	$\frac{1}{2}x^2+x$	8
orering	$\frac{x(x+2)(x^5-x^3+x^2-1)}{2(x+1)(x^4-x^3+x-1)}$	38

input
```
int((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{1}{2}\,x^2+x$$

input
```
integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="fricas")
```

output
```
1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{x^2}{2}+x$$

input
```
integrate((x**5-x**3+x**2-1)/(x**4-x**3+x-1),x)
```

output
```
x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{1}{2}\,x^2+x$$

input
```
integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="maxima")
```

output
```
1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{1}{2}\,x^2+x$$

input
```
integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="giac")
```

output
```
1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{x\,(x+2)}{2}$$

input
```
int((x^2 - x^3 + x^5 - 1)/(x - x^3 + x^4 - 1),x)
```

output
```
(x*(x + 2))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4}\,dx=\frac{x(x+2)}{2}$$

input
```
int((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x)
```

output
```
(x*(x + 2))/2
```

3.83 $\int \log(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 8

$$\int \log(x)\,dx = -x + x\log(x)$$

output
```
-x+x*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = -x + x\log(x)$$

input
```
Integrate[Log[x],x]
```

output
```
-x + x*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\,dx$$

$$\Big\downarrow \text{ 2732}$$

$$x\log(x) - x$$

input
```
Int[Log[x],x]
```

output
```
-x + x*Log[x]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
lookup	$\ln(x) x - x$	9
default	$\ln(x) x - x$	9
norman	$\ln(x) x - x$	9
risch	$\ln(x) x - x$	9
parallelrisch	$\ln(x) x - x$	9
parts	$\ln(x) x - x$	9
orering	$\ln(x) x - x$	9

input
```
int(ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)*x-x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\, dx = x \log(x) - x$$

input
```
integrate(log(x),x, algorithm="fricas")
```

output
```
x*log(x) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \log(x)\,dx = x\log{(x)} - x$$

input
```
integrate(ln(x),x)
```

output
```
x*log(x) - x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log{(x)} - x$$

input
```
integrate(log(x),x, algorithm="maxima")
```

output
```
x*log(x) - x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log{(x)} - x$$

input
```
integrate(log(x),x, algorithm="giac")
```

output
```
x*log(x) - x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x)\,dx = x\left(\ln\left(x\right) - 1\right)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x)\,dx = x(\log(x) - 1)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

3.84 $\int \frac{1}{1-e^{-x}} dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{1}{1-e^{-x}} dx = \log(-1+e^x)$$

output
```
ln(-1+exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-e^{-x}} dx = \log(-1+e^x)$$

input
```
Integrate[(1 - E^(-x))^(-1),x]
```

output
```
Log[-1 + E^x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-e^{-x}}dx$$

$$\Big\downarrow \; 2720$$

$$-\int \frac{e^x}{1-e^{-x}}de^{-x}$$

$$\Big\downarrow \; 47$$

$$-\int e^x de^{-x}-\int \frac{1}{1-e^{-x}}de^{-x}$$

$$\Big\downarrow \; 14$$

$$-\int \frac{1}{1-e^{-x}}de^{-x}-\log \left(e^{-x}\right)$$

$$\Big\downarrow \; 16$$

$$\log \left(1-e^{-x}\right)-\log \left(e^{-x}\right)$$

input
```
Int[(1 - E^(-x))^(-1),x]
```

output
```
-Log[E^(-x)] + Log[1 - E^(-x)]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

method	result	size
norman	$x + \ln(-1 + e^{-x})$	10
risch	$x + \ln(-1 + e^{-x})$	10
parallelrisch	$x + \ln(-1 + e^{-x})$	10
derivativedivides	$\ln(-1 + e^{-x}) - \ln(e^{-x})$	16
default	$\ln(-1 + e^{-x}) - \ln(e^{-x})$	16

input
```
int(1/(1-exp(-x)),x,method=_RETURNVERBOSE)
```

output
```
x+ln(-1+exp(-x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-e^{-x}}\,dx = x + \log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(1-exp(-x)),x, algorithm="fricas")
```

output
```
x + log(e^(-x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{1-e^{-x}}\,dx = x + \log\left(-1 + e^{-x}\right)$$

input
```
integrate(1/(1-exp(-x)),x)
```

output
```
x + log(-1 + exp(-x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-e^{-x}}\,dx = x + \log\left(e^{(-x)} - 1\right)$$

input
```
integrate(1/(1-exp(-x)),x, algorithm="maxima")
```

output
```
x + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{1}{1-e^{-x}}\,dx = x + \log\left(\left|e^{(-x)} - 1\right|\right)$$

input
```
integrate(1/(1-exp(-x)),x, algorithm="giac")
```

output
```
x + log(abs(e^(-x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{1-e^{-x}}\,dx = \ln\left(1 - e^{x}\right)$$

input
```
int(-1/(exp(-x) - 1),x)
```

output
```
log(1 - exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-e^{-x}}\,dx = \log(e^{x} - 1)$$

input
```
int(1/(1-exp(-x)),x)
```

output
```
log(e**x - 1)
```

3.85 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{32}(4x - \sin(4x))$$

output
```
1/8*x-1/32*sin(4*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{1}{32}\sin(4x)$$

input
```
Integrate[Cos[x]^2*Sin[x]^2,x]
```

output
```
x/8 - Sin[4*x]/32
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)^2\cos(x)^2dx$$

$$\Big\downarrow \; 3048$$

$$\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

$$\Big\downarrow \; 24$$

$$\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)$$

input
```
Int[Cos[x]^2*Sin[x]^2,x]
```

output
```
-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$-\frac{\cos(x)^3\sin(x)}{4}+\frac{\cos(x)\sin(x)}{8}+\frac{x}{8}$	19
orering	$x\cos(x)^2\sin(x)^2+\frac{\cos(x)\sin(x)^3}{8}-\frac{\cos(x)^3\sin(x)}{8}+\frac{x\left(2\sin(x)^4-12\cos(x)^2\sin(x)^2+2\cos(x)^4\right)}{16}$	54
norman	$\frac{\frac{x}{8}+\frac{7\tan\left(\frac{x}{2}\right)^3}{4}-\frac{7\tan\left(\frac{x}{2}\right)^5}{4}+\frac{\tan\left(\frac{x}{2}\right)^7}{4}+\frac{x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^4}{4}+\frac{x\tan\left(\frac{x}{2}\right)^6}{2}+\frac{x\tan\left(\frac{x}{2}\right)^8}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$	82

input
```
int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x-1/32*sin(4*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

input
```
integrate(cos(x)**2*sin(x)**2,x)
```

output
```
x/8 - sin(2*x)*cos(2*x)/16
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}x - \frac{1}{32}\sin(4x)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
1/8*x - 1/32*sin(4*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
1/8*x - 1/32*sin(4*x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos\left(x\right)\,\sin\left(x\right)^3}{4} - \frac{\cos\left(x\right)\,\sin\left(x\right)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \cos^2(x)\sin^2(x)\,dx = \frac{\cos\left(x\right)\sin\left(x\right)^3}{4} - \frac{\cos\left(x\right)\sin\left(x\right)}{8} + \frac{x}{8}$$

input
```
int(cos(x)^2*sin(x)^2,x)
```

output
```
(2*cos(x)*sin(x)**3 - cos(x)*sin(x) + x)/8
```

3.86 $\int \frac{\pi \sin(\pi\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{\pi \sin\left(\pi\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\pi\sqrt{x}\right)$$

output
```
-2*cos(Pi*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\pi \sin\left(\pi\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\pi\sqrt{x}\right)$$

input
```
Integrate[(Pi*Sin[Pi*Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Cos[Pi*Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {27, 3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\pi \sin \left(\pi \sqrt{x}\right)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 27}$$

$$\pi \int \frac{\sin \left(\pi \sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 3860}$$

$$2 \pi \int \sin \left(\pi \sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2 \pi \int \sin \left(\pi \sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3118}$$

$$-2 \cos \left(\pi \sqrt{x}\right)$$

input
```
Int[(Pi*Sin[Pi*Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Cos[Pi*Sqrt[x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-2\cos\left(\pi\sqrt{x}\right)$	9
default	$-2\cos\left(\pi\sqrt{x}\right)$	9
meijerg	$2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\pi\sqrt{x}\right)}{\sqrt{\pi}}\right)$	21

input
```
int(Pi*sin(Pi*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*cos(Pi*x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\pi \sin\left(\pi\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\pi\sqrt{x}\right)$$

input
```
integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-2*cos(pi*sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\pi \sin\left(\pi\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\pi\sqrt{x}\right)$$

input
```
integrate(pi*sin(pi*x**(1/2))/x**(1/2),x)
```

output
```
-2*cos(pi*sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\pi \sin\left(\pi\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cos\left(\pi\sqrt{x}\right)$$

input
```
integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
-2*cos(pi*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\pi \sin \left(\pi \sqrt{x}\right)}{\sqrt{x}}\,dx = -2 \cos \left(\pi \sqrt{x}\right)$$

input
```
integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
-2*cos(pi*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\pi \sin \left(\pi \sqrt{x}\right)}{\sqrt{x}}\,dx = -2 \cos \left(\Pi \sqrt{x}\right)$$

input
```
int((Pi*sin(Pi*x^(1/2)))/x^(1/2),x)
```

output
```
-2*cos(Pi*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\pi \sin \left(\pi \sqrt{x}\right)}{\sqrt{x}}\,dx = -2 \cos \left(\sqrt{x}\,\pi\right)$$

input
```
int(Pi*sin(Pi*x^(1/2))/x^(1/2),x)
```

output
```
 - 2*cos(sqrt(x)*pi)
```

3.87 $\int \tan^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 6

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

output
```
-x+tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \tan^2(x)\,dx = -\arctan(\tan(x)) + \tan(x)$$

input
```
Integrate[Tan[x]^2,x]
```

output
```
-ArcTan[Tan[x]] + Tan[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\tan(x)-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\tan(x)-x$$

input
```
Int[Tan[x]^2,x]
```

output
```
-x + Tan[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
norman	$-x+\tan \left(x \right)$	7
parallelrisch	$-x+\tan \left(x \right)$	7
derivativedivides	$\tan \left(x \right)-\arctan \left(\tan \left(x \right)\right)$	9
default	$\tan \left(x \right)-\arctan \left(\tan \left(x \right)\right)$	9
risch	$-x+\frac{2 i}{{\mathrm e}^{2 i x}+1}$	17

input
```
int(tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx=-x+\tan \left(x \right)$$

input
```
integrate(tan(x)^2,x, algorithm="fricas")
```

output
```
-x + tan(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \tan^2(x)\,dx = -x + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(tan(x)**2,x)
```

output
```
-x + sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

input
```
integrate(tan(x)^2,x, algorithm="maxima")
```

output
```
-x + tan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = -x + \tan(x)$$

input
```
integrate(tan(x)^2,x, algorithm="giac")
```

output
```
-x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = \tan(x) - x$$

input
```
int(tan(x)^2,x)
```

output
```
tan(x) - x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tan^2(x)\,dx = \tan{(x)} - x$$

input
```
int(tan(x)^2,x)
```

output
```
tan(x) - x
```

3.88 $\int e^{\sqrt[4]{x}}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^{\sqrt[4]{x}}\,dx = 4e^{\sqrt[4]{x}}\left(-6+6\sqrt[4]{x}-3\sqrt{x}+x^{3/4}\right)$$

output
```
4*exp(x^(1/4))*(-6+6*x^(1/4)-3*x^(1/2)+x^(3/4))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int e^{\sqrt[4]{x}}\,dx = e^{\sqrt[4]{x}}\left(-24+24\sqrt[4]{x}-12\sqrt{x}+4x^{3/4}\right)$$

input
```
Integrate[E^x^(1/4),x]
```

output
```
E^x^(1/4)*(-24 + 24*x^(1/4) - 12*Sqrt[x] + 4*x^(3/4))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2636, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sqrt[4]{x}}dx$$
$$\Big\downarrow \text{2636}$$
$$4\int e^{\sqrt[4]{x}}x^{3/4}d\sqrt[4]{x}$$
$$\Big\downarrow \text{2607}$$
$$4\left(e^{\sqrt[4]{x}}x^{3/4}-3\int e^{\sqrt[4]{x}}\sqrt{x}d\sqrt[4]{x}\right)$$
$$\Big\downarrow \text{2607}$$
$$4\left(e^{\sqrt[4]{x}}x^{3/4}-3\left(e^{\sqrt[4]{x}}\sqrt{x}-2\int e^{\sqrt[4]{x}}\sqrt[4]{x}d\sqrt[4]{x}\right)\right)$$
$$\Big\downarrow \text{2607}$$
$$4\left(e^{\sqrt[4]{x}}x^{3/4}-3\left(e^{\sqrt[4]{x}}\sqrt{x}-2\left(e^{\sqrt[4]{x}}\sqrt[4]{x}-\int e^{\sqrt[4]{x}}d\sqrt[4]{x}\right)\right)\right)$$
$$\Big\downarrow \text{2624}$$
$$4\left(e^{\sqrt[4]{x}}x^{3/4}-3\left(e^{\sqrt[4]{x}}\sqrt{x}-2\left(e^{\sqrt[4]{x}}\sqrt[4]{x}-e^{\sqrt[4]{x}}\right)\right)\right)$$

input
```
Int[E^x^(1/4),x]
```

output
```
4*(-3*(-2*(-E^x^(1/4) + E^x^(1/4)*x^(1/4)) + E^x^(1/4)*Sqrt[x]) + E^x^(1/4
)*x^(3/4))
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2636
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Simp[k/d   Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (
c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Int
egerQ[n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
meijerg	$24-\left(-4x^{\frac{3}{4}}+12\sqrt{x}-24x^{\frac{1}{4}}+24\right){\mathrm e}^{x^{\frac{1}{4}}}$	26
derivativedivides	$4\,{\mathrm e}^{x^{\frac{1}{4}}} x^{\frac{3}{4}}-12\sqrt{x}\,{\mathrm e}^{x^{\frac{1}{4}}}+24x^{\frac{1}{4}}{\mathrm e}^{x^{\frac{1}{4}}}-24\,{\mathrm e}^{x^{\frac{1}{4}}}$	35
default	$4\,{\mathrm e}^{x^{\frac{1}{4}}} x^{\frac{3}{4}}-12\sqrt{x}\,{\mathrm e}^{x^{\frac{1}{4}}}+24x^{\frac{1}{4}}{\mathrm e}^{x^{\frac{1}{4}}}-24\,{\mathrm e}^{x^{\frac{1}{4}}}$	35

input
```
int(exp(x^(1/4)),x,method=_RETURNVERBOSE)
```

output
```
24-(-4*x^(3/4)+12*x^(1/2)-24*x^(1/4)+24)*exp(x^(1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int e^{\sqrt[4]{x}}\,dx = 4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\,x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

input
```
integrate(exp(x^(1/4)),x, algorithm="fricas")
```

output
```
4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int e^{\sqrt[4]{x}}\,dx = 4x^{\frac{3}{4}}e^{\sqrt[4]{x}} + 24\sqrt[4]{x}e^{\sqrt[4]{x}} - 12\sqrt{x}e^{\sqrt[4]{x}} - 24e^{\sqrt[4]{x}}$$

input
```
integrate(exp(x**(1/4)),x)
```

output
```
4*x**(3/4)*exp(x**(1/4)) + 24*x**(1/4)*exp(x**(1/4)) - 12*sqrt(x)*exp(x**(
1/4)) - 24*exp(x**(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int e^{\sqrt[4]{x}}\,dx = 4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\,x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

input
```
integrate(exp(x^(1/4)),x, algorithm="maxima")
```

output
```
4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int e^{\sqrt[4]{x}}\,dx = 4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\,x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

input
```
integrate(exp(x^(1/4)),x, algorithm="giac")
```

output
```
4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{\sqrt[4]{x}}\,dx = -4\,x\,\mathrm{e}^{x^{1/4}}\left(\frac{6}{x} + \frac{3}{\sqrt{x}} - \frac{1}{x^{1/4}} - \frac{6}{x^{3/4}}\right)$$

input
```
int(exp(x^(1/4)),x)
```

output
```
-4*x*exp(x^(1/4))*(6/x + 3/x^(1/2) - 1/x^(1/4) - 6/x^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int e^{\sqrt[4]{x}}\,dx = 4e^{x^{\frac{1}{4}}}\left(x^{\frac{3}{4}} + 6x^{\frac{1}{4}} - 3\sqrt{x} - 6\right)$$

input
```
int(exp(x^(1/4)),x)
```

output
```
4*e**(x**(1/4))*(x**(3/4) + 6*x**(1/4) - 3*sqrt(x) - 6)
```

3.89 $\int \cos(x)\cot(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\cot(x)\,dx = -\operatorname{arctanh}(\cos(x)) + \cos(x)$$

output
```
-arctanh(cos(x))+cos(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[x],x]
```

output
```
Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\int \frac{\cos^2(x)}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{262}$$

$$\cos(x)-\int \frac{1}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{219}$$

$$\cos(x)-\operatorname{arctanh}(\cos(x))$$

input
```
Int[Cos[x]*Cot[x],x]
```

output
```
-ArcTanh[Cos[x]] + Cos[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$\cos(x)+\ln(\csc(x)-\cot(x))$	12
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\ln(e^{ix}+1)+\ln(e^{ix}-1)$	34

input
```
int(cos(x)*cot(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+ln(csc(x)-cot(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(8) = 16$.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(x),x, algorithm="fricas")
```

output
```
cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

input
```
integrate(cos(x)*cot(x),x)
```

output
```
log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

input
```
integrate(cos(x)*cot(x),x, algorithm="maxima")
```

output
```
cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \cos(x)\cot(x)\,dx = \cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(-\cos(x) + 1)$$

input
```
integrate(cos(x)*cot(x),x, algorithm="giac")
```

output
```
cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \cos(x)\cot(x)\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

input
```
int(cos(x)*cot(x),x)
```

output
```
log(tan(x/2)) + 2/(tan(x/2)^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \cos(x)\cot(x)\,dx = \cos(x) + \log\left(\tan\left(\frac{x}{2}\right)\right) - 1$$

input
```
int(cos(x)*cot(x),x)
```

output
```
cos(x) + log(tan(x/2)) - 1
```

3.90 $\int \left(2 \log(x)+\log^2(x)\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \left(2 \log(x)+\log^2(x)\right) \, dx=x \log^2(x)$$

output
```
x*ln(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2 \log(x)+\log^2(x)\right) \, dx=x \log^2(x)$$

input
```
Integrate[2*Log[x] + Log[x]^2,x]
```

output
```
x*Log[x]^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\log^2(x)+2\log(x)\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^2(x)$$

input
```
Int[2*Log[x] + Log[x]^2,x]
```

output
```
x*Log[x]^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x\ln(x)^{2}$	7
norman	$x\ln(x)^{2}$	7
risch	$x\ln(x)^{2}$	7
parallelrisch	$x\ln(x)^{2}$	7
parts	$x\ln(x)^{2}$	7
orering	$\left(2\ln(x)+\ln(x)^{2}\right)x-2\ln(x)x$	18

input
```
int(2*ln(x)+ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\log{(x)}^2$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="fricas")
```

output
```
x*log(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\log{(x)}^2$$

input
```
integrate(2*ln(x)+ln(x)**2,x)
```

output
```
x*log(x)**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(6) = 12.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.50

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = \left(\log\left(x\right)^2 - 2\,\log\left(x\right) + 2\right)x + 2\,x\log\left(x\right) - 2\,x$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="maxima")
```

output
```
(log(x)^2 - 2*log(x) + 2)*x + 2*x*log(x) - 2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\log\left(x\right)^2$$

input
```
integrate(2*log(x)+log(x)^2,x, algorithm="giac")
```

output
```
x*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x)+\log^2(x)\right)\,dx = x\ln\left(x\right)^2$$

input
```
int(2*log(x) + log(x)^2,x)
```

output
```
x*log(x)^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(2\log(x) + \log^2(x)\right) \, dx = \log(x)^2 \, x$$

input
```
int(2*log(x)+log(x)^2,x)
```

output
```
log(x)**2*x
```

3.91 $\int \frac{x^3}{1+x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{x^3}{1+x^2} \, dx = \frac{x^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

output
```
1/2*x^2-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^2} \, dx = \frac{x^2}{2} - \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[x^3/(1 + x^2),x]
```

output
```
x^2/2 - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^2+1} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{x^2}{x^2+1} dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2} \int \left(1+\frac{1}{-x^2-1}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(x^2-\log \left(x^2+1\right)\right)$$

input
```
Int[x^3/(1 + x^2),x]
```

output
```
(x^2 - Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
norman	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
meijerg	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
risch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
parallelrisch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15

input
```
int(x^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx=\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{\log(x^2+1)}{2}$$

input
```
integrate(x**3/(x**2+1),x)
```

output
```
x**2/2 - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3/(x^2+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = \frac{x^2}{2} - \frac{\ln(x^2+1)}{2}$$

input
```
int(x^3/(x^2 + 1),x)
```

output
```
x^2/2 - log(x^2 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{1+x^2}\,dx = -\frac{\log(x^2+1)}{2} + \frac{x^2}{2}$$

input
```
int(x^3/(x^2+1),x)
```

output
```
( - log(x**2 + 1) + x**2)/2
```

3.92 $\int \frac{1}{2-2x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{1}{2-2x+x^2} dx = -\arctan(1-x)$$

output
```
arctan(-1+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-2x+x^2} dx = -\arctan(1-x)$$

input
```
Integrate[(2 - 2*x + x^2)^(-1),x]
```

output
```
-ArcTan[1 - x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-2x+2}dx$$

$$\Big\downarrow \text{1082}$$

$$\int \frac{1}{-(1-x)^2-1}d(1-x)$$

$$\Big\downarrow \text{217}$$

$$-\arctan(1-x)$$

input
```
Int[(2 - 2*x + x^2)^(-1),x]
```

output
```
-ArcTan[1 - x]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

method	result	size
default	$\arctan(x-1)$	5
risch	$\arctan(x-1)$	5
parallelrisch	$\frac{i\ln(x-1+i)}{2}-\frac{i\ln(x-1-i)}{2}$	20

input
```
int(1/(x^2-2*x+2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{2-2x+x^2}\,dx=\arctan(x-1)$$

input
```
integrate(1/(x^2-2*x+2),x, algorithm="fricas")
```

output
```
arctan(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{2-2x+x^2}\,dx=\operatorname{atan}(x-1)$$

input
```
integrate(1/(x**2-2*x+2),x)
```

output
```
atan(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{2-2x+x^2}\,dx = \arctan(x-1)$$

input
```
integrate(1/(x^2-2*x+2),x, algorithm="maxima")
```

output
```
arctan(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{2-2x+x^2}\,dx = \arctan(x-1)$$

input
```
integrate(1/(x^2-2*x+2),x, algorithm="giac")
```

output
```
arctan(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{2-2x+x^2}\,dx = \operatorname{atan}(x-1)$$

input
```
int(1/(x^2 - 2*x + 2),x)
```

output
```
atan(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.50

$$\int \frac{1}{2-2x+x^2}\,dx = atan(x-1)$$

input
```
int(1/(x^2-2*x+2),x)
```

output
```
atan(x - 1)
```

3.93 $\int \log(\sin(x))\sin(x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 15

$$\int \log(\sin(x))\sin(x)\,dx = -\text{arctanh}(\cos(x)) + \cos(x) - \cos(x)\log(\sin(x))$$

output
```
-arctanh(cos(x))+cos(x)-cos(x)*ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \log(\sin(x))\sin(x)\,dx = \cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \cos(x)\log(\sin(x))$$

input
```
Integrate[Log[Sin[x]]*Sin[x],x]
```

output
```
Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]] - Cos[x]*Log[Sin[x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3034, 25, 3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\log(\sin(x))\,dx$$

$$\Big\downarrow \text{3034}$$

$$-\int -\cos(x)\cot(x)dx-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{25}$$

$$\int \cos(x)\cot(x)dx-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)dx-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{3072}$$

$$-\int \frac{\cos^2(x)}{1-\cos^2(x)}d\cos(x)-\cos(x)\log(\sin(x))$$

$$\Big\downarrow \text{262}$$

$$-\int \frac{1}{1-\cos^2(x)}d\cos(x)+\cos(x)+\cos(x)(-\log(\sin(x)))$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}(\cos(x))+\cos(x)-\cos(x)\log(\sin(x))$$

input
```
Int[Log[Sin[x]]*Sin[x],x]
```

output
```
-ArcTanh[Cos[x]] + Cos[x] - Cos[x]*Log[Sin[x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result
parallelrisch	$1+\ln\left(\tan\left(\frac{x}{2}\right)\right)-\cos\left(x\right)\left(-1+\ln\left(\sin\left(x\right)\right)\right)$
norman	$\frac{2\tan\left(\frac{x}{2}\right)^{2}\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)+2}{1+\tan\left(\frac{x}{2}\right)^{2}}+\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$
default	$-\frac{e^{ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{2}+\frac{e^{ix}}{2}+\ln\left(e^{ix}-1\right)-\ln\left(e^{ix}+1\right)-\frac{e^{-ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{2}+\frac{e^{-ix}}{2}+\frac{\ln(2)\left(e^{ix}\right.}{2}$
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\ln\left(e^{ix}+1\right)-\frac{ie^{-ix}\operatorname{csgn}(\sin(x))^{2}\operatorname{csgn}\left(ie^{2ix}-i\right)\pi}{4}-\frac{ie^{ix}\pi\operatorname{csgn}\left(ie^{2ix}-i\right)\operatorname{csgn}(\sin(x))^{2}}{4}-\frac{ie^{-ix}\operatorname{cs}}{}$

input
```
int(ln(sin(x))*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1+ln(tan(1/2*x))-cos(x)*(-1+ln(sin(x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \log(\sin(x))\sin(x)\,dx = -\cos\left(x\right)\log\left(\sin\left(x\right)\right)+\cos\left(x\right)-\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(log(sin(x))*sin(x),x, algorithm="fricas")
```

output
```
-cos(x)*log(sin(x)) + cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*co
s(x) + 1/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(15) = 30$.

Time = 0.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 7.00

$$\int \log(\sin(x))\sin(x)\,dx = \frac{2\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log(2)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2}{\tan^2\left(\frac{x}{2}\right)+1}$$

input
```
integrate(ln(sin(x))*sin(x),x)
```

output
```
2*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan(x/2)**2 + 1) + log(tan(
x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2
)**2 + 1) + 2*log(2)*tan(x/2)**2/(tan(x/2)**2 + 1) + 2/(tan(x/2)**2 + 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.93

$$\int \log(\sin(x))\sin(x)\,dx = -\frac{2\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} + \frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right) + \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)$$

input
```
integrate(log(sin(x))*sin(x),x, algorithm="maxima")
```

output
```
-2*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)))/(sin(x)^2/(c
os(x) + 1)^2 + 1) + 2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x)
 + 1)^2 + 1) + log(sin(x)^2/(cos(x) + 1)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \log(\sin(x))\sin(x)\,dx = -\cos(x)\log(\sin(x)) + \cos(x) - \frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(-\cos(x)+1)$$

input
```
integrate(log(sin(x))*sin(x),x, algorithm="giac")
```

output
```
-cos(x)*log(sin(x)) + cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \log(\sin(x))\sin(x)\,dx = \int \ln(\sin(x))\,\sin(x)\,dx$$

input
```
int(log(sin(x))*sin(x),x)
```

output
```
int(log(sin(x))*sin(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \log(\sin(x))\sin(x)\,dx = -\cos(x)\log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right) + \cos(x) + \log\left(\tan\left(\frac{x}{2}\right)^2+1\right) + \log\left(\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+1}\right) - 1$$

input
```
int(log(sin(x))*sin(x),x)
```

output
```
- cos(x)*log((2*tan(x/2))/(tan(x/2)**2 + 1)) + cos(x) + log(tan(x/2)**2 +
1) + log((2*tan(x/2))/(tan(x/2)**2 + 1)) - 1
```

3.94 $\int \frac{x}{1-x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{x}{1-x^4} \, dx = \frac{\operatorname{arctanh}(x^2)}{2}$$

output
```
1/2*arctanh(x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{x}{1-x^4} \, dx = -\frac{1}{4} \log \left(1-x^2\right) + \frac{1}{4} \log \left(1+x^2\right)$$

input
```
Integrate[x/(1 - x^4),x]
```

output
```
-1/4*Log[1 - x^2] + Log[1 + x^2]/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{1-x^4}\,dx$$
$$\Big\downarrow \ 807$$
$$\frac{1}{2}\int \frac{1}{1-x^4}dx^2$$
$$\Big\downarrow \ 219$$
$$\frac{\operatorname{arctanh}\left(x^2\right)}{2}$$

input
```
Int[x/(1 - x^4),x]
```

output
```
ArcTanh[x^2]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}$	7
risch	$\frac{\ln\left(x^{2}+1\right)}{4}-\frac{\ln\left(x^{2}-1\right)}{4}$	18
default	$-\frac{\ln\left(x-1\right)}{4}-\frac{\ln\left(x+1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}$	22
norman	$-\frac{\ln\left(x-1\right)}{4}-\frac{\ln\left(x+1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}$	22
parallelrisch	$-\frac{\ln\left(x-1\right)}{4}-\frac{\ln\left(x+1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}$	22

input
```
int(x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log\left(x^2+1\right)-\frac{1}{4}\log\left(x^2-1\right)$$

input
```
integrate(x/(-x^4+1),x, algorithm="fricas")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{x}{1-x^4}\,dx = -\frac{\log(x^2-1)}{4} + \frac{\log(x^2+1)}{4}$$

input
```
integrate(x/(-x**4+1),x)
```

output
```
-log(x**2 - 1)/4 + log(x**2 + 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log(x^2+1) - \frac{1}{4}\log(x^2-1)$$

input
```
integrate(x/(-x^4+1),x, algorithm="maxima")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log(x^2+1) - \frac{1}{4}\log(|x^2-1|)$$

input
```
integrate(x/(-x^4+1),x, algorithm="giac")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1-x^4}\,dx = \frac{\operatorname{atanh}(x^2)}{2}$$

input
```
int(-x/(x^4 - 1),x)
```

output
```
atanh(x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{x}{1-x^4}\,dx = \frac{\log(x^2+1)}{4} - \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4}$$

input
```
int(x/(-x^4+1),x)
```

output
```
(log(x**2 + 1) - log(x - 1) - log(x + 1))/4
```

3.95 $\int \sqrt{12-3x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)$$

output
```
1/2*3^(1/2)*x*(-x^2+4)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \sqrt{12-3x^2}\,dx=\frac{1}{2}\sqrt{3}\left(x\sqrt{4-x^2}-8\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)\right)$$

input
```
Integrate[Sqrt[12 - 3*x^2],x]
```

output
```
(Sqrt[3]*(x*Sqrt[4 - x^2] - 8*ArcTan[Sqrt[4 - x^2]/(2 + x)]))/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{12-3x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$6\int \frac{1}{\sqrt{12-3x^2}}dx+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

$$\Big\downarrow \text{223}$$

$$2\sqrt{3}\arcsin\left(\frac{x}{2}\right)+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

input
```
Int[Sqrt[12 - 3*x^2],x]
```

output
```
(Sqrt[3]*x*Sqrt[4 - x^2])/2 + 2*Sqrt[3]*ArcSin[x/2]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{x\sqrt{-3x^2+12}}{2}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	23
risch	$-\frac{3x(x^2-4)}{2\sqrt{-3x^2+12}}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	28
meijerg	$\frac{i\sqrt{3}\left(-i\sqrt{\pi}\,x\sqrt{-\frac{x^2}{4}+1}-2i\sqrt{\pi}\,\arcsin\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{-3x^2+12}}{2}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{-3x^2+12}\,\sqrt{3}}{3x}\right)$	37
trager	$\frac{x\sqrt{-3x^2+12}}{2}+2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+3\right)\sqrt{-3x^2+12}+3x\right)$	43

input
```
int((-3*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-3*x^2+12)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sqrt{12-3x^2}\,dx = \frac{1}{2}\sqrt{-3\,x^2+12}x - 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+12}x}{3\,(x^2-4)}\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-3*x^2 + 12)*x - 2*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 12)*x
/(x^2 - 4))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{12-3x^2}\,dx = \frac{\sqrt{3}x\sqrt{4-x^2}}{2} + 2\sqrt{3}\,\text{asin}\left(\frac{x}{2}\right)$$

input
```
integrate((-3*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*x*sqrt(4 - x**2)/2 + 2*sqrt(3)*asin(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \sqrt{12-3x^2}\,dx = \frac{1}{2}\,\sqrt{-3\,x^2+12}x + 2\,\sqrt{3}\arcsin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-3*x^2 + 12)*x + 2*sqrt(3)*arcsin(1/2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt{12-3x^2}\,dx = \frac{1}{2}\,\sqrt{3}\left(\sqrt{-x^2+4}x + 4\,\arcsin\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate((-3*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*(sqrt(-x^2 + 4)*x + 4*arcsin(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sqrt{12-3x^2}\,dx = 2\sqrt{3}\,\mathrm{asin}\left(\frac{x}{2}\right) + \frac{\sqrt{3}\,x\,\sqrt{4-x^2}}{2}$$

input
```
int((12 - 3*x^2)^(1/2),x)
```

output
```
2*3^(1/2)*asin(x/2) + (3^(1/2)*x*(4 - x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sqrt{12-3x^2}\,dx = \frac{\sqrt{3}\,\left(4asin\left(\frac{x}{2}\right) + \sqrt{-x^2+4}\,x\right)}{2}$$

input
```
int((-3*x^2+12)^(1/2),x)
```

output
```
(sqrt(3)*(4*asin(x/2) + sqrt( - x**2 + 4)*x))/2
```

3.96 $\int \sec^5(x)\tan^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \sec^5(x)\tan^3(x)\,dx = -\frac{1}{5}\sec^5(x) + \frac{\sec^7(x)}{7}$$

output
```
-1/5*sec(x)^5+1/7*sec(x)^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^5(x)\tan^3(x)\,dx = -\frac{1}{5}\sec^5(x) + \frac{\sec^7(x)}{7}$$

input
```
Integrate[Sec[x]^5*Tan[x]^3,x]
```

output
```
-1/5*Sec[x]^5 + Sec[x]^7/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\sec^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^3\sec(x)^5dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int -\sec^4(x)\left(1-\sec^2(x)\right)d\sec(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sec^4(x)\left(1-\sec^2(x)\right)d\sec(x)$$

$$\Big\downarrow \text{ 244}$$

$$-\int \left(\sec^4(x)-\sec^6(x)\right)d\sec(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}$$

input
```
Int[Sec[x]^5*Tan[x]^3,x]
```

output
```
-1/5*Sec[x]^5 + Sec[x]^7/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\sec(x)^5}{5}+\frac{\sec(x)^7}{7}$	14
default	$-\frac{\sec(x)^5}{5}+\frac{\sec(x)^7}{7}$	14
risch	$-\frac{32\left(7\,\mathrm{e}^{9ix}-6\,\mathrm{e}^{7ix}+7\,\mathrm{e}^{5ix}\right)}{35\left(\mathrm{e}^{2ix}+1\right)^7}$	34

input
```
int(sec(x)^5*tan(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*sec(x)^5+1/7*sec(x)^7
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^5(x)\tan^3(x)\,dx = -\frac{7\cos(x)^2 - 5}{35\cos(x)^7}$$

input
```
integrate(sec(x)^5*tan(x)^3,x, algorithm="fricas")
```

output
```
-1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^5(x)\tan^3(x)\,dx = \frac{5 - 7\cos^2(x)}{35\cos^7(x)}$$

input
```
integrate(sec(x)**5*tan(x)**3,x)
```

output
```
(5 - 7*cos(x)**2)/(35*cos(x)**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^5(x)\tan^3(x)\,dx = -\frac{7\cos(x)^2 - 5}{35\cos(x)^7}$$

input
```
integrate(sec(x)^5*tan(x)^3,x, algorithm="maxima")
```

output
```
-1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^5(x) \tan^3(x) \, dx = -\frac{7 \cos(x)^2 - 5}{35 \cos(x)^7}$$

input
```
integrate(sec(x)^5*tan(x)^3,x, algorithm="giac")
```

output
```
-1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sec^5(x) \tan^3(x) \, dx = \frac{1}{7 \cos(x)^7} - \frac{1}{5 \cos(x)^5}$$

input
```
int(tan(x)^3/cos(x)^5,x)
```

output
```
1/(7*cos(x)^7) - 1/(5*cos(x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^5(x) \tan^3(x) \, dx = \frac{\sec(x)^5 (5 \tan(x)^2 - 2)}{35}$$

input
```
int(sec(x)^5*tan(x)^3,x)
```

output
```
(sec(x)**5*(5*tan(x)**2 - 2))/35
```

3.97 $\int \frac{1}{1-\sin(x)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{1-\sin(x)} \, dx = \frac{\cos(x)}{1-\sin(x)}$$

output
```
cos(x)/(1-sin(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{1-\sin(x)} \, dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(1 - Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin(x)} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin(x)} dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\cos(x)}{1-\sin(x)}$$

input
```
Int[(1 - Sin[x])^(-1),x]
```

output
```
Cos[x]/(1 - Sin[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
parallelrisch	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

input
```
int(1/(1-sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(1/2*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="fricas")
```

output
```
(cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
integrate(1/(1-sin(x)),x)
```

output
```
-2/(tan(x/2) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

input
```
integrate(1/(1-sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(-1/(sin(x) - 1),x)
```

output
```
-2/(tan(x/2) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{1}{1-\sin(x)}\,dx = -\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(1/(1-sin(x)),x)
```

output
```
( - 2*tan(x/2))/(tan(x/2) - 1)
```

3.98 $\int \frac{1}{x\sqrt{-2+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x*Sqrt[-2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2-2}}dx^2$$

$$\Big\downarrow \text{ 73}$$

$$\int \frac{1}{x^4+2}d\sqrt{x^2-2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[1/(x*Sqrt[-2 + x^2]),x]
```

output
```
ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{2}$	18
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{x^2-2}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	19
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\sqrt{x^2-2}+\mathrm{RootOf}\left(\_Z^2+2\right)}{x}\right)}{2}$	28
meijerg	$\frac{\sqrt{2}\,\sqrt{-\,\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}\,\left((-3\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{x^2}{2}}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}}$	68

input
```
int(1/x/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctan(1/(x^2-2)^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)$$

input
```
integrate(1/x/(x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(x^2 - 2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \begin{cases} \frac{\sqrt{2}i\operatorname{acosh}\left(\frac{\sqrt{2}}{x}\right)}{2} & \text{for}\ \frac{1}{|x^2|}>\frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{x}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(x**2-2)**(1/2),x)
```

output
```
Piecewise((sqrt(2)*I*acosh(sqrt(2)/x)/2, 1/Abs(x**2) > 1/2), (-sqrt(2)*asi
n(sqrt(2)/x)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = -\frac{1}{2}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right)$$

input
```
integrate(1/x/(x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arcsin(sqrt(2)/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{x^2-2}\right)$$

input
```
integrate(1/x/(x^2-2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x^2-2}}{2}\right)}{2}$$

input
```
int(1/(x*(x^2 - 2)^(1/2)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{-2+x^2}}\,dx = \sqrt{2}\,atan\left(\frac{\sqrt{x^2-2}+x}{\sqrt{2}}\right)$$

input
```
int(1/x/(x^2-2)^(1/2),x)
```

output
```
sqrt(2)*atan((sqrt(x**2 - 2) + x)/sqrt(2))
```

3.99 $\int \log(x^2) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 10

$$\int \log(x^2) \, dx = -2x + x \log(x^2)$$

output
```
-2*x+x*ln(x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(x^2) \, dx = -2x + x \log(x^2)$$

input
```
Integrate[Log[x^2],x]
```

output
```
-2*x + x*Log[x^2]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x^2)\,dx$$
$$\Big\downarrow \text{2732}$$
$$x\log(x^2)-2x$$

input
```
Int[Log[x^2],x]
```

output
```
-2*x + x*Log[x^2]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-2x+x\ln(x^2)$	11
norman	$-2x+x\ln(x^2)$	11
risch	$-2x+x\ln(x^2)$	11
parallelrisch	$-2x+x\ln(x^2)$	11
parts	$-2x+x\ln(x^2)$	11
orering	$-2x+x\ln(x^2)$	11

input
```
int(ln(x^2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*ln(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log \left(x^2\right) \, dx = x \log \left(x^2\right) - 2 \, x$$

input
```
integrate(log(x^2),x, algorithm="fricas")
```

output
```
x*log(x^2) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \log \left(x^2\right) \, dx = x \log \left(x^2\right) - 2x$$

input
```
integrate(ln(x**2),x)
```

output
```
x*log(x**2) - 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log \left(x^2\right) \, dx = x \log \left(x^2\right) - 2 \, x$$

input
```
integrate(log(x^2),x, algorithm="maxima")
```

output
```
x*log(x^2) - 2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log \left(x^2\right) \, dx = x \log \left(x^2\right) - 2 \, x$$

input
```
integrate(log(x^2),x, algorithm="giac")
```

output
```
x*log(x^2) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \log \left(x^2\right) \, dx = x \left(\ln \left(x^2\right) - 2\right)$$

input
```
int(log(x^2),x)
```

output
```
x*(log(x^2) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \log \left(x^2\right) \, dx = x\left(\log\left(x^2\right) - 2\right)$$

input
```
int(log(x^2),x)
```

output
```
x*(log(x**2) - 2)
```

3.100 $\int \sin\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = 6\cos\left(\sqrt[3]{x}\right) - 3x^{2/3}\cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)$$

output
```
6*cos(x^(1/3))-3*x^(2/3)*cos(x^(1/3))+6*x^(1/3)*sin(x^(1/3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = -3\left(-2 + x^{2/3}\right)\cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)$$

input
```
Integrate[Sin[x^(1/3)],x]
```

output
```
-3*(-2 + x^(2/3))*Cos[x^(1/3)] + 6*x^(1/3)*Sin[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3842, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt[3]{x}\right) dx$$

$$\Big\downarrow \text{3842}$$

$$3\int x^{2/3}\sin\left(\sqrt[3]{x}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \text{3042}$$

$$3\int x^{2/3}\sin\left(\sqrt[3]{x}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \text{3777}$$

$$3\left(2\int \sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) d\sqrt[3]{x} - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$3\left(2\int \sqrt[3]{x}\sin\left(\sqrt[3]{x}+\frac{\pi}{2}\right) d\sqrt[3]{x} - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$3\left(2\left(\int -\sin\left(\sqrt[3]{x}\right) d\sqrt[3]{x} + \sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(2\left(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - \int \sin\left(\sqrt[3]{x}\right) d\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$3\left(2\left(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) - \int \sin\left(\sqrt[3]{x}\right) d\sqrt[3]{x}\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$3\left(2\left(\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) + \cos\left(\sqrt[3]{x}\right)\right) - x^{2/3}\cos\left(\sqrt[3]{x}\right)\right)$$

input
```
Int[Sin[x^(1/3)],x]
```

output
```
3*(-(x^(2/3)*Cos[x^(1/3)]) + 2*(Cos[x^(1/3)] + x^(1/3)*Sin[x^(1/3)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$6\cos\left(x^{\frac{1}{3}}\right)-3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$	26
default	$6\cos\left(x^{\frac{1}{3}}\right)-3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$	26
meijerg	$12\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^{\frac{2}{3}}}{2}+1\right)\cos\left(x^{\frac{1}{3}}\right)}{2\sqrt{\pi}}+\frac{x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)}{2\sqrt{\pi}}\right)$	40

input
```
int(sin(x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
6*cos(x^(1/3))-3*x^(2/3)*cos(x^(1/3))+6*x^(1/3)*sin(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[3]{x}\right)\,dx=-3\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)+6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(sin(x^(1/3)),x, algorithm="fricas")
```

output
```
-3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = -3x^{\frac{2}{3}}\cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right) + 6\cos\left(\sqrt[3]{x}\right)$$

input
```
integrate(sin(x**(1/3)),x)
```

output
```
-3*x**(2/3)*cos(x**(1/3)) + 6*x**(1/3)*sin(x**(1/3)) + 6*cos(x**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = -3\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) + 6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(sin(x^(1/3)),x, algorithm="maxima")
```

output
```
-3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = -3\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) + 6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(sin(x^(1/3)),x, algorithm="giac")
```

output
```
-3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = 6\,x^{1/3}\,\sin\left(x^{1/3}\right) - 3\,\cos\left(x^{1/3}\right)\,\left(x^{2/3}-2\right)$$

input
```
int(sin(x^(1/3)),x)
```

output
```
6*x^(1/3)*sin(x^(1/3)) - 3*cos(x^(1/3))*(x^(2/3) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sin\left(\sqrt[3]{x}\right)\,dx = -3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right) + 6\cos\left(x^{\frac{1}{3}}\right) + 6x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

input
```
int(sin(x^(1/3)),x)
```

output
```
3*( - x**(2/3)*cos(x**(1/3)) + 2*cos(x**(1/3)) + 2*x**(1/3)*sin(x**(1/3)))
```

3.101 $\int e^{1+x-x^2}(1-2x)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 10

$$\int e^{1+x-x^2}(1-2x)\,dx = e^{1+x-x^2}$$

output
```
exp(-x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{1+x-x^2}(1-2x)\,dx = e^{1+x-x^2}$$

input
```
Integrate[E^(1 + x - x^2)*(1 - 2*x),x]
```

output
```
E^(1 + x - x^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2666}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^2+x+1}(1-2x)\,dx$$

$$\Big\downarrow \text{2666}$$

$$e^{-x^2+x+1}$$

input
```
Int[E^(1 + x - x^2)*(1 - 2*x),x]
```

output
```
E^(1 + x - x^2)
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
gosper	$\mathrm{e}^{-x^{2}+x+1}$	10
derivativedivides	$\mathrm{e}^{-x^{2}+x+1}$	10
default	$\mathrm{e}^{-x^{2}+x+1}$	10
norman	$\mathrm{e}^{-x^{2}+x+1}$	10
risch	$\mathrm{e}^{-x^{2}+x+1}$	10
parallelrisch	$\mathrm{e}^{-x^{2}+x+1}$	10
orering	$-\frac{\mathrm{e}^{-x^{2}+x+1}(1-2x)}{-1+2x}$	24
parts	$-\sqrt{\pi}\,\mathrm{e}^{\frac{5}{4}}\operatorname{erf}\left(x-\frac{1}{2}\right)x+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{5}{4}}\operatorname{erf}\left(x-\frac{1}{2}\right)}{2}+\frac{\mathrm{e}^{\frac{5}{4}}\left(2x\operatorname{erf}\left(x-\frac{1}{2}\right)\sqrt{\pi}-\operatorname{erf}\left(x-\frac{1}{2}\right)\sqrt{\pi}+2\,\mathrm{e}^{-\left(x-\frac{1}{2}\right)^{2}}\right)}{2}$	59

input
```
int(exp(-x^2+x+1)*(1-2*x),x,method=_RETURNVERBOSE)
```

output
```
exp(-x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int e^{1+x-x^2}(1-2x)\,dx=e^{\left(-x^2+x+1\right)}$$

input
```
integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="fricas")
```

output
```
e^(-x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{1+x-x^2}(1-2x)\,dx = e^{-x^2+x+1}$$

input
```
integrate(exp(-x**2+x+1)*(1-2*x),x)
```

output
```
exp(-x**2 + x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int e^{1+x-x^2}(1-2x)\,dx = e^{(-x^2+x+1)}$$

input
```
integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="maxima")
```

output
```
e^(-x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int e^{1+x-x^2}(1-2x)\,dx = e^{(-x^2+x+1)}$$

input
```
integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="giac")
```

output
```
e^(-x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int e^{1+x-x^2}(1-2x)\,dx = \mathrm{e}\,\mathrm{e}^{-x^2}\,\mathrm{e}^x$$

input
```
int(-exp(x - x^2 + 1)*(2*x - 1),x)
```

output
```
exp(1)*exp(-x^2)*exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int e^{1+x-x^2}(1-2x)\,dx = \frac{e^x e}{e^{x^2}}$$

input
```
int(exp(-x^2+x+1)*(1-2*x),x)
```

output
```
(e**x*e)/e**(x**2)
```

3.102 $\int e^{\sqrt{x}}\sqrt{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 4e^{\sqrt{x}} - 4e^{\sqrt{x}}\sqrt{x} + 2e^{\sqrt{x}}x$$

output
```
4*exp(x^(1/2))-4*exp(x^(1/2))*x^(1/2)+2*exp(x^(1/2))*x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.32

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 2\Gamma\left(3,-\sqrt{x}\right)$$

input
```
Integrate[E^Sqrt[x]*Sqrt[x],x]
```

output
```
2*Gamma[3, -Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2645, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sqrt{x}}\sqrt{x}\,dx$$

$$\Big\downarrow \text{2645}$$

$$2\int e^{\sqrt{x}}xd\sqrt{x}$$

$$\Big\downarrow \text{2607}$$

$$2\left(e^{\sqrt{x}}x-2\int e^{\sqrt{x}}\sqrt{x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2607}$$

$$2\left(e^{\sqrt{x}}x-2\left(e^{\sqrt{x}}\sqrt{x}-\int e^{\sqrt{x}}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{2624}$$

$$2\left(e^{\sqrt{x}}x-2\left(e^{\sqrt{x}}\sqrt{x}-e^{\sqrt{x}}\right)\right)$$

input
```
Int[E^Sqrt[x]*Sqrt[x],x]
```

output
```
2*(-2*(-E^Sqrt[x] + E^Sqrt[x]*Sqrt[x]) + E^Sqrt[x]*x)
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2645
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(m +
1) - 1)*F^(a + b*x^(k*n)), x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b,
c, d, m, n}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[0, (m + 1)/n, 5] &&  !Int
egerQ[n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

method	result	size
meijerg	$-4+\frac{2\left(3x-6\sqrt{x}+6\right){\mathrm e}^{\sqrt{x}}}{3}$	19
derivativedivides	$4\,{\mathrm e}^{\sqrt{x}}-4\,{\mathrm e}^{\sqrt{x}}\sqrt{x}+2\,{\mathrm e}^{\sqrt{x}}x$	24
default	$4\,{\mathrm e}^{\sqrt{x}}-4\,{\mathrm e}^{\sqrt{x}}\sqrt{x}+2\,{\mathrm e}^{\sqrt{x}}x$	24

input
```
int(exp(x^(1/2))*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4+2/3*(3*x-6*x^(1/2)+6)*exp(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 2\left(x - 2\sqrt{x} + 2\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

output
```
2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = -4\sqrt{x}e^{\sqrt{x}} + 2xe^{\sqrt{x}} + 4e^{\sqrt{x}}$$

input
```
integrate(exp(x**(1/2))*x**(1/2),x)
```

output
```
-4*sqrt(x)*exp(sqrt(x)) + 2*x*exp(sqrt(x)) + 4*exp(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 2\left(x - 2\sqrt{x} + 2\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

output
```
2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 2\left(x-2\sqrt{x}+2\right)e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))*x^(1/2),x, algorithm="giac")
```

output
```
2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 4\,\mathrm{e}^{\sqrt{x}}+2\,x\,\mathrm{e}^{\sqrt{x}}-4\,\sqrt{x}\,\mathrm{e}^{\sqrt{x}}$$

input
```
int(x^(1/2)*exp(x^(1/2)),x)
```

output
```
4*exp(x^(1/2)) + 2*x*exp(x^(1/2)) - 4*x^(1/2)*exp(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.38

$$\int e^{\sqrt{x}}\sqrt{x}\,dx = 2e^{\sqrt{x}}\left(-2\sqrt{x}+x+2\right)$$

input
```
int(exp(x^(1/2))*x^(1/2),x)
```

output
```
2*e**sqrt(x)*( - 2*sqrt(x) + x + 2)
```

3.103 $\int \cos(3x)\sin(2x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x)\sin(2x)\,dx = \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

output
```
1/2*cos(x)-1/10*cos(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x)\sin(2x)\,dx = \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[3*x]*Sin[2*x],x]
```

output
```
Cos[x]/2 - Cos[5*x]/10
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(3x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(2x)\cos(3x)dx$$
$$\Big\downarrow \text{4772}$$
$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

input
```
Int[Cos[3*x]*Sin[2*x],x]
```

output
```
Cos[x]/2 - Cos[5*x]/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
parallelrisch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}-\frac{2}{5}$	13
orering	$\frac{3\sin(3x)\sin(2x)}{5}+\frac{2\cos(3x)\cos(2x)}{5}$	22
norman	$\frac{-\frac{4\tan(x)^2}{5}-\frac{4\tan\left(\frac{3x}{2}\right)^2}{5}+\frac{12\tan(x)\tan\left(\frac{3x}{2}\right)}{5}}{\left(1+\tan\left(\frac{3x}{2}\right)^2\right)\left(1+\tan(x)^2\right)}$	43

input
```
int(cos(3*x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)-1/10*cos(5*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x)\sin(2x)\,dx = -\frac{8}{5}\cos(x)^5 + 2\cos(x)^3$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 2*cos(x)^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x)\sin(2x)\,dx = \frac{3\sin(2x)\sin(3x)}{5} + \frac{2\cos(2x)\cos(3x)}{5}$$

input
```
integrate(cos(3*x)*sin(2*x),x)
```

output
```
3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\sin(2x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x)\sin(2x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x)\sin(2x)\,dx = 2\cos(x)^3 - \frac{8\cos(x)^5}{5}$$

input
```
int(cos(3*x)*sin(2*x),x)
```

output
```
2*cos(x)^3 - (8*cos(x)^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cos(3x)\sin(2x)\,dx = \frac{2\cos(3x)\cos(2x)}{5} + \frac{3\sin(3x)\sin(2x)}{5}$$

input
```
int(cos(3*x)*sin(2*x),x)
```

output
```
(2*cos(3*x)*cos(2*x) + 3*sin(3*x)*sin(2*x))/5
```

3.104 $\int (1+2\sin(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int (1+2\sin(x))\,dx = x-2\cos(x)$$

output
```
x-2*cos(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1+2\sin(x))\,dx = x-2\cos(x)$$

input
```
Integrate[1 + 2*Sin[x],x]
```

output
```
x - 2*Cos[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2\sin(x)+1)\,dx$$

$$\Big\downarrow \; 2009$$

$$x-2\cos(x)$$

input
```
Int[1 + 2*Sin[x],x]
```

output
```
x - 2*Cos[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x-2\cos\left(x\right)$	7
risch	$x-2\cos\left(x\right)$	7
parts	$x-2\cos\left(x\right)$	7
parallelrisch	$-2\cos\left(x\right)-2+x$	8
orering	$x(1+2\sin\left(x\right))-2\cos\left(x\right)-2x\sin\left(x\right)$	19
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{2}-4}{1+\tan\left(\frac{x}{2}\right)^{2}}$	23

input
```
int(1+2*sin(x),x,method=_RETURNVERBOSE)
```

output
```
x-2*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1 + 2\sin(x))\,dx = x - 2\,\cos\left(x\right)$$

input
```
integrate(1+2*sin(x),x, algorithm="fricas")
```

output
```
x - 2*cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int (1 + 2\sin(x))\,dx = x - 2\cos\left(x\right)$$

input
```
integrate(1+2*sin(x),x)
```

output
```
x - 2*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1 + 2\sin(x))\,dx = x - 2\,\cos(x)$$

input
```
integrate(1+2*sin(x),x, algorithm="maxima")
```

output
```
x - 2*cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1 + 2\sin(x))\,dx = x - 2\,\cos(x)$$

input
```
integrate(1+2*sin(x),x, algorithm="giac")
```

output
```
x - 2*cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1 + 2\sin(x))\,dx = x - 2\,\cos(x)$$

input
```
int(2*sin(x) + 1,x)
```

output
```
x - 2*cos(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int (1+2\sin(x))\,dx = -2\cos(x)+x$$

input
```
int(1+2*sin(x),x)
```

output
```
- 2*cos(x) + x
```

3.105 $\int (1-x)^{2014} x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int (1-x)^{2014} x \, dx = -\frac{(1-x)^{2015}}{2015} + \frac{(1-x)^{2016}}{2016}$$

output
```
-1/2015*(1-x)^2015+1/2016*(1-x)^2016
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12138 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 12138, normalized size of antiderivative = 527.74

$$\int (1-x)^{2014} x \, dx = \text{Result too large to show}$$

input
```
Integrate[(1 - x)^2014*x,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{2014}x\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((1-x)^{2014}-(1-x)^{2015}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-x)^{2016}}{2016}-\frac{(1-x)^{2015}}{2015}$$

input
```
Int[(1 - x)^2014*x,x]
```

output
```
-1/2015*(1 - x)^2015 + (1 - x)^2016/2016
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10075 vs. 2(19) = 38.

Time = 7.77 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

method	result	size
gosper	Expression too large to display	10076
default	Expression too large to display	10077
risch	Expression too large to display	10077
parallelrisch	Expression too large to display	10077
orering	Expression too large to display	10088

input
```
int((1-x)^2014*x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-2)]

Exception generated.

$$\int (1-x)^{2014}x\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((1-x)^2014*x,x, algorithm="fricas")
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12024 vs. 2(12) = 24.

Time = 2.79 (sec) , antiderivative size = 12024, normalized size of antiderivative = 522.78

$$\int (1-x)^{2014} x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)**2014*x,x)
```

output
```
x**2016/2016 - 2014*x**2015/2015 + 2013*x**2014/2 - 2026084*x**2013/3 + 13
58826667*x**2012/4 - 136629987582*x**2011 + 137313103532509*x**2010/30 -
91953985549170536*x**2009/7 + 26377651026988133103*x**2008/8 - 66174915585
42444915874*x**2007/9 + 294992994835264731661117*x**2006/2 - 1344229107990
97740606580164*x**2005/5 + 53876689232818214844524454823*x**2004/12 - 6917
62702790623489451562620638*x**2003 + 2571973087266166342850029070691063*x*
*2002/26 - 39588591248274824267756569403940400*x**2001/3 + 131961937837090
040663501674882660163383*x**2000/80 - 193964593913505209402992927651733959
950*x**1999 + 387541161559807075301489477559812273935075*x**1998/18 - 2262
922530915969121458419278661196219696900*x**1997 + 903358447595910292527771
972191922410542659825*x**1996/4 - 2145475773979272745021812337899023630563
0977010*x**1995 + 3889161470317168646281025630946632817062982660525*x**199
4/2 - 168502105628944447818705199670044187130493533353400*x**1993 + 111885
369941148961252482398045233357984870743212666075*x**1992/8 - 1113818576786
312843600553552092286915421060813044865230*x**1991 + 170499877545918079073
432876733170209565984828166917578371*x**1990/2 - 7347690365544198594596833
03160816263037237362099897570964140*x**1989/117 + 178354148359957950795405
5928245847409394570340210724349640535*x**1988/4 - 305508278474757676047756
49853836796394905428348789090875481890*x**1987 + 1213478574439825631499678
6936077927810352871698504752093214631735*x**1986/6 - 129502922523593280...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. $2(15) = 30$.

Time = 2.52 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

$$\int (1-x)^{2014} x \, dx = \text{Too large to display}$$

input
```
integrate((1-x)^2014*x,x, algorithm="maxima")
```

output
```
1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 1358
826667/4*x^2012 - 136629987582*x^2011 + 1373131035323509/30*x^2010 - 91953
985549170536/7*x^2009 + 26377651026988133103/8*x^2008 - 661749155854244491
5874/9*x^2007 + 294992994835264731661117/2*x^2006 - 1344229107990977406065
80164/5*x^2005 + 5387668923281821484452445482 3/12*x^2004 - 691762702790623
489451562620638*x^2003 + 25719730872661663428500290706910 63/26*x^2002 - 39
588591248274824267756569403940400/3*x^2001 + 131961937837090040663501674 88
2660163383/80*x^2000 - 1939645939135052094029929276517339 59950*x^1999 + 38
754116155980707530148947755981227393507 5/18*x^1998 - 226292253091596912145
841927866119621969690 0*x^1997 + 903358447595910292527771972191922410542659
825/4*x^1996 - 214547577397927274502181233789902363056309770 10*x^1995 + 38
891614703171686462810256309466328170629826605 25/2*x^1994 - 168502105628944
447818705199670044187130493533353400*x^1993 + 1118853699411489612524823980
452333579848707432126660 75/8*x^1992 - 1113818576786312843600553552092286 91
542106081304486523 0*x^1991 + 170499877545918079073432876733170209565984828
166917578371/2*x^1990 - 734769036554419859459683303160816263037237362099 89
7570964140/117*x^1989 + 178354148359957950795405592824584740939457034021 07
24349640535/4*x^1988 - 305508278474757676047756498538367963949054283487890
90875481890*x^1987 + 121347857443982563149967869360779278103528716985047 52
093214631735/6*x^1986 - 129502922523593280492136994687090590798244450 40...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. 2(15) = 30.

Time = 1.68 (sec) , antiderivative size = 10076, normalized size of antiderivative = 438.09

$$\int (1-x)^{2014} x \, dx = \text{Too large to display}$$

input
```
integrate((1-x)^2014*x,x, algorithm="giac")
```

output
```
1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 1358
826667/4*x^2012 - 136629987582*x^2011 + 1373131035323509/30*x^2010 - 91953
985549170536/7*x^2009 + 2637765102698813310 3/8*x^2008 - 661749155854244491
5874/9*x^2007 + 29499299483526473166117/2*x^2006 - 134422910799097740606 5
80164/5*x^2005 + 538766892328182148445244548 23/12*x^2004 - 691762702790623
489451562620638*x^2003 + 2571973087266166342850029070691063/26*x^2002 - 39
5885912482748242677565694039404 00/3*x^2001 + 131961937837090040663501674 88
2660163383/80*x^2000 - 193964593913505209402992927651733959950*x^1999 + 38
7541161559807075301489477559812273935075/18*x^1998 - 226292253091596912145
841927866119621969 6900*x^1997 + 903358447595910292527771972191922410542659
825/4*x^1996 - 214547577397927274502181233789902363056309770 10*x^1995 + 38
8916147031716864628102563094663281706298266052 5/2*x^1994 - 168502105628944
44781870519967004418713049353335 3400*x^1993 + 1118853699411489612524823980
4523335798487074321266607 5/8*x^1992 - 1113818576786312843600553552092286 91
54210608130448652 30*x^1991 + 17049987754591807907343287673317020956598482 8
166917578371/2*x^1990 - 734769036554419859459683303160816263037237362099 89
7570964140/117*x^1989 + 17835414835995795079540559282458474093945703402107
24349640535/4*x^1988 - 305508278474757676047756498538367963949054283487890
90875481890*x^1987 + 121347857443982563149967869360779278103528716985047 52
093214631735/6*x^1986 - 1295029225235932804921369946870905907982444 5040...
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int (1-x)^{2014}x\,dx = \frac{(x-1)^{2015}}{2015} + \frac{(x-1)^{2016}}{2016}$$

input
```
int(x*(x - 1)^2014,x)
```

output
```
(x - 1)^2015/2015 + (x - 1)^2016/2016
```

Reduce [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 10075, normalized size of antiderivative = 438.04

$$\int (1-x)^{2014}x\,dx = \text{Too large to display}$$

input
```
int((1-x)^2014*x,x)
```

output
```
(x**2*(2015*x**2014 - 4060224*x**2013 + 4088644560*x**2012 - 2743479822720
*x**2011 + 1379970009938520*x**2010 - 555023800755103680*x**2009 + 1859329
27231085706672*x**2008 - 533627368938946454515 20*x**2007 + 133940436384840
34227041340*x**2006 - 29868709898637179372288 8640*x**2005 + 5991661716698
02901771527961040*x**2004 - 1092116250329053611603348410 81472*x**2003 + 18
23833683909362208916841844668 1960*x**2002 - 281010612178418236378971574006
0509120*x**2001 + 401845075154465829406888542004771683120*x**2000 - 536061
196374639740444838154984876168 32000*x**1999 + 670076327949175808481128 8047
191717776261974*x**1998 - 78793073197919740184521399042397976146728 8000*x*
*1997 + 874602893408172607540401452956984339816677 26000*x**1996 - 91925344
219880864039532491305486577315015350 56000*x**1995 + 9174147050405026566795
0410407922872325070361187 7000*x**1994 - 871543750808956091573740695150 6929
75301863800491024 00*x**1993 + 7899353645590607580834316779528324847392965 3
41445538000*x**1992 - 6844959935701232937070570103076402987289760509295156
16000*x**1991 + 56813153148716619544785512079408594517557665988527579563 50
0*x**1990 - 45245983753644314857879126614513715993000500771833733319152 00*
x**1989 + 346305711281065128767630984590276676053663104186389631780905520*
x**1988 - 255111809491694575204402042857435406526528812121084436638749408 0
0*x**1987 + 1811293389084388965097821038489352795084749854704403220520941 7
24600*x**1986 - 124104794915129962194823835862249987787240627255584996 5...
```

3.106 $\int \operatorname{arcsinh}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 16

$$\int \operatorname{arcsinh}(x)\,dx = -\sqrt{1+x^2} + x\operatorname{arcsinh}(x)$$

output
```
-(x^2+1)^(1/2)+x*arcsinh(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(x)\,dx = -\sqrt{1+x^2} + x\operatorname{arcsinh}(x)$$

input
```
Integrate[ArcSinh[x],x]
```

output
```
-Sqrt[1 + x^2] + x*ArcSinh[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6187, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{arcsinh}(x) \, dx$$

$$\Big\downarrow \text{6187}$$

$$x\text{arcsinh}(x)-\int \frac{x}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{241}$$

$$x\text{arcsinh}(x)-\sqrt{x^2+1}$$

input
```
Int[ArcSinh[x],x]
```

output
```
-Sqrt[1 + x^2] + x*ArcSinh[x]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 6187
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[
1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
lookup	$-\sqrt{x^2+1}+x \operatorname{arcsinh}(x)$	15
default	$-\sqrt{x^2+1}+x \operatorname{arcsinh}(x)$	15
parts	$-\sqrt{x^2+1}+x \operatorname{arcsinh}(x)$	15
orering	$x \operatorname{arcsinh}(x)+\frac{-x^2-1}{\sqrt{x^2+1}}$	21

input
```
int(arcsinh(x),x,method=_RETURNVERBOSE)
```

output
```
-(x^2+1)^(1/2)+x*arcsinh(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \operatorname{arcsinh}(x)\, dx = x \log \left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

input
```
integrate(arcsinh(x),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \operatorname{arcsinh}(x)\, dx = x \operatorname{asinh}(x)-\sqrt{x^2+1}$$

input
```
integrate(asinh(x),x)
```

output
```
x*asinh(x) - sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{arcsinh}(x)\,dx = x\operatorname{arsinh}(x) - \sqrt{x^2+1}$$

input
```
integrate(arcsinh(x),x, algorithm="maxima")
```

output
```
x*arcsinh(x) - sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \operatorname{arcsinh}(x)\,dx = x\log\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}$$

input
```
integrate(arcsinh(x),x, algorithm="giac")
```

output
```
x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \operatorname{arcsinh}(x)\,dx = x\operatorname{asinh}(x) - \sqrt{x^2+1}$$

input
```
int(asinh(x),x)
```

output
```
x*asinh(x) - (x^2 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \operatorname{arcsinh}(x)\,dx = asinh(x)\,x - \sqrt{x^2+1}$$

input
```
int(asinh(x),x)
```

output
```
asinh(x)*x - sqrt(x**2 + 1)
```

3.107 $\int \frac{x^2}{-1+x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{x^2}{-1+x} \, dx = x + \frac{x^2}{2} + \log(1-x)$$

output
```
x+1/2*x^2+ln(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{-1+x} \, dx = -\frac{3}{2} + x + \frac{x^2}{2} + \log(-1+x)$$

input
```
Integrate[x^2/(-1 + x),x]
```

output
```
-3/2 + x + x^2/2 + Log[-1 + x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x-1} \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x+\frac{1}{x-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+x+\log(1-x)$$

input
```
Int[x^2/(-1 + x),x]
```

output
```
x + x^2/2 + Log[1 - x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2}+x+\ln(x-1)$	12
norman	$\frac{x^2}{2}+x+\ln(x-1)$	12
risch	$\frac{x^2}{2}+x+\ln(x-1)$	12
parallelrisch	$\frac{x^2}{2}+x+\ln(x-1)$	12
meijerg	$\frac{x(3x+6)}{6}+\ln(1-x)$	16

input
```
int(x^2/(x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x+ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{-1+x}\,dx = \frac{1}{2}\,x^2+x+\log(x-1)$$

input
```
integrate(x^2/(-1+x),x, algorithm="fricas")
```

output
```
1/2*x^2 + x + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{-1+x}\,dx = \frac{x^2}{2} + x + \log\left(x-1\right)$$

input
```
integrate(x**2/(-1+x),x)
```

output
```
x**2/2 + x + log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{-1+x}\,dx = \frac{1}{2}\,x^2 + x + \log\left(x-1\right)$$

input
```
integrate(x^2/(-1+x),x, algorithm="maxima")
```

output
```
1/2*x^2 + x + log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{-1+x}\,dx = \frac{1}{2}\,x^2 + x + \log\left(|x-1|\right)$$

input
```
integrate(x^2/(-1+x),x, algorithm="giac")
```

output
```
1/2*x^2 + x + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{-1+x}\,dx = x+\ln(x-1)+\frac{x^2}{2}$$

input
```
int(x^2/(x - 1),x)
```

output
```
x + log(x - 1) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{-1+x}\,dx = \log(x-1)+\frac{x^2}{2}+x$$

input
```
int(x^2/(-1+x),x)
```

output
```
(2*log(x - 1) + x**2 + 2*x)/2
```

3.108 $\int x \arctan(x) \, dx$

Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x) \, dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x)$$

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \arctan(x) \, dx = \frac{1}{2}\left(-x + \left(1 + x^2\right) \arctan(x)\right)$$

input
```
Integrate[x*ArcTan[x],x]
```

output
```
(-x + (1 + x^2)*ArcTan[x])/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-x\right)+\frac{1}{2}x^2\arctan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\arctan(x)+\frac{1}{2}(\arctan(x)-x)$$

input
```
Int[x*ArcTan[x],x]
```

output
```
(x^2*ArcTan[x])/2 + (-x + ArcTan[x])/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^{2}\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^{2}+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^{2}\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^{2}\arctan(x)}{2}$	16
orering	$\arctan\left(x\right)\left(x^{2}+1\right)+\left(-\frac{x^{2}}{2}-\frac{1}{2}\right)\left(\arctan\left(x\right)+\frac{x}{x^{2}+1}\right)$	30
risch	$-\frac{i x^{2}\ln(ix+1)}{4}+\frac{i x^{2}\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

input
```
int(x*arctan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x\arctan(x)\,dx = \frac{1}{2}\left(x^2+1\right)\arctan\left(x\right) - \frac{1}{2}\,x$$

input
```
integrate(x*arctan(x),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{x^2\,\mathrm{atan}\left(x\right)}{2} - \frac{x}{2} + \frac{\mathrm{atan}\left(x\right)}{2}$$

input
```
integrate(x*atan(x),x)
```

output
```
x**2*atan(x)/2 - x/2 + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x\arctan(x)\,dx = \frac{1}{2}\,x^2\arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

input
```
integrate(x*arctan(x),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x)\,dx = \frac{1}{2}\,x^2 \arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

input
```
integrate(x*arctan(x),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x \arctan(x)\,dx = \mathrm{atan}(x)\,\left(\frac{x^2}{2} + \frac{1}{2}\right) - \frac{x}{2}$$

input
```
int(x*atan(x),x)
```

output
```
atan(x)*(x^2/2 + 1/2) - x/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x)\,dx = \frac{atan(x)\,x^2}{2} + \frac{atan(x)}{2} - \frac{x}{2}$$

input
```
int(x*atan(x),x)
```

output
```
(atan(x)*x**2 + atan(x) - x)/2
```

3.109 $\int \frac{1}{-2014-15x+x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{-2014-15x+x^2} dx = \frac{1}{91}\log(53-x) - \frac{1}{91}\log(38+x)$$

output
```
1/91*ln(53-x)-1/91*ln(38+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{-2014-15x+x^2} dx = \frac{1}{91}\log(53-x) - \frac{1}{91}\log(38+x)$$

input
```
Integrate[(-2014 - 15*x + x^2)^(-1),x]
```

output
```
Log[53 - x]/91 - Log[38 + x]/91
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-15x-2014}dx$$

$$\Big\downarrow \text{1081}$$

$$\int \left(-\frac{1}{91(x+38)}-\frac{1}{91(53-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{91}\log(53-x)-\frac{1}{91}\log(x+38)$$

input
```
Int[(-2014 - 15*x + x^2)^(-1),x]
```

output
```
Log[53 - x]/91 - Log[38 + x]/91
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\ln(38+x)}{91}+\frac{\ln(x-53)}{91}$	14
norman	$-\frac{\ln(38+x)}{91}+\frac{\ln(x-53)}{91}$	14
risch	$-\frac{\ln(38+x)}{91}+\frac{\ln(x-53)}{91}$	14
parallelrisch	$-\frac{\ln(38+x)}{91}+\frac{\ln(x-53)}{91}$	14

input
```
int(1/(x^2-15*x-2014),x,method=_RETURNVERBOSE)
```

output
```
-1/91*ln(38+x)+1/91*ln(x-53)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{-2014-15x+x^2}\,dx=-\frac{1}{91}\log\left(x+38\right)+\frac{1}{91}\log\left(x-53\right)$$

input
```
integrate(1/(x^2-15*x-2014),x, algorithm="fricas")
```

output
```
-1/91*log(x + 38) + 1/91*log(x - 53)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{-2014-15x+x^2}\,dx=\frac{\log\left(x-53\right)}{91}-\frac{\log\left(x+38\right)}{91}$$

input
```
integrate(1/(x**2-15*x-2014),x)
```

output
```
log(x - 53)/91 - log(x + 38)/91
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{-2014-15x+x^2}\,dx=-\frac{1}{91}\,\log\left(x+38\right)+\frac{1}{91}\,\log\left(x-53\right)$$

input
```
integrate(1/(x^2-15*x-2014),x, algorithm="maxima")
```

output
```
-1/91*log(x + 38) + 1/91*log(x - 53)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{-2014-15x+x^2}\,dx=-\frac{1}{91}\,\log\left(|x+38|\right)+\frac{1}{91}\,\log\left(|x-53|\right)$$

input
```
integrate(1/(x^2-15*x-2014),x, algorithm="giac")
```

output
```
-1/91*log(abs(x + 38)) + 1/91*log(abs(x - 53))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{-2014-15x+x^2}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{91}-\frac{15}{91}\right)}{91}$$

input
```
int(-1/(15*x - x^2 + 2014),x)
```

output
```
-(2*atanh((2*x)/91 - 15/91))/91
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{-2014-15x+x^2}\,dx=\frac{\log(x-53)}{91}-\frac{\log(x+38)}{91}$$

input
```
int(1/(x^2-15*x-2014),x)
```

output
```
(log(x - 53) - log(x + 38))/91
```

3.110 $\int e^x(-2(1+x)\arctan(x)+\log(1+x^2))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 19

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx=-2e^xx\arctan(x)+e^x\log\left(1+x^2\right)$$

output
```
-2*exp(x)*x*arctan(x)+exp(x)*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx=e^x\left(-2x\arctan(x)+\log\left(1+x^2\right)\right)$$

input
```
Integrate[E^x*(-2*(1 + x)*ArcTan[x] + Log[1 + x^2]),x]
```

output
```
E^x*(-2*x*ArcTan[x] + Log[1 + x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(\log \left(x^2+1\right)-2(x+1) \arctan(x)\right) \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(e^x \log \left(x^2+1\right)-2e^x(x+1) \arctan(x)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$2e^x \arctan(x)-2e^x(x+1) \arctan(x)+e^x \log \left(x^2+1\right)$$

input
```
Int[E^x*(-2*(1 + x)*ArcTan[x] + Log[1 + x^2]),x]
```

output
```
2*E^x*ArcTan[x] - 2*E^x*(1 + x)*ArcTan[x] + E^x*Log[1 + x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result
parallelrisch	$-2\,\mathrm{e}^{x} x \arctan\left(x\right)+\mathrm{e}^{x} \ln\left(x^{2}+1\right)$
orering	$\frac{\left(3x^{2}-4x+3\right)\mathrm{e}^{x}\left(-2(x+1)\arctan(x)+\ln(x^{2}+1)\right)}{\left(x-1\right)^{2}}-\frac{\left(3x^{2}-2x+3\right)\left(\mathrm{e}^{x}\left(-2(x+1)\arctan(x)+\ln(x^{2}+1)\right)+\mathrm{e}^{x}\left(-2\arctan(x)-\right.\right.}{\left(x-1\right)^{2}}$
risch	$-i(i+x)\,\mathrm{e}^{x}\ln\left(i+x\right)+\mathrm{e}^{x}\ln\left(x-i\right)-\frac{i\pi\,\mathrm{csgn}\left(i(x-i)(i+x)\right)^{3}\mathrm{e}^{x}}{2}+ix\,\mathrm{e}^{x}\ln\left(x-i\right)-\frac{i\pi\,\mathrm{csgn}(i(x-i))}{}$

input
```
int(exp(x)*(-2*(x+1)*arctan(x)+ln(x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*x*arctan(x)+exp(x)*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int e^{x}\left(-2(1+x)\arctan(x)+\log\left(1+x^{2}\right)\right)\,dx=-2\,x\arctan\left(x\right)e^{x}+e^{x}\log\left(x^{2}+1\right)$$

input
```
integrate(exp(x)*(-2*(1+x)*arctan(x)+log(x^2+1)),x, algorithm="fricas")
```

output
```
-2*x*arctan(x)*e^x + e^x*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{x}\left(-2(1+x)\arctan(x)+\log\left(1+x^{2}\right)\right)\,dx=-2xe^{x}\,\mathrm{atan}\left(x\right)+e^{x}\log\left(x^{2}+1\right)$$

input
```
integrate(exp(x)*(-2*(1+x)*atan(x)+ln(x**2+1)),x)
```

output
```
-2*x*exp(x)*atan(x) + exp(x)*log(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx=-2\,x\arctan\left(x\right)e^x+e^x\log\left(x^2+1\right)$$

input
```
integrate(exp(x)*(-2*(1+x)*arctan(x)+log(x^2+1)),x, algorithm="maxima")
```

output
```
-2*x*arctan(x)*e^x + e^x*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx$$
$$=-\pi xe^x\operatorname{sgn}(x)+2\,x\arctan\left(\frac{1}{x}\right)e^x+e^x\log\left(x^2+1\right)$$

input
```
integrate(exp(x)*(-2*(1+x)*arctan(x)+log(x^2+1)),x, algorithm="giac")
```

output
```
-pi*x*e^x*sgn(x) + 2*x*arctan(1/x)*e^x + e^x*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx = \mathrm{e}^x\left(\ln\left(x^2+1\right)-2\,x\,\mathrm{atan}(x)\right)$$

input
```
int(exp(x)*(log(x^2 + 1) - 2*atan(x)*(x + 1)),x)
```

output
```
exp(x)*(log(x^2 + 1) - 2*x*atan(x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x\left(-2(1+x)\arctan(x)+\log\left(1+x^2\right)\right)\,dx = e^x\left(-2atan(x)\,x+\log\left(x^2+1\right)\right)$$

input
```
int(exp(x)*(-2*(1+x)*atan(x)+log(x^2+1)),x)
```

output
```
e**x*( - 2*atan(x)*x + log(x**2 + 1))
```

3.111 $\int \arcsin(x)^2 \, dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

output
```
-2*x+2*(-x^2+1)^(1/2)*arcsin(x)+x*arcsin(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \arcsin(x)^2 \, dx = -2x + 2\sqrt{1-x^2} \arcsin(x) + x \arcsin(x)^2$$

input
```
Integrate[ArcSin[x]^2,x]
```

output
```
-2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5130, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x)^2 \, dx$$

$$\Big\downarrow \ 5130$$

$$x \arcsin(x)^2 - 2 \int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$$

$$\Big\downarrow \ 5182$$

$$x \arcsin(x)^2 - 2 \left(\int 1 dx - \sqrt{1-x^2} \arcsin(x) \right)$$

$$\Big\downarrow \ 24$$

$$x \arcsin(x)^2 - 2 \left(x - \sqrt{1-x^2} \arcsin(x) \right)$$

input
```
Int[ArcSin[x]^2,x]
```

output
```
x*ArcSin[x]^2 - 2*(x - Sqrt[1 - x^2]*ArcSin[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5130
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-2x+2\sqrt{-x^{2}+1}\, \arcsin \left(x \right)+x \arcsin \left(x \right)^{2}$	24
orering	$x \arcsin \left(x \right)^{2}+\frac{2 \arcsin \left(x \right)}{\sqrt{-x^{2}+1}}+x \left(x -1\right) \left(x +1\right) \left(\frac{2}{-x^{2}+1}+\frac{2 \arcsin \left(x \right) x}{\left(-x^{2}+1\right)^{\frac{3}{2}}}\right)$	55

input
```
int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x+2*(-x^2+1)^(1/2)*arcsin(x)+x*arcsin(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x \arcsin \left(x\right)^{2} + 2 \sqrt{-x^{2} + 1} \arcsin \left(x\right) - 2 \, x$$

input
```
integrate(arcsin(x)^2,x, algorithm="fricas")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = x\,\mathrm{asin}^2\,(x) - 2x + 2\sqrt{1-x^2}\,\mathrm{asin}\,(x)$$

input
```
integrate(asin(x)**2,x)
```

output
```
x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\,(x)^2 + 2\,\sqrt{-x^2+1}\arcsin\,(x) - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="maxima")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \arcsin(x)^2\,dx = x\arcsin\,(x)^2 + 2\,\sqrt{-x^2+1}\arcsin\,(x) - 2\,x$$

input
```
integrate(arcsin(x)^2,x, algorithm="giac")
```

output
```
x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = 2\,\mathrm{asin}(x)\,\sqrt{1-x^2}+x\left(\mathrm{asin}(x)^2-2\right)$$

input
```
int(asin(x)^2,x)
```

output
```
2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \arcsin(x)^2\,dx = asin(x)^2 x+2\sqrt{-x^2+1}\,asin(x)-2x$$

input
```
int(asin(x)^2,x)
```

output
```
asin(x)**2*x + 2*sqrt( - x**2 + 1)*asin(x) - 2*x
```

3.112 $\int \frac{\sqrt{-1+x^2}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{-1+x^2}}{x} \, dx = \sqrt{-1+x^2} - \arctan\left(\sqrt{-1+x^2}\right)$$

output
```
(x^2-1)^(1/2)-arctan((x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{x} \, dx = \sqrt{-1+x^2} - \arctan\left(\sqrt{-1+x^2}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/x,x]
```

output
```
Sqrt[-1 + x^2] - ArcTan[Sqrt[-1 + x^2]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {243, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{x} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{\sqrt{x^2-1}}{x^2} dx^2$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2} \left(2\sqrt{x^2-1} - \int \frac{1}{x^2\sqrt{x^2-1}} dx^2\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2} \left(2\sqrt{x^2-1} - 2\int \frac{1}{x^4+1} d\sqrt{x^2-1}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2} \left(2\sqrt{x^2-1} - 2\arctan\left(\sqrt{x^2-1}\right)\right)$$

input
```
Int[Sqrt[-1 + x^2]/x,x]
```

output
```
(2*Sqrt[-1 + x^2] - 2*ArcTan[Sqrt[-1 + x^2]])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\sqrt{x^{2}-1}+\arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)$	17
pseudoelliptic	$\sqrt{x^{2}-1}-\arctan\left(\sqrt{x^{2}-1}\right)$	19
trager	$\sqrt{x^{2}-1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)-\sqrt{x^{2}-1}}{x}\right)$	38
meijerg	$-\frac{\sqrt{\operatorname{signum}\left(x^{2}-1\right)}\left(-2\left(2-2\ln\left(2\right)+2\ln\left(x\right)+i\pi\right)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-x^{2}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(x^{2}-1\right)}}$	82

input
```
int((x^2-1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(x^2-1)^(1/2)+arctan(1/(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = \sqrt{x^2-1} - 2\arctan\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(x^2 - 1) - 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = \begin{cases} -\frac{ix}{\sqrt{-1+\frac{1}{x^2}}} - i\,\mathrm{acosh}\left(\frac{1}{x}\right) + \frac{i}{x\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{x}{\sqrt{1-\frac{1}{x^2}}} + \mathrm{asin}\left(\frac{1}{x}\right) - \frac{1}{x\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((x**2-1)**(1/2)/x,x)
```

output
```
Piecewise((-I*x/sqrt(-1 + x**(-2)) - I*acosh(1/x) + I/(x*sqrt(-1 + x**(-2)
)), 1/Abs(x**2) > 1), (x/sqrt(1 - 1/x**2) + asin(1/x) - 1/(x*sqrt(1 - 1/x*
*2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = \sqrt{x^2-1} + \arcsin\left(\frac{1}{|x|}\right)$$

input
```
integrate((x^2-1)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1) + arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = \sqrt{x^2-1} - \arctan\left(\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(x^2 - 1) - arctan(sqrt(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = \sqrt{x^2-1} - \ln\left(\frac{\sqrt{x^2-1}+1\mathrm{i}}{x}\right)\,1\mathrm{i}$$

input
```
int((x^2 - 1)^(1/2)/x,x)
```

output
```
(x^2 - 1)^(1/2) - log(((x^2 - 1)^(1/2) + 1i)/x)*1i
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+x^2}}{x}\,dx = -2atan\left(\sqrt{x^2-1}+x\right)+\sqrt{x^2-1}$$

input
```
int((x^2-1)^(1/2)/x,x)
```

output
```
- 2*atan(sqrt(x**2 - 1) + x) + sqrt(x**2 - 1)
```

3.113 $\int x \sec^2(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x \sec^2(4x)\,dx = \frac{1}{16}\log(\cos(4x)) + \frac{1}{4}x\tan(4x)$$

output
```
1/16*ln(cos(4*x))+1/4*x*tan(4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x \sec^2(4x)\,dx = \frac{1}{16}\log(\cos(4x)) + \frac{1}{4}x\tan(4x)$$

input
```
Integrate[x*Sec[4*x]^2,x]
```

output
```
Log[Cos[4*x]]/16 + (x*Tan[4*x])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^2(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \csc\left(4x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{1}{4}\int -\tan(4x)dx + \frac{1}{4}x\tan(4x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}x\tan(4x) - \frac{1}{4}\int \tan(4x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x\tan(4x) - \frac{1}{4}\int \tan(4x)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{4}x\tan(4x) + \frac{1}{16}\log(\cos(4x))$$

input
```
Int[x*Sec[4*x]^2,x]
```

output
```
Log[Cos[4*x]]/16 + (x*Tan[4*x])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\ln(\cos(4x))}{16}+\frac{x\tan(4x)}{4}$	16
default	$\frac{\ln(\cos(4x))}{16}+\frac{x\tan(4x)}{4}$	16
risch	$-\frac{ix}{2}+\frac{ix}{2\,\mathrm{e}^{8ix}+2}+\frac{\ln(\mathrm{e}^{8ix}+1)}{16}$	29
norman	$-\frac{x\tan(2x)}{2\left(\tan(2x)^2-1\right)}+\frac{\ln(\tan(2x)-1)}{16}+\frac{\ln(\tan(2x)+1)}{16}-\frac{\ln\left(1+\tan(2x)^2\right)}{16}$	48
parallelrisch	$\frac{\cos(4x)\ln(\tan(2x)-1)+\cos(4x)\ln(\tan(2x)+1)-\cos(4x)\ln\left(\sec(2x)^2\right)+4x\sin(4x)}{16\cos(4x)}$	54

input
```
int(x*sec(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*ln(cos(4*x))+1/4*x*tan(4*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int x\sec^2(4x)\,dx = \frac{\cos(4x)\log(-\cos(4x)) + 4x\sin(4x)}{16\cos(4x)}$$

input
```
integrate(x*sec(4*x)^2,x, algorithm="fricas")
```

output
```
1/16*(cos(4*x)*log(-cos(4*x)) + 4*x*sin(4*x))/cos(4*x)
```

Sympy [F]

$$\int x\sec^2(4x)\,dx = \int x\sec^2(4x)\,dx$$

input
```
integrate(x*sec(4*x)**2,x)
```

output
```
Integral(x*sec(4*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.89

$$\int x\sec^2(4x)\,dx = \frac{(\cos(8x)^2 + \sin(8x)^2 + 2\cos(8x) + 1)\log(\cos(8x)^2 + \sin(8x)^2 + 2\cos(8x) + 1) + 16x\sin(8x)}{32(\cos(8x)^2 + \sin(8x)^2 + 2\cos(8x) + 1)}$$

input
```
integrate(x*sec(4*x)^2,x, algorithm="maxima")
```

output
```
1/32*((cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1)*log(cos(8*x)^2 + sin(8*x)
^2 + 2*cos(8*x) + 1) + 16*x*sin(8*x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x
) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(15) = 30$.

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 5.42

$$\int x\sec^2(4x)\,dx = \frac{\log\left(\frac{4\left(\tan(2x)^4-2\tan(2x)^2+1\right)}{\tan(2x)^4+2\tan(2x)^2+1}\right)\tan(2x)^2 - 16x\tan(2x) - \log\left(\frac{4\left(\tan(2x)^4-2\tan(2x)^2+1\right)}{\tan(2x)^4+2\tan(2x)^2+1}\right)}{32\left(\tan(2x)^2-1\right)}$$

input
```
integrate(x*sec(4*x)^2,x, algorithm="giac")
```

output
```
1/32*(log(4*(tan(2*x)^4 - 2*tan(2*x)^2 + 1)/(tan(2*x)^4 + 2*tan(2*x)^2 + 1
))*tan(2*x)^2 - 16*x*tan(2*x) - log(4*(tan(2*x)^4 - 2*tan(2*x)^2 + 1)/(tan
(2*x)^4 + 2*tan(2*x)^2 + 1)))/(tan(2*x)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sec^2(4x)\,dx = \frac{\ln(\cos(4x))}{16} + \frac{x\tan(4x)}{4}$$

input
```
int(x/cos(4*x)^2,x)
```

output
```
log(cos(4*x))/16 + (x*tan(4*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.89

$$\int x\sec^2(4x)\,dx$$
$$= \frac{-\cos(4x)\log\left(\tan(2x)^2+1\right)+\cos(4x)\log(\tan(2x)-1)+\cos(4x)\log(\tan(2x)+1)+4\sin(4x)\,x}{16\cos(4x)}$$

input
```
int(x*sec(4*x)^2,x)
```

output
```
( - cos(4*x)*log(tan(2*x)**2 + 1) + cos(4*x)*log(tan(2*x) - 1) + cos(4*x)*
log(tan(2*x) + 1) + 4*sin(4*x)*x)/(16*cos(4*x))
```

3.114 $\int \frac{2}{6-11x+6x^2-x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{2}{6-11x+6x^2-x^3} \, dx = -\log(1-x)+2\log(2-x)-\log(3-x)$$

output
```
-ln(1-x)+2*ln(2-x)-ln(3-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{2}{6-11x+6x^2-x^3} \, dx = -2\left(-\log(2-x)+\frac{1}{2}\log\left(3-4x+x^2\right)\right)$$

input
```
Integrate[2/(6 - 11*x + 6*x^2 - x^3),x]
```

output
```
-2*(-Log[2 - x] + Log[3 - 4*x + x^2]/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {27, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{-x^3+6x^2-11x+6}dx$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{-x^3+6x^2-11x+6}dx$$

$$\Big\downarrow \text{2462}$$

$$2\int \left(\frac{1}{x-2}-\frac{1}{2(x-1)}-\frac{1}{2(x-3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{1}{2}\log(1-x)+\log(2-x)-\frac{1}{2}\log(3-x)\right)$$

input
```
Int[2/(6 - 11*x + 6*x^2 - x^3),x]
```

output
```
2*(-1/2*Log[1 - x] + Log[2 - x] - Log[3 - x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$2\ln(x-2)-\ln(x^2-4x+3)$	19
default	$-\ln(-3+x)+2\ln(x-2)-\ln(x-1)$	20
norman	$-\ln(-3+x)+2\ln(x-2)-\ln(x-1)$	20
parallelrisch	$-\ln(-3+x)+2\ln(x-2)-\ln(x-1)$	20

input
```
int(2/(-x^3+6*x^2-11*x+6),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x-2)-ln(x^2-4*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = -\log\left(x^2-4\,x+3\right)+2\,\log\left(x-2\right)$$

input
```
integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="fricas")
```

output
```
-log(x^2 - 4*x + 3) + 2*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = 2\log(x-2) - \log(x^2-4x+3)$$

input
```
integrate(2/(-x**3+6*x**2-11*x+6),x)
```

output
```
2*log(x - 2) - log(x**2 - 4*x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = -\log(x-1) + 2\log(x-2) - \log(x-3)$$

input
```
integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="maxima")
```

output
```
-log(x - 1) + 2*log(x - 2) - log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = -\log(|x-1|) + 2\log(|x-2|) - \log(|x-3|)$$

input
```
integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="giac")
```

output
```
-log(abs(x - 1)) + 2*log(abs(x - 2)) - log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = 2\,\ln\left(x-2\right)-\ln\left(x^2-4\,x+3\right)$$

input
```
int(-2/(11*x - 6*x^2 + x^3 - 6),x)
```

output
```
2*log(x - 2) - log(x^2 - 4*x + 3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{2}{6-11x+6x^2-x^3}\,dx = -\log(x-3)+2\log(x-2)-\log(x-1)$$

input
```
int(2/(-x^3+6*x^2-11*x+6),x)
```

output
```
- log(x - 3) + 2*log(x - 2) - log(x - 1)
```

3.115 $\int \frac{1}{1-\log(1-x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{1-\log(1-x)}\,dx = e\,\text{ExpIntegralEi}(-1+\log(1-x))$$

output
```
exp(1)*Ei(-1+ln(1-x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\log(1-x)}\,dx = e\,\text{ExpIntegralEi}(-1+\log(1-x))$$

input
```
Integrate[(1 - Log[1 - x])^(-1),x]
```

output
```
E*ExpIntegralEi[-1 + Log[1 - x]]
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2836, 2736, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\log(1-x)}dx$$

$$\Big\downarrow \text{2836}$$

$$-\int \frac{1}{1-\log(1-x)}d(1-x)$$

$$\Big\downarrow \text{2736}$$

$$-\int \frac{1-x}{x}d\log(1-x)$$

$$\Big\downarrow \text{2609}$$

$$e\,\text{ExpIntegralEi}(-x)$$

input
```
Int[(1 - Log[1 - x])^(-1),x]
```

output
```
E*ExpIntegralEi[-x]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2736
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[1/(n*c^(1
/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b
, c, p}, x] && IntegerQ[1/n]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$-\mathrm{e}\ \mathrm{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17
default	$-\mathrm{e}\ \mathrm{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17
risch	$-\mathrm{e}\ \mathrm{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17

input
```
int(1/(1-ln(1-x)),x,method=_RETURNVERBOSE)
```

output
```
-exp(1)*Ei(1,1-ln(1-x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\log(1-x)}\,dx = e\,\mathrm{log\_integral}\left(-(x-1)e^{(-1)}\right)$$

input
```
integrate(1/(1-log(1-x)),x, algorithm="fricas")
```

output
```
e*log_integral(-(x - 1)*e^(-1))
```

Sympy [F]

$$\int \frac{1}{1-\log(1-x)}\,dx = -\int \frac{1}{\log(1-x)-1}\,dx$$

input
```
integrate(1/(1-ln(1-x)),x)
```

output
```
-Integral(1/(log(1 - x) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{1}{1-\log(1-x)}\,dx = -eE_1(-\log(-x+1)+1)$$

input
```
integrate(1/(1-log(1-x)),x, algorithm="maxima")
```

output
```
-e*exp_integral_e(1, -log(-x + 1) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{1-\log(1-x)}\,dx = \mathrm{Ei}(\log(-x+1)-1)\,e$$

input
```
integrate(1/(1-log(1-x)),x, algorithm="giac")
```

output
```
Ei(log(-x + 1) - 1)*e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1-\log(1-x)}\,dx = -\int \frac{1}{\ln (1-x)-1}\,dx$$

input
```
int(-1/(log(1 - x) - 1),x)
```

output
```
-int(1/(log(1 - x) - 1), x)
```

Reduce [F]

$$\int \frac{1}{1-\log(1-x)}\,dx = -\left(\int \frac{x}{\log (1-x)\,x-\log (1-x)-x+1}dx\right)+\log(\log(1-x)-1)$$

input
```
int(1/(1-log(1-x)),x)
```

output
```
 - int(x/(log( - x + 1)*x - log( - x + 1) - x + 1),x) + log(log( - x + 1)
- 1)
```

3.116 $\int \sqrt{x+\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx = -\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}$$

output
```
-1/(x+(x^2+1)^(1/2))^(1/2)+1/3*(x+(x^2+1)^(1/2))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx = -\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}$$

input
```
Integrate[Sqrt[x + Sqrt[1 + x^2]],x]
```

output
```
-(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^2+1}+x}\,dx$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{1}{2}\int \frac{\left(x+\sqrt{x^2+1}\right)^2+1}{\left(x+\sqrt{x^2+1}\right)^{3/2}}d\left(x+\sqrt{x^2+1}\right)$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{2}\int \left(\sqrt{x+\sqrt{x^2+1}}+\frac{1}{\left(x+\sqrt{x^2+1}\right)^{3/2}}\right)d\left(x+\sqrt{x^2+1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{2}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

input
```
Int[Sqrt[x + Sqrt[1 + x^2]],x]
```

output
```
(-2/Sqrt[x + Sqrt[1 + x^2]] + (2*(x + Sqrt[1 + x^2])^(3/2))/3)/2
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result	size
meijerg	$\frac{\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{\frac{3}{2}}\left(-\frac{1}{x^{2}}+1\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}+\frac{16\sqrt{\pi}\,\sqrt{2}\,\sqrt{x}\,\sqrt{\frac{1}{x^{2}}+1}\,\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}}{8\sqrt{\pi}}$	57

input
```
int((x+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/Pi^(1/2)*(16/3*Pi^(1/2)*2^(1/2)*x^(3/2)*(-1/x^2+1)*cosh(1/2*arcsinh(1/
x))+16/3*Pi^(1/2)*2^(1/2)*x^(1/2)*(1/x^2+1)^(1/2)*sinh(1/2*arcsinh(1/x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\frac{2}{3}\left(2\,x-\sqrt{x^2+1}\right)\sqrt{x+\sqrt{x^2+1}}$$

input
```
integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*x - sqrt(x^2 + 1))*sqrt(x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\frac{4x\sqrt{x+\sqrt{x^2+1}}}{3}-\frac{2\sqrt{x+\sqrt{x^2+1}}\sqrt{x^2+1}}{3}$$

input
```
integrate((x+(x**2+1)**(1/2))**(1/2),x)
```

output
```
4*x*sqrt(x + sqrt(x**2 + 1))/3 - 2*sqrt(x + sqrt(x**2 + 1))*sqrt(x**2 + 1)
/3
```

Maxima [F]

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\int \sqrt{x+\sqrt{x^2+1}}\,dx$$

input
```
integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\int \sqrt{x+\sqrt{x^2+1}}\,dx$$

input
```
integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\int \sqrt{x+\sqrt{x^2+1}}\,dx$$

input
```
int((x + (x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int((x + (x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \sqrt{x+\sqrt{1+x^2}}\,dx=\frac{2\sqrt{\sqrt{x^2+1}+x}\,\left(-\sqrt{x^2+1}+2x\right)}{3}$$

input
```
int((x+(x^2+1)^(1/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(x**2 + 1) + x)*( - sqrt(x**2 + 1) + 2*x))/3
```

3.117 $\int \frac{1}{2+\cos(x)} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 20

$$\int \frac{1}{2+\cos(x)} \, dx = \frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(1/3*tan(1/2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+\cos(x)} \, dx = \frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(2 + Cos[x])^(-1),x]
```

output
```
(2*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+2}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{x}{\sqrt{3}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

input
```
Int[(2 + Cos[x])^(-1),x]
```

output
```
x/Sqrt[3] - (2*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	16
risch	$\frac{i\sqrt{3}\,\ln\left(e^{ix}+2+\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(e^{ix}+2-\sqrt{3}\right)}{3}$	38

input
```
int(1/(2+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*tan(1/2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{2+\cos(x)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(x)+\sqrt{3}}{3\sin(x)}\right)$$

input
```
integrate(1/(2+cos(x)),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{1}{2+\cos(x)}\,dx = \frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

input
```
integrate(1/(2+cos(x)),x)
```

output
```
2*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{2+\cos(x)}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin\left(x\right)}{3\left(\cos\left(x\right)+1\right)}\right)$$

input
```
integrate(1/(2+cos(x)),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{2+\cos(x)}\,dx = \frac{1}{3}\,\sqrt{3}\left(x+2\,\arctan\left(-\frac{\sqrt{3}\sin\left(x\right)-\sin\left(x\right)}{\sqrt{3}\cos\left(x\right)+\sqrt{3}-\cos\left(x\right)+1}\right)\right)$$

input
```
integrate(1/(2+cos(x)),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqr
t(3) - cos(x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{1}{2+\cos(x)}\,dx = \frac{2\sqrt{3}\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

input
```
int(1/(cos(x) + 2),x)
```

output
```
(2*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (2*3^(1/2)*atan((3^(1/2)*tan(x/2))/
3))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+\cos(x)}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{3}$$

input
```
int(1/(2+cos(x)),x)
```

output
```
(2*sqrt(3)*atan(tan(x/2)/sqrt(3)))/3
```

3.118 $\int (\cos^4(x) - \sin^4(x)) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 5

$$\int (\cos^4(x) - \sin^4(x)) \, dx = \cos(x) \sin(x)$$

output
```
cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int (\cos^4(x) - \sin^4(x)) \, dx = \frac{1}{2} \sin(2x)$$

input
```
Integrate[Cos[x]^4 - Sin[x]^4,x]
```

output
```
Sin[2*x]/2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(5) = 10.

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 5.80, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^4(x)-\sin^4(x)\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{4}\sin^3(x)\cos(x)+\frac{3}{4}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^4 - Sin[x]^4,x]
```

output
```
(3*Cos[x]*Sin[x])/4 + (Cos[x]^3*Sin[x])/4 + (Cos[x]*Sin[x]^3)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{\sin(2x)}{2}$	7
parallelrisch	$\frac{\sin(2x)}{2}$	7
orering	$\cos(x)^3\sin(x)+\cos(x)\sin(x)^3$	16
default	$\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}$	28
parts	$\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}$	28
norman	$\frac{2\tan\left(\frac{x}{2}\right)^3-2\tan\left(\frac{x}{2}\right)^5-2\tan\left(\frac{x}{2}\right)^7+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$	43

input
```
int(cos(x)^4-sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int\left(\cos^4(x)-\sin^4(x)\right)\,dx=\cos(x)\sin(x)$$

input
```
integrate(cos(x)^4-sin(x)^4,x, algorithm="fricas")
```

output
```
cos(x)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 5.80

$$\int \left(\cos^4(x) - \sin^4(x)\right)\, dx = \frac{\sin^3(x)\cos(x)}{4} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{4}$$

input
```
integrate(cos(x)**4-sin(x)**4,x)
```

output
```
sin(x)**3*cos(x)/4 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \left(\cos^4(x) - \sin^4(x)\right)\, dx = \frac{1}{2}\sin(2x)$$

input
```
integrate(cos(x)^4-sin(x)^4,x, algorithm="maxima")
```

output
```
1/2*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \left(\cos^4(x) - \sin^4(x)\right)\, dx = \frac{1}{2}\sin(2x)$$

input
```
integrate(cos(x)^4-sin(x)^4,x, algorithm="giac")
```

output
```
1/2*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \left(\cos^4(x) - \sin^4(x)\right) \, dx = \frac{\sin(2x)}{2}$$

input
```
int(cos(x)^4 - sin(x)^4,x)
```

output
```
sin(2*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \left(\cos^4(x) - \sin^4(x)\right) \, dx = \cos(x)\sin(x)\left(\cos(x)^2 + \sin(x)^2\right)$$

input
```
int(cos(x)^4-sin(x)^4,x)
```

output
```
cos(x)*sin(x)*(cos(x)**2 + sin(x)**2)
```

3.119 $\int \frac{x}{\sqrt{2+4x}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{\sqrt{2+4x}} \, dx = \frac{1}{6}(-1+x)\sqrt{2+4x}$$

output
```
1/6*(-1+x)*(2+4*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{2+4x}} \, dx = \frac{1}{3}(-1+x)\sqrt{\frac{1}{2}+x}$$

input
```
Integrate[x/Sqrt[2 + 4*x],x]
```

output
```
((-1 + x)*Sqrt[1/2 + x])/3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x+2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{4}\sqrt{4x+2}-\frac{1}{2\sqrt{4x+2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(2x+1)^{3/2}}{6\sqrt{2}}-\frac{\sqrt{2x+1}}{2\sqrt{2}}$$

input
```
Int[x/Sqrt[2 + 4*x],x]
```

output
```
-1/2*Sqrt[1 + 2*x]/Sqrt[2] + (1 + 2*x)^(3/2)/(6*Sqrt[2])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{(x-1)\sqrt{2+4x}}{6}$	13
trager	$\frac{\left(\frac{x}{3}-\frac{1}{3}\right)\sqrt{2+4x}}{2}$	15
gosper	$\frac{(1+2x)(x-1)}{3\sqrt{2+4x}}$	18
risch	$\frac{(1+2x)(x-1)}{3\sqrt{2+4x}}$	18
orering	$\frac{(1+2x)(x-1)}{3\sqrt{2+4x}}$	18
derivativedivides	$\frac{(2+4x)^{\frac{3}{2}}}{24}-\frac{\sqrt{2+4x}}{4}$	20
default	$\frac{(2+4x)^{\frac{3}{2}}}{24}-\frac{\sqrt{2+4x}}{4}$	20
meijerg	$\frac{\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(-8x+8)\sqrt{1+2x}}{6}\right)}{8\sqrt{\pi}}$	32

input
```
int(x/(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x-1)*(2+4*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{1}{6}\,\sqrt{4\,x+2}(x-1)$$

input
```
integrate(x/(2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(4*x + 2)*(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{\sqrt{2}x\sqrt{2x+1}}{6} - \frac{\sqrt{2}\sqrt{2x+1}}{6}$$

input
```
integrate(x/(2+4*x)**(1/2),x)
```

output
```
sqrt(2)*x*sqrt(2*x + 1)/6 - sqrt(2)*sqrt(2*x + 1)/6
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{1}{24}\,(4\,x+2)^{\frac{3}{2}} - \frac{1}{4}\,\sqrt{4\,x+2}$$

input
```
integrate(x/(2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(4*x + 2)^(3/2) - 1/4*sqrt(4*x + 2)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{1}{12}\,\sqrt{2}\Big((2\,x+1)^{\frac{3}{2}} - 3\,\sqrt{2\,x+1}\Big)$$

input
```
integrate(x/(2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*((2*x + 1)^(3/2) - 3*sqrt(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{\sqrt{4\,x+2}\,(4\,x-4)}{24}$$

input
```
int(x/(4*x + 2)^(1/2),x)
```

output
```
((4*x + 2)^(1/2)*(4*x - 4))/24
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{2+4x}}\,dx = \frac{\sqrt{2x+1}\,\sqrt{2}\,(x-1)}{6}$$

input
```
int(x/(2+4*x)^(1/2),x)
```

output
```
(sqrt(2*x + 1)*sqrt(2)*(x - 1))/6
```

3.120 $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = 2\sin\left(\sqrt{x}\right)$$

output
```
2*sin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = 2\sin\left(\sqrt{x}\right)$$

input
```
Integrate[Cos[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sin[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 3861}$$

$$2\int \cos\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3117}$$

$$2\sin\left(\sqrt{x}\right)$$

input
```
Int[Cos[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sin[Sqrt[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)$	7
default	$2\sin\left(\sqrt{x}\right)$	7
meijerg	$2\sin\left(\sqrt{x}\right)$	7

input
```
int(cos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*sin(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\, dx = 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(cos(x**(1/2))/x**(1/2),x)
```

output
```
2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = 2\,\sin(\sqrt{x})$$

input
```
int(cos(x^(1/2))/x^(1/2),x)
```

output
```
2*sin(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = 2\sin(\sqrt{x})$$

input
```
int(cos(x^(1/2))/x^(1/2),x)
```

output
```
2*sin(sqrt(x))
```

3.121 $\int \sec(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \sec(x)\,dx = \operatorname{arctanh}(\sin(x))$$

output
```
arctanh(sin(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sec(x)\,dx = \coth^{-1}(\sin(x))$$

input
```
Integrate[Sec[x],x]
```

output
```
ArcCoth[Sin[x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4257}$$

$$\text{arctanh}(\sin(x))$$

input
```
Int[Sec[x],x]
```

output
```
ArcTanh[Sin[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

method	result	size
lookup	$\ln \left(\sec \left(x\right)+\tan \left(x\right)\right)$	7
default	$\ln \left(\sec \left(x\right)+\tan \left(x\right)\right)$	7
norman	$-\ln \left(\tan \left(\frac{x}{2}\right)-1\right)+\ln \left(1+\tan \left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln \left(\tan \left(\frac{x}{2}\right)-1\right)+\ln \left(1+\tan \left(\frac{x}{2}\right)\right)$	18
risch	$-\ln \left({\mathrm e}^{i x}-i\right)+\ln \left(i+{\mathrm e}^{i x}\right)$	22

input
```
int(sec(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)+tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\,dx = \frac{1}{2}\,\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\,\log\left(-\sin\left(x\right) + 1\right)$$

input
```
integrate(sec(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x)\,dx = -\frac{\log\left(\sin\left(x\right) - 1\right)}{2} + \frac{\log\left(\sin\left(x\right) + 1\right)}{2}$$

input
```
integrate(sec(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

$$\int \sec(x)\,dx = \log\left(\sec\left(x\right) + \tan\left(x\right)\right)$$

input
```
integrate(sec(x),x, algorithm="maxima")
```

output
```
log(sec(x) + tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(3) = 6$.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 8.33

$$\int \sec(x)\,dx = \frac{1}{4}\log\left(\left|\frac{1}{\sin\left(x\right)} + \sin\left(x\right) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin\left(x\right)} + \sin\left(x\right) - 2\right|\right)$$

input
```
integrate(sec(x),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \sec(x)\,dx = \ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

input
```
int(1/cos(x),x)
```

output
```
log(1/cos(x)) + log(sin(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x)\,dx = -\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)$$

input
```
int(sec(x),x)
```

output
```
- log(tan(x/2) - 1) + log(tan(x/2) + 1)
```

3.122 $\int e^{\sin(x)} \cos(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 4

$$\int e^{\sin(x)} \cos(x) \, dx = e^{\sin(x)}$$

output
```
exp(sin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^{\sin(x)} \cos(x) \, dx = e^{\sin(x)}$$

input
```
Integrate[E^Sin[x]*Cos[x],x]
```

output
```
E^Sin[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sin(x)}\cos(x)\,dx$$

$$\Big\downarrow \text{4834}$$

$$\int e^{\sin(x)}d\sin(x)$$

$$\Big\downarrow \text{2624}$$

$$e^{\sin(x)}$$

input
```
Int[E^Sin[x]*Cos[x],x]
```

output
```
E^Sin[x]
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\mathrm{e}^{\sin(x)}$	4
default	$\mathrm{e}^{\sin(x)}$	4
risch	$\mathrm{e}^{\sin(x)}$	4
parallelrisch	$\mathrm{e}^{\sin(x)}$	4
norman	$\frac{\tan\left(\frac{x}{2}\right)^{2} \mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}+\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	54

input
```
int(exp(sin(x))*cos(x),x,method=_RETURNVERBOSE)
```

output
```
exp(sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sin(x)} \cos(x)\, dx = e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x),x, algorithm="fricas")
```

output
```
e^sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sin(x)} \cos(x)\, dx = e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x),x)
```

output
```
exp(sin(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sin(x)} \cos(x)\, dx = e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x),x, algorithm="maxima")
```

output
```
e^sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sin(x)} \cos(x)\, dx = e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x),x, algorithm="giac")
```

output
```
e^sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sin(x)} \cos(x) \, dx = \mathrm{e}^{\sin(x)}$$

input
```
int(exp(sin(x))*cos(x),x)
```

output
```
exp(sin(x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^{\sin(x)} \cos(x) \, dx = e^{\sin(x)}$$

input
```
int(exp(sin(x))*cos(x),x)
```

output
```
e**sin(x)
```

3.123 $\int x \log^2(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x \log^2(x) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

input
```
Integrate[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2(x) \, dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{2}x^2 \log^2(x) - \int x \log(x) dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

input
```
Int[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
orering	$\frac{7x^2\ln(x)^2}{8}-\frac{3x^2\left(2\ln(x)+\ln(x)^2\right)}{8}+\frac{x^3\left(\frac{2}{x}+\frac{2\ln(x)}{x}\right)}{8}$	43

input
```
int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="fricas")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{x^2\log{(x)}^2}{2} - \frac{x^2\log{(x)}}{2} + \frac{x^2}{4}$$

input
```
integrate(x*ln(x)**2,x)
```

output
```
x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{1}{4}\left(2\,\log{(x)}^2 - 2\,\log{(x)} + 1\right)x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx = \frac{1}{2}\,x^2\log{(x)}^2 - \frac{1}{2}\,x^2\log{(x)} + \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{x^2\left(2\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x\log^2(x)\,dx = \frac{x^2\left(2\log(x)^2 - 2\log(x) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x**2*(2*log(x)**2 - 2*log(x) + 1))/4
```

3.124 $\int \frac{1}{5+4\sqrt{x}+x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx=-4\arctan\left(2+\sqrt{x}\right)+\log\left(5+4\sqrt{x}+x\right)$$

output
```
-4*arctan(2+x^(1/2))+ln(5+4*x^(1/2)+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx=-4\arctan\left(2+\sqrt{x}\right)+\log\left(5+4\sqrt{x}+x\right)$$

input
```
Integrate[(5 + 4*Sqrt[x] + x)^(-1),x]
```

output
```
-4*ArcTan[2 + Sqrt[x]] + Log[5 + 4*Sqrt[x] + x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1680, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+4\sqrt{x}+5}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{x+4\sqrt{x}+5}d\sqrt{x}$$

$$\Big\downarrow \text{1142}$$

$$2\left(\frac{1}{2}\int \frac{2(\sqrt{x}+2)}{x+4\sqrt{x}+5}d\sqrt{x}-2\int \frac{1}{x+4\sqrt{x}+5}d\sqrt{x}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\int \frac{\sqrt{x}+2}{x+4\sqrt{x}+5}d\sqrt{x}-2\int \frac{1}{x+4\sqrt{x}+5}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(4\int \frac{1}{-x-4}d(2\sqrt{x}+4)+\int \frac{\sqrt{x}+2}{x+4\sqrt{x}+5}d\sqrt{x}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\int \frac{\sqrt{x}+2}{x+4\sqrt{x}+5}d\sqrt{x}-2\arctan\left(\frac{1}{2}(2\sqrt{x}+4)\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\log\left(x+4\sqrt{x}+5\right)-2\arctan\left(\frac{1}{2}(2\sqrt{x}+4)\right)\right)$$

input
```
Int[(5 + 4*Sqrt[x] + x)^(-1),x]
```

output
```
2*(-2*ArcTan[(4 + 2*Sqrt[x])/2] + Log[5 + 4*Sqrt[x] + x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-4\arctan\left(2+\sqrt{x}\right)+\ln\left(5+4\sqrt{x}+x\right)$
default	$2\arctan\left(\frac{x}{4}-\frac{3}{4}\right)+\frac{\ln(x^2-6x+25)}{2}+\frac{\ln(5+4\sqrt{x}+x)}{2}-2\arctan\left(2+\sqrt{x}\right)-\frac{\ln(5+x-4\sqrt{x})}{2}-2$
trager	$\mathrm{RootOf}\left(\_Z^2-2\_Z+5\right)\ln\left(5+4\sqrt{x}+x\right)-\ln\Big(79\,\mathrm{RootOf}\left(\_Z^2-2\_Z+5\right)^2 x-$

input
```
int(1/(5+4*x^(1/2)+x),x,method=_RETURNVERBOSE)
```

output
```
-4*arctan(2+x^(1/2))+ln(5+4*x^(1/2)+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx=-4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

input
```
integrate(1/(5+4*x^(1/2)+x),x, algorithm="fricas")
```

output
```
-4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx=\log\left(4\sqrt{x}+x+5\right)-4\,\mathrm{atan}\left(\sqrt{x}+2\right)$$

input
```
integrate(1/(5+4*x**(1/2)+x),x)
```

output
```
log(4*sqrt(x) + x + 5) - 4*atan(sqrt(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx = -4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

input
```
integrate(1/(5+4*x^(1/2)+x),x, algorithm="maxima")
```

output
```
-4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx = -4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

input
```
integrate(1/(5+4*x^(1/2)+x),x, algorithm="giac")
```

output
```
-4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx = \ln\left(x+4\sqrt{x}+5\right)-4\,\mathrm{atan}\left(\sqrt{x}+2\right)$$

input
```
int(1/(x + 4*x^(1/2) + 5),x)
```

output
```
log(x + 4*x^(1/2) + 5) - 4*atan(x^(1/2) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{5+4\sqrt{x}+x}\,dx = -4atan\left(\sqrt{x}+2\right)+\log\left(4\sqrt{x}+x+5\right)$$

input
```
int(1/(5+4*x^(1/2)+x),x)
```

output
```
- 4*atan(sqrt(x) + 2) + log(4*sqrt(x) + x + 5)
```

3.125 $\int 2015^x\,dx$

Optimal result

Integrand size = 3, antiderivative size = 8

$$\int 2015^x\,dx=\frac{2015^x}{\log(2015)}$$

output
```
2015^x/ln(2015)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx=\frac{2015^x}{\log(2015)}$$

input
```
Integrate[2015^x,x]
```

output
```
2015^x/Log[2015]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2015^x\,dx$$

$$\Big\downarrow \ 2624$$

$$\frac{2015^x}{\log(2015)}$$

input
```
Int[2015^x,x]
```

output
```
2015^x/Log[2015]
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
gosper	$\frac{2015^x}{\ln(2015)}$	9
derivativedivides	$\frac{2015^x}{\ln(2015)}$	9
default	$\frac{2015^x}{\ln(2015)}$	9
parallelrisch	$\frac{2015^x}{\ln(2015)}$	9
norman	$\frac{e^{x\ln(2015)}}{\ln(2015)}$	11
orering	$\frac{2015^x}{\ln(5)+\ln(13)+\ln(31)}$	14
meijerg	$-\frac{1-e^{x\ln(2015)}}{\ln(2015)}$	16
risch	$\frac{31^x 13^x 5^x}{\ln(5)+\ln(13)+\ln(31)}$	20

input
```
int(2015^x,x,method=_RETURNVERBOSE)
```

output
```
2015^x/ln(2015)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx = \frac{2015^x}{\log(2015)}$$

input
```
integrate(2015^x,x, algorithm="fricas")
```

output
```
2015^x/log(2015)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int 2015^x\,dx = \frac{2015^x}{\log(2015)}$$

input
```
integrate(2015**x,x)
```

output
```
2015**x/log(2015)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx = \frac{2015^x}{\log(2015)}$$

input
```
integrate(2015^x,x, algorithm="maxima")
```

output
```
2015^x/log(2015)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx = \frac{2015^x}{\log(2015)}$$

input
```
integrate(2015^x,x, algorithm="giac")
```

output
```
2015^x/log(2015)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx = \frac{2015^x}{\ln(2015)}$$

input
```
int(2015^x,x)
```

output
```
2015^x/log(2015)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2015^x\,dx = \frac{2015^x}{\log(2015)}$$

input
```
int(2015^x,x)
```

output
```
2015**x/log(2015)
```

3.126 $\int \frac{x}{(-3+x)(5+x)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{x}{(-3+x)(5+x)^2}\,dx = -\frac{5}{8(5+x)} - \frac{3}{32}\text{arctanh}\left(\frac{1+x}{4}\right)$$

output
```
-5/(40+8*x)-3/32*arctanh(1/4+1/4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x}{(-3+x)(5+x)^2}\,dx = -\frac{5}{8(5+x)} + \frac{3}{64}\log(3-x) - \frac{3}{64}\log(5+x)$$

input
```
Integrate[x/((-3 + x)*(5 + x)^2),x]
```

output
```
-5/(8*(5 + x)) + (3*Log[3 - x])/64 - (3*Log[5 + x])/64
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-3)(x+5)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{3}{64(x+5)}+\frac{5}{8(x+5)^2}+\frac{3}{64(x-3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{8(x+5)}+\frac{3}{64}\log(3-x)-\frac{3}{64}\log(x+5)$$

input
```
Int[x/((-3 + x)*(5 + x)^2),x]
```

output
```
-5/(8*(5 + x)) + (3*Log[3 - x])/64 - (3*Log[5 + x])/64
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{5}{8(5+x)}-\frac{3\ln(5+x)}{64}+\frac{3\ln(-3+x)}{64}$	21
norman	$-\frac{5}{8(5+x)}-\frac{3\ln(5+x)}{64}+\frac{3\ln(-3+x)}{64}$	21
risch	$-\frac{5}{8(5+x)}-\frac{3\ln(5+x)}{64}+\frac{3\ln(-3+x)}{64}$	21
parallelrisch	$-\frac{3\ln(5+x)x-3\ln(-3+x)x+40+15\ln(5+x)-15\ln(-3+x)}{64(5+x)}$	36

input
```
int(x/(-3+x)/(5+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/8/(5+x)-3/64*ln(5+x)+3/64*ln(-3+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(-3+x)(5+x)^2}\,dx=-\frac{3\,(x+5)\log{(x+5)}-3\,(x+5)\log{(x-3)}+40}{64\,(x+5)}$$

input
```
integrate(x/(-3+x)/(5+x)^2,x, algorithm="fricas")
```

output
```
-1/64*(3*(x + 5)*log(x + 5) - 3*(x + 5)*log(x - 3) + 40)/(x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-3+x)(5+x)^2}\,dx=\frac{3\log{(x-3)}}{64}-\frac{3\log{(x+5)}}{64}-\frac{5}{8x+40}$$

input
```
integrate(x/(-3+x)/(5+x)**2,x)
```

output
```
3*log(x - 3)/64 - 3*log(x + 5)/64 - 5/(8*x + 40)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{(-3+x)(5+x)^2} dx = -\frac{5}{8(x+5)} - \frac{3}{64} \log(x+5) + \frac{3}{64} \log(x-3)$$

input
```
integrate(x/(-3+x)/(5+x)^2,x, algorithm="maxima")
```

output
```
-5/8/(x + 5) - 3/64*log(x + 5) + 3/64*log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{x}{(-3+x)(5+x)^2} dx = -\frac{5}{8(x+5)} + \frac{3}{64} \log\left(\left|-\frac{8}{x+5} + 1\right|\right)$$

input
```
integrate(x/(-3+x)/(5+x)^2,x, algorithm="giac")
```

output
```
-5/8/(x + 5) + 3/64*log(abs(-8/(x + 5) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-3+x)(5+x)^2} dx = -\frac{3 \ln\left(\frac{x+5}{x-3}\right)}{64} - \frac{5}{8(x+5)}$$

input
```
int(x/((x - 3)*(x + 5)^2),x)
```

output
```
- (3*log((x + 5)/(x - 3)))/64 - 5/(8*(x + 5))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{x}{(-3+x)(5+x)^2}\,dx = \frac{3\log(x-3)\,x+15\log(x-3)-3\log(x+5)\,x-15\log(x+5)+8x}{64x+320}$$

input
```
int(x/(-3+x)/(5+x)^2,x)
```

output
```
(3*log(x - 3)*x + 15*log(x - 3) - 3*log(x + 5)*x - 15*log(x + 5) + 8*x)/(6
4*(x + 5))
```

3.127 $\int \frac{\log(1+\log(x))}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{\log(1+\log(x))}{x} \, dx = -\log(x) + (1+\log(x))\log(1+\log(x))$$

output
```
-ln(x)+(1+ln(x))*ln(1+ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+\log(x))}{x} \, dx = -\log(x) + (1+\log(x))\log(1+\log(x))$$

input
```
Integrate[Log[1 + Log[x]]/x,x]
```

output
```
-Log[x] + (1 + Log[x])*Log[1 + Log[x]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3039, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x)+1)}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log(\log(x)+1)d\log(x)$$

$$\Big\downarrow \text{2836}$$

$$\int \log(\log(x)+1)d(\log(x)+1)$$

$$\Big\downarrow \text{2732}$$

$$-\log(x)+(\log(x)+1)\log(\log(x)+1)-1$$

input
```
Int[Log[1 + Log[x]]/x,x]
```

output
```
-1 - Log[x] + (1 + Log[x])*Log[1 + Log[x]]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$(1+\ln(x))\ln(1+\ln(x))-1-\ln(x)$	17
default	$(1+\ln(x))\ln(1+\ln(x))-1-\ln(x)$	17
norman	$-\ln(x)+\ln(1+\ln(x))\ln(x)+\ln(1+\ln(x))$	19
risch	$-\ln(x)+\ln(1+\ln(x))\ln(x)+\ln(1+\ln(x))$	19

input
```
int(ln(1+ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
(1+ln(x))*ln(1+ln(x))-1-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+\log(x))}{x}\,dx = (\log(x)+1)\log(\log(x)+1)-\log(x)$$

input
```
integrate(log(1+log(x))/x,x, algorithm="fricas")
```

output
```
(log(x) + 1)*log(log(x) + 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{\log(1+\log(x))}{x}\,dx = \log(x)\log(\log(x)+1) - \log(x) + \log(\log(x)+1)$$

input
```
integrate(ln(1+ln(x))/x,x)
```

output
```
log(x)*log(log(x) + 1) - log(x) + log(log(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{\log(1+\log(x))}{x}\,dx = (\log(x)+1)\log(\log(x)+1) - \log(x) - 1$$

input
```
integrate(log(1+log(x))/x,x, algorithm="maxima")
```

output
```
(log(x) + 1)*log(log(x) + 1) - log(x) - 1
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{\log(1+\log(x))}{x}\,dx = (\log(x)+1)\log(\log(x)+1) - \log(x) - 1$$

input
```
integrate(log(1+log(x))/x,x, algorithm="giac")
```

output
```
(log(x) + 1)*log(log(x) + 1) - log(x) - 1
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\log(1+\log(x))}{x} \, dx = \ln(\ln(x)+1) - \ln(x) + \ln(\ln(x)+1)\, \ln(x)$$

input
```
int(log(log(x) + 1)/x,x)
```

output
```
log(log(x) + 1) - log(x) + log(log(x) + 1)*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\log(1+\log(x))}{x} \, dx = \log(\log(x)+1)\log(x) + \log(\log(x)+1) - \log(x)$$

input
```
int(log(1+log(x))/x,x)
```

output
```
log(log(x) + 1)*log(x) + log(log(x) + 1) - log(x)
```

3.128 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

input
```
Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(x)-\sin(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

input
```
Int[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$2\sqrt{\cos\left(x\right)\cot\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(e^{2ix}+1\right)^{2}e^{-ix}}{e^{2ix}-1}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}$	51

input
```
int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \int \sqrt{-\sin(x)+\csc(x)}\,dx$$

input
```
integrate((csc(x)-sin(x))**(1/2),x)
```

output
```
Integral(sqrt(-sin(x) + csc(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 14.46

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$
$$= \frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\right.\right.}{}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
(((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(
x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin
(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) -
((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(
1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)^(1/4))
```

Giac [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\int \sqrt{\csc\left(x\right)-\sin\left(x\right)}\,dx$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\frac{2\left|\cos\left(x\right)\right|}{\cos\left(x\right)\,\sqrt{\frac{1}{\sin(x)}}}$$

input
```
int((1/sin(x) - sin(x))^(1/2),x)
```

output
```
(2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Reduce [F]

$$\int \sqrt{\csc(x) - \sin(x)}\, dx = \int \sqrt{\csc(x) - \sin(x)} dx$$

input
```
int((csc(x)-sin(x))^(1/2),x)
```

output
```
int(sqrt(csc(x) - sin(x)),x)
```

3.129 $\int \frac{1}{\sqrt{25+x^2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = \operatorname{arcsinh}\left(\frac{x}{5}\right)$$

output
```
arcsinh(1/5*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = -\log\left(-x+\sqrt{25+x^2}\right)$$

input
```
Integrate[1/Sqrt[25 + x^2],x]
```

output
```
-Log[-x + Sqrt[25 + x^2]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+25}} \, dx$$

$$\Big\downarrow \; 222$$

$$\text{arcsinh}\left(\frac{x}{5}\right)$$

input
```
Int[1/Sqrt[25 + x^2],x]
```

output
```
ArcSinh[x/5]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\text{arcsinh}\left(\frac{x}{5}\right)$	5
meijerg	$\text{arcsinh}\left(\frac{x}{5}\right)$	5
trager	$\ln\left(x+\sqrt{x^2+25}\right)$	11
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^2+25}}{x}\right)$	13

input
```
int(1/(x^2+25)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+25}\right)$$

input
```
integrate(1/(x^2+25)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 25))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{5}\right)$$

input
```
integrate(1/(x**2+25)**(1/2),x)
```

output
```
asinh(x/5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = \operatorname{arsinh}\left(\frac{1}{5}\,x\right)$$

input
```
integrate(1/(x^2+25)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(4) = 8$.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+25}x - \frac{25}{2}\,\log\left(-x+\sqrt{x^2+25}\right)$$

input
```
integrate(1/(x^2+25)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 25)*x - 25/2*log(-x + sqrt(x^2 + 25))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{25+x^2}}\,dx = \operatorname{asinh}\left(\frac{x}{5}\right)$$

input
```
int(1/(x^2 + 25)^(1/2),x)
```

output
```
asinh(x/5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{25+x^2}}\,dx=\log\left(\frac{\sqrt{x^2+25}}{5}+\frac{x}{5}\right)$$

input
```
int(1/(x^2+25)^(1/2),x)
```

output
```
log((sqrt(x**2 + 25) + x)/5)
```

3.130 $\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{1}{\log(x)}+\log(x)$$

output

```
1/ln(x)+ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{1}{\log(x)}+\log(x)$$

input

```
Integrate[(-1 + Log[x]^2)/(x*Log[x]^2),x]
```

output

```
Log[x]^(-1) + Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3039, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)-1}{x\log^2(x)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int -\frac{1-\log^2(x)}{\log^2(x)}d\log(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\log^2(x)}{\log^2(x)}d\log(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\frac{1}{\log^2(x)}-1\right)d\log(x)$$

$$\Big\downarrow \text{2009}$$

$$\log(x)+\frac{1}{\log(x)}$$

input
```
Int[(-1 + Log[x]^2)/(x*Log[x]^2),x]
```

output
```
Log[x]^(-1) + Log[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{1}{\ln(x)}+\ln\left(x\right)$	8
default	$\frac{1}{\ln(x)}+\ln\left(x\right)$	8
risch	$\frac{1}{\ln(x)}+\ln\left(x\right)$	8
parts	$\frac{1}{\ln(x)}+\ln\left(x\right)$	8
norman	$\frac{1+\ln(x)^2}{\ln(x)}$	12
parallelrisch	$\frac{1+\ln(x)^2}{\ln(x)}$	12

input
```
int((-1+ln(x)^2)/x/ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/ln(x)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{\log(x)^2+1}{\log(x)}$$

input
```
integrate((-1+log(x)^2)/x/log(x)^2,x, algorithm="fricas")
```

output
```
(log(x)^2 + 1)/log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \log(x) + \frac{1}{\log(x)}$$

input
```
integrate((-1+ln(x)**2)/x/ln(x)**2,x)
```

output
```
log(x) + 1/log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{1}{\log(x)} + \log(x)$$

input
```
integrate((-1+log(x)^2)/x/log(x)^2,x, algorithm="maxima")
```

output
```
1/log(x) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{1}{\log(x)}+\log(x)$$

input
```
integrate((-1+log(x)^2)/x/log(x)^2,x, algorithm="giac")
```

output
```
1/log(x) + log(x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \ln(x)+\frac{1}{\ln(x)}$$

input
```
int((log(x)^2 - 1)/(x*log(x)^2),x)
```

output
```
log(x) + 1/log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx = \frac{\log(x)^2+1}{\log(x)}$$

input
```
int((-1+log(x)^2)/x/log(x)^2,x)
```

output
```
(log(x)**2 + 1)/log(x)
```

3.131 $\int e^{3x} \arctan(e^x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int e^{3x} \arctan(e^x) \, dx = -\frac{e^{2x}}{6} + \frac{1}{3} e^{3x} \arctan(e^x) + \frac{1}{6} \log\left(1 + e^{2x}\right)$$

output
```
-1/6*exp(2*x)+1/3*exp(3*x)*arctan(exp(x))+1/6*ln(1+exp(2*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int e^{3x} \arctan(e^x) \, dx = \frac{1}{6} \left(-e^{2x} + 2e^{3x} \arctan(e^x) + \log\left(1 + e^{2x}\right)\right)$$

input
```
Integrate[E^(3*x)*ArcTan[E^x],x]
```

output
```
(-E^(2*x) + 2*E^(3*x)*ArcTan[E^x] + Log[1 + E^(2*x)])/6
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5730, 27, 2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x} \arctan\left(e^x\right) dx$$

$$\Big\downarrow \text{5730}$$

$$\frac{1}{3}e^{3x} \arctan\left(e^x\right) - \int \frac{e^{4x}}{3\left(1+e^{2x}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}e^{3x} \arctan\left(e^x\right) - \frac{1}{3}\int \frac{e^{4x}}{1+e^{2x}}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{1}{3}e^{3x} \arctan\left(e^x\right) - \frac{1}{6}\int \frac{e^{2x}}{1+e^{2x}}de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}e^{3x} \arctan\left(e^x\right) - \frac{1}{6}\int \left(1+\frac{1}{-1-e^{2x}}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}e^{3x} \arctan\left(e^x\right) + \frac{1}{6}\left(\log\left(e^{2x}+1\right) - e^{2x}\right)$$

input

```
Int[E^(3*x)*ArcTan[E^x],x]
```

output

```
(E^(3*x)*ArcTan[E^x])/3 + (-E^(2*x) + Log[1 + E^(2*x)])/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{{\mathrm e}^{2 x}}{6}+\frac{{\mathrm e}^{3 x} \arctan \left({\mathrm e}^{x}\right)}{3}+\frac{\ln \left(1+{\mathrm e}^{2 x}\right)}{6}$	26
risch	$-\frac{i {\mathrm e}^{3 x} \ln \left(1+i {\mathrm e}^{x}\right)}{6}+\frac{i {\mathrm e}^{3 x} \ln \left(1-i {\mathrm e}^{x}\right)}{6}-\frac{{\mathrm e}^{2 x}}{6}+\frac{\ln \left(1+{\mathrm e}^{2 x}\right)}{6}$	47

input
```
int(exp(3*x)*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(x)^3*arctan(exp(x))-1/6*exp(x)^2+1/6*ln(exp(x)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{3x}\arctan\left(e^x\right)\,dx=\frac{1}{3}\arctan\left(e^x\right)e^{(3x)}-\frac{1}{6}e^{(2x)}+\frac{1}{6}\log\left(e^{(2x)}+1\right)$$

input
```
integrate(exp(3*x)*arctan(exp(x)),x, algorithm="fricas")
```

output
```
1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int e^{3x}\arctan\left(e^x\right)\,dx=\frac{e^{3x}\operatorname{atan}\left(e^x\right)}{3}-\frac{e^{2x}}{6}+\frac{\log\left(e^{2x}+1\right)}{6}$$

input
```
integrate(exp(3*x)*atan(exp(x)),x)
```

output
```
exp(3*x)*atan(exp(x))/3 - exp(2*x)/6 + log(exp(2*x) + 1)/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{3x}\arctan\left(e^x\right)\,dx=\frac{1}{3}\arctan\left(e^x\right)e^{(3x)}-\frac{1}{6}e^{(2x)}+\frac{1}{6}\log\left(e^{(2x)}+1\right)$$

input
```
integrate(exp(3*x)*arctan(exp(x)),x, algorithm="maxima")
```

output
```
1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{3x} \arctan\left(e^{x}\right) dx = \frac{1}{3} \arctan\left(e^{x}\right) e^{(3x)} - \frac{1}{6} e^{(2x)} + \frac{1}{6} \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(exp(3*x)*arctan(exp(x)),x, algorithm="giac")
```

output
```
1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{3x} \arctan\left(e^{x}\right) dx = \frac{\ln\left(e^{2x} + 1\right)}{6} - \frac{e^{2x}}{6} + \frac{\mathrm{atan}(e^{x})\, e^{3x}}{3}$$

input
```
int(atan(exp(x))*exp(3*x),x)
```

output
```
log(exp(2*x) + 1)/6 - exp(2*x)/6 + (atan(exp(x))*exp(3*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{3x} \arctan\left(e^{x}\right) dx = \frac{e^{3x} atan(e^{x})}{3} - \frac{e^{2x}}{6} + \frac{\log(e^{2x} + 1)}{6}$$

input
```
int(exp(3*x)*atan(exp(x)),x)
```

output
```
(2*e**(3*x)*atan(e**x) - e**(2*x) + log(e**(2*x) + 1))/6
```

3.132 $\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx=\sqrt{2}x+\frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}-2\cos(x)\sin(x)}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}+2\cos(x)\sin(x)}\right)}{\sqrt{2}}$$

output
```
x*2^(1/2)+1/2*arctan((1-2*cos(x)^2)/(1+2^(1/2)-2*cos(x)*sin(x)))*2^(1/2)-1
/2*arctan((1-2*cos(x)^2)/(1+2^(1/2)+2*cos(x)*sin(x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.45

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx=\frac{-\arctan\left(1-\sqrt{2}\tan(x)\right)+\arctan\left(1+\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

input
```
Integrate[(Cos[x]^4 + Sin[x]^4)^(-1),x]
```

output
```
(-ArcTan[1 - Sqrt[2]*Tan[x]] + ArcTan[1 + Sqrt[2]*Tan[x]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4889, 1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^4(x)+\cos^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^4+\cos(x)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)+1}{\tan^4(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\int \frac{1}{\tan^2(x)-\sqrt{2}\tan(x)+1}d\tan(x)+\frac{1}{2}\int \frac{1}{\tan^2(x)+\sqrt{2}\tan(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{1}{-\left(1-\sqrt{2}\tan(x)\right)^2-1}d\left(1-\sqrt{2}\tan(x)\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}\tan(x)+1\right)^2-1}d\left(\sqrt{2}\tan(x)+1\right)}{\sqrt{2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\sqrt{2}\tan(x)+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

input
```
Int[(Cos[x]^4 + Sin[x]^4)^(-1),x]
```

output
```
-(ArcTan[1 - Sqrt[2]*Tan[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Tan[x]]/Sqrt[2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{4ix}-2\sqrt{2}+3\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(x)^2+\tan(x)\sqrt{2}+1}{\tan(x)^2-\tan(x)\sqrt{2}+1}\right)+2\arctan\left(\tan(x)\sqrt{2}+1\right)+2\arctan\left(\tan(x)\sqrt{2}-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(x)^2-\tan(x)\sqrt{2}+1}{\tan(x)^2+\tan(x)\sqrt{2}+1}\right)+2\arctan\left(\right.\right.}{8}$

input
```
int(1/(cos(x)^4+sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*2^(1/2)*ln(exp(4*I*x)+2*2^(1/2)+3)-1/4*I*2^(1/2)*ln(exp(4*I*x)-2*2^(
1/2)+3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx=-\frac{1}{4}\sqrt{2}\arctan\left(\frac{6\sqrt{2}\cos(x)^4-6\sqrt{2}\cos(x)^2+\sqrt{2}}{4\left(2\cos(x)^3-\cos(x)\right)\sin(x)}\right)$$

input
```
integrate(1/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(6*sqrt(2)*cos(x)^4 - 6*sqrt(2)*cos(x)^2 + sqrt(2)
)/((2*cos(x)^3 - cos(x))*sin(x)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(cos(x)**4+sin(x)**4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\tan(x)\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\tan(x)\right)\right)$$

input
```
integrate(1/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*tan(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*tan(x)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.68

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx = \frac{1}{2}\sqrt{2}\left(2x+\arctan\left(-\frac{\sqrt{2}\sin(4x)-\sin(4x)}{\sqrt{2}\cos(4x)+\sqrt{2}-\cos(4x)+1}\right)\right)$$

input
```
integrate(1/(cos(x)^4+sin(x)^4),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(2*x + arctan(-(sqrt(2)*sin(4*x) - sin(4*x))/(sqrt(2)*cos(4*x)
 + sqrt(2) - cos(4*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx = \sqrt{2}\,(x-\operatorname{atan}(\tan(x))) + \frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)+\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)^3}{2}+\frac{\sqrt{2}\tan(x)}{2}\right)\right)}{2}$$

input
```
int(1/(cos(x)^4 + sin(x)^4),x)
```

output
```
2^(1/2)*(x - atan(tan(x))) + (2^(1/2)*(atan((2^(1/2)*tan(x))/2) + atan((2^
(1/2)*tan(x)^3)/2 + (2^(1/2)*tan(x))/2)))/2
```

Reduce [F]

$$\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx = \int \frac{1}{\cos(x)^4+\sin(x)^4}dx$$

input
```
int(1/(cos(x)^4+sin(x)^4),x)
```

output
```
int(1/(cos(x)**4 + sin(x)**4),x)
```

3.133 $\int \frac{1+e^x}{1-e^x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+e^x}{1-e^x} \, dx = x - 2\log\left(1-e^x\right)$$

output
```
x-2*ln(1-exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+e^x}{1-e^x} \, dx = \log\left(e^x\right) - 2\log\left(-1+e^x\right)$$

input
```
Integrate[(1 + E^x)/(1 - E^x),x]
```

output
```
Log[E^x] - 2*Log[-1 + E^x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+1}{1-e^x} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^{-x}(e^x+1)}{1-e^x} de^x$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(e^{-x}-\frac{2}{e^x-1}\right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\log(e^x)-2\log(1-e^x)$$

input
```
Int[(1 + E^x)/(1 - E^x),x]
```

output
```
Log[E^x] - 2*Log[1 - E^x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
norman	$x-2\ln\left(\mathrm{e}^{x}-1\right)$	10
risch	$x-2\ln\left(\mathrm{e}^{x}-1\right)$	10
parallelrisch	$x-2\ln\left(\mathrm{e}^{x}-1\right)$	10
derivativedivides	$-2\ln\left(\mathrm{e}^{x}-1\right)+\ln\left(\mathrm{e}^{x}\right)$	12
default	$-2\ln\left(\mathrm{e}^{x}-1\right)+\ln\left(\mathrm{e}^{x}\right)$	12

input
```
int((1+exp(x))/(1-exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(exp(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(e^{x} - 1\right)$$

input
```
integrate((1+exp(x))/(1-exp(x)),x, algorithm="fricas")
```

output
```
x - 2*log(e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\log\left(e^x-1\right)$$

input
```
integrate((1+exp(x))/(1-exp(x)),x)
```

output
```
x - 2*log(exp(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(e^x-1\right)$$

input
```
integrate((1+exp(x))/(1-exp(x)),x, algorithm="maxima")
```

output
```
x - 2*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{1-e^x}\,dx = x - 2\,\log\left(|e^x-1|\right)$$

input
```
integrate((1+exp(x))/(1-exp(x)),x, algorithm="giac")
```

output
```
x - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1+e^x}{1-e^x}\,dx = x-2\,\ln\left(\mathrm{e}^x-1\right)$$

input
```
int(-(exp(x) + 1)/(exp(x) - 1),x)
```

output
```
x - 2*log(exp(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{1-e^x}\,dx = -2\log(e^x-1)+x$$

input
```
int((1+exp(x))/(1-exp(x)),x)
```

output
```
- 2*log(e**x - 1) + x
```

3.134 $\int \tan^4(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(x)\,dx = x - \tan(x) + \frac{\tan^3(x)}{3}$$

output
```
x-tan(x)+1/3*tan(x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(x)\,dx = \arctan(\tan(x)) - \tan(x) + \frac{\tan^3(x)}{3}$$

input
```
Integrate[Tan[x]^4,x]
```

output
```
ArcTan[Tan[x]] - Tan[x] + Tan[x]^3/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^4dx$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^3(x)}{3}-\int \tan^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^3(x)}{3}-\int \tan(x)^2dx$$

$$\Big\downarrow \text{ 3954}$$

$$\int 1dx+\frac{\tan^3(x)}{3}-\tan(x)$$

$$\Big\downarrow \text{ 24}$$

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

input
```
Int[Tan[x]^4,x]
```

output
```
x - Tan[x] + Tan[x]^3/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
norman	$x-\tan\left(x\right)+\frac{\tan\left(x\right)^{3}}{3}$	13
parallelrisch	$x-\tan\left(x\right)+\frac{\tan\left(x\right)^{3}}{3}$	13
derivativedivides	$\frac{\tan\left(x\right)^{3}}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
default	$\frac{\tan\left(x\right)^{3}}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
risch	$x-\frac{4i\left(3\,\mathrm{e}^{4ix}+3\,\mathrm{e}^{2ix}+2\right)}{3\left(\mathrm{e}^{2ix}+1\right)^{3}}$	31

input
```
int(tan(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tan(x)+1/3*tan(x)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan(x)^3 + x - \tan(x)$$

input
```
integrate(tan(x)^4,x, algorithm="fricas")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(x)\,dx = x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(tan(x)**4,x)
```

output
```
x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan(x)^3 + x - \tan(x)$$

input
```
integrate(tan(x)^4,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{1}{3}\tan(x)^3 + x - \tan(x)$$

input
```
integrate(tan(x)^4,x, algorithm="giac")
```

output
```
1/3*tan(x)^3 + x - tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{\tan(x)^3}{3} - \tan(x) + x$$

input
```
int(tan(x)^4,x)
```

output
```
x - tan(x) + tan(x)^3/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(x)\,dx = \frac{\tan(x)^3}{3} - \tan(x) + x$$

input
```
int(tan(x)^4,x)
```

output
```
(tan(x)**3 - 3*tan(x) + 3*x)/3
```

3.135 $\int \sin(x)\tan^2(x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \sec(x)$$

output
```
cos(x)+sec(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \sec(x)$$

input
```
Integrate[Sin[x]*Tan[x]^2,x]
```

output
```
Cos[x] + Sec[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(x)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\int \left(1-\cos^2(x)\right)\sec^2(x)d\cos(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\sec^2(x)-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)+\sec(x)$$

input
```
Int[Sin[x]*Tan[x]^2,x]
```

output
```
Cos[x] + Sec[x]
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 1.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

method	result	size
default	$\frac{\sin(x)^4}{\cos(x)} + \left(2 + \sin\left(x\right)^2\right)\cos\left(x\right)$	20
risch	$\frac{e^{3ix}+7\cos(x)+5i\sin(x)}{2\,e^{2ix}+2}$	27

input
```
int(sin(x)*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \sin(x)\tan^2(x)\,dx = \frac{\cos(x)^2 + 1}{\cos(x)}$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
```

output
```
(cos(x)^2 + 1)/cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \frac{1}{\cos(x)}$$

input
```
integrate(sin(x)*tan(x)**2,x)
```

output
```
cos(x) + 1/cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \frac{1}{\cos(x)} + \cos(x)$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
```

output
```
1/cos(x) + cos(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x) \tan^2(x) \, dx = \frac{1}{\cos(x)} + \cos(x)$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="giac")
```

output
```
1/cos(x) + cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x) \tan^2(x) \, dx = \cos(x) + \frac{1}{\cos(x)}$$

input
```
int(sin(x)*tan(x)^2,x)
```

output
```
cos(x) + 1/cos(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \sin(x) \tan^2(x) \, dx = \frac{-2\cos(x) - \sin(x)^2 + 2}{\cos(x)}$$

input
```
int(sin(x)*tan(x)^2,x)
```

output
```
( - 2*cos(x) - sin(x)**2 + 2)/cos(x)
```

3.136 $\int \frac{1+x}{3+2x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{1}{2}\log\left(3+2x+x^2\right)$$

output
```
1/2*ln(x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{1}{2}\log\left(3+2x+x^2\right)$$

input
```
Integrate[(1 + x)/(3 + 2*x + x^2),x]
```

output
```
Log[3 + 2*x + x^2]/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^2+2x+3}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
Int[(1 + x)/(3 + 2*x + x^2),x]
```

output
```
Log[3 + 2*x + x^2]/2
```

Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(x^{2}+2x+3\right)}{2}$	12
norman	$\frac{\ln\left(x^{2}+2x+3\right)}{2}$	12
risch	$\frac{\ln\left(x^{2}+2x+3\right)}{2}$	12
parallelrisch	$\frac{\ln\left(x^{2}+2x+3\right)}{2}$	12

input
```
int((x+1)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{1}{2}\,\log\left(x^2+2\,x+3\right)$$

input
```
integrate((1+x)/(x^2+2*x+3),x, algorithm="fricas")
```

output
```
1/2*log(x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{\log\left(x^2+2x+3\right)}{2}$$

input
```
integrate((1+x)/(x**2+2*x+3),x)
```

output
```
log(x**2 + 2*x + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{1}{2}\,\log\left(x^2+2\,x+3\right)$$

input
```
integrate((1+x)/(x^2+2*x+3),x, algorithm="maxima")
```

output
```
1/2*log(x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{1}{2}\,\log\left(x^2+2\,x+3\right)$$

input
```
integrate((1+x)/(x^2+2*x+3),x, algorithm="giac")
```

output
```
1/2*log(x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{\ln\left(x^2+2\,x+3\right)}{2}$$

input
```
int((x + 1)/(2*x + x^2 + 3),x)
```

output
```
log(2*x + x^2 + 3)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+x}{3+2x+x^2}\,dx = \frac{\log(x^2+2x+3)}{2}$$

input
```
int((1+x)/(x^2+2*x+3),x)
```

output
```
log(x**2 + 2*x + 3)/2
```

3.137 $\int \tanh(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

output
```
ln(cosh(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

input
```
Integrate[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)dx$$

$$\Big\downarrow \text{3956}$$

$$\log(\cosh(x))$$

input
```
Int[Tanh[x],x]
```

output
```
Log[Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\ln(\cosh(x))$	4
derivativedivides	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(1+e^{2x})$	12
parallelrisch	$-\ln(1-\tanh(x))-x$	14

input
```
int(tanh(x),x,method=_RETURNVERBOSE)
```

output
```
ln(cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x)\,dx = -x + \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(tanh(x),x, algorithm="fricas")
```

output
```
-x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x)\,dx = x - \log(\tanh(x) + 1)$$

input
```
integrate(tanh(x),x)
```

output
```
x - log(tanh(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

input
```
integrate(tanh(x),x, algorithm="maxima")
```

output
```
log(cosh(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x)\,dx = -x + \log\left(e^{(2x)} + 1\right)$$

input
```
integrate(tanh(x),x, algorithm="giac")
```

output
```
-x + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x)\,dx = \ln(\cosh(x))$$

input
```
int(tanh(x),x)
```

output
```
log(cosh(x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \tanh(x)\,dx = \log\left(e^{2x}+1\right) - x$$

input
```
int(tanh(x),x)
```

output
```
log(e**(2*x) + 1) - x
```

3.138 $\int (-x + x^3) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int (-x + x^3) \, dx = -\frac{x^2}{2} + \frac{x^4}{4}$$

output
```
-1/2*x^2+1/4*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (-x + x^3) \, dx = -\frac{x^2}{2} + \frac{x^4}{4}$$

input
```
Integrate[-x + x^3,x]
```

output
```
-1/2*x^2 + x^4/4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3 - x)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4} - \frac{x^2}{2}$$

input
```
Int[-x + x^3,x]
```

output
```
-1/2*x^2 + x^4/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{x^2(x^2-2)}{4}$	11
default	$-\frac{1}{2}x^2+\frac{1}{4}x^4$	12
norman	$-\frac{1}{2}x^2+\frac{1}{4}x^4$	12
risch	$-\frac{1}{2}x^2+\frac{1}{4}x^4$	12
parallelrisch	$-\frac{1}{2}x^2+\frac{1}{4}x^4$	12
parts	$-\frac{1}{2}x^2+\frac{1}{4}x^4$	12
orering	$\frac{x(x^2-2)(x^3-x)}{4(x-1)(x+1)}$	26

input
```
int(x^3-x,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(x^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-x+x^3)\, dx = \frac{1}{4}\, x^4 - \frac{1}{2}\, x^2$$

input
```
integrate(x^3-x,x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int (-x + x^3) \, dx = \frac{x^4}{4} - \frac{x^2}{2}$$

input
```
integrate(x**3-x,x)
```

output
```
x**4/4 - x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-x + x^3) \, dx = \frac{1}{4} x^4 - \frac{1}{2} x^2$$

input
```
integrate(x^3-x,x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/2*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-x + x^3) \, dx = \frac{1}{4} x^4 - \frac{1}{2} x^2$$

input
```
integrate(x^3-x,x, algorithm="giac")
```

output
```
1/4*x^4 - 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int (-x + x^3) \, dx = \frac{x^2 (x^2 - 2)}{4}$$

input
```
int(x^3 - x,x)
```

output
```
(x^2*(x^2 - 2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int (-x + x^3) \, dx = \frac{x^2(x^2 - 2)}{4}$$

input
```
int(x^3-x,x)
```

output
```
(x**2*(x**2 - 2))/4
```

3.139 $\int \log(\sqrt{x})\,dx$

Optimal result

Integrand size = 6, antiderivative size = 14

$$\int \log\left(\sqrt{x}\right)\,dx = -\frac{x}{2} + x\log\left(\sqrt{x}\right)$$

output
```
-1/2*x+1/2*x*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{1}{2}(-x + x\log(x))$$

input
```
Integrate[Log[Sqrt[x]],x]
```

output
```
(-x + x*Log[x])/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 2732}$$

$$x \log\left(\sqrt{x}\right) - \frac{x}{2}$$

input
```
Int[Log[Sqrt[x]],x]
```

output
```
-1/2*x + x*Log[Sqrt[x]]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

method	result	size
lookup	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
default	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
norman	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
risch	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
parallelrisch	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
parts	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10
orering	$-\frac{x}{2}+\frac{\ln(x)x}{2}$	10

input
```
int(1/2*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/2*ln(x)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x\log\left(x\right) - \frac{1}{2}\,x$$

input
```
integrate(1/2*log(x),x, algorithm="fricas")
```

output
```
1/2*x*log(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \log \left(\sqrt{x}\right) \, dx = \frac{x \log \left(x\right)}{2} - \frac{x}{2}$$

input
```
integrate(1/2*ln(x),x)
```

output
```
x*log(x)/2 - x/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log \left(\sqrt{x}\right) \, dx = \frac{1}{2} \, x \log \left(x\right) - \frac{1}{2} \, x$$

input
```
integrate(1/2*log(x),x, algorithm="maxima")
```

output
```
1/2*x*log(x) - 1/2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \log \left(\sqrt{x}\right) \, dx = \frac{1}{2} \, x \log \left(x\right) - \frac{1}{2} \, x$$

input
```
integrate(1/2*log(x),x, algorithm="giac")
```

output
```
1/2*x*log(x) - 1/2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{x\left(\ln\left(x\right)-1\right)}{2}$$

input
```
int(log(x)/2,x)
```

output
```
(x*(log(x) - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \log\left(\sqrt{x}\right)\,dx = \frac{x(\log(x)-1)}{2}$$

input
```
int(1/2*log(x),x)
```

output
```
(x*(log(x) - 1))/2
```

3.140 $\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 11

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx = e^{e^{-x}+e^x}$$

output
```
exp(exp(1)^(-x)+exp(1)^x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx = e^{e^{-x}+e^x}$$

input
```
Integrate[-E^(E^(-x) + E^x - x) + E^(E^(-x) + E^x + x),x]
```

output
```
E^(E^(-x) + E^x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{x+e^{-x}+e^x}-e^{-x+e^{-x}+e^x}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\text{Subst}\left(\int e^{x+\frac{1}{x}}dx,x,e^x\right)-\text{Subst}\left(\int \frac{e^{x+\frac{1}{x}}}{x^2}dx,x,e^x\right)$$

input
```
Int[-E^(E^(-x) + E^x - x) + E^(E^(-x) + E^x + x),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
risch	$\mathrm{e}^{\mathrm{e}^{-x}+\mathrm{e}^{x}}$	9
norman	$\mathrm{e}^{x} \mathrm{e}^{\mathrm{e}^{-x}+\mathrm{e}^{x}-x}$	15

input
```
int(-exp(exp(1)^(-x)+exp(1)^x-x)+exp(exp(1)^(-x)+exp(1)^x+x),x,method=_RET
URNVERBOSE)
```

output
```
exp(exp(-x)+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right)\,dx=e^{\left(\left(xe^x+e^{(2\,x)}+1\right)e^{(-x)}-x\right)}$$

input
```
integrate(-exp(exp(1)^(-x)+exp(1)^x-x)+exp(exp(1)^(-x)+exp(1)^x+x),x, algo
rithm="fricas")
```

output
```
e^((x*e^x + e^(2*x) + 1)*e^(-x) - x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right)\,dx=e^xe^{-x+e^x+e^{-x}}$$

input
```
integrate(-exp(exp(1)**(-x)+exp(1)**x-x)+exp(exp(1)**(-x)+exp(1)**x+x),x)
```

output
```
exp(x)*exp(-x + exp(x) + exp(-x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right)\,dx=e^{\left(e^{(-x)}+e^x\right)}$$

input
```
integrate(-exp(exp(1)^(-x)+exp(1)^x-x)+exp(exp(1)^(-x)+exp(1)^x+x),x, algo
rithm="maxima")
```

output
```
e^(e^(-x) + e^x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx = e^{\left(\left(e^{(2x)}+1\right)e^{(-x)}\right)}$$

input
```
integrate(-exp(exp(1)^(-x)+exp(1)^x-x)+exp(exp(1)^(-x)+exp(1)^x+x),x, algo
rithm="giac")
```

output
```
e^((e^(2*x) + 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx = e^{e^{-x}+e^x}$$

input
```
int(exp(x + exp(-x) + exp(x)) - exp(exp(-x) - x + exp(x)),x)
```

output
```
exp(exp(-x) + exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx = e^{\frac{e^{2x}+1}{e^x}}$$

input
```
int(-exp(exp(1)^(-x)+exp(1)^x-x)+exp(exp(1)^(-x)+exp(1)^x+x),x)
```

output
```
e**((e**(2*x) + 1)/e**x)
```

3.141 $\int \frac{\log(\log(x))}{x\log(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{1}{2}\log^2(\log(x))$$

output
```
1/2*ln(ln(x))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{1}{2}\log^2(\log(x))$$

input
```
Integrate[Log[Log[x]]/(x*Log[x]),x]
```

output
```
Log[Log[x]]^2/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3039, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(\log(x))}{x\log(x)}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \frac{\log(\log(x))}{\log(x)}d\log(x)$$

$$\Big\downarrow \text{2738}$$

$$\frac{1}{2}\log^2(\log(x))$$

input
```
Int[Log[Log[x]]/(x*Log[x]),x]
```

output
```
Log[Log[x]]^2/2
```

Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\ln(x))^2}{2}$	8
default	$\frac{\ln(\ln(x))^2}{2}$	8
norman	$\frac{\ln(\ln(x))^2}{2}$	8
risch	$\frac{\ln(\ln(x))^2}{2}$	8

input
```
int(ln(ln(x))/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(ln(x))^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{1}{2}\,\log\left(\log\left(x\right)\right)^2$$

input
```
integrate(log(log(x))/x/log(x),x, algorithm="fricas")
```

output
```
1/2*log(log(x))^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{\log\left(\log\left(x\right)\right)^2}{2}$$

input
```
integrate(ln(ln(x))/x/ln(x),x)
```

output
```
log(log(x))**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{1}{2}\log\left(\log\left(x\right)\right)^2$$

input
```
integrate(log(log(x))/x/log(x),x, algorithm="maxima")
```

output
```
1/2*log(log(x))^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{1}{2}\log\left(\log\left(x\right)\right)^2$$

input
```
integrate(log(log(x))/x/log(x),x, algorithm="giac")
```

output
```
1/2*log(log(x))^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{\ln\left(\ln\left(x\right)\right)^2}{2}$$

input
```
int(log(log(x))/(x*log(x)),x)
```

output
```
log(log(x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\log(\log(x))}{x\log(x)}\,dx = \frac{\log(\log(x))^2}{2}$$

input
```
int(log(log(x))/x/log(x),x)
```

output
```
log(log(x))**2/2
```

3.142 $\int \frac{1}{1+\tan^2(x)} dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{1}{1+\tan^2(x)}\,dx=\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^2(x)}\,dx=\frac{x}{2}+\frac{1}{4}\sin(2x)$$

input
```
Integrate[(1 + Tan[x]^2)^(-1),x]
```

output
```
x/2 + Sin[2*x]/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4140, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan^2(x)+1} dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)^2+1} dx$$
$$\Big\downarrow \text{4140}$$
$$\int \cos^2(x) dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$
$$\Big\downarrow \text{3115}$$
$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[(1 + Tan[x]^2)^(-1),x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
derivativedivides	$\frac{\tan(x)}{2+2\tan(x)^{2}}+\frac{\arctan(\tan(x))}{2}$	19
default	$\frac{\tan(x)}{2+2\tan(x)^{2}}+\frac{\arctan(\tan(x))}{2}$	19
parallelrisch	$\frac{x\tan(x)^{2}+x+\tan(x)}{2+2\tan(x)^{2}}$	21
norman	$\frac{\frac{x}{2}+\frac{x\tan(x)^{2}}{2}+\frac{\tan(x)}{2}}{1+\tan(x)^{2}}$	25

input
```
int(1/(1+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="fricas")
```

output
```
1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan^2(x)}{2\tan^2(x)+2} + \frac{x}{2\tan^2(x)+2} + \frac{\tan(x)}{2\tan^2(x)+2}$$

input
```
integrate(1/(1+tan(x)**2),x)
```

output
```
x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2
+ 2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}\,x + \frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^2+1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="giac")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

input
```
int(1/(tan(x)^2 + 1),x)
```

output
```
x/2 + sin(2*x)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{\tan\left(x\right)^2 x+\tan\left(x\right)+x}{2\tan\left(x\right)^2+2}$$

input
```
int(1/(1+tan(x)^2),x)
```

output
```
(tan(x)**2*x + tan(x) + x)/(2*(tan(x)**2 + 1))
```

3.143 $\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right) dx = 9\sqrt{9-x^{2/3}} - \frac{1}{3}\left(9-x^{2/3}\right)^{3/2} + x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)$$

output
```
9*(9-x^(2/3))^(1/2)-1/3*(9-x^(2/3))^(3/2)+x*arcsin(1/3*x^(1/3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right) dx = -\frac{1}{3}\left(-18-x^{2/3}\right)\sqrt{9-x^{2/3}} + x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)$$

input
```
Integrate[ArcSin[x^(1/3)/3],x]
```

output
```
-1/3*((-18 - x^(2/3))*Sqrt[9 - x^(2/3)]) + x*ArcSin[x^(1/3)/3]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5339, 27, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)dx$$

$$\Big\downarrow \; 5339$$

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\int \frac{\sqrt[3]{x}}{3\sqrt{9-x^{2/3}}}dx$$

$$\Big\downarrow \; 27$$

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{3}\int \frac{\sqrt[3]{x}}{\sqrt{9-x^{2/3}}}dx$$

$$\Big\downarrow \; 798$$

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{2}\int \frac{x^{2/3}}{\sqrt{9-x^{2/3}}}dx^{2/3}$$

$$\Big\downarrow \; 53$$

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{2}\int \left(\frac{9}{\sqrt{9-x^{2/3}}}-\sqrt{9-x^{2/3}}\right)dx^{2/3}$$

$$\Big\downarrow \; 2009$$

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)+\frac{1}{2}\left(18\sqrt{9-x^{2/3}}-\frac{2}{3}\left(9-x^{2/3}\right)^{3/2}\right)$$

input
```
Int[ArcSin[x^(1/3)/3],x]
```

output
```
(18*Sqrt[9 - x^(2/3)] - (2*(9 - x^(2/3))^(3/2))/3)/2 + x*ArcSin[x^(1/3)/3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5339
```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$x \arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+x^{\frac{2}{3}}\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}+18\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}$	34
default	$x \arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+x^{\frac{2}{3}}\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}+18\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}$	34
parts	$x \arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+\frac{x^{\frac{2}{3}}\sqrt{9-x^{\frac{2}{3}}}}{3}+6\sqrt{9-x^{\frac{2}{3}}}$	35

input
```
int(arcsin(1/3*x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
x*arcsin(1/3*x^(1/3))+x^(2/3)*(-1/9*x^(2/3)+1)^(1/2)+18*(-1/9*x^(2/3)+1)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = x\arcsin\left(\frac{1}{3}\,x^{\frac{1}{3}}\right) + \frac{1}{3}\left(x^{\frac{2}{3}} + 18\right)\sqrt{-x^{\frac{2}{3}} + 9}$$

input
```
integrate(arcsin(1/3*x^(1/3)),x, algorithm="fricas")
```

output
```
x*arcsin(1/3*x^(1/3)) + 1/3*(x^(2/3) + 18)*sqrt(-x^(2/3) + 9)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = \frac{x^{\frac{2}{3}}\sqrt{9 - x^{\frac{2}{3}}}}{3} + x\,\mathrm{asin}\left(\frac{\sqrt[3]{x}}{3}\right) + 6\sqrt{9 - x^{\frac{2}{3}}}$$

input
```
integrate(asin(1/3*x**(1/3)),x)
```

output
```
x**(2/3)*sqrt(9 - x**(2/3))/3 + x*asin(x**(1/3)/3) + 6*sqrt(9 - x**(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = x\arcsin\left(\frac{1}{3}\,x^{\frac{1}{3}}\right) + x^{\frac{2}{3}}\sqrt{-\frac{1}{9}\,x^{\frac{2}{3}} + 1} + 18\,\sqrt{-\frac{1}{9}\,x^{\frac{2}{3}} + 1}$$

input
```
integrate(arcsin(1/3*x^(1/3)),x, algorithm="maxima")
```

output
```
x*arcsin(1/3*x^(1/3)) + x^(2/3)*sqrt(-1/9*x^(2/3) + 1) + 18*sqrt(-1/9*x^(2
/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = x^{\frac{1}{3}}\left(x^{\frac{2}{3}}-9\right)\arcsin\left(\frac{1}{3}\,x^{\frac{1}{3}}\right)-9\left(-\frac{1}{9}\,x^{\frac{2}{3}}+1\right)^{\frac{3}{2}} + 9\,x^{\frac{1}{3}}\arcsin\left(\frac{1}{3}\,x^{\frac{1}{3}}\right)+27\sqrt{-\frac{1}{9}\,x^{\frac{2}{3}}+1}$$

input
```
integrate(arcsin(1/3*x^(1/3)),x, algorithm="giac")
```

output
```
x^(1/3)*(x^(2/3) - 9)*arcsin(1/3*x^(1/3)) - 9*(-1/9*x^(2/3) + 1)^(3/2) + 9
*x^(1/3)*arcsin(1/3*x^(1/3)) + 27*sqrt(-1/9*x^(2/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = x\,\mathrm{asin}\left(\frac{x^{1/3}}{3}\right)+\frac{\sqrt{9-x^{2/3}}\,\left(x^{2/3}+18\right)}{3}$$

input
```
int(asin(x^(1/3)/3),x)
```

output
```
x*asin(x^(1/3)/3) + ((9 - x^(2/3))^(1/2)*(x^(2/3) + 18))/3
```

Reduce [F]

$$\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx = \int asin\left(\frac{x^{\frac{1}{3}}}{3}\right)dx$$

input
```
int(asin(1/3*x^(1/3)),x)
```

output
```
int(asin(x**(1/3)/3),x)
```

3.144 $\int \log(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 8

$$\int \log(x)\,dx = -x + x\log(x)$$

output
```
-x+x*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = -x + x\log(x)$$

input
```
Integrate[Log[x],x]
```

output
```
-x + x*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x)\,dx$$

$$\Big\downarrow \text{2732}$$

$$x\log(x) - x$$

input
```
Int[Log[x],x]
```

output
```
-x + x*Log[x]
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
lookup	$\ln(x)x - x$	9
default	$\ln(x)x - x$	9
norman	$\ln(x)x - x$	9
risch	$\ln(x)x - x$	9
parallelrisch	$\ln(x)x - x$	9
parts	$\ln(x)x - x$	9
orering	$\ln(x)x - x$	9

input
```
int(ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)*x-x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(log(x),x, algorithm="fricas")
```

output
```
x*log(x) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(ln(x),x)
```

output
```
x*log(x) - x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(log(x),x, algorithm="maxima")
```

output
```
x*log(x) - x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x)\,dx = x\log(x) - x$$

input
```
integrate(log(x),x, algorithm="giac")
```

output
```
x*log(x) - x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x) \, dx = x \left(\ln\left(x\right) - 1\right)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x) \, dx = x(\log(x) - 1)$$

input
```
int(log(x),x)
```

output
```
x*(log(x) - 1)
```

3.145 $\int e^x(\cos(x) - \sin(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int e^x(\cos(x) - \sin(x))\,dx = e^x\cos(x)$$

output
```
exp(x)*cos(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x(\cos(x) - \sin(x))\,dx = e^x\cos(x)$$

input
```
Integrate[E^x*(Cos[x] - Sin[x]),x]
```

output
```
E^x*Cos[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x(\cos(x)-\sin(x))\,dx$$

$$\Big\downarrow \; 2726$$

$$e^x\cos(x)$$

input
```
Int[E^x*(Cos[x] - Sin[x]),x]
```

output
```
E^x*Cos[x]
```

Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$e^{x} \cos \left(x \right)$	6
parallelrisch	$e^{x} \cos \left(x \right)$	6
parts	$e^{x} \cos \left(x \right)$	6
risch	$\frac{e^{(1+i) x}}{2}+\frac{e^{(1-i) x}}{2}$	18
norman	$\frac{-e^{x} \tan \left(\frac{x}{2}\right)^{2}+e^{x}}{1+\tan \left(\frac{x}{2}\right)^{2}}$	25
orering	$\frac{e^{x} \left(\cos \left(x \right)-\sin \left(x \right)\right)}{2}-\frac{e^{x} \left(-\cos \left(x \right)-\sin \left(x \right)\right)}{2}$	26

input
```
int(exp(x)*(cos(x)-sin(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x(\cos(x) - \sin(x))\,dx = \cos\left(x\right)e^{x}$$

input
```
integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="fricas")
```

output
```
cos(x)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x(\cos(x)-\sin(x))\,dx = e^x\cos(x)$$

input
```
integrate(exp(x)*(cos(x)-sin(x)),x)
```

output
```
exp(x)*cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(5) = 10$.

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.50

$$\int e^x(\cos(x)-\sin(x))\,dx = \frac{1}{2}\left(\cos(x)+\sin(x)\right)e^x + \frac{1}{2}\left(\cos(x)-\sin(x)\right)e^x$$

input
```
integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="maxima")
```

output
```
1/2*(cos(x) + sin(x))*e^x + 1/2*(cos(x) - sin(x))*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(5) = 10$.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.50

$$\int e^x(\cos(x)-\sin(x))\,dx = \frac{1}{2}\left(\cos(x)+\sin(x)\right)e^x + \frac{1}{2}\left(\cos(x)-\sin(x)\right)e^x$$

input
```
integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="giac")
```

output
```
1/2*(cos(x) + sin(x))*e^x + 1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x(\cos(x) - \sin(x))\,dx = \mathrm{e}^x\,\cos(x)$$

input
```
int(exp(x)*(cos(x) - sin(x)),x)
```

output
```
exp(x)*cos(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x(\cos(x) - \sin(x))\,dx = e^x\cos(x)$$

input
```
int(exp(x)*(cos(x)-sin(x)),x)
```

output
```
e**x*cos(x)
```

3.146 $\int e^{-x^2} x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int e^{-x^2} x^3 \, dx = -\frac{e^{-x^2}}{2} - \frac{1}{2} e^{-x^2} x^2$$

output
```
-1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x^2} x^3 \, dx = -\frac{1}{2} e^{-x^2} \left(1 + x^2\right)$$

input
```
Integrate[x^3/E^x^2,x]
```

output
```
-1/2*(1 + x^2)/E^x^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^2} x^3 \, dx$$

$$\Big\downarrow \text{2641}$$

$$\int e^{-x^2} x dx - \frac{1}{2} e^{-x^2} x^2$$

$$\Big\downarrow \text{2638}$$

$$-\frac{1}{2} e^{-x^2} x^2 - \frac{e^{-x^2}}{2}$$

input
```
Int[x^3/E^x^2,x]
```

output
```
-1/2*1/E^x^2 - x^2/(2*E^x^2)
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{(x^2+1)\mathrm{e}^{-x^2}}{2}$	14
orering	$-\frac{(x^2+1)\mathrm{e}^{-x^2}}{2}$	14
norman	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^2}$	15
risch	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^2}$	15
parallelrisch	$\frac{(-x^2-1)\mathrm{e}^{-x^2}}{2}$	16
meijerg	$\frac{1}{2}-\frac{(2x^2+2)\mathrm{e}^{-x^2}}{4}$	18
derivativedivides	$-\frac{\mathrm{e}^{-x^2}}{2}-\frac{x^2\mathrm{e}^{-x^2}}{2}$	21
default	$-\frac{\mathrm{e}^{-x^2}}{2}-\frac{x^2\mathrm{e}^{-x^2}}{2}$	21

input
```
int(x^3/exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2+1)/exp(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int e^{-x^2}x^3\,dx = \frac{(-x^2-1)\,e^{-x^2}}{2}$$

input
```
integrate(x**3/exp(x**2),x)
```

output
```
(-x**2 - 1)*exp(-x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\,(x^2+1)e^{(-x^2)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="maxima")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{1}{2}\,(x^2+1)e^{(-x^2)}$$

input
```
integrate(x^3/exp(x^2),x, algorithm="giac")
```

output
```
-1/2*(x^2 + 1)*e^(-x^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int e^{-x^2}x^3\,dx = -\frac{\mathrm{e}^{-x^2}\,(x^2+1)}{2}$$

input
```
int(x^3*exp(-x^2),x)
```

output
```
-(exp(-x^2)*(x^2 + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int e^{-x^2}x^3\,dx = \frac{-x^2-1}{2e^{x^2}}$$

input
```
int(x^3/exp(x^2),x)
```

output
```
( - (x**2 + 1))/(2*e**(x**2))
```

3.147 $\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 10

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx=\left(e^{x^2}+x\right)\cos(x)$$

output
```
(exp(x^2)+x)*cos(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx=\frac{1}{2}e^{-ix}\left(1+e^{2ix}\right)\left(e^{x^2}+x\right)$$

input
```
Integrate[(1 + 2*E^x^2*x)*Cos[x] - (E^x^2 + x)*Sin[x],x]
```

output
```
((1 + E^((2*I)*x))*(E^x^2 + x))/(2*E^(I*x))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\left(2e^{x^2}x+1\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\int e^{x^2}x\cos(x)dx-\frac{1}{4}i\sqrt[4]{e}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(2x-i)\right)+\frac{1}{4}i\sqrt[4]{e}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(2x+i)\right)+x\cos(x)$$

input
```
Int[(1 + 2*E^x^2*x)*Cos[x] - (E^x^2 + x)*Sin[x],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 116.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\left(e^{x^2}+x\right)\cos\left(x\right)$
risch	$\frac{e^{x(x-i)}}{2}+\frac{e^{x(i+x)}}{2}+x\cos\left(x\right)$
norman	$\frac{x-x\tan\left(\frac{x}{2}\right)^2-e^{x^2}\tan\left(\frac{x}{2}\right)^2+e^{x^2}}{1+\tan\left(\frac{x}{2}\right)^2}$
orering	$\frac{2x\left(24x^6+44x^4+54x^2+31\right)\left(\left(1+2\,e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)}{16x^8+44x^6+112x^4+119x^2-12}-\frac{4\left(4x^8+12x^6+11x^4+7x^2+2\right)\left(\left(4\,e^{x^2}x^2+2\,e^{x^2}\right)\cos(\right.\right.}{16x^8+44x^6+112x^4+}$

input
```
int((1+2*exp(x^2)*x)*cos(x)-(exp(x^2)+x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(exp(x^2)+x)*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx = x\cos\left(x\right)+\cos\left(x\right)e^{\left(x^2\right)}$$

input
```
integrate((1+2*exp(x^2)*x)*cos(x)-(exp(x^2)+x)*sin(x),x, algorithm="fricas
")
```

output
```
x*cos(x) + cos(x)*e^(x^2)
```

Sympy [F]

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx = \int \left(-\left(x+e^{x^2}\right)\sin\left(x\right)+\left(2xe^{x^2}+1\right)\cos\left(x\right)\right)\,dx$$

input
```
integrate((1+2*exp(x**2)*x)*cos(x)-(exp(x**2)+x)*sin(x),x)
```

output
```
Integral(-(x + exp(x**2))*sin(x) + (2*x*exp(x**2) + 1)*cos(x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 473, normalized size of antiderivative = 47.30

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx = \text{Too large to display}$$

input
```
integrate((1+2*exp(x^2)*x)*cos(x)-(exp(x^2)+x)*sin(x),x, algorithm="maxima
")
```

output
```
-1/4*sqrt(pi)*(erf(I*x + 1/2) - erf(I*x - 1/2))*e^(1/4) + x*cos(x) + 1/8*(
2*(16*x^4 + 8*x^2 + 1)^(1/4)*(e^(x^2 + I*x - 1/4) + e^(x^2 - I*x - 1/4) +
e^(conjugate(x)^2 + I*conjugate(x) - 1/4) + e^(conjugate(x)^2 - I*conjugat
e(x) - 1/4))*e^(1/4) - ((2*(I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/
4))) - 1) - I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - I*sq
rt(pi)*(erf(sqrt(-x^2 + I*x + 1/4)) - 1) + I*sqrt(pi)*(erf(sqrt(-x^2 - I*x
 + 1/4)) - 1))*x - sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4))) - 1) -
 sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - sqrt(pi)*(erf(sqr
t(-x^2 + I*x + 1/4)) - 1) - sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4)) - 1))*co
s(1/2*arctan2(4*x, -4*x^2 + 1)) - (2*(sqrt(pi)*(conjugate(erf(sqrt(-x^2 +
I*x + 1/4))) - 1) + sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1)
+ sqrt(pi)*(erf(sqrt(-x^2 + I*x + 1/4)) - 1) + sqrt(pi)*(erf(sqrt(-x^2 - I
*x + 1/4)) - 1))*x + I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4))) -
1) - I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - I*sqrt(pi)*
(erf(sqrt(-x^2 + I*x + 1/4)) - 1) + I*sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4)
) - 1))*sin(1/2*arctan2(4*x, -4*x^2 + 1)))*e^(1/4))/(16*x^4 + 8*x^2 + 1)^(
1/4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx=x\cos\left(x\right)+\cos\left(x\right)e^{\left(x^2\right)}$$

input
```
integrate((1+2*exp(x^2)*x)*cos(x)-(exp(x^2)+x)*sin(x),x, algorithm="giac")
```

output
```
x*cos(x) + cos(x)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx=\cos\left(x\right)\,\left(x+\mathrm{e}^{x^2}\right)$$

input
```
int(cos(x)*(2*x*exp(x^2) + 1) - sin(x)*(x + exp(x^2)),x)
```

output
```
cos(x)*(x + exp(x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx=\cos\left(x\right)\left(e^{x^2}+x\right)$$

input
```
int((1+2*exp(x^2)*x)*cos(x)-(exp(x^2)+x)*sin(x),x)
```

output
```
cos(x)*(e**(x**2) + x)
```

3.148 $\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right) \left(1+\sqrt[3]{x}+\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right) \left(1+\sqrt[3]{x}+\sqrt{x}\right) \, dx=2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+3x+\frac{6x^{7/6}}{7}+\frac{3x^{4/3}}{4}+\frac{2x^{3/2}}{3}$$

output
```
2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+3*x+6/7*x^(7/6)+3/4*x^(4/3)+2/3*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right) \left(1+\sqrt[3]{x}+\sqrt{x}\right) \, dx = \frac{1}{420}\left(840\sqrt{x}+630x^{2/3}+504x^{5/6}+1260x + 360x^{7/6}+315x^{4/3}+280x^{3/2}\right)$$

input
```
Integrate[(1 + 1/Sqrt[x] + x^(-1/3))*(1 + x^(1/3) + Sqrt[x]),x]
```

output
```
(840*Sqrt[x] + 630*x^(2/3) + 504*x^(5/6) + 1260*x + 360*x^(7/6) + 315*x^(4
/3) + 280*x^(3/2))/420
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {7267, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt{x}}+1\right)\left(\sqrt{x}+\sqrt[3]{x}+1\right)dx$$

$$\Big\downarrow \text{7267}$$

$$6\int \left(\sqrt{x}+\sqrt[6]{x}+1\right)\left(\sqrt{x}+\sqrt[3]{x}+1\right)\sqrt[3]{x}d\sqrt[6]{x}$$

$$\Big\downarrow \text{7293}$$

$$6\int \left(x^{4/3}+x^{7/6}+x+3x^{5/6}+x^{2/3}+\sqrt{x}+\sqrt[3]{x}\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{2009}$$

$$6\left(\frac{x^{3/2}}{9}+\frac{x^{4/3}}{8}+\frac{x^{7/6}}{7}+\frac{x^{5/6}}{5}+\frac{x^{2/3}}{4}+\frac{x}{2}+\frac{\sqrt{x}}{3}\right)$$

input
```
Int[(1 + 1/Sqrt[x] + x^(-1/3))*(1 + x^(1/3) + Sqrt[x]),x]
```

output
```
6*(Sqrt[x]/3 + x^(2/3)/4 + x^(5/6)/5 + x/2 + x^(7/6)/7 + x^(4/3)/8 + x^(3/
2)/9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$2\sqrt{x}+\frac{3x^{\frac{2}{3}}}{2}+\frac{6x^{\frac{5}{6}}}{5}+3x+\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{4}{3}}}{4}+\frac{2x^{\frac{3}{2}}}{3}$$

input
```
int((1+1/x^(1/2)+1/x^(1/3))*(1+x^(1/3)+x^(1/2)),x)
```

output
```
2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+3*x+6/7*x^(7/6)+3/4*x^(4/3)+2/3*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

$$\int\left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx=\frac{2}{3}\,(x+3)\sqrt{x}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((1+1/x^(1/2)+1/x^(1/3))*(1+x^(1/3)+x^(1/2)),x, algorithm="fricas
")
```

output
```
2/3*(x + 3)*sqrt(x) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*
x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx=\frac{6x^{\frac{7}{6}}}{7}+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{4}{3}}}{4}+\frac{3x^{\frac{2}{3}}}{2}+\frac{2x^{\frac{3}{2}}}{3}+2\sqrt{x}+3x$$

input
```
integrate((1+1/x**(1/2)+1/x**(1/3))*(1+x**(1/3)+x**(1/2)),x)
```

output
```
6*x**(7/6)/7 + 6*x**(5/6)/5 + 3*x**(4/3)/4 + 3*x**(2/3)/2 + 2*x**(3/2)/3 +
 2*sqrt(x) + 3*x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}+2\,\sqrt{x}$$

input
```
integrate((1+1/x^(1/2)+1/x^(1/3))*(1+x^(1/3)+x^(1/2)),x, algorithm="maxima
")
```

output
```
2/3*x^(3/2) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*x^(2/3)
+ 2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}+2\,\sqrt{x}$$

input
```
integrate((1+1/x^(1/2)+1/x^(1/3))*(1+x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*x^(2/3)
+ 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx = 3\,x+2\,\sqrt{x}+\frac{3\,x^{2/3}}{2}+\frac{2\,x^{3/2}}{3}+\frac{3\,x^{4/3}}{4}+\frac{6\,x^{5/6}}{5}+\frac{6\,x^{7/6}}{7}$$

input
```
int((1/x^(1/2) + 1/x^(1/3) + 1)*(x^(1/2) + x^(1/3) + 1),x)
```

output
```
3*x + 2*x^(1/2) + (3*x^(2/3))/2 + (2*x^(3/2))/3 + (3*x^(4/3))/4 + (6*x^(5/
6))/5 + (6*x^(7/6))/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)\left(1+\sqrt[3]{x}+\sqrt{x}\right)\,dx = \frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}+\frac{3x^{\frac{4}{3}}}{4}+\frac{2\sqrt{x}\,x}{3}+2\sqrt{x}+3x$$

input
```
int((1+1/x^(1/2)+1/x^(1/3))*(1+x^(1/3)+x^(1/2)),x)
```

output
```
(504*x**(5/6) + 360*x**(1/6)*x + 630*x**(2/3) + 315*x**(1/3)*x + 280*sqrt(
x)*x + 840*sqrt(x) + 1260*x)/420
```

3.149 $\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = \frac{1}{2}\sin^2(\sin(x))$$

output
```
1/2*sin(sin(x))^2
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{1}{2}\cos^2(\sin(x))$$

input
```
Integrate[Cos[x]*Cos[Sin[x]]*Sin[Sin[x]],x]
```

output
```
-1/2*Cos[Sin[x]]^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4834, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\sin(x))\cos(x)\cos(\sin(x))\,dx$$

$$\Big\downarrow \text{4834}$$

$$\int \sin(\sin(x))\cos(\sin(x))d\sin(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(\sin(x))\cos(\sin(x))d\sin(x)$$

$$\Big\downarrow \text{3044}$$

$$\int \sin(\sin(x))d\sin(\sin(x))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\sin^2(\sin(x))$$

input
```
Int[Cos[x]*Cos[Sin[x]]*Sin[Sin[x]],x]
```

output
```
Sin[Sin[x]]^2/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\sin(\sin(x))^2}{2}$	8
default	$\frac{\sin(\sin(x))^2}{2}$	8
risch	$-\frac{\cos(2\sin(x))}{4}$	8
parallelrisch	$-\frac{3}{4}-\frac{\cos(2\sin(x))}{4}$	10
norman	$\frac{2\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan(\frac{x}{2})^2}\right)^2+2\tan(\frac{x}{2})^2\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan(\frac{x}{2})^2}\right)^2}{\left(1+\tan(\frac{x}{2})^2\right)\left(1+\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan(\frac{x}{2})^2}\right)^2\right)^2}$	81

input
```
int(cos(x)*cos(sin(x))*sin(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(sin(x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{1}{2}\cos\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2$$

input
```
integrate(cos(x)*cos(sin(x))*sin(sin(x)),x, algorithm="fricas")
```

output
```
-1/2*cos(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{\cos^2(\sin(x))}{2}$$

input
```
integrate(cos(x)*cos(sin(x))*sin(sin(x)),x)
```

output
```
-cos(sin(x))**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{1}{2}\cos\left(\sin\left(x\right)\right)^2$$

input
```
integrate(cos(x)*cos(sin(x))*sin(sin(x)),x, algorithm="maxima")
```

output
```
-1/2*cos(sin(x))^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{1}{2}\cos\left(\sin\left(x\right)\right)^2$$

input
```
integrate(cos(x)*cos(sin(x))*sin(sin(x)),x, algorithm="giac")
```

output
```
-1/2*cos(sin(x))^2
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = \frac{\sin\left(\sin\left(x\right)\right)^2}{2}$$

input
```
int(cos(sin(x))*sin(sin(x))*cos(x),x)
```

output
```
sin(sin(x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx = -\frac{\cos(\sin(x))^2}{2}$$

input
```
int(cos(x)*cos(sin(x))*sin(sin(x)),x)
```

output
```
( - cos(sin(x))**2)/2
```

3.150 $\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right) \, dx=-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$$

output
```
-cos(x)/x-sin(x)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right) \, dx=-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$$

input
```
Integrate[(-Cos[x] + Sin[x])/x + (Cos[x] + Sin[x])/x^2,x]
```

output
```
-(Cos[x]/x) - Sin[x]/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{\sin(x)+\cos(x)}{x^2}+\frac{\sin(x)-\cos(x)}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sin(x)}{x}-\frac{\cos(x)}{x}$$

input
```
Int[(-Cos[x] + Sin[x])/x + (Cos[x] + Sin[x])/x^2,x]
```

output
```
-(Cos[x]/x) - Sin[x]/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 2.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
parallelrisch	$\frac{-\cos(x)-\sin(x)}{x}$
default	$-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$
parts	$-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$
risch	$\frac{i(2i\cos(x)+2i\sin(x))}{2x}$
norman	$\frac{-1+\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)x}$
orering	$-\frac{2\left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^{2}}\right)}{x}-\frac{\cos(x)+\sin(x)}{x}+\frac{-\cos(x)+\sin(x)}{x^{2}}-\frac{\cos(x)-\sin(x)}{x^{2}}+\frac{2\cos(x)+2\sin(x)}{x^{3}}$
meijerg	$-\frac{\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{Ci}(x)}{\sqrt{\pi}}\right)}{2}+\mathrm{Si}\left(x\right)+\frac{\sqrt{\pi}\left(-\frac{4\cos(x)}{x\sqrt{\pi}}-\frac{4\,\mathrm{Si}(x)}{\sqrt{\pi}}\right)}{4}+\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\right.}{}$

input
```
int((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-cos(x)-sin(x))/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^{2}}\right)\,dx=-\frac{\cos\left(x\right)+\sin\left(x\right)}{x}$$

input
```
integrate((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x^2,x, algorithm="fricas")
```

output
```
-(cos(x) + sin(x))/x
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right)\,dx = -\log{(x)} + \frac{\log{(x^2)}}{2} - \frac{\sin{(x)}}{x} - \frac{\cos{(x)}}{x}$$

input
```
integrate((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x**2,x)
```

output
```
-log(x) + log(x**2)/2 - sin(x)/x - cos(x)/x
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\begin{aligned}&\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right)\,dx\\ &= -\left(\frac{1}{2}i+\frac{1}{2}\right)\,\mathrm{Ei}(i\,x)+\left(\frac{1}{2}i-\frac{1}{2}\right)\,\mathrm{Ei}(-i\,x)\\ &\quad-\left(\frac{1}{2}i-\frac{1}{2}\right)\,\Gamma(-1,i\,x)+\left(\frac{1}{2}i+\frac{1}{2}\right)\,\Gamma(-1,-i\,x)\end{aligned}$$

input
```
integrate((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x^2,x, algorithm="maxima")
```

output
```
-(1/2*I + 1/2)*Ei(I*x) + (1/2*I - 1/2)*Ei(-I*x) - (1/2*I - 1/2)*gamma(-1,
I*x) + (1/2*I + 1/2)*gamma(-1, -I*x)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right)\,dx$$
$$=\frac{x\,\mathrm{Ci}\,(x)-x\,\mathrm{Si}\,(x)-\cos\,(x)-\sin\,(x)}{x}-\mathrm{Ci}\,(x)+\mathrm{Si}\,(x)$$

input
```
integrate((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x^2,x, algorithm="giac")
```

output
```
(x*cos_integral(x) - x*sin_integral(x) - cos(x) - sin(x))/x - cos_integral
(x) + sin_integral(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right)\,dx=-\frac{\sqrt{2}\,\sin\,\left(x+\frac{\pi}{4}\right)}{x}$$

input
```
int((cos(x) + sin(x))/x^2 - (cos(x) - sin(x))/x,x)
```

output
```
-(2^(1/2)*sin(x + pi/4))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right)\,dx=\frac{-\cos\,(x)-\sin\,(x)}{x}$$

input
```
int((-cos(x)+sin(x))/x+(cos(x)+sin(x))/x^2,x)
```

output
```
( - (cos(x) + sin(x)))/x
```

3.151 $\int x^3\sqrt{1+x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^3\sqrt{1+x^2}\,dx=-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}$$

output
```
-1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx=\frac{1}{15}\left(1+x^2\right)^{3/2}\left(-2+3x^2\right)$$

input
```
Integrate[x^3*Sqrt[1 + x^2],x]
```

output
```
((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2 \sqrt{x^2+1}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\left(x^2+1\right)^{3/2}-\sqrt{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(x^2+1\right)^{5/2}-\frac{2}{3}\left(x^2+1\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[1 + x^2],x]
```

output
```
((-2*(1 + x^2)^(3/2))/3 + (2*(1 + x^2)^(5/2))/5)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
pseudoelliptic	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
orering	$\frac{(x^2+1)^{\frac{3}{2}}(3x^2-2)}{15}$	17
risch	$\frac{(3x^4+x^2-2)\sqrt{x^2+1}}{15}$	20
trager	$\left(\frac{1}{5}x^4+\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{x^2+1}$	21
default	$\frac{x^2(x^2+1)^{\frac{3}{2}}}{5}-\frac{2(x^2+1)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(x^2+1)^{\frac{3}{2}}(-3x^2+2)}{15}}{4\sqrt{\pi}}$	31

input
```
int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{15}\left(3\,x^4+x^2-2\right)\sqrt{x^2+1}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^3\sqrt{1+x^2}\,dx = \frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

input
```
integrate(x**3*(x**2+1)**(1/2),x)
```

output
```
x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{1+x^2}\,dx = \frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{1+x^2}\,dx = \sqrt{x^2+1}\left(\frac{x^4}{5}+\frac{x^2}{15}-\frac{2}{15}\right)$$

input
```
int(x^3*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int x^3\sqrt{1+x^2}\,dx = \frac{\sqrt{x^2+1}\,(3x^4+x^2-2)}{15}$$

input
```
int(x^3*(x^2+1)^(1/2),x)
```

output
```
(sqrt(x**2 + 1)*(3*x**4 + x**2 - 2))/15
```

3.152 $\int \frac{x}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{x}{1+x^2+x^4} \, dx = \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+x^4} \, dx = \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x/(1 + x^2 + x^4),x]
```

output
```
ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4-3}d\left(2x^2+1\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 + x^2 + x^4),x]
```

output
```
ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19

input
```
int(x/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**4+x**2+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+x^4} \, dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(x/(x^2 + x^4 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x}{1+x^2+x^4} \, dx = \frac{\sqrt{3}\left(atan\left(\frac{2x-1}{\sqrt{3}}\right)-atan\left(\frac{2x+1}{\sqrt{3}}\right)\right)}{3}$$

input
```
int(x/(x^4+x^2+1),x)
```

output
```
(sqrt(3)*(atan((2*x - 1)/sqrt(3)) - atan((2*x + 1)/sqrt(3))))/3
```

3.153 $\int e^{e^{2016x}+6048x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int e^{e^{2016x}+6048x}\,dx=\frac{e^{e^{2016x}}}{1008}-\frac{e^{e^{2016x}+2016x}}{1008}+\frac{e^{e^{2016x}+4032x}}{2016}$$

output
```
1/1008*exp(exp(1)^(2016*x))-1/1008*exp(exp(1)^(2016*x)+2016*x)+1/2016*exp(
exp(1)^(2016*x)+4032*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int e^{e^{2016x}+6048x}\,dx=\frac{e^{e^{2016x}}\left(2-2e^{2016x}+e^{4032x}\right)}{2016}$$

input
```
Integrate[E^(E^(2016*x) + 6048*x),x]
```

output
```
(E^E^(2016*x)*(2 - 2*E^(2016*x) + E^(4032*x)))/2016
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2720, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{6048x+e^{2016x}}\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int e^{4032x+e^{2016x}}de^{2016x}}{2016}$$

$$\Big\downarrow \ 2607$$

$$\frac{e^{4032x+e^{2016x}}-2\int e^{2016x+e^{2016x}}de^{2016x}}{2016}$$

$$\Big\downarrow \ 2607$$

$$\frac{e^{4032x+e^{2016x}}-2\left(e^{2016x+e^{2016x}}-\int e^{e^{2016x}}de^{2016x}\right)}{2016}$$

$$\Big\downarrow \ 2624$$

$$\frac{e^{4032x+e^{2016x}}-2\left(e^{2016x+e^{2016x}}-e^{e^{2016x}}\right)}{2016}$$

input
```
Int[E^(E^(2016*x) + 6048*x),x]
```

output
```
(E^(E^(2016*x) + 4032*x) - 2*(-E^E^(2016*x) + E^(E^(2016*x) + 2016*x)))/20
16
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{\left({\mathrm e}^{4032 x}-2 \,{\mathrm e}^{2016 x}+2\right) {\mathrm e}^{{\mathrm e}^{2016 x}}}{2016}$	20

input
```
int(exp(exp(1)^(2016*x)+6048*x),x,method=_RETURNVERBOSE)
```

output
```
1/2016*(exp(4032*x)-2*exp(2016*x)+2)*exp(exp(2016*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.45

$$\int e^{e^{2016x}+6048x}\,dx = \frac{1}{2016}\left(e^{(4032\,x)} - 2\,e^{(2016\,x)} + 2\right)e^{\left(e^{(2016\,x)}\right)}$$

input
```
integrate(exp(exp(1)^(2016*x)+6048*x),x, algorithm="fricas")
```

output
```
1/2016*(e^(4032*x) - 2*e^(2016*x) + 2)*e^(e^(2016*x))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int e^{e^{2016x}+6048x}\,dx = \frac{e^{4032x}e^{e^{2016x}}}{2016} - \frac{e^{2016x}e^{e^{2016x}}}{1008} + \frac{e^{e^{2016x}}}{1008}$$

input
```
integrate(exp(exp(1)**(2016*x)+6048*x),x)
```

output
```
exp(4032*x)*exp(exp(2016*x))/2016 - exp(2016*x)*exp(exp(2016*x))/1008 + ex
p(exp(2016*x))/1008
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.45

$$\int e^{e^{2016x}+6048x}\,dx = \frac{1}{2016}\left(e^{(4032\,x)} - 2\,e^{(2016\,x)} + 2\right)e^{\left(e^{(2016\,x)}\right)}$$

input
```
integrate(exp(exp(1)^(2016*x)+6048*x),x, algorithm="maxima")
```

output
```
1/2016*(e^(4032*x) - 2*e^(2016*x) + 2)*e^(e^(2016*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int e^{e^{2016x}+6048x}\,dx$$
$$=\frac{1}{2016}\left(e^{\left(10080\,x+e^{(2016\,x)}\right)}-2\,e^{\left(8064\,x+e^{(2016\,x)}\right)}+2\,e^{\left(6048\,x+e^{(2016\,x)}\right)}\right)e^{(-6048\,x)}$$

input
```
integrate(exp(exp(1)^(2016*x)+6048*x),x, algorithm="giac")
```

output
```
1/2016*(e^(10080*x + e^(2016*x)) - 2*e^(8064*x + e^(2016*x)) + 2*e^(6048*x
 + e^(2016*x)))*e^(-6048*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int e^{e^{2016x}+6048x}\,dx=\frac{\mathrm{e}^{\mathrm{e}^{2016\,x}}}{1008}-\frac{\mathrm{e}^{2016\,x}\,\mathrm{e}^{\mathrm{e}^{2016\,x}}}{1008}+\frac{\mathrm{e}^{4032\,x}\,\mathrm{e}^{\mathrm{e}^{2016\,x}}}{2016}$$

input
```
int(exp(6048*x + exp(2016*x)),x)
```

output
```
exp(exp(2016*x))/1008 - (exp(2016*x)*exp(exp(2016*x)))/1008 + (exp(4032*x)
*exp(exp(2016*x)))/2016
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int e^{e^{2016x}+6048x}\,dx=\frac{e^{e^{2016x}}\left(e^{4032x}-2e^{2016x}+2\right)}{2016}$$

input
```
int(exp(exp(1)^(2016*x)+6048*x),x)
```

output
```
(e**(e**(2016*x))*(e**(4032*x) - 2*e**(2016*x) + 2))/2016
```

3.154 $\int (1-\cot(x))\,dx$

Optimal result

Integrand size = 6, antiderivative size = 7

$$\int (1-\cot(x))\,dx = x-\log(\sin(x))$$

output
```
x-ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1-\cot(x))\,dx = x-\log(\sin(x))$$

input
```
Integrate[1 - Cot[x],x]
```

output
```
x - Log[Sin[x]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\cot(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$x-\log(\sin(x))$$

input
```
Int[1 - Cot[x],x]
```

output
```
x - Log[Sin[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$x-\ln\left(\sin\left(x\right)\right)$	8
parts	$x-\ln\left(\sin\left(x\right)\right)$	8
derivativedivides	$\frac{\ln\left(1+\cot\left(x\right)^{2}\right)}{2}-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	17
norman	$x-\ln\left(\tan\left(x\right)\right)+\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{2}$	17
risch	$x+ix-\ln\left(\mathrm{e}^{2ix}-1\right)$	17
parallelrisch	$-\ln\left(\tan\left(x\right)\right)-\ln\left(\frac{1}{\sqrt{\sec\left(x\right)^{2}}}\right)+x$	17

input
```
int(1-cot(x),x,method=_RETURNVERBOSE)
```

output
```
x-ln(sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.86

$$\int (1-\cot(x))\,dx = x-\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

input
```
integrate(1-cot(x),x, algorithm="fricas")
```

output
```
x - 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int (1-\cot(x))\,dx = x - \log(\sin(x))$$

input
```
integrate(1-cot(x),x)
```

output
```
x - log(sin(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1-\cot(x))\,dx = x - \log(\sin(x))$$

input
```
integrate(1-cot(x),x, algorithm="maxima")
```

output
```
x - log(sin(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int (1-\cot(x))\,dx = x - \log(|\sin(x)|)$$

input
```
integrate(1-cot(x),x, algorithm="giac")
```

output
```
x - log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int (1-\cot(x))\,dx = x\,(1+1\mathrm{i}) - \ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)$$

input
```
int(1 - cot(x),x)
```

output
```
x*(1 + 1i) - log(exp(x*2i) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int (1-\cot(x))\,dx = \log\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \log\left(\tan\left(\frac{x}{2}\right)\right) + x$$

input
```
int(1-cot(x),x)
```

output
```
log(tan(x/2)**2 + 1) - log(tan(x/2)) + x
```

3.155 $\int \frac{1}{1-x+x^2-x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{1-x+x^2-x^3} \, dx = \frac{\arctan(x)}{2} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log\left(1+x^2\right)$$

output
```
1/2*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x+x^2-x^3} \, dx = \frac{\arctan(x)}{2} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[(1 - x + x^2 - x^3)^(-1),x]
```

output
```
ArcTan[x]/2 - Log[1 - x]/2 + Log[1 + x^2]/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x^3+x^2-x+1} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{x+1}{2\left(x^2+1\right)}-\frac{1}{2(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(x)}{2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(1-x)$$

input
```
Int[(1 - x + x^2 - x^3)^(-1),x]
```

output
```
ArcTan[x]/2 - Log[1 - x]/2 + Log[1 + x^2]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}-\frac{\ln(x-1)}{2}$	20
risch	$\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}-\frac{\ln(x-1)}{2}$	20
parallelrisch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(i+x)}{4}+\frac{i\ln(i+x)}{4}$	38

input
```
int(1/(-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2+1)+1/2*arctan(x)-1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{1-x+x^2-x^3}\,dx = \frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x-1)$$

input
```
integrate(1/(-x^3+x^2-x+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{1-x+x^2-x^3}\,dx = -\frac{\log(x-1)}{2}+\frac{\log(x^2+1)}{4}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(-x**3+x**2-x+1),x)
```

output
```
-log(x - 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{1-x+x^2-x^3}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x-1)$$

input
```
integrate(1/(-x^3+x^2-x+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-x+x^2-x^3}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate(1/(-x^3+x^2-x+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{1-x+x^2-x^3}\,dx = -\frac{\ln(x-1)}{2} + \ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(-1/(x - x^2 + x^3 - 1),x)
```

output
```
log(x - 1i)*(1/4 - 1i/4) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{1-x+x^2-x^3}\,dx = \frac{atan(x)}{2} + \frac{\log(x^2+1)}{4} - \frac{\log(x-1)}{2}$$

input
```
int(1/(-x^3+x^2-x+1),x)
```

output
```
(2*atan(x) + log(x**2 + 1) - 2*log(x - 1))/4
```

3.156 $\int \frac{1}{2+\cosh(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 20

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{2\coth^{-1}\left(\sqrt{3}\coth\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

output
```
2/3*arccoth(3^(1/2)*coth(1/2*x))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{2\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(2 + Cosh[x])^(-1),x]
```

output
```
(2*ArcTanh[Tanh[x/2]/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{2+\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{x}{\sqrt{3}}-\frac{2\text{arctanh}\left(\frac{\sinh(x)}{\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

input
```
Int[(2 + Cosh[x])^(-1),x]
```

output
```
x/Sqrt[3] - (2*ArcTanh[Sinh[x]/(2 + Sqrt[3] + Cosh[x])])/Sqrt[3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\sqrt{3}\, \operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{\sqrt{3}\, \ln\left(e^{x}+2-\sqrt{3}\right)}{3}-\frac{\sqrt{3}\, \ln\left(e^{x}+2+\sqrt{3}\right)}{3}$	30

input
```
int(1/(2+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctanh(1/3*tanh(1/2*x)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{1}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)+\sqrt{3}-2}{\cosh\left(x\right)+2}\right)$$

input
```
integrate(1/(2+cosh(x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) + sqrt
(3) - 2)/(cosh(x) + 2))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{2+\cosh(x)}\,dx = -\frac{\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3} + \frac{\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

input
```
integrate(1/(2+cosh(x)),x)
```

output
```
-sqrt(3)*log(tanh(x/2) - sqrt(3))/3 + sqrt(3)*log(tanh(x/2) + sqrt(3))/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{1}{2+\cosh(x)}\,dx = -\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-2}{\sqrt{3}+e^{(-x)}+2}\right)$$

input
```
integrate(1/(2+cosh(x)),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*log(-(sqrt(3) - e^(-x) - 2)/(sqrt(3) + e^(-x) + 2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{x}-2}{\sqrt{3}+e^{x}+2}\right)$$

input
```
integrate(1/(2+cosh(x)),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*log(-(sqrt(3) - e^x - 2)/(sqrt(3) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{\sqrt{3}\left(\ln\left(-2\,\mathrm{e}^x-\frac{\sqrt{3}\,(4\,\mathrm{e}^x+2)}{3}\right)-\ln\left(\frac{\sqrt{3}\,(4\,\mathrm{e}^x+2)}{3}-2\,\mathrm{e}^x\right)\right)}{3}$$

input
```
int(1/(cosh(x) + 2),x)
```

output
```
(3^(1/2)*(log(- 2*exp(x) - (3^(1/2)*(4*exp(x) + 2))/3) - log((3^(1/2)*(4*e
xp(x) + 2))/3 - 2*exp(x))))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{2+\cosh(x)}\,dx = \frac{\sqrt{3}\left(\log\left(e^x-\sqrt{3}+2\right)-\log\left(e^x+\sqrt{3}+2\right)\right)}{3}$$

input
```
int(1/(2+cosh(x)),x)
```

output
```
(sqrt(3)*(log(e**x - sqrt(3) + 2) - log(e**x + sqrt(3) + 2)))/3
```

3.157 $\int \frac{x^2}{\sqrt{2+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2\sqrt{2+x^3}}{3}$$

output
```
2/3*(x^3+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2\sqrt{2+x^3}}{3}$$

input
```
Integrate[x^2/Sqrt[2 + x^3],x]
```

output
```
(2*Sqrt[2 + x^3])/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^3+2}}dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{2\sqrt{x^3+2}}{3}$$

input
```
Int[x^2/Sqrt[2 + x^3],x]
```

output
```
(2*Sqrt[2 + x^3])/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{2\sqrt{x^3+2}}{3}$	10
derivativedivides	$\frac{2\sqrt{x^3+2}}{3}$	10
default	$\frac{2\sqrt{x^3+2}}{3}$	10
trager	$\frac{2\sqrt{x^3+2}}{3}$	10
risch	$\frac{2\sqrt{x^3+2}}{3}$	10
elliptic	$\frac{2\sqrt{x^3+2}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{x^3+2}}{3}$	10
orering	$\frac{2\sqrt{x^3+2}}{3}$	10
meijerg	$\frac{\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^3}{2}}\right)}{3\sqrt{\pi}}$	29

input
```
int(x^2/(x^3+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^3+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2}{3}\sqrt{x^3+2}$$

input
```
integrate(x^2/(x^3+2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(x^3 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2\sqrt{x^3+2}}{3}$$

input
```
integrate(x**2/(x**3+2)**(1/2),x)
```

output
```
2*sqrt(x**3 + 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3+2}$$

input
```
integrate(x^2/(x^3+2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(x^3 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3+2}$$

input
```
integrate(x^2/(x^3+2)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x^3 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2\sqrt{x^3+2}}{3}$$

input
```
int(x^2/(x^3 + 2)^(1/2),x)
```

output
```
(2*(x^3 + 2)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\sqrt{2+x^3}}\,dx = \frac{2\sqrt{x^3+2}}{3}$$

input
```
int(x^2/(x^3+2)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 2))/3
```

3.158 $\int \frac{\log(x)}{x^2} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 13

$$\int \frac{\log(x)}{x^2} \, dx = -\frac{1}{x} - \frac{\log(x)}{x}$$

output
```
-1/x-ln(x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^2} \, dx = -\frac{1}{x} - \frac{\log(x)}{x}$$

input
```
Integrate[Log[x]/x^2,x]
```

output
```
-x^(-1) - Log[x]/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{x^2} \, dx$$

$$\Big\downarrow \text{2741}$$

$$-\frac{1}{x}-\frac{\log(x)}{x}$$

input
```
Int[Log[x]/x^2,x]
```

output
```
-x^(-1) - Log[x]/x
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
norman	$\frac{-1-\ln(x)}{x}$	11
parallelrisch	$\frac{-1-\ln(x)}{x}$	11
default	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14
risch	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14
parts	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14
orering	$-\frac{3\ln(x)}{x}-\left(\frac{1}{x^3}-\frac{2\ln(x)}{x^3}\right)x^2$	25

input
```
int(ln(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-1-ln(x))/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x^2}\,dx = -\frac{\log(x)+1}{x}$$

input
```
integrate(log(x)/x^2,x, algorithm="fricas")
```

output
```
-(log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{\log(x)}{x^2}\,dx = -\frac{\log(x)}{x} - \frac{1}{x}$$

input
```
integrate(ln(x)/x**2,x)
```

output
```
-log(x)/x - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^2}\,dx = -\frac{\log(x)}{x} - \frac{1}{x}$$

input
```
integrate(log(x)/x^2,x, algorithm="maxima")
```

output
```
-log(x)/x - 1/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{x^2}\,dx = -\frac{\log(x)}{x} - \frac{1}{x}$$

input
```
integrate(log(x)/x^2,x, algorithm="giac")
```

output
```
-log(x)/x - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\log(x)}{x^2}\, dx = -\frac{\ln(x) + 1}{x}$$

input
```
int(log(x)/x^2,x)
```

output
```
-(log(x) + 1)/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\log(x)}{x^2}\, dx = \frac{-\log(x) - 1}{x}$$

input
```
int(log(x)/x^2,x)
```

output
```
( - (log(x) + 1))/x
```

3.159 $\int \operatorname{sech}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

output
```
arctan(sinh(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = -\cot^{-1}(\sinh(x))$$

input
```
Integrate[Sech[x],x]
```

output
```
-ArcCot[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2} + ix\right) dx$$

$$\Big\downarrow \text{4257}$$

$$\arctan(\sinh(x))$$

input
```
Int[Sech[x],x]
```

output
```
ArcTan[Sinh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(e^{x}+i)-i\ln(e^{x}-i)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$	23

input
```
int(sech(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \operatorname{sech}(x)\,dx = 2 \arctan(\cosh(x)+\sinh(x))$$

input
```
integrate(sech(x),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \operatorname{sech}(x)\,dx = 2\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x),x)
```

output
```
2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x)\,dx = \arctan\left(\sinh\left(x\right)\right)$$

input
```
integrate(sech(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\,\arctan\left(e^{x}\right)$$

input
```
integrate(sech(x),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \mathrm{sech}(x)\,dx = 2\,\mathrm{atan}(\mathrm{e}^x)$$

input
```
int(1/cosh(x),x)
```

output
```
2*atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

$$\int \mathrm{sech}(x)\,dx = 2atan(e^x)$$

input
```
int(sech(x),x)
```

output
```
2*atan(e**x)
```

3.160 $\int e^{x^2} x^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int e^{x^2} x^3 \, dx = -\frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2$$

output
```
-1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} e^{x^2} \left(-1 + x^2\right)$$

input
```
Integrate[E^x^2*x^3,x]
```

output
```
(E^x^2*(-1 + x^2))/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x^3 \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2} e^{x^2} x^2 - \int e^{x^2} x dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{2} e^{x^2} x^2 - \frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x^3,x]
```

output
```
-1/2*E^x^2 + (E^x^2*x^2)/2
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(x^2-1)\,\mathrm{e}^{x^2}}{2}$	12
orering	$\frac{(x^2-1)\,\mathrm{e}^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)\,\mathrm{e}^{x^2}}{4}$	16
derivativedivides	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
default	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
norman	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parts	$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)\,x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\,\mathrm{erfi}(x)}{3}-\frac{2\left(-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

input
```
int(exp(x^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2-1)*exp(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx=\frac{1}{2}\,(x^2-1)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int e^{x^2}x^3\,dx=\frac{(x^2-1)\,e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x**3,x)
```

output
```
(x**2 - 1)*exp(x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx=\frac{1}{2}\,(x^2-1)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx=\frac{1}{2}\,(x^2-1)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{e^{x^2} (x^2 - 1)}{2}$$

input
```
int(x^3*exp(x^2),x)
```

output
```
(exp(x^2)*(x^2 - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int e^{x^2} x^3 \, dx = \frac{e^{x^2}(x^2 - 1)}{2}$$

input
```
int(exp(x^2)*x^3,x)
```

output
```
(e**(x**2)*(x**2 - 1))/2
```

3.161 $\int \frac{1}{x\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = \arctan\left(\sqrt{-1+x^2}\right)$$

output
```
arctan((x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = \arctan\left(\sqrt{-1+x^2}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 + x^2]),x]
```

output
```
ArcTan[Sqrt[-1 + x^2]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^2-1}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{x^2-1}}dx^2$$
$$\Big\downarrow \text{73}$$
$$\int \frac{1}{x^4+1}d\sqrt{x^2-1}$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(\sqrt{x^2-1}\right)$$

input
```
Int[1/(x*Sqrt[-1 + x^2]),x]
```

output
```
ArcTan[Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x^2-1}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)$	11
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^2-1}}{x}\right)$	27
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left((2\ln(x)-2\ln(2)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)\right)}{2\sqrt{\pi}\sqrt{\operatorname{signum}(x^2-1)}}$	61

input
```
int(1/x/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = 2\arctan\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(1/x/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = \begin{cases} i\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ -\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(x**2-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(1/x), 1/Abs(x**2) > 1), (-asin(1/x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = -\arcsin\left(\frac{1}{|x|}\right)$$

input
```
integrate(1/x/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = \arctan\left(\sqrt{x^2-1}\right)$$

input
```
integrate(1/x/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = \operatorname{atan}\left(\sqrt{x^2-1}\right)$$

input
```
int(1/(x*(x^2 - 1)^(1/2)),x)
```

output
```
atan((x^2 - 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt{-1+x^2}}\,dx = 2atan\left(\sqrt{x^2-1}+x\right)$$

input
```
int(1/x/(x^2-1)^(1/2),x)
```

output
```
2*atan(sqrt(x**2 - 1) + x)
```

3.162 $\int \frac{1}{x(1+x^2)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{x(1+x^2)}\,dx = \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
ln(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x^2)}\,dx = \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[1/(x*(1 + x^2)),x]
```

output
```
Log[x] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (x^2+1)} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (x^2+1)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2} \left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2} \left(\log \left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2} \left(\log \left(x^2\right)-\log \left(x^2+1\right)\right)$$

input
```
Int[1/(x*(1 + x^2)),x]
```

output
```
(Log[x^2] - Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
norman	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
meijerg	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
risch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
parallelrisch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12

input
```
int(1/x/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x(1+x^2)}\,dx = \log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}$$

input
```
integrate(1/x/(x**2+1),x)
```

output
```
log(x) - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^2)}\,dx = \ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

input
```
int(1/(x*(x^2 + 1)),x)
```

output
```
log(x) - log(x^2 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(1+x^2)}\,dx = -\frac{\log(x^2+1)}{2}+\log(x)$$

input
```
int(1/x/(x^2+1),x)
```

output
```
( - log(x**2 + 1) + 2*log(x))/2
```

3.163 $\int \operatorname{arccosh}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 21

$$\int \operatorname{arccosh}(x)\,dx=-\sqrt{-1+x}\sqrt{1+x}+x\operatorname{arccosh}(x)$$

output
```
-(-1+x)^(1/2)*(1+x)^(1/2)+x*arccosh(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \operatorname{arccosh}(x)\,dx=-\sqrt{-1+x}\sqrt{1+x}+x\operatorname{arccosh}(x)$$

input
```
Integrate[ArcCosh[x],x]
```

output
```
-(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6294, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arccosh}(x) \, dx$$

$$\Big\downarrow \ 6294$$

$$x\operatorname{arccosh}(x) - \int \frac{x}{\sqrt{x-1}\sqrt{x+1}}dx$$

$$\Big\downarrow \ 83$$

$$x\operatorname{arccosh}(x) - \sqrt{x-1}\sqrt{x+1}$$

input
```
Int[ArcCosh[x],x]
```

output
```
-(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 6294
```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Simp[b*c*n   Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt
[1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
lookup	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\operatorname{arccosh}\left(x\right)$	18
default	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\operatorname{arccosh}\left(x\right)$	18
parts	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\operatorname{arccosh}\left(x\right)$	18
orering	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\operatorname{arccosh}\left(x\right)$	18

input
```
int(arccosh(x),x,method=_RETURNVERBOSE)
```

output
```
-(x-1)^(1/2)*(x+1)^(1/2)+x*arccosh(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \operatorname{arccosh}(x)\,dx = x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

input
```
integrate(arccosh(x),x, algorithm="fricas")
```

output
```
x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [F]

$$\int \operatorname{arccosh}(x)\,dx = \int \operatorname{acosh}\left(x\right)dx$$

input
```
integrate(acosh(x),x)
```

output
```
Integral(acosh(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \operatorname{arccosh}(x)\,dx = x\operatorname{arcosh}(x) - \sqrt{x^2-1}$$

input
```
integrate(arccosh(x),x, algorithm="maxima")
```

output
```
x*arccosh(x) - sqrt(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \operatorname{arccosh}(x)\,dx = x\log\left(x+\sqrt{x^2-1}\right) - \sqrt{x^2-1}$$

input
```
integrate(arccosh(x),x, algorithm="giac")
```

output
```
x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \operatorname{arccosh}(x)\,dx = x\operatorname{acosh}(x) - \sqrt{x-1}\,\sqrt{x+1}$$

input
```
int(acosh(x),x)
```

output
```
x*acosh(x) - (x - 1)^(1/2)*(x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \operatorname{arccosh}(x)\,dx = acosh(x)\,x - \sqrt{x^2 - 1}$$

input
```
int(acosh(x),x)
```

output
```
acosh(x)*x - sqrt(x**2 - 1)
```

3.164 $\int e^{-3-5x-2x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int e^{-3-5x-2x^2} \, dx = \frac{\sqrt[8]{e}\sqrt{\pi}\text{erf}\left(\frac{5+4x}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
1/4*exp(1/8)*Pi^(1/2)*erf(1/4*(5+4*x)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int e^{-3-5x-2x^2} \, dx = \frac{\sqrt[8]{e}\sqrt{\pi}\text{erf}\left(\frac{5+4x}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Integrate[E^(-3 - 5*x - 2*x^2),x]
```

output
```
(E^(1/8)*Sqrt[Pi]*Erf[(5 + 4*x)/(2*Sqrt[2])])/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x^2-5x-3} \, dx$$

$$\Big\downarrow \text{2664}$$

$$\sqrt[8]{e} \int e^{-\frac{1}{8}(4x+5)^2} dx$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt[8]{e}\sqrt{\pi}\text{erf}\left(\frac{4x+5}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[E^(-3 - 5*x - 2*x^2),x]
```

output
```
(E^(1/8)*Sqrt[Pi]*Erf[(5 + 4*x)/(2*Sqrt[2])])/(2*Sqrt[2])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sqrt{\pi}\, \mathrm{e}^{\frac{1}{8}} \sqrt{2}\, \mathrm{erf}\left(x\sqrt{2}+\frac{5\sqrt{2}}{4}\right)}{4}$	23
risch	$\frac{\sqrt{\pi}\, \mathrm{e}^{\frac{1}{8}} \sqrt{2}\, \mathrm{erf}\left(x\sqrt{2}+\frac{5\sqrt{2}}{4}\right)}{4}$	23

input
```
int(exp(-2*x^2-5*x-3),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(1/8)*2^(1/2)*erf(x*2^(1/2)+5/4*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int e^{-3-5x-2x^2}\, dx = \frac{1}{4}\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{4}\,\sqrt{2}(4\,x+5)\right)e^{\frac{1}{8}}$$

input
```
integrate(exp(-2*x^2-5*x-3),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(1/4*sqrt(2)*(4*x + 5))*e^(1/8)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int e^{-3-5x-2x^2}\, dx = \frac{\sqrt{2}\sqrt{\pi}e^{\frac{1}{8}}\,\mathrm{erf}\left(\sqrt{2}x+\frac{5\sqrt{2}}{4}\right)}{4}$$

input
```
integrate(exp(-2*x**2-5*x-3),x)
```

output
```
sqrt(2)*sqrt(pi)*exp(1/8)*erf(sqrt(2)*x + 5*sqrt(2)/4)/4
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int e^{-3-5x-2x^2}\,dx = \frac{1}{4}\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}x + \frac{5}{4}\sqrt{2}\right)e^{\frac{1}{8}}$$

input
```
integrate(exp(-2*x^2-5*x-3),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(sqrt(2)*x + 5/4*sqrt(2))*e^(1/8)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int e^{-3-5x-2x^2}\,dx = \frac{1}{4}\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}(4x+5)\right)e^{\frac{1}{8}}$$

input
```
integrate(exp(-2*x^2-5*x-3),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(pi)*erf(1/4*sqrt(2)*(4*x + 5))*e^(1/8)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int e^{-3-5x-2x^2}\,dx = \frac{\sqrt{2}\,\sqrt{\pi}\,e^{1/8}\operatorname{erf}\left(\sqrt{2}\,x + \frac{5\sqrt{2}}{4}\right)}{4}$$

input
```
int(exp(- 5*x - 2*x^2 - 3),x)
```

output
```
(2^(1/2)*pi^(1/2)*exp(1/8)*erf(2^(1/2)*x + (5*2^(1/2))/4))/4
```

Reduce [F]

$$\int e^{-3-5x-2x^2}\,dx = \frac{\int \frac{1}{e^{2x^2+5x}}dx}{e^3}$$

input
```
int(exp(-2*x^2-5*x-3),x)
```

output
```
int(1/e**(2*x**2 + 5*x),x)/e**3
```

3.165 $\int \sin\left(\sqrt{x}\right) dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin\left(\sqrt{x}\right) dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin\left(\sqrt{x}\right) dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
Integrate[Sin[Sqrt[x]],x]
```

output
```
-2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{3842}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3777}$$

$$2\left(\int \cos\left(\sqrt{x}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\int \sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$2\left(\sin\left(\sqrt{x}\right)-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

input
```
Int[Sin[Sqrt[x]],x]
```

output
```
2*(-(Sqrt[x]*Cos[Sqrt[x]]) + Sin[Sqrt[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
default	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	28

input
```
int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="fricas")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x} \cos \left(\sqrt{x}\right) + 2 \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x**(1/2)),x)
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin \left(\sqrt{x}\right) \, dx = -2 \sqrt{x} \cos \left(\sqrt{x}\right) + 2 \sin \left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="maxima")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = 2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*( - sqrt(x)*cos(sqrt(x)) + sin(sqrt(x)))
```

3.166 $\int \frac{1}{\left(\frac{1}{x}+x\right)^2} dx$

Optimal result

Integrand size = 7, antiderivative size = 19

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

input
```
Integrate[(x^(-1) + x)^(-2),x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2027, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\frac{1}{x}\right)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2}{\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

input
```
Int[(x^(-1) + x)^(-2),x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(i+x)x^2+i\ln(x-i)-i\ln(i+x)+2x}{4(x^2+1)}$	52

input
```
int(1/(1/x+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx=\frac{(x^2+1)\arctan(x)-x}{2\,(x^2+1)}$$

input
```
integrate(1/(1/x+x)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx=-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(1/x+x)**2,x)
```

output
```
-x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx=-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan(x)$$

input
```
integrate(1/(1/x+x)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx=-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan(x)$$

input
```
integrate(1/(1/x+x)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx = \frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

input
```
int(1/(x + 1/x)^2,x)
```

output
```
atan(x)/2 - x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{1}{x}+x\right)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) - x}{2x^2+2}$$

input
```
int(1/(1/x+x)^2,x)
```

output
```
(atan(x)*x**2 + atan(x) - x)/(2*(x**2 + 1))
```

3.167 $\int \frac{e^{-x}(2+x)}{x^3} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 10

$$\int \frac{e^{-x}(2+x)}{x^3} \, dx = -\frac{e^{-x}}{x^2}$$

output
```
-1/exp(x)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}(2+x)}{x^3} \, dx = -\frac{e^{-x}}{x^2}$$

input
```
Integrate[(2 + x)/(E^x*x^3),x]
```

output
```
-(1/(E^x*x^2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2627}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}(x+2)}{x^3} \, dx$$

$$\Big\downarrow \text{2627}$$

$$-\frac{e^{-x}}{x^2}$$

input
```
Int[(2 + x)/(E^x*x^3),x]
```

output
```
-(1/(E^x*x^2))
```

Defintions of rubi rules used

rule 2627
```
Int[(F_)^(v_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)), x_Symbol] :>
 Simp[g*(d + e*x)^(m + 1)*(F^v/(D[v, x]*e*Log[F])), x] /; FreeQ[{F, d, e, f
, g, m}, x] && LinearQ[v, x] && EqQ[e*g*(m + 1) - D[v, x]*(e*f - d*g)*Log[F
], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-x}}{x^2}$	10
default	$-\frac{e^{-x}}{x^2}$	10
norman	$-\frac{e^{-x}}{x^2}$	10
risch	$-\frac{e^{-x}}{x^2}$	10
parallelrisch	$-\frac{e^{-x}}{x^2}$	10
orering	$-\frac{e^{-x}}{x^2}$	10
meijerg	$\frac{1}{x}-\frac{1}{2}+\frac{2-2x}{2x}-\frac{e^{-x}}{x}-\frac{1}{x^2}+\frac{9x^2-12x+6}{6x^2}-\frac{(3-3x)e^{-x}}{3x^2}$	59

input
```
int((x+2)/exp(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/exp(x)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\frac{e^{(-x)}}{x^2}$$

input
```
integrate((2+x)/exp(x)/x^3,x, algorithm="fricas")
```

output
```
-e^(-x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\frac{e^{-x}}{x^2}$$

input
```
integrate((2+x)/exp(x)/x**3,x)
```

output
```
-exp(-x)/x**2
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\Gamma(-1,x) - 2\,\Gamma(-2,x)$$

input
```
integrate((2+x)/exp(x)/x^3,x, algorithm="maxima")
```

output
```
-gamma(-1, x) - 2*gamma(-2, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\frac{e^{(-x)}}{x^2}$$

input
```
integrate((2+x)/exp(x)/x^3,x, algorithm="giac")
```

output
```
-e^(-x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\frac{\mathrm{e}^{-x}}{x^2}$$

input
```
int((exp(-x)*(x + 2))/x^3,x)
```

output
```
-exp(-x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}(2+x)}{x^3}\,dx = -\frac{1}{e^x x^2}$$

input
```
int((2+x)/exp(x)/x^3,x)
```

output
```
( - 1)/(e**x*x**2)
```

3.168 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -\arcsin(1-2x)$$

output
```
arcsin(-1+2*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/Sqrt[(1 - x)*x],x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2048, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(1-x)x}}dx$$
$$\Big\downarrow \text{ 2048}$$
$$\int \frac{1}{\sqrt{x-x^2}}dx$$
$$\Big\downarrow \text{ 1090}$$
$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin(1-2x)$$

input
```
Int[1/Sqrt[(1 - x)*x],x]
```

output
```
-ArcSin[1 - 2*x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	siz
default	$\arcsin\left(-1+2x\right)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x}\right)$	36

input
```
int(1/((1-x)*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1+2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -2\,\arctan\left(\frac{\sqrt{-x^2+x}}{x-1}\right)$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + x)/(x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \operatorname{asin}(2x-1)$$

input
```
integrate(1/((1-x)*x)**(1/2),x)
```

output
```
asin(2*x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \arcsin(2\,x-1)$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \frac{1}{4}\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\arcsin(2\,x-1)$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \operatorname{asin}(2\,x-1)$$

input
```
int(1/(-x*(x - 1))^(1/2),x)
```

output
```
asin(2*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -2\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i$$

input
```
int(1/((1-x)*x)^(1/2),x)
```

output
```
 - 2*log(sqrt( - x + 1) + sqrt(x)*i)*i
```

3.169 $\int e^{-x}\tanh(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int e^{-x}\tanh(x)\,dx = e^{-x} + 2\arctan\left(e^{x}\right)$$

output
```
exp(-x)+2*arctan(exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{-x}\tanh(x)\,dx = e^{-x} + 2\arctan\left(e^{x}\right)$$

input
```
Integrate[Tanh[x]/E^x,x]
```

output
```
E^(-x) + 2*ArcTan[E^x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2720, 25, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\tanh(x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{-2x}\left(1-e^{2x}\right)}{e^{2x}+1}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{e^{-2x}\left(1-e^{2x}\right)}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{359}$$

$$2\int \frac{1}{1+e^{2x}}de^x+e^{-x}$$

$$\Big\downarrow \text{216}$$

$$2\arctan\left(e^x\right)+e^{-x}$$

input
```
Int[Tanh[x]/E^x,x]
```

output
```
E^(-x) + 2*ArcTan[E^x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

method	result	size
default	$2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2}{1+\tanh\left(\frac{x}{2}\right)}$	19
risch	$\mathrm{e}^{-x}+i\ln\left(\mathrm{e}^{x}+i\right)-i\ln\left(\mathrm{e}^{x}-i\right)$	24

input
```
int(tanh(x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(tanh(1/2*x))+2/(1+tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int e^{-x}\tanh(x)\,dx = \frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

input
```
integrate(tanh(x)/exp(x),x, algorithm="fricas")
```

output
```
(2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int e^{-x}\tanh(x)\,dx = \int e^{-x}\tanh\left(x\right)dx$$

input
```
integrate(tanh(x)/exp(x),x)
```

output
```
Integral(exp(-x)*tanh(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{-x}\tanh(x)\,dx = -2\arctan\left(e^{(-x)}\right)+e^{(-x)}$$

input
```
integrate(tanh(x)/exp(x),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x)) + e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int e^{-x}\tanh(x)\,dx = 2\,\arctan\left(e^{x}\right) + e^{(-x)}$$

input
```
integrate(tanh(x)/exp(x),x, algorithm="giac")
```

output
```
2*arctan(e^x) + e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{-x}\tanh(x)\,dx = \mathrm{e}^{-x} - 2\,\mathrm{atan}\left(\mathrm{e}^{-x}\right)$$

input
```
int(exp(-x)*tanh(x),x)
```

output
```
exp(-x) - 2*atan(exp(-x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int e^{-x}\tanh(x)\,dx = \frac{2e^{x}\,atan(e^{x}) + 1}{e^{x}}$$

input
```
int(tanh(x)/exp(x),x)
```

output
```
(2*e**x*atan(e**x) + 1)/e**x
```

3.170 $\int \sqrt{1+\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sqrt{1+\sin(x)}\,dx=-\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

output
```
-2*cos(x)/(1+sin(x))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \sqrt{1+\sin(x)}\,dx=\frac{2\left(-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1+\sin(x)}}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sqrt[1 + Sin[x]],x]
```

output
```
(2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)+1}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

input
```
Int[Sqrt[1 + Sin[x]],x]
```

output
```
(-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{2(-1+\sin(x))\sqrt{1+\sin(x)}}{\cos(x)}$	17
risch	$-\frac{i\sqrt{2}\,\sqrt{2+2\sin(x)}\,(\mathrm{e}^{ix}-i)\,(i+\mathrm{e}^{ix})}{\mathrm{e}^{2ix}+2i\,\mathrm{e}^{ix}-1}$	48

input
```
int((1+sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1+sin(x))*(1+sin(x))^(1/2)/cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\left(\cos\left(x\right)-\sin\left(x\right)+1\right)\sqrt{\sin\left(x\right)+1}}{\cos\left(x\right)+\sin\left(x\right)+1}$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F]

$$\int \sqrt{1+\sin(x)}\,dx = \int \sqrt{\sin\left(x\right)+1}\,dx$$

input
```
integrate((1+sin(x))**(1/2),x)
```

output
```
Integral(sqrt(sin(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\sin(x)}\,dx = \int \sqrt{\sin(x)+1}\,dx$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \sqrt{1+\sin(x)}\,dx = 2\sqrt{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)$$

input
```
integrate((1+sin(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \sqrt{1+\sin(x)}\,dx = \frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$$

input
```
int((sin(x) + 1)^(1/2),x)
```

output
```
(2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)
```

Reduce [F]

$$\int \sqrt{1+\sin(x)}\, dx = \int \sqrt{\sin(x)+1}dx$$

input
```
int((1+sin(x))^(1/2),x)
```

output
```
int(sqrt(sin(x) + 1),x)
```

3.171 $\int \frac{1}{1+\sqrt{x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

output
```
2*x^(1/2)-2*ln(1+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

input
```
Integrate[(1 + Sqrt[x])^(-1),x]
```

output
```
2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+1}\,dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{\sqrt{x}+1}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(1+\frac{1}{-\sqrt{x}-1}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\sqrt{x}-\log\left(\sqrt{x}+1\right)\right)$$

input
```
Int[(1 + Sqrt[x])^(-1),x]
```

output
```
2*(Sqrt[x] - Log[1 + Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$2\sqrt{x}-2\ln\left(\sqrt{x}+1\right)$	15
meijerg	$2\sqrt{x}-2\ln\left(\sqrt{x}+1\right)$	15
trager	$2\sqrt{x}-\ln\left(2\sqrt{x}+1+x\right)$	18
default	$2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)-\ln\left(x-1\right)$	27

input
```
int(1/(x^(1/2)+1),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)-2*ln(x^(1/2)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\sqrt{x}-2\,\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x**(1/2)),x)
```

output
```
2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right) + 2$$

input
```
integrate(1/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\sqrt{x} - 2\,\ln\left(\sqrt{x}+1\right)$$

input
```
int(1/(x^(1/2) + 1),x)
```

output
```
2*x^(1/2) - 2*log(x^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

input
```
int(1/(1+x^(1/2)),x)
```

output
```
2*(sqrt(x) - log(sqrt(x) + 1))
```

3.172 $\int e^{-x^2} \sin^2\left(\frac{\pi}{4}+x\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int e^{-x^2} \sin^2\left(\frac{\pi}{4}+x\right) \, dx = \frac{1}{4}\sqrt{\pi}\text{erf}(x) - \frac{\sqrt{\pi}\text{erfi}(1-ix)}{8e} - \frac{\sqrt{\pi}\text{erfi}(1+ix)}{8e}$$

output
```
1/4*Pi^(1/2)*erf(x)+1/8*Pi^(1/2)*erfi(-1+I*x)*exp(-1)-1/8*Pi^(1/2)*erfi(1+
I*x)*exp(-1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int e^{-x^2} \sin^2\left(\frac{\pi}{4}+x\right) \, dx = \frac{\sqrt{\pi}(2e\text{erf}(x) - \text{erfi}(1-ix) - \text{erfi}(1+ix))}{8e}$$

input
```
Integrate[Sin[Pi/4 + x]^2/E^x^2,x]
```

output
```
(Sqrt[Pi]*(2*E*Erf[x] - Erfi[1 - I*x] - Erfi[1 + I*x]))/(8*E)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4975, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^2}\sin^2\left(x+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \text{ 4975}$$
$$\int \left(\frac{e^{-x^2}}{2}+\frac{1}{4}ie^{-x^2-2ix}-\frac{1}{4}ie^{-x^2+2ix}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{4}\sqrt{\pi}\text{erf}(x)-\frac{\sqrt{\pi}\text{erfi}(1-ix)}{8e}-\frac{\sqrt{\pi}\text{erfi}(1+ix)}{8e}$$

input
```
Int[Sin[Pi/4 + x]^2/E^x^2,x]
```

output
```
(Sqrt[Pi]*Erf[x])/4 - (Sqrt[Pi]*Erfi[1 - I*x])/(8*E) - (Sqrt[Pi]*Erfi[1 +
I*x])/(8*E)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4975
```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-1}\operatorname{erf}(i+x)}{8}-\frac{i\sqrt{\pi}\,\mathrm{e}^{-1}\operatorname{erf}(x-i)}{8}+\frac{\sqrt{\pi}\,\operatorname{erf}(x)}{4}$	35

input
```
int(sin(1/4*Pi+x)^2/exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*Pi^(1/2)*exp(-1)*erf(I+x)-1/8*I*Pi^(1/2)*exp(-1)*erf(x-I)+1/4*Pi^(1/
2)*erf(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{-x^2}\sin^2\left(\frac{\pi}{4}+x\right)\,dx$$
$$=\frac{1}{8}\sqrt{\pi}\Big(\Big(\operatorname{erf}\left(-x+i\right)e^{\left(\frac{1}{2}i\pi-1\right)}+2\operatorname{erf}\left(x\right)\Big)e^{\left(\frac{1}{2}i\pi+1\right)}-\operatorname{erf}\left(x+i\right)\Big)e^{\left(-\frac{1}{2}i\pi-1\right)}$$

input
```
integrate(sin(1/4*pi+x)^2/exp(x^2),x, algorithm="fricas")
```

output
```
1/8*sqrt(pi)*((erf(-x + I)*e^(1/2*I*pi - 1) + 2*erf(x))*e^(1/2*I*pi + 1) -
 erf(x + I))*e^(-1/2*I*pi - 1)
```

Sympy [F]

$$\int e^{-x^2}\sin^2\left(\frac{\pi}{4}+x\right)\,dx=\int e^{-x^2}\sin^2\left(x+\frac{\pi}{4}\right)\,dx$$

input
```
integrate(sin(1/4*pi+x)**2/exp(x**2),x)
```

output
```
Integral(exp(-x**2)*sin(x + pi/4)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

$$\int e^{-x^2}\sin^2\left(\frac{\pi}{4}+x\right)\,dx=\frac{1}{8}\sqrt{\pi}(2\,\mathrm{erf}\,(x)\,e+i\,\mathrm{erf}\,(x+i)-i\,\mathrm{erf}\,(x-i))e^{(-1)}$$

input
```
integrate(sin(1/4*pi+x)^2/exp(x^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*(2*erf(x)*e + I*erf(x + I) - I*erf(x - I))*e^(-1)
```

Giac [F]

$$\int e^{-x^2}\sin^2\left(\frac{\pi}{4}+x\right)\,dx=\int e^{(-x^2)}\sin\left(\frac{1}{4}\,\pi+x\right)^2\,dx$$

input
```
integrate(sin(1/4*pi+x)^2/exp(x^2),x, algorithm="giac")
```

output
```
integrate(e^(-x^2)*sin(1/4*pi + x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-x^2}\sin^2\left(\frac{\pi}{4}+x\right)\,dx=\int \mathrm{e}^{-x^2}\sin\left(\frac{\Pi}{4}+x\right)^2\,dx$$

input
```
int(exp(-x^2)*sin(Pi/4 + x)^2,x)
```

output
```
int(exp(-x^2)*sin(Pi/4 + x)^2, x)
```

Reduce [F]

$$\int e^{-x^2} \sin^2\left(\frac{\pi}{4}+x\right) dx = \int \frac{\sin\left(\frac{\pi}{4}+x\right)^2}{e^{x^2}} dx$$

input
```
int(sin(1/4*Pi+x)^2/exp(x^2),x)
```

output
```
int(sin((pi + 4*x)/4)**2/e**(x**2),x)
```

3.173 $\int 3e^{-2x^3-x^6}x^2(1+x^3)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 39

$$\int 3e^{-2x^3-x^6}x^2(1+x^3)^2\,dx=-\frac{1}{2}e^{-2x^3-x^6}(1+x^3)+\frac{1}{4}e\sqrt{\pi}\operatorname{erf}(1+x^3)$$

output
```
-1/2*exp(-x^6-2*x^3)*(x^3+1)+1/4*exp(1)*Pi^(1/2)*erf(x^3+1)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int 3e^{-2x^3-x^6}x^2(1+x^3)^2\,dx=\frac{1}{4}\left(-2e^{-x^3(2+x^3)}(1+x^3)+e\sqrt{\pi}\operatorname{erf}(1+x^3)\right)$$

input
```
Integrate[3*E^(-2*x^3 - x^6)*x^2*(1 + x^3)^2,x]
```

output
```
((-2*(1 + x^3))/E^(x^3*(2 + x^3)) + E*Sqrt[Pi]*Erf[1 + x^3])/4
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {27, 7266, 2667, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3e^{-x^6-2x^3}x^2\left(x^3+1\right)^2dx$$

$$\Big\downarrow \text{27}$$

$$3\int e^{-x^6-2x^3}x^2\left(x^3+1\right)^2dx$$

$$\Big\downarrow \text{7266}$$

$$\int e^{-x^6-2x^3}\left(x^3+1\right)^2dx^3$$

$$\Big\downarrow \text{2667}$$

$$\frac{1}{2}\int e^{-x^6-2x^3}dx^3-\frac{1}{2}e^{-x^6-2x^3}\left(x^3+1\right)$$

$$\Big\downarrow \text{2664}$$

$$\frac{1}{2}e\int e^{-\left(x^3+1\right)^2}dx^3-\frac{1}{2}e^{-x^6-2x^3}\left(x^3+1\right)$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{4}e\sqrt{\pi}\text{erf}\left(x^3+1\right)-\frac{1}{2}e^{-x^6-2x^3}\left(x^3+1\right)$$

input
```
Int[3*E^(-2*x^3 - x^6)*x^2*(1 + x^3)^2,x]
```

output
```
-1/2*(E^(-2*x^3 - x^6)*(1 + x^3)) + (E*Sqrt[Pi]*Erf[1 + x^3])/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2667
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2
), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m,
1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int 3\,\mathrm{e}^{-x^{6}-2x^{3}} x^{2} \left(x^{3}+1\right)^{2} dx$$

input
```
int(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x)
```

output
```
int(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx=\frac{1}{4}\sqrt{\pi}\operatorname{erf}\left(x^3+1\right)e-\frac{1}{2}\left(x^3+1\right)e^{\left(-x^6-2\,x^3\right)}$$

input
```
integrate(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*erf(x^3 + 1)*e - 1/2*(x^3 + 1)*e^(-x^6 - 2*x^3)
```

Sympy [F]

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx=3\left(\int x^2e^{-2x^3}e^{-x^6}\,dx+\int 2x^5e^{-2x^3}e^{-x^6}\,dx+\int x^8e^{-2x^3}e^{-x^6}\,dx\right)$$

input
```
integrate(3*exp(-x**6-2*x**3)*x**2*(x**3+1)**2,x)
```

output
```
3*(Integral(x**2*exp(-2*x**3)*exp(-x**6), x) + Integral(2*x**5*exp(-2*x**3
)*exp(-x**6), x) + Integral(x**8*exp(-2*x**3)*exp(-x**6), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.51

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx = \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}\left(x^3+1\right)e + \frac{1}{2}i\left(\frac{i\left(x^3+1\right)^3\Gamma\left(\frac{3}{2},\left(x^3+1\right)^2\right)}{\left(\left(x^3+1\right)^2\right)^{\frac{3}{2}}} - \frac{i\sqrt{\pi}\left(x^3+1\right)\left(\mathrm{erf}\left(\sqrt{\left(x^3+1\right)^2}\right)-1\right)}{\sqrt{\left(x^3+1\right)^2}} - 2i\,e^{\left(-\left(x^3+1\right)^2\right)}\right)e + i\left(\frac{i\sqrt{\pi}\left(x^3+1\right)\left(\mathrm{erf}\left(\sqrt{\left(x^3+1\right)^2}\right)-1\right)}{\sqrt{\left(x^3+1\right)^2}} + i\,e^{\left(-\left(x^3+1\right)^2\right)}\right)e$$

input
```
integrate(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*erf(x^3 + 1)*e + 1/2*I*(I*(x^3 + 1)^3*gamma(3/2, (x^3 + 1)^2)
/((x^3 + 1)^2)^(3/2) - I*sqrt(pi)*(x^3 + 1)*(erf(sqrt((x^3 + 1)^2)) - 1)/s
qrt((x^3 + 1)^2) - 2*I*e^(-(x^3 + 1)^2))*e + I*(I*sqrt(pi)*(x^3 + 1)*(erf(
sqrt((x^3 + 1)^2)) - 1)/sqrt((x^3 + 1)^2) + I*e^(-(x^3 + 1)^2))*e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx = \frac{1}{4}\sqrt{\pi}\,\mathrm{erf}\left(x^3+1\right)e - \frac{1}{2}\left(x^3+1\right)e^{\left(-x^6-2\,x^3\right)}$$

input
```
integrate(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(x^3 + 1)*e - 1/2*(x^3 + 1)*e^(-x^6 - 2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx=\frac{\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}\left(x^3+1\right)}{4}-\frac{x^3\,\mathrm{e}^{-x^6-2\,x^3}}{2}-\frac{\mathrm{e}^{-x^6-2\,x^3}}{2}$$

input
```
int(3*x^2*exp(- 2*x^3 - x^6)*(x^3 + 1)^2,x)
```

output
```
(pi^(1/2)*exp(1)*erf(x^3 + 1))/4 - (x^3*exp(- 2*x^3 - x^6))/2 - exp(- 2*x^
3 - x^6)/2
```

Reduce [F]

$$\int 3e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx=\frac{3e^{x^6+2x^3}\left(\int \frac{x^2}{e^{x^6+2x^3}}dx\right)-x^3-1}{2e^{x^6+2x^3}}$$

input
```
int(3*exp(-x^6-2*x^3)*x^2*(x^3+1)^2,x)
```

output
```
(3*e**(x**6 + 2*x**3)*int(x**2/e**(x**6 + 2*x**3),x) - x**3 - 1)/(2*e**(x*
*6 + 2*x**3))
```

3.174 $\int e^{2x} \cos(3x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{2x} \cos(3x) \, dx = \frac{2}{13} e^{2x} \cos(3x) + \frac{3}{13} e^{2x} \sin(3x)$$

output
```
2/13*exp(2*x)*cos(3*x)+3/13*exp(2*x)*sin(3*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{2x} \cos(3x) \, dx = \frac{1}{13} e^{2x} (2\cos(3x) + 3\sin(3x))$$

input
```
Integrate[E^(2*x)*Cos[3*x],x]
```

output
```
(E^(2*x)*(2*Cos[3*x] + 3*Sin[3*x]))/13
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2x}\cos(3x)\,dx$$

$$\Big\downarrow \text{ 4933}$$

$$\frac{3}{13}e^{2x}\sin(3x)+\frac{2}{13}e^{2x}\cos(3x)$$

input
```
Int[E^(2*x)*Cos[3*x],x]
```

output
```
(2*E^(2*x)*Cos[3*x])/13 + (3*E^(2*x)*Sin[3*x])/13
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{2x}(2\cos(3x)+3\sin(3x))}{13}$	20
default	$\frac{2\,e^{2x}\cos(3x)}{13}+\frac{3\,e^{2x}\sin(3x)}{13}$	22
orering	$\frac{2\,e^{2x}\cos(3x)}{13}+\frac{3\,e^{2x}\sin(3x)}{13}$	22
risch	$\frac{e^{(2+3i)x}}{13}-\frac{3ie^{(2+3i)x}}{26}+\frac{e^{(2-3i)x}}{13}+\frac{3ie^{(2-3i)x}}{26}$	36
norman	$\frac{\frac{6\,e^{2x}\tan\left(\frac{3x}{2}\right)}{13}-\frac{2\,e^{2x}\tan\left(\frac{3x}{2}\right)^{2}}{13}+\frac{2\,e^{2x}}{13}}{1+\tan\left(\frac{3x}{2}\right)^{2}}$	41

input
```
int(exp(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/13*exp(2*x)*(2*cos(3*x)+3*sin(3*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{2x}\cos(3x)\,dx=\frac{2}{13}\cos(3\,x)\,e^{(2\,x)}+\frac{3}{13}\,e^{(2\,x)}\sin(3\,x)$$

input
```
integrate(exp(2*x)*cos(3*x),x, algorithm="fricas")
```

output
```
2/13*cos(3*x)*e^(2*x) + 3/13*e^(2*x)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{2x}\cos(3x)\,dx=\frac{3e^{2x}\sin(3x)}{13}+\frac{2e^{2x}\cos(3x)}{13}$$

input
```
integrate(exp(2*x)*cos(3*x),x)
```

output
```
3*exp(2*x)*sin(3*x)/13 + 2*exp(2*x)*cos(3*x)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{13}\,(2\,\cos\left(3\,x\right) + 3\,\sin\left(3\,x\right))e^{(2\,x)}$$

input
```
integrate(exp(2*x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/13*(2*cos(3*x) + 3*sin(3*x))*e^(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos(3x)\,dx = \frac{1}{13}\,(2\,\cos\left(3\,x\right) + 3\,\sin\left(3\,x\right))e^{(2\,x)}$$

input
```
integrate(exp(2*x)*cos(3*x),x, algorithm="giac")
```

output
```
1/13*(2*cos(3*x) + 3*sin(3*x))*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos(3x)\,dx = \frac{\mathrm{e}^{2\,x}\,\left(2\,\cos\left(3\,x\right) + 3\,\sin\left(3\,x\right)\right)}{13}$$

input
```
int(cos(3*x)*exp(2*x),x)
```

output
```
(exp(2*x)*(2*cos(3*x) + 3*sin(3*x)))/13
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{2x}\cos(3x)\,dx=\frac{e^{2x}(2\cos(3x)+3\sin(3x))}{13}$$

input
```
int(exp(2*x)*cos(3*x),x)
```

output
```
(e**(2*x)*(2*cos(3*x) + 3*sin(3*x)))/13
```

3.175 $\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 7

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx=-\cos^{\cos(x)}(x)$$

output
```
-cos(x)^cos(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx=-\cos^{\cos(x)}(x)$$

input
```
Integrate[Cos[x]^Cos[x]*(1 + Log[Cos[x]])*Sin[x],x]
```

output
```
-Cos[x]^Cos[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4835, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^{\cos(x)}(x)(\log(\cos(x))+1)\,dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \cos^{\cos(x)}(x)(\log(\cos(x))+1)d\cos(x)$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\log(\cos(x))\cos^{\cos(x)}(x)+\cos^{\cos(x)}(x)\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\cos^{\cos(x)}(x)$$

input
```
Int[Cos[x]^Cos[x]*(1 + Log[Cos[x]])*Sin[x],x]
```

output
```
-Cos[x]^Cos[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 7.88 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\cos\left(x\right)^{\cos(x)}$	8
default	$-\cos\left(x\right)^{\cos(x)}$	8
parallelrisch	$-\cos\left(x\right)^{\cos(x)}$	8
norman	$\frac{-\tan\left(\frac{x}{2}\right)^{2} e^{\frac{\left(1-\tan\left(\frac{x}{2}\right)^{2}\right)\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}-e^{\frac{\left(1-\tan\left(\frac{x}{2}\right)^{2}\right)\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	111

input
```
int(cos(x)^cos(x)*(1+ln(cos(x)))*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)^cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos\left(x\right)^{\cos\left(x\right)}$$

input
```
integrate(cos(x)^cos(x)*(1+log(cos(x)))*sin(x),x, algorithm="fricas")
```

output
```
-cos(x)^cos(x)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos^{\cos(x)}(x)$$

input
```
integrate(cos(x)**cos(x)*(1+ln(cos(x)))*sin(x),x)
```

output
```
-cos(x)**cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos(x)^{\cos(x)}$$

input
```
integrate(cos(x)^cos(x)*(1+log(cos(x)))*sin(x),x, algorithm="maxima")
```

output
```
-cos(x)^cos(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos(x)^{\cos(x)}$$

input
```
integrate(cos(x)^cos(x)*(1+log(cos(x)))*sin(x),x, algorithm="giac")
```

output
```
-cos(x)^cos(x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos(x)^{\cos(x)}$$

input
```
int(cos(x)^cos(x)*sin(x)*(log(cos(x)) + 1),x)
```

output
```
-cos(x)^cos(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx = -\cos(x)^{\cos(x)}$$

input
```
int(cos(x)^cos(x)*(1+log(cos(x)))*sin(x),x)
```

output
```
 - cos(x)**cos(x)
```

3.176 $\int \frac{e^x}{2+e^x} dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{e^x}{2+e^x} dx = \log(2+e^x)$$

output
```
ln(2+exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{2+e^x} dx = \log(2+e^x)$$

input
```
Integrate[E^x/(2 + E^x),x]
```

output
```
Log[2 + E^x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^x+2} \, dx$$

$$\Big\downarrow \text{2676}$$

$$\int \frac{1}{e^x+2} de^x$$

$$\Big\downarrow \text{16}$$

$$\log \left(e^x+2\right)$$

input
```
Int[E^x/(2 + E^x),x]
```

output
```
Log[2 + E^x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(2 + e^x)$	6
default	$\ln(2 + e^x)$	6
norman	$\ln(2 + e^x)$	6
risch	$\ln(2 + e^x)$	6
parallelrisch	$\ln(2 + e^x)$	6

input
```
int(exp(x)/(2+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(2+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2 + e^x} \, dx = \log(e^x + 2)$$

input
```
integrate(exp(x)/(2+exp(x)),x, algorithm="fricas")
```

output
```
log(e^x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2 + e^x} \, dx = \log(e^x + 2)$$

input
```
integrate(exp(x)/(2+exp(x)),x)
```

output
```
log(exp(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2+e^x}\,dx = \log\left(e^x+2\right)$$

input
```
integrate(exp(x)/(2+exp(x)),x, algorithm="maxima")
```

output
```
log(e^x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2+e^x}\,dx = \log\left(e^x+2\right)$$

input
```
integrate(exp(x)/(2+exp(x)),x, algorithm="giac")
```

output
```
log(e^x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{2+e^x}\,dx = \ln\left(\mathrm{e}^x+2\right)$$

input
```
int(exp(x)/(exp(x) + 2),x)
```

output
```
log(exp(x) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{2+e^x}\,dx = \log(e^x + 2)$$

input
```
int(exp(x)/(2+exp(x)),x)
```

output
```
log(e**x + 2)
```

3.177 $\int \sin(2018x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 8

$$\int \sin(2018x)\,dx = -\frac{\cos(2018x)}{2018}$$

output
```
-1/2018*cos(2018*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sin(2018x)\,dx = -\frac{\cos(2018x)}{2018}$$

input
```
Integrate[Sin[2018*x],x]
```

output
```
-1/2018*Cos[2018*x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2018x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(2018x)dx$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{\cos(2018x)}{2018}$$

input
```
Int[Sin[2018*x],x]
```

output
```
-1/2018*Cos[2018*x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\cos(2018x)}{2018}$	7
default	$-\frac{\cos(2018x)}{2018}$	7
risch	$-\frac{\cos(2018x)}{2018}$	7
orering	$-\frac{\cos(2018x)}{2018}$	7
parallelrisch	$-\frac{\cos(2018x)}{2018}-\frac{1}{2018}$	9
norman	$-\frac{1}{1009\left(1+\tan(1009x)^2\right)}$	13
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2018x)}{\sqrt{\pi}}\right)}{2018}$	19

input
```
int(sin(2018*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2018*cos(2018*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sin(2018x)\,dx = -\frac{1}{2018}\cos(2018\,x)$$

input
```
integrate(sin(2018*x),x, algorithm="fricas")
```

output
```
-1/2018*cos(2018*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sin(2018x)\,dx = -\frac{\cos(2018x)}{2018}$$

input
```
integrate(sin(2018*x),x)
```

output
```
-cos(2018*x)/2018
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sin(2018x)\,dx = -\frac{1}{2018}\cos(2018\,x)$$

input
```
integrate(sin(2018*x),x, algorithm="maxima")
```

output
```
-1/2018*cos(2018*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sin(2018x)\,dx = -\frac{1}{2018}\cos(2018\,x)$$

input
```
integrate(sin(2018*x),x, algorithm="giac")
```

output
```
-1/2018*cos(2018*x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sin(2018x)\,dx = -\frac{\cos(2018\,x)}{2018}$$

input
```
int(sin(2018*x),x)
```

output
```
-cos(2018*x)/2018
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sin(2018x)\,dx = -\frac{\cos(2018x)}{2018}$$

input
```
int(sin(2018*x),x)
```

output
```
( - cos(2018*x))/2018
```

3.178 $\int \frac{1}{\cot(x)+\tan(x)}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx = \frac{\sin^2(x)}{2}$$

output
```
1/2*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx = -\frac{1}{2}\cos^2(x)$$

input
```
Integrate[(Cot[x] + Tan[x])^(-1),x]
```

output
```
-1/2*Cos[x]^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4853, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan(x)+\cot(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)+\cot(x)}dx$$
$$\Big\downarrow \text{4853}$$
$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)$$
$$\Big\downarrow \text{241}$$
$$-\frac{1}{2\left(\tan^2(x)+1\right)}$$

input
```
Int[(Cot[x] + Tan[x])^(-1),x]
```

output
```
-1/2*1/(1 + Tan[x]^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\cos(x)^2}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$-\frac{\cos(2x)}{4}-\frac{1}{4}$	9
norman	$-\frac{1}{2\left(1+\tan(x)^2\right)}$	11

input
```
int(1/(cot(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx=\frac{\tan{(x)}^2-1}{4\left(\tan{(x)}^2+1\right)}$$

input
```
integrate(1/(cot(x)+tan(x)),x, algorithm="fricas")
```

output
```
1/4*(tan(x)^2 - 1)/(tan(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx = -\frac{1}{2\tan^2(x)+2}$$

input
```
integrate(1/(cot(x)+tan(x)),x)
```

output
```
-1/(2*tan(x)**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx = -\frac{1}{2\left(\tan(x)^2+1\right)}$$

input
```
integrate(1/(cot(x)+tan(x)),x, algorithm="maxima")
```

output
```
-1/2/(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx = -\frac{1}{2}\cos(x)^2$$

input
```
integrate(1/(cot(x)+tan(x)),x, algorithm="giac")
```

output
```
-1/2*cos(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx=\frac{\sin{(x)}^2}{2}$$

input
```
int(1/(cot(x) + tan(x)),x)
```

output
```
sin(x)^2/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\cot(x)+\tan(x)}\,dx=\frac{\sin{(x)}^2}{2}$$

input
```
int(1/(cot(x)+tan(x)),x)
```

output
```
sin(x)**2/2
```

3.179 $\int \frac{x^5}{2+x^{12}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^5}{2+x^{12}} \, dx = \frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

output
```
1/12*arctan(1/2*x^6*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{2+x^{12}} \, dx = \frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

input
```
Integrate[x^5/(2 + x^12),x]
```

output
```
ArcTan[x^6/Sqrt[2]]/(6*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^{12}+2} \, dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{6} \int \frac{1}{x^{12}+2} dx^6$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

input
```
Int[x^5/(2 + x^12),x]
```

output
```
ArcTan[x^6/Sqrt[2]]/(6*Sqrt[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^6\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15
meijerg	$\frac{\arctan\left(\frac{x^6\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15
risch	$\frac{\arctan\left(\frac{x^6\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15

input
```
int(x^5/(x^12+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*arctan(1/2*x^6*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^6\right)$$

input
```
integrate(x^5/(x^12+2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^6}{2}\right)}{12}$$

input
```
integrate(x**5/(x**12+2),x)
```

output
```
sqrt(2)*atan(sqrt(2)*x**6/2)/12
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^6\right)$$

input
```
integrate(x^5/(x^12+2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^6\right)$$

input
```
integrate(x^5/(x^12+2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^6}{2}\right)}{12}$$

input
```
int(x^5/(x^12 + 2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x^6)/2))/12
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 416, normalized size of antiderivative = 21.89

$$\int \frac{x^5}{2+x^{12}}\,dx = \frac{\sqrt{2}\left(-\sqrt{-\sqrt{3}+2}\,2^{\frac{1}{6}}\sqrt{3}\,atan\left(\frac{\left(\sqrt{6}\,2^{\frac{1}{12}}+\sqrt{2}\,2^{\frac{1}{12}}-4x\right)2^{\frac{11}{12}}}{4\sqrt{-\sqrt{3}+2}}\right)-\sqrt{-\sqrt{3}+2}\,2^{\frac{1}{6}}\,atan\left(\frac{\left(\sqrt{6}\,2^{\frac{1}{12}}+\sqrt{2}\,2^{\frac{1}{12}}-4x\right)2^{\frac{11}{12}}}{4\sqrt{-\sqrt{3}+2}}\right)-$$

input
```
int(x^5/(x^12+2),x)
```

output
```
(sqrt(2)*( - sqrt( - sqrt(3) + 2)*2**(1/6)*sqrt(3)*atan((sqrt(6)*2**(1/12)
 + sqrt(2)*2**(1/12) - 4*x)/(2*sqrt( - sqrt(3) + 2)*2**(1/12))) - sqrt( -
sqrt(3) + 2)*2**(1/6)*atan((sqrt(6)*2**(1/12) + sqrt(2)*2**(1/12) - 4*x)/(
2*sqrt( - sqrt(3) + 2)*2**(1/12))) - sqrt( - sqrt(3) + 2)*2**(1/6)*sqrt(3)
*atan((sqrt(6)*2**(1/12) + sqrt(2)*2**(1/12) + 4*x)/(2*sqrt( - sqrt(3) + 2
)*2**(1/12))) - sqrt( - sqrt(3) + 2)*2**(1/6)*atan((sqrt(6)*2**(1/12) + sq
rt(2)*2**(1/12) + 4*x)/(2*sqrt( - sqrt(3) + 2)*2**(1/12))) + 2**(2/3)*atan
((sqrt(2)*2**(1/12) - 2*x)/(sqrt(2)*2**(1/12))) + 2**(2/3)*atan((sqrt(2)*2
**(1/12) + 2*x)/(sqrt(2)*2**(1/12))) - sqrt( - sqrt(3) + 2)*2**(1/6)*sqrt(
3)*atan((2*sqrt( - sqrt(3) + 2)*2**(1/12) - 4*x)/(sqrt(6)*2**(1/12) + sqrt
(2)*2**(1/12))) - sqrt( - sqrt(3) + 2)*2**(1/6)*atan((2*sqrt( - sqrt(3) +
2)*2**(1/12) - 4*x)/(sqrt(6)*2**(1/12) + sqrt(2)*2**(1/12))) - sqrt( - sqr
t(3) + 2)*2**(1/6)*sqrt(3)*atan((2*sqrt( - sqrt(3) + 2)*2**(1/12) + 4*x)/(
sqrt(6)*2**(1/12) + sqrt(2)*2**(1/12))) - sqrt( - sqrt(3) + 2)*2**(1/6)*at
an((2*sqrt( - sqrt(3) + 2)*2**(1/12) + 4*x)/(sqrt(6)*2**(1/12) + sqrt(2)*2
**(1/12)))))/(12*2**(2/3))
```

3.180 $\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 5

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx=\cosh(x)\sin(x)$$

output
```
cosh(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx=\cosh(x)\sin(x)$$

input
```
Integrate[Cos[x]*Cosh[x] + Sin[x]*Sinh[x],x]
```

output
```
Cosh[x]*Sin[x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)\sinh(x)+\cos(x)\cosh(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\int \sin(x)\sinh(x)dx+\int \cos(x)\cosh(x)dx$$

input
```
Int[Cos[x]*Cosh[x] + Sin[x]*Sinh[x],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 6.88 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result
orering	$\cosh(x)\sin(x)$
default	$\frac{e^{x}\sin(x)}{2}+\frac{e^{-x}\sin(x)}{2}$
parts	$\frac{e^{x}\sin(x)}{2}+\frac{e^{-x}\sin(x)}{2}$
risch	$\frac{i e^{(-1-i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{i e^{(-1+i)x}}{4}+\frac{i e^{(1-i)x}}{4}$
meijerg	$\pi^{\frac{3}{2}}\left(\frac{e^{x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{x}\sin(x)}{4\pi^{\frac{3}{2}}}-\frac{e^{-x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\sin(x)}{4\pi^{\frac{3}{2}}}\right)+\pi^{\frac{3}{2}}\left(-\frac{e^{x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{x}\sin(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\sin(x)}{4\pi^{\frac{3}{2}}}\right)$

input
```
int(cos(x)*cosh(x)+sin(x)*sinh(x),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)*sin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(5) = 10$.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 6.80

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx = \frac{2\cosh(x)\sin(x)\sinh(x)+\sin(x)\sinh(x)^2+\left(\cosh(x)^2+1\right)\sin(x)}{2\left(\cosh(x)+\sinh(x)\right)}$$

input
```
integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(x)*sin(x)*sinh(x) + sin(x)*sinh(x)^2 + (cosh(x)^2 + 1)*sin(x))
/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx = \sin(x)\cosh(x)$$

input
```
integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x)
```

output
```
sin(x)*cosh(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(5) = 10$.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 10.40

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx = \frac{1}{4}\left(\left(e^{(2x)}-1\right)\cos(x)+\left(e^{(2x)}+1\right)\sin(x)\right)e^{(-x)} - \frac{1}{4}\left(\left(e^{(2x)}-1\right)\cos(x)-\left(e^{(2x)}+1\right)\sin(x)\right)e^{(-x)}$$

input
```
integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="maxima")
```

output
```
1/4*((e^(2*x) - 1)*cos(x) + (e^(2*x) + 1)*sin(x))*e^(-x) - 1/4*((e^(2*x) -
 1)*cos(x) - (e^(2*x) + 1)*sin(x))*e^(-x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(5) = 10$.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 9.00

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx = \frac{1}{4}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{(-x)} - \frac{1}{4}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{(-x)} + \frac{1}{4}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{x} - \frac{1}{4}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

input
```
integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="giac")
```

output
```
1/4*(cos(x) + sin(x))*e^(-x) - 1/4*(cos(x) - sin(x))*e^(-x) + 1/4*(cos(x)
+ sin(x))*e^x - 1/4*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx = \cosh(x)\,\sin\left(x\right)$$

input
```
int(cos(x)*cosh(x) + sin(x)*sinh(x),x)
```

output
```
cosh(x)*sin(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (\cos(x)\cosh(x)+\sin(x)\sinh(x))\,dx=\cosh(x)\sin(x)$$

input
```
int(cos(x)*cosh(x)+sin(x)*sinh(x),x)
```

output
```
cosh(x)*sin(x)
```

3.181 $\int \frac{e^x+\cos(x)}{e^x+\sin(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 7

$$\int \frac{e^x+\cos(x)}{e^x+\sin(x)}\,dx=\log\left(e^x+\sin(x)\right)$$

output
```
ln(exp(x)+sin(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x+\cos(x)}{e^x+\sin(x)}\,dx=\log\left(e^x+\sin(x)\right)$$

input
```
Integrate[(E^x + Cos[x])/(E^x + Sin[x]),x]
```

output
```
Log[E^x + Sin[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+\cos(x)}{e^x+\sin(x)}\,dx$$
$$\Big\downarrow \text{7235}$$
$$\log\left(e^x+\sin(x)\right)$$

input
```
Int[(E^x + Cos[x])/(E^x + Sin[x]),x]
```

output
```
Log[E^x + Sin[x]]
```

Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln\left(\mathrm{e}^{x}+\sin\left(x\right)\right)$	7
default	$\ln\left(\mathrm{e}^{x}+\sin\left(x\right)\right)$	7
risch	$-ix+\ln\left(\mathrm{e}^{2ix}+2i\mathrm{e}^{(1+i)x}-1\right)$	23
parallelrisch	$-\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)+\ln\left(2\tan\left(\frac{x}{2}\right)+\sec\left(\frac{x}{2}\right)^{2}\mathrm{e}^{x}\right)$	28
norman	$-\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)+\ln\left(\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}+\mathrm{e}^{x}+2\tan\left(\frac{x}{2}\right)\right)$	32

input
```
int((exp(x)+cos(x))/(exp(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)}\, dx = \log\left(e^x + \sin\left(x\right)\right)$$

input
```
integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="fricas")
```

output
```
log(e^x + sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)}\, dx = \log\left(e^x + \sin\left(x\right)\right)$$

input
```
integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x)
```

output
```
log(exp(x) + sin(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)}\,dx = \log\left(e^x + \sin\left(x\right)\right)$$

input
```
integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="maxima")
```

output
```
log(e^x + sin(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 11.00

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)}\,dx = \frac{1}{2}\,\log\left(\frac{4\,\left(e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^4 + 4\,e^x\tan\left(\frac{1}{2}\,x\right)^3 + 2\,e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^2 + 4\,e^x\tan\left(\frac{1}{2}\,x\right) + 4\,\tan\left(\frac{1}{2}\,x\right)^2 + e^{(2\,x)}\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)$$

input
```
integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*log(4*(e^(2*x)*tan(1/2*x)^4 + 4*e^x*tan(1/2*x)^3 + 2*e^(2*x)*tan(1/2*x
)^2 + 4*e^x*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(2*x))/(tan(1/2*x)^4 + 2*tan(1
/2*x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)} \, dx = \ln\left(\mathrm{e}^x + \sin\left(x\right)\right)$$

input
```
int((cos(x) + exp(x))/(exp(x) + sin(x)),x)
```

output
```
log(exp(x) + sin(x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x + \cos(x)}{e^x + \sin(x)} \, dx = \log(e^x + \sin\left(x\right))$$

input
```
int((exp(x)+cos(x))/(exp(x)+sin(x)),x)
```

output
```
log(e**x + sin(x))
```

3.182 $\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

output
```
cos(cos(sin(x)))
```

Mathematica [A] (verified)

Time = 7.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
Integrate[Cos[x]*Sin[Cos[Sin[x]]]*Sin[Sin[x]],x]
```

output
```
Cos[Cos[Sin[x]]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4834, 4835, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\sin(x))\cos(x)\sin(\cos(\sin(x)))\,dx$$

$$\Big\downarrow \text{ 4834}$$

$$\int \sin(\sin(x))\sin(\cos(\sin(x)))d\sin(x)$$

$$\Big\downarrow \text{ 4835}$$

$$-\int \sin(\cos(\sin(x)))d\cos(\sin(x))$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \sin(\cos(\sin(x)))d\cos(\sin(x))$$

$$\Big\downarrow \text{ 3118}$$

$$\cos(\cos(\sin(x)))$$

input
```
Int[Cos[x]*Sin[Cos[Sin[x]]]*Sin[Sin[x]],x]
```

output
```
Cos[Cos[Sin[x]]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 9.56 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\cos(\cos(\sin(x)))$	5
default	$\cos(\cos(\sin(x)))$	5
risch	$\cos(\cos(\sin(x)))$	5
parallelrisch	$-1+\cos(\cos(\sin(x)))$	7

input
```
int(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
cos(cos(sin(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(4) = 8$.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 11.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos\left(\frac{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2-1}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

input
```
integrate(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x, algorithm="fricas")
```

output
```
cos((tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))^2 - 1)/(tan(tan(1/2*x)/(tan(1/2*x)
^2 + 1))^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
integrate(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x)
```

output
```
cos(cos(sin(x)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
integrate(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x, algorithm="maxima")
```

output
```
cos(cos(sin(x)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
integrate(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x, algorithm="giac")
```

output
```
cos(cos(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
int(sin(sin(x))*sin(cos(sin(x)))*cos(x),x)
```

output
```
cos(cos(sin(x)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(\cos(\sin(x)))\sin(\sin(x))\,dx = \cos(\cos(\sin(x)))$$

input
```
int(cos(x)*sin(cos(sin(x)))*sin(sin(x)),x)
```

output
```
cos(cos(sin(x)))
```

3.183 $\int \frac{1}{1+\sin(x)} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{1+\sin(x)} \, dx = -\frac{\cos(x)}{1+\sin(x)}$$

output
```
-cos(x)/(1+sin(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{1+\sin(x)} \, dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(1 + Sin[x])^(-1),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+1} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3127}$$
$$-\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[(1 + Sin[x])^(-1),x]
```

output
```
-(Cos[x]/(1 + Sin[x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
parallelrisch	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

input
```
int(1/(1+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(1+tan(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos\left(x\right) - \sin\left(x\right) + 1}{\cos\left(x\right) + \sin\left(x\right) + 1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="fricas")
```

output
```
-(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

input
```
integrate(1/(1+sin(x)),x)
```

output
```
-2/(tan(x/2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

input
```
integrate(1/(1+sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(sin(x) + 1),x)
```

output
```
-2/(tan(x/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1}{1+\sin(x)} \, dx = \frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(1+sin(x)),x)
```

output
```
(2*tan(x/2))/(tan(x/2) + 1)
```

3.184 $\int \frac{\cos(x)}{1-\cos(2x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{\cos(x)}{1-\cos(2x)} \, dx = -\frac{\csc(x)}{2}$$

output
```
-1/2*csc(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1-\cos(2x)} \, dx = -\frac{\csc(x)}{2}$$

input
```
Integrate[Cos[x]/(1 - Cos[2*x]),x]
```

output
```
-1/2*Csc[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4856, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{1-\cos(2x)} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{1-\cos(2x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{\csc^2(x)}{2}d\sin(x)$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\csc(x)}{2}$$

input
```
Int[Cos[x]/(1 - Cos[2*x]),x]
```

output
```
-1/2*Csc[x]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\csc(x)}{2}$	5
risch	$-\frac{i e^{ix}}{e^{2ix}-1}$	18

input
```
int(cos(x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csc(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = -\frac{1}{2\,\sin{(x)}}$$

input
```
integrate(cos(x)/(1-cos(2*x)),x, algorithm="fricas")
```

output
```
-1/2/sin(x)
```

Sympy [F]

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = -\int \frac{\cos(x)}{\cos(2x)-1}\,dx$$

input
```
integrate(cos(x)/(1-cos(2*x)),x)
```

output
```
-Integral(cos(x)/(cos(2*x) - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(4) = 8$.

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 7.00

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = -\frac{\cos(x)\sin(2x)-\cos(2x)\sin(x)+\sin(x)}{\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1}$$

input
```
integrate(cos(x)/(1-cos(2*x)),x, algorithm="maxima")
```

output
```
-(cos(x)*sin(2*x) - cos(2*x)*sin(x) + sin(x))/(cos(2*x)^2 + sin(2*x)^2 - 2
*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = -\frac{1}{2\sin(x)}$$

input
```
integrate(cos(x)/(1-cos(2*x)),x, algorithm="giac")
```

output
```
-1/2/sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = -\frac{1}{2\sin(x)}$$

input
```
int(-cos(x)/(cos(2*x) - 1),x)
```

output
```
-1/(2*sin(x))
```

Reduce [F]

$$\int \frac{\cos(x)}{1-\cos(2x)}\,dx = \frac{2\cos(2x)\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(x)^2+\tan(x)^2}dx\right)+\cos(2x)\sin(x)+\cos(2x)\,x-2\left(\int \frac{1}{\tan\left(\frac{x}{2}\right)^2\tan(x)^2+\tan(x)^2}dx\right)-\mathrm{s}}{2\cos(2x)-2}$$

input
```
int(cos(x)/(1-cos(2*x)),x)
```

output
```
(2*cos(2*x)*int(1/(tan(x/2)**2*tan(x)**2 + tan(x)**2),x) + cos(2*x)*sin(x)
 + cos(2*x)*x - 2*int(1/(tan(x/2)**2*tan(x)**2 + tan(x)**2),x) - sin(2*x)
- sin(x) - x)/(2*(cos(2*x) - 1))
```

3.185 $\int e^x \left(\frac{1}{x}+\log(x)\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int e^x \left(\frac{1}{x}+\log(x)\right) \, dx = e^x \log(x)$$

output
```
exp(x)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x \left(\frac{1}{x}+\log(x)\right) \, dx = e^x \log(x)$$

input
```
Integrate[E^x*(x^(-1) + Log[x]),x]
```

output
```
E^x*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(\frac{1}{x}+\log(x)\right) \, dx$$

$$\Big\downarrow \; 2726$$

$$e^x \log(x)$$

input
```
Int[E^x*(x^(-1) + Log[x]),x]
```

output
```
E^x*Log[x]
```

Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Maple [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
norman	$e^{x} \ln \left(x \right)$	6
risch	$e^{x} \ln \left(x \right)$	6
parallelrisch	$e^{x} \ln \left(x \right)$	6
orering	$\frac{\left(-1+2 x \right) e^{x} \left(\frac{1}{x}+\ln \left(x \right)\right)}{x -1}-\frac{x \left(e^{x} \left(\frac{1}{x}+\ln \left(x \right)\right)+e^{x} \left(-\frac{1}{x^{2}}+\frac{1}{x}\right)\right)}{x -1}$	51

input
```
int(exp(x)*(1/x+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \left(\frac{1}{x} + \log(x)\right) \, dx = e^x \log{(x)}$$

input
```
integrate(exp(x)*(1/x+log(x)),x, algorithm="fricas")
```

output
```
e^x*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \left(\frac{1}{x} + \log(x)\right) \, dx = e^x \log{(x)}$$

input
```
integrate(exp(x)*(1/x+ln(x)),x)
```

output
```
exp(x)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \left(\frac{1}{x} + \log(x)\right) dx = e^x \log(x)$$

input
```
integrate(exp(x)*(1/x+log(x)),x, algorithm="maxima")
```

output
```
e^x*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \left(\frac{1}{x} + \log(x)\right) dx = e^x \log(x)$$

input
```
integrate(exp(x)*(1/x+log(x)),x, algorithm="giac")
```

output
```
e^x*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \left(\frac{1}{x} + \log(x)\right) dx = \mathrm{e}^x \ln(x)$$

input
```
int(exp(x)*(log(x) + 1/x),x)
```

output
```
exp(x)*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x\left(\frac{1}{x}+\log(x)\right)\,dx=e^x\log(x)$$

input
```
int(exp(x)*(1/x+log(x)),x)
```

output
```
e**x*log(x)
```

3.186 $\int \tanh^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 6

$$\int \tanh^2(x)\,dx = x - \tanh(x)$$

output
```
x-tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \tanh^2(x)\,dx = \operatorname{arctanh}(\tanh(x)) - \tanh(x)$$

input
```
Integrate[Tanh[x]^2,x]
```

output
```
ArcTanh[Tanh[x]] - Tanh[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\tan(ix)^2 dx$$
$$\Big\downarrow \; 25$$
$$-\int \tan(ix)^2 dx$$
$$\Big\downarrow \; 3954$$
$$\int 1dx-\tanh(x)$$
$$\Big\downarrow \; 24$$
$$x-\tanh(x)$$

input
```
Int[Tanh[x]^2,x]
```

output
```
x - Tanh[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$x - \tanh(x)$	7
risch	$x + \frac{2}{1+e^{2x}}$	13
derivativedivides	$-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	20
default	$-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	20

input
```
int(tanh(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x-tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \tanh^2(x)\,dx = \frac{(x+1)\cosh(x) - \sinh(x)}{\cosh(x)}$$

input
```
integrate(tanh(x)^2,x, algorithm="fricas")
```

output
```
((x + 1)*cosh(x) - sinh(x))/cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \tanh^2(x)\,dx = x - \tanh(x)$$

input
```
integrate(tanh(x)**2,x)
```

output
```
x - tanh(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \tanh^2(x)\,dx = x - \frac{2}{e^{(-2x)} + 1}$$

input
```
integrate(tanh(x)^2,x, algorithm="maxima")
```

output
```
x - 2/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \tanh^2(x)\,dx = x + \frac{2}{e^{(2x)} + 1}$$

input
```
integrate(tanh(x)^2,x, algorithm="giac")
```

output
```
x + 2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tanh^2(x)\,dx = x - \tanh(x)$$

input
```
int(tanh(x)^2,x)
```

output
```
x - tanh(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \tanh^2(x)\,dx = -\tanh(x) + x$$

input
```
int(tanh(x)^2,x)
```

output
```
- tanh(x) + x
```

3.187 $\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 7

$$\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx = x-2\log(\sin(x))$$

output
```
x-2*ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx = x-2\log(\sin(x))$$

input
```
Integrate[(-Sin[x]^2 + Sin[2*x])/(-Cos[x]^2 + Cos[2*x]),x]
```

output
```
x - 2*Log[Sin[x]]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(7) = 14.

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4889, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)-\sin^2(x)}{\cos(2x)-\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2x)-\sin(x)^2}{\cos(2x)-\cos(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int -\frac{(2-\tan(x))\cot(x)}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot(x)(2-\tan(x))}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{523}$$

$$-\int \left(2\cot(x)+\frac{-2\tan(x)-1}{\tan^2(x)+1}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\arctan(\tan(x))+\log\left(\tan^2(x)+1\right)-2\log(\tan(x))$$

input
```
Int[(-Sin[x]^2 + Sin[2*x])/(-Cos[x]^2 + Cos[2*x]),x]
```

output
```
ArcTan[Tan[x]] - 2*Log[Tan[x]] + Log[1 + Tan[x]^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$x-2\ln\left(\sin\left(x\right)\right)$	8
risch	$2ix-2\ln\left(\mathrm{e}^{2ix}-1\right)+x$	17

input
```
int((-sin(x)^2+sin(2*x))/(-cos(x)^2+cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx = x-2\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate((-sin(x)^2+sin(2*x))/(-cos(x)^2+cos(2*x)),x, algorithm="fricas")
```

output
```
x - 2*log(1/2*sin(x))
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx = x-\log\left(\cos^2(x)-1\right)$$

input
```
integrate((-sin(x)**2+sin(2*x))/(-cos(x)**2+cos(2*x)),x)
```

output
```
x - log(cos(x)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 5.14

$$\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx = x-\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) - \log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate((-sin(x)^2+sin(2*x))/(-cos(x)^2+cos(2*x)),x, algorithm="maxima")
```

output
```
x - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \frac{-\sin^2(x) + \sin(2x)}{-\cos^2(x) + \cos(2x)}\,dx = x + 2\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right) - 2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate((-sin(x)^2+sin(2*x))/(-cos(x)^2+cos(2*x)),x, algorithm="giac")
```

output
```
x + 2*log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.71

$$\int \frac{-\sin^2(x) + \sin(2x)}{-\cos^2(x) + \cos(2x)}\,dx = -2\,\ln\left(\tan(x)\right) + \ln\left(\tan(x) - \mathrm{i}\right)\left(1 - \frac{1}{2}\mathrm{i}\right) + \ln\left(\tan(x) + 1\mathrm{i}\right)\left(1 + \frac{1}{2}\mathrm{i}\right)$$

input
```
int((sin(2*x) - sin(x)^2)/(cos(2*x) - cos(x)^2),x)
```

output
```
log(tan(x) - 1i)*(1 - 1i/2) - 2*log(tan(x)) + log(tan(x) + 1i)*(1 + 1i/2)
```

Reduce [F]

$$\int \frac{-\sin^2(x) + \sin(2x)}{-\cos^2(x) + \cos(2x)}\,dx = -\left(\int \frac{\sin(x)^2}{\cos(2x) - \cos(x)^2}dx\right) + \int \frac{\sin(2x)}{\cos(2x) - \cos(x)^2}dx$$

input
```
int((-sin(x)^2+sin(2*x))/(-cos(x)^2+cos(2*x)),x)
```

output
```
 - int(sin(x)**2/(cos(2*x) - cos(x)**2),x) + int(sin(2*x)/(cos(2*x) - cos(
x)**2),x)
```

3.188 $\int \frac{1}{x^{9/25}+x^{41/25}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{x^{9/25}+x^{41/25}}\,dx=\frac{25}{16}\arctan\left(x^{16/25}\right)$$

output
```
25/16*arctan(x^(16/25))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{9/25}+x^{41/25}}\,dx=\frac{25}{16}\arctan\left(x^{16/25}\right)$$

input
```
Integrate[(x^(9/25) + x^(41/25))^(-1),x]
```

output
```
(25*ArcTan[x^(16/25)])/16
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 864, 807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{41/25}+x^{9/25}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{x^{9/25}\left(x^{32/25}+1\right)}dx$$

$$\Big\downarrow \text{864}$$

$$25\int \frac{x^{3/5}}{x^{32/25}+1}d\sqrt[25]{x}$$

$$\Big\downarrow \text{807}$$

$$\frac{25}{16}\int \frac{1}{x^{2/25}+1}dx^{16/25}$$

$$\Big\downarrow \text{216}$$

$$\frac{25}{16}\arctan\left(x^{16/25}\right)$$

input
```
Int[(x^(9/25) + x^(41/25))^(-1),x]
```

output
```
(25*ArcTan[x^(16/25)])/16
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(1/(x^(9/25)+x^(41/25)),x)
```

output
```
int(1/(x^(9/25)+x^(41/25)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{9/25}+x^{41/25}}\,dx=\frac{25}{16}\arctan\left(x^{\frac{16}{25}}\right)$$

input
```
integrate(1/(x^(9/25)+x^(41/25)),x, algorithm="fricas")
```

output
```
25/16*arctan(x^(16/25))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/25}+x^{41/25}}\,dx=\text{Timed out}$$

input
```
integrate(1/(x**(9/25)+x**(41/25)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{9/25}+x^{41/25}}\,dx=\int \frac{1}{x^{\frac{41}{25}}+x^{\frac{9}{25}}}\,dx$$

input
```
integrate(1/(x^(9/25)+x^(41/25)),x, algorithm="maxima")
```

output
```
25/16*x^(16/25) - integrate(x^(23/25)/(x^(32/25) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{9/25} + x^{41/25}} \, dx = \frac{25}{16} \arctan\left(x^{\frac{16}{25}}\right)$$

input
```
integrate(1/(x^(9/25)+x^(41/25)),x, algorithm="giac")
```

output
```
25/16*arctan(x^(16/25))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{9/25} + x^{41/25}} \, dx = \frac{25 \operatorname{atan}\left(x^{16/25}\right)}{16}$$

input
```
int(1/(x^(9/25) + x^(41/25)),x)
```

output
```
(25*atan(x^(16/25)))/16
```

Reduce [F]

$$\int \frac{1}{x^{9/25} + x^{41/25}} \, dx = \int \frac{1}{x^{\frac{9}{25}} + x^{\frac{41}{25}}} dx$$

input
```
int(1/(x^(9/25)+x^(41/25)),x)
```

output
```
int(1/(x^(9/25)+x^(41/25)),x)
```

3.189 $\int \frac{\cos(x)}{2-\cos^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \arctan(\sin(x))$$

output
```
arctan(sin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \arctan(\sin(x))$$

input
```
Integrate[Cos[x]/(2 - Cos[x]^2),x]
```

output
```
ArcTan[Sin[x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3665, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{2-\cos^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{2-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{3665}$$
$$\int \frac{1}{\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{216}$$
$$\arctan(\sin(x))$$

input
```
Int[Cos[x]/(2 - Cos[x]^2),x]
```

output
```
ArcTan[Sin[x]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$\arctan(\sin(x))$	4
parallelrisch	$\frac{i\left(\ln\left(\sec\left(\frac{x}{2}\right)^2-2i\tan\left(\frac{x}{2}\right)\right)-\ln\left(\sec\left(\frac{x}{2}\right)^2+2i\tan\left(\frac{x}{2}\right)\right)\right)}{2}$	37
risch	$-\frac{i\ln\left(e^{2ix}+2\,e^{ix}-1\right)}{2}+\frac{i\ln\left(e^{2ix}-2\,e^{ix}-1\right)}{2}$	38

input
```
int(cos(x)/(2-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \arctan(\sin(x))$$

input
```
integrate(cos(x)/(2-cos(x)^2),x, algorithm="fricas")
```

output
```
arctan(sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(3) = 6$.

Time = 10.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 73.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \frac{1607521\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857} + \frac{1136689\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857} - \frac{195025\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857} - \frac{275807\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857}$$

input
```
integrate(cos(x)/(2-cos(x)**2),x)
```

output
```
1607521*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor
((x/2 - pi/2)/pi))/(470832*sqrt(2) + 665857) + 1136689*sqrt(2)*sqrt(3 - 2*
sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/
(470832*sqrt(2) + 665857) - 195025*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x
/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(470832*sqrt(2) + 66
5857) - 275807*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + p
i*floor((x/2 - pi/2)/pi))/(470832*sqrt(2) + 665857)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(2-cos(x)^2),x, algorithm="maxima")
```

output
```
arctan(sin(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \arctan(\sin(x))$$

input
```
integrate(cos(x)/(2-cos(x)^2),x, algorithm="giac")
```

output
```
arctan(sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = \mathrm{atan}(\sin(x))$$

input
```
int(-cos(x)/(cos(x)^2 - 2),x)
```

output
```
atan(sin(x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 15.33

$$\int \frac{\cos(x)}{2-\cos^2(x)}\,dx = -atan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2}+1}\right) - \frac{\log\left(-\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)+i\right)i}{2} + \frac{\log\left(\sqrt{2}\,i+\tan\left(\frac{x}{2}\right)-i\right)i}{2}$$

input
```
int(cos(x)/(2-cos(x)^2),x)
```

output
```
( - 2*atan(tan(x/2)/(sqrt(2) + 1)) - log( - sqrt(2)*i + tan(x/2) + i)*i +
log(sqrt(2)*i + tan(x/2) - i)*i)/2
```

3.190 $\int \frac{1}{(1+x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

output
```
x/(x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{1+x^2}}$$

input
```
Integrate[(1 + x^2)^(-3/2),x]
```

output
```
x/Sqrt[1 + x^2]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 208$$

$$\frac{x}{\sqrt{x^2+1}}$$

input
```
Int[(1 + x^2)^(-3/2),x]
```

output
```
x/Sqrt[1 + x^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x}{\sqrt{x^2+1}}$	10
default	$\frac{x}{\sqrt{x^2+1}}$	10
trager	$\frac{x}{\sqrt{x^2+1}}$	10
meijerg	$\frac{x}{\sqrt{x^2+1}}$	10
risch	$\frac{x}{\sqrt{x^2+1}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x^2+1}}$	10
orering	$\frac{x}{\sqrt{x^2+1}}$	10

input
```
int(1/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x^2+\sqrt{x^2+1}x+1}{x^2+1}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x**2+1)**(3/2),x)
```

output
```
x/sqrt(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}} \, dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
x/sqrt(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
int(1/(x^2 + 1)^(3/2),x)
```

output
```
x/(x^2 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{\sqrt{x^2+1}\,x+x^2+1}{x^2+1}$$

input
```
int(1/(x^2+1)^(3/2),x)
```

output
```
(sqrt(x**2 + 1)*x + x**2 + 1)/(x**2 + 1)
```

3.191 $\int \frac{1}{\sqrt{x^{3/2}-x^2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}} dx = 4 \arctan \left(\frac{x}{\sqrt{x^{3/2}-x^2}} \right)$$

output
```
4*arctan(x/(x^(3/2)-x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}} dx = 4 \arctan \left(\frac{x}{\sqrt{x^{3/2}-x^2}} \right)$$

input
```
Integrate[1/Sqrt[x^(3/2) - x^2],x]
```

output
```
4*ArcTan[x/Sqrt[x^(3/2) - x^2]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1914, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}dx$$

$$\Big\downarrow \text{ 1914}$$

$$4\int \frac{1}{\frac{x^2}{x^{3/2}-x^2}+1}d\frac{x}{\sqrt{x^{3/2}-x^2}}$$

$$\Big\downarrow \text{ 216}$$

$$4\arctan\left(\frac{x}{\sqrt{x^{3/2}-x^2}}\right)$$

input
```
Int[1/Sqrt[x^(3/2) - x^2],x]
```

output
```
4*ArcTan[x/Sqrt[x^(3/2) - x^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

method	result	size
meijerg	$4 \arcsin \left(x^{\frac{1}{4}}\right)$	7
derivativedivides	$\frac{2 \sqrt{x}\, \sqrt{-\sqrt{x}\, \left(-1+\sqrt{x}\right)}\, \arcsin \left(2 \sqrt{x}-1\right)}{\sqrt{x^{\frac{3}{2}}-x^{2}}}$	37
default	$\frac{2 x \left(-1+\sqrt{x}\right) \ln \left(\sqrt{x}-\frac{1}{2}+\sqrt{x-\sqrt{x}}\right)}{\sqrt{x^{\frac{3}{2}}-x^{2}}\, \sqrt{\sqrt{x}\, \left(-1+\sqrt{x}\right)}}$	46

input
```
int(1/(x^(3/2)-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*arcsin(x^(1/4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx = -4 \arctan \left(\frac{\sqrt{-x^{2}+x^{\frac{3}{2}}}\left(x+\sqrt{x}\right)}{x^{2}-x}\right)$$

input
```
integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-4*arctan(sqrt(-x^2 + x^(3/2))*(x + sqrt(x))/(x^2 - x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx=\int \frac{1}{\sqrt{x^{\frac{3}{2}}-x^2}}\,dx$$

input
```
integrate(1/(x**(3/2)-x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(x**(3/2) - x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx=\int \frac{1}{\sqrt{-x^2+x^{\frac{3}{2}}}}\,dx$$

input
```
integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^2 + x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx=\left(\pi+2\arcsin\left(2\sqrt{x}-1\right)\right)\operatorname{sgn}(x)$$

input
```
integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="giac")
```

output
```
(pi + 2*arcsin(2*sqrt(x) - 1))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx=\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx$$

input
```
int(1/(x^(3/2) - x^2)^(1/2),x)
```

output
```
int(1/(x^(3/2) - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx=4asin\left(x^{\frac{1}{4}}\right)$$

input
```
int(1/(x^(3/2)-x^2)^(1/2),x)
```

output
```
4*asin(x**(1/4))
```

3.192 $\int \frac{-1+x}{x+x^2 \log(x)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{-1+x}{x+x^2 \log(x)} \, dx = \log \left(\frac{1}{x}+\log(x)\right)$$

output
```
ln(1/x+ln(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{-1+x}{x+x^2 \log(x)} \, dx = -\log(x)+\log(1+x\log(x))$$

input
```
Integrate[(-1 + x)/(x + x^2*Log[x]),x]
```

output
```
-Log[x] + Log[1 + x*Log[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3041, 7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{x^2 \log(x)+x} \, dx$$

$$\Big\downarrow \text{3041}$$

$$\int \frac{x-1}{x^2 \left(\frac{1}{x}+\log(x)\right)}dx$$

$$\Big\downarrow \text{7235}$$

$$\log\left(\frac{1}{x}+\log(x)\right)$$

input
```
Int[(-1 + x)/(x + x^2*Log[x]),x]
```

output
```
Log[x^(-1) + Log[x]]
```

Defintions of rubi rules used

rule 3041
```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
risch	$\ln\left(\frac{1}{x}+\ln\left(x\right)\right)$	8
default	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13
norman	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13
parallelrisch	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13

input
```
int((-1+x)/(x+x^2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1/x+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx=\log\left(\frac{x\log\left(x\right)+1}{x}\right)$$

input
```
integrate((-1+x)/(x+x^2*log(x)),x, algorithm="fricas")
```

output
```
log((x*log(x) + 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx=\log\left(\log\left(x\right)+\frac{1}{x}\right)$$

input
```
integrate((-1+x)/(x+x**2*ln(x)),x)
```

output
```
log(log(x) + 1/x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx = \log\left(\frac{x\log(x)+1}{x}\right)$$

input
```
integrate((-1+x)/(x+x^2*log(x)),x, algorithm="maxima")
```

output
```
log((x*log(x) + 1)/x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx = \log(x\log(x)+1) - \log(x)$$

input
```
integrate((-1+x)/(x+x^2*log(x)),x, algorithm="giac")
```

output
```
log(x*log(x) + 1) - log(x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx = \ln(x\ \ln(x)+1) - \ln(x)$$

input
```
int((x - 1)/(x + x^2*log(x)),x)
```

output
```
log(x*log(x) + 1) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{-1+x}{x+x^2\log(x)}\,dx=\log(\log(x)\,x+1)-\log(x)$$

input
```
int((-1+x)/(x+x^2*log(x)),x)
```

output
```
log(log(x)*x + 1) - log(x)
```

3.193 $\int \csc(x)\sec(x)\,dx$

Optimal result

Integrand size = 5, antiderivative size = 3

$$\int \csc(x)\sec(x)\,dx = \log(\tan(x))$$

output
```
ln(tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \csc(x)\sec(x)\,dx = -\log(\cos(x)) + \log(\sin(x))$$

input
```
Integrate[Csc[x]*Sec[x],x]
```

output
```
-Log[Cos[x]] + Log[Sin[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(x)\sec(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)\sec(x)dx$$

$$\Big\downarrow \text{3100}$$

$$\int \cot(x)d\tan(x)$$

$$\Big\downarrow \text{14}$$

$$\log(\tan(x))$$

input
```
Int[Csc[x]*Sec[x],x]
```

output
```
Log[Tan[x]]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	$\ln(\tan(x))$	4
risch	$-\ln(e^{2ix}+1)+\ln(e^{2ix}-1)$	20
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	25
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	25

input
```
int(csc(x)*sec(x),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(3) = 6.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

$$\int \csc(x)\sec(x)\,dx = -\frac{1}{2}\log\left(\cos(x)^2\right)+\frac{1}{2}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

input
```
integrate(csc(x)*sec(x),x, algorithm="fricas")
```

output
```
-1/2*log(cos(x)^2) + 1/2*log(-1/4*cos(x)^2 + 1/4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \csc(x)\sec(x)\,dx = -\frac{\log\left(\sin^2(x) - 1\right)}{2} + \log\left(\sin(x)\right)$$

input
```
integrate(csc(x)*sec(x),x)
```

output
```
-log(sin(x)**2 - 1)/2 + log(sin(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \csc(x)\sec(x)\,dx = -\frac{1}{2}\log\left(\sin(x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(x)^2\right)$$

input
```
integrate(csc(x)*sec(x),x, algorithm="maxima")
```

output
```
-1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \csc(x)\sec(x)\,dx = -\frac{1}{2}\log\left(-\sin(x)^2 + 1\right) + \log\left(|\sin(x)|\right)$$

input
```
integrate(csc(x)*sec(x),x, algorithm="giac")
```

output
```
-1/2*log(-sin(x)^2 + 1) + log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \csc(x)\sec(x)\,dx = \ln(\tan(x))$$

input
```
int(1/(cos(x)*sin(x)),x)
```

output
```
log(tan(x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 8.00

$$\int \csc(x)\sec(x)\,dx = -\log\left(\tan\left(\frac{x}{2}\right)-1\right)-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(csc(x)*sec(x),x)
```

output
```
- log(tan(x/2) - 1) - log(tan(x/2) + 1) + log(tan(x/2))
```

3.194 $\int \tan(\cos(x))\,dx$

Optimal result

Integrand size = 3, antiderivative size = 3

$$\int \tan(\cos(x))\,dx = \text{Int}(\tan(\cos(x)),x)$$

output
```
Defer(Int)(tan(cos(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan(\cos(x))\,dx$$

input
```
Integrate[Tan[Cos[x]],x]
```

output
```
Integrate[Tan[Cos[x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(\cos(x))\,dx$$

$$\Big\downarrow \; 4902$$

$$2\int \frac{\tan\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 7276$$

$$2\int \left(\frac{i\tan\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{2\left(i-\tan\left(\frac{x}{2}\right)\right)}+\frac{i\tan\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{2\left(\tan\left(\frac{x}{2}\right)+i\right)}\right)d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{2}i\int \frac{\tan\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{i-\tan\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)+\frac{1}{2}i\int \frac{\tan\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan\left(\frac{x}{2}\right)+i}d\tan\left(\frac{x}{2}\right)\right)$$

input
```
Int[Tan[Cos[x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tan(\cos(x))\,dx$$

input
```
int(tan(cos(x)),x)
```

output
```
int(tan(cos(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan(\cos(x))\,dx$$

input
```
integrate(tan(cos(x)),x, algorithm="fricas")
```

output
```
integral(tan(cos(x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan(\cos(x))\,dx$$

input
```
integrate(tan(cos(x)),x)
```

output
```
Integral(tan(cos(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan\left(\cos\left(x\right)\right)\,dx$$

input
```
integrate(tan(cos(x)),x, algorithm="maxima")
```

output
```
integrate(tan(cos(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan\left(\cos\left(x\right)\right)\,dx$$

input
```
integrate(tan(cos(x)),x, algorithm="giac")
```

output
```
integrate(tan(cos(x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan(\cos\left(x\right))\,dx$$

input
```
int(tan(cos(x)),x)
```

output
```
int(tan(cos(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \tan(\cos(x))\,dx = \int \tan\left(\cos\left(x\right)\right)dx$$

input
```
int(tan(cos(x)),x)
```

output
```
int(tan(cos(x)),x)
```

3.195 $\int \frac{1+x}{x(x+\log(x))}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log(x+\log(x))$$

output
```
ln(x+ln(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log(x+\log(x))$$

input
```
Integrate[(1 + x)/(x*(x + Log[x])),x]
```

output
```
Log[x + Log[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x(x+\log(x))} \, dx$$
$$\Big\downarrow \text{7235}$$
$$\log(x+\log(x))$$

input
```
Int[(1 + x)/(x*(x + Log[x])),x]
```

output
```
Log[x + Log[x]]
```

Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\ln(x+\ln(x))$	6
norman	$\ln(x+\ln(x))$	6
risch	$\ln(x+\ln(x))$	6
parallelrisch	$\ln(x+\ln(x))$	6

input
```
int((1+x)/x/(x+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(x+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right)$$

input
```
integrate((1+x)/x/(x+log(x)),x, algorithm="fricas")
```

output
```
log(x + log(x))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log\left(x+\log\left(x\right)\right)$$

input
```
integrate((1+x)/x/(x+ln(x)),x)
```

output
```
log(x + log(x))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log(x+\log(x))$$

input
```
integrate((1+x)/x/(x+log(x)),x, algorithm="maxima")
```

output
```
log(x + log(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log(x+\log(x))$$

input
```
integrate((1+x)/x/(x+log(x)),x, algorithm="giac")
```

output
```
log(x + log(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \ln(x+\ln(x))$$

input
```
int((x + 1)/(x*(x + log(x))),x)
```

output
```
log(x + log(x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{x(x+\log(x))}\,dx = \log(\log(x)+x)$$

input
```
int((1+x)/x/(x+log(x)),x)
```

output
```
log(log(x) + x)
```

3.196 $\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 6

$$\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx=2\sinh\left(e^x\right)$$

output
```
2*sinh(exp(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx=-e^{-e^x}+e^{e^x}$$

input
```
Integrate[E^(-E^x + x) + E^(E^x + x),x]
```

output
```
-E^(-E^x) + E^E^x
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{x-e^x}+e^{x+e^x}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^{e^x}-e^{-e^x}$$

input
```
Int[E^(-E^x + x) + E^(E^x + x),x]
```

output
```
-E^(-E^x) + E^E^x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

method	result	size
default	$-{\rm e}^{-{\rm e}^{x}}+{\rm e}^{{\rm e}^{x}}$	12
parts	$-{\rm e}^{-{\rm e}^{x}}+{\rm e}^{{\rm e}^{x}}$	12
risch	$-\left({\rm e}^{-{\rm e}^{x}+x}-{\rm e}^{{\rm e}^{x}+x}\right) {\rm e}^{-x}$	22

input
```
int(exp(-exp(1)^x+x)+exp(exp(1)^x+x),x,method=_RETURNVERBOSE)
```

output
```
-1/exp(exp(x))+exp(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(5) = 10.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx=-\left(e^{(2\,x)}-e^{(2\,x+2\,e^x)}\right)e^{(-2\,x-e^x)}$$

input
```
integrate(exp(-exp(1)^x+x)+exp(exp(1)^x+x),x, algorithm="fricas")
```

output
```
-(e^(2*x) - e^(2*x + 2*e^x))*e^(-2*x - e^x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx=\text{Timed out}$$

input
```
integrate(exp(-exp(1)**x+x)+exp(exp(1)**x+x),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.83

$$\int \left(e^{-e^x+x} + e^{e^x+x}\right) dx = -e^{(-e^x)} + e^{(e^x)}$$

input
```
integrate(exp(-exp(1)^x+x)+exp(exp(1)^x+x),x, algorithm="maxima")
```

output
```
-e^(-e^x) + e^(e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.83

$$\int \left(e^{-e^x+x} + e^{e^x+x}\right) dx = -e^{(-e^x)} + e^{(e^x)}$$

input
```
integrate(exp(-exp(1)^x+x)+exp(exp(1)^x+x),x, algorithm="giac")
```

output
```
-e^(-e^x) + e^(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \left(e^{-e^x+x} + e^{e^x+x}\right) dx = 2\sinh(e^x)$$

input
```
int(exp(x + exp(x)) + exp(x - exp(x)),x)
```

output
```
2*sinh(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \left(e^{-e^x+x}+e^{e^x+x}\right)\,dx=\frac{e^{2e^x}-1}{e^{e^x}}$$

input
```
int(exp(-exp(1)^x+x)+exp(exp(1)^x+x),x)
```

output
```
(e**(2*e**x) - 1)/e**(e**x)
```

3.197 $\int \frac{1}{1-x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{1-x^2} \, dx = \operatorname{arctanh}(x)$$

output
```
arctanh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{1}{1-x^2} \, dx = -\frac{1}{2} \log(1-x) + \frac{1}{2} \log(1+x)$$

input
```
Integrate[(1 - x^2)^(-1),x]
```

output
```
-1/2*Log[1 - x] + Log[1 + x]/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-x^2} \, dx$$

$$\Big\downarrow \; 219$$

$$\text{arctanh}(x)$$

input
```
Int[(1 - x^2)^(-1),x]
```

output
```
ArcTanh[x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

input
```
int(1/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(1/(-x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1}{1-x^2}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate(1/(-x**2+1),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate(1/(-x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate(1/(-x^2+1),x, algorithm="giac")
```

output
```
1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-x^2}\,dx = \operatorname{atanh}(x)$$

input
```
int(-1/(x^2 - 1),x)
```

output
```
atanh(x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1}{1-x^2}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
int(1/(-x^2+1),x)
```

output
```
( - log(x - 1) + log(x + 1))/2
```

3.198 $\int 2^{\log(x)}\,dx$

Optimal result

Integrand size = 4, antiderivative size = 12

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{1+\log(2)}$$

output
```
2^ln(x)*x/(1+ln(2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{1+\log(2)}$$

input
```
Integrate[2^Log[x],x]
```

output
```
(2^Log[x]*x)/(1 + Log[2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2704, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{\log(x)}\,dx$$
$$\Big\downarrow \text{ 2704}$$
$$\int x^{\log(2)}dx$$
$$\Big\downarrow \text{ 15}$$
$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

input
```
Int[2^Log[x],x]
```

output
```
x^(1 + Log[2])/(1 + Log[2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2^{\ln(x)} x}{1+\ln(2)}$	13
risch	$\frac{x\, x^{\ln(2)}}{1+\ln(2)}$	13
parallelrisch	$\frac{2^{\ln(x)} x}{1+\ln(2)}$	13
norman	$\frac{x\, \mathrm{e}^{\ln(x)\ln(2)}}{1+\ln(2)}$	15

input
```
int(2^ln(x),x,method=_RETURNVERBOSE)
```

output
```
2^ln(x)*x/(1+ln(2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int 2^{\log(x)}\,dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

input
```
integrate(2^log(x),x, algorithm="fricas")
```

output
```
x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{\log(2)+1}$$

input
```
integrate(2**ln(x),x)
```

output
```
2**log(x)*x/(log(2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int 2^{\log(x)}\,dx = \frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

input
```
integrate(2^log(x),x, algorithm="maxima")
```

output
```
2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int 2^{\log(x)}\,dx = \frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

input
```
integrate(2^log(x),x, algorithm="giac")
```

output
```
x*e^(log(2)*log(x))/(log(2) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int 2^{\log(x)}\,dx = \frac{x^{\ln(2)+1}}{\ln(2)+1}$$

input
```
int(2^log(x),x)
```

output
```
x^(log(2) + 1)/(log(2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int 2^{\log(x)}\,dx = \frac{2^{\log(x)}x}{\log(2)+1}$$

input
```
int(2^log(x),x)
```

output
```
(2**log(x)*x)/(log(2) + 1)
```

3.199 $\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx = \frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)+\frac{1}{3}\sin(3x)-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$$

output
```
1/2*cos(x)-1/10*cos(5*x)+1/3*sin(3*x)-1/4034*sin(2017*x)+1/4042*sin(2021*x
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx = \frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)+\frac{1}{3}\sin(3x)-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$$

input
```
Integrate[Cos[3*x] + Sin[2*x]*(Cos[3*x] - Sin[2019*x]),x]
```

output
```
Cos[x]/2 - Cos[5*x]/10 + Sin[3*x]/3 - Sin[2017*x]/4034 + Sin[2021*x]/4042
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cos(3x) + \sin(2x)(\cos(3x) - \sin(2019x)))\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\sin(3x) - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042} + \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x)$$

input
```
Int[Cos[3*x] + Sin[2*x]*(Cos[3*x] - Sin[2019*x]),x]
```

output
```
Cos[x]/2 - Cos[5*x]/10 + Sin[3*x]/3 - Sin[2017*x]/4034 + Sin[2021*x]/4042
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result
default	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}+\frac{\sin(3x)}{3}-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$
risch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}+\frac{\sin(3x)}{3}-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$
parts	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}+\frac{\sin(3x)}{3}-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$
parallelrisch	$\frac{\frac{2\left(-8152714+(-10095\tan(x)-8152714)\tan\left(\frac{2019x}{2}\right)^{2}+10\left(\tan(x)^{2}-1\right)\tan\left(\frac{2019x}{2}\right)+10095\tan(x)\right)\tan\left(\frac{3x}{2}\right)^{2}}{20381785}+\frac{2\left(\tan(x)^{2}+\frac{18\tan(x)}{5}+1\right.}{}}{\left(1+\tan(x)^{2}\right)\left(1+\tan(\right.}$
orering	$\frac{\sin(3x)}{3}-\frac{19940000842106\cos(2x)(\cos(3x)-\sin(2019x))}{16616686391449}+\frac{29910131706719\sin(2x)(-3\sin(3x)-2019\cos(2019x))}{83083431957245}-2$

input
```
int(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)-1/10*cos(5*x)+1/3*sin(3*x)-1/4034*sin(2017*x)+1/4042*sin(2021*x
)
```

Fricas [F(-1)]

Timed out.

$$\int(\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx=\text{Timed out}$$

input
```
integrate(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int (\cos(3x) + \sin(2x)(\cos(3x) - \sin(2019x)))\,dx = \frac{3\sin(2x)\sin(3x)}{5} + \frac{2019\sin(2x)\cos(2019x)}{4076357} + \frac{\sin(3x)}{3} - \frac{2\sin(2019x)\cos(2x)}{4076357} + \frac{2\cos(2x)\cos(3x)}{5}$$

input
```
integrate(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x)
```

output
```
3*sin(2*x)*sin(3*x)/5 + 2019*sin(2*x)*cos(2019*x)/4076357 + sin(3*x)/3 - 2
*sin(2019*x)*cos(2*x)/4076357 + 2*cos(2*x)*cos(3*x)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (\cos(3x) + \sin(2x)(\cos(3x) - \sin(2019x)))\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x) + \frac{1}{4042}\sin(2021x) - \frac{1}{4034}\sin(2017x) + \frac{1}{3}\sin(3x)$$

input
```
integrate(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x) + 1/4042*sin(2021*x) - 1/4034*sin(2017*x) + 1/
3*sin(3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx = -\frac{1}{10}\cos(5x)+\frac{1}{2}\cos(x)+\frac{1}{4042}\sin(2021x)-\frac{1}{4034}\sin(2017x)+\frac{1}{3}\sin(3x)$$

input
```
integrate(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x, algorithm="giac")
```

output
```
-1/10*cos(5*x) + 1/2*cos(x) + 1/4042*sin(2021*x) - 1/4034*sin(2017*x) + 1/
3*sin(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx = \frac{\sin(3x)}{3}-\frac{\cos(5x)}{10}-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}+\frac{\cos(x)}{2}$$

input
```
int(cos(3*x) + sin(2*x)*(cos(3*x) - sin(2019*x)),x)
```

output
```
sin(3*x)/3 - cos(5*x)/10 - sin(2017*x)/4034 + sin(2021*x)/4042 + cos(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx = \frac{2019\cos(2019x)\sin(2x)}{4076357}+\frac{2\cos(3x)\cos(2x)}{5} - \frac{2\cos(2x)\sin(2019x)}{4076357}+\frac{3\sin(3x)\sin(2x)}{5}+\frac{\sin(3x)}{3}$$

input
```
int(cos(3*x)+sin(2*x)*(cos(3*x)-sin(2019*x)),x)
```

output
```
(30285*cos(2019*x)*sin(2*x) + 24458142*cos(3*x)*cos(2*x) - 30*cos(2*x)*sin
(2019*x) + 36687213*sin(3*x)*sin(2*x) + 20381785*sin(3*x))/61145355
```

3.200 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin(\sin(\sin(x)))$$

output
```
sin(sin(sin(x)))
```

Mathematica [A] (verified)

Time = 4.56 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin(\sin(\sin(x)))$$

input
```
Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

output
```
Sin[Sin[Sin[x]]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4834, 4834, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$$

$$\Big\downarrow \text{4834}$$

$$\int \cos(\sin(x))\cos(\sin(\sin(x)))d\sin(x)$$

$$\Big\downarrow \text{4834}$$

$$\int \cos(\sin(\sin(x)))d\sin(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\sin(\sin(x))+\frac{\pi}{2}\right)d\sin(\sin(x))$$

$$\Big\downarrow \text{3117}$$

$$\sin(\sin(\sin(x)))$$

input
```
Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

output
```
Sin[Sin[Sin[x]]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 9.92 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\sin(\sin(\sin(x)))$	5
default	$\sin(\sin(\sin(x)))$	5
risch	$\sin(\sin(\sin(x)))$	5
parallelrisch	$\sin(\sin(\sin(x)))$	5

input
```
int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x,method=_RETURNVERBOSE)
```

output
```
sin(sin(sin(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(4) = 8.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 10.25

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\frac{2\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
```

output
```
sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1
))^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

output
```
sin(sin(sin(x)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
```

output
```
sin(sin(sin(x)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

output
```
sin(sin(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)
```

output
```
sin(sin(sin(x)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

output
```
sin(sin(sin(x)))
```

3.201 $\int \frac{e^{-\frac{2019}{4x^2}}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2} dx = -\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

output
```
-1/2019*2019^(1/2)*Pi^(1/2)*erf(1/2*2019^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2} dx = -\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

input
```
Integrate[1/(E^(2019/(4*x^2))*x^2),x]
```

output
```
-(Sqrt[Pi/2019]*Erf[Sqrt[2019]/(2*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2640, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2} \, dx$$

$$\Big\downarrow \text{2640}$$

$$-\int e^{-\frac{2019}{4x^2}} d\frac{1}{x}$$

$$\Big\downarrow \text{2634}$$

$$-\sqrt{\frac{\pi}{2019}} \text{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

input
```
Int[1/(E^(2019/(4*x^2))*x^2),x]
```

output
```
-(Sqrt[Pi/2019]*Erf[Sqrt[2019]/(2*x)])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
default	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
meijerg	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
risch	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18

input
```
int(1/exp(2019/4/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2019*2019^(1/2)*Pi^(1/2)*erf(1/2*2019^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = -\frac{1}{2019}\,\sqrt{2019}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{2019}}{2\,x}\right)$$

input
```
integrate(1/exp(2019/4/x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/2019*sqrt(2019)*sqrt(pi)*erf(1/2*sqrt(2019)/x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = -\frac{\sqrt{2019}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$$

input
```
integrate(1/exp(2019/4/x**2)/x**2,x)
```

output
```
-sqrt(2019)*sqrt(pi)*erf(sqrt(2019)/(2*x))/2019
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = -\frac{\sqrt{2019}\sqrt{\pi}\sqrt{x^2}\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{2019}\sqrt{\frac{1}{x^2}}\right)-1\right)}{2019\,x}$$

input
```
integrate(1/exp(2019/4/x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/2019*sqrt(2019)*sqrt(pi)*sqrt(x^2)*(erf(1/2*sqrt(2019)*sqrt(x^(-2))) -
1)/x
```

Giac [F]

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = \int \frac{e^{\left(-\frac{2019}{4\,x^2}\right)}}{x^2}\,dx$$

input
```
integrate(1/exp(2019/4/x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(e^(-2019/4/x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = -\frac{\sqrt{2019}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{2019}}{2\,x}\right)}{2019}$$

input
```
int(exp(-2019/(4*x^2))/x^2,x)
```

output
```
-(2019^(1/2)*pi^(1/2)*erf(2019^(1/2)/(2*x)))/2019
```

Reduce [F]

$$\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx = \frac{2019e^{\frac{2019}{4x^2}}\left(\int \frac{1}{e^{\frac{2019}{4x^2}}x^4}dx\right)x-2}{2e^{\frac{2019}{4x^2}}x}$$

input
```
int(1/exp(2019/4/x^2)/x^2,x)
```

output
```
(2019*e**(2019/(4*x**2))*int(1/(e**(2019/(4*x**2))*x**4),x)*x - 2)/(2*e**(
2019/(4*x**2))*x)
```

3.202 $\int \sin \left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x}\cos \left(\sqrt{x}\right) + 2\sin \left(\sqrt{x}\right)$$

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin \left(\sqrt{x}\right) \, dx = -2\sqrt{x}\cos \left(\sqrt{x}\right) + 2\sin \left(\sqrt{x}\right)$$

input
```
Integrate[Sin[Sqrt[x]],x]
```

output
```
-2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{3842}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3777}$$

$$2\left(\int \cos\left(\sqrt{x}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\int \sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$2\left(\sin\left(\sqrt{x}\right)-\sqrt{x}\cos\left(\sqrt{x}\right)\right)$$

input
```
Int[Sin[Sqrt[x]],x]
```

output
```
2*(-(Sqrt[x]*Cos[Sqrt[x]]) + Sin[Sqrt[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3842
```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
default	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

input
```
int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="fricas")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x**(1/2)),x)
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\,dx = -2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="maxima")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\, dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
integrate(sin(x^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin\left(\sqrt{x}\right)\, dx = 2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \sin\left(\sqrt{x}\right)\, dx = -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

input
```
int(sin(x^(1/2)),x)
```

output
```
2*( - sqrt(x)*cos(sqrt(x)) + sin(sqrt(x)))
```

3.203 $\int \frac{\sqrt{x}}{1+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\sqrt{x}}{1+x} \, dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

output
```
2*x^(1/2)-2*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+x} \, dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[Sqrt[x]/(1 + x),x]
```

output
```
2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x+1} \, dx$$

$$\Big\downarrow \text{60}$$

$$2\sqrt{x}-\int \frac{1}{\sqrt{x}(x+1)}dx$$

$$\Big\downarrow \text{73}$$

$$2\sqrt{x}-2\int \frac{1}{x+1}d\sqrt{x}$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x]/(1 + x),x]
```

output
```
2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$	13
default	$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$	13
meijerg	$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$	13
risch	$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$	13
trager	$2\sqrt{x}-\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)$	36

input
```
int(x^(1/2)/(1+x),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)-2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(x**(1/2)/(1+x),x)
```

output
```
2*sqrt(x) - 2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x),x, algorithm="giac")
```

output
```
2*sqrt(x) - 2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{1+x}\,dx = 2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input
```
int(x^(1/2)/(x + 1),x)
```

output
```
2*x^(1/2) - 2*atan(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}}{1+x}\,dx = -2atan\left(\sqrt{x}\right) + 2\sqrt{x}$$

input
```
int(x^(1/2)/(1+x),x)
```

output
```
2*( - atan(sqrt(x)) + sqrt(x))
```

3.204 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) + \frac{1}{16}\sin(4x) + \frac{1}{24}\sin(6x)$$

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) + \frac{1}{16}\sin(4x) + \frac{1}{24}\sin(6x)$$

input
```
Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cos(2x)\cos(3x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

input

```
Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output

```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$
orering	$x\cos(x)\cos(2x)\cos(3x)+\frac{5\sin(x)\cos(2x)\cos(3x)}{48}-\frac{\cos(x)\sin(2x)\cos(3x)}{48}+\frac{11\cos(x)\cos(2x)\sin(3x)}{48}+$

input
```
int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int\cos(x)\cos(2x)\cos(3x)\,dx=\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

output
```
1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.82 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = -\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} + \frac{3\sin(x)\sin(2x)\sin(3x)}{8} + \frac{\sin(x)\cos(2x)\cos(3x)}{3} + \frac{5\sin(2x)\cos(x)\cos(3x)}{24}$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

output
```
-x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*
sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + 3*sin(x)*sin(2*x)*sin(3
*x)/8 + sin(x)*cos(2*x)*cos(3*x)/3 + 5*sin(2*x)*cos(x)*cos(3*x)/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin(6\,x) + \frac{1}{16}\,\sin(4\,x) + \frac{1}{8}\,\sin(2\,x)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

input
```
int(cos(2*x)*cos(3*x)*cos(x),x)
```

output
```
x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \frac{\cos\left(3x\right)\cos\left(2x\right)\cos\left(x\right)x}{4} + \frac{\cos\left(3x\right)\cos\left(2x\right)\sin\left(x\right)}{3} \\ &+ \frac{5\cos\left(3x\right)\cos\left(x\right)\sin\left(2x\right)}{24} \\ &- \frac{\cos\left(3x\right)\sin\left(2x\right)\sin\left(x\right)x}{4} + \frac{\cos\left(2x\right)\sin\left(3x\right)\sin\left(x\right)x}{4} \\ &+ \frac{\cos\left(x\right)\sin\left(3x\right)\sin\left(2x\right)x}{4} + \frac{3\sin\left(3x\right)\sin\left(2x\right)\sin\left(x\right)}{8}\end{aligned}$$

input
```
int(cos(x)*cos(2*x)*cos(3*x),x)
```

output
```
(6*cos(3*x)*cos(2*x)*cos(x)*x + 8*cos(3*x)*cos(2*x)*sin(x) + 5*cos(3*x)*co
s(x)*sin(2*x) - 6*cos(3*x)*sin(2*x)*sin(x)*x + 6*cos(2*x)*sin(3*x)*sin(x)*
x + 6*cos(x)*sin(3*x)*sin(2*x)*x + 9*sin(3*x)*sin(2*x)*sin(x))/24
```

3.205 $\int e^{-x^{2n}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int e^{-x^{2n}}\,dx=-\frac{x\,\text{ExpIntegralE}\left(1-\frac{1}{2n},x^{2n}\right)}{2n}$$

output
```
-1/2*x*Ei(1-1/2/n,x^(2*n))/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int e^{-x^{2n}}\,dx=-\frac{x\left(x^{2n}\right)^{-\frac{1}{2}/n}\Gamma\left(\frac{1}{2n},x^{2n}\right)}{2n}$$

input
```
Integrate[E^(-x^(2*n)),x]
```

output
```
-1/2*(x*Gamma[1/(2*n), x^(2*n)])/(n*(x^(2*n))^(1/(2*n)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^{2n}}\,dx$$

$$\Big\downarrow \; 2637$$

$$-\frac{x\left(x^{2n}\right)^{-\frac{1}{2}/n}\Gamma\left(\frac{1}{2n},x^{2n}\right)}{2n}$$

input
```
Int[E^(-x^(2*n)),x]
```

output
```
-1/2*(x*Gamma[1/(2*n), x^(2*n)])/(n*(x^(2*n))^(1/(2*n)))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.04

method	result
meijerg	$\frac{\frac{4n^{2}x^{-2n+1}\left(2n\,x^{2n}+2n+1\right)\left(x^{2n}\right)^{-\frac{2n+1}{4n}}\mathrm{e}^{-\frac{x^{2n}}{2}}\operatorname{WhittakerM}\left(\frac{1}{2n}-\frac{2n+1}{4n},\frac{2n+1}{4n}+\frac{1}{2},x^{2n}\right)}{(2n+1)(4n+1)}+\frac{2n\,x^{-2n+1}(2n+1)\left(x^{2n}\right)^{-\frac{2n+1}{4n}}\mathrm{e}^{-\frac{x^{2n}}{2}}\operatorname{Whitta}}{4n+1}}{2n}$

input
```
int(exp(-x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*(4*n^2*x^(-2*n+1)*(2*n*x^(2*n)+2*n+1)/(2*n+1)/(4*n+1)*(x^(2*n))^(-1/
4*(2*n+1)/n)*exp(-1/2*x^(2*n))*WhittakerM(1/2/n-1/4*(2*n+1)/n,1/4*(2*n+1)/
n+1/2,x^(2*n))+2*n*x^(-2*n+1)*(2*n+1)/(4*n+1)*(x^(2*n))^(-1/4*(2*n+1)/n)*e
xp(-1/2*x^(2*n))*WhittakerM(1/2/n-1/4*(2*n+1)/n+1,1/4*(2*n+1)/n+1/2,x^(2*n
)))
```

Fricas [F]

$$\int e^{-x^{2n}}\,dx = \int e^{\left(-x^{2\,n}\right)}\,dx$$

input
```
integrate(exp(-x^(2*n)),x, algorithm="fricas")
```

output
```
integral(e^(-x^(2*n)), x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int e^{-x^{2n}}\,dx = \frac{\Gamma\left(\frac{1}{2n}\right)\gamma\left(\frac{1}{2n},x^{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)}$$

input
```
integrate(exp(-x**(2*n)),x)
```

output
```
gamma(1/(2*n))*lowergamma(1/(2*n), x**(2*n))/(4*n**2*gamma(1 + 1/(2*n)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int e^{-x^{2n}}\,dx = -\frac{x\Gamma\left(\frac{1}{2n}, x^{2n}\right)}{2n(x^{2n})^{\frac{1}{2n}}}$$

input
```
integrate(exp(-x^(2*n)),x, algorithm="maxima")
```

output
```
-1/2*x*gamma(1/2/n, x^(2*n))/(n*(x^(2*n))^(1/2/n))
```

Giac [F]

$$\int e^{-x^{2n}}\,dx = \int e^{\left(-x^{2n}\right)}\,dx$$

input
```
integrate(exp(-x^(2*n)),x, algorithm="giac")
```

output
```
integrate(e^(-x^(2*n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-x^{2n}}\,dx = \int \mathrm{e}^{-x^{2n}}\,dx$$

input
```
int(exp(-x^(2*n)),x)
```

output
```
int(exp(-x^(2*n)), x)
```

Reduce [F]

$$\int e^{-x^{2n}}\,dx=\int \frac{1}{e^{x^{2n}}}dx$$

input
```
int(exp(-x^(2*n)),x)
```

output
```
int(1/e**(x**(2*n)),x)
```

3.206 $\int e\,dx$

Optimal result

Integrand size = 1, antiderivative size = 3

$$\int e\,dx = ex$$

output
```
exp(1)*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
Integrate[E,x]
```

output
```
E*x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e\,dx$$

$$\Big\downarrow \; 24$$

$$ex$$

input
```
Int[E,x]
```

output
```
E*x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

method	result	size
default	ex	5
norman	ex	5
risch	ex	5
parallelrisch	ex	5
orering	ex	5

input
```
int(exp(1),x,method=_RETURNVERBOSE)
```

output
```
exp(1)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="fricas")
```

output
```
x*e
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
integrate(exp(1),x)
```

output
```
E*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="maxima")
```

output
```
x*e
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="giac")
```

output
```
x*e
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = x\,\mathrm{e}$$

input
```
int(exp(1),x)
```

output
```
x*exp(1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
int(exp(1),x)
```

output
```
e*x
```

3.207 $\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}\,dx = -\frac{2}{39}\log\left(\cos\left(\frac{39x}{2}\right)\right)$$

output
```
-2/39*ln(cos(39/2*x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 11.91

$$\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}\,dx = -\frac{2}{39}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{2}{39}\log(1-2\cos(x)+2\cos(2x) -2\cos(3x)+2\cos(4x)-2\cos(5x)+2\cos(6x)-2\cos(7x) +2\cos(8x)-2\cos(9x)+2\cos(10x)-2\cos(11x) +2\cos(12x)-2\cos(13x)+2\cos(14x)-2\cos(15x) +2\cos(16x)-2\cos(17x)+2\cos(18x)-2\cos(19x))$$

input
```
Integrate[(Sin[19*x] + Sin[20*x])/(Cos[19*x] + Cos[20*x]),x]
```

output
```
(-2*Log[Cos[x/2]])/39 - (2*Log[1 - 2*Cos[x] + 2*Cos[2*x] - 2*Cos[3*x] + 2*
Cos[4*x] - 2*Cos[5*x] + 2*Cos[6*x] - 2*Cos[7*x] + 2*Cos[8*x] - 2*Cos[9*x]
+ 2*Cos[10*x] - 2*Cos[11*x] + 2*Cos[12*x] - 2*Cos[13*x] + 2*Cos[14*x] - 2*
Cos[15*x] + 2*Cos[16*x] - 2*Cos[17*x] + 2*Cos[18*x] - 2*Cos[19*x]])/39
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. $2(11) = 22$.

Time = 2.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 12.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3042, 4901, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{\sin(19x)}{\cos(19x)+\cos(20x)}+\frac{\sin(20x)}{\cos(19x)+\cos(20x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{39}\log\left(-64\cos^6(x)+32\cos^5(x)+80\cos^4(x)-32\cos^3(x)-24\cos^2(x)+6\cos(x)+1\right)-\frac{2}{39}\log(1-2\cos(x))-\frac{1}{39}\log(\cos(x)+1)-\frac{2}{39}\log\left(4096\cos^{12}(x)+2048\cos^{11}(x)-12288\cos^{10}(x)-6144\cos^9(x)+13568\cos^8(x)+6784\cos^7(x)-6592\text{co}\right.$$

input
```
Int[(Sin[19*x] + Sin[20*x])/(Cos[19*x] + Cos[20*x]),x]
```

output
```
(-2*Log[1 - 2*Cos[x]])/39 - Log[1 + Cos[x]]/39 - (2*Log[1 + 6*Cos[x] - 24*
Cos[x]^2 - 32*Cos[x]^3 + 80*Cos[x]^4 + 32*Cos[x]^5 - 64*Cos[x]^6])/39 - (2
*Log[1 - 24*Cos[x] - 48*Cos[x]^2 + 632*Cos[x]^3 + 1264*Cos[x]^4 - 3296*Cos
[x]^5 - 6592*Cos[x]^6 + 6784*Cos[x]^7 + 13568*Cos[x]^8 - 6144*Cos[x]^9 - 1
2288*Cos[x]^10 + 2048*Cos[x]^11 + 4096*Cos[x]^12])/39
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
risch	$ix-\frac{2\ln\left(e^{39ix}+1\right)}{39}$	16
parallelrisch	$\ln\left(\left(\sec\left(10x\right)^{2}\right)^{\frac{1}{39}}\right)+\ln\left(\left(\sec\left(\frac{19x}{2}\right)^{2}\right)^{\frac{1}{39}}\right)+\ln\left(\frac{1}{\left(\tan\left(\frac{19x}{2}\right)\tan\left(10x\right)-1\right)^{\frac{2}{39}}}\right)$	34

input
```
int((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-2/39*ln(exp(39*I*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(7) = 14$.

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 11.18

$$\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)}\,dx = -\frac{1}{39}\log\left(137438953472\cos(x)^{39} - 1340029796352\cos(x)^{37} + 6030134083584\cos(x)^{35} - 16610786017280\cos(x)^{33} + 31323196489728\cos(x)^{31} - 42839077552128\cos(x)^{29} + 43920872439808\cos(x)^{27} - 34411219255296\cos(x)^{25} + 20813237452800\cos(x)^{23} - 9751387176960\cos(x)^{21} + 3530674667520\cos(x)^{19} - 980106117120\cos(x)^{17} + 205701283840\cos(x)^{15} - 31950643200\cos(x)^{13} + 3560214528\cos(x)^{11} - 271960832\cos(x)^{9} + 13302432\cos(x)^{7} - 373464\cos(x)^{5} + 4940\cos(x)^{3} - \frac{39}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x, algorithm="fricas
")
```

output
```
-1/39*log(137438953472*cos(x)^39 - 1340029796352*cos(x)^37 + 6030134083584
*cos(x)^35 - 16610786017280*cos(x)^33 + 31323196489728*cos(x)^31 - 4283907
7552128*cos(x)^29 + 43920872439808*cos(x)^27 - 34411219255296*cos(x)^25 +
20813237452800*cos(x)^23 - 9751387176960*cos(x)^21 + 3530674667520*cos(x)^
19 - 980106117120*cos(x)^17 + 205701283840*cos(x)^15 - 31950643200*cos(x)^
13 + 3560214528*cos(x)^11 - 271960832*cos(x)^9 + 13302432*cos(x)^7 - 37346
4*cos(x)^5 + 4940*cos(x)^3 - 39/2*cos(x) + 1/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(19x) + \sin(20x)}{\cos(19x) + \cos(20x)}\,dx = \text{Timed out}$$

input
```
integrate((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. 2(7) = 14.

Time = 0.22 (sec) , antiderivative size = 2527, normalized size of antiderivative = 229.73

$$\int \frac{\sin(19x) + \sin(20x)}{\cos(19x) + \cos(20x)}\,dx = \text{Too large to display}$$

input
```
integrate((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x, algorithm="maxima
")
```

output
```
-1/39*log(2*(cos(23*x) - cos(21*x) - cos(20*x) + cos(18*x) + cos(17*x) - c
os(15*x) - cos(14*x) + cos(12*x) - cos(10*x) - cos(9*x) + cos(7*x) + cos(6
*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(24*x) + cos(24*x)^2 - 2*(cos(2
1*x) + cos(20*x) - cos(18*x) - cos(17*x) + cos(15*x) + cos(14*x) - cos(12*
x) + cos(10*x) + cos(9*x) - cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - co
s(x) - 1)*cos(23*x) + cos(23*x)^2 + 2*(cos(20*x) - cos(18*x) - cos(17*x) +
 cos(15*x) + cos(14*x) - cos(12*x) + cos(10*x) + cos(9*x) - cos(7*x) - cos
(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(21*x) + cos(21*x)^2 - 2*(cos
(18*x) + cos(17*x) - cos(15*x) - cos(14*x) + cos(12*x) - cos(10*x) - cos(9
*x) + cos(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(20*x) +
cos(20*x)^2 + 2*(cos(17*x) - cos(15*x) - cos(14*x) + cos(12*x) - cos(10*x)
 - cos(9*x) + cos(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(
18*x) + cos(18*x)^2 - 2*(cos(15*x) + cos(14*x) - cos(12*x) + cos(10*x) + c
os(9*x) - cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(17*x
) + cos(17*x)^2 + 2*(cos(14*x) - cos(12*x) + cos(10*x) + cos(9*x) - cos(7*
x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(15*x) + cos(15*x)^2
- 2*(cos(12*x) - cos(10*x) - cos(9*x) + cos(7*x) + cos(6*x) - cos(4*x) - c
os(3*x) + cos(x) + 1)*cos(14*x) + cos(14*x)^2 - 2*(cos(10*x) + cos(9*x) -
cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(12*x) + cos(12
*x)^2 + 2*(cos(9*x) - cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(7) = 14$.

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 12.18

$$\int \frac{\sin(19x) + \sin(20x)}{\cos(19x) + \cos(20x)}\,dx = -\frac{1}{39}\log(\cos(x) + 1)$$
$$-\frac{2}{39}\log\left(\left|4096\cos(x)^{12} + 2048\cos(x)^{11} - 12288\cos(x)^{10} - 6144\cos(x)^{9} + 13568\cos(x)^{8} + 6784\, c\right.\right.$$
$$-\frac{2}{39}\log\left(\left|64\cos(x)^{6} - 32\cos(x)^{5} - 80\cos(x)^{4} + 32\cos(x)^{3} + 24\cos(x)^{2} - 6\cos(x) - 1\right|\right)$$
$$-\frac{2}{39}\log(|2\cos(x) - 1|)$$

input
```
integrate((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x, algorithm="giac")
```

output
```
-1/39*log(cos(x) + 1) - 2/39*log(abs(4096*cos(x)^12 + 2048*cos(x)^11 - 122
88*cos(x)^10 - 6144*cos(x)^9 + 13568*cos(x)^8 + 6784*cos(x)^7 - 6592*cos(x
)^6 - 3296*cos(x)^5 + 1264*cos(x)^4 + 632*cos(x)^3 - 48*cos(x)^2 - 24*cos(
x) + 1)) - 2/39*log(abs(64*cos(x)^6 - 32*cos(x)^5 - 80*cos(x)^4 + 32*cos(x
)^3 + 24*cos(x)^2 - 6*cos(x) - 1)) - 2/39*log(abs(2*cos(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sin(19x) + \sin(20x)}{\cos(19x) + \cos(20x)}\,dx = x\,1\mathrm{i} - \frac{2\,\ln\left(\mathrm{e}^{x\,39\mathrm{i}} + 1\right)}{39}$$

input
```
int((sin(19*x) + sin(20*x))/(cos(19*x) + cos(20*x)),x)
```

output
```
x*1i - (2*log(exp(x*39i) + 1))/39
```

Reduce [F]

$$\int \frac{\sin(19x) + \sin(20x)}{\cos(19x) + \cos(20x)}\,dx = \frac{\left(\int \frac{\sin(19x)}{\cos(20x)+\cos(19x)}dx\right)}{20} - \frac{\log(\cos(20x) + \cos(19x))}{20}$$

input
```
int((sin(19*x)+sin(20*x))/(cos(19*x)+cos(20*x)),x)
```

output
```
(int(sin(19*x)/(cos(20*x) + cos(19*x)),x) - log(cos(20*x) + cos(19*x)))/20
```

3.208 $\int e^x(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 8

$$\int e^x\left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx=e^x\cos(x)\sin(x)$$

output
```
exp(x)*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int e^x\left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx=\frac{1}{2}e^x\sin(2x)$$

input
```
Integrate[E^x*(Cos[x]^2 + Cos[x]*Sin[x] - Sin[x]^2),x]
```

output
```
(E^x*Sin[2*x])/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(-\sin^2(x)+\cos^2(x)+\sin(x)\cos(x)\right)\,dx$$

$$\Big\downarrow \text{2726}$$

$$e^x\sin(x)\cos(x)$$

input
```
Int[E^x*(Cos[x]^2 + Cos[x]*Sin[x] - Sin[x]^2),x]
```

output
```
E^x*Cos[x]*Sin[x]
```

Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$\frac{e^{x} \sin(2x)}{2}$	9
risch	$-\frac{i e^{(1+2i)x}}{4}+\frac{i e^{(1-2i)x}}{4}$	20
norman	$\frac{2 e^{x} \tan\left(\frac{x}{2}\right)-2 e^{x} \tan\left(\frac{x}{2}\right)^{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	31
default	$\frac{(\cos(x)+2\sin(x))e^{x}\cos(x)}{5}+\frac{e^{x}(\sin(2x)-2\cos(2x))}{10}-\frac{(\sin(x)-2\cos(x))e^{x}\sin(x)}{5}$	43
parts	$\frac{(\cos(x)+2\sin(x))e^{x}\cos(x)}{5}+\frac{e^{x}(\sin(2x)-2\cos(2x))}{10}-\frac{(\sin(x)-2\cos(x))e^{x}\sin(x)}{5}$	43
orering	$\frac{e^{x}\left(\cos(x)^{2}+\cos(x)\sin(x)-\sin(x)^{2}\right)}{5}-\frac{e^{x}\left(-4\cos(x)\sin(x)-\sin(x)^{2}+\cos(x)^{2}\right)}{5}$	43

input
```
int(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{x}\left(\cos^{2}(x)+\cos(x)\sin(x)-\sin^{2}(x)\right)\,dx=\cos(x)\,e^{x}\sin(x)$$

input
```
integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="fricas")
```

output
```
cos(x)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int e^x \left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx = e^x\sin(x)\cos(x)$$

input
```
integrate(exp(x)*(cos(x)**2+cos(x)*sin(x)-sin(x)**2),x)
```

output
```
exp(x)*sin(x)*cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int e^x \left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx$$
$$= -\frac{1}{10}\,(2\,\cos(2\,x)-\sin(2\,x))e^x+\frac{1}{5}\,\cos(2\,x)\,e^x+\frac{2}{5}\,e^x\sin(2\,x)$$

input
```
integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="maxima")
```

output
```
-1/10*(2*cos(2*x) - sin(2*x))*e^x + 1/5*cos(2*x)*e^x + 2/5*e^x*sin(2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 4.12

$$\int e^x \left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx$$
$$= -\frac{1}{10}\,(2\,\cos(2\,x)-\sin(2\,x))e^x+\frac{1}{5}\,(\cos(2\,x)+2\,\sin(2\,x))e^x$$

input
```
integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="giac")
```

output
```
-1/10*(2*cos(2*x) - sin(2*x))*e^x + 1/5*(cos(2*x) + 2*sin(2*x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int e^x \left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx=\frac{\sin\left(2\,x\right)\,\mathrm{e}^x}{2}$$

input
```
int(exp(x)*(cos(x)*sin(x) + cos(x)^2 - sin(x)^2),x)
```

output
```
(sin(2*x)*exp(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int e^x \left(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x)\right)\,dx=e^x\cos\left(x\right)\sin\left(x\right)$$

input
```
int(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x)
```

output
```
e**x*cos(x)*sin(x)
```

3.209 $\int \csc \left(\frac{\pi}{4}+x\right) \sin(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \csc \left(\frac{\pi}{4}+x\right) \sin(x) \, dx = \frac{x}{\sqrt{2}} - \frac{\log \left(\sin \left(\frac{\pi}{4}+x\right)\right)}{\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/2*ln(sin(1/4*Pi+x))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \csc \left(\frac{\pi}{4}+x\right) \sin(x) \, dx = \left(-\frac{1}{4}-\frac{i}{4}\right) (-1)^{3/4} (2x - 2\text{arctanh}(\cot(x)) - \log(\cos(2x)))$$

input
```
Integrate[Csc[Pi/4 + x]*Sin[x],x]
```

output
```
(-1/4 - I/4)*(-1)^(3/4)*(2*x - 2*ArcTanh[Cot[x]] - Log[Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5093, 24, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc\left(x+\frac{\pi}{4}\right)dx$$

$$\Big\downarrow \text{ 5093}$$

$$\frac{\int 1dx}{\sqrt{2}}-\frac{\int \cot\left(x+\frac{\pi}{4}\right)dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{\sqrt{2}}-\frac{\int \cot\left(x+\frac{\pi}{4}\right)dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{\sqrt{2}}-\frac{\int -\tan\left(x+\frac{3\pi}{4}\right)dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \tan\left(x+\frac{3\pi}{4}\right)dx}{\sqrt{2}}+\frac{x}{\sqrt{2}}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{x}{\sqrt{2}}-\frac{\log\left(\sin\left(x+\frac{\pi}{4}\right)\right)}{\sqrt{2}}$$

input
```
Int[Csc[Pi/4 + x]*Sin[x],x]
```

output
```
x/Sqrt[2] - Log[Sin[Pi/4 + x]]/Sqrt[2]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5093
```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Simp[Sin[v - w]   Int[Cot[w]*Csc[w]
^(n - 1), x], x] + Simp[Cos[v - w]   Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0
] && FreeQ[v - w, x] && NeQ[w, v]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

method	result	size
default	$\sqrt{2}\left(-\frac{\ln(1+\tan(x))}{2}+\frac{\ln\left(1+\tan(x)^{2}\right)}{4}+\frac{\arctan(\tan(x))}{2}\right)$	27
risch	$\frac{x\sqrt{2}}{2}+\frac{i\sqrt{2}\,x}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\right)}{2}$	29

input
```
int(csc(1/4*Pi+x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(-1/2*ln(1+tan(x))+1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx = \frac{1}{2}\sqrt{2}x - \frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sin\left(\frac{1}{4}\pi+x\right)\right)$$

input
```
integrate(csc(1/4*pi+x)*sin(x),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*x - 1/2*sqrt(2)*log(1/2*sin(1/4*pi + x))
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx = \frac{\sqrt{2}x}{2} - \frac{\sqrt{2}\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{2}$$

input
```
integrate(csc(1/4*pi+x)*sin(x),x)
```

output
```
sqrt(2)*x/2 - sqrt(2)*log(sin(x) + cos(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx = \frac{1}{2}\sqrt{2}x - \frac{1}{4}\sqrt{2}\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\sin\left(2\,x\right)+1\right)$$

input
```
integrate(csc(1/4*pi+x)*sin(x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*x - 1/4*sqrt(2)*log(cos(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx = \frac{1}{8}\sqrt{2}(\pi+4x)+\frac{1}{2}\sqrt{2}\log\left(\tan\left(\frac{1}{8}\pi+\frac{1}{2}x\right)^2+1\right) - \frac{1}{2}\sqrt{2}\log\left(\left|\tan\left(\frac{1}{8}\pi+\frac{1}{2}x\right)\right|\right)$$

input
```
integrate(csc(1/4*pi+x)*sin(x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(pi + 4*x) + 1/2*sqrt(2)*log(tan(1/8*pi + 1/2*x)^2 + 1) - 1/2*
sqrt(2)*log(abs(tan(1/8*pi + 1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx = x\,e^{\frac{\Pi\,1i}{4}} - \frac{e^{-\frac{\Pi\,1i}{2}}\ln\left(e^{\frac{\Pi\,1i}{2}}\,e^{x\,2i}-1\right)\left(e^{\frac{\Pi\,1i}{4}}\,2i - e^{\frac{\Pi\,3i}{4}}\,2i\right)}{4}$$

input
```
int(sin(x)/sin(Pi/4 + x),x)
```

output
```
x*exp((Pi*1i)/4) - (exp(-(Pi*1i)/2)*log(exp((Pi*1i)/2)*exp(x*2i) - 1)*(exp
((Pi*1i)/4)*2i - exp((Pi*3i)/4)*2i))/4
```

Reduce [F]

$$\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx=\int \csc\left(\frac{\pi}{4}+x\right)\sin\left(x\right)dx$$

input
```
int(csc(1/4*Pi+x)*sin(x),x)
```

output
```
int(csc((pi + 4*x)/4)*sin(x),x)
```

3.210 $\int \frac{1}{\sqrt[3]{x}+x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx=\frac{3}{2}\log\left(1+x^{2/3}\right)$$

output
```
3/2*ln(1+x^(2/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx=\frac{3}{2}\log\left(1+x^{2/3}\right)$$

input
```
Integrate[(x^(1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(2/3)])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+\sqrt[3]{x}}dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{1}{\left(x^{2/3}+1\right)\sqrt[3]{x}}dx$$

$$\Big\downarrow \ 792$$

$$\frac{3}{2}\log\left(x^{2/3}+1\right)$$

input
```
Int[(x^(1/3) + x)^(-1),x]
```

output
```
(3*Log[1 + x^(2/3)])/2
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
trager	$\frac{\ln\left(3x^{\frac{2}{3}}+3x^{\frac{4}{3}}+x^2+1\right)}{2}$	19
default	$\frac{\ln(x^2+1)}{2}+\ln\left(1+x^{\frac{2}{3}}\right)-\frac{\ln\left(x^{\frac{4}{3}}-x^{\frac{2}{3}}+1\right)}{2}$	29

input
```
int(1/(x^(1/3)+x),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(1+x^(2/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx=\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x),x, algorithm="fricas")
```

output
```
3/2*log(x^(2/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx = \frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

input
```
integrate(1/(x**(1/3)+x),x)
```

output
```
3*log(x**(2/3) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx = \frac{3}{2}\,\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x),x, algorithm="maxima")
```

output
```
3/2*log(x^(2/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx = \frac{3}{2}\,\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x),x, algorithm="giac")
```

output
```
3/2*log(x^(2/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx=\frac{3\,\ln\left(x^{2/3}+1\right)}{2}$$

input
```
int(1/(x + x^(1/3)),x)
```

output
```
(3*log(x^(2/3) + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[3]{x}+x}\,dx=\frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

input
```
int(1/(x^(1/3)+x),x)
```

output
```
(3*log(x**(2/3) + 1))/2
```

3.211 $\int x^{1+x^2}(1+2\log(x))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 5

$$\int x^{1+x^2}(1+2\log(x))\,dx = x^{x^2}$$

output
```
x^(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{1+x^2}(1+2\log(x))\,dx = x^{x^2}$$

input
```
Integrate[x^(1 + x^2)*(1 + 2*Log[x]),x]
```

output
```
x^x^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{x^2+1}(2\log(x)+1)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^{x^2+1}+2x^{x^2+1}\log(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int x^{x^2+1}dx-2\int \frac{\int x^{x^2+1}dx}{x}dx+2\log(x)\int x^{x^2+1}dx$$

input
```
Int[x^(1 + x^2)*(1 + 2*Log[x]),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.40

method	result	size
risch	$\frac{x^{x^2+1}}{x}$	12
parallelrisch	$\frac{x^{x^2+1}}{x}$	12

input
```
int(x^(x^2+1)*(1+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x^(x^2+1)/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int x^{1+x^2}(1+2\log(x))\,dx=\frac{x^{x^2+1}}{x}$$

input
```
integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="fricas")
```

output
```
x^(x^2 + 1)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int x^{1+x^2}(1+2\log(x))\,dx=\frac{x^{x^2+1}}{x}$$

input
```
integrate(x**(x**2+1)*(1+2*ln(x)),x)
```

output
```
x**(x**2 + 1)/x
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{1+x^2}(1+2\log(x))\,dx=x^{(x^2)}$$

input
```
integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="maxima")
```

output
```
x^(x^2)
```

Giac [F]

$$\int x^{1+x^2}(1+2\log(x))\,dx = \int x^{x^2+1}(2\,\log(x)+1)\,dx$$

input
```
integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="giac")
```

output
```
integrate(x^(x^2 + 1)*(2*log(x) + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{1+x^2}(1+2\log(x))\,dx = x^{x^2}$$

input
```
int(x^(x^2 + 1)*(2*log(x) + 1),x)
```

output
```
x^(x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{1+x^2}(1+2\log(x))\,dx = x^{x^2}$$

input
```
int(x^(x^2+1)*(1+2*log(x)),x)
```

output
```
x**(x**2)
```

3.212 $\int \frac{-1+2x^3}{x(1+x^3)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{-1+2x^3}{x\left(1+x^3\right)}\,dx = -\log(x)+\log\left(1+x^3\right)$$

output
```
-ln(x)+ln(x^3+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^3}{x\left(1+x^3\right)}\,dx = -\log(x)+\log\left(1+x^3\right)$$

input
```
Integrate[(-1 + 2*x^3)/(x*(1 + x^3)),x]
```

output
```
-Log[x] + Log[1 + x^3]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {948, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-1}{x\left(x^3+1\right)}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int -\frac{1-2x^3}{x^3\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{3}\int \frac{1-2x^3}{x^3\left(x^3+1\right)}dx^3$$

$$\Big\downarrow \; 86$$

$$-\frac{1}{3}\int \left(\frac{1}{x^3}-\frac{3}{x^3+1}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(3\log\left(x^3+1\right)-\log\left(x^3\right)\right)$$

input
```
Int[(-1 + 2*x^3)/(x*(1 + x^3)),x]
```

output
```
(-Log[x^3] + 3*Log[1 + x^3])/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
meijerg	$-\ln(x)+\ln(x^3+1)$	12
risch	$-\ln(x)+\ln(x^3+1)$	12
default	$\ln(x^2-x+1)-\ln(x)+\ln(1+x)$	19
norman	$\ln(x^2-x+1)-\ln(x)+\ln(1+x)$	19
parallelrisch	$\ln(x^2-x+1)-\ln(x)+\ln(1+x)$	19

input
```
int((2*x^3-1)/x/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^3}{x\left(1+x^3\right)}\,dx=\log\left(x^3+1\right)-\log\left(x\right)$$

input
```
integrate((2*x^3-1)/x/(x^3+1),x, algorithm="fricas")
```

output
```
log(x^3 + 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+2x^3}{x\left(1+x^3\right)}\,dx=-\log\left(x\right)+\log\left(x^3+1\right)$$

input
```
integrate((2*x**3-1)/x/(x**3+1),x)
```

output
```
-log(x) + log(x**3 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-1+2x^3}{x\left(1+x^3\right)}\,dx=\log\left(x^3+1\right)-\frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((2*x^3-1)/x/(x^3+1),x, algorithm="maxima")
```

output
```
log(x^3 + 1) - 1/3*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-1+2x^3}{x(1+x^3)}\,dx = \log\left(\left|x^3+1\right|\right) - \log\left(|x|\right)$$

input
```
integrate((2*x^3-1)/x/(x^3+1),x, algorithm="giac")
```

output
```
log(abs(x^3 + 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^3}{x(1+x^3)}\,dx = \ln\left(x^3+1\right) - \ln\left(x\right)$$

input
```
int((2*x^3 - 1)/(x*(x^3 + 1)),x)
```

output
```
log(x^3 + 1) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{-1+2x^3}{x(1+x^3)}\,dx = \log\left(x^2-x+1\right) + \log(x+1) - \log(x)$$

input
```
int((2*x^3-1)/x/(x^3+1),x)
```

output
```
log(x**2 - x + 1) + log(x + 1) - log(x)
```

3.213 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = -\log \left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x^2],x]
```

output
```
-Log[-x + Sqrt[1 + x^2]]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \ 222$$

$$\text{arcsinh}(x)$$

input
```
Int[1/Sqrt[1 + x^2],x]
```

output
```
ArcSinh[x]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\text{arcsinh}\left(x\right)$	3
meijerg	$\text{arcsinh}\left(x\right)$	3
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^2+1}\right)$	15

input
```
int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
integrate(1/(x**2+1)**(1/2),x)
```

output
```
asinh(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
int(1/(x^2 + 1)^(1/2),x)
```

output
```
asinh(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \log\left(\sqrt{x^2+1}+x\right)$$

input
```
int(1/(x^2+1)^(1/2),x)
```

output
```
log(sqrt(x**2 + 1) + x)
```

3.214 $\int \frac{\log(2x)}{x\log(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(x) + \log(2)\log(\log(x))$$

output
```
ln(x)+ln(2)*ln(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(x) + \log(2)\log(\log(x))$$

input
```
Integrate[Log[2*x]/(x*Log[x]),x]
```

output
```
Log[x] + Log[2]*Log[Log[x]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2813, 3001}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(2x)}{x\log(x)}dx$$
$$\Big\downarrow \text{2813}$$
$$\log(2x)\log(\log(x)) - \int \frac{\log(\log(x))}{x}dx$$
$$\Big\downarrow \text{3001}$$
$$-\log(\log(x))\log(x) + \log(x) + \log(2x)\log(\log(x))$$

input
```
Int[Log[2*x]/(x*Log[x]),x]
```

output
```
Log[x] - Log[x]*Log[Log[x]] + Log[2*x]*Log[Log[x]]
```

Defintions of rubi rules used

rule 2813
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Simp[(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[Simp
lifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r},
x] &&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

rule 3001
```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\ln(x)+\ln(2)\ln(\ln(x))$	10
risch	$\ln(x)+\ln(2)\ln(\ln(x))$	10

input
```
int(ln(2*x)/x/ln(x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+ln(2)*ln(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(2)\log(\log(x)) + \log(x)$$

input
```
integrate(log(2*x)/x/log(x),x, algorithm="fricas")
```

output
```
log(2)*log(log(x)) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(x) + \log(2)\log(\log(x))$$

input
```
integrate(ln(2*x)/x/ln(x),x)
```

output
```
log(x) + log(2)*log(log(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(2x)\log(\log(x)) - \log(x)\log(\log(x)) + \log(x)$$

input
```
integrate(log(2*x)/x/log(x),x, algorithm="maxima")
```

output
```
log(2*x)*log(log(x)) - log(x)*log(log(x)) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \log(2)\log(|\log(x)|) + \log(x)$$

input
```
integrate(log(2*x)/x/log(x),x, algorithm="giac")
```

output
```
log(2)*log(abs(log(x))) + log(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \ln(x) + \ln(\ln(x))\ln(2)$$

input
```
int(log(2*x)/(x*log(x)),x)
```

output
```
log(x) + log(log(x))*log(2)
```

Reduce [F]

$$\int \frac{\log(2x)}{x\log(x)}\,dx = \int \frac{\log(2x)}{\log(x)\,x}dx$$

input
```
int(log(2*x)/x/log(x),x)
```

output
```
int(log(2*x)/(log(x)*x),x)
```

3.215 $\int \frac{1}{1+e^x} dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{1+e^x} dx = x - \log(1+e^x)$$

output
```
x-ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+e^x} dx = -2\text{arctanh}(1+2e^x)$$

input
```
Integrate[(1 + E^x)^(-1),x]
```

output
```
-2*ArcTanh[1 + 2*E^x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^x+1}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{e^{-x}}{e^x+1}de^x$$

$$\Big\downarrow \text{47}$$

$$\int e^{-x}de^x-\int \frac{1}{1+e^x}de^x$$

$$\Big\downarrow \text{14}$$

$$\log\left(e^x\right)-\int \frac{1}{1+e^x}de^x$$

$$\Big\downarrow \text{16}$$

$$\log\left(e^x\right)-\log\left(e^x+1\right)$$

input
```
Int[(1 + E^x)^(-1),x]
```

output
```
Log[E^x] - Log[1 + E^x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
norman	$x - \ln(1 + e^x)$	10
risch	$x - \ln(1 + e^x)$	10
parallelrisch	$x - \ln(1 + e^x)$	10
derivativedivides	$\ln(e^x) - \ln(1 + e^x)$	12
default	$\ln(e^x) - \ln(1 + e^x)$	12

input
```
int(1/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="fricas")
```

output
```
x - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x)
```

output
```
x - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="maxima")
```

output
```
x - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \log\left(e^x + 1\right)$$

input
```
integrate(1/(1+exp(x)),x, algorithm="giac")
```

output
```
x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+e^x}\,dx = x - \ln\left(\mathrm{e}^x + 1\right)$$

input
```
int(1/(exp(x) + 1),x)
```

output
```
x - log(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+e^x}\,dx = -\log(e^x + 1) + x$$

input
```
int(1/(1+exp(x)),x)
```

output
```
- log(e**x + 1) + x
```

3.216 $\int \frac{\log(x)\log(\log(x))}{x}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = -\frac{1}{4}\log^2(x) + \frac{1}{2}\log^2(x)\log(\log(x))$$

output
```
-1/4*ln(x)^2+1/2*ln(x)^2*ln(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = -\frac{1}{4}\log^2(x) + \frac{1}{2}\log^2(x)\log(\log(x))$$

input
```
Integrate[(Log[x]*Log[Log[x]])/x,x]
```

output
```
-1/4*Log[x]^2 + (Log[x]^2*Log[Log[x]])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3039, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\int \log(x)\log(\log(x))d\log(x)$$

$$\Big\downarrow \text{2741}$$

$$\frac{1}{2}\log^2(x)\log(\log(x))-\frac{\log^2(x)}{4}$$

input
```
Int[(Log[x]*Log[Log[x]])/x,x]
```

output
```
-1/4*Log[x]^2 + (Log[x]^2*Log[Log[x]])/2
```

Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
default	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
norman	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
risch	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17

input
```
int(ln(x)*ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x)^2+1/2*ln(x)^2*ln(ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{1}{2}\log(x)^2\log(\log(x)) - \frac{1}{4}\log(x)^2$$

input
```
integrate(log(x)*log(log(x))/x,x, algorithm="fricas")
```

output
```
1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{\log(x)^2\log(\log(x))}{2} - \frac{\log(x)^2}{4}$$

input
```
integrate(ln(x)*ln(ln(x))/x,x)
```

output
```
log(x)**2*log(log(x))/2 - log(x)**2/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{1}{2}\log\left(x\right)^2\log\left(\log\left(x\right)\right) - \frac{1}{4}\log\left(x\right)^2$$

input
```
integrate(log(x)*log(log(x))/x,x, algorithm="maxima")
```

output
```
1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{1}{2}\log\left(x\right)^2\log\left(\log\left(x\right)\right) - \frac{1}{4}\log\left(x\right)^2$$

input
```
integrate(log(x)*log(log(x))/x,x, algorithm="giac")
```

output
```
1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{\ln\left(x\right)^2\,\left(2\,\ln\left(\ln\left(x\right)\right)-1\right)}{4}$$

input
```
int((log(log(x))*log(x))/x,x)
```

output
```
(log(x)^2*(2*log(log(x)) - 1))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{\log(x)\log(\log(x))}{x}\,dx = \frac{\log(x)^2\,(2\log(\log(x)) - 1)}{4}$$

input
```
int(log(x)*log(log(x))/x,x)
```

output
```
(log(x)**2*(2*log(log(x)) - 1))/4
```

3.217 $\int \log \left(\frac{1+x}{1-x}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \log \left(\frac{1+x}{1-x}\right) dx = 2\log(1-x) + (1+x)\log\left(\frac{1+x}{1-x}\right)$$

output
```
2*ln(1-x)+(1+x)*ln((1+x)/(1-x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log \left(\frac{1+x}{1-x}\right) dx = 2\log(1-x) + (1+x)\log\left(\frac{1+x}{1-x}\right)$$

input
```
Integrate[Log[(1 + x)/(1 - x)],x]
```

output
```
2*Log[1 - x] + (1 + x)*Log[(1 + x)/(1 - x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2935, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(\frac{x+1}{1-x}\right) dx$$

$$\Big\downarrow \text{ 2935}$$

$$(x+1)\log\left(\frac{x+1}{1-x}\right) - 2\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$2\log(1-x) + (x+1)\log\left(\frac{x+1}{1-x}\right)$$

input
```
Int[Log[(1 + x)/(1 - x)],x]
```

output
```
2*Log[1 - x] + (1 + x)*Log[(1 + x)/(1 - x)]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2935
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Simp[B*n*p*((b*c - a*d)/b)   Int[(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, 
x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$x \ln\left(\frac{1+x}{1-x}\right)+\ln\left(x^{2}-1\right)$	22
parts	$x \ln\left(\frac{1+x}{1-x}\right)+\ln\left(\left(-1+x\right)\left(1+x\right)\right)$	24
meijerg	$\frac{(2x+2)\ln(1+x)}{2}+\frac{(-2x+2)\ln(1-x)}{2}$	26
parallelrisch	$x \ln\left(-\frac{1+x}{-1+x}\right)+2\ln\left(-1+x\right)+\ln\left(-\frac{1+x}{-1+x}\right)$	32
derivativedivides	$-2\ln\left(-\frac{2}{-1+x}\right)-\ln\left(-1-\frac{2}{-1+x}\right)\left(-1-\frac{2}{-1+x}\right)\left(-1+x\right)$	36
default	$-2\ln\left(-\frac{2}{-1+x}\right)-\ln\left(-1-\frac{2}{-1+x}\right)\left(-1-\frac{2}{-1+x}\right)\left(-1+x\right)$	36

input
```
int(ln((1+x)/(1-x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln((1+x)/(1-x))+ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = x\log\left(-\frac{x+1}{x-1}\right)+\log\left(x^{2}-1\right)$$

input
```
integrate(log((1+x)/(1-x)),x, algorithm="fricas")
```

output
```
x*log(-(x + 1)/(x - 1)) + log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = x\log\left(\frac{x+1}{1-x}\right) + \log\left(x^2-1\right)$$

input
```
integrate(ln((1+x)/(1-x)),x)
```

output
```
x*log((x + 1)/(1 - x)) + log(x**2 - 1)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = x\log\left(-\frac{x+1}{x-1}\right) + \log\left(x+1\right) + \log\left(x-1\right)$$

input
```
integrate(log((1+x)/(1-x)),x, algorithm="maxima")
```

output
```
x*log(-(x + 1)/(x - 1)) + log(x + 1) + log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.28

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = \frac{2\log\left(-\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}-1} + 2\log\left(\frac{|-x-1|}{|x-1|}\right) - 2\log\left(\left|-\frac{x+1}{x-1}+1\right|\right)$$

input
```
integrate(log((1+x)/(1-x)),x, algorithm="giac")
```

output
```
2*log(-(((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1)
 + 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) - 1) + 2*log(abs(-x - 1
)/abs(x - 1)) - 2*log(abs(-(x + 1)/(x - 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = \ln\left(x^2-1\right) + x\,\ln\left(-\frac{x+1}{x-1}\right)$$

input
```
int(log(-(x + 1)/(x - 1)),x)
```

output
```
log(x^2 - 1) + x*log(-(x + 1)/(x - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \log\left(\frac{1+x}{1-x}\right)\,dx = 2\log(x-1) + \log\left(\frac{-x-1}{x-1}\right)x + \log\left(\frac{-x-1}{x-1}\right)$$

input
```
int(log((1+x)/(1-x)),x)
```

output
```
2*log(x - 1) + log(( - x - 1)/(x - 1))*x + log(( - x - 1)/(x - 1))
```

3.218 $\int \frac{1}{(-1+x)^2+x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{(-1+x)^2+x^2} \, dx = -\arctan(1-2x)$$

output
```
arctan(-1+2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1+x)^2+x^2} \, dx = \arctan(1+2(-1+x))$$

input
```
Integrate[((-1 + x)^2 + x^2)^(-1),x]
```

output
```
ArcTan[1 + 2*(-1 + x)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2080, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+(x-1)^2}\,dx$$

$$\Big\downarrow \text{ 2080}$$

$$\int \frac{1}{2x^2-2x+1}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\int \frac{1}{-(1-2x)^2-1}d(1-2x)$$

$$\Big\downarrow \text{ 217}$$

$$-\arctan(1-2x)$$

input
```
Int[((-1 + x)^2 + x^2)^(-1),x]
```

output
```
-ArcTan[1 - 2*x]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2080
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\arctan(-1+2x)$	7
risch	$\arctan(-1+2x)$	7
parallelrisch	$-\frac{i\ln\left(x-\frac{1}{2}-\frac{i}{2}\right)}{2}+\frac{i\ln\left(x-\frac{1}{2}+\frac{i}{2}\right)}{2}$	20

input
```
int(1/((-1+x)^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(-1+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)^2+x^2}\,dx=\arctan(2\,x-1)$$

input
```
integrate(1/((-1+x)^2+x^2),x, algorithm="fricas")
```

output
```
arctan(2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{(-1+x)^2+x^2}\,dx = \operatorname{atan}(2x-1)$$

input
```
integrate(1/((-1+x)**2+x**2),x)
```

output
```
atan(2*x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)^2+x^2}\,dx = \arctan(2\,x-1)$$

input
```
integrate(1/((-1+x)^2+x^2),x, algorithm="maxima")
```

output
```
arctan(2*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)^2+x^2}\,dx = \arctan(2\,x-1)$$

input
```
integrate(1/((-1+x)^2+x^2),x, algorithm="giac")
```

output
```
arctan(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)^2+x^2}\,dx = \operatorname{atan}(2\,x-1)$$

input
```
int(1/((x - 1)^2 + x^2),x)
```

output
```
atan(2*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)^2+x^2}\,dx = atan(2x-1)$$

input
```
int(1/((-1+x)^2+x^2),x)
```

output
```
atan(2*x - 1)
```

3.219 $\int \sqrt{x\sqrt{x^{3/2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}$$

output
```
8/15*x*(x*(x^(3/2))^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}$$

input
```
Integrate[Sqrt[x*Sqrt[x^(3/2)]],x]
```

output
```
(8*x*Sqrt[x*Sqrt[x^(3/2)]])/15
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {7267, 7270, 21, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x\sqrt{x^{3/2}}}dx$$

$$\Big\downarrow \text{ 7267}$$

$$2\int \sqrt{x}\sqrt{x\sqrt{x^{3/2}}}d\sqrt{x}$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{2\sqrt{x\sqrt{x^{3/2}}}\int x\sqrt[4]{x^{3/2}}d\sqrt{x}}{\sqrt{x}\sqrt[4]{x^{3/2}}}$$

$$\Big\downarrow \text{ 21}$$

$$\frac{2\sqrt{x\sqrt{x^{3/2}}}\int \sqrt[8]{x}dx^{3/2}}{3\sqrt{x}\sqrt[4]{x^{3/2}}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{8\sqrt[8]{x}\sqrt{x\sqrt{x^{3/2}}}}{15\sqrt[4]{x^{3/2}}}$$

input
```
Int[Sqrt[x*Sqrt[x^(3/2)]],x]
```

output
```
(8*x^(1/8)*Sqrt[x*Sqrt[x^(3/2)]])/(15*(x^(3/2))^(1/4))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13
derivativedivides	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13
default	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13
orering	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13

input
```
int((x*(x^(3/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*x*(x*(x^(3/2))^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8}{15}\sqrt{\sqrt{x^{\frac{3}{2}}}x}x$$

input
```
integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/15*sqrt(sqrt(x^(3/2))*x)*x
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$$

input
```
integrate((x*(x**(3/2))**(1/2))**(1/2),x)
```

output
```
8*x*sqrt(x*sqrt(x**(3/2)))/15
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.25

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8}{15}\,x^{\frac{15}{8}}$$

input
```
integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/15*x^(15/8)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:gen.cc:simplify/tmp.type!=_EXT Erro
r: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8\,x\,\sqrt{x\,\sqrt{x^{3/2}}}}{15}$$

input
```
int((x*(x^(3/2))^(1/2))^(1/2),x)
```

output
```
(8*x*(x*(x^(3/2))^(1/2))^(1/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.25

$$\int \sqrt{x\sqrt{x^{3/2}}}\,dx = \frac{8x^{\frac{15}{8}}}{15}$$

input
```
int((x*(x^(3/2))^(1/2))^(1/2),x)
```

output
```
(8*x**(7/8)*x)/15
```

3.220 $\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx=\frac{1}{5}\cos^5(x)\sin^5(x)$$

output
```
1/5*cos(x)^5*sin(x)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx$$
$$=\frac{1}{256}\sin(2x)-\frac{1}{512}\sin(6x)+\frac{\sin(10x)}{2560}$$

input
```
Integrate[Cos[x]^4*(Cos[x] - Sin[x])*Sin[x]^4*(Cos[x] + Sin[x]),x]
```

output
```
Sin[2*x]/256 - Sin[6*x]/512 + Sin[10*x]/2560
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4889, 356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(x)\cos^4(x)(\cos(x)-\sin(x))(\sin(x)+\cos(x))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)^4\cos(x)^4(\cos(x)-\sin(x))(\sin(x)+\cos(x))dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{\tan^4(x)\left(1-\tan^2(x)\right)}{\left(\tan^2(x)+1\right)^6}d\tan(x)$$

$$\Big\downarrow \ 356$$

$$\frac{\tan^5(x)}{5\left(\tan^2(x)+1\right)^5}$$

input
```
Int[Cos[x]^4*(Cos[x] - Sin[x])*Sin[x]^4*(Cos[x] + Sin[x]),x]
```

output
```
Tan[x]^5/(5*(1 + Tan[x]^2)^5)
```

Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 6.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result
risch	$\frac{\sin(10x)}{2560}-\frac{\sin(6x)}{512}+\frac{\sin(2x)}{256}$
parallelrisch	$\frac{\left(\sin\left(\frac{5x}{2}\right)-5\sin\left(\frac{3x}{2}\right)+10\sin\left(\frac{x}{2}\right)\right)\left(\cos(5x)+5\cos(3x)+10\cos(x)\right)\left(\cos\left(\frac{5x}{2}\right)+5\cos\left(\frac{3x}{2}\right)+10\cos\left(\frac{x}{2}\right)\right)}{640}$
default	$-\frac{\sin(x)^3\cos(x)^7}{10}-\frac{3\sin(x)\cos(x)^7}{80}+\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{\cos(x)^5\sin(x)^5}{10}+\frac{\sin(x)^3\cos(x)^5}{16}+$
parts	$-\frac{\sin(x)^3\cos(x)^7}{10}-\frac{3\sin(x)\cos(x)^7}{80}+\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160}+\frac{\cos(x)^5\sin(x)^5}{10}+\frac{\sin(x)^3\cos(x)^5}{16}+$
orering	Expression too large to display

input
```
int(cos(x)^4*(cos(x)-sin(x))*sin(x)^4*(cos(x)+sin(x)),x,method=_RETURNVERB
OSE)
```

output
```
1/2560*sin(10*x)-1/512*sin(6*x)+1/256*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx$$
$$=\frac{1}{5}\left(\cos(x)^9-2\cos(x)^7+\cos(x)^5\right)\sin(x)$$

input
```
integrate(cos(x)^4*(cos(x)-sin(x))*sin(x)^4*(cos(x)+sin(x)),x, algorithm="
fricas")
```

output
```
1/5*(cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx=\frac{\sin^5(x)\cos^5(x)}{5}$$

input
```
integrate(cos(x)**4*(cos(x)-sin(x))*sin(x)**4*(cos(x)+sin(x)),x)
```

output
```
sin(x)**5*cos(x)**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx=\frac{1}{160}\sin(2x)^5$$

input
```
integrate(cos(x)^4*(cos(x)-sin(x))*sin(x)^4*(cos(x)+sin(x)),x, algorithm="
maxima")
```

output
```
1/160*sin(2*x)^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx = \frac{1}{2560}\sin(10x) - \frac{1}{512}\sin(6x) + \frac{1}{256}\sin(2x)$$

input
```
integrate(cos(x)^4*(cos(x)-sin(x))*sin(x)^4*(cos(x)+sin(x)),x, algorithm="
giac")
```

output
```
1/2560*sin(10*x) - 1/512*sin(6*x) + 1/256*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx = \frac{\cos(x)^5\sin(x)\left(\cos(x)^2-1\right)^2}{5}$$

input
```
int(cos(x)^4*sin(x)^4*(cos(x) + sin(x))*(cos(x) - sin(x)),x)
```

output
```
(cos(x)^5*sin(x)*(cos(x)^2 - 1)^2)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx = \frac{\cos(x)^5\sin(x)^5}{5}$$

input
```
int(cos(x)^4*(cos(x)-sin(x))*sin(x)^4*(cos(x)+sin(x)),x)
```

output
```
(cos(x)**5*sin(x)**5)/5
```

3.221 $\int \log(1+x^2)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \log(1+x^2)\,dx = -2x + 2\arctan(x) + x\log(1+x^2)$$

output
```
-2*x+2*arctan(x)+x*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log(1+x^2)\,dx = -2x + 2\arctan(x) + x\log(1+x^2)$$

input
```
Integrate[Log[1 + x^2],x]
```

output
```
-2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{ 2898}$$

$$x \log \left(x^2+1\right)-2 \int \frac{x^2}{x^2+1}dx$$

$$\Big\downarrow \text{ 262}$$

$$x \log \left(x^2+1\right)-2 \left(x-\int \frac{1}{x^2+1}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$x \log \left(x^2+1\right)-2 (x-\arctan (x))$$

input
```
Int[Log[1 + x^2],x]
```

output
```
-2*(x - ArcTan[x]) + x*Log[1 + x^2]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
risch	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
parts	$-2x+2\arctan\left(x\right)+x\ln\left(x^{2}+1\right)$	17
meijerg	$-2x+\frac{2x\arctan\left(\sqrt{x^{2}}\right)}{\sqrt{x^{2}}}+x\ln\left(x^{2}+1\right)$	27
parallelrisch	$-2i\ln\left(x-i\right)+i\ln\left(x^{2}+1\right)+x\ln\left(x^{2}+1\right)-2x$	30

input
```
int(ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x+2*arctan(x)+x*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log\left(1+x^{2}\right)\,dx=x\log\left(x^{2}+1\right)-2\,x+2\,\arctan\left(x\right)$$

input
```
integrate(log(x^2+1),x, algorithm="fricas")
```

output
```
x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \log \left(1+x^2\right) \, dx = x \log \left(x^2+1\right) - 2x + 2 \operatorname{atan}(x)$$

input
```
integrate(ln(x**2+1),x)
```

output
```
x*log(x**2 + 1) - 2*x + 2*atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(1+x^2\right) \, dx = x \log \left(x^2+1\right) - 2\, x + 2\, \arctan \left(x\right)$$

input
```
integrate(log(x^2+1),x, algorithm="maxima")
```

output
```
x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(1+x^2\right) \, dx = x \log \left(x^2+1\right) - 2\, x + 2\, \arctan \left(x\right)$$

input
```
integrate(log(x^2+1),x, algorithm="giac")
```

output
```
x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(1+x^2\right) \, dx = 2 \operatorname{atan}(x) - 2 x + x \ln \left(x^2+1\right)$$

input
```
int(log(x^2 + 1),x)
```

output
```
2*atan(x) - 2*x + x*log(x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \log \left(1+x^2\right) \, dx = 2atan(x) + \log\left(x^2+1\right) x - 2x$$

input
```
int(log(x^2+1),x)
```

output
```
2*atan(x) + log(x**2 + 1)*x - 2*x
```

3.222 $\int \frac{1+2x}{1+2x+2x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{1+2x}{1+2x+2x^2} \, dx = \frac{1}{2} \log \left(1+2x+2x^2\right)$$

output
```
1/2*ln(2*x^2+2*x+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{1+2x+2x^2} \, dx = \frac{1}{2} \log \left(1+2x+2x^2\right)$$

input
```
Integrate[(1 + 2*x)/(1 + 2*x + 2*x^2),x]
```

output
```
Log[1 + 2*x + 2*x^2]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{2x^2+2x+1}\,dx$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\log\left(2x^2+2x+1\right)$$

input
```
Int[(1 + 2*x)/(1 + 2*x + 2*x^2),x]
```

output
```
Log[1 + 2*x + 2*x^2]/2
```

Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x^2+x+\frac{1}{2}\right)}{2}$	10
default	$\frac{\ln\left(2x^2+2x+1\right)}{2}$	14
norman	$\frac{\ln\left(2x^2+2x+1\right)}{2}$	14
risch	$\frac{\ln\left(2x^2+2x+1\right)}{2}$	14

input
```
int((1+2*x)/(2*x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+x+1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{1}{2}\,\log\left(2\,x^2+2\,x+1\right)$$

input
```
integrate((1+2*x)/(2*x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/2*log(2*x^2 + 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{\log\left(2x^2+2x+1\right)}{2}$$

input
```
integrate((1+2*x)/(2*x**2+2*x+1),x)
```

output
```
log(2*x**2 + 2*x + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{1}{2}\,\log\left(2\,x^2+2\,x+1\right)$$

input
```
integrate((1+2*x)/(2*x^2+2*x+1),x, algorithm="maxima")
```

output
```
1/2*log(2*x^2 + 2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{1}{2}\,\log\left(2\,x^2+2\,x+1\right)$$

input
```
integrate((1+2*x)/(2*x^2+2*x+1),x, algorithm="giac")
```

output
```
1/2*log(2*x^2 + 2*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{\ln\left(x^2+x+\frac{1}{2}\right)}{2}$$

input
```
int((2*x + 1)/(2*x + 2*x^2 + 1),x)
```

output
```
log(x + x^2 + 1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+2x}{1+2x+2x^2}\,dx = \frac{\log(2x^2+2x+1)}{2}$$

input
```
int((1+2*x)/(2*x^2+2*x+1),x)
```

output
```
log(2*x**2 + 2*x + 1)/2
```

3.223 $\int \frac{\arcsin(x)}{x^3}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int \frac{\arcsin(x)}{x^3}\,dx = -\frac{\sqrt{1-x^2}}{2x} - \frac{\arcsin(x)}{2x^2}$$

output
```
-1/2*(-x^2+1)^(1/2)/x-1/2*arcsin(x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{\arcsin(x)}{x^3}\,dx = -\frac{x\sqrt{1-x^2}+\arcsin(x)}{2x^2}$$

input
```
Integrate[ArcSin[x]/x^3,x]
```

output
```
-1/2*(x*Sqrt[1 - x^2] + ArcSin[x])/x^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5138, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(x)}{x^3} \, dx$$

$$\Big\downarrow \text{5138}$$

$$\frac{1}{2} \int \frac{1}{x^2\sqrt{1-x^2}}dx - \frac{\arcsin(x)}{2x^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\arcsin(x)}{2x^2} - \frac{\sqrt{1-x^2}}{2x}$$

input
```
Int[ArcSin[x]/x^3,x]
```

output
```
-1/2*Sqrt[1 - x^2]/x - ArcSin[x]/(2*x^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{-x^{2}+1}}{2x}-\frac{\arcsin \left(x \right)}{2x^{2}}$	23
parts	$-\frac{\sqrt{-x^{2}+1}}{2x}-\frac{\arcsin \left(x \right)}{2x^{2}}$	23
orering	$\frac{\left(\frac{3}{2}x^{3}-2x \right) \arcsin \left(x \right)}{x^{3}}+\frac{x^{2} \left(-1+x \right) \left(1+x \right) \left(\frac{1}{\sqrt{-x^{2}+1}\, x^{3}}-\frac{3 \arcsin \left(x \right)}{x^{4}}\right)}{2}$	49

input
```
int(arcsin(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^2+1)^(1/2)/x-1/2*arcsin(x)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{\arcsin(x)}{x^3}\,dx = -\frac{\sqrt{-x^2+1}x+\arcsin\left(x\right)}{2\,x^2}$$

input
```
integrate(arcsin(x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-x^2 + 1)*x + arcsin(x))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\arcsin(x)}{x^3}\,dx = \frac{\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{asin}\left(x\right)}{2x^2}$$

input
```
integrate(asin(x)/x**3,x)
```

output
```
Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
/2 - asin(x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\arcsin(x)}{x^3}\,dx = -\frac{\sqrt{-x^2+1}}{2\,x} - \frac{\arcsin\left(x\right)}{2\,x^2}$$

input
```
integrate(arcsin(x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 1)/x - 1/2*arcsin(x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{\arcsin(x)}{x^3}\,dx = \frac{x}{4\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{4\,x} - \frac{\arcsin\left(x\right)}{2\,x^2}$$

input
```
integrate(arcsin(x)/x^3,x, algorithm="giac")
```

output
```
1/4*x/(sqrt(-x^2 + 1) - 1) - 1/4*(sqrt(-x^2 + 1) - 1)/x - 1/2*arcsin(x)/x^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(x)}{x^3} \, dx = \int \frac{\operatorname{asin}(x)}{x^3} \, dx$$

input
```
int(asin(x)/x^3,x)
```

output
```
int(asin(x)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\arcsin(x)}{x^3} \, dx = \frac{-asin(x) - \sqrt{-x^2+1}\, x}{2x^2}$$

input
```
int(asin(x)/x^3,x)
```

output
```
( - (asin(x) + sqrt( - x**2 + 1)*x))/(2*x**2)
```

3.224 $\int \cos(\cos(x))\sin(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \cos(\cos(x))\sin(2x)\,dx = -2\cos(\cos(x)) - 2\cos(x)\sin(\cos(x))$$

output
```
-2*cos(cos(x))-2*cos(x)*sin(cos(x))
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(2x)\,dx = -2\cos(\cos(x)) - 2\cos(x)\sin(\cos(x))$$

input
```
Integrate[Cos[Cos[x]]*Sin[2*x],x]
```

output
```
-2*Cos[Cos[x]] - 2*Cos[x]*Sin[Cos[x]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {4879, 27, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(\cos(x))\,dx$$

$$\Big\downarrow \text{4879}$$

$$-\int 2\cos(x)\cos(\cos(x))d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-2\int \cos(x)\cos(\cos(x))d\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$-2\int \cos(x)\sin\left(\cos(x)+\frac{\pi}{2}\right)d\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$-2\left(\int -\sin(\cos(x))d\cos(x)+\cos(x)\sin(\cos(x))\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(\cos(x)\sin(\cos(x))-\int \sin(\cos(x))d\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-2\left(\cos(x)\sin(\cos(x))-\int \sin(\cos(x))d\cos(x)\right)$$

$$\Big\downarrow \text{3118}$$

$$-2(\cos(\cos(x))+\cos(x)\sin(\cos(x)))$$

input
```
Int[Cos[Cos[x]]*Sin[2*x],x]
```

output
```
-2*(Cos[Cos[x]] + Cos[x]*Sin[Cos[x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

Maple [A] (verified)

Time = 12.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
risch	$\sin(x-\cos(x))-\sin(x+\cos(x))-2\cos(\cos(x))$	21

input
```
int(cos(cos(x))*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
sin(x-cos(x))-sin(x+cos(x))-2*cos(cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.62

$$\int \cos(\cos(x))\sin(2x)\,dx = 2\,\cos\left(x\right)\sin\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right) - 2\,\cos\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

input
```
integrate(cos(cos(x))*sin(2*x),x, algorithm="fricas")
```

output
```
2*cos(x)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) - 2*cos((tan(1/2*x)^2
- 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \cos(\cos(x))\sin(2x)\,dx = -2\sin\left(\cos\left(x\right)\right)\cos\left(x\right) - 2\cos\left(\cos\left(x\right)\right)$$

input
```
integrate(cos(cos(x))*sin(2*x),x)
```

output
```
-2*sin(cos(x))*cos(x) - 2*cos(cos(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(2x)\,dx = -2\,\cos(x)\sin(\cos(x)) - 2\,\cos(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(2*x),x, algorithm="maxima")
```

output
```
-2*cos(x)*sin(cos(x)) - 2*cos(cos(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(2x)\,dx = -2\,\cos(x)\sin(\cos(x)) - 2\,\cos(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(2*x),x, algorithm="giac")
```

output
```
-2*cos(x)*sin(cos(x)) - 2*cos(cos(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \cos(\cos(x))\sin(2x)\,dx = -4\cos\left(\frac{\cos(x)}{2}\right)^2 - 4\,\sin\left(\frac{\cos(x)}{2}\right)\,\cos(x)\,\cos\left(\frac{\cos(x)}{2}\right)$$

input
```
int(cos(cos(x))*sin(2*x),x)
```

output
```
- 4*cos(cos(x)/2)^2 - 4*cos(cos(x)/2)*sin(cos(x)/2)*cos(x)
```

Reduce [F]

$$\int \cos(\cos(x))\sin(2x)\,dx = \int \cos\left(\cos\left(x\right)\right)\sin\left(2x\right)dx$$

input
```
int(cos(cos(x))*sin(2*x),x)
```

output
```
int(cos(cos(x))*sin(2*x),x)
```

3.225 $\int -\sin(x-\sin(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int -\sin(x-\sin(x))\,dx = -\text{Int}(\sin(x-\sin(x)),x)$$

output
```
-Defer(Int)(sin(x-sin(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int -\sin(x-\sin(x))\,dx = \int -\sin(x-\sin(x))\,dx$$

input
```
Integrate[-Sin[x - Sin[x]],x]
```

output
```
-Integrate[Sin[x - Sin[x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\sin(x-\sin(x))\,dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(x-\sin(x))dx$$

$$\Big\downarrow \text{7299}$$

$$-\int \sin(x-\sin(x))dx$$

input
```
Int[-Sin[x - Sin[x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int -\sin\left(x-\sin\left(x\right)\right)dx$$

input
```
int(-sin(x-sin(x)),x)
```

output
```
int(-sin(x-sin(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int -\sin(x-\sin(x))\,dx=\int -\sin\left(x-\sin\left(x\right)\right)\,dx$$

input
```
integrate(-sin(x-sin(x)),x, algorithm="fricas")
```

output
```
integral(sin(-x + sin(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int -\sin(x-\sin(x))\,dx=-\int \sin\left(x-\sin\left(x\right)\right)dx$$

input
```
integrate(-sin(x-sin(x)),x)
```

output
```
-Integral(sin(x - sin(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int -\sin(x-\sin(x))\,dx=\int -\sin\left(x-\sin\left(x\right)\right)\,dx$$

input
```
integrate(-sin(x-sin(x)),x, algorithm="maxima")
```

output
```
integrate(sin(-x + sin(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int -\sin(x - \sin(x))\, dx = \int -\sin\left(x - \sin\left(x\right)\right)\, dx$$

input
```
integrate(-sin(x-sin(x)),x, algorithm="giac")
```

output
```
integrate(-sin(x - sin(x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int -\sin(x - \sin(x))\, dx = \int -\sin\left(x - \sin\left(x\right)\right)\, dx$$

input
```
int(-sin(x - sin(x)),x)
```

output
```
int(-sin(x - sin(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int -\sin(x-\sin(x))\,dx = \int \sin\left(\sin\left(x\right)-x\right)dx$$

input
```
int(-sin(x-sin(x)),x)
```

output
```
int(sin(sin(x) - x),x)
```

3.226 $\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\text{Int}\left(\frac{1}{1+\tan^{2\sqrt{505}}(x)},x\right)$$

output
```
Defer(Int)(1/(1+tan(x)^(2*505^(1/2))),x)
```

Mathematica [N/A]

Not integrable

Time = 6.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$$

input
```
Integrate[(1 + Tan[x]^(2*Sqrt[505]))^(-1),x]
```

output
```
Integrate[(1 + Tan[x]^(2*Sqrt[505]))^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan^{2\sqrt{505}}(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)^{2\sqrt{505}}+1}dx$$
$$\Big\downarrow \text{4145}$$
$$\int \frac{1}{\tan^{2\sqrt{505}}(x)+1}dx$$

input
```
Int[(1 + Tan[x]^(2*Sqrt[505]))^(-1),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+\tan(x)^{2\sqrt{505}}}dx$$

input
```
int(1/(1+tan(x)^(2*505^(1/2))),x)
```

output
```
int(1/(1+tan(x)^(2*505^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\int \frac{1}{\tan\left(x\right)^{2\,\sqrt{505}}+1}\,dx$$

input
```
integrate(1/(1+tan(x)^(2*505^(1/2))),x, algorithm="fricas")
```

output
```
integral(1/(tan(x)^(2*sqrt(505)) + 1), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\int \frac{1}{\tan^{2\sqrt{505}}\left(x\right)+1}\,dx$$

input
```
integrate(1/(1+tan(x)**(2*505**(1/2))),x)
```

output
```
Integral(1/(tan(x)**(2*sqrt(505)) + 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 1038, normalized size of antiderivative = 74.14

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\int \frac{1}{\tan\left(x\right)^{2\,\sqrt{505}}+1}\,dx$$

input
```
integrate(1/(1+tan(x)^(2*505^(1/2))),x, algorithm="maxima")
```

output
```
-(-1)^(sqrt(101)*sqrt(5))*integrate(((-1)^(sqrt(101)*sqrt(5))*cos(2*sqrt(1
01)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arctan2(sin(
x), -cos(x) + 1))^2*e^(2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos
(x) + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)) +
(-1)^(sqrt(101)*sqrt(5))*e^(2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1
))*sin(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(
5)*arctan2(sin(x), -cos(x) + 1))^2 + cos(2*sqrt(101)*sqrt(5)*arctan2(sin(2
*x), cos(2*x) + 1))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) -
2*sqrt(101)*sqrt(5)*arctan2(sin(x), -cos(x) + 1))*e^(sqrt(101)*sqrt(5)*log
(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1)) + e^(sqrt(101)*sqrt(5)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + s
qrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*sin(2*sqrt(101)*
sqrt(5)*arctan2(sin(2*x), cos(2*x) + 1))*sin(2*sqrt(101)*sqrt(5)*arctan2(s
in(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arctan2(sin(x), -cos(x) + 1)))/(2
*(-1)^(sqrt(101)*sqrt(5))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(2*x), cos(2*
x) + 1))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)
*sqrt(5)*arctan2(sin(x), -cos(x) + 1))*e^(sqrt(101)*sqrt(5)*log(cos(2*x...
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx=\int \frac{1}{\tan\left(x\right)^{2\sqrt{505}}+1}\,dx$$

input
```
integrate(1/(1+tan(x)^(2*505^(1/2))),x, algorithm="giac")
```

output
```
integrate(1/(tan(x)^(2*sqrt(505)) + 1), x)
```

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx = \int \frac{1}{\tan\left(x\right)^{2\,\sqrt{505}}+1}\,dx$$

input
```
int(1/(tan(x)^(2*505^(1/2)) + 1),x)
```

output
```
int(1/(tan(x)^(2*505^(1/2)) + 1), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx = \int \frac{1}{\tan\left(x\right)^{2\sqrt{505}}+1}dx$$

input
```
int(1/(1+tan(x)^(2*505^(1/2))),x)
```

output
```
int(1/(tan(x)**(2*sqrt(505)) + 1),x)
```

3.227 $\int (1-x)^{2020} x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int (1-x)^{2020} x \, dx = -\frac{(1-x)^{2021}}{2021} + \frac{(1-x)^{2022}}{2022}$$

output
```
-1/2021*(1-x)^2021+1/2022*(1-x)^2022
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11128 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 11128, normalized size of antiderivative = 483.83

$$\int (1-x)^{2020} x \, dx = \text{Result too large to show}$$

input
```
Integrate[(1 - x)^2020*x,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{2020} x \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left((1-x)^{2020}-(1-x)^{2021}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-x)^{2022}}{2022}-\frac{(1-x)^{2021}}{2021}$$

input
```
Int[(1 - x)^2020*x,x]
```

output
```
-1/2021*(1 - x)^2021 + (1 - x)^2022/2022
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10105 vs. $2(19) = 38$.

Time = 8.50 (sec) , antiderivative size = 10106, normalized size of antiderivative = 439.39

method	result	size
gosper	Expression too large to display	10106
default	Expression too large to display	10107
risch	Expression too large to display	10107
parallelrisch	Expression too large to display	10107
orering	Expression too large to display	10118

input
```
int((1-x)^2020*x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-2)]

Exception generated.

$$\int (1-x)^{2020}x\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((1-x)^2020*x,x, algorithm="fricas")
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11171 vs. $2(12) = 24$.

Time = 2.73 (sec) , antiderivative size = 11171, normalized size of antiderivative = 485.70

$$\int (1-x)^{2020} x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)**2020*x,x)
```

output
```
x**2022/2022 - 2020*x**2021/2021 + 2019*x**2020/2 - 2038180*x**2019/3 + 68
5507705*x**2018/2 - 138266870112*x**2017 + 278745941561035*x**2016/6 - 133
73165093095968*x**2015 + 336669348217418073 0*x**2014 - 2259050769046992147
760*x**2013/3 + 1515069674844631864481 38*x**2012 - 276982214792851707236 85
360*x**2011 + 13918352848288375491988868066*x**2010/3 - 716973434466946772
290161333280*x**2009 + 10283444995365327274902478 5712470*x**2008 - 4127773
7963817952963643626311862816*x**2007/3 + 172506503704752592591034895112430
2995*x**2006 - 20345614934698381583634515569589335 5528*x**2005 + 679543369
61055157115435769332338918346705*x**2004/3 - 238793864924060382048318075 55
99043532230840*x**2003 + 23903259915015703881888236558399683502277231 3*x**
2002 - 68329158777556135034683015049498345643691036000*x**2001/3 + 2070580
051371710322595432011477108583912261365977*x**2000 - 17996036899496170800 8
1637844544560519675793639960 00*x**1999 + 449450908977098425459030187708672
3741383462986800075 0*x**1998/3 - 11967376535522886579990092210473624419580
82554435826640*x**1997 + 918726060354771330699460983850587266686763964103 3
3163850*x**1996 - 20365089221038364860461105557164855112824957167732656091
280*x**1995/3 + 48342830587578972051848087019380740255600717970820742741 25
50*x**1994 - 33223185208528449708566031169292044668543059434670067575119 20
0*x**1993 + 66180568257082844244940588021945029439554707856182335518238061
30*x**1992/3 - 1416833098494499231566155690748323651597511524000665751 6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10106 vs. 2(15) = 30.

Time = 2.68 (sec) , antiderivative size = 10106, normalized size of antiderivative = 439.39

$$\int (1-x)^{2020} x\,dx = \text{Too large to display}$$

input
```
integrate((1-x)^2020*x,x, algorithm="maxima")
```

output
```
1/2022*x^2022 - 2020/2021*x^2021 + 2019/2*x^2020 - 2038180/3*x^2019 + 6855
07705/2*x^2018 - 138266870112*x^2017 + 278745941561035/6*x^2016 - 13373165
093095968*x^2015 + 3366693482174180730*x^2014 - 2259050769046992147760/3*x
^2013 + 151506967484463186448138*x^2012 - 27698221479285170723685360*x^201
1 + 13918352848288375491988868066/3*x^2010 - 716973434466946772290161333280*x^2009 + 102834449953653272749024785712470*x^2008 - 412777379638179529636
43626311862816/3*x^2007 + 1725065037047525925910348951124302995*x^2006 -
203456149346983815836345155695893355528*x^2005 + 679543369610551571154357
69332338918346705/3*x^2004 - 2387938649240603820483180755599043532230840*x^
2003 + 2390325991501570388188823655839968350227723 13*x^2002 - 683291587775
56135034683015049498345643691036000/3*x^2001 + 207058005137171032259543201
14771085839122613659 77*x^2000 - 179960368994961708008163784454456051967579
363996000*x^1999 + 449450908977098425459030187708672374138346298680007 50/3
*x^1998 - 1196737653552288657999009221047362441958082554435826640*x^1997 +
 91872606035477133069946098385058726668676396410333163850*x^1996 - 2036508
922103836486046110555716485511282495716773265609128 0/3*x^1995 + 4834283058
757897205184808701938074025560071797082074274125 50*x^1994 - 33223185208528
44970856603116929204466854305943467006757511920 0*x^1993 + 661805682570828 4
4244940588021945029439554707856182335518238061 30/3*x^1992 - 14168330984944
99231566155690748323651597511524000665751695193 60800*x^1991 + 176218572...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10106 vs. $2(15) = 30$.

Time = 1.02 (sec) , antiderivative size = 10106, normalized size of antiderivative = 439.39

$$\int (1-x)^{2020} x \, dx = \text{Too large to display}$$

input
```
integrate((1-x)^2020*x,x, algorithm="giac")
```

output
```
1/2022*x^2022 - 2020/2021*x^2021 + 2019/2*x^2020 - 2038180/3*x^2019 + 6855
07705/2*x^2018 - 138266870112*x^2017 + 278745941561035/6*x^2016 - 13373165
093095968*x^2015 + 3366693482174180730*x^2014 - 2259050769046992147760/3*x
^2013 + 151506967484463186448138*x^2012 - 27698221479285170723685360*x^201
1 + 13918352848288375491988868066/3*x^2010 - 716973434466946772290161333 28
0*x^2009 + 10283444995365327274902478571247 0*x^2008 - 412777379638179529 63
643626311862816/3*x^2007 + 17250650370475259259103489511243029 95*x^2006 -
20345614934698381583634515569589335552 8*x^2005 + 679543369610551571154357 6
9332338918346705/3*x^2004 - 23879386492406038204831807555990435322308 40*x^
2003 + 239032599150157038818882365583996835022772313*x^2002 - 683291587775
561350346830150494983456436910360 00/3*x^2001 + 207058005137171032259543201
1477108583912261365977*x^2000 - 179960368994961708008163784454456051967579
363996000*x^1999 + 44945090897709842545903018770867237413834629868000750/3
*x^1998 - 11967376535522886579990092210473624419580825544358266 40*x^1997 +
 918726060354771330699460983850587266686763964103331638 50*x^1996 - 2036508
922103836486046110555716485511282495716773265609128 0/3*x^1995 + 4834283058
75789720518480870193807402556007179708207427412550*x^1994 - 33223185208528
44970856603116929204466854305943467006757511920 0*x^1993 + 6618056825708284
4244940588021945029439554707856182335518238061 30/3*x^1992 - 14168330984944
9923156615569074832365159751152400066575169519360800*x^1991 + 176218572...
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int (1-x)^{2020}x\,dx = \frac{(x-1)^{2021}}{2021} + \frac{(x-1)^{2022}}{2022}$$

input
```
int(x*(x - 1)^2020,x)
```

output
```
(x - 1)^2021/2021 + (x - 1)^2022/2022
```

Reduce [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 10105, normalized size of antiderivative = 439.35

$$\int (1-x)^{2020}x\,dx = \text{Too large to display}$$

input
```
int((1-x)^2020*x,x)
```

output

```
(x**2*(2021*x**2020 - 4084440*x**2019 + 4125283389*x**2018 - 2776315039720
*x**2017 + 1400650593594855*x**2016 - 565022310571623744*x**2015 + 1898474
49640565034695*x**2014 - 54648930972663135585216*x**2013 + 137578649805524
66934277260*x**2012 - 3077175041260436542039875040*x**2011 + 6191274653604
94401819230907756*x**2010 - 113187729542682637325852723596320*x**2009 + 18
958940005707403829914604591574164*x**2008 - 292988469495866824098639726231
8055360*x**2007 + 420229072026505860264525323872151581140*x**2006 - 562266
358783664799012390201552091822656664*x**2005 + 7049412721423306890247456395
509321465553690*x**2004 - 831415822972774178030222697635351753417661936*x*
*2003 + 92564271908849126485419294939122786981637602570*x**2002 - 97582205
48453056369459339796886778630807102888080*x**2001 + 9767976331883490331658
87669429110874440828191726606*x**2000 - 9307483694548319956203360768173436
9511936319451544000*x**1999 + 84613466978885421082939742884847680980312674
06133103374*x**1998 - 7354012094038892112304569949493253870355383029538221
52000*x**1997 + 6122213534667905286327198063081189091220449641321669361550
0*x**1996 - 4890422945210592613943947219459646819288909951564937002947680*
x**1995 + 375433913404947956159278073098803854299932684227762881412798700*
x**1994 - 2774038774279429284813653677911433605135496670585570842536141712
0*x**1993 + 19755114016857914128893923737739265858638262116047607402391438
98100*x**1992 - 1357652838736135856529661608641275074403038077435207136...
```

3.228 $\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{1}{4}\log\left(4+\sec^4(x)\right)$$

output
```
1/4*ln(4+sec(x)^4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = -\log(\cos(x))+\frac{1}{4}\log\left(1+4\cos^4(x)\right)$$

input
```
Integrate[(Sec[x]^4*Tan[x])/(4 + Sec[x]^4),x]
```

output
```
-Log[Cos[x]] + Log[1 + 4*Cos[x]^4]/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4839, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec^4(x)}{\sec^4(x)+4}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(x)\sec(x)^4}{\sec(x)^4+4}dx$$

$$\Big\downarrow \ 4839$$

$$-\int \frac{\sec(x)}{4\cos^4(x)+1}d\cos(x)$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{4}\int \frac{\sec(x)}{4\cos^4(x)+1}d\cos^4(x)$$

$$\Big\downarrow \ 47$$

$$\frac{1}{4}\left(4\int \frac{1}{4\cos^4(x)+1}d\cos^4(x)-\int \sec(x)d\cos^4(x)\right)$$

$$\Big\downarrow \ 14$$

$$\frac{1}{4}\left(4\int \frac{1}{4\cos^4(x)+1}d\cos^4(x)-\log\left(\cos^4(x)\right)\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{4}\left(\log\left(4\cos^4(x)+1\right)-\log\left(\cos^4(x)\right)\right)$$

input
```
Int[(Sec[x]^4*Tan[x])/(4 + Sec[x]^4),x]
```

output
```
(-Log[Cos[x]^4] + Log[1 + 4*Cos[x]^4])/4
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\ln\left(4+\sec(x)^4\right)}{4}$	10
default	$\frac{\ln\left(4+\sec(x)^4\right)}{4}$	10
risch	$-\ln\left(e^{2ix}+1\right)+\frac{\ln\left(e^{8ix}+4\,e^{6ix}+10\,e^{4ix}+4\,e^{2ix}+1\right)}{4}$	43

input
```
int(sec(x)^4*tan(x)/(4+sec(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(4+sec(x)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{1}{4}\log\left(4\cos(x)^4+1\right)-\log\left(-\cos(x)\right)$$

input
```
integrate(sec(x)^4*tan(x)/(4+sec(x)^4),x, algorithm="fricas")
```

output
```
1/4*log(4*cos(x)^4 + 1) - log(-cos(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{\log\left(\sec^2(x)-2\sec(x)+2\right)}{4}+\frac{\log\left(\sec^2(x)+2\sec(x)+2\right)}{4}$$

input
```
integrate(sec(x)**4*tan(x)/(4+sec(x)**4),x)
```

output
```
log(sec(x)**2 - 2*sec(x) + 2)/4 + log(sec(x)**2 + 2*sec(x) + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{1}{4}\,\log\left(\sec\left(x\right)^4+4\right)$$

input
```
integrate(sec(x)^4*tan(x)/(4+sec(x)^4),x, algorithm="maxima")
```

output
```
1/4*log(sec(x)^4 + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{1}{4}\,\log\left(4\,\cos\left(x\right)^4+1\right)-\frac{1}{4}\,\log\left(\cos\left(x\right)^4\right)$$

input
```
integrate(sec(x)^4*tan(x)/(4+sec(x)^4),x, algorithm="giac")
```

output
```
1/4*log(4*cos(x)^4 + 1) - 1/4*log(cos(x)^4)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{\ln\left(\tan(x)^4+2\tan(x)^2+5\right)}{4}$$

input
```
int(tan(x)/(cos(x)^4*(1/cos(x)^4 + 4)),x)
```

output
```
log(2*tan(x)^2 + tan(x)^4 + 5)/4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx = \frac{\log\left(\sec(x)^2-2\sec(x)+2\right)}{4}+\frac{\log\left(\sec(x)^2+2\sec(x)+2\right)}{4}$$

input
```
int(sec(x)^4*tan(x)/(4+sec(x)^4),x)
```

output
```
(log(sec(x)**2 - 2*sec(x) + 2) + log(sec(x)**2 + 2*sec(x) + 2))/4
```

3.229 $\int x^{2x}(2+2\log(x))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int x^{2x}(2+2\log(x))\,dx = x^{2x}$$

output
```
x^(2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2x}$$

input
```
Integrate[x^(2*x)*(2 + 2*Log[x]),x]
```

output
```
x^(2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2x}(2\log(x)+2)\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int 2x^{2x}(\log(x)+1)dx$$

$$\Big\downarrow \text{27}$$

$$2\int x^{2x}(\log(x)+1)dx$$

$$\Big\downarrow \text{7293}$$

$$2\int \left(\log(x)x^{2x}+x^{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^{2x}$$

input
```
Int[x^(2*x)*(2 + 2*Log[x]),x]
```

output
```
x^(2*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	x^{2x}	6
risch	x^{2x}	6
parallelrisch	x^{2x}	6
norman	$e^{2x\ln(x)}$	7

input
```
int(x^(2*x)*(2+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x^(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2\,x}$$

input
```
integrate(x^(2*x)*(2+2*log(x)),x, algorithm="fricas")
```

output
```
x^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int x^{2x}(2+2\log(x))\,dx = x^{2x}$$

input
```
integrate(x**(2*x)*(2+2*ln(x)),x)
```

output
```
x**(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2\,x}$$

input
```
integrate(x^(2*x)*(2+2*log(x)),x, algorithm="maxima")
```

output
```
x^(2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2\,x}$$

input
```
integrate(x^(2*x)*(2+2*log(x)),x, algorithm="giac")
```

output
```
x^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2\,x}$$

input
```
int(x^(2*x)*(2*log(x) + 2),x)
```

output
```
x^(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int x^{2x}(2+2\log(x))\,dx = x^{2x}$$

input
```
int(x^(2*x)*(2+2*log(x)),x)
```

output
```
x**(2*x)
```

3.230 $\int \sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{-x^{2}+1}}{2}+\frac{\arcsin\left(x\right)}{2}$	18
risch	$-\frac{x\left(x^{2}-1\right)}{2\sqrt{-x^{2}+1}}+\frac{\arcsin\left(x\right)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^{2}+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^{2}+1}-2i\sqrt{\pi}\,\arcsin\left(x\right)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^{2}+1}}{2}+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)}{2}$	41

input
```
int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 + asin(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \frac{x\sqrt{1-x^2}}{2}$$

input
```
int((1 - x^2)^(1/2),x)
```

output
```
asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \sqrt{1-x^2}\,dx = \frac{asin(x)}{2} + \frac{\sqrt{-x^2+1}\,x}{2}$$

input
```
int((-x^2+1)^(1/2),x)
```

output
```
(asin(x) + sqrt( - x**2 + 1)*x)/2
```

3.231 $\int e^{-x^4} x^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int e^{-x^4} x^5 \, dx = -\frac{1}{4} e^{-x^4} x^2 + \frac{1}{8} \sqrt{\pi} \operatorname{erf}\left(x^2\right)$$

output
```
-1/4*x^2/exp(x^4)+1/8*Pi^(1/2)*erf(x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int e^{-x^4} x^5 \, dx = -\frac{1}{4} e^{-x^4} x^2 + \frac{1}{8} \sqrt{\pi} \operatorname{erf}\left(x^2\right)$$

input
```
Integrate[x^5/E^x^4,x]
```

output
```
-1/4*x^2/E^x^4 + (Sqrt[Pi]*Erf[x^2])/8
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2641, 2640, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^4} x^5 \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2}\int e^{-x^4} x dx - \frac{1}{4}e^{-x^4} x^2$$

$$\Big\downarrow \text{2640}$$

$$\frac{1}{4}\int e^{-x^4} dx^2 - \frac{1}{4}e^{-x^4} x^2$$

$$\Big\downarrow \text{2634}$$

$$\frac{1}{8}\sqrt{\pi}\text{erf}(x^2) - \frac{1}{4}e^{-x^4} x^2$$

input
```
Int[x^5/E^x^4,x]
```

output
```
-1/4*x^2/E^x^4 + (Sqrt[Pi]*Erf[x^2])/8
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
meijerg	$-\frac{x^{2} \mathrm{e}^{-x^{4}}}{4}+\frac{\sqrt{\pi}\, \operatorname{erf}\left(x^{2}\right)}{8}$	22

input
```
int(x^5/exp(x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*exp(-x^4)+1/8*Pi^(1/2)*erf(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int e^{-x^4} x^5 \, dx = -\frac{1}{4} \, x^{2} e^{\left(-x^{4}\right)} + \frac{1}{8} \, \sqrt{\pi} \operatorname{erf}\left(x^{2}\right)$$

input
```
integrate(x^5/exp(x^4),x, algorithm="fricas")
```

output
```
-1/4*x^2*e^(-x^4) + 1/8*sqrt(pi)*erf(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int e^{-x^4}x^5\,dx=-\frac{x^2e^{-x^4}}{4}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(x^2\right)}{8}$$

input
```
integrate(x**5/exp(x**4),x)
```

output
```
-x**2*exp(-x**4)/4 + sqrt(pi)*erf(x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int e^{-x^4}x^5\,dx=-\frac{1}{4}\,x^2e^{\left(-x^4\right)}+\frac{1}{8}\,\sqrt{\pi}\,\mathrm{erf}\left(x^2\right)$$

input
```
integrate(x^5/exp(x^4),x, algorithm="maxima")
```

output
```
-1/4*x^2*e^(-x^4) + 1/8*sqrt(pi)*erf(x^2)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int e^{-x^4}x^5\,dx=-\frac{\sqrt{\pi}\left(\frac{2\,\sqrt{x^4}e^{\left(-x^4\right)}}{\sqrt{\pi}}-\mathrm{erf}\left(\sqrt{x^4}\right)\right)|x|}{8\,x}$$

input
```
integrate(x^5/exp(x^4),x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*(2*sqrt(x^4)*e^(-x^4)/sqrt(pi) - erf(sqrt(x^4)))*abs(x)/x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int e^{-x^4}x^5\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}(x^2)}{8} - \frac{x^2\,\mathrm{e}^{-x^4}}{4}$$

input
```
int(x^5*exp(-x^4),x)
```

output
```
(pi^(1/2)*erf(x^2))/8 - (x^2*exp(-x^4))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int e^{-x^4}x^5\,dx = -\frac{x^2}{4e^{x^4}}$$

input
```
int(x^5/exp(x^4),x)
```

output
```
( - x**2)/(4*e**(x**4))
```

3.232 $\int \frac{1+\cos(x)}{x+\sin(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 5

$$\int \frac{1+\cos(x)}{x+\sin(x)} \, dx = \log(x+\sin(x))$$

output
```
ln(x+sin(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)} \, dx = \log(x+\sin(x))$$

input
```
Integrate[(1 + Cos[x])/(x + Sin[x]),x]
```

output
```
Log[x + Sin[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)+1}{x+\sin(x)}\,dx$$

$$\Big\downarrow \; 7235$$

$$\log(x+\sin(x))$$

input
```
Int[(1 + Cos[x])/(x + Sin[x]),x]
```

output
```
Log[x + Sin[x]]
```

Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln\left(x+\sin\left(x\right)\right)$	6
default	$\ln\left(x+\sin\left(x\right)\right)$	6
risch	$-ix+\ln\left(\mathrm{e}^{2ix}+2ix\,\mathrm{e}^{ix}-1\right)$	23
parallelrisch	$-\ln\left(\sec\left(\frac{x}{2}\right)^{2}\right)+\ln\left(2\tan\left(\frac{x}{2}\right)+x\sec\left(\frac{x}{2}\right)^{2}\right)$	27
norman	$-\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)+\ln\left(\tan\left(\frac{x}{2}\right)^{2}x+x+2\tan\left(\frac{x}{2}\right)\right)$	30

input
```
int((1+cos(x))/(x+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(x+sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx = \log\left(x+\sin\left(x\right)\right)$$

input
```
integrate((1+cos(x))/(x+sin(x)),x, algorithm="fricas")
```

output
```
log(x + sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx = \log\left(x+\sin\left(x\right)\right)$$

input
```
integrate((1+cos(x))/(x+sin(x)),x)
```

output
```
log(x + sin(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx = \log\left(x+\sin\left(x\right)\right)$$

input
```
integrate((1+cos(x))/(x+sin(x)),x, algorithm="maxima")
```

output
```
log(x + sin(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(5) = 10.

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 14.40

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx$$
$$= \frac{1}{2}\log\left(\frac{4\left(x^2\tan\left(\frac{1}{2}x\right)^4+2x^2\tan\left(\frac{1}{2}x\right)^2+4x\tan\left(\frac{1}{2}x\right)^3+x^2+4x\tan\left(\frac{1}{2}x\right)+4\tan\left(\frac{1}{2}x\right)^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

input
```
integrate((1+cos(x))/(x+sin(x)),x, algorithm="giac")
```

output
```
1/2*log(4*(x^2*tan(1/2*x)^4 + 2*x^2*tan(1/2*x)^2 + 4*x*tan(1/2*x)^3 + x^2
+ 4*x*tan(1/2*x) + 4*tan(1/2*x)^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx = \ln\left(x+\sin\left(x\right)\right)$$

input
```
int((cos(x) + 1)/(x + sin(x)),x)
```

output
```
log(x + sin(x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos(x)}{x+\sin(x)}\,dx = \log(\sin\left(x\right)+x)$$

input
```
int((1+cos(x))/(x+sin(x)),x)
```

output
```
log(sin(x) + x)
```

3.233 $\int \frac{\cot^{-1}(x)+\arctan(x)}{x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 57

$$\int \frac{\cot^{-1}(x)+\arctan(x)}{x} \, dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2,-\frac{i}{x}\right)+\frac{1}{2} i \operatorname{PolyLog}\left(2,\frac{i}{x}\right) +\frac{1}{2} i \operatorname{PolyLog}(2,-i x)-\frac{1}{2} i \operatorname{PolyLog}(2,i x)$$

output
```
-1/2*I*polylog(2,-I/x)+1/2*I*polylog(2,I/x)+1/2*I*polylog(2,-I*x)-1/2*I*po
lylog(2,I*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(x)+\arctan(x)}{x} \, dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2,-\frac{i}{x}\right)+\frac{1}{2} i \operatorname{PolyLog}\left(2,\frac{i}{x}\right) +\frac{1}{2} i \operatorname{PolyLog}(2,-i x)-\frac{1}{2} i \operatorname{PolyLog}(2,i x)$$

input
```
Integrate[(ArcCot[x] + ArcTan[x])/x,x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/x] + (I/2)*PolyLog[2, I/x] + (I/2)*PolyLog[2, (-I
)*x] - (I/2)*PolyLog[2, I*x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)+\cot^{-1}(x)}{x}\,dx$$

$$\Big\downarrow \; 2010$$

$$\int \left(\frac{\arctan(x)}{x}+\frac{\cot^{-1}(x)}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}i\operatorname{PolyLog}\left(2,-\frac{i}{x}\right)+\frac{1}{2}i\operatorname{PolyLog}\left(2,\frac{i}{x}\right)+\frac{1}{2}i\operatorname{PolyLog}(2,-ix)-\frac{1}{2}i\operatorname{PolyLog}(2,ix)$$

input
```
Int[(ArcCot[x] + ArcTan[x])/x,x]
```

output

```
(-1/2*I)*PolyLog[2, (-I)/x] + (I/2)*PolyLog[2, I/x] + (I/2)*PolyLog[2, (-I
)*x] - (I/2)*PolyLog[2, I*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.21

method	result	size
default	$\ln(x)\,\mathrm{arccot}(x)+\arctan(x)\ln(x)$	12
parts	$\ln(x)\,\mathrm{arccot}(x)+\arctan(x)\ln(x)$	12

input
```
int((arccot(x)+arctan(x))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(x)+arctan(x)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{\cot^{-1}(x)+\arctan(x)}{x}\,dx = -\frac{1}{2}\,\pi\log\left(x\right)$$

input
```
integrate((arccot(x)+arctan(x))/x,x, algorithm="fricas")
```

output
```
-1/2*pi*log(x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(x) + \arctan(x)}{x}\,dx = \int \frac{\operatorname{acot}(x) + \operatorname{atan}(x)}{x}\,dx$$

input
```
integrate((acot(x)+atan(x))/x,x)
```

output
```
Integral((acot(x) + atan(x))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.16

$$\int \frac{\cot^{-1}(x) + \arctan(x)}{x}\,dx = (\arctan(x) + \arctan(1, x))\log(x)$$

input
```
integrate((arccot(x)+arctan(x))/x,x, algorithm="maxima")
```

output
```
(arctan(x) + arctan2(1, x))*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{\cot^{-1}(x) + \arctan(x)}{x}\,dx = -\frac{1}{2}\pi\log(x)$$

input
```
integrate((arccot(x)+arctan(x))/x,x, algorithm="giac")
```

output
```
-1/2*pi*log(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(x) + \arctan(x)}{x} \, dx = \int \frac{\mathrm{atan}(x) + \mathrm{acot}(x)}{x} \, dx$$

input
```
int((atan(x) + acot(x))/x,x)
```

output
```
int((atan(x) + acot(x))/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(x) + \arctan(x)}{x} \, dx = \int \frac{acot(x)}{x} dx + \int \frac{atan(x)}{x} dx$$

input
```
int((acot(x)+atan(x))/x,x)
```

output
```
int(acot(x)/x,x) + int(atan(x)/x,x)
```

3.234 $\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{1}{2}(-x+e^x\sinh(x))$$

output
```
-1/2*x+1/2*exp(x)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -\frac{x}{2}+\frac{\cosh^2(x)}{2}+\frac{1}{4}\sinh(2x)$$

input
```
Integrate[Sinh[x]/(Cosh[x] - Sinh[x]),x]
```

output
```
-1/2*x + Cosh[x]^2/2 + Sinh[2*x]/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3560, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{i\sin(ix)+\cos(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\cos(ix)+i\sin(ix)}dx$$

$$\Big\downarrow \text{3560}$$

$$-i\left(\frac{i\sinh(x)}{2(\cosh(x)-\sinh(x))}-\frac{i\int 1dx}{2}\right)$$

$$\Big\downarrow \text{24}$$

$$-i\left(\frac{i\sinh(x)}{2(\cosh(x)-\sinh(x))}-\frac{ix}{2}\right)$$

input
```
Int[Sinh[x]/(Cosh[x] - Sinh[x]),x]
```

output
```
(-I)*((-1/2*I)*x + ((I/2)*Sinh[x])/(Cosh[x] - Sinh[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3560
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c
 + d*x])^n/(2*b*d*n*Sin[c + d*x]^n)), x] + Simp[1/(2*b)   Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Sin[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{2 x}}{4}$	11
parallelrisch	$-\frac{x}{2}+\frac{\cosh \left(2 x \right)}{4}+\frac{1}{4}+\frac{\sinh \left(2 x \right)}{4}$	18
default	$-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{\tanh \left(\frac{x}{2}\right)-1}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{2}$	36
orering	$\frac{\left(\frac{1}{2}+x \right) \sinh \left(x \right)}{\cosh \left(x \right)-\sinh \left(x \right)}-\frac{x \left(\frac{\cosh \left(x \right)}{\cosh \left(x \right)-\sinh \left(x \right)}-\frac{\sinh \left(x \right) \left(\sinh \left(x \right)-\cosh \left(x \right)\right)}{\left(\cosh \left(x \right)-\sinh \left(x \right)\right)^{2}}\right)}{2}$	53

input
```
int(sinh(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4*exp(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -\frac{(2\,x-1)\cosh\left(x\right)-(2\,x+1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

output
```
-1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{x\sinh\left(x\right)}{-2\sinh\left(x\right)+2\cosh\left(x\right)}-\frac{x\cosh\left(x\right)}{-2\sinh\left(x\right)+2\cosh\left(x\right)} + \frac{\cosh\left(x\right)}{-2\sinh\left(x\right)+2\cosh\left(x\right)}$$

input
```
integrate(sinh(x)/(cosh(x)-sinh(x)),x)
```

output
```
x*sinh(x)/(-2*sinh(x) + 2*cosh(x)) - x*cosh(x)/(-2*sinh(x) + 2*cosh(x)) +
cosh(x)/(-2*sinh(x) + 2*cosh(x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -\frac{1}{2}\,x + \frac{1}{4}\,e^{(2\,x)}$$

input
```
integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

output
```
-1/2*x + 1/4*e^(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = -\frac{1}{2}\,x + \frac{1}{4}\,e^{(2\,x)}$$

input
```
integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
```

output
```
-1/2*x + 1/4*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{4} - \frac{x}{2}$$

input
```
int(sinh(x)/(cosh(x) - sinh(x)),x)
```

output
```
exp(2*x)/4 - x/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx = \frac{e^{2x}}{4}-\frac{x}{2}$$

input
```
int(sinh(x)/(cosh(x)-sinh(x)),x)
```

output
```
(e**(2*x) - 2*x)/4
```

3.235 $\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{3}(-1+x)^{3/2}-\frac{1}{5}(-1+x)^{5/2}-\frac{1}{3}(1+x)^{3/2}+\frac{1}{5}(1+x)^{5/2}$$

output
```
-1/3*(-1+x)^(3/2)-1/5*(-1+x)^(5/2)-1/3*(1+x)^(3/2)+1/5*(1+x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{15}\left((1+x)^{3/2}(-2+3x)+\sqrt{-1+x}\left(2+x-3x^2\right)\right)$$

input
```
Integrate[x/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
((1 + x)^(3/2)*(-2 + 3*x) + Sqrt[-1 + x]*(2 + x - 3*x^2))/15
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2529, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x-1}+\sqrt{x+1}}dx$$

$$\Big\downarrow \text{2529}$$

$$\frac{1}{2}\int x\sqrt{x+1}dx-\frac{1}{2}\int \sqrt{x-1}xdx$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left((x+1)^{3/2}-\sqrt{x+1}\right)dx-\frac{1}{2}\int \left((x-1)^{3/2}+\sqrt{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2}{5}(x-1)^{5/2}-\frac{2}{3}(x-1)^{3/2}\right)+\frac{1}{2}\left(\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}\right)$$

input
```
Int[x/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
((-2*(-1 + x)^(3/2))/3 - (2*(-1 + x)^(5/2))/5)/2 + ((-2*(1 + x)^(3/2))/3 +
 (2*(1 + x)^(5/2))/5)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{(-1+x)^{\frac{3}{2}}}{3}-\frac{(-1+x)^{\frac{5}{2}}}{5}-\frac{(1+x)^{\frac{3}{2}}}{3}+\frac{(1+x)^{\frac{5}{2}}}{5}$	30

input
```
int(x/((-1+x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-1+x)^(3/2)-1/5*(-1+x)^(5/2)-1/3*(1+x)^(3/2)+1/5*(1+x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}-\frac{1}{15}\left(3\,x^2-x-2\right)\sqrt{x-1}$$

input
```
integrate(x/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/15*(3*x^2 + x - 2)*sqrt(x + 1) - 1/15*(3*x^2 - x - 2)*sqrt(x - 1)
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \int \frac{x}{\sqrt{x-1}+\sqrt{x+1}}\,dx$$

input
```
integrate(x/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
Integral(x/(sqrt(x - 1) + sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \int \frac{x}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

input
```
integrate(x/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{5}\,(x+1)^{\frac{5}{2}} - \frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{15}\,((3\,x-4)(x+1)+2)\sqrt{x-1}$$

input
```
integrate(x/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
1/5*(x + 1)^(5/2) - 1/3*(x + 1)^(3/2) - 1/15*((3*x - 4)*(x + 1) + 2)*sqrt(
x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{x\sqrt{x-1}}{15}+\frac{x\sqrt{x+1}}{15}+\frac{2\sqrt{x-1}}{15}-\frac{2\sqrt{x+1}}{15}-\frac{x^2\sqrt{x-1}}{5}+\frac{x^2\sqrt{x+1}}{5}$$

input
```
int(x/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

output
```
(x*(x - 1)^(1/2))/15 + (x*(x + 1)^(1/2))/15 + (2*(x - 1)^(1/2))/15 - (2*(x
 + 1)^(1/2))/15 - (x^2*(x - 1)^(1/2))/5 + (x^2*(x + 1)^(1/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{\sqrt{x-1}\,x^2}{5}+\frac{\sqrt{x-1}\,x}{15}+\frac{2\sqrt{x-1}}{15}+\frac{\sqrt{x+1}\,x^2}{5}+\frac{\sqrt{x+1}\,x}{15}-\frac{2\sqrt{x+1}}{15}$$

input
```
int(x/((-1+x)^(1/2)+(1+x)^(1/2)),x)
```

output
```
( - 3*sqrt(x - 1)*x**2 + sqrt(x - 1)*x + 2*sqrt(x - 1) + 3*sqrt(x + 1)*x**
2 + sqrt(x + 1)*x - 2*sqrt(x + 1))/15
```

3.236 $\int \cos(x+\cos(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 5

$$\int \cos(x+\cos(x))\,dx = \text{Int}(\cos(x+\cos(x)),x)$$

output
```
Defer(Int)(cos(x+cos(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx = \int \cos(x+\cos(x))\,dx$$

input
```
Integrate[Cos[x + Cos[x]],x]
```

output
```
Integrate[Cos[x + Cos[x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x+\cos(x))\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cos(x+\cos(x))dx$$

input
```
Int[Cos[x + Cos[x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos\left(x+\cos\left(x\right)\right)dx$$

input
```
int(cos(x+cos(x)),x)
```

output
```
int(cos(x+cos(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx=\int \cos\left(x+\cos\left(x\right)\right)\,dx$$

input
```
integrate(cos(x+cos(x)),x, algorithm="fricas")
```

output
```
integral(cos(x + cos(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx=\int \cos\left(x+\cos\left(x\right)\right)dx$$

input
```
integrate(cos(x+cos(x)),x)
```

output
```
Integral(cos(x + cos(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx=\int \cos\left(x+\cos\left(x\right)\right)\,dx$$

input
```
integrate(cos(x+cos(x)),x, algorithm="maxima")
```

output
```
integrate(cos(x + cos(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx = \int \cos\left(x+\cos\left(x\right)\right)\,dx$$

input
```
integrate(cos(x+cos(x)),x, algorithm="giac")
```

output
```
integrate(cos(x + cos(x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx = \int \cos\left(x+\cos\left(x\right)\right)\,dx$$

input
```
int(cos(x + cos(x)),x)
```

output
```
int(cos(x + cos(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \cos(x+\cos(x))\,dx=\int \cos\left(\cos\left(x\right)+x\right)dx$$

input
```
int(cos(x+cos(x)),x)
```

output
```
int(cos(cos(x) + x),x)
```

3.237 $\int x^3 \sin(x^2)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

input
```
Integrate[x^3*Sin[x^2],x]
```

output
```
-1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin \left(x^2\right) \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2} \int x^2 \sin \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int x^2 \sin \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2} \left(\int \cos \left(x^2\right) dx^2 - x^2 \cos \left(x^2\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \left(\int \sin \left(x^2 + \frac{\pi}{2}\right) dx^2 - x^2 \cos \left(x^2\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2} \left(\sin \left(x^2\right) - x^2 \cos \left(x^2\right)\right)$$

input
```
Int[x^3*Sin[x^2],x]
```

output
```
(-(x^2*Cos[x^2]) + Sin[x^2])/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
parallelrisch	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi}\left(-\frac{x^2\cos(x^2)}{2\sqrt{\pi}}+\frac{\sin(x^2)}{2\sqrt{\pi}}\right)$	27
orering	$\frac{5\sin(x^2)}{4}-\frac{3x^2\sin(x^2)+2x^4\cos(x^2)}{4x^2}$	32
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\tan\left(\frac{x^2}{2}\right)^2}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan\left(\frac{x^2}{2}\right)^2}$	39
parts	$\frac{x^3\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)}{2}-\frac{3\pi^2\left(\frac{2\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x^3}{3\pi^{\frac{3}{2}}}+\frac{2x^2\cos(x^2)}{3\pi^2}-\frac{2\sin(x^2)}{3\pi^2}\right)}{4}$	69

input
```
int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3\sin\left(x^2\right)\,dx=-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="fricas")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

input
```
integrate(x**3*sin(x**2),x)
```

output
```
-x**2*cos(x**2)/2 + sin(x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="maxima")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="giac")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = \frac{\sin \left(x^2\right)}{2} - \frac{x^2 \cos \left(x^2\right)}{2}$$

input
```
int(x^3*sin(x^2),x)
```

output
```
sin(x^2)/2 - (x^2*cos(x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{\cos \left(x^2\right) x^2}{2} + \frac{\sin \left(x^2\right)}{2}$$

input
```
int(x^3*sin(x^2),x)
```

output
```
( - cos(x**2)*x**2 + sin(x**2))/2
```

3.238 $\int \frac{x}{1-x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{x}{1-x^4} \, dx = \frac{\operatorname{arctanh}(x^2)}{2}$$

output
```
1/2*arctanh(x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{x}{1-x^4} \, dx = -\frac{1}{4} \log \left(1-x^2\right) + \frac{1}{4} \log \left(1+x^2\right)$$

input
```
Integrate[x/(1 - x^4),x]
```

output
```
-1/4*Log[1 - x^2] + Log[1 + x^2]/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{1-x^4} dx$$
$$\Big\downarrow \text{ 807}$$
$$\frac{1}{2}\int \frac{1}{1-x^4}dx^2$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\operatorname{arctanh}(x^2)}{2}$$

input
```
Int[x/(1 - x^4),x]
```

output
```
ArcTanh[x^2]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x^2)}{2}$	7
risch	$-\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}$	18
default	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22
norman	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22
parallelrisch	$-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	22

input
```
int(x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

input
```
integrate(x/(-x^4+1),x, algorithm="fricas")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{x}{1-x^4}\,dx = -\frac{\log(x^2-1)}{4} + \frac{\log(x^2+1)}{4}$$

input
```
integrate(x/(-x**4+1),x)
```

output
```
-log(x**2 - 1)/4 + log(x**2 + 1)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

input
```
integrate(x/(-x^4+1),x, algorithm="maxima")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{1-x^4}\,dx = \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(\left|x^2-1\right|\right)$$

input
```
integrate(x/(-x^4+1),x, algorithm="giac")
```

output
```
1/4*log(x^2 + 1) - 1/4*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1-x^4} \, dx = \frac{\operatorname{atanh}(x^2)}{2}$$

input
```
int(-x/(x^4 - 1),x)
```

output
```
atanh(x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{x}{1-x^4} \, dx = \frac{\log(x^2+1)}{4} - \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4}$$

input
```
int(x/(-x^4+1),x)
```

output
```
(log(x**2 + 1) - log(x - 1) - log(x + 1))/4
```

3.239 $\int \operatorname{sech}^2(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

output
```
tanh(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

input
```
Integrate[Sech[x]^2,x]
```

output
```
Tanh[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ix\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$i\int 1d(-i\tanh(x))$$

$$\Big\downarrow \text{24}$$

$$\tanh(x)$$

input
```
Int[Sech[x]^2,x]
```

output
```
Tanh[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\tanh(x)$	3
parallelrisch	$\frac{\sinh(x)}{\cosh(x)}$	8
risch	$-\frac{2}{1+e^{2x}}$	11

input
```
int(sech(x)^2,x,method=_RETURNVERBOSE)
```

output
```
tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(2) = 4.

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \operatorname{sech}^2(x)\,dx = -\frac{2}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

input
```
integrate(sech(x)^2,x, algorithm="fricas")
```

output
```
-2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \operatorname{sech}^2(x)\,dx = \int \operatorname{sech}^2(x)\,dx$$

input
```
integrate(sech(x)**2,x)
```

output
```
Integral(sech(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \operatorname{sech}^2(x)\,dx = \frac{2}{e^{(-2\,x)} + 1}$$

input
```
integrate(sech(x)^2,x, algorithm="maxima")
```

output
```
2/(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \operatorname{sech}^2(x)\,dx = -\frac{2}{e^{(2\,x)} + 1}$$

input
```
integrate(sech(x)^2,x, algorithm="giac")
```

output
```
-2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

input
```
int(1/cosh(x)^2,x)
```

output
```
tanh(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \operatorname{sech}^2(x)\,dx = \frac{2e^{2x}}{e^{2x}+1}$$

input
```
int(sech(x)^2,x)
```

output
```
(2*e**(2*x))/(e**(2*x) + 1)
```

3.240 $\int \left(e^{e^x}-e^{e^x-x}\right) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \left(e^{e^x}-e^{e^x-x}\right) \, dx=e^{e^x-x}$$

output
```
exp(exp(1)^x-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \left(e^{e^x}-e^{e^x-x}\right) \, dx=e^{e^x-x}$$

input
```
Integrate[E^E^x - E^(E^x - x),x]
```

output
```
E^(E^x - x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{e^x}-e^{e^x-x}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^{e^x-x}$$

input
```
Int[E^E^x - E^(E^x - x),x]
```

output
```
E^(E^x - x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
risch	$\mathrm{e}^{\mathrm{e}^{x}-x}$	8
norman	$\mathrm{e}^{\mathrm{e}^{x}} \mathrm{e}^{-x}$	9
default	$\mathrm{e}^{\mathrm{e}^{x}} \mathrm{e}^{-x}$	26
parts	$\mathrm{e}^{\mathrm{e}^{x}} \mathrm{e}^{-x}$	26

input
```
int(exp(exp(1)^x)-exp(exp(1)^x-x),x,method=_RETURNVERBOSE)
```

output
```
exp(exp(x)-x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(e^{e^x} - e^{e^x - x}\right) dx = e^{\left(-x + e^x\right)}$$

input
```
integrate(exp(exp(1)^x)-exp(exp(1)^x-x),x, algorithm="fricas")
```

output
```
e^(-x + e^x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(e^{e^x} - e^{e^x - x}\right) dx = e^{-x} e^{e^x}$$

input
```
integrate(exp(exp(1)**x)-exp(exp(1)**x-x),x)
```

output
```
exp(-x)*exp(exp(x))
```

Maxima [F]

$$\int \left(e^{e^x} - e^{e^x - x}\right) dx = \int -e^{\left(-x + e^x\right)} + e^{\left(e^x\right)}\, dx$$

input
```
integrate(exp(exp(1)^x)-exp(exp(1)^x-x),x, algorithm="maxima")
```

output
```
Ei(e^x) - integrate(e^(-x + e^x), x)
```

Giac [F]

$$\int \left(e^{e^x}-e^{e^x-x}\right)\,dx=\int -e^{(-x+e^x)}+e^{(e^x)}\,dx$$

input
```
integrate(exp(exp(1)^x)-exp(exp(1)^x-x),x, algorithm="giac")
```

output
```
integrate(-e^(-x + e^x) + e^(e^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \left(e^{e^x}-e^{e^x-x}\right)\,dx=\mathrm{e}^{\mathrm{e}^x-x}$$

input
```
int(exp(exp(x)) - exp(exp(x) - x),x)
```

output
```
exp(exp(x) - x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \left(e^{e^x}-e^{e^x-x}\right)\,dx=\frac{e^{e^x}}{e^x}$$

input
```
int(exp(exp(1)^x)-exp(exp(1)^x-x),x)
```

output
```
e**(e**x)/e**x
```

3.241 $\int \sqrt{1-\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt{1-\sqrt{x}}\,dx = -\frac{4}{3}\left(1-\sqrt{x}\right)^{3/2}+\frac{4}{5}\left(1-\sqrt{x}\right)^{5/2}$$

output
```
-4/3*(1-x^(1/2))^(3/2)+4/5*(1-x^(1/2))^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sqrt{1-\sqrt{x}}\,dx = -\frac{4}{15}\left(1-\sqrt{x}\right)^{3/2}\left(2+3\sqrt{x}\right)$$

input
```
Integrate[Sqrt[1 - Sqrt[x]],x]
```

output
```
(-4*(1 - Sqrt[x])^(3/2)*(2 + 3*Sqrt[x]))/15
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\sqrt{x}}\,dx$$

$$\Big\downarrow \text{774}$$

$$2\int \sqrt{1-\sqrt{x}}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\sqrt{1-\sqrt{x}}-\left(1-\sqrt{x}\right)^{3/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2}{5}\left(1-\sqrt{x}\right)^{5/2}-\frac{2}{3}\left(1-\sqrt{x}\right)^{3/2}\right)$$

input
```
Int[Sqrt[1 - Sqrt[x]],x]
```

output
```
2*((-2*(1 - Sqrt[x])^(3/2))/3 + (2*(1 - Sqrt[x])^(5/2))/5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{4(1-\sqrt{x})^{\frac{3}{2}}}{3}+\frac{4(1-\sqrt{x})^{\frac{5}{2}}}{5}$	24
default	$-\frac{4(1-\sqrt{x})^{\frac{3}{2}}}{3}+\frac{4(1-\sqrt{x})^{\frac{5}{2}}}{5}$	24
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1-\sqrt{x})^{\frac{3}{2}}(3\sqrt{x}+2)}{15}}{\sqrt{\pi}}$	33

input
```
int((1-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(1-x^(1/2))^(3/2)+4/5*(1-x^(1/2))^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sqrt{1-\sqrt{x}}\,dx = \frac{4}{15}\left(3\,x-\sqrt{x}-2\right)\sqrt{-\sqrt{x}+1}$$

input
```
integrate((1-x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/15*(3*x - sqrt(x) - 2)*sqrt(-sqrt(x) + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 306, normalized size of antiderivative = 8.74

$$\int \sqrt{1-\sqrt{x}}\,dx = \begin{cases} -\frac{12ix^{\frac{7}{2}}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^2} + \frac{4ix^{\frac{5}{2}}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^2} - \frac{8x^{\frac{5}{2}}}{-15x^{\frac{5}{2}}+15x^2} + \frac{16ix^3\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^2} - \frac{8ix^2\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^2} + \frac{8x^2}{-15x^{\frac{5}{2}}+15x^2} & \text{for } \left|\sqrt{x}\right| > 1 \\ -\frac{12x^{\frac{7}{2}}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^2} + \frac{4x^{\frac{5}{2}}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^2} - \frac{8x^{\frac{5}{2}}}{-15x^{\frac{5}{2}}+15x^2} + \frac{16x^3\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^2} - \frac{8x^2\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^2} + \frac{8x^2}{-15x^{\frac{5}{2}}+15x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((1-x**(1/2))**(1/2),x)
```

output
```
Piecewise((-12*I*x**(7/2)*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) + 4*I
*x**(5/2)*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) - 8*x**(5/2)/(-15*x**
(5/2) + 15*x**2) + 16*I*x**3*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) -
8*I*x**2*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) + 8*x**2/(-15*x**(5/2)
 + 15*x**2), Abs(sqrt(x)) > 1), (-12*x**(7/2)*sqrt(1 - sqrt(x))/(-15*x**(5
/2) + 15*x**2) + 4*x**(5/2)*sqrt(1 - sqrt(x))/(-15*x**(5/2) + 15*x**2) - 8
*x**(5/2)/(-15*x**(5/2) + 15*x**2) + 16*x**3*sqrt(1 - sqrt(x))/(-15*x**(5/
2) + 15*x**2) - 8*x**2*sqrt(1 - sqrt(x))/(-15*x**(5/2) + 15*x**2) + 8*x**2
/(-15*x**(5/2) + 15*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt{1-\sqrt{x}}\,dx = \frac{4}{5}\left(-\sqrt{x}+1\right)^{\frac{5}{2}} - \frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1-x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/5*(-sqrt(x) + 1)^(5/2) - 4/3*(-sqrt(x) + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \sqrt{1-\sqrt{x}}\,dx = \frac{4}{5}\left(\sqrt{x}-1\right)^{2}\sqrt{-\sqrt{x}+1} - \frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1-x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/5*(sqrt(x) - 1)^2*sqrt(-sqrt(x) + 1) - 4/3*(-sqrt(x) + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.29

$$\int \sqrt{1-\sqrt{x}}\,dx = x\,{}_2\mathrm{F}_1\left(-\frac{1}{2}, 2;\ 3;\ \sqrt{x}\right)$$

input
```
int((1 - x^(1/2))^(1/2),x)
```

output
```
x*hypergeom([-1/2, 2], 3, x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \sqrt{1-\sqrt{x}}\,dx = \frac{4\sqrt{-\sqrt{x}+1}\left(-\sqrt{x}+3x-2\right)}{15}$$

input
```
int((1-x^(1/2))^(1/2),x)
```

output
```
(4*sqrt( - sqrt(x) + 1)*( - sqrt(x) + 3*x - 2))/15
```

3.242 $\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}} \, dx = 6x - 6\log\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)$$

output
```
6*x-6*ln(1+x+1/2*x^2+1/6*x^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}} \, dx = 6\left(x - \log\left(6+6x+3x^2+x^3\right)\right)$$

input
```
Integrate[x^3/(1 + x + x^2/2 + x^3/6),x]
```

output
```
6*(x - Log[6 + 6*x + 3*x^2 + x^3])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. 2(24) = 48.

Time = 0.67 (sec) , antiderivative size = 194, normalized size of antiderivative = 8.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2490, 2485, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\frac{x^3}{6}+\frac{x^2}{2}+x+1}dx$$

$$\Big\downarrow \text{ 2490}$$

$$\int \frac{x^3}{\frac{1}{6}(x+1)^3+\frac{x+1}{2}+\frac{1}{3}}d(x+1)$$

$$\Big\downarrow \text{ 2485}$$

$$\frac{1}{36}\int \frac{216x^3}{\left(x+\frac{1-\left(-1+\sqrt{2}\right)^{2/3}}{\sqrt[3]{-1+\sqrt{2}}}+1\right)\left((x+1)^2-\frac{\left(1-\left(-1+\sqrt{2}\right)^{2/3}\right)(x+1)}{\sqrt[3]{-1+\sqrt{2}}}+\left(-1+\sqrt{2}\right)^{2/3}+\frac{1}{\left(-1+\sqrt{2}\right)^{2/3}}+1\right)}d(x+1)$$

$$\Big\downarrow \text{ 27}$$

$$-6\int -\frac{x^3}{\left(x-\sqrt[3]{-1+\sqrt{2}}+\frac{1}{\sqrt[3]{-1+\sqrt{2}}}+1\right)\left((x+1)^2-\frac{\left(1-\left(-1+\sqrt{2}\right)^{2/3}\right)(x+1)}{\sqrt[3]{-1+\sqrt{2}}}+\left(-1+\sqrt{2}\right)^{2/3}+\frac{1}{\left(-1+\sqrt{2}\right)^{2/3}}\right.}d(x+1)$$

$$\Big\downarrow \text{ 1200}$$

$$-6\int \left(\frac{2\left(\left(3-2\sqrt{2}+\left(-1+\sqrt{2}\right)^{2/3}-\left(-1+\sqrt{2}\right)^{4/3}\right)(x+1)-\left(1-\sqrt{2}\right)\left(1-\sqrt[3]{-1+\sqrt{2}}-\right.\right.}{\left(1-\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}\right)\left(\left(-1+\sqrt{2}\right)^{2/3}(x+1)^2-\left(1-\sqrt{2}+\sqrt[3]{-1+\sqrt{2}}\right)(x+1)+\left(-\right.\right.}\right.d(x+1)$$

$$\Big\downarrow \text{ 2009}$$

$$-6\left(-x+\log\left(\sqrt[3]{\sqrt{2}-1}(x+1)-\left(\sqrt{2}-1\right)^{2/3}+1\right)+\frac{2(1-\sqrt{2})\left(1-\sqrt[3]{\sqrt{2}-1}-\left(\sqrt{2}-1\right)^{2/3}\right)\log\left(\left(\sqrt{2}-\right.\right.}{\left(1-\sqrt{2}+\sqrt[3]{\right.}}\right.$$

input
```
Int[x^3/(1 + x + x^2/2 + x^3/6),x]
```

output
```
-6*(-1 - x + Log[1 - (-1 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(1/3)*(1 + x)]
+ (2*(1 - Sqrt[2])*(1 - (-1 + Sqrt[2])^(1/3) - (-1 + Sqrt[2])^(2/3))*Log[1
 + (-1 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(4/3) - (1 - Sqrt[2] + (-1 + Sqrt
[2])^(1/3))*(1 + x) + (-1 + Sqrt[2])^(2/3)*(1 + x)^2])/((1 - Sqrt[2] + (-1
 + Sqrt[2])^(1/3))*(1 - (-1 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(4/3))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$6x - 6\ln\left(x^3 + 3x^2 + 6x + 6\right)$	21
norman	$6x - 6\ln\left(x^3 + 3x^2 + 6x + 6\right)$	21
risch	$6x - 6\ln\left(x^3 + 3x^2 + 6x + 6\right)$	21
parallelrisch	$6x - 6\ln\left(x^3 + 3x^2 + 6x + 6\right)$	21

input
```
int(x^3/(1+x+1/2*x^2+1/6*x^3),x,method=_RETURNVERBOSE)
```

output
```
6*x-6*ln(x^3+3*x^2+6*x+6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1 + x + \frac{x^2}{2} + \frac{x^3}{6}}\, dx = 6\,x - 6\,\log\left(x^3 + 3\,x^2 + 6\,x + 6\right)$$

input
```
integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="fricas")
```

output
```
6*x - 6*log(x^3 + 3*x^2 + 6*x + 6)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx = 6x - 6\log\left(x^3+3x^2+6x+6\right)$$

input
```
integrate(x**3/(1+x+1/2*x**2+1/6*x**3),x)
```

output
```
6*x - 6*log(x**3 + 3*x**2 + 6*x + 6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx = 6\,x - 6\,\log\left(x^3+3\,x^2+6\,x+6\right)$$

input
```
integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="maxima")
```

output
```
6*x - 6*log(x^3 + 3*x^2 + 6*x + 6)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx = 6\,x - 6\,\log\left(\left|x^3+3\,x^2+6\,x+6\right|\right)$$

input
```
integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="giac")
```

output
```
6*x - 6*log(abs(x^3 + 3*x^2 + 6*x + 6))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx = 6\,x-6\,\ln\left(x^3+3\,x^2+6\,x+6\right)$$

input
```
int(x^3/(x + x^2/2 + x^3/6 + 1),x)
```

output
```
6*x - 6*log(6*x + 3*x^2 + x^3 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx = -6\log\left(x^3+3x^2+6x+6\right)+6x$$

input
```
int(x^3/(1+x+1/2*x^2+1/6*x^3),x)
```

output
```
6*( - log(x**3 + 3*x**2 + 6*x + 6) + x)
```

3.243 $\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 5

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=-2\cos(\sin(x))$$

output
```
-2*cos(sin(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=-2\cos(\sin(x))$$

input
```
Integrate[-Sin[x - Sin[x]] + Sin[x + Sin[x]],x]
```

output
```
-2*Cos[Sin[x]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x+\sin(x))-\sin(x-\sin(x)))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\int \sin(x+\sin(x))dx-\int \sin(x-\sin(x))dx$$

input
```
Int[-Sin[x - Sin[x]] + Sin[x + Sin[x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 11.89 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-2\cos\left(\sin\left(x\right)\right)$	6
default	$-2\cos\left(\sin\left(x\right)\right)$	6
risch	$-2\cos\left(\sin\left(x\right)\right)$	6

input
```
int(-sin(x-sin(x))+sin(x+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*cos(sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(5) = 10$.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 13.00

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx = -2\cos(x)\cos\left(\frac{x\tan\left(\frac{1}{2}x\right)^2+x+2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right) - 2\sin(x)\sin\left(\frac{x\tan\left(\frac{1}{2}x\right)^2+x+2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

input
```
integrate(-sin(x-sin(x))+sin(x+sin(x)),x, algorithm="fricas")
```

output
```
-2*cos(x)*cos((x*tan(1/2*x)^2 + x + 2*tan(1/2*x))/(tan(1/2*x)^2 + 1)) - 2*
sin(x)*sin((x*tan(1/2*x)^2 + x + 2*tan(1/2*x))/(tan(1/2*x)^2 + 1))
```

Sympy [F]

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx = \int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx$$

input
```
integrate(-sin(x-sin(x))+sin(x+sin(x)),x)
```

output
```
Integral(-sin(x - sin(x)) + sin(x + sin(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=-2\cos(\sin(x))$$

input
```
integrate(-sin(x-sin(x))+sin(x+sin(x)),x, algorithm="maxima")
```

output
```
-2*cos(sin(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=-2\cos(\sin(x))$$

input
```
integrate(-sin(x-sin(x))+sin(x+sin(x)),x, algorithm="giac")
```

output
```
-2*cos(sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=-2\cos(\sin(x))$$

input
```
int(sin(x + sin(x)) - sin(x - sin(x)),x)
```

output
```
-2*cos(sin(x))
```

Reduce [F]

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx=\int \sin(\sin(x)-x)\,dx+\int \sin(\sin(x)+x)\,dx$$

input
```
int(-sin(x-sin(x))+sin(x+sin(x)),x)
```

output
```
int(sin(sin(x) - x),x) + int(sin(sin(x) + x),x)
```

3.244 $\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx=\frac{1}{3}\sec^3(x)\tan^3(x)$$

output
```
1/3*sec(x)^3*tan(x)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx=\frac{1}{3}\sec^3(x)\tan^3(x)$$

input
```
Integrate[Sec[x]^5*Tan[x]^2 + Sec[x]^3*Tan[x]^4,x]
```

output
```
(Sec[x]^3*Tan[x]^3)/3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tan^2(x)\sec^5(x)+\tan^4(x)\sec^3(x)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\tan(x)\sec^5(x)+\frac{1}{6}\tan^3(x)\sec^3(x)-\frac{1}{6}\tan(x)\sec^3(x)$$

input
```
Int[Sec[x]^5*Tan[x]^2 + Sec[x]^3*Tan[x]^4,x]
```

output
```
-1/6*(Sec[x]^3*Tan[x]) + (Sec[x]^5*Tan[x])/6 + (Sec[x]^3*Tan[x]^3)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

method	result	size
risch	$\frac{8i\left(e^{9ix}-3\,e^{7ix}+3\,e^{5ix}-e^{3ix}\right)}{3\left(e^{2ix}+1\right)^{6}}$	40
default	$\frac{\sin(x)^{5}}{6\cos(x)^{6}}+\frac{\sin(x)^{5}}{24\cos(x)^{4}}-\frac{\sin(x)^{5}}{48\cos(x)^{2}}-\frac{\sin(x)^{3}}{48}+\frac{\sin(x)^{3}}{6\cos(x)^{6}}+\frac{\sin(x)^{3}}{8\cos(x)^{4}}+\frac{\sin(x)^{3}}{16\cos(x)^{2}}$	68
parts	$\frac{\sin(x)^{5}}{6\cos(x)^{6}}+\frac{\sin(x)^{5}}{24\cos(x)^{4}}-\frac{\sin(x)^{5}}{48\cos(x)^{2}}-\frac{\sin(x)^{3}}{48}+\frac{\sin(x)^{3}}{6\cos(x)^{6}}+\frac{\sin(x)^{3}}{8\cos(x)^{4}}+\frac{\sin(x)^{3}}{16\cos(x)^{2}}$	68

input
```
int(sec(x)^5*tan(x)^2+sec(x)^3*tan(x)^4,x,method=_RETURNVERBOSE)
```

output
```
8/3*I/(exp(2*I*x)+1)^6*(exp(9*I*x)-3*exp(7*I*x)+3*exp(5*I*x)-exp(3*I*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx=-\frac{\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}{3\,\cos\left(x\right)^{6}}$$

input
```
integrate(sec(x)^5*tan(x)^2+sec(x)^3*tan(x)^4,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(10) = 20.

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 6.67

$$\begin{aligned}\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx&=\frac{-3\sin^5(x)-8\sin^3(x)+3\sin(x)}{48\sin^6(x)-144\sin^4(x)+144\sin^2(x)-48}\\&\quad+\frac{3\sin^5(x)-8\sin^3(x)-3\sin(x)}{48\sin^6(x)-144\sin^4(x)+144\sin^2(x)-48}\end{aligned}$$

input
```
integrate(sec(x)**5*tan(x)**2+sec(x)**3*tan(x)**4,x)
```

output
```
(-3*sin(x)**5 - 8*sin(x)**3 + 3*sin(x))/(48*sin(x)**6 - 144*sin(x)**4 + 14
4*sin(x)**2 - 48) + (3*sin(x)**5 - 8*sin(x)**3 - 3*sin(x))/(48*sin(x)**6 -
 144*sin(x)**4 + 144*sin(x)**2 - 48)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 6.58

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx = -\frac{3\,\sin(x)^5+8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)} + \frac{3\,\sin(x)^5-8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)}$$

input
```
integrate(sec(x)^5*tan(x)^2+sec(x)^3*tan(x)^4,x, algorithm="maxima")
```

output
```
-1/48*(3*sin(x)^5 + 8*sin(x)^3 - 3*sin(x))/(sin(x)^6 - 3*sin(x)^4 + 3*sin(
x)^2 - 1) + 1/48*(3*sin(x)^5 - 8*sin(x)^3 - 3*sin(x))/(sin(x)^6 - 3*sin(x)
^4 + 3*sin(x)^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.58

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx = -\frac{3\,\sin(x)^5+8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^2-1\right)^3} + \frac{3\,\sin(x)^5-8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^2-1\right)^3}$$

input
```
integrate(sec(x)^5*tan(x)^2+sec(x)^3*tan(x)^4,x, algorithm="giac")
```

output
```
-1/48*(3*sin(x)^5 + 8*sin(x)^3 - 3*sin(x))/(sin(x)^2 - 1)^3 + 1/48*(3*sin(
x)^5 - 8*sin(x)^3 - 3*sin(x))/(sin(x)^2 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx=-\frac{\sin\left(x\right)^3}{3\,\left(\sin\left(x\right)^2-1\right)^3}$$

input
```
int(tan(x)^4/cos(x)^3 + tan(x)^2/cos(x)^5,x)
```

output
```
-sin(x)^3/(3*(sin(x)^2 - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx=-\frac{\sin\left(x\right)^3}{3\sin\left(x\right)^6-9\sin\left(x\right)^4+9\sin\left(x\right)^2-3}$$

input
```
int(sec(x)^5*tan(x)^2+sec(x)^3*tan(x)^4,x)
```

output
```
( - sin(x)**3)/(3*(sin(x)**6 - 3*sin(x)**4 + 3*sin(x)**2 - 1))
```

3.245 $\int (1+\log(x))\log(\log(x))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 10

$$\int (1+\log(x))\log(\log(x))\,dx = x(-1+\log(x)\log(\log(x)))$$

output
```
x*(-1+ln(x)*ln(ln(x)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int (1+\log(x))\log(\log(x))\,dx = -x+x\log(\log(x))+x(-1+\log(x))\log(\log(x))$$

input
```
Integrate[(1 + Log[x])*Log[Log[x]],x]
```

output
```
-x + x*Log[Log[x]] + x*(-1 + Log[x])*Log[Log[x]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\log(x)+1)\log(\log(x))\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int (\log(x)\log(\log(x))+\log(\log(x)))dx$$

$$\Big\downarrow \text{2009}$$

$$\int \log(x)\log(\log(x))dx-\text{LogIntegral}(x)+x\log(\log(x))$$

input
```
Int[(1 + Log[x])*Log[Log[x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
norman	$x\ln(x)\ln(\ln(x))-x$	12
risch	$x\ln(x)\ln(\ln(x))-x$	12
parallelrisch	$x\ln(x)\ln(\ln(x))-x$	12
default	$(-1+\ln(x))\,x\ln(\ln(x))-x+x\ln(\ln(x))$	19
parts	$(-1+\ln(x))\,x\ln(\ln(x))-x+x\ln(\ln(x))$	19

input
```
int((1+ln(x))*ln(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
x*ln(x)*ln(ln(x))-x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int (1+\log(x))\log(\log(x))\,dx = x\log(x)\log(\log(x)) - x$$

input
```
integrate((1+log(x))*log(log(x)),x, algorithm="fricas")
```

output
```
x*log(x)*log(log(x)) - x
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (1+\log(x))\log(\log(x))\,dx = x\log(x)\log(\log(x)) - x$$

input
```
integrate((1+ln(x))*ln(ln(x)),x)
```

output
```
x*log(x)*log(log(x)) - x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int (1+\log(x))\log(\log(x))\,dx = (x(\log(x)-1)+x)\log(\log(x)) - x$$

input
```
integrate((1+log(x))*log(log(x)),x, algorithm="maxima")
```

output
```
(x*(log(x) - 1) + x)*log(log(x)) - x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int (1+\log(x))\log(\log(x))\,dx = x\log\left(x\right)\log\left(\log\left(x\right)\right) - x$$

input
```
integrate((1+log(x))*log(log(x)),x, algorithm="giac")
```

output
```
x*log(x)*log(log(x)) - x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int (1+\log(x))\log(\log(x))\,dx = x\,\ln\left(\ln\left(x\right)\right)\,\ln\left(x\right) - x$$

input
```
int(log(log(x))*(log(x) + 1),x)
```

output
```
x*log(log(x))*log(x) - x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (1+\log(x))\log(\log(x))\,dx = x(\log(\log(x))\log(x) - 1)$$

input
```
int((1+log(x))*log(log(x)),x)
```

output
```
x*(log(log(x))*log(x) - 1)
```

3.246 $\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\right.$

Optimal result

Integrand size = 37, antiderivative size = 3

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)\,dx=3x$$

output
```
3*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)\,dx=3x$$

input
```
Integrate[(1 + Cos[x])^(-1) + (1 + Cot[x])^(-1) + (1 + Csc[x])^(-1) + (1 +
 Sec[x])^(-1) + (1 + Sin[x])^(-1) + (1 + Tan[x])^(-1),x]
```

output
```
3*x
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 14.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\sin(x)+1}+\frac{1}{\cos(x)+1}+\frac{1}{\tan(x)+1}+\frac{1}{\cot(x)+1}+\frac{1}{\csc(x)+1}+\frac{1}{\sec(x)+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3x+\frac{\sin(x)}{\cos(x)+1}-\frac{\cos(x)}{\sin(x)+1}+\frac{\cot(x)}{\csc(x)+1}-\frac{\tan(x)}{\sec(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1) + (1 + Cot[x])^(-1) + (1 + Csc[x])^(-1) + (1 + Sec[x
])^(-1) + (1 + Sin[x])^(-1) + (1 + Tan[x])^(-1),x]
```

output
```
3*x + Cot[x]/(1 + Csc[x]) + Sin[x]/(1 + Cos[x]) - Cos[x]/(1 + Sin[x]) - Ta
n[x]/(1 + Sec[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result
risch	$3x$
norman	$\frac{3x+3x\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}$
orering	$x\left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)$
default	$-\frac{\ln(1+\cot(x))}{2}+\frac{\ln\left(1+\cot(x)^2\right)}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}(\cot(x))}{2}+4\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{\ln(1+\tan(x))}{2}-\frac{\ln\left(1+\tan(x)^2\right)}{4}$
parts	$-\frac{\ln(1+\cot(x))}{2}+\frac{\ln\left(1+\cot(x)^2\right)}{4}-\frac{\pi}{4}+\frac{\operatorname{arccot}(\cot(x))}{2}+4\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{\ln(1+\tan(x))}{2}-\frac{\ln\left(1+\tan(x)^2\right)}{4}$

input
```
int(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))+1/(1+
tan(x)),x,method=_RETURNVERBOSE)
```

output
```
3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)\,dx=3\,x$$

input
```
integrate(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))
+1/(1+tan(x)),x, algorithm="fricas")
```

output
```
3*x
```

Sympy [F]

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))
+1/(1+tan(x)),x)
```

output
```
Integral((sin(x)*cos(x)*tan(x)*cot(x)*csc(x) + sin(x)*cos(x)*tan(x)*cot(x)
*sec(x) + 2*sin(x)*cos(x)*tan(x)*cot(x) + sin(x)*cos(x)*tan(x)*csc(x)*sec(
x) + 2*sin(x)*cos(x)*tan(x)*csc(x) + 2*sin(x)*cos(x)*tan(x)*sec(x) + 3*sin
(x)*cos(x)*tan(x) + sin(x)*cos(x)*cot(x)*csc(x)*sec(x) + 2*sin(x)*cos(x)*c
ot(x)*csc(x) + 2*sin(x)*cos(x)*cot(x)*sec(x) + 3*sin(x)*cos(x)*cot(x) + 2*
sin(x)*cos(x)*csc(x)*sec(x) + 3*sin(x)*cos(x)*csc(x) + 3*sin(x)*cos(x)*sec
(x) + 4*sin(x)*cos(x) + sin(x)*tan(x)*cot(x)*csc(x)*sec(x) + 2*sin(x)*tan(
x)*cot(x)*csc(x) + 2*sin(x)*tan(x)*cot(x)*sec(x) + 3*sin(x)*tan(x)*cot(x)
+ 2*sin(x)*tan(x)*csc(x)*sec(x) + 3*sin(x)*tan(x)*csc(x) + 3*sin(x)*tan(x)
*sec(x) + 4*sin(x)*tan(x) + 2*sin(x)*cot(x)*csc(x)*sec(x) + 3*sin(x)*cot(x
)*csc(x) + 3*sin(x)*cot(x)*sec(x) + 4*sin(x)*cot(x) + 3*sin(x)*csc(x)*sec(
x) + 4*sin(x)*csc(x) + 4*sin(x)*sec(x) + 5*sin(x) + cos(x)*tan(x)*cot(x)*c
sc(x)*sec(x) + 2*cos(x)*tan(x)*cot(x)*csc(x) + 2*cos(x)*tan(x)*cot(x)*sec(
x) + 3*cos(x)*tan(x)*cot(x) + 2*cos(x)*tan(x)*csc(x)*sec(x) + 3*cos(x)*tan
(x)*csc(x) + 3*cos(x)*tan(x)*sec(x) + 4*cos(x)*tan(x) + 2*cos(x)*cot(x)*cs
c(x)*sec(x) + 3*cos(x)*cot(x)*csc(x) + 3*cos(x)*cot(x)*sec(x) + 4*cos(x)*c
ot(x) + 3*cos(x)*csc(x)*sec(x) + 4*cos(x)*csc(x) + 4*cos(x)*sec(x) + 5*cos
(x) + 2*tan(x)*cot(x)*csc(x)*sec(x) + 3*tan(x)*cot(x)*csc(x) + 3*tan(x)*co
t(x)*sec(x) + 4*tan(x)*cot(x) + 3*tan(x)*csc(x)*sec(x) + 4*tan(x)*csc(x) +
 4*tan(x)*sec(x) + 5*tan(x) + 3*cot(x)*csc(x)*sec(x) + 4*cot(x)*csc(x) ...
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)dx = x+4\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))
+1/(1+tan(x)),x, algorithm="maxima")
```

output
```
x + 4*arctan(sin(x)/(cos(x) + 1))
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 13.33

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)dx = 3x-\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}-\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))
+1/(1+tan(x)),x, algorithm="giac")
```

output
```
3*x - 2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1)) - tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)\,dx=3\,x$$

input
```
int(1/(1/sin(x) + 1) + 1/(cos(x) + 1) + 1/(cot(x) + 1) + 1/(sin(x) + 1) +
1/(tan(x) + 1) + 1/(1/cos(x) + 1),x)
```

output
```
3*x
```

Reduce [B] (verification not implemented)

Time = 16.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 18.33

$$\int \left(\frac{1}{1+\cos(x)}+\frac{1}{1+\cot(x)}+\frac{1}{1+\csc(x)}+\frac{1}{1+\sec(x)}+\frac{1}{1+\sin(x)}+\frac{1}{1+\tan(x)}\right)\,dx=\frac{\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)}{2}-\frac{\log\left(\tan(x)^2+1\right)}{4}-\frac{\log\left(-\sqrt{2}+\tan\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)-1\right)}{2}+\frac{\log(\tan(x)+1)}{2}+3x$$

input
```
int(1/(1+cos(x))+1/(1+cot(x))+1/(1+csc(x))+1/(1+sec(x))+1/(1+sin(x))+1/(1+
tan(x)),x)
```

output
```
(2*log(tan(x/2)**2 + 1) - log(tan(x)**2 + 1) - 2*log( - sqrt(2) + tan(x/2)
 - 1) - 2*log(sqrt(2) + tan(x/2) - 1) + 2*log(tan(x) + 1) + 12*x)/4
```

3.247 $\int \frac{1}{\sqrt{x-x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\arcsin(1-2x)$$

output
```
arcsin(-1+2*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{x-x^2}} \, dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/Sqrt[x - x^2],x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-x^2}}\,dx$$
$$\Big\downarrow \text{1090}$$
$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$
$$\Big\downarrow \text{223}$$
$$-\arcsin(1-2x)$$

input
```
Int[1/Sqrt[x - x^2],x]
```

output
```
-ArcSin[1 - 2*x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	siz
default	$\arcsin\left(-1+2x\right)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-x(-1+x)}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	36

input
```
int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1+2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+x}}{x-1}\right)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + x)/(x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}(2x-1)$$

input
```
integrate(1/(-x**2+x)**(1/2),x)
```

output
```
asin(2*x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \arcsin(2\,x-1)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \frac{1}{4}\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\arcsin(2\,x-1)$$

input
```
integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = \operatorname{asin}(2\,x-1)$$

input
```
int(1/(x - x^2)^(1/2),x)
```

output
```
asin(2*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{x-x^2}}\,dx = -2\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i$$

input
```
int(1/(-x^2+x)^(1/2),x)
```

output
```
- 2*log(sqrt( - x + 1) + sqrt(x)*i)*i
```

3.248 $\int \frac{1}{1+\cos^2(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^2(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$
$$\Big\downarrow \text{3660}$$
$$-\int \frac{1}{2\cot^2(x)+1}d\cot(x)$$
$$\Big\downarrow \text{216}$$
$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{\sqrt{2}}$$

input
```
Int[(1 + Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{2}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{4}$	40

input
```
int(1/(1+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+\cos^2(x)}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos{(x)}^2-\sqrt{2}}{4\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

input
```
integrate(1/(1+cos(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.38

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

input
```
int(1/(cos(x)^2 + 1),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\left(-atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{2}$$

input
```
int(1/(1+cos(x)^2),x)
```

output
```
(sqrt(2)*( - atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + atan((sqrt(2) + 2*tan(
x/2))/sqrt(2))))/2
```

3.249 $\int \frac{\log(1+x)}{x^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 18

$$\int \frac{\log(1+x)}{x^2} dx = \log(x) - \log(1+x) - \frac{\log(1+x)}{x}$$

output
```
ln(x)-ln(1+x)-ln(1+x)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2} dx = \log(x) - \log(1+x) - \frac{\log(1+x)}{x}$$

input
```
Integrate[Log[1 + x]/x^2,x]
```

output
```
Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x+1)}{x^2} \, dx$$

$$\Big\downarrow \; 2842$$

$$\int \frac{1}{x(x+1)}dx - \frac{\log(x+1)}{x}$$

$$\Big\downarrow \; 47$$

$$\int \frac{1}{x}dx - \int \frac{1}{x+1}dx - \frac{\log(x+1)}{x}$$

$$\Big\downarrow \; 14$$

$$-\int \frac{1}{x+1}dx + \log(x) - \frac{\log(x+1)}{x}$$

$$\Big\downarrow \; 16$$

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

input
```
Int[Log[1 + x]/x^2,x]
```

output
```
Log[x] - Log[1 + x] - Log[1 + x]/x
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\ln(x) - \frac{\ln(1+x)(1+x)}{x}$	16
default	$\ln(x) - \frac{\ln(1+x)(1+x)}{x}$	16
meijerg	$\ln(x) - \frac{(2x+2)\ln(1+x)}{2x}$	18
risch	$\ln(x) - \ln(1+x) - \frac{\ln(1+x)}{x}$	19
parts	$\ln(x) - \ln(1+x) - \frac{\ln(1+x)}{x}$	19
norman	$\frac{-\ln(1+x)x-\ln(1+x)}{x} + \ln(x)$	22
parallelrisch	$\frac{x\ln(x)-\ln(1+x)x-\ln(1+x)}{x}$	23

input
```
int(ln(1+x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-ln(1+x)*(1+x)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\log(1+x)}{x^2}\,dx = -\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

input
```
integrate(log(1+x)/x^2,x, algorithm="fricas")
```

output
```
-((x + 1)*log(x + 1) - x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\log(1+x)}{x^2}\,dx = \log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

input
```
integrate(ln(1+x)/x**2,x)
```

output
```
log(x) - log(x + 1) - log(x + 1)/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2}\,dx = -\frac{\log(x+1)}{x} - \log(x+1) + \log(x)$$

input
```
integrate(log(1+x)/x^2,x, algorithm="maxima")
```

output
```
-log(x + 1)/x - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log(1+x)}{x^2}\,dx = -\frac{\log(x+1)}{x} - \log(|x+1|) + \log(|x|)$$

input
```
integrate(log(1+x)/x^2,x, algorithm="giac")
```

output
```
-log(x + 1)/x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+x)}{x^2}\,dx = -\ln\left(\frac{1}{x}+1\right) - \frac{\ln(x+1)}{x}$$

input
```
int(log(x + 1)/x^2,x)
```

output
```
- log(1/x + 1) - log(x + 1)/x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\log(1+x)}{x^2}\,dx = \frac{-\log(x+1)\,x - \log(x+1) + \log(x)\,x}{x}$$

input
```
int(log(1+x)/x^2,x)
```

output
```
( - log(x + 1)*x - log(x + 1) + log(x)*x)/x
```

3.250 $\int \sqrt{1-\arccos(\sin(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx=-\frac{1}{2}\left(\arccos(\sin(x))\sqrt{1-\arccos(\sin(x))^2}-2\arctan\left(\frac{\sqrt{1-\arccos(\sin(x))^2}}{1+\arccos(\sin(x))}\right)\right)\sqrt{\cos^2(x)}\sec(x)$$

output
```
-1/2*(arccos(sin(x))*(1-arccos(sin(x))^2)^(1/2)-2*arctan((1-arccos(sin(x))
^2)^(1/2)/(1+arccos(sin(x)))))*(cos(x)^2)^(1/2)*sec(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx=-\frac{1}{2}\left(\arccos(\sin(x))\sqrt{1-\arccos(\sin(x))^2}-2\arctan\left(\frac{\sqrt{1-\arccos(\sin(x))^2}}{1+\arccos(\sin(x))}\right)\right)\sqrt{\cos^2(x)}\sec(x)$$

input
```
Integrate[Sqrt[1 - ArcCos[Sin[x]]^2],x]
```

output
```
-1/2*((ArcCos[Sin[x]]*Sqrt[1 - ArcCos[Sin[x]]^2] - 2*ArcTan[Sqrt[1 - ArcCo
s[Sin[x]]^2]/(1 + ArcCos[Sin[x]])])*Sqrt[Cos[x]^2]*Sec[x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sqrt{1-\arccos(\sin(x))^2}dx$$

input
```
Int[Sqrt[1 - ArcCos[Sin[x]]^2],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \sqrt{1-\arccos\left(\sin\left(x\right)\right)^{2}}dx$$

input
```
int((1-arccos(sin(x))^2)^(1/2),x)
```

output
```
int((1-arccos(sin(x))^2)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = -\frac{1}{8}\,(\pi-2\,x)\sqrt{-\pi^2+4\,\pi x-4\,x^2+4} - \frac{1}{2}\arctan\left(-\frac{(\pi-2\,x)\sqrt{-\pi^2+4\,\pi x-4\,x^2+4}}{\pi^2-4\,\pi x+4\,x^2-4}\right)$$

input
```
integrate((1-arccos(sin(x))^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(pi - 2*x)*sqrt(-pi^2 + 4*pi*x - 4*x^2 + 4) - 1/2*arctan(-(pi - 2*x)*
sqrt(-pi^2 + 4*pi*x - 4*x^2 + 4)/(pi^2 - 4*pi*x + 4*x^2 - 4))
```

Sympy [F]

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = \int \sqrt{1-\operatorname{acos}^2{(\sin{(x)})}}\,dx$$

input
```
integrate((1-acos(sin(x))**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - acos(sin(x))**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = -\frac{1}{4}\,(\pi-2\,x)\sqrt{-\frac{1}{4}\,\pi^2+\pi x-x^2+1} + \frac{1}{2}\arctan\left(-\frac{1}{2}\,\pi+x, \sqrt{-\frac{1}{4}\,\pi^2+\pi x-x^2+1}\right)$$

input
```
integrate((1-arccos(sin(x))^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(pi - 2*x)*sqrt(-1/4*pi^2 + pi*x - x^2 + 1) + 1/2*arctan2(-1/2*pi + x
, sqrt(-1/4*pi^2 + pi*x - x^2 + 1))
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((1-arccos(sin(x))^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = \int \sqrt{1-\mathrm{acos}\left(\sin\left(x\right)\right)^2}\,dx$$

input
```
int((1 - acos(sin(x))^2)^(1/2),x)
```

output
```
int((1 - acos(sin(x))^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx = \int \sqrt{-acos\left(\sin\left(x\right)\right)^2+1}dx$$

input
```
int((1-acos(sin(x))^2)^(1/2),x)
```

output
```
int(sqrt( - acos(sin(x))**2 + 1),x)
```

3.251 $\int (-2+x)(-1+x)x(1+x)(2+x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = 2x^2 - \frac{5x^4}{4} + \frac{x^6}{6}$$

output
```
2*x^2-5/4*x^4+1/6*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = 2x^2 - \frac{5x^4}{4} + \frac{x^6}{6}$$

input
```
Integrate[(-2 + x)*(-1 + x)*x*(1 + x)*(2 + x),x]
```

output
```
2*x^2 - (5*x^4)/4 + x^6/6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2109, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-2)(x-1)x(x+1)(x+2)\,dx$$

$$\Big\downarrow \text{ 2109}$$

$$\int \left(x^5-5x^3+4x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^6}{6}-\frac{5x^4}{4}+2x^2$$

input
```
Int[(-2 + x)*(-1 + x)*x*(1 + x)*(2 + x),x]
```

output
```
2*x^2 - (5*x^4)/4 + x^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2109
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
default	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
norman	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
risch	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
parallelrisch	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
orering	$\frac{x^2(2x^4-15x^2+24)}{12}$	18

input
```
int((-2+x)*(-1+x)*x*(1+x)*(2+x),x,method=_RETURNVERBOSE)
```

output
```
2*x^2-5/4*x^4+1/6*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{1}{6}\,x^6 - \frac{5}{4}\,x^4 + 2\,x^2$$

input
```
integrate((-2+x)*(-1+x)*x*(1+x)*(2+x),x, algorithm="fricas")
```

output
```
1/6*x^6 - 5/4*x^4 + 2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{x^6}{6} - \frac{5x^4}{4} + 2x^2$$

input
```
integrate((-2+x)*(-1+x)*x*(1+x)*(2+x),x)
```

output
```
x**6/6 - 5*x**4/4 + 2*x**2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{1}{6}\,x^6 - \frac{5}{4}\,x^4 + 2\,x^2$$

input
```
integrate((-2+x)*(-1+x)*x*(1+x)*(2+x),x, algorithm="maxima")
```

output
```
1/6*x^6 - 5/4*x^4 + 2*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{1}{6}\,x^6 - \frac{5}{4}\,x^4 + 2\,x^2$$

input
```
integrate((-2+x)*(-1+x)*x*(1+x)*(2+x),x, algorithm="giac")
```

output
```
1/6*x^6 - 5/4*x^4 + 2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{x^2\,(2\,x^4-15\,x^2+24)}{12}$$

input
```
int(x*(x - 1)*(x + 1)*(x - 2)*(x + 2),x)
```

output
```
(x^2*(2*x^4 - 15*x^2 + 24))/12
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (-2+x)(-1+x)x(1+x)(2+x)\,dx = \frac{x^2(2x^4-15x^2+24)}{12}$$

input
```
int((-2+x)*(-1+x)*x*(1+x)*(2+x),x)
```

output
```
(x**2*(2*x**4 - 15*x**2 + 24))/12
```

3.252 $\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 21

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = \left(1+x^3\right)^{2/3}+x\left(1+x^3\right)^{2/3}$$

output
```
(x^3+1)^(2/3)+x*(x^3+1)^(2/3)
```

Mathematica [A] (verified)

Time = 8.57 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = (1+x)\left(1+x^3\right)^{2/3}$$

input
```
Integrate[(1 + 2*x^2 + 3*x^3)/(1 + x^3)^(1/3),x]
```

output
```
(1 + x)*(1 + x^3)^(2/3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^3+2x^2+1}{\sqrt[3]{x^3+1}}dx$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(\frac{3x^3}{\sqrt[3]{x^3+1}}+\frac{1}{\sqrt[3]{x^3+1}}+\frac{2x^2}{\sqrt[3]{x^3+1}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\left(x^3+1\right)^{2/3}x+\left(x^3+1\right)^{2/3}$$

input
```
Int[(1 + 2*x^2 + 3*x^3)/(1 + x^3)^(1/3),x]
```

output
```
(1 + x^3)^(2/3) + x*(1 + x^3)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
trager	$(1+x)\left(x^{3}+1\right)^{\frac{2}{3}}$	12
risch	$(1+x)\left(x^{3}+1\right)^{\frac{2}{3}}$	12
gosper	$\frac{(1+x)^{2}\left(x^{2}-x+1\right)}{\left(x^{3}+1\right)^{\frac{1}{3}}}$	22
orering	$\frac{(1+x)\left(x^{2}-x+1\right)\left(3x^{3}+2x^{2}+1\right)}{\left(3x^{2}-x+1\right)\left(x^{3}+1\right)^{\frac{1}{3}}}$	44
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^{3}\right)+\frac{3x^{4}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{4}{3}\right],\left[\frac{7}{3}\right],-x^{3}\right)}{4}+\frac{2x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1\right],[2],-x^{3}\right)}{3}$	47

input
```
int((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*(x^3+1)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx=\left(x^{3}+1\right)^{\frac{2}{3}}(x+1)$$

input
```
integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
(x^3 + 1)^(2/3)*(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{\Gamma\left(\frac{7}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \left(x^3+1\right)^{\frac{2}{3}}$$

input
```
integrate((3*x**3+2*x**2+1)/(x**3+1)**(1/3),x)
```

output
```
x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi))/gamma(7/3)
 + x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3)) + (x**3 + 1)**(2/3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = \left(x^3+1\right)^{\frac{2}{3}} + \frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2\left(\frac{x^3+1}{x^3}-1\right)}$$

input
```
integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
(x^3 + 1)^(2/3) + (x^3 + 1)^(2/3)/(x^2*((x^3 + 1)/x^3 - 1))
```

Giac [F]

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = \int \frac{3\,x^3+2\,x^2+1}{\left(x^3+1\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate((3*x^3 + 2*x^2 + 1)/(x^3 + 1)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = \left(x^3+1\right)^{2/3}\,(x+1)$$

input
```
int((2*x^2 + 3*x^3 + 1)/(x^3 + 1)^(1/3),x)
```

output
```
(x^3 + 1)^(2/3)*(x + 1)
```

Reduce [F]

$$\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}}\,dx = 3\left(\int \frac{x^3}{\left(x^3+1\right)^{\frac{1}{3}}}dx\right)+2\left(\int \frac{x^2}{\left(x^3+1\right)^{\frac{1}{3}}}dx\right)+\int \frac{1}{\left(x^3+1\right)^{\frac{1}{3}}}dx$$

input
```
int((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x)
```

output
```
3*int(x**3/(x**3 + 1)**(1/3),x) + 2*int(x**2/(x**3 + 1)**(1/3),x) + int(1/
(x**3 + 1)**(1/3),x)
```

3.253 $\int \csc^4(x)\sec^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \csc^4(x)\sec^4(x)\,dx = -8\cot(2x) - \frac{8}{3}\cot^3(2x)$$

output
```
-8*cot(2*x)-8/3*cot(2*x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \csc^4(x)\sec^4(x)\,dx = -\frac{8\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x) + \frac{8\tan(x)}{3} + \frac{1}{3}\sec^2(x)\tan(x)$$

input
```
Integrate[Csc[x]^4*Sec[x]^4,x]
```

output
```
(-8*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (8*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(x)\sec^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^4\sec(x)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\int \left(\tan^2(x)+1\right)^3\cot^4(x)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^2(x)+\cot^4(x)+3\cot^2(x)+3\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^3(x)}{3}+3\tan(x)-\frac{1}{3}\cot^3(x)-3\cot(x)$$

input
```
Int[Csc[x]^4*Sec[x]^4,x]
```

output
```
-3*Cot[x] - Cot[x]^3/3 + 3*Tan[x] + Tan[x]^3/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

method	result	size
risch	$\frac{32i\left(3\,{\mathrm e}^{4ix}-1\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}\left({\mathrm e}^{2ix}+1\right)^{3}}$	31
default	$\frac{1}{3\sin\left(x\right)^{3}\cos\left(x\right)^{3}}-\frac{2}{3\sin\left(x\right)^{3}\cos\left(x\right)}+\frac{8}{3\cos\left(x\right)\sin\left(x\right)}-\frac{16\cot\left(x\right)}{3}$	36
norman	$\frac{\frac{1}{24}+\frac{5\tan\left(\frac{x}{2}\right)^{2}}{4}-\frac{91\tan\left(\frac{x}{2}\right)^{4}}{8}+\frac{35\tan\left(\frac{x}{2}\right)^{6}}{2}-\frac{91\tan\left(\frac{x}{2}\right)^{8}}{8}+\frac{5\tan\left(\frac{x}{2}\right)^{10}}{4}+\frac{\tan\left(\frac{x}{2}\right)^{12}}{24}}{\tan\left(\frac{x}{2}\right)^{3}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)^{3}}$	68
parallelrisch	$\frac{\tan\left(\frac{x}{2}\right)^{9}+30\tan\left(\frac{x}{2}\right)^{7}-273\tan\left(\frac{x}{2}\right)^{5}+\cot\left(\frac{x}{2}\right)^{3}+420\tan\left(\frac{x}{2}\right)^{3}+30\cot\left(\frac{x}{2}\right)-273\tan\left(\frac{x}{2}\right)}{24\left(\tan\left(\frac{x}{2}\right)-1\right)^{3}\left(1+\tan\left(\frac{x}{2}\right)\right)^{3}}$	68

input
```
int(csc(x)^4*sec(x)^4,x,method=_RETURNVERBOSE)
```

output
```
32/3*I*(3*exp(4*I*x)-1)/(exp(2*I*x)-1)^3/(exp(2*I*x)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \csc^4(x)\sec^4(x)\,dx = -\frac{16\,\cos(x)^6 - 24\,\cos(x)^4 + 6\,\cos(x)^2 + 1}{3\,\left(\cos(x)^5 - \cos(x)^3\right)\sin(x)}$$

input
```
integrate(csc(x)^4*sec(x)^4,x, algorithm="fricas")
```

output
```
-1/3*(16*cos(x)^6 - 24*cos(x)^4 + 6*cos(x)^2 + 1)/((cos(x)^5 - cos(x)^3)*s
in(x))
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \csc^4(x)\sec^4(x)\,dx = -\frac{16\cos(2x)}{3\sin(2x)} - \frac{8\cos(2x)}{3\sin^3(2x)}$$

input
```
integrate(csc(x)**4*sec(x)**4,x)
```

output
```
-16*cos(2*x)/(3*sin(2*x)) - 8*cos(2*x)/(3*sin(2*x)**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \csc^4(x)\sec^4(x)\,dx = \frac{1}{3}\,\tan(x)^3 - \frac{9\,\tan(x)^2 + 1}{3\,\tan(x)^3} + 3\,\tan(x)$$

input
```
integrate(csc(x)^4*sec(x)^4,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3 - 1/3*(9*tan(x)^2 + 1)/tan(x)^3 + 3*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \csc^4(x)\sec^4(x)\,dx = -\frac{8\left(3\,\tan\left(2\,x\right)^2+1\right)}{3\,\tan\left(2\,x\right)^3}$$

input
```
integrate(csc(x)^4*sec(x)^4,x, algorithm="giac")
```

output
```
-8/3*(3*tan(2*x)^2 + 1)/tan(2*x)^3
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \csc^4(x)\sec^4(x)\,dx = -\frac{24\,\cos\left(2\,x\right)-16\cos\left(2\,x\right)^3}{3\,\sin\left(2\,x\right)-3\cos\left(2\,x\right)^2\,\sin\left(2\,x\right)}$$

input
```
int(1/(cos(x)^4*sin(x)^4),x)
```

output
```
-(24*cos(2*x) - 16*cos(2*x)^3)/(3*sin(2*x) - 3*cos(2*x)^2*sin(2*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

$$\int \csc^4(x)\sec^4(x)\,dx = \frac{16\sin\left(x\right)^6-24\sin\left(x\right)^4+6\sin\left(x\right)^2+1}{3\cos\left(x\right)\sin\left(x\right)^3\left(\sin\left(x\right)^2-1\right)}$$

input
```
int(csc(x)^4*sec(x)^4,x)
```

output
```
(16*sin(x)**6 - 24*sin(x)**4 + 6*sin(x)**2 + 1)/(3*cos(x)*sin(x)**3*(sin(x
)**2 - 1))
```

3.254 $\int \frac{x+\sin(x)}{1+\cos(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{x+\sin(x)}{1+\cos(x)} dx = x\tan\left(\frac{x}{2}\right)$$

output
```
x*tan(1/2*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x+\sin(x)}{1+\cos(x)} dx = x\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(x + Sin[x])/(1 + Cos[x]),x]
```

output
```
x*Tan[x/2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4877, 3042, 3146, 16, 3799, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sin(x)}{\cos(x)+1}\,dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{x}{\cos(x)+1}dx+\int \frac{\sin(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{\sin\left(x+\frac{\pi}{2}\right)+1}dx+\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{1-\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{x}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-\int \frac{1}{\cos(x)+1}d\cos(x)$$

$$\Big\downarrow \text{16}$$

$$\int \frac{x}{\sin\left(x+\frac{\pi}{2}\right)+1}dx-\log(\cos(x)+1)$$

$$\Big\downarrow \text{3799}$$

$$\frac{1}{2}\int x\sec^2\left(\frac{x}{2}\right)dx-\log(\cos(x)+1)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int x\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)^2dx-\log(\cos(x)+1)$$

$$\Big\downarrow \text{4672}$$

$$\frac{1}{2}\left(2\int -\tan\left(\frac{x}{2}\right)dx+2x\tan\left(\frac{x}{2}\right)\right)-\log(\cos(x)+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(2x\tan\left(\frac{x}{2}\right)-2\int\tan\left(\frac{x}{2}\right)dx\right)-\log(\cos(x)+1)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2x\tan\left(\frac{x}{2}\right)-2\int\tan\left(\frac{x}{2}\right)dx\right)-\log(\cos(x)+1)$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\left(2x\tan\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)-\log(\cos(x)+1)$$

input
```
Int[(x + Sin[x])/(1 + Cos[x]),x]
```

output
```
-Log[1 + Cos[x]] + (4*Log[Cos[x/2]] + 2*x*Tan[x/2])/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
lookup	$x \tan\left(\frac{x}{2}\right)$	7
default	$x \tan\left(\frac{x}{2}\right)$	7
norman	$x \tan\left(\frac{x}{2}\right)$	7
parallelrisch	$x \tan\left(\frac{x}{2}\right)$	7
risch	$-ix + \frac{2ix}{e^{ix}+1}$	19

input
```
int((x+sin(x))/(1+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
x*tan(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = \frac{x\sin(x)}{\cos(x)+1}$$

input
```
integrate((x+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

output
```
x*sin(x)/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = x\tan\left(\frac{x}{2}\right)$$

input
```
integrate((x+sin(x))/(1+cos(x)),x)
```

output
```
x*tan(x/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(6) = 12.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 7.62

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = \frac{\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+2x\sin(x)}{\cos(x)^2+\sin(x)^2+2\cos(x)+1} - \log(\cos(x)+1)$$

input
```
integrate((x+sin(x))/(1+cos(x)),x, algorithm="maxima")
```

output
```
((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1) + 2*x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = x\tan\left(\frac{1}{2}x\right)$$

input
```
integrate((x+sin(x))/(1+cos(x)),x, algorithm="giac")
```

output
```
x*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = x\tan\left(\frac{x}{2}\right)$$

input
```
int((x + sin(x))/(cos(x) + 1),x)
```

output
```
x*tan(x/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x+\sin(x)}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)x$$

input
```
int((x+sin(x))/(1+cos(x)),x)
```

output
```
tan(x/2)*x
```

3.255 $\int \cosh^2(x)\sinh^3(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cosh^2(x)\sinh^3(x)\,dx = -\frac{1}{3}\cosh^3(x) + \frac{\cosh^5(x)}{5}$$

output
```
-1/3*cosh(x)^3+1/5*cosh(x)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \cosh^2(x)\sinh^3(x)\,dx = -\frac{\cosh(x)}{8} - \frac{1}{48}\cosh(3x) + \frac{1}{80}\cosh(5x)$$

input
```
Integrate[Cosh[x]^2*Sinh[x]^3,x]
```

output
```
-1/8*Cosh[x] - Cosh[3*x]/48 + Cosh[5*x]/80
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 26, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(x)\cosh^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\sin(ix)^3\cos(ix)^2dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \cos(ix)^2\sin(ix)^3dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\int \cosh^2(x)\left(1-\cosh^2(x)\right)d\cosh(x)$$

$$\Big\downarrow \text{ 244}$$

$$-\int \left(\cosh^2(x)-\cosh^4(x)\right)d\cosh(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh^5(x)}{5}-\frac{\cosh^3(x)}{3}$$

input
```
Int[Cosh[x]^2*Sinh[x]^3,x]
```

output
```
-1/3*Cosh[x]^3 + Cosh[x]^5/5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 10.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\cosh(x)^3}{3}+\frac{\cosh(x)^5}{5}$	14
default	$-\frac{\cosh(x)^3}{3}+\frac{\cosh(x)^5}{5}$	14
orering	$\frac{\cosh(x)^3\sinh(x)^2}{3}-\frac{2\cosh(x)^5}{15}$	18
risch	$\frac{e^{5x}}{160}-\frac{e^{3x}}{96}-\frac{e^{x}}{16}-\frac{e^{-x}}{16}-\frac{e^{-3x}}{96}+\frac{e^{-5x}}{160}$	36

input
```
int(cosh(x)^2*sinh(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*cosh(x)^3+1/5*cosh(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int \cosh^2(x)\sinh^3(x)\,dx = \frac{1}{80}\cosh(x)^5 + \frac{1}{16}\cosh(x)\sinh(x)^4 - \frac{1}{48}\cosh(x)^3 + \frac{1}{16}\left(2\cosh(x)^3 - \cosh(x)\right)\sinh(x)^2 - \frac{1}{8}\cosh(x)$$

input
```
integrate(cosh(x)^2*sinh(x)^3,x, algorithm="fricas")
```

output
```
1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 1/48*cosh(x)^3 + 1/16*(2*cosh(x)
^3 - cosh(x))*sinh(x)^2 - 1/8*cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cosh^2(x)\sinh^3(x)\,dx = \frac{\sinh^2(x)\cosh^3(x)}{3} - \frac{2\cosh^5(x)}{15}$$

input
```
integrate(cosh(x)**2*sinh(x)**3,x)
```

output
```
sinh(x)**2*cosh(x)**3/3 - 2*cosh(x)**5/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \cosh^2(x)\sinh^3(x)\,dx = -\frac{1}{480}\left(5\,e^{(-2\,x)} + 30\,e^{(-4\,x)} - 3\right)e^{(5\,x)} - \frac{1}{16}\,e^{(-x)} - \frac{1}{96}\,e^{(-3\,x)} + \frac{1}{160}\,e^{(-5\,x)}$$

input
```
integrate(cosh(x)^2*sinh(x)^3,x, algorithm="maxima")
```

output
```
-1/480*(5*e^(-2*x) + 30*e^(-4*x) - 3)*e^(5*x) - 1/16*e^(-x) - 1/96*e^(-3*x
) + 1/160*e^(-5*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \cosh^2(x)\sinh^3(x)\,dx = -\frac{1}{480}\left(30\,e^{(4\,x)} + 5\,e^{(2\,x)} - 3\right)e^{(-5\,x)} + \frac{1}{160}\,e^{(5\,x)} - \frac{1}{96}\,e^{(3\,x)} - \frac{1}{16}\,e^{x}$$

input
```
integrate(cosh(x)^2*sinh(x)^3,x, algorithm="giac")
```

output
```
-1/480*(30*e^(4*x) + 5*e^(2*x) - 3)*e^(-5*x) + 1/160*e^(5*x) - 1/96*e^(3*x
) - 1/16*e^x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cosh^2(x)\sinh^3(x)\,dx = \frac{\cosh(x)^3\,\left(3\cosh(x)^2-5\right)}{15}$$

input
```
int(cosh(x)^2*sinh(x)^3,x)
```

output
```
(cosh(x)^3*(3*cosh(x)^2 - 5))/15
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.71

$$\int \cosh^2(x)\sinh^3(x)\,dx = \frac{3e^{10x}-5e^{8x}-30e^{6x}-30e^{4x}-5e^{2x}+3}{480e^{5x}}$$

input
```
int(cosh(x)^2*sinh(x)^3,x)
```

output
```
(3*e**(10*x) - 5*e**(8*x) - 30*e**(6*x) - 30*e**(4*x) - 5*e**(2*x) + 3)/(4
80*e**(5*x))
```

3.256 $\int 3^{2^x} 4^x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int 3^{2^x} 4^x \, dx = -\frac{3^{2^x}}{\log(2)\log^2(3)} + \frac{2^x 3^{2^x}}{\log(2)\log(3)}$$

output
```
-3^(2^x)/ln(2)/ln(3)^2+2^x*3^(2^x)/ln(2)/ln(3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int 3^{2^x} 4^x \, dx = \frac{3^{2^x}(-1 + 2^x \log(3))}{\log(2)\log^2(3)}$$

input
```
Integrate[3^2^x*4^x,x]
```

output
```
(3^2^x*(-1 + 2^x*Log[3]))/(Log[2]*Log[3]^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3^{2^x} 4^x \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int 2^x 3^{2^x} d2^x}{\log(2)}$$

$$\Big\downarrow \text{2607}$$

$$\frac{\frac{2^x 3^{2^x}}{\log(3)} - \frac{\int 3^{2^x} d2^x}{\log(3)}}{\log(2)}$$

$$\Big\downarrow \text{2624}$$

$$\frac{\frac{2^x 3^{2^x}}{\log(3)} - \frac{3^{2^x}}{\log^2(3)}}{\log(2)}$$

input
```
Int[3^2^x*4^x,x]
```

output
```
(-(3^2^x/Log[3]^2) + (2^x*3^2^x)/Log[3])/Log[2]
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{(2^x \ln(3)-1)3^{2^x}}{\ln(2)\ln(3)^2}$	23
norman	$\frac{e^{x\ln(2)} e^{e^{x\ln(2)}\ln(3)}}{\ln(2)\ln(3)} - \frac{e^{e^{x\ln(2)}\ln(3)}}{\ln(2)\ln(3)^2}$	44

input
```
int(3^(2^x)*4^x,x,method=_RETURNVERBOSE)
```

output
```
(2^x*ln(3)-1)/ln(2)/ln(3)^2*3^(2^x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int 3^{2^x} 4^x \, dx = \frac{(2^x \log(3) - 1)3^{(2^x)}}{\log(3)^2 \log(2)}$$

input
```
integrate(3^(2^x)*4^x,x, algorithm="fricas")
```

output
```
(2^x*log(3) - 1)*3^(2^x)/(log(3)^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int 3^{2^x}4^x\,dx = \frac{(2^x\log(3)-1)\,e^{2^x\log(3)}}{\log(2)\log(3)^2}$$

input
```
integrate(3**(2**x)*4**x,x)
```

output
```
(2**x*log(3) - 1)*exp(2**x*log(3))/(log(2)*log(3)**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int 3^{2^x}4^x\,dx = -\frac{4^x\Gamma\left(2,-4^{\frac{1}{2}x}\log(3)\right)}{4^x\log(3)^2\log(2)}$$

input
```
integrate(3^(2^x)*4^x,x, algorithm="maxima")
```

output
```
-4^x*gamma(2, -4^(1/2*x)*log(3))/(4^x*log(3)^2*log(2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int 3^{2^x}4^x\,dx = \frac{2^x e^{(2^x\log(3)+2x\log(2))}\log(3)-e^{(2^x\log(3)+2x\log(2))}}{2^{2x}\log(3)^2\log(2)}$$

input
```
integrate(3^(2^x)*4^x,x, algorithm="giac")
```

output
```
(2^x*e^(2^x*log(3) + 2*x*log(2))*log(3) - e^(2^x*log(3) + 2*x*log(2)))/(2^
(2*x)*log(3)^2*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int 3^{2^x} 4^x \, dx = \frac{3^{2^x} (2^x \ln(3) - 1)}{\ln(2) \ln(3)^2}$$

input
```
int(3^(2^x)*4^x,x)
```

output
```
(3^(2^x)*(2^x*log(3) - 1))/(log(2)*log(3)^2)
```

Reduce [F]

$$\int 3^{2^x} 4^x \, dx = \int 3^{2^x} 4^x dx$$

input
```
int(3^(2^x)*4^x,x)
```

output
```
int(3**(2**x)*4**x,x)
```

3.257 $\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx = \arctan(\cos(x)+\sin(x))$$

output
```
arctan(cos(x)+sin(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx = \arctan(\cos(x)+\sin(x))$$

input
```
Integrate[(Cos[x] - Sin[x])/(2 + Sin[2*x]),x]
```

output
```
ArcTan[Cos[x] + Sin[x]]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 217, normalized size of antiderivative = 36.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)-\sin(x)}{\sin(2x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)-\sin(x)}{\sin(2x)+2}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{\cos(x)}{\sin(2x)+2}-\frac{\sin(x)}{\sin(2x)+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(3+i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)-\sqrt{3}+i}{\sqrt{2\left(1+i\sqrt{3}\right)}}\right)+\frac{1}{6}\left(3-i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)-\sqrt{3}+i}{\sqrt{2\left(1+i\sqrt{3}\right)}}\right)-\frac{1}{6}\left(3+i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)+\sqrt{3}+i}{\sqrt{2\left(1-i\sqrt{3}\right)}}\right)-\frac{1}{6}\left(3-i\sqrt{3}\right)\arctan\left(\frac{-2i\tan\left(\frac{x}{2}\right)+\sqrt{3}+i}{\sqrt{2\left(1-i\sqrt{3}\right)}}\right)$$

input
```
Int[(Cos[x] - Sin[x])/(2 + Sin[2*x]),x]
```

output
```
((3 - I*Sqrt[3])*ArcTan[(I - Sqrt[3] - (2*I)*Tan[x/2])/Sqrt[2*(1 + I*Sqrt[
3])]])/6 + ((3 + I*Sqrt[3])*ArcTan[(I - Sqrt[3] - (2*I)*Tan[x/2])/Sqrt[2*(
1 + I*Sqrt[3])]])/6 - ((3 - I*Sqrt[3])*ArcTan[(I + Sqrt[3] - (2*I)*Tan[x/2
])/Sqrt[2*(1 - I*Sqrt[3])]])/6 - ((3 + I*Sqrt[3])*ArcTan[(I + Sqrt[3] - (2
*I)*Tan[x/2])/Sqrt[2*(1 - I*Sqrt[3])]])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 7.33

method	result
default	$\frac{i \ln\left(\tan\left(\frac{x}{2}\right)^{2}+\left(-1-i\right) \tan\left(\frac{x}{2}\right)-i\right)}{2}-\frac{i \ln\left(\tan\left(\frac{x}{2}\right)^{2}+\left(-1+i\right) \tan\left(\frac{x}{2}\right)+i\right)}{2}$
risch	$\frac{i \ln\left({\mathrm e}^{2 i x}+\left(-1+i\right) {\mathrm e}^{i x}+i\right)}{2}-\frac{i \ln\left({\mathrm e}^{2 i x}+\left(1-i\right) {\mathrm e}^{i x}+i\right)}{2}$
parts	$\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{4}-2 \_Z^{3}+2 \_Z^{2}+2 \_Z +1\right)}\frac{\left(-\_R^{2}+1\right) \ln\left(\tan\left(\frac{x}{2}\right)-\_R \right)}{2 \_R^{3}-3 \_R^{2}+2 \_R +1}\right)}{2}-\left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{4}-2 \_Z^{3}+2 \_Z^{2}+2 \_Z +1\right)}\right.$

input
```
int((cos(x)-sin(x))/(2+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(tan(1/2*x)^2-(1+I)*tan(1/2*x)-I)-1/2*I*ln(tan(1/2*x)^2+(-1+I)*tan
(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx = \frac{1}{2}\arctan\left(\frac{\cos(x)\sin(x)}{\cos(x)+\sin(x)}\right)$$

input
```
integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="fricas")
```

output
```
1/2*arctan(cos(x)*sin(x)/(cos(x) + sin(x)))
```

Sympy [F]

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx = \int \frac{-\sin(x)+\cos(x)}{\sin(2x)+2}\,dx$$

input
```
integrate((cos(x)-sin(x))/(2+sin(2*x)),x)
```

output
```
Integral((-sin(x) + cos(x))/(sin(2*x) + 2), x)
```

Maxima [F]

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx = \int \frac{\cos(x)-\sin(x)}{\sin(2x)+2}\,dx$$

input
```
integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="maxima")
```

output
```
integrate((cos(x) - sin(x))/(sin(2*x) + 2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(6) = 12$.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.83

$$\int \frac{\cos(x) - \sin(x)}{2 + \sin(2x)}\,dx = \arctan\left(\frac{1}{2}\tan\left(\frac{1}{2}x\right)^3 - \frac{3}{2}\tan\left(\frac{1}{2}x\right)^2 + \frac{3}{2}\tan\left(\frac{1}{2}x\right) + \frac{1}{2}\right) - \arctan\left(\tan\left(\frac{1}{2}x\right) - 2\right)$$

input
```
integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="giac")
```

output
```
arctan(1/2*tan(1/2*x)^3 - 3/2*tan(1/2*x)^2 + 3/2*tan(1/2*x) + 1/2) - arcta
n(tan(1/2*x) - 2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.83

$$\int \frac{\cos(x) - \sin(x)}{2 + \sin(2x)}\,dx = \mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2} - \frac{3\tan\left(\frac{x}{2}\right)^2}{2} + \frac{3\tan\left(\frac{x}{2}\right)}{2} + \frac{1}{2}\right) - \mathrm{atan}\left(\tan\left(\frac{x}{2}\right) - 2\right)$$

input
```
int((cos(x) - sin(x))/(sin(2*x) + 2),x)
```

output
```
atan((3*tan(x/2))/2 - (3*tan(x/2)^2)/2 + tan(x/2)^3/2 + 1/2) - atan(tan(x/
2) - 2)
```

Reduce [F]

$$\int \frac{\cos(x) - \sin(x)}{2 + \sin(2x)}\, dx = \int \frac{\cos(x)}{\sin(2x) + 2} dx - \left(\int \frac{\sin(x)}{\sin(2x) + 2} dx\right)$$

input
```
int((cos(x)-sin(x))/(2+sin(2*x)),x)
```

output
```
int(cos(x)/(sin(2*x) + 2),x) - int(sin(x)/(sin(2*x) + 2),x)
```

3.258 $\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx=\frac{\tan(1+\log(x))}{x}$$

output
```
tan(1+ln(x))/x
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx=\frac{\sec(1)\sec(1+\log(x))\sin(\log(x))}{x}+\frac{\tan(1)}{x}$$

input
```
Integrate[(Sec[1 + Log[x]]^2 - Tan[1 + Log[x]])/x^2,x]
```

output
```
(Sec[1]*Sec[1 + Log[x]]*Sin[Log[x]])/x + Tan[1]/x
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 9.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(\log(x)+1)-\tan(\log(x)+1)}{x^2} \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\frac{\sec^2(\log(x)+1)}{x^2}-\frac{\tan(\log(x)+1)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2i\,\text{Hypergeometric2F1}\left(\frac{i}{2},1,1+\frac{i}{2},-e^{2i}x^{2i}\right)}{x}-\left(\frac{4}{5}+\frac{8i}{5}\right)e^{2i}x^{-1+2i}\,\text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i}x^{2i}\right)-\frac{i}{x}$$

input
```
Int[(Sec[1 + Log[x]]^2 - Tan[1 + Log[x]])/x^2,x]
```

output
```
(-I)/x + ((2*I)*Hypergeometric2F1[I/2, 1, 1 + I/2, -(E^(2*I)*x^(2*I))])/x
- ((4/5 + (8*I)/5)*E^(2*I)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -(E^(2*I
)*x^(2*I))])/x^(1 - 2*I)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

method	result	size
risch	$-\frac{i}{x}+\frac{2i}{x\left(x^{2i} \mathrm{e}^{2i}+1\right)}$	28

input
```
int((sec(1+ln(x))^2-tan(1+ln(x)))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-I/x+2*I/x/((x^I)^2*exp(2*I)+1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx=\frac{\sin\left(\log\left(x\right)+1\right)}{x\cos\left(\log\left(x\right)+1\right)}$$

input
```
integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="fricas")
```

output
```
sin(log(x) + 1)/(x*cos(log(x) + 1))
```

Sympy [F]

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx=\int \frac{-\tan\left(\log\left(x\right)+1\right)+\sec^{2}\left(\log\left(x\right)+1\right)}{x^{2}}\,dx$$

input
```
integrate((sec(1+ln(x))**2-tan(1+ln(x)))/x**2,x)
```

output
```
Integral((-tan(log(x) + 1) + sec(log(x) + 1)**2)/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(9) = 18$.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.00

$$\int \frac{\sec^2(1+\log(x)) - \tan(1+\log(x))}{x^2}\,dx = \frac{2\,\sin\left(2\,\log\left(x\right)+2\right)}{x\cos\left(2\,\log\left(x\right)+2\right)^2 + x\sin\left(2\,\log\left(x\right)+2\right)^2 + 2\,x\cos\left(2\,\log\left(x\right)+2\right) + x}$$

input
```
integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="maxima")
```

output
```
2*sin(2*log(x) + 2)/(x*cos(2*log(x) + 2)^2 + x*sin(2*log(x) + 2)^2 + 2*x*c
os(2*log(x) + 2) + x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5161 vs. $2(9) = 18$.

Time = 1.57 (sec) , antiderivative size = 5161, normalized size of antiderivative = 573.44

$$\int \frac{\sec^2(1+\log(x)) - \tan(1+\log(x))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="giac")
```

output
```
1/4*(13*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)*x) + 42*tan(1)^4*tan(l
og(x))^10/((tan(log(x))^2 - 1)^2*x) + 84*tan(1)^4*tan(log(x))^11/((tan(log
(x))^2 - 1)^3*x) - 88*tan(1)^4*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) -
17*tan(1)^4*tan(log(x))^7/x - 4*tan(1)^3*tan(log(x))^8/x - 62*tan(1)^4*tan
(log(x))^8/((tan(log(x))^2 - 1)*x) + 22*tan(1)^3*tan(log(x))^9/((tan(log(x
))^2 - 1)*x) - 102*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)^2*x) - 6*ta
n(1)^3*tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) + 520*tan(1)^4*tan(log(x))
^10/((tan(log(x))^2 - 1)^3*x) - 168*tan(1)^3*tan(log(x))^11/((tan(log(x))^
2 - 1)^3*x) + 248*tan(1)^4*tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) - 88*t
an(1)^3*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) + 20*tan(1)^4*tan(log(x))
^6/x - 22*tan(1)^3*tan(log(x))^7/x - 17*tan(1)^4*tan(log(x))^7/((tan(log(x
))^2 - 1)*x) - 70*tan(1)^3*tan(log(x))^8/((tan(log(x))^2 - 1)*x) - 740*tan
(1)^4*tan(log(x))^8/((tan(log(x))^2 - 1)^2*x) + 17*tan(1)^2*tan(log(x))^9/
((tan(log(x))^2 - 1)*x) + 488*tan(1)^3*tan(log(x))^9/((tan(log(x))^2 - 1)^
2*x) - 548*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)^3*x) - 56*tan(1)^2*
tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) + 40*tan(1)^3*tan(log(x))^10/((ta
n(log(x))^2 - 1)^3*x) + 560*tan(1)^4*tan(log(x))^10/((tan(log(x))^2 - 1)^4
*x) + 36*tan(1)^2*tan(log(x))^11/((tan(log(x))^2 - 1)^3*x) - 256*tan(1)^3*
tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) + 32*tan(1)^2*tan(log(x))^12/((ta
n(log(x))^2 - 1)^4*x) - 22*tan(1)^4*tan(log(x))^5/x + 36*tan(1)^3*tan(l...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx=\int -\frac{\tan(\ln(x)+1)-\frac{1}{\cos(\ln(x)+1)^2}}{x^2}\,dx$$

input
```
int(-(tan(log(x) + 1) - 1/cos(log(x) + 1)^2)/x^2,x)
```

output
```
int(-(tan(log(x) + 1) - 1/cos(log(x) + 1)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx$$
$$= \frac{-\left(\int \frac{\tan(\log(x)+1)}{x^2}dx\right)x+2\left(\int \frac{\sec(\log(x)+1)^2\tan(\log(x)+1)}{x^2}dx\right)x-\sec\left(\log(x)+1\right)^2}{x}$$

input
```
int((sec(1+log(x))^2-tan(1+log(x)))/x^2,x)
```

output
```
( - int(tan(log(x) + 1)/x**2,x)*x + 2*int((sec(log(x) + 1)**2*tan(log(x) +
 1))/x**2,x)*x - sec(log(x) + 1)**2)/x
```

3.259 $\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx=2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}-\frac{\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}$$

output
```
2*x*(ln(1/x)/x)^(1/2)-2^(1/2)*Pi^(1/2)*erf(1/2*ln(1/x)^(1/2)*2^(1/2))*(ln(
1/x)/x)^(1/2)/(1/x)^(1/2)/ln(1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx=\left(2x-\frac{\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}\right)\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}$$

input
```
Integrate[Sqrt[Log[x^(-1)]/x],x]
```

output
```
(2*x - (Sqrt[2*Pi]*Erf[Sqrt[Log[x^(-1)]]/Sqrt[2]])/(Sqrt[x^(-1)]*Sqrt[Log[
x^(-1)]]))*Sqrt[Log[x^(-1)]/x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {7270, 2742, 2747, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}dx$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\int \frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{x}}dx}{\sqrt{\log\left(\frac{1}{x}\right)}}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\left(\int \frac{1}{\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}}dx+2\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}\right)}{\sqrt{\log\left(\frac{1}{x}\right)}}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\left(2\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}-\sqrt{\frac{1}{x}}\sqrt{x}\int \frac{1}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}d\log\left(\frac{1}{x}\right)\right)}{\sqrt{\log\left(\frac{1}{x}\right)}}$$

$$\Big\downarrow \text{ 2611}$$

$$\frac{\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\left(2\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}-2\sqrt{\frac{1}{x}}\sqrt{x}\int \frac{1}{\sqrt{\frac{1}{x}}}d\sqrt{\log\left(\frac{1}{x}\right)}\right)}{\sqrt{\log\left(\frac{1}{x}\right)}}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\left(2\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}-\sqrt{2\pi}\sqrt{\frac{1}{x}}\sqrt{x}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)\right)}{\sqrt{\log\left(\frac{1}{x}\right)}}$$

input
```
Int[Sqrt[Log[x^(-1)]/x],x]
```

output
```
(Sqrt[x]*(-(Sqrt[2*Pi]*Sqrt[x^(-1)]*Sqrt[x]*Erf[Sqrt[Log[x^(-1)]]/Sqrt[2]]
) + 2*Sqrt[x]*Sqrt[Log[x^(-1)]])*Sqrt[Log[x^(-1)]/x])/Sqrt[Log[x^(-1)]]
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [F]

$$\int \sqrt{\frac{\ln\left(\frac{1}{x}\right)}{x}}dx$$

input
```
int((ln(1/x)/x)^(1/2),x)
```

output
```
int((ln(1/x)/x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((log(1/x)/x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = \int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx$$

input
```
integrate((ln(1/x)/x)**(1/2),x)
```

output
```
Integral(sqrt(log(1/x)/x), x)
```

Maxima [F]

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = \int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx$$

input
```
integrate((log(1/x)/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-log(x)/x), x)
```

Giac [F]

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = \int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx$$

input
```
integrate((log(1/x)/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(log(1/x)/x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = \int \sqrt{\frac{\ln\left(\frac{1}{x}\right)}{x}}\,dx$$

input
```
int((log(1/x)/x)^(1/2),x)
```

output
```
int((log(1/x)/x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx = i\left(2\sqrt{x}\,\sqrt{\log(x)} - \left(\int \frac{\sqrt{x}\,\sqrt{\log(x)}}{\log(x)\,x}dx\right)\right)$$

input
```
int((log(1/x)/x)^(1/2),x)
```

output
```
i*(2*sqrt(x)*sqrt(log(x)) - int((sqrt(x)*sqrt(log(x)))/(log(x)*x),x))
```

3.260 $\int (1-\cos(x))^5 \cos(5x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int (1-\cos(x))^5 \cos(5x)\,dx = -\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45}{64}\sin(2x)+\frac{5}{4}\sin(3x) -\frac{105}{64}\sin(4x)+\frac{63}{40}\sin(5x)-\frac{35}{32}\sin(6x)+\frac{15}{28}\sin(7x) -\frac{45}{256}\sin(8x)+\frac{5}{144}\sin(9x)-\frac{1}{320}\sin(10x)$$

output
```
-1/32*x+5/16*sin(x)-45/64*sin(2*x)+5/4*sin(3*x)-105/64*sin(4*x)+63/40*sin(
5*x)-35/32*sin(6*x)+15/28*sin(7*x)-45/256*sin(8*x)+5/144*sin(9*x)-1/320*si
n(10*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int (1-\cos(x))^5 \cos(5x)\,dx = -\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45}{64}\sin(2x)+\frac{5}{4}\sin(3x) -\frac{105}{64}\sin(4x)+\frac{63}{40}\sin(5x)-\frac{35}{32}\sin(6x)+\frac{15}{28}\sin(7x) -\frac{45}{256}\sin(8x)+\frac{5}{144}\sin(9x)-\frac{1}{320}\sin(10x)$$

input
```
Integrate[(1 - Cos[x])^5*Cos[5*x],x]
```

output
```
-1/32*x + (5*Sin[x])/16 - (45*Sin[2*x])/64 + (5*Sin[3*x])/4 - (105*Sin[4*x
])/64 + (63*Sin[5*x])/40 - (35*Sin[6*x])/32 + (15*Sin[7*x])/28 - (45*Sin[8
*x])/256 + (5*Sin[9*x])/144 - Sin[10*x]/320
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\cos(x))^5 \cos(5x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (1-\cos(x))^5 \cos(5x)dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(-\cos(5x)\cos^5(x)+5\cos(5x)\cos^4(x)-10\cos(5x)\cos^3(x)+10\cos(5x)\cos^2(x)-5\cos(5x)\cos(x)+\cos(5x)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45}{64}\sin(2x)+\frac{5}{4}\sin(3x)-\frac{105}{64}\sin(4x)+\frac{63}{40}\sin(5x)-\frac{35}{32}\sin(6x)+\frac{15}{28}\sin(7x)-\frac{45}{256}\sin(8x)+\frac{5}{144}\sin(9x)-\frac{1}{320}\sin(10x)$$

input
```
Int[(1 - Cos[x])^5*Cos[5*x],x]
```

output
```
-1/32*x + (5*Sin[x])/16 - (45*Sin[2*x])/64 + (5*Sin[3*x])/4 - (105*Sin[4*x
])/64 + (63*Sin[5*x])/40 - (35*Sin[6*x])/32 + (15*Sin[7*x])/28 - (45*Sin[8
*x])/256 + (5*Sin[9*x])/144 - Sin[10*x]/320
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 238.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result
default	$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45\sin(2x)}{64}+\frac{5\sin(3x)}{4}-\frac{105\sin(4x)}{64}+\frac{63\sin(5x)}{40}-\frac{35\sin(6x)}{32}+\frac{15\sin(7x)}{28}-\frac{45\sin(8x)}{256}+$
risch	$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45\sin(2x)}{64}+\frac{5\sin(3x)}{4}-\frac{105\sin(4x)}{64}+\frac{63\sin(5x)}{40}-\frac{35\sin(6x)}{32}+\frac{15\sin(7x)}{28}-\frac{45\sin(8x)}{256}+$
parallelrisch	$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45\sin(2x)}{64}+\frac{5\sin(3x)}{4}-\frac{105\sin(4x)}{64}+\frac{63\sin(5x)}{40}-\frac{35\sin(6x)}{32}+\frac{15\sin(7x)}{28}-\frac{45\sin(8x)}{256}+$
parts	$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45\sin(2x)}{64}+\frac{5\sin(3x)}{4}-\frac{105\sin(4x)}{64}+\frac{63\sin(5x)}{40}-\frac{35\sin(6x)}{32}+\frac{15\sin(7x)}{28}-\frac{45\sin(8x)}{256}+$
orering	Expression too large to display

input
```
int((1-cos(x))^5*cos(5*x),x,method=_RETURNVERBOSE)
```

output
```
-1/32*x+5/16*sin(x)-45/64*sin(2*x)+5/4*sin(3*x)-105/64*sin(4*x)+63/40*sin(
5*x)-35/32*sin(6*x)+15/28*sin(7*x)-45/256*sin(8*x)+5/144*sin(9*x)-1/320*si
n(10*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int (1-\cos(x))^5\cos(5x)\,dx =$$
$$-\frac{1}{10080}\left(16128\,\cos\left(x\right)^{9} - 89600\,\cos\left(x\right)^{8} + 194544\,\cos\left(x\right)^{7} - 188800\,\cos\left(x\right)^{6} + 33768\,\cos\left(x\right)^{5} + 9398\right.$$
$$-\frac{1}{32}\,x$$

input
```
integrate((1-cos(x))^5*cos(5*x),x, algorithm="fricas")
```

output
```
-1/10080*(16128*cos(x)^9 - 89600*cos(x)^8 + 194544*cos(x)^7 - 188800*cos(x
)^6 + 33768*cos(x)^5 + 93984*cos(x)^4 - 83790*cos(x)^3 + 24512*cos(x)^2 +
315*cos(x) - 1376)*sin(x) - 1/32*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(83) = 166.

Time = 3.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.69

$$\int (1-\cos(x))^5\cos(5x)\,dx = -\frac{x\sin^5(x)\sin(5x)}{32} - \frac{5x\sin^4(x)\cos(x)\cos(5x)}{32} + \frac{5x\sin^3(x)\sin(5x)\cos^2(x)}{16} + \frac{5x\sin^2(x)\cos^3(x)\cos(5x)}{16} - \frac{5x\sin(x)\sin(5x)\cos^4(x)}{32} - \frac{x\cos^5(x)\cos(5x)}{32} - \frac{\sin^5(x)\cos(5x)}{64} + \frac{3\sin^4(x)\sin(5x)\cos(x)}{64} + \frac{8\sin^4(x)\sin(5x)}{63} + \frac{40\sin^3(x)\cos(x)\cos(5x)}{63} - \frac{5\sin^3(x)\cos(5x)}{32} + \frac{\sin^2(x)\sin(5x)\cos^3(x)}{6} - \frac{4\sin^2(x)\sin(5x)\cos^2(x)}{3} + \frac{25\sin^2(x)\sin(5x)\cos(x)}{32} - \frac{4\sin^2(x)\sin(5x)}{21} + \frac{55\sin(x)\cos^4(x)\cos(5x)}{192} - \frac{100\sin(x)\cos^3(x)\cos(5x)}{63} + \frac{55\sin(x)\cos^2(x)\cos(5x)}{32} - \frac{20\sin(x)\cos(x)\cos(5x)}{21} + \frac{5\sin(x)\cos(5x)}{24} - \frac{241\sin(5x)\cos^5(x)}{960} + \frac{83\sin(5x)\cos^4(x)}{63} - \frac{75\sin(5x)\cos^3(x)}{32} + \frac{46\sin(5x)\cos^2(x)}{21} - \frac{25\sin(5x)\cos(x)}{24} + \frac{\sin(5x)}{5}$$

input
```
integrate((1-cos(x))**5*cos(5*x),x)
```

output
```
-x*sin(x)**5*sin(5*x)/32 - 5*x*sin(x)**4*cos(x)*cos(5*x)/32 + 5*x*sin(x)**
3*sin(5*x)*cos(x)**2/16 + 5*x*sin(x)**2*cos(x)**3*cos(5*x)/16 - 5*x*sin(x)
*sin(5*x)*cos(x)**4/32 - x*cos(x)**5*cos(5*x)/32 - sin(x)**5*cos(5*x)/64 +
 3*sin(x)**4*sin(5*x)*cos(x)/64 + 8*sin(x)**4*sin(5*x)/63 + 40*sin(x)**3*c
os(x)*cos(5*x)/63 - 5*sin(x)**3*cos(5*x)/32 + sin(x)**2*sin(5*x)*cos(x)**3
/6 - 4*sin(x)**2*sin(5*x)*cos(x)**2/3 + 25*sin(x)**2*sin(5*x)*cos(x)/32 -
4*sin(x)**2*sin(5*x)/21 + 55*sin(x)*cos(x)**4*cos(5*x)/192 - 100*sin(x)*co
s(x)**3*cos(5*x)/63 + 55*sin(x)*cos(x)**2*cos(5*x)/32 - 20*sin(x)*cos(x)*c
os(5*x)/21 + 5*sin(x)*cos(5*x)/24 - 241*sin(5*x)*cos(x)**5/960 + 83*sin(5*
x)*cos(x)**4/63 - 75*sin(5*x)*cos(x)**3/32 + 46*sin(5*x)*cos(x)**2/21 - 25
*sin(5*x)*cos(x)/24 + sin(5*x)/5
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int (1-\cos(x))^5\cos(5x)\,dx = \frac{80}{9}\sin(x)^9 - \frac{380}{7}\sin(x)^7 - \frac{1}{20}\sin(2x)^5 + \frac{501}{5}\sin(x)^5 + \frac{71}{16}\sin(2x)^3 - \frac{212}{3}\sin(x)^3 - \frac{1}{32}x - \frac{45}{256}\sin(8x) - \frac{105}{64}\sin(4x) - 4\sin(2x) + 16\sin(x)$$

input
```
integrate((1-cos(x))^5*cos(5*x),x, algorithm="maxima")
```

output
```
80/9*sin(x)^9 - 380/7*sin(x)^7 - 1/20*sin(2*x)^5 + 501/5*sin(x)^5 + 71/16*
sin(2*x)^3 - 212/3*sin(x)^3 - 1/32*x - 45/256*sin(8*x) - 105/64*sin(4*x) -
 4*sin(2*x) + 16*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int (1-\cos(x))^5\cos(5x)\,dx = -\frac{1}{32}x - \frac{1}{320}\sin(10x) + \frac{5}{144}\sin(9x) - \frac{45}{256}\sin(8x) + \frac{15}{28}\sin(7x) - \frac{35}{32}\sin(6x) + \frac{63}{40}\sin(5x) - \frac{105}{64}\sin(4x) + \frac{5}{4}\sin(3x) - \frac{45}{64}\sin(2x) + \frac{5}{16}\sin(x)$$

input
```
integrate((1-cos(x))^5*cos(5*x),x, algorithm="giac")
```

output
```
-1/32*x - 1/320*sin(10*x) + 5/144*sin(9*x) - 45/256*sin(8*x) + 15/28*sin(7
*x) - 35/32*sin(6*x) + 63/40*sin(5*x) - 105/64*sin(4*x) + 5/4*sin(3*x) - 4
5/64*sin(2*x) + 5/16*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int (1-\cos(x))^5\cos(5x)\,dx = -\frac{8\sin(x)\cos(x)^9}{5}+\frac{80\sin(x)\cos(x)^8}{9} -\frac{193\sin(x)\cos(x)^7}{10}+\frac{1180\sin(x)\cos(x)^6}{63} -\frac{67\sin(x)\cos(x)^5}{20}-\frac{979\sin(x)\cos(x)^4}{105} +\frac{133\sin(x)\cos(x)^3}{16}-\frac{766\sin(x)\cos(x)^2}{315} -\frac{\sin(x)\cos(x)}{32}-\frac{x}{32}+\frac{43\sin(x)}{315}$$

input
```
int(-cos(5*x)*(cos(x) - 1)^5,x)
```

output
```
(43*sin(x))/315 - x/32 - (cos(x)*sin(x))/32 - (766*cos(x)^2*sin(x))/315 +
(133*cos(x)^3*sin(x))/16 - (979*cos(x)^4*sin(x))/105 - (67*cos(x)^5*sin(x)
)/20 + (1180*cos(x)^6*sin(x))/63 - (193*cos(x)^7*sin(x))/10 + (80*cos(x)^8
*sin(x))/9 - (8*cos(x)^9*sin(x))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.08

$$\int (1-\cos(x))^5\cos(5x)\,dx = -\frac{\cos(5x)\cos(x)\sin(x)^4 x}{2} + \frac{20\cos(5x)\cos(x)\sin(x)^3}{9} + \frac{3\cos(5x)\cos(x)\sin(x)^2 x}{8} - \frac{160\cos(5x)\cos(x)\sin(x)}{63} - \frac{\cos(5x)\cos(x)x}{32} - \frac{15059\cos(5x)\sin(x)^5}{1008} + \frac{33395\cos(5x)\sin(x)^3}{2016} - \frac{160\cos(5x)\sin(x)}{63} + \frac{74791\cos(x)\sin(5x)\sin(x)^4}{5040} - \frac{25583\cos(x)\sin(5x)\sin(x)^2}{3360} - \frac{27073\cos(x)\sin(5x)}{10080} - \frac{\sin(5x)\sin(x)^5 x}{2} + \frac{25\sin(5x)\sin(x)^4}{9} + \frac{5\sin(5x)\sin(x)^3 x}{8} - \frac{400\sin(5x)\sin(x)^2}{63} - \frac{5\sin(5x)\sin(x)x}{32} + \frac{1168\sin(5x)}{315}$$

input
```
int((1-cos(x))^5*cos(5*x),x)
```

output
```
( - 5040*cos(5*x)*cos(x)*sin(x)**4*x + 22400*cos(5*x)*cos(x)*sin(x)**3 + 3
780*cos(5*x)*cos(x)*sin(x)**2*x - 25600*cos(5*x)*cos(x)*sin(x) - 315*cos(5
*x)*cos(x)*x - 150590*cos(5*x)*sin(x)**5 + 166975*cos(5*x)*sin(x)**3 - 256
00*cos(5*x)*sin(x) + 149582*cos(x)*sin(5*x)*sin(x)**4 - 76749*cos(x)*sin(5
*x)*sin(x)**2 - 27073*cos(x)*sin(5*x) - 5040*sin(5*x)*sin(x)**5*x + 28000*
sin(5*x)*sin(x)**4 + 6300*sin(5*x)*sin(x)**3*x - 64000*sin(5*x)*sin(x)**2
- 1575*sin(5*x)*sin(x)*x + 37376*sin(5*x))/10080
```

3.261 $\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{24x}{25}-\frac{7}{25}\log(4\cos(x)+3\sin(x))$$

output
```
24/25*x-7/25*ln(4*cos(x)+3*sin(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{24x}{25}-\frac{7}{25}\log(4\cos(x)+3\sin(x))$$

input
```
Integrate[(3*Cos[x] + 4*Sin[x])/(4*Cos[x] + 3*Sin[x]),x]
```

output
```
(24*x)/25 - (7*Log[4*Cos[x] + 3*Sin[x]])/25
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4\sin(x)+3\cos(x)}{3\sin(x)+4\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{4\sin(x)+3\cos(x)}{3\sin(x)+4\cos(x)}dx$$

$$\Big\downarrow \text{3612}$$

$$\frac{24x}{25}-\frac{7}{25}\log(3\sin(x)+4\cos(x))$$

input
```
Int[(3*Cos[x] + 4*Sin[x])/(4*Cos[x] + 3*Sin[x]),x]
```

output
```
(24*x)/25 - (7*Log[4*Cos[x] + 3*Sin[x]])/25
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{24x}{25}+\frac{7ix}{25}-\frac{7\ln\left(\mathrm{e}^{2ix}+\frac{7}{25}+\frac{24i}{25}\right)}{25}$	21
default	$\frac{7\ln\left(1+\tan\left(x\right)^{2}\right)}{50}+\frac{24\arctan\left(\tan\left(x\right)\right)}{25}-\frac{7\ln\left(4+3\tan\left(x\right)\right)}{25}$	25
parallelrisch	$\ln\left(\frac{262144^{\frac{24}{25}}}{131072\left(\left(2\tan\left(\frac{x}{2}\right)+1\right)^{7}\right)^{\frac{1}{25}}}\right)+\ln\left(\frac{1}{\left(\tan\left(\frac{x}{2}\right)-2\right)^{\frac{7}{25}}}\right)+\ln\left(\left(\sec\left(\frac{x}{2}\right)^{2}\right)^{\frac{7}{25}}\right)+\frac{24x}{25}$	41
norman	$\frac{\frac{24x}{25}+\frac{24\tan\left(\frac{x}{2}\right)^{2}x}{25}}{1+\tan\left(\frac{x}{2}\right)^{2}}-\frac{7\ln\left(\tan\left(\frac{x}{2}\right)-2\right)}{25}-\frac{7\ln\left(2\tan\left(\frac{x}{2}\right)+1\right)}{25}+\frac{7\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{25}$	57

input
```
int((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
24/25*x+7/25*I*x-7/25*ln(exp(2*I*x)+7/25+24/25*I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{24}{25}\,x-\frac{7}{25}\,\log\left(-2\,\cos\left(x\right)-\frac{3}{2}\,\sin\left(x\right)\right)$$

input
```
integrate((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x, algorithm="fricas")
```

output
```
24/25*x - 7/25*log(-2*cos(x) - 3/2*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{24x}{25}-\frac{7\log\left(3\sin\left(x\right)+4\cos\left(x\right)\right)}{25}$$

input
```
integrate((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x)
```

output
```
24*x/25 - 7*log(3*sin(x) + 4*cos(x))/25
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{48}{25}\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)-\frac{7}{25}\log\left(\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)-\frac{7}{25}\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-2\right)+\frac{7}{25}\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

input
```
integrate((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x, algorithm="maxima")
```

output
```
48/25*arctan(sin(x)/(cos(x) + 1)) - 7/25*log(2*sin(x)/(cos(x) + 1) + 1) -
7/25*log(sin(x)/(cos(x) + 1) - 2) + 7/25*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx=\frac{24}{25}x+\frac{7}{50}\log\left(\tan\left(x\right)^{2}+1\right)-\frac{7}{25}\log\left(\left|3\tan\left(x\right)+4\right|\right)$$

input
```
integrate((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x, algorithm="giac")
```

output
```
24/25*x + 7/50*log(tan(x)^2 + 1) - 7/25*log(abs(3*tan(x) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx = \frac{24\,x}{25}-\frac{7\,\ln\left(\tan\left(\frac{x}{2}\right)^2-\frac{3\tan\left(\frac{x}{2}\right)}{2}-1\right)}{25}+\frac{7\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)}{25}$$

input
```
int((3*cos(x) + 4*sin(x))/(4*cos(x) + 3*sin(x)),x)
```

output
```
(24*x)/25 - (7*log(tan(x/2)^2 - (3*tan(x/2))/2 - 1))/25 + (7*log(tan(x/2)^
2 + 1))/25
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx = -\frac{7\log(4\cos(x)+3\sin(x))}{25}+\frac{24x}{25}$$

input
```
int((3*cos(x)+4*sin(x))/(4*cos(x)+3*sin(x)),x)
```

output
```
( - 7*log(4*cos(x) + 3*sin(x)) + 24*x)/25
```

3.262 $\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 63

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = -\sqrt{3}x-\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}(1+x)\sqrt{4-(1+x)^2}-2\arcsin\left(\frac{x}{2}\right)+2\arcsin\left(\frac{1+x}{2}\right)$$

output
```
-x*3^(1/2)-1/2*x*(-x^2+4)^(1/2)+1/2*(1+x)*(4-(1+x)^2)^(1/2)-2*arcsin(1/2*x
)+2*arcsin(1/2+1/2*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.41

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = -\sqrt{3}x-\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}(1+x)\sqrt{3-2x-x^2}+4\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)-4\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)$$

input
```
Integrate[-Sqrt[3] - Sqrt[4 - x^2] + Sqrt[4 - (1 + x)^2],x]
```

output
```
-(Sqrt[3]*x) - (x*Sqrt[4 - x^2])/2 + ((1 + x)*Sqrt[3 - 2*x - x^2])/2 + 4*A
rcTan[Sqrt[4 - x^2]/(2 + x)] - 4*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{4-x^2}+\sqrt{4-(x+1)^2}-\sqrt{3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-2\arcsin\left(\frac{x}{2}\right)+2\arcsin\left(\frac{x+1}{2}\right)-\frac{1}{2}\sqrt{4-x^2}x-\sqrt{3}x+\frac{1}{2}(x+1)\sqrt{4-(x+1)^2}$$

input
```
Int[-Sqrt[3] - Sqrt[4 - x^2] + Sqrt[4 - (1 + x)^2],x]
```

output
```
-(Sqrt[3]*x) - (x*Sqrt[4 - x^2])/2 + ((1 + x)*Sqrt[4 - (1 + x)^2])/2 - 2*A
rcSin[x/2] + 2*ArcSin[(1 + x)/2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{(-2x-2)\sqrt{-x^2-2x+3}}{4}+2\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)-\sqrt{3}\,x-\frac{x\sqrt{-x^2+4}}{2}-2\arcsin\left(\frac{x}{2}\right)$	53
parts	$-\frac{(-2x-2)\sqrt{-x^2-2x+3}}{4}+2\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)-\sqrt{3}\,x-\frac{x\sqrt{-x^2+4}}{2}-2\arcsin\left(\frac{x}{2}\right)$	53

input
```
int(-3^(1/2)-(-x^2+4)^(1/2)+(4-(1+x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*x-2)*(-x^2-2*x+3)^(1/2)+2*arcsin(1/2+1/2*x)-3^(1/2)*x-1/2*x*(-x^2
+4)^(1/2)-2*arcsin(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = \frac{1}{2}\,\sqrt{-x^2-2\,x+3}(x+1) - \sqrt{3}x-\frac{1}{2}\,\sqrt{-x^2+4}x - 2\,\arctan\left(\frac{\sqrt{-x^2-2\,x+3}(x+1)}{x^2+2\,x-3}\right) + 4\,\arctan\left(\frac{\sqrt{-x^2+4}-2}{x}\right)$$

input
```
integrate(-3^(1/2)-(-x^2+4)^(1/2)+(4-(1+x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 - 2*x + 3)*(x + 1) - sqrt(3)*x - 1/2*sqrt(-x^2 + 4)*x - 2*ar
ctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 4*arctan((sqrt(-x^2 +
 4) - 2)/x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.89

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right) dx$$
$$= -\frac{x\sqrt{4-x^2}}{2}-\sqrt{3}x + \begin{cases} \frac{i(x+1)^3}{2\sqrt{(x+1)^2-4}}-\frac{2i(x+1)}{\sqrt{(x+1)^2-4}}-2i\operatorname{acosh}\left(\frac{x}{2}+\frac{1}{2}\right) & \text{for } \left|(x+1)^2\right|>4 \\ 2\operatorname{asin}\left(\frac{x}{2}+\frac{1}{2}\right)-\frac{(x+1)^3}{2\sqrt{4-(x+1)^2}}+\frac{2(x+1)}{\sqrt{4-(x+1)^2}} & \text{otherwise} \end{cases} - 2\operatorname{asin}\left(\frac{x}{2}\right)$$

input
```
integrate(-3**(1/2)-(-x**2+4)**(1/2)+(4-(1+x)**2)**(1/2),x)
```

output
```
-x*sqrt(4 - x**2)/2 - sqrt(3)*x + Piecewise((I*(x + 1)**3/(2*sqrt((x + 1)*
*2 - 4)) - 2*I*(x + 1)/sqrt((x + 1)**2 - 4) - 2*I*acosh(x/2 + 1/2), Abs((x
 + 1)**2) > 4), (2*asin(x/2 + 1/2) - (x + 1)**3/(2*sqrt(4 - (x + 1)**2)) +
 2*(x + 1)/sqrt(4 - (x + 1)**2), True)) - 2*asin(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right) dx = -\sqrt{3}x+\frac{1}{2}\sqrt{-x^2-2x+3}x - \frac{1}{2}\sqrt{-x^2+4}x+\frac{1}{2}\sqrt{-x^2-2x+3} - 2\arcsin\left(\frac{1}{2}x\right)-2\arcsin\left(-\frac{1}{2}x-\frac{1}{2}\right)$$

input
```
integrate(-3^(1/2)-(-x^2+4)^(1/2)+(4-(1+x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(3)*x + 1/2*sqrt(-x^2 - 2*x + 3)*x - 1/2*sqrt(-x^2 + 4)*x + 1/2*sqrt(
-x^2 - 2*x + 3) - 2*arcsin(1/2*x) - 2*arcsin(-1/2*x - 1/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = \frac{1}{2}\sqrt{-x^2-2x+3}(x+1)-\sqrt{3}x - \frac{1}{2}\sqrt{-x^2+4}x - 2\arcsin\left(\frac{1}{2}x\right) + 2\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)$$

input
```
integrate(-3^(1/2)-(-x^2+4)^(1/2)+(4-(1+x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 2*x + 3)*(x + 1) - sqrt(3)*x - 1/2*sqrt(-x^2 + 4)*x - 2*ar
csin(1/2*x) + 2*arcsin(1/2*x + 1/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = \int \sqrt{4-(x+1)^2}-\sqrt{3}-\sqrt{4-x^2}\,dx$$

input
```
int((4 - (x + 1)^2)^(1/2) - 3^(1/2) - (4 - x^2)^(1/2),x)
```

output
```
int((4 - (x + 1)^2)^(1/2) - 3^(1/2) - (4 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right)\,dx = -2asin\left(\frac{x}{2}\right)-\frac{\sqrt{-x^2+4}\,x}{2} - \sqrt{3}\,x+\int \sqrt{-x^2-2x+3}dx$$

input
```
int(-3^(1/2)-(-x^2+4)^(1/2)+(4-(1+x)^2)^(1/2),x)
```

output
```
( - 4*asin(x/2) - sqrt( - x**2 + 4)*x - 2*sqrt(3)*x + 2*int(sqrt( - x**2 -
 2*x + 3),x))/2
```

3.263 $\int x^2 \sin(\log(x)) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int x^2 \sin(\log(x)) \, dx = -\frac{1}{10} x^3 \cos(\log(x)) + \frac{3}{10} x^3 \sin(\log(x))$$

output
```
-1/10*x^3*cos(ln(x))+3/10*x^3*sin(ln(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^2 \sin(\log(x)) \, dx = -\frac{1}{10} x^3 \cos(\log(x)) + \frac{3}{10} x^3 \sin(\log(x))$$

input
```
Integrate[x^2*Sin[Log[x]],x]
```

output
```
-1/10*(x^3*Cos[Log[x]]) + (3*x^3*Sin[Log[x]])/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin(\log(x))\,dx$$

$$\Big\downarrow \text{4988}$$

$$\frac{3}{10}x^3\sin(\log(x)) - \frac{1}{10}x^3\cos(\log(x))$$

input
```
Int[x^2*Sin[Log[x]],x]
```

output
```
-1/10*(x^3*Cos[Log[x]]) + (3*x^3*Sin[Log[x]])/10
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$\left(-\frac{1}{20}-\frac{3i}{20}\right)x^3x^i+\left(-\frac{1}{20}+\frac{3i}{20}\right)x^3x^{-i}$	26
norman	$\frac{-\frac{x^3}{10}+\frac{3x^3\tan\left(\frac{\ln(x)}{2}\right)}{5}+\frac{x^3\tan\left(\frac{\ln(x)}{2}\right)^2}{10}}{1+\tan\left(\frac{\ln(x)}{2}\right)^2}$	41

input
```
int(x^2*sin(ln(x)),x,method=_RETURNVERBOSE)
```

output
```
(-1/20-3/20*I)*x^3*x^I+(-1/20+3/20*I)*x^3/(x^I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^2\sin(\log(x))\,dx=-\frac{1}{10}\,x^3\cos\left(\log\left(x\right)\right)+\frac{3}{10}\,x^3\sin\left(\log\left(x\right)\right)$$

input
```
integrate(x^2*sin(log(x)),x, algorithm="fricas")
```

output
```
-1/10*x^3*cos(log(x)) + 3/10*x^3*sin(log(x))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^2\sin(\log(x))\,dx=\frac{3x^3\sin\left(\log\left(x\right)\right)}{10}-\frac{x^3\cos\left(\log\left(x\right)\right)}{10}$$

input
```
integrate(x**2*sin(ln(x)),x)
```

output
```
3*x**3*sin(log(x))/10 - x**3*cos(log(x))/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x^2 \sin(\log(x))\,dx = -\frac{1}{10}\,x^3(\cos(\log(x)) - 3\,\sin(\log(x)))$$

input
```
integrate(x^2*sin(log(x)),x, algorithm="maxima")
```

output
```
-1/10*x^3*(cos(log(x)) - 3*sin(log(x)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^2 \sin(\log(x))\,dx = -\frac{1}{10}\,x^3\cos(\log(x)) + \frac{3}{10}\,x^3\sin(\log(x))$$

input
```
integrate(x^2*sin(log(x)),x, algorithm="giac")
```

output
```
-1/10*x^3*cos(log(x)) + 3/10*x^3*sin(log(x))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x^2 \sin(\log(x))\,dx = -\frac{\sqrt{10}\,x^3\,\cos(\mathrm{atan}(3) + \ln(x))}{10}$$

input
```
int(x^2*sin(log(x)),x)
```

output
```
-(10^(1/2)*x^3*cos(atan(3) + log(x)))/10
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x^2 \sin(\log(x))\,dx = \frac{x^3(-\cos(\log(x)) + 3\sin(\log(x)))}{10}$$

input
```
int(x^2*sin(log(x)),x)
```

output
```
(x**3*( - cos(log(x)) + 3*sin(log(x))))/10
```

3.264 $\int e^{-x}(36x^5 - 12x^6 + x^7)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{-x}(36x^5 - 12x^6 + x^7)\,dx = -720e^{-x} - 720e^{-x}x - 360e^{-x}x^2 - 120e^{-x}x^3 - 30e^{-x}x^4 - 6e^{-x}x^5 + 5e^{-x}x^6 - e^{-x}x^7$$

output
```
-720/exp(x)-720*x/exp(x)-360*x^2/exp(x)-120*x^3/exp(x)-30*x^4/exp(x)-6*x^5
/exp(x)+5*x^6/exp(x)-x^7/exp(x)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int e^{-x}(36x^5 - 12x^6 + x^7)\,dx = e^{-x}(-720 - 720x - 360x^2 - 120x^3 - 30x^4 - 6x^5 + 5x^6 - x^7)$$

input
```
Integrate[(36*x^5 - 12*x^6 + x^7)/E^x,x]
```

output
```
(-720 - 720*x - 360*x^2 - 120*x^3 - 30*x^4 - 6*x^5 + 5*x^6 - x^7)/E^x
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2028, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\left(x^7-12x^6+36x^5\right)dx$$

$$\Big\downarrow \text{2028}$$

$$\int e^{-x}x^5\left(x^2-12x+36\right)dx$$

$$\Big\downarrow \text{2626}$$

$$\int \left(e^{-x}x^7-12e^{-x}x^6+36e^{-x}x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{-x}x^7+5e^{-x}x^6-6e^{-x}x^5-30e^{-x}x^4-120e^{-x}x^3-360e^{-x}x^2-720e^{-x}x-720e^{-x}$$

input
```
Int[(36*x^5 - 12*x^6 + x^7)/E^x,x]
```

output
```
-720/E^x - (720*x)/E^x - (360*x^2)/E^x - (120*x^3)/E^x - (30*x^4)/E^x - (6
*x^5)/E^x + (5*x^6)/E^x - x^7/E^x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] && !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

method	result
gosper	$-\left(x^{7}-5x^{6}+6x^{5}+30x^{4}+120x^{3}+360x^{2}+720x+720\right) \mathrm{e}^{-x}$
parallelrisch	$-\left(x^{7}-5x^{6}+6x^{5}+30x^{4}+120x^{3}+360x^{2}+720x+720\right) \mathrm{e}^{-x}$
norman	$\left(-x^{7}+5x^{6}-6x^{5}-30x^{4}-120x^{3}-360x^{2}-720x-720\right) \mathrm{e}^{-x}$
risch	$\left(-x^{7}+5x^{6}-6x^{5}-30x^{4}-120x^{3}-360x^{2}-720x-720\right) \mathrm{e}^{-x}$
orering	$-\frac{\left(x^{7}-5x^{6}+6x^{5}+30x^{4}+120x^{3}+360x^{2}+720x+720\right) \left(x^{7}-12x^{6}+36x^{5}\right) \mathrm{e}^{-x}}{\left(-6+x\right)^{2} x^{5}}$
default	$-720\,\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-360x^{2}\mathrm{e}^{-x}-120x^{3}\mathrm{e}^{-x}-30\,\mathrm{e}^{-x}x^{4}-6\,\mathrm{e}^{-x}x^{5}+5\,\mathrm{e}^{-x}x^{6}-\mathrm{e}^{-x}x^{7}$
meijerg	$720-\frac{\left(8x^{7}+56x^{6}+336x^{5}+1680x^{4}+6720x^{3}+20160x^{2}+40320x+40320\right) \mathrm{e}^{-x}}{8}+\frac{12\left(7x^{6}+42x^{5}+210x^{4}+840x^{3}+2520x^{2}+\right.}{7}$

input
```
int((x^7-12*x^6+36*x^5)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-(x^7-5*x^6+6*x^5+30*x^4+120*x^3+360*x^2+720*x+720)/exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx = -\left(x^{7}-5\,x^{6}+6\,x^{5}+30\,x^{4}+120\,x^{3}+360\,x^{2}+720\,x+720\right)e^{\left(-x\right)}$$

input
```
integrate((x^7-12*x^6+36*x^5)/exp(x),x, algorithm="fricas")
```

output
```
-(x^7 - 5*x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx=\left(-x^7+5x^6-6x^5-30x^4-120x^3-360x^2-720x-720\right)e^{-x}$$

input
```
integrate((x**7-12*x**6+36*x**5)/exp(x),x)
```

output
```
(-x**7 + 5*x**6 - 6*x**5 - 30*x**4 - 120*x**3 - 360*x**2 - 720*x - 720)*ex
p(-x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\begin{aligned}&\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx\\&=-\left(x^7+7\,x^6+42\,x^5+210\,x^4+840\,x^3+2520\,x^2+5040\,x+5040\right)e^{(-x)}\\&\quad+12\,\left(x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)e^{(-x)}\\&\quad-36\,\left(x^5+5\,x^4+20\,x^3+60\,x^2+120\,x+120\right)e^{(-x)}\end{aligned}$$

input
```
integrate((x^7-12*x^6+36*x^5)/exp(x),x, algorithm="maxima")
```

output
```
-(x^7 + 7*x^6 + 42*x^5 + 210*x^4 + 840*x^3 + 2520*x^2 + 5040*x + 5040)*e^(
-x) + 12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x) -
 36*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx = -\left(x^7-5\,x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)e^{(-x)}$$

input
```
integrate((x^7-12*x^6+36*x^5)/exp(x),x, algorithm="giac")
```

output
```
-(x^7 - 5*x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx = -\mathrm{e}^{-x}\,\left(x^7-5\,x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)$$

input
```
int(exp(-x)*(36*x^5 - 12*x^6 + x^7),x)
```

output
```
-exp(-x)*(720*x + 360*x^2 + 120*x^3 + 30*x^4 + 6*x^5 - 5*x^6 + x^7 + 720)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int e^{-x}\left(36x^5-12x^6+x^7\right)\,dx = \frac{-x^7+5x^6-6x^5-30x^4-120x^3-360x^2-720x-720}{e^x}$$

input
```
int((x^7-12*x^6+36*x^5)/exp(x),x)
```

output
```
( - x**7 + 5*x**6 - 6*x**5 - 30*x**4 - 120*x**3 - 360*x**2 - 720*x - 720)/
e**x
```

3.265 $\int \arccos(x) \arcsin(x) \, dx$

Optimal result

Integrand size = 5, antiderivative size = 38

$$\int \arccos(x) \arcsin(x) \, dx = 2x - \sqrt{1-x^2} \arcsin(x) + \arccos(x) \left(\sqrt{1-x^2} + x \arcsin(x)\right)$$

output
```
2*x-(-x^2+1)^(1/2)*arcsin(x)+arccos(x)*((-x^2+1)^(1/2)+x*arcsin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \arccos(x) \arcsin(x) \, dx = 2x - \sqrt{1-x^2} \arcsin(x) + \arccos(x) \left(\sqrt{1-x^2} + x \arcsin(x)\right)$$

input
```
Integrate[ArcCos[x]*ArcSin[x],x]
```

output
```
2*x - Sqrt[1 - x^2]*ArcSin[x] + ArcCos[x]*(Sqrt[1 - x^2] + x*ArcSin[x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(x)\arcsin(x)\,dx$$

$$\Big\downarrow \text{5300}$$

$$\int \arccos(x)\arcsin(x)dx$$

input
```
Int[ArcCos[x]*ArcSin[x],x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

method	result
orering	$x\arccos(x)\arcsin(x) - \frac{\arcsin(x)}{\sqrt{-x^2+1}} + \frac{\arccos(x)}{\sqrt{-x^2+1}} + (-1+x)(1+x)x\left(-\frac{\arcsin(x)x}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{-x^2+1} + \frac{\arccos(x)}{(-x^2+1)}\right.$

input
```
int(arccos(x)*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
x*arccos(x)*arcsin(x)-1/(-x^2+1)^(1/2)*arcsin(x)+arccos(x)/(-x^2+1)^(1/2)+
(-1+x)*(1+x)*x*(-1/(-x^2+1)^(3/2)*arcsin(x)*x-2/(-x^2+1)+arccos(x)/(-x^2+1
)^(3/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \arccos(x)\arcsin(x)\,dx = \frac{1}{2}\,\pi x \arccos\left(x\right) - x \arccos\left(x\right)^{2} - \frac{1}{2}\left(\pi - 4\,\arccos\left(x\right)\right)\sqrt{-x^{2} + 1} + 2\,x$$

input
```
integrate(arccos(x)*arcsin(x),x, algorithm="fricas")
```

output
```
1/2*pi*x*arccos(x) - x*arccos(x)^2 - 1/2*(pi - 4*arccos(x))*sqrt(-x^2 + 1)
 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \arccos(x)\arcsin(x)\,dx = x\operatorname{acos}\left(x\right)\operatorname{asin}\left(x\right) + 2x + \sqrt{1 - x^{2}}\operatorname{acos}\left(x\right) - \sqrt{1 - x^{2}}\operatorname{asin}\left(x\right)$$

input
```
integrate(acos(x)*asin(x),x)
```

output
```
x*acos(x)*asin(x) + 2*x + sqrt(1 - x**2)*acos(x) - sqrt(1 - x**2)*asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \arccos(x)\arcsin(x)\,dx = \left(x \arcsin\left(x\right) + \sqrt{-x^{2} + 1}\right) \arccos\left(x\right) - \sqrt{-x^{2} + 1} \arcsin\left(x\right) + 2\,x$$

input
```
integrate(arccos(x)*arcsin(x),x, algorithm="maxima")
```

output
```
(x*arcsin(x) + sqrt(-x^2 + 1))*arccos(x) - sqrt(-x^2 + 1)*arcsin(x) + 2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.29

$$\int \arccos(x) \arcsin(x) \, dx = -\pi(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} x \arccos(x) \left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor + \frac{1}{2}\pi(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} x \arccos(x) - (-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} x \arccos(x)^2 + \pi\sqrt{-x^2+1}(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} \left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor - \frac{1}{2}\pi\sqrt{-x^2+1}(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} + 2\sqrt{-x^2+1}(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} \arccos(x) + 2(-1)^{\left\lfloor -\frac{\arccos(x)}{\pi}+1\right\rfloor} x$$

input
```
integrate(arccos(x)*arcsin(x),x, algorithm="giac")
```

output
```
-pi*(-1)^floor(-arccos(x)/pi + 1)*x*arccos(x)*floor(-arccos(x)/pi + 1) + 1
/2*pi*(-1)^floor(-arccos(x)/pi + 1)*x*arccos(x) - (-1)^floor(-arccos(x)/pi
 + 1)*x*arccos(x)^2 + pi*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1)*floo
r(-arccos(x)/pi + 1) - 1/2*pi*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1)
 + 2*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1)*arccos(x) + 2*(-1)^floor
(-arccos(x)/pi + 1)*x
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(x) \arcsin(x) \, dx = \int \operatorname{acos}(x) \operatorname{asin}(x) \, dx$$

input
```
int(acos(x)*asin(x),x)
```

output
```
int(acos(x)*asin(x), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \arccos(x)\arcsin(x)\,dx = acos(x)\,asin(x)\,x + \sqrt{-x^2+1}\,acos(x) - \sqrt{-x^2+1}\,asin(x) + 2x$$

input

```
int(acos(x)*asin(x),x)
```

output

```
acos(x)*asin(x)*x + sqrt( - x**2 + 1)*acos(x) - sqrt( - x**2 + 1)*asin(x)
+ 2*x
```

3.266 $\int (1+6x-7x^2+4x^3-x^4)^n \, dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int (1+6x-7x^2+4x^3-x^4)^n \, dx = 3^n(-1+x) \operatorname{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2(-1+x)^2}{1+\sqrt{13}},\frac{2(-1+x)^2}{-1+\sqrt{13}}\right)$$

output
```
3^n*(-1+x)*AppellF1(1/2,-n,-n,3/2,2*(-1+x)^2/(-1+13^(1/2)),-2*(-1+x)^2/(1+
13^(1/2)))
```

Mathematica [F]

$$\int (1+6x-7x^2+4x^3-x^4)^n \, dx = \int (1+6x-7x^2+4x^3-x^4)^n \, dx$$

input
```
Integrate[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n,x]
```

output
```
Integrate[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n, x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. $2(52) = 104$.

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2458, 1418, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4+4x^3-7x^2+6x+1\right)^n dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(-(x-1)^4-(x-1)^2+3\right)^n d(x-1)$$

$$\Big\downarrow \text{1418}$$

$$\left(\frac{2(x-1)^2}{1-\sqrt{13}}+1\right)^{-n}\left(\frac{2(x-1)^2}{1+\sqrt{13}}+1\right)^{-n}\left(-(x-1)^4-(x-1)^2+3\right)^n\int \left(\frac{2(x-1)^2}{1-\sqrt{13}}+1\right)^n\left(\frac{2(x-1)^2}{1+\sqrt{13}}+1\right)^n d(x-1)$$

$$\Big\downarrow \text{333}$$

$$(x-1)\left(\frac{2(x-1)^2}{1-\sqrt{13}}+1\right)^{-n}\left(\frac{2(x-1)^2}{1+\sqrt{13}}+1\right)^{-n}\left(-(x-1)^4-(x-1)^2+3\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2(x-1)^2}{1-\sqrt{13}},-\right.$$

input
```
Int[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n,x]
```

output
```
((3 - (-1 + x)^2 - (-1 + x)^4)^n*(-1 + x)*AppellF1[1/2, -n, -n, 3/2, (-2*(
-1 + x)^2)/(1 - Sqrt[13]), (-2*(-1 + x)^2)/(1 + Sqrt[13])])/((1 + (2*(-1 +
 x)^2)/(1 - Sqrt[13]))^n*(1 + (2*(-1 + x)^2)/(1 + Sqrt[13]))^n)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1418
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p]))   Int[(1
 + 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b,
c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [F]

$$\int \left(-x^{4}+4x^{3}-7x^{2}+6x+1\right)^{n}dx$$

input
```
int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
```

output
```
int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
```

Fricas [F]

$$\int \left(1+6x-7x^2+4x^3-x^4\right)^n\,dx=\int \left(-x^4+4\,x^3-7\,x^2+6\,x+1\right)^n\,dx$$

input
```
integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="fricas")
```

output
```
integral((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Sympy [F]

$$\int \left(1+6x-7x^2+4x^3-x^4\right)^n\,dx=\int \left(-x^4+4x^3-7x^2+6x+1\right)^n\,dx$$

input
```
integrate((-x**4+4*x**3-7*x**2+6*x+1)**n,x)
```

output
```
Integral((-x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n, x)
```

Maxima [F]

$$\int \left(1+6x-7x^2+4x^3-x^4\right)^n\,dx=\int \left(-x^4+4\,x^3-7\,x^2+6\,x+1\right)^n\,dx$$

input
```
integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Giac [F]

$$\int (1+6x-7x^2+4x^3-x^4)^n \, dx = \int (-x^4+4\,x^3-7\,x^2+6\,x+1)^n \, dx$$

input
```
integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+6x-7x^2+4x^3-x^4)^n \, dx = \int (-x^4+4\,x^3-7\,x^2+6\,x+1)^n \, dx$$

input
```
int((6*x - 7*x^2 + 4*x^3 - x^4 + 1)^n,x)
```

output
```
int((6*x - 7*x^2 + 4*x^3 - x^4 + 1)^n, x)
```

Reduce [F]

$$\int (1+6x-7x^2+4x^3-x^4)^n \, dx$$
$$= \frac{6(-x^4+4x^3-7x^2+6x+1)^n x - 7(-x^4+4x^3-7x^2+6x+1)^n - 264\Big(\int \frac{(-x^4+4x^3-7x^2+6x+}{4n\,x^4-16n\,x^3+x^4+28n\,x^2-4x^3-24n}}{}$$

input
```
int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
```

output

```
(6*( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n*x - 7*( - x**4 + 4*x**3 - 7*x*
*2 + 6*x + 1)**n - 264*int(( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n/(4*n*x
**4 - 16*n*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x
- 1),x)*n**2 - 66*int(( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n/(4*n*x**4 -
 16*n*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x - 1),
x)*n + 16*int((( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n*x**3)/(4*n*x**4 -
16*n*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x - 1),x
)*n**2 + 4*int((( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n*x**3)/(4*n*x**4 -
 16*n*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x - 1),
x)*n - 40*int((( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n*x)/(4*n*x**4 - 16*
n*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x - 1),x)*n
**2 - 10*int((( - x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n*x)/(4*n*x**4 - 16*n
*x**3 + 28*n*x**2 - 24*n*x - 4*n + x**4 - 4*x**3 + 7*x**2 - 6*x - 1),x)*n)
/(6*(4*n + 1))
```

3.267 $\int \frac{x^4}{\sqrt{1-x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{x^4}{\sqrt{1-x}}\,dx=-2\sqrt{1-x}+\frac{8}{3}(1-x)^{3/2}-\frac{12}{5}(1-x)^{5/2}+\frac{8}{7}(1-x)^{7/2}-\frac{2}{9}(1-x)^{9/2}$$

output
```
-2*(1-x)^(1/2)+8/3*(1-x)^(3/2)-12/5*(1-x)^(5/2)+8/7*(1-x)^(7/2)-2/9*(1-x)^
(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{\sqrt{1-x}}\,dx=-\frac{2}{315}\sqrt{1-x}\left(128+64x+48x^2+40x^3+35x^4\right)$$

input
```
Integrate[x^4/Sqrt[1 - x],x]
```

output
```
(-2*Sqrt[1 - x]*(128 + 64*x + 48*x^2 + 40*x^3 + 35*x^4))/315
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-x}} \, dx$$

$$\Big\downarrow \text{53}$$

$$\int \left((1-x)^{7/2}-4(1-x)^{5/2}+6(1-x)^{3/2}-4\sqrt{1-x}+\frac{1}{\sqrt{1-x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{9}(1-x)^{9/2}+\frac{8}{7}(1-x)^{7/2}-\frac{12}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

input
```
Int[x^4/Sqrt[1 - x],x]
```

output
```
-2*Sqrt[1 - x] + (8*(1 - x)^(3/2))/3 - (12*(1 - x)^(5/2))/5 + (8*(1 - x)^(
7/2))/7 - (2*(1 - x)^(9/2))/9
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

method	result	size
trager	$\left(-\frac{2}{9}x^{4}-\frac{16}{63}x^{3}-\frac{32}{105}x^{2}-\frac{128}{315}x-\frac{256}{315}\right)\sqrt{1-x}$	29
gosper	$-\frac{2\sqrt{1-x}\,(35x^{4}+40x^{3}+48x^{2}+64x+128)}{315}$	30
pseudoelliptic	$-\frac{2\sqrt{1-x}\,(35x^{4}+40x^{3}+48x^{2}+64x+128)}{315}$	30
risch	$\frac{2(-1+x)(35x^{4}+40x^{3}+48x^{2}+64x+128)}{315\sqrt{1-x}}$	33
orering	$\frac{2(-1+x)(35x^{4}+40x^{3}+48x^{2}+64x+128)}{315\sqrt{1-x}}$	33
meijerg	$-\frac{-\frac{256\sqrt{\pi}}{315}+\frac{\sqrt{\pi}\left(70x^{4}+80x^{3}+96x^{2}+128x+256\right)\sqrt{1-x}}{315}}{\sqrt{\pi}}$	44
derivativedivides	$-2\sqrt{1-x}+\frac{8(1-x)^{\frac{3}{2}}}{3}-\frac{12(1-x)^{\frac{5}{2}}}{5}+\frac{8(1-x)^{\frac{7}{2}}}{7}-\frac{2(1-x)^{\frac{9}{2}}}{9}$	47
default	$-2\sqrt{1-x}+\frac{8(1-x)^{\frac{3}{2}}}{3}-\frac{12(1-x)^{\frac{5}{2}}}{5}+\frac{8(1-x)^{\frac{7}{2}}}{7}-\frac{2(1-x)^{\frac{9}{2}}}{9}$	47

input
```
int(x^4/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/9*x^4-16/63*x^3-32/105*x^2-128/315*x-256/315)*(1-x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = -\frac{2}{315}\left(35\,x^4+40\,x^3+48\,x^2+64\,x+128\right)\sqrt{-x+1}$$

input
```
integrate(x^4/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*x^4 + 40*x^3 + 48*x^2 + 64*x + 128)*sqrt(-x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = \begin{cases} -\frac{2ix^4\sqrt{x-1}}{9} - \frac{16ix^3\sqrt{x-1}}{63} - \frac{32ix^2\sqrt{x-1}}{105} - \frac{128ix\sqrt{x-1}}{315} - \frac{256i\sqrt{x-1}}{315} & \text{for } |x| > 1 \\ -\frac{2x^4\sqrt{1-x}}{9} - \frac{16x^3\sqrt{1-x}}{63} - \frac{32x^2\sqrt{1-x}}{105} - \frac{128x\sqrt{1-x}}{315} - \frac{256\sqrt{1-x}}{315} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4/(1-x)**(1/2),x)
```

output
```
Piecewise((-2*I*x**4*sqrt(x - 1)/9 - 16*I*x**3*sqrt(x - 1)/63 - 32*I*x**2*
sqrt(x - 1)/105 - 128*I*x*sqrt(x - 1)/315 - 256*I*sqrt(x - 1)/315, Abs(x)
> 1), (-2*x**4*sqrt(1 - x)/9 - 16*x**3*sqrt(1 - x)/63 - 32*x**2*sqrt(1 - x
)/105 - 128*x*sqrt(1 - x)/315 - 256*sqrt(1 - x)/315, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = -\frac{2}{9}(-x+1)^{\frac{9}{2}} + \frac{8}{7}(-x+1)^{\frac{7}{2}} - \frac{12}{5}(-x+1)^{\frac{5}{2}} + \frac{8}{3}(-x+1)^{\frac{3}{2}} - 2\sqrt{-x+1}$$

input
```
integrate(x^4/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*(-x + 1)^(9/2) + 8/7*(-x + 1)^(7/2) - 12/5*(-x + 1)^(5/2) + 8/3*(-x +
 1)^(3/2) - 2*sqrt(-x + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = -\frac{2}{9}\,(x-1)^4\sqrt{-x+1} - \frac{8}{7}\,(x-1)^3\sqrt{-x+1} - \frac{12}{5}\,(x-1)^2\sqrt{-x+1} + \frac{8}{3}\,(-x+1)^{\frac{3}{2}} - 2\,\sqrt{-x+1}$$

input
```
integrate(x^4/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-2/9*(x - 1)^4*sqrt(-x + 1) - 8/7*(x - 1)^3*sqrt(-x + 1) - 12/5*(x - 1)^2*
sqrt(-x + 1) + 8/3*(-x + 1)^(3/2) - 2*sqrt(-x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = \frac{8\,(1-x)^{3/2}}{3} - 2\,\sqrt{1-x} - \frac{12\,(1-x)^{5/2}}{5} + \frac{8\,(1-x)^{7/2}}{7} - \frac{2\,(1-x)^{9/2}}{9}$$

input
```
int(x^4/(1 - x)^(1/2),x)
```

output
```
(8*(1 - x)^(3/2))/3 - 2*(1 - x)^(1/2) - (12*(1 - x)^(5/2))/5 + (8*(1 - x)^
(7/2))/7 - (2*(1 - x)^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{\sqrt{1-x}}\,dx = \frac{2\sqrt{1-x}\,(-35x^4-40x^3-48x^2-64x-128)}{315}$$

input
```
int(x^4/(1-x)^(1/2),x)
```

output
```
(2*sqrt( - x + 1)*( - 35*x**4 - 40*x**3 - 48*x**2 - 64*x - 128))/315
```

3.268 $\int \sin(x^{-n})\,dx$

Optimal result

Integrand size = 6, antiderivative size = 43

$$\int \sin(x^{-n})\,dx = \frac{ix\left(-\operatorname{ExpIntegralE}\left(1+\frac{1}{n},-ix^{-n}\right)+\operatorname{ExpIntegralE}\left(1+\frac{1}{n},ix^{-n}\right)\right)}{2n}$$

output
```
1/2*I*x*(-Ei(1+1/n,-I/(x^n))+Ei(1+1/n,I/(x^n)))/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \sin(x^{-n})\,dx = -\frac{ix\left((-ix^{-n})^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-ix^{-n}\right)-(ix^{-n})^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},ix^{-n}\right)\right)}{2n}$$

input
```
Integrate[Sin[x^(-n)],x]
```

output
```
((-1/2*I)*x*(((-I)/x^n)^n^(-1)*Gamma[-n^(-1), (-I)/x^n] - (I/x^n)^n^(-1)*G
amma[-n^(-1), I/x^n]))/n
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3846, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(x^{-n}\right) \, dx$$

$$\Big\downarrow \text{ 3846}$$

$$\frac{1}{2} i \int e^{-i x^{-n}} dx - \frac{1}{2} i \int e^{i x^{-n}} dx$$

$$\Big\downarrow \text{ 2637}$$

$$\frac{i x \left(i x^{-n}\right)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, i x^{-n}\right)}{2 n} - \frac{i x \left(-i x^{-n}\right)^{\frac{1}{n}} \Gamma\left(-\frac{1}{n}, -i x^{-n}\right)}{2 n}$$

input
```
Int[Sin[x^(-n)],x]
```

output
```
((-1/2*I)*x*((-I)/x^n)^n^(-1)*Gamma[-n^(-1), (-I)/x^n])/n + ((I/2)*x*(I/x^
n)^n^(-1)*Gamma[-n^(-1), I/x^n])/n
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

rule 3846
```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Simp[I/2   I
nt[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Simp[I/2   Int[E^(c*I + d*I*(e +
f*x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
meijerg	$-\frac{x^{-n+1} \operatorname{hypergeom}\left(\left[\frac{1}{2}-\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}-\frac{1}{2n}\right],-\frac{x^{-2n}}{4}\right)}{n-1}$	40

input
```
int(sin(x^(-n)),x,method=_RETURNVERBOSE)
```

output
```
-1/(n-1)*x^(-n+1)*hypergeom([1/2-1/2/n],[3/2,3/2-1/2/n],-1/4*x^(-2*n))
```

Fricas [F]

$$\int \sin \left(x^{-n}\right) \, dx = \int \sin \left(\frac{1}{x^n}\right) \, dx$$

input
```
integrate(sin(x^(-n)),x, algorithm="fricas")
```

output
```
integral(sin(1/(x^n)), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.79 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sin \left(x^{-n}\right) \, dx = -\frac{x^{1-n}\Gamma\left(\frac{1}{2}-\frac{1}{2n}\right) {}_1F_2\left(\begin{matrix} \frac{1}{2}-\frac{1}{2n} \\ \frac{3}{2},\frac{3}{2}-\frac{1}{2n} \end{matrix}\middle| -\frac{x^{-2n}}{4}\right)}{2n\Gamma\left(\frac{3}{2}-\frac{1}{2n}\right)}$$

input
```
integrate(sin(x**(-n)),x)
```

output
```
-x**(1 - n)*gamma(1/2 - 1/(2*n))*hyper((1/2 - 1/(2*n),), (3/2, 3/2 - 1/(2*
n)), -1/(4*x**(2*n)))/(2*n*gamma(3/2 - 1/(2*n)))
```

Maxima [F]

$$\int \sin\left(x^{-n}\right)\,dx = \int \sin\left(\frac{1}{x^n}\right)\,dx$$

input
```
integrate(sin(x^(-n)),x, algorithm="maxima")
```

output
```
integrate(sin(1/(x^n)), x)
```

Giac [F]

$$\int \sin\left(x^{-n}\right)\,dx = \int \sin\left(\frac{1}{x^n}\right)\,dx$$

input
```
integrate(sin(x^(-n)),x, algorithm="giac")
```

output
```
integrate(sin(1/(x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin\left(x^{-n}\right)\,dx = \int \sin\left(\frac{1}{x^n}\right)\,dx$$

input
```
int(sin(1/x^n),x)
```

output
```
int(sin(1/x^n), x)
```

Reduce [F]

$$\int \sin\left(x^{-n}\right)\,dx = \int \sin\left(\frac{1}{x^n}\right)dx$$

input
```
int(sin(x^(-n)),x)
```

output
```
int(sin(1/x**n),x)
```

3.269 $\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right) dx = -\frac{x^2}{4}+\frac{1}{4}x\sqrt{4-3x^2}+\frac{\arcsin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}}$$

output
```
-1/4*x^2+1/4*x*(-3*x^2+4)^(1/2)+1/3*arcsin(1/2*x*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right) dx = \frac{1}{2}\left(-\frac{x^2}{2}+\frac{1}{2}x\sqrt{4-3x^2}+\frac{4\arctan\left(\frac{\sqrt{3}x}{-2+\sqrt{4-3x^2}}\right)}{\sqrt{3}}\right)$$

input
```
Integrate[(-x + Sqrt[4 - 3*x^2])/2,x]
```

output
```
(-1/2*x^2 + (x*Sqrt[4 - 3*x^2])/2 + (4*ArcTan[(Sqrt[3]*x)/(-2 + Sqrt[4 - 3
*x^2])])/Sqrt[3])/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2}\left(\sqrt{4-3x^2}-x\right) dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \left(\sqrt{4-3x^2}-x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\arcsin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}}-\frac{x^2}{2}+\frac{1}{2}\sqrt{4-3x^2}x\right)$$

input
```
Int[(-x + Sqrt[4 - 3*x^2])/2,x]
```

output
```
(-1/2*x^2 + (x*Sqrt[4 - 3*x^2])/2 + (2*ArcSin[(Sqrt[3]*x)/2])/Sqrt[3])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}+\frac{\arcsin\left(\frac{\sqrt{3}\,x}{2}\right)\sqrt{3}}{3}$	31
parts	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}+\frac{\arcsin\left(\frac{\sqrt{3}\,x}{2}\right)\sqrt{3}}{3}$	31
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}+\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+3\right)\sqrt{-3x^2+4}+3x\right)}{3}$	48

input
```
int(-1/2*x+1/2*(-3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/4*x*(-3*x^2+4)^(1/2)+1/3*arcsin(1/2*3^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx = -\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x - \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4}-2\,\sqrt{3}}{3\,x}\right)$$

input
```
integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x - 2/3*sqrt(3)*arctan(1/3*(sqrt(3)*sqrt(-
3*x^2 + 4) - 2*sqrt(3))/x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx=-\frac{x^2}{4}+\frac{x\sqrt{4-3x^2}}{4}+\frac{\sqrt{3}\operatorname{asin}\left(\frac{\sqrt{3}x}{2}\right)}{3}$$

input
```
integrate(-1/2*x+1/2*(-3*x**2+4)**(1/2),x)
```

output
```
-x**2/4 + x*sqrt(4 - 3*x**2)/4 + sqrt(3)*asin(sqrt(3)*x/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx=-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x+\frac{1}{3}\,\sqrt{3}\arcsin\left(\frac{1}{2}\,\sqrt{3}x\right)$$

input
```
integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x + 1/3*sqrt(3)*arcsin(1/2*sqrt(3)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx=-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x+\frac{1}{3}\,\sqrt{3}\arcsin\left(\frac{1}{2}\,\sqrt{3}x\right)$$

input
```
integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
-1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x + 1/3*sqrt(3)*arcsin(1/2*sqrt(3)*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx=\frac{\sqrt{3}\left(\operatorname{asin}\left(\frac{\sqrt{3}\,x}{2}\right)+\frac{3\,x\,\sqrt{\frac{4}{3}-x^2}}{4}\right)}{3}-\frac{x^2}{4}$$

input
```
int((4 - 3*x^2)^(1/2)/2 - x/2,x)
```

output
```
(3^(1/2)*(asin((3^(1/2)*x)/2) + (3*x*(4/3 - x^2)^(1/2))/4))/3 - x^2/4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx=\frac{\sqrt{3}\,asin\left(\frac{\sqrt{3}\,x}{2}\right)}{3}+\frac{\sqrt{-3x^2+4}\,x}{4}-\frac{x^2}{4}$$

input
```
int(-1/2*x+1/2*(-3*x^2+4)^(1/2),x)
```

output
```
(4*sqrt(3)*asin((sqrt(3)*x)/2) + 3*sqrt( - 3*x**2 + 4)*x - 3*x**2)/12
```

3.270 $\int (1-3x^2+x^4)^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int (1-3x^2+x^4)^n\,dx = x\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}(1-3x^2+x^4)^n\operatorname{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},\frac{2x^2}{3+\sqrt{5}},\frac{2x^2}{3-\sqrt{5}}\right)$$

output
```
x*(x^4-3*x^2+1)^n*AppellF1(1/2,-n,-n,3/2,2*x^2/(3-5^(1/2)),2*x^2/(3+5^(1/2
)))/((1-2*x^2/(3-5^(1/2)))^n)/((1-2*x^2/(3+5^(1/2)))^n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.43

$$\int (1-3x^2+x^4)^n\,dx = \left(3+\sqrt{5}\right)^n x\left(-\left(3+\sqrt{5}-2x^2\right)^2\right)^{-n}\left(-3-\sqrt{5}+2x^2\right)^n\left(-3+\sqrt{5}+2x^2\right)^n\left(\frac{\left(-3+\sqrt{5}+2x^2\right)^2}{-3+\sqrt{5}}\right)^{-n}(1-3x^2+x^4)^n\operatorname{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2x^2}{-3+\sqrt{5}},\frac{2x^2}{3+\sqrt{5}}\right)$$

input
```
Integrate[(1 - 3*x^2 + x^4)^n,x]
```

output
```
((3 + Sqrt[5])^n*x*(-3 - Sqrt[5] + 2*x^2)^n*(-3 + Sqrt[5] + 2*x^2)^n*(1 -
3*x^2 + x^4)^n*AppellF1[1/2, -n, -n, 3/2, (-2*x^2)/(-3 + Sqrt[5]), (2*x^2)
/(3 + Sqrt[5])])/((-(3 + Sqrt[5] - 2*x^2)^2)^n*((-3 + Sqrt[5] + 2*x^2)^2/(
-3 + Sqrt[5]))^n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1418, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4-3x^2+1\right)^n\,dx$$

$$\Big\downarrow \text{1418}$$

$$\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}\left(x^4-3x^2+1\right)^n\int\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^n\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^n dx$$

$$\Big\downarrow \text{333}$$

$$x\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}\left(x^4-3x^2+1\right)^n\text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},\frac{2x^2}{3-\sqrt{5}},\frac{2x^2}{3+\sqrt{5}}\right)$$

input
```
Int[(1 - 3*x^2 + x^4)^n,x]
```

output
```
(x*(1 - 3*x^2 + x^4)^n*AppellF1[1/2, -n, -n, 3/2, (2*x^2)/(3 - Sqrt[5]), (
2*x^2)/(3 + Sqrt[5])])/((1 - (2*x^2)/(3 - Sqrt[5]))^n*(1 - (2*x^2)/(3 + Sq
rt[5]))^n)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1418
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p]))   Int[(1
 + 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b,
c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \left(x^4-3x^2+1\right)^n dx$$

input
```
int((x^4-3*x^2+1)^n,x)
```

output
```
int((x^4-3*x^2+1)^n,x)
```

Fricas [F]

$$\int \left(1-3x^2+x^4\right)^n dx=\int \left(x^4-3\,x^2+1\right)^n dx$$

input
```
integrate((x^4-3*x^2+1)^n,x, algorithm="fricas")
```

output
```
integral((x^4 - 3*x^2 + 1)^n, x)
```

Sympy [F]

$$\int \left(1-3x^2+x^4\right)^n \, dx = \int \left(x^4-3x^2+1\right)^n \, dx$$

input
```
integrate((x**4-3*x**2+1)**n,x)
```

output
```
Integral((x**4 - 3*x**2 + 1)**n, x)
```

Maxima [F]

$$\int \left(1-3x^2+x^4\right)^n \, dx = \int \left(x^4-3\,x^2+1\right)^n \, dx$$

input
```
integrate((x^4-3*x^2+1)^n,x, algorithm="maxima")
```

output
```
integrate((x^4 - 3*x^2 + 1)^n, x)
```

Giac [F]

$$\int \left(1-3x^2+x^4\right)^n \, dx = \int \left(x^4-3\,x^2+1\right)^n \, dx$$

input
```
integrate((x^4-3*x^2+1)^n,x, algorithm="giac")
```

output
```
integrate((x^4 - 3*x^2 + 1)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-3x^2+x^4\right)^n\,dx=\int \left(x^4-3\,x^2+1\right)^n\,dx$$

input
```
int((x^4 - 3*x^2 + 1)^n,x)
```

output
```
int((x^4 - 3*x^2 + 1)^n, x)
```

Reduce [F]

$$\int \left(1-3x^2+x^4\right)^n\,dx$$
$$=\frac{\left(x^4-3x^2+1\right)^n x+16\left(\int \frac{\left(x^4-3x^2+1\right)^n}{4n\,x^4+x^4-12n\,x^2-3x^2+4n+1}dx\right)n^2+4\left(\int \frac{\left(x^4-3x^2+1\right)^n}{4n\,x^4+x^4-12n\,x^2-3x^2+4n+1}dx\right)n-24\left(\int \frac{}{4n}\right.}{4n+1}$$

input
```
int((x^4-3*x^2+1)^n,x)
```

output
```
((x**4 - 3*x**2 + 1)**n*x + 16*int((x**4 - 3*x**2 + 1)**n/(4*n*x**4 - 12*n
*x**2 + 4*n + x**4 - 3*x**2 + 1),x)*n**2 + 4*int((x**4 - 3*x**2 + 1)**n/(4
*n*x**4 - 12*n*x**2 + 4*n + x**4 - 3*x**2 + 1),x)*n - 24*int(((x**4 - 3*x*
*2 + 1)**n*x**2)/(4*n*x**4 - 12*n*x**2 + 4*n + x**4 - 3*x**2 + 1),x)*n**2
- 6*int(((x**4 - 3*x**2 + 1)**n*x**2)/(4*n*x**4 - 12*n*x**2 + 4*n + x**4 -
 3*x**2 + 1),x)*n)/(4*n + 1)
```

3.271 $\int \frac{(1+e^{-x})x}{-1+e^x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = e^{-x}+e^{-x}x-x^2+2x\log(1-e^x)+2\,\text{PolyLog}(2,e^x)$$

output
```
exp(-x)+x/exp(x)-x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = e^{-x}(1+x-e^x x^2)+2x\log(1-e^x)+2\,\text{PolyLog}(2,e^x)$$

input
```
Integrate[((1 + E^(-x))*x)/(-1 + E^x),x]
```

output
```
(1 + x - E^x*x^2)/E^x + 2*x*Log[1 - E^x] + 2*PolyLog[2, E^x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2684, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e^{-x}+1)\,x}{e^x-1}\,dx$$

$$\Big\downarrow \; 2684$$

$$\int \left(\frac{2x}{e^x-1}-e^{-x}x\right)dx$$

$$\Big\downarrow \; 2009$$

$$2\,\text{PolyLog}\left(2,e^x\right)-x^2+e^{-x}x+e^{-x}+2x\log\left(1-e^x\right)$$

input
```
Int[((1 + E^(-x))*x)/(-1 + E^x),x]
```

output
```
E^(-x) + x/E^x - x^2 + 2*x*Log[1 - E^x] + 2*PolyLog[2, E^x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2684
```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
risch	$(1+x)\,\mathrm{e}^{-x}-x^{2}+2x\ln\left(1-\mathrm{e}^{x}\right)+2\,\mathrm{polylog}\left(2,\mathrm{e}^{x}\right)$	31
default	$x\,\mathrm{e}^{-x}+2x\ln\left(1-\mathrm{e}^{x}\right)-x^{2}+\mathrm{e}^{-x}+2\,\mathrm{polylog}\left(2,\mathrm{e}^{x}\right)$	33

input
```
int((1+exp(-x))*x/(-1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
(1+x)/exp(x)-x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = -\left(x^2e^x-2\,xe^x\log\left(-e^x+1\right)-2\,\mathrm{Li}_2(e^x)\,e^x-x-1\right)e^{(-x)}$$

input
```
integrate((1+exp(-x))*x/(-1+exp(x)),x, algorithm="fricas")
```

output
```
-(x^2*e^x - 2*x*e^x*log(-e^x + 1) - 2*dilog(e^x)*e^x - x - 1)*e^(-x)
```

Sympy [F]

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = \int \frac{x(e^x+1)\,e^{-x}}{e^x-1}\,dx$$

input
```
integrate((1+exp(-x))*x/(-1+exp(x)),x)
```

output
```
Integral(x*(exp(x) + 1)*exp(-x)/(exp(x) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = -x^2 + (x+1)e^{(-x)} + 2\,x\log\left(-e^x + 1\right) + 2\,\mathrm{Li}_2(e^x)$$

input
```
integrate((1+exp(-x))*x/(-1+exp(x)),x, algorithm="maxima")
```

output
```
-x^2 + (x + 1)*e^(-x) + 2*x*log(-e^x + 1) + 2*dilog(e^x)
```

Giac [F]

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = \int \frac{x\left(e^{(-x)}+1\right)}{e^x-1}\,dx$$

input
```
integrate((1+exp(-x))*x/(-1+exp(x)),x, algorithm="giac")
```

output
```
integrate(x*(e^(-x) + 1)/(e^x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = \int \frac{x\,(\mathrm{e}^{-x}+1)}{\mathrm{e}^x-1}\,dx$$

input
```
int((x*(exp(-x) + 1))/(exp(x) - 1),x)
```

output
```
int((x*(exp(-x) + 1))/(exp(x) - 1), x)
```

Reduce [F]

$$\int \frac{(1+e^{-x})\,x}{-1+e^x}\,dx = \frac{2e^x\left(\int \frac{x}{e^{2x}-e^x}dx\right)-x-1}{e^x}$$

input
```
int((1+exp(-x))*x/(-1+exp(x)),x)
```

output
```
(2*e**x*int(x/(e**(2*x) - e**x),x) - x - 1)/e**x
```

3.272 $\int e^{-x}x^4\sin(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int e^{-x}x^4\sin(x)\,dx = \frac{1}{2}e^{-x}\left(-\left(\left(-6+x^2(6+x(4+x))\right)\cos(x)\right)+\left(6+x\left(12+6x-x^3\right)\right)\sin(x)\right)$$

output
```
1/2*(-(-6+x^2*(6+x*(4+x)))*cos(x)+(6+x*(-x^3+6*x+12))*sin(x))/exp(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int e^{-x}x^4\sin(x)\,dx = \frac{1}{2}e^{-x}\left(-\left(\left(-6+6x^2+4x^3+x^4\right)\cos(x)\right)+\left(6+12x+6x^2-x^4\right)\sin(x)\right)$$

input
```
Integrate[(x^4*Sin[x])/E^x,x]
```

output
```
(-((-6 + 6*x^2 + 4*x^3 + x^4)*Cos[x]) + (6 + 12*x + 6*x^2 - x^4)*Sin[x])/(
2*E^x)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. $2(44) = 88$.

Time = 0.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4968, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x} x^4 \sin(x) \, dx$$

$$\Big\downarrow \text{4968}$$

$$-4 \int -\frac{1}{2} x^3 \left(e^{-x} \cos(x) + e^{-x} \sin(x)\right) dx - \frac{1}{2} e^{-x} x^4 \sin(x) - \frac{1}{2} e^{-x} x^4 \cos(x)$$

$$\Big\downarrow \text{27}$$

$$2 \int x^3 \left(e^{-x} \cos(x) + e^{-x} \sin(x)\right) dx - \frac{1}{2} e^{-x} x^4 \sin(x) - \frac{1}{2} e^{-x} x^4 \cos(x)$$

$$\Big\downarrow \text{2010}$$

$$2 \int \left(e^{-x} \cos(x) x^3 + e^{-x} \sin(x) x^3\right) dx - \frac{1}{2} e^{-x} x^4 \sin(x) - \frac{1}{2} e^{-x} x^4 \cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2} e^{-x} x^4 \sin(x) - \frac{1}{2} e^{-x} x^4 \cos(x) + 2\left(-e^{-x} x^3 \cos(x) + \frac{3}{2} e^{-x} x^2 \sin(x) - \frac{3}{2} e^{-x} x^2 \cos(x) + 3 e^{-x} x \sin(x) + \frac{3}{2} e^{-x} \sin(x) + \frac{3}{2} e^{-x} \cos(x)\right)$$

input
```
Int[(x^4*Sin[x])/E^x,x]
```

output
```
-1/2*(x^4*Cos[x])/E^x - (x^4*Sin[x])/(2*E^x) + 2*((3*Cos[x])/(2*E^x) - (3*
x^2*Cos[x])/(2*E^x) - (x^3*Cos[x])/E^x + (3*Sin[x])/(2*E^x) + (3*x*Sin[x])
/E^x + (3*x^2*Sin[x])/(2*E^x))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4968
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m   u, x] - Simp[f*m   Int[(f*x)^(m - 1)*u, x], x]] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result
parallelrisch	$-\frac{\left(\left(x^{4}+4x^{3}+6x^{2}-6\right)\cos\left(x\right)+\sin\left(x\right)\left(x^{4}-6x^{2}-12x-6\right)\right){\mathrm e}^{-x}}{2}$
default	$\left(-\frac{1}{2}x^{4}-2x^{3}-3x^{2}+3\right){\mathrm e}^{-x}\cos\left(x\right)+\left(-\frac{1}{2}x^{4}+3x^{2}+6x+3\right){\mathrm e}^{-x}\sin\left(x\right)$
risch	$\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^{4}+2ix^{3}+2x^{3}+6ix^{2}+6ix-6x-6\right){\mathrm e}^{\left(-1+i\right)x}+\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^{4}-2ix^{3}+2x^{3}\right.$
orering	$-\frac{\left(x^{5}-8x^{3}-18x^{2}-6x+12\right)\sin\left(x\right){\mathrm e}^{-x}}{x}-\frac{\left(x^{4}+4x^{3}+6x^{2}-6\right)\left(4x^{3}\sin\left(x\right){\mathrm e}^{-x}+x^{4}\cos\left(x\right){\mathrm e}^{-x}-x^{4}\sin\left(x\right){\mathrm e}^{-x}\right)}{2x^{4}}$
norman	$\frac{\left(3-3x^{2}-2x^{3}-\frac{x^{4}}{2}-3\tan\left(\frac{x}{2}\right)^{2}+12x\tan\left(\frac{x}{2}\right)+6x^{2}\tan\left(\frac{x}{2}\right)+3x^{2}\tan\left(\frac{x}{2}\right)^{2}+2x^{3}\tan\left(\frac{x}{2}\right)^{2}-x^{4}\tan\left(\frac{x}{2}\right)+\frac{x^{4}\tan\left(\frac{x}{2}\right)^{2}}{2}+6\tan\left(\frac{x}{2}\right)\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}$

input
```
int(x^4*sin(x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((x^4+4*x^3+6*x^2-6)*cos(x)+sin(x)*(x^4-6*x^2-12*x-6))*exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int e^{-x}x^4\sin(x)\,dx = -\frac{1}{2}\left(x^4+4\,x^3+6\,x^2-6\right)\cos\left(x\right)e^{(-x)} - \frac{1}{2}\left(x^4-6\,x^2-12\,x-6\right)e^{(-x)}\sin\left(x\right)$$

input
```
integrate(x^4*sin(x)/exp(x),x, algorithm="fricas")
```

output
```
-1/2*(x^4 + 4*x^3 + 6*x^2 - 6)*cos(x)*e^(-x) - 1/2*(x^4 - 6*x^2 - 12*x - 6
)*e^(-x)*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.

Time = 0.97 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int e^{-x}x^4\sin(x)\,dx = -\frac{x^4e^{-x}\sin\left(x\right)}{2} - \frac{x^4e^{-x}\cos\left(x\right)}{2} - 2x^3e^{-x}\cos\left(x\right) + 3x^2e^{-x}\sin\left(x\right) - 3x^2e^{-x}\cos\left(x\right) + 6xe^{-x}\sin\left(x\right) + 3e^{-x}\sin\left(x\right) + 3e^{-x}\cos\left(x\right)$$

input
```
integrate(x**4*sin(x)/exp(x),x)
```

output
```
-x**4*exp(-x)*sin(x)/2 - x**4*exp(-x)*cos(x)/2 - 2*x**3*exp(-x)*cos(x) + 3
*x**2*exp(-x)*sin(x) - 3*x**2*exp(-x)*cos(x) + 6*x*exp(-x)*sin(x) + 3*exp(
-x)*sin(x) + 3*exp(-x)*cos(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int e^{-x}x^4\sin(x)\,dx$$
$$= -\frac{1}{2}\left(\left(x^4+4\,x^3+6\,x^2-6\right)\cos\left(x\right)+\left(x^4-6\,x^2-12\,x-6\right)\sin\left(x\right)\right)e^{(-x)}$$

input
```
integrate(x^4*sin(x)/exp(x),x, algorithm="maxima")
```

output
```
-1/2*((x^4 + 4*x^3 + 6*x^2 - 6)*cos(x) + (x^4 - 6*x^2 - 12*x - 6)*sin(x))*
e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int e^{-x}x^4\sin(x)\,dx$$
$$= -\frac{1}{2}\left(\left(x^4+4\,x^3+6\,x^2-6\right)\cos\left(x\right)+\left(x^4-6\,x^2-12\,x-6\right)\sin\left(x\right)\right)e^{(-x)}$$

input
```
integrate(x^4*sin(x)/exp(x),x, algorithm="giac")
```

output
```
-1/2*((x^4 + 4*x^3 + 6*x^2 - 6)*cos(x) + (x^4 - 6*x^2 - 12*x - 6)*sin(x))*
e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int e^{-x}x^4\sin(x)\,dx$$
$$= \frac{\mathrm{e}^{-x}\left(6\,\cos\left(x\right)+6\,\sin\left(x\right)-6\,x^2\,\cos\left(x\right)-4\,x^3\,\cos\left(x\right)-x^4\,\cos\left(x\right)+6\,x^2\,\sin\left(x\right)-x^4\,\sin\left(x\right)+12\,x\,\sin}{2}$$

input
```
int(x^4*exp(-x)*sin(x),x)
```

output
```
(exp(-x)*(6*cos(x) + 6*sin(x) - 6*x^2*cos(x) - 4*x^3*cos(x) - x^4*cos(x) +
 6*x^2*sin(x) - x^4*sin(x) + 12*x*sin(x)))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int e^{-x}x^4\sin(x)\,dx$$
$$= \frac{-\cos(x)\,x^4 - 4\cos(x)\,x^3 - 6\cos(x)\,x^2 + 6\cos(x) - \sin(x)\,x^4 + 6\sin(x)\,x^2 + 12\sin(x)\,x + 6\sin(x)}{2e^x}$$

input
```
int(x^4*sin(x)/exp(x),x)
```

output
```
( - cos(x)*x**4 - 4*cos(x)*x**3 - 6*cos(x)*x**2 + 6*cos(x) - sin(x)*x**4 +
 6*sin(x)*x**2 + 12*sin(x)*x + 6*sin(x))/(2*e**x)
```

3.273 $\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 56

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \sqrt{-4+x^2}+\frac{10}{3}\left(-4+x^2\right)^{3/2}+3\left(-4+x^2\right)^{5/2}+\left(-4+x^2\right)^{7/2}+\frac{1}{9}\left(-4+x^2\right)^{9/2}$$

output
```
(x^2-4)^(1/2)+10/3*(x^2-4)^(3/2)+3*(x^2-4)^(5/2)+(x^2-4)^(7/2)+1/9*(x^2-4)
^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{1}{9}\sqrt{-4+x^2}\left(1-10x^2+15x^4-7x^6+x^8\right)$$

input
```
Integrate[(-3*(-3*x + x^3) + (-3*x + x^3)^3)/Sqrt[-4 + x^2],x]
```

output
```
(Sqrt[-4 + x^2]*(1 - 10*x^2 + 15*x^4 - 7*x^6 + x^8))/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2342, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-3x)^3-3(x^3-3x)}{\sqrt{x^2-4}}dx$$

$$\Big\downarrow \text{2342}$$

$$\int \frac{x(x^8-9x^6+27x^4-30x^2+9)}{\sqrt{x^2-4}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{x^8-9x^6+27x^4-30x^2+9}{\sqrt{x^2-4}}dx^2$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{2}\int \left((x^2-4)^{7/2}+7(x^2-4)^{5/2}+15(x^2-4)^{3/2}+10\sqrt{x^2-4}+\frac{1}{\sqrt{x^2-4}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{9}(x^2-4)^{9/2}+2(x^2-4)^{7/2}+6(x^2-4)^{5/2}+\frac{20}{3}(x^2-4)^{3/2}+2\sqrt{x^2-4}\right)$$

input
```
Int[(-3*(-3*x + x^3) + (-3*x + x^3)^3)/Sqrt[-4 + x^2],x]
```

output
```
(2*Sqrt[-4 + x^2] + (20*(-4 + x^2)^(3/2))/3 + 6*(-4 + x^2)^(5/2) + 2*(-4 +
 x^2)^(7/2) + (2*(-4 + x^2)^(9/2))/9)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(x^8-7x^6+15x^4-10x^2+1)\sqrt{x^2-4}}{9}$
pseudoelliptic	$\frac{(x^8-7x^6+15x^4-10x^2+1)\sqrt{x^2-4}}{9}$
trager	$\left(\frac{1}{9}x^8-\frac{7}{9}x^6+\frac{5}{3}x^4-\frac{10}{9}x^2+\frac{1}{9}\right)\sqrt{x^2-4}$
gosper	$\frac{(x^8-7x^6+15x^4-10x^2+1)(-2+x)(2+x)}{9\sqrt{x^2-4}}$
default	$\frac{x^8\sqrt{x^2-4}}{9}-\frac{7x^6\sqrt{x^2-4}}{9}+\frac{5x^4\sqrt{x^2-4}}{3}-\frac{10x^2\sqrt{x^2-4}}{9}+\frac{\sqrt{x^2-4}}{9}$
orering	$\frac{(x^8-7x^6+15x^4-10x^2+1)(-2+x)(2+x)\left(-3x^3+9x+(x^3-3x)^3\right)}{9x(x^2-3)(x^6-6x^4+9x^2-3)\sqrt{x^2-4}}$
meijerg	$-\frac{120\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(x^2+8)\sqrt{1-\frac{x^2}{4}}}{6}\right)}{\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}}-\frac{9\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1-\frac{x^2}{4}}\right)}{\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{4}\right)}}-\frac{256\ldots}{}$

input
```
int((-3*x^3+9*x+(x^3-3*x)^3)/(x^2-4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(x^8-7*x^6+15*x^4-10*x^2+1)*(x^2-4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{1}{9}\left(x^8-7\,x^6+15\,x^4-10\,x^2+1\right)\sqrt{x^2-4}$$

input
```
integrate((-3*x^3+9*x+(x^3-3*x)^3)/(x^2-4)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(x^8 - 7*x^6 + 15*x^4 - 10*x^2 + 1)*sqrt(x^2 - 4)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{x^8\sqrt{x^2-4}}{9}-\frac{7x^6\sqrt{x^2-4}}{9}+\frac{5x^4\sqrt{x^2-4}}{3}-\frac{10x^2\sqrt{x^2-4}}{9}+\frac{\sqrt{x^2-4}}{9}$$

input
```
integrate((-3*x**3+9*x+(x**3-3*x)**3)/(x**2-4)**(1/2),x)
```

output
```
x**8*sqrt(x**2 - 4)/9 - 7*x**6*sqrt(x**2 - 4)/9 + 5*x**4*sqrt(x**2 - 4)/3
- 10*x**2*sqrt(x**2 - 4)/9 + sqrt(x**2 - 4)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{1}{9}\sqrt{x^2-4}x^8 - \frac{7}{9}\sqrt{x^2-4}x^6 + \frac{5}{3}\sqrt{x^2-4}x^4 - \frac{10}{9}\sqrt{x^2-4}x^2 + \frac{1}{9}\sqrt{x^2-4}$$

input
```
integrate((-3*x^3+9*x+(x^3-3*x)^3)/(x^2-4)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(x^2 - 4)*x^8 - 7/9*sqrt(x^2 - 4)*x^6 + 5/3*sqrt(x^2 - 4)*x^4 - 10
/9*sqrt(x^2 - 4)*x^2 + 1/9*sqrt(x^2 - 4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{1}{9}\left(x^2-4\right)^{\frac{9}{2}} + \left(x^2-4\right)^{\frac{7}{2}} + 3\left(x^2-4\right)^{\frac{5}{2}} + \frac{10}{3}\left(x^2-4\right)^{\frac{3}{2}} + \sqrt{x^2-4}$$

input
```
integrate((-3*x^3+9*x+(x^3-3*x)^3)/(x^2-4)^(1/2),x, algorithm="giac")
```

output
```
1/9*(x^2 - 4)^(9/2) + (x^2 - 4)^(7/2) + 3*(x^2 - 4)^(5/2) + 10/3*(x^2 - 4)
^(3/2) + sqrt(x^2 - 4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \sqrt{x^2-4}\left(\frac{x^8}{9}-\frac{7\,x^6}{9}+\frac{5\,x^4}{3}-\frac{10\,x^2}{9}+\frac{1}{9}\right)$$

input
```
int(-((3*x - x^3)^3 - 9*x + 3*x^3)/(x^2 - 4)^(1/2),x)
```

output
```
(x^2 - 4)^(1/2)*((5*x^4)/3 - (10*x^2)/9 - (7*x^6)/9 + x^8/9 + 1/9)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.50

$$\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx = \frac{\sqrt{x^2-4}\,(x^8-7x^6+15x^4-10x^2+1)}{9}$$

input
```
int((-3*x^3+9*x+(x^3-3*x)^3)/(x^2-4)^(1/2),x)
```

output
```
(sqrt(x**2 - 4)*(x**8 - 7*x**6 + 15*x**4 - 10*x**2 + 1))/9
```

3.274 $\int \frac{1}{(1+x^2)^3} dx$

Optimal result

Integrand size = 7, antiderivative size = 31

$$\int \frac{1}{(1+x^2)^3} dx = \frac{x}{4(1+x^2)^2} + \frac{3x}{8(1+x^2)} + \frac{3\arctan(x)}{8}$$

output
```
1/4*x/(x^2+1)^2+3*x/(8*x^2+8)+3/8*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(1+x^2)^3} dx = \frac{1}{8}\left(\frac{x(5+3x^2)}{(1+x^2)^2} + 3\arctan(x)\right)$$

input
```
Integrate[(1 + x^2)^(-3),x]
```

output
```
((x*(5 + 3*x^2))/(1 + x^2)^2 + 3*ArcTan[x])/8
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3}{4}\int \frac{1}{(x^2+1)^2}dx+\frac{x}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2\,(x^2+1)}\right)+\frac{x}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{4}\left(\frac{\arctan(x)}{2}+\frac{x}{2\,(x^2+1)}\right)+\frac{x}{4\,(x^2+1)^2}$$

input
```
Int[(1 + x^2)^(-3),x]
```

output
```
x/(4*(1 + x^2)^2) + (3*(x/(2*(1 + x^2)) + ArcTan[x]/2))/4
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
meijerg	$\frac{x(3x^2+5)}{8(x^2+1)^2}+\frac{3\arctan(x)}{8}$	23
risch	$\frac{\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^2}+\frac{3\arctan(x)}{8}$	23
default	$\frac{x}{4(x^2+1)^2}+\frac{3x}{8(x^2+1)}+\frac{3\arctan(x)}{8}$	26
parallelrisch	$-\frac{3i\ln(x-i)x^4-3i\ln(i+x)x^4+6i\ln(x-i)x^2-6i\ln(i+x)x^2-6x^3+3i\ln(x-i)-3i\ln(i+x)-10x}{16(x^2+1)^2}$	79

input
```
int(1/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(3*x^2+5)/(x^2+1)^2+3/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+x^2)^3}\,dx=\frac{3\,x^3+3\,(x^4+2\,x^2+1)\arctan{(x)}+5\,x}{8\,(x^4+2\,x^2+1)}$$

input
```
integrate(1/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/8*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{(1+x^2)^3}\,dx = \frac{3x^3+5x}{8x^4+16x^2+8} + \frac{3\,\mathrm{atan}\,(x)}{8}$$

input
```
integrate(1/(x**2+1)**3,x)
```

output
```
(3*x**3 + 5*x)/(8*x**4 + 16*x**2 + 8) + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1+x^2)^3}\,dx = \frac{3\,x^3+5\,x}{8\,(x^4+2\,x^2+1)} + \frac{3}{8}\arctan(x)$$

input
```
integrate(1/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/8*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) + 3/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1+x^2)^3}\,dx = \frac{3\,x^3+5\,x}{8\,(x^2+1)^2} + \frac{3}{8}\arctan(x)$$

input
```
integrate(1/(x^2+1)^3,x, algorithm="giac")
```

output
```
1/8*(3*x^3 + 5*x)/(x^2 + 1)^2 + 3/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{(1+x^2)^3} \, dx = \frac{3 \operatorname{atan}(x)}{8} + x \left(\frac{1}{4 (x^2+1)^2} + \frac{3}{8 x^2+8} \right)$$

input
```
int(1/(x^2 + 1)^3,x)
```

output
```
(3*atan(x))/8 + x*(1/(4*(x^2 + 1)^2) + 3/(8*x^2 + 8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{(1+x^2)^3} \, dx = \frac{3atan(x) x^4 + 6atan(x) x^2 + 3atan(x) + 3x^3 + 5x}{8x^4 + 16x^2 + 8}$$

input
```
int(1/(x^2+1)^3,x)
```

output
```
(3*atan(x)*x**4 + 6*atan(x)*x**2 + 3*atan(x) + 3*x**3 + 5*x)/(8*(x**4 + 2*
x**2 + 1))
```

3.275 $\int \log\left(\sqrt{3}+\tan(x)\right)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 108

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx = -\frac{1}{2}i\left(\left(\log\left(\frac{i-\tan(x)}{i+\sqrt{3}}\right)-\log\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)\right)\log\left(\sqrt{3}+\tan(x)\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{3}+\tan(x)}{-i+\sqrt{3}}\right)+\operatorname{PolyLog}\left(2,\frac{\sqrt{3}+\tan(x)}{i+\sqrt{3}}\right)\right)$$

output
```
-1/2*I*((ln((I-tan(x))/(3^(1/2)+I))-ln((I+tan(x))/(I-3^(1/2))))*ln(3^(1/2)
+tan(x))-polylog(2,(3^(1/2)+tan(x))/(-I+3^(1/2)))+polylog(2,(3^(1/2)+tan(x
))/(3^(1/2)+I)))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \log \left(\sqrt{3}+\tan(x)\right) \, dx = -\frac{1}{2} i \log \left(\frac{i-\tan(x)}{i+\sqrt{3}}\right) \log \left(\sqrt{3}+\tan(x)\right) + \frac{1}{2} i \log \left(\frac{i+\tan(x)}{i-\sqrt{3}}\right) \log \left(\sqrt{3}+\tan(x)\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2,-\frac{\sqrt{3}+\tan(x)}{i-\sqrt{3}}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2,\frac{\sqrt{3}+\tan(x)}{i+\sqrt{3}}\right)$$

input
```
Integrate[Log[Sqrt[3] + Tan[x]],x]
```

output
```
(-1/2*I)*Log[(I - Tan[x])/(I + Sqrt[3])]*Log[Sqrt[3] + Tan[x]] + (I/2)*Log
[(I + Tan[x])/(I - Sqrt[3])]*Log[Sqrt[3] + Tan[x]] + (I/2)*PolyLog[2, -((S
qrt[3] + Tan[x])/(I - Sqrt[3]))] - (I/2)*PolyLog[2, (Sqrt[3] + Tan[x])/(I
+ Sqrt[3])]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(\tan(x)+\sqrt{3}\right) \, dx$$

$$\Big\downarrow \text{3028}$$

$$x \log \left(\tan(x)+\sqrt{3}\right) - \int \frac{x \sec^2(x)}{\tan(x)+\sqrt{3}} dx$$

$$\Big\downarrow \text{7299}$$

$$x \log \left(\tan(x)+\sqrt{3}\right) - \int \frac{x \sec^2(x)}{\tan(x)+\sqrt{3}} dx$$

input
```
Int[Log[Sqrt[3] + Tan[x]],x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{i \ln\left(\sqrt{3}+\tan(x)\right) \ln\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i \ln\left(\sqrt{3}+\tan(x)\right) \ln\left(\frac{-i-\tan(x)}{-i+\sqrt{3}}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-i-\tan(x)}{-i+\sqrt{3}}\right)}{2}$
default	$-\frac{i \ln\left(\sqrt{3}+\tan(x)\right) \ln\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i \ln\left(\sqrt{3}+\tan(x)\right) \ln\left(\frac{-i-\tan(x)}{-i+\sqrt{3}}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{-i-\tan(x)}{-i+\sqrt{3}}\right)}{2}$
risch	$\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{e^{2ix}+1}\right)\mathrm{csgn}\left(\frac{i\left(\sqrt{3}\,e^{2ix}-ie^{2ix}+\sqrt{3}+i\right)}{e^{2ix}+1}\right)^{2} x}{2}+x\ln\left(\sqrt{3}\,e^{2ix}-ie^{2ix}+\sqrt{3}+i\right)+i\ln\left(e^{ix}\right)\ln$

input
```
int(ln(3^(1/2)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*ln(3^(1/2)+tan(x))*ln((I-tan(x))/(3^(1/2)+I))+1/2*I*ln(3^(1/2)+tan(
x))*ln((-I-tan(x))/(-I+3^(1/2)))-1/2*I*dilog((I-tan(x))/(3^(1/2)+I))+1/2*I
*dilog((-I-tan(x))/(-I+3^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.43

$$\begin{aligned}
&\int \log\left(\sqrt{3}+\tan(x)\right)\,dx \\
&= x\log\left(\sqrt{3}+\tan(x)\right) \\
&\quad -\frac{1}{2}x\log\left(\frac{\left(i\sqrt{3}+1\right)\tan(x)^2+2\left(\sqrt{3}+i\right)\tan(x)-i\sqrt{3}+3}{2\left(\tan(x)^2+1\right)}\right) \\
&\quad -\frac{1}{2}x\log\left(\frac{\left(-i\sqrt{3}+1\right)\tan(x)^2+2\left(\sqrt{3}-i\right)\tan(x)+i\sqrt{3}+3}{2\left(\tan(x)^2+1\right)}\right) \\
&\quad +\frac{1}{2}x\log\left(-\frac{2\left(i\tan(x)-1\right)}{\tan(x)^2+1}\right)+\frac{1}{2}x\log\left(-\frac{2\left(-i\tan(x)-1\right)}{\tan(x)^2+1}\right) \\
&\quad +\frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{\left(i\sqrt{3}+1\right)\tan(x)^2+2\left(\sqrt{3}+i\right)\tan(x)-i\sqrt{3}+3}{2\left(\tan(x)^2+1\right)}+1\right) \\
&\quad -\frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{\left(-i\sqrt{3}+1\right)\tan(x)^2+2\left(\sqrt{3}-i\right)\tan(x)+i\sqrt{3}+3}{2\left(\tan(x)^2+1\right)}+1\right) \\
&\quad +\frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2\left(i\tan(x)-1\right)}{\tan(x)^2+1}+1\right)-\frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2\left(-i\tan(x)-1\right)}{\tan(x)^2+1}+1\right)
\end{aligned}$$

input
```
integrate(log(3^(1/2)+tan(x)),x, algorithm="fricas")
```

output
```
x*log(sqrt(3) + tan(x)) - 1/2*x*log(1/2*((I*sqrt(3) + 1)*tan(x)^2 + 2*(sqr
t(3) + I)*tan(x) - I*sqrt(3) + 3)/(tan(x)^2 + 1)) - 1/2*x*log(1/2*((-I*sqr
t(3) + 1)*tan(x)^2 + 2*(sqrt(3) - I)*tan(x) + I*sqrt(3) + 3)/(tan(x)^2 + 1
)) + 1/2*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x)
 - 1)/(tan(x)^2 + 1)) + 1/4*I*dilog(-1/2*((I*sqrt(3) + 1)*tan(x)^2 + 2*(sq
rt(3) + I)*tan(x) - I*sqrt(3) + 3)/(tan(x)^2 + 1) + 1) - 1/4*I*dilog(-1/2*
((-I*sqrt(3) + 1)*tan(x)^2 + 2*(sqrt(3) - I)*tan(x) + I*sqrt(3) + 3)/(tan(
x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/4*I*
dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx=\int \log\left(\tan\left(x\right)+\sqrt{3}\right)\,dx$$

input
```
integrate(ln(3**(1/2)+tan(x)),x)
```

output
```
Integral(log(tan(x) + sqrt(3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx=\frac{1}{2}\arctan\left(\frac{1}{4}\sqrt{3}+\frac{1}{4}\tan\left(x\right),\frac{1}{4}\sqrt{3}\tan\left(x\right)+\frac{3}{4}\right)\log\left(\tan\left(x\right)^{2}+1\right)-\frac{1}{2}x\log\left(\frac{1}{4}\tan\left(x\right)^{2}+\frac{1}{2}\sqrt{3}\tan\left(x\right)+\frac{3}{4}\right)+x\log\left(\sqrt{3}+\tan\left(x\right)\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{\left(\sqrt{3}+i\right)\tan\left(x\right)-i\sqrt{3}+1}{2i\sqrt{3}+2}\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{\left(\sqrt{3}-i\right)\tan\left(x\right)+i\sqrt{3}+1}{2i\sqrt{3}-2}\right)$$

input
```
integrate(log(3^(1/2)+tan(x)),x, algorithm="maxima")
```

output
```
1/2*arctan2(1/4*sqrt(3) + 1/4*tan(x), 1/4*sqrt(3)*tan(x) + 3/4)*log(tan(x)
^2 + 1) - 1/2*x*log(1/4*tan(x)^2 + 1/2*sqrt(3)*tan(x) + 3/4) + x*log(sqrt(
3) + tan(x)) + 1/2*I*dilog(-((sqrt(3) + I)*tan(x) - I*sqrt(3) + 1)/(2*I*sq
rt(3) + 2)) - 1/2*I*dilog(((sqrt(3) - I)*tan(x) + I*sqrt(3) + 1)/(2*I*sqrt
(3) - 2))
```

Giac [F]

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx=\int \log\left(\sqrt{3}+\tan\left(x\right)\right)\,dx$$

input
```
integrate(log(3^(1/2)+tan(x)),x, algorithm="giac")
```

output
```
integrate(log(sqrt(3) + tan(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx=\int \ln\left(\tan(x)+\sqrt{3}\right)\,dx$$

input
```
int(log(tan(x) + 3^(1/2)),x)
```

output
```
int(log(tan(x) + 3^(1/2)), x)
```

Reduce [F]

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx=\int \log\left(\sqrt{3}+\tan\left(x\right)\right)dx$$

input
```
int(log(3^(1/2)+tan(x)),x)
```

output
```
int(log(sqrt(3) + tan(x)),x)
```

3.276 $\int \sqrt{(\cos(20x) + 3\cos(21x) + \cos(22x))^2 + (\sin(20x)$

Optimal result

Integrand size = 39, antiderivative size = 32

$$\int \sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)+3\sin(21x)+\sin(22x))^2}\,dx = \frac{2(3+2\cos(x))\left(3\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\sin(x)\right)}{\sqrt{(3+2\cos(x))^2}}$$

output
```
2*(3+2*cos(x))*(3*arctan(tan(1/2*x))+sin(x))/((3+2*cos(x))^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)+3\sin(21x)+\sin(22x))^2}\,dx = \frac{\sqrt{(3+2\cos(x))^2}(3x+2\sin(x))}{3+2\cos(x)}$$

input
```
Integrate[Sqrt[(Cos[20*x] + 3*Cos[21*x] + Cos[22*x])^2 + (Sin[20*x] + 3*Si
n[21*x] + Sin[22*x])^2],x]
```

output
```
(Sqrt[(3 + 2*Cos[x])^2]*(3*x + 2*Sin[x]))/(3 + 2*Cos[x])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(32) = 64.

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4902, 2058, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(\sin(20x)+3\sin(21x)+\sin(22x))^2+(\cos(20x)+3\cos(21x)+\cos(22x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{(\sin(20x)+3\sin(21x)+\sin(22x))^2+(\cos(20x)+3\cos(21x)+\cos(22x))^2}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{\sqrt{\frac{\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}}}{\tan^2\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2058}$$

$$\frac{2\left(\tan^2\left(\frac{x}{2}\right)+1\right)\sqrt{\frac{\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}}\int \frac{\tan^2\left(\frac{x}{2}\right)+5}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}d\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+5}$$

$$\Big\downarrow \text{298}$$

$$\frac{2\left(\tan^2\left(\frac{x}{2}\right)+1\right)\sqrt{\frac{\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}}\left(3\int \frac{1}{\tan^2\left(\frac{x}{2}\right)+1}d\tan\left(\frac{x}{2}\right)+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+5}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\tan^2\left(\frac{x}{2}\right)+1\right)\sqrt{\frac{\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}}\left(3\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+5}$$

input
```
Int[Sqrt[(Cos[20*x] + 3*Cos[21*x] + Cos[22*x])^2 + (Sin[20*x] + 3*Sin[21*x
] + Sin[22*x])^2],x]
```

output
```
(2*(1 + Tan[x/2]^2)*Sqrt[(5 + Tan[x/2]^2)^2/(1 + Tan[x/2]^2)^2]*(3*ArcTan[
Tan[x/2]] + (2*Tan[x/2])/(1 + Tan[x/2]^2)))/(5 + Tan[x/2]^2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] && !FalseQ[FunctionOfTrig[u, x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

method	result	size
default	$\operatorname{csgn}\left(3+2\cos\left(x\right)\right)\left(3x+2\sin\left(x\right)\right)$	17
risch	$\frac{3\sqrt{\left(\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1\right)^{2}\mathrm{e}^{-2ix}}\,\mathrm{e}^{ix}x}{\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1}-\frac{i\sqrt{\left(\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1\right)^{2}\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1}+\frac{i\sqrt{\left(\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1\right)^{2}\mathrm{e}^{-2ix}}}{\mathrm{e}^{2ix}+3\,\mathrm{e}^{ix}+1}$	141

input
```
int(((cos(20*x)+3*cos(21*x)+cos(22*x))^2+(sin(20*x)+3*sin(21*x)+sin(22*x))
^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(3+2*cos(x))*(3*x+2*sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.25

$$\int\sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)+3\sin(21x)+\sin(22x))^2}\,dx$$
$$=3\,x+2\,\sin\left(x\right)$$

input
```
integrate(((cos(20*x)+3*cos(21*x)+cos(22*x))^2+(sin(20*x)+3*sin(21*x)+sin(
22*x))^2)^(1/2),x, algorithm="fricas")
```

output
```
3*x + 2*sin(x)
```

Sympy [F]

$$\int \sqrt{(\cos(20x) + 3\cos(21x) + \cos(22x))^2 + (\sin(20x) + 3\sin(21x) + \sin(22x))^2}\, dx$$
$$= \int \sqrt{(\sin(20x) + 3\sin(21x) + \sin(22x))^2 + (\cos(20x) + 3\cos(21x) + \cos(22x))^2}\, dx$$

input
```
integrate(((cos(20*x)+3*cos(21*x)+cos(22*x))**2+(sin(20*x)+3*sin(21*x)+sin
(22*x))**2)**(1/2),x)
```

output
```
Integral(sqrt((sin(20*x) + 3*sin(21*x) + sin(22*x))**2 + (cos(20*x) + 3*co
s(21*x) + cos(22*x))**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.25

$$\int \sqrt{(\cos(20x) + 3\cos(21x) + \cos(22x))^2 + (\sin(20x) + 3\sin(21x) + \sin(22x))^2}\, dx$$
$$= 3x + 2\sin(x)$$

input
```
integrate(((cos(20*x)+3*cos(21*x)+cos(22*x))^2+(sin(20*x)+3*sin(21*x)+sin(
22*x))^2)^(1/2),x, algorithm="maxima")
```

output
```
3*x + 2*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{(\cos(20x) + 3\cos(21x) + \cos(22x))^2 + (\sin(20x) + 3\sin(21x) + \sin(22x))^2}\, dx$$
$$= -6\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor + 3x + \frac{4\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

input
```
integrate(((cos(20*x)+3*cos(21*x)+cos(22*x))^2+(sin(20*x)+3*sin(21*x)+sin(
22*x))^2)^(1/2),x, algorithm="giac")
```

output
```
-6*pi*floor(1/2*x/pi + 1/2) + 3*x + 4*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.25

$$\int \sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)+3\sin(21x)+\sin(22x))^2}\,dx$$
$$= 3\,x+2\,\sin\left(x\right)$$

input
```
int(((cos(20*x) + 3*cos(21*x) + cos(22*x))^2 + (sin(20*x) + 3*sin(21*x) +
sin(22*x))^2)^(1/2),x)
```

output
```
3*x + 2*sin(x)
```

Reduce [F]

$$\int \sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)+3\sin(21x)+\sin(22x))^2}\,dx$$
$$= \int \sqrt{\cos\left(22x\right)^2+6\cos\left(22x\right)\cos\left(21x\right)+2\cos\left(22x\right)\cos\left(20x\right)+9\cos\left(21x\right)^2+6\cos\left(21x\right)\cos\left(20x\right)+\mathrm{co}}$$

input
```
int(((cos(20*x)+3*cos(21*x)+cos(22*x))^2+(sin(20*x)+3*sin(21*x)+sin(22*x))
^2)^(1/2),x)
```

output
```
int(sqrt(cos(22*x)**2 + 6*cos(22*x)*cos(21*x) + 2*cos(22*x)*cos(20*x) + 9*
cos(21*x)**2 + 6*cos(21*x)*cos(20*x) + cos(20*x)**2 + sin(22*x)**2 + 6*sin
(22*x)*sin(21*x) + 2*sin(22*x)*sin(20*x) + 9*sin(21*x)**2 + 6*sin(21*x)*si
n(20*x) + sin(20*x)**2),x)
```

3.277 $\int \frac{e^{-2x}\sin(3x)}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = \frac{1}{2}i(\text{ExpIntegralEi}((-2-3i)x) - \text{ExpIntegralEi}((-2+3i)x))$$

output
```
1/2*I*(Ei((-2-3*I)*x)-Ei((-2+3*I)*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = -\frac{1}{2}i(-\text{ExpIntegralEi}((-2-3i)x) + \text{ExpIntegralEi}((-2+3i)x))$$

input
```
Integrate[Sin[3*x]/(E^(2*x)*x),x]
```

output
```
(-1/2*I)*(-ExpIntegralEi[(-2 - 3*I)*x] + ExpIntegralEi[(-2 + 3*I)*x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{e^{-2x}\sin(3x)}{x}dx$$

input
```
Int[Sin[3*x]/(E^(2*x)*x),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{i\,\mathrm{expIntegral}_1((2-3i)x)}{2} - \frac{i\,\mathrm{expIntegral}_1((2+3i)x)}{2}$	22

input
```
int(sin(3*x)/exp(2*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*Ei(1,(2-3*I)*x)-1/2*I*Ei(1,(2+3*I)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = -\frac{1}{2}i\,\mathrm{Ei}((3i-2)\ x) + \frac{1}{2}i\,\mathrm{Ei}(-(3i+2)\ x)$$

input
```
integrate(sin(3*x)/exp(2*x)/x,x, algorithm="fricas")
```

output
```
-1/2*I*Ei((3*I - 2)*x) + 1/2*I*Ei(-(3*I + 2)*x)
```

Sympy [F]

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = \int \frac{e^{-2x}\sin{(3x)}}{x}\,dx$$

input
```
integrate(sin(3*x)/exp(2*x)/x,x)
```

output
```
Integral(exp(-2*x)*sin(3*x)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = -\frac{1}{4}i\,\mathrm{Ei}((3i-2)\ x) + \frac{1}{4}i\,\mathrm{Ei}(-(3i+2)\ x) + \frac{1}{4}i\,\overline{\mathrm{Ei}\left((3i-2)\ x\right)} - \frac{1}{4}i\,\overline{\mathrm{Ei}\left(-(3i+2)\ x\right)}$$

input
```
integrate(sin(3*x)/exp(2*x)/x,x, algorithm="maxima")
```

output
```
-1/4*I*Ei((3*I - 2)*x) + 1/4*I*Ei(-(3*I + 2)*x) + 1/4*I*conjugate(Ei((3*I
- 2)*x)) - 1/4*I*conjugate(Ei(-(3*I + 2)*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = -\frac{1}{2}i\,\mathrm{Ei}((3i-2)\ x) + \frac{1}{2}i\,\mathrm{Ei}(-(3i+2)\ x)$$

input
```
integrate(sin(3*x)/exp(2*x)/x,x, algorithm="giac")
```

output
```
-1/2*I*Ei((3*I - 2)*x) + 1/2*I*Ei(-(3*I + 2)*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = \frac{\mathrm{ei}(x\,(-2-3\mathrm{i}))\ 1\mathrm{i}}{2} - \frac{\mathrm{ei}(x\,(-2+3\mathrm{i}))\ 1\mathrm{i}}{2}$$

input
```
int((sin(3*x)*exp(-2*x))/x,x)
```

output
```
(ei(x*(- 2 - 3i))*1i)/2 - (ei(x*(- 2 + 3i))*1i)/2
```

Reduce [F]

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx = \int \frac{\sin(3x)}{e^{2x}x}dx$$

input
```
int(sin(3*x)/exp(2*x)/x,x)
```

output
```
int(sin(3*x)/(e**(2*x)*x),x)
```

3.278 $\int (1-x)^{2/3} \sqrt[3]{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int (1-x)^{2/3} \sqrt[3]{x} \, dx = \frac{1}{6}(1-x)^{2/3} \sqrt[3]{x} - \frac{1}{2}(1-x)^{5/3} \sqrt[3]{x} + \frac{\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1 + \frac{\sqrt[3]{1-x}}{\sqrt[3]{x}}\right) + \frac{\log(x)}{18}$$

output
```
1/6*(1-x)^(2/3)*x^(1/3)-1/2*(1-x)^(5/3)*x^(1/3)-1/9*arctan(-1/3*3^(1/2)+2/
3*(1-x)^(1/3)*3^(1/2)/x^(1/3))*3^(1/2)+1/6*ln(1+(1-x)^(1/3)/x^(1/3))+1/18*
ln(x)
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int (1-x)^{2/3} \sqrt[3]{x} \, dx = \frac{1}{18}\left(3(1-x)^{2/3}\sqrt[3]{x}(-2+3x) + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{2\sqrt[3]{1-x} - \sqrt[3]{x}}\right) + 2\log\left(\sqrt[3]{1-x} + \sqrt[3]{x}\right) - \log\left(\right.\right.$$

input
```
Integrate[(1 - x)^(2/3)*x^(1/3),x]
```

output
```
(3*(1 - x)^(2/3)*x^(1/3)*(-2 + 3*x) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(1/3))/(
2*(1 - x)^(1/3) - x^(1/3))] + 2*Log[(1 - x)^(1/3) + x^(1/3)] - Log[(1 - x)
^(2/3) + x^(2/3) - (-((-1 + x)*x))^(1/3)])/18
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{2/3} \sqrt[3]{x} \, dx$$

$$\Big\downarrow \; 60$$

$$\frac{1}{6} \int \frac{(1-x)^{2/3}}{x^{2/3}}dx - \frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{6}\left(\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x}x^{2/3}}dx + (1-x)^{2/3}\sqrt[3]{x}\right) - \frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}$$

$$\Big\downarrow \; 72$$

$$\frac{1}{6}\left(\frac{2}{3}\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x}}\right) + \frac{3}{2}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x}}+1\right) + \frac{\log(x)}{2}\right) + (1-x)^{2/3}\sqrt[3]{x}\right) - \frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}$$

input
```
Int[(1 - x)^(2/3)*x^(1/3),x]
```

output
```
-1/2*((1 - x)^(5/3)*x^(1/3)) + ((1 - x)^(2/3)*x^(1/3) + (2*(Sqrt[3]*ArcTan
[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*x^(1/3))] + (3*Log[1 + (1 - x)^(1/
3)/x^(1/3)])/2 + Log[x]/2))/3)/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.13

method	result	size
meijerg	$\frac{3x^{\frac{4}{3}} \operatorname{hypergeom}\left(\left[-\frac{2}{3},\frac{4}{3}\right],\left[\frac{7}{3}\right],x\right)}{4}$	13
risch	$-\frac{(-2+3x)x^{\frac{1}{3}}(-1+x)\left(x^2(1-x)\right)^{\frac{1}{3}}}{6\left(-x^2(-1+x)\right)^{\frac{1}{3}}(1-x)^{\frac{1}{3}}}+\frac{\operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)\left(x^2(1-x)\right)^{\frac{1}{3}}}{3x^{\frac{1}{3}}(1-x)^{\frac{1}{3}}}$	73

input
```
int((1-x)^(2/3)*x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(4/3)*hypergeom([-2/3,4/3],[7/3],x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = \frac{1}{6}(3x-2)x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}} - \frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}}{3(x-1)}\right) - \frac{1}{18}\log\left(\frac{x-x^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}-1}{x-1}\right) + \frac{1}{9}\log\left(-\frac{x-x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}-1}{x-1}\right)$$

input
```
integrate((1-x)^(2/3)*x^(1/3),x, algorithm="fricas")
```

output
```
1/6*(3*x - 2)*x^(1/3)*(-x + 1)^(2/3) - 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x
- 1) + 2*sqrt(3)*x^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 1/18*log((x - x^(2/3)*
(-x + 1)^(1/3) + x^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 1/9*log(-(x - x^(1
/3)*(-x + 1)^(2/3) - 1)/(x - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = \frac{x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|xe^{2i\pi}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((1-x)**(2/3)*x**(1/3),x)
```

output
```
x**(4/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x*exp_polar(2*I*pi))/gamma(
7/3)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}-1\right)\right)+\frac{\frac{(-x+1)^{\frac{2}{3}}}{x^{\frac{2}{3}}}-\frac{2(-x+1)^{\frac{5}{3}}}{x^{\frac{5}{3}}}}{6\left(\frac{(x-1)^2}{x^2}-\frac{2(x-1)}{x}+1\right)}+\frac{1}{9}\log\left(\frac{(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}+1\right)-\frac{1}{18}\log\left(-\frac{(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}+\frac{(-x+1)^{\frac{2}{3}}}{x^{\frac{2}{3}}}+1\right)$$

input
```
integrate((1-x)^(2/3)*x^(1/3),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x + 1)^(1/3)/x^(1/3) - 1)) + 1/6*((-x
 + 1)^(2/3)/x^(2/3) - 2*(-x + 1)^(5/3)/x^(5/3))/((x - 1)^2/x^2 - 2*(x - 1)
/x + 1) + 1/9*log((-x + 1)^(1/3)/x^(1/3) + 1) - 1/18*log(-(-x + 1)^(1/3)/x
^(1/3) + (-x + 1)^(2/3)/x^(2/3) + 1)
```

Giac [F]

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}\,dx$$

input
```
integrate((1-x)^(2/3)*x^(1/3),x, algorithm="giac")
```

output
```
integrate(x^(1/3)*(-x + 1)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = \int x^{1/3}\,(1-x)^{2/3}\,dx$$

input
```
int(x^(1/3)*(1 - x)^(2/3),x)
```

output
```
int(x^(1/3)*(1 - x)^(2/3), x)
```

Reduce [F]

$$\int (1-x)^{2/3}\sqrt[3]{x}\,dx = \frac{x^{\frac{4}{3}}(1-x)^{\frac{2}{3}}}{2} - \frac{x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}}{3} - \frac{\left(\int \frac{(1-x)^{\frac{2}{3}}}{x^{\frac{5}{3}}-x^{\frac{2}{3}}}dx\right)}{9}$$

input
```
int((1-x)^(2/3)*x^(1/3),x)
```

output
```
(9*x**(1/3)*( - x + 1)**(2/3)*x - 6*x**(1/3)*( - x + 1)**(2/3) - 2*int(( -
 x + 1)**(2/3)/(x**(2/3)*x - x**(2/3)),x))/18
```

3.279 $\int e\,dx$

Optimal result

Integrand size = 1, antiderivative size = 3

$$\int e\,dx = ex$$

output
```
exp(1)*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
Integrate[E,x]
```

output
```
E*x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e\,dx$$
$$\Big\downarrow \; 24$$
$$ex$$

input
```
Int[E,x]
```

output
```
E*x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

method	result	size
default	ex	5
norman	ex	5
risch	ex	5
parallelrisch	ex	5
orering	ex	5

input
```
int(exp(1),x,method=_RETURNVERBOSE)
```

output
```
exp(1)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="fricas")
```

output
```
x*e
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
integrate(exp(1),x)
```

output
```
E*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="maxima")
```

output
```
x*e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = xe$$

input
```
integrate(exp(1),x, algorithm="giac")
```

output
```
x*e
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e\,dx = x\,\mathrm{e}$$

input
```
int(exp(1),x)
```

output
```
x*exp(1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e\,dx = ex$$

input
```
int(exp(1),x)
```

output
```
e*x
```

3.280 $\int \operatorname{sech}(x)\,dx$

Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

output
```
arctan(sinh(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = -\cot^{-1}(\sinh(x))$$

input
```
Integrate[Sech[x],x]
```

output
```
-ArcCot[Sinh[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ix\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\arctan(\sinh(x))$$

input
```
Int[Sech[x],x]
```

output
```
ArcTan[Sinh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(e^x+i)-i\ln(e^x-i)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$	23

input
```
int(sech(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \operatorname{sech}(x)\,dx = 2\arctan(\cosh(x)+\sinh(x))$$

input
```
integrate(sech(x),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \operatorname{sech}(x)\,dx = 2\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate(sech(x),x)
```

output
```
2*atan(tanh(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(x)\,dx = \arctan\left(\sinh\left(x\right)\right)$$

input
```
integrate(sech(x),x, algorithm="maxima")
```

output
```
arctan(sinh(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2 \arctan\left(e^{x}\right)$$

input
```
integrate(sech(x),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \operatorname{sech}(x)\,dx = 2\,\operatorname{atan}(\mathrm{e}^x)$$

input
```
int(1/cosh(x),x)
```

output
```
2*atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(x)\,dx = 2atan(e^x)$$

input
```
int(sech(x),x)
```

output
```
2*atan(e**x)
```

3.281 $\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 7

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(1+e^x))$$

output
```
ln(ln(1+exp(x)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(1+e^x))$$

input
```
Integrate[E^x/((1 + E^x)*Log[1 + E^x]),x]
```

output
```
Log[Log[1 + E^x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2720, 2837, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{(e^x+1)\log(e^x+1)}dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{(e^x+1)\log(e^x+1)}de^x$$

$$\Big\downarrow \text{2837}$$

$$\int \frac{e^{-x}}{\log(e^x+1)}d(e^x+1)$$

$$\Big\downarrow \text{2739}$$

$$\int e^{-x}d\log(e^x+1)$$

$$\Big\downarrow \text{14}$$

$$\log(\log(e^x+1))$$

input
```
Int[E^x/((1 + E^x)*Log[1 + E^x]),x]
```

output
```
Log[Log[1 + E^x]]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(\ln(1+e^x))$	7
default	$\ln(\ln(1+e^x))$	7
norman	$\ln(\ln(1+e^x))$	7
risch	$\ln(\ln(1+e^x))$	7
parallelrisch	$\ln(\ln(1+e^x))$	7

input
```
int(exp(x)/(1+exp(x))/ln(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(ln(1+exp(x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(e^x+1))$$

input
```
integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="fricas")
```

output
```
log(log(e^x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(e^x+1))$$

input
```
integrate(exp(x)/(1+exp(x))/ln(1+exp(x)),x)
```

output
```
log(log(exp(x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(e^x+1))$$

input
```
integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="maxima")
```

output
```
log(log(e^x + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(|\log(e^x+1)|)$$

input
```
integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="giac")
```

output
```
log(abs(log(e^x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \ln(\ln(\mathrm{e}^x+1))$$

input
```
int(exp(x)/(log(exp(x) + 1)*(exp(x) + 1)),x)
```

output
```
log(log(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx = \log(\log(e^x+1))$$

input
```
int(exp(x)/(1+exp(x))/log(1+exp(x)),x)
```

output
```
log(log(e**x + 1))
```

3.282 $\int (1 - x + x^2 - x^3 + x^4) (1 + x + x^2 + x^3 + x^4) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 30

$$\int (1 - x + x^2 - x^3 + x^4) (1 + x + x^2 + x^3 + x^4) \, dx = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9}$$

output
```
x+1/3*x^3+1/5*x^5+1/7*x^7+1/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (1 - x + x^2 - x^3 + x^4) (1 + x + x^2 + x^3 + x^4) \, dx = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9}$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)*(1 + x + x^2 + x^3 + x^4),x]
```

output
```
x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4-x^3+x^2-x+1\right) \left(x^4+x^3+x^2+x+1\right) \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int \left(x^8+x^6+x^4+x^2+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^9}{9}+\frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)*(1 + x + x^2 + x^3 + x^4),x]
```

output
```
x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
gosper	$x+\frac{1}{3}x^{3}+\frac{1}{5}x^{5}+\frac{1}{7}x^{7}+\frac{1}{9}x^{9}$	23
default	$x+\frac{1}{3}x^{3}+\frac{1}{5}x^{5}+\frac{1}{7}x^{7}+\frac{1}{9}x^{9}$	23
norman	$x+\frac{1}{3}x^{3}+\frac{1}{5}x^{5}+\frac{1}{7}x^{7}+\frac{1}{9}x^{9}$	23
risch	$x+\frac{1}{3}x^{3}+\frac{1}{5}x^{5}+\frac{1}{7}x^{7}+\frac{1}{9}x^{9}$	23
parallelrisch	$x+\frac{1}{3}x^{3}+\frac{1}{5}x^{5}+\frac{1}{7}x^{7}+\frac{1}{9}x^{9}$	23
orering	$\frac{x\left(35x^{8}+45x^{6}+63x^{4}+105x^{2}+315\right)}{315}$	26

input
```
int((x^4-x^3+x^2-x+1)*(x^4+x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*x^3+1/5*x^5+1/7*x^7+1/9*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^{2}-x^{3}+x^{4}\right)\left(1+x+x^{2}+x^{3}+x^{4}\right)\,dx=\frac{1}{9}\,x^{9}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5}+\frac{1}{3}\,x^{3}+x$$

input
```
integrate((x^4-x^3+x^2-x+1)*(x^4+x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \left(1-x+x^2-x^3+x^4\right)\left(1+x+x^2+x^3+x^4\right)\,dx=\frac{x^9}{9}+\frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x$$

input
```
integrate((x**4-x**3+x**2-x+1)*(x**4+x**3+x**2+x+1),x)
```

output
```
x**9/9 + x**7/7 + x**5/5 + x**3/3 + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right)\left(1+x+x^2+x^3+x^4\right)\,dx=\frac{1}{9}\,x^9+\frac{1}{7}\,x^7+\frac{1}{5}\,x^5+\frac{1}{3}\,x^3+x$$

input
```
integrate((x^4-x^3+x^2-x+1)*(x^4+x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right)\left(1+x+x^2+x^3+x^4\right)\,dx=\frac{1}{9}\,x^9+\frac{1}{7}\,x^7+\frac{1}{5}\,x^5+\frac{1}{3}\,x^3+x$$

input
```
integrate((x^4-x^3+x^2-x+1)*(x^4+x^3+x^2+x+1),x, algorithm="giac")
```

output
```
1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right)\left(1+x+x^2+x^3+x^4\right)\,dx=\frac{x^9}{9}+\frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x$$

input
```
int((x^2 - x - x^3 + x^4 + 1)*(x + x^2 + x^3 + x^4 + 1),x)
```

output
```
x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(1-x+x^2-x^3+x^4\right)\left(1+x+x^2+x^3+x^4\right)\,dx$$
$$=\frac{x(35x^8+45x^6+63x^4+105x^2+315)}{315}$$

input
```
int((x^4-x^3+x^2-x+1)*(x^4+x^3+x^2+x+1),x)
```

output
```
(x*(35*x**8 + 45*x**6 + 63*x**4 + 105*x**2 + 315))/315
```

3.283 $\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{x^2}{10}-\frac{5x^3}{36}+\frac{7x^4}{96}-\frac{x^5}{60}+\frac{x^6}{720}$$

output
```
1/10*x^2-5/36*x^3+7/96*x^4-1/60*x^5+1/720*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{1}{120}\left(12x^2-\frac{50x^3}{3}+\frac{35x^4}{4}-2x^5+\frac{x^6}{6}\right)$$

input
```
Integrate[((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)*x)/120,x]
```

output
```
(12*x^2 - (50*x^3)/3 + (35*x^4)/4 - 2*x^5 + x^6/6)/120
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {27, 2109, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{120}(x-4)(x-3)(x-2)(x-1)x\,dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{120}\int (1-x)(2-x)(3-x)(4-x)xdx$$

$$\Big\downarrow \text{2109}$$

$$\frac{1}{120}\int \left(x^5-10x^4+35x^3-50x^2+24x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{120}\left(\frac{x^6}{6}-2x^5+\frac{35x^4}{4}-\frac{50x^3}{3}+12x^2\right)$$

input
```
Int[((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)*x)/120,x]
```

output
```
(12*x^2 - (50*x^3)/3 + (35*x^4)/4 - 2*x^5 + x^6/6)/120
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2109
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
orering	$\frac{(2x^3-16x^2+41x-36)x^2(-4+x)}{1440}$	24
gosper	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
default	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
norman	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
risch	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
parallelrisch	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27

input
```
int(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x,method=_RETURNVERBOSE)
```

output
```
1/1440*(2*x^3-16*x^2+41*x-36)*x^2*(-4+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{1}{720}\,x^6-\frac{1}{60}\,x^5+\frac{7}{96}\,x^4-\frac{5}{36}\,x^3+\frac{1}{10}\,x^2$$

input
```
integrate(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x, algorithm="fricas")
```

output
```
1/720*x^6 - 1/60*x^5 + 7/96*x^4 - 5/36*x^3 + 1/10*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{x^6}{720} - \frac{x^5}{60} + \frac{7x^4}{96} - \frac{5x^3}{36} + \frac{x^2}{10}$$

input
```
integrate(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x)
```

output
```
x**6/720 - x**5/60 + 7*x**4/96 - 5*x**3/36 + x**2/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{1}{720}\,x^6 - \frac{1}{60}\,x^5 + \frac{7}{96}\,x^4 - \frac{5}{36}\,x^3 + \frac{1}{10}\,x^2$$

input
```
integrate(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x, algorithm="maxima")
```

output
```
1/720*x^6 - 1/60*x^5 + 7/96*x^4 - 5/36*x^3 + 1/10*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{1}{720}\left(x^2-4\,x\right)^3 + \frac{1}{160}\left(x^2-4\,x\right)^2$$

input
```
integrate(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x, algorithm="giac")
```

output
```
1/720*(x^2 - 4*x)^3 + 1/160*(x^2 - 4*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{x^6}{720} - \frac{x^5}{60} + \frac{7\,x^4}{96} - \frac{5\,x^3}{36} + \frac{x^2}{10}$$

input
```
int((x*(x - 1)*(x - 2)*(x - 3)*(x - 4))/120,x)
```

output
```
x^2/10 - (5*x^3)/36 + (7*x^4)/96 - x^5/60 + x^6/720
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx = \frac{x^2(2x^4 - 24x^3 + 105x^2 - 200x + 144)}{1440}$$

input
```
int(1/120*(-4+x)*(-3+x)*(-2+x)*(-1+x)*x,x)
```

output
```
(x**2*(2*x**4 - 24*x**3 + 105*x**2 - 200*x + 144))/1440
```

3.284 $\int (x+x\cos(x)+\sin(x)+\cos(x)\sin(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int (x+x\cos(x)+\sin(x)+\cos(x)\sin(x))\,dx=\frac{1}{2}(x+\sin(x))^2$$

output
```
1/2*(x+sin(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int (x+x\cos(x)+\sin(x)+\cos(x)\sin(x))\,dx=\frac{x^2}{2}-\frac{\cos^2(x)}{2}+x\sin(x)$$

input
```
Integrate[x + x*Cos[x] + Sin[x] + Cos[x]*Sin[x],x]
```

output
```
x^2/2 - Cos[x]^2/2 + x*Sin[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sin(x)+x\cos(x)+\sin(x)\cos(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+\frac{\sin^2(x)}{2}+x\sin(x)$$

input
```
Int[x + x*Cos[x] + Sin[x] + Cos[x]*Sin[x],x]
```

output
```
x^2/2 + x*Sin[x] + Sin[x]^2/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

method	result
default	$\frac{x^2}{2}+x\sin\left(x\right)+\frac{\sin(x)^2}{2}$
risch	$\frac{x^2}{2}+x\sin\left(x\right)-\frac{\cos(2x)}{4}$
parts	$\frac{x^2}{2}-\frac{\cos(x)^2}{2}+x\sin\left(x\right)$
norman	$\frac{x^2\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)^2+\frac{x^2}{2}+2x\tan\left(\frac{x}{2}\right)+2x\tan\left(\frac{x}{2}\right)^3+\frac{x^2\tan\left(\frac{x}{2}\right)^4}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$
orering	$\frac{(81x^4+540x^2+140)(x+x\cos(x)+\sin(x)+\cos(x)\sin(x))}{18x(9x^2+10)}-\frac{(81x^2+10)\left(1+2\cos(x)-x\sin(x)-\sin(x)^2+\cos(x)^2\right)}{4(9x^2+10)}+\frac{5(81x^4+108}{}$

input
```
int(x+x*cos(x)+sin(x)+cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x*sin(x)+1/2*sin(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int (x+x\cos(x)+\sin(x)+\cos(x)\sin(x))\,dx=\frac{1}{2}\,x^2-\frac{1}{2}\,\cos\left(x\right)^2+x\sin\left(x\right)$$

input
```
integrate(x+x*cos(x)+sin(x)+cos(x)*sin(x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int (x + x\cos(x) + \sin(x) + \cos(x)\sin(x))\, dx = \frac{x^2}{2} + x\sin(x) + \frac{\sin^2(x)}{2}$$

input
```
integrate(x+x*cos(x)+sin(x)+cos(x)*sin(x),x)
```

output
```
x**2/2 + x*sin(x) + sin(x)**2/2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int (x + x\cos(x) + \sin(x) + \cos(x)\sin(x))\, dx = \frac{1}{2}x^2 - \frac{1}{2}\cos(x)^2 + x\sin(x)$$

input
```
integrate(x+x*cos(x)+sin(x)+cos(x)*sin(x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int (x + x\cos(x) + \sin(x) + \cos(x)\sin(x))\, dx = \frac{1}{2}x^2 - \frac{1}{2}\cos(x)^2 + x\sin(x)$$

input
```
integrate(x+x*cos(x)+sin(x)+cos(x)*sin(x),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (x + x\cos(x) + \sin(x) + \cos(x)\sin(x))\,dx = \frac{(x + \sin(x))^2}{2}$$

input
```
int(x + sin(x) + cos(x)*sin(x) + x*cos(x),x)
```

output
```
(x + sin(x))^2/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int (x + x\cos(x) + \sin(x) + \cos(x)\sin(x))\,dx = -\frac{\cos(x)^2}{2} + \sin(x)\,x + \frac{x^2}{2}$$

input
```
int(x+x*cos(x)+sin(x)+cos(x)*sin(x),x)
```

output
```
( - cos(x)**2 + 2*sin(x)*x + x**2)/2
```

3.285 $\int (\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+$

Optimal result

Integrand size = 25, antiderivative size = 12

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx = -x-2\cot(x)+2\tan(x)$$

output
```
-x-2*cot(x)+2*tan(x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx = -x-2\cot(x)+2\tan(x)$$

input
```
Integrate[Cos[x]^2 + Cot[x]^2 + Csc[x]^2 + Sec[x]^2 + Sin[x]^2 + Tan[x]^2,
x]
```

output
```
-x - 2*Cot[x] + 2*Tan[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^2(x)+\cos^2(x)+\tan^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)\right) dx$$

$$\Big\downarrow \; 2009$$

$$-x+2\tan(x)-2\cot(x)$$

input
```
Int[Cos[x]^2 + Cot[x]^2 + Csc[x]^2 + Sec[x]^2 + Sin[x]^2 + Tan[x]^2,x]
```

output
```
-x - 2*Cot[x] + 2*Tan[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

method	result	size
default	$x-2\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(x\right)\right)+2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	24
parts	$x-2\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(x\right)\right)+2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	24
risch	$-\frac{4i}{{\mathrm e}^{2ix}-1}+\frac{4i}{{\mathrm e}^{2ix}+1}-x$	29
parallelrisch	$\frac{\left(-4\cos\left(x\right)+4\right)\cot\left(\frac{x}{2}\right)-x\cos\left(x\right)-\sec\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)}{\cos\left(x\right)}$	33

input
```
int(cos(x)^2+cot(x)^2+csc(x)^2+sec(x)^2+sin(x)^2+tan(x)^2,x,method=_RETURN
VERBOSE)
```

output
```
x-2*cot(x)+1/2*Pi-arccot(cot(x))+2*tan(x)-arctan(tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx = -\frac{x\cos(x)\sin(x)+4\cos(x)^2-2}{\cos(x)\sin(x)}$$

input
```
integrate(cos(x)^2+cot(x)^2+csc(x)^2+sec(x)^2+sin(x)^2+tan(x)^2,x, algorit
hm="fricas")
```

output
```
-(x*cos(x)*sin(x) + 4*cos(x)^2 - 2)/(cos(x)*sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx = -x+\frac{2\sin(x)}{\cos(x)}-\frac{2\cos(x)}{\sin(x)}$$

input
```
integrate(cos(x)**2+cot(x)**2+csc(x)**2+sec(x)**2+sin(x)**2+tan(x)**2,x)
```

output
```
-x + 2*sin(x)/cos(x) - 2*cos(x)/sin(x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx$$
$$= -x-\frac{2}{\tan(x)}+2\,\tan(x)$$

input
```
integrate(cos(x)^2+cot(x)^2+csc(x)^2+sec(x)^2+sin(x)^2+tan(x)^2,x, algorit
hm="maxima")
```

output
```
-x - 2/tan(x) + 2*tan(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx$$
$$= -x-\frac{1}{2\,\tan\left(\frac{1}{2}\,x\right)}-\frac{1}{\tan(x)}+\frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)+2\,\tan(x)$$

input
```
integrate(cos(x)^2+cot(x)^2+csc(x)^2+sec(x)^2+sin(x)^2+tan(x)^2,x, algorit
hm="giac")
```

output
```
-x - 1/2/tan(1/2*x) - 1/tan(x) + 1/2*tan(1/2*x) + 2*tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx=2\tan(x)-x-\frac{2}{\tan(x)}$$

input
```
int(1/cos(x)^2 + cos(x)^2 + cot(x)^2 + 1/sin(x)^2 + sin(x)^2 + tan(x)^2,x)
```

output
```
2*tan(x) - x - 2/tan(x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \left(\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\tan^2(x)\right)\,dx$$
$$=\frac{-\cos(x)^2-\cos(x)\cot(x)\sin(x)+\cos(x)\sin(x)\tan(x)-\cos(x)\sin(x)x+\sin(x)^2}{\cos(x)\sin(x)}$$

input
```
int(cos(x)^2+cot(x)^2+csc(x)^2+sec(x)^2+sin(x)^2+tan(x)^2,x)
```

output
```
( - cos(x)**2 - cos(x)*cot(x)*sin(x) + cos(x)*sin(x)*tan(x) - cos(x)*sin(x
)*x + sin(x)**2)/(cos(x)*sin(x))
```

3.286 $\int e^{\log^2(x)}(1+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int e^{\log^2(x)}(1+2\log(x))\,dx=e^{\log^2(x)}x$$

output
```
exp(ln(x)^2)*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int e^{\log^2(x)}(1+2\log(x))\,dx=e^{\log^2(x)}x$$

input
```
Integrate[E^Log[x]^2*(1 + 2*Log[x]),x]
```

output
```
E^Log[x]^2*x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\log^2(x)}(2\log(x)+1)\,dx$$

$$\Big\downarrow \text{2726}$$

$$xe^{\log^2(x)}$$

input
```
Int[E^Log[x]^2*(1 + 2*Log[x]),x]
```

output
```
E^Log[x]^2*x
```

Defintions of rubi rules used

rule 2726
```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
norman	$e^{\ln(x)^2} x$	8
risch	$e^{\ln(x)^2} x$	8
parallelrisch	$e^{\ln(x)^2} x$	8

input
```
int(exp(ln(x)^2)*(1+2*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(ln(x)^2)*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = xe^{\left(\log(x)^2\right)}$$

input
```
integrate(exp(log(x)^2)*(1+2*log(x)),x, algorithm="fricas")
```

output
```
x*e^(log(x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = xe^{\log(x)^2}$$

input
```
integrate(exp(ln(x)**2)*(1+2*ln(x)),x)
```

output
```
x*exp(log(x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = xe^{\left(\log(x)^2\right)}$$

input
```
integrate(exp(log(x)^2)*(1+2*log(x)),x, algorithm="maxima")
```

output
```
x*e^(log(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = xe^{\left(\log(x)^2\right)}$$

input
```
integrate(exp(log(x)^2)*(1+2*log(x)),x, algorithm="giac")
```

output
```
x*e^(log(x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = x\,\mathrm{e}^{\ln(x)^2}$$

input
```
int(exp(log(x)^2)*(2*log(x) + 1),x)
```

output
```
x*exp(log(x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int e^{\log^2(x)}(1+2\log(x))\,dx = e^{\log(x)^2}x$$

input
```
int(exp(log(x)^2)*(1+2*log(x)),x)
```

output
```
e**(log(x)**2)*x
```

3.287 $\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+\right.\right.$

Optimal result

Integrand size = 43, antiderivative size = 1

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

output
```
0
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

input
```
Integrate[(1 - x)^3 + (x - x^2)^3 - 3*(1 - x)*(x - x^2)*(-1 + x^2) + (-1 +
 x^2)^3,x]
```

output
```
0
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3 vs. 2(1) = 2.

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 3.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3(x-x^2)\left(x^2-1\right)(1-x)+\left(x-x^2\right)^3+\left(x^2-1\right)^3+(1-x)^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}$$

input
```
Int[(1 - x)^3 + (x - x^2)^3 - 3*(1 - x)*(x - x^2)*(-1 + x^2) + (-1 + x^2)^
3,x]
```

output
```
-1/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	0	2
norman	0	2
meijerg	0	2
risch	$-\frac{1}{4}$	2
parallelrisch	0	2

input
```
int((1-x)^3+(-x^2+x)^3-3*(1-x)*(-x^2+x)*(x^2-1)+(x^2-1)^3,x,method=_RETURN
VERBOSE)
```

output
```
0
```

Fricas [F]

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx$$
$$=\int -\left(x^2-x\right)^3+\left(x^2-1\right)^3-3\left(x^2-x\right)\left(x^2-1\right)(x-1)-(x-1)^3\,dx$$

input
```
integrate((1-x)^3+(-x^2+x)^3-3*(1-x)*(-x^2+x)*(x^2-1)+(x^2-1)^3,x, algorit
hm="fricas")
```

output
```
0
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

input
```
integrate((1-x)**3+(-x**2+x)**3-3*(1-x)*(-x**2+x)*(x**2-1)+(x**2-1)**3,x)
```

output
```
0
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

input
```
integrate((1-x)^3+(-x^2+x)^3-3*(1-x)*(-x^2+x)*(x^2-1)+(x^2-1)^3,x, algorit
hm="maxima")
```

output
```
0
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 26.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx = -\frac{1}{4}\,(x-1)^4+\frac{1}{4}\,x^4-x^3+\frac{3}{2}\,x^2-x$$

input
```
integrate((1-x)^3+(-x^2+x)^3-3*(1-x)*(-x^2+x)*(x^2-1)+(x^2-1)^3,x, algorit
hm="giac")
```

output
```
-1/4*(x - 1)^4 + 1/4*x^4 - x^3 + 3/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

input
```
int((x - x^2)^3 - (x - 1)^3 + (x^2 - 1)^3 + 3*(x - x^2)*(x^2 - 1)*(x - 1),
x)
```

output
```
0
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left((1-x)^3+\left(x-x^2\right)^3-3(1-x)\left(x-x^2\right)\left(-1+x^2\right)+\left(-1+x^2\right)^3\right)\,dx=0$$

input
```
int((1-x)^3+(-x^2+x)^3-3*(1-x)*(-x^2+x)*(x^2-1)+(x^2-1)^3,x)
```

output
```
0
```

3.288 $\int (\cos^6(x) + 3\cos^2(x)\sin^2(x) + \sin^6(x)) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1

$$\int \left(\cos^6(x) + 3\cos^2(x)\sin^2(x) + \sin^6(x)\right) \, dx = x$$

output
```
x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x) + 3\cos^2(x)\sin^2(x) + \sin^6(x)\right) \, dx = x$$

input
```
Integrate[Cos[x]^6 + 3*Cos[x]^2*Sin[x]^2 + Sin[x]^6,x]
```

output
```
x
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 50.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^6(x)+\cos^6(x)+3\sin^2(x)\cos^2(x)\right) dx$$

$$\Big\downarrow \ 2009$$

$$x+\frac{1}{6}\sin(x)\cos^5(x)-\frac{13}{24}\sin(x)\cos^3(x)-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)+\frac{3}{8}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^6 + 3*Cos[x]^2*Sin[x]^2 + Sin[x]^6,x]
```

output
```
x + (3*Cos[x]*Sin[x])/8 - (13*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x])/6 -
(5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result
risch	x
orering	$x\left(\cos\left(x\right)^{6}+3\cos\left(x\right)^{2}\sin\left(x\right)^{2}+\sin\left(x\right)^{6}\right)$
default	$\frac{\left(\cos(x)^{5}+\frac{5\cos(x)^{3}}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+x-\frac{\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}-\frac{3\cos(x)^{3}\sin(x)}{4}+\frac{3\cos(x)\sin(x)}{8}$
parts	$\frac{\left(\cos(x)^{5}+\frac{5\cos(x)^{3}}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+x-\frac{\left(\sin(x)^{5}+\frac{5\sin(x)^{3}}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}-\frac{3\cos(x)^{3}\sin(x)}{4}+\frac{3\cos(x)\sin(x)}{8}$
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{12}+6\tan\left(\frac{x}{2}\right)^{2}x+15\tan\left(\frac{x}{2}\right)^{4}x+20\tan\left(\frac{x}{2}\right)^{6}x+15\tan\left(\frac{x}{2}\right)^{8}x+6\tan\left(\frac{x}{2}\right)^{10}x}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{6}}$

input
```
int(cos(x)^6+3*cos(x)^2*sin(x)^2+sin(x)^6,x,method=_RETURNVERBOSE)
```

output
```
x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x)+3\cos^2(x)\sin^2(x)+\sin^6(x)\right)\,dx=x$$

input
```
integrate(cos(x)^6+3*cos(x)^2*sin(x)^2+sin(x)^6,x, algorithm="fricas")
```

output
```
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(0) = 0$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 58.00

$$\int \left(\cos^6(x)+3\cos^2(x)\sin^2(x)+\sin^6(x)\right)\,dx = x-\frac{\sin^5(x)\cos(x)}{6}-\frac{5\sin^3(x)\cos(x)}{24}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}-\frac{3\sin(2x)\cos(2x)}{16}$$

input
```
integrate(cos(x)**6+3*cos(x)**2*sin(x)**2+sin(x)**6,x)
```

output
```
x - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 + sin(x)*cos(x)**5/6 + 5*si
n(x)*cos(x)**3/24 - 3*sin(2*x)*cos(2*x)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x)+3\cos^2(x)\sin^2(x)+\sin^6(x)\right)\,dx = x$$

input
```
integrate(cos(x)^6+3*cos(x)^2*sin(x)^2+sin(x)^6,x, algorithm="maxima")
```

output
```
x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x)+3\cos^2(x)\sin^2(x)+\sin^6(x)\right)\,dx = x$$

input
```
integrate(cos(x)^6+3*cos(x)^2*sin(x)^2+sin(x)^6,x, algorithm="giac")
```

output
```
x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x) + 3\cos^2(x)\sin^2(x) + \sin^6(x)\right) dx = x$$

input
```
int(cos(x)^6 + sin(x)^6 + 3*cos(x)^2*sin(x)^2,x)
```

output
```
x
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^6(x) + 3\cos^2(x)\sin^2(x) + \sin^6(x)\right) dx = x$$

input
```
int(cos(x)^6+3*cos(x)^2*sin(x)^2+sin(x)^6,x)
```

output
```
x
```

3.289 $\int e^x x^e (1+e+x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int e^x x^e (1+e+x)\,dx = e^x x^{1+e}$$

output
```
exp(x)*x^(1+exp(1))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^x x^e (1+e+x)\,dx = e^x x^{1+e}$$

input
```
Integrate[E^x*x^E*(1 + E + x),x]
```

output
```
E^x*x^(1 + E)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2627}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x^e (x+e+1)\,dx$$

$$\Big\downarrow \text{2627}$$

$$e^x x^{1+e}$$

input
```
Int[E^x*x^E*(1 + E + x),x]
```

output
```
E^x*x^(1 + E)
```

Defintions of rubi rules used

rule 2627
```
Int[(F_)^(v_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)), x_Symbol] :>
 Simp[g*(d + e*x)^(m + 1)*(F^v/(D[v, x]*e*Log[F])), x] /; FreeQ[{F, d, e, f
, g, m}, x] && LinearQ[v, x] && EqQ[e*g*(m + 1) - D[v, x]*(e*f - d*g)*Log[F
], 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result
risch	$x\,\mathrm{e}^{x} x^{\mathrm{e}}$
parallelrisch	$x\,\mathrm{e}^{x} x^{\mathrm{e}}$
gosper	$\mathrm{e}^{x} x^{1+\mathrm{e}}$
norman	$x\,\mathrm{e}^{x} \mathrm{e}^{\mathrm{e}\ln(x)}$
meijerg	$-(-1)^{-\mathrm{e}} \left(x^{\mathrm{e}}(-1)^{\mathrm{e}}\,\mathrm{e}\Gamma(\mathrm{e})\,(-x)^{-\mathrm{e}} - x^{\mathrm{e}}(-1)^{\mathrm{e}}\,\mathrm{e}^{x} - x^{\mathrm{e}}(-1)^{\mathrm{e}}\,\mathrm{e}(-x)^{-\mathrm{e}}\,\Gamma(\mathrm{e},-x)\right) - (-1)^{-\mathrm{e}}\,\mathrm{e}\big(x^{\mathrm{e}}$

input
```
int(exp(x)*x^exp(1)*(1+exp(1)+x),x,method=_RETURNVERBOSE)
```

output
```
x*exp(x)*x^exp(1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{x}x^{e}(1+e+x)\,dx = xx^{e}e^{x}$$

input
```
integrate(exp(x)*x^exp(1)*(1+exp(1)+x),x, algorithm="fricas")
```

output
```
x*x^e*e^x
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{x}x^{e}(1+e+x)\,dx = xx^{e}e^{x}$$

input
```
integrate(exp(x)*x**exp(1)*(1+exp(1)+x),x)
```

output
```
x*x**E*exp(x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 9.00

$$\int e^x x^e (1 + e + x) \, dx = -(-x)^{-e-1} x^{e+1} e \Gamma(e+1, -x) - (-x)^{-e-2} x^{e+2} \Gamma(e+2, -x) - (-x)^{-e-1} x^{e+1} \Gamma(e+1, -x)$$

input
```
integrate(exp(x)*x^exp(1)*(1+exp(1)+x),x, algorithm="maxima")
```

output
```
-(-x)^(-e - 1)*x^(e + 1)*e*gamma(e + 1, -x) - (-x)^(-e - 2)*x^(e + 2)*gamm
a(e + 2, -x) - (-x)^(-e - 1)*x^(e + 1)*gamma(e + 1, -x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^x x^e (1 + e + x) \, dx = x x^e e^x$$

input
```
integrate(exp(x)*x^exp(1)*(1+exp(1)+x),x, algorithm="giac")
```

output
```
x*x^e*e^x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^x x^e (1 + e + x) \, dx = x^{e+1} \, e^x$$

input
```
int(x^exp(1)*exp(x)*(x + exp(1) + 1),x)
```

output
```
x^(exp(1) + 1)*exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^x x^e (1 + e + x) \, dx = x^e e^x x$$

input
```
int(exp(x)*x^exp(1)*(1+exp(1)+x),x)
```

output
```
x**e*e**x*x
```

3.290 $\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx=-x+\sqrt{2}\sqrt{x}\sqrt{1+x}-\sqrt{2}\text{arcsinh}\left(\sqrt{x}\right)+\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)$$

output
```
-x+2^(1/2)*x^(1/2)*(1+x)^(1/2)-2^(1/2)*arcsinh(x^(1/2))+2^(1/2)*arctanh(1/
2*x*2^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx=-x+\sqrt{2}\sqrt{\frac{x}{1+x}}(1+x)-\frac{\log\left(\sqrt{2}-x\right)}{\sqrt{2}}+\frac{\log\left(\sqrt{2}+x\right)}{\sqrt{2}}+\frac{\sqrt{2}\sqrt{\frac{x}{1+x}}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[2]*Sqrt[x/(1 + x)] + x^2/(2 - x^2),x]
```

output
```
-x + Sqrt[2]*Sqrt[x/(1 + x)]*(1 + x) - Log[Sqrt[2] - x]/Sqrt[2] + Log[Sqrt
[2] + x]/Sqrt[2] + (Sqrt[2]*Sqrt[x/(1 + x)]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqr
t[1 + x]])/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{2-x^2}+\sqrt{2}\sqrt{\frac{x}{x+1}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{2}\text{arcsinh}\left(\sqrt{x}\right)+\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)-x+\sqrt{2}\sqrt{x+1}\sqrt{x}$$

input
```
Int[Sqrt[2]*Sqrt[x/(1 + x)] + x^2/(2 - x^2),x]
```

output
```
-x + Sqrt[2]*Sqrt[x]*Sqrt[1 + x] - Sqrt[2]*ArcSinh[Sqrt[x]] + Sqrt[2]*ArcT
anh[x/Sqrt[2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{2}\,\sqrt{\frac{x}{1+x}}\,(1+x)\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)}{2\sqrt{(1+x)x}}-x+\sqrt{2}\,\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)$
trager	$1-x+\sqrt{2}\,(1+x)\,\sqrt{\frac{x}{1+x}}+\frac{\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{x}{1+x}}\,x^2-2\sqrt{2}\,x^2-2\sqrt{\frac{x}{1+x}}\,x^2-2\sqrt{2}\,\sqrt{\frac{x}{1+x}}+x\sqrt{2}+2x\sqrt{\frac{x}{1+x}}+2x^2+\sqrt{2}+4\sqrt{\frac{x}{1+x}}-3x-}{x\sqrt{2}-\sqrt{2}+x-2}\right.}{2}$

input
```
int(2^(1/2)*(x/(1+x))^(1/2)+x^2/(-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2))
)/((1+x)*x)^(1/2)-x+2^(1/2)*arctanh(1/2*x*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int\left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)dx=\sqrt{2}(x+1)\sqrt{\frac{x}{x+1}}$$
$$+\frac{1}{2}\sqrt{2}\log\left(-\frac{6\,x^3+19\,x^2+2\,\sqrt{2}(2\,x^3+7\,x^2+7\,x+2)-2\left(3\,x^3+11\,x^2+2\,\sqrt{2}(x^3+4\,x^2+5\,x+2)\right.}{x^2-2}\right.$$
$$-x$$

input
```
integrate(2^(1/2)*(x/(1+x))^(1/2)+x^2/(-x^2+2),x, algorithm="fricas")
```

output
```
sqrt(2)*(x + 1)*sqrt(x/(x + 1)) + 1/2*sqrt(2)*log(-(6*x^3 + 19*x^2 + 2*sqr
t(2)*(2*x^3 + 7*x^2 + 7*x + 2) - 2*(3*x^3 + 11*x^2 + 2*sqrt(2)*(x^3 + 4*x^
2 + 5*x + 2) + 14*x + 6)*sqrt(x/(x + 1)) + 20*x + 6)/(x^2 - 2)) - x
```

Sympy [F]

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx=\int \frac{\sqrt{2}x^2\sqrt{\frac{x}{x+1}}-x^2-2\sqrt{2}\sqrt{\frac{x}{x+1}}}{x^2-2}\,dx$$

input
```
integrate(2**(1/2)*(x/(1+x))**(1/2)+x**2/(-x**2+2),x)
```

output
```
Integral((sqrt(2)*x**2*sqrt(x/(x + 1)) - x**2 - 2*sqrt(2)*sqrt(x/(x + 1)))
/(x**2 - 2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\begin{aligned}
&\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx \\
&=-\frac{1}{2}\sqrt{2}\left(\frac{2\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}+\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)\right) \\
&\quad-\frac{1}{2}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)-x
\end{aligned}$$

input
```
integrate(2^(1/2)*(x/(1+x))^(1/2)+x^2/(-x^2+2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*(2*sqrt(x/(x + 1))/(x/(x + 1) - 1) + log(sqrt(x/(x + 1)) + 1)
 - log(sqrt(x/(x + 1)) - 1)) - 1/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))
) - x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right) dx$$
$$=\frac{1}{2}\sqrt{2}\left(\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+2\sqrt{x^2+x}\operatorname{sgn}(x+1)\right)$$
$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}\right|}{\left|2x+2\sqrt{2}\right|}\right)-x$$

input
```
integrate(2^(1/2)*(x/(1+x))^(1/2)+x^2/(-x^2+2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + 2*sqrt(x^2
+ x)*sgn(x + 1)) - 1/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2
))) - x
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.86

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right) dx$$
$$=-\sqrt{2}\operatorname{atanh}\left(\frac{96\sqrt{\frac{2x}{x+1}}}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}\right.$$
$$-\frac{80\left(\frac{2x}{x+1}\right)^{3/2}}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}$$
$$+\frac{128}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}$$
$$\left.-\frac{224x}{(x+1)\left(64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}\right)}\right)-\frac{\sqrt{\frac{2x}{x+1}}-1}{\frac{x}{x+1}-1}$$

input
```
int(2^(1/2)*(x/(x + 1))^(1/2) - x^2/(x^2 - 2),x)
```

output
```
- 2^(1/2)*atanh((96*((2*x)/(x + 1))^(1/2))/(64*2^(1/2)*((2*x)/(x + 1))^(1/
2) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/2)*x)/(x +
1)) - (80*((2*x)/(x + 1))^(3/2))/(64*2^(1/2)*((2*x)/(x + 1))^(1/2) - 56*2^
(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/2)*x)/(x + 1)) + 128/
(64*2^(1/2)*((2*x)/(x + 1))^(1/2) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*
2^(1/2) - (160*2^(1/2)*x)/(x + 1)) - (224*x)/((x + 1)*(64*2^(1/2)*((2*x)/(
x + 1))^(1/2) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/
2)*x)/(x + 1)))) - (((2*x)/(x + 1))^(1/2) - 1)/(x/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}}+\frac{x^2}{2-x^2}\right)\,dx = \sqrt{x}\,\sqrt{x+1}\,\sqrt{2}-\frac{\sqrt{2}\log\left(-\sqrt{2}+x\right)}{2} - \sqrt{2}\log\left(\sqrt{x+1}+\sqrt{x}\right)+\frac{\sqrt{2}\log\left(\sqrt{2}+x\right)}{2}-x$$

input
```
int(2^(1/2)*(x/(1+x))^(1/2)+x^2/(-x^2+2),x)
```

output
```
(2*sqrt(x)*sqrt(x + 1)*sqrt(2) - sqrt(2)*log( - sqrt(2) + x) - 2*sqrt(2)*l
og(sqrt(x + 1) + sqrt(x)) + sqrt(2)*log(sqrt(2) + x) - 2*x)/2
```

3.291 $\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \log(x) + \frac{\log\left(1+x^{2022}\right)}{2022}$$

output
```
ln(x)+1/2022*ln(x^2022+1)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \log(x) + \frac{\log\left(1+x^{2022}\right)}{2022}$$

input
```
Integrate[(1 + 2*x^2022)/(x + x^2023),x]
```

output
```
Log[x] + Log[1 + x^2022]/2022
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^{2022}+1}{x^{2023}+x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{2x^{2022}+1}{x(x^{2022}+1)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{2x^{2022}+1}{x^{2022}(x^{2022}+1)}dx^{2022}}{2022}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{1}{x^{2022}}+\frac{1}{x^{2022}+1}\right)dx^{2022}}{2022}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x^{2022})+\log(x^{2022}+1)}{2022}$$

input
```
Int[(1 + 2*x^2022)/(x + x^2023),x]
```

output
```
(Log[x^2022] + Log[1 + x^2022])/2022
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2022+1)/(x^2023+x),x)
```

output
```
int((2*x^2022+1)/(x^2023+x),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \text{Timed out}$$

input
```
integrate((2*x^2022+1)/(x^2023+x),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \text{Timed out}$$

input
```
integrate((2*x**2022+1)/(x**2023+x),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \frac{1}{2022}\log\left(x^{2022}+1\right)+\log\left(x\right)$$

input
```
integrate((2*x^2022+1)/(x^2023+x),x, algorithm="maxima")
```

output
```
1/2022*log(x^2022 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \frac{1}{2022}\log\left(x^{2022}+1\right)+\frac{1}{2022}\log\left(x^{2022}\right)$$

input
```
integrate((2*x^2022+1)/(x^2023+x),x, algorithm="giac")
```

output
```
1/2022*log(x^2022 + 1) + 1/2022*log(x^2022)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \int \frac{2\,x^{2022}+1}{x^{2023}+x}\,dx$$

input
```
int((2*x^2022 + 1)/(x + x^2023),x)
```

output
```
int((2*x^2022 + 1)/(x + x^2023), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1385, normalized size of antiderivative = 106.54

$$\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx = \text{Too large to display}$$

input
```
int((2*x^2022+1)/(x^2023+x),x)
```

output

```
(log(x**2016 - x**2010 + x**2004 - x**1998 + x**1992 - x**1986 + x**1980 -
 x**1974 + x**1968 - x**1962 + x**1956 - x**1950 + x**1944 - x**1938 + x**
1932 - x**1926 + x**1920 - x**1914 + x**1908 - x**1902 + x**1896 - x**1890
 + x**1884 - x**1878 + x**1872 - x**1866 + x**1860 - x**1854 + x**1848 - x
**1842 + x**1836 - x**1830 + x**1824 - x**1818 + x**1812 - x**1806 + x**18
00 - x**1794 + x**1788 - x**1782 + x**1776 - x**1770 + x**1764 - x**1758 +
 x**1752 - x**1746 + x**1740 - x**1734 + x**1728 - x**1722 + x**1716 - x**
1710 + x**1704 - x**1698 + x**1692 - x**1686 + x**1680 - x**1674 + x**1668
 - x**1662 + x**1656 - x**1650 + x**1644 - x**1638 + x**1632 - x**1626 + x
**1620 - x**1614 + x**1608 - x**1602 + x**1596 - x**1590 + x**1584 - x**15
78 + x**1572 - x**1566 + x**1560 - x**1554 + x**1548 - x**1542 + x**1536 -
 x**1530 + x**1524 - x**1518 + x**1512 - x**1506 + x**1500 - x**1494 + x**
1488 - x**1482 + x**1476 - x**1470 + x**1464 - x**1458 + x**1452 - x**1446
 + x**1440 - x**1434 + x**1428 - x**1422 + x**1416 - x**1410 + x**1404 - x
**1398 + x**1392 - x**1386 + x**1380 - x**1374 + x**1368 - x**1362 + x**13
56 - x**1350 + x**1344 - x**1338 + x**1332 - x**1326 + x**1320 - x**1314 +
 x**1308 - x**1302 + x**1296 - x**1290 + x**1284 - x**1278 + x**1272 - x**
1266 + x**1260 - x**1254 + x**1248 - x**1242 + x**1236 - x**1230 + x**1224
 - x**1218 + x**1212 - x**1206 + x**1200 - x**1194 + x**1188 - x**1182 + x
**1176 - x**1170 + x**1164 - x**1158 + x**1152 - x**1146 + x**1140 - x*...
```

3.292 $\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 9

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx=\sin(20x)\sin(23x)$$

output
```
sin(20*x)*sin(23*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx=\frac{1}{2}\cos(3x)-\frac{1}{2}\cos(43x)$$

input
```
Integrate[3*Cos[23*x]*Sin[20*x] + 20*Sin[43*x],x]
```

output
```
Cos[3*x]/2 - Cos[43*x]/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (20\sin(43x) + 3\sin(20x)\cos(23x))\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\cos(3x) - \frac{1}{2}\cos(43x)$$

input
```
Int[3*Cos[23*x]*Sin[20*x] + 20*Sin[43*x],x]
```

output
```
Cos[3*x]/2 - Cos[43*x]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

method	result
default	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
risch	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
parts	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
orering	$\frac{23\sin(20x)\sin(23x)}{43}+\frac{20\cos(23x)\cos(20x)}{43}-\frac{20\cos(43x)}{43}$
parallelrisch	$\frac{-\frac{40}{43}+\frac{40\left(-\tan(10x)^2-\tan\left(\frac{43x}{2}\right)^2-2\right)\tan\left(\frac{23x}{2}\right)^2}{43}+\frac{92\tan(10x)\left(1+\tan\left(\frac{43x}{2}\right)^2\right)\tan\left(\frac{23x}{2}\right)}{43}-\frac{40\tan\left(\frac{43x}{2}\right)^2\tan(10x)^2}{43}-\frac{80\tan(10x)^2}{43}}{\left(1+\tan\left(\frac{23x}{2}\right)^2\right)\left(1+\tan(10x)^2\right)\left(1+\tan\left(\frac{43x}{2}\right)^2\right)}$

input
```
int(3*cos(23*x)*sin(20*x)+20*sin(43*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(3*x)-1/2*cos(43*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(9) = 18.

Time = 2.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 14.56

$$\int (3\cos(23x)\sin(20x) + 20\sin(43x))\,dx = -2199023255552\cos(x)^{43} + 23639499997184\cos(x)^{41} - 118197499985920\cos(x)^{39} + 364934781206528\cos(x)^{37} - 778995398344704\cos(x)^{35} + 1219742794776576\cos(x)^{33} - 1450504945139712\cos(x)^{31} + 1338263491051520\cos(x)^{29} - 970241031012352\cos(x)^{27} + 556461767786496\cos(x)^{25} - 252937167175680\cos(x)^{23} + 90899294453760\cos(x)^{21} - 25657058918400\cos(x)^{19} + 5624816762880\cos(x)^{17} - 942087536640\cos(x)^{15} + 117760942080\cos(x)^{13} - 10631196160\cos(x)^{11} + 661443200\cos(x)^{9} - 26457728\cos(x)^{7} + 609224\cos(x)^{5} - 6620\cos(x)^{3} + 20\cos(x)$$

input

```
integrate(3*cos(23*x)*sin(20*x)+20*sin(43*x),x, algorithm="fricas")
```

output

```
-2199023255552*cos(x)^43 + 23639499997184*cos(x)^41 - 118197499985920*cos(
x)^39 + 364934781206528*cos(x)^37 - 778995398344704*cos(x)^35 + 1219742794
776576*cos(x)^33 - 1450504945139712*cos(x)^31 + 1338263491051520*cos(x)^29
 - 970241031012352*cos(x)^27 + 556461767786496*cos(x)^25 - 252937167175680
*cos(x)^23 + 90899294453760*cos(x)^21 - 25657058918400*cos(x)^19 + 5624816
762880*cos(x)^17 - 942087536640*cos(x)^15 + 117760942080*cos(x)^13 - 10631
196160*cos(x)^11 + 661443200*cos(x)^9 - 26457728*cos(x)^7 + 609224*cos(x)^
5 - 6620*cos(x)^3 + 20*cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx = \frac{23\sin{\left(20x \right)}\sin{\left(23x \right)}}{43} + \frac{20\cos{\left(20x \right)}\cos{\left(23x \right)}}{43} - \frac{20\cos{\left(43x \right)}}{43}$$

input
```
integrate(3*cos(23*x)*sin(20*x)+20*sin(43*x),x)
```

output
```
23*sin(20*x)*sin(23*x)/43 + 20*cos(20*x)*cos(23*x)/43 - 20*cos(43*x)/43
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx = -\frac{1}{2}\cos\left(43\,x\right) + \frac{1}{2}\cos\left(3\,x\right)$$

input
```
integrate(3*cos(23*x)*sin(20*x)+20*sin(43*x),x, algorithm="maxima")
```

output
```
-1/2*cos(43*x) + 1/2*cos(3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx = -\frac{1}{2}\cos\left(43\,x\right) + \frac{1}{2}\cos\left(3\,x\right)$$

input
```
integrate(3*cos(23*x)*sin(20*x)+20*sin(43*x),x, algorithm="giac")
```

output
```
-1/2*cos(43*x) + 1/2*cos(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx=\frac{\cos(3\,x)}{2}-\frac{\cos(43\,x)}{2}$$

input
```
int(20*sin(43*x) + 3*cos(23*x)*sin(20*x),x)
```

output
```
cos(3*x)/2 - cos(43*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.00

$$\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx=-\frac{20\cos(43x)}{43}+\frac{20\cos(23x)\cos(20x)}{43}+\frac{23\sin(23x)\sin(20x)}{43}$$

input
```
int(3*cos(23*x)*sin(20*x)+20*sin(43*x),x)
```

output
```
( - 20*cos(43*x) + 20*cos(23*x)*cos(20*x) + 23*sin(23*x)*sin(20*x))/43
```

3.293 $\int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx = \frac{1}{2}(x-\log(2e^x+\cos(x)+\sin(x)))$$

output
```
1/2*x-1/2*ln(2*exp(x)+cos(x)+sin(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\frac{1}{2}\log(2e^x+\cos(x)+\sin(x))$$

input
```
Integrate[Sin[x]/(2*E^x + Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[2*E^x + Cos[x] + Sin[x]]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{2e^x + \sin(x) + \cos(x)} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\sin(x)}{2e^x + \sin(x) + \cos(x)} dx$$

input
```
Int[Sin[x]/(2*E^x + Cos[x] + Sin[x]),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{\ln\left(\mathrm{e}^{2 i x}+(2+2 i) \mathrm{e}^{(1+i) x}+i\right)}{2}$	30
parallelrisch	$\ln\left(\frac{2}{\sqrt{(4 \,\mathrm{e}^{x}-2) \sec\left(\frac{x}{2}\right)^{2}+4 \tan\left(\frac{x}{2}\right)+4}}\right)+\ln\left(\sqrt{\sec\left(\frac{x}{2}\right)^{2}}\right)+\frac{x}{2}$	40
norman	$\frac{\frac{x}{2}+\frac{x \tan\left(\frac{x}{2}\right)^{2}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}+\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{2}-\frac{\ln\left(2 \,\mathrm{e}^{x} \tan\left(\frac{x}{2}\right)^{2}-\tan\left(\frac{x}{2}\right)^{2}+2 \,\mathrm{e}^{x}+2 \tan\left(\frac{x}{2}\right)+1\right)}{2}$	70

input
```
int(sin(x)/(2*exp(x)+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*I*x-1/2*ln(exp(2*I*x)+(2+2*I)*exp((1+I)*x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\, dx = \frac{1}{2}\, x - \frac{1}{2}\, \log\left(\cos\left(x\right) + 2\, e^{x} + \sin\left(x\right)\right)$$

input
```
integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*log(cos(x) + 2*e^x + sin(x))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\, dx = \frac{x}{2} - \frac{\log\left(2e^{x} + \sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

input
```
integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x)
```

output
```
x/2 - log(2*exp(x) + sin(x) + cos(x))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(16) = 32.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.79

$$\begin{aligned}\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\, dx = \frac{1}{2}\, x - \frac{1}{4}\, \log\Big(&8\, \cos\left(x\right)^{2} e^{\left(2\, x\right)} + 8\, e^{\left(2\, x\right)} \sin\left(x\right)^{2} \\ &+ 4\left(\cos\left(x\right) e^{x} - e^{x} \sin\left(x\right)\right) \cos\left(2\, x\right) + \cos\left(2\, x\right)^{2} \\ &+ 4\, \cos\left(x\right) e^{x} + 2\left(2\, \cos\left(x\right) e^{x} + 2\, e^{x} \sin\left(x\right) + 1\right) \sin\left(2\, x\right) \\ &+ \sin\left(2\, x\right)^{2} + 4\, e^{x} \sin\left(x\right) + 1\Big)\end{aligned}$$

input
```
integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
1/2*x - 1/4*log(8*cos(x)^2*e^(2*x) + 8*e^(2*x)*sin(x)^2 + 4*(cos(x)*e^x -
e^x*sin(x))*cos(2*x) + cos(2*x)^2 + 4*cos(x)*e^x + 2*(2*cos(x)*e^x + 2*e^x
*sin(x) + 1)*sin(2*x) + sin(2*x)^2 + 4*e^x*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 6.26

$$\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\,dx = \frac{1}{2}\,x$$
$$-\frac{1}{4}\,\log\left(\frac{2\left(4\,e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^4 - 4\,e^x\tan\left(\frac{1}{2}\,x\right)^4 + 8\,e^x\tan\left(\frac{1}{2}\,x\right)^3 + \tan\left(\frac{1}{2}\,x\right)^4 + 8\,e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^2 - 4\,\mathrm{ta}\right.}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)}\right.$$

input
```
integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*x - 1/4*log(2*(4*e^(2*x)*tan(1/2*x)^4 - 4*e^x*tan(1/2*x)^4 + 8*e^x*tan
(1/2*x)^3 + tan(1/2*x)^4 + 8*e^(2*x)*tan(1/2*x)^2 - 4*tan(1/2*x)^3 + 8*e^x
*tan(1/2*x) + 2*tan(1/2*x)^2 + 4*e^(2*x) + 4*e^x + 4*tan(1/2*x) + 1)/(tan(
1/2*x)^4 + 2*tan(1/2*x)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\,dx = \frac{x}{2} - \frac{\ln\left(2\,\mathrm{e}^x + \sqrt{2}\,\cos\left(x - \frac{\pi}{4}\right)\right)}{2}$$

input
```
int(sin(x)/(cos(x) + 2*exp(x) + sin(x)),x)
```

output
```
x/2 - log(2*exp(x) + 2^(1/2)*cos(x - pi/4))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sin(x)}{2e^x + \cos(x) + \sin(x)}\,dx = -\frac{\log(\cos{(x)} + 2e^x + \sin{(x)})}{2} + \frac{x}{2}$$

input
```
int(sin(x)/(2*exp(x)+cos(x)+sin(x)),x)
```

output
```
( - log(cos(x) + 2*e**x + sin(x)) + x)/2
```

3.294 $\int \log^{-\log(e\pi)}(x) \log\left(\frac{x}{\pi}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 9

$$\int \log^{-\log(e\pi)}(x) \log\left(\frac{x}{\pi}\right) dx = x \log^{-\log(\pi)}(x)$$

output
```
x/(ln(x)^ln(Pi))
```

Mathematica [F]

$$\int \log^{-\log(e\pi)}(x) \log\left(\frac{x}{\pi}\right) dx = \int \log^{-\log(e\pi)}(x) \log\left(\frac{x}{\pi}\right) dx$$

input
```
Integrate[Log[x/Pi]/Log[x]^Log[E*Pi],x]
```

output
```
Integrate[Log[x/Pi]/Log[x]^Log[E*Pi], x]
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 8.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2808, 25, 2033, 3039, 7111}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx$$

$$\Big\downarrow \text{2808}$$

$$(-\log(x))^{1+\log(\pi)}\log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\Gamma(-\log(\pi),-\log(x))-\int -\frac{\Gamma(-\log(\pi),-\log(x))(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)}{x}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\Gamma(-\log(\pi),-\log(x))(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)}{x}dx+(-\log(x))^{1+\log(\pi)}\log\left(\frac{x}{\pi}\right)\log^{-\log(e\pi)}(x)\Gamma(-\log(\pi),-\log(x))$$

$$\Big\downarrow \text{2033}$$

$$(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)\int \frac{\Gamma(-\log(\pi),-\log(x))}{x}dx+(-\log(x))^{1+\log(\pi)}\log\left(\frac{x}{\pi}\right)\log^{-\log(e\pi)}(x)\Gamma(-\log(\pi),-\log(x))$$

$$\Big\downarrow \text{3039}$$

$$(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)\int \Gamma(-\log(\pi),-\log(x))d\log(x)+(-\log(x))^{1+\log(\pi)}\log\left(\frac{x}{\pi}\right)\log^{-\log(e\pi)}(x)\Gamma(-\log(\pi),-\log(x))$$

$$\Big\downarrow \text{7111}$$

$$(-\log(x))^{1+\log(\pi)}\log\left(\frac{x}{\pi}\right)\log^{-\log(e\pi)}(x)\Gamma(-\log(\pi),-\log(x))+(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)(\Gamma(1-\log(\pi),-\log(x))+\log(x)\Gamma(-\log(\pi),-\log(x)))$$

input
```
Int[Log[x/Pi]/Log[x]^Log[E*Pi],x]
```

output
```
((-Log[x])^Log[Pi]*(Gamma[1 - Log[Pi], -Log[x]] + Gamma[-Log[Pi], -Log[x]]
*Log[x]))/Log[x]^Log[Pi] + (Gamma[-Log[Pi], -Log[x]]*(-Log[x])^(1 + Log[Pi
])*Log[x/Pi])/Log[x]^Log[E*Pi]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2033
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + n)
*((b*v)^n/(a*v)^n)   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&
 !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

rule 2808
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Simp[
(d + e*Log[f*x^r])   u, x] - Simp[e*r   Int[SimplifyIntegrand[u/x, x], x],
x]] /; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 7111
```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Maple [F]

$$\int \ln \left(\frac{x}{\pi}\right) \ln (x)^{-\ln(\mathrm{e}\pi)}\, dx$$

input
```
int(ln(x/Pi)/(ln(x)^ln(exp(1)*Pi)),x)
```

output
```
int(ln(x/Pi)/(ln(x)^ln(exp(1)*Pi)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(9) = 18$.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = \frac{x\log(\pi) + x\log\left(\frac{x}{\pi}\right)}{\left(\log(\pi) + \log\left(\frac{x}{\pi}\right)\right)^{\log(\pi)+1}}$$

input
```
integrate(log(x/pi)/(log(x)^log(exp(1)*pi)),x, algorithm="fricas")
```

output
```
(x*log(pi) + x*log(x/pi))/(log(pi) + log(x/pi))^(log(pi) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(7) = 14$.

Time = 19.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 6.22

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = -\frac{(-\log(x))^{1+\log(\pi)}\log(\pi)\Gamma(-\log(\pi), -\log(x))}{\log(x)^{1+\log(\pi)}} + \frac{(-\log(x))^{\log(\pi)}\,\Gamma(1-\log(\pi), -\log(x))}{\log(x)^{\log(\pi)}}$$

input
```
integrate(ln(x/pi)/(ln(x)**ln(exp(1)*pi)),x)
```

output
```
-(-log(x))**(1 + log(pi))*log(pi)*log(x)**(-log(pi) - 1)*uppergamma(-log(p
i), -log(x)) + (-log(x))**log(pi)*uppergamma(1 - log(pi), -log(x))/log(x)*
*log(pi)
```

Maxima [F]

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = \int \frac{\log\left(\frac{x}{\pi}\right)}{\log(x)^{\log(\pi e)}}\,dx$$

input
```
integrate(log(x/pi)/(log(x)^log(exp(1)*pi)),x, algorithm="maxima")
```

output
```
integrate(log(x)^(-log(pi*e))*log(x/pi), x)
```

Giac [F]

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = \int \frac{\log\left(\frac{x}{\pi}\right)}{\log(x)^{\log(\pi e)}}\,dx$$

input
```
integrate(log(x/pi)/(log(x)^log(exp(1)*pi)),x, algorithm="giac")
```

output
```
integrate(log(x/pi)/log(x)^log(pi*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = \int \frac{\ln\left(\frac{x}{\Pi}\right)}{\ln(x)^{\ln(\Pi\,\mathrm{e})}}\,dx$$

input
```
int(log(x/Pi)/log(x)^log(Pi*exp(1)),x)
```

output
```
int(log(x/Pi)/log(x)^log(Pi*exp(1)), x)
```

Reduce [F]

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx = \int \frac{\log\left(\frac{x}{\pi}\right)}{\log(x)^{\log(e\pi)}}dx$$

input
```
int(log(x/Pi)/(log(x)^log(exp(1)*Pi)),x)
```

output
```
int(log(x/pi)/log(x)**log(e*pi),x)
```

3.295 $\int (x + 8x^2 + 10x^3 + 5x^4 + x^5) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int (x + 8x^2 + 10x^3 + 5x^4 + x^5) \, dx = \frac{x^2}{2} + \frac{8x^3}{3} + \frac{5x^4}{2} + x^5 + \frac{x^6}{6}$$

output
```
1/2*x^2+8/3*x^3+5/2*x^4+x^5+1/6*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (x + 8x^2 + 10x^3 + 5x^4 + x^5) \, dx = \frac{x^2}{2} + \frac{8x^3}{3} + \frac{5x^4}{2} + x^5 + \frac{x^6}{6}$$

input
```
Integrate[x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x]
```

output
```
x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^5+5x^4+10x^3+8x^2+x\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^6}{6}+x^5+\frac{5x^4}{2}+\frac{8x^3}{3}+\frac{x^2}{2}$$

input
```
Int[x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x]
```

output
```
x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x^2(x^4+6x^3+15x^2+16x+3)}{6}$	24
default	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
norman	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
risch	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
parallelrisch	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
parts	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
orering	$\frac{x(x^4+6x^3+15x^2+16x+3)(x^5+5x^4+10x^3+8x^2+x)}{6x^4+30x^3+60x^2+48x+6}$	62

input
```
int(x^5+5*x^4+10*x^3+8*x^2+x,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(x^4+6*x^3+15*x^2+16*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int (x+8x^2+10x^3+5x^4+x^5)\,dx=\frac{1}{6}\,x^6+x^5+\frac{5}{2}\,x^4+\frac{8}{3}\,x^3+\frac{1}{2}\,x^2$$

input
```
integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="fricas")
```

output
```
1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(x + 8x^2 + 10x^3 + 5x^4 + x^5\right) dx = \frac{x^6}{6} + x^5 + \frac{5x^4}{2} + \frac{8x^3}{3} + \frac{x^2}{2}$$

input
```
integrate(x**5+5*x**4+10*x**3+8*x**2+x,x)
```

output
```
x**6/6 + x**5 + 5*x**4/2 + 8*x**3/3 + x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(x + 8x^2 + 10x^3 + 5x^4 + x^5\right) dx = \frac{1}{6}x^6 + x^5 + \frac{5}{2}x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2$$

input
```
integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="maxima")
```

output
```
1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(x + 8x^2 + 10x^3 + 5x^4 + x^5\right) dx = \frac{1}{6}x^6 + x^5 + \frac{5}{2}x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2$$

input
```
integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="giac")
```

output
```
1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(x+8x^2+10x^3+5x^4+x^5\right)\,dx=\frac{x^6}{6}+x^5+\frac{5\,x^4}{2}+\frac{8\,x^3}{3}+\frac{x^2}{2}$$

input
```
int(x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x)
```

output
```
x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(x+8x^2+10x^3+5x^4+x^5\right)\,dx=\frac{x^2(x^4+6x^3+15x^2+16x+3)}{6}$$

input
```
int(x^5+5*x^4+10*x^3+8*x^2+x,x)
```

output
```
(x**2*(x**4 + 6*x**3 + 15*x**2 + 16*x + 3))/6
```

3.296 $\int \frac{(1+x)(4+x)}{(2+x)(3+x)} dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)} dx = x - 2\log(2+x) + 2\log(3+x)$$

output
```
x-2*ln(2+x)+2*ln(3+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)} dx = x - 2\log(2+x) + 2\log(3+x)$$

input
```
Integrate[((1 + x)*(4 + x))/((2 + x)*(3 + x)),x]
```

output
```
x - 2*Log[2 + x] + 2*Log[3 + x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)(x+4)}{(x+2)(x+3)} \, dx$$

$$\Big\downarrow \; 159$$

$$\int \left(\frac{2}{x+3}-\frac{2}{x+2}+1\right) dx$$

$$\Big\downarrow \; 2009$$

$$x-2\log(x+2)+2\log(x+3)$$

input
```
Int[((1 + x)*(4 + x))/((2 + x)*(3 + x)),x]
```

output
```
x - 2*Log[2 + x] + 2*Log[3 + x]
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$x-2\ln\left(2+x\right)+2\ln\left(3+x\right)$	15
norman	$x-2\ln\left(2+x\right)+2\ln\left(3+x\right)$	15
risch	$x-2\ln\left(2+x\right)+2\ln\left(3+x\right)$	15
parallelrisch	$x-2\ln\left(2+x\right)+2\ln\left(3+x\right)$	15

input
```
int((1+x)*(4+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
```

output
```
x-2*ln(2+x)+2*ln(3+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = x+2\,\log\left(x+3\right)-2\,\log\left(x+2\right)$$

input
```
integrate((1+x)*(4+x)/(2+x)/(3+x),x, algorithm="fricas")
```

output
```
x + 2*log(x + 3) - 2*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = x-2\log\left(x+2\right)+2\log\left(x+3\right)$$

input
```
integrate((1+x)*(4+x)/(2+x)/(3+x),x)
```

output
```
x - 2*log(x + 2) + 2*log(x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = x + 2\,\log\left(x+3\right) - 2\,\log\left(x+2\right)$$

input
```
integrate((1+x)*(4+x)/(2+x)/(3+x),x, algorithm="maxima")
```

output
```
x + 2*log(x + 3) - 2*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = x + 2\,\log\left(|x+3|\right) - 2\,\log\left(|x+2|\right)$$

input
```
integrate((1+x)*(4+x)/(2+x)/(3+x),x, algorithm="giac")
```

output
```
x + 2*log(abs(x + 3)) - 2*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = x + 4\,\mathrm{atanh}(2\,x+5)$$

input
```
int(((x + 1)*(x + 4))/((x + 2)*(x + 3)),x)
```

output
```
x + 4*atanh(2*x + 5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx = 2\log(x+3) - 2\log(x+2) + x$$

input
```
int((1+x)*(4+x)/(2+x)/(3+x),x)
```

output
```
2*log(x + 3) - 2*log(x + 2) + x
```

3.297 $\int x \cot(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 39

$$\int x\cot(x)\,dx=-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

output
```
-1/2*I*x^2+x*ln(1-exp(2*I*x))-1/2*I*polylog(2,exp(2*I*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int x\cot(x)\,dx=x\log\left(1-e^{2ix}\right)-\frac{1}{2}i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)$$

input
```
Integrate[x*Cot[x],x]
```

output
```
x*Log[1 - E^((2*I)*x)] - (I/2)*(x^2 + PolyLog[2, E^((2*I)*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.750, Rules used = {3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4200}$$

$$2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx-\frac{ix^2}{2}$$

$$\Big\downarrow \text{25}$$

$$-2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)-\frac{ix^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)-\frac{ix^2}{2}$$

input
```
Int[x*Cot[x],x]
```

output
```
(-1/2*I)*x^2 - (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + PolyLog[2, E^((2*I)*x
)]/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
parts	$x \ln \left(1-\mathrm{e}^{2 i x}\right)-\frac{i\left(x^{2}+\operatorname{polylog}\left(2, \mathrm{e}^{2 i x}\right)\right)}{2}$	28
risch	$-\frac{i x^{2}}{2}+x \ln \left(1+\mathrm{e}^{i x}\right)-i \operatorname{polylog}\left(2,-\mathrm{e}^{i x}\right)+x \ln \left(1-\mathrm{e}^{i x}\right)-i \operatorname{polylog}\left(2, \mathrm{e}^{i x}\right)$	52

input
```
int(x*cot(x),x,method=_RETURNVERBOSE)
```

output
```
x*ln(1-exp(2*I*x))-1/2*I*(x^2+polylog(2,exp(2*I*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(24) = 48.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int x \cot(x)\, dx = \frac{1}{2}\, x \log\left(-\cos\left(2\, x\right) + i\, \sin\left(2\, x\right) + 1\right) + \frac{1}{2}\, x \log\left(-\cos\left(2\, x\right) - i\, \sin\left(2\, x\right) + 1\right) - \frac{1}{4} i\, \mathrm{Li}_2(\cos\left(2\, x\right) + i\, \sin\left(2\, x\right)) + \frac{1}{4} i\, \mathrm{Li}_2(\cos\left(2\, x\right) - i\, \sin\left(2\, x\right))$$

input
```
integrate(x*cot(x),x, algorithm="fricas")
```

output
```
1/2*x*log(-cos(2*x) + I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) - I*sin(2*x) +
 1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x)) + 1/4*I*dilog(cos(2*x) - I*sin(2*
x))
```

Sympy [F]

$$\int x\cot(x)\,dx = \int x\cot(x)\,dx$$

input
```
integrate(x*cot(x),x)
```

output
```
Integral(x*cot(x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\begin{aligned}\int x\cot(x)\,dx =& -\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin(x),\cos(x)+1\right)\\ &- i\,x\arctan\left(\sin(x),-\cos(x)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\\ &+ \frac{1}{2}\,x\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) - i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) - i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\end{aligned}$$

input
```
integrate(x*cot(x),x, algorithm="maxima")
```

output
```
-1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
 + 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) - I*dilog(-e^(I*x)) - I*dilog(e^(I*x))
```

Giac [F]

$$\int x\cot(x)\,dx = \int x\cot(x)\,dx$$

input
```
integrate(x*cot(x),x, algorithm="giac")
```

output
```
integrate(x*cot(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot(x)\,dx = \int x\cot(x)\,dx$$

input
```
int(x*cot(x),x)
```

output
```
int(x*cot(x), x)
```

Reduce [F]

$$\int x\cot(x)\,dx = \int \cot(x)\,x dx$$

input
```
int(x*cot(x),x)
```

output
```
int(cot(x)*x,x)
```

3.298 $\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = e^{\frac{1}{x}+x}\left(4+\frac{1}{x^2}-\frac{2}{x}-2x+x^2\right)$$

output
```
exp(1/x+x)*(4+1/x^2-2/x-2*x+x^2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = e^{\frac{1}{x}+x}\left(4+\frac{1}{x^2}-\frac{2}{x}-2x+x^2\right)$$

input
```
Integrate[(E^(x^(-1) + x)*(-1 - x^2 + x^4 + x^6))/x^4,x]
```

output
```
E^(x^(-1) + x)*(4 + x^(-2) - 2/x - 2*x + x^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x+\frac{1}{x}}\left(x^6+x^4-x^2-1\right)}{x^4}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{e^{x+\frac{1}{x}}\left(x^2-1\right)\left(x^2+1\right)^2}{x^4}dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(-\frac{e^{x+\frac{1}{x}}}{x^4}+e^{x+\frac{1}{x}}x^2-\frac{e^{x+\frac{1}{x}}}{x^2}+e^{x+\frac{1}{x}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\int \frac{e^{x+\frac{1}{x}}}{x^4}dx-\int \frac{e^{x+\frac{1}{x}}}{x^2}dx+\int e^{x+\frac{1}{x}}x^2dx+\int e^{x+\frac{1}{x}}dx$$

input
```
Int[(E^(x^(-1) + x)*(-1 - x^2 + x^4 + x^6))/x^4,x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

method	result	size
gosper	$\frac{e^{\frac{x^2+1}{x}}\left(x^4-2x^3+4x^2-2x+1\right)}{x^2}$	33
risch	$\frac{e^{\frac{x^2+1}{x}}\left(x^4-2x^3+4x^2-2x+1\right)}{x^2}$	33
norman	$\frac{x\,e^{\frac{1}{x}+x}+x^5e^{\frac{1}{x}+x}+4x^3e^{\frac{1}{x}+x}-2x^4e^{\frac{1}{x}+x}-2\,e^{\frac{1}{x}+x}x^2}{x^3}$	57
orering	$\frac{\left(x^4-2x^3+4x^2-2x+1\right)e^{\frac{1}{x}+x}\left(x^6+x^4-x^2-1\right)}{x^2(-1+x)(1+x)\left(x^2+1\right)^2}$	59
parallelrisch	$\frac{e^{\frac{x^2+1}{x}}x^4-2\,e^{\frac{x^2+1}{x}}x^3+4\,e^{\frac{x^2+1}{x}}x^2-2x\,e^{\frac{x^2+1}{x}}+e^{\frac{x^2+1}{x}}}{x^2}$	73

input
```
int(exp(1/x+x)*(x^6+x^4-x^2-1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
exp((x^2+1)/x)*(x^4-2*x^3+4*x^2-2*x+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx=\frac{(x^4-2\,x^3+4\,x^2-2\,x+1)e^{\left(\frac{x^2+1}{x}\right)}}{x^2}$$

input
```
integrate(exp(1/x+x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="fricas")
```

output
```
(x^4 - 2*x^3 + 4*x^2 - 2*x + 1)*e^((x^2 + 1)/x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = \frac{(x^4-2x^3+4x^2-2x+1)\,e^{x+\frac{1}{x}}}{x^2}$$

input
```
integrate(exp(1/x+x)*(x**6+x**4-x**2-1)/x**4,x)
```

output
```
(x**4 - 2*x**3 + 4*x**2 - 2*x + 1)*exp(x + 1/x)/x**2
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = \frac{(x^4-2\,x^3+4\,x^2-2\,x+1)e^{\left(x+\frac{1}{x}\right)}}{x^2}$$

input
```
integrate(exp(1/x+x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="maxima")
```

output
```
(x^4 - 2*x^3 + 4*x^2 - 2*x + 1)*e^(x + 1/x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(23) = 46.

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = \frac{x^4e^{\left(\frac{x^2+1}{x}\right)}-2\,x^3e^{\left(\frac{x^2+1}{x}\right)}+4\,x^2e^{\left(\frac{x^2+1}{x}\right)}-2\,xe^{\left(\frac{x^2+1}{x}\right)}+e^{\left(\frac{x^2+1}{x}\right)}}{x^2}$$

input
```
integrate(exp(1/x+x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="giac")
```

output
```
(x^4*e^((x^2 + 1)/x) - 2*x^3*e^((x^2 + 1)/x) + 4*x^2*e^((x^2 + 1)/x) - 2*x
*e^((x^2 + 1)/x) + e^((x^2 + 1)/x))/x^2
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = \frac{\mathrm{e}^{x+\frac{1}{x}}\,(x^4-2\,x^3+4\,x^2-2\,x+1)}{x^2}$$

input
```
int(-(exp(x + 1/x)*(x^2 - x^4 - x^6 + 1))/x^4,x)
```

output
```
(exp(x + 1/x)*(4*x^2 - 2*x - 2*x^3 + x^4 + 1))/x^2
```

Reduce [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx = \frac{e^{\frac{x^2+1}{x}}(x^4-2x^3+4x^2-2x+1)}{x^2}$$

input
```
int(exp(1/x+x)*(x^6+x^4-x^2-1)/x^4,x)
```

output
```
(e**((x**2 + 1)/x)*(x**4 - 2*x**3 + 4*x**2 - 2*x + 1))/x**2
```

3.299 $\int \frac{1}{\sqrt{(-1+x)(1+x)^3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}} dx = -\frac{(1-x)(1+x)}{\sqrt{-((1-x)(1+x)^3)}}$$

output
```
-(1-x)*(1+x)/(-(1-x)*(1+x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}} dx = \frac{(-1+x)(1+x)}{\sqrt{(-1+x)(1+x)^3}}$$

input
```
Integrate[1/Sqrt[(-1 + x)*(1 + x)^3],x]
```

output
```
((-1 + x)*(1 + x))/Sqrt[(-1 + x)*(1 + x)^3]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7270, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(x-1)(x+1)^3}}dx$$
$$\Big\downarrow \text{7270}$$
$$\frac{\sqrt{x-1}(x+1)^{3/2}\int \frac{1}{\sqrt{x-1}(x+1)^{3/2}}dx}{\sqrt{-\left((1-x)(x+1)^3\right)}}$$
$$\Big\downarrow \text{48}$$
$$\frac{(x-1)(x+1)}{\sqrt{-\left((1-x)(x+1)^3\right)}}$$

input
```
Int[1/Sqrt[(-1 + x)*(1 + x)^3],x]
```

output
```
((-1 + x)*(1 + x))/Sqrt[-((1 - x)*(1 + x)^3)]
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(-1+x)(1+x)}{\sqrt{(-1+x)(1+x)^{3}}}$	19
risch	$\frac{(-1+x)(1+x)}{\sqrt{(-1+x)(1+x)^{3}}}$	19
orering	$\frac{(-1+x)(1+x)}{\sqrt{(-1+x)(1+x)^{3}}}$	19
trager	$\frac{\sqrt{x^{4}+2x^{3}-2x-1}}{(1+x)^{2}}$	22
default	$\frac{\sqrt{(-1+x)(1+x)}\,\sqrt{x^{2}-1}}{\sqrt{(-1+x)(1+x)^{3}}}$	29

input
```
int(1/((-1+x)*(1+x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*(1+x)/((-1+x)*(1+x)^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \frac{x^2+2\,x+\sqrt{x^4+2\,x^3-2\,x-1}+1}{x^2+2\,x+1}$$

input
```
integrate(1/((-1+x)*(1+x)^3)^(1/2),x, algorithm="fricas")
```

output
```
(x^2 + 2*x + sqrt(x^4 + 2*x^3 - 2*x - 1) + 1)/(x^2 + 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)^3}}\,dx$$

input
```
integrate(1/((-1+x)*(1+x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt((x - 1)*(x + 1)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \int \frac{1}{\sqrt{(x+1)^3(x-1)}}\,dx$$

input
```
integrate(1/((-1+x)*(1+x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((x + 1)^3*(x - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}\left(x+1\right)}$$

input
```
integrate(1/((-1+x)*(1+x)^3)^(1/2),x, algorithm="giac")
```

output
```
2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \frac{x^2-1}{\sqrt{(x-1)\,(x+1)^3}}$$

input
```
int(1/((x - 1)*(x + 1)^3)^(1/2),x)
```

output
```
(x^2 - 1)/((x - 1)*(x + 1)^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{(-1+x)(1+x)^3}}\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}+x+1}{x+1}$$

input
```
int(1/((-1+x)*(1+x)^3)^(1/2),x)
```

output
```
(sqrt(x + 1)*sqrt(x - 1) + x + 1)/(x + 1)
```

3.300 $\int x \sin^4(x) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 44

$$\int x \sin^4(x) \, dx = \frac{3x^2}{16} - \frac{3}{8} x \cos(x) \sin(x) + \frac{3 \sin^2(x)}{16} - \frac{1}{4} x \cos(x) \sin^3(x) + \frac{\sin^4(x)}{16}$$

output
```
3/16*x^2-3/8*x*cos(x)*sin(x)+3/16*sin(x)^2-1/4*x*cos(x)*sin(x)^3+1/16*sin(
x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int x \sin^4(x) \, dx = \frac{3x^2}{16} - \frac{1}{8} \cos(2x) + \frac{1}{128} \cos(4x) - \frac{1}{4} x \sin(2x) + \frac{1}{32} x \sin(4x)$$

input
```
Integrate[x*Sin[x]^4,x]
```

output
```
(3*x^2)/16 - Cos[2*x]/8 + Cos[4*x]/128 - (x*Sin[2*x])/4 + (x*Sin[4*x])/32
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3791, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^4(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x \sin(x)^4 dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4} \int x \sin^2(x) dx + \frac{\sin^4(x)}{16} - \frac{1}{4} x \sin^3(x) \cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4} \int x \sin(x)^2 dx + \frac{\sin^4(x)}{16} - \frac{1}{4} x \sin^3(x) \cos(x)$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4} \left(\frac{\int x dx}{2} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x) \right) + \frac{\sin^4(x)}{16} - \frac{1}{4} x \sin^3(x) \cos(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4} \left(\frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x) \right) + \frac{\sin^4(x)}{16} - \frac{1}{4} x \sin^3(x) \cos(x)$$

input
```
Int[x*Sin[x]^4,x]
```

output
```
-1/4*(x*Cos[x]*Sin[x]^3) + Sin[x]^4/16 + (3*(x^2/4 - (x*Cos[x]*Sin[x])/2 +
 Sin[x]^2/4))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result
risch	$\frac{3x^2}{16}+\frac{\cos(4x)}{128}+\frac{x\sin(4x)}{32}-\frac{\cos(2x)}{8}-\frac{\sin(2x)x}{4}$
default	$x\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-\frac{3x^2}{16}+\frac{\left(2\cos(x)^2-5\right)^2}{64}$
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)^2}{4}+\frac{3\tan\left(\frac{x}{2}\right)^6}{4}+\frac{5\tan\left(\frac{x}{2}\right)^4}{2}+\frac{3x^2}{16}-\frac{3x\tan\left(\frac{x}{2}\right)}{4}-\frac{11x\tan\left(\frac{x}{2}\right)^3}{4}+\frac{11x\tan\left(\frac{x}{2}\right)^5}{4}+\frac{3x\tan\left(\frac{x}{2}\right)^7}{4}+\frac{3x^2\tan\left(\frac{x}{2}\right)^2}{4}+\frac{9x^2\tan\left(\frac{x}{2}\right)^4}{8}+\frac{3x^2 \mathrm{t}}{}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$
orering	$\frac{(8x^4+15x^2+5)\sin(x)^4}{16x^2}-\frac{5(2x^2+1)\left(\sin(x)^4+4x\cos(x)\sin(x)^3\right)}{16x^2}+\frac{5(x^2+1)\left(8\sin(x)^3\cos(x)-4x\sin(x)^4+12x\cos(x)^2\sin(x)^2\right.}{32x}$

input
```
int(x*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/16*x^2+1/128*cos(4*x)+1/32*x*sin(4*x)-1/8*cos(2*x)-1/4*sin(2*x)*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x \sin^4(x)\, dx = \frac{1}{16} \cos(x)^4 + \frac{3}{16} x^2 - \frac{5}{16} \cos(x)^2 + \frac{1}{8} \left(2 x \cos(x)^3 - 5 x \cos(x)\right) \sin(x)$$

input
```
integrate(x*sin(x)^4,x, algorithm="fricas")
```

output
```
1/16*cos(x)^4 + 3/16*x^2 - 5/16*cos(x)^2 + 1/8*(2*x*cos(x)^3 - 5*x*cos(x))
*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.89

$$\int x \sin^4(x)\, dx = \frac{3x^2 \sin^4(x)}{16} + \frac{3x^2 \sin^2(x) \cos^2(x)}{8} + \frac{3x^2 \cos^4(x)}{16} - \frac{5x \sin^3(x) \cos(x)}{8} - \frac{3x \sin(x) \cos^3(x)}{8} + \frac{5 \sin^4(x)}{32} - \frac{3 \cos^4(x)}{32}$$

input
```
integrate(x*sin(x)**4,x)
```

output
```
3*x**2*sin(x)**4/16 + 3*x**2*sin(x)**2*cos(x)**2/8 + 3*x**2*cos(x)**4/16 -
 5*x*sin(x)**3*cos(x)/8 - 3*x*sin(x)*cos(x)**3/8 + 5*sin(x)**4/32 - 3*cos(
x)**4/32
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int x \sin^4(x)\, dx = \frac{3}{16} x^2 + \frac{1}{32} x \sin(4x) - \frac{1}{4} x \sin(2x) + \frac{1}{128} \cos(4x) - \frac{1}{8} \cos(2x)$$

input
```
integrate(x*sin(x)^4,x, algorithm="maxima")
```

output
```
3/16*x^2 + 1/32*x*sin(4*x) - 1/4*x*sin(2*x) + 1/128*cos(4*x) - 1/8*cos(2*x
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int x\sin^4(x)\,dx=\frac{3}{16}\,x^2+\frac{1}{32}\,x\sin(4\,x)-\frac{1}{4}\,x\sin(2\,x)+\frac{1}{128}\,\cos(4\,x)-\frac{1}{8}\,\cos(2\,x)$$

input
```
integrate(x*sin(x)^4,x, algorithm="giac")
```

output
```
3/16*x^2 + 1/32*x*sin(4*x) - 1/4*x*sin(2*x) + 1/128*cos(4*x) - 1/8*cos(2*x
)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x\sin^4(x)\,dx=\frac{\cos(2\,x)^2}{64}-\frac{x\,\sin(2\,x)}{4}-\frac{\cos(2\,x)}{8}+\frac{3\,x^2}{16}+\frac{x\,\cos(2\,x)\,\sin(2\,x)}{16}$$

input
```
int(x*sin(x)^4,x)
```

output
```
cos(2*x)^2/64 - (x*sin(2*x))/4 - cos(2*x)/8 + (3*x^2)/16 + (x*cos(2*x)*sin
(2*x))/16
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int x\sin^4(x)\,dx = -\frac{\cos(x)\sin(x)^3 x}{4} - \frac{3\cos(x)\sin(x)x}{8} + \frac{\sin(x)^4}{16} + \frac{3\sin(x)^2}{16} + \frac{3x^2}{16} - \frac{3}{16}$$

input
```
int(x*sin(x)^4,x)
```

output
```
( - 4*cos(x)*sin(x)**3*x - 6*cos(x)*sin(x)*x + sin(x)**4 + 3*sin(x)**2 + 3
*x**2 - 3)/16
```

3.301 $\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 6\log(\cos(x)) + 2\log(\sin(x))$$

output
```
6*ln(cos(x))+2*ln(sin(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 6\log(\cos(x)) + 2\log(\sin(x))$$

input
```
Integrate[Cos[3*x]*Csc[x]^2*Sec[x]^3*Sin[2*x],x]
```

output
```
6*Log[Cos[x]] + 2*Log[Sin[x]]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4865, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(3x)\csc^2(x)\sec^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\cos(3x)\csc(x)^2\sec(x)^3dx$$

$$\Big\downarrow \text{4865}$$

$$\int \frac{2\left(1-4\sin^2(x)\right)\csc(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\csc(x)\left(1-4\sin^2(x)\right)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{354}$$

$$\int \frac{\left(1-4\sin^2(x)\right)\csc(x)}{1-\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{3}{\sin^2(x)-1}+\csc(x)\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\log\left(\sin^2(x)\right)+3\log\left(1-\sin^2(x)\right)$$

input
```
Int[Cos[3*x]*Csc[x]^2*Sec[x]^3*Sin[2*x],x]
```

output
```
Log[Sin[x]^2] + 3*Log[1 - Sin[x]^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4865
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((-n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
 /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$-6\ln\left(\tan\left(x\right)\right)+8\ln\left(\sin\left(x\right)\right)$$

input
```
int(cos(3*x)*csc(x)^2*sec(x)^3*sin(2*x),x)
```

output
```
-6*ln(tan(x))+8*ln(sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 3\,\log\left(\cos\left(x\right)^{2}\right)+\log\left(-\frac{1}{4}\,\cos\left(x\right)^{2}+\frac{1}{4}\right)$$

input
```
integrate(cos(3*x)*csc(x)^2*sec(x)^3*sin(2*x),x, algorithm="fricas")
```

output
```
3*log(cos(x)^2) + log(-1/4*cos(x)^2 + 1/4)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = \text{Timed out}$$

input
```
integrate(cos(3*x)*csc(x)**2*sec(x)**3*sin(2*x),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 3\log\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right) + \log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(3*x)*csc(x)^2*sec(x)^3*sin(2*x),x, algorithm="maxima")
```

output
```
3*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + log(cos(x)^2 + sin(x)^2
+ 2*cos(x) + 1) + log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 3\log\left(-\sin(x)^2 + 1\right) + 2\log\left(|\sin(x)|\right)$$

input
```
integrate(cos(3*x)*csc(x)^2*sec(x)^3*sin(2*x),x, algorithm="giac")
```

output
```
3*log(-sin(x)^2 + 1) + 2*log(abs(sin(x)))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = 6\ln\left(\cos(x)\right) + \ln\left(\sin(x)^2\right)$$

input
```
int((cos(3*x)*sin(2*x))/(cos(x)^3*sin(x)^2),x)
```

output
```
6*log(cos(x)) + log(sin(x)^2)
```

Reduce [F]

$$\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx = \int \cos{(3x)}\csc{(x)}^2\sec{(x)}^3\sin{(2x)}\,dx$$

input
```
int(cos(3*x)*csc(x)^2*sec(x)^3*sin(2*x),x)
```

output
```
int(cos(3*x)*csc(x)**2*sec(x)**3*sin(2*x),x)
```

3.302 $\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 13

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right) dx=\sqrt{2}x\sqrt{\log(x)}$$

output
```
2^(1/2)*x*ln(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right) dx=\sqrt{2}x\sqrt{\log(x)}$$

input
```
Integrate[1/(Sqrt[2]*Sqrt[Log[x]]) + Sqrt[2]*Sqrt[Log[x]],x]
```

output
```
Sqrt[2]*x*Sqrt[Log[x]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{2}\sqrt{\log(x)}+\frac{1}{\sqrt{2}\sqrt{\log(x)}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\sqrt{2}x\sqrt{\log(x)}$$

input
```
Int[1/(Sqrt[2]*Sqrt[Log[x]]) + Sqrt[2]*Sqrt[Log[x]],x]
```

output
```
Sqrt[2]*x*Sqrt[Log[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{\sqrt{2}}{2\sqrt{\ln(x)}}+\sqrt{2}\,\sqrt{\ln(x)}\right) dx$$

input
```
int(1/2*2^(1/2)/ln(x)^(1/2)+2^(1/2)*ln(x)^(1/2),x)
```

output
```
int(1/2*2^(1/2)/ln(x)^(1/2)+2^(1/2)*ln(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/2*2^(1/2)/log(x)^(1/2)+2^(1/2)*log(x)^(1/2),x, algorithm="fric
as")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right)\,dx =\frac{\sqrt{2}\sqrt{\pi}\sqrt{-\log{(x)}}\operatorname{erfc}\left(\sqrt{-\log{(x)}}\right)}{2\sqrt{\log{(x)}}}+\frac{\sqrt{2}\left(x\sqrt{-\log{(x)}}+\frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-\log{(x)}}\right)}{2}\right)\sqrt{\log{(x)}}}{\sqrt{-\log{(x)}}}$$

input
```
integrate(1/2*2**(1/2)/ln(x)**(1/2)+2**(1/2)*ln(x)**(1/2),x)
```

output
```
sqrt(2)*sqrt(pi)*sqrt(-log(x))*erfc(sqrt(-log(x)))/(2*sqrt(log(x))) + sqrt
(2)*(x*sqrt(-log(x)) + sqrt(pi)*erfc(sqrt(-log(x)))/2)*sqrt(log(x))/sqrt(-
log(x))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right)\,dx=-\frac{1}{2}i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{\log\left(x\right)}\right)-\frac{1}{2}\,\sqrt{2}\left(-i\,\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{\log\left(x\right)}\right)-2\,x\sqrt{\log\left(x\right)}\right)$$

input
```
integrate(1/2*2^(1/2)/log(x)^(1/2)+2^(1/2)*log(x)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*I*sqrt(2)*sqrt(pi)*erf(I*sqrt(log(x))) - 1/2*sqrt(2)*(-I*sqrt(pi)*erf
(I*sqrt(log(x))) - 2*x*sqrt(log(x)))
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right)\,dx=\frac{1}{2}i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-i\,\sqrt{\log\left(x\right)}\right)-\frac{1}{2}\,\sqrt{2}\left(i\,\sqrt{\pi}\,\mathrm{erf}\left(-i\,\sqrt{\log\left(x\right)}\right)-2\,x\sqrt{\log\left(x\right)}\right)$$

input
```
integrate(1/2*2^(1/2)/log(x)^(1/2)+2^(1/2)*log(x)^(1/2),x, algorithm="giac
")
```

output
```
1/2*I*sqrt(2)*sqrt(pi)*erf(-I*sqrt(log(x))) - 1/2*sqrt(2)*(I*sqrt(pi)*erf(
-I*sqrt(log(x))) - 2*x*sqrt(log(x)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}} + \sqrt{2}\sqrt{\log(x)}\right) dx = \sqrt{2}\,x\,\sqrt{\ln(x)}$$

input
```
int(2^(1/2)/(2*log(x)^(1/2)) + 2^(1/2)*log(x)^(1/2),x)
```

output
```
2^(1/2)*x*log(x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}} + \sqrt{2}\sqrt{\log(x)}\right) dx = \sqrt{\log(x)}\,\sqrt{2}\,x$$

input
```
int(1/2*2^(1/2)/log(x)^(1/2)+2^(1/2)*log(x)^(1/2),x)
```

output
```
sqrt(log(x))*sqrt(2)*x
```

3.303 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

output
```
-x+tan(x)+ln(cos(x))*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x)) \sec^2(x) \, dx = -x + \tan(x) + \log(\cos(x)) \tan(x)$$

input
```
Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3034, 25, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x) \log(\cos(x)) \, dx$$

$$\Big\downarrow \text{3034}$$

$$\tan(x) \log(\cos(x)) - \int -\tan^2(x) dx$$

$$\Big\downarrow \text{25}$$

$$\int \tan^2(x) dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2 dx + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{3954}$$

$$-\int 1 dx + \tan(x) + \tan(x) \log(\cos(x))$$

$$\Big\downarrow \text{24}$$

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

input
```
Int[Log[Cos[x]]*Sec[x]^2,x]
```

output
```
-x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3034
```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Simp[Log[u]   w, x
] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w,
x]] /; InverseFunctionFreeQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result
parallelrisch	$\frac{\ln(\cos(x))\sin(x)-x\cos(x)+\sin(x)}{\cos(x)}$
norman	$\frac{x-x\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(e^{2ix}+1\right)e^{-ix}\right)}{2}-\frac{1}{2}}{e^{2ix}+1}-\frac{\ln\left(e^{2ix}+1\right)}{4}+\frac{\ln(2)}{2\,e^{2ix}+2}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{e^{2ix}+1}+\frac{\pi\ \mathrm{csgn}\left(i\left(e^{2ix}+1\right)\right)\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)-\pi\ \mathrm{csgn}\left(i\left(e^{2ix}+1\right)\right)\mathrm{csgn}\left(i\cos(x)\right)^{2}-\pi\ \mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(i\,\mathrm{co}}{e^{2ix}+1}$

input
```
int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(cos(x))*sin(x)-x*cos(x)+sin(x))/cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \log(\cos(x))\sec^2(x)\,dx = -x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

input
```
integrate(ln(cos(x))*sec(x)**2,x)
```

output
```
-x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 7.83

$$\int \log(\cos(x))\sec^2(x)\,dx = -\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

output
```
-2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x
)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(
x) + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \log(\cos(x))\sec^2(x)\,dx = \log(\cos(x))\tan(x) - x + \tan(x)$$

input
```
integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

output
```
log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

$$\int \log(\cos(x))\sec^2(x)\,dx = \tan(x) - 2x + \ln(\cos(x))\tan(x) + \ln(\cos(x))\,1i - \ln(\cos(2x) + 1 + \sin(2x)\,1i)\,1i$$

input
```
int(log(cos(x))/cos(x)^2,x)
```

output
```
log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(c
os(x))*tan(x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \log(\cos(x))\sec^2(x)\,dx = \frac{-\cos(x)\,x+\log\left(\frac{-\tan\left(\frac{x}{2}\right)^2+1}{\tan\left(\frac{x}{2}\right)^2+1}\right)\sin(x)+\sin(x)}{\cos(x)}$$

input
```
int(log(cos(x))*sec(x)^2,x)
```

output
```
( - cos(x)*x + log(( - tan(x/2)**2 + 1)/(tan(x/2)**2 + 1))*sin(x) + sin(x)
)/cos(x)
```

3.304 $\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 19

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx=\frac{1}{1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}}$$

output

```
1/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-1+x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx=\frac{1}{1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}}$$

input

```
Integrate[(5/(-5 + x)^6 + 3/(-3 + x)^4 + (-1 + x)^(-2))/(1 + (-5 + x)^(-5)
 + (-3 + x)^(-3) + (-1 + x)^(-1))^2,x]
```

output
```
(1 + (-5 + x)^(-5) + (-3 + x)^(-3) + (-1 + x)^(-1))^(-1)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{3}{(x-3)^4}+\frac{5}{(x-5)^6}+\frac{1}{(x-1)^2}}{\left(\frac{1}{(x-3)^3}+\frac{1}{(x-5)^5}+\frac{1}{x-1}+1\right)^2}dx$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{\frac{1}{(x-3)^3}+\frac{1}{(x-5)^5}+\frac{1}{x-1}+1}$$

input
```
Int[(5/(-5 + x)^6 + 3/(-3 + x)^4 + (-1 + x)^(-2))/(1 + (-5 + x)^(-5) + (-3
 + x)^(-3) + (-1 + x)^(-1))^2,x]
```

output
```
(1 + (-5 + x)^(-5) + (-3 + x)^(-3) + (-1 + x)^(-1))^(-1)
```

Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.53

method	result
gosper	$-\frac{x^8-34x^7+503x^6-4228x^5+22076x^4-73260x^3+150661x^2-175054x+87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
default	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
risch	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
parallelrisch	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
norman	$\frac{-x^{17}+69x^{16}-2229x^{15}+44761x^{14}-625602x^{13}+6455858x^{12}-50913715x^{11}+313280159x^{10}-1521750430x^9+5864557678x^8}{(-5+x)^5(-3+x)^3(-1+x)(x^9-34x^8+502x^7-4201}$
orering	$-\frac{(x^8-34x^7+503x^6-4228x^5+22076x^4-73260x^3+150661x^2-175054x+87527)(x^9-34x^8+502x^7-4201x^6+21774x^5-714}{(-5+x)^4(x^{10}-42x^9+792x^8-8824x^7+64259x^6-319090x^5+1091895x^4-2536140x^3+3817380x^2-3357}$

input
```
int((5/(-5+x)^6+3/(-3+x)^4+1/(-1+x)^2)/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-1+x))^
2,x,method=_RETURNVERBOSE)
```

output
```
-(x^8-34*x^7+503*x^6-4228*x^5+22076*x^4-73260*x^3+150661*x^2-175054*x+8752
7)/(x^9-34*x^8+502*x^7-4201*x^6+21774*x^5-71474*x^4+144740*x^3-164339*x^2+
78071*x+3152)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.47

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx = -\frac{x^8-34\,x^7+503\,x^6-4228\,x^5+22076\,x^4-73260\,x^3+150661\,x^2-175054\,x+87527}{x^9-34\,x^8+502\,x^7-4201\,x^6+21774\,x^5-71474\,x^4+144740\,x^3-164339\,x^2+78071\,x+3152}$$

input
```
integrate((5/(-5+x)^6+3/(-3+x)^4+1/(-1+x)^2)/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-
1+x))^2,x, algorithm="fricas")
```

output
```
-(x^8 - 34*x^7 + 503*x^6 - 4228*x^5 + 22076*x^4 - 73260*x^3 + 150661*x^2 -
 175054*x + 87527)/(x^9 - 34*x^8 + 502*x^7 - 4201*x^6 + 21774*x^5 - 71474*
x^4 + 144740*x^3 - 164339*x^2 + 78071*x + 3152)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(20) = 40$.

Time = 0.63 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.32

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx = \frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$$

input
```
integrate((5/(-5+x)**6+3/(-3+x)**4+1/(-1+x)**2)/(1+1/(-5+x)**5+1/(-3+x)**3
+1/(-1+x))**2,x)
```

output
```
(-x**8 + 34*x**7 - 503*x**6 + 4228*x**5 - 22076*x**4 + 73260*x**3 - 150661
*x**2 + 175054*x - 87527)/(x**9 - 34*x**8 + 502*x**7 - 4201*x**6 + 21774*x
**5 - 71474*x**4 + 144740*x**3 - 164339*x**2 + 78071*x + 3152)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx = \frac{1}{\frac{1}{x-1}+\frac{1}{(x-3)^3}+\frac{1}{(x-5)^5}+1}$$

input
```
integrate((5/(-5+x)^6+3/(-3+x)^4+1/(-1+x)^2)/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-
1+x))^2,x, algorithm="maxima")
```

output
```
1/(1/(x - 1) + 1/(x - 3)^3 + 1/(x - 5)^5 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.47

$$\int \frac{\frac{5}{(-5+x)^6} + \frac{3}{(-3+x)^4} + \frac{1}{(-1+x)^2}}{\left(1 + \frac{1}{(-5+x)^5} + \frac{1}{(-3+x)^3} + \frac{1}{-1+x}\right)^2} dx = -\frac{x^8 - 34x^7 + 503x^6 - 4228x^5 + 22076x^4 - 73260x^3 + 150661x^2 - 175054x + 87527}{x^9 - 34x^8 + 502x^7 - 4201x^6 + 21774x^5 - 71474x^4 + 144740x^3 - 164339x^2 + 78071x + 3152}$$

input
```
integrate((5/(-5+x)^6+3/(-3+x)^4+1/(-1+x)^2)/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-
1+x))^2,x, algorithm="giac")
```

output
```
-(x^8 - 34*x^7 + 503*x^6 - 4228*x^5 + 22076*x^4 - 73260*x^3 + 150661*x^2 -
 175054*x + 87527)/(x^9 - 34*x^8 + 502*x^7 - 4201*x^6 + 21774*x^5 - 71474*
x^4 + 144740*x^3 - 164339*x^2 + 78071*x + 3152)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.47

$$\int \frac{\frac{5}{(-5+x)^6} + \frac{3}{(-3+x)^4} + \frac{1}{(-1+x)^2}}{\left(1 + \frac{1}{(-5+x)^5} + \frac{1}{(-3+x)^3} + \frac{1}{-1+x}\right)^2} dx = -\frac{x^8 - 34x^7 + 503x^6 - 4228x^5 + 22076x^4 - 73260x^3 + 150661x^2 - 175054x + 87527}{x^9 - 34x^8 + 502x^7 - 4201x^6 + 21774x^5 - 71474x^4 + 144740x^3 - 164339x^2 + 78071x + 3152}$$

input
```
int((1/(x - 1)^2 + 3/(x - 3)^4 + 5/(x - 5)^6)/(1/(x - 1) + 1/(x - 3)^3 + 1
/(x - 5)^5 + 1)^2,x)
```

output
```
-(150661*x^2 - 175054*x - 73260*x^3 + 22076*x^4 - 4228*x^5 + 503*x^6 - 34*
x^7 + x^8 + 87527)/(78071*x - 164339*x^2 + 144740*x^3 - 71474*x^4 + 21774*
x^5 - 4201*x^6 + 502*x^7 - 34*x^8 + x^9 + 3152)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.63

$$\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx$$
$$=\frac{-x^9+654x^7-12901x^6+121978x^5-679110x^4+2346100x^3-4958135x^2+5873765x-29790}{34x^9-1156x^8+17068x^7-142834x^6+740316x^5-2430116x^4+4921160x^3-5587526x^2+2654414x}$$

input
```
int((5/(-5+x)^6+3/(-3+x)^4+1/(-1+x)^2)/(1+1/(-5+x)^5+1/(-3+x)^3+1/(-1+x))^
2,x)
```

output
```
( - x**9 + 654*x**7 - 12901*x**6 + 121978*x**5 - 679110*x**4 + 2346100*x**
3 - 4958135*x**2 + 5873765*x - 2979070)/(34*(x**9 - 34*x**8 + 502*x**7 - 4
201*x**6 + 21774*x**5 - 71474*x**4 + 144740*x**3 - 164339*x**2 + 78071*x +
 3152))
```

3.305 $\int \csc(x)\sin(23x)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 86

$$\int \csc(x)\sin(23x)\,dx = x+\sin(2x)+\frac{1}{2}\sin(4x)+\frac{1}{3}\sin(6x)+\frac{1}{4}\sin(8x) +\frac{1}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{1}{7}\sin(14x)+\frac{1}{8}\sin(16x) +\frac{1}{9}\sin(18x)+\frac{1}{10}\sin(20x)+\frac{1}{11}\sin(22x)$$

output
```
x+sin(2*x)+1/2*sin(4*x)+1/3*sin(6*x)+1/4*sin(8*x)+1/5*sin(10*x)+1/6*sin(12
*x)+1/7*sin(14*x)+1/8*sin(16*x)+1/9*sin(18*x)+1/10*sin(20*x)+1/11*sin(22*x
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \csc(x)\sin(23x)\,dx = x+\sin(2x)+\frac{1}{2}\sin(4x)+\frac{1}{3}\sin(6x)+\frac{1}{4}\sin(8x) +\frac{1}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{1}{7}\sin(14x)+\frac{1}{8}\sin(16x) +\frac{1}{9}\sin(18x)+\frac{1}{10}\sin(20x)+\frac{1}{11}\sin(22x)$$

input
```
Integrate[Csc[x]*Sin[23*x],x]
```

output
```
x + Sin[2*x] + Sin[4*x]/2 + Sin[6*x]/3 + Sin[8*x]/4 + Sin[10*x]/5 + Sin[12
*x]/6 + Sin[14*x]/7 + Sin[16*x]/8 + Sin[18*x]/9 + Sin[20*x]/10 + Sin[22*x]
/11
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(86) = 172$.

Time = 2.01 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.21, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 3.429$, Rules used = {3042, 4889, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 1471, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(23x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(23x)}{\sin(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{-\tan^{22}(x)+253\tan^{20}(x)-8855\tan^{18}(x)+100947\tan^{16}(x)-490314\tan^{14}(x)+1144066\tan^{12}(x)-13520}{\left(\tan^2(x)+1\right)^{12}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}-$$
$$\frac{1}{22}\int \frac{2\left(11\tan^{20}(x)-2794\tan^{18}(x)+100199\tan^{16}(x)-1210616\tan^{14}(x)+6604070\tan^{12}(x)-19188796\tan^{1}\right.}{\left(\tan^2(x)+\right.}$$

$$\Big\downarrow \text{27}$$

$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}-$$
$$\frac{1}{11}\int \frac{11\tan^{20}(x)-2794\tan^{18}(x)+100199\tan^{16}(x)-1210616\tan^{14}(x)+6604070\tan^{12}(x)-19188796\tan^{10}(\ldots}{\left(\tan^2(x)+1\right)\ldots}$$

$\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{20}\int \frac{4\left(-55\tan^{18}(x)+14025\tan^{16}(x)-515020\tan^{14}(x)+6568100\tan^{12}(x)-39588450\tan^{10}(x)+1355\ldots}{\left(\tan^2(x)+1\ldots}\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\int \frac{-55\tan^{18}(x)+14025\tan^{16}(x)-515020\tan^{14}(x)+6568100\tan^{12}(x)-39588450\tan^{10}(x)+1355324\ldots}{\left(\tan^2(x)+1\right)^{10}\ldots}\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{899022848\tan(x)}{9\left(\tan^2(x)+1\right)^9}-\frac{1}{18}\int \frac{2\left(495\tan^{16}(x)-126720\tan^{14}(x)+4761900\tan^{12}(x)-63874800\tan^{10}(x)+\ldots}{\ldots}\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{899022848\tan(x)}{9\left(\tan^2(x)+1\right)^9}-\frac{1}{9}\int \frac{495\tan^{16}(x)-126720\tan^{14}(x)+4761900\tan^{12}(x)-63874800\tan^{10}(x)+42\ldots}{\ldots}\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(\frac{1}{16}\int \frac{240\left(-33\tan^{14}(x)+8481\tan^{12}(x)-325941\tan^{10}(x)+4584261\tan^{8}(x)-32595651\tan^{6}(x)+\ldots}{\left(\tan^2(x)+1\right)^8}\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\int\frac{-33\tan^{14}(x)+8481\tan^{12}(x)-325941\tan^{10}(x)+4584261\tan^{8}(x)-32595651\tan^{6}(x)+141}{\left(\tan^2(x)+1\right)^8}\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\big\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{322500608\tan(x)}{7\left(\tan^2(x)+1\right)^7}-\frac{1}{14}\int\frac{2\left(231\tan^{12}(x)-59598\tan^{10}(x)+2341185\tan^{8}(x)-34431012\tan^{6}\right.}{\left(\tan^2(x)+\right.}\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{322500608\tan(x)}{7\left(\tan^2(x)+1\right)^7}-\frac{1}{7}\int\frac{231\tan^{12}(x)-59598\tan^{10}(x)+2341185\tan^{8}(x)-34431012\tan^{6}(x)}{\left(\tan^2(x)+1\right)}\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\big\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{1}{12}\int\frac{44\left(-63\tan^{10}(x)+16317\tan^{8}(x)-654822\tan^{6}(x)+10045098\tan^{4}(x)-81663435\tan\right.}{\left(\tan^2(x)+1\right)^6}\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\int\frac{-63\tan^{10}(x)+16317\tan^{8}(x)-654822\tan^{6}(x)+10045098\tan^{4}(x)-81663435\tan^{2}(}{\left(\tan^2(x)+1\right)^6}\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\big\downarrow$ 2345

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-\frac{1}{10}\int\frac{90\left(7\tan^{8}(x)-1820\tan^{6}(x)+74578\tan^{4}(x)-1190700\tan^{2}(\right.}{\left(\tan^2(x)+1\right)^5}\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-9\int \frac{7\tan^8(x)-1820\tan^6(x)+74578\tan^4(x)-1190700\tan^2(x)+}{\left(\tan^2(x)+1\right)^5}\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 2345$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-9\left(\frac{178880\tan(x)}{\left(\tan^2(x)+1\right)^4}-\frac{1}{8}\int \frac{56\left(-\tan^6(x)+261\tan^4(x)-10915\tan^2(\right.}{\left(\tan^2(x)+1\right)^4}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-9\left(\frac{178880\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\int \frac{-\tan^6(x)+261\tan^4(x)-10915\tan^2(x)-}{\left(\tan^2(x)+1\right)^4}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 2345$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-9\left(\frac{178880\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\left(\frac{6656\tan(x)}{3\left(\tan^2(x)+1\right)^3}-\frac{1}{6}\int \frac{2\left(3\tan^4(x)-786\,\mathrm{t}\right.}{\left(\tan^2(x)\right.}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{\left(\tan^2(x)+1\right)^5}-9\left(\frac{178880\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\left(\frac{6656\tan(x)}{3\left(\tan^2(x)+1\right)^3}-\frac{1}{3}\int \frac{3\tan^4(x)-786\tan}{\left(\tan^2(x)+\right.}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$$\Big\downarrow \; 1471$$

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{(\tan^2(x)+1)^5}-9\left(\frac{178880\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{1}{3}\left(\frac{1}{4}\int\frac{12(3-\tan^2(x))}{(\tan^2(x)+1)^2}d\tan(x)-\frac{260\,\mathrm{t}}{(\tan^2(x}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{(\tan^2(x)+1)^5}-9\left(\frac{178880\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{1}{3}\left(3\int\frac{3-\tan^2(x)}{(\tan^2(x)+1)^2}d\tan(x)-\frac{260\,\mathrm{tan}}{(\tan^2(x)}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\Big\downarrow$ 298

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{(\tan^2(x)+1)^5}-9\left(\frac{178880\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{1}{3}\left(3\left(\int\frac{1}{\tan^2(x)+1}d\tan(x)+\frac{2\tan(x)}{\tan^2(x)+1}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

$\Big\downarrow$ 216

$$\frac{1}{11}\left(\frac{1}{5}\left(\frac{1}{9}\left(15\left(\frac{1}{7}\left(\frac{11}{3}\left(\frac{10100480\tan(x)}{(\tan^2(x)+1)^5}-9\left(\frac{178880\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{1}{3}\left(3\left(\arctan(\tan(x))+\frac{2\tan(x)}{\tan^2(x)+1}\right)-\frac{}{(\mathrm{t}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{2097152\tan(x)}{11\left(\tan^2(x)+1\right)^{11}}$$

input
```
Int[Csc[x]*Sin[23*x],x]
```

output
```
(2097152*Tan[x])/(11*(1 + Tan[x]^2)^11) + ((-49545216*Tan[x])/(5*(1 + Tan[
x]^2)^10) + ((899022848*Tan[x])/(9*(1 + Tan[x]^2)^9) + ((-1009455104*Tan[x
])/(1 + Tan[x]^2)^8 + 15*((322500608*Tan[x])/(7*(1 + Tan[x]^2)^7) + ((-415
607296*Tan[x])/(3*(1 + Tan[x]^2)^6) + (11*((10100480*Tan[x])/(1 + Tan[x]^2
)^5 - 9*((178880*Tan[x])/(1 + Tan[x]^2)^4 - 7*((6656*Tan[x])/(3*(1 + Tan[x
]^2)^3) + ((-260*Tan[x])/(1 + Tan[x]^2)^2 + 3*(ArcTan[Tan[x]] + (2*Tan[x])
/(1 + Tan[x]^2)))/3))))/3)/7))/9)/5)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

method	result
risch	$x+\sin(2x)+\frac{\sin(4x)}{2}+\frac{\sin(6x)}{3}+\frac{\sin(8x)}{4}+\frac{\sin(10x)}{5}+\frac{\sin(12x)}{6}+\frac{\sin(14x)}{7}+\frac{\sin(16x)}{8}+\frac{\sin(18x)}{9}+\frac{\sin(20x)}{10}$

input
```
int(csc(x)*sin(23*x),x,method=_RETURNVERBOSE)
```

output
```
x+sin(2*x)+1/2*sin(4*x)+1/3*sin(6*x)+1/4*sin(8*x)+1/5*sin(10*x)+1/6*sin(12
*x)+1/7*sin(14*x)+1/8*sin(16*x)+1/9*sin(18*x)+1/10*sin(20*x)+1/11*sin(22*x
)
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \csc(x)\sin(23x)\,dx = \frac{2}{3465}\left(330301440\cos(x)^{21}-1560674304\cos(x)^{19}+3146579968\cos(x)^{17}-3533092864\cos(x)^{15}+24\right. + x$$

input
```
integrate(csc(x)*sin(23*x),x, algorithm="fricas")
```

output
```
2/3465*(330301440*cos(x)^21 - 1560674304*cos(x)^19 + 3146579968*cos(x)^17
- 3533092864*cos(x)^15 + 2418754560*cos(x)^13 - 1039018240*cos(x)^11 + 277
763200*cos(x)^9 - 44272800*cos(x)^7 + 3843840*cos(x)^5 - 150150*cos(x)^3 +
 3465*cos(x))*sin(x) + x
```

Sympy [A] (verification not implemented)

Time = 73.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \csc(x)\sin(23x)\,dx = x - \frac{1024\sin^{11}(2x)}{11} + \frac{2560\sin^{9}(2x)}{9} - \frac{2304\sin^{7}(2x)}{7} + \frac{896\sin^{5}(2x)}{5} - \frac{140\sin^{3}(2x)}{3} + 6\sin(2x) + \frac{8\sin^{5}(4x)}{5} - \frac{8\sin^{3}(4x)}{3} + \frac{3\sin(4x)}{2} + \frac{\sin(8x)}{4} + \frac{\sin(16x)}{8}$$

input
```
integrate(csc(x)*sin(23*x),x)
```

output
```
x - 1024*sin(2*x)**11/11 + 2560*sin(2*x)**9/9 - 2304*sin(2*x)**7/7 + 896*s
in(2*x)**5/5 - 140*sin(2*x)**3/3 + 6*sin(2*x) + 8*sin(4*x)**5/5 - 8*sin(4*
x)**3/3 + 3*sin(4*x)/2 + sin(8*x)/4 + sin(16*x)/8
```

Maxima [F(-1)]

Timed out.

$$\int \csc(x)\sin(23x)\,dx = \text{Timed out}$$

input
```
integrate(csc(x)*sin(23*x),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \csc(x)\sin(23x)\,dx = x + \frac{1}{11}\sin(22x) + \frac{1}{10}\sin(20x) + \frac{1}{9}\sin(18x) + \frac{1}{8}\sin(16x) + \frac{1}{7}\sin(14x) + \frac{1}{6}\sin(12x) + \frac{1}{5}\sin(10x) + \frac{1}{4}\sin(8x) + \frac{1}{3}\sin(6x) + \frac{1}{2}\sin(4x) + \sin(2x)$$

input
```
integrate(csc(x)*sin(23*x),x, algorithm="giac")
```

output
```
x + 1/11*sin(22*x) + 1/10*sin(20*x) + 1/9*sin(18*x) + 1/8*sin(16*x) + 1/7*
sin(14*x) + 1/6*sin(12*x) + 1/5*sin(10*x) + 1/4*sin(8*x) + 1/3*sin(6*x) +
1/2*sin(4*x) + sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \csc(x)\sin(23x)\,dx = x+\sin\left(2\,x\right)+\frac{\sin\left(4\,x\right)}{2}+\frac{\sin\left(6\,x\right)}{3}+\frac{\sin\left(8\,x\right)}{4}+\frac{\sin\left(10\,x\right)}{5}+\frac{\sin\left(12\,x\right)}{6}+\frac{\sin\left(14\,x\right)}{7}+\frac{\sin\left(16\,x\right)}{8}+\frac{\sin\left(18\,x\right)}{9}+\frac{\sin\left(20\,x\right)}{10}+\frac{\sin\left(22\,x\right)}{11}$$

input
```
int(sin(23*x)/sin(x),x)
```

output
```
x + sin(2*x) + sin(4*x)/2 + sin(6*x)/3 + sin(8*x)/4 + sin(10*x)/5 + sin(12
*x)/6 + sin(14*x)/7 + sin(16*x)/8 + sin(18*x)/9 + sin(20*x)/10 + sin(22*x)
/11
```

Reduce [F]

$$\int \csc(x)\sin(23x)\,dx = \int \csc\left(x\right)\sin\left(23x\right)dx$$

input
```
int(csc(x)*sin(23*x),x)
```

output
```
int(csc(x)*sin(23*x),x)
```

3.306 $\int \frac{(1-x)^2x^4}{1+x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{(1-x)^2x^4}{1+x^2} \, dx = x^2 - \frac{x^4}{2} + \frac{x^5}{5} - \log\left(1+x^2\right)$$

output
```
x^2-1/2*x^4+1/5*x^5-ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(1-x)^2x^4}{1+x^2} \, dx = x^2 - \frac{x^4}{2} + \frac{x^5}{5} - \log\left(1+x^2\right)$$

input
```
Integrate[((1 - x)^2*x^4)/(1 + x^2),x]
```

output
```
x^2 - x^4/2 + x^5/5 - Log[1 + x^2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {525, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^2x^4}{x^2+1}dx$$

$$\Big\downarrow \text{ 525}$$

$$\int -\frac{2x^5}{x^2+1}dx+\frac{x^5}{5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5}{5}-2\int \frac{x^5}{x^2+1}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{x^5}{5}-\int \frac{x^4}{x^2+1}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{x^5}{5}-\int \left(x^2+\frac{1}{x^2+1}-1\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^5}{5}-\frac{x^4}{2}+x^2-\log\left(x^2+1\right)$$

input
```
Int[((1 - x)^2*x^4)/(1 + x^2),x]
```

output
```
x^2 - x^4/2 + x^5/5 - Log[1 + x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$x^{2}-\frac{x^{4}}{2}+\frac{x^{5}}{5}-\ln \left(x^{2}+1\right)$	23
norman	$x^{2}-\frac{x^{4}}{2}+\frac{x^{5}}{5}-\ln \left(x^{2}+1\right)$	23
risch	$x^{2}-\frac{x^{4}}{2}+\frac{x^{5}}{5}-\ln \left(x^{2}+1\right)$	23
parallelrisch	$x^{2}-\frac{x^{4}}{2}+\frac{x^{5}}{5}-\ln \left(x^{2}+1\right)$	23
meijerg	$-\frac{x \left(-5 x^{2}+15\right)}{15}+\frac{x^{2} \left(-3 x^{2}+6\right)}{6}-\ln \left(x^{2}+1\right)+\frac{x \left(21 x^{4}-35 x^{2}+105\right)}{105}$	47

input
```
int((1-x)^2*x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x^2-1/2*x^4+1/5*x^5-ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^2x^4}{1+x^2}\,dx = \frac{1}{5}\,x^5 - \frac{1}{2}\,x^4 + x^2 - \log\left(x^2+1\right)$$

input
```
integrate((1-x)^2*x^4/(x^2+1),x, algorithm="fricas")
```

output
```
1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{(1-x)^2x^4}{1+x^2}\,dx = \frac{x^5}{5} - \frac{x^4}{2} + x^2 - \log\left(x^2+1\right)$$

input
```
integrate((1-x)**2*x**4/(x**2+1),x)
```

output
```
x**5/5 - x**4/2 + x**2 - log(x**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^2 x^4}{1+x^2} dx = \frac{1}{5} x^5 - \frac{1}{2} x^4 + x^2 - \log(x^2+1)$$

input
```
integrate((1-x)^2*x^4/(x^2+1),x, algorithm="maxima")
```

output
```
1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^2 x^4}{1+x^2} dx = \frac{1}{5} x^5 - \frac{1}{2} x^4 + x^2 - \log(x^2+1)$$

input
```
integrate((1-x)^2*x^4/(x^2+1),x, algorithm="giac")
```

output
```
1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^2 x^4}{1+x^2} dx = x^2 - \ln(x^2+1) - \frac{x^4}{2} + \frac{x^5}{5}$$

input
```
int((x^4*(x - 1)^2)/(x^2 + 1),x)
```

output
```
x^2 - log(x^2 + 1) - x^4/2 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(1-x)^2x^4}{1+x^2}\,dx = -\log\left(x^2+1\right)+\frac{x^5}{5}-\frac{x^4}{2}+x^2$$

input
```
int((1-x)^2*x^4/(x^2+1),x)
```

output
```
( - 10*log(x**2 + 1) + 2*x**5 - 5*x**4 + 10*x**2)/10
```

3.307 $\int x^{-\log(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int x^{-\log(x)}\,dx = -\frac{1}{2}\sqrt[4]{e}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}-\log(x)\right)$$

output
```
1/2*exp(1/4)*Pi^(1/2)*erf(-1/2+ln(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^{-\log(x)}\,dx = \frac{1}{2}\sqrt[4]{e}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}(-1+2\log(x))\right)$$

input
```
Integrate[x^(-Log[x]),x]
```

output
```
(E^(1/4)*Sqrt[Pi]*Erf[(-1 + 2*Log[x])/2])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\log(x)}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int x^{-\log(x)}dx$$

input
```
Int[x^(-Log[x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int x^{-\ln(x)}dx$$

input
```
int(x^(-ln(x)),x)
```

output
```
int(x^(-ln(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x^{-\log(x)}\,dx = \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}\left(\log(x) - \frac{1}{2}\right)e^{\frac{1}{4}}$$

input
```
integrate(x^(-log(x)),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erf(log(x) - 1/2)*e^(1/4)
```

Sympy [F]

$$\int x^{-\log(x)}\,dx = \int x^{-\log{(x)}}\,dx$$

input
```
integrate(x**(-ln(x)),x)
```

output
```
Integral(x**(-log(x)), x)
```

Maxima [F]

$$\int x^{-\log(x)}\,dx = \int \frac{1}{x^{\log(x)}}\,dx$$

input
```
integrate(x^(-log(x)),x, algorithm="maxima")
```

output
```
integrate(1/(x^log(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x^{-\log(x)}\,dx = \frac{1}{2}\,\sqrt{\pi}\,\mathrm{erf}\left(\log{(x)} - \frac{1}{2}\right)e^{\frac{1}{4}}$$

input
```
integrate(x^(-log(x)),x, algorithm="giac")
```

output
```
1/2*sqrt(pi)*erf(log(x) - 1/2)*e^(1/4)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-\log(x)}\,dx = \int e^{-\ln(x)^2}\,dx$$

input
```
int(1/x^log(x),x)
```

output
```
int(exp(-log(x)^2), x)
```

Reduce [F]

$$\int x^{-\log(x)}\,dx = \int \frac{1}{x^{\log(x)}}dx$$

input
```
int(x^(-log(x)),x)
```

output
```
int(1/x**log(x),x)
```

3.308 $\int \frac{1-2x}{x^{2/3}(1+x)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1-2x}{x^{2/3}(1+x)^2} \, dx = \frac{3\sqrt[3]{x}}{1+x}$$

output
```
3*x^(1/3)/(1+x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1-2x}{x^{2/3}(1+x)^2} \, dx = \frac{3\sqrt[3]{x}}{1+x}$$

input
```
Integrate[(1 - 2*x)/(x^(2/3)*(1 + x)^2),x]
```

output
```
(3*x^(1/3))/(1 + x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x}{x^{2/3}(x+1)^2} \, dx$$

$$\Big\downarrow \; 83$$

$$\frac{3\sqrt[3]{x}}{x+1}$$

input
```
Int[(1 - 2*x)/(x^(2/3)*(1 + x)^2),x]
```

output
```
(3*x^(1/3))/(1 + x)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{3x^{\frac{1}{3}}}{1+x}$
trager	$\frac{3x^{\frac{1}{3}}}{1+x}$
risch	$\frac{3x^{\frac{1}{3}}}{1+x}$
orering	$-\frac{3x^{\frac{1}{3}}(1-2x)}{(1+x)(-1+2x)}$
derivativedivides	$-\frac{1}{1+x^{\frac{1}{3}}}-\frac{-x^{\frac{1}{3}}-1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}$
default	$-\frac{1}{1+x^{\frac{1}{3}}}-\frac{-x^{\frac{1}{3}}-1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}$
meijerg	$\frac{3x^{\frac{1}{3}}}{3+3x}+\frac{2\ln\left(1+x^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{3}}}{2-x^{\frac{1}{3}}}\right)}{3}+\frac{2x^{\frac{1}{3}}}{1+x}-\frac{2x^{\frac{1}{3}}\left(\frac{\ln\left(1+x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right.}{2x^{\frac{1}{3}}}\right.}{3}$

input
```
int((1-2*x)/x^(2/3)/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx=\frac{3\,x^{\frac{1}{3}}}{x+1}$$

input
```
integrate((1-2*x)/x^(2/3)/(1+x)^2,x, algorithm="fricas")
```

output
```
3*x^(1/3)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx = \frac{3\sqrt[3]{x}}{x+1}$$

input
```
integrate((1-2*x)/x**(2/3)/(1+x)**2,x)
```

output
```
3*x**(1/3)/(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx = \frac{3\,x^{\frac{1}{3}}}{x+1}$$

input
```
integrate((1-2*x)/x^(2/3)/(1+x)^2,x, algorithm="maxima")
```

output
```
3*x^(1/3)/(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx = \frac{3\,x^{\frac{1}{3}}}{x+1}$$

input
```
integrate((1-2*x)/x^(2/3)/(1+x)^2,x, algorithm="giac")
```

output
```
3*x^(1/3)/(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx = \frac{3\,x^{1/3}}{x+1}$$

input
```
int(-(2*x - 1)/(x^(2/3)*(x + 1)^2),x)
```

output
```
(3*x^(1/3))/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx = \frac{3x^{\frac{1}{3}}}{x+1}$$

input
```
int((1-2*x)/x^(2/3)/(1+x)^2,x)
```

output
```
(3*x**(1/3))/(x + 1)
```

3.309 $\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx = \frac{x}{2} + \frac{\text{FresnelC}\left(2^{3/4}x\right)}{2\ 2^{3/4}}$$

output
```
1/2*x+1/4*FresnelC(2^(3/4)*x)*2^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx = \frac{1}{4}\left(2x + \sqrt[4]{2}\,\text{FresnelC}\left(2^{3/4}x\right)\right)$$

input
```
Integrate[Cos[(Pi*x^2)/Sqrt[2]]^2,x]
```

output
```
(2*x + 2^(1/4)*FresnelC[2^(3/4)*x])/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3839, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx$$

$$\Big\downarrow \ 3839$$

$$\int \left(\frac{1}{2}\cos\left(\sqrt{2}\pi x^2\right)+\frac{1}{2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\text{FresnelC}\left(2^{3/4}x\right)}{2\ 2^{3/4}}+\frac{x}{2}$$

input
```
Int[Cos[(Pi*x^2)/Sqrt[2]]^2,x]
```

output
```
x/2 + FresnelC[2^(3/4)*x]/(2*2^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3839
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

method	result	size
default	$\frac{x}{2}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{\pi}\,x}{\sqrt{\pi\sqrt{2}}}\right)}{4\sqrt{\pi\sqrt{2}}}$	34
risch	$\frac{x}{2}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{i\pi\sqrt{2}}\,x\right)}{8\sqrt{i\pi\sqrt{2}}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i\pi\sqrt{2}}\,x\right)}{8\sqrt{-i\pi\sqrt{2}}}$	57

input
```
int(cos(1/2*Pi*x^2*2^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*2^(1/2)*Pi^(1/2)/(Pi*2^(1/2))^(1/2)*FresnelC(2*Pi^(1/2)/(Pi*2^(1
/2))^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right)\,dx=\frac{1}{4}\cdot 2^{\frac{1}{4}}\,\mathrm{C}\left(2^{\frac{3}{4}}x\right)+\frac{1}{2}\,x$$

input
```
integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="fricas")
```

output
```
1/4*2^(1/4)*fresnel_cos(2^(3/4)*x) + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right)\,dx = \frac{x}{2} + \frac{\sqrt[4]{2}C\left(2^{\frac{3}{4}}x\right)\Gamma\left(\frac{1}{4}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(cos(1/2*pi*x**2*2**(1/2))**2,x)
```

output
```
x/2 + 2**(1/4)*fresnelc(2**(3/4)*x)*gamma(1/4)/(16*gamma(5/4))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right)\,dx = -\frac{2^{\frac{1}{4}}\pi^2\left((i-1)\,\mathrm{erf}\left(\sqrt{i\sqrt{2}\pi}x\right)-(i+1)\,\mathrm{erf}\left(\sqrt{-i\sqrt{2}\pi}x\right)\right)-8\,\pi^2x}{16\,\pi^2}$$

input
```
integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="maxima")
```

output
```
-1/16*(2^(1/4)*pi^2*((I - 1)*erf(sqrt(I*sqrt(2)*pi)*x) - (I + 1)*erf(sqrt(
-I*sqrt(2)*pi)*x)) - 8*pi^2*x)/pi^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx = -\left(\frac{1}{16}i + \frac{1}{16}\right) \cdot 2^{\frac{1}{4}} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\sqrt{2}\pi}x\right) + \left(\frac{1}{16}i - \frac{1}{16}\right) \cdot 2^{\frac{1}{4}} \operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\sqrt{2}\pi}x\right) + \frac{1}{2}x$$

input
```
integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="giac")
```

output
```
-(1/16*I + 1/16)*2^(1/4)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(sqrt(2)*pi)*x) + (
1/16*I - 1/16)*2^(1/4)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(sqrt(2)*pi)*x) + 1/
2*x
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx = \int \cos\left(\frac{\sqrt{2}\,\Pi\,x^2}{2}\right)^2 dx$$

input
```
int(cos((2^(1/2)*Pi*x^2)/2)^2,x)
```

output
```
int(cos((2^(1/2)*Pi*x^2)/2)^2, x)
```

Reduce [F]

$$\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx = \int \cos\left(\frac{\sqrt{2}\,\pi\, x^2}{2}\right)^2 dx$$

input
```
int(cos(1/2*Pi*x^2*2^(1/2))^2,x)
```

output
```
int(cos((sqrt(2)*pi*x**2)/2)**2,x)
```

3.310 $\int \frac{1}{1+\cos(x)+\sin(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

output
```
ln(1+tan(1/2*x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.67

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = -\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(1 + Cos[x] + Sin[x])^(-1),x]
```

output
```
-Log[Cos[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\cos(x)+1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(x)+\cos(x)+1}dx$$

$$\Big\downarrow \ 3603$$

$$2\int \frac{1}{2\tan\left(\frac{x}{2}\right)+2}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \ 16$$

$$\log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
Int[(1 + Cos[x] + Sin[x])^(-1),x]
```

output
```
Log[1 + Tan[x/2]]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	8
parallelrisch	$\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	8
norman	$\ln\left(2+2\tan\left(\frac{x}{2}\right)\right)$	10
risch	$\ln\left(\mathrm{e}^{ix}+i\right)-\ln\left(1+\mathrm{e}^{ix}\right)$	21

input
```
int(1/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+tan(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx=-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(\sin(x)+1\right)$$

input
```
integrate(1/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(sin(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
integrate(1/(1+cos(x)+sin(x)),x)
```

output
```
log(tan(x/2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)$$

input
```
integrate(1/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
log(sin(x)/(cos(x) + 1) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

input
```
integrate(1/(1+cos(x)+sin(x)),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int(1/(cos(x) + sin(x) + 1),x)
```

output
```
log(tan(x/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{1+\cos(x)+\sin(x)}\,dx = \log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int(1/(1+cos(x)+sin(x)),x)
```

output
```
log(tan(x/2) + 1)
```

3.311 $\int \tan^5(x)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 22

$$\int \tan^5(x)\,dx = -\log(\cos(x)) - \frac{\tan^2(x)}{2} + \frac{\tan^4(x)}{4}$$

output
```
-ln(cos(x))-1/2*tan(x)^2+1/4*tan(x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \tan^5(x)\,dx = -\log(\cos(x)) - \sec^2(x) + \frac{\sec^4(x)}{4}$$

input
```
Integrate[Tan[x]^5,x]
```

output
```
-Log[Cos[x]] - Sec[x]^2 + Sec[x]^4/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^5dx$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^4(x)}{4}-\int \tan^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^4(x)}{4}-\int \tan(x)^3dx$$

$$\Big\downarrow \text{ 3954}$$

$$\int \tan(x)dx+\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)dx+\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

input
```
Int[Tan[x]^5,x]
```

output
```
-Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}+\frac{\ln\left(1+\tan(x)^2\right)}{2}$	23
default	$\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}+\frac{\ln\left(1+\tan(x)^2\right)}{2}$	23
norman	$\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}+\frac{\ln\left(1+\tan(x)^2\right)}{2}$	23
parallelrisch	$\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}+\frac{\ln\left(1+\tan(x)^2\right)}{2}$	23
risch	$ix-\frac{4(e^{6ix}+e^{4ix}+e^{2ix})}{(e^{2ix}+1)^4}-\ln\left(e^{2ix}+1\right)$	43

input
```
int(tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)^4-1/2*tan(x)^2+1/2*ln(1+tan(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \tan^5(x)\,dx = \frac{1}{4}\tan{(x)}^4 - \frac{1}{2}\tan{(x)}^2 - \frac{1}{2}\log\left(\frac{1}{\tan{(x)}^2+1}\right)$$

input
```
integrate(tan(x)^5,x, algorithm="fricas")
```

output
```
1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \tan^5(x)\,dx = -\frac{4\cos^2{(x)}-1}{4\cos^4{(x)}} - \log{(\cos{(x)})}$$

input
```
integrate(tan(x)**5,x)
```

output
```
-(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \tan^5(x)\,dx = \frac{4\sin{(x)}^2-3}{4\left(\sin{(x)}^4-2\sin{(x)}^2+1\right)} - \frac{1}{2}\log\left(\sin{(x)}^2-1\right)$$

input
```
integrate(tan(x)^5,x, algorithm="maxima")
```

output
```
1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \tan^5(x)\,dx = \frac{1}{4}\tan(x)^4 - \frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

input
```
integrate(tan(x)^5,x, algorithm="giac")
```

output
```
1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*log(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \tan^5(x)\,dx = \frac{\tan(x)^4}{4} - \frac{\tan(x)^2}{2} - \ln(\cos(x))$$

input
```
int(tan(x)^5,x)
```

output
```
tan(x)^4/4 - tan(x)^2/2 - log(cos(x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \tan^5(x)\,dx = \frac{\log\left(\tan(x)^2 + 1\right)}{2} + \frac{\tan(x)^4}{4} - \frac{\tan(x)^2}{2}$$

input
```
int(tan(x)^5,x)
```

output
```
(2*log(tan(x)**2 + 1) + tan(x)**4 - 2*tan(x)**2)/4
```

3.312 $\int \sqrt{1+\frac{1}{x}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \sqrt{1+\frac{1}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

output
```
(1+1/x)^(1/2)*x+arctanh((1+1/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\frac{1}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

input
```
Integrate[Sqrt[1 + x^(-1)],x]
```

output
```
Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {773, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1}{x}+1}\,dx$$

$$\Big\downarrow \; 773$$

$$-\int \sqrt{1+\frac{1}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \; 51$$

$$\sqrt{\frac{1}{x}+1}x-\frac{1}{2}\int \frac{x}{\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \; 73$$

$$\sqrt{\frac{1}{x}+1}x-\int \frac{1}{\frac{1}{x^2}-1}d\sqrt{1+\frac{1}{x}}$$

$$\Big\downarrow \; 220$$

$$\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\right)+\sqrt{\frac{1}{x}+1}x$$

input
```
Int[Sqrt[1 + x^(-1)],x]
```

output
```
Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

method	result	size
trager	$\sqrt{-\frac{-1-x}{x}}\, x+\frac{\ln\left(2\sqrt{-\frac{-1-x}{x}}\, x+2x+1\right)}{2}$	39
default	$\frac{\sqrt{\frac{1+x}{x}}\, x\left(2\sqrt{x^{2}+x}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\right)}{2\sqrt{x(1+x)}}$	41
risch	$x\sqrt{\frac{1+x}{x}}+\frac{\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\sqrt{\frac{1+x}{x}}\, \sqrt{x(1+x)}}{2+2x}$	47

input
```
int((1+1/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-1-x)/x)^(1/2)*x+1/2*ln(2*(-(-1-x)/x)^(1/2)*x+2*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \sqrt{1+\frac{1}{x}}\,dx = x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate((1+1/x)^(1/2),x, algorithm="fricas")
```

output
```
x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \sqrt{1+\frac{1}{x}}\,dx = \sqrt{x}\sqrt{x+1} + \operatorname{asinh}\left(\sqrt{x}\right)$$

input
```
integrate((1+1/x)**(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) + asinh(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{1+\frac{1}{x}}\,dx = x\sqrt{\frac{1}{x}+1}+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}-1\right)$$

input
```
integrate((1+1/x)^(1/2),x, algorithm="maxima")
```

output
```
x*sqrt(1/x + 1) + 1/2*log(sqrt(1/x + 1) + 1) - 1/2*log(sqrt(1/x + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \sqrt{1+\frac{1}{x}}\,dx = -\frac{1}{2}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\sqrt{x^2+x}\operatorname{sgn}(x)$$

input
```
integrate((1+1/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \sqrt{1+\frac{1}{x}}\,dx = x\sqrt{\frac{1}{x}+1}+\frac{x\ln\left(x+\sqrt{x^2+x}+\frac{1}{2}\right)\sqrt{\frac{1}{x}+1}}{2\sqrt{x^2+x}}$$

input
```
int((1/x + 1)^(1/2),x)
```

output
```
x*(1/x + 1)^(1/2) + (x*log(x + (x + x^2)^(1/2) + 1/2)*(1/x + 1)^(1/2))/(2*
(x + x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sqrt{1+\frac{1}{x}}\,dx = \sqrt{x}\,\sqrt{x+1}+\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int((1+1/x)^(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) + log(sqrt(x + 1) + sqrt(x))
```

3.313 $\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx = 2e^{\cos(x)} \cos\left(\frac{x}{2} + \sin(x)\right) \sin\left(\frac{x}{2}\right)$$

output
```
2*exp(cos(x))*cos(1/2*x+sin(x))*sin(1/2*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx = 2e^{\cos(x)} \cos\left(\frac{x}{2} + \sin(x)\right) \sin\left(\frac{x}{2}\right)$$

input
```
Integrate[E^Cos[x]*Cos[2*x + Sin[x]],x]
```

output
```
2*E^Cos[x]*Cos[x/2 + Sin[x]]*Sin[x/2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\cos(x)} \cos(2x + \sin(x))\, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int e^{\cos(x)} \cos(2x + \sin(x)) dx$$

input
```
Int[E^Cos[x]*Cos[2*x + Sin[x]],x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

method	result	size
risch	$-\frac{i e^{ix} e^{e^{ix}}}{2}+\frac{i e^{e^{ix}}}{2}+\frac{i e^{e^{-ix}} e^{-ix}}{2}-\frac{i e^{e^{-ix}}}{2}$	52

input
```
int(exp(cos(x))*cos(2*x+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)*exp(exp(I*x))+1/2*I*exp(exp(I*x))+1/2*I*exp(1/exp(I*x))*ex
p(-I*x)-1/2*I*exp(1/exp(I*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.00

$$\int e^{\cos(x)}\cos(2x+\sin(x))\,dx = (2\cos(x)-1)\cos\left(\frac{2\left(x\tan\left(\frac{1}{2}x\right)^2+x+\tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)e^{\cos(x)}\sin(x) - \left(2\cos(x)^2-\cos(x)-1\right)e^{\cos(x)}\sin\left(\frac{2\left(x\tan\left(\frac{1}{2}x\right)^2+x+\tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

input
```
integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="fricas")
```

output
```
(2*cos(x) - 1)*cos(2*(x*tan(1/2*x)^2 + x + tan(1/2*x))/(tan(1/2*x)^2 + 1))
*e^cos(x)*sin(x) - (2*cos(x)^2 - cos(x) - 1)*e^cos(x)*sin(2*(x*tan(1/2*x)^
2 + x + tan(1/2*x))/(tan(1/2*x)^2 + 1))
```

Sympy [F]

$$\int e^{\cos(x)}\cos(2x+\sin(x))\,dx = \int e^{\cos(x)}\cos(2x+\sin(x))\,dx$$

input
```
integrate(exp(cos(x))*cos(2*x+sin(x)),x)
```

output
```
Integral(exp(cos(x))*cos(2*x + sin(x)), x)
```

Maxima [F]

$$\int e^{\cos(x)}\cos(2x+\sin(x))\,dx=\int \cos\left(2\,x+\sin\left(x\right)\right)e^{\cos\left(x\right)}\,dx$$

input
```
integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="maxima")
```

output
```
1/4*e^cos(x)*sin(2*x + sin(x)) + 1/2*e^cos(x)*sin(x + sin(x)) - 1/2*e^cos(
x)*sin(sin(x)) - 1/4*integrate(cos(3*x + sin(x))*e^cos(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(16) = 32.

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 6.48

$$\int e^{\cos(x)}\cos(2x+\sin(x))\,dx$$
$$=\frac{\cos\left(x\right)^{3}e^{\cos\left(x\right)}\sin\left(2\,x+\sin\left(x\right)\right)-\cos\left(2\,x+\sin\left(x\right)\right)\cos\left(x\right)^{2}e^{\cos\left(x\right)}\sin\left(x\right)+\cos\left(x\right)e^{\cos\left(x\right)}\sin\left(2\,x+\sin\left(\right.\right.}{}$$

input
```
integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="giac")
```

output
```
(cos(x)^3*e^cos(x)*sin(2*x + sin(x)) - cos(2*x + sin(x))*cos(x)^2*e^cos(x)
*sin(x) + cos(x)*e^cos(x)*sin(2*x + sin(x))*sin(x)^2 - cos(2*x + sin(x))*e
^cos(x)*sin(x)^3 - cos(x)^2*e^cos(x)*sin(2*x + sin(x)) + 2*cos(2*x + sin(x
))*cos(x)*e^cos(x)*sin(x) + e^cos(x)*sin(2*x + sin(x))*sin(x)^2)/(cos(x)^4
 + 2*cos(x)^2*sin(x)^2 + sin(x)^4)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx = \mathrm{e}^{\cos(x)} (\sin(x + \sin(x)) - \sin(\sin(x)))$$

input
```
int(exp(cos(x))*cos(2*x + sin(x)),x)
```

output
```
exp(cos(x))*(sin(x + sin(x)) - sin(sin(x)))
```

Reduce [F]

$$\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx = \int e^{\cos(x)} \cos(\sin(x) + 2x) \, dx$$

input
```
int(exp(cos(x))*cos(2*x+sin(x)),x)
```

output
```
int(e**cos(x)*cos(sin(x) + 2*x),x)
```

3.314 $\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 17

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = 3\log(x)-\log\left(x+2x^3+\log(x)\right)$$

output
```
3*ln(x)-ln(x+2*x^3+ln(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = 3\log(x)-\log\left(x+2x^3+\log(x)\right)$$

input
```
Integrate[(-1 + 2*x + 3*Log[x])/(x^2 + 2*x^4 + x*Log[x]),x]
```

output
```
3*Log[x] - Log[x + 2*x^3 + Log[x]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3\log(x)-1}{2x^4+x^2+x\log(x)}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{-6x^3-x-1}{x\left(2x^3+x+\log(x)\right)}+\frac{3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\log(x)-\log\left(2x^3+x+\log(x)\right)$$

input
```
Int[(-1 + 2*x + 3*Log[x])/(x^2 + 2*x^4 + x*Log[x]),x]
```

output
```
3*Log[x] - Log[x + 2*x^3 + Log[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
default	$3\ln(x)-\ln(x+2x^3+\ln(x))$	18
norman	$3\ln(x)-\ln(x+2x^3+\ln(x))$	18
risch	$3\ln(x)-\ln(x+2x^3+\ln(x))$	18
parallelrisch	$-\ln\left(x^3+\frac{x}{2}+\frac{\ln(x)}{2}\right)+3\ln(x)$	20

input
```
int((-1+2*x+3*ln(x))/(x^2+2*x^4+x*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
3*ln(x)-ln(x+2*x^3+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = -\log\left(2\,x^3+x+\log(x)\right)+3\,\log(x)$$

input
```
integrate((-1+2*x+3*log(x))/(x^2+2*x^4+x*log(x)),x, algorithm="fricas")
```

output
```
-log(2*x^3 + x + log(x)) + 3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = 3\log(x)-\log\left(2x^3+x+\log(x)\right)$$

input
```
integrate((-1+2*x+3*ln(x))/(x**2+2*x**4+x*ln(x)),x)
```

output
```
3*log(x) - log(2*x**3 + x + log(x))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = -\log\left(2\,x^3+x+\log\left(x\right)\right)+3\,\log\left(x\right)$$

input
```
integrate((-1+2*x+3*log(x))/(x^2+2*x^4+x*log(x)),x, algorithm="maxima")
```

output
```
-log(2*x^3 + x + log(x)) + 3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = -\log\left(-2\,x^3-x-\log\left(x\right)\right)+3\,\log\left(x\right)$$

input
```
integrate((-1+2*x+3*log(x))/(x^2+2*x^4+x*log(x)),x, algorithm="giac")
```

output
```
-log(-2*x^3 - x - log(x)) + 3*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = 3\,\ln\left(x\right)-\ln\left(x+\ln\left(x\right)+2\,x^3\right)$$

input
```
int((2*x + 3*log(x) - 1)/(x*log(x) + x^2 + 2*x^4),x)
```

output
```
3*log(x) - log(x + log(x) + 2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx = -\log\left(\log(x)+2x^3+x\right)+3\log(x)$$

input
```
int((-1+2*x+3*log(x))/(x^2+2*x^4+x*log(x)),x)
```

output
```
 - log(log(x) + 2*x**3 + x) + 3*log(x)
```

3.315 $\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx=-\frac{2\left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\left(1+2x+\pi\sqrt{x}\sqrt{1+x}\right)}{-4+\pi^2}$$

output
```
-2*(-x^(1/2)+(1+x)^(1/2))^Pi*(1+2*x+Pi*x^(1/2)*(1+x)^(1/2))/(Pi^2-4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(45) = 90$.

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx$$
$$=\frac{2\sqrt{1+x}\left(-\sqrt{x}+\sqrt{1+x}\right)^{-1+\pi}\left(1-2(-2+\pi)x-2(-2+\pi)x^2+(-2+\pi)\sqrt{x}\sqrt{1+x}+2(-2+\pi)x^{3/2}\right.}{(-2+\pi)(2+\pi)\left(-1-x+\sqrt{x}\sqrt{1+x}\right)}$$

input
```
Integrate[(-Sqrt[x] + Sqrt[1 + x])^Pi,x]
```

output
```
(2*Sqrt[1 + x]*(-Sqrt[x] + Sqrt[1 + x])^(-1 + Pi)*(1 - 2*(-2 + Pi)*x - 2*(
-2 + Pi)*x^2 + (-2 + Pi)*Sqrt[x]*Sqrt[1 + x] + 2*(-2 + Pi)*x^(3/2)*Sqrt[1
+ x]))/((-2 + Pi)*(2 + Pi)*(-1 - x + Sqrt[x]*Sqrt[1 + x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {7296, 2544, 25, 335, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{x+1}-\sqrt{x}\right)^{\pi} dx$$

$$\Big\downarrow \text{7296}$$

$$2\int \sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)^{\pi} d\sqrt{x}$$

$$\Big\downarrow \text{2544}$$

$$\frac{1}{2}\int -\left((1-x)x^{\frac{1}{2}(-3+\pi)}(x+1)\right) d\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int (1-x)x^{\frac{1}{2}(-3+\pi)}(x+1)d\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$\Big\downarrow \text{335}$$

$$-\frac{1}{2}\int x^{\frac{1}{2}(-3+\pi)}\left(1-x^2\right) d\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$\Big\downarrow \text{802}$$

$$-\frac{1}{2}\int \left(x^{\frac{1}{2}(-3+\pi)}-x^{\frac{1+\pi}{2}}\right) d\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^{\frac{1}{2}(\pi-2)}}{2-\pi}+\frac{x^{\frac{2+\pi}{2}}}{2+\pi}\right)$$

input
```
Int[(-Sqrt[x] + Sqrt[1 + x])^Pi,x]
```

output
```
(x^((-2 + Pi)/2)/(2 - Pi) + x^((2 + Pi)/2)/(2 + Pi))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2544
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Simp[1/(2^(m + 1)*e^(m + 1))   Subst[Int[x^(n - m -
 2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
 c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [F]

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx$$

input
```
int((-x^(1/2)+(1+x)^(1/2))^Pi,x)
```

output
```
int((-x^(1/2)+(1+x)^(1/2))^Pi,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx = -\frac{2\left(\pi\sqrt{x+1}\sqrt{x}+2\,x+1\right)\left(\sqrt{x+1}-\sqrt{x}\right)^{\pi}}{\pi^{2}-4}$$

input
```
integrate((-x^(1/2)+(1+x)^(1/2))^pi,x, algorithm="fricas")
```

output
```
-2*(pi*sqrt(x + 1)*sqrt(x) + 2*x + 1)*(sqrt(x + 1) - sqrt(x))^pi/(pi^2 - 4
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4362 vs. 2(39) = 78.

Time = 4.55 (sec) , antiderivative size = 4362, normalized size of antiderivative = 96.93

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx = \text{Too large to display}$$

input
```
integrate((-x**(1/2)+(1+x)**(1/2))**pi,x)
```

output
```
Piecewise((4*x**(13/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x
)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2)
- 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + 2*pi*x**(13/2)*sq
rt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x
**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2)
+ pi**2*x**4*gamma(1 + pi/2)) - 2*pi*x**(13/2)*cosh(asinh(sqrt(x)) + pi*as
inh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(
1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 4*x**(1
3/2)*cosh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gam
ma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2
*x**4*gamma(1 + pi/2)) + 6*x**(11/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + p
i*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*ga
mma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + 2*p
i*x**(11/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1
 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*ga
mma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 4*pi*x**(11/2)*cosh(asinh(sq
rt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi*
*2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi
/2)) - 6*x**(11/2)*cosh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2
)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(...
```

Maxima [F]

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx=\int \left(\sqrt{x+1}-\sqrt{x}\right)^{\pi}\,dx$$

input
```
integrate((-x^(1/2)+(1+x)^(1/2))^pi,x, algorithm="maxima")
```

output
```
integrate((sqrt(x + 1) - sqrt(x))^pi, x)
```

Giac [F]

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi} dx = \int \left(\sqrt{x+1}-\sqrt{x}\right)^{\pi} dx$$

input
```
integrate((-x^(1/2)+(1+x)^(1/2))^pi,x, algorithm="giac")
```

output
```
integrate((sqrt(x + 1) - sqrt(x))^pi, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi} dx = \int \left(\sqrt{x+1}-\sqrt{x}\right)^{\Pi} dx$$

input
```
int(((x + 1)^(1/2) - x^(1/2))^Pi,x)
```

output
```
int(((x + 1)^(1/2) - x^(1/2))^Pi, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi} dx = \frac{-2\sqrt{x}\,\sqrt{x+1}\,\pi-4x-2}{\left(\sqrt{x+1}+\sqrt{x}\right)^{\pi}(\pi^2-4)}$$

input
```
int((-x^(1/2)+(1+x)^(1/2))^Pi,x)
```

output
```
(2*( - sqrt(x)*sqrt(x + 1)*pi - 2*x - 1))/((sqrt(x + 1) + sqrt(x))**pi*(pi
**2 - 4))
```

3.316 $\int \left(-2+\left(-2+\left(-2+(-2+x^2)^2\right)^2\right)^2\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \left(-2+\left(-2+\left(-2+(-2+x^2)^2\right)^2\right)^2\right)\,dx = 2x-\frac{64x^3}{3}+\frac{336x^5}{5}-96x^7+\frac{220x^9}{3}-32x^{11}+8x^{13}-\frac{16x^{15}}{15}+\frac{x^{17}}{17}$$

output
```
2*x-64/3*x^3+336/5*x^5-96*x^7+220/3*x^9-32*x^11+8*x^13-16/15*x^15+1/17*x^1
7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(-2+\left(-2+\left(-2+(-2+x^2)^2\right)^2\right)^2\right)\,dx = 2x-\frac{64x^3}{3}+\frac{336x^5}{5}-96x^7+\frac{220x^9}{3}-32x^{11}+8x^{13}-\frac{16x^{15}}{15}+\frac{x^{17}}{17}$$

input
```
Integrate[-2 + (-2 + (-2 + (-2 + x^2)^2)^2)^2,x]
```

output
```
2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
 (16*x^15)/15 + x^17/17
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\left(\left(\left(x^2-2\right)^2-2\right)^2-2\right)^2-2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{17}}{17}-\frac{16x^{15}}{15}+8x^{13}-32x^{11}+\frac{220x^9}{3}-96x^7+\frac{336x^5}{5}-\frac{64x^3}{3}+2x$$

input
```
Int[-2 + (-2 + (-2 + (-2 + x^2)^2)^2)^2,x]
```

output
```
2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
 (16*x^15)/15 + x^17/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	s
default	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	4
norman	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	4
risch	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	4
parallelrisch	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	4
parts	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	4
gosper	$\frac{x\left(15x^{16}-272x^{14}+2040x^{12}-8160x^{10}+18700x^{8}-24480x^{6}+17136x^{4}-5440x^{2}+510\right)}{255}$	4
orering	$\frac{x\left(15x^{16}-272x^{14}+2040x^{12}-8160x^{10}+18700x^{8}-24480x^{6}+17136x^{4}-5440x^{2}+510\right)\left(-2+\left(-2+\left(-2+\left(x^{2}-2\right)^{2}\right)^{2}\right)^{2}\right)}{255x^{16}-4080x^{14}+26520x^{12}-89760x^{10}+168300x^{8}-171360x^{6}+85680x^{4}-16320x^{2}+510}$	1

input
```
int(-2+(-2+(-2+(x^2-2)^2)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-64/3*x^3+336/5*x^5-96*x^7+220/3*x^9-32*x^11+8*x^13-16/15*x^15+1/17*x^1
7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^{2}\right)^{2}\right)^{2}\right)^{2}\right)\,dx$$
$$=\frac{1}{17}\,x^{17}-\frac{16}{15}\,x^{15}+8\,x^{13}-32\,x^{11}+\frac{220}{3}\,x^{9}-96\,x^{7}+\frac{336}{5}\,x^{5}-\frac{64}{3}\,x^{3}+2\,x$$

input
```
integrate(-2+(-2+(-2+(x^2-2)^2)^2)^2,x, algorithm="fricas")
```

output
```
1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
 - 64/3*x^3 + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^2\right)^2\right)^2\right)^2\right)\,dx = \frac{x^{17}}{17}-\frac{16x^{15}}{15}+8x^{13}-32x^{11}+\frac{220x^9}{3}-96x^7+\frac{336x^5}{5}-\frac{64x^3}{3}+2x$$

input
```
integrate(-2+(-2+(-2+(x**2-2)**2)**2)**2,x)
```

output
```
x**17/17 - 16*x**15/15 + 8*x**13 - 32*x**11 + 220*x**9/3 - 96*x**7 + 336*x
**5/5 - 64*x**3/3 + 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^2\right)^2\right)^2\right)^2\right)\,dx = \frac{1}{17}\,x^{17}-\frac{16}{15}\,x^{15}+8\,x^{13}-32\,x^{11}+\frac{220}{3}\,x^9-96\,x^7+\frac{336}{5}\,x^5-\frac{64}{3}\,x^3+2\,x$$

input
```
integrate(-2+(-2+(-2+(x^2-2)^2)^2)^2,x, algorithm="maxima")
```

output
```
1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
 - 64/3*x^3 + 2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^2\right)^2\right)^2\right)^2\right)\,dx = \frac{1}{17}\,x^{17}-\frac{16}{15}\,x^{15}+8\,x^{13}-32\,x^{11}+\frac{220}{3}\,x^{9}-96\,x^{7}+\frac{336}{5}\,x^{5}-\frac{64}{3}\,x^{3}+2\,x$$

input
```
integrate(-2+(-2+(-2+(x^2-2)^2)^2)^2,x, algorithm="giac")
```

output
```
1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
 - 64/3*x^3 + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^2\right)^2\right)^2\right)^2\right)\,dx = \frac{x^{17}}{17}-\frac{16\,x^{15}}{15}+8\,x^{13}-32\,x^{11}+\frac{220\,x^9}{3}-96\,x^7+\frac{336\,x^5}{5}-\frac{64\,x^3}{3}+2\,x$$

input
```
int((((x^2 - 2)^2 - 2)^2 - 2)^2 - 2,x)
```

output
```
2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
 (16*x^15)/15 + x^17/17
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(-2+\left(-2+\left(-2+\left(-2+x^2\right)^2\right)^2\right)^2\right) dx$$
$$= \frac{x(15x^{16}-272x^{14}+2040x^{12}-8160x^{10}+18700x^8-24480x^6+17136x^4-5440x^2+510)}{255}$$

input
```
int(-2+(-2+(-2+(x^2-2)^2)^2)^2,x)
```

output
```
(x*(15*x**16 - 272*x**14 + 2040*x**12 - 8160*x**10 + 18700*x**8 - 24480*x*
*6 + 17136*x**4 - 5440*x**2 + 510))/255
```

3.317 $\int \sin(4\arctan(x))\,dx$

Optimal result

Integrand size = 5, antiderivative size = 18

$$\int \sin(4\arctan(x))\,dx = -\frac{4}{1+x^2} - 2\log\left(1+x^2\right)$$

output
```
-4/(x^2+1)-2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sin(4\arctan(x))\,dx = -\frac{4}{1+x^2} - 2\log\left(1+x^2\right)$$

input
```
Integrate[Sin[4*ArcTan[x]],x]
```

output
```
-4/(1 + x^2) - 2*Log[1 + x^2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4\arctan(x))\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \sin(4\arctan(x))dx$$

input
```
Int[Sin[4*ArcTan[x]],x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{2i}{i+x}-2\ln\left(i+x\right)+\frac{2i}{x-i}-2\ln\left(x-i\right)$	34

input
```
int(sin(4*arctan(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*I/(I+x)-2*ln(I+x)+2*I/(x-I)-2*ln(x-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sin(4\arctan(x))\,dx = -\frac{2\left((x^2+1)\log\left(x^2+1\right)+2\right)}{x^2+1}$$

input
```
integrate(sin(4*arctan(x)),x, algorithm="fricas")
```

output
```
-2*((x^2 + 1)*log(x^2 + 1) + 2)/(x^2 + 1)
```

Sympy [F]

$$\int \sin(4\arctan(x))\,dx = \int \sin\left(4\operatorname{atan}\left(x\right)\right)\,dx$$

input
```
integrate(sin(4*atan(x)),x)
```

output
```
Integral(sin(4*atan(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sin(4\arctan(x))\,dx = -\frac{2\left((x^2+1)\log\left(x^2+1\right)+2\right)}{x^2+1}$$

input
```
integrate(sin(4*arctan(x)),x, algorithm="maxima")
```

output
```
-2*((x^2 + 1)*log(x^2 + 1) + 2)/(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sin(4\arctan(x))\,dx = \frac{2\,(x^2-1)}{x^2+1} - 2\,\log\left(x^2+1\right)$$

input
```
integrate(sin(4*arctan(x)),x, algorithm="giac")
```

output
```
2*(x^2 - 1)/(x^2 + 1) - 2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sin(4\arctan(x))\,dx = -2\,\ln\left(x^2+1\right) - \frac{4}{x^2+1}$$

input
```
int(sin(4*atan(x)),x)
```

output
```
- 2*log(x^2 + 1) - 4/(x^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \sin(4\arctan(x))\,dx = \frac{-2\log(x^2+1)\,x^2 - 2\log(x^2+1) + 4x^2}{x^2+1}$$

input
```
int(sin(4*atan(x)),x)
```

output
```
(2*( - log(x**2 + 1)*x***2 - log(x**2 + 1) + 2*x***2))/(x**2 + 1)
```

3.318 $\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = -\frac{\arctan\left(\frac{1-2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+\sqrt[3]{\tan(x)}\right)-\frac{1}{6}\log(1+\tan(x))-\frac{\sqrt[3]{\tan(x)}}{1+\tan(x)}$$

output
```
-1/3*arctan(1/3*(1-2*tan(x)^(1/3))*3^(1/2))*3^(1/2)+1/2*ln(1+tan(x)^(1/3))
-1/6*ln(1+tan(x))-tan(x)^(1/3)/(1+tan(x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = \frac{\arctan\left(\frac{-1+2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(1+\sqrt[3]{\tan(x)}\right)-\frac{1}{6}\log\left(1-\sqrt[3]{\tan(x)}+\tan^{\frac{2}{3}}(x)\right)+\left(-1+\frac{\sin(x)}{\cos(x)+\sin(x)}\right)\sqrt[3]{\tan(x)}$$

input
```
Integrate[Tan[x]^(1/3)/(Cos[x] + Sin[x])^2,x]
```

output
```
ArcTan[(-1 + 2*Tan[x]^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 + Tan[x]^(1/3)]/3 -
Log[1 - Tan[x]^(1/3) + Tan[x]^(2/3)]/6 + (-1 + Sin[x]/(Cos[x] + Sin[x]))*T
an[x]^(1/3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4889, 51, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\tan(x)}}{(\sin(x)+\cos(x))^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\sqrt[3]{\tan(x)}}{(\tan(x)+1)^2}d\tan(x)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\int \frac{1}{\tan^{\frac{2}{3}}(x)(\tan(x)+1)}d\tan(x)-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}$$

$$\Big\downarrow \text{70}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{\tan^{\frac{2}{3}}(x)-\sqrt[3]{\tan(x)}+1}d\sqrt[3]{\tan(x)}+\frac{3}{2}\int \frac{1}{\sqrt[3]{\tan(x)}+1}d\sqrt[3]{\tan(x)}-\frac{1}{2}\log(\tan(x)+1)\right)-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{\tan^{\frac{2}{3}}(x)-\sqrt[3]{\tan(x)}+1}d\sqrt[3]{\tan(x)}+\frac{3}{2}\log\left(\sqrt[3]{\tan(x)}+1\right)-\frac{1}{2}\log(\tan(x)+1)\right)-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}$$

$\big\downarrow$ 1083

$$\frac{1}{3}\left(-3\int \frac{1}{-\tan^{\frac{2}{3}}(x)-3}d\left(2\sqrt[3]{\tan(x)}-1\right)+\frac{3}{2}\log\left(\sqrt[3]{\tan(x)}+1\right)-\frac{1}{2}\log(\tan(x)+1)\right)-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\tan(x)}-1}{\sqrt{3}}\right)+\frac{3}{2}\log\left(\sqrt[3]{\tan(x)}+1\right)-\frac{1}{2}\log(\tan(x)+1)\right)-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}$$

input
```
Int[Tan[x]^(1/3)/(Cos[x] + Sin[x])^2,x]
```

output
```
(Sqrt[3]*ArcTan[(-1 + 2*Tan[x]^(1/3))/Sqrt[3]] + (3*Log[1 + Tan[x]^(1/3)])
/2 - Log[1 + Tan[x]]/2)/3 - Tan[x]^(1/3)/(1 + Tan[x])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [F]

$$\int \frac{\tan(x)^{\frac{1}{3}}}{(\cos(x)+\sin(x))^2}dx$$

input
```
int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

output
```
int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(48) = 96$.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = \frac{2\left(\sqrt{3}\cos\left(x\right)+\sqrt{3}\sin\left(x\right)\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-\left(\cos\left(x\right)+\sin\left(x\right)\right)\log\left(\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{2}{3}}-\left(\frac{\sin(x)}{\cos(x)}\right.\right.}{6\left(\cos\left(x\right)+\sin\left(x\right)\right)}$$

input
```
integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(sqrt(3)*cos(x) + sqrt(3)*sin(x))*arctan(2/3*sqrt(3)*(sin(x)/cos(x)
)^(1/3) - 1/3*sqrt(3)) - (cos(x) + sin(x))*log((sin(x)/cos(x))^(2/3) - (si
n(x)/cos(x))^(1/3) + 1) + 2*(cos(x) + sin(x))*log((sin(x)/cos(x))^(1/3) +
1) - 6*(sin(x)/cos(x))^(1/3)*cos(x))/(cos(x) + sin(x))
```

Sympy [F]

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = \int \frac{\sqrt[3]{\tan\left(x\right)}}{\left(\sin\left(x\right)+\cos\left(x\right)\right)^2}\,dx$$

input
```
integrate(tan(x)**(1/3)/(cos(x)+sin(x))**2,x)
```

output
```
Integral(tan(x)**(1/3)/(sin(x) + cos(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(x)^{\frac{1}{3}}-1\right)\right) - \frac{\tan(x)^{\frac{1}{3}}}{\tan(x)+1} - \frac{1}{6}\log\left(\tan(x)^{\frac{2}{3}}-\tan(x)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\tan(x)^{\frac{1}{3}}+1\right)$$

input
```
integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x)^(1/3) - 1)) - tan(x)^(1/3)/(tan(x
) + 1) - 1/6*log(tan(x)^(2/3) - tan(x)^(1/3) + 1) + 1/3*log(tan(x)^(1/3) +
 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(x)^{\frac{1}{3}}-1\right)\right) - \frac{\tan(x)^{\frac{1}{3}}}{\tan(x)+1} - \frac{1}{6}\log\left(\tan(x)^{\frac{2}{3}}-\tan(x)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left|\tan(x)^{\frac{1}{3}}+1\right|\right)$$

input
```
integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x)^(1/3) - 1)) - tan(x)^(1/3)/(tan(x
) + 1) - 1/6*log(tan(x)^(2/3) - tan(x)^(1/3) + 1) + 1/3*log(abs(tan(x)^(1/
3) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx=\int \frac{\tan(x)^{1/3}}{\left(\cos\left(x\right)+\sin\left(x\right)\right)^2}\,dx$$

input
```
int(tan(x)^(1/3)/(cos(x) + sin(x))^2,x)
```

output
```
int(tan(x)^(1/3)/(cos(x) + sin(x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2}\,dx=\int \frac{\tan\left(x\right)^{\frac{1}{3}}}{\cos\left(x\right)^2+2\cos\left(x\right)\sin\left(x\right)+\sin\left(x\right)^2}dx$$

input
```
int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

output
```
int(tan(x)**(1/3)/(cos(x)**2 + 2*cos(x)*sin(x) + sin(x)**2),x)
```

3.319 $\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3$

Optimal result

Integrand size = 47, antiderivative size = 64

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= 7x+4\sin(2x)-\frac{1}{2}\sin(4x)-\frac{4}{3}\sin(6x)-\sin(8x)$$
$$-\frac{2}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{2}{7}\sin(14x)+\frac{1}{8}\sin(16x)$$

output
```
7*x+4*sin(2*x)-1/2*sin(4*x)-4/3*sin(6*x)-sin(8*x)-2/5*sin(10*x)+1/6*sin(12
*x)+2/7*sin(14*x)+1/8*sin(16*x)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= 7x+4\sin(2x)-\frac{1}{2}\sin(4x)-\frac{4}{3}\sin(6x)-\sin(8x)$$
$$-\frac{2}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{2}{7}\sin(14x)+\frac{1}{8}\sin(16x)$$

input
```
Integrate[Csc[x]^2*Csc[6*x]^2*Csc[10*x]^2*Csc[15*x]^2*Sin[2*x]^2*Sin[3*x]^
2*Sin[5*x]^2*Sin[30*x]^2,x]
```

output
```
7*x + 4*Sin[2*x] - Sin[4*x]/2 - (4*Sin[6*x])/3 - Sin[8*x] - (2*Sin[10*x])/
5 + Sin[12*x]/6 + (2*Sin[14*x])/7 + Sin[16*x]/8
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. $2(64) = 128$.

Time = 2.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.383$, Rules used = {3042, 4889, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 2345, 27, 1471, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)^2\sin(3x)^2\sin(5x)^2\sin(30x)^2\csc(x)^2\csc(6x)^2\csc(10x)^2\csc(15x)^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^8(x)-28\tan^6(x)+134\tan^4(x)-92\tan^2(x)+1\right)^2}{\left(\tan^2(x)+1\right)^9}d\tan(x)$$

$$\Big\downarrow \text{2345}$$

$$\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}-\frac{1}{16}\int \frac{16\left(-\tan^{14}(x)+57\tan^{12}(x)-1109\tan^{10}(x)+8797\tan^8(x)-31907\tan^6(x)+56619\tan^4(x)-65351\tan\right.}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}-$$
$$\int \frac{-\tan^{14}(x)+57\tan^{12}(x)-1109\tan^{10}(x)+8797\tan^8(x)-31907\tan^6(x)+56619\tan^4(x)-65351\tan^2(x)+}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{14}\int \frac{2\left(7\tan^{12}(x)-406\tan^{10}(x)+8169\tan^8(x)-69748\tan^6(x)+293097\tan^4(x)-689430\tan^2(x)+55303\right)}{\left(\tan^2(x)+1\right)^7}$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{7\tan^{12}(x)-406\tan^{10}(x)+8169\tan^8(x)-69748\tan^6(x)+293097\tan^4(x)-689430\tan^2(x)+55303}{\left(\tan^2(x)+1\right)^7}d\tan$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{7}\left(\frac{279040\tan(x)}{3\left(\tan^2(x)+1\right)^6}-\frac{1}{12}\int \frac{4\left(-21\tan^{10}(x)+1239\tan^8(x)-25746\tan^6(x)+234990\tan^4(x)-1114281\tan^2(\right.}{\left(\tan^2(x)+1\right)^6}\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{279040\tan(x)}{3\left(\tan^2(x)+1\right)^6}-\frac{1}{3}\int \frac{-21\tan^{10}(x)+1239\tan^8(x)-25746\tan^6(x)+234990\tan^4(x)-1114281\tan^2(x)-}{\left(\tan^2(x)+1\right)^6}\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{1}{10}\int \frac{6\left(35\tan^8(x)-2100\tan^6(x)+45010\tan^4(x)-436660\tan^2(x)+59683\right)}{\left(\tan^2(x)+1\right)^5}d\tan(x)-\frac{744704\tan(x}{5\left(\tan^2(x)+1\right.}\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\int \frac{35\tan^8(x)-2100\tan^6(x)+45010\tan^4(x)-436660\tan^2(x)+59683}{\left(\tan^2(x)+1\right)^5}d\tan(x)-\frac{744704\tan(x)}{5\left(\tan^2(x)+1\right)^5}\right)\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 2345$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{\left(\tan^2(x)+1\right)^4}-\frac{1}{8}\int \frac{56\left(-5\tan^6(x)+305\tan^4(x)-6735\tan^2(x)+1179\right)}{\left(\tan^2(x)+1\right)^4}d\tan(x)\right)-\frac{744704\tan}{5\left(\tan^2(x)\right.}\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\int \frac{-5\tan^6(x)+305\tan^4(x)-6735\tan^2(x)+1179}{\left(\tan^2(x)+1\right)^4}d\tan(x)\right)-\frac{744704\tan(x)}{5\left(\tan^2(x)+1\right)^5}\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 2345$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\left(\frac{4112\tan(x)}{3\left(\tan^2(x)+1\right)^3}-\frac{1}{6}\int \frac{10\left(3\tan^4(x)-186\tan^2(x)+115\right)}{\left(\tan^2(x)+1\right)^3}d\tan(x)\right)\right)-\frac{7447}{5\left(\tan\right.}\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\left(\frac{4112\tan(x)}{3\left(\tan^2(x)+1\right)^3}-\frac{5}{3}\int \frac{3\tan^4(x)-186\tan^2(x)+115}{\left(\tan^2(x)+1\right)^3}d\tan(x)\right)\right)-\frac{744704\tan}{5\left(\tan^2(x)\right.}\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 1471$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{\left(\tan^2(x)+1\right)^4}-7\left(\frac{4112\tan(x)}{3\left(\tan^2(x)+1\right)^3}-\frac{5}{3}\left(\frac{76\tan(x)}{\left(\tan^2(x)+1\right)^2}-\frac{1}{4}\int -\frac{12\left(\tan^2(x)+13\right)}{\left(\tan^2(x)+1\right)^2}d\tan(x)\right)\right)\right)\right.\right.$$
$$\frac{83968\tan(x)}{7\left(\tan^2(x)+1\right)^7}+\frac{4096\tan(x)}{\left(\tan^2(x)+1\right)^8}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{4112\tan(x)}{3(\tan^2(x)+1)^3}-\frac{5}{3}\left(3\int\frac{\tan^2(x)+13}{(\tan^2(x)+1)^2}d\tan(x)+\frac{76\tan(x)}{(\tan^2(x)+1)^2}\right)\right)\right)-\frac{7}{5\ \cdots}\right.\right.$$
$$\frac{83968\tan(x)}{7(\tan^2(x)+1)^7}+\frac{4096\tan(x)}{(\tan^2(x)+1)^8}$$

$$\Big\downarrow \; 298$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{4112\tan(x)}{3(\tan^2(x)+1)^3}-\frac{5}{3}\left(3\left(7\int\frac{1}{\tan^2(x)+1}d\tan(x)+\frac{6\tan(x)}{\tan^2(x)+1}\right)+\frac{76\tan(\cdots}{(\tan^2(x)\cdots}\right.\right.\right.\right.\right.$$
$$\frac{83968\tan(x)}{7(\tan^2(x)+1)^7}+\frac{4096\tan(x)}{(\tan^2(x)+1)^8}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{3}{5}\left(\frac{67936\tan(x)}{(\tan^2(x)+1)^4}-7\left(\frac{4112\tan(x)}{3(\tan^2(x)+1)^3}-\frac{5}{3}\left(3\left(7\arctan(\tan(x))+\frac{6\tan(x)}{\tan^2(x)+1}\right)+\frac{76\tan(x)}{(\tan^2(x)+1)^2}\right)\cdots\right.\right.\right.\right.$$
$$\frac{83968\tan(x)}{7(\tan^2(x)+1)^7}+\frac{4096\tan(x)}{(\tan^2(x)+1)^8}$$

input
```
Int[Csc[x]^2*Csc[6*x]^2*Csc[10*x]^2*Csc[15*x]^2*Sin[2*x]^2*Sin[3*x]^2*Sin[
5*x]^2*Sin[30*x]^2,x]
```

output
```
(4096*Tan[x])/(1 + Tan[x]^2)^8 - (83968*Tan[x])/(7*(1 + Tan[x]^2)^7) + ((2
79040*Tan[x])/(3*(1 + Tan[x]^2)^6) + ((-744704*Tan[x])/(5*(1 + Tan[x]^2)^5
) + (3*((67936*Tan[x])/(1 + Tan[x]^2)^4 - 7*((4112*Tan[x])/(3*(1 + Tan[x]^
2)^3) - (5*((76*Tan[x])/(1 + Tan[x]^2)^2 + 3*(7*ArcTan[Tan[x]] + (6*Tan[x]
)/(1 + Tan[x]^2))))/3)))/5)/3)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(52) = 104$.

Time = 0.81 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.56

$$4096\left(\cos(x)^{15} + \frac{15\cos(x)^{13}}{14} + \frac{65\cos(x)^{11}}{56} + \frac{143\cos(x)^{9}}{112} + \frac{1287\cos(x)^{7}}{896} + \frac{429\cos(x)^{5}}{256} + \frac{2145\cos(x)^{3}}{1024}\right.$$

input
```
int(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^2*sin(
5*x)^2*sin(30*x)^2,x)
```

output
```
4096*(cos(x)^15+15/14*cos(x)^13+65/56*cos(x)^11+143/112*cos(x)^9+1287/896*
cos(x)^7+429/256*cos(x)^5+2145/1024*cos(x)^3+6435/2048*cos(x))*sin(x)+7*x-
16384*(cos(x)^13+13/12*cos(x)^11+143/120*cos(x)^9+429/320*cos(x)^7+1001/64
0*cos(x)^5+1001/512*cos(x)^3+3003/1024*cos(x))*sin(x)+78848/3*(cos(x)^11+1
1/10*cos(x)^9+99/80*cos(x)^7+231/160*cos(x)^5+231/128*cos(x)^3+693/256*cos
(x))*sin(x)-108544/5*(cos(x)^9+9/8*cos(x)^7+21/16*cos(x)^5+105/64*cos(x)^3
+315/128*cos(x))*sin(x)+9920*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*c
os(x))*sin(x)-7616/3*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+368*(cos(x
)^3+3/2*cos(x))*sin(x)-32*cos(x)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= \frac{2}{105}\left(215040\cos(x)^{15} - 629760\cos(x)^{13} + 697600\cos(x)^{11} - 372352\cos(x)^{9} + 101904\cos(x)^{7} - 143\right.$$
$$+ 7\,x$$

input
```
integrate(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^
2*sin(5*x)^2*sin(30*x)^2,x, algorithm="fricas")
```

output
```
2/105*(215040*cos(x)^15 - 629760*cos(x)^13 + 697600*cos(x)^11 - 372352*cos
(x)^9 + 101904*cos(x)^7 - 14392*cos(x)^5 + 1330*cos(x)^3 + 315*cos(x))*sin
(x) + 7*x
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= \text{Timed out}$$

input
```
integrate(csc(x)**2*csc(6*x)**2*csc(10*x)**2*csc(15*x)**2*sin(2*x)**2*sin(
3*x)**2*sin(5*x)**2*sin(30*x)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= 7\,x + \frac{1}{8}\,\sin\left(16\,x\right) + \frac{2}{7}\,\sin\left(14\,x\right) + \frac{1}{6}\,\sin\left(12\,x\right) - \frac{2}{5}\,\sin\left(10\,x\right)$$
$$- \sin\left(8\,x\right) - \frac{4}{3}\,\sin\left(6\,x\right) - \frac{1}{2}\,\sin\left(4\,x\right) + 4\,\sin\left(2\,x\right)$$

input
```
integrate(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^
2*sin(5*x)^2*sin(30*x)^2,x, algorithm="maxima")
```

output
```
7*x + 1/8*sin(16*x) + 2/7*sin(14*x) + 1/6*sin(12*x) - 2/5*sin(10*x) - sin(
8*x) - 4/3*sin(6*x) - 1/2*sin(4*x) + 4*sin(2*x)
```

Giac [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx = 7\,x$$
$$+ \frac{2\,\left(315\,\tan\left(x\right)^{15} + 3535\,\tan\left(x\right)^{13} + 203\,\tan\left(x\right)^{11} + 60919\,\tan\left(x\right)^{9} - 71031\,\tan\left(x\right)^{7} + 74613\,\tan\left(x\right)^{5}}{105\,\left(\tan\left(x\right)^{2} + 1\right)^{8}}$$

input
```
integrate(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^
2*sin(5*x)^2*sin(30*x)^2,x, algorithm="giac")
```

output
```
7*x + 2/105*(315*tan(x)^15 + 3535*tan(x)^13 + 203*tan(x)^11 + 60919*tan(x)
^9 - 71031*tan(x)^7 + 74613*tan(x)^5 - 5775*tan(x)^3 - 315*tan(x))/(tan(x)
^2 + 1)^8
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= 4096\sin(x)\cos(x)^{15} - \frac{83968\sin(x)\cos(x)^{13}}{7} + \frac{279040\sin(x)\cos(x)^{11}}{21} - \frac{744704\sin(x)\cos(x)^9}{105} + \frac{67936\sin(x)\cos(x)^7}{35} - \frac{4112\sin(x)\cos(x)^5}{15} + \frac{76\sin(x)\cos(x)^3}{3} + 6\sin(x)\cos(x) + 7x$$

input
```
int((sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2)/(sin(6*x)^2*sin(10*x)^2
*sin(15*x)^2*sin(x)^2),x)
```

output
```
7*x + 6*cos(x)*sin(x) + (76*cos(x)^3*sin(x))/3 - (4112*cos(x)^5*sin(x))/15
 + (67936*cos(x)^7*sin(x))/35 - (744704*cos(x)^9*sin(x))/105 + (279040*cos
(x)^11*sin(x))/21 - (83968*cos(x)^13*sin(x))/7 + 4096*cos(x)^15*sin(x)
```

Reduce [F]

$$\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x)\sin^2(5x)\sin^2(30x)\,dx$$
$$= \int \csc(x)^2\csc(6x)^2\csc(10x)^2\csc(15x)^2\sin(2x)^2\sin(3x)^2\sin(5x)^2\sin(30x)^2\,dx$$

input
```
int(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^2*sin(
5*x)^2*sin(30*x)^2,x)
```

output
```
int(csc(x)^2*csc(6*x)^2*csc(10*x)^2*csc(15*x)^2*sin(2*x)^2*sin(3*x)^2*sin(
5*x)^2*sin(30*x)^2,x)
```

3.320 $\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx=\frac{1}{8}\left(\frac{2x\left(5+x^2+5\sqrt{1+x^2+x^4}\right)}{\left(1-x^2+\sqrt{1+x^2+x^4}\right)\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}\right)\right)$$

output
```
1/4*x*(5+x^2+5*(x^4+x^2+1)^(1/2))/(1-x^2+(x^4+x^2+1)^(1/2))/(1+x^2+(x^4+x^
2+1)^(1/2))^(1/2)+3/8*2^(1/2)*arctanh(2^(1/2)*x/(1+x^2+(x^4+x^2+1)^(1/2))^
(1/2))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx=\frac{1}{8}\left(\frac{2x\left(5+x^2+5\sqrt{1+x^2+x^4}\right)}{\left(1-x^2+\sqrt{1+x^2+x^4}\right)\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}\right)\right)$$

input
```
Integrate[Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]],x]
```

output
```
((2*x*(5 + x^2 + 5*Sqrt[1 + x^2 + x^4]))/((1 - x^2 + Sqrt[1 + x^2 + x^4])*
Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]]) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[
1 + x^2 + Sqrt[1 + x^2 + x^4]]])/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+\sqrt{x^4+x^2+1}+1}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sqrt{x^2+\sqrt{x^4+x^2+1}+1}dx$$

input
```
Int[Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]],x]
```

output
```
$Aborted
```

Maple [F]

$$\int \sqrt{1+x^2+\sqrt{x^4+x^2+1}}dx$$

input
```
int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

output
```
int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{3\sqrt{2}x\log\left(-\frac{64\,x^5+16\,x^3+16\,\sqrt{x^4+x^2+1}(4\,x^3-x)+4\left(\sqrt{2}\sqrt{x^4+x^2+1}(8\,x^2-5)+\sqrt{2}(8\,x^4-x^2+5)\right)\sqrt{x^2+\sqrt{x^4+x^2+1}+1}+25\,x}{x}\right)+8}{32\,x}$$

input
```
integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/32*(3*sqrt(2)*x*log(-(64*x^5 + 16*x^3 + 16*sqrt(x^4 + x^2 + 1)*(4*x^3 -
x) + 4*(sqrt(2)*sqrt(x^4 + x^2 + 1)*(8*x^2 - 5) + sqrt(2)*(8*x^4 - x^2 + 5
))*sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1) + 25*x)/x) + 8*(3*x^2 - sqrt(x^4 +
x^2 + 1) + 1)*sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1))/x
```

Sympy [F]

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx = \int \sqrt{x^2+\sqrt{x^4+x^2+1}+1}\,dx$$

input
```
integrate((1+x**2+(x**4+x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(x**2 + sqrt(x**4 + x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx = \int \sqrt{x^2+\sqrt{x^4+x^2+1}+1}\,dx$$

input
```
integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{1}{16}\sqrt{2}\Big(2\sqrt{x^2+x+1}(2x+1)-3\log\left(-2x+2\sqrt{x^2+x+1}-1\right)\Big)$$
$$+\frac{1}{16}\sqrt{2}\Big(2\sqrt{x^2-x+1}(2x-1)-3\log\left(-2x+2\sqrt{x^2-x+1}+1\right)\Big)$$

input
```
integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(2*sqrt(x^2 + x + 1)*(2*x + 1) - 3*log(-2*x + 2*sqrt(x^2 + x
+ 1) - 1)) + 1/16*sqrt(2)*(2*sqrt(x^2 - x + 1)*(2*x - 1) - 3*log(-2*x + 2*
sqrt(x^2 - x + 1) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx = \int \sqrt{\sqrt{x^4+x^2+1}+x^2+1}\,dx$$

input
```
int(((x^2 + x^4 + 1)^(1/2) + x^2 + 1)^(1/2),x)
```

output
```
int(((x^2 + x^4 + 1)^(1/2) + x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$$
$$=\frac{\sqrt{2}\left(4\sqrt{x^2+x+1}\,x+2\sqrt{x^2+x+1}+8\left(\int \frac{\sqrt{x^2+x+1}\,\sqrt{x^4+x^2+1}}{x^2+x+1}dx\right)+3\log\left(\frac{2\sqrt{x^2+x+1}+2x+1}{\sqrt{3}}\right)\right)}{16}$$

input
```
int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

output
```
(sqrt(2)*(4*sqrt(x**2 + x + 1)*x + 2*sqrt(x**2 + x + 1) + 8*int((sqrt(x**2
 + x + 1)*sqrt(x**4 + x**2 + 1))/(x**2 + x + 1),x) + 3*log((2*sqrt(x**2 +
x + 1) + 2*x + 1)/sqrt(3))))/16
```

3.321 $\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx = -\frac{1}{20}\log\left(23-x^{10}\right)+\frac{1}{20}\log\left(25-x^{10}\right)$$

output
```
-1/20*ln(-x^10+23)+1/20*ln(-x^10+25)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx = -\frac{1}{20}\log\left(23-x^{10}\right)+\frac{1}{20}\log\left(25-x^{10}\right)$$

input
```
Integrate[x^9/(575 - 48*x^10 + x^20),x]
```

output
```
-1/20*Log[23 - x^10] + Log[25 - x^10]/20
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1690, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^{20}-48x^{10}+575}\,dx$$

$$\Big\downarrow \text{1690}$$

$$\frac{1}{10}\int \frac{1}{x^{20}-48x^{10}+575}dx^{10}$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{10}\int \left(\frac{1}{2(23-x^{10})}-\frac{1}{2(25-x^{10})}\right)dx^{10}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}\left(\frac{1}{2}\log\left(25-x^{10}\right)-\frac{1}{2}\log\left(23-x^{10}\right)\right)$$

input
```
Int[x^9/(575 - 48*x^10 + x^20),x]
```

output
```
(-1/2*Log[23 - x^10] + Log[25 - x^10]/2)/10
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\ln(x^{10}-23)}{20}+\frac{\ln(x^{10}-25)}{20}$	18
risch	$-\frac{\ln(x^{10}-23)}{20}+\frac{\ln(x^{10}-25)}{20}$	18
norman	$\frac{\ln(x^5-5)}{20}+\frac{\ln(x^5+5)}{20}-\frac{\ln(x^{10}-23)}{20}$	26
parallelrisch	$\frac{\ln(x^5-5)}{20}+\frac{\ln(x^5+5)}{20}-\frac{\ln(x^{10}-23)}{20}$	26

input
```
int(x^9/(x^20-48*x^10+575),x,method=_RETURNVERBOSE)
```

output
```
-1/20*ln(x^10-23)+1/20*ln(x^10-25)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx=-\frac{1}{20}\log\left(x^{10}-23\right)+\frac{1}{20}\log\left(x^{10}-25\right)$$

input
```
integrate(x^9/(x^20-48*x^10+575),x, algorithm="fricas")
```

output
```
-1/20*log(x^10 - 23) + 1/20*log(x^10 - 25)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx = \frac{\log(x^{10}-25)}{20} - \frac{\log(x^{10}-23)}{20}$$

input
```
integrate(x**9/(x**20-48*x**10+575),x)
```

output
```
log(x**10 - 25)/20 - log(x**10 - 23)/20
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx = -\frac{1}{20}\log\left(x^{10}-23\right) + \frac{1}{20}\log\left(x^{10}-25\right)$$

input
```
integrate(x^9/(x^20-48*x^10+575),x, algorithm="maxima")
```

output
```
-1/20*log(x^10 - 23) + 1/20*log(x^10 - 25)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx = -\frac{1}{20}\log\left(|x^{10}-23|\right) + \frac{1}{20}\log\left(|x^{10}-25|\right)$$

input
```
integrate(x^9/(x^20-48*x^10+575),x, algorithm="giac")
```

output
```
-1/20*log(abs(x^10 - 23)) + 1/20*log(abs(x^10 - 25))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx=\frac{\operatorname{atan}\left(\frac{x^{10}\,1006772302081\mathrm{i}-21354610286400\mathrm{i}}{2807924963544\,x^{10}-66383426822975}\right)\,1\mathrm{i}}{10}$$

input
```
int(x^9/(x^20 - 48*x^10 + 575),x)
```

output
```
(atan((x^10*1006772302081i - 21354610286400i)/(2807924963544*x^10 - 663834
26822975))*1i)/10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx=-\frac{\log(x^{10}-23)}{20}+\frac{\log(x^5-5)}{20}+\frac{\log(x^5+5)}{20}$$

input
```
int(x^9/(x^20-48*x^10+575),x)
```

output
```
( - log(x**10 - 23) + log(x**5 - 5) + log(x**5 + 5))/20
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file