Computer Algebra Independent Integration Tests

Summer 2024

367-Blake-2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**111**]. This is test number [367].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.10 (110)	0.90 (1)
Maple	99.10 (110)	0.90 (1)
Mupad	96.40 (107)	3.60 (4)
Fricas	87.39 (97)	12.61 (14)
Sympy	70.27 (78)	29.73 (33)
Rubi	65.77 (73)	34.23 (38)
Giac	65.77 (73)	34.23 (38)
Reduce	43.24 (48)	56.76 (63)
Maxima	37.84 (42)	62.16 (69)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

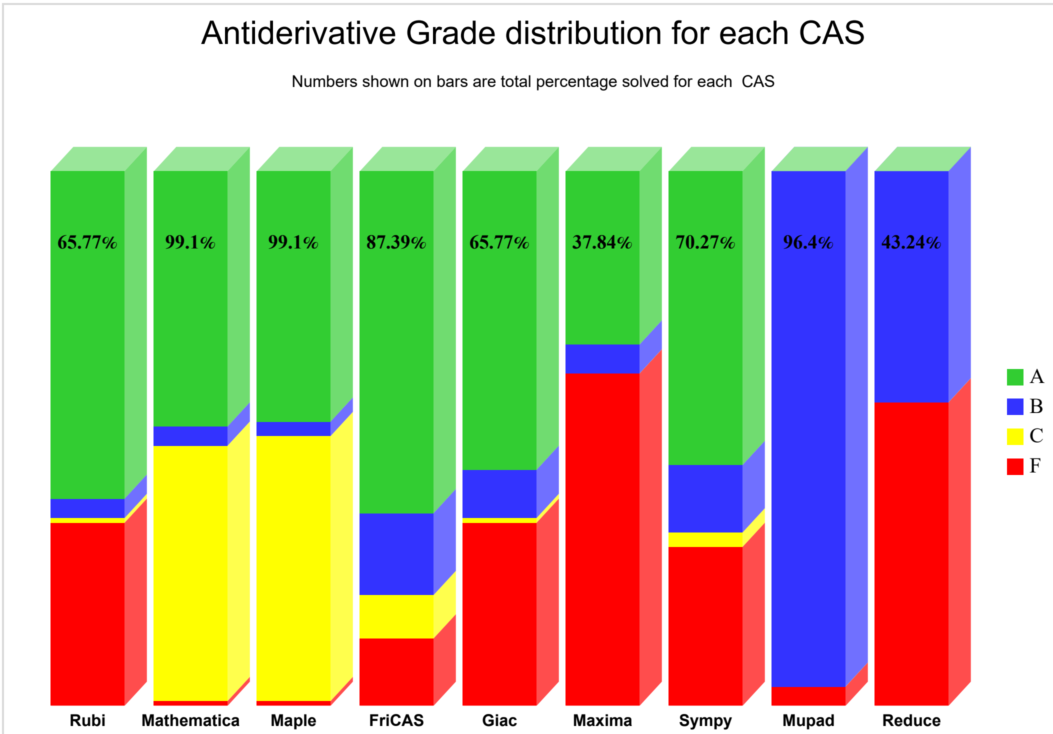
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

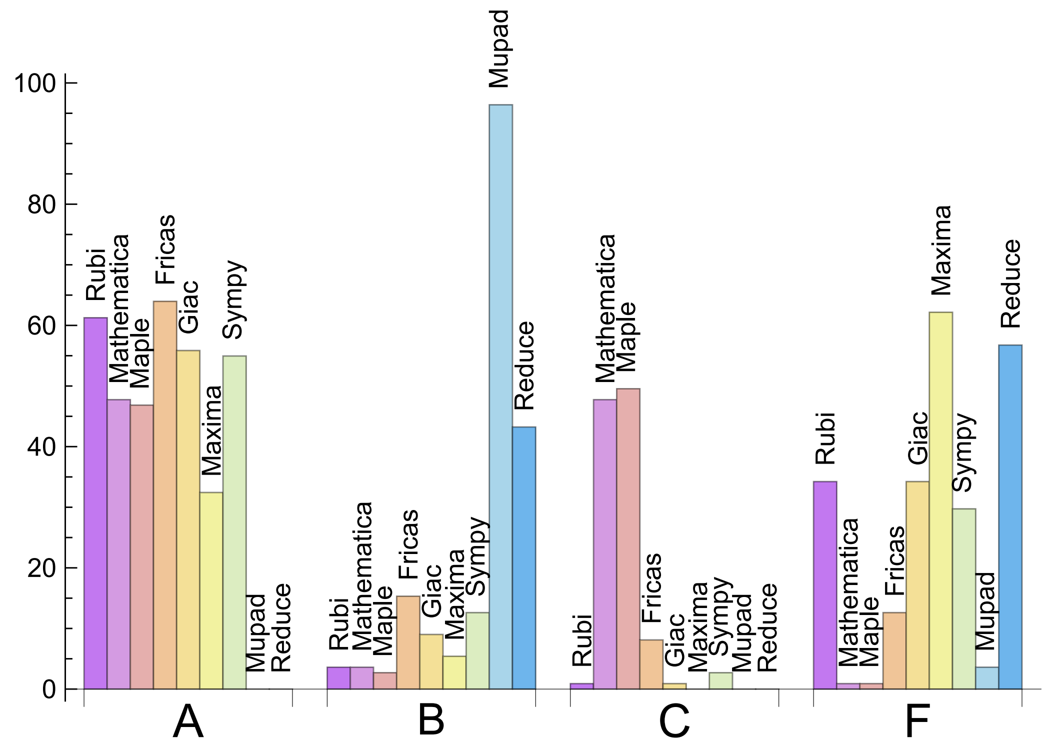
System	% A grade	% B grade	% C grade	% F grade
Fricas	63.964	15.315	8.108	12.613
Rubi	61.261	3.604	0.901	34.234
Giac	55.856	9.009	0.901	34.234
Sympy	54.955	12.613	2.703	29.730
Mathematica	47.748	3.604	47.748	0.901
Maple	46.847	2.703	49.550	0.901
Maxima	32.432	5.405	0.000	62.162
Mupad	0.000	96.396	0.000	3.604
Reduce	0.000	43.243	0.000	56.757

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	1	0.00	100.00	0.00
Maple	1	0.00	100.00	0.00
Mupad	4	0.00	100.00	0.00
Fricas	14	21.43	78.57	0.00
Sympy	33	0.00	60.61	39.39
Rubi	38	100.00	0.00	0.00
Giac	38	28.95	10.53	60.53
Reduce	63	100.00	0.00	0.00
Maxima	69	95.65	0.00	4.35

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.12
Mathematica	0.16
Giac	0.19
Fricas	0.32
Maple	0.34
Rubi	0.78
Sympy	0.91
Reduce	4.24
Mupad	9.07

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	72.66	0.59	54.50	0.62
Maxima	157.40	1.37	120.00	0.95
Rubi	164.37	1.28	131.00	1.01
Mathematica	164.95	1.36	135.50	0.96
Giac	203.52	1.24	102.00	0.90
Sympy	214.36	1.40	90.00	0.85
Reduce	308.73	2.22	121.00	1.42
Mupad	850.80	4.92	235.00	1.41
Fricas	24697.14	82.64	153.00	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

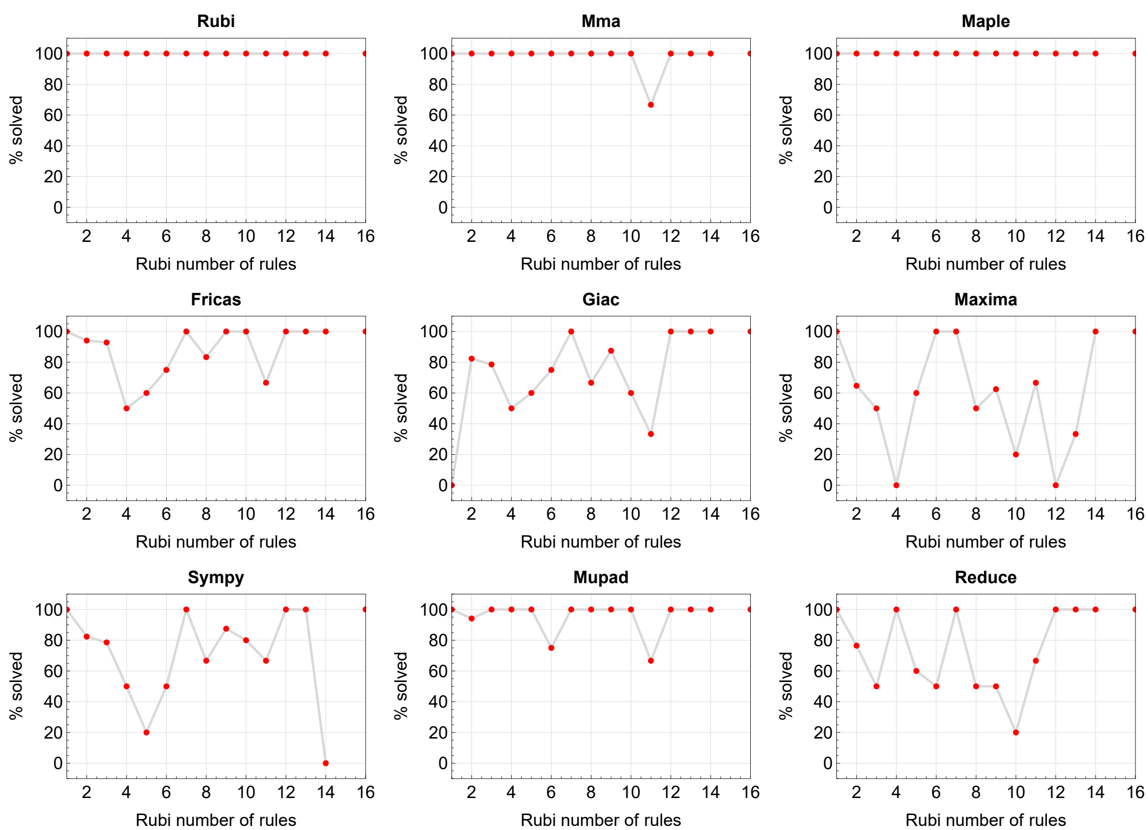


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

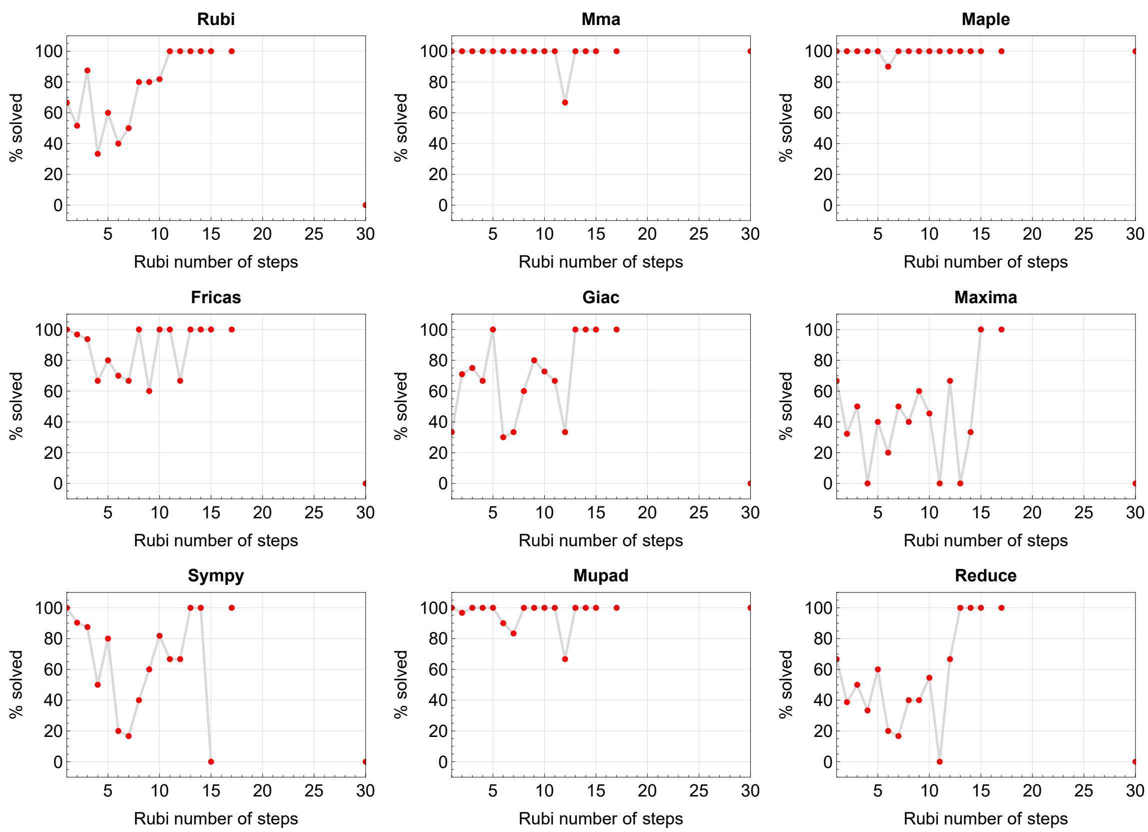


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

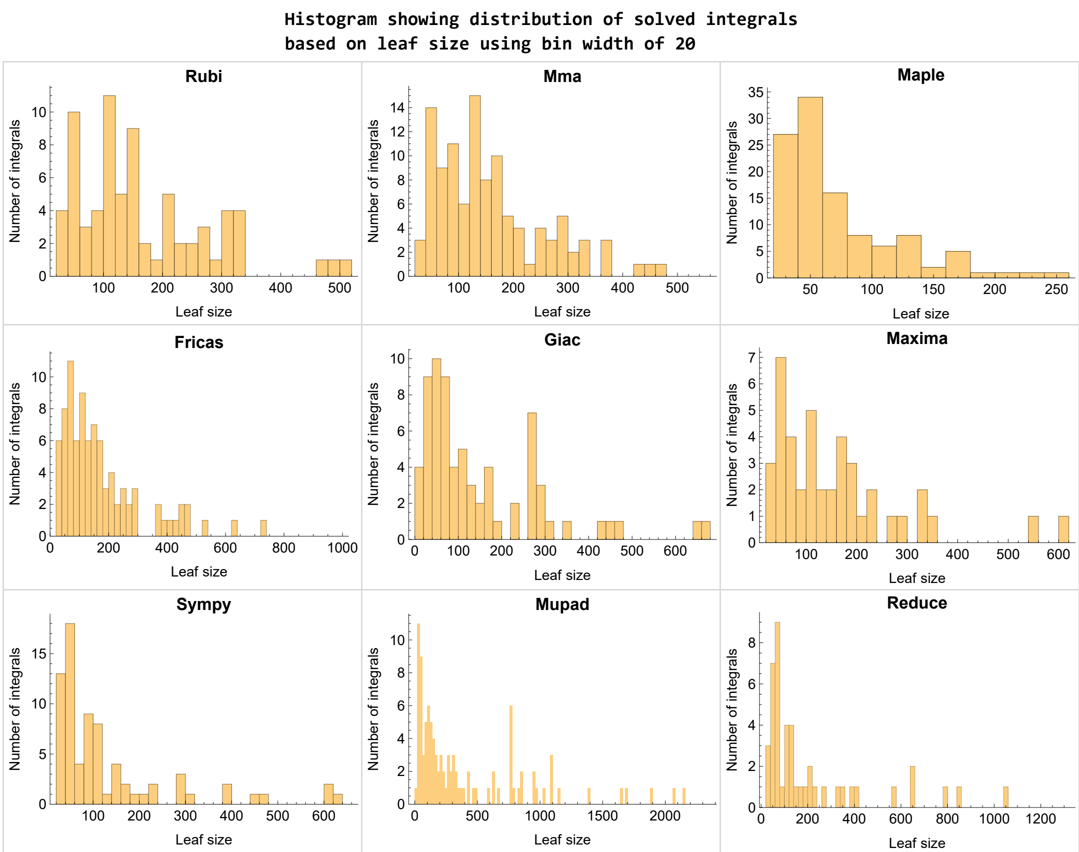


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

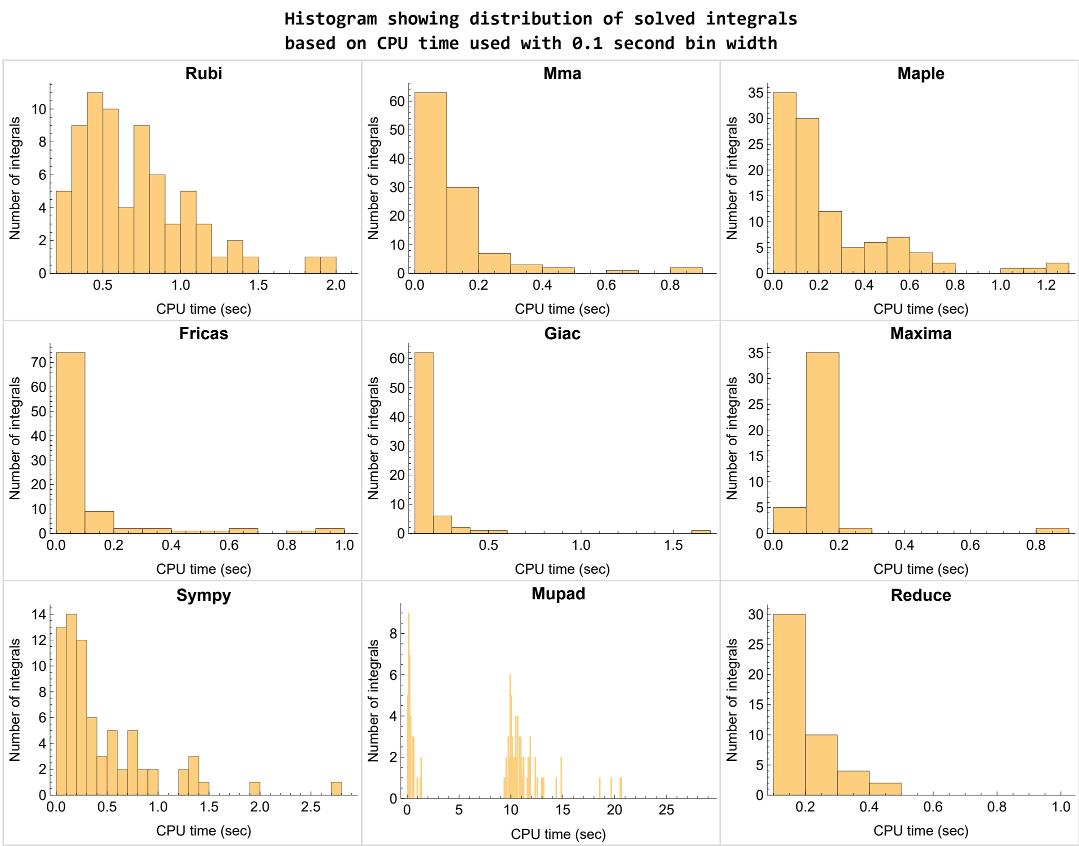


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

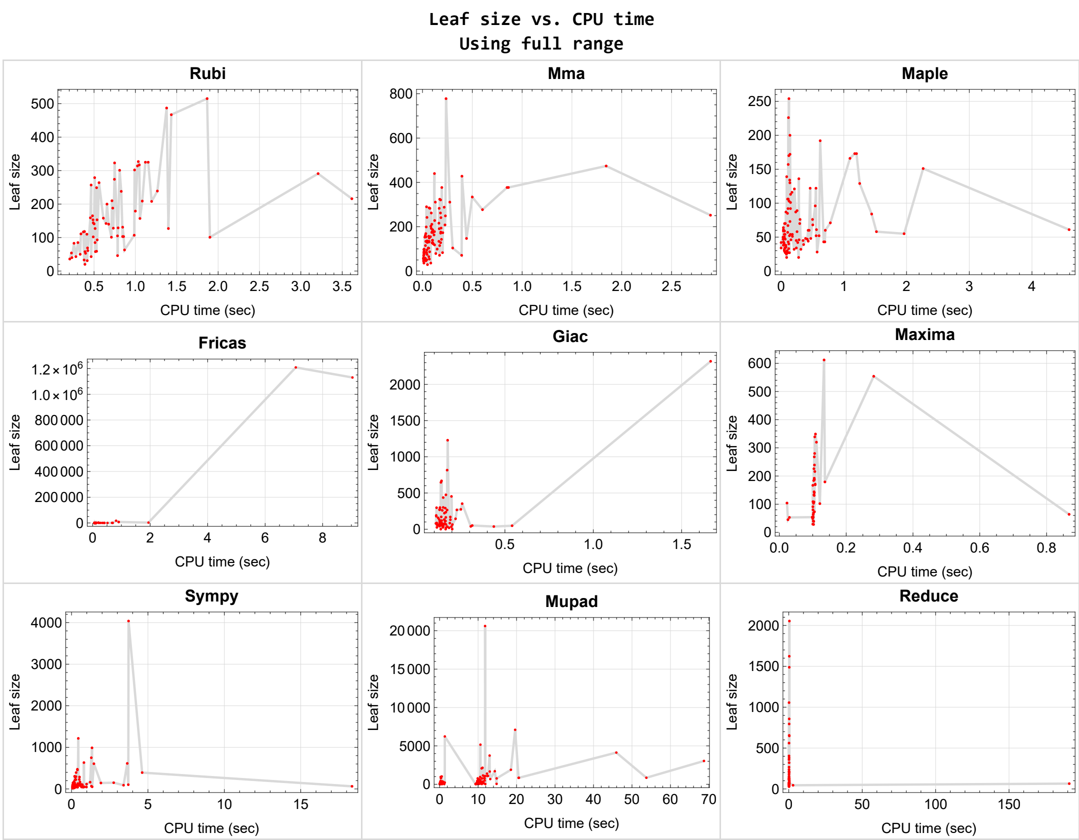


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{38, 61, 62\}$

Mathematica $\{41, 99\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

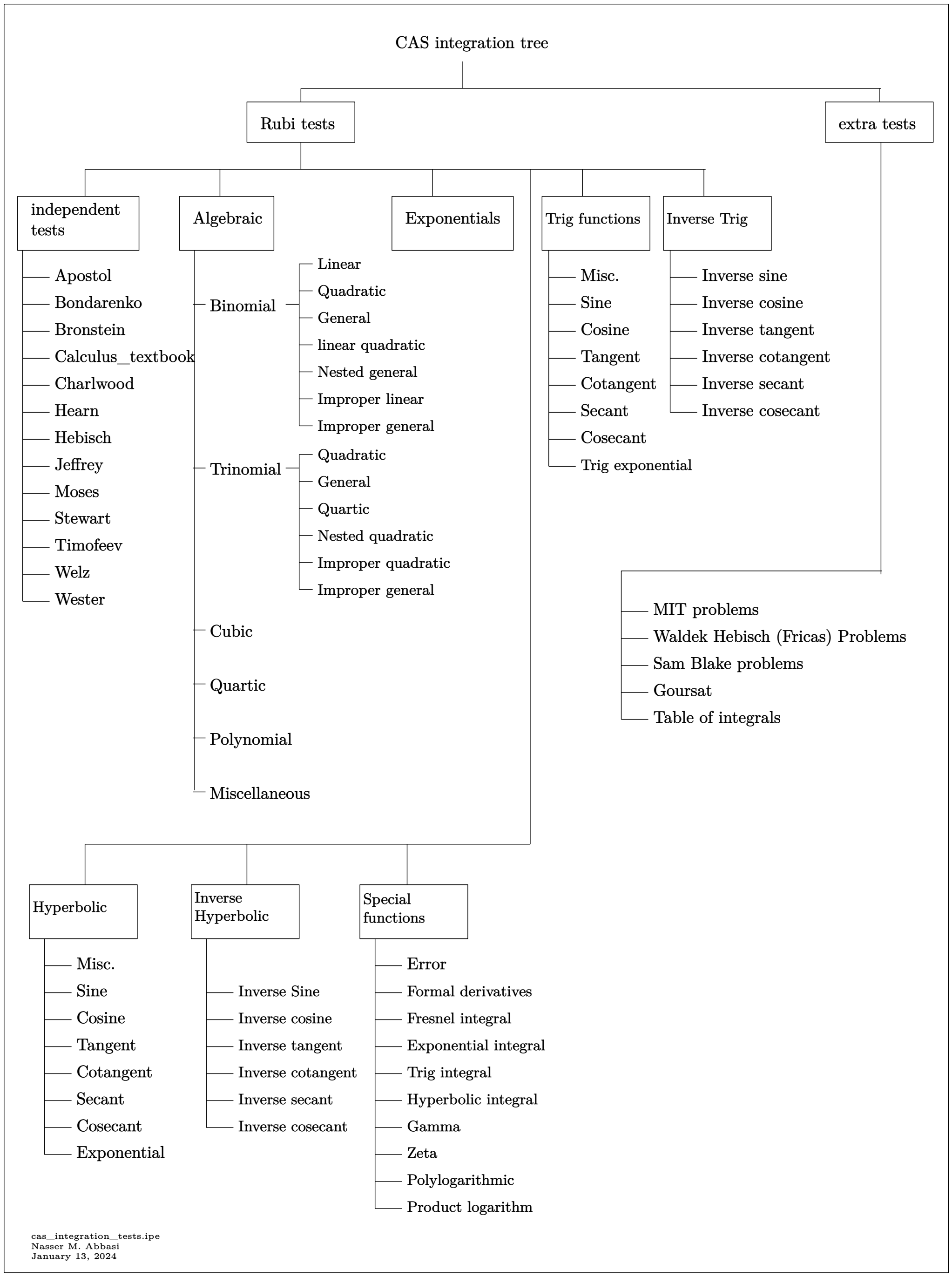
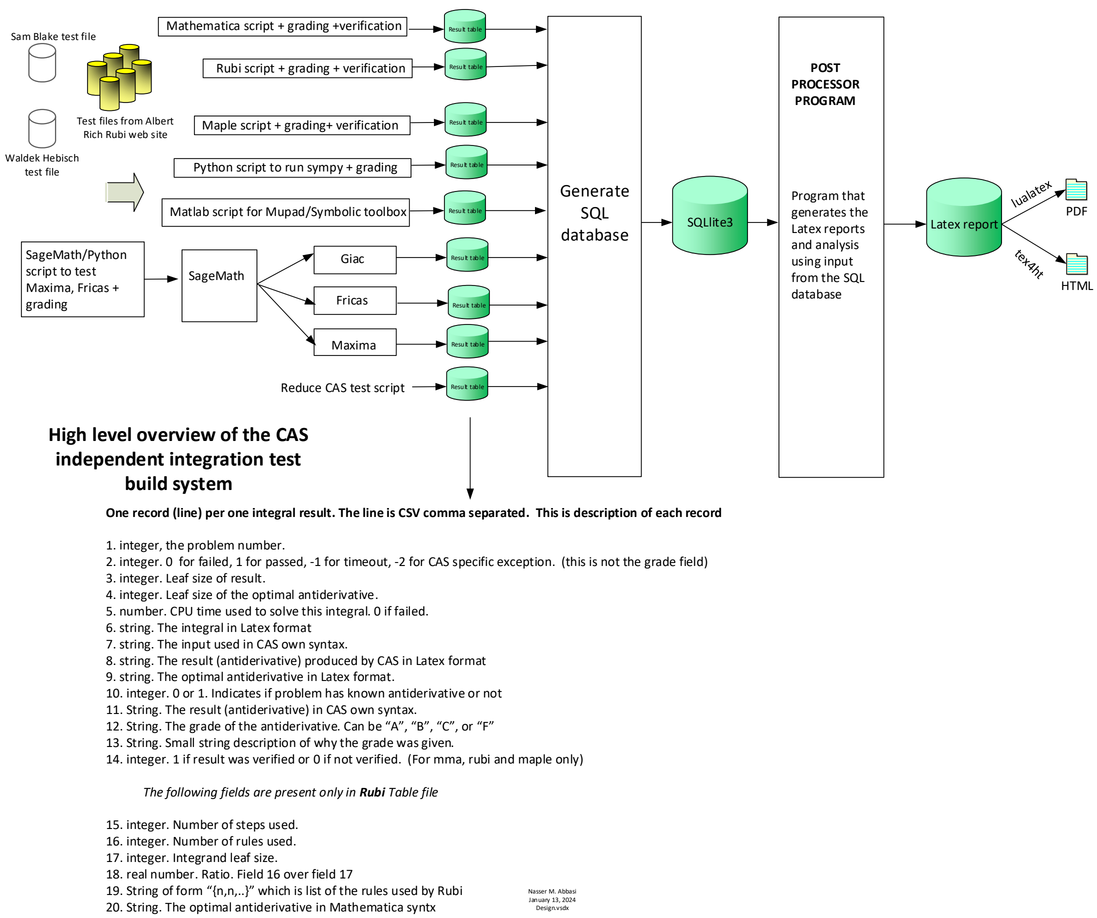


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 4, 5, 6, 12, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 39, 40, 41, 42, 44, 45, 46, 50, 54, 58, 59, 60, 61, 62, 63, 64, 65, 67, 70, 74, 75, 79, 80, 84, 85, 86, 87, 93, 96, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade { 7, 47, 49, 83 }

C grade { 38 }

F normal fail { 3, 8, 9, 10, 11, 13, 14, 17, 34, 35, 43, 48, 51, 52, 53, 55, 56, 57, 66, 68, 69, 71, 72, 73, 76, 77, 78, 81, 82, 88, 89, 90, 91, 92, 94, 95, 97, 99 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 4, 5, 6, 12, 13, 14, 18, 19, 20, 21, 23, 25, 26, 27, 28, 29, 30, 32, 33, 37, 38, 39, 40, 42, 43, 44, 45, 46, 48, 52, 54, 64, 79, 80, 82, 84, 85, 87, 88, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade { 7, 47, 63, 83 }

C grade { 1, 3, 8, 9, 10, 11, 15, 16, 17, 22, 24, 34, 35, 36, 41, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99 }

F normal fail { }

F(-1) timedout fail { 31 }

F(-2) exception fail { }

Maple

A grade { 2, 4, 5, 6, 7, 12, 13, 14, 19, 20, 21, 23, 24, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 63, 65, 70, 80, 82, 83, 84, 85, 88, 106, 107, 108, 109 }

B grade { 48, 50, 52 }

C grade { 1, 3, 8, 9, 10, 11, 15, 16, 17, 18, 22, 25, 31, 33, 56, 57, 59, 60, 61, 62, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 110, 111 }

F normal fail { }

F(-1) timedout fail { 99 }

F(-2) exception fail { }

Fricas

A grade { 1, 4, 6, 9, 10, 11, 12, 17, 19, 21, 23, 24, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 53, 54, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 102, 105, 106 }

B grade { 13, 14, 15, 16, 22, 26, 29, 31, 46, 55, 58, 59, 60, 69, 80, 82, 111 }

C grade { 18, 25, 100, 101, 103, 104, 107, 109, 110 }

F normal fail { 2, 50, 98 }

F(-1) timedout fail { 3, 5, 7, 8, 20, 48, 49, 51, 52, 99, 108 }

F(-2) exception fail { }

Maxima

A grade { 2, 4, 6, 12, 18, 19, 21, 23, 25, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 42, 45, 54, 70, 79, 80, 84, 85, 100, 101, 102, 105, 106, 107, 108, 109, 111 }

B grade { 5, 7, 20, 26, 40, 41 }

C grade { }

F normal fail { 1, 3, 8, 9, 10, 11, 13, 14, 15, 16, 17, 22, 33, 34, 35, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 110 }

F(-1) timedout fail { }

F(-2) exception fail { 24, 43, 44 }

Giac

A grade { 2, 4, 6, 11, 12, 15, 18, 19, 20, 21, 23, 24, 27, 28, 29, 30, 31, 32, 33, 36, 38, 41, 42, 43, 45, 47, 54, 60, 62, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 82, 83, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 97, 100, 101, 102, 103, 104, 106, 110, 111 }

B grade { 3, 25, 26, 48, 59, 63, 64, 107, 108, 109 }

C grade { 105 }

F normal fail { 13, 14, 17, 22, 52, 69, 81, 86, 92, 93, 99 }

F(-1) timedout fail { 10, 35, 51, 77 }

F(-2) exception fail { 1, 5, 7, 8, 9, 16, 34, 37, 39, 40, 44, 46, 49, 50, 53, 55, 56, 57, 58, 61, 65, 80, 98 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 7, 99, 108 }

F(-2) exception fail { }

Sympy

A grade { 4, 6, 7, 11, 15, 16, 17, 18, 21, 22, 25, 31, 33, 36, 37, 38, 39, 40, 42, 44, 45, 60, 61, 62, 63, 64, 66, 67, 68, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 91, 92, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 110, 111 }

B grade { 2, 10, 13, 14, 23, 26, 28, 41, 46, 65, 69, 90, 93, 106 }

C grade { 12, 32, 75 }

F normal fail { }

F(-1) timedout fail { 5, 20, 24, 27, 29, 30, 34, 35, 43, 49, 52, 53, 55, 56, 57, 58, 99, 107, 108, 109 }

F(-2) exception fail { 1, 3, 8, 9, 19, 47, 48, 50, 51, 54, 59, 83, 98 }

Reduce

A grade { }

B grade { 4, 6, 7, 12, 19, 20, 21, 23, 25, 26, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 62, 63, 64, 70, 79, 80, 84, 85, 87, 93, 100, 102, 103, 104, 106, 107, 108, 109, 110, 111 }

C grade { }

F normal fail { 1, 2, 3, 5, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 22, 24, 27, 34, 35, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 86, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 101, 105 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-2)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	301	122	96	0	281	0	0	257	1021
N.S.	1	1.31	0.53	0.42	0.00	1.23	0.00	0.00	1.12	4.46
time (sec)	N/A	0.809	0.109	0.544	0.000	0.090	0.000	0.000	0.168	0.555

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	111	95	129	102	0	287	84	781	116
N.S.	1	1.28	1.09	1.48	1.17	0.00	3.30	0.97	8.98	1.33
time (sec)	N/A	0.341	0.044	1.253	0.121	0.000	0.522	0.143	0.180	10.674

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-2)	B	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	228	151	0	0	0	2319	0	4129
N.S.	1	0.00	1.58	1.05	0.00	0.00	0.00	16.10	0.00	28.67
time (sec)	N/A	0.000	0.122	2.265	0.000	0.000	0.000	1.664	1.041	45.930

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	50	52	53	56	46	54	52	56
N.S.	1	0.91	0.91	0.95	0.96	1.02	0.84	0.98	0.95	1.02
time (sec)	N/A	0.333	0.039	0.133	0.030	0.067	0.977	0.150	0.164	0.190

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	84	612	0	0	0	0	0
N.S.	1	1.00	1.03	0.83	6.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.710	0.303	1.444	0.134	0.000	0.000	0.000	6.035	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	59	71	61	64	112	66	64	118	50
N.S.	1	0.83	1.00	0.86	0.90	1.58	0.93	0.90	1.66	0.70
time (sec)	N/A	0.531	0.393	4.592	0.867	0.075	1.301	0.203	0.164	14.808

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	F(-1)	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	216	474	43	554	0	63	0	95	0
N.S.	1	2.84	6.24	0.57	7.29	0.00	0.83	0.00	1.25	0.00
time (sec)	N/A	3.617	1.844	0.679	0.282	0.000	18.356	0.000	0.389	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-2)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	0	174	122	0	0	0	0	0	1698
N.S.	1	0.00	0.89	0.63	0.00	0.00	0.00	0.00	0.00	8.71
time (sec)	N/A	0.000	0.104	0.556	0.000	0.000	0.000	0.000	0.226	14.344

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	F(-2)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	0	131	89	0	113	0	0	518	224
N.S.	1	0.00	0.71	0.48	0.00	0.61	0.00	0.00	2.80	1.21
time (sec)	N/A	0.000	0.065	0.255	0.000	0.079	0.000	0.000	0.170	1.344

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	B	F(-1)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	0	211	48	0	456	4038	0	741	266
N.S.	1	0.00	0.35	0.08	0.00	0.77	6.79	0.00	1.25	0.45
time (sec)	N/A	0.000	0.094	0.473	0.000	0.096	3.730	0.000	0.182	10.833

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	0	129	70	0	253	80	66	303	166
N.S.	1	0.00	0.55	0.30	0.00	1.09	0.34	0.28	1.30	0.71
time (sec)	N/A	0.000	0.048	0.068	0.000	0.094	0.550	0.151	0.166	0.102

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	141	127	87	108	129	607	116	125	110
N.S.	1	1.04	0.93	0.64	0.79	0.95	4.46	0.85	0.92	0.81
time (sec)	N/A	0.501	0.104	0.077	0.104	0.079	1.463	0.134	0.164	0.341

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	B	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	0	199	92	0	7957	150	0	145	771
N.S.	1	0.00	0.71	0.33	0.00	28.32	0.53	0.00	0.52	2.74
time (sec)	N/A	0.000	0.038	0.110	0.000	0.908	2.759	0.000	0.162	9.900

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	B	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	0	192	121	0	7153	614	0	139	294
N.S.	1	0.00	1.00	0.63	0.00	37.26	3.20	0.00	0.72	1.53
time (sec)	N/A	0.000	0.088	0.212	0.000	0.903	3.645	0.000	0.173	9.806

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	144	86	61	0	471	41	25	85	146
N.S.	1	0.83	0.50	0.35	0.00	2.72	0.24	0.14	0.49	0.84
time (sec)	N/A	0.490	0.015	0.037	0.000	0.083	0.263	0.115	0.188	10.036

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	238	79	43	0	295	46	0	65	151
N.S.	1	1.23	0.41	0.22	0.00	1.52	0.24	0.00	0.34	0.78
time (sec)	N/A	0.829	0.017	0.038	0.000	0.699	0.651	0.000	0.173	9.785

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	0	85	59	0	155	41	0	83	146
N.S.	1	0.00	0.47	0.32	0.00	0.85	0.23	0.00	0.46	0.80
time (sec)	N/A	0.000	0.013	0.036	0.000	0.079	0.213	0.000	0.164	9.922

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	264	216	45	192	1208573	41	260	21	283
N.S.	1	1.08	0.89	0.18	0.79	4953.17	0.17	1.07	0.09	1.16
time (sec)	N/A	0.560	0.164	0.350	0.104	7.066	0.102	0.146	0.162	10.852

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	159	262	122	141	432	0	133	2054	251
N.S.	1	1.13	1.86	0.87	1.00	3.06	0.00	0.94	14.57	1.78
time (sec)	N/A	0.456	0.180	0.463	0.105	0.334	0.000	0.128	0.421	10.822

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	58	65	58	133	0	0	56	72	358
N.S.	1	0.85	0.96	0.85	1.96	0.00	0.00	0.82	1.06	5.26
time (sec)	N/A	0.391	0.063	1.520	0.103	0.000	0.000	0.137	0.446	11.271

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	29	32	28	35	36	31	28	28
N.S.	1	0.98	0.71	0.78	0.68	0.85	0.88	0.76	0.68	0.68
time (sec)	N/A	0.233	0.050	0.315	0.102	0.063	0.170	0.149	0.168	11.881

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	323	50	46	0	537	31	0	22	674
N.S.	1	0.91	0.14	0.13	0.00	1.52	0.09	0.00	0.06	1.90
time (sec)	N/A	0.748	0.023	0.157	0.000	0.078	0.138	0.000	0.204	13.160

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	153	134	134	167	443	751	170	404	266
N.S.	1	0.87	0.76	0.76	0.95	2.52	4.27	0.97	2.30	1.51
time (sec)	N/A	0.488	0.165	0.137	0.100	0.084	1.294	0.140	0.162	10.365

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	118	135	139	0	407	0	131	396	7097
N.S.	1	1.01	1.15	1.19	0.00	3.48	0.00	1.12	3.38	60.66
time (sec)	N/A	0.385	0.052	0.089	0.000	0.266	0.000	0.191	0.164	19.634

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	186	106	144	17307	95	268	154	196
N.S.	1	1.00	1.59	0.91	1.23	147.92	0.81	2.29	1.32	1.68
time (sec)	N/A	0.372	0.102	0.104	0.104	0.816	0.384	0.130	0.166	0.263

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	115	85	131	170	162	222	118	306
N.S.	1	1.00	1.35	1.00	1.54	2.00	1.91	2.61	1.39	3.60
time (sec)	N/A	0.305	0.029	0.102	0.104	0.080	1.216	0.157	0.168	0.994

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	279	277	166	187	3389	0	301	1145	3037
N.S.	1	1.11	1.10	0.66	0.75	13.50	0.00	1.20	4.56	12.10
time (sec)	N/A	0.507	0.602	1.098	0.105	0.198	0.000	0.151	0.193	68.702

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	165	103	101	109	379	989	102	359	425
N.S.	1	1.45	0.90	0.89	0.96	3.32	8.68	0.89	3.15	3.73
time (sec)	N/A	0.481	0.063	0.150	0.101	0.085	1.339	0.131	0.157	9.773

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	262	226	226	1179	0	299	561	6231
N.S.	1	1.00	1.02	0.88	0.88	4.59	0.00	1.16	2.18	24.25
time (sec)	N/A	0.465	0.088	0.117	0.103	0.116	0.000	0.132	0.173	1.354

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	110	104	103	104	151	0	119	212	97
N.S.	1	0.99	0.94	0.93	0.94	1.36	0.00	1.07	1.91	0.87
time (sec)	N/A	0.414	0.058	0.126	0.023	0.517	0.000	0.142	0.158	9.993

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	C	A	B	A	A	B	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	249	0	116	280	476	168	297	1056	795
N.S.	1	0.94	0.00	0.44	1.06	1.80	0.63	1.12	3.98	3.00
time (sec)	N/A	0.531	0.000	0.173	0.106	0.080	0.554	0.113	0.179	0.346

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	54	54	73	69	58	394	75	77	167
N.S.	1	0.98	0.98	1.33	1.25	1.05	7.16	1.36	1.40	3.04
time (sec)	N/A	0.227	0.027	0.141	0.102	0.077	0.301	0.133	0.182	0.223

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	83	75	34	0	96	26	79	795	137
N.S.	1	1.05	0.95	0.43	0.00	1.22	0.33	1.00	10.06	1.73
time (sec)	N/A	0.258	0.046	0.060	0.000	0.073	0.182	0.130	0.265	9.707

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	176	60	0	117	0	0	687	761
N.S.	1	0.00	2.02	0.69	0.00	1.34	0.00	0.00	7.90	8.75
time (sec)	N/A	0.000	0.183	0.447	0.000	0.090	0.000	0.000	0.197	11.708

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F(-1)	F(-1)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	171	60	0	114	0	0	687	761
N.S.	1	0.00	2.09	0.73	0.00	1.39	0.00	0.00	8.38	9.28
time (sec)	N/A	0.000	0.182	0.431	0.000	0.086	0.000	0.000	0.184	11.622

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	334	32	29	53	51	51	58	29
N.S.	1	1.00	9.28	0.89	0.81	1.47	1.42	1.42	1.61	0.81
time (sec)	N/A	0.206	0.500	0.207	0.100	0.066	0.799	0.315	0.149	0.316

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	42	54	66	56	0	66	39
N.S.	1	1.00	1.26	0.98	1.26	1.53	1.30	0.00	1.53	0.91
time (sec)	N/A	0.281	0.069	0.265	0.098	0.069	0.760	0.000	0.145	0.511

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	515	46	43	63	72	44	45	73	42
N.S.	1	10.10	0.90	0.84	1.24	1.41	0.86	0.88	1.43	0.82
time (sec)	N/A	1.867	0.034	0.697	0.101	0.068	0.377	0.150	0.181	0.651

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	42	54	66	56	0	66	39
N.S.	1	1.00	1.26	0.98	1.26	1.53	1.30	0.00	1.53	0.91
time (sec)	N/A	0.465	0.026	0.000	0.101	0.065	0.764	0.000	0.160	0.002

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	127	83	44	179	65	90	0	78	40
N.S.	1	1.27	0.83	0.44	1.79	0.65	0.90	0.00	0.78	0.40
time (sec)	N/A	1.399	0.201	0.432	0.136	0.069	3.403	0.000	0.152	1.216

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	288	20	91	33	42	19	34	19
N.S.	1	1.00	14.40	1.00	4.55	1.65	2.10	0.95	1.70	0.95
time (sec)	N/A	0.390	0.218	0.279	0.100	0.072	0.765	0.184	0.156	9.402

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	107	79	61	103	122	102	74	129	76
N.S.	1	1.35	1.00	0.77	1.30	1.54	1.29	0.94	1.63	0.96
time (sec)	N/A	0.986	0.134	0.564	0.102	0.066	3.714	0.156	0.158	10.246

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	36	28	0	55	0	37	53	27
N.S.	1	0.00	1.00	0.78	0.00	1.53	0.00	1.03	1.47	0.75
time (sec)	N/A	0.000	0.083	0.574	0.000	0.072	0.000	0.437	0.178	0.695

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	71	39	0	59	49	0	61	42
N.S.	1	1.09	1.22	0.67	0.00	1.02	0.84	0.00	1.05	0.72
time (sec)	N/A	0.869	0.171	0.359	0.000	0.074	0.830	0.000	0.159	9.950

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	46	36	35	45	45	41	35	45	34
N.S.	1	1.24	0.97	0.95	1.22	1.22	1.11	0.95	1.22	0.92
time (sec)	N/A	0.784	0.014	0.077	0.026	0.058	0.161	0.149	2.903	9.323

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	239	143	106	0	225	296	0	43	120
N.S.	1	1.76	1.05	0.78	0.00	1.65	2.18	0.00	0.32	0.88
time (sec)	N/A	1.266	0.190	0.114	0.000	0.090	0.292	0.000	200.010	9.544

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	103	139	34	0	33	0	35	207	34
N.S.	1	2.51	3.39	0.83	0.00	0.80	0.00	0.85	5.05	0.83
time (sec)	N/A	0.857	0.100	0.243	0.000	0.066	0.000	0.174	0.163	10.976

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F(-1)	F(-2)	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	0	191	52	0	0	0	66	65	1659
N.S.	1	0.00	1.00	0.27	0.00	0.00	0.00	0.35	0.34	8.69
time (sec)	N/A	0.000	0.168	0.605	0.000	0.000	0.000	0.166	191.015	13.033

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	291	194	60	0	0	0	0	687	1086
N.S.	1	3.46	2.31	0.71	0.00	0.00	0.00	0.00	8.18	12.93
time (sec)	N/A	3.209	0.187	0.705	0.000	0.000	0.000	0.000	0.272	12.586

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-2)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	153	118	192	0	0	0	0	107	1889
N.S.	1	1.31	1.01	1.64	0.00	0.00	0.00	0.00	0.91	16.15
time (sec)	N/A	0.536	0.121	0.623	0.000	0.000	0.000	0.000	0.268	18.533

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-2)	F(-1)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	208	52	0	0	0	0	439	850
N.S.	1	0.00	2.77	0.69	0.00	0.00	0.00	0.00	5.85	11.33
time (sec)	N/A	0.000	0.226	0.558	0.000	0.000	0.000	0.000	0.480	53.735

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F(-1)	F(-1)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	0	321	71	0	0	0	0	685	761
N.S.	1	0.00	1.57	0.35	0.00	0.00	0.00	0.00	3.36	3.73
time (sec)	N/A	0.000	0.182	0.785	0.000	0.000	0.000	0.000	0.276	14.824

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	178	48	0	104	0	0	687	761
N.S.	1	0.00	2.51	0.68	0.00	1.46	0.00	0.00	9.68	10.72
time (sec)	N/A	0.000	0.120	0.390	0.000	0.086	0.000	0.000	0.201	10.972

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	33	51	40	39	23	0	39	138	47
N.S.	1	0.97	1.50	1.18	1.15	0.68	0.00	1.15	4.06	1.38
time (sec)	N/A	0.380	0.043	0.122	0.103	0.071	0.000	0.307	0.173	9.579

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	B	F(-1)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	174	46	0	104	0	0	687	761
N.S.	1	0.00	2.60	0.69	0.00	1.55	0.00	0.00	10.25	11.36
time (sec)	N/A	0.000	0.093	0.286	0.000	0.088	0.000	0.000	0.196	11.058

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	F(-1)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	377	173	0	197	0	0	0	841
N.S.	1	0.00	1.94	0.89	0.00	1.02	0.00	0.00	0.00	4.34
time (sec)	N/A	0.000	0.849	1.202	0.000	0.410	0.000	0.000	0.363	20.618

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	F(-1)	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	377	173	0	198	0	0	0	837
N.S.	1	0.00	2.13	0.98	0.00	1.12	0.00	0.00	0.00	4.73
time (sec)	N/A	0.000	0.862	1.174	0.000	0.391	0.000	0.000	0.334	20.519

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	101	163	47	0	200	0	0	687	1086
N.S.	1	1.36	2.20	0.64	0.00	2.70	0.00	0.00	9.28	14.68
time (sec)	N/A	1.901	0.116	0.417	0.000	0.088	0.000	0.000	0.253	12.376

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	208	134	87	0	291	0	816	596	160
N.S.	1	1.68	1.08	0.70	0.00	2.35	0.00	6.58	4.81	1.29
time (sec)	N/A	1.196	0.066	0.233	0.000	0.083	0.000	0.173	0.220	10.691

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	131	84	59	0	129	26	25	79	265
N.S.	1	1.30	0.83	0.58	0.00	1.28	0.26	0.25	0.78	2.62
time (sec)	N/A	0.846	0.013	0.054	0.000	0.072	0.163	0.149	0.171	0.201

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	105	89	28	0	78	87	0	116	201
N.S.	1	0.97	0.82	0.26	0.00	0.72	0.81	0.00	1.07	1.86
time (sec)	N/A	0.784	0.019	0.070	0.000	0.065	0.116	0.000	0.184	11.069

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	58	55	47	0	74	99	86	70	49
N.S.	1	0.50	0.48	0.41	0.00	0.64	0.86	0.75	0.61	0.43
time (sec)	N/A	0.517	0.009	0.033	0.000	0.065	0.076	0.114	0.170	0.086

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	31	57	25	0	24	22	55	47	24
N.S.	1	1.82	3.35	1.47	0.00	1.41	1.29	3.24	2.76	1.41
time (sec)	N/A	0.418	0.015	0.084	0.000	0.067	0.063	0.138	0.202	0.071

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	202	38	0	126	148	267	267	69
N.S.	1	1.00	1.28	0.24	0.00	0.80	0.94	1.69	1.69	0.44
time (sec)	N/A	0.613	0.084	0.075	0.000	0.074	0.096	0.228	0.176	9.947

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	179	151	136	0	157	457	0	76	2153
N.S.	1	1.27	1.07	0.96	0.00	1.11	3.24	0.00	0.54	15.27
time (sec)	N/A	1.000	0.069	0.283	0.000	0.128	0.388	0.000	200.031	11.155

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	0	155	58	0	196	117	84	365	629
N.S.	1	0.00	0.86	0.32	0.00	1.08	0.65	0.46	2.02	3.48
time (sec)	N/A	0.000	0.080	0.254	0.000	0.214	0.985	0.188	0.202	10.375

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	115	29	0	88	104	144	26	118
N.S.	1	1.03	0.93	0.23	0.00	0.71	0.84	1.16	0.21	0.95
time (sec)	N/A	0.733	0.172	0.091	0.000	0.068	0.100	0.221	200.010	0.163

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	78	30	0	69	90	1	63	321
N.S.	1	0.00	0.79	0.30	0.00	0.70	0.91	0.01	0.64	3.24
time (sec)	N/A	0.000	0.016	0.097	0.000	0.098	0.185	0.202	0.215	10.059

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	B	B	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	178	54	0	211	473	0	455	951
N.S.	1	0.00	1.23	0.37	0.00	1.46	3.26	0.00	3.14	6.56
time (sec)	N/A	0.000	0.097	0.179	0.000	0.112	0.381	0.000	0.259	10.535

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	69	99	54	53	53	70	53	51	83
N.S.	1	0.81	1.16	0.64	0.62	0.62	0.82	0.62	0.60	0.98
time (sec)	N/A	0.421	0.147	0.050	0.100	0.068	0.085	0.124	0.178	0.115

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	136	92	0	162	228	48	372	384
N.S.	1	0.00	0.89	0.60	0.00	1.06	1.49	0.31	2.43	2.51
time (sec)	N/A	0.000	0.044	0.149	0.000	0.096	0.506	0.541	0.310	0.685

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	0	131	54	0	158	109	115	271	629
N.S.	1	0.00	0.87	0.36	0.00	1.05	0.72	0.76	1.79	4.17
time (sec)	N/A	0.000	0.042	0.148	0.000	0.087	0.638	0.160	0.175	10.299

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	0	75	31	0	87	29	1	74	305
N.S.	1	0.00	0.77	0.32	0.00	0.90	0.30	0.01	0.76	3.14
time (sec)	N/A	0.000	0.014	0.050	0.000	0.073	0.160	0.137	0.171	0.151

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	129	153	33	0	91	141	1	57	305
N.S.	1	0.77	0.92	0.20	0.00	0.54	0.84	0.01	0.34	1.83
time (sec)	N/A	0.791	0.023	0.057	0.000	0.072	0.208	0.164	0.180	10.099

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	209	133	45	0	109	51	22	205	131
N.S.	1	1.12	0.72	0.24	0.00	0.59	0.27	0.12	1.10	0.70
time (sec)	N/A	1.083	0.053	0.241	0.000	0.077	1.349	0.167	0.183	10.471

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	45	38	0	68	100	79	21	361
N.S.	1	0.00	0.43	0.36	0.00	0.65	0.95	0.75	0.20	3.44
time (sec)	N/A	0.000	0.010	0.036	0.000	0.068	0.102	0.159	0.172	10.599

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F(-1)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	0	285	55	0	171	224	0	56	81
N.S.	1	0.00	0.87	0.17	0.00	0.52	0.69	0.00	0.17	0.25
time (sec)	N/A	0.000	0.067	1.961	0.000	0.134	0.283	0.000	200.030	10.428

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	0	440	70	0	365	102	353	397	477
N.S.	1	0.00	0.91	0.14	0.00	0.75	0.21	0.73	0.82	0.99
time (sec)	N/A	0.000	0.120	0.297	0.000	0.109	0.710	0.258	0.243	10.623

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	102	149	20	80	68	80	80	211	37
N.S.	1	1.10	1.60	0.22	0.86	0.73	0.86	0.86	2.27	0.40
time (sec)	N/A	0.490	0.067	0.088	0.101	0.065	0.068	0.109	0.231	10.070

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	128	68	73	157	92	0	1488	483
N.S.	1	1.00	1.38	0.73	0.78	1.69	0.99	0.00	16.00	5.19
time (sec)	N/A	0.538	0.182	0.505	0.105	0.069	0.457	0.000	0.324	11.722

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	0	97	64	0	163	46	0	98	181
N.S.	1	0.00	0.48	0.32	0.00	0.80	0.23	0.00	0.48	0.89
time (sec)	N/A	0.000	0.014	0.036	0.000	0.075	0.213	0.000	0.176	10.159

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	58	50	0	105	56	77	95	85
N.S.	1	0.00	1.00	0.86	0.00	1.81	0.97	1.33	1.64	1.47
time (sec)	N/A	0.000	0.028	0.053	0.000	0.066	0.057	0.136	0.182	0.216

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	103	139	34	0	33	0	35	207	34
N.S.	1	2.51	3.39	0.83	0.00	0.80	0.00	0.85	5.05	0.83
time (sec)	N/A	0.845	0.100	0.000	0.000	0.069	0.000	0.197	0.177	0.002

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	55	39	49	60	97	55	57	42
N.S.	1	1.09	0.98	0.70	0.88	1.07	1.73	0.98	1.02	0.75
time (sec)	N/A	0.425	0.027	0.043	0.100	0.065	0.260	0.151	0.166	0.182

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	53	49	41	40	40	54	42	39	52
N.S.	1	0.88	0.82	0.68	0.67	0.67	0.90	0.70	0.65	0.87
time (sec)	N/A	0.398	0.015	0.047	0.101	0.073	0.093	0.130	0.167	0.135

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	157	69	54	0	153	41	0	55	146
N.S.	1	0.90	0.40	0.31	0.00	0.88	0.24	0.00	0.32	0.84
time (sec)	N/A	1.055	0.011	0.062	0.000	0.085	0.399	0.000	0.163	0.284

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	140	127	47	0	132	42	134	179	126
N.S.	1	1.32	1.20	0.44	0.00	1.25	0.40	1.26	1.69	1.19
time (sec)	N/A	0.677	0.097	0.066	0.000	0.077	0.201	0.157	0.164	10.195

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	0	147	130	0	229	282	57	301	1389
N.S.	1	0.00	1.00	0.88	0.00	1.56	1.92	0.39	2.05	9.45
time (sec)	N/A	0.000	0.442	0.140	0.000	0.115	0.243	0.119	0.204	12.316

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	0	169	43	0	162	180	163	271	1081
N.S.	1	0.00	0.80	0.20	0.00	0.76	0.85	0.77	1.28	5.10
time (sec)	N/A	0.000	0.033	0.113	0.000	0.079	0.167	0.165	0.242	11.808

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	B	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	0	249	51	0	273	634	235	580	969
N.S.	1	0.00	0.86	0.18	0.00	0.94	2.18	0.81	1.99	3.33
time (sec)	N/A	0.000	0.230	0.180	0.000	0.097	0.811	0.166	0.231	11.652

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	0	87	76	0	89	143	113	683	117
N.S.	1	0.00	0.70	0.61	0.00	0.71	1.14	0.90	5.46	0.94
time (sec)	N/A	0.000	0.072	0.501	0.000	0.078	1.922	0.164	0.271	11.543

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	0	84	67	0	217	60	0	530	1145
N.S.	1	0.00	0.21	0.17	0.00	0.54	0.15	0.00	1.31	2.83
time (sec)	N/A	0.000	0.041	0.109	0.000	0.102	0.234	0.000	0.273	11.271

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	142	59	50	0	93	1216	0	198	91
N.S.	1	1.30	0.54	0.46	0.00	0.85	11.16	0.00	1.82	0.83
time (sec)	N/A	0.645	0.010	0.031	0.000	0.084	0.443	0.000	0.241	10.437

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	242	33	0	177	218	156	517	959
N.S.	1	0.00	1.05	0.14	0.00	0.77	0.94	0.68	2.24	4.15
time (sec)	N/A	0.000	0.045	0.121	0.000	0.089	0.231	0.122	0.185	0.277

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	0	290	57	0	254	304	272	636	131
N.S.	1	0.00	0.80	0.16	0.00	0.70	0.84	0.75	1.75	0.36
time (sec)	N/A	0.000	0.040	0.105	0.000	0.083	0.200	0.130	0.190	10.423

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	85	99	70	0	66	100	64	108	76
N.S.	1	1.09	1.27	0.90	0.00	0.85	1.28	0.82	1.38	0.97
time (sec)	N/A	0.512	0.017	0.055	0.000	0.065	0.075	0.115	0.172	0.110

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	A	A	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	0	161	29	0	142	29	169	237	593
N.S.	1	0.00	0.64	0.12	0.00	0.57	0.12	0.68	0.95	2.37
time (sec)	N/A	0.000	0.028	0.054	0.000	0.075	0.191	0.191	0.174	10.024

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	127	173	76	0	0	0	0	377	2068
N.S.	1	1.14	1.56	0.68	0.00	0.00	0.00	0.00	3.40	18.63
time (sec)	N/A	0.513	0.204	0.302	0.000	0.000	0.000	0.000	0.286	10.913

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F(-1)	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	252	0	0	0	0	0	101	0
N.S.	1	0.00	1.81	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.000	2.891	180.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	200	134	27	169	112	20	179	112	33
N.S.	1	1.03	0.69	0.14	0.87	0.57	0.10	0.92	0.57	0.17
time (sec)	N/A	0.657	0.033	0.109	0.107	0.070	0.071	0.112	0.190	0.099

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	317	311	27	239	1130235	20	275	11	235
N.S.	1	0.76	0.75	0.06	0.57	2710.40	0.05	0.66	0.03	0.56
time (sec)	N/A	1.048	0.129	0.133	0.105	9.037	0.067	0.133	0.237	0.596

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	210	154	27	184	216	20	190	124	219
N.S.	1	0.98	0.72	0.13	0.86	1.00	0.09	0.88	0.58	1.02
time (sec)	N/A	0.717	0.035	0.094	0.103	0.065	0.069	0.111	0.254	9.905

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	302	324	27	0	248	20	437	321	110
N.S.	1	0.54	0.58	0.05	0.00	0.45	0.04	0.78	0.58	0.20
time (sec)	N/A	0.990	0.173	0.067	0.000	0.074	0.076	0.151	0.246	9.912

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	467	778	27	0	639	20	647	649	435
N.S.	1	0.56	0.94	0.03	0.00	0.77	0.02	0.78	0.78	0.52
time (sec)	N/A	1.434	0.236	0.069	0.000	0.674	0.086	0.138	0.352	0.270

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	325	241	157	216	395	129	274	59	320
N.S.	1	1.28	0.95	0.62	0.85	1.56	0.51	1.08	0.23	1.26
time (sec)	N/A	1.121	0.185	0.113	0.106	0.091	0.409	0.251	0.198	10.145

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	188	157	172	171	139	393	291	139	3743
N.S.	1	0.84	0.70	0.77	0.77	0.62	1.76	1.30	0.62	16.78
time (sec)	N/A	0.726	0.062	0.142	0.107	0.094	4.621	0.162	0.162	12.996

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	274	283	170	268	1503	0	475	237	5162
N.S.	1	0.96	1.00	0.60	0.94	5.29	0.00	1.67	0.83	18.18
time (sec)	N/A	0.746	0.075	0.123	0.105	0.115	0.000	0.166	0.166	10.611

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	487	428	254	339	0	0	1229	1623	0
N.S.	1	0.97	0.85	0.50	0.67	0.00	0.00	2.44	3.23	0.00
time (sec)	N/A	1.378	0.396	0.125	0.106	0.000	0.000	0.176	0.335	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	327	311	200	349	2801	0	668	655	20619
N.S.	1	0.95	0.90	0.58	1.01	8.12	0.00	1.94	1.90	59.77
time (sec)	N/A	1.035	0.274	0.141	0.108	1.942	0.000	0.141	0.282	11.817

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	325	377	36	0	273	29	453	380	118
N.S.	1	0.58	0.67	0.06	0.00	0.48	0.05	0.80	0.67	0.21
time (sec)	N/A	1.155	0.194	0.075	0.000	0.079	0.116	0.197	0.218	0.185

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	314	283	113	320	728	119	265	857	208
N.S.	1	1.35	1.22	0.49	1.38	3.14	0.51	1.14	3.69	0.90
time (sec)	N/A	1.025	0.168	0.151	0.111	0.084	0.530	0.173	0.298	0.381

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [104] had the largest ratio of [1.44443999999999995]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	11	1.31	52	0.212
2	A	9	8	1.28	50	0.160
3	F	0	0	N/A	0.000	N/A
4	A	6	6	0.91	105	0.057
5	A	7	6	1.00	179	0.034
6	A	2	2	0.83	107	0.019
7	B	12	11	2.84	231	0.048
8	F	0	0	N/A	0.000	N/A
9	F	0	0	N/A	0.000	N/A
10	F	0	0	N/A	0.000	N/A
11	F	0	0	N/A	0.000	N/A
12	A	2	2	1.04	38	0.053
13	F	0	0	N/A	0.000	N/A
14	F	0	0	N/A	0.000	N/A
15	A	2	2	0.83	33	0.061
16	A	3	3	1.23	30	0.100
17	F	0	0	N/A	0.000	N/A
18	A	10	9	1.08	11	0.818
19	A	6	5	1.13	28	0.179
20	A	5	5	0.85	82	0.061

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	3	0.98	60	0.050
22	A	10	9	0.91	19	0.474
23	A	5	5	0.87	29	0.172
24	A	4	3	1.01	30	0.100
25	A	3	3	1.00	26	0.115
26	A	3	3	1.00	22	0.136
27	A	9	9	1.11	36	0.250
28	A	2	2	1.45	26	0.077
29	A	2	2	1.00	24	0.083
30	A	3	3	0.99	28	0.107
31	A	12	11	0.94	21	0.524
32	A	3	3	0.98	23	0.130
33	A	5	4	1.05	16	0.250
34	F	0	0	N/A	0.000	N/A
35	F	0	0	N/A	0.000	N/A
36	A	3	2	1.00	34	0.059
37	A	1	1	1.00	67	0.015
38	C	2	2	10.10	90	0.022
39	A	1	1	1.00	67	0.015
40	A	8	8	1.27	122	0.066
41	A	3	3	1.00	46	0.065
42	A	3	3	1.35	93	0.032
43	F	0	0	N/A	0.000	N/A
44	A	10	10	1.09	119	0.084
45	A	7	6	1.24	44	0.136
46	A	3	3	1.76	41	0.073
47	B	8	8	2.51	86	0.093
48	F	0	0	N/A	0.000	N/A
49	B	6	5	3.46	115	0.043
50	A	4	4	1.31	60	0.067
51	F	0	0	N/A	0.000	N/A
52	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	F	0	0	N/A	0.000	N/A
54	A	7	6	0.97	54	0.111
55	F	0	0	N/A	0.000	N/A
56	F	0	0	N/A	0.000	N/A
57	F	0	0	N/A	0.000	N/A
58	A	6	5	1.36	79	0.063
59	A	11	10	1.68	68	0.147
60	A	2	2	1.30	31	0.065
61	A	11	10	0.97	34	0.294
62	A	10	9	0.50	20	0.450
63	A	2	2	1.82	53	0.038
64	A	2	2	1.00	29	0.069
65	A	2	2	1.27	74	0.027
66	F	0	0	N/A	0.000	N/A
67	A	11	10	1.03	24	0.417
68	F	0	0	N/A	0.000	N/A
69	F	0	0	N/A	0.000	N/A
70	A	8	7	0.81	18	0.389
71	F	0	0	N/A	0.000	N/A
72	F	0	0	N/A	0.000	N/A
73	F	0	0	N/A	0.000	N/A
74	A	3	3	0.77	30	0.100
75	A	3	3	1.12	57	0.053
76	F	0	0	N/A	0.000	N/A
77	F	0	0	N/A	0.000	N/A
78	F	0	0	N/A	0.000	N/A
79	A	10	9	1.10	9	1.000
80	A	2	2	1.00	43	0.047
81	F	0	0	N/A	0.000	N/A
82	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
83	B	8	8	2.51	86	0.093
84	A	2	2	1.09	23	0.087
85	A	2	2	0.88	24	0.083
86	A	2	2	0.90	28	0.071
87	A	10	9	1.32	25	0.360
88	F	0	0	N/A	0.000	N/A
89	F	0	0	N/A	0.000	N/A
90	F	0	0	N/A	0.000	N/A
91	F	0	0	N/A	0.000	N/A
92	F	0	0	N/A	0.000	N/A
93	A	9	8	1.30	22	0.364
94	F	0	0	N/A	0.000	N/A
95	F	0	0	N/A	0.000	N/A
96	A	3	3	1.09	38	0.079
97	F	0	0	N/A	0.000	N/A
98	A	3	3	1.14	38	0.079
99	F	0	0	N/A	0.000	N/A
100	A	9	8	1.03	9	0.889
101	A	10	9	0.76	9	1.000
102	A	10	9	0.98	9	1.000
103	A	13	12	0.54	9	1.333
104	A	14	13	0.56	9	1.444
105	A	10	10	1.28	24	0.417
106	A	14	13	0.84	24	0.542
107	A	2	2	0.96	33	0.061
108	A	2	2	0.97	59	0.034
109	A	15	14	0.95	24	0.583
110	A	14	13	0.58	13	1.000
111	A	17	16	1.35	14	1.143

CHAPTER 3

Listing of integrals

3.1 $\int \frac{2a(-b^3+2\sqrt{2}a^2bx^2)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 229

$$\int \frac{2a(-b^3+2\sqrt{2}a^2bx^2)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx = -\sqrt{-1+2\sqrt[4]{2}}\arctan\left(\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{2^{3/4}\sqrt{-1+2\sqrt[4]{2}}b}\right) + \sqrt{-1+2\sqrt[4]{2}}\arctan\left(\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b+4ax}{2^{3/4}\sqrt{-1+2\sqrt[4]{2}}b}\right) + \frac{1}{2}\sqrt{1+2\sqrt[4]{2}}\log\left(b^2-\sqrt{1+2\sqrt[4]{2}}abx+\sqrt[4]{2}a^2x^2\right) - \frac{1}{2}$$

output
```
-(-1+2*2^(1/4))^(1/2)*arctan(1/2*(2^(3/4)*(1+2*2^(1/4))^(1/2)*b-4*a*x)*2^(
1/4)/(-1+2*2^(1/4))^(1/2)/b)+(-1+2*2^(1/4))^(1/2)*arctan(1/2*(2^(3/4)*(1+2
*2^(1/4))^(1/2)*b+4*a*x)*2^(1/4)/(-1+2*2^(1/4))^(1/2)/b)+1/2*(1+2*2^(1/4))
^(1/2)*ln(b^2-(1+2*2^(1/4))^(1/2)*a*b*x+2^(1/4)*a^2*x^2)-1/2*(1+2*2^(1/4))
^(1/2)*ln(b^2+(1+2*2^(1/4))^(1/2)*a*b*x+2^(1/4)*a^2*x^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.53

$$\int \frac{2a(-b^3+2\sqrt{2}a^2bx^2)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx = (1+i)\left(\sqrt{i+\sqrt{-1+4\sqrt{2}}}\arctan\left(\frac{(1+i)\sqrt[4]{2}ax}{\sqrt{-i+\sqrt{-1+4\sqrt{2}}}b}\right) - \sqrt{-i+\sqrt{-1+4\sqrt{2}}}\text{arctanh}\left(\frac{(1+i)\sqrt[4]{2}ax}{\sqrt{i+\sqrt{-1+4\sqrt{2}}}b}\right)\right)$$

input
```
Integrate[(2*a*(-b^3 + 2*Sqrt[2]*a^2*b*x^2))/(b^4 - a^2*b^2*x^2 + Sqrt[2]*
a^4*x^4),x]
```

output
```
(1 + I)*(Sqrt[I + Sqrt[-1 + 4*Sqrt[2]]]*ArcTan[((1 + I)*2^(1/4)*a*x)/(Sqrt
[-I + Sqrt[-1 + 4*Sqrt[2]]]*b)] - Sqrt[-I + Sqrt[-1 + 4*Sqrt[2]]]*ArcTanh[
((1 + I)*2^(1/4)*a*x)/(Sqrt[I + Sqrt[-1 + 4*Sqrt[2]]]*b)])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {27, 25, 27, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a(2\sqrt{2}a^2bx^2-b^3)}{\sqrt{2}a^4x^4-a^2b^2x^2+b^4}\,dx$$

$$\Big\downarrow \text{27}$$

$$2a\int -\frac{b(b^2-2\sqrt{2}a^2x^2)}{b^4-a^2x^2b^2+\sqrt{2}a^4x^4}dx$$

$$\Big\downarrow \; 25$$

$$-2a\int \frac{b\left(b^2-2\sqrt{2}a^2x^2\right)}{b^4-a^2x^2b^2+\sqrt{2}a^4x^4}dx$$

$$\Big\downarrow \; 27$$

$$-2ab\int \frac{b^2-2\sqrt{2}a^2x^2}{b^4-a^2x^2b^2+\sqrt{2}a^4x^4}dx$$

$$\Big\downarrow \; 1483$$

$$-2ab\left(\frac{\int \frac{\sqrt{1+2\sqrt[4]{2}}b^2\left(2^{3/4}b-2\sqrt{1+2\sqrt[4]{2}}ax\right)}{a\left(\frac{2^{3/4}b^2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2\right)}dx}{2\sqrt{1+2\sqrt[4]{2}}ab^3}+\frac{\int \frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b^2\left(b+\sqrt[4]{2}\sqrt{1+2\sqrt[4]{2}}ax\right)}{a\left(\frac{2^{3/4}b^2}{a^2}+\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2\right)}dx}{2\sqrt{1+2\sqrt[4]{2}}ab^3}\right)$$

$$\Big\downarrow \; 27$$

$$-2ab\left(\frac{\int \frac{2^{3/4}b-2\sqrt{1+2\sqrt[4]{2}}ax}{\frac{2^{3/4}b^2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx}{2a^2b}+\frac{\int \frac{b+\sqrt[4]{2}\sqrt{1+2\sqrt[4]{2}}ax}{\frac{2^{3/4}b^2}{a^2}+\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx}{\sqrt[4]{2}a^2b}\right)$$

$$\Big\downarrow \; 1142$$

$$-2ab\left(\frac{-\frac{1}{2}\left(4-2^{3/4}\right)b\int \frac{1}{\frac{2^{3/4}b^2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx-\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}a\int -\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{a\left(\frac{2^{3/4}b^2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2\right)}dx}{2a^2b}+\frac{\frac{1}{2}\left(1\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$-2ab\left(\frac{\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}a\int\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{a\left(\frac{2^{3/4}b2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2\right)}dx-\frac{1}{2}\left(4-2^{3/4}\right)b\int\frac{1}{\frac{2^{3/4}b2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx}{2a^2b}+\frac{\frac{1}{2}\left(1-2\right.}{}\right.$$

$\downarrow$ 27

$$-2ab\left(\frac{\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}\int\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{\frac{2^{3/4}b2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx-\frac{1}{2}\left(4-2^{3/4}\right)b\int\frac{1}{\frac{2^{3/4}b2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx}{2a^2b}+\frac{\frac{1}{2}\left(1-2\sqrt[4]{2}\right)b}{}\right.$$

$\downarrow$ 1083

$$-2ab\left(\frac{\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}\int\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{\frac{2^{3/4}b2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx+\left(4-2^{3/4}\right)b\int\frac{1}{\frac{2\sqrt{2}\left(1-2\sqrt[4]{2}\right)b^2}{a^2}-\left(4x-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b}{a}\right)^2}d\left(4x-\frac{2^{3}}{}\right.}{2a^2b}\right.$$

$\downarrow$ 217

$$-2ab\left(\frac{\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}\int\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b-4ax}{\frac{2^{3/4}b^2}{a^2}-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}xb}{a}+2x^2}dx-\frac{(4-2^{3/4})a\arctan\left(\frac{a\left(4x-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b}{a}\right)}{2^{3/4}\sqrt{2\sqrt[4]{2}-1}b}\right)}{2^{3/4}\sqrt{2\sqrt[4]{2}-1}}}{2a^2b}+\frac{\frac{\sqrt{1+2\sqrt[4]{2}}\int\frac{2^{3/4}\sqrt{}}{\frac{2^{3/4}b^2}{a^2}+\frac{2^{3/}}{}}}{2\ 2^{3/}}}{}\right.$$

$\downarrow$ 1103

$$-2ab\left(\frac{-\frac{1}{2}\sqrt{1+2\sqrt[4]{2}}a\log\left(\sqrt[4]{2}a^2x^2-\sqrt{1+2\sqrt[4]{2}}abx+b^2\right)-\frac{(4-2^{3/4})a\arctan\left(\frac{a\left(4x-\frac{2^{3/4}\sqrt{1+2\sqrt[4]{2}}b}{a}\right)}{2^{3/4}\sqrt{2\sqrt[4]{2}-1}b}\right)}{2^{3/4}\sqrt{2\sqrt[4]{2}-1}}}{2a^2b}+\frac{\sqrt{1+2\sqrt[4]{2}}a}{}\right.$$

input
```
Int[(2*a*(-b^3 + 2*Sqrt[2]*a^2*b*x^2))/(b^4 - a^2*b^2*x^2 + Sqrt[2]*a^4*x^
4),x]
```

output
```
-2*a*b*((-(((4 - 2^(3/4))*a*ArcTan[(a*(-((2^(3/4)*Sqrt[1 + 2*2^(1/4)]*b)/a
) + 4*x))/(2^(3/4)*Sqrt[-1 + 2*2^(1/4)]*b)])/(2^(3/4)*Sqrt[-1 + 2*2^(1/4)]
)) - (Sqrt[1 + 2*2^(1/4)]*a*Log[b^2 - Sqrt[1 + 2*2^(1/4)]*a*b*x + 2^(1/4)*
a^2*x^2])/2)/(2*a^2*b) + (((1 - 2*2^(1/4))*a*ArcTan[(a*((2^(3/4)*Sqrt[1 +
2*2^(1/4)]*b)/a + 4*x))/(2^(3/4)*Sqrt[-1 + 2*2^(1/4)]*b)])/(2^(3/4)*Sqrt[-
1 + 2*2^(1/4)]) + (Sqrt[1 + 2*2^(1/4)]*a*Log[b^2 + Sqrt[1 + 2*2^(1/4)]*a*b
*x + 2^(1/4)*a^2*x^2])/(2*2^(3/4)))/(2^(1/4)*a^2*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.42

method	result
risch	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(2a^4\_Z^4-\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)a^2b^2\_Z^2+\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)b^4\right)}\frac{\left(-b^2+2\sqrt{2}\,a^2\_R^2\right)\ln\left(x-\_R\right)}{-b^2\_R+2\sqrt{2}\,a^2\_R^3}\right)}{a}$
default	$2ab\sqrt{2}\left(\frac{\frac{\left(-b^2\sqrt{2}\,\mathrm{csgn}\left(a^2\right)a^2-2\sqrt{2}\,a^2\sqrt{\sqrt{2}\,b^4}\right)\sqrt{2}\,\ln\left(\sqrt{2}\,\mathrm{csgn}\left(a^2\right)a^2x^2+x\sqrt{\sqrt{2}\left(a^2b^2+2\sqrt{\sqrt{2}\,a^4b^4}\right)}+\sqrt{\sqrt{2}\,b^4}\right)}{4\,\mathrm{csgn}\left(a^2\right)a^2}+\frac{2\left(-b^2\sqrt{\sqrt{2}\left(a^2b^2+2\sqrt{\right.}\right.}\right.}{}}{2\sqrt{\sqrt{2}\left(a^2\right.}}\right.$

input
```
int(2*a*(-b^3+2*2^(1/2)*a^2*b*x^2)/(b^4-a^2*b^2*x^2+2^(1/2)*a^4*x^4),x,met
hod=_RETURNVERBOSE)
```

output
```
1/a*b*sum((-b^2+2*2^(1/2)*a^2*_R^2)/(-b^2*_R+2*2^(1/2)*a^2*_R^3)*ln(x-_R),
_R=RootOf(2*a^4*_Z^4-RootOf(_Z^2-2,index=1)*a^2*b^2*_Z^2+RootOf(_Z^2-2,ind
ex=1)*b^4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.23

$$\int \frac{2a\left(-b^3+2\sqrt{2}a^2bx^2\right)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx$$
$$=\sqrt{2\cdot 2^{\frac{1}{4}}-1}\arctan\left(\frac{\left(4\sqrt{2}b+2\cdot 2^{\frac{1}{4}}\left(4\sqrt{2}b+b\right)+b\right)\sqrt{2\cdot 2^{\frac{1}{4}}+1}\sqrt{2\cdot 2^{\frac{1}{4}}-1}+2\left(2\sqrt{2}ax+16\,ax+2\right.}{31\,b}\right.$$
$$-\sqrt{2\cdot 2^{\frac{1}{4}}-1}\arctan\left(\frac{\left(4\sqrt{2}b+2\cdot 2^{\frac{1}{4}}\left(4\sqrt{2}b+b\right)+b\right)\sqrt{2\cdot 2^{\frac{1}{4}}+1}\sqrt{2\cdot 2^{\frac{1}{4}}-1}-2\left(2\sqrt{2}ax+16\,ax+\right.}{31\,b}\right.$$
$$-\frac{1}{2}\sqrt{2\cdot 2^{\frac{1}{4}}+1}\log\left(2^{\frac{3}{4}}abx\sqrt{2\cdot 2^{\frac{1}{4}}+1}+2\,a^2x^2+2^{\frac{3}{4}}b^2\right)$$
$$+\frac{1}{2}\sqrt{2\cdot 2^{\frac{1}{4}}+1}\log\left(-2^{\frac{3}{4}}abx\sqrt{2\cdot 2^{\frac{1}{4}}+1}+2\,a^2x^2+2^{\frac{3}{4}}b^2\right)$$

input
```
integrate(2*a*(-b^3+2*2^(1/2)*a^2*b*x^2)/(b^4-a^2*b^2*x^2+2^(1/2)*a^4*x^4)
,x, algorithm="fricas")
```

output
```
sqrt(2*2^(1/4) - 1)*arctan(1/31*((4*sqrt(2)*b + 2*2^(1/4)*(4*sqrt(2)*b + b
) + b)*sqrt(2*2^(1/4) + 1)*sqrt(2*2^(1/4) - 1) + 2*(2*sqrt(2)*a*x + 16*a*x
 + 2^(1/4)*(4*sqrt(2)*a*x + a*x))*sqrt(2*2^(1/4) - 1))/b) - sqrt(2*2^(1/4)
 - 1)*arctan(1/31*((4*sqrt(2)*b + 2*2^(1/4)*(4*sqrt(2)*b + b) + b)*sqrt(2*
2^(1/4) + 1)*sqrt(2*2^(1/4) - 1) - 2*(2*sqrt(2)*a*x + 16*a*x + 2^(1/4)*(4*
sqrt(2)*a*x + a*x))*sqrt(2*2^(1/4) - 1))/b) - 1/2*sqrt(2*2^(1/4) + 1)*log(
2^(3/4)*a*b*x*sqrt(2*2^(1/4) + 1) + 2*a^2*x^2 + 2^(3/4)*b^2) + 1/2*sqrt(2*
2^(1/4) + 1)*log(-2^(3/4)*a*b*x*sqrt(2*2^(1/4) + 1) + 2*a^2*x^2 + 2^(3/4)*
b^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{2a(-b^3 + 2\sqrt{2}a^2bx^2)}{b^4 - a^2b^2x^2 + \sqrt{2}a^4x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate(2*a*(-b**3+2*2**(1/2)*a**2*b*x**2)/(b**4-a**2*b**2*x**2+2**(1/2)
*a**4*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(_t**4 - 4*sqrt(2)*_t**2 + 8) conta
ins an element of the set of generators.
```

Maxima [F]

$$\int \frac{2a(-b^3 + 2\sqrt{2}a^2bx^2)}{b^4 - a^2b^2x^2 + \sqrt{2}a^4x^4}\,dx = \int \frac{2\left(2\sqrt{2}a^2bx^2 - b^3\right)a}{\sqrt{2}a^4x^4 - a^2b^2x^2 + b^4}\,dx$$

input
```
integrate(2*a*(-b^3+2*2^(1/2)*a^2*b*x^2)/(b^4-a^2*b^2*x^2+2^(1/2)*a^4*x^4)
,x, algorithm="maxima")
```

output
```
2*a*integrate((2*sqrt(2)*a^2*b*x^2 - b^3)/(sqrt(2)*a^4*x^4 - a^2*b^2*x^2 +
 b^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2a(-b^3 + 2\sqrt{2}a^2bx^2)}{b^4 - a^2b^2x^2 + \sqrt{2}a^4x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(2*a*(-b^3+2*2^(1/2)*a^2*b*x^2)/(b^4-a^2*b^2*x^2+2^(1/2)*a^4*x^4)
,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[16]%%%}+%%%{%%%{%%{[-68114827739917379197O,584447570846
596707587
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1021, normalized size of antiderivative = 4.46

$$\int \frac{2a\left(-b^3+2\sqrt{2}a^2bx^2\right)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx = \text{Too large to display}$$

input
```
int(-(2*a*(b^3 - 2*2^(1/2)*a^2*b*x^2))/(b^4 + 2^(1/2)*a^4*x^4 - a^2*b^2*x^
2),x)
```

output
```
atan(((((2*2^(1/2)*b^7 - 16*b^7)/a^7 + (x*(2*2^(1/2)*b^6 - 16*b^6)*(1/2 -
(2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2))/a^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1
/2))^(1/2))/4)^(1/2) + (x*(32*2^(1/2)*b^6 - 8*b^6))/a^6)*(1/2 - (2^(1/2)*(
2 - 8*2^(1/2))^(1/2))/4)^(1/2)*1i - (((2*2^(1/2)*b^7 - 16*b^7)/a^7 - (x*(2
*2^(1/2)*b^6 - 16*b^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2))/a^
6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2) - (x*(32*2^(1/2)*b^6 -
8*b^6))/a^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2)*1i)/((2*(32*2
^(1/2)*b^7 - 8*b^7))/a^7 + (((2*2^(1/2)*b^7 - 16*b^7)/a^7 + (x*(2*2^(1/2)*
b^6 - 16*b^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2))/a^6)*(1/2 -
 (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2) + (x*(32*2^(1/2)*b^6 - 8*b^6))/a
^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2) + (((2*2^(1/2)*b^7 - 1
6*b^7)/a^7 - (x*(2*2^(1/2)*b^6 - 16*b^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(
1/2))/4)^(1/2))/a^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2) - (x*
(32*2^(1/2)*b^6 - 8*b^6))/a^6)*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(
1/2)))*(1/2 - (2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4)^(1/2)*2i + atan(((((2*2^(
1/2)*b^7 - 16*b^7)/a^7 + (x*(2*2^(1/2)*b^6 - 16*b^6)*((2^(1/2)*(2 - 8*2^(1
/2))^(1/2))/4 + 1/2)^(1/2))/a^6)*((2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4 + 1/2)
^(1/2) + (x*(32*2^(1/2)*b^6 - 8*b^6))/a^6)*((2^(1/2)*(2 - 8*2^(1/2))^(1/2)
)/4 + 1/2)^(1/2)*1i - (((2*2^(1/2)*b^7 - 16*b^7)/a^7 - (x*(2*2^(1/2)*b^6 -
 16*b^6)*((2^(1/2)*(2 - 8*2^(1/2))^(1/2))/4 + 1/2)^(1/2))/a^6)*((2^(1/2...
```

Reduce [F]

$$\int \frac{2a(-b^3+2\sqrt{2}a^2bx^2)}{b^4-a^2b^2x^2+\sqrt{2}a^4x^4}\,dx = 2ab\left(\sqrt{2}\left(\int \frac{x^4}{2a^8x^8-a^4b^4x^4+2a^2b^6x^2-b^8}dx\right)a^4b^2 - 2\sqrt{2}\left(\int \frac{x^2}{2a^8x^8-a^4b^4x^4+2a^2b^6x^2-b^8}dx\right)a^2b^4 + 4\left(\int \frac{x^6}{2a^8x^8-a^4b^4x^4+2a^2b^6x^2-b^8}dx\right)a^6 - \left(\int \frac{x^2}{2a^8x^8-a^4b^4x^4+2a^2b^6x^2-b^8}dx\right)a^2b^4 + \left(\int \frac{1}{2a^8x^8-a^4b^4x^4+2a^2b^6x^2-b^8}dx\right)b^6\right)$$

input
```
int(2*a*(-b^3+2*2^(1/2)*a^2*b*x^2)/(b^4-a^2*b^2*x^2+2^(1/2)*a^4*x^4),x)
```

output
```
2*a*b*(sqrt(2)*int(x**4/(2*a**8*x**8 - a**4*b**4*x**4 + 2*a**2*b**6*x**2 -
 b**8),x)*a**4*b**2 - 2*sqrt(2)*int(x**2/(2*a**8*x**8 - a**4*b**4*x**4 + 2
*a**2*b**6*x**2 - b**8),x)*a**2*b**4 + 4*int(x**6/(2*a**8*x**8 - a**4*b**4
*x**4 + 2*a**2*b**6*x**2 - b**8),x)*a**6 - int(x**2/(2*a**8*x**8 - a**4*b*
*4*x**4 + 2*a**2*b**6*x**2 - b**8),x)*a**2*b**4 + int(1/(2*a**8*x**8 - a**
4*b**4*x**4 + 2*a**2*b**6*x**2 - b**8),x)*b**6)
```

3.2 $\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 87

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx = -\sqrt{-3+4\sqrt{2}}\arctan\left(\frac{\sqrt{3}c-2\sqrt{2}ax}{\sqrt{-3+4\sqrt{2}}c}\right) + \frac{1}{2}\sqrt{3}\log\left(c^2-\sqrt{3}acx+\sqrt{2}a^2x^2\right)$$

output
```
-(-3+4*2^(1/2))^(1/2)*arctan((c*3^(1/2)-2*2^(1/2)*a*x)/(-3+4*2^(1/2))^(1/2
)/c)+1/2*3^(1/2)*ln(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*x^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx = a\left(\frac{\sqrt{-3+4\sqrt{2}}\arctan\left(\frac{-\sqrt{3}c+2\sqrt{2}ax}{\sqrt{-3+4\sqrt{2}}c}\right)}{a} + \frac{\sqrt{3}\log\left(c^2-\sqrt{3}acx+\sqrt{2}a^2x^2\right)}{2a}\right)$$

input
```
Integrate[(a*(-3*c + 2*Sqrt[2]*c + Sqrt[6]*a*x))/(c^2 - Sqrt[3]*a*c*x + Sq
rt[2]*a^2*x^2),x]
```

output
```
a*((Sqrt[-3 + 4*Sqrt[2]]*ArcTan[(-(Sqrt[3]*c) + 2*Sqrt[2]*a*x)/(Sqrt[-3 +
4*Sqrt[2]]*c)])/a + (Sqrt[3]*Log[c^2 - Sqrt[3]*a*c*x + Sqrt[2]*a^2*x^2])/(
2*a))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {27, 25, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\left(\sqrt{6}ax+2\sqrt{2}c-3c\right)}{\sqrt{2}a^2x^2-\sqrt{3}acx+c^2}dx$$

$$\Big\downarrow \textbf{27}$$

$$a\int -\frac{\left(3-2\sqrt{2}\right)c-\sqrt{6}ax}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx$$

$$\Big\downarrow \textbf{25}$$

$$-a\int \frac{\left(3-2\sqrt{2}\right)c-\sqrt{6}ax}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx$$

$$\Big\downarrow \textbf{1142}$$

$$-a\left(\frac{1}{2}\left(3-4\sqrt{2}\right)c\int \frac{1}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx-\frac{\sqrt{3}\int -\frac{a\left(\sqrt{3}c-2\sqrt{2}ax\right)}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx}{2a}\right)$$

$$\Big\downarrow \textbf{25}$$

$$-a\left(\frac{1}{2}\left(3-4\sqrt{2}\right)c\int \frac{1}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx+\frac{\sqrt{3}\int \frac{a\left(\sqrt{3}c-2\sqrt{2}ax\right)}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx}{2a}\right)$$

$$\Big\downarrow \textbf{27}$$

$$-a\left(\frac{1}{2}\left(3-4\sqrt{2}\right)c\int \frac{1}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}c-2\sqrt{2}ax}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx\right)$$

$$\Big\downarrow \textbf{1083}$$

$$-a\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}c-2\sqrt{2}ax}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx-\left(3-4\sqrt{2}\right)c\int \frac{1}{\left(3-4\sqrt{2}\right)a^2c^2-\left(2\sqrt{2}a^2x-\sqrt{3}ac\right)^2}d\left(2\sqrt{2}a^2x-\sqrt{3}a\right.\right.$$

$$\Big\downarrow \textbf{217}$$

$$-a\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}c-2\sqrt{2}ax}{c^2-\sqrt{3}axc+\sqrt{2}a^2x^2}dx+\frac{\left(3-4\sqrt{2}\right)\arctan\left(\frac{2\sqrt{2}a^2x-\sqrt{3}ac}{\sqrt{4\sqrt{2}-3}ac}\right)}{\sqrt{4\sqrt{2}-3}a}\right)$$

$$\Big\downarrow \textbf{1103}$$

$$-a\left(\frac{\left(3-4\sqrt{2}\right)\arctan\left(\frac{2\sqrt{2}a^2x-\sqrt{3}ac}{\sqrt{4\sqrt{2}-3}ac}\right)}{\sqrt{4\sqrt{2}-3}a}-\frac{\sqrt{3}\log\left(\sqrt{2}a^2x^2-\sqrt{3}acx+c^2\right)}{2a}\right)$$

input
```
Int[(a*(-3*c + 2*Sqrt[2]*c + Sqrt[6]*a*x))/(c^2 - Sqrt[3]*a*c*x + Sqrt[2]*
a^2*x^2),x]
```

output
```
-(a*(((3 - 4*Sqrt[2])*ArcTan[(-(Sqrt[3]*a*c) + 2*Sqrt[2]*a^2*x)/(Sqrt[-3 +
 4*Sqrt[2]]*a*c)])/(Sqrt[-3 + 4*Sqrt[2]]*a) - (Sqrt[3]*Log[c^2 - Sqrt[3]*a
*c*x + Sqrt[2]*a^2*x^2])/(2*a)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

method	result	size
default	$a\sqrt{2}\left(\frac{\sqrt{6}\,\ln\left(\sqrt{3}\,\sqrt{2}\,acx-2x^{2}a^{2}-\sqrt{2}\,c^{2}\right)}{4a}+\frac{2\left(-\frac{\sqrt{6}\,\sqrt{2}\,\sqrt{3}\,c}{4}-2\sqrt{2}\,c+3c\right)\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,ac-4x\,a^{2}}{\sqrt{8\sqrt{2}\,c^{2}a^{2}-6a^{2}c^{2}}}\right)}{\sqrt{8\sqrt{2}\,c^{2}a^{2}-6a^{2}c^{2}}}\right)$	129

input
```
int(a*(-3*c+2*2^(1/2)*c+6^(1/2)*a*x)/(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*x^2),x
,method=_RETURNVERBOSE)
```

output
```
a*2^(1/2)*(1/4*6^(1/2)/a*ln(3^(1/2)*2^(1/2)*a*c*x-2*x^2*a^2-2^(1/2)*c^2)+2
*(-1/4*6^(1/2)*2^(1/2)*3^(1/2)*c-2*2^(1/2)*c+3*c)/(8*2^(1/2)*c^2*a^2-6*a^2
*c^2)^(1/2)*arctan((3^(1/2)*2^(1/2)*a*c-4*x*a^2)/(8*2^(1/2)*c^2*a^2-6*a^2*
c^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx=\int \frac{\left(\sqrt{6}ax+2\,\sqrt{2}c-3\,c\right)a}{\sqrt{2}a^2x^2-\sqrt{3}acx+c^2}\,dx$$

input
```
integrate(a*(-3*c+2*2^(1/2)*c+6^(1/2)*a*x)/(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*
x^2),x, algorithm="fricas")
```

output
```
0
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(76) = 152$.

Time = 0.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.30

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx=\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{2}\sqrt{315-236\sqrt{2}}}{2\left(-8+3\sqrt{2}\right)}\right)\log\left(-\frac{24243\sqrt{3}c}{-34472a+24243\sqrt{2}a}+\frac{17236\sqrt{6}c}{-34472a+24243\sqrt{2}a}+x+\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{2}\sqrt{31}}{2\left(-8\right.}\right.\right.$$
$$+\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{2}\sqrt{315-236\sqrt{2}}}{2\left(-8+3\sqrt{2}\right)}\right)\log\left(-\frac{24243\sqrt{3}c}{-34472a+24243\sqrt{2}a}+\frac{17236\sqrt{6}c}{-34472a+24243\sqrt{2}a}+x+\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{2}\sqrt{31}}{2\left(-8\right.}\right.\right.$$

input
```
integrate(a*(-3*c+2*2**(1/2)*c+6**(1/2)*a*x)/(c**2-3**(1/2)*a*c*x+2**(1/2)
*a**2*x**2),x)
```

output
```
(sqrt(3)/2 - sqrt(2)*sqrt(315 - 236*sqrt(2))/(2*(-8 + 3*sqrt(2))))*log(-24
243*sqrt(3)*c/(-34472*a + 24243*sqrt(2)*a) + 17236*sqrt(6)*c/(-34472*a + 2
4243*sqrt(2)*a) + x + (sqrt(3)/2 - sqrt(2)*sqrt(315 - 236*sqrt(2))/(2*(-8
+ 3*sqrt(2))))*(-24*sqrt(2)*c/(-48*a + 41*sqrt(2)*a) + 41*c/(-48*a + 41*sq
rt(2)*a))) + (sqrt(3)/2 + sqrt(2)*sqrt(315 - 236*sqrt(2))/(2*(-8 + 3*sqrt(
2))))*log(-24243*sqrt(3)*c/(-34472*a + 24243*sqrt(2)*a) + 17236*sqrt(6)*c/
(-34472*a + 24243*sqrt(2)*a) + x + (sqrt(3)/2 + sqrt(2)*sqrt(315 - 236*sqr
t(2))/(2*(-8 + 3*sqrt(2))))*(-24*sqrt(2)*c/(-48*a + 41*sqrt(2)*a) + 41*c/(
-48*a + 41*sqrt(2)*a)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx = \frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\left(\sqrt{6}\sqrt{3}-6\,\sqrt{2}+8\right)\arctan\left(\frac{2\,\sqrt{2}a^2x-\sqrt{3}ac}{ac\sqrt{4\,\sqrt{2}-3}}\right)}{a\sqrt{4\,\sqrt{2}-3}}+\frac{\sqrt{6}\sqrt{2}\log\left(\sqrt{2}a^2x^2-\sqrt{3}acx+c^2\right)}{a}\right)$$

input
```
integrate(a*(-3*c+2*2^(1/2)*c+6^(1/2)*a*x)/(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*
x^2),x, algorithm="maxima")
```

output
```
1/4*a*(2*sqrt(2)*(sqrt(6)*sqrt(3) - 6*sqrt(2) + 8)*arctan((2*sqrt(2)*a^2*x
 - sqrt(3)*a*c)/(a*c*sqrt(4*sqrt(2) - 3)))/(a*sqrt(4*sqrt(2) - 3)) + sqrt(
6)*sqrt(2)*log(sqrt(2)*a^2*x^2 - sqrt(3)*a*c*x + c^2)/a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx = -\frac{1}{2}\,a\left(\frac{2\left(3\,\sqrt{2}-8\right)\arctan\left(\frac{4\,ax-\sqrt{6}c}{c\sqrt{8\,\sqrt{2}-6}}\right)}{a\sqrt{8\,\sqrt{2}-6}}-\frac{\sqrt{3}\log\left(4\,a^2x^2-2\,\sqrt{6}acx+2\,\sqrt{2}c^2\right)}{a}\right)$$

input
```
integrate(a*(-3*c+2*2^(1/2)*c+6^(1/2)*a*x)/(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*
x^2),x, algorithm="giac")
```

output
```
-1/2*a*(2*(3*sqrt(2) - 8)*arctan((4*a*x - sqrt(6)*c)/(c*sqrt(8*sqrt(2) - 6
)))/(a*sqrt(8*sqrt(2) - 6)) - sqrt(3)*log(4*a^2*x^2 - 2*sqrt(6)*a*c*x + 2*
sqrt(2)*c^2)/a)
```

Mupad [B] (verification not implemented)

Time = 10.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

$$\int \frac{a\left(-3c+2\sqrt{2}c+\sqrt{6}ax\right)}{c^2-\sqrt{3}acx+\sqrt{2}a^2x^2}\,dx = \ln\left(\sqrt{6}\,c-4\,a\,x+\sqrt{2}\,c\,\sqrt{3-4\,\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}-\frac{3\,\sqrt{2}}{2\,\sqrt{6-8\,\sqrt{2}}}+\frac{4}{\sqrt{6-8\,\sqrt{2}}}\right)+\ln\left(4\,a\,x-\sqrt{6}\,c+\sqrt{2}\,c\,\sqrt{3-4\,\sqrt{2}}\right)\left(\frac{3\,\sqrt{2}}{2\,\sqrt{6-8\,\sqrt{2}}}+\frac{\sqrt{3}}{2}-\frac{4}{\sqrt{6-8\,\sqrt{2}}}\right)$$

input
```
int((a*(2*2^(1/2)*c - 3*c + 6^(1/2)*a*x))/(c^2 + 2^(1/2)*a^2*x^2 - 3^(1/2)
*a*c*x),x)
```

output
```
log(6^(1/2)*c - 4*a*x + 2^(1/2)*c*(3 - 4*2^(1/2))^(1/2))*(3^(1/2)/2 - (3*2
^(1/2))/(2*(6 - 8*2^(1/2))^(1/2)) + 4/(6 - 8*2^(1/2))^(1/2)) + log(4*a*x -
 6^(1/2)*c + 2^(1/2)*c*(3 - 4*2^(1/2))^(1/2))*((3*2^(1/2))/(2*(6 - 8*2^(1/
2))^(1/2)) + 3^(1/2)/2 - 4/(6 - 8*2^(1/2))^(1/2))
```

Reduce [F]

$$\int \frac{a\left(-3c+2\sqrt{2}\,c+\sqrt{6}\,ax\right)}{c^2-\sqrt{3}\,acx+\sqrt{2}\,a^2x^2}\,dx$$
$$= 2\sqrt{6}\left(\int \frac{x^5}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^6c^2$$
$$- 3\sqrt{6}\left(\int \frac{x^3}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^4c^4$$
$$+ 3\sqrt{6}\left(\int \frac{x}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^2c^6$$
$$- 3\sqrt{3}\left(\int \frac{x^5}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^6c^2$$
$$- \frac{7\sqrt{3}\left(\int \frac{x^3}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^4c^4}{2}$$
$$- \frac{3\sqrt{3}\left(\int \frac{x}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^2c^6}{2}$$
$$+ \frac{\sqrt{3}\log(4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8)}{8}$$
$$- 4\sqrt{2}\left(\int \frac{x^4}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^5c^3$$
$$+ 2\sqrt{2}\left(\int \frac{1}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a\,c^7$$
$$+ 8\left(\int \frac{x^6}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^7c$$
$$- 18\left(\int \frac{x^4}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^5c^3$$
$$+ 5\left(\int \frac{x^2}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a^3c^5$$
$$- 3\left(\int \frac{1}{4a^8x^8-12a^6c^2x^6+5a^4c^4x^4-6a^2c^6x^2+c^8}dx\right)a\,c^7$$

input
```
int(a*(-3*c+2*2^(1/2)*c+6^(1/2)*a*x)/(c^2-3^(1/2)*a*c*x+2^(1/2)*a^2*x^2),x
)
```

output
```
(16*sqrt(6)*int(x**5/(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 -
 6*a**2*c**6*x**2 + c**8),x)*a**6*c**2 - 24*sqrt(6)*int(x**3/(4*a**8*x**8
- 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a**4*
c**4 + 24*sqrt(6)*int(x/(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**
4 - 6*a**2*c**6*x**2 + c**8),x)*a**2*c**6 - 24*sqrt(3)*int(x**5/(4*a**8*x*
*8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a*
*6*c**2 - 28*sqrt(3)*int(x**3/(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c*
*4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a**4*c**4 - 12*sqrt(3)*int(x/(4*a**8
*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)
*a**2*c**6 + sqrt(3)*log(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**
4 - 6*a**2*c**6*x**2 + c**8) - 32*sqrt(2)*int(x**4/(4*a**8*x**8 - 12*a**6*
c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a**5*c**3 + 16*
sqrt(2)*int(1/(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2
*c**6*x**2 + c**8),x)*a*c**7 + 64*int(x**6/(4*a**8*x**8 - 12*a**6*c**2*x**
6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a**7*c - 144*int(x**4/(
4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c*
*8),x)*a**5*c**3 + 40*int(x**2/(4*a**8*x**8 - 12*a**6*c**2*x**6 + 5*a**4*c
**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a**3*c**5 - 24*int(1/(4*a**8*x**8 -
 12*a**6*c**2*x**6 + 5*a**4*c**4*x**4 - 6*a**2*c**6*x**2 + c**8),x)*a*c**7
)/8
```

3.3 $\int \frac{-\sqrt{3}b-2ax+4\sqrt{2}ax+3b^2x+2\sqrt{3}abx^2-2\sqrt{6}abx^2+2\sqrt{2}a^2x^3}{1-2\sqrt{3}bx-ax^2+3b^2x^2+\sqrt{3}abx^3+\sqrt{2}a^2x^4}\,dx$

Optimal result

Integrand size = 116, antiderivative size = 144

$$\int \frac{-\sqrt{3}b-2ax+4\sqrt{2}ax+3b^2x+2\sqrt{3}abx^2-2\sqrt{6}abx^2+2\sqrt{2}a^2x^3}{1-2\sqrt{3}bx-ax^2+3b^2x^2+\sqrt{3}abx^3+\sqrt{2}a^2x^4}\,dx$$
$$=\sqrt{-1+4\sqrt{2}}\arctan\left(\frac{-\sqrt{2}+\sqrt{6}bx+4ax^2}{4\left(\sqrt{-\frac{1}{8}+\frac{1}{\sqrt{2}}}-\sqrt{-\frac{3}{8}+\frac{3}{\sqrt{2}}}bx\right)}\right)$$
$$+\frac{1}{2}\log\left(\frac{\sqrt{2}-2\sqrt{6}bx-\sqrt{2}ax^2+3\sqrt{2}b^2x^2+\sqrt{6}abx^3+2a^2x^4}{a^2}\right)$$

output
```
(-1+4*2^(1/2))^(1/2)*arctan((-2^(1/2)+6^(1/2)*b*x+4*a*x^2)/((-2+8*2^(1/2))
^(1/2)-(-6+24*2^(1/2))^(1/2)*b*x))+1/2*ln((2^(1/2)-2*6^(1/2)*b*x-2^(1/2)*a
*x^2+3*b^2*x^2*2^(1/2)+6^(1/2)*a*b*x^3+2*a^2*x^4)/a^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.58

$$\int \frac{-\sqrt{3}b-2ax+4\sqrt{2}ax+3b^2x+2\sqrt{3}abx^2-2\sqrt{6}abx^2+2\sqrt{2}a^2x^3}{1-2\sqrt{3}bx-ax^2+3b^2x^2+\sqrt{3}abx^3+\sqrt{2}a^2x^4}\,dx$$
$$= -\text{RootSum}\Big[1-2\sqrt{3}b\#1-a\#1^2+3b^2\#1^2+\sqrt{3}ab\#1^3$$
$$+\sqrt{2}a^2\#1^4\&, \frac{\sqrt{3}b\log(x-\#1)+2a\log(x-\#1)\#1-4\sqrt{2}a\log(x-\#1)\#1-3b^2\log(x-\#1)\#1-2}{-2\sqrt{3}b-2a\#1+6b^2\#1+3\sqrt{3}a}$$

input
```
Integrate[(-(Sqrt[3]*b) - 2*a*x + 4*Sqrt[2]*a*x + 3*b^2*x + 2*Sqrt[3]*a*b*
x^2 - 2*Sqrt[6]*a*b*x^2 + 2*Sqrt[2]*a^2*x^3)/(1 - 2*Sqrt[3]*b*x - a*x^2 +
3*b^2*x^2 + Sqrt[3]*a*b*x^3 + Sqrt[2]*a^2*x^4),x]
```

output
```
-RootSum[1 - 2*Sqrt[3]*b*#1 - a*#1^2 + 3*b^2*#1^2 + Sqrt[3]*a*b*#1^3 + Sqr
t[2]*a^2*#1^4 & , (Sqrt[3]*b*Log[x - #1] + 2*a*Log[x - #1]*#1 - 4*Sqrt[2]*
a*Log[x - #1]*#1 - 3*b^2*Log[x - #1]*#1 - 2*Sqrt[3]*a*b*Log[x - #1]*#1^2 +
 2*Sqrt[6]*a*b*Log[x - #1]*#1^2 - 2*Sqrt[2]*a^2*Log[x - #1]*#1^3)/(-2*Sqrt
[3]*b - 2*a*#1 + 6*b^2*#1 + 3*Sqrt[3]*a*b*#1^2 + 4*Sqrt[2]*a^2*#1^3) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{2}a^2x^3-2\sqrt{6}abx^2+2\sqrt{3}abx^2+4\sqrt{2}ax-2ax+3b^2x-\sqrt{3}b}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-ax^2+3b^2x^2-2\sqrt{3}bx+1}\,dx$$
$$\Big\downarrow 6$$
$$\int \frac{2\sqrt{2}a^2x^3-2\sqrt{6}abx^2+2\sqrt{3}abx^2+\left(4\sqrt{2}-2\right)ax+3b^2x-\sqrt{3}b}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-ax^2+3b^2x^2-2\sqrt{3}bx+1}dx$$
$$\Big\downarrow 6$$

$$\int \frac{2\sqrt{2}a^2x^3+x\left(\left(4\sqrt{2}-2\right)a+3b^2\right)-2\sqrt{6}abx^2+2\sqrt{3}abx^2-\sqrt{3}b}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-ax^2+3b^2x^2-2\sqrt{3}bx+1}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{2\sqrt{2}a^2x^3+x\left(\left(4\sqrt{2}-2\right)a+3b^2\right)+\left(2\sqrt{3}-2\sqrt{6}\right)abx^2-\sqrt{3}b}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-ax^2+3b^2x^2-2\sqrt{3}bx+1}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{2\sqrt{2}a^2x^3+x\left(\left(4\sqrt{2}-2\right)a+3b^2\right)+\left(2\sqrt{3}-2\sqrt{6}\right)abx^2-\sqrt{3}b}{\sqrt{2}a^2x^4+x^2\left(3b^2-a\right)+\sqrt{3}abx^3-2\sqrt{3}bx+1}dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{\int \frac{2\left(\sqrt{6}\left(1-4\sqrt{2}\right)bx^2a^3+2\left(8-\sqrt{2}\right)xa^3\right)}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}dx}{4\sqrt{2}a^2}+\frac{1}{2}\log\left(\sqrt{2}a^2x^4-x^2\left(a-3b^2\right)+\sqrt{3}abx^3-2\sqrt{3}bx+1\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{6}\left(1-4\sqrt{2}\right)bx^2a^3+2\left(8-\sqrt{2}\right)xa^3}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}dx}{2\sqrt{2}a^2}+\frac{1}{2}\log\left(\sqrt{2}a^2x^4-x^2\left(a-3b^2\right)+\sqrt{3}abx^3-2\sqrt{3}bx+1\right)$$

$$\Big\downarrow \text{ 2027}$$

$$\frac{\int \frac{x\left(\sqrt{6}\left(1-4\sqrt{2}\right)bxa^3+2\left(8-\sqrt{2}\right)a^3\right)}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}dx}{2\sqrt{2}a^2}+\frac{1}{2}\log\left(\sqrt{2}a^2x^4-x^2\left(a-3b^2\right)+\sqrt{3}abx^3-2\sqrt{3}bx+1\right)$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\int \left(\frac{\sqrt{6}\left(1-4\sqrt{2}\right)bx^2a^3}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}+\frac{2\left(8-\sqrt{2}\right)xa^3}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}\right)dx}{2\sqrt{2}a^2}+\frac{1}{2}\log\left(\sqrt{2}a^2x^4-x^2\left(a-3b^2\right)+\sqrt{3}abx^3-2\sqrt{3}bx+1\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(8-\sqrt{2})\,a^3\int\frac{x}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}dx+\sqrt{6}(1-4\sqrt{2})\,a^3b\int\frac{x^2}{\sqrt{2}a^2x^4+\sqrt{3}abx^3-(a-3b^2)x^2-2\sqrt{3}bx+1}dx}{2\sqrt{2}a^2}+\frac{1}{2}\log\left(\sqrt{2}a^2x^4-x^2(a-3b^2)+\sqrt{3}abx^3-2\sqrt{3}bx+1\right)$$

input
```
Int[(-(Sqrt[3]*b) - 2*a*x + 4*Sqrt[2]*a*x + 3*b^2*x + 2*Sqrt[3]*a*b*x^2 -
2*Sqrt[6]*a*b*x^2 + 2*Sqrt[2]*a^2*x^3)/(1 - 2*Sqrt[3]*b*x - a*x^2 + 3*b^2*
x^2 + Sqrt[3]*a*b*x^3 + Sqrt[2]*a^2*x^4),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sqrt{2}\left(\sum_{\_R=\mathrm{RootOf}\left(1+\sqrt{2}\,a^2\_Z^4+\sqrt{3}\,ab\_Z^3+\left(3b^2-a\right)\_Z^2-2\sqrt{3}\,b\_Z\right)}\frac{\left(4a^2\_R^3+\sqrt{2}\left(-2\sqrt{3}\,ab\_R^2\left(\sqrt{2}-1\right)-\_R\left(-4\sqrt{2}\,a-3b^2+2a\right)\right.\right.}{6b^2\_R-2a\_R+3\sqrt{3}\,ab\_R^2+4\sqrt{2}\,a^2\_R^3-}\right.}{2}$

input
```
int((-3^(1/2)*b-2*a*x+4*2^(1/2)*a*x+3*b^2*x+2*3^(1/2)*a*b*x^2-2*6^(1/2)*a*
b*x^2+2*2^(1/2)*a^2*x^3)/(1-2*3^(1/2)*b*x-a*x^2+3*b^2*x^2+3^(1/2)*a*b*x^3+
2^(1/2)*a^2*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*sum((4*a^2*_R^3+2^(1/2)*(-2*3^(1/2)*a*b*_R^2*(2^(1/2)-1)-_R*(-
4*2^(1/2)*a-3*b^2+2*a)-3^(1/2)*b))/(6*b^2*_R-2*a*_R+3*3^(1/2)*a*b*_R^2+4*2
^(1/2)*a^2*_R^3-2*3^(1/2)*b)*ln(x-_R),_R=RootOf(1+2^(1/2)*a^2*_Z^4+3^(1/2)
*a*b*_Z^3+(3*b^2-a)*_Z^2-2*3^(1/2)*b*_Z))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{-\sqrt{3}b - 2ax + 4\sqrt{2}ax + 3b^2x + 2\sqrt{3}abx^2 - 2\sqrt{6}abx^2 + 2\sqrt{2}a^2x^3}{1 - 2\sqrt{3}bx - ax^2 + 3b^2x^2 + \sqrt{3}abx^3 + \sqrt{2}a^2x^4}\,dx = \text{Timed out}$$

input
```
integrate((-3^(1/2)*b-2*a*x+4*2^(1/2)*a*x+3*b^2*x+2*3^(1/2)*a*b*x^2-2*6^(1
/2)*a*b*x^2+2*2^(1/2)*a^2*x^3)/(1-2*3^(1/2)*b*x-a*x^2+3*b^2*x^2+3^(1/2)*a*
b*x^3+2^(1/2)*a^2*x^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-\sqrt{3}b - 2ax + 4\sqrt{2}ax + 3b^2x + 2\sqrt{3}abx^2 - 2\sqrt{6}abx^2 + 2\sqrt{2}a^2x^3}{1 - 2\sqrt{3}bx - ax^2 + 3b^2x^2 + \sqrt{3}abx^3 + \sqrt{2}a^2x^4}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((-3**(1/2)*b-2*a*x+4*2**(1/2)*a*x+3*b**2*x+2*3**(1/2)*a*b*x**2-2
*6**(1/2)*a*b*x**2+2*2**(1/2)*a**2*x**3)/(1-2*3**(1/2)*b*x-a*x**2+3*b**2*x
**2+3**(1/2)*a*b*x**3+2**(1/2)*a**2*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(128*_t**4*a**2 - 768*_t**4*a*b**2
+ 144*sqrt(2)*_t**4*a*b**2 - 432*sqrt(2)*_t**4*b**4 + 1233*_t**4*b**4 - 51
2*sqrt(2)*_t**3*a**2 - 128*_t**3*a**2 + 192*_t**3*a*b**2 + 1368*sqrt(2)*_t
**3*a*b**2 - 12
```

Maxima [F]

$$\int \frac{-\sqrt{3}b - 2ax + 4\sqrt{2}ax + 3b^2x + 2\sqrt{3}abx^2 - 2\sqrt{6}abx^2 + 2\sqrt{2}a^2x^3}{1 - 2\sqrt{3}bx - ax^2 + 3b^2x^2 + \sqrt{3}abx^3 + \sqrt{2}a^2x^4}\,dx$$
$$= \int \frac{2\sqrt{2}a^2x^3 - 2\sqrt{6}abx^2 + 2\sqrt{3}abx^2 + 3b^2x + 4\sqrt{2}ax - 2ax - \sqrt{3}b}{\sqrt{2}a^2x^4 + \sqrt{3}abx^3 + 3b^2x^2 - ax^2 - 2\sqrt{3}bx + 1}\,dx$$

input
```
integrate((-3^(1/2)*b-2*a*x+4*2^(1/2)*a*x+3*b^2*x+2*3^(1/2)*a*b*x^2-2*6^(1
/2)*a*b*x^2+2*2^(1/2)*a^2*x^3)/(1-2*3^(1/2)*b*x-a*x^2+3*b^2*x^2+3^(1/2)*a*
b*x^3+2^(1/2)*a^2*x^4),x, algorithm="maxima")
```

output
```
integrate((2*sqrt(2)*a^2*x^3 - 2*sqrt(6)*a*b*x^2 + 2*sqrt(3)*a*b*x^2 + 3*b
^2*x + 4*sqrt(2)*a*x - 2*a*x - sqrt(3)*b)/(sqrt(2)*a^2*x^4 + sqrt(3)*a*b*x
^3 + 3*b^2*x^2 - a*x^2 - 2*sqrt(3)*b*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2319 vs. $2(113) = 226$.

Time = 1.66 (sec) , antiderivative size = 2319, normalized size of antiderivative = 16.10

$$\int \frac{-\sqrt{3}b - 2ax + 4\sqrt{2}ax + 3b^2x + 2\sqrt{3}abx^2 - 2\sqrt{6}abx^2 + 2\sqrt{2}a^2x^3}{1 - 2\sqrt{3}bx - ax^2 + 3b^2x^2 + \sqrt{3}abx^3 + \sqrt{2}a^2x^4}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((-3^(1/2)*b-2*a*x+4*2^(1/2)*a*x+3*b^2*x+2*3^(1/2)*a*b*x^2-2*6^(1
/2)*a*b*x^2+2*2^(1/2)*a^2*x^3)/(1-2*3^(1/2)*b*x-a*x^2+3*b^2*x^2+3^(1/2)*a*
b*x^3+2^(1/2)*a^2*x^4),x, algorithm="giac")
```

output
```
sqrt(a^4*(5396*sqrt(2) - 5441))*(arctan(2*(27636341338012491152696093987*(
sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^7 - 5230211714540
0708898809898241333*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 544
1))^6 + 328430321341231094421127562357562*(sqrt(6) + sqrt(3) + sqrt(2) + s
qrt(5396*sqrt(2) - 5441))^5 - 446297368309840706129438434620479336*(sqrt(6
) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^4 + 2848431849895069212
060254831863634696*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441
))^3 + 2850541588971381834052436922303878348368*(sqrt(6) + sqrt(3) + sqrt(
2) + sqrt(5396*sqrt(2) - 5441))^2 - 95624413143988939361244970804996794279
36*sqrt(6) - 9562441314398893936124497080499679427936*sqrt(3) - 9562441314
398893936124497080499679427936*sqrt(2) - 956244131439889393612449708049967
9427936*sqrt(5396*sqrt(2) - 5441) - 34477728300671528566665149819572174997
57088)/(368450358337850127539543314507*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt
(5396*sqrt(2) - 5441))^7 + 2115716933455990134676622929690*(sqrt(6) + sqrt
(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^6 + 334211186648388039806413891
9013736*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^5 - 6735
244973173027461677719411135868*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sq
rt(2) - 5441))^4 - 17950209296288329441229873576913097616*(sqrt(6) + sqrt(
3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^3 - 8404418848071122883364005272
6594356240*(sqrt(6) + sqrt(3) + sqrt(2) + sqrt(5396*sqrt(2) - 5441))^2 ...
```

Mupad [B] (verification not implemented)

Time = 45.93 (sec) , antiderivative size = 4129, normalized size of antiderivative = 28.67

$$\int \frac{-\sqrt{3}b - 2ax + 4\sqrt{2}ax + 3b^2x + 2\sqrt{3}abx^2 - 2\sqrt{6}abx^2 + 2\sqrt{2}a^2x^3}{1 - 2\sqrt{3}bx - ax^2 + 3b^2x^2 + \sqrt{3}abx^3 + \sqrt{2}a^2x^4}\,dx$$
$$= \text{Too large to display}$$

input
```
int((3*b^2*x - 2*a*x - 3^(1/2)*b + 2*2^(1/2)*a^2*x^3 + 4*2^(1/2)*a*x + 2*3
^(1/2)*a*b*x^2 - 2*6^(1/2)*a*b*x^2)/(3*b^2*x^2 - a*x^2 + 2^(1/2)*a^2*x^4 -
 2*3^(1/2)*b*x + 3^(1/2)*a*b*x^3 + 1),x)
```

output

```
symsum(log(root(768*2^(1/2)*a^3*b^2*z^4 + 576*2^(1/2)*a^2*b^4*z^4 - 3168*a
^3*b^2*z^4 - 2376*a^2*b^4*z^4 - 4224*2^(1/2)*a^4*z^4 + 2048*a^4*z^4 - 1536
*2^(1/2)*a^3*b^2*z^3 - 1152*2^(1/2)*a^2*b^4*z^3 + 6336*a^3*b^2*z^3 + 4752*
a^2*b^4*z^3 + 8448*2^(1/2)*a^4*z^3 - 4096*a^4*z^3 + 4344*2^(1/2)*3^(1/2)*6
^(1/2)*a^2*b^4*z^2 - 2944*2^(1/2)*3^(1/2)*6^(1/2)*a^3*b^2*z^2 + 1188*2^(1/
2)*3^(1/2)*6^(1/2)*a*b^6*z^2 + 5696*3^(1/2)*6^(1/2)*a^3*b^2*z^2 - 2016*3^(
1/2)*6^(1/2)*a^2*b^4*z^2 - 576*3^(1/2)*6^(1/2)*a*b^6*z^2 - 216*2^(1/2)*3^(
1/2)*6^(1/2)*b^8*z^2 + 1728*2^(1/2)*a*b^6*z^2 + 1728*3^(1/2)*6^(1/2)*b^8*z
^2 - 22656*2^(1/2)*a^3*b^2*z^2 + 1872*2^(1/2)*a^2*b^4*z^2 - 26136*a^2*b^4*
z^2 + 17568*a^3*b^2*z^2 - 7128*a*b^6*z^2 - 5184*2^(1/2)*b^8*z^2 - 128*2^(1
/2)*a^4*z^2 + 1296*b^8*z^2 - 14848*a^4*z^2 - 10608*2^(1/2)*3^(1/2)*6^(1/2)
*a^2*b^4*z + 7680*2^(1/2)*3^(1/2)*6^(1/2)*a^3*b^2*z - 468*2^(1/2)*3^(1/2)*
6^(1/2)*a*b^6*z - 5184*3^(1/2)*6^(1/2)*a*b^6*z + 216*2^(1/2)*3^(1/2)*6^(1/
2)*b^8*z + 10464*3^(1/2)*6^(1/2)*a^2*b^4*z - 9856*3^(1/2)*6^(1/2)*a^3*b^2*
z + 35904*2^(1/2)*a^3*b^2*z - 26640*2^(1/2)*a^2*b^4*z + 15552*2^(1/2)*a*b^
6*z - 1728*3^(1/2)*6^(1/2)*b^8*z + 2808*a*b^6*z + 5184*2^(1/2)*b^8*z - 409
6*2^(1/2)*a^4*z + 61344*a^2*b^4*z - 49152*a^3*b^2*z - 1296*b^8*z + 16896*a
^4*z + 8256*2^(1/2)*3^(1/2)*6^(1/2)*a^2*b^4 - 576*2^(1/2)*3^(1/2)*6^(1/2)*
a^3*b^2 + 1872*2^(1/2)*3^(1/2)*6^(1/2)*a*b^6 + 4608*3^(1/2)*6^(1/2)*a^3*b^
2 + 1824*3^(1/2)*6^(1/2)*a^2*b^4 + 6696*3^(1/2)*6^(1/2)*a*b^6 + 4320*2^...
```

Reduce [F]

$$\int \frac{-\sqrt{3}b-2ax+4\sqrt{2}ax+3b^2x+2\sqrt{3}abx^2-2\sqrt{6}abx^2+2\sqrt{2}a^2x^3}{1-2\sqrt{3}bx-ax^2+3b^2x^2+\sqrt{3}abx^3+\sqrt{2}a^2x^4}\,dx$$
$$= \text{too large to display}$$

input

```
int((-3^(1/2)*b-2*a*x+4*2^(1/2)*a*x+3*b^2*x+2*3^(1/2)*a*b*x^2-2*6^(1/2)*a*
b*x^2+2*2^(1/2)*a^2*x^3)/(1-2*3^(1/2)*b*x-a*x^2+3*b^2*x^2+3^(1/2)*a*b*x^3+
2^(1/2)*a^2*x^4),x)
```

output
```
( - 48*sqrt(6)*int(x**12/(4*a**8*x**16 - 12*a**6*b**2*x**14 - 4*a**6*x**12
 + 72*a**5*b**2*x**12 + 8*a**5*x**10 - 27*a**4*b**4*x**12 - 78*a**4*b**2*x
**10 - 3*a**4*x**8 - 36*a**3*b**4*x**10 + 24*a**3*b**2*x**8 - 4*a**3*x**6
- 54*a**2*b**6*x**10 + 90*a**2*b**4*x**8 - 42*a**2*b**2*x**6 + 6*a**2*x**4
 + 108*a*b**6*x**8 - 108*a*b**4*x**6 + 36*a*b**2*x**4 - 4*a*x**2 + 81*b**8
*x**8 - 108*b**6*x**6 + 54*b**4*x**4 - 12*b**2*x**2 + 1),x)*a**6*b + 96*sq
rt(6)*int(x**12/(4*a**8*x**16 - 12*a**6*b**2*x**14 - 4*a**6*x**12 + 72*a**
5*b**2*x**12 + 8*a**5*x**10 - 27*a**4*b**4*x**12 - 78*a**4*b**2*x**10 - 3*
a**4*x**8 - 36*a**3*b**4*x**10 + 24*a**3*b**2*x**8 - 4*a**3*x**6 - 54*a**2
*b**6*x**10 + 90*a**2*b**4*x**8 - 42*a**2*b**2*x**6 + 6*a**2*x**4 + 108*a*
b**6*x**8 - 108*a*b**4*x**6 + 36*a*b**2*x**4 - 4*a*x**2 + 81*b**8*x**8 - 1
08*b**6*x**6 + 54*b**4*x**4 - 12*b**2*x**2 + 1),x)*a**5*b**3 + 160*sqrt(6)
*int(x**10/(4*a**8*x**16 - 12*a**6*b**2*x**14 - 4*a**6*x**12 + 72*a**5*b**
2*x**12 + 8*a**5*x**10 - 27*a**4*b**4*x**12 - 78*a**4*b**2*x**10 - 3*a**4*
x**8 - 36*a**3*b**4*x**10 + 24*a**3*b**2*x**8 - 4*a**3*x**6 - 54*a**2*b**6
*x**10 + 90*a**2*b**4*x**8 - 42*a**2*b**2*x**6 + 6*a**2*x**4 + 108*a*b**6*
x**8 - 108*a*b**4*x**6 + 36*a*b**2*x**4 - 4*a*x**2 + 81*b**8*x**8 - 108*b*
*6*x**6 + 54*b**4*x**4 - 12*b**2*x**2 + 1),x)*a**5*b + 24*sqrt(6)*int(x**1
0/(4*a**8*x**16 - 12*a**6*b**2*x**14 - 4*a**6*x**12 + 72*a**5*b**2*x**12 +
 8*a**5*x**10 - 27*a**4*b**4*x**12 - 78*a**4*b**2*x**10 - 3*a**4*x**8 -...
```

3.4 $\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc(-c^2+b^2x^2-2acx^2-a^2x^4)^2}\,dx$

Optimal result

Integrand size = 105, antiderivative size = 55

$$\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)^2}\,dx = \frac{-b^2x+acx+a^2x^3}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)}$$

output
```
(a^2*x^3+a*c*x-b^2*x)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)^2}\,dx = \frac{x(b^2-a(c+ax^2))}{bc\left(c^2-b^2x^2+2acx^2+a^2x^4\right)}$$

input
```
Integrate[(b^2*c^2 - a*c^3 + b^4*x^2 - 3*a*b^2*c*x^2 - a^2*c^2*x^2 - 2*a^2
*b^2*x^4 + a^3*c*x^4 + a^4*x^6)/(b*c*(-c^2 + b^2*x^2 - 2*a*c*x^2 - a^2*x^4
)^2),x]
```

output
```
(x*(b^2 - a*(c + a*x^2)))/(b*c*(c^2 - b^2*x^2 + 2*a*c*x^2 + a^2*x^4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {6, 6, 6, 6, 27, 2204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^4x^6+a^3cx^4-2a^2b^2x^4-a^2c^2x^2-3ab^2cx^2-ac^3+b^4x^2+b^2c^2}{bc\left(-a^2x^4-2acx^2+b^2x^2-c^2\right)^2}dx$$

$$\Big\downarrow \text{6}$$

$$\int \frac{a^4x^6+a^3cx^4-2a^2b^2x^4-a^2c^2x^2-3ab^2cx^2-ac^3+b^4x^2+b^2c^2}{bc\left(-a^2x^4+x^2\left(b^2-2ac\right)-c^2\right)^2}dx$$

$$\Big\downarrow \text{6}$$

$$\int \frac{a^4x^6+a^3cx^4-2a^2b^2x^4-a^2c^2x^2+x^2\left(b^4-3ab^2c\right)-ac^3+b^2c^2}{bc\left(-a^2x^4+x^2\left(b^2-2ac\right)-c^2\right)^2}dx$$

$$\Big\downarrow \text{6}$$

$$\int \frac{a^4x^6+a^3cx^4-2a^2b^2x^4+x^2\left(-a^2c^2-3ab^2c+b^4\right)-ac^3+b^2c^2}{bc\left(-a^2x^4+x^2\left(b^2-2ac\right)-c^2\right)^2}dx$$

$$\Big\downarrow \text{6}$$

$$\int \frac{a^4x^6+x^2\left(-a^2c^2-3ab^2c+b^4\right)+x^4\left(a^3c-2a^2b^2\right)-ac^3+b^2c^2}{bc\left(-a^2x^4+x^2\left(b^2-2ac\right)-c^2\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^4x^6-a^2\left(2b^2-ac\right)x^4+\left(b^4-3acb^2-a^2c^2\right)x^2+c^2\left(b^2-ac\right)}{\left(a^2x^4-\left(b^2-2ac\right)x^2+c^2\right)^2}dx}{bc}$$

$$\Big\downarrow \text{2204}$$

$$\frac{x\left(-a^2x^2-ac+b^2\right)}{bc\left(a^2x^4-x^2\left(b^2-2ac\right)+c^2\right)}$$

input
```
Int[(b^2*c^2 - a*c^3 + b^4*x^2 - 3*a*b^2*c*x^2 - a^2*c^2*x^2 - 2*a^2*b^2*x
^4 + a^3*c*x^4 + a^4*x^6)/(b*c*(-c^2 + b^2*x^2 - 2*a*c*x^2 - a^2*x^4)^2),x
]
```

output
```
(x*(b^2 - a*c - a^2*x^2))/(b*c*(c^2 - (b^2 - 2*a*c)*x^2 + a^2*x^4))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{-ax-b}{2a\,x^2+2bx+2c}+\frac{-ax+b}{2a\,x^2-2bx+2c}}{bc}$	52
gosper	$-\frac{x\left(x^2a^2+ac-b^2\right)}{\left(a^2x^4+2ac\,x^2-b^2x^2+c^2\right)cb}$	54
risch	$\frac{-a^2x^3+\left(-ac+b^2\right)x}{bc\left(a^2x^4+2ac\,x^2-b^2x^2+c^2\right)}$	55
norman	$\frac{-\frac{a^2x^3}{bc}-\frac{\left(ac-b^2\right)x}{cb}}{a^2x^4+2ac\,x^2-b^2x^2+c^2}$	63
parallelrisch	$\frac{-a^4x^3-a^3cx+a^2b^2x}{bc\,a^2\left(a^2x^4+2ac\,x^2-b^2x^2+c^2\right)}$	63
orering	$-\frac{\left(a\,x^2-bx+c\right)\left(a\,x^2+bx+c\right)x\left(x^2a^2+ac-b^2\right)}{bc\left(-a^2x^4-2ac\,x^2+b^2x^2-c^2\right)^2}$	77

input
```
int((a^4*x^6+a^3*c*x^4-2*a^2*b^2*x^4-a^2*c^2*x^2-3*a*b^2*c*x^2+b^4*x^2-a*c
^3+b^2*c^2)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)^2,x,method=_RETURNVERBOSE
)
```

output
```
1/b/c*(1/2*(-a*x-b)/(a*x^2+b*x+c)+1/2*(-a*x+b)/(a*x^2-b*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)^2}\,dx$$
$$=-\frac{a^2x^3-(b^2-ac)x}{a^2bcx^4+bc^3-(b^3c-2\,abc^2)x^2}$$

input
```
integrate((a^4*x^6+a^3*c*x^4-2*a^2*b^2*x^4-a^2*c^2*x^2-3*a*b^2*c*x^2+b^4*x
^2-a*c^3+b^2*c^2)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)^2,x, algorithm="fri
cas")
```

output
```
-(a^2*x^3 - (b^2 - a*c)*x)/(a^2*b*c*x^4 + b*c^3 - (b^3*c - 2*a*b*c^2)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)^2}\,dx = \frac{-a^2x^3+x(-ac+b^2)}{a^2bcx^4+bc^3+x^2\cdot(2abc^2-b^3c)}$$

input
```
integrate((a**4*x**6+a**3*c*x**4-2*a**2*b**2*x**4-a**2*c**2*x**2-3*a*b**2*
c*x**2+b**4*x**2-a*c**3+b**2*c**2)/b/c/(-a**2*x**4-2*a*c*x**2+b**2*x**2-c*
*2)**2,x)
```

output
```
(-a**2*x**3 + x*(-a*c + b**2))/(a**2*b*c*x**4 + b*c**3 + x**2*(2*a*b*c**2
- b**3*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{b^2c^2-ac^3+b^4x^2-3ab^2cx^2-a^2c^2x^2-2a^2b^2x^4+a^3cx^4+a^4x^6}{bc\left(-c^2+b^2x^2-2acx^2-a^2x^4\right)^2}\,dx = -\frac{a^2x^3-(b^2-ac)x}{(a^2x^4-(b^2-2\,ac)x^2+c^2)bc}$$

input
```
integrate((a^4*x^6+a^3*c*x^4-2*a^2*b^2*x^4-a^2*c^2*x^2-3*a*b^2*c*x^2+b^4*x
^2-a*c^3+b^2*c^2)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)^2,x, algorithm="max
ima")
```

output
```
-(a^2*x^3 - (b^2 - a*c)*x)/((a^2*x^4 - (b^2 - 2*a*c)*x^2 + c^2)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{b^2c^2 - ac^3 + b^4x^2 - 3ab^2cx^2 - a^2c^2x^2 - 2a^2b^2x^4 + a^3cx^4 + a^4x^6}{bc\left(-c^2 + b^2x^2 - 2acx^2 - a^2x^4\right)^2}\,dx = -\frac{a^2x^3 - b^2x + acx}{(a^2x^4 - b^2x^2 + 2\,acx^2 + c^2)bc}$$

input
```
integrate((a^4*x^6+a^3*c*x^4-2*a^2*b^2*x^4-a^2*c^2*x^2-3*a*b^2*c*x^2+b^4*x
^2-a*c^3+b^2*c^2)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)^2,x, algorithm="gia
c")
```

output
```
-(a^2*x^3 - b^2*x + a*c*x)/((a^2*x^4 - b^2*x^2 + 2*a*c*x^2 + c^2)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{b^2c^2 - ac^3 + b^4x^2 - 3ab^2cx^2 - a^2c^2x^2 - 2a^2b^2x^4 + a^3cx^4 + a^4x^6}{bc\left(-c^2 + b^2x^2 - 2acx^2 - a^2x^4\right)^2}\,dx = -\frac{x\left(a\,c - b^2\right) + a^2\,x^3}{b\,c^3 - x^2\,\left(b^3\,c - 2\,a\,b\,c^2\right) + a^2\,b\,c\,x^4}$$

input
```
int(-(a*c^3 - b^2*c^2 - a^4*x^6 - b^4*x^2 - a^3*c*x^4 + 2*a^2*b^2*x^4 + a^
2*c^2*x^2 + 3*a*b^2*c*x^2)/(b*c*(c^2 + a^2*x^4 - b^2*x^2 + 2*a*c*x^2)^2),x
)
```

output
```
-(x*(a*c - b^2) + a^2*x^3)/(b*c^3 - x^2*(b^3*c - 2*a*b*c^2) + a^2*b*c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{b^2c^2 - ac^3 + b^4x^2 - 3ab^2cx^2 - a^2c^2x^2 - 2a^2b^2x^4 + a^3cx^4 + a^4x^6}{bc\left(-c^2 + b^2x^2 - 2acx^2 - a^2x^4\right)^2}\,dx = \frac{x(-a^2x^2 - ac + b^2)}{bc\left(a^2x^4 + 2ac\,x^2 - b^2x^2 + c^2\right)}$$

input
```
int((a^4*x^6+a^3*c*x^4-2*a^2*b^2*x^4-a^2*c^2*x^2-3*a*b^2*c*x^2+b^4*x^2-a*c
^3+b^2*c^2)/b/c/(-a^2*x^4-2*a*c*x^2+b^2*x^2-c^2)^2,x)
```

output
```
(x*( - a**2*x**2 - a*c + b**2))/(b*c*(a**2*x**4 + 2*a*c*x**2 - b**2*x**2 +
 c**2))
```

3.5 $\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}\right.\right.}{}$

Optimal result

Integrand size = 179, antiderivative size = 101

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^{3}}$$

output
```
(18+6*2^(3/4)*3^(1/4)-10*2^(1/4)*3^(3/4)+4*6^(1/2)+(30-6^(3/4)*(2*2^(1/2)+
3*3^(1/2)+6^(3/4)))*x^2)/(114*6^(1/2)-114*2^(1/4)*3^(3/4)*x^2+342*x^4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^{3}}$$

input
```
Integrate[(3*x^3*(-16 + 20*2^(3/4)*3^(1/4) - 8*2^(1/4)*3^(3/4) - 12*Sqrt[6
] + (36 + 12*2^(3/4)*3^(1/4) + 18*2^(1/4)*3^(3/4) - 30*Sqrt[6])*x^2 + (-54
 - 18*2^(3/4)*3^(1/4) + 30*2^(1/4)*3^(3/4) - 12*Sqrt[6])*x^4 + (-30 + 9*2^
(3/4)*3^(1/4) + 4*2^(1/4)*3^(3/4) + 6*Sqrt[6])*x^6))/(19*(Sqrt[6] - 2^(1/4
)*3^(3/4)*x^2 + 3*x^4)^3),x]
```

output
```
(3*(4 - 5*2^(3/4)*3^(1/4) + 2*2^(1/4)*3^(3/4) + 3*Sqrt[6] - (6 + 2*2^(3/4)
*3^(1/4) + 3*2^(1/4)*3^(3/4) - 5*Sqrt[6])*x^2))/(19*(18 - 9*2^(3/4)*3^(1/4
)*x^2 + 9*Sqrt[6]*x^4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {27, 25, 2019, 1578, 27, 1223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^3\left(\left(-30+9\ 2^{3/4}\sqrt[4]{3}+4\sqrt[4]{2}3^{3/4}+6\sqrt{6}\right)x^6+\left(-54-18\ 2^{3/4}\sqrt[4]{3}+30\sqrt[4]{2}3^{3/4}-12\sqrt{6}\right)x^4+\left(36+12\ 2^{3/4}\right.\right.}{19\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^3}$$

$$\Big\downarrow 27$$

$$\frac{3}{19}\int -\frac{x^3\left(\left(30-6^{3/4}\left(2\sqrt{2}+3\sqrt{3}+6^{3/4}\right)\right)x^6+6\left(9+\sqrt{3}\left(2\sqrt{2}-5\sqrt[4]{6}+6^{3/4}\right)\right)x^4-6\left(6+\sqrt[4]{6}\left(2\sqrt{2}+3\sqrt{3}-\right.\right.\right.}{\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^3}$$

$$\Big\downarrow 25$$

$$-\frac{3}{19}\int \frac{x^3\left(\left(30-6^{3/4}\left(2\sqrt{2}+3\sqrt{3}+6^{3/4}\right)\right)x^6+6\left(9+\sqrt{3}\left(2\sqrt{2}-5\sqrt[4]{6}+6^{3/4}\right)\right)x^4-6\left(6+\sqrt[4]{6}\left(2\sqrt{2}+3\sqrt{3}-\right.\right.\right.}{\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^3}$$

$$\Big\downarrow 2019$$

$$-\frac{3}{19}\int \frac{x^3\left(\left(10-4\sqrt[4]{\frac{2}{3}}-3\ 2^{3/4}\sqrt[4]{3}-2\sqrt{6}\right)x^2+4\ 2^{3/4}\sqrt[4]{3}+8\sqrt{\frac{2}{3}}-20\sqrt[4]{\frac{2}{3}}+12\right)}{\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^2}dx$$

$$\Big\downarrow \text{1578}$$

$$-\frac{3}{38}\int \frac{x^2\left(\left(30-6^{3/4}\left(2\sqrt{2}+3\sqrt{3}+6^{3/4}\right)\right)x^2+4\left(9+3\ 2^{3/4}\sqrt[4]{3}-5\sqrt[4]{2}3^{3/4}+2\sqrt{6}\right)\right)}{3\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^2}dx^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{38}\int \frac{x^2\left(\left(30-6^{3/4}\left(2\sqrt{2}+3\sqrt{3}+6^{3/4}\right)\right)x^2+4\left(9+3\ 2^{3/4}\sqrt[4]{3}-5\sqrt[4]{2}3^{3/4}+2\sqrt{6}\right)\right)}{\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)^2}dx^2$$

$$\Big\downarrow \text{1223}$$

$$\frac{\left(30-6^{3/4}\left(2\sqrt{2}+3\sqrt{3}+6^{3/4}\right)\right)x^2+2\left(9+3\ 2^{3/4}\sqrt[4]{3}-5\sqrt[4]{2}3^{3/4}+2\sqrt{6}\right)}{114\left(3x^4-\sqrt[4]{2}3^{3/4}x^2+\sqrt{6}\right)}$$

input
```
Int[(3*x^3*(-16 + 20*2^(3/4)*3^(1/4) - 8*2^(1/4)*3^(3/4) - 12*Sqrt[6] + (3
6 + 12*2^(3/4)*3^(1/4) + 18*2^(1/4)*3^(3/4) - 30*Sqrt[6])*x^2 + (-54 - 18*
2^(3/4)*3^(1/4) + 30*2^(1/4)*3^(3/4) - 12*Sqrt[6])*x^4 + (-30 + 9*2^(3/4)*
3^(1/4) + 4*2^(1/4)*3^(3/4) + 6*Sqrt[6])*x^6))/(19*(Sqrt[6] - 2^(1/4)*3^(3
/4)*x^2 + 3*x^4)^3),x]
```

output
```
(2*(9 + 3*2^(3/4)*3^(1/4) - 5*2^(1/4)*3^(3/4) + 2*Sqrt[6]) + (30 - 6^(3/4)
*(2*Sqrt[2] + 3*Sqrt[3] + 6^(3/4)))*x^2)/(114*(Sqrt[6] - 2^(1/4)*3^(3/4)*x
^2 + 3*x^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1223
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b^2*e*g*(p + 2) - 2*a*c*e*
g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3), 0] && NeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2\left(\left(\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}}}{3}+\frac{\sqrt{3}\,\sqrt{2}}{2}+\frac{3\,2^{\frac{3}{4}} 3^{\frac{1}{4}}}{4}-\frac{5}{2}\right)x^{2}-\frac{2^{\frac{3}{4}} 3^{\frac{1}{4}}}{2}-\frac{\sqrt{3}\,\sqrt{2}}{3}-\frac{3}{2}+\frac{5\,2^{\frac{1}{4}} 3^{\frac{3}{4}}}{6}\right)}{57\left(-\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{2}}{3}+\frac{\sqrt{3}\,\sqrt{2}}{3}+x^{4}\right)}$
risch	$\frac{\frac{1}{19}+\frac{3\left(-\frac{2^{\frac{3}{4}} 3^{\frac{1}{4}}}{6}+\frac{5}{9}-\frac{2\,2^{\frac{1}{4}} 3^{\frac{3}{4}}}{27}-\frac{\sqrt{3}\,\sqrt{2}}{9}\right)x^{2}}{19}-\frac{5\,2^{\frac{1}{4}} 3^{\frac{3}{4}}}{171}+\frac{2\sqrt{3}\,\sqrt{2}}{171}+\frac{2^{\frac{3}{4}} 3^{\frac{1}{4}}}{57}}{-\frac{2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{2}}{3}+\frac{\sqrt{3}\,\sqrt{2}}{3}+x^{4}}$
gosper	$-\frac{\left(x^{2} 24^{\frac{3}{4}}-12x^{4}-2\sqrt{24}\right)\left(24^{\frac{3}{4}}-12x^{2}\right)\left(9\,2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{6}+4\,2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{6}-18\,2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{4}+30\,2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{4}+6\sqrt{6}\,x^{6}+12\,2^{\frac{3}{4}} 3^{\frac{1}{4}} x^{2}+18\,2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{2}-30x}{304\left(24^{\frac{3}{4}}-6x^{2}\right)\left(2^{\frac{1}{4}} 3^{\frac{3}{4}} x^{2}-3x^{4}-\sqrt{6}\right)^{3}}$

input
```
int(3/19*x^3*(-16+20*2^(3/4)*3^(1/4)-8*2^(1/4)*3^(3/4)-12*6^(1/2)+(36+12*2
^(3/4)*3^(1/4)+18*2^(1/4)*3^(3/4)-30*6^(1/2))*x^2+(-54-18*2^(3/4)*3^(1/4)+
30*2^(1/4)*3^(3/4)-12*6^(1/2))*x^4+(-30+9*2^(3/4)*3^(1/4)+4*2^(1/4)*3^(3/4
)+6*6^(1/2))*x^6)/(6^(1/2)-2^(1/4)*3^(3/4)*x^2+3*x^4)^3,x,method=_RETURNVE
RBOSE)
```

output

```
-2/57*((1/3*2^(1/4)*3^(3/4)+1/2*3^(1/2)*2^(1/2)+3/4*2^(3/4)*3^(1/4)-5/2)*x
^2-1/2*2^(3/4)*3^(1/4)-1/3*3^(1/2)*2^(1/2)-3/2+5/6*2^(1/4)*3^(3/4))/(-1/3*
2^(1/4)*3^(3/4)*x^2+1/3*3^(1/2)*2^(1/2)+x^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input

```
integrate(3/19*x^3*(-16+20*2^(3/4)*3^(1/4)-8*2^(1/4)*3^(3/4)-12*6^(1/2)+(3
6+12*2^(3/4)*3^(1/4)+18*2^(1/4)*3^(3/4)-30*6^(1/2))*x^2+(-54-18*2^(3/4)*3^
(1/4)+30*2^(1/4)*3^(3/4)-12*6^(1/2))*x^4+(-30+9*2^(3/4)*3^(1/4)+4*2^(1/4)*
3^(3/4)+6*6^(1/2))*x^6)/(6^(1/2)-2^(1/4)*3^(3/4)*x^2+3*x^4)^3,x, algorithm
="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input

```
integrate(3/19*x**3*(-16+20*2**(3/4)*3**(1/4)-8*2**(1/4)*3**(3/4)-12*6**(1
/2)+(36+12*2**(3/4)*3**(1/4)+18*2**(1/4)*3**(3/4)-30*6**(1/2))*x**2+(-54-1
8*2**(3/4)*3**(1/4)+30*2**(1/4)*3**(3/4)-12*6**(1/2))*x**4+(-30+9*2**(3/4)
*3**(1/4)+4*2**(1/4)*3**(3/4)+6*6**(1/2))*x**6)/(6**(1/2)-2**(1/4)*3**(3/4
)*x**2+3*x**4)**3,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 612, normalized size of antiderivative = 6.06

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input
```
integrate(3/19*x^3*(-16+20*2^(3/4)*3^(1/4)-8*2^(1/4)*3^(3/4)-12*6^(1/2)+(3
6+12*2^(3/4)*3^(1/4)+18*2^(1/4)*3^(3/4)-30*6^(1/2))*x^2+(-54-18*2^(3/4)*3^
(1/4)+30*2^(1/4)*3^(3/4)-12*6^(1/2))*x^4+(-30+9*2^(3/4)*3^(1/4)+4*2^(1/4)*
3^(3/4)+6*6^(1/2))*x^6)/(6^(1/2)-2^(1/4)*3^(3/4)*x^2+3*x^4)^3,x, algorithm
="maxima")
```

output
```
3/19*(6*sqrt(6)*3^(3/4)*2^(1/4) + sqrt(6)*(3*3^(3/4)*2^(1/4) - 10*sqrt(3)*
sqrt(2) - 4*3^(1/4)*2^(3/4) + 6) + 8*3^(3/4)*2^(1/4) + 6*sqrt(3)*sqrt(2) -
 27*3^(1/4)*2^(3/4) - 12*sqrt(6) + 60)*arctan((6*x^2 - 3^(3/4)*2^(1/4))/sq
rt(-3*sqrt(3)*sqrt(2) + 12*sqrt(6)))/((4*sqrt(6)*sqrt(3)*sqrt(2) - 51)*sqr
t(-3*sqrt(3)*sqrt(2) + 12*sqrt(6))) - 1/228*(6*(sqrt(6)*(15*3^(3/4)*2^(1/4
) - 10*sqrt(3)*sqrt(2) + 28*3^(1/4)*2^(3/4) - 18) - 6*sqrt(6)*(3*3^(3/4)*2
^(1/4) + 1) - 92*3^(3/4)*2^(1/4) + 30*sqrt(3)*sqrt(2) - 72*3^(1/4)*2^(3/4)
 - 60*sqrt(6) + 330)*x^6 - 3*(4*sqrt(6)*3^(3/4)*2^(1/4) + sqrt(6)*(30*3^(3
/4)*2^(1/4) + 69*sqrt(3)*sqrt(2) - 70*3^(1/4)*2^(3/4) + 92) - 2*sqrt(6)*(3
^(3/4)*2^(1/4) + 27*sqrt(3)*sqrt(2) + 96) + 500*3^(3/4)*2^(1/4) - 44*sqrt(
3)*sqrt(2) - 312*3^(1/4)*2^(3/4) - 738)*x^4 + 2*(60*sqrt(6)*sqrt(3)*sqrt(2
) + sqrt(6)*(16*3^(3/4)*2^(1/4) + 12*sqrt(3)*sqrt(2) + 99*3^(1/4)*2^(3/4)
+ 420) - 12*sqrt(6)*(7*3^(3/4)*2^(1/4) + 6*3^(1/4)*2^(3/4) - 30) - 378*3^(
3/4)*2^(1/4) - 240*sqrt(3)*sqrt(2) - 12*3^(1/4)*2^(3/4) - 1080)*x^2 + 4*sq
rt(6)*(10*3^(3/4)*2^(1/4) - 7*sqrt(3)*sqrt(2) + 36*3^(1/4)*2^(3/4) + 63) -
 4*sqrt(6)*(5*3^(3/4)*2^(1/4) - 2*sqrt(3)*sqrt(2) - 3*3^(1/4)*2^(3/4)) + 6
*sqrt(6)*(3^(1/4)*2^(3/4) + 48) - 108*3^(3/4)*2^(1/4) - 216*sqrt(3)*sqrt(2
) - 600*3^(1/4)*2^(3/4) + 552)/(3*(4*sqrt(6)*sqrt(3)*sqrt(2) - 51)*x^8 - 6
*(4*sqrt(6)*3^(1/4)*2^(3/4) - 17*3^(3/4)*2^(1/4))*x^6 - 3*(sqrt(3)*sqrt(2)
 + 26*sqrt(6))*x^4 + 2*(17*sqrt(6)*3^(3/4)*2^(1/4) - 24*3^(1/4)*2^(3/4)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input

```
integrate(3/19*x^3*(-16+20*2^(3/4)*3^(1/4)-8*2^(1/4)*3^(3/4)-12*6^(1/2)+(3
6+12*2^(3/4)*3^(1/4)+18*2^(1/4)*3^(3/4)-30*6^(1/2))*x^2+(-54-18*2^(3/4)*3^
(1/4)+30*2^(1/4)*3^(3/4)-12*6^(1/2))*x^4+(-30+9*2^(3/4)*3^(1/4)+4*2^(1/4)*
3^(3/4)+6*6^(1/2))*x^6)/(6^(1/2)-2^(1/4)*3^(3/4)*x^2+3*x^4)^3,x, algorithm
="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{133056,[12]%%%}+%%%{%%{[-459,0,0,0,12717,0,0,0,193023,0,0,
0,360639]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input

```
int(-(3*x^3*(8*2^(1/4)*3^(3/4) - 20*2^(3/4)*3^(1/4) + 12*6^(1/2) - x^6*(4*
2^(1/4)*3^(3/4) + 9*2^(3/4)*3^(1/4) + 6*6^(1/2) - 30) - x^2*(18*2^(1/4)*3^
(3/4) + 12*2^(3/4)*3^(1/4) - 30*6^(1/2) + 36) + x^4*(18*2^(3/4)*3^(1/4) -
30*2^(1/4)*3^(3/4) + 12*6^(1/2) + 54) + 16))/(19*(6^(1/2) + 3*x^4 - 2^(1/4
)*3^(3/4)*x^2)^3),x)
```

output

```
\text{Hanged}
```

Reduce [F]

$$\int \frac{3x^3\left(-16+20\ 2^{3/4}\sqrt[4]{3}-8\sqrt[4]{2}3^{3/4}-12\sqrt{6}+\left(36+12\ 2^{3/4}\sqrt[4]{3}+18\sqrt[4]{2}3^{3/4}-30\sqrt{6}\right)x^2+\left(-54-18\ 2\right.\right.}{19\left(\sqrt{6}-\sqrt[4]{2}3^{3/4}x^2+3x^4\right)^3}$$

input
```
int(3/19*x^3*(-16+20*2^(3/4)*3^(1/4)-8*2^(1/4)*3^(3/4)-12*6^(1/2)+(36+12*2
^(3/4)*3^(1/4)+18*2^(1/4)*3^(3/4)-30*6^(1/2))*x^2+(-54-18*2^(3/4)*3^(1/4)+
30*2^(1/4)*3^(3/4)-12*6^(1/2))*x^4+(-30+9*2^(3/4)*3^(1/4)+4*2^(1/4)*3^(3/4
)+6*6^(1/2))*x^6)/(6^(1/2)-2^(1/4)*3^(3/4)*x^2+3*x^4)^3,x)
```

output
```
(4374*sqrt(6)*2**(1/4)*3**(3/4)*int(x**43/(2916*2**(1/4)*3**(3/4)*x**46 +
21384*2**(1/4)*3**(3/4)*x**38 - 14256*2**(1/4)*3**(3/4)*x**30 + 9504*2**(1
/4)*3**(3/4)*x**22 + 576*2**(1/4)*3**(3/4)*x**14 - 384*2**(1/4)*3**(3/4)*x
**6 + 9720*2**(3/4)*3**(1/4)*x**42 + 73872*2**(3/4)*3**(1/4)*x**34 - 31104
*2**(3/4)*3**(1/4)*x**26 - 2880*2**(3/4)*3**(1/4)*x**18 + 1152*2**(3/4)*3*
*(1/4)*x**10 - 8748*sqrt(6)*x**44 - 53136*sqrt(6)*x**36 + 40176*sqrt(6)*x*
*28 - 5184*sqrt(6)*x**20 - 1728*sqrt(6)*x**12 - 729*x**48 - 18954*x**40 -
87480*x**32 - 648*x**24 + 9504*x**16 + 864*x**8 - 64),x) - 8748*sqrt(6)*2*
*(1/4)*3**(3/4)*int(x**41/(2916*2**(1/4)*3**(3/4)*x**46 + 21384*2**(1/4)*3
**(3/4)*x**38 - 14256*2**(1/4)*3**(3/4)*x**30 + 9504*2**(1/4)*3**(3/4)*x**
22 + 576*2**(1/4)*3**(3/4)*x**14 - 384*2**(1/4)*3**(3/4)*x**6 + 9720*2**(3
/4)*3**(1/4)*x**42 + 73872*2**(3/4)*3**(1/4)*x**34 - 31104*2**(3/4)*3**(1/
4)*x**26 - 2880*2**(3/4)*3**(1/4)*x**18 + 1152*2**(3/4)*3**(1/4)*x**10 - 8
748*sqrt(6)*x**44 - 53136*sqrt(6)*x**36 + 40176*sqrt(6)*x**28 - 5184*sqrt(
6)*x**20 - 1728*sqrt(6)*x**12 - 729*x**48 - 18954*x**40 - 87480*x**32 - 64
8*x**24 + 9504*x**16 + 864*x**8 - 64),x) + 19440*sqrt(6)*2**(1/4)*3**(3/4)
*int(x**39/(2916*2**(1/4)*3**(3/4)*x**46 + 21384*2**(1/4)*3**(3/4)*x**38 -
 14256*2**(1/4)*3**(3/4)*x**30 + 9504*2**(1/4)*3**(3/4)*x**22 + 576*2**(1/
4)*3**(3/4)*x**14 - 384*2**(1/4)*3**(3/4)*x**6 + 9720*2**(3/4)*3**(1/4)*x*
*42 + 73872*2**(3/4)*3**(1/4)*x**34 - 31104*2**(3/4)*3**(1/4)*x**26 - 2...
```

3.6 $\int \frac{x^2(\sqrt{6}-12x^3)(\sqrt{6}-3x^3)^2(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16})}{3(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16})^2}\,dx$

Optimal result

Integrand size = 107, antiderivative size = 71

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx = \frac{-2\sqrt{6}x^3+18x^6-9\sqrt{6}x^9+9x^{12}}{-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}}$$

output
```
(-2*6^(1/2)*x^3+18*x^6-9*6^(1/2)*x^9+9*x^12)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^
10+12*6^(1/2)*x^13-9*x^16)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx = \frac{x^3\left(2\sqrt{6}-18x^3+9\sqrt{6}x^6-9x^9\right)}{3+4x^4-8\sqrt{6}x^7+36x^{10}-12\sqrt{6}x^{13}+9x^{16}}$$

input
```
Integrate[(x^2*(Sqrt[6] - 12*x^3)*(Sqrt[6] - 3*x^3)^2*(9 - 4*x^4 + 8*Sqrt[
6]*x^7 - 36*x^10 + 12*Sqrt[6]*x^13 - 9*x^16))/(3*(-3 - 4*x^4 + 8*Sqrt[6]*x
^7 - 36*x^10 + 12*Sqrt[6]*x^13 - 9*x^16)^2),x]
```

output
```
(x^3*(2*Sqrt[6] - 18*x^3 + 9*Sqrt[6]*x^6 - 9*x^9))/(3 + 4*x^4 - 8*Sqrt[6]*
x^7 + 36*x^10 - 12*Sqrt[6]*x^13 + 9*x^16)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {27, 2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(\sqrt{6}-12 x^3\right) \left(\sqrt{6}-3 x^3\right)^2 \left(-9 x^{16}+12 \sqrt{6} x^{13}-36 x^{10}+8 \sqrt{6} x^7-4 x^4+9\right)}{3 \left(-9 x^{16}+12 \sqrt{6} x^{13}-36 x^{10}+8 \sqrt{6} x^7-4 x^4-3\right)^2} \, dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{3} \int \frac{x^2 \left(\sqrt{6}-12 x^3\right) \left(\sqrt{6}-3 x^3\right)^2 \left(-9 x^{16}+12 \sqrt{6} x^{13}-36 x^{10}+8 \sqrt{6} x^7-4 x^4+9\right)}{\left(9 x^{16}-12 \sqrt{6} x^{13}+36 x^{10}-8 \sqrt{6} x^7+4 x^4+3\right)^2} dx$$
$$\Big\downarrow \text{ 2023}$$
$$\frac{x^3 \left(\sqrt{6}-3 x^3\right)^3}{3 \left(9 x^{16}-12 \sqrt{6} x^{13}+36 x^{10}-8 \sqrt{6} x^7+4 x^4+3\right)}$$

input
```
Int[(x^2*(Sqrt[6] - 12*x^3)*(Sqrt[6] - 3*x^3)^2*(9 - 4*x^4 + 8*Sqrt[6]*x^7
 - 36*x^10 + 12*Sqrt[6]*x^13 - 9*x^16))/(3*(-3 - 4*x^4 + 8*Sqrt[6]*x^7 - 3
6*x^10 + 12*Sqrt[6]*x^13 - 9*x^16)^2),x]
```

output
```
(x^3*(Sqrt[6] - 3*x^3)^3)/(3*(3 + 4*x^4 - 8*Sqrt[6]*x^7 + 36*x^10 - 12*Sqr
t[6]*x^13 + 9*x^16))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\frac{2\sqrt{6}\,x^{3}}{9}-2x^{6}+\sqrt{6}\,x^{9}-x^{12}}{\frac{1}{3}+\frac{4x^{4}}{9}-\frac{8\sqrt{6}\,x^{7}}{9}+4x^{10}-\frac{4\sqrt{6}\,x^{13}}{3}+x^{16}}$	61
risch	$\frac{\frac{2\sqrt{6}\,x^{3}}{3}-6x^{6}+3\sqrt{6}\,x^{9}-3x^{12}}{1+\frac{4x^{4}}{3}-\frac{8\sqrt{6}\,x^{7}}{3}+12x^{10}-4\sqrt{6}\,x^{13}+3x^{16}}$	63
parallelrisch	$\frac{27x^{12}-27\sqrt{6}\,x^{9}+54x^{6}-6\sqrt{6}\,x^{3}}{-9-12x^{4}+24\sqrt{6}\,x^{7}-108x^{10}+36\sqrt{6}\,x^{13}-27x^{16}}$	65
orering	$-\frac{x^{3}\left(\sqrt{6}-3x^{3}\right)^{3}\left(9-4x^{4}+8\sqrt{6}\,x^{7}-36x^{10}+12\sqrt{6}\,x^{13}-9x^{16}\right)}{\left(-3-4x^{4}+8\sqrt{6}\,x^{7}-36x^{10}+12\sqrt{6}\,x^{13}-9x^{16}\right)\left(-3x^{8}+2x^{5}\sqrt{6}-2x^{2}-3\right)\left(-3x^{4}+3+\sqrt{6}\,x\right)\left(-3x^{4}+\sqrt{6}\,x-3\right)}$	135
gosper	$-\frac{3x^{3}\left(-9x^{9}+9\sqrt{6}\,x^{6}-18x^{3}+2\sqrt{6}\right)\left(9-4x^{4}+8\sqrt{6}\,x^{7}-36x^{10}+12\sqrt{6}\,x^{13}-9x^{16}\right)}{\left(-3-4x^{4}+8\sqrt{6}\,x^{7}-36x^{10}+12\sqrt{6}\,x^{13}-9x^{16}\right)\left(-3x^{4}+\sqrt{6}\,x-3\right)\left(-3x^{4}+3+\sqrt{6}\,x\right)\left(-3x^{8}+2x^{5}\sqrt{6}-2x^{2}-3\right)}$	148

input
```
int(1/3*x^2*(6^(1/2)-12*x^3)*(6^(1/2)-3*x^3)^2*(9-4*x^4+8*6^(1/2)*x^7-36*x
^10+12*6^(1/2)*x^13-9*x^16)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^10+12*6^(1/2)*x^1
3-9*x^16)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/9*6^(1/2)*x^3-2*x^6+6^(1/2)*x^9-x^12)/(1/3+4/9*x^4-8/9*6^(1/2)*x^7+4*x^
10-4/3*6^(1/2)*x^13+x^16)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx = -\frac{81\,x^{28}-162\,x^{22}+108\,x^{16}+27\,x^{12}-24\,x^{10}+54\,x^6+\sqrt{6}(27\,x^{25}-54\,x^{19}+36\,x^{13}-27\,x^9-8\,x^7-6}{81\,x^{32}-216\,x^{26}+216\,x^{20}+54\,x^{16}-96\,x^{14}+216\,x^{10}+16\,x^8+24\,x^4+9}$$

input
```
integrate(1/3*x^2*(6^(1/2)-12*x^3)*(6^(1/2)-3*x^3)^2*(9-4*x^4+8*6^(1/2)*x^
7-36*x^10+12*6^(1/2)*x^13-9*x^16)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^10+12*6^(1/
2)*x^13-9*x^16)^2,x, algorithm="fricas")
```

output
```
-(81*x^28 - 162*x^22 + 108*x^16 + 27*x^12 - 24*x^10 + 54*x^6 + sqrt(6)*(27
*x^25 - 54*x^19 + 36*x^13 - 27*x^9 - 8*x^7 - 6*x^3))/(81*x^32 - 216*x^26 +
 216*x^20 + 54*x^16 - 96*x^14 + 216*x^10 + 16*x^8 + 24*x^4 + 9)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx = \frac{-81x^{12}+81\sqrt{6}x^9-162x^6+18\sqrt{6}x^3}{81x^{16}-108\sqrt{6}x^{13}+324x^{10}-72\sqrt{6}x^7+36x^4+27}$$

input
```
integrate(1/3*x**2*(6**(1/2)-12*x**3)*(6**(1/2)-3*x**3)**2*(9-4*x**4+8*6**
(1/2)*x**7-36*x**10+12*6**(1/2)*x**13-9*x**16)/(-3-4*x**4+8*6**(1/2)*x**7-
36*x**10+12*6**(1/2)*x**13-9*x**16)**2,x)
```

output
```
(-81*x**12 + 81*sqrt(6)*x**9 - 162*x**6 + 18*sqrt(6)*x**3)/(81*x**16 - 108
*sqrt(6)*x**13 + 324*x**10 - 72*sqrt(6)*x**7 + 36*x**4 + 27)
```

Maxima [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{x^2(\sqrt{6}-12x^3)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx$$
$$=-\frac{9\,x^{12}-9\,\sqrt{6}x^9+18\,x^6-2\,\sqrt{6}x^3}{9\,x^{16}-12\,\sqrt{6}x^{13}+36\,x^{10}-8\,\sqrt{6}x^7+4\,x^4+3}$$

input
```
integrate(1/3*x^2*(6^(1/2)-12*x^3)*(6^(1/2)-3*x^3)^2*(9-4*x^4+8*6^(1/2)*x^
7-36*x^10+12*6^(1/2)*x^13-9*x^16)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^10+12*6^(1/
2)*x^13-9*x^16)^2,x, algorithm="maxima")
```

output
```
-(9*x^12 - 9*sqrt(6)*x^9 + 18*x^6 - 2*sqrt(6)*x^3)/(9*x^16 - 12*sqrt(6)*x^
13 + 36*x^10 - 8*sqrt(6)*x^7 + 4*x^4 + 3)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{x^2(\sqrt{6}-12x^3)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx$$
$$=-\frac{9\,x^{12}-9\,\sqrt{6}x^9+18\,x^6-2\,\sqrt{6}x^3}{9\,x^{16}-12\,\sqrt{6}x^{13}+36\,x^{10}-8\,\sqrt{6}x^7+4\,x^4+3}$$

input
```
integrate(1/3*x^2*(6^(1/2)-12*x^3)*(6^(1/2)-3*x^3)^2*(9-4*x^4+8*6^(1/2)*x^
7-36*x^10+12*6^(1/2)*x^13-9*x^16)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^10+12*6^(1/
2)*x^13-9*x^16)^2,x, algorithm="giac")
```

output
```
-(9*x^12 - 9*sqrt(6)*x^9 + 18*x^6 - 2*sqrt(6)*x^3)/(9*x^16 - 12*sqrt(6)*x^
13 + 36*x^10 - 8*sqrt(6)*x^7 + 4*x^4 + 3)
```

Mupad [B] (verification not implemented)

Time = 14.81 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx$$
$$=\frac{x^3\left(\sqrt{6}-3\,x^3\right)^3}{3\,\left(9\,x^{16}-12\,\sqrt{6}\,x^{13}+36\,x^{10}-8\,\sqrt{6}\,x^7+4\,x^4+3\right)}$$

input

```
int((x^2*(6^(1/2) - 3*x^3)^2*(6^(1/2) - 12*x^3)*(8*6^(1/2)*x^7 + 12*6^(1/2
)*x^13 - 4*x^4 - 36*x^10 - 9*x^16 + 9))/(3*(4*x^4 - 12*6^(1/2)*x^13 - 8*6^
(1/2)*x^7 + 36*x^10 + 9*x^16 + 3)^2),x)
```

output

```
(x^3*(6^(1/2) - 3*x^3)^3)/(3*(4*x^4 - 12*6^(1/2)*x^13 - 8*6^(1/2)*x^7 + 36
*x^10 + 9*x^16 + 3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{x^2\left(\sqrt{6}-12x^3\right)\left(\sqrt{6}-3x^3\right)^2\left(9-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)}{3\left(-3-4x^4+8\sqrt{6}x^7-36x^{10}+12\sqrt{6}x^{13}-9x^{16}\right)^2}\,dx$$
$$=\frac{x^3\left(-27\sqrt{6}\,x^{22}+54\sqrt{6}\,x^{16}-36\sqrt{6}\,x^{10}+27\sqrt{6}\,x^6+8\sqrt{6}\,x^4+6\sqrt{6}-81x^{25}+162x^{19}-108x^{13}-27x^9\right.}{81x^{32}-216x^{26}+216x^{20}+54x^{16}-96x^{14}+216x^{10}+16x^8+24x^4+9}$$

input

```
int(1/3*x^2*(6^(1/2)-12*x^3)*(6^(1/2)-3*x^3)^2*(9-4*x^4+8*6^(1/2)*x^7-36*x
^10+12*6^(1/2)*x^13-9*x^16)/(-3-4*x^4+8*6^(1/2)*x^7-36*x^10+12*6^(1/2)*x^1
3-9*x^16)^2,x)
```

output

```
(x**3*( - 27*sqrt(6)*x**22 + 54*sqrt(6)*x**16 - 36*sqrt(6)*x**10 + 27*sqrt
(6)*x**6 + 8*sqrt(6)*x**4 + 6*sqrt(6) - 81*x**25 + 162*x**19 - 108*x**13 -
 27*x**9 + 24*x**7 - 54*x**3))/(81*x**32 - 216*x**26 + 216*x**20 + 54*x**1
6 - 96*x**14 + 216*x**10 + 16*x**8 + 24*x**4 + 9)
```

3.7 $\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3(2-\sqrt{3})}-8\sqrt{2(2-\sqrt{3})}x^2-6\sqrt{2}x^4+2\sqrt{\cdots}\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+\cdots\right.}$

Optimal result

Integrand size = 231, antiderivative size = 76

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4\cdots\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3\cdots\right.}$$
$$=\frac{24-12\sqrt{3}-6\sqrt{2\left(7-4\sqrt{3}\right)}x^2}{24-24\sqrt{3}+24\sqrt{2\left(7-4\sqrt{3}\right)}x^2-24x^4+12\sqrt{3}x^4}$$

output
```
(24-12*3^(1/2)-6*(2*2^(1/2)-6^(1/2))*x^2)/(24-24*3^(1/2)+24*(2*2^(1/2)-6^(
1/2))*x^2-24*x^4+12*3^(1/2)*x^4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 474 vs. 2(76) = 152.

Time = 1.84 (sec) , antiderivative size = 474, normalized size of antiderivative = 6.24

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3\right.}$$

$$= \frac{\sqrt{3}\left(-1148625788556528844066159033262220068722206008007247900413395522084959231874596984\right.}{\left(-3220874966854814188580851119+185957302913975\right.}$$

input
```
Integrate[(2*Sqrt[3]*x*(-8*Sqrt[6] + 16*Sqrt[2 - Sqrt[3]] + 8*Sqrt[3*(2 -
Sqrt[3])] - 8*Sqrt[2*(2 - Sqrt[3])]*x^2 - 6*Sqrt[2]*x^4 + 2*Sqrt[6]*x^4 +
12*Sqrt[2 - Sqrt[3]]*x^4 + 4*Sqrt[3*(2 - Sqrt[3])]*x^4 - 4*Sqrt[2*(2 - Sqr
t[3])]*x^6 + Sqrt[2 - Sqrt[3]]*x^8))/((2 + 2*Sqrt[3] - 2*Sqrt[2]*x^2 + x^4
)^2*(-4*Sqrt[3] + 4*Sqrt[3*(2 - Sqrt[3])]*x^2 - 3*x^4 + Sqrt[3]*x^4)),x]
```

output
```
(Sqrt[3]*(-114862578855652884406615903326222006872220600800724790041339552
2084959231874596984923161348 + 663159408221258093117811630391460877479645
4336019989320654459513074427058646148003568219 6*Sqrt[3] + 2427331901676353
754741698431482826362303147824400221220041813899086749383703572034026895 24
*Sqrt[12 - 6*Sqrt[3]] - 420426218053622717643631246206799161400925575248 59
806852405554487509637053318451200209663 0*Sqrt[4 - 2*Sqrt[3]] + (3889612146
2883788897674083034114494258257092366439089553544950693950792157205737203 1
6532049*Sqrt[2] - 224566861970283358046513942360937998182446110133680116 25
087242668622458099456707351467765 73*Sqrt[6] - 1643943526585545309302265021
514972484744442185423605911246526728964400863501192436128705424*Sqrt[6 - 3
*Sqrt[3]] + 28473937128201218516280032846674083987877558315784167110926141
44413117794707229269533951680*Sqrt[2 - Sqrt[3]])*x^2))/((-322087496685481 4
18858085119 + 185957302913975393256650292*Sqrt[3] + 680650854823697161725 3
5224*Sqrt[12 - 6*Sqrt[3]] - 117892186276983133050206984*Sqrt[4 - 2*Sqrt[3]
])*(-252086908569103685779 + 145542444521551451775*Sqrt[3] + 5327207380047
3568424*Sqrt[12 - 6*Sqrt[3]] - 92269938446981769376*Sqrt[4 - 2*Sqrt[3]])*(
-5651986252505 + 3263175794250*Sqrt[3] + 1192503998950*Sqrt[12 - 6*Sqrt[3]
] - 2065477490966*Sqrt[4 - 2*Sqrt[3]])*(-662849748 + 382696633*Sqrt[3] + 1
37980318*Sqrt[12 - 6*Sqrt[3]] - 238988562*Sqrt[4 - 2*Sqrt[3]])*(-3194627 +
 1844361*Sqrt[3] + 659824*Sqrt[12 - 6*Sqrt[3]] - 1142984*Sqrt[4 - 2*Sqr...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(76) = 152$.

Time = 3.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {6, 6, 6, 6, 27, 7266, 2126, 2191, 27, 2191, 27}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{3}x\left(\sqrt{2-\sqrt{3}}x^8-4\sqrt{2\left(2-\sqrt{3}\right)}x^6+4\sqrt{3\left(2-\sqrt{3}\right)}x^4+12\sqrt{2-\sqrt{3}}x^4+2\sqrt{6}x^4-6\sqrt{2}x^4-8\sqrt{2\left(2-\sqrt{\right.}\right.}}{\left(x^4-2\sqrt{2}x^2+2\sqrt{3}+2\right)^2\left(\sqrt{3}x^4-3x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-4\sqrt{\right.}}$$

$$\Big\downarrow 6$$

$$\int \frac{2\sqrt{3}x\left(\sqrt{2-\sqrt{3}}x^8-4\sqrt{2\left(2-\sqrt{3}\right)}x^6+4\sqrt{3\left(2-\sqrt{3}\right)}x^4+12\sqrt{2-\sqrt{3}}x^4+2\sqrt{6}x^4-6\sqrt{2}x^4-8\sqrt{2\left(2-\cdots\right.}\right.}{\left(x^4-2\sqrt{2}x^2+2\sqrt{3}+2\right)^2\left(\left(\sqrt{3}-3\right)x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-4\cdots\right.}$$

$\downarrow$ 6

$$\int \frac{2\sqrt{3}x\left(\sqrt{2-\sqrt{3}}x^8-4\sqrt{2\left(2-\sqrt{3}\right)}x^6+\left(2\sqrt{6}-6\sqrt{2}\right)x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^4+12\sqrt{2-\sqrt{3}}x^4-8\sqrt{2\left(2-\cdots\right.}\right.}{\left(x^4-2\sqrt{2}x^2+2\sqrt{3}+2\right)^2\left(\left(\sqrt{3}-3\right)x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-4\cdots\right.}$$

$\downarrow$ 6

$$\int \frac{2\sqrt{3}x\left(\sqrt{2-\sqrt{3}}x^8-4\sqrt{2\left(2-\sqrt{3}\right)}x^6+\left(12\sqrt{2-\sqrt{3}}+4\sqrt{3\left(2-\sqrt{3}\right)}\right)x^4+\left(2\sqrt{6}-6\sqrt{2}\right)x^4-8\sqrt{2\left(2-\cdots\right.}\right.}{\left(x^4-2\sqrt{2}x^2+2\sqrt{3}+2\right)^2\left(\left(\sqrt{3}-3\right)x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-4\cdots\right.}$$

$\downarrow$ 6

$$\int \frac{2\sqrt{3}x\left(\sqrt{2-\sqrt{3}}x^8-4\sqrt{2\left(2-\sqrt{3}\right)}x^6+\left(-6\sqrt{2}+2\sqrt{6}+12\sqrt{2-\sqrt{3}}+4\sqrt{3\left(2-\sqrt{3}\right)}\right)x^4-8\sqrt{2\left(2-\cdots\right.}\right.}{\left(x^4-2\sqrt{2}x^2+2\sqrt{3}+2\right)^2\left(\left(\sqrt{3}-3\right)x^4+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-4\sqrt{\cdots}\right.}$$

$\downarrow$ 27

$$2\sqrt{3}\int \frac{x\left(-\sqrt{2-\sqrt{3}}x^8+4\sqrt{2\left(2-\sqrt{3}\right)}x^6+2\sqrt{2}\left(3-\sqrt{3}-\sqrt{12-6\sqrt{3}}-3\sqrt{4-2\sqrt{3}}\right)x^4+8\sqrt{2\left(2-\sqrt{3}\right)}x\cdots\right.}{\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2\left(\left(3-\sqrt{3}\right)x^4-4\sqrt{3\left(2-\sqrt{3}\right)}x^2+4\cdots\right.}$$

$\downarrow$ 7266

$$\sqrt{3}\int \frac{-\sqrt{2-\sqrt{3}}x^8+4\sqrt{2\left(2-\sqrt{3}\right)}x^6+2\sqrt{2}\left(3-\sqrt{3}-\sqrt{12-6\sqrt{3}}-3\sqrt{4-2\sqrt{3}}\right)x^4+8\sqrt{2\left(2-\sqrt{3}\right)}x^2+\cdots}{\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2\left(\left(3-\sqrt{3}\right)x^4-4\sqrt{3\left(2-\sqrt{3}\right)}x^2+4\sqrt{3}\right)\cdots}$$

$\downarrow$ 2126

$$\frac{\sqrt{3}\int \frac{-\sqrt{2-\sqrt{3}}x^8+4\sqrt{2\left(2-\sqrt{3}\right)}x^6+2\sqrt{2}\left(3-\sqrt{3}-\sqrt{12-6\sqrt{3}}-3\sqrt{4-2\sqrt{3}}\right)x^4+8\sqrt{2\left(2-\sqrt{3}\right)}x^2+8\left(\sqrt{6}-\sqrt{6-3\sqrt{3}}-2\sqrt{2-\sqrt{3}}\right)}{\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^3}dx^2}{3-\sqrt{3}}$$

$\downarrow$ 2191

$$\frac{\sqrt{3}\left(\frac{\int -\frac{16\left(\sqrt{3\left(2-\sqrt{3}\right)}x^4-2\sqrt{6\left(2-\sqrt{3}\right)}x^2-3\left(\sqrt{2}+\sqrt{6}\right)+6\sqrt{2-\sqrt{3}}+8\sqrt{6-3\sqrt{3}}\right)}{\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}dx^2}{16\sqrt{3}}+\frac{\left(3\sqrt{2}-\sqrt{6}-2\sqrt{6-3\sqrt{3}}\right)x^2+2\left(3\sqrt{4-2\sqrt{3}}-\sqrt{3}\left(2-\sqrt{4-2\cdots\right.\right.}{\sqrt{3}\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}\cdots\right.}{3-\sqrt{3}}$$

$\downarrow$ 27

$$\frac{\sqrt{3}\left(\frac{\left(3\sqrt{2}-\sqrt{6}-2\sqrt{6-3\sqrt{3}}\right)x^2+2\left(3\sqrt{4-2\sqrt{3}}-\sqrt{3}\left(2-\sqrt{4-2\sqrt{3}}\right)\right)}{\sqrt{3}\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}x^4-2\sqrt{6\left(2-\sqrt{3}\right)}x^2-3\left(\sqrt{2}+\sqrt{6}\right)+6\sqrt{2-\sqrt{3}}+8\sqrt{6-3\sqrt{3}}}{\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}dx^2}{\sqrt{3}}\right)}{3-\sqrt{3}}$$

$\downarrow$ 2191

$$\frac{\sqrt{3}\left(\frac{\left(3\sqrt{2}-\sqrt{6}-2\sqrt{6-3\sqrt{3}}\right)x^2+2\left(3\sqrt{4-2\sqrt{3}}-\sqrt{3}\left(2-\sqrt{4-2\sqrt{3}}\right)\right)}{\sqrt{3}\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}-\frac{\frac{\int -\frac{6\sqrt{2}\left(1+\sqrt{3}-\sqrt{12-6\sqrt{3}}-2\sqrt{4-2\sqrt{3}}\right)}{x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)}dx^2}{8\sqrt{3}}+\frac{\sqrt{\frac{3}{2}}\left(\sqrt{2}\left(1+\sqrt{3}-\sqrt{12-6\sqrt{3}}\right)\cdots\right.}{2\left(x^4-2\sqrt{2}x^2\cdots\right.}}{\sqrt{3}}\cdots\right.}{3-\sqrt{3}}$$

$\downarrow$ 27

$$\frac{\sqrt{3}\left(\frac{\left(3\sqrt{2}-\sqrt{6}-2\sqrt{6-3\sqrt{3}}\right)x^2+2\left(3\sqrt{4-2\sqrt{3}}-\sqrt{3}\left(2-\sqrt{4-2\sqrt{3}}\right)\right)}{\sqrt{3}\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)^2}-\frac{\sqrt{2}\left(1+\sqrt{3}-\sqrt{12-6\sqrt{3}}\right)-\left(1+\sqrt{3}-\sqrt{12-6\sqrt{3}}\right)x^2}{2\sqrt{2}\left(x^4-2\sqrt{2}x^2+2\left(1+\sqrt{3}\right)\right)}\right)}{3-\sqrt{3}}$$

input
```
Int[(2*Sqrt[3]*x*(-8*Sqrt[6] + 16*Sqrt[2 - Sqrt[3]] + 8*Sqrt[3*(2 - Sqrt[3
])] - 8*Sqrt[2*(2 - Sqrt[3])]*x^2 - 6*Sqrt[2]*x^4 + 2*Sqrt[6]*x^4 + 12*Sqr
t[2 - Sqrt[3]]*x^4 + 4*Sqrt[3*(2 - Sqrt[3])]*x^4 - 4*Sqrt[2*(2 - Sqrt[3])]
*x^6 + Sqrt[2 - Sqrt[3]]*x^8))/((2 + 2*Sqrt[3] - 2*Sqrt[2]*x^2 + x^4)^2*(-
4*Sqrt[3] + 4*Sqrt[3*(2 - Sqrt[3])]*x^2 - 3*x^4 + Sqrt[3]*x^4)),x]
```

output
```
(Sqrt[3]*((2*(3*Sqrt[4 - 2*Sqrt[3]] - Sqrt[3]*(2 - Sqrt[4 - 2*Sqrt[3]])) +
 (3*Sqrt[2] - Sqrt[6] - 2*Sqrt[6 - 3*Sqrt[3]])*x^2)/(Sqrt[3]*(2*(1 + Sqrt[
3]) - 2*Sqrt[2]*x^2 + x^4)^2) - (Sqrt[2]*(1 + Sqrt[3] - Sqrt[12 - 6*Sqrt[3
]]) - (1 + Sqrt[3] - Sqrt[12 - 6*Sqrt[3]])*x^2)/(2*Sqrt[2]*(2*(1 + Sqrt[3]
) - 2*Sqrt[2]*x^2 + x^4))))/(3 - Sqrt[3])
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2126
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(c/f)^p   Int[Px*(d + e*x + f*x^2)^(p +
 q), x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && PolyQ[Px, x] && EqQ[c*
d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !Inte
gerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result
risch	$\frac{2\sqrt{3}\left(\frac{\sqrt{2}\sqrt{3}\,x^{2}}{12}-\frac{\sqrt{3}}{6}\right)}{2+2\sqrt{3}-2\sqrt{2}\,x^{2}+x^{4}}$
default	$-\frac{\sqrt{3}\,\sqrt{2}\,\left(\left(\sqrt{3}-1\right)x^{2}+\sqrt{2}-\sqrt{3}\,\sqrt{2}\right)}{2\left(-3+\sqrt{3}\right)\left(2+2\sqrt{3}-2\sqrt{2}\,x^{2}+x^{4}\right)}$
parallelrisch	$-\frac{\sqrt{3}\,\left(x^{8}\sqrt{2}\,\sqrt{6}\,\sqrt{3}+24-9\sqrt{3}\,x^{8}+42\sqrt{2}\,\sqrt{3}\,x^{6}-14x^{6}\sqrt{6}\,\sqrt{3}+3x^{8}+6x^{6}\sqrt{2}-18\sqrt{6}\,x^{6}-24\sqrt{2}\,\sqrt{3}\,x^{2}-8x^{2}\sqrt{6}\,\sqrt{3}+8\sqrt{6}\,\sqrt{3}\,\sqrt{2}\cdots\right.}{72\left(-x^{4}+2\sqrt{2}\,x^{2}-2\sqrt{3}-2\right)^{2}}$
gosper	$-\frac{\left(-x^{2}+\sqrt{2}\right)\sqrt{3}\,\left(\sqrt{2}\,x^{8}-\sqrt{6}\,x^{8}+8\sqrt{3}\,x^{6}-8x^{6}+12\sqrt{2}\,x^{4}-12\sqrt{6}\,x^{4}+16\sqrt{3}\,x^{2}-16x^{2}-8\sqrt{2}+8\sqrt{6}\right)}{2\left(-x^{4}+2\sqrt{2}\,x^{2}-2\sqrt{3}-2\right)\left(-x^{4}+2\sqrt{2}\,x^{2}+2\sqrt{3}-2\right)\left(\sqrt{3}\,x^{4}-3x^{4}+6\sqrt{2}\,x^{2}-2\sqrt{6}\,x^{2}-4\sqrt{3}\right)}$

input
```
int(2*3^(1/2)*x*(-4*6^(1/2)+4*2^(1/2)-8*(3^(1/2)-1)*x^2-6*2^(1/2)*x^4+2*6^
(1/2)*x^4+12*(1/2*6^(1/2)-1/2*2^(1/2))*x^4+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^4
-4*(3^(1/2)-1)*x^6+(1/2*6^(1/2)-1/2*2^(1/2))*x^8)/(2+2*3^(1/2)-2*2^(1/2)*x
^2+x^4)^2/(-4*3^(1/2)+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^2-3*x^4+3^(1/2)*x^4),x
,method=_RETURNVERBOSE)
```

output
```
2*3^(1/2)*(1/12*2^(1/2)*3^(1/2)*x^2-1/6*3^(1/2))/(2+2*3^(1/2)-2*2^(1/2)*x^
2+x^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^{2}-6\sqrt{2}x^{4}+2\sqrt{6}x^{4}+12\sqrt{2-\sqrt{3}}x^{4}\cdots\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^{2}+x^{4}\right)^{2}\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^{2}-3\cdots\right.}$$

= Timed out

input
```
integrate(2*3^(1/2)*x*(-4*6^(1/2)+4*2^(1/2)-8*(3^(1/2)-1)*x^2-6*2^(1/2)*x^
4+2*6^(1/2)*x^4+12*(1/2*6^(1/2)-1/2*2^(1/2))*x^4+4*(3/2*2^(1/2)-1/2*6^(1/2
))*x^4-4*(3^(1/2)-1)*x^6+(1/2*6^(1/2)-1/2*2^(1/2))*x^8)/(2+2*3^(1/2)-2*2^(
1/2)*x^2+x^4)^2/(-4*3^(1/2)+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^2-3*x^4+3^(1/2)*
x^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3\right.}$$

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input
```
integrate(2*3**(1/2)*x*(-4*6**(1/2)+4*2**(1/2)-8*(3**(1/2)-1)*x**2-6*2**(1
/2)*x**4+2*6**(1/2)*x**4+12*(1/2*6**(1/2)-1/2*2**(1/2))*x**4+4*(3/2*2**(1/
2)-1/2*6**(1/2))*x**4-4*(3**(1/2)-1)*x**6+(1/2*6**(1/2)-1/2*2**(1/2))*x**8
)/(2+2*3**(1/2)-2*2**(1/2)*x**2+x**4)**2/(-4*3**(1/2)+4*(3/2*2**(1/2)-1/2*
6**(1/2))*x**2-3*x**4+3**(1/2)*x**4),x)
```

output
```
-(x**2*(-54567305088918325173851069823287372660371022840022464733663230138
34002086560494644585837460250822552784182692025735315573748238551705640375
65486988312005865671634234779703889861953362813643197499390540916862725858
46757687337485231674401989261528405637315606216926966045264716293396592352
23383331687932479778626028846196746440625127681767883166364268384068762702
35122400178224887716549955749309172654293474389760498397384047729251494541
22074040366883850829324452942498933894171898568540165914669010674616900354
72610943591666237*sqrt(2) + 3150444828203943090284248911440571141311610235
57927923814788993224130403308803652390052013806250541842135615440095453307
29810338021537142641531454574510385463575917958185798198894457914728653307
11748390510068228739206974952443761825512734425571121684282060865386373072
16820207586829630712626609081512864977709977845141231382773808453476510528
83770100571040684361560756644358956214767189301969185262851364247808438684
78213287754621448169485372656127223315321009931780038233901403015539579995
172649170704644595639876157101524442*sqrt(6)) + 10913461017783665034770213
96465747453207420456800449294673264602766800417312098928917167492050164510
55683653840514706311474964771034112807513097397662401173134326846955940777
972390672562728639499987810818337254517169351537467497046334880397852305681
12746312124338539320905294325867931847044676666337586495955725205769239349
28812502553635357663327285367681375254047024480035644977543309991149861...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 554, normalized size of antiderivative = 7.29

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3\right.}$$

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input
```
integrate(2*3^(1/2)*x*(-4*6^(1/2)+4*2^(1/2)-8*(3^(1/2)-1)*x^2-6*2^(1/2)*x^
4+2*6^(1/2)*x^4+12*(1/2*6^(1/2)-1/2*2^(1/2))*x^4+4*(3/2*2^(1/2)-1/2*6^(1/2
))*x^4-4*(3^(1/2)-1)*x^6+(1/2*6^(1/2)-1/2*2^(1/2))*x^8)/(2+2*3^(1/2)-2*2^(
1/2)*x^2+x^4)^2/(-4*3^(1/2)+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^2-3*x^4+3^(1/2)*
x^4),x, algorithm="maxima")
```

output
```
-1/288*sqrt(3)*(2*3^(3/4)*sqrt(2)*(sqrt(6)*(31*sqrt(3) + 57) + sqrt(6)*(25
*sqrt(3) + 39) - 96*sqrt(3)*sqrt(2) - 168*sqrt(2))*arctan(1/6*3^(3/4)*sqrt
(2)*(x^2 - sqrt(2)))/(sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2)) - 4*sqrt(3) -
6) - 9*(sqrt(6)*(5*sqrt(3)*sqrt(2) + 9*sqrt(2)) + 3*sqrt(6)*(sqrt(3)*sqrt(
2) + 5*sqrt(2)) - 48*sqrt(3) - 48)*log(x^4*(sqrt(3) - 3) - 2*x^2*(sqrt(6)
- 3*sqrt(2)) - 4*sqrt(3))/(sqrt(6)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2)) - 6*sqr
t(3) - 12) + 9*(sqrt(6)*(5*sqrt(3)*sqrt(2) + 9*sqrt(2)) + 3*sqrt(6)*(sqrt(
3)*sqrt(2) + 5*sqrt(2)) - 48*sqrt(3) - 48)*log(x^4 - 2*sqrt(2)*x^2 + 2*sqr
t(3) + 2)/(sqrt(6)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2)) - 6*sqrt(3) - 12) + 18*
(sqrt(6)*(43*sqrt(3) + 63) + 2*sqrt(6)*(17*sqrt(3) + 39) + 3*sqrt(6)*(sqrt
(3) + 1) - 144*sqrt(3)*sqrt(2) - 240*sqrt(2))*arctan((x^2*(sqrt(3) - 3) -
sqrt(6) + 3*sqrt(2))/sqrt(6*sqrt(6)*sqrt(2) + 12*sqrt(3) - 36))/((sqrt(6)*
(2*sqrt(3)*sqrt(2) + 3*sqrt(2)) - 6*sqrt(3) - 12)*sqrt(6*sqrt(6)*sqrt(2) +
 12*sqrt(3) - 36)) - 96*((sqrt(6)*(sqrt(3) - 3) + 3*sqrt(3)*sqrt(2) - 3*sq
rt(2))*x^2 + sqrt(6)*(5*sqrt(3)*sqrt(2) + 9*sqrt(2)) - 18*sqrt(3) - 30)/((
sqrt(6)*(sqrt(3)*sqrt(2) + sqrt(2)) - 2*sqrt(3) - 6)*x^4 - 4*(sqrt(6)*(sqr
t(3) + 1) - sqrt(3)*sqrt(2) - 3*sqrt(2))*x^2 + 4*sqrt(6)*(sqrt(3)*sqrt(2)
+ 2*sqrt(2)) - 16*sqrt(3) - 24))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4\right.}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3\right.}$$

= Exception raised: TypeError

input
```
integrate(2*3^(1/2)*x*(-4*6^(1/2)+4*2^(1/2)-8*(3^(1/2)-1)*x^2-6*2^(1/2)*x^
4+2*6^(1/2)*x^4+12*(1/2*6^(1/2)-1/2*2^(1/2))*x^4+4*(3/2*2^(1/2)-1/2*6^(1/2
))*x^4-4*(3^(1/2)-1)*x^6+(1/2*6^(1/2)-1/2*2^(1/2))*x^8)/(2+2*3^(1/2)-2*2^(
1/2)*x^2+x^4)^2/(-4*3^(1/2)+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^2-3*x^4+3^(1/2)*
x^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:   ***   Warning: increasing stack s
ize to 4096000.   ***   Warning: increasing stack size to 4096000.   ***   W
arning: i
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3}$$

$= \text{Hanged}$

input
```
int(-(2*3^(1/2)*x*(12*x^4*(2^(1/2)/2 - 6^(1/2)/2) - 4*x^4*((3*2^(1/2))/2 -
 6^(1/2)/2) + x^8*(2^(1/2)/2 - 6^(1/2)/2) - 4*2^(1/2) + 4*6^(1/2) + 6*2^(1
/2)*x^4 - 2*6^(1/2)*x^4 + 8*x^2*(3^(1/2) - 1) + 4*x^6*(3^(1/2) - 1)))/((2*
3^(1/2) - 2*2^(1/2)*x^2 + x^4 + 2)^2*(4*x^2*((3*2^(1/2))/2 - 6^(1/2)/2) -
4*3^(1/2) + 3^(1/2)*x^4 - 3*x^4)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \frac{2\sqrt{3}x\left(-8\sqrt{6}+16\sqrt{2-\sqrt{3}}+8\sqrt{3\left(2-\sqrt{3}\right)}-8\sqrt{2\left(2-\sqrt{3}\right)}x^2-6\sqrt{2}x^4+2\sqrt{6}x^4+12\sqrt{2-\sqrt{3}}x^4}{\left(2+2\sqrt{3}-2\sqrt{2}x^2+x^4\right)^2\left(-4\sqrt{3}+4\sqrt{3\left(2-\sqrt{3}\right)}x^2-3}$$

$$= \frac{-8\sqrt{6}\,x^{10}-32\sqrt{6}\,x^6+192\sqrt{6}\,x^2-48\sqrt{3}\,x^8+64\sqrt{3}\,x^4-128\sqrt{3}+4\sqrt{2}\,x^{14}-24\sqrt{2}\,x^{10}-320\sqrt{2}\,x^2+}{8x^{16}-64x^{12}-2560x^4+512}$$

input

```
int(2*3^(1/2)*x*(-4*6^(1/2)+4*2^(1/2)-8*(3^(1/2)-1)*x^2-6*2^(1/2)*x^4+2*6^
(1/2)*x^4+12*(1/2*6^(1/2)-1/2*2^(1/2))*x^4+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^4
-4*(3^(1/2)-1)*x^6+(1/2*6^(1/2)-1/2*2^(1/2))*x^8)/(2+2*3^(1/2)-2*2^(1/2)*x
^2+x^4)^2/(-4*3^(1/2)+4*(3/2*2^(1/2)-1/2*6^(1/2))*x^2-3*x^4+3^(1/2)*x^4),x
)
```

output

```
( - 8*sqrt(6)*x**10 - 32*sqrt(6)*x**6 + 192*sqrt(6)*x**2 - 48*sqrt(3)*x**8
 + 64*sqrt(3)*x**4 - 128*sqrt(3) + 4*sqrt(2)*x**14 - 24*sqrt(2)*x**10 - 32
0*sqrt(2)*x**2 + x**16 - 48*x**8 + 64*x**4 + 192)/(8*(x**16 - 8*x**12 - 32
0*x**4 + 64))
```

3.8 $\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx$

Optimal result

Integrand size = 83, antiderivative size = 195

$$\begin{aligned}
&\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx \\
&= -\sqrt{-1+4\sqrt{2}}\arctan\left(\sqrt{\frac{113}{279}+\frac{80\sqrt{2}}{279}}+\sqrt{\frac{8}{93}+\frac{32\sqrt{2}}{93}}x\right) \\
&\quad -\sqrt{-1+4\sqrt{2}}\arctan\left(\sqrt{\frac{49}{31}+\frac{196\sqrt{2}}{31}}-\sqrt{\frac{108}{31}+\frac{432\sqrt{2}}{31}}x-\sqrt{\frac{236}{31}+\frac{200\sqrt{2}}{31}}x^2\right. \\
&\quad \left.-\sqrt{\frac{24}{31}+\frac{96\sqrt{2}}{31}}x^3\right)+\frac{1}{2}\log\left(\sqrt{2}-2\sqrt{6}x+2\sqrt{2}x^2+\sqrt{6}x^3+2x^4\right)
\end{aligned}$$

output
```
-(-1+4*2^(1/2))^(1/2)*arctan(1/93*(3503+2480*2^(1/2))^(1/2)+2/93*(186+744*
2^(1/2))^(1/2)*x)+(-1+4*2^(1/2))^(1/2)*arctan(-7/31*(31+124*2^(1/2))^(1/2)
+6/31*(93+372*2^(1/2))^(1/2)*x+2/31*(1829+1550*2^(1/2))^(1/2)*x^2+2/31*(18
6+744*2^(1/2))^(1/2)*x^3)+1/2*ln(2^(1/2)-2*x*6^(1/2)+2*x^2*2^(1/2)+6^(1/2)
*x^3+2*x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89

$$\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx$$
$$= -\text{RootSum}\Bigg[1-2\sqrt{3}\#1+2\#1^2+\sqrt{3}\#1^3$$
$$+\sqrt{2}\#1^4\&,\frac{\sqrt{3}\log(x-\#1)-\log(x-\#1)\#1-4\sqrt{2}\log(x-\#1)\#1-2\sqrt{3}\log(x-\#1)\#1^2+2\sqrt{6}\,\mathrm{lo}}{-2\sqrt{3}+4\#1+3\sqrt{3}\#1^2+4\sqrt{2}\#1^3}$$

input
```
Integrate[(-Sqrt[3] + x + 4*Sqrt[2]*x + 2*Sqrt[3]*x^2 - 2*Sqrt[6]*x^2 + 2*
Sqrt[2]*x^3)/(1 - 2*Sqrt[3]*x + 2*x^2 + Sqrt[3]*x^3 + Sqrt[2]*x^4),x]
```

output
```
-RootSum[1 - 2*Sqrt[3]*#1 + 2*#1^2 + Sqrt[3]*#1^3 + Sqrt[2]*#1^4 & , (Sqrt
[3]*Log[x - #1] - Log[x - #1]*#1 - 4*Sqrt[2]*Log[x - #1]*#1 - 2*Sqrt[3]*Lo
g[x - #1]*#1^2 + 2*Sqrt[6]*Log[x - #1]*#1^2 - 2*Sqrt[2]*Log[x - #1]*#1^3)/
(-2*Sqrt[3] + 4*#1 + 3*Sqrt[3]*#1^2 + 4*Sqrt[2]*#1^3) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{2}x^3-2\sqrt{6}x^2+2\sqrt{3}x^2+4\sqrt{2}x+x-\sqrt{3}}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}\,dx$$
$$\Big\downarrow \text{ 6}$$
$$\int \frac{2\sqrt{2}x^3-2\sqrt{6}x^2+2\sqrt{3}x^2+\left(1+4\sqrt{2}\right)x-\sqrt{3}}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx$$
$$\Big\downarrow \text{ 6}$$
$$\int \frac{2\sqrt{2}x^3+\left(2\sqrt{3}-2\sqrt{6}\right)x^2+\left(1+4\sqrt{2}\right)x-\sqrt{3}}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{\int \frac{2\left(\sqrt{6}\left(1-4\sqrt{2}\right)x^2+2\left(8-\sqrt{2}\right)x\right)}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx}{4\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{6}\left(1-4\sqrt{2}\right)x^2+2\left(8-\sqrt{2}\right)x}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1\right)$$

$$\Big\downarrow \text{2027}$$

$$\frac{\int \frac{x\left(\sqrt{6}\left(1-4\sqrt{2}\right)x+2\left(8-\sqrt{2}\right)\right)}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(-\frac{\sqrt{6}\left(-1+4\sqrt{2}\right)x^2}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}-\frac{2\left(-8+\sqrt{2}\right)x}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}\right)dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(8-\sqrt{2})\int \frac{x}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx+\sqrt{6}(1-4\sqrt{2})\int \frac{x^2}{\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+\sqrt{3}x^3+2x^2-2\sqrt{3}x+1\right)$$

input
```
Int[(-Sqrt[3] + x + 4*Sqrt[2]*x + 2*Sqrt[3]*x^2 - 2*Sqrt[6]*x^2 + 2*Sqrt[2
]*x^3)/(1 - 2*Sqrt[3]*x + 2*x^2 + Sqrt[3]*x^3 + Sqrt[2]*x^4),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.63

method	result
default	$\frac{\sqrt{2}\left(\sum_{\_R=\mathrm{RootOf}\left(\sqrt{3}\,\sqrt{2}\,\_Z^{3}+2\_Z^{4}-2\sqrt{3}\,\sqrt{2}\,\_Z+2\sqrt{2}\,\_Z^{2}+\sqrt{2}\right)}\frac{\left(4\sqrt{2}\,\_R^{3}+2\sqrt{3}\,\sqrt{2}\,\_R^{2}\left(\sqrt{2}-2\right)+\sqrt{2}\left(8+\sqrt{2}\right)\_R-2\sqrt{3}\right)\ln\left(x-\_\right.}{8\_R^{3}+\sqrt{2}\left(3\sqrt{3}\,\_R^{2}-2\sqrt{3}+4\_R\right)}\right.}{2}$

input
```
int((-3^(1/2)+x+4*2^(1/2)*x+2*3^(1/2)*x^2-2*6^(1/2)*x^2+2*2^(1/2)*x^3)/(1-
2*3^(1/2)*x+2*x^2+3^(1/2)*x^3+2^(1/2)*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*sum((4*2^(1/2)*_R^3+2*3^(1/2)*2^(1/2)*_R^2*(2^(1/2)-2)+2^(1/2)
*(8+2^(1/2))*_R-2*3^(1/2))/(8*_R^3+2^(1/2)*(3*3^(1/2)*_R^2-2*3^(1/2)+4*_R)
)*ln(x-_R),_R=RootOf(3^(1/2)*2^(1/2)*_Z^3+2*_Z^4-2*3^(1/2)*2^(1/2)*_Z+2*2^
(1/2)*_Z^2+2^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx=\text{Timed out}$$

input
```
integrate((-3^(1/2)+x+4*2^(1/2)*x+2*3^(1/2)*x^2-2*6^(1/2)*x^2+2*2^(1/2)*x^
3)/(1-2*3^(1/2)*x+2*x^2+3^(1/2)*x^3+2^(1/2)*x^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-\sqrt{3} + x + 4\sqrt{2}x + 2\sqrt{3}x^2 - 2\sqrt{6}x^2 + 2\sqrt{2}x^3}{1 - 2\sqrt{3}x + 2x^2 + \sqrt{3}x^3 + \sqrt{2}x^4} \, dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((-3**(1/2)+x+4*2**(1/2)*x+2*3**(1/2)*x**2-2*6**(1/2)*x**2+2*2**(
1/2)*x**3)/(1-2*3**(1/2)*x+2*x**2+3**(1/2)*x**3+2**(1/2)*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(-593*_t**4 + 288*sqrt(2)*_t**4 - 1
360*sqrt(2)*_t**3 + 1196*_t**3 - 1044*_t**2 + 244*sqrt(2)*_t**2 - 800*_t +
 584*sqrt(2)*_t - 136 + 96*sqrt(2)) contains an element of the set of gene
rators.
```

Maxima [F]

$$\int \frac{-\sqrt{3} + x + 4\sqrt{2}x + 2\sqrt{3}x^2 - 2\sqrt{6}x^2 + 2\sqrt{2}x^3}{1 - 2\sqrt{3}x + 2x^2 + \sqrt{3}x^3 + \sqrt{2}x^4} \, dx$$
$$= \int \frac{2\sqrt{2}x^3 - 2\sqrt{6}x^2 + 2\sqrt{3}x^2 + 4\sqrt{2}x + x - \sqrt{3}}{\sqrt{2}x^4 + \sqrt{3}x^3 + 2x^2 - 2\sqrt{3}x + 1} \, dx$$

input
```
integrate((-3^(1/2)+x+4*2^(1/2)*x+2*3^(1/2)*x^2-2*6^(1/2)*x^2+2*2^(1/2)*x^
3)/(1-2*3^(1/2)*x+2*x^2+3^(1/2)*x^3+2^(1/2)*x^4),x, algorithm="maxima")
```

output
```
integrate((2*sqrt(2)*x^3 - 2*sqrt(6)*x^2 + 2*sqrt(3)*x^2 + 4*sqrt(2)*x + x
 - sqrt(3))/(sqrt(2)*x^4 + sqrt(3)*x^3 + 2*x^2 - 2*sqrt(3)*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-3^(1/2)+x+4*2^(1/2)*x+2*3^(1/2)*x^2-2*6^(1/2)*x^2+2*2^(1/2)*x^
3)/(1-2*3^(1/2)*x+2*x^2+3^(1/2)*x^3+2^(1/2)*x^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Precision problem choosing root in
common_EXT, current precision 14Precision problem choosing root in common_
EXT, curr
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 1698, normalized size of antiderivative = 8.71

$$\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx = \text{Too large to display}$$

input
```
int((x + 4*2^(1/2)*x - 3^(1/2) + 2*2^(1/2)*x^3 + 2*3^(1/2)*x^2 - 2*6^(1/2)
*x^2)/(2^(1/2)*x^4 - 2*3^(1/2)*x + 3^(1/2)*x^3 + 2*x^2 + 1),x)
```

output
```
symsum(log(163*x + (291*3^(1/2)*root(2880*2^(1/2)*z^4 + 3496*z^4 - 5760*2^
(1/2)*z^3 - 6992*z^3 - 2372*2^(1/2)*3^(1/2)*6^(1/2)*z^2 - 4832*3^(1/2)*6^(
1/2)*z^2 + 24368*2^(1/2)*z^2 + 29248*z^2 + 3180*2^(1/2)*3^(1/2)*6^(1/2)*z
+ 6304*3^(1/2)*6^(1/2)*z - 25904*2^(1/2)*z - 30600*z - 13872*2^(1/2)*3^(1/
2)*6^(1/2) - 19608*3^(1/2)*6^(1/2) + 64584*2^(1/2) + 90224, z, k))/4 + (23
1*6^(1/2)*root(2880*2^(1/2)*z^4 + 3496*z^4 - 5760*2^(1/2)*z^3 - 6992*z^3 -
 2372*2^(1/2)*3^(1/2)*6^(1/2)*z^2 - 4832*3^(1/2)*6^(1/2)*z^2 + 24368*2^(1/
2)*z^2 + 29248*z^2 + 3180*2^(1/2)*3^(1/2)*6^(1/2)*z + 6304*3^(1/2)*6^(1/2)
*z - 25904*2^(1/2)*z - 30600*z - 13872*2^(1/2)*3^(1/2)*6^(1/2) - 19608*3^(
1/2)*6^(1/2) + 64584*2^(1/2) + 90224, z, k))/4 - (1209*root(2880*2^(1/2)*z
^4 + 3496*z^4 - 5760*2^(1/2)*z^3 - 6992*z^3 - 2372*2^(1/2)*3^(1/2)*6^(1/2)
*z^2 - 4832*3^(1/2)*6^(1/2)*z^2 + 24368*2^(1/2)*z^2 + 29248*z^2 + 3180*2^(
1/2)*3^(1/2)*6^(1/2)*z + 6304*3^(1/2)*6^(1/2)*z - 25904*2^(1/2)*z - 30600*
z - 13872*2^(1/2)*3^(1/2)*6^(1/2) - 19608*3^(1/2)*6^(1/2) + 64584*2^(1/2)
+ 90224, z, k)*x)/4 + (337*2^(1/2)*x)/4 - 39*3^(1/2) - (27*6^(1/2))/4 - 45
*3^(1/2)*root(2880*2^(1/2)*z^4 + 3496*z^4 - 5760*2^(1/2)*z^3 - 6992*z^3 -
2372*2^(1/2)*3^(1/2)*6^(1/2)*z^2 - 4832*3^(1/2)*6^(1/2)*z^2 + 24368*2^(1/2
)*z^2 + 29248*z^2 + 3180*2^(1/2)*3^(1/2)*6^(1/2)*z + 6304*3^(1/2)*6^(1/2)*
z - 25904*2^(1/2)*z - 30600*z - 13872*2^(1/2)*3^(1/2)*6^(1/2) - 19608*3^(1
/2)*6^(1/2) + 64584*2^(1/2) + 90224, z, k)^2 + (153*3^(1/2)*root(2880*2...
```

Reduce [F]

$$\int \frac{-\sqrt{3}+x+4\sqrt{2}x+2\sqrt{3}x^2-2\sqrt{6}x^2+2\sqrt{2}x^3}{1-2\sqrt{3}x+2x^2+\sqrt{3}x^3+\sqrt{2}x^4}\,dx = \text{too large to display}$$

input
```
int((-3^(1/2)+x+4*2^(1/2)*x+2*3^(1/2)*x^2-2*6^(1/2)*x^2+2*2^(1/2)*x^3)/(1-
2*3^(1/2)*x+2*x^2+3^(1/2)*x^3+2^(1/2)*x^4),x)
```

output

```
(48*sqrt(6)*int(x**12/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**
8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) + 184*sqrt(6)*int(x**10/(4*x**16
- 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**
2 + 1),x) - 896*sqrt(6)*int(x**8/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**1
0 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) + 1088*sqrt(6)*int(x**
6/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x*
*4 - 16*x**2 + 1),x) - 408*sqrt(6)*int(x**4/(4*x**16 - 12*x**14 + 41*x**12
 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) + 48*sqrt(6
)*int(x**2/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x**
6 + 96*x**4 - 16*x**2 + 1),x) - 64*sqrt(3)*int(x**14/(4*x**16 - 12*x**14 +
 41*x**12 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) +
144*sqrt(3)*int(x**12/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**
8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) - 232*sqrt(3)*int(x**10/(4*x**16
- 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**
2 + 1),x) + 352*sqrt(3)*int(x**8/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**1
0 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) - 136*sqrt(3)*int(x**6
/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x**6 + 96*x**
4 - 16*x**2 + 1),x) - 232*sqrt(3)*int(x**4/(4*x**16 - 12*x**14 + 41*x**12
- 160*x**10 + 300*x**8 - 262*x**6 + 96*x**4 - 16*x**2 + 1),x) + 80*sqrt(3)
*int(x**2/(4*x**16 - 12*x**14 + 41*x**12 - 160*x**10 + 300*x**8 - 262*x...
```

3.9 $\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 185

$$\begin{aligned}
&\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx \\
&= -\sqrt{-1+4\sqrt{2}}\arctan\left(\sqrt{\frac{41}{31}+\frac{40\sqrt{2}}{31}}+\sqrt{\frac{8}{31}+\frac{32\sqrt{2}}{31}}x\right) \\
&\quad -\sqrt{-1+4\sqrt{2}}\arctan\left(\sqrt{\frac{9}{31}+\frac{36\sqrt{2}}{31}}-\sqrt{\frac{4}{31}+\frac{16\sqrt{2}}{31}}x-\sqrt{\frac{76}{31}+\frac{56\sqrt{2}}{31}}x^2\right. \\
&\quad \left.-\sqrt{\frac{8}{31}+\frac{32\sqrt{2}}{31}}x^3\right)+\frac{1}{2}\log\left(\sqrt{2}-2\sqrt{2}x+\sqrt{2}x^3+2x^4\right)
\end{aligned}$$

output
```
-(-1+4*2^(1/2))^(1/2)*arctan(1/31*(1271+1240*2^(1/2))^(1/2)+2/31*(62+248*2
^(1/2))^(1/2)*x)+(-1+4*2^(1/2))^(1/2)*arctan(-3/31*(31+124*2^(1/2))^(1/2)+
2/31*(31+124*2^(1/2))^(1/2)*x+2/31*(589+434*2^(1/2))^(1/2)*x^2+2/31*(62+24
8*2^(1/2))^(1/2)*x^3)+1/2*ln(2^(1/2)-2*x*2^(1/2)+x^3*2^(1/2)+2*x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.71

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx = \text{RootSum}\Big[1-2\#1+\#1^3 +\sqrt{2}\#1^4\&, \frac{-\log(x-\#1)-\log(x-\#1)\#1+4\sqrt{2}\log(x-\#1)\#1+2\log(x-\#1)\#1^2-2\sqrt{2}\log(x}{-2+3\#1^2+4\sqrt{2}\#1^3}$$

input
```
Integrate[(-1 - x + 4*Sqrt[2]*x + 2*x^2 - 2*Sqrt[2]*x^2 + 2*Sqrt[2]*x^3)/(
1 - 2*x + x^3 + Sqrt[2]*x^4),x]
```

output
```
RootSum[1 - 2*#1 + #1^3 + Sqrt[2]*#1^4 & , (-Log[x - #1] - Log[x - #1]*#1
+ 4*Sqrt[2]*Log[x - #1]*#1 + 2*Log[x - #1]*#1^2 - 2*Sqrt[2]*Log[x - #1]*#1
^2 + 2*Sqrt[2]*Log[x - #1]*#1^3)/(-2 + 3*#1^2 + 4*Sqrt[2]*#1^3) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{2}x^3-2\sqrt{2}x^2+2x^2+4\sqrt{2}x-x-1}{\sqrt{2}x^4+x^3-2x+1}\,dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{2\sqrt{2}x^3-2\sqrt{2}x^2+2x^2+\left(4\sqrt{2}-1\right)x-1}{\sqrt{2}x^4+x^3-2x+1}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{2\sqrt{2}x^3+\left(2-2\sqrt{2}\right)x^2+\left(4\sqrt{2}-1\right)x-1}{\sqrt{2}x^4+x^3-2x+1}dx$$

$$\Big\downarrow \; 2525$$

$$\frac{\int \frac{2\left(2\left(8-\sqrt{2}\right)x-\left(8-\sqrt{2}\right)x^2\right)}{\sqrt{2}x^4+x^3-2x+1}dx}{4\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+x^3-2x+1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2\left(8-\sqrt{2}\right)x-\left(8-\sqrt{2}\right)x^2}{\sqrt{2}x^4+x^3-2x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+x^3-2x+1\right)$$

$$\Big\downarrow \text{2027}$$

$$\frac{\int \frac{x\left(\left(-8+\sqrt{2}\right)x+2\left(8-\sqrt{2}\right)\right)}{\sqrt{2}x^4+x^3-2x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+x^3-2x+1\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\frac{\left(-8+\sqrt{2}\right)x^2}{\sqrt{2}x^4+x^3-2x+1}-\frac{2\left(-8+\sqrt{2}\right)x}{\sqrt{2}x^4+x^3-2x+1}\right)dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+x^3-2x+1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(8-\sqrt{2}\right)\int \frac{x}{\sqrt{2}x^4+x^3-2x+1}dx-\left(8-\sqrt{2}\right)\int \frac{x^2}{\sqrt{2}x^4+x^3-2x+1}dx}{2\sqrt{2}}+\frac{1}{2}\log\left(\sqrt{2}x^4+x^3-2x+1\right)$$

input

```
Int[(-1 - x + 4*Sqrt[2]*x + 2*x^2 - 2*Sqrt[2]*x^2 + 2*Sqrt[2]*x^3)/(1 - 2*
x + x^3 + Sqrt[2]*x^4),x]
```

output

```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\sqrt{2}\left(\sum_{\_R=\operatorname{RootOf}\left(\sqrt{2}\_Z^3+2\_Z^4-2\sqrt{2}\_Z+\sqrt{2}\right)}\frac{\left(-2+\sqrt{2}\left(4\_R^3+2\_R^2\left(\sqrt{2}-2\right)+\_R\left(8-\sqrt{2}\right)\right)\right)\ln\left(x-\_R\right)}{8\_R^3+\sqrt{2}\left(3\_R^2-2\right)}\right)}{2}$	89

input
```
int((-1-x+4*2^(1/2)*x+2*x^2-2*2^(1/2)*x^2+2*2^(1/2)*x^3)/(1-2*x+x^3+2^(1/2
)*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*sum((-2+2^(1/2)*(4*_R^3+2*_R^2*(2^(1/2)-2)+_R*(8-2^(1/2))))/(8
*_R^3+2^(1/2)*(3*_R^2-2))*ln(x-_R),_R=RootOf(2^(1/2)*_Z^3+2*_Z^4-2*2^(1/2)
*_Z+2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx$$
$$= \sqrt{4\sqrt{2}-1}\arctan\left(\frac{1}{31}\left(16\,x^3+18\,x^2+2\,\sqrt{2}(x^3+5\,x^2+4\,x-6)+2\,x-3\right)\sqrt{4\,\sqrt{2}-1}\right)$$
$$-\sqrt{4\,\sqrt{2}-1}\arctan\left(\frac{1}{31}\left(2\,\sqrt{2}(x+3)+16\,x+17\right)\sqrt{4\,\sqrt{2}-1}\right)$$
$$+\frac{1}{2}\log\left(2\,x^4+\sqrt{2}(x^3-2\,x+1)\right)$$

input
```
integrate((-1-x+4*2^(1/2)*x+2*x^2-2*2^(1/2)*x^2+2*2^(1/2)*x^3)/(1-2*x+x^3+
2^(1/2)*x^4),x, algorithm="fricas")
```

output
```
sqrt(4*sqrt(2) - 1)*arctan(1/31*(16*x^3 + 18*x^2 + 2*sqrt(2)*(x^3 + 5*x^2
+ 4*x - 6) + 2*x - 3)*sqrt(4*sqrt(2) - 1)) - sqrt(4*sqrt(2) - 1)*arctan(1/
31*(2*sqrt(2)*(x + 3) + 16*x + 17)*sqrt(4*sqrt(2) - 1)) + 1/2*log(2*x^4 +
sqrt(2)*(x^3 - 2*x + 1))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((-1-x+4*2**(1/2)*x+2*x**2-2*2**(1/2)*x**2+2*2**(1/2)*x**3)/(1-2*
x+x**3+2**(1/2)*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(9*_t**4 - 204*_t**3 + 12*sqrt(2)*_
t**2 + 1204*_t**2 - 544*_t - 136*sqrt(2)*_t + 32*sqrt(2) + 72) contains an
 element of the set of generators.
```

Maxima [F]

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx = \int \frac{2\sqrt{2}x^3-2\sqrt{2}x^2+2\,x^2+4\sqrt{2}x-x-1}{\sqrt{2}x^4+x^3-2\,x+1}\,dx$$

input
```
integrate((-1-x+4*2^(1/2)*x+2*x^2-2*2^(1/2)*x^2+2*2^(1/2)*x^3)/(1-2*x+x^3+
2^(1/2)*x^4),x, algorithm="maxima")
```

output
```
integrate((2*sqrt(2)*x^3 - 2*sqrt(2)*x^2 + 2*x^2 + 4*sqrt(2)*x - x - 1)/(s
qrt(2)*x^4 + x^3 - 2*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-1-x+4*2^(1/2)*x+2*x^2-2*2^(1/2)*x^2+2*2^(1/2)*x^3)/(1-2*x+x^3+
2^(1/2)*x^4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,0]:[1,0,-2]%%},[4]%%%}+%%%{1,[3]%%%}+%%%{-2,[1]%%%}+
%%%{1,[0]
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.21

$$\int \frac{-1-x+4\sqrt{2}x+2x^2-2\sqrt{2}x^2+2\sqrt{2}x^3}{1-2x+x^3+\sqrt{2}x^4}\,dx = \ln\left(\frac{63\,x}{4}-\sqrt{2}\,x-\frac{x\,\sqrt{1-4\,\sqrt{2}}}{4}+\sqrt{2}+\frac{\sqrt{2}\,x^2}{2}+\frac{31\,x^2\,\sqrt{1-4\,\sqrt{2}}}{4}+\frac{15\,x^2}{4}-2^{1/4}\,\sqrt{\sqrt{2}-8}+\frac{2^{3/4}\,\sqrt{\sqrt{2}-8}}{8}+\sqrt{2}\,x\,\sqrt{1-4\,\sqrt{2}}-\frac{63}{4}\right)\left(\frac{\sqrt{2}\,\sqrt{\sqrt{2}\,\left(\sqrt{2}-8\right)}}{4}+\frac{1}{2}\right) -\ln\left(\frac{63\,x}{4}-\sqrt{2}\,x+\frac{x\,\sqrt{1-4\,\sqrt{2}}}{4}+\sqrt{2}+\frac{\sqrt{2}\,x^2}{2}-\frac{31\,x^2\,\sqrt{1-4\,\sqrt{2}}}{4}+\frac{15\,x^2}{4}+2^{1/4}\,\sqrt{\sqrt{2}-8}-\frac{2^{3/4}\,\sqrt{\sqrt{2}-}}{8}\right.$$

input
```
int(-(x - 4*2^(1/2)*x + 2*2^(1/2)*x^2 - 2*2^(1/2)*x^3 - 2*x^2 + 1)/(2^(1/2
)*x^4 - 2*x + x^3 + 1),x)
```

output

```
log((63*x)/4 - 2^(1/2)*x - (x*(1 - 4*2^(1/2))^(1/2))/4 + 2^(1/2) + (2^(1/2
)*x^2)/2 + (31*x^2*(1 - 4*2^(1/2))^(1/2))/4 + (15*x^2)/4 - 2^(1/4)*(2^(1/2
) - 8)^(1/2) + (2^(3/4)*(2^(1/2) - 8)^(1/2))/8 + 2^(1/2)*x*(1 - 4*2^(1/2))
^(1/2) - 63/4)*((2^(1/2)*(2^(1/2)*(2^(1/2) - 8))^(1/2))/4 + 1/2) - log((63
*x)/4 - 2^(1/2)*x + (x*(1 - 4*2^(1/2))^(1/2))/4 + 2^(1/2) + (2^(1/2)*x^2)/
2 - (31*x^2*(1 - 4*2^(1/2))^(1/2))/4 + (15*x^2)/4 + 2^(1/4)*(2^(1/2) - 8)^
(1/2) - (2^(3/4)*(2^(1/2) - 8)^(1/2))/8 - 2^(1/2)*x*(1 - 4*2^(1/2))^(1/2)
- 63/4)*((2^(1/2)*(2^(1/2)*(2^(1/2) - 8))^(1/2))/4 - 1/2)
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{-1-x+4\sqrt{2}\,x+2x^2-2\sqrt{2}\,x^2+2\sqrt{2}\,x^3}{1-2x+x^3+\sqrt{2}\,x^4}\,dx \\
&= \sqrt{2}\left(\int \frac{x^5}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -\sqrt{2}\left(\int \frac{x^4}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -6\sqrt{2}\left(\int \frac{x^3}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad +10\sqrt{2}\left(\int \frac{x^2}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -4\sqrt{2}\left(\int \frac{x}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad +4\left(\int \frac{x^7}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -4\left(\int \frac{x^6}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad +6\left(\int \frac{x^5}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad +\int \frac{x^4}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx \\
&\quad +5\left(\int \frac{x^3}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -4\left(\int \frac{x^2}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad -\left(\int \frac{x}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx\right) \\
&\quad +\int \frac{1}{2x^8-x^6+4x^4-2x^3-4x^2+4x-1}dx
\end{aligned}
$$

input
```
int((-1-x+4*2^(1/2)*x+2*x^2-2*2^(1/2)*x^2+2*2^(1/2)*x^3)/(1-2*x+x^3+2^(1/2
)*x^4),x)
```

output
```
sqrt(2)*int(x**5/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) -
 sqrt(2)*int(x**4/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x)
- 6*sqrt(2)*int(x**3/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),
x) + 10*sqrt(2)*int(x**2/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x -
 1),x) - 4*sqrt(2)*int(x/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x -
 1),x) + 4*int(x**7/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x
) - 4*int(x**6/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) + 6
*int(x**5/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) + int(x*
*4/(2*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) + 5*int(x**3/(2
*x**8 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) - 4*int(x**2/(2*x**8
 - x**6 + 4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) - int(x/(2*x**8 - x**6 +
4*x**4 - 2*x**3 - 4*x**2 + 4*x - 1),x) + int(1/(2*x**8 - x**6 + 4*x**4 - 2
*x**3 - 4*x**2 + 4*x - 1),x)
```

3.10 $\int \frac{x(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$

Optimal result

Integrand size = 94, antiderivative size = 595

$$\int \frac{x\left(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5\right)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$$

$$= -\sqrt{-\frac{1}{4}+\frac{\sqrt{5}}{2}}\arctan\left(\frac{1}{3\sqrt{\frac{19}{209+76\sqrt{5}-12\sqrt{5\left(61+46\sqrt{5}\right)}}}}-\sqrt{\frac{20}{57}+\frac{40\sqrt{5}}{57}}x\right)$$
$$+\sqrt{-\frac{1}{4}+\frac{\sqrt{5}}{2}}\arctan\left(\frac{1}{3\sqrt{\frac{19}{209+76\sqrt{5}+12\sqrt{5\left(61+46\sqrt{5}\right)}}}}+\sqrt{\frac{20}{57}+\frac{40\sqrt{5}}{57}}x\right)$$
$$+\sqrt{-\frac{1}{4}+\frac{\sqrt{5}}{2}}\arctan\left(\sqrt{\frac{1}{19}\left(57+76\sqrt{5}-12\sqrt{61+46\sqrt{5}}\right)}-\sqrt{\frac{108}{19}+\frac{216\sqrt{5}}{19}}x\right.$$
$$\left.+2\sqrt{\frac{2}{19}\left(97+23\sqrt{5}-3\sqrt{5\left(61+46\sqrt{5}\right)}\right)}x^2-\sqrt{\frac{60}{19}+\frac{120\sqrt{5}}{19}}x^3\right)$$
$$+\sqrt{-\frac{1}{4}+\frac{\sqrt{5}}{2}}\arctan\left(\sqrt{\frac{1}{19}\left(57+76\sqrt{5}+12\sqrt{61+46\sqrt{5}}\right)}-\sqrt{\frac{108}{19}+\frac{216\sqrt{5}}{19}}x\right.$$
$$\left.-2\sqrt{\frac{2}{19}\left(97+23\sqrt{5}+3\sqrt{5\left(61+46\sqrt{5}\right)}\right)}x^2-\sqrt{\frac{60}{19}+\frac{120\sqrt{5}}{19}}x^3\right)$$
$$+\sqrt{\frac{1}{16}+\frac{\sqrt{5}}{8}}\log\left(\sqrt{5}-2\sqrt{15}x+\left(3\sqrt{5}+\sqrt{5\left(1+2\sqrt{5}\right)}\right)x^2-\sqrt{15\left(1+2\sqrt{5}\right)}x^3\right.$$
$$\left.+5x^4\right)-\sqrt{\frac{1}{16}+\frac{\sqrt{5}}{8}}\log\left(\sqrt{5}-2\sqrt{15}x+\left(3\sqrt{5}-\sqrt{5\left(1+2\sqrt{5}\right)}\right)x^2\right.$$
$$\left.+\sqrt{15\left(1+2\sqrt{5}\right)}x^3+5x^4\right)$$

output
```
1/2*(-1+2*5^(1/2))^(1/2)*arctan(-1/57*19^(1/2)*(209+76*5^(1/2)-12*(305+230
*5^(1/2))^(1/2))^(1/2)+2/57*(285+570*5^(1/2))^(1/2)*x)+1/2*(-1+2*5^(1/2))^
(1/2)*arctan(1/57*19^(1/2)*(209+76*5^(1/2)+12*(305+230*5^(1/2))^(1/2))^(1/
2)+2/57*(285+570*5^(1/2))^(1/2)*x)-1/2*(-1+2*5^(1/2))^(1/2)*arctan(-1/19*(
1083+1444*5^(1/2)-228*(61+46*5^(1/2))^(1/2))^(1/2)+6/19*(57+114*5^(1/2))^(
1/2)*x-2/19*(3686+874*5^(1/2)-114*(305+230*5^(1/2))^(1/2))^(1/2)*x^2+2/19*
(285+570*5^(1/2))^(1/2)*x^3)-1/2*(-1+2*5^(1/2))^(1/2)*arctan(-1/19*(1083+1
444*5^(1/2)+228*(61+46*5^(1/2))^(1/2))^(1/2)+6/19*(57+114*5^(1/2))^(1/2)*x
+2/19*(3686+874*5^(1/2)+114*(305+230*5^(1/2))^(1/2))^(1/2)*x^2+2/19*(285+5
70*5^(1/2))^(1/2)*x^3)+1/4*(1+2*5^(1/2))^(1/2)*ln(5^(1/2)-2*15^(1/2)*x+(3*
5^(1/2)+(5+10*5^(1/2))^(1/2))*x^2-(15+30*5^(1/2))^(1/2)*x^3+5*x^4)-1/4*(1+
2*5^(1/2))^(1/2)*ln(5^(1/2)-2*15^(1/2)*x+(3*5^(1/2)-(5+10*5^(1/2))^(1/2))*
x^2+(15+30*5^(1/2))^(1/2)*x^3+5*x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.35

$$\int \frac{x(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$$
$$=\frac{1}{2}\text{RootSum}\Big[1-4\sqrt{3}\#1+18\#1^2-12\sqrt{3}\#1^3+8\#1^4+2\sqrt{3}\#1^5-3\#1^6$$
$$+5\#1^8\&,\frac{2\log(x-\#1)\#1-5\sqrt{3}\log(x-\#1)\#1^2+12\log(x-\#1)\#1^3-3\sqrt{3}\log(x-\#1)\#1^4-20}{-2\sqrt{3}+18\#1-18\sqrt{3}\#1^2+16\#1^3+5\sqrt{3}\#1^4-9\#1^5+}$$

input
```
Integrate[(x*(2 - 5*Sqrt[3]*x + 12*x^2 - 3*Sqrt[3]*x^3 - 20*x^4 + 10*Sqrt[
3]*x^5))/(1 - 4*Sqrt[3]*x + 18*x^2 - 12*Sqrt[3]*x^3 + 8*x^4 + 2*Sqrt[3]*x^
5 - 3*x^6 + 5*x^8),x]
```

output
```
RootSum[1 - 4*Sqrt[3]*#1 + 18*#1^2 - 12*Sqrt[3]*#1^3 + 8*#1^4 + 2*Sqrt[3]*
#1^5 - 3*#1^6 + 5*#1^8 & , (2*Log[x - #1]*#1 - 5*Sqrt[3]*Log[x - #1]*#1^2
+ 12*Log[x - #1]*#1^3 - 3*Sqrt[3]*Log[x - #1]*#1^4 - 20*Log[x - #1]*#1^5 +
 10*Sqrt[3]*Log[x - #1]*#1^6)/(-2*Sqrt[3] + 18*#1 - 18*Sqrt[3]*#1^2 + 16*#
1^3 + 5*Sqrt[3]*#1^4 - 9*#1^5 + 20*#1^7) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(10\sqrt{3}x^5-20x^4-3\sqrt{3}x^3+12x^2-5\sqrt{3}x+2)}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{10\sqrt{3}x^6}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}-\frac{20x^5}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-\right.$$

$$\Big\downarrow \text{2009}$$

$$2\int \frac{x}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx-$$
$$5\sqrt{3}\int \frac{x^2}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx+$$
$$12\int \frac{x^3}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx-$$
$$3\sqrt{3}\int \frac{x^4}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx-$$
$$20\int \frac{x^5}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx+$$
$$10\sqrt{3}\int \frac{x^6}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}dx$$

input
```
Int[(x*(2 - 5*Sqrt[3]*x + 12*x^2 - 3*Sqrt[3]*x^3 - 20*x^4 + 10*Sqrt[3]*x^5
))/(1 - 4*Sqrt[3]*x + 18*x^2 - 12*Sqrt[3]*x^3 + 8*x^4 + 2*Sqrt[3]*x^5 - 3*
x^6 + 5*x^8),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.08

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+5\right)}\_R\ln\left(5x^2+\left(\sqrt{3}\,\_R^3-\sqrt{3}\,\_R\right)x-\_R^3+\_R\right)\right)}{2}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^8+2\sqrt{3}\,\_Z^5-3\_Z^6-12\sqrt{3}\,\_Z^3+8\_Z^4-4\sqrt{3}\,\_Z+18\_Z^2+1\right)}\frac{\left(-20\_R^5+12\_R^3+2\_R+\sqrt{3}\left(10\_R^6-3\_R^4-5\right.\right.}{20\_R^7-9\_R^5+16\_R^3+18\_R+\sqrt{3}\left(5\_R\right.}\right.}{2}$

input
```
int(x*(2-5*3^(1/2)*x+12*x^2-3*3^(1/2)*x^3-20*x^4+10*3^(1/2)*x^5)/(1-4*3^(1
/2)*x+18*x^2-12*3^(1/2)*x^3+8*x^4+2*3^(1/2)*x^5-3*x^6+5*x^8),x,method=_RET
URNVERBOSE)
```

output
```
1/2*sum(_R*ln(5*x^2+(3^(1/2)*_R^3-3^(1/2)*_R)*x-_R^3+_R),_R=RootOf(_Z^4-_Z
^2+5))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.77

$$\int \frac{x(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$$
$$= -\frac{1}{2}\sqrt{2\sqrt{5}-1}\arctan\left(\frac{1}{19}\left(6x^2+2\sqrt{5}(6x^2-1)-1\right)\sqrt{2\sqrt{5}+1}\sqrt{2\sqrt{5}-1} + \frac{2}{19}\left(10x^2+\sqrt{3}(10x^3+3x)+\sqrt{5}\left(x^2+\sqrt{3}(x^3+6x)-6\right)-3\right)\sqrt{2\sqrt{5}-1}\right)$$
$$+ \frac{1}{2}\sqrt{2\sqrt{5}-1}\arctan\left(\frac{1}{19}\left(6x^2+2\sqrt{5}(6x^2-1)-1\right)\sqrt{2\sqrt{5}+1}\sqrt{2\sqrt{5}-1} - \frac{2}{19}\left(10x^2+\sqrt{3}(10x^3+3x)+\sqrt{5}\left(x^2+\sqrt{3}(x^3+6x)-6\right)-3\right)\sqrt{2\sqrt{5}-1}\right)$$
$$+ \frac{1}{2}\sqrt{2\sqrt{5}-1}\arctan\left(\frac{1}{19}\left(2\sqrt{5}+1\right)^{\frac{3}{2}}\sqrt{2\sqrt{5}-1} + \frac{2}{57}\left(\sqrt{5}\left(\sqrt{3}x+1\right)+10\sqrt{3}x+10\right)\sqrt{2\sqrt{5}-1}\right)$$
$$- \frac{1}{2}\sqrt{2\sqrt{5}-1}\arctan\left(\frac{1}{19}\left(2\sqrt{5}+1\right)^{\frac{3}{2}}\sqrt{2\sqrt{5}-1} - \frac{2}{57}\left(\sqrt{5}\left(\sqrt{3}x+1\right)+10\sqrt{3}x+10\right)\sqrt{2\sqrt{5}-1}\right)$$
$$- \frac{1}{4}\sqrt{2\sqrt{5}+1}\log\left(5x^4+\sqrt{5}\left(\sqrt{3}x^3-x^2\right)\sqrt{2\sqrt{5}+1}+\sqrt{5}\left(3x^2-2\sqrt{3}x+1\right)\right)$$
$$+ \frac{1}{4}\sqrt{2\sqrt{5}+1}\log\left(5x^4-\sqrt{5}\left(\sqrt{3}x^3-x^2\right)\sqrt{2\sqrt{5}+1}+\sqrt{5}\left(3x^2-2\sqrt{3}x+1\right)\right)$$

input
```
integrate(x*(2-5*3^(1/2)*x+12*x^2-3*3^(1/2)*x^3-20*x^4+10*3^(1/2)*x^5)/(1-
4*3^(1/2)*x+18*x^2-12*3^(1/2)*x^3+8*x^4+2*3^(1/2)*x^5-3*x^6+5*x^8),x, algo
rithm="fricas")
```

output
```
-1/2*sqrt(2*sqrt(5) - 1)*arctan(1/19*(6*x^2 + 2*sqrt(5)*(6*x^2 - 1) - 1)*s
qrt(2*sqrt(5) + 1)*sqrt(2*sqrt(5) - 1) + 2/19*(10*x^2 + sqrt(3)*(10*x^3 +
3*x) + sqrt(5)*(x^2 + sqrt(3)*(x^3 + 6*x) - 6) - 3)*sqrt(2*sqrt(5) - 1)) +
 1/2*sqrt(2*sqrt(5) - 1)*arctan(1/19*(6*x^2 + 2*sqrt(5)*(6*x^2 - 1) - 1)*s
qrt(2*sqrt(5) + 1)*sqrt(2*sqrt(5) - 1) - 2/19*(10*x^2 + sqrt(3)*(10*x^3 +
3*x) + sqrt(5)*(x^2 + sqrt(3)*(x^3 + 6*x) - 6) - 3)*sqrt(2*sqrt(5) - 1)) +
 1/2*sqrt(2*sqrt(5) - 1)*arctan(1/19*(2*sqrt(5) + 1)^(3/2)*sqrt(2*sqrt(5)
- 1) + 2/57*(sqrt(5)*(sqrt(3)*x + 1) + 10*sqrt(3)*x + 10)*sqrt(2*sqrt(5) -
 1)) - 1/2*sqrt(2*sqrt(5) - 1)*arctan(1/19*(2*sqrt(5) + 1)^(3/2)*sqrt(2*sq
rt(5) - 1) - 2/57*(sqrt(5)*(sqrt(3)*x + 1) + 10*sqrt(3)*x + 10)*sqrt(2*sqr
t(5) - 1)) - 1/4*sqrt(2*sqrt(5) + 1)*log(5*x^4 + sqrt(5)*(sqrt(3)*x^3 - x^
2)*sqrt(2*sqrt(5) + 1) + sqrt(5)*(3*x^2 - 2*sqrt(3)*x + 1)) + 1/4*sqrt(2*s
qrt(5) + 1)*log(5*x^4 - sqrt(5)*(sqrt(3)*x^3 - x^2)*sqrt(2*sqrt(5) + 1) +
sqrt(5)*(3*x^2 - 2*sqrt(3)*x + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4038 vs. $2(491) = 982$.

Time = 3.73 (sec) , antiderivative size = 4038, normalized size of antiderivative = 6.79

$$\int \frac{x\left(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5\right)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2-5*3**(1/2)*x+12*x**2-3*3**(1/2)*x**3-20*x**4+10*3**(1/2)*x*
*5)/(1-4*3**(1/2)*x+18*x**2-12*3**(1/2)*x**3+8*x**4+2*3**(1/2)*x**5-3*x**6
+5*x**8),x)
```

output
```
(2*atan(20*sqrt(3)*x/(-3*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) + 3
*sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))) - 24*
sqrt(5)*sqrt(1 + 2*sqrt(5))/(-3*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(
5)) + 3*sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))
) - 9*sqrt(1 + 2*sqrt(5))/(-3*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)
) + 3*sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)))
+ 20/(-3*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) + 3*sqrt(4*sqrt(5)
+ 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))) + 9*sqrt(1 + 2*sqrt(5
))*sqrt(4*sqrt(5) + 21)/(-3*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))
+ 3*sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)))) -
 2*atan(20*sqrt(3)*x**3/(-sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) +
sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))) - x**2
*(-18*sqrt(1 + 2*sqrt(5))*sqrt(4*sqrt(5) + 21)/(-sqrt(-2*sqrt(4*sqrt(5) +
21) + 1 + 6*sqrt(5)) + sqrt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) +
 1 + 6*sqrt(5))) - 20/(-sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) + sq
rt(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))) + 18*sqr
t(1 + 2*sqrt(5))/(-sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) + sqrt(4*
sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5))) + 48*sqrt(5)*
sqrt(1 + 2*sqrt(5))/(-sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)) + sqrt
(4*sqrt(5) + 21)*sqrt(-2*sqrt(4*sqrt(5) + 21) + 1 + 6*sqrt(5)))) - x*(-...
```

Maxima [F]

$$\int \frac{x(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx = \int \frac{(10\sqrt{3}x^5-20x^4-3\sqrt{3}x^3+12x^2-5\sqrt{3}x+2)x}{5x^8-3x^6+2\sqrt{3}x^5+8x^4-12\sqrt{3}x^3+18x^2-4\sqrt{3}x+1}\,dx$$

input
```
integrate(x*(2-5*3^(1/2)*x+12*x^2-3*3^(1/2)*x^3-20*x^4+10*3^(1/2)*x^5)/(1-
4*3^(1/2)*x+18*x^2-12*3^(1/2)*x^3+8*x^4+2*3^(1/2)*x^5-3*x^6+5*x^8),x, algo
rithm="maxima")
```

output
```
integrate((10*sqrt(3)*x^5 - 20*x^4 - 3*sqrt(3)*x^3 + 12*x^2 - 5*sqrt(3)*x
+ 2)*x/(5*x^8 - 3*x^6 + 2*sqrt(3)*x^5 + 8*x^4 - 12*sqrt(3)*x^3 + 18*x^2 -
4*sqrt(3)*x + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x\left(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5\right)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx=\text{Timed out}$$

input
```
integrate(x*(2-5*3^(1/2)*x+12*x^2-3*3^(1/2)*x^3-20*x^4+10*3^(1/2)*x^5)/(1-
4*3^(1/2)*x+18*x^2-12*3^(1/2)*x^3+8*x^4+2*3^(1/2)*x^5-3*x^6+5*x^8),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.45

$$\int \frac{x\left(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5\right)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$$
$$=\sum_{k=1}^{8}\ln\left(-\frac{\left(16\,\mathrm{root}\left(z^8-\frac{z^6}{2}+\frac{11\,z^4}{16}-\frac{5\,z^2}{32}+\frac{25}{256},z,k\right)^4-4\,\mathrm{root}\left(z^8-\frac{z^6}{2}+\frac{11\,z^4}{16}-\frac{5\,z^2}{32}+\frac{25}{256},z,k\right)^2+5\right)}{\cdots}\right.$$
$$-\frac{z^6}{2}+\frac{11\,z^4}{16}-\frac{5\,z^2}{32}+\frac{25}{256},z,k\Big)$$

input
```
int(-(x*(5*3^(1/2)*x + 3*3^(1/2)*x^3 - 10*3^(1/2)*x^5 - 12*x^2 + 20*x^4 -
2))/(2*3^(1/2)*x^5 - 12*3^(1/2)*x^3 - 4*3^(1/2)*x + 18*x^2 + 8*x^4 - 3*x^6
 + 5*x^8 + 1),x)
```

output
```
symsum(log(-(13718*(16*root(z^8 - z^6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/25
6, z, k)^4 - 4*root(z^8 - z^6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k)
^2 + 5)*(8559440*x + 156497*3^(1/2)*root(z^8 - z^6/2 + (11*z^4)/16 - (5*z^
2)/32 + 25/256, z, k) - 2573211*root(z^8 - z^6/2 + (11*z^4)/16 - (5*z^2)/3
2 + 25/256, z, k)*x - 1767270*3^(1/2) + 5079720*3^(1/2)*root(z^8 - z^6/2 +
 (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k)^2 - 7464368*3^(1/2)*root(z^8 - z
^6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k)^3 - 18353520*root(z^8 - z^
6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k)^2*x + 30736704*root(z^8 - z
^6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k)^3*x))/48828125)*root(z^8 -
 z^6/2 + (11*z^4)/16 - (5*z^2)/32 + 25/256, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{x(2-5\sqrt{3}x+12x^2-3\sqrt{3}x^3-20x^4+10\sqrt{3}x^5)}{1-4\sqrt{3}x+18x^2-12\sqrt{3}x^3+8x^4+2\sqrt{3}x^5-3x^6+5x^8}\,dx$$
$$= 50\sqrt{3}\left(\int \frac{x^{14}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-45\sqrt{3}\left(\int \frac{x^{12}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$+104\sqrt{3}\left(\int \frac{x^{10}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-93\sqrt{3}\left(\int \frac{x^{8}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-24\sqrt{3}\left(\int \frac{x^{6}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-21\sqrt{3}\left(\int \frac{x^{4}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$+3\sqrt{3}\left(\int \frac{x^{2}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-100\left(\int \frac{x^{13}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$+60\left(\int \frac{x^{11}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$+192\left(\int \frac{x^{9}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-228\left(\int \frac{x^{7}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-4\left(\int \frac{x^{5}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$-12\left(\int \frac{x^{3}}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$
$$+2\left(\int \frac{x}{25x^{16}-30x^{14}+89x^{12}+120x^{10}+110x^8-102x^6+52x^4-12x^2+1}dx\right)$$

input
```
int(x*(2-5*3^(1/2)*x+12*x^2-3*3^(1/2)*x^3-20*x^4+10*3^(1/2)*x^5)/(1-4*3^(1
/2)*x+18*x^2-12*3^(1/2)*x^3+8*x^4+2*3^(1/2)*x^5-3*x^6+5*x^8),x)
```

output
```
50*sqrt(3)*int(x**14/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**
8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) - 45*sqrt(3)*int(x**12/(25*x**16
- 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**
2 + 1),x) + 104*sqrt(3)*int(x**10/(25*x**16 - 30*x**14 + 89*x**12 + 120*x*
*10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) - 93*sqrt(3)*int(x**
8/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x
**4 - 12*x**2 + 1),x) - 24*sqrt(3)*int(x**6/(25*x**16 - 30*x**14 + 89*x**1
2 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) - 21*sqrt(
3)*int(x**4/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 102*x
**6 + 52*x**4 - 12*x**2 + 1),x) + 3*sqrt(3)*int(x**2/(25*x**16 - 30*x**14
+ 89*x**12 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) -
 100*int(x**13/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 10
2*x**6 + 52*x**4 - 12*x**2 + 1),x) + 60*int(x**11/(25*x**16 - 30*x**14 + 8
9*x**12 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) + 19
2*int(x**9/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 102*x*
*6 + 52*x**4 - 12*x**2 + 1),x) - 228*int(x**7/(25*x**16 - 30*x**14 + 89*x*
*12 + 120*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) - 4*int(
x**5/(25*x**16 - 30*x**14 + 89*x**12 + 120*x**10 + 110*x**8 - 102*x**6 + 5
2*x**4 - 12*x**2 + 1),x) - 12*int(x**3/(25*x**16 - 30*x**14 + 89*x**12 + 1
20*x**10 + 110*x**8 - 102*x**6 + 52*x**4 - 12*x**2 + 1),x) + 2*int(x/(2...
```

3.11 $\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 233

$$
\begin{aligned}
&\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx \\
&= \frac{-182-570x-285x^2+285x^3}{2394(1+x)} \\
&\quad + \sqrt{\frac{147447819270109}{193432722516432}+\frac{27432480848221}{13816623036888\sqrt{7}}}\operatorname{arctanh}\left(3\sqrt{\frac{1}{53}\left(9-2\sqrt{7}\right)} \right. \\
&\qquad \left. + \sqrt{\frac{7}{53}\left(9-2\sqrt{7}\right)}x\right) \\
&\quad - \frac{\sqrt{\frac{1}{53}\left(147447819270109-54864961696442\sqrt{7}\right)}\operatorname{arctanh}\left(\sqrt{\frac{81}{53}+\frac{18\sqrt{7}}{53}}-\sqrt{\frac{63}{53}+\frac{14\sqrt{7}}{53}}x\right)}{1910412} \\
&\quad + \frac{1226\log(1+x)}{9747}+\frac{13309\log\left(2\sqrt{7}-6\sqrt{7}x+7x^2\right)}{272916\sqrt{7}} \\
&\quad - \frac{13309\log\left(-2\sqrt{7}+6\sqrt{7}x+7x^2\right)}{272916\sqrt{7}}+\frac{456517\log\left(-4+24x-36x^2+7x^4\right)}{1910412}
\end{aligned}
$$

output
```
(285*x^3-285*x^2-570*x-182)/(2394+2394*x)+1/101251836*(7814734421315777+29
07842969911426*7^(1/2))^(1/2)*arctanh(3/53*(477-106*7^(1/2))^(1/2)+1/53*(3
339-742*7^(1/2))^(1/2)*x)+1/101251836*(7814734421315777-2907842969911426*7
^(1/2))^(1/2)*arctanh(-3/53*(477+106*7^(1/2))^(1/2)+1/53*(3339+742*7^(1/2)
)^(1/2)*x)+1226/9747*ln(1+x)+13309/1910412*ln(2*7^(1/2)-6*7^(1/2)*x+7*x^2)
*7^(1/2)-13309/1910412*ln(-2*7^(1/2)+6*7^(1/2)*x+7*x^2)*7^(1/2)+456517/191
0412*ln(7*x^4-36*x^2+24*x-4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.55

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx$$
$$=\frac{114\left(-855-570x+285x^2-\frac{182}{1+x}\right)+34328\log(1+x)+\text{RootSum}\left[-57+68\#1+6\#1^2-28\#1^3+7\#\right.}{272916}$$

input
```
Integrate[(-9 + 7*x + x^2 - 6*x^4 - 8*x^5 + 5*x^6 + 5*x^7)/(-12 + 48*x + 2
4*x^2 - 144*x^3 - 87*x^4 + 42*x^5 + 21*x^6),x]
```

output
```
(114*(-855 - 570*x + 285*x^2 - 182/(1 + x)) + 34328*Log[1 + x] + RootSum[-
57 + 68*#1 + 6*#1^2 - 28*#1^3 + 7*#1^4 & , (-2125483*Log[1 + x - #1] + 366
1146*Log[1 + x - #1]*#1 - 2206771*Log[1 + x - #1]*#1^2 + 456517*Log[1 + x
- #1]*#1^3)/(17 + 3*#1 - 21*#1^2 + 7*#1^3) & ])/272916
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^7+5x^6-8x^5-6x^4+x^2+7x-9}{21x^6+42x^5-87x^4-144x^3+24x^2+48x-12}\,dx$$
$$\Big\downarrow \; 2462$$

$$\int \left(\frac{456517x^3 - 837220x^2 + 617155x - 214591}{68229\,(7x^4 - 36x^2 + 24x - 4)} + \frac{5x}{21} + \frac{1226}{9747(x+1)} + \frac{13}{171(x+1)^2} - \frac{5}{21}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{12537391 \int \frac{x}{7x^4-36x^2+24x-4}dx}{477603} - \frac{837220 \int \frac{x^2}{7x^4-36x^2+24x-4}dx}{68229} - \frac{4241239\sqrt{\frac{1}{742}\,(270+7\sqrt{7})}\,\text{arctanh}\left(\frac{2-3x}{\sqrt{9-2\sqrt{7}}x}\right)}{955206} + \frac{4241239\sqrt{\frac{1}{742}\,(270-7\sqrt{7})}\,\text{arctanh}\left(\frac{2-3x}{\sqrt{9+2\sqrt{7}}x}\right)}{955206} + \frac{5x^2}{42} - \frac{4241239\log\left(-\frac{-\sqrt{7}x^2-6x+2}{x^2}\right)}{1273608\sqrt{7}} + \frac{4241239\log\left(\frac{\sqrt{7}x^2-6x+2}{x^2}\right)}{1273608\sqrt{7}} + \frac{456517\log\left(7x^4-36x^2+24x-4\right)}{1910412} - \frac{5x}{21} - \frac{13}{171(x+1)} + \frac{1226\log(x+1)}{9747}$$

input
```
Int[(-9 + 7*x + x^2 - 6*x^4 - 8*x^5 + 5*x^6 + 5*x^7)/(-12 + 48*x + 24*x^2
- 144*x^3 - 87*x^4 + 42*x^5 + 21*x^6),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.30

method	result
risch	$\frac{5x^2}{42} - \frac{5x}{21} - \frac{13}{171(1+x)} + \frac{\left(\sum_{\_R=\text{RootOf}\left(444087\_Z^4-1354942456\_Z^3-177348969358\_Z^2+2046980307370824\_Z-961594993216030509\right.}\right.}{}$
default	$\frac{5x^2}{42} - \frac{5x}{21} + \frac{\left(\sum_{\_R=\text{RootOf}\left(7\_Z^4-36\_Z^2+24\_Z-4\right)} \frac{\left(456517\_R^3-837220\_R^2+617155\_R-214591\right)\ln\left(x-\_R\right)}{7\_R^3-18\_R+6}\right)}{272916} - \frac{13}{171(1-}$

input
```
int((5*x^7+5*x^6-8*x^5-6*x^4+x^2+7*x-9)/(21*x^6+42*x^5-87*x^4-144*x^3+24*x
^2+48*x-12),x,method=_RETURNVERBOSE)
```

output
```
5/42*x^2-5/21*x-13/171/(1+x)+1/3192*sum(_R*ln(-695955842235177857946 9*_R^3
+20437113244618150785324361*_R^2+14202653843620853496116406745*_R+79859139
537525817600504002845 94*x-307589347577220926972997014705 61),_R=RootOf(4440
87*_Z^4-1354942456*_Z^3-177348969358*_Z^2+2046980307370824*_Z-961594993216
030509))+1226/9747*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.09

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx = \frac{454860\,x^3-454860\,x^2-\left(26618\sqrt{7}(x+1)+(x+1)\sqrt{\frac{54864961696442}{53}\sqrt{7}+\frac{147447819270109}{53}}-913034\,x-\right.}{}$$

input
```
integrate((5*x^7+5*x^6-8*x^5-6*x^4+x^2+7*x-9)/(21*x^6+42*x^5-87*x^4-144*x^
3+24*x^2+48*x-12),x, algorithm="fricas")
```

output
```
1/3820824*(454860*x^3 - 454860*x^2 - (26618*sqrt(7)*(x + 1) + (x + 1)*sqrt
(54864961696442/53*sqrt(7) + 147447819270109/53) - 913034*x - 913034)*log(
sqrt(54864961696442/53*sqrt(7) + 147447819270109/53)*(13611035*sqrt(7) - 5
1143176) + 24883066697517*x + 10664171441793*sqrt(7)) - (26618*sqrt(7)*(x
+ 1) - (x + 1)*sqrt(54864961696442/53*sqrt(7) + 147447819270109/53) - 9130
34*x - 913034)*log(-sqrt(54864961696442/53*sqrt(7) + 147447819270109/53)*(
13611035*sqrt(7) - 51143176) + 24883066697517*x + 10664171441793*sqrt(7))
+ (26618*sqrt(7)*(x + 1) + (x + 1)*sqrt(-54864961696442/53*sqrt(7) + 14744
7819270109/53) + 913034*x + 913034)*log((13611035*sqrt(7) + 51143176)*sqrt
(-54864961696442/53*sqrt(7) + 147447819270109/53) + 24883066697517*x - 106
64171441793*sqrt(7)) + (26618*sqrt(7)*(x + 1) - (x + 1)*sqrt(-548649616964
42/53*sqrt(7) + 147447819270109/53) + 913034*x + 913034)*log(-(13611035*sq
rt(7) + 51143176)*sqrt(-54864961696442/53*sqrt(7) + 147447819270109/53) +
24883066697517*x - 10664171441793*sqrt(7)) + 480592*(x + 1)*log(x + 1) - 9
09720*x - 290472)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx = \frac{5x^2}{42}-\frac{5x}{21}+\frac{1226\log(x+1)}{9747} + \text{RootSum}\Big(106688654585856t^4-101978389008384t^3-4181702014336t^2+15120814828224t-2225 - \frac{13}{171x+171}$$

input
```
integrate((5*x**7+5*x**6-8*x**5-6*x**4+x**2+7*x-9)/(21*x**6+42*x**5-87*x**
4-144*x**3+24*x**2+48*x-12),x)
```

output
```
5*x**2/42 - 5*x/21 + 1226*log(x + 1)/9747 + RootSum(106688654585856*_t**4
- 101978389008384*_t**3 - 4181702014336*_t**2 + 15120814828224*_t - 222531
1647577, Lambda(_t, _t*log(-45913112888165767304845537165609636162866522l5
41419008*_t**4/643874660611987289835864594285862394112626071202699 + 42061
13204517900909457069756788688708158701945996736OO*_t**3/643874660611987289
835864594285862394112626071202699 + 347847091576168880046838594601145291l0
782934404083232*_t**2/643874660611987289835864594285862394112626071202699
- 61416748387941039117552316594189231141973353111718092*_t/643874660611987
289835864594285862394112626071202699 + x + 709657678105704374767900504419б
835327775046553423155/643874660611987289835864594285862394112626071202699)
)) - 13/(171*x + 171)
```

Maxima [F]

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx = \int \frac{5\,x^7+5\,x^6-8\,x^5-6\,x^4+x^2+7\,x-9}{3\,(7\,x^6+14\,x^5-29\,x^4-48\,x^3+8\,x^2+16\,x-4)}\,dx$$

input
```
integrate((5*x^7+5*x^6-8*x^5-6*x^4+x^2+7*x-9)/(21*x^6+42*x^5-87*x^4-144*x^
3+24*x^2+48*x-12),x, algorithm="maxima")
```

output
```
5/42*x^2 - 5/21*x - 13/171/(x + 1) + 1/68229*integrate((456517*x^3 - 83722
0*x^2 + 617155*x - 214591)/(7*x^4 - 36*x^2 + 24*x - 4), x) + 1226/9747*log
(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.28

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx$$
$$= \frac{5}{42}\,x^2 - \frac{5}{21}\,x - \frac{13}{171\,(x+1)} + 0.596528807208595\,\log\left(x+2.56275423511000\right) - \frac{13759974409}{21724250058}\,\log\left(x-0.294967397054000\right) + 0.0728202222081972\,\log\left(x-0.406029874333000\right) - 0.0359566512100309\,\log\left(x-1.86175696372000\right) + \frac{456517}{1910412}\,\log\left(\left|7\,x^4-36\,x^2+24\,x-4\right|\right) + \frac{1226}{9747}\,\log\left(\left|x+1\right|\right)$$

input
```
integrate((5*x^7+5*x^6-8*x^5-6*x^4+x^2+7*x-9)/(21*x^6+42*x^5-87*x^4-144*x^
3+24*x^2+48*x-12),x, algorithm="giac")
```

output
```
5/42*x^2 - 5/21*x - 13/171/(x + 1) + 0.596528807208595*log(x + 2.562754235
11000) - 13759974409/21724250058*log(x - 0.294967397054000) + 0.0728202222
081972*log(x - 0.406029874333000) - 0.0359566512100309*log(x - 1.861756963
72000) + 456517/1910412*log(abs(7*x^4 - 36*x^2 + 24*x - 4)) + 1226/9747*lo
g(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx$$
$$= \frac{1226\ln(x+1)}{9747}-\frac{5x}{21}-\frac{13}{171\,(x+1)}+\left(\sum_{k=1}^{4}\ln\left(-\frac{58710137727601\,x}{3539939466861}\right.\right.$$
$$-\mathrm{root}\left(z^4-\frac{456517\,z^3}{477603}-\frac{4667078141\,z^2}{119072159136}+\frac{11250606271\,z}{79381439424}-\frac{2225311647577}{106688654585856},z,k\right)\left(\frac{2244714273\,x}{6067327}-\mathrm{r}\right.$$
$$\left.+\frac{17559825878003}{1179979822287}\right)\mathrm{root}\left(z^4-\frac{456517\,z^3}{477603}-\frac{4667078141\,z^2}{119072159136}+\frac{11250606271\,z}{79381439424}\right.$$
$$\left.\left.-\frac{2225311647577}{106688654585856},z,k\right)\right)+\frac{5\,x^2}{42}$$

input
```
int((7*x + x^2 - 6*x^4 - 8*x^5 + 5*x^6 + 5*x^7 - 9)/(48*x + 24*x^2 - 144*x
^3 - 87*x^4 + 42*x^5 + 21*x^6 - 12),x)
```

output
```
(1226*log(x + 1))/9747 - (5*x)/21 - 13/(171*(x + 1)) + symsum(log(17559825
878003/1179979822287 - root(z^4 - (456517*z^3)/477603 - (4667078141*z^2)/1
19072159136 + (11250606271*z)/79381439424 - 2225311647577/106688654585856,
 z, k)*((2244714273*x)/6067327 - root(z^4 - (456517*z^3)/477603 - (4667078
141*z^2)/119072159136 + (11250606271*z)/79381439424 - 2225311647577/106688
654585856, z, k)*((8839526135207188*x)/1179979822287 + root(z^4 - (456517*
z^3)/477603 - (4667078141*z^2)/119072159136 + (11250606271*z)/79381439424
- 2225311647577/106688654585856, z, k)*(root(z^4 - (456517*z^3)/477603 - (
4667078141*z^2)/119072159136 + (11250606271*z)/79381439424 - 2225311647577
/106688654585856, z, k)*((465882880*x)/16807 - 160128512/16807) - (4939301
29216*x)/15647317 + 1455193577408/140825853) - 779319434401232/39332660742
9) + 61696879052062/3539939466861) - (58710137727601*x)/3539939466861)*roo
t(z^4 - (456517*z^3)/477603 - (4667078141*z^2)/119072159136 + (11250606271
*z)/79381439424 - 2225311647577/106688654585856, z, k), k, 1, 4) + (5*x^2)
/42
```

Reduce [F]

$$\int \frac{-9+7x+x^2-6x^4-8x^5+5x^6+5x^7}{-12+48x+24x^2-144x^3-87x^4+42x^5+21x^6}\,dx$$
$$= \frac{-242454\left(\int \frac{x^2}{7x^6+14x^5-29x^4-48x^3+8x^2+16x-4}dx\right)x-242454\left(\int \frac{x^2}{7x^6+14x^5-29x^4-48x^3+8x^2+16x-4}dx\right)+396750\left(\int\right.}{}$$

input
```
int((5*x^7+5*x^6-8*x^5-6*x^4+x^2+7*x-9)/(21*x^6+42*x^5-87*x^4-144*x^3+24*x
^2+48*x-12),x)
```

output
```
( - 242454*int(x**2/(7*x**6 + 14*x**5 - 29*x**4 - 48*x**3 + 8*x**2 + 16*x
- 4),x)*x - 242454*int(x**2/(7*x**6 + 14*x**5 - 29*x**4 - 48*x**3 + 8*x**2
 + 16*x - 4),x) + 396750*int(x/(7*x**6 + 14*x**5 - 29*x**4 - 48*x**3 + 8*x
**2 + 16*x - 4),x)*x + 396750*int(x/(7*x**6 + 14*x**5 - 29*x**4 - 48*x**3
+ 8*x**2 + 16*x - 4),x) - 123842*int(1/(7*x**6 + 14*x**5 - 29*x**4 - 48*x*
*3 + 8*x**2 + 16*x - 4),x)*x - 123842*int(1/(7*x**6 + 14*x**5 - 29*x**4 -
48*x**3 + 8*x**2 + 16*x - 4),x) + 1882*log(7*x**4 - 36*x**2 + 24*x - 4)*x
+ 1882*log(7*x**4 - 36*x**2 + 24*x - 4) + 422*log(x + 1)*x + 422*log(x + 1
) + 875*x**3 - 875*x**2 - 14578*x)/(7350*(x + 1))
```

3.12 $\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx = -\frac{4643\sqrt{\frac{3}{29}}\arctan\left(\frac{1}{29}\left(\sqrt{87}+2\sqrt{87}x\right)\right)}{235924} - \frac{11657\text{arctanh}\left(\frac{1}{5}\left(5\sqrt{5}+2\sqrt{5}x\right)\right)}{28618\sqrt{5}} - \frac{1385\sqrt{\frac{5}{2}}\text{arctanh}\left(\frac{1}{10}\left(2\sqrt{10}+3\sqrt{10}x\right)\right)}{13858} + \frac{5575\log\left(5+5x+x^2\right)}{57236} - \frac{4599\log\left(8+3x+3x^2\right)}{471848} - \frac{4859\log\left(-2+4x+3x^2\right)}{55432}$$

output
```
-4643/6841796*87^(1/2)*arctan(1/29*87^(1/2)+2/29*87^(1/2)*x)-11657/143090*
arctanh(5^(1/2)+2/5*x*5^(1/2))*5^(1/2)-1385/27716*10^(1/2)*arctanh(1/5*10^
(1/2)+3/10*10^(1/2)*x)+5575/57236*ln(x^2+5*x+5)-4599/471848*ln(3*x^2+3*x+8
)-4859/55432*ln(3*x^2+4*x-2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6} \, dx$$
$$= \frac{-1903630\sqrt{87}\arctan\left(\sqrt{\frac{3}{29}}(1+2x)\right)+50605\left(-4859+1385\sqrt{10}\right)\log\left(-2+\sqrt{10}-3x\right)+9802(27875}{}$$

input
```
Integrate[(-9 + 4*x)/(-160 + 100*x + 528*x^2 + 562*x^3 + 360*x^4 + 132*x^5
 + 18*x^6),x]
```

output
```
(-1903630*Sqrt[87]*ArcTan[Sqrt[3/29]*(1 + 2*x)] + 50605*(-4859 + 1385*Sqrt
[10])*Log[-2 + Sqrt[10] - 3*x] + 9802*(27875 + 11657*Sqrt[5])*Log[-5 + Sqr
t[5] - 2*x] + 9802*(27875 - 11657*Sqrt[5])*Log[5 + Sqrt[5] + 2*x] - 50605*
(4859 + 1385*Sqrt[10])*Log[2 + Sqrt[10] + 3*x] - 27341055*Log[8 + 3*x + 3*
x^2])/2805136360
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x-9}{18x^6+132x^5+360x^4+562x^3+528x^2+100x-160} \, dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{4132-14577x}{27716\,(3x^2+4x-2)}+\frac{5575x+19766}{28618\,(x^2+5x+5)}-\frac{3(4599x+4621)}{235924\,(3x^2+3x+8)}\right) dx$$
$$\Big\downarrow \text{2009}$$

$$
-\frac{4643\sqrt{\frac{3}{29}}\arctan\left(\sqrt{\frac{3}{29}}(2x+1)\right)}{235924}-\frac{4599\log\left(3x^2+3x+8\right)}{471848}+\frac{\left(27875+11657\sqrt{5}\right)\log\left(2x-\sqrt{5}+5\right)}{286180}+\frac{\left(27875-11657\sqrt{5}\right)\log\left(2x+\sqrt{5}+5\right)}{286180}-\frac{\left(4859-1385\sqrt{10}\right)\log\left(3x-\sqrt{10}+2\right)}{55432}-\frac{\left(4859+1385\sqrt{10}\right)\log\left(3x+\sqrt{10}+2\right)}{55432}
$$

input
```
Int[(-9 + 4*x)/(-160 + 100*x + 528*x^2 + 562*x^3 + 360*x^4 + 132*x^5 + 18*
x^6),x]
```

output
```
(-4643*Sqrt[3/29]*ArcTan[Sqrt[3/29]*(1 + 2*x)])/235924 + ((27875 + 11657*S
qrt[5])*Log[5 - Sqrt[5] + 2*x])/286180 + ((27875 - 11657*Sqrt[5])*Log[5 +
Sqrt[5] + 2*x])/286180 - ((4859 - 1385*Sqrt[10])*Log[2 - Sqrt[10] + 3*x])/
55432 - ((4859 + 1385*Sqrt[10])*Log[2 + Sqrt[10] + 3*x])/55432 - (4599*Log
[8 + 3*x + 3*x^2])/471848
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

method	result
default	$-\frac{4599\ln\left(3x^2+3x+8\right)}{471848}-\frac{4643\sqrt{87}\,\operatorname{arctan}\left(\frac{(6x+3)\sqrt{87}}{87}\right)}{6841796}-\frac{4859\ln\left(3x^2+4x-2\right)}{55432}-\frac{1385\sqrt{10}\,\operatorname{arctanh}\left(\frac{(6x+4)\sqrt{10}}{20}\right)}{27716}+\frac{5575\ln}{}$
risch	$-\frac{4859\ln\left(3x+2-\sqrt{10}\right)}{55432}+\frac{1385\ln\left(3x+2-\sqrt{10}\right)\sqrt{10}}{55432}-\frac{4859\ln\left(3x+2+\sqrt{10}\right)}{55432}-\frac{1385\ln\left(3x+2+\sqrt{10}\right)\sqrt{10}}{55432}-\frac{4599\ln\left(36x^2\right.}{47184}$

input
```
int((-9+4*x)/(18*x^6+132*x^5+360*x^4+562*x^3+528*x^2+100*x-160),x,method=_
RETURNVERBOSE)
```

output
```
-4599/471848*ln(3*x^2+3*x+8)-4643/6841796*87^(1/2)*arctan(1/87*(6*x+3)*87^
(1/2))-4859/55432*ln(3*x^2+4*x-2)-1385/27716*10^(1/2)*arctanh(1/20*(6*x+4)
*10^(1/2))+5575/57236*ln(x^2+5*x+5)-11657/143090*5^(1/2)*arctanh(1/5*(2*x+
5)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx = -\frac{4643}{235924}\sqrt{\frac{3}{29}}\arctan\left(\sqrt{\frac{3}{29}}(2x+1)\right) + \frac{1385}{27716}\sqrt{\frac{5}{2}}\log\left(\frac{9x^2-4\sqrt{\frac{5}{2}}(3x+2)+12x+14}{3x^2+4x-2}\right) + \frac{11657}{286180}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(2x+5)+10x+15}{x^2+5x+5}\right) - \frac{4859}{55432}\log\left(3x^2+4x-2\right) - \frac{4599}{471848}\log\left(3x^2+3x+8\right) + \frac{5575}{57236}\log\left(x^2+5x+5\right)$$

input
```
integrate((-9+4*x)/(18*x^6+132*x^5+360*x^4+562*x^3+528*x^2+100*x-160),x, a
lgorithm="fricas")
```

output
```
-4643/235924*sqrt(3/29)*arctan(sqrt(3/29)*(2*x + 1)) + 1385/27716*sqrt(5/2
)*log((9*x^2 - 4*sqrt(5/2)*(3*x + 2) + 12*x + 14)/(3*x^2 + 4*x - 2)) + 116
57/286180*sqrt(5)*log((2*x^2 - sqrt(5)*(2*x + 5) + 10*x + 15)/(x^2 + 5*x +
 5)) - 4859/55432*log(3*x^2 + 4*x - 2) - 4599/471848*log(3*x^2 + 3*x + 8)
+ 5575/57236*log(x^2 + 5*x + 5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 607, normalized size of antiderivative = 4.46

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx = \text{Too large to display}$$

input
```
integrate((-9+4*x)/(18*x**6+132*x**5+360*x**4+562*x**3+528*x**2+100*x-160)
,x)
```

output
```
(-4859/55432 + 1385*sqrt(10)/55432)*log(x - 565732077701203774025061970019
4805156569203471427645811546509113/252100455284058577303741432538493999692
939149495175852498358660 + 10129684555407887655447955898819029261969643296
41065155913001056*(-4859/55432 + 1385*sqrt(10)/55432)**3/20830180650528857
3610641935712026233112192876847260919505 + 1115822485043302606994913180995
6848865243153859426216115022336*(-4859/55432 + 1385*sqrt(10)/55432)**4/181
9169110146186876199606238551695769179817791132745363677 - 8334560637989254
78043887426932196724464384639955568721749230454784*(-4859/55432 + 1385*sqr
t(10)/55432)**5/5457507330438560628598818715655087307539453373398236091031
 + 2859308122032441774983768929838611722250974892588615417986265808*(-4859
/55432 + 1385*sqrt(10)/55432)**2/27287536652192803142994093578275436537697
266866991180455155 + 99903036978949839125398451490478271133212666483986468
6241723*sqrt(10)/182021989374771535959380095695663537684432598913484370034
916) + (-4599/471848 - 4643*sqrt(87)*I/13683592)*log(x - 11430107533137461
758559528951195647416490440834837303840396269261/2145928265710644962902579
511120351363239896662776009085900662740 + 1012968455540788765544795589819
02926196964329641065155913001056*(-4599/471848 - 4643*sqrt(87)*I/13683592)
**3/208301806505288573610641935712026233112192876847260919505 - 9990303697
89498391253984514904782711332126664839864686241723*sqrt(87)*I/134033856785
71764790905622619532670586680379758885260282386220 + 111582248504330260...
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx$$
$$= -\frac{4643}{6841796}\sqrt{87}\arctan\left(\frac{1}{29}\sqrt{87}(2x+1)\right)+\frac{1385}{55432}\sqrt{10}\log\left(\frac{3x-\sqrt{10}+2}{3x+\sqrt{10}+2}\right)$$
$$+\frac{11657}{286180}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+5}{2x+\sqrt{5}+5}\right)-\frac{4859}{55432}\log\left(3x^2+4x-2\right)$$
$$-\frac{4599}{471848}\log\left(3x^2+3x+8\right)+\frac{5575}{57236}\log\left(x^2+5x+5\right)$$

input
```
integrate((-9+4*x)/(18*x^6+132*x^5+360*x^4+562*x^3+528*x^2+100*x-160),x, a
lgorithm="maxima")
```

output
```
-4643/6841796*sqrt(87)*arctan(1/29*sqrt(87)*(2*x + 1)) + 1385/55432*sqrt(1
0)*log((3*x - sqrt(10) + 2)/(3*x + sqrt(10) + 2)) + 11657/286180*sqrt(5)*l
og((2*x - sqrt(5) + 5)/(2*x + sqrt(5) + 5)) - 4859/55432*log(3*x^2 + 4*x -
 2) - 4599/471848*log(3*x^2 + 3*x + 8) + 5575/57236*log(x^2 + 5*x + 5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx$$
$$= -\frac{4643}{6841796}\sqrt{87}\arctan\left(\frac{1}{29}\sqrt{87}(2x+1)\right)+\frac{1385}{55432}\sqrt{10}\log\left(\frac{|6x-2\sqrt{10}+4|}{|6x+2\sqrt{10}+4|}\right)$$
$$+\frac{11657}{286180}\sqrt{5}\log\left(\frac{|2x-\sqrt{5}+5|}{|2x+\sqrt{5}+5|}\right)-\frac{4599}{471848}\log\left(3x^2+3x+8\right)$$
$$-\frac{4859}{55432}\log\left(|3x^2+4x-2|\right)+\frac{5575}{57236}\log\left(|x^2+5x+5|\right)$$

input
```
integrate((-9+4*x)/(18*x^6+132*x^5+360*x^4+562*x^3+528*x^2+100*x-160),x, a
lgorithm="giac")
```

output
```
-4643/6841796*sqrt(87)*arctan(1/29*sqrt(87)*(2*x + 1)) + 1385/55432*sqrt(1
0)*log(abs(6*x - 2*sqrt(10) + 4)/abs(6*x + 2*sqrt(10) + 4)) + 11657/286180
*sqrt(5)*log(abs(2*x - sqrt(5) + 5)/abs(2*x + sqrt(5) + 5)) - 4599/471848*
log(3*x^2 + 3*x + 8) - 4859/55432*log(abs(3*x^2 + 4*x - 2)) + 5575/57236*l
og(abs(x^2 + 5*x + 5))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx$$
$$= \ln\left(x-\frac{\sqrt{10}}{3}+\frac{2}{3}\right)\left(\frac{1385\sqrt{10}}{55432}-\frac{4859}{55432}\right)$$
$$-\ln\left(x+\frac{\sqrt{10}}{3}+\frac{2}{3}\right)\left(\frac{1385\sqrt{10}}{55432}+\frac{4859}{55432}\right)$$
$$+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{5}{2}\right)\left(\frac{11657\sqrt{5}}{286180}+\frac{5575}{57236}\right)$$
$$-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{5}{2}\right)\left(\frac{11657\sqrt{5}}{286180}-\frac{5575}{57236}\right)$$
$$+\ln\left(x+\frac{1}{2}-\frac{\sqrt{87}\,1\mathrm{i}}{6}\right)\left(-\frac{4599}{471848}+\frac{\sqrt{87}\,4643\mathrm{i}}{13683592}\right)$$
$$-\ln\left(x+\frac{1}{2}+\frac{\sqrt{87}\,1\mathrm{i}}{6}\right)\left(\frac{4599}{471848}+\frac{\sqrt{87}\,4643\mathrm{i}}{13683592}\right)$$

input
```
int((4*x - 9)/(100*x + 528*x^2 + 562*x^3 + 360*x^4 + 132*x^5 + 18*x^6 - 16
0),x)
```

output
```
log(x - 10^(1/2)/3 + 2/3)*((1385*10^(1/2))/55432 - 4859/55432) - log(x + 1
0^(1/2)/3 + 2/3)*((1385*10^(1/2))/55432 + 4859/55432) + log(x - 5^(1/2)/2
+ 5/2)*((11657*5^(1/2))/286180 + 5575/57236) - log(x + 5^(1/2)/2 + 5/2)*((
11657*5^(1/2))/286180 - 5575/57236) + log(x - (87^(1/2)*1i)/6 + 1/2)*((87^
(1/2)*4643i)/13683592 - 4599/471848) - log(x + (87^(1/2)*1i)/6 + 1/2)*((87
^(1/2)*4643i)/13683592 + 4599/471848)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{-9+4x}{-160+100x+528x^2+562x^3+360x^4+132x^5+18x^6}\,dx$$
$$= -\frac{4643\sqrt{87}\,atan\left(\frac{6x+3}{\sqrt{87}}\right)}{6841796} + \frac{1385\sqrt{10}\log\left(-\sqrt{10}+3x+2\right)}{55432} - \frac{1385\sqrt{10}\log\left(\sqrt{10}+3x+2\right)}{55432} + \frac{11657\sqrt{5}\log\left(-\sqrt{5}+2x+5\right)}{286180} - \frac{11657\sqrt{5}\log\left(\sqrt{5}+2x+5\right)}{286180} - \frac{4859\log\left(-\sqrt{10}+3x+2\right)}{55432} + \frac{5575\log\left(-\sqrt{5}+2x+5\right)}{57236} - \frac{4859\log\left(\sqrt{10}+3x+2\right)}{55432} + \frac{5575\log\left(\sqrt{5}+2x+5\right)}{57236} - \frac{4599\log(3x^2+3x+8)}{471848}$$

input
```
int((-9+4*x)/(18*x^6+132*x^5+360*x^4+562*x^3+528*x^2+100*x-160),x)
```

output
```
( - 1903630*sqrt(87)*atan((6*x + 3)/sqrt(87)) + 70087925*sqrt(10)*log( - s
qrt(10) + 3*x + 2) - 70087925*sqrt(10)*log(sqrt(10) + 3*x + 2) + 114261914
*sqrt(5)*log( - sqrt(5) + 2*x + 5) - 114261914*sqrt(5)*log(sqrt(5) + 2*x +
 5) - 245889695*log( - sqrt(10) + 3*x + 2) + 273230750*log( - sqrt(5) + 2*
x + 5) - 245889695*log(sqrt(10) + 3*x + 2) + 273230750*log(sqrt(5) + 2*x +
 5) - 27341055*log(3*x**2 + 3*x + 8))/2805136360
```

3.13 $\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 281

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx = \frac{27331243886191\sqrt{\frac{33}{590\left(838674112505756+48951578419025\sqrt{295}\right)}}\arctan\left(\sqrt{-\frac{551}{231}+\frac{40\sqrt{295}}{231}}+2\sqrt{\frac{10}{231}\left(-8+\sqrt{295}\right)}\right.}{} + \frac{\text{RootSum}\left[2+9\#1+2\#1^2-9\#1^3+3\#1^4\&,\frac{-13555697\log(x-\#1)-6761246\log(x-\#1)\#1+3434127\log(x-\#1)}{9+4\#1-27\#1^2+12\#1^3}\right.}{22703142}$$

output
```
79454293/66*33^(1/2)/(494817726378396040+28881431267224750*295^(1/2))^(1/2
)*arctan(1/231*(-127281+9240*295^(1/2))^(1/2)+2/231*(-18480+2310*295^(1/2)
)^(1/2)*x)+1/13394853780*(3463724084648772280+202170018870573250*295^(1/2)
)^(1/2)*arctanh(-1/231*(127281+9240*295^(1/2))^(1/2)+2/231*(18480+2310*295
^(1/2))^(1/2)*x)-1/26789707560*(168408125+14036453*295^(1/2))*ln(10+(-5-29
5^(1/2))*x+10*x^2)-1/26789707560*(168408125-14036453*295^(1/2))*ln(10+(-5+
295^(1/2))*x+10*x^2)+1/22703142*RootSum(_Z1 -> 3*_Z1^4-9*_Z1^3+2*_Z1^2+9*_
Z1+2,_Z1 -> (-13555697*ln(x-_Z1)-6761246*ln(x-_Z1)*_Z1+3434127*ln(x-_Z1)*_
Z1^2+1712625*ln(x-_Z1)*_Z1^3)/(12*_Z1^3-27*_Z1^2+4*_Z1+9))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.71

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$$
$$= \frac{\text{RootSum}\left[2+9\#1+2\#1^2-9\#1^3+3\#1^4\&, \frac{-13555697\log(x-\#1)-6761246\log(x-\#1)\#1+3434127\log(x-\#1)\#1}{9+4\#1-27\#1^2+12\#1^3}\right.}{22703142}$$
$$- \frac{\text{RootSum}\left[10-10\#1-7\#1^2-10\#1^3+10\#1^4\&, \frac{11682512\log(x-\#1)+26807235\log(x-\#1)\#1+22864590\log(x-}{-5-7\#1-15\#1^2+20\#1}\right.}{45406284}$$

input
```
Integrate[(-7 - 4*x - 5*x^2)/(20 + 70*x - 84*x^2 - 193*x^3 + 36*x^4 + 103*
x^5 + 89*x^6 - 120*x^7 + 30*x^8),x]
```

output
```
RootSum[2 + 9*#1 + 2*#1^2 - 9*#1^3 + 3*#1^4 & , (-13555697*Log[x - #1] - 6
761246*Log[x - #1]*#1 + 3434127*Log[x - #1]*#1^2 + 1712625*Log[x - #1]*#1^
3)/(9 + 4*#1 - 27*#1^2 + 12*#1^3) & ]/22703142 - RootSum[10 - 10*#1 - 7*#1
^2 - 10*#1^3 + 10*#1^4 & , (11682512*Log[x - #1] + 26807235*Log[x - #1]*#1
 + 22864590*Log[x - #1]*#1^2 + 5708750*Log[x - #1]*#1^3)/(-5 - 7*#1 - 15*#
1^2 + 20*#1^3) & ]/45406284
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-5x^2-4x-7}{30x^8-120x^7+89x^6+103x^5+36x^4-193x^3-84x^2+70x+20}\,dx$$
$$\Big\downarrow \ 2462$$
$$\int \left(\frac{-5708750x^3-22864590x^2-26807235x-11682512}{22703142\,(10x^4-10x^3-7x^2-10x+10)}+\frac{1712625x^3+3434127x^2-6761246x-13555697}{22703142\,(3x^4-9x^3+2x^2+9x+2)}\right)$$
$$\Big\downarrow \ 2009$$

$$-\frac{59360663\int\frac{1}{3x^4-9x^3+2x^2+9x+2}dx}{90812568}-\frac{7332121\int\frac{x}{3x^4-9x^3+2x^2+9x+2}dx}{22703142}+\frac{9716711\int\frac{x^2}{3x^4-9x^3+2x^2+9x+2}dx}{30270856}-\frac{\left(11105503\sqrt{5}-2875418\sqrt{59}\right)\arctan\left(\frac{-20x-\sqrt{295}+5}{\sqrt{10\left(8+\sqrt{295}\right)}}\right)}{3243306\sqrt{590\left(8+\sqrt{295}\right)}}-\frac{\left(11105503\sqrt{5}+2875418\sqrt{59}\right)\operatorname{arctanh}\left(\frac{-20x+\sqrt{295}+5}{\sqrt{10\left(\sqrt{295}-8\right)}}\right)}{3243306\sqrt{590\left(\sqrt{295}-8\right)}}-\frac{\left(168408125-14036453\sqrt{295}\right)\log\left(10x^2-\left(5-\sqrt{295}\right)x+10\right)}{26789707560}-\frac{\left(168408125+14036453\sqrt{295}\right)\log\left(10x^2-\left(5+\sqrt{295}\right)x+10\right)}{26789707560}+\frac{570875\log\left(3x^4-9x^3+2x^2+9x+2\right)}{90812568}$$

input
```
Int[(-7 - 4*x - 5*x^2)/(20 + 70*x - 84*x^2 - 193*x^3 + 36*x^4 + 103*x^5 +
89*x^6 - 120*x^7 + 30*x^8),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.33

method	result
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(2892539212794\_Z^4-72733471125\_Z^3-15447799570\_Z^2+845714817\_Z-11926899\right)}\_R\ln\left(4047485732\right.\right.$
default	$\frac{\left(14036453\sqrt{295}-168408125\right)\ln\left(\sqrt{295}\,x+10x^2-5x+10\right)}{26789707560}+\frac{\left(26939741\sqrt{295}-344634104-\frac{\left(14036453\sqrt{295}-168408125\right)\left(-5+\sqrt{295}\right.}{20}\right.}{669742689\sqrt{80+10\sqrt{295}}}$

input
```
int((-5*x^2-4*x-7)/(30*x^8-120*x^7+89*x^6+103*x^5+36*x^4-193*x^3-84*x^2+70
*x+20),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(40474857325016669451516767681788*_R^3+242378729226029010825665838б
0*_R^2-1155408599958166938987224783*_R+9967012827048091416020589*x-2238451
671241308994328263 5),_R=RootOf(2892539212794*_Z^4-72733471125*_Z^3-1544779
9570*_Z^2+845714817*_Z-11926899))+sum(_R*ln(-14862206786925373021561035749
4360*_R^3-51877771525353151103750026695 00*_R^2+14114149205061283933954091 5
1337*_R+6050214123593112330770975000*x-18173503750350565666162165 0),_R=Roo
tOf(1095350773005720*_Z^4+27542812027500*_Z^3-10490512063449*_Z^2+56698109
100*_Z-200672500))
```

Fricas [B] (verification not implemented)

Default grade assigned because unable to parse optimal

Time = 0.91 (sec) , antiderivative size = 7957, normalized size of antiderivative = 28.32

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((-5*x^2-4*x-7)/(30*x^8-120*x^7+89*x^6+103*x^5+36*x^4-193*x^3-84*
x^2+70*x+20),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Default grade assigned because unable to parse optimal

Time = 2.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.53

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((-5*x**2-4*x-7)/(30*x**8-120*x**7+89*x**6+103*x**5+36*x**4-193*x
**3-84*x**2+70*x+20),x)
```

output
```
-RootSum(2892539212794*_t**4 + 72733471125*_t**3 - 15447799570*_t**2 - 845
714817*_t - 11926899, Lambda(_t, _t*log(-125321034572133108556905574656481
30376764004458547980354328503286333368883028804271998285360510110808520862
8132942226839246843430340765774*_t**7/761387234033768129319773871117548342
21231842929823943546301087592150460424717449154725674297626622822292107379
21905738369661875 + 809315046154666812991652724725301121212071603089692100
41686801073063917906649687075863206584045141473191257211034476523361475490
57532*_t**6/30455489361350725172790954844701933688492737171929577418520435
0368601841698869796618902697190506491289168429516876229534786475 + 4047988
77530009335272429088791616466981271868409535633781796907803640739748459997
39313782685930947558169610366342536537085760206962937991*_t**5/15227744680
67536258639547742235096684424636858596478870926021751843009208494348983094
51348595253245644584214758438114767393237500 + 384090448206105367031357014
22344894886037857818575423307719396406438057749510748614539407769863295174
1590197284034701457516906557863961*_t**4/609109787227014503455819096894038
67376985474343859154837040870073720368339773959323780539438101298257833685
903375245906957295000 - 91494097986588684421891279403635119694971092419708
38495163208768750167466870253697286070190988800815165861948482456984690284
981302499*_t**3/9136646808405217551837286453410580106547821151578873225556
130511058055250966093898567080915715194738675052885506286886043594250 -...
```

Maxima [F]

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$$
$$= \int -\frac{5\,x^2+4\,x+7}{30\,x^8-120\,x^7+89\,x^6+103\,x^5+36\,x^4-193\,x^3-84\,x^2+70\,x+20}\,dx$$

input
```
integrate((-5*x^2-4*x-7)/(30*x^8-120*x^7+89*x^6+103*x^5+36*x^4-193*x^3-84*
x^2+70*x+20),x, algorithm="maxima")
```

output
```
-integrate((5*x^2 + 4*x + 7)/(30*x^8 - 120*x^7 + 89*x^6 + 103*x^5 + 36*x^4
 - 193*x^3 - 84*x^2 + 70*x + 20), x)
```

Giac [F]

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx = \int -\frac{5\,x^2+4\,x+7}{30\,x^8-120\,x^7+89\,x^6+103\,x^5+36\,x^4-193\,x^3-84\,x^2+70\,x+20}\,dx$$

input
```
integrate((-5*x^2-4*x-7)/(30*x^8-120*x^7+89*x^6+103*x^5+36*x^4-193*x^3-84*
x^2+70*x+20),x, algorithm="giac")
```

output
```
integrate(-(5*x^2 + 4*x + 7)/(30*x^8 - 120*x^7 + 89*x^6 + 103*x^5 + 36*x^4
 - 193*x^3 - 84*x^2 + 70*x + 20), x)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.74

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx = \text{Too large to display}$$

input
```
int(-(4*x + 5*x^2 + 7)/(70*x - 84*x^2 - 193*x^3 + 36*x^4 + 103*x^5 + 89*x^
6 - 120*x^7 + 30*x^8 + 20),x)
```

output
```
symsum(log((34396962819377653*root(z^4 + (570875*z^3)/22703142 - (53880390
67*z^2)/562583858760 + (5338805*z)/103140374106 - 1433375/7823934092898, z
, k)^2*x)/2214337500000000 - (398341*x)/32805000000 - (808108441*root(z^4
+ (570875*z^3)/22703142 - (5388039067*z^2)/562583858760 + (5338805*z)/1031
40374106 - 1433375/7823934092898, z, k)*x)/22781250000 - (44761017427*root
(z^4 + (570875*z^3)/22703142 - (5388039067*z^2)/562583858760 + (5338805*z)
/103140374106 - 1433375/7823934092898, z, k))/3280500000000 - (39954030731
81761061*root(z^4 + (570875*z^3)/22703142 - (5388039067*z^2)/562583858760
+ (5338805*z)/103140374106 - 1433375/7823934092898, z, k)^3*x)/22143375000
00000 + (5417924663881855601 9*root(z^4 + (570875*z^3)/22703142 - (53880390
67*z^2)/562583858760 + (5338805*z)/103140374106 - 1433375/7823934092898, z
, k)^4*x)/738112500000000 - (4701169234988061179 33*root(z^4 + (570875*z^3)
/22703142 - (5388039067*z^2)/562583858760 + (5338805*z)/103140374106 - 143
3375/7823934092898, z, k)^5*x)/492075000000000 - (132539568714382002226 9*r
oot(z^4 + (570875*z^3)/22703142 - (5388039067*z^2)/562583858760 + (5338805
*z)/103140374106 - 1433375/7823934092898, z, k)^6*x)/328050000000000 + (88
14932846054050405921*root(z^4 + (570875*z^3)/22703142 - (5388039067*z^2)/5
62583858760 + (5338805*z)/103140374106 - 1433375/7823934092898, z, k)^7*x)
/82012500000000 + (32150255721172381*root(z^4 + (570875*z^3)/22703142 - (5
388039067*z^2)/562583858760 + (5338805*z)/103140374106 - 1433375/782393...
```

Reduce [F]

$$\int \frac{-7-4x-5x^2}{20+70x-84x^2-193x^3+36x^4+103x^5+89x^6-120x^7+30x^8}\,dx$$
$$= -5\left(\int \frac{x^2}{30x^8-120x^7+89x^6+103x^5+36x^4-193x^3-84x^2+70x+20}dx\right) - 4\left(\int \frac{x}{30x^8-120x^7+89x^6+103x^5+36x^4-193x^3-84x^2+70x+20}dx\right) - 7\left(\int \frac{1}{30x^8-120x^7+89x^6+103x^5+36x^4-193x^3-84x^2+70x+20}dx\right)$$

input
```
int((-5*x^2-4*x-7)/(30*x^8-120*x^7+89*x^6+103*x^5+36*x^4-193*x^3-84*x^2+70
*x+20),x)
```

output
```
 - 5*int(x**2/(30*x**8 - 120*x**7 + 89*x**6 + 103*x**5 + 36*x**4 - 193*x**
3 - 84*x**2 + 70*x + 20),x) - 4*int(x/(30*x**8 - 120*x**7 + 89*x**6 + 103*
x**5 + 36*x**4 - 193*x**3 - 84*x**2 + 70*x + 20),x) - 7*int(1/(30*x**8 - 1
20*x**7 + 89*x**6 + 103*x**5 + 36*x**4 - 193*x**3 - 84*x**2 + 70*x + 20),x
)
```

3.14 $\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 192

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx$$
$$= -\frac{13\left(280\sqrt{5}\text{arctanh}\left(\frac{1}{5}\left(-\sqrt{5}+2\sqrt{5}x\right)\right)+279\log\left(-1-x+x^2\right)\right)}{201590}$$
$$+\frac{\frac{200723\text{arctanh}\left(\frac{1}{12}\left(3\sqrt{6}+5\sqrt{6}x\right)\right)}{2\sqrt{6}}-\frac{1411}{2}\log\left(-3+6x+5x^2\right)}{4440490}$$
$$+\frac{\text{RootSum}\left[9+\#1+5\#1^2+7\#1^3+8\#1^4\&,\frac{-3484952\log(x-\#1)+2016354\log(x-\#1)\#1+5342081\log(x-\#1)\#1}{1+10\#1+21\#1^2+32\#1^3}\right.}{49593262}$$

output

```
-364/20159*5^(1/2)*arctanh(-1/5*5^(1/2)+2/5*x*5^(1/2))-3627/201590*ln(x^2-
x-1)+200723/53285880*arctanh(1/4*6^(1/2)+5/12*x*6^(1/2))*6^(1/2)-1411/8880
980*ln(5*x^2+6*x-3)+1/49593262*RootSum(_Z1 -> 8*_Z1^4+7*_Z1^3+5*_Z1^2+_Z1+
9,_Z1 -> (-3484952*ln(x-_Z1)+2016354*ln(x-_Z1)*_Z1+5342081*ln(x-_Z1)*_Z1^2
+14402552*ln(x-_Z1)*_Z1^3)/(32*_Z1^3+21*_Z1^2+10*_Z1+1))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx$$
$$= \frac{-1061\left(16932+200723\sqrt{6}\right)\log\left(-3+2\sqrt{6}-5x\right)+7291752\left(-279+140\sqrt{5}\right)\log\left(1+\sqrt{5}-2x\right)-729}{}$$

input
```
Integrate[(-3 + 10*x + 3*x^2)/((3 - 6*x - 5*x^2)*(-2 - 2*x + 2*x^2)*(-9 -
x - 5*x^2 - 7*x^3 - 8*x^4)),x]
```

output
```
(-1061*(16932 + 200723*Sqrt[6])*Log[-3 + 2*Sqrt[6] - 5*x] + 7291752*(-279
+ 140*Sqrt[5])*Log[1 + Sqrt[5] - 2*x] - 7291752*(279 + 140*Sqrt[5])*Log[-1
 + Sqrt[5] + 2*x] + 1061*(-16932 + 200723*Sqrt[6])*Log[3 + 2*Sqrt[6] + 5*x
] + 2280*RootSum[9 + #1 + 5*#1^2 + 7*#1^3 + 8*#1^4 & , (-3484952*Log[x - #
1] + 2016354*Log[x - #1]*#1 + 5342081*Log[x - #1]*#1^2 + 14402552*Log[x -
#1]*#1^3)/(1 + 10*#1 + 21*#1^2 + 32*#1^3) & ])/113072637360
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+10x-3}{(-5x^2-6x+3)(2x^2-2x-2)(-8x^4-7x^3-5x^2-x-9)}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-7055x-204956}{4440490\,(5x^2+6x-3)}-\frac{13(558x-979)}{201590\,(x^2-x-1)}+\frac{14402552x^3+5342081x^2+2016354x-3484952}{49593262\,(8x^4+7x^3+5x^2+x+9)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15740127\int \frac{1}{8x^4+7x^3+5x^2+x+9}dx}{198373048}-\frac{4968887\int \frac{x}{8x^4+7x^3+5x^2+x+9}dx}{99186524}-\frac{16438375\int \frac{x^2}{8x^4+7x^3+5x^2+x+9}dx}{198373048}+\frac{1800319\log\left(8x^4+7x^3+5x^2+x+9\right)}{198373048}-\frac{13(279+140\sqrt{5})\log\left(-2x-\sqrt{5}+1\right)}{201590}-\frac{(16932+200723\sqrt{6})\log\left(5x-2\sqrt{6}+3\right)}{106571760}-\frac{(16932-200723\sqrt{6})\log\left(5x+2\sqrt{6}+3\right)}{106571760}-\frac{13(279-140\sqrt{5})\log\left(-2\sqrt{5}x+\sqrt{5}+5\right)}{201590}$$

input
```
Int[(-3 + 10*x + 3*x^2)/((3 - 6*x - 5*x^2)*(-2 - 2*x + 2*x^2)*(-9 - x - 5*
x^2 - 7*x^3 - 8*x^4)),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

method	result
default	$-\frac{1411\ln\left(5x^2+6x-3\right)}{8880980}+\frac{200723\sqrt{6}\,\operatorname{arctanh}\left(\frac{(10x+6)\sqrt{6}}{24}\right)}{53285880}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(8\_Z^4+7\_Z^3+5\_Z^2+\_Z+9\right)}\frac{\left(14402552\_R^3+53420\right.}{32\_}\right.}{49593262}$
risch	$-\frac{3627\ln\left(2x-1-\sqrt{5}\right)}{201590}+\frac{182\ln\left(2x-1-\sqrt{5}\right)\sqrt{5}}{20159}-\frac{3627\ln\left(2x-1+\sqrt{5}\right)}{201590}-\frac{182\ln\left(2x-1+\sqrt{5}\right)\sqrt{5}}{20159}+\left(\sum_{\_R=\operatorname{RootOf}\left(178075\right.}\right.$

input
```
int((3*x^2+10*x-3)/(-5*x^2-6*x+3)/(2*x^2-2*x-2)/(-8*x^4-7*x^3-5*x^2-x-9),x
,method=_RETURNVERBOSE)
```

output
```
-1411/8880980*ln(5*x^2+6*x-3)+200723/53285880*6^(1/2)*arctanh(1/24*(10*x+6
)*6^(1/2))+1/49593262*sum((14402552*_R^3+5342081*_R^2+2016354*_R-3484952)/
(32*_R^3+21*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(8*_Z^4+7*_Z^3+5*_Z^2+_Z+9))-3
627/201590*ln(x^2-x-1)-364/20159*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))
```

Fricas [B] (verification not implemented)

Default grade assigned because unable to parse optimal

Time = 0.90 (sec) , antiderivative size = 7153, normalized size of antiderivative = 37.26

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx = \text{Too large to display}$$

input
```
integrate((3*x^2+10*x-3)/(-5*x^2-6*x+3)/(2*x^2-2*x-2)/(-8*x^4-7*x^3-5*x^2-
x-9),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Default grade assigned because unable to parse optimal

Time = 3.65 (sec) , antiderivative size = 614, normalized size of antiderivative = 3.20

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx = \text{Too large to display}$$

input
```
integrate((3*x**2+10*x-3)/(-5*x**2-6*x+3)/(2*x**2-2*x-2)/(-8*x**4-7*x**3-5
*x**2-x-9),x)
```

output
```
(-3627/201590 + 182*sqrt(5)/20159)*log(x - 7816968523551351214667646383007
53721845232330177464546232672194295446830470550489649253834859974570445426
187353921638239732376237733008992460449/1301879540764276453642608484059842
12275170347009805730569006712147771231640986061044696868419159144875112159
22643400111062434057152992943122750 - 110511693241465043554921249310046330
75178084621679785729849010802577064395919813589917831937898064474292593358
277865775478786122162428015622296272*(-3627/201590 + 182*sqrt(5)/20159)**3
/9328303338423750019210559547584452268781049055164081647552442845605359675
7600343578608908590542348516871872499387757022676192460488509825 - 1136715
65536163371969545962221470606624017522511147219850394502118185183349901720
978352082792567090336303186163785756708371750091886352003469370752000*(-36
27/201590 + 182*sqrt(5)/20159)**6/6027519080212269243182207707669953773673
90862025986814149542460792961702126032989277165255508119790416710560765274
7376849846282062334481 - 2146509329679775723240687726962270423805652333166
17065712584962517103534415010225359745109939806106475409915599442399270045
26582921511224027689100738560*(-3627/201590 + 182*sqrt(5)/20159)**7/783577
48042759500161368700199709399057760812063378285839440519903085021276384288
606031483216055572754172372899485715899048001666810348253 + 29552543239354
24393158216944489652185301971336093339276036024919660852629469172273357081
7451830075531464776448955519207367975044103221228338578629888*(-3627/20...
```

Maxima [F]

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)\,(-2-2x+2x^2)\,(-9-x-5x^2-7x^3-8x^4)}\,dx = \int \frac{3\,x^2+10\,x-3}{2\,(8\,x^4+7\,x^3+5\,x^2+x+9)(5\,x^2+6\,x-3)(x^2-x-1)}\,dx$$

input
```
integrate((3*x^2+10*x-3)/(-5*x^2-6*x+3)/(2*x^2-2*x-2)/(-8*x^4-7*x^3-5*x^2-
x-9),x, algorithm="maxima")
```

output
```
-200723/106571760*sqrt(6)*log((5*x - 2*sqrt(6) + 3)/(5*x + 2*sqrt(6) + 3))
 + 182/20159*sqrt(5)*log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) + 1/4959
3262*integrate((14402552*x^3 + 5342081*x^2 + 2016354*x - 3484952)/(8*x^4 +
 7*x^3 + 5*x^2 + x + 9), x) - 1411/8880980*log(5*x^2 + 6*x - 3) - 3627/201
590*log(x^2 - x - 1)
```

Giac [F]

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx = \int \frac{3x^2+10x-3}{2(8x^4+7x^3+5x^2+x+9)(5x^2+6x-3)(x^2-x-1)}\,dx$$

input
```
integrate((3*x^2+10*x-3)/(-5*x^2-6*x+3)/(2*x^2-2*x-2)/(-8*x^4-7*x^3-5*x^2-
x-9),x, algorithm="giac")
```

output
```
integrate(1/2*(3*x^2 + 10*x - 3)/((8*x^4 + 7*x^3 + 5*x^2 + x + 9)*(5*x^2 +
 6*x - 3)*(x^2 - x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.53

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx = \text{Too large to display}$$

input
```
int(-(10*x + 3*x^2 - 3)/((2*x - 2*x^2 + 2)*(6*x + 5*x^2 - 3)*(x + 5*x^2 +
7*x^3 + 8*x^4 + 9)),x)
```

output
```
log(x - 5^(1/2)/2 - 1/2)*((182*5^(1/2))/20159 - 3627/201590) - log(x + 5^(
1/2)/2 - 1/2)*((182*5^(1/2))/20159 + 3627/201590) - log(x - (2*6^(1/2))/5
+ 3/5)*((200723*6^(1/2))/106571760 + 1411/8880980) + log(x + (2*6^(1/2))/5
 + 3/5)*((200723*6^(1/2))/106571760 - 1411/8880980) + symsum(log((12260027
*x)/26214400000000 - root(z^4 - (1800319*z^3)/49593262 + (570527537199*z^2
)/1112970646785926 - (59271808291*z)/17807530348574816 + 4685120/556485323
392963, z, k)*((15968387919198813*x)/3355443200000000000 + root(z^4 - (18
00319*z^3)/49593262 + (570527537199*z^2)/1112970646785926 - (59271808291*z
)/17807530348574816 + 4685120/556485323392963, z, k)*(root(z^4 - (1800319*
z^3)/49593262 + (570527537199*z^2)/1112970646785926 - (59271808291*z)/1780
7530348574816 + 4685120/556485323392963, z, k)*((479072916770444818863*x)/1
677721600000000000 + root(z^4 - (1800319*z^3)/49593262 + (570527537199*z^2
)/1112970646785926 - (59271808291*z)/17807530348574816 + 4685120/556485323
392963, z, k)*(root(z^4 - (1800319*z^3)/49593262 + (570527537199*z^2)/1112
970646785926 - (59271808291*z)/17807530348574816 + 4685120/556485323392963
, z, k)*((431229927213556357411*x)/1048576000000000000 + root(z^4 - (18003
19*z^3)/49593262 + (570527537199*z^2)/1112970646785926 - (59271808291*z)/1
7807530348574816 + 4685120/556485323392963, z, k)*((1614192635121054173529
0189*x)/2097152000000000000 - root(z^4 - (1800319*z^3)/49593262 + (5705275
37199*z^2)/1112970646785926 - (59271808291*z)/17807530348574816 + 46851...
```

Reduce [F]

$$\int \frac{-3+10x+3x^2}{(3-6x-5x^2)(-2-2x+2x^2)(-9-x-5x^2-7x^3-8x^4)}\,dx$$
$$= \frac{3\left(\int \frac{x^2}{40x^8+43x^7-80x^6-112x^5-21x^4+x^3-114x^2-24x+27}dx\right)}{2} + 5\left(\int \frac{x}{40x^8+43x^7-80x^6-112x^5-21x^4+x^3-114x^2-24x+27}dx\right) - \frac{3\left(\int \frac{1}{40x^8+43x^7-80x^6-112x^5-21x^4+x^3-114x^2-24x+27}dx\right)}{2}$$

input
```
int((3*x^2+10*x-3)/(-5*x^2-6*x+3)/(2*x^2-2*x-2)/(-8*x^4-7*x^3-5*x^2-x-9),x
)
```

output
```
(3*int(x**2/(40*x**8 + 43*x**7 - 80*x**6 - 112*x**5 - 21*x**4 + x**3 - 114
*x**2 - 24*x + 27),x) + 10*int(x/(40*x**8 + 43*x**7 - 80*x**6 - 112*x**5 -
 21*x**4 + x**3 - 114*x**2 - 24*x + 27),x) - 3*int(1/(40*x**8 + 43*x**7 -
80*x**6 - 112*x**5 - 21*x**4 + x**3 - 114*x**2 - 24*x + 27),x))/2
```

3.15 $\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 173

$$\begin{aligned}\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx &= \sqrt{\frac{21557}{5772}+\frac{1405}{481\sqrt{3}}}\operatorname{arctanh}\left(\sqrt{\frac{113}{481}+\frac{64\sqrt{3}}{481}}-4\sqrt{\frac{1}{481}\left(23-4\sqrt{3}\right)}x\right)\\ &\quad -\frac{1}{2}\sqrt{\frac{21557-5620\sqrt{3}}{1443}}\operatorname{arctanh}\left(\sqrt{\frac{1}{481}\left(113-64\sqrt{3}\right)}-\sqrt{\frac{368}{481}+\frac{64\sqrt{3}}{481}}x\right)\\ &\quad -\frac{1}{4}\sqrt{3}\log\left(-2-2x-\sqrt{3}x+2x^2\right)+\frac{1}{4}\sqrt{3}\log\left(-2-2x+\sqrt{3}x+2x^2\right)\end{aligned}$$

output
```
-1/2886*(31106751+8109660*3^(1/2))^(1/2)*arctanh(-1/481*(54353+30784*3^(1/
2))^(1/2)+4/481*(11063-1924*3^(1/2))^(1/2)*x)+1/2886*(31106751-8109660*3^(
1/2))^(1/2)*arctanh(-1/481*(54353-30784*3^(1/2))^(1/2)+4/481*(11063+1924*3
^(1/2))^(1/2)*x)-1/4*3^(1/2)*ln(-2-2*x-x*3^(1/2)+2*x^2)+1/4*3^(1/2)*ln(-2-
2*x+x*3^(1/2)+2*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.50

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx = \frac{1}{2}\text{RootSum}\left[4+8\#1-7\#1^2-8\#1^3 + 4\#1^4\&, \frac{-10\log(x-\#1)+9\log(x-\#1)\#1+4\log(x-\#1)\#1^2}{4-7\#1-12\#1^2+8\#1^3}\&\right]$$

input
```
Integrate[(10 - 9*x - 4*x^2)/(-4 - 8*x + 7*x^2 + 8*x^3 - 4*x^4),x]
```

output
```
RootSum[4 + 8*#1 - 7*#1^2 - 8*#1^3 + 4*#1^4 & , (-10*Log[x - #1] + 9*Log[x
 - #1]*#1 + 4*Log[x - #1]*#1^2)/(4 - 7*#1 - 12*#1^2 + 8*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-4x^2-9x+10}{-4x^4+8x^3+7x^2-8x-4}\,dx$$

$$\Big\downarrow \text{2492}$$

$$-\frac{1}{4}\int \left(\frac{2\left(-6x+5\sqrt{3}+19\right)}{\sqrt{3}\left(-2x^2+\left(2+\sqrt{3}\right)x+2\right)}-\frac{2\left(-6x-5\sqrt{3}+19\right)}{\sqrt{3}\left(-2x^2+\left(2-\sqrt{3}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-2\sqrt{\frac{21557-5620\sqrt{3}}{1443}}\operatorname{arctanh}\left(\frac{-4x-\sqrt{3}+2}{\sqrt{23-4\sqrt{3}}}\right)+2\sqrt{\frac{21557+5620\sqrt{3}}{1443}}\operatorname{arctanh}\left(\frac{-4x+\sqrt{3}+2}{\sqrt{23+4\sqrt{3}}}\right)+\sqrt{3}\log\right.$$

input
```
Int[(10 - 9*x - 4*x^2)/(-4 - 8*x + 7*x^2 + 8*x^3 - 4*x^4),x]
```

output
```
(-2*Sqrt[(21557 - 5620*Sqrt[3])/1443]*ArcTanh[(2 - Sqrt[3] - 4*x)/Sqrt[23
- 4*Sqrt[3]]] + 2*Sqrt[(21557 + 5620*Sqrt[3])/1443]*ArcTanh[(2 + Sqrt[3] -
 4*x)/Sqrt[23 + 4*Sqrt[3]]] + Sqrt[3]*Log[2 + (2 - Sqrt[3])*x - 2*x^2] - S
qrt[3]*Log[2 + (2 + Sqrt[3])*x - 2*x^2])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-8\_Z^3-7\_Z^2+8\_Z+4\right)}\frac{\left(4\_R^2+9\_R-10\right)\ln\left(x-\_R\right)}{8\_R^3-12\_R^2-7\_R+4}\right)}{2}$	61
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-8\_Z^3-7\_Z^2+8\_Z+4\right)}\frac{\left(4\_R^2+9\_R-10\right)\ln\left(x-\_R\right)}{8\_R^3-12\_R^2-7\_R+4}\right)}{2}$	61

input
```
int((-4*x^2-9*x+10)/(-4*x^4+8*x^3+7*x^2-8*x-4),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((4*_R^2+9*_R-10)/(8*_R^3-12*_R^2-7*_R+4)*ln(x-_R),_R=RootOf(4*_Z^4
-8*_Z^3-7*_Z^2+8*_Z+4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(116) = 232.

Time = 0.08 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.72

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4} \, dx = \text{Too large to display}$$

input
```
integrate((-4*x^2-9*x+10)/(-4*x^4+8*x^3+7*x^2-8*x-4),x, algorithm="fricas"
)
```

output
```
1/4*(sqrt(1/2810*(877*sqrt(481) - 21557)*sqrt(877/2886*sqrt(481) + 21557/2
886) - 877/2886*sqrt(481) + 30215/2886) + sqrt(877/2886*sqrt(481) + 21557/
2886))*log((1443*(1477387*sqrt(481) + 6680613)*sqrt(877/2886*sqrt(481) + 2
1557/2886) + 4151457470*sqrt(481) + 66441092770)*sqrt(1/2810*(877*sqrt(481
) - 21557)*sqrt(877/2886*sqrt(481) + 21557/2886) - 877/2886*sqrt(481) + 30
215/2886) + 33927808*(163*sqrt(481) - 3848)*sqrt(877/2886*sqrt(481) + 2155
7/2886) + 190674280960*x - 95337140480) - 1/4*(sqrt(1/2810*(877*sqrt(481)
- 21557)*sqrt(877/2886*sqrt(481) + 21557/2886) - 877/2886*sqrt(481) + 3021
5/2886) - sqrt(877/2886*sqrt(481) + 21557/2886))*log(-(1443*(1477387*sqrt(
481) + 6680613)*sqrt(877/2886*sqrt(481) + 21557/2886) + 4151457470*sqrt(48
1) + 66441092770)*sqrt(1/2810*(877*sqrt(481) - 21557)*sqrt(877/2886*sqrt(4
81) + 21557/2886) - 877/2886*sqrt(481) + 30215/2886) + 33927808*(163*sqrt(
481) - 3848)*sqrt(877/2886*sqrt(481) + 21557/2886) + 190674280960*x - 9533
7140480) - 1/4*(sqrt(-1/2810*(877*sqrt(481) - 21557)*sqrt(877/2886*sqrt(48
1) + 21557/2886) - 877/2886*sqrt(481) + 30215/2886) + sqrt(877/2886*sqrt(4
81) + 21557/2886))*log((1443*(1477387*sqrt(481) + 6680613)*sqrt(877/2886*s
qrt(481) + 21557/2886) - 4151457470*sqrt(481) - 66441092770)*sqrt(-1/2810*
(877*sqrt(481) - 21557)*sqrt(877/2886*sqrt(481) + 21557/2886) - 877/2886*s
qrt(481) + 30215/2886) - 33927808*(163*sqrt(481) - 3848)*sqrt(877/2886*sqr
t(481) + 21557/2886) + 190674280960*x - 95337140480) + 1/4*(sqrt(-1/281...
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.24

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx$$
$$=\operatorname{RootSum}\left(34632t^4-77658t^2+25290t+13127,\left(t\mapsto t\log\left(\frac{2131869441t^3}{929833988}+\frac{4840845087t^2}{1859667976}-\frac{308471}{92983}\right.\right.\right.$$

input
```
integrate((-4*x**2-9*x+10)/(-4*x**4+8*x**3+7*x**2-8*x-4),x)
```

output
```
RootSum(34632*_t**4 - 77658*_t**2 + 25290*_t + 13127, Lambda(_t, _t*log(21
31869441*_t**3/929833988 + 4840845087*_t**2/1859667976 - 3084714377*_t/929
833988 + x - 8044277969/3719335952)))
```

Maxima [F]

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx=\int \frac{4\,x^2+9\,x-10}{4\,x^4-8\,x^3-7\,x^2+8\,x+4}\,dx$$

input
```
integrate((-4*x^2-9*x+10)/(-4*x^4+8*x^3+7*x^2-8*x-4),x, algorithm="maxima"
)
```

output
```
integrate((4*x^2 + 9*x - 10)/(4*x^4 - 8*x^3 - 7*x^2 + 8*x + 4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.14

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx=1.14860948933000\log(x+0.935253839597000)$$
$$-1.59718546836000\log(x+0.434654180756000)$$
$$-0.282584085545000\log(x-1.06922843581000)$$
$$+0.731160064576000\log(x-2.30067958454000)$$

input
```
integrate((-4*x^2-9*x+10)/(-4*x^4+8*x^3+7*x^2-8*x-4),x, algorithm="giac")
```

output
```
1.14860948933000*log(x + 0.935253839597000) - 1.59718546836000*log(x + 0.4
34654180756000) - 0.282584085545000*log(x - 1.06922843581000) + 0.73116006
4576000*log(x - 2.30067958454000)
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.84

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx = \sum_{k=1}^{4}\ln\left(\frac{163\,\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)}{4} + \frac{2037\,x}{64} - \frac{\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)x\,1611}{32} - \frac{\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)^2 x\,1221}{16} + \frac{\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)^3 x\,891}{8} - \frac{117\,\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)^2}{8} + \frac{51\,\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)^3}{2} - \frac{857}{32}\right)\mathrm{root}\left(z^4-\frac{12943\,z^2}{5772}+\frac{1405\,z}{1924}+\frac{13127}{34632},z,k\right)$$

input
```
int((9*x + 4*x^2 - 10)/(8*x - 7*x^2 - 8*x^3 + 4*x^4 + 4),x)
```

output
```
symsum(log((163*root(z^4 - (12943*z^2)/5772 + (1405*z)/1924 + 13127/34632,
 z, k))/4 + (2037*x)/64 - (1611*root(z^4 - (12943*z^2)/5772 + (1405*z)/192
4 + 13127/34632, z, k)*x)/32 - (1221*root(z^4 - (12943*z^2)/5772 + (1405*z
)/1924 + 13127/34632, z, k)^2*x)/16 + (891*root(z^4 - (12943*z^2)/5772 + (
1405*z)/1924 + 13127/34632, z, k)^3*x)/8 - (117*root(z^4 - (12943*z^2)/577
2 + (1405*z)/1924 + 13127/34632, z, k)^2)/8 + (51*root(z^4 - (12943*z^2)/5
772 + (1405*z)/1924 + 13127/34632, z, k)^3)/2 - 857/32)*root(z^4 - (12943*
z^2)/5772 + (1405*z)/1924 + 13127/34632, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{10-9x-4x^2}{-4-8x+7x^2+8x^3-4x^4}\,dx = 4\left(\int \frac{x^2}{4x^4-8x^3-7x^2+8x+4}dx\right) + 9\left(\int \frac{x}{4x^4-8x^3-7x^2+8x+4}dx\right) - 10\left(\int \frac{1}{4x^4-8x^3-7x^2+8x+4}dx\right)$$

input
```
int((-4*x^2-9*x+10)/(-4*x^4+8*x^3+7*x^2-8*x-4),x)
```

output
```
4*int(x**2/(4*x**4 - 8*x**3 - 7*x**2 + 8*x + 4),x) + 9*int(x/(4*x**4 - 8*x
**3 - 7*x**2 + 8*x + 4),x) - 10*int(1/(4*x**4 - 8*x**3 - 7*x**2 + 8*x + 4)
,x)
```

3.16 $\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 194

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = \sqrt{78+\frac{169}{9\sqrt[3]{3}}+81\sqrt[3]{3}}\arctan\left(\frac{1}{3}\sqrt{3+4\sqrt[3]{3}+4\,3^{2/3}} - \sqrt{4+\frac{28}{3\,3^{2/3}}+\frac{20}{3\sqrt[3]{3}}}x\right)-10\log(x) + \left(\frac{13}{3\,3^{2/3}}-3\,3^{2/3}\right)\log\left(\frac{1}{2}\left(1+\sqrt[3]{3}+3^{2/3}\right)+x\right)+\frac{1}{6}\left(-\frac{13}{3^{2/3}}+9\,3^{2/3}\right)\log\left(\frac{1}{2}\left(-1+\sqrt[3]{3}\right)+\frac{1}{2}\left(2-\sqrt[3]{3}-3^{2/\right.\right.$$

output

```
-1/9*(6318+507*3^(2/3)+6561*3^(1/3))^(1/2)*arctan(-1/3*(3+4*3^(1/3)+4*3^(2
/3))^(1/2)+2/3*(9+7*3^(1/3)+5*3^(2/3))^(1/2)*x)-10*ln(x)+(13/9*3^(1/3)-3*3
^(2/3))*ln(1/2+1/2*3^(1/3)+1/2*3^(2/3)+x)+1/6*(-13/3*3^(1/3)+9*3^(2/3))*ln
(-1/2+1/2*3^(1/3)+1/2*(2-3^(1/3)-3^(2/3))*x+x^2)+10/3*ln(2*x^3+3*x^2-3*x+1
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.41

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = -10\log(x)+\frac{1}{3}\text{RootSum}\left[1-3\#1+3\#1^2+2\#1^3\&,\frac{-37\log(x-\#1)+34\log(x-\#1)\#1+20\log(x-\#1)\#1^2}{-1+2\#1+2\#1^2}\&\right]$$

input
```
Integrate[(-10 - 7*x + 4*x^2)/(x - 3*x^2 + 3*x^3 + 2*x^4),x]
```

output
```
-10*Log[x] + RootSum[1 - 3*#1 + 3*#1^2 + 2*#1^3 & , (-37*Log[x - #1] + 34*
Log[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(-1 + 2*#1 + 2*#1^2) & ]/3
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2026, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-7x-10}{2x^4+3x^3-3x^2+x}\,dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{4x^2-7x-10}{x\left(2x^3+3x^2-3x+1\right)}dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{20x^2+34x-37}{2x^3+3x^2-3x+1}-\frac{10}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\left(13+27\sqrt[3]{3}\right)\arctan\left(\frac{4x-3^{2/3}-\sqrt[3]{3}+2}{\sqrt{3\left(-6+3\sqrt[3]{3}+3^{2/3}\right)}}\right)}{\sqrt[6]{3}\sqrt{-6+3\sqrt[3]{3}+3^{2/3}}\left(3+3\sqrt[3]{3}+3^{2/3}\right)}+\frac{\left(34+7\sqrt[3]{3}+7\ 3^{2/3}\right)\log\left(-2x^2-\left(2-\sqrt[3]{3}-3^{2/3}\right)x-\sqrt[3]{3}+1\right)}{3\left(3+3\sqrt[3]{3}+3^{2/3}\right)}+\frac{10}{3}\log\left(2x^3+3x^2-3x+1\right)-10\log(x)-\frac{2\left(34+7\sqrt[3]{3}+7\ 3^{2/3}\right)\log\left(2x+3^{2/3}+\sqrt[3]{3}+1\right)}{3\left(3+3\sqrt[3]{3}+3^{2/3}\right)}$$

input
```
Int[(-10 - 7*x + 4*x^2)/(x - 3*x^2 + 3*x^3 + 2*x^4),x]
```

output
```
(-2*(13 + 27*3^(1/3))*ArcTan[(2 - 3^(1/3) - 3^(2/3) + 4*x)/Sqrt[3*(-6 + 3*
3^(1/3) + 3^(2/3))]])/(3^(1/6)*Sqrt[-6 + 3*3^(1/3) + 3^(2/3)]*(3 + 3*3^(1/
3) + 3^(2/3))) - 10*Log[x] - (2*(34 + 7*3^(1/3) + 7*3^(2/3))*Log[1 + 3^(1/
3) + 3^(2/3) + 2*x])/(3*(3 + 3*3^(1/3) + 3^(2/3))) + ((34 + 7*3^(1/3) + 7*
3^(2/3))*Log[1 - 3^(1/3) - (2 - 3^(1/3) - 3^(2/3))*x - 2*x^2])/(3*(3 + 3*3
^(1/3) + 3^(2/3))) + (10*Log[1 - 3*x + 3*x^2 + 2*x^3])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.22

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{3}-270\_Z^{2}+5859\_Z+16262\right)}\_R\ln\left(180\_R^{2}-6627\_R+61246x+121013\right)\right)}{3}-10\ln\left(x\right)$	43
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{3}+3\_Z^{2}-3\_Z+1\right)}\frac{\left(20\_R^{2}+34\_R-37\right)\ln\left(x-\_R\right)}{2\_R^{2}+2\_R-1}\right)}{3}-10\ln\left(x\right)$	56

input
```
int((4*x^2-7*x-10)/(2*x^4+3*x^3-3*x^2+x),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(_R*ln(180*_R^2-6627*_R+61246*x+121013),_R=RootOf(9*_Z^3-270*_Z^2+5
859*_Z+16262))-10*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(136) = 272.

Time = 0.70 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.52

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx$$
$$= \frac{1}{27}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)\log\left(\frac{20}{9}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)^2+61246\,x-\frac{28717}{3}\cdot 9^{\frac{2}{3}}+59643\cdot 9^{\frac{1}{3}}+54743\right)$$
$$-\frac{1}{54}\left(13\cdot 9^{\frac{2}{3}}+27\sqrt{-\frac{1}{243}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)^2+\frac{260}{27}\cdot 9^{\frac{2}{3}}-60\cdot 9^{\frac{1}{3}}-\frac{368}{3}}-81\cdot 9^{\frac{1}{3}}-180\right)\log\left(+3\sqrt{-\frac{1}{243}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)^2+\frac{260}{27}\cdot 9^{\frac{2}{3}}-60\cdot 9^{\frac{1}{3}}-\frac{368}{3}}\left(260\cdot 9^{\frac{2}{3}}-1620\cdot 9^{\frac{1}{3}}+3027\right)+122492\,x+\frac{28717}{3}\cdot 9^{\frac{2}{3}}-59643\cdot 9^{\frac{1}{3}}+37126\right)$$
$$-\frac{1}{54}\left(13\cdot 9^{\frac{2}{3}}-27\sqrt{-\frac{1}{243}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)^2+\frac{260}{27}\cdot 9^{\frac{2}{3}}-60\cdot 9^{\frac{1}{3}}-\frac{368}{3}}-81\cdot 9^{\frac{1}{3}}-180\right)\log\left(-3\sqrt{-\frac{1}{243}\left(13\cdot 9^{\frac{2}{3}}-81\cdot 9^{\frac{1}{3}}+90\right)^2+\frac{260}{27}\cdot 9^{\frac{2}{3}}-60\cdot 9^{\frac{1}{3}}-\frac{368}{3}}\left(260\cdot 9^{\frac{2}{3}}-1620\cdot 9^{\frac{1}{3}}+3027\right)+122492\,x+\frac{28717}{3}\cdot 9^{\frac{2}{3}}-59643\cdot 9^{\frac{1}{3}}+37126\right)-10\log\left(x\right)$$

input
```
integrate((4*x^2-7*x-10)/(2*x^4+3*x^3-3*x^2+x),x, algorithm="fricas")
```

output
```
1/27*(13*9^(2/3) - 81*9^(1/3) + 90)*log(20/9*(13*9^(2/3) - 81*9^(1/3) + 90
)^2 + 61246*x - 28717/3*9^(2/3) + 59643*9^(1/3) + 54743) - 1/54*(13*9^(2/3
) + 27*sqrt(-1/243*(13*9^(2/3) - 81*9^(1/3) + 90)^2 + 260/27*9^(2/3) - 60*
9^(1/3) - 368/3) - 81*9^(1/3) - 180)*log(-20/9*(13*9^(2/3) - 81*9^(1/3) +
90)^2 + 3*sqrt(-1/243*(13*9^(2/3) - 81*9^(1/3) + 90)^2 + 260/27*9^(2/3) -
60*9^(1/3) - 368/3)*(260*9^(2/3) - 1620*9^(1/3) + 3027) + 122492*x + 28717
/3*9^(2/3) - 59643*9^(1/3) + 37126) - 1/54*(13*9^(2/3) - 27*sqrt(-1/243*(1
3*9^(2/3) - 81*9^(1/3) + 90)^2 + 260/27*9^(2/3) - 60*9^(1/3) - 368/3) - 81
*9^(1/3) - 180)*log(-20/9*(13*9^(2/3) - 81*9^(1/3) + 90)^2 - 3*sqrt(-1/243
*(13*9^(2/3) - 81*9^(1/3) + 90)^2 + 260/27*9^(2/3) - 60*9^(1/3) - 368/3)*(
260*9^(2/3) - 1620*9^(1/3) + 3027) + 122492*x + 28717/3*9^(2/3) - 59643*9^
(1/3) + 37126) - 10*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = -10\log(x) + \text{RootSum}\left(243t^3-2430t^2+17577t+16262, \left(t \mapsto t\log\left(\frac{117082989t^3}{39534782968} - \frac{62553465t^2}{19767391484} - \frac{43643416}{39534782}\right.\right.\right.$$

input
```
integrate((4*x**2-7*x-10)/(2*x**4+3*x**3-3*x**2+x),x)
```

output
```
-10*log(x) + RootSum(243*_t**3 - 2430*_t**2 + 17577*_t + 16262, Lambda(_t,
 _t*log(117082989*_t**3/39534782968 - 62553465*_t**2/19767391484 - 4364341
677*_t/39534782968 + x + 42975132615/19767391484)))
```

Maxima [F]

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = \int \frac{4\,x^2-7\,x-10}{2\,x^4+3\,x^3-3\,x^2+x}\,dx$$

input
```
integrate((4*x^2-7*x-10)/(2*x^4+3*x^3-3*x^2+x),x, algorithm="maxima")
```

output
```
integrate((20*x^2 + 34*x - 37)/(2*x^3 + 3*x^2 - 3*x + 1), x) - 10*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((4*x^2-7*x-10)/(2*x^4+3*x^3-3*x^2+x),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -10*
ln(abs(sageVARx))+((7/221184*rootof([[-3,0,8640,0,-5308416],[1,0,-3456,0,2
985984,0,254803968]])-162)/(6*(1/2654208*rootof([[-3,0,8640,0,-5308416],[1
,0,-3456,0,2985984,
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.78

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = \left(\sum_{k=1}^{3}\ln\left(-\frac{721\,\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)}{4} - \frac{1301\,x}{8} + \frac{\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)\,x\,859}{16} + \frac{\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)^2\,x\,4407}{16} - \frac{\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)^3\,x\,27}{2} - \frac{1077\,\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)^2}{16} - \frac{81\,\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)^3}{16} - \frac{535}{4}\right)\,\mathrm{root}\left(z^3-10\,z^2+\frac{217\,z}{3}+\frac{16262}{243},z,k\right)\right) - 10\,\ln\left(x\right)$$

input
```
int(-(7*x - 4*x^2 + 10)/(x - 3*x^2 + 3*x^3 + 2*x^4),x)
```

output
```
symsum(log((859*root(z^3 - 10*z^2 + (217*z)/3 + 16262/243, z, k)*x)/16 - (
1301*x)/8 - (721*root(z^3 - 10*z^2 + (217*z)/3 + 16262/243, z, k))/4 + (44
07*root(z^3 - 10*z^2 + (217*z)/3 + 16262/243, z, k)^2*x)/16 - (27*root(z^3
 - 10*z^2 + (217*z)/3 + 16262/243, z, k)^3*x)/2 - (1077*root(z^3 - 10*z^2
+ (217*z)/3 + 16262/243, z, k)^2)/16 - (81*root(z^3 - 10*z^2 + (217*z)/3 +
 16262/243, z, k)^3)/16 - 535/4)*root(z^3 - 10*z^2 + (217*z)/3 + 16262/243
, z, k), k, 1, 3) - 10*log(x)
```

Reduce [F]

$$\int \frac{-10-7x+4x^2}{x-3x^2+3x^3+2x^4}\,dx = -14\left(\int \frac{1}{2x^4+3x^3-3x^2+x}dx\right) + \int \frac{1}{2x^3+3x^2-3x+1}dx - \frac{4\log(2x^3+3x^2-3x+1)}{3} + 4\log(x)$$

input
```
int((4*x^2-7*x-10)/(2*x^4+3*x^3-3*x^2+x),x)
```

output
```
( - 42*int(1/(2*x**4 + 3*x**3 - 3*x**2 + x),x) + 3*int(1/(2*x**3 + 3*x**2
- 3*x + 1),x) - 4*log(2*x**3 + 3*x**2 - 3*x + 1) + 12*log(x))/3
```

3.17 $\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4} dx$

Optimal result

Integrand size = 33, antiderivative size = 182

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4} dx = \sqrt{\frac{529}{836}+\frac{83}{38\sqrt{11}}}\arctan\left(\sqrt{-\frac{13}{19}+\frac{10\sqrt{11}}{19}}+\sqrt{\frac{20}{19}+\frac{8\sqrt{11}}{19}}x\right)+\sqrt{-\frac{529}{836}+\frac{83}{38\sqrt{11}}}\operatorname{arctanh}\left(\sqrt{\frac{13}{19}+\frac{10\sqrt{11}}{19}}-\sqrt{-\frac{20}{19}+\frac{8\sqrt{11}}{19}}x\right)-\frac{\log\left(5+\sqrt{11}-4x-2\sqrt{11}x+2x^2\right)}{4\sqrt{11}}+\frac{\log\left(5-\sqrt{11}-4x+2\sqrt{11}x+2x^2\right)}{4\sqrt{11}}$$

output
```
1/418*(110561+34694*11^(1/2))^(1/2)*arctan(1/19*(-247+190*11^(1/2))^(1/2)+
2/19*(95+38*11^(1/2))^(1/2)*x)-1/418*(-110561+34694*11^(1/2))^(1/2)*arctan
h(-1/19*(247+190*11^(1/2))^(1/2)+2/19*(-95+38*11^(1/2))^(1/2)*x)-1/44*ln(5
+11^(1/2)-4*x-2*11^(1/2)*x+2*x^2)*11^(1/2)+1/44*ln(5-11^(1/2)-4*x+2*11^(1/
2)*x+2*x^2)*11^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \frac{1}{2}\text{RootSum}\left[7+2\#1-4\#1^2-8\#1^3+2\#1^4\&,\frac{8\log(x-\#1)-6\log(x-\#1)\#1+\log(x-\#1)\#1^2}{1-4\#1-12\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(-8 + 6*x - x^2)/(-7 - 2*x + 4*x^2 + 8*x^3 - 2*x^4),x]
```

output
```
RootSum[7 + 2*#1 - 4*#1^2 - 8*#1^3 + 2*#1^4 & , (8*Log[x - #1] - 6*Log[x -
 #1]*#1 + Log[x - #1]*#1^2)/(1 - 4*#1 - 12*#1^2 + 4*#1^3) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+6x-8}{-2x^4+8x^3+4x^2-2x-7}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^2}{2x^4-8x^3-4x^2+2x+7}-\frac{6x}{2x^4-8x^3-4x^2+2x+7}+\frac{8}{2x^4-8x^3-4x^2+2x+7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$8\int \frac{1}{2x^4-8x^3-4x^2+2x+7}dx-6\int \frac{x}{2x^4-8x^3-4x^2+2x+7}dx+\int \frac{x^2}{2x^4-8x^3-4x^2+2x+7}dx$$

input
```
Int[(-8 + 6*x - x^2)/(-7 - 2*x + 4*x^2 + 8*x^3 - 2*x^4),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-8\_Z^3-4\_Z^2+2\_Z+7\right)}\frac{\left(\_R^2-6\_R+8\right)\ln\left(x-\_R\right)}{4\_R^3-12\_R^2-4\_R+1}\right)}{2}$	59
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-8\_Z^3-4\_Z^2+2\_Z+7\right)}\frac{\left(\_R^2-6\_R+8\right)\ln\left(x-\_R\right)}{4\_R^3-12\_R^2-4\_R+1}\right)}{2}$	59

input
```
int((-x^2+6*x-8)/(-2*x^4+8*x^3+4*x^2-2*x-7),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2-6*_R+8)/(4*_R^3-12*_R^2-4*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4-8*
_Z^3-4*_Z^2+2*_Z+7))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx$$
$$= -\frac{1}{44}\left(\sqrt{11}+11\sqrt{\frac{166}{209}\sqrt{11}-\frac{529}{209}}\right)\log\left(\left(59\sqrt{11}+176\right)\sqrt{\frac{166}{209}\sqrt{11}-\frac{529}{209}}+70\,x-35\sqrt{11}-70\right)$$
$$-\frac{1}{44}\left(\sqrt{11}-11\sqrt{\frac{166}{209}\sqrt{11}-\frac{529}{209}}\right)\log\left(-\left(59\sqrt{11}+176\right)\sqrt{\frac{166}{209}\sqrt{11}-\frac{529}{209}}+70\,x-35\sqrt{11}-70\right)$$
$$-\frac{1}{2}\sqrt{\frac{166}{209}\sqrt{11}+\frac{529}{209}}\arctan\left(\frac{1}{35}\left(2\sqrt{11}(3\,x-14)-44\,x+77\right)\sqrt{\frac{166}{209}\sqrt{11}+\frac{529}{209}}\right)$$
$$+\frac{1}{44}\sqrt{11}\log\left(2\,x^2+\sqrt{11}(2\,x-1)-4\,x+5\right)$$

input
```
integrate((-x^2+6*x-8)/(-2*x^4+8*x^3+4*x^2-2*x-7),x, algorithm="fricas")
```

output
```
-1/44*(sqrt(11) + 11*sqrt(166/209*sqrt(11) - 529/209))*log((59*sqrt(11) +
176)*sqrt(166/209*sqrt(11) - 529/209) + 70*x - 35*sqrt(11) - 70) - 1/44*(s
qrt(11) - 11*sqrt(166/209*sqrt(11) - 529/209))*log(-(59*sqrt(11) + 176)*sq
rt(166/209*sqrt(11) - 529/209) + 70*x - 35*sqrt(11) - 70) - 1/2*sqrt(166/2
09*sqrt(11) + 529/209)*arctan(1/35*(2*sqrt(11)*(3*x - 14) - 44*x + 77)*sqr
t(166/209*sqrt(11) + 529/209)) + 1/44*sqrt(11)*log(2*x^2 + sqrt(11)*(2*x -
 1) - 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.23

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \operatorname{RootSum}\left(147136t^4+44880t^2+7304t-37,\left(t\mapsto t\log\left(-\frac{30824992t^3}{430955}-\frac{1699588t^2}{430955}-\frac{8685072t}{430955}+x\right.\right.\right.$$

input
```
integrate((-x**2+6*x-8)/(-2*x**4+8*x**3+4*x**2-2*x-7),x)
```

output
```
RootSum(147136*_t**4 + 44880*_t**2 + 7304*_t - 37, Lambda(_t, _t*log(-3082
4992*_t**3/430955 - 1699588*_t**2/430955 - 8685072*_t/430955 + x - 3675607
/861910)))
```

Maxima [F]

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \int \frac{x^2-6\,x+8}{2\,x^4-8\,x^3-4\,x^2+2\,x+7}\,dx$$

input
```
integrate((-x^2+6*x-8)/(-2*x^4+8*x^3+4*x^2-2*x-7),x, algorithm="maxima")
```

output
```
integrate((x^2 - 6*x + 8)/(2*x^4 - 8*x^3 - 4*x^2 + 2*x + 7), x)
```

Giac [F]

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \int \frac{x^2-6\,x+8}{2\,x^4-8\,x^3-4\,x^2+2\,x+7}\,dx$$

input
```
integrate((-x^2+6*x-8)/(-2*x^4+8*x^3+4*x^2-2*x-7),x, algorithm="giac")
```

output
```
integrate((x^2 - 6*x + 8)/(2*x^4 - 8*x^3 - 4*x^2 + 2*x + 7), x)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \sum_{k=1}^{4} \ln\left(370\,\mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)+\frac{3\,x}{8} - \frac{\mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)\,x\,479}{2} + \mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)^2\,x\,1100 - \mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)^3\,x\,132 + \frac{363\,\mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)^2}{2} + 2684\,\mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)^3+\frac{25}{16}\right)\,\mathrm{root}\left(z^4+\frac{255\,z^2}{836}+\frac{83\,z}{1672}-\frac{37}{147136},z,k\right)$$

input
```
int((x^2 - 6*x + 8)/(2*x - 4*x^2 - 8*x^3 + 2*x^4 + 7),x)
```

output
```
symsum(log(370*root(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k) +
 (3*x)/8 - (479*root(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k)*
x)/2 + 1100*root(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k)^2*x
- 132*root(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k)^3*x + (363
*root(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k)^2)/2 + 2684*roo
t(z^4 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k)^3 + 25/16)*root(z^4
 + (255*z^2)/836 + (83*z)/1672 - 37/147136, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{-8+6x-x^2}{-7-2x+4x^2+8x^3-2x^4}\,dx = \int \frac{x^2}{2x^4-8x^3-4x^2+2x+7}dx - 6\left(\int \frac{x}{2x^4-8x^3-4x^2+2x+7}dx\right) + 8\left(\int \frac{1}{2x^4-8x^3-4x^2+2x+7}dx\right)$$

input
```
int((-x^2+6*x-8)/(-2*x^4+8*x^3+4*x^2-2*x-7),x)
```

output
```
int(x**2/(2*x**4 - 8*x**3 - 4*x**2 + 2*x + 7),x) - 6*int(x/(2*x**4 - 8*x**
3 - 4*x**2 + 2*x + 7),x) + 8*int(1/(2*x**4 - 8*x**3 - 4*x**2 + 2*x + 7),x)
```

3.18 $\int \frac{x}{(a^3+x^5)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 244

$$\int \frac{x}{(a^3+x^5)^2} \, dx = \frac{x^2}{5a^3(a^3+x^5)} - \frac{3\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a^{3/5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a^{3/5}}\right)}{50a^{24/5}} + \frac{3\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a^{3/5}\right)+4x}{\sqrt{10-2\sqrt{5}}a^{3/5}}\right)}{25a^{24/5}} - \frac{3\log\left(a^{3/5}+x\right)}{25a^{24/5}} + \frac{3(1+\sqrt{5})\log\left(a^{6/5}+\frac{1}{2}\left(-1+\sqrt{5}\right)a^{3/5}x+x^2\right)}{100a^{24/5}} - \frac{3(-1+\sqrt{5})\log\left(a^{6/5}-\frac{1}{2}\left(1+\sqrt{5}\right)a^{3/5}x+x^2\right)}{100a^{24/5}}$$

output
```
1/5*x^2/a^3/(x^5+a^3)-3/50*(10-2*5^(1/2))^(1/2)*arctan(((5^(1/2)-1)*a^(3/5
)+4*x)/(10+2*5^(1/2))^(1/2)/a^(3/5))/a^(24/5)+3/50*(10+2*5^(1/2))^(1/2)*ar
ctan((-(5^(1/2)+1)*a^(3/5)+4*x)/(10-2*5^(1/2))^(1/2)/a^(3/5))/a^(24/5)-3/2
5*ln(a^(3/5)+x)/a^(24/5)+3/100*(5^(1/2)+1)*ln(a^(6/5)+1/2*(5^(1/2)-1)*a^(3
/5)*x+x^2)/a^(24/5)-3/100*(5^(1/2)-1)*ln(a^(6/5)-1/2*(5^(1/2)+1)*a^(3/5)*x
+x^2)/a^(24/5)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.89

$$\int \frac{x}{(a^3+x^5)^2}\,dx$$
$$= \frac{\frac{20a^{9/5}x^2}{a^3+x^5} - 6\sqrt{10-2\sqrt{5}}\arctan\left(\frac{\left(-1+\sqrt{5}\right)a^{3/5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}a^{3/5}}\right) + 6\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)a^{3/5}\right)+4x}{\sqrt{10-2\sqrt{5}}a^{3/5}}\right) - 12\log}{}$$

input
```
Integrate[x/(a^3 + x^5)^2,x]
```

output
```
((20*a^(9/5)*x^2)/(a^3 + x^5) - 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[
5])*a^(3/5) + 4*x)/(Sqrt[2*(5 + Sqrt[5])]*a^(3/5))] + 6*Sqrt[2*(5 + Sqrt[5
])]*ArcTan[(-((1 + Sqrt[5])*a^(3/5)) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a^(3/5))
] - 12*Log[a^(3/5) + x] + 3*(1 + Sqrt[5])*Log[a^(6/5) + ((-1 + Sqrt[5])*a^
(3/5)*x)/2 + x^2] - 3*(-1 + Sqrt[5])*Log[a^(6/5) - ((1 + Sqrt[5])*a^(3/5)*
x)/2 + x^2])/(100*a^(24/5))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {819, 822, 16, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^3+x^5)^2}\,dx$$
$$\Big\downarrow \text{ 819}$$
$$\frac{3\int \frac{x}{x^5+a^3}dx}{5a^3} + \frac{x^2}{5a^3(a^3+x^5)}$$
$$\Big\downarrow \text{ 822}$$

$$\frac{3\left(\frac{2\int \frac{\left(1+\sqrt{5}\right)x+\left(1-\sqrt{5}\right)a^{3/5}}{2\left(2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}\right)}dx}{5a^{9/5}}+\frac{2\int \frac{\left(1-\sqrt{5}\right)x+\left(1+\sqrt{5}\right)a^{3/5}}{2\left(2x^2-\left(1+\sqrt{5}\right)a^{3/5}x+2a^{6/5}\right)}dx}{5a^{9/5}}-\frac{\int \frac{1}{x+a^{3/5}}dx}{5a^{9/5}}\right)}{5a^3}+\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$\Big\downarrow$ 16

$$\frac{3\left(\frac{2\int \frac{\left(1+\sqrt{5}\right)x+\left(1-\sqrt{5}\right)a^{3/5}}{2\left(2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}\right)}dx}{5a^{9/5}}+\frac{2\int \frac{\left(1-\sqrt{5}\right)x+\left(1+\sqrt{5}\right)a^{3/5}}{2\left(2x^2-\left(1+\sqrt{5}\right)a^{3/5}x+2a^{6/5}\right)}dx}{5a^{9/5}}-\frac{\log\left(a^{3/5}+x\right)}{5a^{9/5}}\right)}{5a^3}+\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\int \frac{\left(1+\sqrt{5}\right)x+\left(1-\sqrt{5}\right)a^{3/5}}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx}{5a^{9/5}}+\frac{\int \frac{\left(1-\sqrt{5}\right)x+\left(1+\sqrt{5}\right)a^{3/5}}{2x^2-\left(1+\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx}{5a^{9/5}}-\frac{\log\left(a^{3/5}+x\right)}{5a^{9/5}}\right)}{5a^3}+\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$\Big\downarrow$ 1142

$$\frac{3\left(\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int -\frac{\left(1-\sqrt{5}\right)a^{3/5}-4x}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx-\sqrt{5}a^{3/5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx}{5a^{9/5}}+\frac{\sqrt{5}a^{3/5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int}{5a^{9/5}}\right.}{5a^3}$$

$$\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{-\sqrt{5}a^{3/5}\int \frac{1}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a^{3/5}-4x}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx}{5a^{9/5}}+\frac{\sqrt{5}a^{3/5}\int \frac{1}{2x^2-\left(1+\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx-\frac{1}{4}\left(1-\sqrt{5}\right)\int}{5a^{9/5}}\right.}{5a^3}$$

$$\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$\Big\downarrow$ 1083

$$3\left(\frac{2\sqrt{5}a^{3/5}\int \frac{1}{-\left(4x-\left(1-\sqrt{5}\right)a^{3/5}\right)^2-2\left(5+\sqrt{5}\right)a^{6/5}}d\left(4x-\left(1-\sqrt{5}\right)a^{3/5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a^{3/5}-4x}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx}{5a^{9/5}}+\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1+\right.}{2x^2-\left(1+\right.}}{}\right.$$
$$\overline{5a^3}$$

$$\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$$\Big\downarrow \; 217$$

$$3\left(\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)a^{3/5}-4x}{2x^2-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}}dx-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a^{3/5}}{\sqrt{2\left(5+\sqrt{5}\right)}a^{3/5}}\right)}{5a^{9/5}}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a^{3/5}}{\sqrt{2\left(5-\sqrt{5}\right)}a^{3/5}}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{}{2x^2\ldots}}{5a^{9/5}}\right.$$
$$\overline{5a^3}$$

$$\frac{x^2}{5a^3\left(a^3+x^5\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{x^2}{5a^3\left(a^3+x^5\right)}+$$
$$3\left(\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(-\left(1-\sqrt{5}\right)a^{3/5}x+2a^{6/5}+2x^2\right)-\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4x-\left(1-\sqrt{5}\right)a^{3/5}}{\sqrt{2\left(5+\sqrt{5}\right)}a^{3/5}}\right)}{5a^{9/5}}+\frac{\sqrt{\frac{10}{5-\sqrt{5}}}\arctan\left(\frac{4x-\left(1+\sqrt{5}\right)a^{3/5}}{\sqrt{2\left(5-\sqrt{5}\right)}a^{3/5}}\right)+\frac{1}{4}\left(1-\sqrt{5}\right)\log}{5a^{9/5}}\right.$$
$$\overline{5a^3}$$

input
```
Int[x/(a^3 + x^5)^2,x]
```

output
```
x^2/(5*a^3*(a^3 + x^5)) + (3*(-1/5*Log[a^(3/5) + x]/a^(9/5) + (-(Sqrt[10/(
5 + Sqrt[5])]*ArcTan[(-((1 - Sqrt[5])*a^(3/5)) + 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a^(3/5))]) + ((1 + Sqrt[5])*Log[2*a^(6/5) - (1 - Sqrt[5])*a^(3/5)*x + 2
*x^2])/4)/(5*a^(9/5)) + (Sqrt[10/(5 - Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a^
(3/5)) + 4*x)/(Sqrt[2*(5 - Sqrt[5])]*a^(3/5))] + ((1 - Sqrt[5])*Log[2*a^(6
/5) - (1 + Sqrt[5])*a^(3/5)*x + 2*x^2])/4)/(5*a^(9/5))))/(5*a^3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 822
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; -(-r)^(m + 1)/(a*n*s^m)   Int[1/(r + s*x), x]
 + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b}
, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

method	result	size
risch	$\frac{x^2}{5a^3(x^5+a^3)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^5+a^3\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{25a^3}$	45
default	Expression too large to display	1273

input
```
int(x/(x^5+a^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^2/a^3/(x^5+a^3)+3/25/a^3*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^5+a^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.07 (sec) , antiderivative size = 1208573, normalized size of antiderivative = 4953.17

$$\int \frac{x}{(a^3+x^5)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(x^5+a^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.17

$$\int \frac{x}{(a^3+x^5)^2}\,dx = \frac{x^2}{5a^6+5a^3x^5} + \operatorname{RootSum}\left(9765625t^5a^{24}+243, \left(t \mapsto t\log\left(-\frac{15625t^3a^{15}}{27}+x\right)\right)\right)$$

input
```
integrate(x/(x**5+a**3)**2,x)
```

output
```
x**2/(5*a**6 + 5*a**3*x**5) + RootSum(9765625*_t**5*a**24 + 243, Lambda(_t
, _t*log(-15625*_t**3*a**15/27 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

$$\int \frac{x}{(a^3+x^5)^2}\,dx = \frac{x^2}{5\,(a^3x^5+a^6)} - \frac{3\left(\frac{2\sqrt{5}\arctan\left(\frac{a^{\frac{3}{5}}(\sqrt{5}-1)+4x}{a^{\frac{3}{5}}\sqrt{2\sqrt{5}+10}}\right)}{a^{\frac{9}{5}}\sqrt{2\sqrt{5}+10}} - \frac{2\sqrt{5}\arctan\left(-\frac{a^{\frac{3}{5}}(\sqrt{5}+1)-4x}{a^{\frac{3}{5}}\sqrt{-2\sqrt{5}+10}}\right)}{a^{\frac{9}{5}}\sqrt{-2\sqrt{5}+10}} + \frac{\log\left(x+a^{\frac{3}{5}}\right)}{a^{\frac{9}{5}}} + \frac{\log\left(-a^{\frac{3}{5}}x\left(\sqrt{5}+1\right)+2x^2+2a^{\frac{6}{5}}\right)}{a^{\frac{9}{5}}\left(\sqrt{5}+1\right)} - \frac{\log\left(a\right.}{}\right.}{25\,a^3}$$

input
```
integrate(x/(x^5+a^3)^2,x, algorithm="maxima")
```

output
```
1/5*x^2/(a^3*x^5 + a^6) - 3/25*(2*sqrt(5)*arctan((a^(3/5)*(sqrt(5) - 1) +
4*x)/(a^(3/5)*sqrt(2*sqrt(5) + 10)))/(a^(9/5)*sqrt(2*sqrt(5) + 10)) - 2*sq
rt(5)*arctan(-(a^(3/5)*(sqrt(5) + 1) - 4*x)/(a^(3/5)*sqrt(-2*sqrt(5) + 10)
))/(a^(9/5)*sqrt(-2*sqrt(5) + 10)) + log(x + a^(3/5))/a^(9/5) + log(-a^(3/
5)*x*(sqrt(5) + 1) + 2*x^2 + 2*a^(6/5))/(a^(9/5)*(sqrt(5) + 1)) - log(a^(3
/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*a^(6/5))/(a^(9/5)*(sqrt(5) - 1)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07

$$\int \frac{x}{(a^3+x^5)^2}\,dx = \frac{x^2}{5\,(x^5+a^3)a^3} - \frac{3\,(-a^3)^{\frac{2}{5}}\,(\sqrt{5}-1)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}(-a^3)^{\frac{1}{5}}+(-a^3)^{\frac{1}{5}}\right)+(-a^3)^{\frac{2}{5}}\right)}{100\,a^6} + \frac{3\,(-a^3)^{\frac{2}{5}}\,(\sqrt{5}+1)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}(-a^3)^{\frac{1}{5}}-(-a^3)^{\frac{1}{5}}\right)+(-a^3)^{\frac{2}{5}}\right)}{100\,a^6} + \frac{3\,(-a^3)^{\frac{2}{5}}\,\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{(-a^3)^{\frac{1}{5}}(\sqrt{5}-1)-4\,x}{(-a^3)^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}\right)}{50\,a^6} - \frac{3\,(-a^3)^{\frac{2}{5}}\,\sqrt{2\sqrt{5}+10}\arctan\left(\frac{(-a^3)^{\frac{1}{5}}(\sqrt{5}+1)+4\,x}{(-a^3)^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}\right)}{50\,a^6} - \frac{3\,(-a^3)^{\frac{2}{5}}\log\left(\left|x-(-a^3)^{\frac{1}{5}}\right|\right)}{25\,a^6}$$

input

```
integrate(x/(x^5+a^3)^2,x, algorithm="giac")
```

output

```
1/5*x^2/((x^5 + a^3)*a^3) - 3/100*(-a^3)^(2/5)*(sqrt(5) - 1)*log(x^2 + 1/2
*x*(sqrt(5)*(-a^3)^(1/5) + (-a^3)^(1/5)) + (-a^3)^(2/5))/a^6 + 3/100*(-a^3
)^(2/5)*(sqrt(5) + 1)*log(x^2 - 1/2*x*(sqrt(5)*(-a^3)^(1/5) - (-a^3)^(1/5)
) + (-a^3)^(2/5))/a^6 + 3/50*(-a^3)^(2/5)*sqrt(-2*sqrt(5) + 10)*arctan(-((
-a^3)^(1/5)*(sqrt(5) - 1) - 4*x)/((-a^3)^(1/5)*sqrt(2*sqrt(5) + 10)))/a^6
- 3/50*(-a^3)^(2/5)*sqrt(2*sqrt(5) + 10)*arctan(((-a^3)^(1/5)*(sqrt(5) + 1
) + 4*x)/((-a^3)^(1/5)*sqrt(-2*sqrt(5) + 10)))/a^6 - 3/25*(-a^3)^(2/5)*log
(abs(x - (-a^3)^(1/5)))/a^6
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.16

$$\int \frac{x}{(a^3+x^5)^2}\,dx$$
$$= \frac{3(-1)^{1/5}\ln\left(x-(-1)^{3/5}a^{3/5}\right)}{25a^{24/5}} + \frac{x^2}{5a^3\,(a^3+x^5)}$$
$$- \frac{(-1)^{1/5}\ln\left(\frac{81x}{625a^{12}}+\frac{81(-1)^{3/5}\left(\sqrt{2}\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{40000a^{57/5}}\right)\left(\frac{3\sqrt{-2\sqrt{5}-10}}{100}-\frac{3\sqrt{5}}{100}+\frac{3}{100}\right)}{a^{24/5}}$$
$$- \frac{(-1)^{1/5}\ln\left(\frac{81x}{625a^{12}}+\frac{81(-1)^{3/5}\left(\sqrt{5}+\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{40000a^{57/5}}\right)\left(\frac{3\sqrt{5}}{100}+\frac{3\sqrt{2\sqrt{5}-10}}{100}+\frac{3}{100}\right)}{a^{24/5}}$$
$$- \frac{(-1)^{1/5}\ln\left(\frac{81x}{625a^{12}}+\frac{81(-1)^{3/5}\left(\sqrt{5}-\sqrt{2}\sqrt{\sqrt{5}-5}+1\right)^3}{40000a^{57/5}}\right)\left(\frac{3\sqrt{5}}{100}-\frac{3\sqrt{2\sqrt{5}-10}}{100}+\frac{3}{100}\right)}{a^{24/5}}$$
$$+ \frac{(-1)^{1/5}\ln\left(\frac{81x}{625a^{12}}-\frac{81(-1)^{3/5}\left(\sqrt{2}\sqrt{-\sqrt{5}-5}+\sqrt{5}-1\right)^3}{40000a^{57/5}}\right)\left(\frac{3\sqrt{5}}{100}+\frac{3\sqrt{-2\sqrt{5}-10}}{100}-\frac{3}{100}\right)}{a^{24/5}}$$

input
```
int(x/(a^3 + x^5)^2,x)
```

output
```
(3*(-1)^(1/5)*log(x - (-1)^(3/5)*a^(3/5)))/(25*a^(24/5)) + x^2/(5*a^3*(a^3
 + x^5)) - ((-1)^(1/5)*log((81*x)/(625*a^12) + (81*(-1)^(3/5)*(2^(1/2)*(-
5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/(40000*a^(57/5)))*((3*(- 2*5^(1/2) -
10)^(1/2))/100 - (3*5^(1/2))/100 + 3/100))/a^(24/5) - ((-1)^(1/5)*log((81*
x)/(625*a^12) + (81*(-1)^(3/5)*(5^(1/2) + 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)
^3)/(40000*a^(57/5)))*((3*5^(1/2))/100 + (3*(2*5^(1/2) - 10)^(1/2))/100 +
3/100))/a^(24/5) - ((-1)^(1/5)*log((81*x)/(625*a^12) + (81*(-1)^(3/5)*(5^(
1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/(40000*a^(57/5)))*((3*5^(1/2))/
100 - (3*(2*5^(1/2) - 10)^(1/2))/100 + 3/100))/a^(24/5) + ((-1)^(1/5)*log(
(81*x)/(625*a^12) - (81*(-1)^(3/5)*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2
) - 1)^3)/(40000*a^(57/5)))*((3*5^(1/2))/100 + (3*(- 2*5^(1/2) - 10)^(1/2)
)/100 - 3/100))/a^(24/5)
```

Reduce [F]

$$\int \frac{x}{(a^3+x^5)^2}\,dx=\int \frac{x}{x^{10}+2a^3x^5+a^6}dx$$

input
```
int(x/(x^5+a^3)^2,x)
```

output
```
int(x/(a**6 + 2*a**3*x**5 + x**10),x)
```

3.19 $\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = \frac{-2\sqrt{2}a^3b+\sqrt{2}a^3b^2x^3+3x^6}{12a^6b^2x^3\left(\sqrt{2}a^3b-x^6\right)}+\frac{\operatorname{arctanh}\left(\frac{x^3}{\sqrt[4]{2}a^{3/2}\sqrt{b}}\right)}{4\sqrt[4]{2}a^{15/2}b^{5/2}} + \frac{\log(x)}{2a^6b}-\frac{\log\left(-\sqrt{2}a^3b+x^6\right)}{12a^6b}$$

output
```
1/12*(-2*2^(1/2)*a^3*b+2^(1/2)*a^3*b^2*x^3+3*x^6)/a^6/b^2/x^3/(2^(1/2)*a^3
*b-x^6)+1/8*arctanh(1/2*x^3*2^(3/4)/a^(3/2)/b^(1/2))*2^(3/4)/a^(15/2)/b^(5
/2)+1/2*ln(x)/a^6/b-1/12*ln(-2^(1/2)*a^3*b+x^6)/a^6/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.86

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx$$
$$= \frac{-\frac{8a^{3/2}\sqrt{b}}{x^3}+\frac{4a^{3/2}\sqrt{b}\left(\sqrt{2}a^3b^2+x^3\right)}{\sqrt{2}a^3b-x^6}+24a^{3/2}b^{3/2}\log(x)-3\ 2^{3/4}\log\left(2\sqrt{a}\sqrt[6]{b}-2^{11/12}x\right)+3\ 2^{3/4}\log\left(2\sqrt{a}\sqrt[6]{b}\right.}{}$$

input
```
Integrate[(1 + b*x^3)/(x^4*(-(Sqrt[2]*a^3*b) + x^6)^2),x]
```

output
```
((-8*a^(3/2)*Sqrt[b])/x^3 + (4*a^(3/2)*Sqrt[b]*(Sqrt[2]*a^3*b^2 + x^3))/(S
qrt[2]*a^3*b - x^6) + 24*a^(3/2)*b^(3/2)*Log[x] - 3*2^(3/4)*Log[2*Sqrt[a]*
b^(1/6) - 2^(11/12)*x] + 3*2^(3/4)*Log[2*Sqrt[a]*b^(1/6) + 2^(11/12)*x] +
3*2^(3/4)*Log[2*a*b^(1/3) - 2^(11/12)*Sqrt[a]*b^(1/6)*x + 2^(5/6)*x^2] - 3
*2^(3/4)*Log[2*a*b^(1/3) + 2^(11/12)*Sqrt[a]*b^(1/6)*x + 2^(5/6)*x^2] - 4*
a^(3/2)*b^(3/2)*Log[-(Sqrt[2]*a^3*b) + x^6])/(48*a^(15/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1803, 532, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^3+1}{x^4\left(x^6-\sqrt{2}a^3b\right)^2}dx$$

$$\Big\downarrow \text{1803}$$

$$\frac{1}{3}\int \frac{bx^3+1}{x^6\left(\sqrt{2}a^3b-x^6\right)^2}dx^3$$

$$\Big\downarrow \text{532}$$

$$\frac{1}{3}\left(\frac{\sqrt{2}a^3b^2+x^3}{4a^6b^2\left(\sqrt{2}a^3b-x^6\right)}-\frac{\int -\frac{\frac{\sqrt{2}x^6}{a^3b}+4bx^3+4}{2x^6\left(\sqrt{2}a^3b-x^6\right)}dx^3}{2\sqrt{2}a^3b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\int \frac{\frac{\sqrt{2}x^6}{a^3b}+4bx^3+4}{x^6\left(\sqrt{2}a^3b-x^6\right)}dx^3}{4\sqrt{2}a^3b}+\frac{\sqrt{2}a^3b^2+x^3}{4a^6b^2\left(\sqrt{2}a^3b-x^6\right)}\right)$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{3}\left(\frac{\int \left(\frac{\sqrt{2}(2bx^3+3)}{a^3b\left(\sqrt{2}a^3b-x^6\right)}+\frac{2\sqrt{2}}{a^3x^3}+\frac{2\sqrt{2}}{a^3bx^6}\right)dx^3}{4\sqrt{2}a^3b}+\frac{\sqrt{2}a^3b^2+x^3}{4a^6b^2\left(\sqrt{2}a^3b-x^6\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\frac{3\sqrt[4]{2}\text{arctanh}\left(\frac{x^3}{\sqrt[4]{2}a^{3/2}\sqrt{b}}\right)}{a^{9/2}b^{3/2}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^3b-x^6\right)}{a^3}-\frac{2\sqrt{2}}{a^3bx^3}+\frac{2\sqrt{2}\log(x^3)}{a^3}}{4\sqrt{2}a^3b}+\frac{\sqrt{2}a^3b^2+x^3}{4a^6b^2\left(\sqrt{2}a^3b-x^6\right)}\right)$$

input
```
Int[(1 + b*x^3)/(x^4*(-(Sqrt[2]*a^3*b) + x^6)^2),x]
```

output
```
((Sqrt[2]*a^3*b^2 + x^3)/(4*a^6*b^2*(Sqrt[2]*a^3*b - x^6)) + ((-2*Sqrt[2])
/(a^3*b*x^3) + (3*2^(1/4)*ArcTanh[x^3/(2^(1/4)*a^(3/2)*Sqrt[b])])/(a^(9/2)
*b^(3/2)) + (2*Sqrt[2]*Log[x^3])/a^3 - (Sqrt[2]*Log[Sqrt[2]*a^3*b - x^6])/
a^3)/(4*Sqrt[2]*a^3*b))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{1}{6a^6b^2x^3}+\frac{\ln(x)}{2a^6b}-\frac{\sqrt{2}\left(\frac{-\frac{x^3}{4}-\frac{\sqrt{2}\,a^3b^2}{4}}{-\frac{x^6\sqrt{2}}{2}+a^3b}+\frac{b\sqrt{2}\,\ln\left(x^6\sqrt{2}-2a^3b\right)}{4}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{x^3}{a\sqrt{ab\sqrt{2}}}\right)}{4a\sqrt{ab\sqrt{2}}}\right)}{6a^6b^2}$	122

input
```
int((b*x^3+1)/x^4/(-2^(1/2)*a^3*b+x^6)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^6/b^2/x^3+1/2*ln(x)/a^6/b-1/6*2^(1/2)/a^6/b^2*((-1/4*x^3-1/4*2^(1/2
)*a^3*b^2)/(-1/2*x^6*2^(1/2)+a^3*b)+1/4*b*2^(1/2)*ln(x^6*2^(1/2)-2*a^3*b)-
3/4*2^(1/2)/a/(a*b*2^(1/2))^(1/2)*arctanh(x^3/a/(a*b*2^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.06

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx$$
$$= \left[-\frac{4\,a^7b^3x^3+6\,ax^{12}-8\,a^7b^2-3\,\sqrt{\frac{1}{2}}(x^{15}-2\,a^6b^2x^3)\sqrt{\frac{\sqrt{2}}{ab}}\log\left(\frac{x^{12}+2\,\sqrt{2}a^3bx^6+2\,a^6b^2+2\,\sqrt{\frac{1}{2}}\left(\sqrt{2}a^2bx^9+2\,a^5b^2x^3\right)}{x^{12}-2\,a^6b^2}\right.}{24\,(a^7b^2x^{15}}\right.$$
$$-\frac{2\,a^7b^3x^3+3\,ax^{12}-4\,a^7b^2+3\,\sqrt{\frac{1}{2}}(x^{15}-2\,a^6b^2x^3)\sqrt{-\frac{\sqrt{2}}{ab}}\arctan\left(\frac{\sqrt{\frac{1}{2}}x^3\sqrt{-\frac{\sqrt{2}}{ab}}}{a}\right)+(abx^{15}-2\,a^7b^3x^3)\,\mathrm{l}}{12\,(a^7b^2x^{15}-2\,a^{13}b^4x^3)}$$

input
```
integrate((b*x^3+1)/x^4/(-2^(1/2)*a^3*b+x^6)^2,x, algorithm="fricas")
```

output
```
[-1/24*(4*a^7*b^3*x^3 + 6*a*x^12 - 8*a^7*b^2 - 3*sqrt(1/2)*(x^15 - 2*a^6*b
^2*x^3)*sqrt(sqrt(2)/(a*b))*log((x^12 + 2*sqrt(2)*a^3*b*x^6 + 2*a^6*b^2 +
2*sqrt(1/2)*(sqrt(2)*a^2*b*x^9 + 2*a^5*b^2*x^3)*sqrt(sqrt(2)/(a*b)))/(x^12
 - 2*a^6*b^2)) + 2*(a*b*x^15 - 2*a^7*b^3*x^3)*log(x^6 - sqrt(2)*a^3*b) - 1
2*(a*b*x^15 - 2*a^7*b^3*x^3)*log(x) + 2*sqrt(2)*(a^4*b^2*x^9 + a^4*b*x^6))
/(a^7*b^2*x^15 - 2*a^13*b^4*x^3), -1/12*(2*a^7*b^3*x^3 + 3*a*x^12 - 4*a^7*
b^2 + 3*sqrt(1/2)*(x^15 - 2*a^6*b^2*x^3)*sqrt(-sqrt(2)/(a*b))*arctan(sqrt(
1/2)*x^3*sqrt(-sqrt(2)/(a*b))/a) + (a*b*x^15 - 2*a^7*b^3*x^3)*log(x^6 - sq
rt(2)*a^3*b) - 6*(a*b*x^15 - 2*a^7*b^3*x^3)*log(x) + sqrt(2)*(a^4*b^2*x^9
+ a^4*b*x^6))/(a^7*b^2*x^15 - 2*a^13*b^4*x^3)]
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((b*x**3+1)/x**4/(-2**(1/2)*a**3*b+x**6)**2,x)
```

output
```
Exception raised: PolynomialError >> 1/(606928896*sqrt(2)*_t**6*a**45*b**1
5 + 151732224*sqrt(2)*_t**5*a**39*b**14 + 12644352*sqrt(2)*_t**4*a**33*b**
13 - 24385536*_t**4*a**30*b**10 + 351232*sqrt(2)*_t**3*a**27*b**12 - 40642
56*_t**3*a**24*
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = -\frac{\sqrt{2}a^3b^2x^3+3\,x^6-2\,\sqrt{2}a^3b}{12\,\left(a^6b^2x^9-\sqrt{2}a^9b^3x^3\right)}-\frac{\log\left(x^6-\sqrt{2}a^3b\right)}{12\,a^6b}+\frac{\log\left(x^3\right)}{6\,a^6b}-\frac{2^{\frac{3}{4}}\log\left(\frac{x^3-\sqrt{\sqrt{2}ab}a}{x^3+\sqrt{\sqrt{2}ab}a}\right)}{16\,\sqrt{ab}a^7b^2}$$

input
```
integrate((b*x^3+1)/x^4/(-2^(1/2)*a^3*b+x^6)^2,x, algorithm="maxima")
```

output
```
-1/12*(sqrt(2)*a^3*b^2*x^3 + 3*x^6 - 2*sqrt(2)*a^3*b)/(a^6*b^2*x^9 - sqrt(
2)*a^9*b^3*x^3) - 1/12*log(x^6 - sqrt(2)*a^3*b)/(a^6*b) + 1/6*log(x^3)/(a^
6*b) - 1/16*2^(3/4)*log((x^3 - sqrt(sqrt(2)*a*b)*a)/(x^3 + sqrt(sqrt(2)*a*
b)*a))/(sqrt(a*b)*a^7*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = -\frac{\log\left(x^6-\sqrt{2}a^3b\right)}{12\,a^6b}+\frac{\log\left(|x|\right)}{2\,a^6b}-\frac{\sqrt{2}a^3b^2x^3+3\,x^6-2\,\sqrt{2}a^3b}{12\,\left(x^9-\sqrt{2}a^3bx^3\right)a^6b^2}-\frac{\arctan\left(\frac{x^3}{\sqrt{ab}\sqrt{-\sqrt{2}}|a|}\right)}{4\,\sqrt{ab}a^6b^2\sqrt{-\sqrt{2}}|a|}$$

input
```
integrate((b*x^3+1)/x^4/(-2^(1/2)*a^3*b+x^6)^2,x, algorithm="giac")
```

output
```
-1/12*log(x^6 - sqrt(2)*a^3*b)/(a^6*b) + 1/2*log(abs(x))/(a^6*b) - 1/12*(s
qrt(2)*a^3*b^2*x^3 + 3*x^6 - 2*sqrt(2)*a^3*b)/((x^9 - sqrt(2)*a^3*b*x^3)*a
^6*b^2) - 1/4*arctan(x^3/(sqrt(a*b)*sqrt(-sqrt(2))*abs(a)))/(sqrt(a*b)*a^6
*b^2*sqrt(-sqrt(2))*abs(a))
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.78

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = \frac{\ln(x)}{2\,a^6\,b} - \frac{\frac{\sqrt{2}\,x^3}{12\,a^3} - \frac{\sqrt{2}}{6\,a^3\,b} + \frac{x^6}{4\,a^6\,b^2}}{x^9 - \sqrt{2}\,a^3\,b\,x^3} + \frac{\sqrt{2}\,\ln\left(3\,2^{1/4}\,\sqrt{a^{15}\,b^5} - 14\,\sqrt{2}\,a^9\,b^4 + 3\,a^6\,b^2\,x^3 - 14\,2^{1/4}\,b\,x^3\,\sqrt{a^{15}\,b^5}\right)\left(3\,\sqrt{\sqrt{2}\,a^{15}\,b^5} - 2\,\sqrt{2}\,a^9\,b^4\right)}{48\,a^{15}\,b^5} - \frac{\sqrt{2}\,\ln\left(3\,2^{1/4}\,\sqrt{a^{15}\,b^5} + 14\,\sqrt{2}\,a^9\,b^4 - 3\,a^6\,b^2\,x^3 - 14\,2^{1/4}\,b\,x^3\,\sqrt{a^{15}\,b^5}\right)\left(3\,\sqrt{\sqrt{2}\,a^{15}\,b^5} + 2\,\sqrt{2}\,a^9\,b^4\right)}{48\,a^{15}\,b^5}$$

input
```
int((b*x^3 + 1)/(x^4*(x^6 - 2^(1/2)*a^3*b)^2),x)
```

output
```
log(x)/(2*a^6*b) - ((2^(1/2)*x^3)/(12*a^3) - 2^(1/2)/(6*a^3*b) + x^6/(4*a^
6*b^2))/(x^9 - 2^(1/2)*a^3*b*x^3) + (2^(1/2)*log(3*2^(1/4)*(a^15*b^5)^(1/2
) - 14*2^(1/2)*a^9*b^4 + 3*a^6*b^2*x^3 - 14*2^(1/4)*b*x^3*(a^15*b^5)^(1/2)
)*(3*(2^(1/2)*a^15*b^5)^(1/2) - 2*2^(1/2)*a^9*b^4))/(48*a^15*b^5) - (2^(1/
2)*log(3*2^(1/4)*(a^15*b^5)^(1/2) + 14*2^(1/2)*a^9*b^4 - 3*a^6*b^2*x^3 - 1
4*2^(1/4)*b*x^3*(a^15*b^5)^(1/2))*(3*(2^(1/2)*a^15*b^5)^(1/2) + 2*2^(1/2)*
a^9*b^4))/(48*a^15*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 2054, normalized size of antiderivative = 14.57

$$\int \frac{1+bx^3}{x^4\left(-\sqrt{2}a^3b+x^6\right)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+1)/x^4/(-2^(1/2)*a^3*b+x^6)^2,x)
```

output
```
( - 12*sqrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*atan((b**(1/6)*sqrt(a)*2
**(1/12)*sqrt(3) - 2*x)/(b**(1/6)*sqrt(a)*2**(1/12)))*a**6*b**2*x**3 + 6*s
qrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*atan((b**(1/6)*sqrt(a)*2**(1/12)
*sqrt(3) - 2*x)/(b**(1/6)*sqrt(a)*2**(1/12)))*x**15 + 12*sqrt(b)*sqrt(a)*2
**(2/3)*2**(1/12)*atan((b**(1/6)*sqrt(a)*2**(1/12)*sqrt(3) - 2*x)/(b**(1/6
)*sqrt(a)*2**(1/12)))*a**6*b**2*x**3 - 6*sqrt(b)*sqrt(a)*2**(2/3)*2**(1/12
)*atan((b**(1/6)*sqrt(a)*2**(1/12)*sqrt(3) - 2*x)/(b**(1/6)*sqrt(a)*2**(1/
12)))*x**15 + 12*sqrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*atan((b**(1/6)
*sqrt(a)*2**(1/12)*sqrt(3) + 2*x)/(b**(1/6)*sqrt(a)*2**(1/12)))*a**6*b**2*
x**3 - 6*sqrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*atan((b**(1/6)*sqrt(a)
*2**(1/12)*sqrt(3) + 2*x)/(b**(1/6)*sqrt(a)*2**(1/12)))*x**15 - 12*sqrt(b)
*sqrt(a)*2**(2/3)*2**(1/12)*atan((b**(1/6)*sqrt(a)*2**(1/12)*sqrt(3) + 2*x
)/(b**(1/6)*sqrt(a)*2**(1/12)))*a**6*b**2*x**3 + 6*sqrt(b)*sqrt(a)*2**(2/3
)*2**(1/12)*atan((b**(1/6)*sqrt(a)*2**(1/12)*sqrt(3) + 2*x)/(b**(1/6)*sqrt
(a)*2**(1/12)))*x**15 - 12*sqrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*atan
(x/(b**(1/6)*sqrt(a)*2**(1/12)))*a**6*b**2*x**3 + 6*sqrt(b)*sqrt(a)*sqrt(2
)*2**(1/6)*2**(1/12)*atan(x/(b**(1/6)*sqrt(a)*2**(1/12)))*x**15 + 12*sqrt(
b)*sqrt(a)*2**(2/3)*2**(1/12)*atan(x/(b**(1/6)*sqrt(a)*2**(1/12)))*a**6*b*
*2*x**3 - 6*sqrt(b)*sqrt(a)*2**(2/3)*2**(1/12)*atan(x/(b**(1/6)*sqrt(a)*2*
*(1/12)))*x**15 + 6*sqrt(b)*sqrt(a)*sqrt(2)*2**(1/6)*2**(1/12)*log( - b...
```

3.20 $\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx$

Optimal result

Integrand size = 82, antiderivative size = 68

$$\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx=\frac{-2\sqrt{10}a^3b+\sqrt{2}a^3b^2x^3+3\sqrt{5}x^6}{3a^6b^2x^3\left(\sqrt{2}a^3b-x^6\right)}$$

output
```
1/3*(-2*10^(1/2)*a^3*b+2^(1/2)*a^3*b^2*x^3+3*5^(1/2)*x^6)/a^6/b^2/x^3/(2^(
1/2)*a^3*b-x^6)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx=\frac{-\frac{2\sqrt{5}}{3x^3}+\frac{-\sqrt{2}a^3b^2-\sqrt{5}x^3}{3\left(-\sqrt{2}a^3b+x^6\right)}}{a^6b^2}$$

input
```
Integrate[(4*Sqrt[5]*a^6*b^2 - 3*Sqrt[10]*a^3*b*x^6 + 2*Sqrt[2]*a^3*b^2*x^
9 + 3*Sqrt[5]*x^12)/(a^6*b^2*x^4*(Sqrt[2]*a^3*b - x^6)^2),x]
```

output
```
((-2*Sqrt[5])/(3*x^3) + (-(Sqrt[2]*a^3*b^2) - Sqrt[5]*x^3)/(3*(-(Sqrt[2]*a
^3*b) + x^6)))/(a^6*b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {27, 2368, 27, 281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4\sqrt{5}a^6b^2 + 2\sqrt{2}a^3b^2x^9 - 3\sqrt{10}a^3bx^6 + 3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b - x^6\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\sqrt{5}x^{12}+2\sqrt{2}a^3b^2x^9-3\sqrt{10}a^3bx^6+4\sqrt{5}a^6b^2}{x^4\left(\sqrt{2}a^3b-x^6\right)^2}dx}{a^6b^2}$$

$$\Big\downarrow \text{2368}$$

$$\frac{\frac{x\left(bx^5+\sqrt{5}x^2\right)}{3\left(\sqrt{2}a^3b-x^6\right)} - \frac{\int -\frac{12a^3b\left(2\sqrt{5}a^3b-\sqrt{10}x^6\right)}{x^4\left(\sqrt{2}a^3b-x^6\right)}dx}{6\sqrt{2}a^3b}}{a^6b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{2}\int \frac{2\sqrt{5}a^3b-\sqrt{10}x^6}{x^4\left(\sqrt{2}a^3b-x^6\right)}dx + \frac{x\left(bx^5+\sqrt{5}x^2\right)}{3\left(\sqrt{2}a^3b-x^6\right)}}{a^6b^2}$$

$$\Big\downarrow \text{281}$$

$$\frac{2\sqrt{5}\int \frac{1}{x^4}dx + \frac{x\left(bx^5+\sqrt{5}x^2\right)}{3\left(\sqrt{2}a^3b-x^6\right)}}{a^6b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{x\left(bx^5+\sqrt{5}x^2\right)}{3\left(\sqrt{2}a^3b-x^6\right)} - \frac{2\sqrt{5}}{3x^3}}{a^6b^2}$$

input
```
Int[(4*Sqrt[5]*a^6*b^2 - 3*Sqrt[10]*a^3*b*x^6 + 2*Sqrt[2]*a^3*b^2*x^9 + 3*
Sqrt[5]*x^12)/(a^6*b^2*x^4*(Sqrt[2]*a^3*b - x^6)^2),x]
```

output
```
((-2*Sqrt[5])/(3*x^3) + (x*(Sqrt[5]*x^2 + b*x^5))/(3*(Sqrt[2]*a^3*b - x^6)
))/(a^6*b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{-\frac{2\sqrt{5}}{3x^{3}}+\frac{\sqrt{2}\left(\frac{\sqrt{5}\,x^{3}}{2}+\frac{\sqrt{2}\,a^{3}b^{2}}{2}\right)}{-\frac{3x^{6}\sqrt{2}}{2}+3a^{3}b}}{a^{6}b^{2}}$	58
risch	$\frac{\frac{\sqrt{2}\,\sqrt{5}\,x^{6}}{2}+\frac{a^{3}b^{2}x^{3}}{3}-\frac{2\sqrt{5}\,a^{3}b}{3}}{a^{6}b^{2}x^{3}\left(-\frac{x^{6}\sqrt{2}}{2}+a^{3}b\right)}$	59
parallelrisch	$-\frac{-\sqrt{2}\,a^{3}b^{2}x^{3}-3\sqrt{5}\,x^{6}+2\sqrt{2}\,a^{3}b\sqrt{5}}{3a^{6}b^{2}x^{3}\left(\sqrt{2}\,a^{3}b-x^{6}\right)}$	63
orering	$\frac{\left(\sqrt{5}\,\sqrt{2}\,a^{3}b^{2}x^{3}+15x^{6}-10\sqrt{2}\,a^{3}b\right)\left(4\sqrt{5}\,a^{6}b^{2}-3\sqrt{10}\,a^{3}b\,x^{6}+2\sqrt{2}\,a^{3}b^{2}x^{9}+3\sqrt{5}\,x^{12}\right)}{3x^{3}\left(\sqrt{2}\,a^{3}b-x^{6}\right)\left(2\sqrt{5}\,\sqrt{2}\,a^{3}b^{2}x^{9}+15x^{12}-15\sqrt{2}\,a^{3}b\,x^{6}+20a^{6}b^{2}\right)a^{6}b^{2}}$	150
gosper	$-\frac{\left(x^{6}\sqrt{2}-2a^{3}b\right)\left(2\sqrt{5}\,a^{3}b^{2}x^{3}+15x^{6}\sqrt{2}-20a^{3}b\right)\left(4\sqrt{5}\,a^{6}b^{2}-3\sqrt{10}\,a^{3}b\,x^{6}+2\sqrt{2}\,a^{3}b^{2}x^{9}+3\sqrt{5}\,x^{12}\right)}{6x^{3}\left(2\sqrt{5}\,\sqrt{2}\,a^{3}b^{2}x^{9}+15x^{12}-15\sqrt{2}\,a^{3}b\,x^{6}+20a^{6}b^{2}\right)a^{6}b^{2}\left(\sqrt{2}\,a^{3}b-x^{6}\right)^{2}}$	162

input
```
int((4*5^(1/2)*a^6*b^2-3*10^(1/2)*a^3*b*x^6+2*2^(1/2)*a^3*b^2*x^9+3*5^(1/2
)*x^12)/a^6/b^2/x^4/(2^(1/2)*a^3*b-x^6)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^6/b^2*(-2/3*5^(1/2)/x^3+1/3*2^(1/2)*(1/2*5^(1/2)*x^3+1/2*2^(1/2)*a^3*b
^2)/(-1/2*x^6*2^(1/2)+a^3*b))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((4*5^(1/2)*a^6*b^2-3*10^(1/2)*a^3*b*x^6+2*2^(1/2)*a^3*b^2*x^9+3*
5^(1/2)*x^12)/a^6/b^2/x^4/(2^(1/2)*a^3*b-x^6)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4\sqrt{5}a^6b^2 - 3\sqrt{10}a^3bx^6 + 2\sqrt{2}a^3b^2x^9 + 3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b - x^6\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*5**(1/2)*a**6*b**2-3*10**(1/2)*a**3*b*x**6+2*2**(1/2)*a**3*b*
*2*x**9+3*5**(1/2)*x**12)/a**6/b**2/x**4/(2**(1/2)*a**3*b-x**6)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(57) = 114$.

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \frac{4\sqrt{5}a^6b^2 - 3\sqrt{10}a^3bx^6 + 2\sqrt{2}a^3b^2x^9 + 3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b - x^6\right)^2}\,dx = \frac{\frac{3\cdot 2^{\frac{3}{4}}\left(\sqrt{10}\sqrt{2}-2\sqrt{5}\right)\log\left(\frac{x^3-\sqrt{\sqrt{2}ab}a}{x^3+\sqrt{\sqrt{2}ab}a}\right)}{\sqrt{ab}a} - \frac{4\left(4\sqrt{2}a^3b^2x^3-3\left(\sqrt{10}\sqrt{2}-6\sqrt{5}\right)x^6-8\sqrt{5}\sqrt{2}a^3b\right)}{x^9-\sqrt{2}a^3bx^3}}{48\,a^6b^2}$$

input
```
integrate((4*5^(1/2)*a^6*b^2-3*10^(1/2)*a^3*b*x^6+2*2^(1/2)*a^3*b^2*x^9+3*
5^(1/2)*x^12)/a^6/b^2/x^4/(2^(1/2)*a^3*b-x^6)^2,x, algorithm="maxima")
```

output
```
1/48*(3*2^(3/4)*(sqrt(10)*sqrt(2) - 2*sqrt(5))*log((x^3 - sqrt(sqrt(2)*a*b
)*a)/(x^3 + sqrt(sqrt(2)*a*b)*a))/(sqrt(a*b)*a) - 4*(4*sqrt(2)*a^3*b^2*x^3
 - 3*(sqrt(10)*sqrt(2) - 6*sqrt(5))*x^6 - 8*sqrt(5)*sqrt(2)*a^3*b)/(x^9 -
sqrt(2)*a^3*b*x^3))/(a^6*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx=-\frac{2\,a^3b^2x^3+3\,\sqrt{10}x^6-4\,\sqrt{5}a^3b}{3\left(\sqrt{2}x^9-2\,a^3bx^3\right)a^6b^2}$$

input
```
integrate((4*5^(1/2)*a^6*b^2-3*10^(1/2)*a^3*b*x^6+2*2^(1/2)*a^3*b^2*x^9+3*
5^(1/2)*x^12)/a^6/b^2/x^4/(2^(1/2)*a^3*b-x^6)^2,x, algorithm="giac")
```

output
```
-1/3*(2*a^3*b^2*x^3 + 3*sqrt(10)*x^6 - 4*sqrt(5)*a^3*b)/((sqrt(2)*x^9 - 2*
a^3*b*x^3)*a^6*b^2)
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.26

$$\begin{aligned}
&\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx \\
&=\frac{2\,\sqrt{2}\,\sqrt{5}}{3\,\left(a^3\,b\,x^9-\sqrt{2}\,a^6\,b^2\,x^3\right)}-\frac{3\,\sqrt{5}\,x^6}{2\,\left(a^6\,b^2\,x^9-\sqrt{2}\,a^9\,b^3\,x^3\right)} \\
&-\frac{\sqrt{2}\,x^3}{3\,\left(a^3\,x^9-\sqrt{2}\,a^6\,b\,x^3\right)}+\frac{\sqrt{2}\,\sqrt{10}\,x^6}{4\,\left(a^6\,b^2\,x^9-\sqrt{2}\,a^9\,b^3\,x^3\right)} \\
&-\frac{2^{3/4}\,\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,\sqrt{5}\,x^3\,135\mathrm{i}}{2\,a^{39/2}\,b^{13/2}\,\left(\frac{135\,\sqrt{5}}{a^{18}\,b^6}-\frac{135\,\sqrt{2}\,\sqrt{10}}{2\,a^{18}\,b^6}\right)}-\frac{2^{1/4}\,\sqrt{10}\,x^3\,135\mathrm{i}}{2\,a^{39/2}\,b^{13/2}\,\left(\frac{135\,\sqrt{5}}{a^{18}\,b^6}-\frac{135\,\sqrt{2}\,\sqrt{10}}{2\,a^{18}\,b^6}\right)}\right)\,1\mathrm{i}}{4\,a^{15/2}\,b^{5/2}} \\
&+\frac{2^{1/4}\,\sqrt{10}\,\mathrm{atan}\left(\frac{2^{3/4}\,\sqrt{5}\,x^3\,135\mathrm{i}}{2\,a^{39/2}\,b^{13/2}\,\left(\frac{135\,\sqrt{5}}{a^{18}\,b^6}-\frac{135\,\sqrt{2}\,\sqrt{10}}{2\,a^{18}\,b^6}\right)}-\frac{2^{1/4}\,\sqrt{10}\,x^3\,135\mathrm{i}}{2\,a^{39/2}\,b^{13/2}\,\left(\frac{135\,\sqrt{5}}{a^{18}\,b^6}-\frac{135\,\sqrt{2}\,\sqrt{10}}{2\,a^{18}\,b^6}\right)}\right)\,1\mathrm{i}}{4\,a^{15/2}\,b^{5/2}}
\end{aligned}$$

input
```
int((3*5^(1/2)*x^12 + 4*5^(1/2)*a^6*b^2 + 2*2^(1/2)*a^3*b^2*x^9 - 3*10^(1/
2)*a^3*b*x^6)/(a^6*b^2*x^4*(x^6 - 2^(1/2)*a^3*b)^2),x)
```

output
```
(2*2^(1/2)*5^(1/2))/(3*(a^3*b*x^9 - 2^(1/2)*a^6*b^2*x^3)) - (3*5^(1/2)*x^6
)/(2*(a^6*b^2*x^9 - 2^(1/2)*a^9*b^3*x^3)) - (2^(1/2)*x^3)/(3*(a^3*x^9 - 2^
(1/2)*a^6*b*x^3)) + (2^(1/2)*10^(1/2)*x^6)/(4*(a^6*b^2*x^9 - 2^(1/2)*a^9*b
^3*x^3)) - (2^(3/4)*5^(1/2)*atan((2^(3/4)*5^(1/2)*x^3*135i)/(2*a^(39/2)*b^
(13/2)*((135*5^(1/2))/(a^18*b^6) - (135*2^(1/2)*10^(1/2))/(2*a^18*b^6))) -
 (2^(1/4)*10^(1/2)*x^3*135i)/(2*a^(39/2)*b^(13/2)*((135*5^(1/2))/(a^18*b^6
) - (135*2^(1/2)*10^(1/2))/(2*a^18*b^6))))*1i)/(4*a^(15/2)*b^(5/2)) + (2^(
1/4)*10^(1/2)*atan((2^(3/4)*5^(1/2)*x^3*135i)/(2*a^(39/2)*b^(13/2)*((135*5
^(1/2))/(a^18*b^6) - (135*2^(1/2)*10^(1/2))/(2*a^18*b^6))) - (2^(1/4)*10^(
1/2)*x^3*135i)/(2*a^(39/2)*b^(13/2)*((135*5^(1/2))/(a^18*b^6) - (135*2^(1/
2)*10^(1/2))/(2*a^18*b^6))))*1i)/(4*a^(15/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{4\sqrt{5}a^6b^2-3\sqrt{10}a^3bx^6+2\sqrt{2}a^3b^2x^9+3\sqrt{5}x^{12}}{a^6b^2x^4\left(\sqrt{2}a^3b-x^6\right)^2}\,dx$$
$$=\frac{\sqrt{10}\,a^3b\,x^6-4\sqrt{5}\,a^6b^2+3\sqrt{5}\,x^{12}+\sqrt{2}\,a^3b^2x^9+b\,x^{15}}{3a^6b^2x^3\left(-x^{12}+2a^6b^2\right)}$$

input
```
int((4*5^(1/2)*a^6*b^2-3*10^(1/2)*a^3*b*x^6+2*2^(1/2)*a^3*b^2*x^9+3*5^(1/2
)*x^12)/a^6/b^2/x^4/(2^(1/2)*a^3*b-x^6)^2,x)
```

output
```
(sqrt(10)*a**3*b*x**6 - 4*sqrt(5)*a**6*b**2 + 3*sqrt(5)*x**12 + sqrt(2)*a*
*3*b**2*x**9 + b*x**15)/(3*a**6*b**2*x**3*(2*a**6*b**2 - x**12))
```

3.21 $\int \frac{(\sqrt{5}-ax)(-2a^3+\sqrt{6}x^6)}{a^3bx^3((-2+\sqrt{6})a^3+(-3+\sqrt{6})x^6)}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 41

$$\int \frac{(\sqrt{5}-ax)(-2a^3+\sqrt{6}x^6)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=\frac{\sqrt{\frac{5}{2}\left(5+2\sqrt{6}\right)}-2ax-\sqrt{6}ax}{a^3bx^2}$$

output
```
(1/2*30^(1/2)+5^(1/2)-2*a*x-6^(1/2)*a*x)/a^3/b/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{(\sqrt{5}-ax)(-2a^3+\sqrt{6}x^6)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=\frac{\left(1+\sqrt{\frac{3}{2}}\right)\left(\sqrt{5}-2ax\right)}{a^3bx^2}$$

input
```
Integrate[((Sqrt[5] - a*x)*(-2*a^3 + Sqrt[6]*x^6))/(a^3*b*x^3*((-2 + Sqrt[
6])*a^3 + (-3 + Sqrt[6])*x^6)),x]
```

output
```
((1 + Sqrt[3/2])*(Sqrt[5] - 2*a*x))/(a^3*b*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {27, 281, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sqrt{5}-ax)\left(\sqrt{6}x^6-2a^3\right)}{a^3bx^3\left(\left(\sqrt{6}-2\right)a^3+\left(\sqrt{6}-3\right)x^6\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(\sqrt{5}-ax\right)\left(2a^3-\sqrt{6}x^6\right)}{x^3\left(\left(3-\sqrt{6}\right)x^6+\left(2-\sqrt{6}\right)a^3\right)}dx}{a^3b}$$

$$\Big\downarrow \text{ 281}$$

$$-\frac{\sqrt{6}\int \frac{\sqrt{5}-ax}{x^3}dx}{\left(3-\sqrt{6}\right)a^3b}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{\frac{3}{10}}\left(\sqrt{5}-ax\right)^2}{\left(3-\sqrt{6}\right)a^3bx^2}$$

input
```
Int[((Sqrt[5] - a*x)*(-2*a^3 + Sqrt[6]*x^6))/(a^3*b*x^3*((-2 + Sqrt[6])*a^
3 + (-3 + Sqrt[6])*x^6)),x]
```

output
```
(Sqrt[3/10]*(Sqrt[5] - a*x)^2)/((3 - Sqrt[6])*a^3*b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{-\left(2+\sqrt{6}\right)ax+\frac{\sqrt{6}\,\sqrt{5}}{2}+\sqrt{5}}{a^{3}b\,x^{2}}$	32
default	$\frac{-\frac{2a}{\left(-2+\sqrt{6}\right)x}+\frac{\sqrt{5}}{\left(-2+\sqrt{6}\right)x^{2}}}{a^{3}b}$	36
parallelrisch	$\frac{-2\sqrt{6}\,ax+\sqrt{6}\,\sqrt{5}-4ax+2\sqrt{5}}{2a^{3}b\,x^{2}}$	36
norman	$\frac{\frac{\sqrt{6}\,\sqrt{5}+2\sqrt{5}}{2ab}-\frac{\left(2+\sqrt{6}\right)x}{b}}{x^{2}a^{2}}$	41
orering	$-\frac{\left(-2ax+\sqrt{5}\right)\left(-2a^{3}+\sqrt{6}\,x^{6}\right)}{2x^{2}a^{3}b\left(\left(-2+\sqrt{6}\right)a^{3}+\left(-3+\sqrt{6}\right)x^{6}\right)}$	54
gosper	$-\frac{\left(-2a^{3}+\sqrt{6}\,x^{6}\right)\left(-2ax+\sqrt{5}\right)}{2\left(\sqrt{6}\,x^{6}-3x^{6}+a^{3}\sqrt{6}-2a^{3}\right)b\,a^{3}x^{2}}$	60

input
```
int((5^(1/2)-a*x)*(-2*a^3+6^(1/2)*x^6)/a^3/b/x^3/((-2+6^(1/2))*a^3+(-3+6^(
1/2))*x^6),x,method=_RETURNVERBOSE)
```

output
```
1/a^3/b*(-(2+6^(1/2))*a*x+1/2*6^(1/2)*5^(1/2)+5^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=-\frac{4\,ax+\sqrt{6}\left(2\,ax-\sqrt{5}\right)-2\,\sqrt{5}}{2\,a^3bx^2}$$

input
```
integrate((5^(1/2)-a*x)*(-2*a^3+x^6*6^(1/2))/a^3/b/x^3/((-2+6^(1/2))*a^3+(
-3+6^(1/2))*x^6),x, algorithm="fricas")
```

output
```
-1/2*(4*a*x + sqrt(6)*(2*a*x - sqrt(5)) - 2*sqrt(5))/(a^3*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=\frac{-2\sqrt{6}ax+\sqrt{30}}{2x^2\left(-\sqrt{6}a^3b+3a^3b\right)}$$

input
```
integrate((5**(1/2)-a*x)*(-2*a**3+x**6*6**(1/2))/a**3/b/x**3/((-2+6**(1/2)
)*a**3+(-3+6**(1/2))*x**6),x)
```

output
```
(-2*sqrt(6)*a*x + sqrt(30))/(2*x**2*(-sqrt(6)*a**3*b + 3*a**3*b))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=-\frac{2\,ax-\sqrt{5}}{a^3bx^2\left(\sqrt{6}-2\right)}$$

input
```
integrate((5^(1/2)-a*x)*(-2*a^3+x^6*6^(1/2))/a^3/b/x^3/((-2+6^(1/2))*a^3+(
-3+6^(1/2))*x^6),x, algorithm="maxima")
```

output
```
-(2*a*x - sqrt(5))/(a^3*b*x^2*(sqrt(6) - 2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=-\frac{2\,ax\left(\sqrt{6}+2\right)-\sqrt{30}-2\,\sqrt{5}}{2\,a^3bx^2}$$

input
```
integrate((5^(1/2)-a*x)*(-2*a^3+x^6*6^(1/2))/a^3/b/x^3/((-2+6^(1/2))*a^3+(
-3+6^(1/2))*x^6),x, algorithm="giac")
```

output
```
-1/2*(2*a*x*(sqrt(6) + 2) - sqrt(30) - 2*sqrt(5))/(a^3*b*x^2)
```

Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=\frac{\sqrt{5}+\frac{\sqrt{30}}{2}-a\,x\,\left(\sqrt{6}+2\right)}{a^3\,b\,x^2}$$

input
```
int(-((6^(1/2)*x^6 - 2*a^3)*(a*x - 5^(1/2)))/(a^3*b*x^3*(a^3*(6^(1/2) - 2)
 + x^6*(6^(1/2) - 3))),x)
```

output
```
(5^(1/2) + 30^(1/2)/2 - a*x*(6^(1/2) + 2))/(a^3*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{(\sqrt{5}-ax)\left(-2a^3+\sqrt{6}x^6\right)}{a^3bx^3\left(\left(-2+\sqrt{6}\right)a^3+\left(-3+\sqrt{6}\right)x^6\right)}\,dx=\frac{\sqrt{30}+2\sqrt{5}-2\sqrt{6}\,ax-4ax}{2a^3b\,x^2}$$

input
```
int((5^(1/2)-a*x)*(-2*a^3+x^6*6^(1/2))/a^3/b/x^3/((-2+6^(1/2))*a^3+(-3+6^(
1/2))*x^6),x)
```

output
```
(sqrt(30) + 2*sqrt(5) - 2*sqrt(6)*a*x - 4*a*x)/(2*a**3*b*x**2)
```

3.22 $\int \frac{1}{-a^6-a^3x^3+x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 354

$$\int \frac{1}{-a^6-a^3x^3+x^6}\,dx = \frac{2^{2/3}\arctan\left(\frac{a-2\sqrt[3]{\frac{2}{-1+\sqrt{5}}}x}{\sqrt{3}a}\right)}{\sqrt{15}\left(-1+\sqrt{5}\right)^{2/3}a^5} - \frac{2^{2/3}\arctan\left(\frac{a+2\sqrt[3]{\frac{2}{1+\sqrt{5}}}x}{\sqrt{3}a}\right)}{\sqrt{15}\left(1+\sqrt{5}\right)^{2/3}a^5} + \frac{2^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3\sqrt{5}\left(1+\sqrt{5}\right)^{2/3}a^5} - \frac{2^{2/3}\log\left(\sqrt[3]{-1+\sqrt{5}}a+\sqrt[3]{2}x\right)}{3\sqrt{5}\left(-1+\sqrt{5}\right)^{2/3}a^5} + \frac{\log\left(\left(-1+\sqrt{5}\right)^{2/3}a^2-\sqrt[3]{2\left(-1+\sqrt{5}\right)}ax+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{5}\left(-1+\sqrt{5}\right)^{2/3}a^5} - \frac{\log\left(\left(1+\sqrt{5}\right)^{2/3}a^2+\sqrt[3]{2\left(1+\sqrt{5}\right)}ax+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{5}\left(1+\sqrt{5}\right)^{2/3}a^5}$$

output
```
1/15*2^(2/3)*arctan(1/3*(a-2*2^(1/3)*(1/(5^(1/2)-1))^(1/3)*x)*3^(1/2)/a)*1
5^(1/2)/(5^(1/2)-1)^(2/3)/a^5-1/15*2^(2/3)*arctan(1/3*(a+2*2^(1/3)*(1/(5^(
1/2)+1))^(1/3)*x)*3^(1/2)/a)*15^(1/2)/(5^(1/2)+1)^(2/3)/a^5+1/15*2^(2/3)*l
n((5^(1/2)+1)^(1/3)*a-2^(1/3)*x)*5^(1/2)/(5^(1/2)+1)^(2/3)/a^5-1/15*2^(2/3
)*ln((5^(1/2)-1)^(1/3)*a+2^(1/3)*x)*5^(1/2)/(5^(1/2)-1)^(2/3)/a^5+1/30*ln(
(5^(1/2)-1)^(2/3)*a^2-(-2+2*5^(1/2))^(1/3)*a*x+2^(2/3)*x^2)*2^(2/3)*5^(1/2
)/(5^(1/2)-1)^(2/3)/a^5-1/30*ln((5^(1/2)+1)^(2/3)*a^2+(2+2*5^(1/2))^(1/3)*
a*x+2^(2/3)*x^2)*2^(2/3)*5^(1/2)/(5^(1/2)+1)^(2/3)/a^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.14

$$\int \frac{1}{-a^6 - a^3x^3 + x^6} \, dx = -\frac{1}{3}\text{RootSum}\left[a^6 + a^3\#1^3 - \#1^6\&, \frac{\log(x - \#1)}{a^3\#1^2 - 2\#1^5}\&\right]$$

input
```
Integrate[(-a^6 - a^3*x^3 + x^6)^(-1),x]
```

output
```
-1/3*RootSum[a^6 + a^3*#1^3 - #1^6 & , Log[x - #1]/(a^3*#1^2 - 2*#1^5) & ]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1685, 750, 16, 25, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-a^6 - a^3x^3 + x^6} \, dx$$

$$\Big\downarrow \text{ 1685}$$

$$\frac{\int \frac{1}{x^3 - \frac{1}{2}\left(1+\sqrt{5}\right)a^3}dx}{\sqrt{5}a^3} - \frac{\int \frac{1}{x^3 - \frac{1}{2}\left(1-\sqrt{5}\right)a^3}dx}{\sqrt{5}a^3}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\int -\frac{2^{2/3}\sqrt[3]{1+\sqrt{5}}a+x}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}+\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\int \frac{1}{x-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}a}dx}{3a^2}}{\sqrt{5}a^3}-\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\int \frac{2^{2/3}\sqrt[3]{-1+\sqrt{5}}a-x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}a+x}dx}{3a^2}}{\sqrt{5}a^3}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\int -\frac{2^{2/3}\sqrt[3]{1+\sqrt{5}}a+x}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}+\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}}{\sqrt{5}a^3}-\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\int \frac{2^{2/3}\sqrt[3]{-1+\sqrt{5}}a-x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\log\left(\sqrt[3]{\sqrt{5}-1}a+\sqrt[3]{2}x\right)}{3a^2}}{\sqrt{5}a^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\int \frac{2^{2/3}\sqrt[3]{1+\sqrt{5}}a+x}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}}{\sqrt{5}a^3}-\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\int \frac{2^{2/3}\sqrt[3]{-1+\sqrt{5}}a-x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\log\left(\sqrt[3]{\sqrt{5}-1}a+\sqrt[3]{2}x\right)}{3a^2}}{\sqrt{5}a^3}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\left(\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}a\int\frac{1}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx+\frac{1}{2}\int\frac{\cdots}{\left(\frac{1}{2}\left(1+\cdots\right.\right.}\right.}{3a^2}}{\sqrt{5}a^3}$$

$$\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\left(\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{5}-1\right)}a\int\frac{1}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx-\frac{1}{2}\int-\frac{\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}a-2x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x\cdots}\right.}{3a^2}}{\sqrt{5}a^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\left(\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}a\int\frac{1}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx+\frac{1}{2}\int\frac{\cdots}{\left(\frac{1}{2}\left(1+\cdots\right.\right.}\right.}{3a^2}}{\sqrt{5}a^3}$$

$$\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\left(\frac{3}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{5}-1\right)}a\int\frac{1}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx+\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}a-2x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}\cdots\right.}{3a^2}}{\sqrt{5}a^3}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\left(\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}a+2x}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx-3\int\frac{1}{-\left(\frac{2\sqrt[3]{\frac{2}{1+\sqrt{5}}}x}{a}+1\right)^2-3 \cdots}\right.}{3a^2}}{\sqrt{5}a^3}$$

$$\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\left(\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}a-2x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx+3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{\frac{2}{-1+\sqrt{5}}}x}{a}\right)^2-3}d\left(1-\frac{2\sqrt[3]{\frac{2}{-1+\sqrt{5}}}x}{a}\right)\right)}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-\cdots}\right.}{\cdots}}{\sqrt{5}a^3}$$

$\Big\downarrow$ 217

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\left(\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}a+2x}{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}a^2+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}xa+x^2}dx+\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{\frac{2}{1+\sqrt{5}}}x}{a}+1}{\sqrt{3}}\right.\cdots\right.}{3a^2}}{\sqrt{5}a^3}$$

$$\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\left(\frac{1}{2}\int\frac{\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}a-2x}{\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)^{2/3}a^2-\sqrt[3]{\frac{1}{2}\left(-1+\sqrt{5}\right)}xa+x^2}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\frac{2}{\sqrt{5}-1}}x}{a}}{\sqrt{3}}\right)\right)}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\log\left(\sqrt[3]{\sqrt{5}-1}a+\cdots\right.}{3a^2}}{\sqrt{5}a^3}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\log\left(\sqrt[3]{1+\sqrt{5}}a-\sqrt[3]{2}x\right)}{3a^2}-\frac{\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\left(\frac{1}{2}\log\left(\left(1+\sqrt{5}\right)^{2/3}a^2+\sqrt[3]{2\left(1+\sqrt{5}\right)}ax+2^{2/3}x^2\right)+\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\frac{2}{1+\sqrt{5}}}}{\frac{a}{\sqrt{3}}}\right.\right.}{3a^2}}{\sqrt{5}a^3}$$

$$\frac{\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\left(-\frac{1}{2}\log\left(\left(\sqrt{5}-1\right)^{2/3}a^2-\sqrt[3]{2\left(\sqrt{5}-1\right)}ax+2^{2/3}x^2\right)-\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{\frac{2}{\sqrt{5}-1}}x}{a}}{\sqrt{3}}\right)\right)}{3a^2}+\frac{\left(\frac{2}{\sqrt{5}-1}\right)^{2/3}\log\left(\sqrt[3]{\sqrt{5}-1}a\right.}{3a^2}}{\sqrt{5}a^3}$$

input
```
Int[(-a^6 - a^3*x^3 + x^6)^(-1),x]
```

output
```
-((((2/(-1 + Sqrt[5]))^(2/3)*Log[(-1 + Sqrt[5])^(1/3)*a + 2^(1/3)*x])/(3*a
^2) + ((2/(-1 + Sqrt[5]))^(2/3)*(-(Sqrt[3]*ArcTan[(1 - (2*(2/(-1 + Sqrt[5]
))^(1/3)*x)/a)/Sqrt[3]]) - Log[(-1 + Sqrt[5])^(2/3)*a^2 - (2*(-1 + Sqrt[5]
))^(1/3)*a*x + 2^(2/3)*x^2]/2))/(3*a^2))/(Sqrt[5]*a^3)) + (((2/(1 + Sqrt[5
]))^(2/3)*Log[(1 + Sqrt[5])^(1/3)*a - 2^(1/3)*x])/(3*a^2) - ((2/(1 + Sqrt[
5]))^(2/3)*(Sqrt[3]*ArcTan[(1 + (2*(2/(1 + Sqrt[5]))^(1/3)*x)/a)/Sqrt[3]]
+ Log[(1 + Sqrt[5])^(2/3)*a^2 + (2*(1 + Sqrt[5]))^(1/3)*a*x + 2^(2/3)*x^2]
/2))/(3*a^2))/(Sqrt[5]*a^3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.13

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-a^{3}\_Z^{3}-a^{6}\right)}\frac{\ln\left(x-\_R\right)}{-2\_R^{5}+\_R^{2}a^{3}}\right)}{3}$	46
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-a^{3}\_Z^{3}-a^{6}\right)}\frac{\ln\left(x-\_R\right)}{-2\_R^{5}+\_R^{2}a^{3}}\right)}{3}$	46

input
```
int(1/(-a^6-a^3*x^3+x^6),x,method=_RETURNVERBOSE)
```

output
```
-1/3*sum(1/(-2*_R^5+_R^2*a^3)*ln(x-_R),_R=RootOf(_Z^6-_Z^3*a^3-a^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(255) = 510.

Time = 0.08 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.52

$$\int \frac{1}{-a^6-a^3x^3+x^6}\,dx=\text{Too large to display}$$

input
```
integrate(1/(-a^6-a^3*x^3+x^6),x, algorithm="fricas")
```

output

```
-1/6*(1/10)^(1/3)*(sqrt(-3) + 1)*(-(3*sqrt(1/5)*a^15*sqrt(a^(-30)) + 1)/a^
15)^(1/3)*log(5*(1/10)^(1/3)*(sqrt(-3)*a^6 + a^6 - sqrt(1/5)*(sqrt(-3)*a^2
1 + a^21)*sqrt(a^(-30)))*(-(3*sqrt(1/5)*a^15*sqrt(a^(-30)) + 1)/a^15)^(1/3
) + 4*x) + 1/6*(1/10)^(1/3)*(sqrt(-3) - 1)*(-(3*sqrt(1/5)*a^15*sqrt(a^(-30
)) + 1)/a^15)^(1/3)*log(-5*(1/10)^(1/3)*(sqrt(-3)*a^6 - a^6 - sqrt(1/5)*(s
qrt(-3)*a^21 - a^21)*sqrt(a^(-30)))*(-(3*sqrt(1/5)*a^15*sqrt(a^(-30)) + 1)
/a^15)^(1/3) + 4*x) - 1/6*(1/10)^(1/3)*(sqrt(-3) + 1)*((3*sqrt(1/5)*a^15*s
qrt(a^(-30)) - 1)/a^15)^(1/3)*log(5*(1/10)^(1/3)*(sqrt(-3)*a^6 + a^6 + sqr
t(1/5)*(sqrt(-3)*a^21 + a^21)*sqrt(a^(-30)))*((3*sqrt(1/5)*a^15*sqrt(a^(-3
0)) - 1)/a^15)^(1/3) + 4*x) + 1/6*(1/10)^(1/3)*(sqrt(-3) - 1)*((3*sqrt(1/5
)*a^15*sqrt(a^(-30)) - 1)/a^15)^(1/3)*log(-5*(1/10)^(1/3)*(sqrt(-3)*a^6 -
a^6 + sqrt(1/5)*(sqrt(-3)*a^21 - a^21)*sqrt(a^(-30)))*((3*sqrt(1/5)*a^15*s
qrt(a^(-30)) - 1)/a^15)^(1/3) + 4*x) + 1/3*(1/10)^(1/3)*(-(3*sqrt(1/5)*a^1
5*sqrt(a^(-30)) + 1)/a^15)^(1/3)*log(5*(1/10)^(1/3)*(sqrt(1/5)*a^21*sqrt(a
^(-30)) - a^6)*(-(3*sqrt(1/5)*a^15*sqrt(a^(-30)) + 1)/a^15)^(1/3) + 2*x) +
 1/3*(1/10)^(1/3)*((3*sqrt(1/5)*a^15*sqrt(a^(-30)) - 1)/a^15)^(1/3)*log(-5
*(1/10)^(1/3)*(sqrt(1/5)*a^21*sqrt(a^(-30)) + a^6)*((3*sqrt(1/5)*a^15*sqrt
(a^(-30)) - 1)/a^15)^(1/3) + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.09

$$\int \frac{1}{-a^6 - a^3x^3 + x^6}\,dx = \frac{\text{RootSum}\left(91125t^6 + 675t^3 - 1, \left(t \mapsto t\log\left(-675t^4a - 10ta + x\right)\right)\right)}{a^5}$$

input

```
integrate(1/(-a**6-a**3*x**3+x**6),x)
```

output

```
RootSum(91125*_t**6 + 675*_t**3 - 1, Lambda(_t, _t*log(-675*_t**4*a - 10*_
t*a + x)))/a**5
```

Maxima [F]

$$\int \frac{1}{-a^6 - a^3x^3 + x^6}\,dx = \int -\frac{1}{a^6 + a^3x^3 - x^6}\,dx$$

input
```
integrate(1/(-a^6-a^3*x^3+x^6),x, algorithm="maxima")
```

output
```
-integrate(1/(a^6 + a^3*x^3 - x^6), x)
```

Giac [F]

$$\int \frac{1}{-a^6 - a^3x^3 + x^6}\,dx = \int -\frac{1}{a^6 + a^3x^3 - x^6}\,dx$$

input
```
integrate(1/(-a^6-a^3*x^3+x^6),x, algorithm="giac")
```

output
```
integrate(-1/(a^6 + a^3*x^3 - x^6), x)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.90

$$\int \frac{1}{-a^6 - a^3x^3 + x^6}\,dx = \text{Too large to display}$$

input
```
int(-1/(a^6 - x^6 + a^3*x^3),x)
```

output
```
log(6*x + (9*2^(2/3)*5^(1/3)*a^6*(3^(1/2)*1i - 1)*(3*5^(1/2) - 5)^(1/3)*(1
/a^15)^(1/3)*(5^(1/2) + (2^(1/3)*5^(2/3)*a^9*x*(3*5^(1/2) - 5)^(2/3)*(1/a^
15)^(2/3))/6 + (2^(1/3)*3^(1/2)*5^(2/3)*a^9*x*(3*5^(1/2) - 5)^(2/3)*(1/a^1
5)^(2/3)*1i)/6 - 5))/40)*((3^(1/2)*1i)/2 - 1/2)*((3*5^(1/2) - 5)/(1350*a^1
5))^(1/3) - log(6*x - (9*2^(2/3)*5^(1/3)*a^6*(3^(1/2)*1i + 1)*(3*5^(1/2) -
 5)^(1/3)*(1/a^15)^(1/3)*(5^(1/2) + (2^(1/3)*5^(2/3)*a^9*x*(3*5^(1/2) - 5)
^(2/3)*(1/a^15)^(2/3))/6 - (2^(1/3)*3^(1/2)*5^(2/3)*a^9*x*(3*5^(1/2) - 5)^
(2/3)*(1/a^15)^(2/3)*1i)/6 - 5))/40)*((3^(1/2)*1i)/2 + 1/2)*((3*5^(1/2) -
5)/(1350*a^15))^(1/3) + log(6*x - (9*2^(2/3)*5^(1/3)*a^6*(3^(1/2)*1i - 1)*
(3*5^(1/2) + 5)^(1/3)*(-1/a^15)^(1/3)*(5^(1/2) - (2^(1/3)*5^(2/3)*a^9*x*(3
*5^(1/2) + 5)^(2/3)*(-1/a^15)^(2/3))/6 - (2^(1/3)*3^(1/2)*5^(2/3)*a^9*x*(3
*5^(1/2) + 5)^(2/3)*(-1/a^15)^(2/3)*1i)/6 + 5))/40)*((3^(1/2)*1i)/2 - 1/2)
*(-(3*5^(1/2) + 5)/(1350*a^15))^(1/3) - log(6*x + (9*2^(2/3)*5^(1/3)*a^6*(
3^(1/2)*1i + 1)*(3*5^(1/2) + 5)^(1/3)*(-1/a^15)^(1/3)*(5^(1/2) - (2^(1/3)*
5^(2/3)*a^9*x*(3*5^(1/2) + 5)^(2/3)*(-1/a^15)^(2/3))/6 + (2^(1/3)*3^(1/2)*
5^(2/3)*a^9*x*(3*5^(1/2) + 5)^(2/3)*(-1/a^15)^(2/3)*1i)/6 + 5))/40)*((3^(1
/2)*1i)/2 + 1/2)*(-(3*5^(1/2) + 5)/(1350*a^15))^(1/3) + (1350^(2/3)*log(40
50*x - 1350*5^(1/2)*x - 450*2^(2/3)*5^(1/3)*a^21*(3*5^(1/2) - 5)^(1/3)*(1/
a^15)^(4/3) + 45*2^(2/3)*5^(1/3)*a^21*(3*5^(1/2) - 5)^(4/3)*(1/a^15)^(4/3)
)*(3*5^(1/2) - 5)^(1/3)*(1/a^15)^(1/3))/1350 + (1350^(2/3)*log(4050*x +...
```

Reduce [F]

$$\int \frac{1}{-a^6-a^3x^3+x^6}\,dx=-\left(\int \frac{1}{a^6+a^3x^3-x^6}dx\right)$$

input
```
int(1/(-a^6-a^3*x^3+x^6),x)
```

output
```
 - int(1/(a**6 + a**3*x**3 - x**6),x)
```

3.23 $\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx = \frac{-6b^5-8ab^4x-9ab^3x^2-3a^2b^3x^2-20a^2b^2x^3+9a^2bx^4+3a^3bx^4+15a^3x^5}{12b^5x^4\left(2b^2-ax^2\right)} + \frac{5a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{2}b}\right)}{4\sqrt{2}b^6} + \frac{(3a^2+a^3)\log(x)}{4b^6} + \frac{(-3a^2-a^3)\log\left(-2b^2+ax^2\right)}{8b^6}$$

output
```
1/12*(3*a^3*b*x^4+15*a^3*x^5-3*a^2*b^3*x^2-20*a^2*b^2*x^3+9*a^2*b*x^4-8*a*
b^4*x-9*a*b^3*x^2-6*b^5)/b^5/x^4/(-a*x^2+2*b^2)+5/8*a^(5/2)*arctanh(1/2*a^
(1/2)*x*2^(1/2)/b)*2^(1/2)/b^6+1/4*(a^3+3*a^2)*ln(x)/b^6+1/8*(-a^3-3*a^2)*
ln(a*x^2-2*b^2)/b^6
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx =$$
$$-\frac{\frac{6b^4}{x^4}+\frac{8ab^3}{x^3}+\frac{3a(4+a)b^2}{x^2}+\frac{24a^2b}{x}+\frac{3a^2b((2+a)b+2ax)}{-2b^2+ax^2}-15\sqrt{2}a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{2}b}\right)-6a^2(3+a)\log(x)+3a^2(3}{24b^6}$$

input
```
Integrate[(2*b + a*x)^2/(x*(-2*b^2*x^2 + a*x^4)^2),x]
```

output
```
-1/24*((6*b^4)/x^4 + (8*a*b^3)/x^3 + (3*a*(4 + a)*b^2)/x^2 + (24*a^2*b)/x
+ (3*a^2*b*((2 + a)*b + 2*a*x))/(-2*b^2 + a*x^2) - 15*Sqrt[2]*a^(5/2)*ArcT
anh[(Sqrt[a]*x)/(Sqrt[2]*b)] - 6*a^2*(3 + a)*Log[x] + 3*a^2*(3 + a)*Log[2*
b^2 - a*x^2])/b^6
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {9, 532, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+2b)^2}{x\left(ax^4-2b^2x^2\right)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(ax+2b)^2}{x^5\left(2b^2-ax^2\right)^2}dx$$

$$\Big\downarrow \; 532$$

$$\frac{a^2((a+2)b+2ax)}{8b^5\left(2b^2-ax^2\right)}-\frac{\int -\frac{\frac{a^3x^5}{b^3}+\frac{a^2(a+2)x^4}{b^2}+\frac{4a^2x^3}{b}+2a(a+2)x^2+8abx+8b^2}{x^5\left(2b^2-ax^2\right)}dx}{4b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{a^3x^5}{b^3}+\frac{a^2(a+2)x^4}{b^2}+\frac{4a^2x^3}{b}+2a(a+2)x^2+8abx+8b^2}{x^5(2b^2-ax^2)}dx}{4b^2}+\frac{a^2((a+2)b+2ax)}{8b^5(2b^2-ax^2)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{(5b+(a+3)x)a^3}{b^4(2b^2-ax^2)}+\frac{(a+3)a^2}{b^4x}+\frac{4a^2}{b^3x^2}+\frac{(a+4)a}{b^2x^3}+\frac{4a}{bx^4}+\frac{4}{x^5}\right)dx}{4b^2}+\frac{a^2((a+2)b+2ax)}{8b^5(2b^2-ax^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2((a+2)b+2ax)}{8b^5(2b^2-ax^2)}+$$
$$\frac{\frac{5a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{2}b}\right)}{\sqrt{2}b^4}+\frac{(a+3)a^2\log(x)}{b^4}-\frac{4a^2}{b^3x}-\frac{(a+3)a^2\log(2b^2-ax^2)}{2b^4}-\frac{(a+4)a}{2b^2x^2}-\frac{4a}{3bx^3}-\frac{1}{x^4}}{4b^2}$$

input
```
Int[(2*b + a*x)^2/(x*(-2*b^2*x^2 + a*x^4)^2),x]
```

output
```
(a^2*((2 + a)*b + 2*a*x))/(8*b^5*(2*b^2 - a*x^2)) + (-x^(-4) - (4*a)/(3*b*
x^3) - (a*(4 + a))/(2*b^2*x^2) - (4*a^2)/(b^3*x) + (5*a^(5/2)*ArcTanh[(Sqr
t[a]*x)/(Sqrt[2]*b)])/(Sqrt[2]*b^4) + (a^2*(3 + a)*Log[x])/b^4 - (a^2*(3 +
 a)*Log[2*b^2 - a*x^2])/(2*b^4))/(4*b^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a^{3}\left(\frac{b x+\frac{b^{2}(a+2)}{2 a}}{a\,x^{2}-2 b^{2}}+\frac{(3+a)\ln\left(a\,x^{2}-2 b^{2}\right)}{2 a}-\frac{5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a}\,x\sqrt{2}}{2 b}\right)}{2\sqrt{a}}\right)}{4 b^{6}}-\frac{1}{4 b^{2} x^{4}}-\frac{a}{3 b^{3} x^{3}}-\frac{a^{2}}{b^{5} x}-\frac{a(4+a)}{8 b^{4} x^{2}}+\frac{a^{2}(3+a)\ln(x)}{4 b^{6}}$
risch	$\frac{-\frac{5 a^{3} x^{5}}{4 b^{5}}-\frac{a^{2}(3+a) x^{4}}{4 b^{4}}+\frac{5 a^{2} x^{3}}{3 b^{3}}+\frac{a(3+a) x^{2}}{4 b^{2}}+\frac{2 a x}{3 b}+\frac{1}{2}}{x^{4}\left(a\,x^{2}-2 b^{2}\right)}+\frac{5 a^{\frac{5}{2}}\ln\left(\left(\left(3\sqrt{2}\,a^{\frac{11}{2}}+9\sqrt{2}\,a^{\frac{9}{2}}-5 a^{5}\right) x-5\sqrt{2}\,a^{\frac{9}{2}} b+6 a^{5} b+18 a^{4} b\right)\sqrt{2}}{16 b^{6}}-\frac{a^{3}\ln\left(\right.}{}$

input
```
int((a*x+2*b)^2/x/(a*x^4-2*b^2*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/b^6*((b*x+1/2*b^2*(a+2)/a)/(a*x^2-2*b^2)+1/2*(3+a)/a*ln(a*x^2-2*b
^2)-5/2*2^(1/2)/a^(1/2)*arctanh(1/2*a^(1/2)*x*2^(1/2)/b))-1/4/b^2/x^4-1/3*
a/b^3/x^3-a^2/b^5/x-1/8*a*(4+a)/b^4/x^2+1/4*a^2*(3+a)/b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.52

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx$$
$$= \left[-\frac{30\,a^3bx^5-40\,a^2b^3x^3-16\,ab^5x-6\,(a^2+3\,a)b^4x^2+6\,(a^3+3\,a^2)b^2x^4-12\,b^6-15\,\sqrt{\tfrac{1}{2}}(a^3x^6-2\,a^2b^2x\right.$$

$$-\frac{30\,a^3bx^5-40\,a^2b^3x^3-16\,ab^5x-6\,(a^2+3\,a)b^4x^2+6\,(a^3+3\,a^2)b^2x^4-12\,b^6+30\,\sqrt{\tfrac{1}{2}}(a^3x^6-2\,a^2b^2x}{}$$

input
```
integrate((a*x+2*b)^2/x/(a*x^4-2*b^2*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(30*a^3*b*x^5 - 40*a^2*b^3*x^3 - 16*a*b^5*x - 6*(a^2 + 3*a)*b^4*x^2
 + 6*(a^3 + 3*a^2)*b^2*x^4 - 12*b^6 - 15*sqrt(1/2)*(a^3*x^6 - 2*a^2*b^2*x^
4)*sqrt(a)*log((4*sqrt(1/2)*sqrt(a)*b*x + a*x^2 + 2*b^2)/(a*x^2 - 2*b^2))
- 3*(2*(a^3 + 3*a^2)*b^2*x^4 - (a^4 + 3*a^3)*x^6)*log(a*x^2 - 2*b^2) + 6*(
2*(a^3 + 3*a^2)*b^2*x^4 - (a^4 + 3*a^3)*x^6)*log(x))/(a*b^6*x^6 - 2*b^8*x^
4), -1/24*(30*a^3*b*x^5 - 40*a^2*b^3*x^3 - 16*a*b^5*x - 6*(a^2 + 3*a)*b^4*
x^2 + 6*(a^3 + 3*a^2)*b^2*x^4 - 12*b^6 + 30*sqrt(1/2)*(a^3*x^6 - 2*a^2*b^2
*x^4)*sqrt(-a)*arctan(sqrt(1/2)*sqrt(-a)*x/b) - 3*(2*(a^3 + 3*a^2)*b^2*x^4
 - (a^4 + 3*a^3)*x^6)*log(a*x^2 - 2*b^2) + 6*(2*(a^3 + 3*a^2)*b^2*x^4 - (a
^4 + 3*a^3)*x^6)*log(x))/(a*b^6*x^6 - 2*b^8*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(182) = 364$.

Time = 1.29 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.27

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*x+2*b)**2/x/(a*x**4-2*b**2*x**2)**2,x)
```

output
```
a**2*(a + 3)*log(x + (24*a**7*b + 24*a**6*b*(a + 3) + 316*a**6*b - 48*a**5
*b*(a + 3)**2 + 44*a**5*b*(a + 3) + 948*a**5*b - 144*a**4*b*(a + 3)**2 + 2
16*a**4*b*(a + 3) + 648*a**4*b)/(90*a**7 + 415*a**6 + 810*a**5))/(4*b**6)
+ (-a**2*(a + 3)/(8*b**6) - 5*sqrt(2)*sqrt(a**5)/(16*b**6))*log(x + (24*a*
*7*b + 316*a**6*b + 948*a**5*b + 96*a**4*b**7*(-a**2*(a + 3)/(8*b**6) - 5*
sqrt(2)*sqrt(a**5)/(16*b**6)) + 648*a**4*b + 176*a**3*b**7*(-a**2*(a + 3)/
(8*b**6) - 5*sqrt(2)*sqrt(a**5)/(16*b**6)) + 864*a**2*b**7*(-a**2*(a + 3)/
(8*b**6) - 5*sqrt(2)*sqrt(a**5)/(16*b**6)) - 768*a*b**13*(-a**2*(a + 3)/(8
*b**6) - 5*sqrt(2)*sqrt(a**5)/(16*b**6))**2 - 2304*b**13*(-a**2*(a + 3)/(8
*b**6) - 5*sqrt(2)*sqrt(a**5)/(16*b**6))**2)/(90*a**7 + 415*a**6 + 810*a**
5)) + (-a**2*(a + 3)/(8*b**6) + 5*sqrt(2)*sqrt(a**5)/(16*b**6))*log(x + (2
4*a**7*b + 316*a**6*b + 948*a**5*b + 96*a**4*b**7*(-a**2*(a + 3)/(8*b**6)
+ 5*sqrt(2)*sqrt(a**5)/(16*b**6)) + 648*a**4*b + 176*a**3*b**7*(-a**2*(a +
 3)/(8*b**6) + 5*sqrt(2)*sqrt(a**5)/(16*b**6)) + 864*a**2*b**7*(-a**2*(a +
 3)/(8*b**6) + 5*sqrt(2)*sqrt(a**5)/(16*b**6)) - 768*a*b**13*(-a**2*(a + 3
)/(8*b**6) + 5*sqrt(2)*sqrt(a**5)/(16*b**6))**2 - 2304*b**13*(-a**2*(a + 3
)/(8*b**6) + 5*sqrt(2)*sqrt(a**5)/(16*b**6))**2)/(90*a**7 + 415*a**6 + 810
*a**5)) + (-15*a**3*x**5 + 20*a**2*b**2*x**3 + 8*a*b**4*x + 6*b**5 + x**4*
(-3*a**3*b - 9*a**2*b) + x**2*(3*a**2*b**3 + 9*a*b**3))/(12*a*b**5*x**6 -
24*b**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx = -\frac{15\,a^3x^5-20\,a^2b^2x^3-8\,ab^4x-3\,(a^2+3\,a)b^3x^2+3\,(a^3+3\,a^2)bx^4-6\,b^5}{12\,(ab^5x^6-2\,b^7x^4)} - \frac{5\,\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}b-ax}{\sqrt{2}\sqrt{a}b+ax}\right)}{16\,b^6} - \frac{(a^3+3\,a^2)\log\left(ax^2-2\,b^2\right)}{8\,b^6} + \frac{(a^3+3\,a^2)\log\left(x\right)}{4\,b^6}$$

input
```
integrate((a*x+2*b)^2/x/(a*x^4-2*b^2*x^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(15*a^3*x^5 - 20*a^2*b^2*x^3 - 8*a*b^4*x - 3*(a^2 + 3*a)*b^3*x^2 + 3
*(a^3 + 3*a^2)*b*x^4 - 6*b^5)/(a*b^5*x^6 - 2*b^7*x^4) - 5/16*sqrt(2)*a^(5/
2)*log(-(sqrt(2)*sqrt(a)*b - a*x)/(sqrt(2)*sqrt(a)*b + a*x))/b^6 - 1/8*(a^
3 + 3*a^2)*log(a*x^2 - 2*b^2)/b^6 + 1/4*(a^3 + 3*a^2)*log(x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97

$$\int \frac{(2b+ax)^2}{x(-2b^2x^2+ax^4)^2}\,dx = -\frac{5\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}ax}{2\sqrt{-ab}}\right)}{8\sqrt{-ab}b^6} - \frac{(a^3+3a^2)\log(ax^2-2b^2)}{8b^6} + \frac{(a^3+3a^2)\log(|x|)}{4b^6} - \frac{15a^3bx^5-20a^2b^3x^3-8ab^5x-6b^6+3(a^3b^2+3a^2b^2)x^4-3(a^2b^4+3ab^4)x^2}{12(ax^2-2b^2)b^6x^4}$$

input
```
integrate((a*x+2*b)^2/x/(a*x^4-2*b^2*x^2)^2,x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*a^3*arctan(1/2*sqrt(2)*a*x/(sqrt(-a)*b))/(sqrt(-a)*b^6) - 1/8
*(a^3 + 3*a^2)*log(a*x^2 - 2*b^2)/b^6 + 1/4*(a^3 + 3*a^2)*log(abs(x))/b^6
- 1/12*(15*a^3*b*x^5 - 20*a^2*b^3*x^3 - 8*a*b^5*x - 6*b^6 + 3*(a^3*b^2 + 3
*a^2*b^2)*x^4 - 3*(a^2*b^4 + 3*a*b^4)*x^2)/((a*x^2 - 2*b^2)*b^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.51

$$\int \frac{(2b+ax)^2}{x(-2b^2x^2+ax^4)^2}\,dx = \frac{\frac{5a^2x^3}{3b^3}-\frac{5a^3x^5}{4b^5}+\frac{x^2(a^2+3a)}{4b^2}+\frac{2ax}{3b}-\frac{ax^4(a^2+3a)}{4b^4}+\frac{1}{2}}{ax^6-2b^2x^4} + \frac{\ln(x)\,(a^3+3a^2)}{4b^6} - \frac{\ln\left(18a^6b+6a^7b-5a^7x-3\sqrt{2}x(a^5)^{3/2}+5\sqrt{2}a^4b\sqrt{a^5}-9\sqrt{2}a^4x\sqrt{a^5}\right)\left(6a^2+2a^3+5\sqrt{2}\sqrt{a\cdots}\right.}{16b^6} - \frac{\ln\left(18a^6b+6a^7b-5a^7x+3\sqrt{2}x(a^5)^{3/2}-5\sqrt{2}a^4b\sqrt{a^5}+9\sqrt{2}a^4x\sqrt{a^5}\right)\left(6a^2+2a^3-5\sqrt{2}\sqrt{a\cdots}\right.}{16b^6}$$

input
```
int((2*b + a*x)^2/(x*(a*x^4 - 2*b^2*x^2)^2),x)
```

output
```
((5*a^2*x^3)/(3*b^3) - (5*a^3*x^5)/(4*b^5) + (x^2*(3*a + a^2))/(4*b^2) + (
2*a*x)/(3*b) - (a*x^4*(3*a + a^2))/(4*b^4) + 1/2)/(a*x^6 - 2*b^2*x^4) + (l
og(x)*(3*a^2 + a^3))/(4*b^6) - (log(18*a^6*b + 6*a^7*b - 5*a^7*x - 3*2^(1/
2)*x*(a^5)^(3/2) + 5*2^(1/2)*a^4*b*(a^5)^(1/2) - 9*2^(1/2)*a^4*x*(a^5)^(1/
2))*(6*a^2 + 2*a^3 + 5*2^(1/2)*(a^5)^(1/2)))/(16*b^6) - (log(18*a^6*b + 6*
a^7*b - 5*a^7*x + 3*2^(1/2)*x*(a^5)^(3/2) - 5*2^(1/2)*a^4*b*(a^5)^(1/2) +
9*2^(1/2)*a^4*x*(a^5)^(1/2))*(6*a^2 + 2*a^3 - 5*2^(1/2)*(a^5)^(1/2)))/(16*
b^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.30

$$\int \frac{(2b+ax)^2}{x\left(-2b^2x^2+ax^4\right)^2}\,dx = \frac{-15\sqrt{a}\,\sqrt{2}\log\left(-\sqrt{a}\,\sqrt{2}\,b+ax\right)a^3x^6+30\sqrt{a}\,\sqrt{2}\log\left(-\sqrt{a}\,\sqrt{2}\,b+ax\right)a^2b^2x^4+15\sqrt{a}\,\sqrt{2}\log\left(\sqrt{a}\,\sqrt{2}\,b\right.}{}$$

input
```
int((a*x+2*b)^2/x/(a*x^4-2*b^2*x^2)^2,x)
```

output
```
( - 15*sqrt(a)*sqrt(2)*log( - sqrt(a)*sqrt(2)*b + a*x)*a**3*x**6 + 30*sqrt
(a)*sqrt(2)*log( - sqrt(a)*sqrt(2)*b + a*x)*a**2*b**2*x**4 + 15*sqrt(a)*sq
rt(2)*log(sqrt(a)*sqrt(2)*b + a*x)*a**3*x**6 - 30*sqrt(a)*sqrt(2)*log(sqrt
(a)*sqrt(2)*b + a*x)*a**2*b**2*x**4 - 6*log( - sqrt(a)*sqrt(2)*b + a*x)*a*
*4*x**6 + 12*log( - sqrt(a)*sqrt(2)*b + a*x)*a**3*b**2*x**4 - 18*log( - sq
rt(a)*sqrt(2)*b + a*x)*a**3*x**6 + 36*log( - sqrt(a)*sqrt(2)*b + a*x)*a**2
*b**2*x**4 - 6*log(sqrt(a)*sqrt(2)*b + a*x)*a**4*x**6 + 12*log(sqrt(a)*sqr
t(2)*b + a*x)*a**3*b**2*x**4 - 18*log(sqrt(a)*sqrt(2)*b + a*x)*a**3*x**6 +
 36*log(sqrt(a)*sqrt(2)*b + a*x)*a**2*b**2*x**4 + 12*log(x)*a**4*x**6 - 24
*log(x)*a**3*b**2*x**4 + 36*log(x)*a**3*x**6 - 72*log(x)*a**2*b**2*x**4 -
6*a**4*x**6 - 60*a**3*b*x**5 - 18*a**3*x**6 + 12*a**2*b**4*x**2 + 80*a**2*
b**3*x**3 + 32*a*b**5*x + 36*a*b**4*x**2 + 24*b**6)/(48*b**6*x**4*(a*x**2
- 2*b**2))
```

3.24 $\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 117

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx=-\frac{2b}{3cx^3}+\frac{b(4b^4-2ac+abc)\operatorname{arctanh}\left(\frac{b^2-ax^3}{\sqrt{b^4-ac}}\right)}{3c^2\sqrt{b^4-ac}}+\frac{(4b^3+ac)\log(x)}{c^2}-\frac{(4b^3+ac)\log\left(c-2b^2x^3+ax^6\right)}{6c^2}$$

output
```
-2/3*b/c/x^3+1/3*b*(4*b^4+a*b*c-2*a*c)*arctanh((-a*x^3+b^2)/(b^4-a*c)^(1/2
))/c^2/(b^4-a*c)^(1/2)+(4*b^3+a*c)*ln(x)/c^2-1/6*(4*b^3+a*c)*ln(a*x^6-2*b^
2*x^3+c)/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx=-\frac{\frac{4bc}{x^3}-6(4b^3+ac)\log(x)+\operatorname{RootSum}\left[c-2b^2\#1^3+a\#1^6\&,\frac{8b^5\log(x-\#1)-2abc\log(x-\#1)+2ab^2c\log(x-\#1)-}{b^2-a\#1^3}\right.}{6c^2}$$

input
```
Integrate[(2*b + a*x^3)/(x^4*(c - 2*b^2*x^3 + a*x^6)),x]
```

output
```
-1/6*((4*b*c)/x^3 - 6*(4*b^3 + a*c)*Log[x] + RootSum[c - 2*b^2*#1^3 + a*#1
^6 & , (8*b^5*Log[x - #1] - 2*a*b*c*Log[x - #1] + 2*a*b^2*c*Log[x - #1] -
4*a*b^3*Log[x - #1]*#1^3 - a^2*c*Log[x - #1]*#1^3)/(b^2 - a*#1^3) & ])/c^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1802, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^3+2b}{x^4\left(ax^6-2b^2x^3+c\right)}dx$$

$$\Big\downarrow \text{ 1802}$$

$$\frac{1}{3}\int \frac{ax^3+2b}{x^6\left(ax^6-2b^2x^3+c\right)}dx^3$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{2b}{cx^6}+\frac{2b\left(4b^4+acb-ac\right)-a\left(4b^3+ac\right)x^3}{c^2\left(ax^6-2b^2x^3+c\right)}+\frac{4b^3+ac}{c^2x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b\left(abc-2ac+4b^4\right)\text{arctanh}\left(\frac{b^2-ax^3}{\sqrt{b^4-ac}}\right)}{c^2\sqrt{b^4-ac}}+\frac{\log\left(x^3\right)\left(ac+4b^3\right)}{c^2}-\frac{\left(ac+4b^3\right)\log\left(ax^6-2b^2x^3+c\right)}{2c^2}-\frac{2b}{cx^3}\right)$$

input
```
Int[(2*b + a*x^3)/(x^4*(c - 2*b^2*x^3 + a*x^6)),x]
```

output
```
((-2*b)/(c*x^3) + (b*(4*b^4 - 2*a*c + a*b*c)*ArcTanh[(b^2 - a*x^3)/Sqrt[b^
4 - a*c]])/(c^2*Sqrt[b^4 - a*c]) + ((4*b^3 + a*c)*Log[x^3])/c^2 - ((4*b^3
+ a*c)*Log[c - 2*b^2*x^3 + a*x^6])/(2*c^2))/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

method	result
default	$\frac{\left(4b^{3}+ac\right)\ln\left(x\right)}{c^{2}}-\frac{2b}{3c\,x^{3}}-\frac{\frac{\left(4a\,b^{3}+a^{2}c\right)\ln\left(a\,x^{6}-2b^{2}x^{3}+c\right)}{2a}-\frac{\left(-8b^{5}-2a\,b^{2}c+2abc+\frac{\left(4a\,b^{3}+a^{2}c\right)b^{2}}{a}\right)\operatorname{arctanh}\left(\frac{2a\,x^{3}-2b^{2}}{2\sqrt{b^{4}-ac}}\right)}{\sqrt{b^{4}-ac}}}{3c^{2}}$
risch	$-\frac{2b}{3c\,x^{3}}+\frac{4\ln\left(x\right)b^{3}}{c^{2}}+\frac{\ln\left(x\right)a}{c}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(-c^{2}b^{4}+a\,c^{3}\right)\_Z^{2}+\left(-8b^{7}-2a\,b^{4}c+8a\,b^{3}c+2a^{2}c^{2}\right)\_Z+4a^{2}b^{3}+a^{3}c+4a^{2}b^{2}\right)}\_R\ln\right.}{}$

input
```
int((a*x^3+2*b)/x^4/(a*x^6-2*b^2*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(4*b^3+a*c)*ln(x)/c^2-2/3*b/c/x^3-1/3/c^2*(1/2*(4*a*b^3+a^2*c)/a*ln(a*x^6-
2*b^2*x^3+c)-(-8*b^5-2*a*b^2*c+2*a*b*c+(4*a*b^3+a^2*c)*b^2/a)/(b^4-a*c)^(1
/2)*arctanh(1/2*(2*a*x^3-2*b^2)/(b^4-a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.48

$$\int \frac{2b+ax^3}{x^4(c-2b^2x^3+ax^6)}\,dx = \left[-\frac{4\,b^5c-(4\,b^5+(ab^2-2\,ab)c)\sqrt{b^4-ac}x^3\log\left(\frac{a^2x^6-2\,ab^2x^3+2\,b^4-ac-2\,\sqrt{b^4-ac}(ax^3-b^2)}{ax^6-2\,b^2x^3+c}\right)+(4\,b^7-a^2c^2+(a\cdots}{6\,(b^4c^2-ac^3)\cdots}\right.$$

$$-\frac{4\,b^5c+2\,(4\,b^5+(ab^2-2\,ab)c)\sqrt{-b^4+ac}x^3\arctan\left(-\frac{\sqrt{-b^4+ac}(ax^3-b^2)}{b^4-ac}\right)+(4\,b^7-a^2c^2+(ab^4-4\,ab^3)\cdots}{6\,(b^4c^2-ac^3)x^3}$$

input
```
integrate((a*x^3+2*b)/x^4/(a*x^6-2*b^2*x^3+c),x, algorithm="fricas")
```

output
```
[-1/6*(4*b^5*c - (4*b^5 + (a*b^2 - 2*a*b)*c)*sqrt(b^4 - a*c)*x^3*log((a^2*
x^6 - 2*a*b^2*x^3 + 2*b^4 - a*c - 2*sqrt(b^4 - a*c)*(a*x^3 - b^2))/(a*x^6
- 2*b^2*x^3 + c)) + (4*b^7 - a^2*c^2 + (a*b^4 - 4*a*b^3)*c)*x^3*log(a*x^6
- 2*b^2*x^3 + c) - 6*(4*b^7 - a^2*c^2 + (a*b^4 - 4*a*b^3)*c)*x^3*log(x) -
4*a*b*c^2)/((b^4*c^2 - a*c^3)*x^3), -1/6*(4*b^5*c + 2*(4*b^5 + (a*b^2 - 2*
a*b)*c)*sqrt(-b^4 + a*c)*x^3*arctan(-sqrt(-b^4 + a*c)*(a*x^3 - b^2)/(b^4 -
 a*c)) + (4*b^7 - a^2*c^2 + (a*b^4 - 4*a*b^3)*c)*x^3*log(a*x^6 - 2*b^2*x^3
 + c) - 6*(4*b^7 - a^2*c^2 + (a*b^4 - 4*a*b^3)*c)*x^3*log(x) - 4*a*b*c^2)/
((b^4*c^2 - a*c^3)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx=\text{Timed out}$$

input
```
integrate((a*x**3+2*b)/x**4/(a*x**6-2*b**2*x**3+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*x^3+2*b)/x^4/(a*x^6-2*b^2*x^3+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^4-4*a*c>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx=-\frac{\left(4\,b^3+ac\right)\log\left(ax^6-2\,b^2x^3+c\right)}{6\,c^2}+\frac{\left(4\,b^3+ac\right)\log\left(|x|\right)}{c^2}+\frac{\left(4\,b^5+ab^2c-2\,abc\right)\arctan\left(\frac{ax^3-b^2}{\sqrt{-b^4+ac}}\right)}{3\,\sqrt{-b^4+ac}c^2}-\frac{4\,b^3x^3+acx^3+2\,bc}{3\,c^2x^3}$$

input
```
integrate((a*x^3+2*b)/x^4/(a*x^6-2*b^2*x^3+c),x, algorithm="giac")
```

output
```
-1/6*(4*b^3 + a*c)*log(a*x^6 - 2*b^2*x^3 + c)/c^2 + (4*b^3 + a*c)*log(abs(
x))/c^2 + 1/3*(4*b^5 + a*b^2*c - 2*a*b*c)*arctan((a*x^3 - b^2)/sqrt(-b^4 +
 a*c))/(sqrt(-b^4 + a*c)*c^2) - 1/3*(4*b^3*x^3 + a*c*x^3 + 2*b*c)/(c^2*x^3
)
```

Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 7097, normalized size of antiderivative = 60.66

$$\int \frac{2b + ax^3}{x^4 (c - 2b^2x^3 + ax^6)} \, dx = \text{Too large to display}$$

input
```
int((2*b + a*x^3)/(x^4*(c + a*x^6 - 2*b^2*x^3)),x)
```

output
```
(log(x)*(a*c + 4*b^3))/c^2 - (2*b)/(3*c*x^3) + (log(((((((((a*c + 4*b^3 + c
^2*(-(b^2*(4*b^4 - 2*a*c + a*b*c)^2)/(c^4*(a*c - b^4)))^(1/2))*((216*a^3*b
^5*(4*b^4 - a*c + a*b*c))/c + (36*a^3*b^4*(a*c + 4*b^3 + c^2*(-(b^2*(4*b^4
 - 2*a*c + a*b*c)^2)/(c^4*(a*c - b^4)))^(1/2))*(b^2*c - 8*b^4*x^3 + 7*a*c*
x^3))/c^2 - (36*a^4*b^3*x^3*(28*a*c - 8*b^4 + 7*a*b*c))/c))/(6*c^2) - (72*
a^4*b^4*(12*b^4 - a*c + 3*a*b*c))/c^2 + (24*a^5*b^2*x^3*(7*a*c + 22*b^4 +
7*a*b*c))/c^2)*(a*c + 4*b^3 + c^2*(-(b^2*(4*b^4 - 2*a*c + a*b*c)^2)/(c^4*(
a*c - b^4)))^(1/2)))/(6*c^2) + (8*a^5*b^3*(36*b^4 - a*c + 9*a*b*c))/c^3 -
(4*a^6*b^2*x^3*(7*a*c + 48*b^3))/c^3)*(a*c + 4*b^3 + c^2*(-(b^2*(4*b^4 - 2
*a*c + a*b*c)^2)/(c^4*(a*c - b^4)))^(1/2)))/(6*c^2) - (8*a^6*b^3*(a*c + 4*
b^3))/c^4 + (16*a^7*b^4*x^3)/c^4)*(((((((a*c + 4*b^3 - c^2*(-(b^2*(4*b^4 -
 2*a*c + a*b*c)^2)/(c^4*(a*c - b^4)))^(1/2))*((216*a^3*b^5*(4*b^4 - a*c +
a*b*c))/c + (36*a^3*b^4*(a*c + 4*b^3 - c^2*(-(b^2*(4*b^4 - 2*a*c + a*b*c)^
2)/(c^4*(a*c - b^4)))^(1/2))*(b^2*c - 8*b^4*x^3 + 7*a*c*x^3))/c^2 - (36*a^
4*b^3*x^3*(28*a*c - 8*b^4 + 7*a*b*c))/c))/(6*c^2) - (72*a^4*b^4*(12*b^4 -
a*c + 3*a*b*c))/c^2 + (24*a^5*b^2*x^3*(7*a*c + 22*b^4 + 7*a*b*c))/c^2)*(a*
c + 4*b^3 - c^2*(-(b^2*(4*b^4 - 2*a*c + a*b*c)^2)/(c^4*(a*c - b^4)))^(1/2)
))/(6*c^2) + (8*a^5*b^3*(36*b^4 - a*c + 9*a*b*c))/c^3 - (4*a^6*b^2*x^3*(7*
a*c + 48*b^3))/c^3)*(a*c + 4*b^3 - c^2*(-(b^2*(4*b^4 - 2*a*c + a*b*c)^2)/(
c^4*(a*c - b^4)))^(1/2)))/(6*c^2) - (8*a^6*b^3*(a*c + 4*b^3))/c^4 + (16...
```

Reduce [F]

$$\int \frac{2b+ax^3}{x^4\left(c-2b^2x^3+ax^6\right)}\,dx$$
$$= \frac{-6\left(\int \frac{1}{-2a\,b^4x^{10}+a^2c\,x^{10}+4b^6x^7-2a\,b^2c\,x^7-2b^4c\,x^4+a\,c^2x^4}dx\right)a^2b^2c^2x^3+12\left(\int \frac{1}{-2a\,b^4x^{10}+a^2c\,x^{10}+4b^6x^7-2a\,b^2c\,x^7-2b^4c\,x^4+a}\right.}{}$$

input

```
int((a*x^3+2*b)/x^4/(a*x^6-2*b^2*x^3+c),x)
```

output

```
( - 6*int(1/(a**2*c*x**10 - 2*a*b**4*x**10 - 2*a*b**2*c*x**7 + a*c**2*x**4
 + 4*b**6*x**7 - 2*b**4*c*x**4),x)*a**2*b**2*c**2*x**3 + 12*int(1/(a**2*c*
x**10 - 2*a*b**4*x**10 - 2*a*b**2*c*x**7 + a*c**2*x**4 + 4*b**6*x**7 - 2*b
**4*c*x**4),x)*a**2*b*c**2*x**3 + 12*int(1/(a**2*c*x**10 - 2*a*b**4*x**10
- 2*a*b**2*c*x**7 + a*c**2*x**4 + 4*b**6*x**7 - 2*b**4*c*x**4),x)*a*b**6*c
*x**3 - 48*int(1/(a**2*c*x**10 - 2*a*b**4*x**10 - 2*a*b**2*c*x**7 + a*c**2
*x**4 + 4*b**6*x**7 - 2*b**4*c*x**4),x)*a*b**5*c*x**3 + 48*int(1/(a**2*c*x
**10 - 2*a*b**4*x**10 - 2*a*b**2*c*x**7 + a*c**2*x**4 + 4*b**6*x**7 - 2*b*
*4*c*x**4),x)*b**9*x**3 - log(a*x**6 - 2*b**2*x**3 + c)*a**2*x**3 + 6*log(
x)*a**2*x**3 - 2*a*b**2)/(6*x**3*(a*c - 2*b**4))
```

3.25 $\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx=\frac{1}{bx}+\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{2}\sqrt{b}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{2}\sqrt{b}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^2}{\sqrt{2}b}\right)}{2\sqrt{2}b}$$

output
```
1/b/x+1/4*a^(1/4)*arctan(1/2*a^(1/4)*x*2^(3/4)/b^(1/2))*2^(3/4)/b^(3/2)-1/
4*a^(1/4)*arctanh(1/2*a^(1/4)*x*2^(3/4)/b^(1/2))*2^(3/4)/b^(3/2)-1/4*a^(1/
2)*arctanh(1/2*a^(1/2)*x^2*2^(1/2)/b)*2^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.59

$$\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx$$
$$=\frac{\frac{16b}{x}+4\,2^{3/4}\sqrt[4]{a}\sqrt{b}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{2}\sqrt{b}}\right)+2^{3/4}\sqrt[4]{a}\left(2\sqrt{b}+2^{3/4}\sqrt[4]{a}b\right)\log\left(2\sqrt{b}-2^{3/4}\sqrt[4]{a}x\right)+2^{3/4}\sqrt[4]{a}\left(-2\sqrt{b}\right.}{16b^2}$$

input
```
Integrate[(2*b + a*x^3)/(-2*b^2*x^2 + a*x^6),x]
```

output
```
((16*b)/x + 4*2^(3/4)*a^(1/4)*Sqrt[b]*ArcTan[(a^(1/4)*x)/(2^(1/4)*Sqrt[b])
] + 2^(3/4)*a^(1/4)*(2*Sqrt[b] + 2^(3/4)*a^(1/4)*b)*Log[2*Sqrt[b] - 2^(3/4
)*a^(1/4)*x] + 2^(3/4)*a^(1/4)*(-2*Sqrt[b] + 2^(3/4)*a^(1/4)*b)*Log[2*Sqrt
[b] + 2^(3/4)*a^(1/4)*x] - 2*Sqrt[2]*Sqrt[a]*b*Log[2*b + Sqrt[2]*Sqrt[a]*x
^2])/(16*b^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2026, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^3+2b}{ax^6-2b^2x^2}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{ax^3+2b}{x^2\left(ax^4-2b^2\right)}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{ax}{ax^4-2b^2}+\frac{2b}{x^2\left(ax^4-2b^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{2}\sqrt{b}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{2}\sqrt{b}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x^2}{\sqrt{2}b}\right)}{2\sqrt{2}b}+\frac{1}{bx}$$

input
```
Int[(2*b + a*x^3)/(-2*b^2*x^2 + a*x^6),x]
```

output
```
1/(b*x) + (a^(1/4)*ArcTan[(a^(1/4)*x)/(2^(1/4)*Sqrt[b])])/(2*2^(1/4)*b^(3/
2)) - (a^(1/4)*ArcTanh[(a^(1/4)*x)/(2^(1/4)*Sqrt[b])])/(2*2^(1/4)*b^(3/2))
 - (Sqrt[a]*ArcTanh[(Sqrt[a]*x^2)/(Sqrt[2]*b)])/(2*Sqrt[2]*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
risch	$\frac{1}{bx}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4b^{6}\_Z^{4}-4a\,b^{4}\_Z^{2}-8a\,b^{2}\_Z+a^{2}b^{2}-2a\right)}\_R\ln\left(\left(-10\_R^{4}b^{6}+9\_R^{2}a\,b^{4}+17\_Ra\,b^{2}-2a^{2}b^{2}+4a\right)x+2b^{5}\_\right.\right.}{4}$
default	$\frac{1}{bx}+\frac{a\left(\frac{b\sqrt{2}\,\ln\left(\frac{-2b^{2}+x^{2}\sqrt{2}\,\sqrt{a\,b^{2}}}{-2b^{2}-x^{2}\sqrt{2}\,\sqrt{a\,b^{2}}}\right)}{8\sqrt{a\,b^{2}}}+\frac{2\arctan\left(\frac{x}{\sqrt{\sqrt{2}\,\sqrt{\frac{b^{2}}{a}}}}\right)-\ln\left(\frac{x+\sqrt{\sqrt{2}\,\sqrt{\frac{b^{2}}{a}}}}{x-\sqrt{\sqrt{2}\,\sqrt{\frac{b^{2}}{a}}}}\right)}{4a\sqrt{\sqrt{2}\,\sqrt{\frac{b^{2}}{a}}}}\right)}{b}$

input
```
int((a*x^3+2*b)/(a*x^6-2*b^2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/b/x+1/4*sum(_R*ln((-10*_R^4*b^6+9*_R^2*a*b^4+17*_R*a*b^2-2*a^2*b^2+4*a)*
x+2*b^5*_R^3+a*b^3*_R),_R=RootOf(4*_Z^4*b^6-4*_Z^2*a*b^4-8*_Z*a*b^2+a^2*b^
2-2*a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 17307, normalized size of antiderivative = 147.92

$$\int \frac{2b + ax^3}{-2b^2x^2 + ax^6}\,dx = \text{Too large to display}$$

input
```
integrate((a*x^3+2*b)/(a*x^6-2*b^2*x^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{2b + ax^3}{-2b^2x^2 + ax^6}\,dx$$
$$= \operatorname{RootSum}\left(1024t^4b^6 - 64t^2ab^4 - 32tab^2 + a^2b^2 - 2a, \left(t \mapsto t\log\left(x + \frac{-128t^3b^5 - 32t^2ab^5 + 12tab^3 + a\right.\right.\right.}{2a^2b^2 + a}$$
$$+ \frac{1}{bx}$$

input
```
integrate((a*x**3+2*b)/(a*x**6-2*b**2*x**2),x)
```

output
```
RootSum(1024*_t**4*b**6 - 64*_t**2*a*b**4 - 32*_t*a*b**2 + a**2*b**2 - 2*a
, Lambda(_t, _t*log(x + (-128*_t**3*b**5 - 32*_t**2*a*b**5 + 12*_t*a*b**3
+ a**2*b**3 + 3*a*b)/(2*a**2*b**2 + a)))) + 1/(b*x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

$$\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx = -\frac{a\left(\frac{\sqrt{2}\log\left(\sqrt{a}x^2+\sqrt{2}b\right)}{\sqrt{a}} - \frac{\sqrt{2}\log\left(\sqrt{a}x^2-\sqrt{2}b\right)}{\sqrt{a}} - \frac{2\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\sqrt{a}x}{2\sqrt{\sqrt{a}b}}\right)}{\sqrt{\sqrt{a}b}\sqrt{a}} - \frac{2^{\frac{3}{4}}\log\left(\frac{\sqrt{a}x-\sqrt{\sqrt{2}\sqrt{a}b}}{\sqrt{a}x+\sqrt{\sqrt{2}\sqrt{a}b}}\right)}{\sqrt{\sqrt{a}b}\sqrt{a}}\right)}{8\,b} + \frac{1}{bx}$$

input
```
integrate((a*x^3+2*b)/(a*x^6-2*b^2*x^2),x, algorithm="maxima")
```

output
```
-1/8*a*(sqrt(2)*log(sqrt(a)*x^2 + sqrt(2)*b)/sqrt(a) - sqrt(2)*log(sqrt(a)
*x^2 - sqrt(2)*b)/sqrt(a) - 2*2^(3/4)*arctan(1/2*2^(3/4)*sqrt(a)*x/sqrt(sq
rt(a)*b))/(sqrt(sqrt(a)*b)*sqrt(a)) - 2^(3/4)*log((sqrt(a)*x - sqrt(sqrt(2
)*sqrt(a)*b))/(sqrt(a)*x + sqrt(sqrt(2)*sqrt(a)*b)))/(sqrt(sqrt(a)*b)*sqrt
(a)))/b + 1/(b*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(82) = 164$.

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.29

$$\int \frac{2b + ax^3}{-2b^2x^2 + ax^6}\,dx = \frac{1}{bx} + \frac{\sqrt{2}\left(2\sqrt{-a^3b^2}ab - 2^{\frac{3}{4}}(-a^3b^2)^{\frac{3}{4}}\right)\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}+2x\right)}{2\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}}\right)}{8a^2b^3} - \frac{\sqrt{2}\left(2\sqrt{-a^3b^2}ab + 2^{\frac{3}{4}}(-a^3b^2)^{\frac{3}{4}}\right)\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}-2x\right)}{2\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}}\right)}{8a^2b^3} + \frac{2^{\frac{1}{4}}(-a^3b^2)^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}x + x^2 + \sqrt{2}\sqrt{-\frac{b^2}{a}}\right)}{8a^2b^3} - \frac{2^{\frac{1}{4}}(-a^3b^2)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-\frac{b^2}{a}\right)^{\frac{1}{4}}x + x^2 + \sqrt{2}\sqrt{-\frac{b^2}{a}}\right)}{8a^2b^3}$$

input

```
integrate((a*x^3+2*b)/(a*x^6-2*b^2*x^2),x, algorithm="giac")
```

output

```
1/(b*x) + 1/8*sqrt(2)*(2*sqrt(-a^3*b^2)*a*b - 2^(3/4)*(-a^3*b^2)^(3/4))*ar
ctan(1/2*2^(1/4)*(2^(3/4)*(-b^2/a)^(1/4) + 2*x)/(-b^2/a)^(1/4))/(a^2*b^3)
- 1/8*sqrt(2)*(2*sqrt(-a^3*b^2)*a*b + 2^(3/4)*(-a^3*b^2)^(3/4))*arctan(-1/
2*2^(1/4)*(2^(3/4)*(-b^2/a)^(1/4) - 2*x)/(-b^2/a)^(1/4))/(a^2*b^3) + 1/8*2
^(1/4)*(-a^3*b^2)^(3/4)*log(2^(3/4)*(-b^2/a)^(1/4)*x + x^2 + sqrt(2)*sqrt(
-b^2/a))/(a^2*b^3) - 1/8*2^(1/4)*(-a^3*b^2)^(3/4)*log(-2^(3/4)*(-b^2/a)^(1
/4)*x + x^2 + sqrt(2)*sqrt(-b^2/a))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.68

$$\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx$$
$$=\left(\sum_{k=1}^{4}\ln\left(a^4\,b^4\,\left(\mathrm{root}\left(1024\,b^6\,z^4-64\,a\,b^4\,z^2-32\,a\,b^2\,z+a^2\,b^2-2\,a,z,k\right)\,b^2\,16-\mathrm{root}\left(1024\,b^6\,z^4-64\,a\,b^4\,z^2\right.\right.\right.\right.$$
$$\left.\left.-64\,a\,b^4\,z^2-32\,a\,b^2\,z+a^2\,b^2-2\,a,z,k\right)\right)+\frac{1}{b\,x}$$

input
```
int((2*b + a*x^3)/(a*x^6 - 2*b^2*x^2),x)
```

output
```
symsum(log(a^4*b^4*(16*root(1024*b^6*z^4 - 64*a*b^4*z^2 - 32*a*b^2*z + a^2
*b^2 - 2*a, z, k)*b^2 - 32*root(1024*b^6*z^4 - 64*a*b^4*z^2 - 32*a*b^2*z +
 a^2*b^2 - 2*a, z, k)^2*b^3*x - 8*root(1024*b^6*z^4 - 64*a*b^4*z^2 - 32*a*
b^2*z + a^2*b^2 - 2*a, z, k)*b*x + a*b*x + 2))*root(1024*b^6*z^4 - 64*a*b^
4*z^2 - 32*a*b^2*z + a^2*b^2 - 2*a, z, k), k, 1, 4) + 1/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{2b+ax^3}{-2b^2x^2+ax^6}\,dx$$
$$=\frac{2\sqrt{b}\,a^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}\,atan\left(\frac{\sqrt{a}\,x2^{\frac{3}{4}}}{2\sqrt{b}\,a^{\frac{1}{4}}}\right)x+\sqrt{b}\,a^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}x-\sqrt{b}\,2^{\frac{1}{4}}\right)x-\sqrt{b}\,a^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}x+\sqrt{b}\,2^{\frac{1}{4}}\right)x+}{8b^2x}$$

input
```
int((a*x^3+2*b)/(a*x^6-2*b^2*x^2),x)
```

output

```
(2*sqrt(b)*a**(1/4)*sqrt(2)*2**(1/4)*atan((sqrt(a)*x)/(sqrt(b)*a**(1/4)*2*
*(1/4)))*x + sqrt(b)*a**(1/4)*sqrt(2)*2**(1/4)*log(a**(1/4)*x - sqrt(b)*2*
*(1/4))*x - sqrt(b)*a**(1/4)*sqrt(2)*2**(1/4)*log(a**(1/4)*x + sqrt(b)*2**
(1/4))*x + sqrt(a)*sqrt(2)*log(a**(1/4)*x - sqrt(b)*2**(1/4))*b*x + sqrt(a
)*sqrt(2)*log(a**(1/4)*x + sqrt(b)*2**(1/4))*b*x - sqrt(a)*sqrt(2)*log(sqr
t(a)*x**2 + sqrt(2)*b)*b*x + 8*b)/(8*b**2*x)
```

3.26 $\int \frac{-b+ax}{-bx^2+ax^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx=-\frac{1}{x}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt[4]{a}\,\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\log(x)}{b}+\frac{a\log\left(b-ax^4\right)}{4b}$$

output
```
-1/x-1/2*a^(1/4)*arctan(a^(1/4)*x/b^(1/4))/b^(1/4)+1/2*a^(1/4)*arctanh(a^(
1/4)*x/b^(1/4))/b^(1/4)-a*ln(x)/b+1/4*a*ln(-a*x^4+b)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx=-\frac{1}{x}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\log(x)}{b}-\frac{\sqrt[4]{a}\log\left(\sqrt[4]{b}-\sqrt[4]{a}x\right)}{4\sqrt[4]{b}}+\frac{\sqrt[4]{a}\log\left(\sqrt[4]{b}+\sqrt[4]{a}x\right)}{4\sqrt[4]{b}}+\frac{a\log\left(b-ax^4\right)}{4b}$$

input
```
Integrate[(-b + a*x)/(-(b*x^2) + a*x^6),x]
```

output
```
-x^(-1) - (a^(1/4)*ArcTan[(a^(1/4)*x)/b^(1/4)])/(2*b^(1/4)) - (a*Log[x])/b
 - (a^(1/4)*Log[b^(1/4) - a^(1/4)*x])/(4*b^(1/4)) + (a^(1/4)*Log[b^(1/4) +
 a^(1/4)*x])/(4*b^(1/4)) + (a*Log[b - a*x^4])/(4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2026, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax-b}{ax^6-bx^2}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{ax-b}{x^2(ax^4-b)}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{a}{x(ax^4-b)}-\frac{b}{x^2(ax^4-b)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}+\frac{a\log\left(b-ax^4\right)}{4b}-\frac{a\log(x)}{b}-\frac{1}{x}$$

input
```
Int[(-b + a*x)/(-(b*x^2) + a*x^6),x]
```

output
```
-x^(-1) - (a^(1/4)*ArcTan[(a^(1/4)*x)/b^(1/4)])/(2*b^(1/4)) + (a^(1/4)*Arc
Tanh[(a^(1/4)*x)/b^(1/4)])/(2*b^(1/4)) - (a*Log[x])/b + (a*Log[b - a*x^4])
/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

method	result
default	$-\frac{1}{x}-\frac{a \ln \left(x \right)}{b}+\frac{a \left(-\frac{b \left(2 \arctan \left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)-\ln \left(\frac{x+\left(\frac{b}{a}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)\right)}{4 a \left(\frac{b}{a}\right)^{\frac{1}{4}}}+\frac{\ln \left(a \,x^{4}-b \right)}{4}\right)}{b}$
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(b^{4} \_Z^{4}-4 a \,b^{3} \_Z^{3}+6 a^{2} b^{2} \_Z^{2}-4 a^{3} b \_Z +a^{4}-a \,b^{3}\right)}{\_R \ln \left(\left(-5 \_R^{4} b^{3}+15 \_R^{3} a \,b^{2}-15 \_R^{2} a^{2} b +5 \_R \,a^{3}\right.\right.}}{4}$

input
```
int((a*x-b)/(a*x^6-b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/x-a*ln(x)/b+a/b*(-1/4*b/a/(b/a)^(1/4)*(2*arctan(x/(b/a)^(1/4))-ln((x+(b
/a)^(1/4))/(x-(b/a)^(1/4))))+1/4*ln(a*x^4-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.00

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = -\frac{4\,ax\log(x)+\left(bx\sqrt{-\sqrt{\frac{a}{b}}}-ax\right)\log\left(ax+b\sqrt{-\sqrt{\frac{a}{b}}}\sqrt{\frac{a}{b}}\right)-\left(bx\sqrt{-\sqrt{\frac{a}{b}}}+ax\right)\log\left(ax-b\sqrt{-\sqrt{\frac{a}{b}}}\sqrt{\frac{a}{b}}\right.}{4\,bx}$$

input
```
integrate((a*x-b)/(a*x^6-b*x^2),x, algorithm="fricas")
```

output
```
-1/4*(4*a*x*log(x) + (b*x*sqrt(-sqrt(a/b)) - a*x)*log(a*x + b*sqrt(-sqrt(a
/b))*sqrt(a/b)) - (b*x*sqrt(-sqrt(a/b)) + a*x)*log(a*x - b*sqrt(-sqrt(a/b)
)*sqrt(a/b)) - (b*x*(a/b)^(1/4) + a*x)*log(a*x + b*(a/b)^(3/4)) + (b*x*(a/
b)^(1/4) - a*x)*log(a*x - b*(a/b)^(3/4)) + 4*b)/(b*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(73) = 146$.

Time = 1.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = -\frac{a\log(x)}{b} + \operatorname{RootSum}\left(256t^4b^4-256t^3ab^3+96t^2a^2b^2-16ta^3b+a^4-ab^3,\left(t\mapsto t\log\left(x+\frac{32000t^4a^2b^4+8000t^3}{\cdots}\right.\right.\right. - \frac{1}{x}$$

input
```
integrate((a*x-b)/(a*x**6-b*x**2),x)
```

output
```
-a*log(x)/b + RootSum(256*_t**4*b**4 - 256*_t**3*a*b**3 + 96*_t**2*a**2*b*
*2 - 16*_t*a**3*b + a**4 - a*b**3, Lambda(_t, _t*log(x + (32000*_t**4*a**2
*b**4 + 8000*_t**3*a**3*b**3 - 64*_t**3*b**6 - 18000*_t**2*a**4*b**2 + 48*
_t**2*a*b**5 + 5500*_t*a**5*b - 12*_t*a**2*b**4 - 500*a**6 - 124*a**3*b**3
)/(625*a**4*b**2 - a*b**5)))) - 1/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = -\frac{a\left(\frac{2\,b\arctan\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{b\log\left(\frac{\sqrt{a}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{a}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}-\log\left(\sqrt{a}x^2+\sqrt{b}\right)-\log\left(\sqrt{a}x^2-\sqrt{b}\right)\right)}{4\,b} - \frac{a\log\left(x\right)}{b} - \frac{1}{x}$$

input
```
integrate((a*x-b)/(a*x^6-b*x^2),x, algorithm="maxima")
```

output
```
-1/4*a*(2*b*arctan(sqrt(a)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))) + b*log((sqrt(a)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*x + sqrt(sq
rt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) - log(sqrt(a)*x^2 + sqrt(
b)) - log(sqrt(a)*x^2 - sqrt(b)))/b - a*log(x)/b - 1/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.61

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = \frac{a\log\left(|ax^4-b|\right)}{4\,b} - \frac{a\log\left(|x|\right)}{b} + \frac{\sqrt{2}(-a^3b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2b} + \frac{\sqrt{2}(-a^3b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2b} - \frac{\sqrt{2}(-a^3b)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(-\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{-\frac{b}{a}}\right)}{8\,a^2b} + \frac{\sqrt{2}(-a^3b)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(-\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{-\frac{b}{a}}\right)}{8\,a^2b} - \frac{1}{x}$$

input

```
integrate((a*x-b)/(a*x^6-b*x^2),x, algorithm="giac")
```

output

```
1/4*a*log(abs(a*x^4 - b))/b - a*log(abs(x))/b + 1/4*sqrt(2)*(-a^3*b)^(3/4)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-b/a)^(1/4))/(-b/a)^(1/4))/(a^2*b) + 1
/4*sqrt(2)*(-a^3*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-b/a)^(1/4))/
(-b/a)^(1/4))/(a^2*b) - 1/8*sqrt(2)*(-a^3*b)^(3/4)*log(x^2 + sqrt(2)*x*(-b
/a)^(1/4) + sqrt(-b/a))/(a^2*b) + 1/8*sqrt(2)*(-a^3*b)^(3/4)*log(x^2 - sqr
t(2)*x*(-b/a)^(1/4) + sqrt(-b/a))/(a^2*b) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.60

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = \frac{\ln\left(b\left(\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}+5\,a\,\sqrt{a\,b^5}+5\,a\,x\left(\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}+a\,b^3\,x\right)\left(a+\sqrt{\frac{\sqrt{a\,b^5}}{b}}\right)}{4\,b}-\frac{a\,\ln(x)}{b}-\frac{1}{x}+\frac{\ln\left(b\left(-\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}-5\,a\,\sqrt{a\,b^5}+5\,a\,x\left(-\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}+a\,b^3\,x\right)\left(a+\sqrt{-\frac{\sqrt{a\,b^5}}{b}}\right)}{4\,b}+\frac{\ln\left(b\left(\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}-5\,a\,\sqrt{a\,b^5}+5\,a\,x\left(\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}-a\,b^3\,x\right)\left(a-\sqrt{\frac{\sqrt{a\,b^5}}{b}}\right)}{4\,b}+\frac{\ln\left(b\left(-\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}+5\,a\,\sqrt{a\,b^5}+5\,a\,x\left(-\frac{\sqrt{a\,b^5}}{b}\right)^{3/2}-a\,b^3\,x\right)\left(a-\sqrt{-\frac{\sqrt{a\,b^5}}{b}}\right)}{4\,b}$$

input
```
int(-(b - a*x)/(a*x^6 - b*x^2),x)
```

output
```
(log(b*((a*b^5)^(1/2)/b)^(3/2) + 5*a*(a*b^5)^(1/2) + 5*a*x*((a*b^5)^(1/2)/
b)^(3/2) + a*b^3*x)*(a + ((a*b^5)^(1/2)/b)^(1/2)))/(4*b) - (a*log(x))/b -
1/x + (log(b*(-(a*b^5)^(1/2)/b)^(3/2) - 5*a*(a*b^5)^(1/2) + 5*a*x*(-(a*b^5
)^(1/2)/b)^(3/2) + a*b^3*x)*(a + (-(a*b^5)^(1/2)/b)^(1/2)))/(4*b) + (log(b
*((a*b^5)^(1/2)/b)^(3/2) - 5*a*(a*b^5)^(1/2) + 5*a*x*((a*b^5)^(1/2)/b)^(3/
2) - a*b^3*x)*(a - ((a*b^5)^(1/2)/b)^(1/2)))/(4*b) + (log(b*(-(a*b^5)^(1/2
)/b)^(3/2) + 5*a*(a*b^5)^(1/2) + 5*a*x*(-(a*b^5)^(1/2)/b)^(3/2) - a*b^3*x)
*(a - (-(a*b^5)^(1/2)/b)^(1/2)))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{-b+ax}{-bx^2+ax^6}\,dx = \frac{-2b^{\frac{3}{4}}a^{\frac{1}{4}}\,atan\left(\frac{\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)x+b^{\frac{3}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}x+b^{\frac{1}{4}}\right)x-b^{\frac{3}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}x-b^{\frac{1}{4}}\right)x+\log\left(a^{\frac{1}{4}}x+b^{\frac{1}{4}}\right)ax+\log\left(a^{\frac{1}{4}}x\right.}{4bx}$$

input
```
int((a*x-b)/(a*x^6-b*x^2),x)
```

output
```
( - 2*b**(3/4)*a**(1/4)*atan((sqrt(a)*x)/(b**(1/4)*a**(1/4)))*x + b**(3/4)
*a**(1/4)*log(a**(1/4)*x + b**(1/4))*x - b**(3/4)*a**(1/4)*log(a**(1/4)*x
- b**(1/4))*x + log(a**(1/4)*x + b**(1/4))*a*x + log(a**(1/4)*x - b**(1/4)
)*a*x + log(sqrt(a)*x**2 + sqrt(b))*a*x - 4*log(x)*a*x - 4*b)/(4*b*x)
```

3.27 $\int \frac{1}{(-a+x^2)(\sqrt{2}a+x^2)(\sqrt{3}a+x^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 251

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \frac{x}{2\sqrt{3}\left(3-\sqrt{2}+\sqrt{3}-\sqrt{6}\right)a^3\left(\sqrt{3}a+x^2\right)}+\frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}\left(1+\sqrt{3}\right)^2a^{7/2}} - \frac{\left(1-\sqrt{2}+2\sqrt{3}\right)\left(5+2\sqrt{6}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}\left(1+\sqrt{3}\right)^2a^{7/2}} + \frac{\left(15-\sqrt{2}+3\sqrt{3}-3\sqrt{6}\right)\left(5+2\sqrt{6}\right)\arctan\left(\frac{x}{\sqrt[4]{3}\sqrt{a}}\right)}{2\,3^{3/4}\left(1+\sqrt{3}\right)^2a^{7/2}}+\frac{\left(1-\sqrt{2}\right)\operatorname{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{\left(1+\sqrt{3}\right)^2a^{7/2}}$$

output

```
1/6*x*3^(1/2)/(3-2^(1/2)+3^(1/2)-6^(1/2))/a^3/(3^(1/2)*a+x^2)+1/2*(1-2^(1/
2))*arctan(1/2*x*2^(3/4)/a^(1/2))*2^(3/4)/(1+3^(1/2))^2/a^(7/2)-1/2*(1-2^(
1/2)+2*3^(1/2))*(5+2*6^(1/2))*arctan(1/2*x*2^(3/4)/a^(1/2))*2^(3/4)/(1+3^(
1/2))^2/a^(7/2)+1/6*(15-2^(1/2)+3*3^(1/2)-3*6^(1/2))*(5+2*6^(1/2))*arctan(
1/3*x*3^(3/4)/a^(1/2))*3^(1/4)/(1+3^(1/2))^2/a^(7/2)+(1-2^(1/2))*arctanh(x
/a^(1/2))/(1+3^(1/2))^2/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \frac{\frac{\sqrt{3}\left(-3+\sqrt{2}-\sqrt{3}+\sqrt{6}\right)\sqrt{a}x}{\left(-10+6\sqrt{2}-5\sqrt{3}+4\sqrt{6}\right)\left(\sqrt{3}a+x^2\right)}+\frac{2\,2^{3/4}\sqrt{3}\left(-2729524173+1929898200\sqrt{2}-1575755281\sqrt{3}+1114323578\sqrt{6}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{-6462963267+4567702969\sqrt{2}-3729513857\sqrt{3}+2638493705\sqrt{6}}+$$

input
```
Integrate[1/((-a + x^2)*(Sqrt[2]*a + x^2)*(Sqrt[3]*a + x^2)^2),x]
```

output
```
((Sqrt[3]*(-3 + Sqrt[2] - Sqrt[3] + Sqrt[6])*Sqrt[a]*x)/((-10 + 6*Sqrt[2]
- 5*Sqrt[3] + 4*Sqrt[6])*(Sqrt[3]*a + x^2)) + (2*2^(3/4)*Sqrt[3]*(-2729524
173 + 1929898200*Sqrt[2] - 1575755281*Sqrt[3] + 1114323578*Sqrt[6])*ArcTan
[x/(2^(1/4)*Sqrt[a])])/(-6462963267 + 4567702969*Sqrt[2] - 3729513857*Sqrt
[3] + 2638493705*Sqrt[6]) + (3^(3/4)*(-93230995 + 65540631*Sqrt[2] - 53513
703*Sqrt[3] + 38061393*Sqrt[6])*ArcTan[x/(3^(1/4)*Sqrt[a])])/(-205326435 +
 145299702*Sqrt[2] - 118636710*Sqrt[3] + 83824166*Sqrt[6]) + (6*(-41772346
09 + 2957309786*Sqrt[2] - 2414633329*Sqrt[3] + 1705348888*Sqrt[6])*ArcTanh
[x/Sqrt[a]])/(-6462963267 + 4567702969*Sqrt[2] - 3729513857*Sqrt[3] + 2638
493705*Sqrt[6]))/(12*a^(7/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {421, 25, 303, 216, 219, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-a)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}dx$$

$$\Big\downarrow \text{ 421}$$

$$\frac{\int -\frac{1}{(a-x^2)\left(x^2+\sqrt{2}a\right)}dx}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{\int \frac{x^2+\left(1+2\sqrt{3}\right)a}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)^2}dx}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{(a-x^2)\left(x^2+\sqrt{2}a\right)}dx}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{\int \frac{x^2+\left(1+2\sqrt{3}\right)a}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)^2}dx}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 303$$

$$-\frac{-\frac{\left(1-\sqrt{2}\right)\int \frac{1}{a-x^2}dx}{a}-\frac{\left(1-\sqrt{2}\right)\int \frac{1}{x^2+\sqrt{2}a}dx}{a}}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{\int \frac{x^2+\left(1+2\sqrt{3}\right)a}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)^2}dx}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 216$$

$$-\frac{\int \frac{x^2+\left(1+2\sqrt{3}\right)a}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)^2}dx}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{-\frac{\left(1-\sqrt{2}\right)\int \frac{1}{a-x^2}dx}{a}-\frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\int \frac{x^2+\left(1+2\sqrt{3}\right)a}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)^2}dx}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{-\frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}-\frac{\left(1-\sqrt{2}\right)\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{3/2}}}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\left(3+\sqrt{3}\right)x}{6\left(\sqrt{2}-\sqrt{3}\right)a\left(\sqrt{3}a+x^2\right)}-\frac{\int \frac{a\left(\left(12-\sqrt{2}+2\sqrt{3}-3\sqrt{6}\right)a-\left(1+\sqrt{3}\right)x^2\right)}{\left(x^2+\sqrt{2}a\right)\left(x^2+\sqrt{3}a\right)}dx}{2\sqrt{3}\left(\sqrt{2}-\sqrt{3}\right)a^2}}{\left(1+\sqrt{3}\right)^2 a^2}-\frac{-\frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}-\frac{\left(1-\sqrt{2}\right)\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{3/2}}}{\left(1+\sqrt{3}\right)^2 a^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{(3+\sqrt{3})x}{6(\sqrt{2}-\sqrt{3})a(\sqrt{3}a+x^2)}-\frac{\int \frac{(12-\sqrt{2}+2\sqrt{3}-3\sqrt{6})a-(1+\sqrt{3})x^2}{(x^2+\sqrt{2}a)(x^2+\sqrt{3}a)}dx}{2\sqrt{3}(\sqrt{2}-\sqrt{3})a}}{(1+\sqrt{3})^2a^2}-\frac{-\frac{(1-\sqrt{2})\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}-\frac{(1-\sqrt{2})\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{3/2}}}{(1+\sqrt{3})^2a^2}$$

$$\Big\downarrow \ 397$$

$$-\frac{\frac{(3+\sqrt{3})x}{6(\sqrt{2}-\sqrt{3})a(\sqrt{3}a+x^2)}-\frac{\frac{(15-\sqrt{2}+3\sqrt{3}-3\sqrt{6})\int \frac{1}{x^2+\sqrt{3}a}dx}{\sqrt{2}-\sqrt{3}}-\frac{2(6+\sqrt{3}(1-\sqrt{2}))\int \frac{1}{x^2+\sqrt{2}a}dx}{\sqrt{2}-\sqrt{3}}}{2\sqrt{3}(\sqrt{2}-\sqrt{3})a}}{(1+\sqrt{3})^2a^2}-\frac{-\frac{(1-\sqrt{2})\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}-\frac{(1-\sqrt{2})\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{3/2}}}{(1+\sqrt{3})^2a^2}$$

$$\Big\downarrow \ 216$$

$$-\frac{\frac{(3+\sqrt{3})x}{6(\sqrt{2}-\sqrt{3})a(\sqrt{3}a+x^2)}-\frac{\frac{(15-\sqrt{2}+3\sqrt{3}-3\sqrt{6})\arctan\left(\frac{x}{\sqrt[4]{3}\sqrt{a}}\right)}{\sqrt[4]{3}(\sqrt{2}-\sqrt{3})\sqrt{a}}-\frac{2^{3/4}(6+\sqrt{3}(1-\sqrt{2}))\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{(\sqrt{2}-\sqrt{3})\sqrt{a}}}{2\sqrt{3}(\sqrt{2}-\sqrt{3})a}}{(1+\sqrt{3})^2a^2}-\frac{-\frac{(1-\sqrt{2})\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt{a}}\right)}{\sqrt[4]{2}a^{3/2}}-\frac{(1-\sqrt{2})\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{3/2}}}{(1+\sqrt{3})^2a^2}$$

input
```
Int[1/((-a + x^2)*(Sqrt[2]*a + x^2)*(Sqrt[3]*a + x^2)^2),x]
```

output
```
-((((3 + Sqrt[3])*x)/(6*(Sqrt[2] - Sqrt[3])*a*(Sqrt[3]*a + x^2)) - (-((2^(
3/4)*(6 + Sqrt[3]*(1 - Sqrt[2]))*ArcTan[x/(2^(1/4)*Sqrt[a])])/((Sqrt[2] -
Sqrt[3])*Sqrt[a])) + ((15 - Sqrt[2] + 3*Sqrt[3] - 3*Sqrt[6])*ArcTan[x/(3^(
1/4)*Sqrt[a])])/(3^(1/4)*(Sqrt[2] - Sqrt[3])*Sqrt[a]))/(2*Sqrt[3]*(Sqrt[2]
 - Sqrt[3])*a))/((1 + Sqrt[3])^2*a^2)) - (-(((1 - Sqrt[2])*ArcTan[x/(2^(1/
4)*Sqrt[a])])/(2^(1/4)*a^(3/2))) - ((1 - Sqrt[2])*ArcTanh[x/Sqrt[a]])/a^(3
/2))/((1 + Sqrt[3])^2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.66

method	result
default	$-\frac{6\left(\frac{\left(\frac{\sqrt{2}}{6}-\frac{1}{6}-\frac{\sqrt{3}}{6}+\frac{\sqrt{3}\sqrt{2}}{18}\right)x}{\sqrt{3}\,a+x^2}+\frac{73\left(\frac{3\sqrt{2}}{73}-\frac{3}{73}-\frac{5\sqrt{3}}{73}+\frac{\sqrt{3}\sqrt{2}}{219}\right)\arctan\left(\frac{x}{\sqrt{\sqrt{3}\,a}}\right)}{6\sqrt{\sqrt{3}\,a}}\right)}{\left(\sqrt{3}\sqrt{2}-2\right)^2\left(1+\sqrt{3}\right)^2a^3}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{a^{\frac{7}{2}}\left(2+\sqrt{2}\right)\left(3+\sqrt{3}\right)^2}-\frac{3\arctan\left(\frac{x}{\sqrt{\sqrt{2}\,a}}\right)}{a^3\left(1+\sqrt{2}\right)\left(\sqrt{3}\sqrt{2}-3\right)^2\sqrt{\sqrt{2}\,a}}$

input
```
int(1/(x^2-a)/(2^(1/2)*a+x^2)/(3^(1/2)*a+x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-6/(3^(1/2)*2^(1/2)-2)^2/(1+3^(1/2))^2/a^3*((1/6*2^(1/2)-1/6-1/6*3^(1/2)+1
/18*3^(1/2)*2^(1/2))*x/(3^(1/2)*a+x^2)+73/6*(3/73*2^(1/2)-3/73-5/73*3^(1/2
)+1/219*3^(1/2)*2^(1/2))/(3^(1/2)*a)^(1/2)*arctan(x/(3^(1/2)*a)^(1/2)))-3*
2^(1/2)/a^(7/2)/(2+2^(1/2))/(3+3^(1/2))^2*arctanh(x/a^(1/2))-3/a^3/(1+2^(1
/2))/(3^(1/2)*2^(1/2)-3)^2/(2^(1/2)*a)^(1/2)*arctan(x/(2^(1/2)*a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3389, normalized size of antiderivative = 13.50

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x^2-a)/(2^(1/2)*a+x^2)/(3^(1/2)*a+x^2)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(x**2-a)/(2**(1/2)*a+x**2)/(3**(1/2)*a+x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx &= -\frac{x}{2\left(\left(\sqrt{3}\sqrt{2}-3\sqrt{3}+3\sqrt{2}-3\right)a^3x^2+3\left(\sqrt{3}(\sqrt{2}-1)+\sqrt{2}-3\right)a^4\right)}\\ &\quad+\frac{3^{\frac{3}{4}}\left(3\sqrt{3}(\sqrt{2}-1)+\sqrt{2}-15\right)\arctan\left(\frac{3^{\frac{3}{4}}x}{3\sqrt{a}}\right)}{6\left(3\sqrt{3}(4\sqrt{2}-3)-11\sqrt{3}+24\sqrt{2}-30\right)a^{\frac{7}{2}}}\\ &\quad+\frac{2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x}{2\sqrt{a}}\right)}{2\left(2\sqrt{3}(\sqrt{2}+2)-5\sqrt{2}-5\right)a^{\frac{7}{2}}}+\frac{\log\left(\frac{x-\sqrt{a}}{x+\sqrt{a}}\right)}{4\left(\sqrt{3}(\sqrt{2}+1)+2\sqrt{2}+2\right)a^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate(1/(x^2-a)/(2^(1/2)*a+x^2)/(3^(1/2)*a+x^2)^2,x, algorithm="maxima
")
```

output
```
-1/2*x/((sqrt(3)*sqrt(2) - 3*sqrt(3) + 3*sqrt(2) - 3)*a^3*x^2 + 3*(sqrt(3)
*(sqrt(2) - 1) + sqrt(2) - 3)*a^4) + 1/6*3^(3/4)*(3*sqrt(3)*(sqrt(2) - 1)
+ sqrt(2) - 15)*arctan(1/3*3^(3/4)*x/sqrt(a))/((3*sqrt(3)*(4*sqrt(2) - 3)
- 11*sqrt(3) + 24*sqrt(2) - 30)*a^(7/2)) + 1/2*2^(3/4)*arctan(1/2*2^(3/4)*
x/sqrt(a))/((2*sqrt(3)*(sqrt(2) + 2) - 5*sqrt(2) - 5)*a^(7/2)) + 1/4*log((
x - sqrt(a))/(x + sqrt(a)))/((sqrt(3)*(sqrt(2) + 1) + 2*sqrt(2) + 2)*a^(7/
2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx$$
$$= \frac{2^{\frac{3}{4}}\left(202\sqrt{3}\sqrt{2}-284\sqrt{3}+359\sqrt{2}-479\right)\arctan\left(\frac{2^{\frac{3}{4}}x}{2\sqrt{a}}\right)}{2\left(888\sqrt{3}\sqrt{2}-1080\sqrt{3}+1320\sqrt{2}-2179\right)a^{\frac{7}{2}}}$$
$$- \frac{\left(437\sqrt{3}\sqrt{2}-893\sqrt{3}+1094\sqrt{2}-1070\right)\arctan\left(\frac{x}{\sqrt{-a}}\right)}{2\left(888\sqrt{3}\sqrt{2}-1080\sqrt{3}+1320\sqrt{2}-2179\right)\sqrt{-a}a^3}$$
$$+ \frac{\left(4\sqrt{3}\sqrt{2}-9\sqrt{3}+10\sqrt{2}-11\right)x}{4\left(x^2+\sqrt{3}a\right)\left(40\sqrt{3}\sqrt{2}-52\sqrt{3}+63\sqrt{2}-99\right)a^3}$$
$$- \frac{\left(6417\sqrt{3}\sqrt{2}-9409\sqrt{3}+11499\sqrt{2}-15753\right)\arctan\left(\frac{40\sqrt{3}\sqrt{2}x-52\sqrt{3}x+63\sqrt{2}x-99x}{\sqrt{-24954\sqrt{3}\sqrt{2}+35451\sqrt{3}-43416\sqrt{2}+61128}\sqrt{a}}\right)}{4\left(40\sqrt{3}\sqrt{2}-52\sqrt{3}+63\sqrt{2}-99\right)\sqrt{-24954\sqrt{3}\sqrt{2}+35451\sqrt{3}-43416\sqrt{2}+61128}a^{\frac{7}{2}}}$$

input
```
integrate(1/(x^2-a)/(2^(1/2)*a+x^2)/(3^(1/2)*a+x^2)^2,x, algorithm="giac")
```

output
```
1/2*2^(3/4)*(202*sqrt(3)*sqrt(2) - 284*sqrt(3) + 359*sqrt(2) - 479)*arctan
(1/2*2^(3/4)*x/sqrt(a))/((888*sqrt(3)*sqrt(2) - 1080*sqrt(3) + 1320*sqrt(2
) - 2179)*a^(7/2)) - 1/2*(437*sqrt(3)*sqrt(2) - 893*sqrt(3) + 1094*sqrt(2)
 - 1070)*arctan(x/sqrt(-a))/((888*sqrt(3)*sqrt(2) - 1080*sqrt(3) + 1320*sq
rt(2) - 2179)*sqrt(-a)*a^3) + 1/4*(4*sqrt(3)*sqrt(2) - 9*sqrt(3) + 10*sqrt
(2) - 11)*x/((x^2 + sqrt(3)*a)*(40*sqrt(3)*sqrt(2) - 52*sqrt(3) + 63*sqrt(
2) - 99)*a^3) - 1/4*(6417*sqrt(3)*sqrt(2) - 9409*sqrt(3) + 11499*sqrt(2) -
 15753)*arctan((40*sqrt(3)*sqrt(2)*x - 52*sqrt(3)*x + 63*sqrt(2)*x - 99*x)
/(sqrt(-24954*sqrt(3)*sqrt(2) + 35451*sqrt(3) - 43416*sqrt(2) + 61128)*sqr
t(a)))/((40*sqrt(3)*sqrt(2) - 52*sqrt(3) + 63*sqrt(2) - 99)*sqrt(-24954*sq
rt(3)*sqrt(2) + 35451*sqrt(3) - 43416*sqrt(2) + 61128)*a^(7/2))
```

Mupad [B] (verification not implemented)

Time = 68.70 (sec) , antiderivative size = 3037, normalized size of antiderivative = 12.10

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(-1/((2^(1/2)*a + x^2)*(3^(1/2)*a + x^2)^2*(a - x^2)),x)
```

output
```
symsum(log(-root(3039952896*2^(1/2)*3^(1/2)*6^(1/2)*a^21*z^6 + 15904014336
*2^(1/2)*3^(1/2)*a^21*z^6 - 2452488192*2^(1/2)*6^(1/2)*a^21*z^6 + 27014529
024*3^(1/2)*6^(1/2)*a^21*z^6 + 4904976384*3^(1/2)*a^21*z^6 - 81043587072*2
^(1/2)*a^21*z^6 - 15904014336*6^(1/2)*a^21*z^6 - 18239717376*a^21*z^6 + 68
281344*2^(1/2)*3^(1/2)*6^(1/2)*a^14*z^4 + 150165504*2^(1/2)*3^(1/2)*a^14*z
^4 + 102274560*2^(1/2)*6^(1/2)*a^14*z^4 - 568320*3^(1/2)*6^(1/2)*a^14*z^4
- 204549120*3^(1/2)*a^14*z^4 - 150165504*6^(1/2)*a^14*z^4 + 1704960*2^(1/2
)*a^14*z^4 - 409688064*a^14*z^4 + 10880*2^(1/2)*3^(1/2)*6^(1/2)*a^7*z^2 +
46120*2^(1/2)*6^(1/2)*a^7*z^2 + 41952*3^(1/2)*6^(1/2)*a^7*z^2 - 22848*2^(1
/2)*3^(1/2)*a^7*z^2 - 126444*2^(1/2)*a^7*z^2 - 91760*3^(1/2)*a^7*z^2 + 231
68*6^(1/2)*a^7*z^2 - 66064*a^7*z^2 - 1, z, k)*(8*x + root(3039952896*2^(1/
2)*3^(1/2)*6^(1/2)*a^21*z^6 + 15904014336*2^(1/2)*3^(1/2)*a^21*z^6 - 24524
88192*2^(1/2)*6^(1/2)*a^21*z^6 + 27014529024*3^(1/2)*6^(1/2)*a^21*z^6 + 49
04976384*3^(1/2)*a^21*z^6 - 81043587072*2^(1/2)*a^21*z^6 - 15904014336*6^(
1/2)*a^21*z^6 - 18239717376*a^21*z^6 + 68281344*2^(1/2)*3^(1/2)*6^(1/2)*a^
14*z^4 + 150165504*2^(1/2)*3^(1/2)*a^14*z^4 + 102274560*2^(1/2)*6^(1/2)*a^
14*z^4 - 568320*3^(1/2)*6^(1/2)*a^14*z^4 - 204549120*3^(1/2)*a^14*z^4 - 15
0165504*6^(1/2)*a^14*z^4 + 1704960*2^(1/2)*a^14*z^4 - 409688064*a^14*z^4 +
 10880*2^(1/2)*3^(1/2)*6^(1/2)*a^7*z^2 + 46120*2^(1/2)*6^(1/2)*a^7*z^2 + 4
1952*3^(1/2)*6^(1/2)*a^7*z^2 - 22848*2^(1/2)*3^(1/2)*a^7*z^2 - 126444*2...
```

Reduce [F]

$$\int \frac{1}{(-a+x^2)\left(\sqrt{2}a+x^2\right)\left(\sqrt{3}a+x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2-a)/(2^(1/2)*a+x^2)/(3^(1/2)*a+x^2)^2,x)
```

output

```
(6*sqrt(a)*sqrt(3)*3**(1/4)*atan(x/(sqrt(a)*3**(1/4)))*a**2 - 2*sqrt(a)*sq
rt(3)*3**(1/4)*atan(x/(sqrt(a)*3**(1/4)))*x**4 - 3*sqrt(a)*sqrt(3)*3**(1/4
)*log(sqrt(a)*3**(1/4) - x)*a**2 + sqrt(a)*sqrt(3)*3**(1/4)*log(sqrt(a)*3*
*(1/4) - x)*x**4 + 3*sqrt(a)*sqrt(3)*3**(1/4)*log(sqrt(a)*3**(1/4) + x)*a*
*2 - sqrt(a)*sqrt(3)*3**(1/4)*log(sqrt(a)*3**(1/4) + x)*x**4 - 144*sqrt(6)
*int(x**4/(18*a**7 - 18*a**6*x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*x
**8 - 8*a**2*x**10 - a*x**12 + x**14),x)*a**7 + 48*sqrt(6)*int(x**4/(18*a*
*7 - 18*a**6*x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*x**8 - 8*a**2*x**
10 - a*x**12 + x**14),x)*a**5*x**4 - 432*sqrt(6)*int(1/(18*a**7 - 18*a**6*
x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*x**8 - 8*a**2*x**10 - a*x**12
+ x**14),x)*a**9 + 144*sqrt(6)*int(1/(18*a**7 - 18*a**6*x**2 - 21*a**5*x**
4 + 21*a**4*x**6 + 8*a**3*x**8 - 8*a**2*x**10 - a*x**12 + x**14),x)*a**7*x
**4 + 144*sqrt(3)*int(x**4/(18*a**7 - 18*a**6*x**2 - 21*a**5*x**4 + 21*a**
4*x**6 + 8*a**3*x**8 - 8*a**2*x**10 - a*x**12 + x**14),x)*a**7 - 48*sqrt(3
)*int(x**4/(18*a**7 - 18*a**6*x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*
x**8 - 8*a**2*x**10 - a*x**12 + x**14),x)*a**5*x**4 + 720*sqrt(3)*int(x**2
/(18*a**7 - 18*a**6*x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*x**8 - 8*a
**2*x**10 - a*x**12 + x**14),x)*a**8 - 240*sqrt(3)*int(x**2/(18*a**7 - 18*
a**6*x**2 - 21*a**5*x**4 + 21*a**4*x**6 + 8*a**3*x**8 - 8*a**2*x**10 - a*x
**12 + x**14),x)*a**6*x**4 - 288*sqrt(3)*int(1/(18*a**7 - 18*a**6*x**2 ...
```

3.28 $\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx = \frac{8a^4x+5abx-2a^3x^3+bx^3}{36a^3(2a^2-ax^2-x^4)} - \frac{(4a^3-11b)\arctan\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)}{108\sqrt{2}a^{7/2}} + \frac{(-5a^3+7b)\,\mathrm{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{54a^{7/2}}$$

output
```
1/36*(-2*a^3*x^3+8*a^4*x+b*x^3+5*a*b*x)/a^3/(-x^4-a*x^2+2*a^2)-1/216*(4*a^
3-11*b)*arctan(1/2*x*2^(1/2)/a^(1/2))*2^(1/2)/a^(7/2)+1/54*(-5*a^3+7*b)*ar
ctanh(x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx = \frac{-\frac{12\sqrt{a}(a^3+b)x}{-a+x^2}+\frac{6\sqrt{a}(4a^3+b)x}{2a+x^2}+\sqrt{2}(-4a^3+11b)\arctan\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)-4(5a^3-7b)\,\mathrm{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{216a^{7/2}}$$

input
```
Integrate[(b + a*x^4)/((-a + x^2)^2*(2*a + x^2)^2),x]
```

output
```
((-12*Sqrt[a]*(a^3 + b)*x)/(-a + x^2) + (6*Sqrt[a]*(4*a^3 + b)*x)/(2*a + x
^2) + Sqrt[2]*(-4*a^3 + 11*b)*ArcTan[x/(Sqrt[2]*Sqrt[a])] - 4*(5*a^3 - 7*b
)*ArcTanh[x/Sqrt[a]])/(216*a^(7/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^4+b}{(x^2-a)^2(2a+x^2)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(-\frac{2(2a^3-b)}{27a^3(a-x^2)}-\frac{2(2a^3-b)}{27a^3(2a+x^2)}+\frac{a^3+b}{9a^2(a-x^2)^2}+\frac{4a^3+b}{9a^2(2a+x^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x(a^3+b)}{18a^3(a-x^2)}+\frac{x(4a^3+b)}{36a^3(2a+x^2)}-\frac{\sqrt{2}(2a^3-b)\arctan\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)}{27a^{7/2}}+\frac{(4a^3+b)\arctan\left(\frac{x}{\sqrt{2}\sqrt{a}}\right)}{36\sqrt{2}a^{7/2}}-\frac{2(2a^3-b)\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{27a^{7/2}}+\frac{(a^3+b)\text{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{18a^{7/2}}$$

input
```
Int[(b + a*x^4)/((-a + x^2)^2*(2*a + x^2)^2),x]
```

output
```
((a^3 + b)*x)/(18*a^3*(a - x^2)) + ((4*a^3 + b)*x)/(36*a^3*(2*a + x^2)) -
(Sqrt[2]*(2*a^3 - b)*ArcTan[x/(Sqrt[2]*Sqrt[a])])/(27*a^(7/2)) + ((4*a^3 +
 b)*ArcTan[x/(Sqrt[2]*Sqrt[a])])/(36*Sqrt[2]*a^(7/2)) - (2*(2*a^3 - b)*Arc
Tanh[x/Sqrt[a]])/(27*a^(7/2)) + ((a^3 + b)*ArcTanh[x/Sqrt[a]])/(18*a^(7/2)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\frac{\left(-\frac{3a^3}{2}-\frac{3b}{2}\right)x}{-x^2+a}+\frac{\left(5a^3-7b\right)\operatorname{arctanh}\left(\frac{x}{\sqrt{a}}\right)}{2\sqrt{a}}}{27a^3}+\frac{\frac{\left(\frac{3a^3}{2}+\frac{3b}{8}\right)x}{\frac{x^2}{2}+a}-\frac{\left(2a^3-\frac{11b}{2}\right)\sqrt{2}\arctan\left(\frac{x\sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}}{27a^3}$
risch	$\frac{-\frac{\left(2a^3-b\right)x^3}{36a^3}+\frac{\left(8a^3+5b\right)x}{36a^2}}{(x^2+2a)(-x^2+a)}+\frac{\sqrt{2}\ln\left(2(-a)^{\frac{21}{2}}-a^{10}x\sqrt{2}\right)a^3}{108(-a)^{\frac{7}{2}}}-\frac{11\sqrt{2}\ln\left(2(-a)^{\frac{21}{2}}-a^{10}x\sqrt{2}\right)b}{432(-a)^{\frac{7}{2}}}-\frac{\sqrt{2}\ln\left(2(-a)^{\frac{21}{2}}+a^{10}x\sqrt{2}\right)a^3}{108(-a)^{\frac{7}{2}}}$

input
```
int((a*x^4+b)/(x^2-a)^2/(x^2+2*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/27/a^3*((-3/2*a^3-3/2*b)*x/(-x^2+a)+1/2*(5*a^3-7*b)/a^(1/2)*arctanh(x/a
^(1/2)))+1/27/a^3*((3/2*a^3+3/8*b)*x/(1/2*x^2+a)-1/4*(2*a^3-11/2*b)*2^(1/2
)/a^(1/2)*arctan(1/2*x*2^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.32

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx$$
$$= \left[\frac{6\,(2\,a^4-ab)x^3+\sqrt{2}(8\,a^5-(4\,a^3-11\,b)x^4-22\,a^2b-(4\,a^4-11\,ab)x^2)\sqrt{a}\arctan\left(\frac{\sqrt{2}x}{2\sqrt{a}}\right)+2\,(10\,a^5-}{216\,(a^4x^4+a^5x^2-2\,a^{}}\right.$$

input
```
integrate((a*x^4+b)/(x^2-a)^2/(x^2+2*a)^2,x, algorithm="fricas")
```

output
```
[1/216*(6*(2*a^4 - a*b)*x^3 + sqrt(2)*(8*a^5 - (4*a^3 - 11*b)*x^4 - 22*a^2
*b - (4*a^4 - 11*a*b)*x^2)*sqrt(a)*arctan(1/2*sqrt(2)*x/sqrt(a)) + 2*(10*a
^5 - (5*a^3 - 7*b)*x^4 - 14*a^2*b - (5*a^4 - 7*a*b)*x^2)*sqrt(a)*log((x^2
+ 2*sqrt(a)*x + a)/(x^2 - a)) - 6*(8*a^5 + 5*a^2*b)*x)/(a^4*x^4 + a^5*x^2
- 2*a^6), 1/432*(12*(2*a^4 - a*b)*x^3 - sqrt(2)*(8*a^5 - (4*a^3 - 11*b)*x^
4 - 22*a^2*b - (4*a^4 - 11*a*b)*x^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*x -
 x^2 + 2*a)/(x^2 + 2*a)) - 8*(10*a^5 - (5*a^3 - 7*b)*x^4 - 14*a^2*b - (5*a
^4 - 7*a*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)*x/a) - 12*(8*a^5 + 5*a^2*b)*x)/(
a^4*x^4 + a^5*x^2 - 2*a^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(109) = 218$.

Time = 1.34 (sec) , antiderivative size = 989, normalized size of antiderivative = 8.68

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*x**4+b)/(x**2-a)**2/(x**2+2*a)**2,x)
```

output
```
-sqrt(2)*sqrt(-1/a**7)*(4*a**3 - 11*b)*log(x + (-4*sqrt(2)*a**14*(-1/a**7)
**(3/2)*(4*a**3 - 11*b)**3/27 + 1016*sqrt(2)*a**13*sqrt(-1/a**7)*(4*a**3 -
 11*b)/27 + 17*sqrt(2)*a**11*b*(-1/a**7)**(3/2)*(4*a**3 - 11*b)**3/108 - 1
444*sqrt(2)*a**10*b*sqrt(-1/a**7)*(4*a**3 - 11*b)/9 + 2081*sqrt(2)*a**7*b*
*2*sqrt(-1/a**7)*(4*a**3 - 11*b)/9 - 12307*sqrt(2)*a**4*b**3*sqrt(-1/a**7)
*(4*a**3 - 11*b)/108)/(160*a**12 - 1144*a**9*b + 2988*a**6*b**2 - 3425*a**
3*b**3 + 1463*b**4))/432 + sqrt(2)*sqrt(-1/a**7)*(4*a**3 - 11*b)*log(x + (
4*sqrt(2)*a**14*(-1/a**7)**(3/2)*(4*a**3 - 11*b)**3/27 - 1016*sqrt(2)*a**1
3*sqrt(-1/a**7)*(4*a**3 - 11*b)/27 - 17*sqrt(2)*a**11*b*(-1/a**7)**(3/2)*(
4*a**3 - 11*b)**3/108 + 1444*sqrt(2)*a**10*b*sqrt(-1/a**7)*(4*a**3 - 11*b)
/9 - 2081*sqrt(2)*a**7*b**2*sqrt(-1/a**7)*(4*a**3 - 11*b)/9 + 12307*sqrt(2
)*a**4*b**3*sqrt(-1/a**7)*(4*a**3 - 11*b)/108)/(160*a**12 - 1144*a**9*b +
2988*a**6*b**2 - 3425*a**3*b**3 + 1463*b**4))/432 - (5*a**3 - 7*b)*sqrt(a*
*(-7))*log(x + (-128*a**14*(5*a**3 - 7*b)**3*(a**(-7))**(3/2)/27 + 4064*a*
*13*(5*a**3 - 7*b)*sqrt(a**(-7))/27 + 136*a**11*b*(5*a**3 - 7*b)**3*(a**(-
7))**(3/2)/27 - 5776*a**10*b*(5*a**3 - 7*b)*sqrt(a**(-7))/9 + 8324*a**7*b*
*2*(5*a**3 - 7*b)*sqrt(a**(-7))/9 - 12307*a**4*b**3*(5*a**3 - 7*b)*sqrt(a*
*(-7))/27)/(160*a**12 - 1144*a**9*b + 2988*a**6*b**2 - 3425*a**3*b**3 + 14
63*b**4))/108 + (5*a**3 - 7*b)*sqrt(a**(-7))*log(x + (128*a**14*(5*a**3 -
7*b)**3*(a**(-7))**(3/2)/27 - 4064*a**13*(5*a**3 - 7*b)*sqrt(a**(-7))/2...
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx = \frac{(2\,a^3-b)x^3-(8\,a^4+5\,ab)x}{36\,(a^3x^4+a^4x^2-2\,a^5)} - \frac{\sqrt{2}(4\,a^3-11\,b)\arctan\left(\frac{\sqrt{2}x}{2\,\sqrt{a}}\right)}{216\,a^{\frac{7}{2}}} + \frac{(5\,a^3-7\,b)\log\left(\frac{x-\sqrt{a}}{x+\sqrt{a}}\right)}{108\,a^{\frac{7}{2}}}$$

input
```
integrate((a*x^4+b)/(x^2-a)^2/(x^2+2*a)^2,x, algorithm="maxima")
```

output
```
1/36*((2*a^3 - b)*x^3 - (8*a^4 + 5*a*b)*x)/(a^3*x^4 + a^4*x^2 - 2*a^5) - 1
/216*sqrt(2)*(4*a^3 - 11*b)*arctan(1/2*sqrt(2)*x/sqrt(a))/a^(7/2) + 1/108*
(5*a^3 - 7*b)*log((x - sqrt(a))/(x + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx = -\frac{\sqrt{2}(4a^3-11b)\arctan\left(\frac{\sqrt{2}x}{2\sqrt{a}}\right)}{216a^{\frac{7}{2}}} + \frac{(5a^3-7b)\arctan\left(\frac{x}{\sqrt{-a}}\right)}{54\sqrt{-a}a^3} + \frac{2a^3x^3-8a^4x-bx^3-5abx}{36(x^4+ax^2-2a^2)a^3}$$

input
```
integrate((a*x^4+b)/(x^2-a)^2/(x^2+2*a)^2,x, algorithm="giac")
```

output
```
-1/216*sqrt(2)*(4*a^3 - 11*b)*arctan(1/2*sqrt(2)*x/sqrt(a))/a^(7/2) + 1/54
*(5*a^3 - 7*b)*arctan(x/sqrt(-a))/(sqrt(-a)*a^3) + 1/36*(2*a^3*x^3 - 8*a^4
*x - b*x^3 - 5*a*b*x)/((x^4 + a*x^2 - 2*a^2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.73

$$\int \frac{b+ax^4}{(-a+x^2)^2(2a+x^2)^2}\,dx$$

$$= \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x\,\sqrt{-a^7}}{324\left(\frac{23\,a\,b}{648}-\frac{a^4}{162}-\frac{85\,b^2}{1296\,a^2}+\frac{209\,b^3}{5184\,a^5}\right)}+\frac{85\sqrt{2}\,b^2\,x\,\sqrt{-a^7}}{2592\left(-\frac{a^{10}}{162}+\frac{23\,a^7\,b}{648}-\frac{85\,a^4\,b^2}{1296}+\frac{209\,a\,b^3}{5184}\right)}-\frac{209\sqrt{2}\,b^3\,x\,\sqrt{-a^7}}{10368\left(-\frac{a^{13}}{162}+\frac{23\,a^{10}\,b}{648}-\frac{85\,a^7\,b^2}{1296}+\frac{209\,a^4 \cdots}{5184 \cdots}\right.\cdots}{216\,a^7}$$

$$-\frac{\mathrm{atanh}\left(\frac{263\,b^2\,x}{1296\sqrt{a^7}\left(\frac{11\,b}{81}-\frac{5\,a^3}{162}-\frac{263\,b^2}{1296\,a^3}+\frac{133\,b^3}{1296\,a^6}\right)}-\frac{133\,b^3\,x}{1296\sqrt{a^7}\left(\frac{11\,a^3\,b}{81}-\frac{5\,a^6}{162}-\frac{263\,b^2}{1296}+\frac{133\,b^3}{1296\,a^3}\right)}+\frac{5\,a^5\,x}{162\sqrt{a^7}\left(\frac{11\,b}{81\,a}-\frac{5\,a^2}{162}-\frac{263\,b^2}{1296\,a^4}+\frac{133 \cdots}{1296 \cdots}\right.\cdots}{54\sqrt{a^7}}$$

$$-\frac{\frac{x\,(8\,a^3+5\,b)}{36\,a^2}+\frac{x^3\,(b-2\,a^3)}{36\,a^3}}{-2\,a^2+a\,x^2+x^4}$$

input
```
int((b + a*x^4)/((a - x^2)^2*(2*a + x^2)^2),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*x*(-a^7)^(1/2))/(324*((23*a*b)/648 - a^4/162 - (85
*b^2)/(1296*a^2) + (209*b^3)/(5184*a^5))) + (85*2^(1/2)*b^2*x*(-a^7)^(1/2)
)/(2592*((209*a*b^3)/5184 + (23*a^7*b)/648 - a^10/162 - (85*a^4*b^2)/1296)
) - (209*2^(1/2)*b^3*x*(-a^7)^(1/2))/(10368*((23*a^10*b)/648 - a^13/162 +
(209*a^4*b^3)/5184 - (85*a^7*b^2)/1296)) + (23*2^(1/2)*b*x*(-a^7)^(1/2))/(
1296*((85*a*b^2)/1296 - (23*a^4*b)/648 + a^7/162 - (209*b^3)/(5184*a^2))))
*(11*b - 4*a^3)*(-a^7)^(1/2))/(216*a^7) - (atanh((263*b^2*x)/(1296*(a^7)^(
1/2)*((11*b)/81 - (5*a^3)/162 - (263*b^2)/(1296*a^3) + (133*b^3)/(1296*a^6
))) - (133*b^3*x)/(1296*(a^7)^(1/2)*((11*a^3*b)/81 - (5*a^6)/162 - (263*b^
2)/1296 + (133*b^3)/(1296*a^3))) + (5*a^5*x)/(162*(a^7)^(1/2)*((11*b)/(81*
a) - (5*a^2)/162 - (263*b^2)/(1296*a^4) + (133*b^3)/(1296*a^7))) - (11*a^2
*b*x)/(81*(a^7)^(1/2)*((11*b)/(81*a) - (5*a^2)/162 - (263*b^2)/(1296*a^4)
+ (133*b^3)/(1296*a^7))))*(7*b - 5*a^3))/(54*(a^7)^(1/2)) - ((x*(5*b + 8*a
^3))/(36*a^2) + (x^3*(b - 2*a^3))/(36*a^3))/(a*x^2 - 2*a^2 + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.15

$$\int \frac{b+ax^4}{(-a+x^2)^2\,(2a+x^2)^2}\,dx = \frac{-8\sqrt{a}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{a}\,\sqrt{2}}\right)a^5+4\sqrt{a}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{a}\,\sqrt{2}}\right)a^4x^2+4\sqrt{a}\,\sqrt{2}\,atan\left(\frac{x}{\sqrt{a}\,\sqrt{2}}\right)a^3x^4+22\sqrt{a}\,\sqrt{2}\,atan\left(\cdots\right.}{\cdots}$$

input
```
int((a*x^4+b)/(x^2-a)^2/(x^2+2*a)^2,x)
```

output

```
( - 8*sqrt(a)*sqrt(2)*atan(x/(sqrt(a)*sqrt(2)))*a**5 + 4*sqrt(a)*sqrt(2)*a
tan(x/(sqrt(a)*sqrt(2)))*a**4*x**2 + 4*sqrt(a)*sqrt(2)*atan(x/(sqrt(a)*sqr
t(2)))*a**3*x**4 + 22*sqrt(a)*sqrt(2)*atan(x/(sqrt(a)*sqrt(2)))*a**2*b - 1
1*sqrt(a)*sqrt(2)*atan(x/(sqrt(a)*sqrt(2)))*a*b*x**2 - 11*sqrt(a)*sqrt(2)*
atan(x/(sqrt(a)*sqrt(2)))*b*x**4 + 20*sqrt(a)*log(sqrt(a) - x)*a**5 - 10*s
qrt(a)*log(sqrt(a) - x)*a**4*x**2 - 10*sqrt(a)*log(sqrt(a) - x)*a**3*x**4
- 28*sqrt(a)*log(sqrt(a) - x)*a**2*b + 14*sqrt(a)*log(sqrt(a) - x)*a*b*x**
2 + 14*sqrt(a)*log(sqrt(a) - x)*b*x**4 - 20*sqrt(a)*log(sqrt(a) + x)*a**5
+ 10*sqrt(a)*log(sqrt(a) + x)*a**4*x**2 + 10*sqrt(a)*log(sqrt(a) + x)*a**3
*x**4 + 28*sqrt(a)*log(sqrt(a) + x)*a**2*b - 14*sqrt(a)*log(sqrt(a) + x)*a
*b*x**2 - 14*sqrt(a)*log(sqrt(a) + x)*b*x**4 + 48*a**5*x - 12*a**4*x**3 +
30*a**2*b*x + 6*a*b*x**3)/(216*a**4*(2*a**2 - a*x**2 - x**4))
```

3.29 $\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx = -\frac{\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \text{arctanh}(x) + \frac{\left(\sqrt{a}-\sqrt{b}\right)(a-b)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)(a-b)\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

output
```
1/4*(a^(1/2)+b^(1/2))*(a-b)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a
^(3/4)/b^(3/4)+1/4*(a^(1/2)+b^(1/2))*(a-b)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1
/4))*2^(1/2)/a^(3/4)/b^(3/4)-arctanh(x)+1/8*(a^(1/2)-b^(1/2))*(a-b)*ln(a^(
1/2)-2^(1/2)*a^(1/4)*b^(1/4)*x+b^(1/2)*x^2)*2^(1/2)/a^(3/4)/b^(3/4)-1/8*(a
^(1/2)-b^(1/2))*(a-b)*ln(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*x+b^(1/2)*x^2)*2^
(1/2)/a^(3/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx$$
$$= \frac{-2\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{}$$

input
```
Integrate[(b + a*x^4)/((-1 + x^2)*(a + b*x^4)),x]
```

output
```
(-2*Sqrt[2]*a^(1/4)*(Sqrt[a] + Sqrt[b])*(a - b)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] + 2*Sqrt[2]*a^(1/4)*(Sqrt[a] + Sqrt[b])*(a - b)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 4*a*b^(3/4)*Log[1 - x] - 4*a*b^(3/4)*Log[1 +
 x] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*(a - b)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*(a -
b)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(8*a*b^(3/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^4+b}{(x^2-1)(a+bx^4)}\,dx$$
$$\Big\downarrow \text{ 2257}$$
$$\int \left(\frac{(x^2+1)(a-b)}{a+bx^4}+\frac{1}{x^2-1}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)(a-b)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)(a-b)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)(a-b)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\operatorname{arctanh}(x)$$

input
```
Int[(b + a*x^4)/((-1 + x^2)*(a + b*x^4)),x]
```

output
```
-1/2*((Sqrt[a] + Sqrt[b])*(a - b)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b])*(a - b)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ArcTanh[x] + ((Sqrt
[a] - Sqrt[b])*(a - b)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*(a - b)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.88

method	result
default	$\frac{\ln(-1+x)}{2}+\frac{(a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{(a-b)\sqrt{2}\left(\ln\left(\frac{x^2}{x^2}\right.\right.}{}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(a^3b^3\_Z^4+\left(4a^4b^2-8a^3b^3+4a^2b^4\right)\_Z^2+a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2a\,b^5+b^6\right)}\_R\ln\left(\left(\left(-2a^3b^3+2a^2b^4\right)\_R^5+(-7\right.\right.\right.$

input
```
int((a*x^4+b)/(x^2-1)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-1+x)+1/8*(a-b)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(
a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*(a-b)/b/(a/b)^(1/4)
*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/
b)^(1/4)*x-1))-1/2*ln(1+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(176) = 352$.

Time = 0.12 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.59

$$\int \frac{b+ax^4}{(-1+x^2)\,(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(a*b*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^
4 - 6*a*b^5 + b^6)/(a^3*b^3)) + 2*a^2 - 4*a*b + 2*b^2)/(a*b))*log((a^5 - 3
*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)*x + (a^3*b^2*sqrt(-(a^6 -
6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a^3*b^3))
 - a^4*b + 3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*sqrt(-(a*b*sqrt(-(a^6 - 6*a^5*b
+ 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a^3*b^3)) + 2*a^2
 - 4*a*b + 2*b^2)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b
^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a^3*b^3)) + 2*a^2 - 4*a*b +
 2*b^2)/(a*b))*log((a^5 - 3*a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - 3*a*b^4 + b^5)
*x - (a^3*b^2*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4
- 6*a*b^5 + b^6)/(a^3*b^3)) - a^4*b + 3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*sqrt(
-(a*b*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^
5 + b^6)/(a^3*b^3)) + 2*a^2 - 4*a*b + 2*b^2)/(a*b))) + 1/4*sqrt((a*b*sqrt(
-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a
^3*b^3)) - 2*a^2 + 4*a*b - 2*b^2)/(a*b))*log((a^5 - 3*a^4*b + 2*a^3*b^2 +
2*a^2*b^3 - 3*a*b^4 + b^5)*x + (a^3*b^2*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2
- 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a^3*b^3)) + a^4*b - 3*a^3*b^2
+ 3*a^2*b^3 - a*b^4)*sqrt((a*b*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)/(a^3*b^3)) - 2*a^2 + 4*a*b - 2*b^2)/(a*b
))) - 1/4*sqrt((a*b*sqrt(-(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate((a*x**4+b)/(x**2-1)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.88

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx$$
$$=\frac{1}{8}(a-b)\left(\frac{2\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right.$$
$$-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(a - b)*(2*sqrt(2)*(sqrt(a) + sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
 + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*(sqrt(a) + sqrt(b))*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + sqrt(2)*(sqrt(a) - s
qrt(b))*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(3/4))) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.16

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}(a-b)+(ab^3)^{\frac{1}{4}}(ab^2-b^3)\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}(a-b)+(ab^3)^{\frac{1}{4}}(ab^2-b^3)\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}(a-b)-(ab^3)^{\frac{1}{4}}(ab^2-b^3)\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}(a-b)-(ab^3)^{\frac{1}{4}}(ab^2-b^3)\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$
$$- \frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(3/4)*(a - b) + (a*b^3)^(1/4)*(a*b^2 - b^3))*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*
((a*b^3)^(3/4)*(a - b) + (a*b^3)^(1/4)*(a*b^2 - b^3))*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(3/
4)*(a - b) - (a*b^3)^(1/4)*(a*b^2 - b^3))*log(x^2 + sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*(a - b) - (a*b^3)^(1/4)*
(a*b^2 - b^3))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/2*
log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 6231, normalized size of antiderivative = 24.25

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
int((b + a*x^4)/((x^2 - 1)*(a + b*x^4)),x)
```

output
```
- atanh(x) - atan((((-(a^3*(-a^3*b^3)^(1/2) - b^3*(-a^3*b^3)^(1/2) + 2*a^2
*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2*(-a^3*b^3)^(1/2) - 3*a^2*b*(-a^3*b^
3)^(1/2))/(16*a^3*b^3))^(1/2)*((x*(32*a*b^8 + 16*b^9 - 272*a^2*b^7 - 320*a
^3*b^6 + 112*a^4*b^5 + 32*a^5*b^4 - 112*a^6*b^3) + (-(a^3*(-a^3*b^3)^(1/2)
 - b^3*(-a^3*b^3)^(1/2) + 2*a^2*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2*(-a^
3*b^3)^(1/2) - 3*a^2*b*(-a^3*b^3)^(1/2))/(16*a^3*b^3))^(1/2)*(128*a^2*b^7
- 64*a*b^8 + 512*a^3*b^6 + 384*a^4*b^5 + 64*a^5*b^4 + x*(-(a^3*(-a^3*b^3)^
(1/2) - b^3*(-a^3*b^3)^(1/2) + 2*a^2*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2
*(-a^3*b^3)^(1/2) - 3*a^2*b*(-a^3*b^3)^(1/2))/(16*a^3*b^3))^(1/2)*(512*a^2
*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)))*(-(a^3*(-a^3*b^3)^(1/2)
- b^3*(-a^3*b^3)^(1/2) + 2*a^2*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2*(-a^3
*b^3)^(1/2) - 3*a^2*b*(-a^3*b^3)^(1/2))/(16*a^3*b^3))^(1/2) + 20*a*b^8 + 4
*b^9 + 20*a^2*b^7 - 76*a^3*b^6 - 84*a^4*b^5 + 60*a^5*b^4 + 60*a^6*b^3 - 4*
a^7*b^2) - x*(8*a*b^8 + 2*a^8*b + 6*b^9 - 12*a^2*b^7 - 16*a^3*b^6 + 8*a^4*
b^5 + 8*a^5*b^4 - 4*a^6*b^3))*(-(a^3*(-a^3*b^3)^(1/2) - b^3*(-a^3*b^3)^(1/
2) + 2*a^2*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2*(-a^3*b^3)^(1/2) - 3*a^2*
b*(-a^3*b^3)^(1/2))/(16*a^3*b^3))^(1/2)*1i - ((-(a^3*(-a^3*b^3)^(1/2) - b^
3*(-a^3*b^3)^(1/2) + 2*a^2*b^4 - 4*a^3*b^3 + 2*a^4*b^2 + 3*a*b^2*(-a^3*b^3
)^(1/2) - 3*a^2*b*(-a^3*b^3)^(1/2))/(16*a^3*b^3))^(1/2)*(20*a*b^8 - (x*(32
*a*b^8 + 16*b^9 - 272*a^2*b^7 - 320*a^3*b^6 + 112*a^4*b^5 + 32*a^5*b^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.18

$$\int \frac{b+ax^4}{(-1+x^2)(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
int((a*x^4+b)/(x^2-1)/(b*x^4+a),x)
```

output

```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b - 2*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b + 2*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*a - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b + 2*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**
(1/4)*sqrt(2)))*a - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b + b**(1/4)*a**(3/4)*sq
rt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a - b**
(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt
(b)*x**2)*b - b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(b)*x**2)*a + b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4
)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b - b**(3/4)*a**(1/4)*sqrt(2)*log( -
 b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a + b**(3/4)*a**(1/
4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b
+ b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +...
```

3.30 $\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = \frac{ab-2b^2x}{2a^2x^2}+\frac{1}{2}\log(-1+x)+\frac{(-a^2b-b^3)\log(x)}{a^3}+\frac{(a+b)\log(1+x)}{2(a-b)}+\frac{(-a^4+a^3b-a^2b^2+ab^3-b^4)\log(a+bx)}{a^3(a-b)}$$

output

```
1/2*(-2*b^2*x+a*b)/a^2/x^2+1/2*ln(-1+x)+(-a^2*b-b^3)*ln(x)/a^3+(a+b)*ln(1+
x)/(2*a-2*b)+(-a^4+a^3*b-a^2*b^2+a*b^3-b^4)*ln(b*x+a)/a^3/(a-b)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = \frac{1}{2}\left(\frac{b}{ax^2}-\frac{2b^2}{a^2x}+\log(1-x)-\frac{2b(a^2+b^2)\log(x)}{a^3}+\frac{(a+b)\log(1+x)}{a-b}-\frac{2(a^4-a^3b+a^2b^2-ab^3+b^4)\log(a+bx)}{a^3(a-b)}\right)$$

input
```
Integrate[(b + a*x^4)/((-1 + x^2)*(a*x^3 + b*x^4)),x]
```

output
```
(b/(a*x^2) - (2*b^2)/(a^2*x) + Log[1 - x] - (2*b*(a^2 + b^2)*Log[x])/a^3 +
 ((a + b)*Log[1 + x])/(a - b) - (2*(a^4 - a^3*b + a^2*b^2 - a*b^3 + b^4)*L
og[a + b*x])/(a^3*(a - b)))/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2026, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^4+b}{(x^2-1)(ax^3+bx^4)}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{ax^4+b}{x^3(x^2-1)(a+bx)}dx$$
$$\Big\downarrow \text{ 2353}$$
$$\int \left(\frac{b^2}{a^2x^2}-\frac{b(a^2+b^2)}{a^3x}-\frac{b(a^4-a^3b+a^2b^2-ab^3+b^4)}{a^3(a-b)(a+bx)}-\frac{b}{ax^3}+\frac{a+b}{2(x+1)(a-b)}+\frac{1}{2(x-1)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{b^2}{a^2x}-\frac{b(a^2+b^2)\log(x)}{a^3}-\frac{(a^4-a^3b+a^2b^2-ab^3+b^4)\log(a+bx)}{a^3(a-b)}+\frac{b}{2ax^2}+\frac{(a+b)\log(x+1)}{2(a-b)}+\frac{1}{2}\log(1-x)$$

input
```
Int[(b + a*x^4)/((-1 + x^2)*(a*x^3 + b*x^4)),x]
```

output
```
b/(2*a*x^2) - b^2/(a^2*x) + Log[1 - x]/2 - (b*(a^2 + b^2)*Log[x])/a^3 + ((
a + b)*Log[1 + x])/(2*(a - b)) - ((a^4 - a^3*b + a^2*b^2 - a*b^3 + b^4)*Lo
g[a + b*x])/(a^3*(a - b))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{b}{2a}-\frac{b^2 x}{a^2}}{x^2}+\frac{(a+b)\ln(1+x)}{2a-2b}-\frac{b(a^2+b^2)\ln(x)}{a^3}-\frac{(a^4-a^3 b+a^2 b^2-a\,b^3+b^4)\ln(bx+a)}{a^3(a-b)}+\frac{\ln(-1+x)}{2}$
default	$\frac{\ln(-1+x)}{2}-\frac{(a^4-a^3 b+a^2 b^2-a\,b^3+b^4)\ln(bx+a)}{a^3(a-b)}-\frac{b^2}{a^2 x}+\frac{b}{2a\,x^2}-\frac{b(a^2+b^2)\ln(x)}{a^3}+\frac{(a+b)\ln(1+x)}{2a-2b}$
risch	$\frac{\frac{b}{2a}-\frac{b^2 x}{a^2}}{x^2}-\frac{b\ln(x)}{a}-\frac{b^3\ln(x)}{a^3}+\frac{\ln(1-x)}{2}+\frac{\ln(1+x)a}{2a-2b}+\frac{\ln(1+x)b}{2a-2b}-\frac{a\ln(-bx-a)}{a-b}+\frac{\ln(-bx-a)b}{a-b}-\frac{\ln(-bx-a)b}{a(a-b)}$
parallelrisch	$-\frac{2\ln(x)x^2a^3b-2\ln(x)x^2a^2b^2+2\ln(x)x^2a\,b^3-2\ln(x)x^2b^4-\ln(-1+x)a^4x^2+\ln(-1+x)a^3x^2b-\ln(1+x)a^4x^2-\ln(1+x)a^3x^2b}{2a^3x^2}$

input
```
int((a*x^4+b)/(x^2-1)/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b/a-b^2/a^2*x)/x^2+1/2*(a+b)/(a-b)*ln(1+x)-b*(a^2+b^2)/a^3*ln(x)-(a^4
-a^3*b+a^2*b^2-a*b^3+b^4)/a^3/(a-b)*ln(b*x+a)+1/2*ln(-1+x)
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.36

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = \frac{a^3b-a^2b^2-2(a^4-a^3b+a^2b^2-ab^3+b^4)x^2\log(bx+a)+(a^4+a^3b)x^2\log(x+1)+(a^4-a^3b)x^2\log (}{2(a^4-a^3b)x^2}$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a*x^3),x, algorithm="fricas")
```

output
```
1/2*(a^3*b - a^2*b^2 - 2*(a^4 - a^3*b + a^2*b^2 - a*b^3 + b^4)*x^2*log(b*x
 + a) + (a^4 + a^3*b)*x^2*log(x + 1) + (a^4 - a^3*b)*x^2*log(x - 1) - 2*(a
^3*b - a^2*b^2 + a*b^3 - b^4)*x^2*log(x) - 2*(a^2*b^2 - a*b^3)*x)/((a^4 -
a^3*b)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate((a*x**4+b)/(x**2-1)/(b*x**4+a*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = -\frac{(a^4-a^3b+a^2b^2-ab^3+b^4)\log(bx+a)}{a^4-a^3b} + \frac{(a+b)\log(x+1)}{2(a-b)} - \frac{(a^2b+b^3)\log(x)}{a^3} - \frac{2b^2x-ab}{2a^2x^2} + \frac{1}{2}\log(x-1)$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a*x^3),x, algorithm="maxima")
```

output
```
-(a^4 - a^3*b + a^2*b^2 - a*b^3 + b^4)*log(b*x + a)/(a^4 - a^3*b) + 1/2*(a
 + b)*log(x + 1)/(a - b) - (a^2*b + b^3)*log(x)/a^3 - 1/2*(2*b^2*x - a*b)/
(a^2*x^2) + 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = -\frac{(a^4b-a^3b^2+a^2b^3-ab^4+b^5)\log(|bx+a|)}{a^4b-a^3b^2} + \frac{(a+b)\log(|x+1|)}{2(a-b)} - \frac{(a^2b+b^3)\log(|x|)}{a^3} - \frac{2ab^2x-a^2b}{2a^3x^2} + \frac{1}{2}\log(|x-1|)$$

input
```
integrate((a*x^4+b)/(x^2-1)/(b*x^4+a*x^3),x, algorithm="giac")
```

output
```
-(a^4*b - a^3*b^2 + a^2*b^3 - a*b^4 + b^5)*log(abs(b*x + a))/(a^4*b - a^3*
b^2) + 1/2*(a + b)*log(abs(x + 1))/(a - b) - (a^2*b + b^3)*log(abs(x))/a^3
 - 1/2*(2*a*b^2*x - a^2*b)/(a^3*x^2) + 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx = \frac{\ln(x-1)}{2} + \frac{\frac{b}{2a}-\frac{b^2x}{a^2}}{x^2} - \ln(a+bx)\left(\frac{a^2b^2-ab^3+b^4}{a^3(a-b)}+1\right) + \frac{\ln(x+1)(a+b)}{2(a-b)} - \frac{b\ln(x)(a^2+b^2)}{a^3}$$

input
```
int((b + a*x^4)/((x^2 - 1)*(a*x^3 + b*x^4)),x)
```

output
```
log(x - 1)/2 + (b/(2*a) - (b^2*x)/a^2)/x^2 - log(a + b*x)*((b^4 - a*b^3 +
a^2*b^2)/(a^3*(a - b)) + 1) + (log(x + 1)*(a + b))/(2*(a - b)) - (b*log(x)
*(a^2 + b^2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.91

$$\int \frac{b+ax^4}{(-1+x^2)(ax^3+bx^4)}\,dx$$
$$= \frac{-2\log(bx+a)\,a^4x^2+2\log(bx+a)\,a^3b\,x^2-2\log(bx+a)\,a^2b^2x^2+2\log(bx+a)\,a\,b^3x^2-2\log(bx+a)\,b}{}$$

input
```
int((a*x^4+b)/(x^2-1)/(b*x^4+a*x^3),x)
```

output
```
( - 2*log(a + b*x)*a**4*x**2 + 2*log(a + b*x)*a**3*b*x**2 - 2*log(a + b*x)
*a**2*b**2*x**2 + 2*log(a + b*x)*a*b**3*x**2 - 2*log(a + b*x)*b**4*x**2 +
log(x - 1)*a**4*x**2 - log(x - 1)*a**3*b*x**2 + log(x + 1)*a**4*x**2 + log
(x + 1)*a**3*b*x**2 - 2*log(x)*a**3*b*x**2 + 2*log(x)*a**2*b**2*x**2 - 2*l
og(x)*a*b**3*x**2 + 2*log(x)*b**4*x**2 + a**3*b - 2*a**2*b**2*x - a**2*b**
2 + 2*a*b**3*x)/(2*a**3*x**2*(a - b))
```

3.31 $\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\begin{aligned}\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx =& -\frac{\left(\frac{a}{b^4}-\frac{b}{a^4}\right)x}{12\left(a^4+b^4x^4\right)^3}+\frac{\left(a^5+11b^5\right)x}{96a^8b^4\left(a^4+b^4x^4\right)^2}\\&+\frac{7\left(a^5+11b^5\right)x}{384a^{12}b^4\left(a^4+b^4x^4\right)}-\frac{7\left(a^5+11b^5\right)\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{256\sqrt{2}a^{15}b^5}\\&+\frac{7\left(a^5+11b^5\right)\arctan\left(1+\frac{\sqrt{2}bx}{a}\right)}{256\sqrt{2}a^{15}b^5}\\&-\frac{7\left(a^5+11b^5\right)\log\left(a^2-\sqrt{2}abx+b^2x^2\right)}{512\sqrt{2}a^{15}b^5}\\&+\frac{7\left(a^5+11b^5\right)\log\left(a^2+\sqrt{2}abx+b^2x^2\right)}{512\sqrt{2}a^{15}b^5}\end{aligned}$$

output

```
-1/12*(a/b^4-b/a^4)*x/(b^4*x^4+a^4)^3+1/96*(a^5+11*b^5)*x/a^8/b^4/(b^4*x^4
+a^4)^2+7/384*(a^5+11*b^5)*x/a^12/b^4/(b^4*x^4+a^4)-7/512*(a^5+11*b^5)*arc
tan(1-2^(1/2)*b*x/a)*2^(1/2)/a^15/b^5+7/512*(a^5+11*b^5)*arctan(1+2^(1/2)*
b*x/a)*2^(1/2)/a^15/b^5-7/1024*(a^5+11*b^5)*ln(a^2-2^(1/2)*a*b*x+b^2*x^2)*
2^(1/2)/a^15/b^5+7/1024*(a^5+11*b^5)*ln(a^2+2^(1/2)*a*b*x+b^2*x^2)*2^(1/2)
/a^15/b^5
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{b+ax^4}{(a^4+b^4x^4)^4}\,dx=\$\text{Aborted}$$

input
```
Integrate[(b + a*x^4)/(a^4 + b^4*x^4)^4,x]
```

output
```
$Aborted
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.524$, Rules used = {910, 749, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^4+b}{(a^4+b^4x^4)^4}dx$$

$$\Big\downarrow \; 910$$

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\int \frac{1}{(a^4+b^4x^4)^3}dx-\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$$\Big\downarrow \; 749$$

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\int \frac{1}{(a^4+b^4x^4)^2}dx}{8a^4}+\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}\right)-\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$$\Big\downarrow \; 749$$

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\int \frac{1}{a^4+b^4x^4}dx}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}\right)-\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$$\Big\downarrow \; 755$$

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\int \frac{a^2-b^2x^2}{a^4+b^4x^4}dx}{2a^2}+\frac{\int \frac{a^2+b^2x^2}{a^4+b^4x^4}dx}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}\right)-\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\Big\downarrow$ 1476

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{\frac{a^2}{b^2}-\frac{\sqrt{2}xa}{b}+x^2}dx}{2b^2}+\frac{\int \frac{1}{\frac{a^2}{b^2}+\frac{\sqrt{2}xa}{b}+x^2}dx}{2b^2}}{2a^2}+\frac{\int \frac{a^2-b^2x^2}{a^4+b^4x^4}dx}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}\right)-\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\Big\downarrow$ 1082

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}bx}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}bx}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}}{2a^2}+\frac{\int \frac{a^2-b^2x^2}{a^4+b^4x^4}dx}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{x}{8a^4 (\cdots\right)$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\Big\downarrow$ 217

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\int \frac{a^2-b^2x^2}{a^4+b^4x^4}dx}{2a^2}+\frac{\frac{\arctan\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}-\frac{\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}\right)-$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\Big\downarrow$ 1479

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}a-2bx}{b\left(\frac{a^2}{b^2}-\frac{\sqrt{2}xa}{b}+x^2\right)}dx}{2\sqrt{2}ab}-\frac{\int -\frac{\sqrt{2}\left(a+\sqrt{2}bx\right)}{b\left(\frac{a^2}{b^2}+\frac{\sqrt{2}xa}{b}+x^2\right)}dx}{2\sqrt{2}ab}}{2a^2}+\frac{\frac{\arctan\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}-\frac{\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}\right.$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\downarrow$ 25

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}a-2bx}{b\left(\frac{a^2}{b^2}-\frac{\sqrt{2}xa}{b}+x^2\right)}dx}{2\sqrt{2}ab}+\frac{\int \frac{\sqrt{2}\left(a+\sqrt{2}bx\right)}{b\left(\frac{a^2}{b^2}+\frac{\sqrt{2}xa}{b}+x^2\right)}dx}{2\sqrt{2}ab}}{2a^2}+\frac{\frac{\arctan\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}-\frac{\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\right.$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}a-2bx}{\frac{a^2}{b^2}-\frac{\sqrt{2}xa}{b}+x^2}dx}{2\sqrt{2}ab^2}+\frac{\int \frac{a+\sqrt{2}bx}{\frac{a^2}{b^2}+\frac{\sqrt{2}xa}{b}+x^2}dx}{2ab^2}}{2a^2}+\frac{\frac{\arctan\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}-\frac{\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}}{2a^2}\right)}{4a^4}+\frac{x}{4a^4(a^4+b^4x^4)}\right)}{8a^4}+\frac{}{8a^4\,(}\right.$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

$\downarrow$ 1103

$$\frac{1}{12}\left(\frac{11b}{a^4}+\frac{a}{b^4}\right)\left(\frac{x}{8a^4\left(a^4+b^4x^4\right)^2}+\frac{7\left(\frac{x}{4a^4(a^4+b^4x^4)}+\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}bx}{a}+1\right)}{\sqrt{2}ab}-\frac{\arctan\left(1-\frac{\sqrt{2}bx}{a}\right)}{\sqrt{2}ab}}{2a^2}+\frac{\frac{\log\left(a^2+\sqrt{2}abx+b^2x^2\right)}{2\sqrt{2}ab}-\frac{\log\left(a^2-\right.}{}}{2a^2}\right)}{4a^4}\right)}{8a^4}\right)$$

$$\frac{x\left(\frac{a}{b^4}-\frac{b}{a^4}\right)}{12\left(a^4+b^4x^4\right)^3}$$

input
```
Int[(b + a*x^4)/(a^4 + b^4*x^4)^4,x]
```

output
```
-1/12*((a/b^4 - b/a^4)*x)/(a^4 + b^4*x^4)^3 + ((a/b^4 + (11*b)/a^4)*(x/(8*
a^4*(a^4 + b^4*x^4)^2) + (7*(x/(4*a^4*(a^4 + b^4*x^4)) + (3*((-(ArcTan[1 -
 (Sqrt[2]*b*x)/a]/(Sqrt[2]*a*b)) + ArcTan[1 + (Sqrt[2]*b*x)/a]/(Sqrt[2]*a*
b))/(2*a^2) + (-1/2*Log[a^2 - Sqrt[2]*a*b*x + b^2*x^2]/(Sqrt[2]*a*b) + Log
[a^2 + Sqrt[2]*a*b*x + b^2*x^2]/(2*Sqrt[2]*a*b))/(2*a^2)))/(4*a^4)))/(8*a^
4))/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{7\left(a^5+11b^5\right)b^4x^9}{384a^{12}}+\frac{3\left(a^5+11b^5\right)x^5}{64a^8}-\frac{\left(7a^5-51b^5\right)x}{128a^4b^4}}{\left(b^4x^4+a^4\right)^3}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(b^4\_Z^4+a^4\right)}\frac{\left(a^5+11b^5\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{512a^{12}b^8}$
default	$\frac{\frac{7\left(a^5+11b^5\right)b^4x^9}{384a^{12}}+\frac{3\left(a^5+11b^5\right)x^5}{64a^8}-\frac{\left(7a^5-51b^5\right)x}{128a^4b^4}}{\left(b^4x^4+a^4\right)^3}+\frac{7(a^5+11b^5)\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a^4}{b^4}}}{x^2-\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a^4}{b^4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}}+1\right.\right.}{1024a^{16}b^4}$

input
```
int((a*x^4+b)/(b^4*x^4+a^4)^4,x,method=_RETURNVERBOSE)
```

output
```
(7/384*(a^5+11*b^5)/a^12*b^4*x^9+3/64/a^8*(a^5+11*b^5)*x^5-1/128*(7*a^5-51
*b^5)/a^4/b^4*x)/(b^4*x^4+a^4)^3+7/512/a^12/b^8*sum((a^5+11*b^5)/_R^3*ln(x
-_R),_R=RootOf(_Z^4*b^4+a^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(235) = 470.

Time = 0.08 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.80

$$\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx$$

$$= \frac{56\left(a^8b^9+11\,a^3b^{14}\right)x^9+144\left(a^{12}b^5+11\,a^7b^{10}\right)x^5+42\sqrt{2}(a^{17}+11\,a^{12}b^5+\left(a^5b^{12}+11\,b^{17}\right)x^{12}+3\left(a^9b^8\right.}{}$$

input
```
integrate((a*x^4+b)/(b^4*x^4+a^4)^4,x, algorithm="fricas")
```

output
```
1/3072*(56*(a^8*b^9 + 11*a^3*b^14)*x^9 + 144*(a^12*b^5 + 11*a^7*b^10)*x^5
+ 42*sqrt(2)*(a^17 + 11*a^12*b^5 + (a^5*b^12 + 11*b^17)*x^12 + 3*(a^9*b^8
+ 11*a^4*b^13)*x^8 + 3*(a^13*b^4 + 11*a^8*b^9)*x^4)*arctan((sqrt(2)*b*x +
a)/a) + 42*sqrt(2)*(a^17 + 11*a^12*b^5 + (a^5*b^12 + 11*b^17)*x^12 + 3*(a^
9*b^8 + 11*a^4*b^13)*x^8 + 3*(a^13*b^4 + 11*a^8*b^9)*x^4)*arctan((sqrt(2)*
b*x - a)/a) + 21*sqrt(2)*(a^17 + 11*a^12*b^5 + (a^5*b^12 + 11*b^17)*x^12 +
 3*(a^9*b^8 + 11*a^4*b^13)*x^8 + 3*(a^13*b^4 + 11*a^8*b^9)*x^4)*log(b^2*x^
2 + sqrt(2)*a*b*x + a^2) - 21*sqrt(2)*(a^17 + 11*a^12*b^5 + (a^5*b^12 + 11
*b^17)*x^12 + 3*(a^9*b^8 + 11*a^4*b^13)*x^8 + 3*(a^13*b^4 + 11*a^8*b^9)*x^
4)*log(b^2*x^2 - sqrt(2)*a*b*x + a^2) - 24*(7*a^16*b - 51*a^11*b^6)*x)/(a^
15*b^17*x^12 + 3*a^19*b^13*x^8 + 3*a^23*b^9*x^4 + a^27*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.63

$$\int \frac{b+ax^4}{(a^4+b^4x^4)^4}\,dx$$
$$= \frac{x^9\cdot(7a^5b^8+77b^{13})+x^5\cdot(18a^9b^4+198a^4b^9)+x(-21a^{13}+153a^8b^5)}{384a^{24}b^4+1152a^{20}b^8x^4+1152a^{16}b^{12}x^8+384a^{12}b^{16}x^{12}}$$
$$+\mathrm{RootSum}\left(68719476736t^4a^{60}b^{20}+2401a^{20}+105644a^{15}b^5+1743126a^{10}b^{10}+12782924a^5b^{15}+35153\right.$$

input
```
integrate((a*x**4+b)/(b**4*x**4+a**4)**4,x)
```

output
```
(x**9*(7*a**5*b**8 + 77*b**13) + x**5*(18*a**9*b**4 + 198*a**4*b**9) + x*(
-21*a**13 + 153*a**8*b**5))/(384*a**24*b**4 + 1152*a**20*b**8*x**4 + 1152*
a**16*b**12*x**8 + 384*a**12*b**16*x**12) + RootSum(68719476736*_t**4*a**6
0*b**20 + 2401*a**20 + 105644*a**15*b**5 + 1743126*a**10*b**10 + 12782924*
a**5*b**15 + 35153041*b**20, Lambda(_t, _t*log(512*_t*a**16*b**4/(7*a**5 +
 77*b**5) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.06

$$\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx = \frac{7\left(a^5b^8+11\,b^{13}\right)x^9+18\left(a^9b^4+11\,a^4b^9\right)x^5-3\left(7\,a^{13}-51\,a^8b^5\right)x}{384\left(a^{12}b^{16}x^{12}+3\,a^{16}b^{12}x^8+3\,a^{20}b^8x^4+a^{24}b^4\right)} + \frac{7\left(\frac{2\sqrt{2}\left(a^5+11\,b^5\right)\arctan\left(\frac{\sqrt{2}\left(2\,b^2x+\sqrt{2}ab\right)}{2\,ab}\right)}{a^3b}+\frac{2\sqrt{2}\left(a^5+11\,b^5\right)\arctan\left(\frac{\sqrt{2}\left(2\,b^2x-\sqrt{2}ab\right)}{2\,ab}\right)}{a^3b}+\frac{\sqrt{2}\left(a^5+11\,b^5\right)\log\left(b^2x^2+\sqrt{2}abx+a^2\right)}{a^3b}\right.}{1024\,a^{12}b^4}$$

input
```
integrate((a*x^4+b)/(b^4*x^4+a^4)^4,x, algorithm="maxima")
```

output
```
1/384*(7*(a^5*b^8 + 11*b^13)*x^9 + 18*(a^9*b^4 + 11*a^4*b^9)*x^5 - 3*(7*a^
13 - 51*a^8*b^5)*x)/(a^12*b^16*x^12 + 3*a^16*b^12*x^8 + 3*a^20*b^8*x^4 + a
^24*b^4) + 7/1024*(2*sqrt(2)*(a^5 + 11*b^5)*arctan(1/2*sqrt(2)*(2*b^2*x +
sqrt(2)*a*b)/(a*b))/(a^3*b) + 2*sqrt(2)*(a^5 + 11*b^5)*arctan(1/2*sqrt(2)*
(2*b^2*x - sqrt(2)*a*b)/(a*b))/(a^3*b) + sqrt(2)*(a^5 + 11*b^5)*log(b^2*x^
2 + sqrt(2)*a*b*x + a^2)/(a^3*b) - sqrt(2)*(a^5 + 11*b^5)*log(b^2*x^2 - sq
rt(2)*a*b*x + a^2)/(a^3*b))/(a^12*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int \frac{b+ax^4}{(a^4+b^4x^4)^4}\,dx = \frac{7\,a^5b^8x^9+77\,b^{13}x^9+18\,a^9b^4x^5+198\,a^4b^9x^5-21\,a^{13}x+153\,a^8b^5x}{384\,(b^4x^4+a^4)^3a^{12}b^4} + \frac{7\,(\sqrt{2}a^5+11\,\sqrt{2}b^5)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}}\right)}{512\,a^{15}b^5} + \frac{7\,(\sqrt{2}a^5+11\,\sqrt{2}b^5)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}}\right)}{512\,a^{15}b^5} + \frac{7\,(\sqrt{2}a^5+11\,\sqrt{2}b^5)\log\left(x^2+\sqrt{2}x\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}+\sqrt{\frac{a^4}{b^4}}\right)}{1024\,a^{15}b^5} - \frac{7\,(\sqrt{2}a^5+11\,\sqrt{2}b^5)\log\left(x^2-\sqrt{2}x\left(\frac{a^4}{b^4}\right)^{\frac{1}{4}}+\sqrt{\frac{a^4}{b^4}}\right)}{1024\,a^{15}b^5}$$

input
```
integrate((a*x^4+b)/(b^4*x^4+a^4)^4,x, algorithm="giac")
```

output
```
1/384*(7*a^5*b^8*x^9 + 77*b^13*x^9 + 18*a^9*b^4*x^5 + 198*a^4*b^9*x^5 - 21
*a^13*x + 153*a^8*b^5*x)/((b^4*x^4 + a^4)^3*a^12*b^4) + 7/512*(sqrt(2)*a^5
 + 11*sqrt(2)*b^5)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a^4/b^4)^(1/4))/(a^4
/b^4)^(1/4))/(a^15*b^5) + 7/512*(sqrt(2)*a^5 + 11*sqrt(2)*b^5)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a^4/b^4)^(1/4))/(a^4/b^4)^(1/4))/(a^15*b^5) + 7/10
24*(sqrt(2)*a^5 + 11*sqrt(2)*b^5)*log(x^2 + sqrt(2)*x*(a^4/b^4)^(1/4) + sq
rt(a^4/b^4))/(a^15*b^5) - 7/1024*(sqrt(2)*a^5 + 11*sqrt(2)*b^5)*log(x^2 -
sqrt(2)*x*(a^4/b^4)^(1/4) + sqrt(a^4/b^4))/(a^15*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.00

$$\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx = \text{Too large to display}$$

input
```
int((b + a*x^4)/(a^4 + b^4*x^4)^4,x)
```

output
```
((3*x^5*(a^5 + 11*b^5))/(64*a^8) - (x*(7*a^5 - 51*b^5))/(128*a^4*b^4) + (7
*b^4*x^9*(a^5 + 11*b^5))/(384*a^12))/(a^12 + b^12*x^12 + 3*a^8*b^4*x^4 + 3
*a^4*b^8*x^8) + ((-1)^(1/4)*atan((((-1)^(1/4)*((49*x*(121*b^14 + 22*a^5*b^
9 + a^10*b^4))/(4096*a^24) - (49*(-1)^(1/4)*(a^5 + 11*b^5)*(11*b^13 + a^5*
b^8))/(4096*a^23*b^5))*(a^5 + 11*b^5)*7i)/(512*a^15*b^5) + ((-1)^(1/4)*((4
9*x*(121*b^14 + 22*a^5*b^9 + a^10*b^4))/(4096*a^24) + (49*(-1)^(1/4)*(a^5
+ 11*b^5)*(11*b^13 + a^5*b^8))/(4096*a^23*b^5))*(a^5 + 11*b^5)*7i)/(512*a^
15*b^5))/((7*(-1)^(1/4)*((49*x*(121*b^14 + 22*a^5*b^9 + a^10*b^4))/(4096*a
^24) - (49*(-1)^(1/4)*(a^5 + 11*b^5)*(11*b^13 + a^5*b^8))/(4096*a^23*b^5))
*(a^5 + 11*b^5))/(512*a^15*b^5) - (7*(-1)^(1/4)*((49*x*(121*b^14 + 22*a^5*
b^9 + a^10*b^4))/(4096*a^24) + (49*(-1)^(1/4)*(a^5 + 11*b^5)*(11*b^13 + a^
5*b^8))/(4096*a^23*b^5))*(a^5 + 11*b^5))/(512*a^15*b^5)))*(a^5 + 11*b^5)*7
i)/(256*a^15*b^5) + (7*(-1)^(1/4)*atan(((7*(-1)^(1/4)*((49*x*(121*b^14 + 2
2*a^5*b^9 + a^10*b^4))/(4096*a^24) - ((-1)^(1/4)*(a^5 + 11*b^5)*(11*b^13 +
 a^5*b^8)*49i)/(4096*a^23*b^5))*(a^5 + 11*b^5))/(512*a^15*b^5) + (7*(-1)^(
1/4)*((49*x*(121*b^14 + 22*a^5*b^9 + a^10*b^4))/(4096*a^24) + ((-1)^(1/4)*
(a^5 + 11*b^5)*(11*b^13 + a^5*b^8)*49i)/(4096*a^23*b^5))*(a^5 + 11*b^5))/(
512*a^15*b^5))/(((-1)^(1/4)*((49*x*(121*b^14 + 22*a^5*b^9 + a^10*b^4))/(40
96*a^24) - ((-1)^(1/4)*(a^5 + 11*b^5)*(11*b^13 + a^5*b^8)*49i)/(4096*a^23*
b^5))*(a^5 + 11*b^5)*7i)/(512*a^15*b^5) - ((-1)^(1/4)*((49*x*(121*b^14 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.98

$$\int \frac{b+ax^4}{\left(a^4+b^4x^4\right)^4}\,dx = \text{Too large to display}$$

input
```
int((a*x^4+b)/(b^4*x^4+a^4)^4,x)
```

output

```
( - 42*sqrt(2)*atan((sqrt(2)*a - 2*b*x)/(sqrt(2)*a))*a**17 - 126*sqrt(2)*a
tan((sqrt(2)*a - 2*b*x)/(sqrt(2)*a))*a**13*b**4*x**4 - 462*sqrt(2)*atan((s
qrt(2)*a - 2*b*x)/(sqrt(2)*a))*a**12*b**5 - 126*sqrt(2)*atan((sqrt(2)*a -
2*b*x)/(sqrt(2)*a))*a**9*b**8*x**8 - 1386*sqrt(2)*atan((sqrt(2)*a - 2*b*x)
/(sqrt(2)*a))*a**8*b**9*x**4 - 42*sqrt(2)*atan((sqrt(2)*a - 2*b*x)/(sqrt(2
)*a))*a**5*b**12*x**12 - 1386*sqrt(2)*atan((sqrt(2)*a - 2*b*x)/(sqrt(2)*a)
)*a**4*b**13*x**8 - 462*sqrt(2)*atan((sqrt(2)*a - 2*b*x)/(sqrt(2)*a))*b**1
7*x**12 + 42*sqrt(2)*atan((sqrt(2)*a + 2*b*x)/(sqrt(2)*a))*a**17 + 126*sqr
t(2)*atan((sqrt(2)*a + 2*b*x)/(sqrt(2)*a))*a**13*b**4*x**4 + 462*sqrt(2)*a
tan((sqrt(2)*a + 2*b*x)/(sqrt(2)*a))*a**12*b**5 + 126*sqrt(2)*atan((sqrt(2
)*a + 2*b*x)/(sqrt(2)*a))*a**9*b**8*x**8 + 1386*sqrt(2)*atan((sqrt(2)*a +
2*b*x)/(sqrt(2)*a))*a**8*b**9*x**4 + 42*sqrt(2)*atan((sqrt(2)*a + 2*b*x)/(
sqrt(2)*a))*a**5*b**12*x**12 + 1386*sqrt(2)*atan((sqrt(2)*a + 2*b*x)/(sqrt
(2)*a))*a**4*b**13*x**8 + 462*sqrt(2)*atan((sqrt(2)*a + 2*b*x)/(sqrt(2)*a)
)*b**17*x**12 - 21*sqrt(2)*log( - sqrt(2)*a*b*x + a**2 + b**2*x**2)*a**17
- 63*sqrt(2)*log( - sqrt(2)*a*b*x + a**2 + b**2*x**2)*a**13*b**4*x**4 - 23
1*sqrt(2)*log( - sqrt(2)*a*b*x + a**2 + b**2*x**2)*a**12*b**5 - 63*sqrt(2)
*log( - sqrt(2)*a*b*x + a**2 + b**2*x**2)*a**9*b**8*x**8 - 693*sqrt(2)*log
( - sqrt(2)*a*b*x + a**2 + b**2*x**2)*a**8*b**9*x**4 - 21*sqrt(2)*log( - s
qrt(2)*a*b*x + a**2 + b**2*x**2)*a**5*b**12*x**12 - 693*sqrt(2)*log( - ...
```

3.32 $\int \frac{b+ax^2}{-a^4+b^4x^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{b+ax^2}{-a^4+b^4x^4} dx = \frac{(a^3-b^3)\arctan\left(\frac{bx}{a}\right)}{2a^3b^3} + \frac{(-a^3-b^3)\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^3b^3}$$

output
```
1/2*(a^3-b^3)*arctan(b*x/a)/a^3/b^3+1/2*(-a^3-b^3)*arctanh(b*x/a)/a^3/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{b+ax^2}{-a^4+b^4x^4} dx = \frac{2(a^3-b^3)\arctan\left(\frac{bx}{a}\right)+(a^3+b^3)(\log(-a+bx)-\log(a+bx))}{4a^3b^3}$$

input
```
Integrate[(b + a*x^2)/(-a^4 + b^4*x^4),x]
```

output
```
(2*(a^3 - b^3)*ArcTan[(b*x)/a] + (a^3 + b^3)*(Log[-a + b*x] - Log[a + b*x]
))/(4*a^3*b^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+b}{b^4x^4-a^4}\,dx$$

$$\Big\downarrow \ 1481$$

$$\frac{1}{2}\left(\frac{b^3}{a^2}+a\right)\int \frac{1}{b^4x^2-a^2b^2}dx+\frac{1}{2}\left(a-\frac{b^3}{a^2}\right)\int \frac{1}{x^2b^4+a^2b^2}dx$$

$$\Big\downarrow \ 218$$

$$\frac{1}{2}\left(\frac{b^3}{a^2}+a\right)\int \frac{1}{b^4x^2-a^2b^2}dx+\frac{\left(a-\frac{b^3}{a^2}\right)\arctan\left(\frac{bx}{a}\right)}{2ab^3}$$

$$\Big\downarrow \ 221$$

$$\frac{\left(a-\frac{b^3}{a^2}\right)\arctan\left(\frac{bx}{a}\right)}{2ab^3}-\frac{\left(\frac{b^3}{a^2}+a\right)\text{arctanh}\left(\frac{bx}{a}\right)}{2ab^3}$$

input
```
Int[(b + a*x^2)/(-a^4 + b^4*x^4),x]
```

output
```
((a - b^3/a^2)*ArcTan[(b*x)/a])/(2*a*b^3) - ((a + b^3/a^2)*ArcTanh[(b*x)/a
])/(2*a*b^3)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

method	result
default	$\frac{\left(-a^{3}-b^{3}\right) \ln \left(b x+a\right)}{4 a^{3} b^{3}}+\frac{\left(a^{3}-b^{3}\right) \arctan \left(\frac{b x}{a}\right)}{2 a^{3} b^{3}}+\frac{\left(a^{3}+b^{3}\right) \ln \left(-b x+a\right)}{4 a^{3} b^{3}}$
parallelrisch	$-\frac{i \ln \left(-i a+b x\right) a^{3}-i \ln \left(-i a+b x\right) b^{3}-i \ln \left(i a+b x\right) a^{3}+i \ln \left(i a+b x\right) b^{3}-\ln \left(b x-a\right) a^{3}-\ln \left(b x-a\right) b^{3}+\ln \left(b x+a\right) a^{3}+\ln \left(b x+a\right) b^{3}}{4 a^{3} b^{3}}$
risch	$-\frac{\ln \left(-b x-a\right)}{4 b^{3}}-\frac{\ln \left(-b x-a\right)}{4 a^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{6} b^{6} \_Z^{2}+a^{6}-2 a^{3} b^{3}+b^{6}\right)}\_R \ln \left(\left(a^{6} b^{4}+b^{10}\right)\_R x-a^{4}\_R^{2} b^{9}-a^{7}+a\, b^{6}\right)}{4}$

input
```
int((a*x^2+b)/(b^4*x^4-a^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-a^3-b^3)/a^3/b^3*ln(b*x+a)+1/2*(a^3-b^3)*arctan(b*x/a)/a^3/b^3+1/4*(
a^3+b^3)/a^3/b^3*ln(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx = \frac{2\left(a^3-b^3\right)\arctan\left(\frac{bx}{a}\right)-\left(a^3+b^3\right)\log\left(bx+a\right)+\left(a^3+b^3\right)\log\left(bx-a\right)}{4\,a^3b^3}$$

input
```
integrate((a*x^2+b)/(b^4*x^4-a^4),x, algorithm="fricas")
```

output
```
1/4*(2*(a^3 - b^3)*arctan(b*x/a) - (a^3 + b^3)*log(b*x + a) + (a^3 + b^3)*
log(b*x - a))/(a^3*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 394, normalized size of antiderivative = 7.16

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx =$$

$$-\frac{i(a-b)(a^2+ab+b^2)\log\left(x+\frac{-3ia^7b^2(a-b)(a^2+ab+b^2)-\frac{ia^4(a-b)^3(a^2+ab+b^2)^3}{b}-iab^8(a-b)(a^2+ab+b^2)}{a^{12}-b^{12}}\right)}{4a^3b^3}$$
$$+\frac{i(a-b)(a^2+ab+b^2)\log\left(x+\frac{3ia^7b^2(a-b)(a^2+ab+b^2)+\frac{ia^4(a-b)^3(a^2+ab+b^2)^3}{b}+iab^8(a-b)(a^2+ab+b^2)}{a^{12}-b^{12}}\right)}{4a^3b^3}$$
$$-\frac{(a+b)(a^2-ab+b^2)\log\left(x+\frac{-3a^7b^2(a+b)(a^2-ab+b^2)+\frac{a^4(a+b)^3(a^2-ab+b^2)^3}{b}-ab^8(a+b)(a^2-ab+b^2)}{a^{12}-b^{12}}\right)}{4a^3b^3}$$
$$+\frac{(a+b)(a^2-ab+b^2)\log\left(x+\frac{3a^7b^2(a+b)(a^2-ab+b^2)-\frac{a^4(a+b)^3(a^2-ab+b^2)^3}{b}+ab^8(a+b)(a^2-ab+b^2)}{a^{12}-b^{12}}\right)}{4a^3b^3}$$

input
```
integrate((a*x**2+b)/(b**4*x**4-a**4),x)
```

output
```
-I*(a - b)*(a**2 + a*b + b**2)*log(x + (-3*I*a**7*b**2*(a - b)*(a**2 + a*b
 + b**2) - I*a**4*(a - b)**3*(a**2 + a*b + b**2)**3/b - I*a*b**8*(a - b)*(
a**2 + a*b + b**2))/(a**12 - b**12))/(4*a**3*b**3) + I*(a - b)*(a**2 + a*b
 + b**2)*log(x + (3*I*a**7*b**2*(a - b)*(a**2 + a*b + b**2) + I*a**4*(a -
b)**3*(a**2 + a*b + b**2)**3/b + I*a*b**8*(a - b)*(a**2 + a*b + b**2))/(a*
*12 - b**12))/(4*a**3*b**3) - (a + b)*(a**2 - a*b + b**2)*log(x + (-3*a**7
*b**2*(a + b)*(a**2 - a*b + b**2) + a**4*(a + b)**3*(a**2 - a*b + b**2)**3
/b - a*b**8*(a + b)*(a**2 - a*b + b**2))/(a**12 - b**12))/(4*a**3*b**3) +
(a + b)*(a**2 - a*b + b**2)*log(x + (3*a**7*b**2*(a + b)*(a**2 - a*b + b**
2) - a**4*(a + b)**3*(a**2 - a*b + b**2)**3/b + a*b**8*(a + b)*(a**2 - a*b
 + b**2))/(a**12 - b**12))/(4*a**3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx = \frac{(a^3-b^3)\arctan\left(\frac{bx}{a}\right)}{2\,a^3b^3} - \frac{(a^3+b^3)\log(bx+a)}{4\,a^3b^3} + \frac{(a^3+b^3)\log(bx-a)}{4\,a^3b^3}$$

input
```
integrate((a*x^2+b)/(b^4*x^4-a^4),x, algorithm="maxima")
```

output
```
1/2*(a^3 - b^3)*arctan(b*x/a)/(a^3*b^3) - 1/4*(a^3 + b^3)*log(b*x + a)/(a^
3*b^3) + 1/4*(a^3 + b^3)*log(b*x - a)/(a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx = \frac{(a^3-b^3)\arctan\left(\frac{bx}{a}\right)}{2\,a^3b^3} - \frac{(a^3b+b^4)\log(|bx+a|)}{4\,a^3b^4} + \frac{(a^3b+b^4)\log(|bx-a|)}{4\,a^3b^4}$$

input
```
integrate((a*x^2+b)/(b^4*x^4-a^4),x, algorithm="giac")
```

output
```
1/2*(a^3 - b^3)*arctan(b*x/a)/(a^3*b^3) - 1/4*(a^3*b + b^4)*log(abs(b*x +
a))/(a^3*b^4) + 1/4*(a^3*b + b^4)*log(abs(b*x - a))/(a^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.04

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx = -\operatorname{atanh}\left(\frac{2\,x\,(4\,a^6\,b^8+4\,b^{14})\,\left(\frac{1}{4\,a^3}+\frac{1}{4\,b^3}\right)}{2\,a^7\,b^4-2\,a\,b^{10}+32\,a^4\,b^{13}\,\left(\frac{1}{4\,a^3}+\frac{1}{4\,b^3}\right)^2}\right)\left(\frac{1}{2\,a^3}+\frac{1}{2\,b^3}\right) - \operatorname{atanh}\left(\frac{2\,x\,(4\,a^6\,b^8+4\,b^{14})\,\left(\frac{1\mathrm{i}}{4\,a^3}-\frac{1\mathrm{i}}{4\,b^3}\right)}{2\,a^7\,b^4-2\,a\,b^{10}+32\,a^4\,b^{13}\,\left(\frac{1\mathrm{i}}{4\,a^3}-\frac{1\mathrm{i}}{4\,b^3}\right)^2}\right)\left(\frac{1\mathrm{i}}{2\,a^3}-\frac{1\mathrm{i}}{2\,b^3}\right)$$

input
```
int(-(b + a*x^2)/(a^4 - b^4*x^4),x)
```

output
```
- atanh((2*x*(4*b^14 + 4*a^6*b^8)*(1/(4*a^3) + 1/(4*b^3)))/(2*a^7*b^4 - 2*
a*b^10 + 32*a^4*b^13*(1/(4*a^3) + 1/(4*b^3))^2))*(1/(2*a^3) + 1/(2*b^3)) -
 atanh((2*x*(4*b^14 + 4*a^6*b^8)*(1i/(4*a^3) - 1i/(4*b^3)))/(2*a^7*b^4 - 2
*a*b^10 + 32*a^4*b^13*(1i/(4*a^3) - 1i/(4*b^3))^2))*(1i/(2*a^3) - 1i/(2*b^
3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{b+ax^2}{-a^4+b^4x^4}\,dx = \frac{2atan\left(\frac{bx}{a}\right)a^3-2atan\left(\frac{bx}{a}\right)b^3+\log(-bx+a)\,a^3+\log(-bx+a)\,b^3-\log(bx+a)\,a^3-\log(bx+a)\,b^3}{4a^3b^3}$$

input
```
int((a*x^2+b)/(b^4*x^4-a^4),x)
```

output
```
(2*atan((b*x)/a)*a**3 - 2*atan((b*x)/a)*b**3 + log(a - b*x)*a**3 + log(a -
 b*x)*b**3 - log(a + b*x)*a**3 - log(a + b*x)*b**3)/(4*a**3*b**3)
```

3.33 $\int \frac{x^3}{-1-x^8+x^{16}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{x^3}{-1-x^8+x^{16}} \, dx = -\frac{1}{4}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x^4\right) - \frac{1}{4}\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^4\right)$$

output
```
-1/40*(10+10*5^(1/2))^(1/2)*arctan(1/2*(2+2*5^(1/2))^(1/2)*x^4)-1/40*(-10+
10*5^(1/2))^(1/2)*arctanh(1/2*(-2+2*5^(1/2))^(1/2)*x^4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{-1-x^8+x^{16}} \, dx = -\frac{\arctan\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x^4\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}} - \frac{\operatorname{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x^4\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Integrate[x^3/(-1 - x^8 + x^16),x]
```

output
```
-1/2*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x^4]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[
Sqrt[2/(1 + Sqrt[5])]*x^4]/(2*Sqrt[10*(1 + Sqrt[5])])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1695, 1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^{16}-x^8-1}\,dx$$

$$\Big\downarrow \text{1695}$$

$$\frac{1}{4}\int \frac{1}{x^{16}-x^8-1}dx^4$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^8+\frac{1}{2}\left(-1-\sqrt{5}\right)}dx^4}{\sqrt{5}}-\frac{\int \frac{1}{x^8+\frac{1}{2}\left(-1+\sqrt{5}\right)}dx^4}{\sqrt{5}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^8+\frac{1}{2}\left(-1-\sqrt{5}\right)}dx^4}{\sqrt{5}}-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x^4\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4}\left(-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}x^4\right)-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x^4\right)\right)$$

input
```
Int[x^3/(-1 - x^8 + x^16),x]
```

output
```
(-(Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x^4]) - Sqrt[2
/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x^4])/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1695
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b
*x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c,
p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(25\_Z^{4}+5\_Z^{2}-1\right)}\_R\ln\left(x^{4}-5\_R^{3}-3\_R\right)\right)}{8}$	34
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x^{4}}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\sqrt{5}\,\arctan\left(\frac{2x^{4}}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}$	60

input
```
int(x^3/(x^16-x^8-1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(x^4-5*_R^3-3*_R),_R=RootOf(25*_Z^4+5*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^3}{-1-x^8+x^{16}}\,dx &= -\frac{1}{4}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\arctan\left(\sqrt{5}x^4\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\right) \\ &\quad -\frac{1}{8}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(2\,x^4+\left(\sqrt{5}+5\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right) \\ &\quad +\frac{1}{8}\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\log\left(2\,x^4-\left(\sqrt{5}+5\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right)\end{aligned}$$

input
```
integrate(x^3/(x^16-x^8-1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/10*sqrt(5) + 1/10)*arctan(sqrt(5)*x^4*sqrt(1/10*sqrt(5) + 1/10
)) - 1/8*sqrt(1/10*sqrt(5) - 1/10)*log(2*x^4 + (sqrt(5) + 5)*sqrt(1/10*sqr
t(5) - 1/10)) + 1/8*sqrt(1/10*sqrt(5) - 1/10)*log(2*x^4 - (sqrt(5) + 5)*sq
rt(1/10*sqrt(5) - 1/10))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.33

$$\int \frac{x^3}{-1-x^8+x^{16}}\,dx \\ = \operatorname{RootSum}\left(102400t^4+320t^2-1,\left(t\mapsto t\log\left(-2560t^3-24t+x^4\right)\right)\right)$$

input
```
integrate(x**3/(x**16-x**8-1),x)
```

output
```
RootSum(102400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-2560*_t**3 - 24*_
t + x**4)))
```

Maxima [F]

$$\int \frac{x^3}{-1-x^8+x^{16}}\,dx = \int \frac{x^3}{x^{16}-x^8-1}\,dx$$

input
```
integrate(x^3/(x^16-x^8-1),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^16 - x^8 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{-1-x^8+x^{16}}\,dx = -\frac{1}{40}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x^4}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{80}\sqrt{10\sqrt{5}-10}\log\left(x^4+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right) + \frac{1}{80}\sqrt{10\sqrt{5}-10}\log\left(\left|x^4-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$

input
```
integrate(x^3/(x^16-x^8-1),x, algorithm="giac")
```

output
```
-1/40*sqrt(10*sqrt(5) + 10)*arctan(x^4/sqrt(1/2*sqrt(5) - 1/2)) - 1/80*sqr
t(10*sqrt(5) - 10)*log(x^4 + sqrt(1/2*sqrt(5) + 1/2)) + 1/80*sqrt(10*sqrt(
5) - 10)*log(abs(x^4 - sqrt(1/2*sqrt(5) + 1/2)))
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \frac{x^3}{-1-x^8+x^{16}}\,dx = \frac{\sqrt{10}\,\mathrm{atanh}\left(\frac{13\sqrt{10}\,x^4\sqrt{-\sqrt{5}-1}}{2\left(13\sqrt{5}+29\right)}+\frac{29\sqrt{5}\sqrt{10}\,x^4\sqrt{-\sqrt{5}-1}}{10\left(13\sqrt{5}+29\right)}\right)\sqrt{-\sqrt{5}-1}}{40} - \frac{\sqrt{10}\,\mathrm{atanh}\left(\frac{13\sqrt{10}\,x^4\sqrt{\sqrt{5}-1}}{2\left(13\sqrt{5}-29\right)}-\frac{29\sqrt{5}\sqrt{10}\,x^4\sqrt{\sqrt{5}-1}}{10\left(13\sqrt{5}-29\right)}\right)\sqrt{\sqrt{5}-1}}{40}$$

input
```
int(-x^3/(x^8 - x^16 + 1),x)
```

output
```
(10^(1/2)*atanh((13*10^(1/2)*x^4*(- 5^(1/2) - 1)^(1/2))/(2*(13*5^(1/2) + 2
9)) + (29*5^(1/2)*10^(1/2)*x^4*(- 5^(1/2) - 1)^(1/2))/(10*(13*5^(1/2) + 29
)))*(- 5^(1/2) - 1)^(1/2))/40 - (10^(1/2)*atanh((13*10^(1/2)*x^4*(5^(1/2)
- 1)^(1/2))/(2*(13*5^(1/2) - 29)) - (29*5^(1/2)*10^(1/2)*x^4*(5^(1/2) - 1)
^(1/2))/(10*(13*5^(1/2) - 29))))*(5^(1/2) - 1)^(1/2))/40
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 795, normalized size of antiderivative = 10.06

$$\int \frac{x^3}{-1-x^8+x^{16}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(x^16-x^8-1),x)
```

output
```
( - 2*sqrt(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*sqrt(5)*ata
n(((sqrt(5) - 1)**(1/8)*sqrt( - sqrt(2) + 2)*2**(1/8) - 2*2**(1/4)*x)/(sqr
t(sqrt(2) + 2)*(sqrt(5) - 1)**(1/8)*2**(1/8))) - 10*sqrt(sqrt(2) + 2)*sqrt
(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*atan(((sqrt(5) - 1)**(1/8)*sqrt( - sqrt
(2) + 2)*2**(1/8) - 2*2**(1/4)*x)/(sqrt(sqrt(2) + 2)*(sqrt(5) - 1)**(1/8)*
2**(1/8))) - 2*sqrt(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*sq
rt(5)*atan(((sqrt(5) - 1)**(1/8)*sqrt( - sqrt(2) + 2)*2**(1/8) + 2*2**(1/4
)*x)/(sqrt(sqrt(2) + 2)*(sqrt(5) - 1)**(1/8)*2**(1/8))) - 10*sqrt(sqrt(2)
+ 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*atan(((sqrt(5) - 1)**(1/8)*sqr
t( - sqrt(2) + 2)*2**(1/8) + 2*2**(1/4)*x)/(sqrt(sqrt(2) + 2)*(sqrt(5) - 1
)**(1/8)*2**(1/8))) + 2*sqrt(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2
) + 2)*sqrt(5)*atan((sqrt(sqrt(2) + 2)*(sqrt(5) - 1)**(1/8)*2**(1/8) - 2*2
**(1/4)*x)/((sqrt(5) - 1)**(1/8)*sqrt( - sqrt(2) + 2)*2**(1/8))) + 10*sqrt
(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) +
2)*(sqrt(5) - 1)**(1/8)*2**(1/8) - 2*2**(1/4)*x)/((sqrt(5) - 1)**(1/8)*sqr
t( - sqrt(2) + 2)*2**(1/8))) + 2*sqrt(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt(
 - sqrt(2) + 2)*sqrt(5)*atan((sqrt(sqrt(2) + 2)*(sqrt(5) - 1)**(1/8)*2**(1
/8) + 2*2**(1/4)*x)/((sqrt(5) - 1)**(1/8)*sqrt( - sqrt(2) + 2)*2**(1/8)))
+ 10*sqrt(sqrt(2) + 2)*sqrt(sqrt(5) - 1)*sqrt( - sqrt(2) + 2)*atan((sqrt(s
qrt(2) + 2)*(sqrt(5) - 1)**(1/8)*2**(1/8) + 2*2**(1/4)*x)/((sqrt(5) - 1...
```

3.34 $\int \frac{-16(105-10\sqrt{21})x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+(896-480\sqrt{21})x^2+(1708-560\sqrt{21})x^4-588(14+5\sqrt{21})x^6+21609}$

Optimal result

Integrand size = 83, antiderivative size = 87

$$\int \frac{-16(105-10\sqrt{21})\,x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+(896-480\sqrt{21})\,x^2+(1708-560\sqrt{21})\,x^4-588\,(14+5\sqrt{21})\,x^6+21609x^8}\,dx$$
$$=\sqrt[4]{\frac{3}{7}}\arctan\left(\sqrt{-\frac{1}{2}+\frac{5}{2\sqrt{21}}}+\frac{7}{2}\sqrt[4]{\frac{7}{3}}x^2\right)-\sqrt[4]{\frac{3}{7}}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}+\frac{5}{2\sqrt{21}}}-\frac{1}{2}3^{3/4}\sqrt[4]{7}x^2\right)$$

output
```
1/7*3^(1/4)*7^(3/4)*arctan(1/42*(-882+210*21^(1/2))^(1/2)+7/6*7^(1/4)*3^(3
/4)*x^2)+1/7*3^(1/4)*7^(3/4)*arctanh(-1/42*(882+210*21^(1/2))^(1/2)+1/2*7^
(1/4)*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int \frac{-16(105-10\sqrt{21})x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+(896-480\sqrt{21})x^2+(1708-560\sqrt{21})x^4-588(14+5\sqrt{21})x^6+21609x^8}\,dx$$
$$= 2\text{RootSum}\Bigg[16+896\#1^2-480\sqrt{21}\#1^2+1708\#1^4-560\sqrt{21}\#1^4-8232\#1^6 - 2940\sqrt{21}\#1^6 +21609\#1^8\&, \frac{105\log(x-\#1)-10\sqrt{21}\log(x-\#1)+147\sqrt{21}\log(x-\#1)\#1^2+147\sqrt{21}\log(x-\#1)}{-224+120\sqrt{21}-854\#1^2+280\sqrt{21}\#1^2+6174\#1^4+2205\sqrt{21}\#1^4-21609\#1^6}$$

input
```
Integrate[(-16*(105 - 10*Sqrt[21])*x - 2352*Sqrt[21]*x^3 - 2352*Sqrt[21]*x
^5)/(16 + (896 - 480*Sqrt[21])*x^2 + (1708 - 560*Sqrt[21])*x^4 - 588*(14 +
 5*Sqrt[21])*x^6 + 21609*x^8),x]
```

output
```
2*RootSum[16 + 896*#1^2 - 480*Sqrt[21]*#1^2 + 1708*#1^4 - 560*Sqrt[21]*#1^
4 - 8232*#1^6 - 2940*Sqrt[21]*#1^6 + 21609*#1^8 & , (105*Log[x - #1] - 10*
Sqrt[21]*Log[x - #1] + 147*Sqrt[21]*Log[x - #1]*#1^2 + 147*Sqrt[21]*Log[x
- #1]*#1^4)/(-224 + 120*Sqrt[21] - 854*#1^2 + 280*Sqrt[21]*#1^2 + 6174*#1^
4 + 2205*Sqrt[21]*#1^4 - 21609*#1^6) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2352\sqrt{21}x^5-2352\sqrt{21}x^3-16(105-10\sqrt{21})x}{21609x^8-588(14+5\sqrt{21})x^6+(1708-560\sqrt{21})x^4+(896-480\sqrt{21})x^2+16}\,dx$$
$$\Big\downarrow \ 2028$$
$$\int \frac{x(-2352\sqrt{21}x^4-2352\sqrt{21}x^2-16(105-10\sqrt{21}))}{21609x^8-588(14+5\sqrt{21})x^6+(1708-560\sqrt{21})x^4+(896-480\sqrt{21})x^2+16}dx$$
$$\Big\downarrow \ 7266$$
$$\frac{1}{2}\int -\frac{16(147\sqrt{21}x^4+147\sqrt{21}x^2+5(21-2\sqrt{21}))}{21609x^8-588(14+5\sqrt{21})x^6+28(61-20\sqrt{21})x^4+32(28-15\sqrt{21})x^2+16}dx^2$$

$$\Big\downarrow \; 27$$

$$-8\int \frac{147\sqrt{21}x^4+147\sqrt{21}x^2+5\left(21-2\sqrt{21}\right)}{21609x^8-588\left(14+5\sqrt{21}\right)x^6+28\left(61-20\sqrt{21}\right)x^4+32\left(28-15\sqrt{21}\right)x^2+16}dx^2$$

$$\Big\downarrow \; 7293$$

$$-8\int \left(\frac{147\sqrt{21}x^4}{21609x^8-588\left(14+5\sqrt{21}\right)x^6+28\left(61-20\sqrt{21}\right)x^4+32\left(28-15\sqrt{21}\right)x^2+16}+\frac{}{21609x^8-588\left(14+\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$-8\left(5\left(21-2\sqrt{21}\right)\int \frac{1}{21609x^8-588\left(14+5\sqrt{21}\right)x^6+28\left(61-20\sqrt{21}\right)x^4+32\left(28-15\sqrt{21}\right)x^2+16}dx^2+14\right.$$

input
```
Int[(-16*(105 - 10*Sqrt[21])*x - 2352*Sqrt[21]*x^3 - 2352*Sqrt[21]*x^5)/(1
6 + (896 - 480*Sqrt[21])*x^2 + (1708 - 560*Sqrt[21])*x^4 - 588*(14 + 5*Sqr
t[21])*x^6 + 21609*x^8),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{21^{\frac{1}{4}}\sqrt{7}\,\operatorname{arctanh}\left(\frac{\left(126x^2-6\sqrt{21}-42\right)\sqrt{7}\,21^{\frac{3}{4}}}{1764}\right)}{7}+\frac{21^{\frac{1}{4}}\sqrt{7}\,\arctan\left(\frac{\left(686x^2-14\sqrt{21}+98\right)\sqrt{7}\,21^{\frac{3}{4}}}{4116}\right)}{7}$	60

input
```
int((-16*(105-10*21^(1/2))*x-2352*21^(1/2)*x^3-2352*21^(1/2)*x^5)/(16+(896
-480*21^(1/2))*x^2+(1708-560*21^(1/2))*x^4-588*(14+5*21^(1/2))*x^6+21609*x
^8),x,method=_RETURNVERBOSE)
```

output
```
1/7*21^(1/4)*7^(1/2)*arctanh(1/1764*(126*x^2-6*21^(1/2)-42)*7^(1/2)*21^(3/
4))+1/7*21^(1/4)*7^(1/2)*arctan(1/4116*(686*x^2-14*21^(1/2)+98)*7^(1/2)*21
^(3/4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{-16\left(105-10\sqrt{21}\right)x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$= -\left(\frac{3}{7}\right)^{\frac{1}{4}}\arctan\left(-\frac{35}{6}\left(\frac{3}{7}\right)^{\frac{3}{4}}x^2-\frac{1}{6}\sqrt{21}\left(\left(\frac{3}{7}\right)^{\frac{1}{4}}(2x^2+1)-\left(\frac{3}{7}\right)^{\frac{3}{4}}\right)\right) - \frac{1}{2}\left(\frac{3}{7}\right)^{\frac{1}{4}}\log\left(441x^2-\sqrt{21}\left(28\left(\frac{3}{7}\right)^{\frac{3}{4}}+35\sqrt{\frac{3}{7}}+15\right)-42\sqrt{\frac{3}{7}}-210\left(\frac{3}{7}\right)^{\frac{1}{4}} - 42\right) + \frac{1}{2}\left(\frac{3}{7}\right)^{\frac{1}{4}}\log\left(441x^2+\sqrt{21}\left(28\left(\frac{3}{7}\right)^{\frac{3}{4}}-35\sqrt{\frac{3}{7}}-15\right)-42\sqrt{\frac{3}{7}} + 210\left(\frac{3}{7}\right)^{\frac{1}{4}}-42\right)$$

input
```
integrate((-16*(105-10*21^(1/2))*x-2352*21^(1/2)*x^3-2352*21^(1/2)*x^5)/(1
6+(896-480*21^(1/2))*x^2+(1708-560*21^(1/2))*x^4-588*(14+5*21^(1/2))*x^6+2
1609*x^8),x, algorithm="fricas")
```

output
```
-(3/7)^(1/4)*arctan(-35/6*(3/7)^(3/4)*x^2 - 1/6*sqrt(21)*((3/7)^(1/4)*(2*x
^2 + 1) - (3/7)^(3/4))) - 1/2*(3/7)^(1/4)*log(441*x^2 - sqrt(21)*(28*(3/7)
^(3/4) + 35*sqrt(3/7) + 15) - 42*sqrt(3/7) - 210*(3/7)^(1/4) - 42) + 1/2*(
3/7)^(1/4)*log(441*x^2 + sqrt(21)*(28*(3/7)^(3/4) - 35*sqrt(3/7) - 15) - 4
2*sqrt(3/7) + 210*(3/7)^(1/4) - 42)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-16(105-10\sqrt{21})\,x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$=\text{Timed out}$$

input
```
integrate((-16*(105-10*21**(1/2))*x-2352*21**(1/2)*x**3-2352*21**(1/2)*x**
5)/(16+(896-480*21**(1/2))*x**2+(1708-560*21**(1/2))*x**4-588*(14+5*21**(1
/2))*x**6+21609*x**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{-16(105-10\sqrt{21})\,x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$=\int -\frac{16\left(147\sqrt{21}x^5+147\sqrt{21}x^3-5\,x\left(2\sqrt{21}-21\right)\right)}{21609\,x^8-588\,x^6\left(5\sqrt{21}+14\right)-28\,x^4\left(20\sqrt{21}-61\right)-32\,x^2\left(15\sqrt{21}-28\right)+16}\,dx$$

input
```
integrate((-16*(105-10*21^(1/2))*x-2352*21^(1/2)*x^3-2352*21^(1/2)*x^5)/(1
6+(896-480*21^(1/2))*x^2+(1708-560*21^(1/2))*x^4-588*(14+5*21^(1/2))*x^6+2
1609*x^8),x, algorithm="maxima")
```

output
```
-16*integrate((147*sqrt(21)*x^5 + 147*sqrt(21)*x^3 - 5*x*(2*sqrt(21) - 21)
)/(21609*x^8 - 588*x^6*(5*sqrt(21) + 14) - 28*x^4*(20*sqrt(21) - 61) - 32*
x^2*(15*sqrt(21) - 28) + 16), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-16(105-10\sqrt{21})\,x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((-16*(105-10*21^(1/2))*x-2352*21^(1/2)*x^3-2352*21^(1/2)*x^5)/(1
6+(896-480*21^(1/2))*x^2+(1708-560*21^(1/2))*x^4-588*(14+5*21^(1/2))*x^6+2
1609*x^8),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueUnable to find common minimal polynomial Er
ror: Bad
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 761, normalized size of antiderivative = 8.75

$$\int \frac{-16(105-10\sqrt{21})\,x-2352\sqrt{21}x^3-2352\sqrt{21}x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int((2352*21^(1/2)*x^3 + 2352*21^(1/2)*x^5 - 16*x*(10*21^(1/2) - 105))/(58
8*x^6*(5*21^(1/2) + 14) + x^2*(480*21^(1/2) - 896) + x^4*(560*21^(1/2) - 1
708) - 21609*x^8 - 16),x)
```

output
```
symsum(log(19808464199328*21^(1/2)*root(19805837694720*21^(1/2)*z^4 - 1258
04864465616*z^4 - 530513509680*21^(1/2) + 3369773155329, z, k) - 235753875
90720*root(19805837694720*21^(1/2)*z^4 - 125804864465616*z^4 - 53051350968
0*21^(1/2) + 3369773155329, z, k) + 1523712435840*21^(1/2) - 1764661242432
0*21^(1/2)*root(19805837694720*21^(1/2)*z^4 - 125804864465616*z^4 - 530513
509680*21^(1/2) + 3369773155329, z, k)^2 + 362865763507968*21^(1/2)*root(1
9805837694720*21^(1/2)*z^4 - 125804864465616*z^4 - 530513509680*21^(1/2) +
 3369773155329, z, k)^3 - 282911370768640*21^(1/2)*root(19805837694720*21^
(1/2)*z^4 - 125804864465616*z^4 - 530513509680*21^(1/2) + 3369773155329, z
, k)^4 + 1465820967604224*21^(1/2)*root(19805837694720*21^(1/2)*z^4 - 1258
04864465616*z^4 - 530513509680*21^(1/2) + 3369773155329, z, k)^5 - 1834667
59168000*21^(1/2)*root(19805837694720*21^(1/2)*z^4 - 125804864465616*z^4 -
 530513509680*21^(1/2) + 3369773155329, z, k)^6 - 17676794551061835*root(1
9805837694720*21^(1/2)*z^4 - 125804864465616*z^4 - 530513509680*21^(1/2) +
 3369773155329, z, k)*x^2 - 651185324124930*21^(1/2)*x^2 + 411391089219480
*root(19805837694720*21^(1/2)*z^4 - 125804864465616*z^4 - 530513509680*21^
(1/2) + 3369773155329, z, k)^2 - 195804904861440*root(19805837694720*21^(1
/2)*z^4 - 125804864465616*z^4 - 530513509680*21^(1/2) + 3369773155329, z,
k)^3 + 3488874721930752*root(19805837694720*21^(1/2)*z^4 - 125804864465616
*z^4 - 530513509680*21^(1/2) + 3369773155329, z, k)^4 - 701859358126080...
```

Reduce [F]

$$\int \frac{-16\left(105-10\sqrt{21}\right)x-2352\sqrt{21}\,x^3-2352\sqrt{21}\,x^5}{16+\left(896-480\sqrt{21}\right)x^2+\left(1708-560\sqrt{21}\right)x^4-588\left(14+5\sqrt{21}\right)x^6+21609x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int((-16*(105-10*21^(1/2))*x-2352*21^(1/2)*x^3-2352*21^(1/2)*x^5)/(16+(896
-480*21^(1/2))*x^2+(1708-560*21^(1/2))*x^4-588*(14+5*21^(1/2))*x^6+21609*x
^8),x)
```

output

```
16*( - 3176523*sqrt(21)*int(x**13/(466948881*x**16 - 355770576*x**14 - 399
33432*x**12 - 58545984*x**10 - 76998992*x**8 - 8492288*x**6 - 3980928*x**4
 + 28672*x**2 + 256),x) - 1966419*sqrt(21)*int(x**11/(466948881*x**16 - 35
5770576*x**14 - 39933432*x**12 - 58545984*x**10 - 76998992*x**8 - 8492288*
x**6 - 3980928*x**4 + 28672*x**2 + 256),x) + 1175118*sqrt(21)*int(x**9/(46
6948881*x**16 - 355770576*x**14 - 39933432*x**12 - 58545984*x**10 - 769989
92*x**8 - 8492288*x**6 - 3980928*x**4 + 28672*x**2 + 256),x) - 773808*sqrt
(21)*int(x**7/(466948881*x**16 - 355770576*x**14 - 39933432*x**12 - 585459
84*x**10 - 76998992*x**8 - 8492288*x**6 - 3980928*x**4 + 28672*x**2 + 256)
,x) - 175784*sqrt(21)*int(x**5/(466948881*x**16 - 355770576*x**14 - 399334
32*x**12 - 58545984*x**10 - 76998992*x**8 - 8492288*x**6 - 3980928*x**4 +
28672*x**2 + 256),x) - 43792*sqrt(21)*int(x**3/(466948881*x**16 - 35577057
6*x**14 - 39933432*x**12 - 58545984*x**10 - 76998992*x**8 - 8492288*x**6 -
 3980928*x**4 + 28672*x**2 + 256),x) + 160*sqrt(21)*int(x/(466948881*x**16
 - 355770576*x**14 - 39933432*x**12 - 58545984*x**10 - 76998992*x**8 - 849
2288*x**6 - 3980928*x**4 + 28672*x**2 + 256),x) - 9075780*int(x**11/(46694
8881*x**16 - 355770576*x**14 - 39933432*x**12 - 58545984*x**10 - 76998992*
x**8 - 8492288*x**6 - 3980928*x**4 + 28672*x**2 + 256),x) - 13073445*int(x
**9/(466948881*x**16 - 355770576*x**14 - 39933432*x**12 - 58545984*x**10 -
 76998992*x**8 - 8492288*x**6 - 3980928*x**4 + 28672*x**2 + 256),x) - 1...
```

3.35 $\int \frac{40(-12+8\sqrt{15})x-5040x^3+4200\sqrt{15}x^5}{4+(560-360\sqrt{15})x^2+5(296-80\sqrt{15})x^4-2100(2+\sqrt{15})x^6+11025x^8}$

Optimal result

Integrand size = 77, antiderivative size = 82

$$\int \frac{40(-12+8\sqrt{15})\,x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$=\sqrt[4]{\frac{3}{5}}\arctan\left(\sqrt{-\frac{1}{2}+\frac{2}{\sqrt{15}}}+\frac{7}{2}\sqrt[4]{\frac{5}{3}}x^2\right)+\sqrt[4]{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{2}{\sqrt{15}}}-\frac{1}{2}3^{3/4}\sqrt[4]{5}x^2\right)$$

output
```
1/5*3^(1/4)*5^(3/4)*arctan(1/30*(-450+120*15^(1/2))^(1/2)+7/6*5^(1/4)*3^(3
/4)*x^2)-1/5*3^(1/4)*5^(3/4)*arctanh(-1/30*(450+120*15^(1/2))^(1/2)+1/2*5^
(1/4)*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.09

$$\int \frac{40(-12+8\sqrt{15})x-5040x^3+4200\sqrt{15}x^5}{4+(560-360\sqrt{15})x^2+5(296-80\sqrt{15})x^4-2100(2+\sqrt{15})x^6+11025x^8}\,dx$$
$$= -\text{RootSum}\Big[4+560\#1^2-360\sqrt{15}\#1^2+1480\#1^4-400\sqrt{15}\#1^4-4200\#1^6 - 2100\sqrt{15}\#1^6 +11025\#1^8\&, \frac{-12\log(x-\#1)+8\sqrt{15}\log(x-\#1)-126\log(x-\#1)\#1^2+105\sqrt{15}\log(x-\#1)\#1}{-28+18\sqrt{15}-148\#1^2+40\sqrt{15}\#1^2+630\#1^4+315\sqrt{15}\#1^4-2205\#1^6}$$

input
```
Integrate[(40*(-12 + 8*Sqrt[15])*x - 5040*x^3 + 4200*Sqrt[15]*x^5)/(4 + (5
60 - 360*Sqrt[15])*x^2 + 5*(296 - 80*Sqrt[15])*x^4 - 2100*(2 + Sqrt[15])*x
^6 + 11025*x^8),x]
```

output
```
-RootSum[4 + 560*#1^2 - 360*Sqrt[15]*#1^2 + 1480*#1^4 - 400*Sqrt[15]*#1^4
- 4200*#1^6 - 2100*Sqrt[15]*#1^6 + 11025*#1^8 & , (-12*Log[x - #1] + 8*Sqr
t[15]*Log[x - #1] - 126*Log[x - #1]*#1^2 + 105*Sqrt[15]*Log[x - #1]*#1^4)/
(-28 + 18*Sqrt[15] - 148*#1^2 + 40*Sqrt[15]*#1^2 + 630*#1^4 + 315*Sqrt[15]
*#1^4 - 2205*#1^6) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4200\sqrt{15}x^5-5040x^3+40(8\sqrt{15}-12)x}{11025x^8-2100(2+\sqrt{15})x^6+5(296-80\sqrt{15})x^4+(560-360\sqrt{15})x^2+4}\,dx$$
$$\Big\downarrow \; 2028$$
$$\int \frac{x(4200\sqrt{15}x^4-5040x^2+40(8\sqrt{15}-12))}{11025x^8-2100(2+\sqrt{15})x^6+5(296-80\sqrt{15})x^4+(560-360\sqrt{15})x^2+4}dx$$
$$\Big\downarrow \; 7266$$
$$\frac{1}{2}\int -\frac{40(-105\sqrt{15}x^4+126x^2+4(3-2\sqrt{15}))}{11025x^8-2100(2+\sqrt{15})x^6+40(37-10\sqrt{15})x^4+40(14-9\sqrt{15})x^2+4}dx^2$$

$$\Big\downarrow \text{27}$$

$$-20\int \frac{-105\sqrt{15}x^4+126x^2+4(3-2\sqrt{15})}{11025x^8-2100\left(2+\sqrt{15}\right)x^6+40\left(37-10\sqrt{15}\right)x^4+40\left(14-9\sqrt{15}\right)x^2+4}dx^2$$

$$\Big\downarrow \text{7293}$$

$$-20\int \left(\frac{105\sqrt{15}x^4}{-11025x^8+2100\left(2+\sqrt{15}\right)x^6-40\left(37-10\sqrt{15}\right)x^4-40\left(14-9\sqrt{15}\right)x^2-4}+\frac{}{11025x^8-2100\left(2+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-20\left(105\sqrt{15}\int \frac{x^4}{-11025x^8+2100\left(2+\sqrt{15}\right)x^6-40\left(37-10\sqrt{15}\right)x^4-40\left(14-9\sqrt{15}\right)x^2-4}dx^2+4\left(3-2\sqrt{\right.}\right.$$

input
```
Int[(40*(-12 + 8*Sqrt[15])*x - 5040*x^3 + 4200*Sqrt[15]*x^5)/(4 + (560 - 3
60*Sqrt[15])*x^2 + 5*(296 - 80*Sqrt[15])*x^4 - 2100*(2 + Sqrt[15])*x^6 + 1
1025*x^8),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{15^{\frac{1}{4}}\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(90x^2-6\sqrt{15}-30\right)\sqrt{5}\,15^{\frac{3}{4}}}{900}\right)}{5}+\frac{15^{\frac{1}{4}}\sqrt{5}\,\arctan\left(\frac{\left(490x^2-14\sqrt{15}+70\right)\sqrt{5}\,15^{\frac{3}{4}}}{2100}\right)}{5}$	60

input
```
int((40*(-12+8*15^(1/2))*x-5040*x^3+4200*15^(1/2)*x^5)/(4+(560-360*15^(1/2
))*x^2+5*(296-80*15^(1/2))*x^4-2100*(2+15^(1/2))*x^6+11025*x^8),x,method=_
RETURNVERBOSE)
```

output
```
-1/5*15^(1/4)*5^(1/2)*arctanh(1/900*(90*x^2-6*15^(1/2)-30)*5^(1/2)*15^(3/4
))+1/5*15^(1/4)*5^(1/2)*arctan(1/2100*(490*x^2-14*15^(1/2)+70)*5^(1/2)*15^
(3/4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$= \left(\frac{3}{5}\right)^{\frac{1}{4}}\arctan\left(\frac{5}{6}\sqrt{15}\left(\frac{3}{5}\right)^{\frac{1}{4}}x^2+\frac{5}{6}\left(\frac{3}{5}\right)^{\frac{3}{4}}\left(2x^2+1\right)-\frac{1}{2}\left(\frac{3}{5}\right)^{\frac{1}{4}}\right)$$
$$+\frac{1}{2}\left(\frac{3}{5}\right)^{\frac{1}{4}}\log\left(63x^2-\sqrt{15}\left(10\left(\frac{3}{5}\right)^{\frac{3}{4}}+5\sqrt{\frac{3}{5}}+3\right)-6\sqrt{\frac{3}{5}}-12\left(\frac{3}{5}\right)^{\frac{1}{4}}-6\right)$$
$$-\frac{1}{2}\left(\frac{3}{5}\right)^{\frac{1}{4}}\log\left(63x^2+\sqrt{15}\left(10\left(\frac{3}{5}\right)^{\frac{3}{4}}-5\sqrt{\frac{3}{5}}-3\right)-6\sqrt{\frac{3}{5}}+12\left(\frac{3}{5}\right)^{\frac{1}{4}}-6\right)$$

input
```
integrate((40*(-12+8*15^(1/2))*x-5040*x^3+4200*15^(1/2)*x^5)/(4+(560-360*1
5^(1/2))*x^2+5*(296-80*15^(1/2))*x^4-2100*(2+15^(1/2))*x^6+11025*x^8),x, a
lgorithm="fricas")
```

output
```
(3/5)^(1/4)*arctan(5/6*sqrt(15)*(3/5)^(1/4)*x^2 + 5/6*(3/5)^(3/4)*(2*x^2 +
 1) - 1/2*(3/5)^(1/4)) + 1/2*(3/5)^(1/4)*log(63*x^2 - sqrt(15)*(10*(3/5)^(
3/4) + 5*sqrt(3/5) + 3) - 6*sqrt(3/5) - 12*(3/5)^(1/4) - 6) - 1/2*(3/5)^(1
/4)*log(63*x^2 + sqrt(15)*(10*(3/5)^(3/4) - 5*sqrt(3/5) - 3) - 6*sqrt(3/5)
 + 12*(3/5)^(1/4) - 6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((40*(-12+8*15**(1/2))*x-5040*x**3+4200*15**(1/2)*x**5)/(4+(560-3
60*15**(1/2))*x**2+5*(296-80*15**(1/2))*x**4-2100*(2+15**(1/2))*x**6+11025
*x**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$= \int \frac{40\left(105\sqrt{15}x^5-126\,x^3+4\,x\left(2\sqrt{15}-3\right)\right)}{11025\,x^8-2100\,x^6\left(\sqrt{15}+2\right)-40\,x^4\left(10\sqrt{15}-37\right)-40\,x^2\left(9\sqrt{15}-14\right)+4}\,dx$$

input
```
integrate((40*(-12+8*15^(1/2))*x-5040*x^3+4200*15^(1/2)*x^5)/(4+(560-360*1
5^(1/2))*x^2+5*(296-80*15^(1/2))*x^4-2100*(2+15^(1/2))*x^6+11025*x^8),x, a
lgorithm="maxima")
```

output
```
40*integrate((105*sqrt(15)*x^5 - 126*x^3 + 4*x*(2*sqrt(15) - 3))/(11025*x^
8 - 2100*x^6*(sqrt(15) + 2) - 40*x^4*(10*sqrt(15) - 37) - 40*x^2*(9*sqrt(1
5) - 14) + 4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((40*(-12+8*15^(1/2))*x-5040*x^3+4200*15^(1/2)*x^5)/(4+(560-360*1
5^(1/2))*x^2+5*(296-80*15^(1/2))*x^4-2100*(2+15^(1/2))*x^6+11025*x^8),x, a
lgorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 761, normalized size of antiderivative = 9.28

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int(-(4200*15^(1/2)*x^5 - 5040*x^3 + 40*x*(8*15^(1/2) - 12))/(5*x^4*(80*15
^(1/2) - 296) + x^2*(360*15^(1/2) - 560) - 11025*x^8 + 2100*x^6*(15^(1/2)
+ 2) - 4),x)
```

output
```
symsum(log(1272777503760*15^(1/2)*root(134752761600*15^(1/2)*z^4 - 7379679
45840*z^4 - 5053228560*15^(1/2) + 27673797969, z, k) - 1265215439 6784*root
(134752761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 2767
3797969, z, k) + 47072975040*15^(1/2) - 1245012050400*15^(1/2)*root(134752
761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 27673797969
, z, k)^2 + 1261817605440*15^(1/2)*root(134752761600*15^(1/2)*z^4 - 737967
945840*z^4 - 5053228560*15^(1/2) + 27673797969, z, k)^3 - 16342805423680*1
5^(1/2)*root(134752761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^
(1/2) + 27673797969, z, k)^4 - 24703130457600*15^(1/2)*root(134752761600*1
5^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 27673797969, z, k)^
5 + 6416990566400*15^(1/2)*root(134752761600*15^(1/2)*z^4 - 737967945840*z
^4 - 5053228560*15^(1/2) + 27673797969, z, k)^6 - 2966711800400910*root(13
4752761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 2767379
7969, z, k)*x^2 - 282772703535840*15^(1/2)*x^2 + 39784538365926*root(13475
2761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 2767379796
9, z, k)^2 - 147474447170400*root(134752761600*15^(1/2)*z^4 - 737967945840
*z^4 - 5053228560*15^(1/2) + 27673797969, z, k)^3 + 259431497859840*root(1
34752761600*15^(1/2)*z^4 - 737967945840*z^4 - 5053228560*15^(1/2) + 276737
97969, z, k)^4 - 399257716838400*root(134752761600*15^(1/2)*z^4 - 73796794
5840*z^4 - 5053228560*15^(1/2) + 27673797969, z, k)^5 + 843958447014400...
```

Reduce [F]

$$\int \frac{40\left(-12+8\sqrt{15}\right)x-5040x^3+4200\sqrt{15}\,x^5}{4+\left(560-360\sqrt{15}\right)x^2+5\left(296-80\sqrt{15}\right)x^4-2100\left(2+\sqrt{15}\right)x^6+11025x^8}\,dx$$
$= \text{Too large to display}$

input
```
int((40*(-12+8*15^(1/2))*x-5040*x^3+4200*15^(1/2)*x^5)/(4+(560-360*15^(1/2
))*x^2+5*(296-80*15^(1/2))*x^4-2100*(2+15^(1/2))*x^6+11025*x^8),x)
```

output
```
40*(1157625*sqrt(15)*int(x**13/(121550625*x**16 - 92610000*x**14 - 1587600
0*x**12 - 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 1618560*x**4 + 4
480*x**2 + 16),x) - 441000*sqrt(15)*int(x**11/(121550625*x**16 - 92610000*
x**14 - 15876000*x**12 - 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 1
618560*x**4 + 4480*x**2 + 16),x) - 21000*sqrt(15)*int(x**9/(121550625*x**1
6 - 92610000*x**14 - 15876000*x**12 - 25284000*x**10 - 27505400*x**8 - 269
6000*x**6 - 1618560*x**4 + 4480*x**2 + 16),x) - 50400*sqrt(15)*int(x**7/(1
21550625*x**16 - 92610000*x**14 - 15876000*x**12 - 25284000*x**10 - 275054
00*x**8 - 2696000*x**6 - 1618560*x**4 + 4480*x**2 + 16),x) - 37900*sqrt(15
)*int(x**5/(121550625*x**16 - 92610000*x**14 - 15876000*x**12 - 25284000*x
**10 - 27505400*x**8 - 2696000*x**6 - 1618560*x**4 + 4480*x**2 + 16),x) +
160*sqrt(15)*int(x**3/(121550625*x**16 - 92610000*x**14 - 15876000*x**12 -
 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 1618560*x**4 + 4480*x**2
+ 16),x) + 32*sqrt(15)*int(x/(121550625*x**16 - 92610000*x**14 - 15876000*
x**12 - 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 1618560*x**4 + 448
0*x**2 + 16),x) + 1918350*int(x**11/(121550625*x**16 - 92610000*x**14 - 15
876000*x**12 - 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 1618560*x**
4 + 4480*x**2 + 16),x) + 1026900*int(x**9/(121550625*x**16 - 92610000*x**1
4 - 15876000*x**12 - 25284000*x**10 - 27505400*x**8 - 2696000*x**6 - 16185
60*x**4 + 4480*x**2 + 16),x) + 682920*int(x**7/(121550625*x**16 - 92610...
```

3.36 $\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 36

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx=-\frac{1+x^2}{2\left(2-\sqrt{3}+\left(2-\sqrt{3}\right)x^2+x^4\right)}$$

output
```
-1/2*(x^2+1)/(2-3^(1/2)+(2-3^(1/2))*x^2+x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 9.28

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx$$
$$=\frac{\left((-2-i)+\sqrt{3}-2x^2\right)\left((1+2i)-i\sqrt{3}+2ix^2\right)\left(x^4\left(-2i+\left((2-i)+\sqrt{3}\right)x^2\right)+2\left(-52+30\sqrt{3}+((-\right.\right.}{}$$

input
```
Integrate[(x^3*(2 + x^2))/(-2 + Sqrt[3] + (-2 + Sqrt[3])*x^2 - x^4)^2,x]
```

output
```
(((-2 - I) + Sqrt[3] - 2*x^2)*((1 + 2*I) - I*Sqrt[3] + (2*I)*x^2)*(x^4*(-2
*I + ((2 - I) + Sqrt[3])*x^2) + 2*(-52 + 30*Sqrt[3] + ((-78 - 7*I) + (45 +
 4*I)*Sqrt[3])*x^2 + ((-40 - 7*I) + (23 + 4*I)*Sqrt[3])*x^4 + ((-7 - 2*I)
+ (4 + I)*Sqrt[3])*x^6)*Log[(2*(-2 + Sqrt[3]) + ((-2 - I) + Sqrt[3])*x^2)/
(2*(-2 + Sqrt[3]) + ((-2 + I) + Sqrt[3])*x^2)] + 2*(-52 + 30*Sqrt[3] + ((-
78 - 7*I) + (45 + 4*I)*Sqrt[3])*x^2 + ((-40 - 7*I) + (23 + 4*I)*Sqrt[3])*x
^4 + ((-7 - 2*I) + (4 + I)*Sqrt[3])*x^6)*Log[(2*(-2 + Sqrt[3]) + ((-2 + I)
 + Sqrt[3])*x^2)/(2*(-2 + Sqrt[3]) + ((-2 - I) + Sqrt[3])*x^2)]))/(8*(2*(-
2 + Sqrt[3]) + ((-2 - I) + Sqrt[3])*x^2)*(-2 + Sqrt[3] + (-2 + Sqrt[3])*x^
2 - x^4)^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1578, 1223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (x^2+2)}{\left(-x^4+\left(\sqrt{3}-2\right) x^2+\sqrt{3}-2\right)^2} dx$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2} \int \frac{x^2 (x^2+2)}{\left(x^4+\left(2-\sqrt{3}\right) x^2-\sqrt{3}+2\right)^2} dx^2$$

$$\Big\downarrow \text{1223}$$

$$-\frac{x^2+1}{2 \left(x^4+\left(2-\sqrt{3}\right) x^2-\sqrt{3}+2\right)}$$

input
```
Int[(x^3*(2 + x^2))/(-2 + Sqrt[3] + (-2 + Sqrt[3])*x^2 - x^4)^2,x]
```

output
```
-1/2*(1 + x^2)/(2 - Sqrt[3] + (2 - Sqrt[3])*x^2 + x^4)
```

Defintions of rubi rules used

rule 1223
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b^2*e*g*(p + 2) - 2*a*c*e*
g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3), 0] && NeQ[p, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{x^{2}+1}{-2x^{4}+2\sqrt{3}\,x^{2}-4x^{2}+2\sqrt{3}-4}$	32
parallelrisch	$\frac{x^{2}+1}{-2x^{4}+2\sqrt{3}\,x^{2}-4x^{2}+2\sqrt{3}-4}$	32
risch	$\frac{-\frac{x^{2}}{2}-\frac{1}{2}}{2-\sqrt{3}+2x^{2}-\sqrt{3}\,x^{2}+x^{4}}$	34
default	$\frac{-x^{2}-1}{4-2\sqrt{3}+4x^{2}-2\sqrt{3}\,x^{2}+2x^{4}}$	35
orering	$\frac{\left(-x^{4}+\sqrt{3}\,x^{2}-2x^{2}+\sqrt{3}-2\right)\left(x^{2}+1\right)}{2\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^{2}-x^{4}\right)^{2}}$	51

input
```
int(x^3*(x^2+2)/(-2+3^(1/2)+(-2+3^(1/2))*x^2-x^4)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/2*(x^2+1)/(-x^4+3^(1/2)*x^2-2*x^2+3^(1/2)-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = -\frac{x^6+3\,x^4+4\,x^2+\sqrt{3}(x^4+2\,x^2+1)+2}{2\,(x^8+4\,x^6+5\,x^4+2\,x^2+1)}$$

input
```
integrate(x^3*(x^2+2)/(-2+3^(1/2)+(-2+3^(1/2))*x^2-x^4)^2,x, algorithm="fr
icas")
```

output
```
-1/2*(x^6 + 3*x^4 + 4*x^2 + sqrt(3)*(x^4 + 2*x^2 + 1) + 2)/(x^8 + 4*x^6 +
5*x^4 + 2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = \frac{x^2\left(-26+15\sqrt{3}\right)-26+15\sqrt{3}}{x^4\cdot\left(52-30\sqrt{3}\right)+x^2\cdot\left(194-112\sqrt{3}\right)-112\sqrt{3}+194}$$

input
```
integrate(x**3*(x**2+2)/(-2+3**(1/2)+(-2+3**(1/2))*x**2-x**4)**2,x)
```

output
```
(x**2*(-26 + 15*sqrt(3)) - 26 + 15*sqrt(3))/(x**4*(52 - 30*sqrt(3)) + x**2
*(194 - 112*sqrt(3)) - 112*sqrt(3) + 194)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = -\frac{x^2+1}{2\left(x^4-x^2\left(\sqrt{3}-2\right)-\sqrt{3}+2\right)}$$

input
```
integrate(x^3*(x^2+2)/(-2+3^(1/2)+(-2+3^(1/2))*x^2-x^4)^2,x, algorithm="ma
xima")
```

output
```
-1/2*(x^2 + 1)/(x^4 - x^2*(sqrt(3) - 2) - sqrt(3) + 2)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = -\frac{x^2\left(15\sqrt{3}+26\right)+15\sqrt{3}+26}{2\left(x^4-x^2\left(\sqrt{3}-2\right)-\sqrt{3}+2\right)\left(15\sqrt{3}+26\right)}$$

input
```
integrate(x^3*(x^2+2)/(-2+3^(1/2)+(-2+3^(1/2))*x^2-x^4)^2,x, algorithm="gi
ac")
```

output
```
-1/2*(x^2*(15*sqrt(3) + 26) + 15*sqrt(3) + 26)/((x^4 - x^2*(sqrt(3) - 2) -
 sqrt(3) + 2)*(15*sqrt(3) + 26))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = \frac{\frac{x^2}{2}+\frac{1}{2}}{-x^4+\left(\sqrt{3}-2\right)\,x^2+\sqrt{3}-2}$$

input
```
int((x^3*(x^2 + 2))/(3^(1/2) - x^4 + x^2*(3^(1/2) - 2) - 2)^2,x)
```

output
```
(x^2/2 + 1/2)/(3^(1/2) - x^4 + x^2*(3^(1/2) - 2) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{x^3(2+x^2)}{\left(-2+\sqrt{3}+\left(-2+\sqrt{3}\right)x^2-x^4\right)^2}\,dx = \frac{-4\sqrt{3}\,x^4-8\sqrt{3}\,x^2-4\sqrt{3}+x^8-7x^4-14x^2-7}{8x^8+32x^6+40x^4+16x^2+8}$$

input
```
int(x^3*(x^2+2)/(-2+3^(1/2)+(-2+3^(1/2))*x^2-x^4)^2,x)
```

output
```
( - 4*sqrt(3)*x**4 - 8*sqrt(3)*x**2 - 4*sqrt(3) + x**8 - 7*x**4 - 14*x**2
- 7)/(8*(x**8 + 4*x**6 + 5*x**4 + 2*x**2 + 1))
```

3.37 $\int \frac{-43-27\sqrt{3}+(110+73\sqrt{3})x^2+(67+46\sqrt{3})x^4+(19+8\sqrt{3})x^6}{(1+(2+\sqrt{3})x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 67, antiderivative size = 43

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x\left(43+27\sqrt{3}+\left(19+8\sqrt{3}\right)x^2\right)}{1+\left(2+\sqrt{3}\right)x^2+x^4}$$

output
```
-x*(43+27*3^(1/2)+(19+8*3^(1/2))*x^2)/(1+(2+3^(1/2))*x^2+x^4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x\left(453+253\sqrt{3}+\left(153+100\sqrt{3}\right)x^2\right)}{\left(3+4\sqrt{3}\right)\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)}$$

input
```
Integrate[(-43 - 27*Sqrt[3] + (110 + 73*Sqrt[3])*x^2 + (67 + 46*Sqrt[3])*x
^4 + (19 + 8*Sqrt[3])*x^6)/(1 + (2 + Sqrt[3])*x^2 + x^4)^2,x]
```

output
```
-((x*(453 + 253*Sqrt[3] + (153 + 100*Sqrt[3])*x^2))/((3 + 4*Sqrt[3])*(1 +
(2 + Sqrt[3])*x^2 + x^4)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.015$, Rules used = $\{2204\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(19+8\sqrt{3}\right)x^6+\left(67+46\sqrt{3}\right)x^4+\left(110+73\sqrt{3}\right)x^2-27\sqrt{3}-43}{\left(x^4+\left(2+\sqrt{3}\right)x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2204}$$

$$-\frac{x\left(\left(19+8\sqrt{3}\right)x^2+27\sqrt{3}+43\right)}{x^4+\left(2+\sqrt{3}\right)x^2+1}$$

input
```
Int[(-43 - 27*Sqrt[3] + (110 + 73*Sqrt[3])*x^2 + (67 + 46*Sqrt[3])*x^4 + (
19 + 8*Sqrt[3])*x^6)/(1 + (2 + Sqrt[3])*x^2 + x^4)^2,x]
```

output
```
-((x*(43 + 27*Sqrt[3] + (19 + 8*Sqrt[3])*x^2))/(1 + (2 + Sqrt[3])*x^2 + x^
4))
```

Defintions of rubi rules used

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\left(-19-8\sqrt{3}\right)x^{3}+\left(-27\sqrt{3}-43\right)x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	42
risch	$\frac{\left(-19-8\sqrt{3}\right)x^{3}+\left(-27\sqrt{3}-43\right)x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	42
parallelrisch	$-\frac{8\sqrt{3}\,x^{3}+19x^{3}+27\sqrt{3}\,x+43x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	45
norman	$\frac{\left(-57-24\sqrt{3}\right)x^{5}+\left(-27\sqrt{3}-43\right)x+\left(-24-19\sqrt{3}\right)x^{3}+\left(-19-8\sqrt{3}\right)x^{7}}{x^{8}+4x^{6}+3x^{4}+4x^{2}+1}$	67
gosper	$-\frac{\left(8\sqrt{3}\,x^{6}+19x^{6}+46\sqrt{3}\,x^{4}+67x^{4}+73\sqrt{3}\,x^{2}+110x^{2}-27\sqrt{3}-43\right)x\left(x^{2}+\sqrt{3}+1\right)}{\left(x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1\right)\left(x^{6}+2\sqrt{3}\,x^{4}+x^{4}+3\sqrt{3}\,x^{2}+2x^{2}-\sqrt{3}-1\right)}$	113
orering	$-\frac{\left(x^{2}+\sqrt{3}+1\right)x\left(x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1\right)\left(-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^{2}+\left(67+46\sqrt{3}\right)x^{4}+\left(19+8\sqrt{3}\right)x^{6}\right)}{\left(x^{6}+2\sqrt{3}\,x^{4}+x^{4}+3\sqrt{3}\,x^{2}+2x^{2}-\sqrt{3}-1\right)\left(1+\left(2+\sqrt{3}\right)x^{2}+x^{4}\right)^{2}}$	121

input
```
int((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3^(1/2)
)*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x,method=_RETURNVERBOSE)
```

output
```
((-19-8*3^(1/2))*x^3+(-27*3^(1/2)-43)*x)/(x^4+3^(1/2)*x^2+2*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^{2}+\left(67+46\sqrt{3}\right)x^{4}+\left(19+8\sqrt{3}\right)x^{6}}{\left(1+\left(2+\sqrt{3}\right)x^{2}+x^{4}\right)^{2}}\,dx$$
$$= -\frac{19\,x^{7}+57\,x^{5}+24\,x^{3}+\sqrt{3}(8\,x^{7}+24\,x^{5}+19\,x^{3}+27\,x)+43\,x}{x^{8}+4\,x^{6}+3\,x^{4}+4\,x^{2}+1}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="fricas")
```

output
```
-(19*x^7 + 57*x^5 + 24*x^3 + sqrt(3)*(8*x^7 + 24*x^5 + 19*x^3 + 27*x) + 43
*x)/(x^8 + 4*x^6 + 3*x^4 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = \frac{x^3\left(-7519-4340\sqrt{3}\right)+x\left(-11859\sqrt{3}-20539\right)}{x^4\cdot\left(132\sqrt{3}+229\right)+x^2\cdot\left(493\sqrt{3}+854\right)+132\sqrt{3}+229}$$

input
```
integrate((-43-27*3**(1/2)+(110+73*3**(1/2))*x**2+(67+46*3**(1/2))*x**4+(1
9+8*3**(1/2))*x**6)/(1+(2+3**(1/2))*x**2+x**4)**2,x)
```

output
```
(x**3*(-7519 - 4340*sqrt(3)) + x*(-11859*sqrt(3) - 20539))/(x**4*(132*sqrt
(3) + 229) + x**2*(493*sqrt(3) + 854) + 132*sqrt(3) + 229)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x^3\left(100\sqrt{3}+153\right)+x\left(253\sqrt{3}+453\right)}{x^4\left(4\sqrt{3}+3\right)+x^2\left(11\sqrt{3}+18\right)+4\sqrt{3}+3}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="maxima")
```

output
```
-(x^3*(100*sqrt(3) + 153) + x*(253*sqrt(3) + 453))/(x^4*(4*sqrt(3) + 3) +
x^2*(11*sqrt(3) + 18) + 4*sqrt(3) + 3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueFrancis algorithm failure for[-1.0,0.0,infi
nity,infi
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$= -\frac{\left(8\sqrt{3}+19\right)x^3+\left(27\sqrt{3}+43\right)x}{x^4+\left(\sqrt{3}+2\right)x^2+1}$$

input
```
int((x^6*(8*3^(1/2) + 19) + x^4*(46*3^(1/2) + 67) + x^2*(73*3^(1/2) + 110)
 - 27*3^(1/2) - 43)/(x^4 + x^2*(3^(1/2) + 2) + 1)^2,x)
```

output
```
-(x^3*(8*3^(1/2) + 19) + x*(27*3^(1/2) + 43))/(x^4 + x^2*(3^(1/2) + 2) + 1
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$=\frac{x\left(-8\sqrt{3}\,x^6-24\sqrt{3}\,x^4-19\sqrt{3}\,x^2-27\sqrt{3}-19x^6-57x^4-24x^2-43\right)}{x^8+4x^6+3x^4+4x^2+1}$$

input
```
int((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3^(1/2)
)*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x)
```

output
```
(x*( - 8*sqrt(3)*x**6 - 24*sqrt(3)*x**4 - 19*sqrt(3)*x**2 - 27*sqrt(3) - 1
9*x**6 - 57*x**4 - 24*x**2 - 43))/(x**8 + 4*x**6 + 3*x**4 + 4*x**2 + 1)
```

3.38 $\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$

Optimal result

Integrand size = 90, antiderivative size = 51

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx = \frac{-2x-\sqrt{3}x^2-3x^3-x^5}{1+3x^2+\sqrt{3}x^3+3x^4+x^6}$$

output
```
(-2*x-3^(1/2)*x^2-3*x^3-x^5)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx = -\frac{x\left(2+\sqrt{3}x+3x^2+x^4\right)}{1+3x^2+\sqrt{3}x^3+3x^4+x^6}$$

input
```
Integrate[(-2 - 2*Sqrt[3]*x - 3*x^2 + 4*Sqrt[3]*x^3 + 7*x^4 + 6*Sqrt[3]*x^
5 + 10*x^6 + 2*Sqrt[3]*x^7 + 6*x^8 + x^10)/(1 + 3*x^2 + Sqrt[3]*x^3 + 3*x^
4 + x^6)^2,x]
```

output
```
-((x*(2 + Sqrt[3]*x + 3*x^2 + x^4))/(1 + 3*x^2 + Sqrt[3]*x^3 + 3*x^4 + x^6
))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.87 (sec) , antiderivative size = 515, normalized size of antiderivative = 10.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}+6x^8+2\sqrt{3}x^7+10x^6+6\sqrt{3}x^5+7x^4+4\sqrt{3}x^3-3x^2-2\sqrt{3}x-2}{\left(x^6+3x^4+\sqrt{3}x^3+3x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2466}$$

$$729\int \left(-\frac{\sqrt[3]{-1}\left(\sqrt{3}\left(1-(-3)^{2/3}\right)-\left(2+\sqrt[3]{-3}\right)x\right)}{2187\sqrt[6]{3}\left(x^2+(-1)^{2/3}\sqrt[6]{3}x+1\right)^2}+\frac{2i}{2187\left(2ix^2-\sqrt[6]{3}\left(i-\sqrt{3}\right)x+2i\right)}+\frac{2i}{2187\left(2ix^2-\sqrt[6]{3}\left(i\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$729\left(\frac{2\arctan\left(\frac{\sqrt[3]{-1}\sqrt[6]{3}-2x}{\sqrt{4-(-1)^{2/3}\sqrt[3]{3}}}\right)}{2187\sqrt{\frac{1}{2}\left(8+\sqrt[3]{3}-i3^{5/6}\right)}}-\frac{2\arctan\left(\frac{2x+(-1)^{2/3}\sqrt[6]{3}}{\sqrt{4+\sqrt[3]{-3}}}\right)}{2187\sqrt{4+\sqrt[3]{-3}}}+\frac{2i\text{arctanh}\left(\frac{\sqrt[6]{3}\left(-\sqrt{3}+i\right)-4ix}{\sqrt{2\left(8+\sqrt[3]{3}-i3^{5/6}\right)}}\right)}{2187\sqrt{\frac{1}{2}\left(8+\sqrt[3]{3}-i3^{5/6}\right)}}+\frac{2i\text{arctanh}\left(\right.}{2187\sqrt{\frac{1}{2}\left(\right.}}\right.$$

input
```
Int[(-2 - 2*Sqrt[3]*x - 3*x^2 + 4*Sqrt[3]*x^3 + 7*x^4 + 6*Sqrt[3]*x^5 + 10
*x^6 + 2*Sqrt[3]*x^7 + 6*x^8 + x^10)/(1 + 3*x^2 + Sqrt[3]*x^3 + 3*x^4 + x^
6)^2,x]
```

output
```
729*((4 - 5*3^(1/3) + 3^(2/3) - 3^(1/6)*(4 - 3^(1/3))*x)/(2187*3^(1/6)*(4
- 3^(1/3))*(1 + 3^(1/6)*x + x^2)) - (6*I - 3*(-1)^(2/3)*3^(1/6) + 2*Sqrt[3
] + 5*3^(5/6) + (-1)^(1/3)*(3 + 3^(2/3)*(2 + (2*I)*Sqrt[3]))*x)/(243*3^(2/
3)*(1 + (-1)^(1/3))^4*(4 - (-1)^(2/3)*3^(1/3))*(1 - (-1)^(1/3)*3^(1/6)*x +
 x^2)) - ((-1)^(1/3)*(4 + 5*(-3)^(1/3) + (-3)^(2/3) - 3^(1/6)*(4*(-1)^(2/3
) - 3^(1/3))*x))/(2187*3^(1/6)*(4 + (-3)^(1/3))*(1 + (-1)^(2/3)*3^(1/6)*x
+ x^2)) + (2*ArcTan[((-1)^(1/3)*3^(1/6) - 2*x)/Sqrt[4 - (-1)^(2/3)*3^(1/3)
]])/(2187*Sqrt[(8 + 3^(1/3) - I*3^(5/6))/2]) - (2*ArcTan[((-1)^(2/3)*3^(1/
6) + 2*x)/Sqrt[4 + (-3)^(1/3)]])/(2187*Sqrt[4 + (-3)^(1/3)]) + (((2*I)/218
7)*ArcTanh[(3^(1/6)*(I - Sqrt[3]) - (4*I)*x)/Sqrt[2*(8 + 3^(1/3) - I*3^(5/
6))]])/Sqrt[(8 + 3^(1/3) - I*3^(5/6))/2] + (((2*I)/2187)*ArcTanh[(3^(1/6)*
(I + Sqrt[3]) - (4*I)*x)/Sqrt[2*(8 + 3^(1/3) + I*3^(5/6))]])/Sqrt[(8 + 3^(
1/3) + I*3^(5/6))/2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{x\left(x^{4}+\sqrt{3}\,x+3x^{2}+2\right)}{1+3x^{2}+\sqrt{3}\,x^{3}+3x^{4}+x^{6}}$	43
orering	$-\frac{x\left(x^{4}+\sqrt{3}\,x+3x^{2}+2\right)}{1+3x^{2}+\sqrt{3}\,x^{3}+3x^{4}+x^{6}}$	43
default	$\frac{-2x-\sqrt{3}\,x^{2}-3x^{3}-x^{5}}{1+3x^{2}+\sqrt{3}\,x^{3}+3x^{4}+x^{6}}$	48
risch	$\frac{-2x-\sqrt{3}\,x^{2}-3x^{3}-x^{5}}{1+3x^{2}+\sqrt{3}\,x^{3}+3x^{4}+x^{6}}$	48
parallelrisch	$\frac{-2x-\sqrt{3}\,x^{2}-3x^{3}-x^{5}}{1+3x^{2}+\sqrt{3}\,x^{3}+3x^{4}+x^{6}}$	48
norman	$\frac{-\sqrt{3}\,x^{2}-\sqrt{3}\,x^{4}-2x-9x^{3}-13x^{5}-14x^{7}-6x^{9}-x^{11}}{x^{12}+6x^{10}+15x^{8}+17x^{6}+15x^{4}+6x^{2}+1}$	79

input
```
int((-2-2*3^(1/2)*x-3*x^2+4*3^(1/2)*x^3+7*x^4+6*3^(1/2)*x^5+10*x^6+2*3^(1/
2)*x^7+6*x^8+x^10)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)^2,x,method=_RETURNVERBO
SE)
```

output
```
-x*(x^4+3^(1/2)*x+3*x^2+2)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=-\frac{x^{11}+6\,x^{9}+14\,x^{7}+13\,x^{5}+9\,x^{3}+\sqrt{3}(x^{4}+x^{2})+2\,x}{x^{12}+6\,x^{10}+15\,x^{8}+17\,x^{6}+15\,x^{4}+6\,x^{2}+1}$$

input
```
integrate((-2-2*3^(1/2)*x-3*x^2+4*3^(1/2)*x^3+7*x^4+6*3^(1/2)*x^5+10*x^6+2
*3^(1/2)*x^7+6*x^8+x^10)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)^2,x, algorithm="f
ricas")
```

output
```
-(x^11 + 6*x^9 + 14*x^7 + 13*x^5 + 9*x^3 + sqrt(3)*(x^4 + x^2) + 2*x)/(x^1
2 + 6*x^10 + 15*x^8 + 17*x^6 + 15*x^4 + 6*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=\frac{-x^5-3x^3-\sqrt{3}x^2-2x}{x^6+3x^4+\sqrt{3}x^3+3x^2+1}$$

input
```
integrate((-2-2*3**(1/2)*x-3*x**2+4*3**(1/2)*x**3+7*x**4+6*3**(1/2)*x**5+1
0*x**6+2*3**(1/2)*x**7+6*x**8+x**10)/(1+3*x**2+3**(1/2)*x**3+3*x**4+x**6)*
*2,x)
```

output
```
(-x**5 - 3*x**3 - sqrt(3)*x**2 - 2*x)/(x**6 + 3*x**4 + sqrt(3)*x**3 + 3*x*
*2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=-\frac{\sqrt{3}x^5+3\sqrt{3}x^3+3x^2+2\sqrt{3}x}{\sqrt{3}x^6+3\sqrt{3}x^4+3x^3+3\sqrt{3}x^2+\sqrt{3}}$$

input
```
integrate((-2-2*3^(1/2)*x-3*x^2+4*3^(1/2)*x^3+7*x^4+6*3^(1/2)*x^5+10*x^6+2
*3^(1/2)*x^7+6*x^8+x^10)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)^2,x, algorithm="m
axima")
```

output
```
-(sqrt(3)*x^5 + 3*sqrt(3)*x^3 + 3*x^2 + 2*sqrt(3)*x)/(sqrt(3)*x^6 + 3*sqrt
(3)*x^4 + 3*x^3 + 3*sqrt(3)*x^2 + sqrt(3))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=-\frac{x^5+3\,x^3+\sqrt{3}x^2+2\,x}{x^6+3\,x^4+\sqrt{3}x^3+3\,x^2+1}$$

input
```
integrate((-2-2*3^(1/2)*x-3*x^2+4*3^(1/2)*x^3+7*x^4+6*3^(1/2)*x^5+10*x^6+2
*3^(1/2)*x^7+6*x^8+x^10)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)^2,x, algorithm="g
iac")
```

output
```
-(x^5 + 3*x^3 + sqrt(3)*x^2 + 2*x)/(x^6 + 3*x^4 + sqrt(3)*x^3 + 3*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=-\frac{x\left(x^4+3\,x^2+\sqrt{3}\,x+2\right)}{x^6+3\,x^4+\sqrt{3}\,x^3+3\,x^2+1}$$

input
```
int((4*3^(1/2)*x^3 - 2*3^(1/2)*x + 6*3^(1/2)*x^5 + 2*3^(1/2)*x^7 - 3*x^2 +
 7*x^4 + 10*x^6 + 6*x^8 + x^10 - 2)/(3^(1/2)*x^3 + 3*x^2 + 3*x^4 + x^6 + 1
)^2,x)
```

output
```
-(x*(3^(1/2)*x + 3*x^2 + x^4 + 2))/(3^(1/2)*x^3 + 3*x^2 + 3*x^4 + x^6 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{-2-2\sqrt{3}x-3x^2+4\sqrt{3}x^3+7x^4+6\sqrt{3}x^5+10x^6+2\sqrt{3}x^7+6x^8+x^{10}}{\left(1+3x^2+\sqrt{3}x^3+3x^4+x^6\right)^2}\,dx$$
$$=\frac{x\left(-\sqrt{3}\,x^3-\sqrt{3}\,x-x^{10}-6x^8-14x^6-13x^4-9x^2-2\right)}{x^{12}+6x^{10}+15x^8+17x^6+15x^4+6x^2+1}$$

input
```
int((-2-2*3^(1/2)*x-3*x^2+4*3^(1/2)*x^3+7*x^4+6*3^(1/2)*x^5+10*x^6+2*3^(1/
2)*x^7+6*x^8+x^10)/(1+3*x^2+3^(1/2)*x^3+3*x^4+x^6)^2,x)
```

output
```
(x*( - sqrt(3)*x**3 - sqrt(3)*x - x**10 - 6*x**8 - 14*x**6 - 13*x**4 - 9*x
**2 - 2))/(x**12 + 6*x**10 + 15*x**8 + 17*x**6 + 15*x**4 + 6*x**2 + 1)
```

3.39 $\int \frac{-43-27\sqrt{3}+(110+73\sqrt{3})x^2+(67+46\sqrt{3})x^4+(19+8\sqrt{3})x^6}{(1+(2+\sqrt{3})x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 67, antiderivative size = 43

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x\left(43+27\sqrt{3}+\left(19+8\sqrt{3}\right)x^2\right)}{1+\left(2+\sqrt{3}\right)x^2+x^4}$$

output

```
-x*(43+27*3^(1/2)+(19+8*3^(1/2))*x^2)/(1+(2+3^(1/2))*x^2+x^4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x\left(453+253\sqrt{3}+\left(153+100\sqrt{3}\right)x^2\right)}{\left(3+4\sqrt{3}\right)\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)}$$

input

```
Integrate[(-43 - 27*Sqrt[3] + (110 + 73*Sqrt[3])*x^2 + (67 + 46*Sqrt[3])*x
^4 + (19 + 8*Sqrt[3])*x^6)/(1 + (2 + Sqrt[3])*x^2 + x^4)^2,x]
```

output
```
-((x*(453 + 253*Sqrt[3] + (153 + 100*Sqrt[3])*x^2))/((3 + 4*Sqrt[3])*(1 +
(2 + Sqrt[3])*x^2 + x^4)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.015, Rules used = {2204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(19+8\sqrt{3}\right)x^6+\left(67+46\sqrt{3}\right)x^4+\left(110+73\sqrt{3}\right)x^2-27\sqrt{3}-43}{\left(x^4+\left(2+\sqrt{3}\right)x^2+1\right)^2}dx$$

$$\Big\downarrow \; 2204$$

$$-\frac{x\left(\left(19+8\sqrt{3}\right)x^2+27\sqrt{3}+43\right)}{x^4+\left(2+\sqrt{3}\right)x^2+1}$$

input
```
Int[(-43 - 27*Sqrt[3] + (110 + 73*Sqrt[3])*x^2 + (67 + 46*Sqrt[3])*x^4 + (
19 + 8*Sqrt[3])*x^6)/(1 + (2 + Sqrt[3])*x^2 + x^4)^2,x]
```

output
```
-((x*(43 + 27*Sqrt[3] + (19 + 8*Sqrt[3])*x^2))/(1 + (2 + Sqrt[3])*x^2 + x^
4))
```

Defintions of rubi rules used

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\left(-19-8\sqrt{3}\right)x^{3}+\left(-27\sqrt{3}-43\right)x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	42
risch	$\frac{\left(-19-8\sqrt{3}\right)x^{3}+\left(-27\sqrt{3}-43\right)x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	42
parallelrisch	$-\frac{8\sqrt{3}\,x^{3}+19x^{3}+27\sqrt{3}\,x+43x}{x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1}$	45
norman	$\frac{\left(-57-24\sqrt{3}\right)x^{5}+\left(-27\sqrt{3}-43\right)x+\left(-24-19\sqrt{3}\right)x^{3}+\left(-19-8\sqrt{3}\right)x^{7}}{x^{8}+4x^{6}+3x^{4}+4x^{2}+1}$	67
gosper	$-\frac{\left(8\sqrt{3}\,x^{6}+19x^{6}+46\sqrt{3}\,x^{4}+67x^{4}+73\sqrt{3}\,x^{2}+110x^{2}-27\sqrt{3}-43\right)x\left(x^{2}+\sqrt{3}+1\right)}{\left(x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1\right)\left(x^{6}+2\sqrt{3}\,x^{4}+x^{4}+3\sqrt{3}\,x^{2}+2x^{2}-\sqrt{3}-1\right)}$	113
orering	$-\frac{\left(x^{2}+\sqrt{3}+1\right)x\left(x^{4}+\sqrt{3}\,x^{2}+2x^{2}+1\right)\left(-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^{2}+\left(67+46\sqrt{3}\right)x^{4}+\left(19+8\sqrt{3}\right)x^{6}\right)}{\left(x^{6}+2\sqrt{3}\,x^{4}+x^{4}+3\sqrt{3}\,x^{2}+2x^{2}-\sqrt{3}-1\right)\left(1+\left(2+\sqrt{3}\right)x^{2}+x^{4}\right)^{2}}$	121

input
```
int((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3^(1/2)
)*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x,method=_RETURNVERBOSE)
```

output
```
((-19-8*3^(1/2))*x^3+(-27*3^(1/2)-43)*x)/(x^4+3^(1/2)*x^2+2*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^{2}+\left(67+46\sqrt{3}\right)x^{4}+\left(19+8\sqrt{3}\right)x^{6}}{\left(1+\left(2+\sqrt{3}\right)x^{2}+x^{4}\right)^{2}}\,dx$$
$$=-\frac{19\,x^{7}+57\,x^{5}+24\,x^{3}+\sqrt{3}(8\,x^{7}+24\,x^{5}+19\,x^{3}+27\,x)+43\,x}{x^{8}+4\,x^{6}+3\,x^{4}+4\,x^{2}+1}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="fricas")
```

output
```
-(19*x^7 + 57*x^5 + 24*x^3 + sqrt(3)*(8*x^7 + 24*x^5 + 19*x^3 + 27*x) + 43
*x)/(x^8 + 4*x^6 + 3*x^4 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = \frac{x^3\left(-7519-4340\sqrt{3}\right)+x\left(-11859\sqrt{3}-20539\right)}{x^4\cdot\left(132\sqrt{3}+229\right)+x^2\cdot\left(493\sqrt{3}+854\right)+132\sqrt{3}+229}$$

input
```
integrate((-43-27*3**(1/2)+(110+73*3**(1/2))*x**2+(67+46*3**(1/2))*x**4+(1
9+8*3**(1/2))*x**6)/(1+(2+3**(1/2))*x**2+x**4)**2,x)
```

output
```
(x**3*(-7519 - 4340*sqrt(3)) + x*(-11859*sqrt(3) - 20539))/(x**4*(132*sqrt
(3) + 229) + x**2*(493*sqrt(3) + 854) + 132*sqrt(3) + 229)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx = -\frac{x^3\left(100\sqrt{3}+153\right)+x\left(253\sqrt{3}+453\right)}{x^4\left(4\sqrt{3}+3\right)+x^2\left(11\sqrt{3}+18\right)+4\sqrt{3}+3}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="maxima")
```

output
```
-(x^3*(100*sqrt(3) + 153) + x*(253*sqrt(3) + 453))/(x^4*(4*sqrt(3) + 3) +
x^2*(11*sqrt(3) + 18) + 4*sqrt(3) + 3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3
^(1/2))*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueFrancis algorithm failure for[-1.0,0.0,infi
nity,infi
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$= -\frac{\left(8\sqrt{3}+19\right)x^3+\left(27\sqrt{3}+43\right)x}{x^4+\left(\sqrt{3}+2\right)x^2+1}$$

input
```
int((x^6*(8*3^(1/2) + 19) + x^4*(46*3^(1/2) + 67) + x^2*(73*3^(1/2) + 110)
 - 27*3^(1/2) - 43)/(x^4 + x^2*(3^(1/2) + 2) + 1)^2,x)
```

output
```
-(x^3*(8*3^(1/2) + 19) + x*(27*3^(1/2) + 43))/(x^4 + x^2*(3^(1/2) + 2) + 1
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{-43-27\sqrt{3}+\left(110+73\sqrt{3}\right)x^2+\left(67+46\sqrt{3}\right)x^4+\left(19+8\sqrt{3}\right)x^6}{\left(1+\left(2+\sqrt{3}\right)x^2+x^4\right)^2}\,dx$$
$$=\frac{x\left(-8\sqrt{3}\,x^6-24\sqrt{3}\,x^4-19\sqrt{3}\,x^2-27\sqrt{3}-19x^6-57x^4-24x^2-43\right)}{x^8+4x^6+3x^4+4x^2+1}$$

input
```
int((-43-27*3^(1/2)+(110+73*3^(1/2))*x^2+(67+46*3^(1/2))*x^4+(19+8*3^(1/2)
)*x^6)/(1+(2+3^(1/2))*x^2+x^4)^2,x)
```

output
```
(x*( - 8*sqrt(3)*x**6 - 24*sqrt(3)*x**4 - 19*sqrt(3)*x**2 - 27*sqrt(3) - 1
9*x**6 - 57*x**4 - 24*x**2 - 43))/(x**8 + 4*x**6 + 3*x**4 + 4*x**2 + 1)
```

3.40 $\int \frac{27\sqrt{3}+(-108-54\sqrt{3})x+(81+54\sqrt{3})x^2-36x^3+(18+3\sqrt{3})x^4-18\sqrt{3}x}{(3\sqrt{3}-6x+\sqrt{3}x^2+x^4)^3}$

Optimal result

Integrand size = 122, antiderivative size = 100

$$\int \frac{27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}x^5+\left(9-12\sqrt{3}\right)x^6+3}{\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)^3}$$
$$= \frac{3+48\sqrt{3}-48x-\sqrt{3}x-48x^2-\sqrt{3}x^2-x^3-16\sqrt{3}x^3}{144+3\sqrt{3}-6x-96\sqrt{3}x+48x^2+\sqrt{3}x^2+x^4+16\sqrt{3}x^4}$$

output
```
(3+48*3^(1/2)-48*x-x*3^(1/2)-48*x^2-3^(1/2)*x^2-x^3-16*3^(1/2)*x^3)/(144+3
*3^(1/2)-6*x-96*x*3^(1/2)+48*x^2+3^(1/2)*x^2+x^4+16*3^(1/2)*x^4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}x^5+\left(9-12\sqrt{3}\right)x^6+3}{\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)^3}$$
$$= -\frac{\sqrt{3}\left(-3-48\sqrt{3}+\left(48+\sqrt{3}\right)x+\left(48+\sqrt{3}\right)x^2+\left(1+16\sqrt{3}\right)x^3\right)}{\left(48+\sqrt{3}\right)\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)}$$

input
```
Integrate[(27*Sqrt[3] + (-108 - 54*Sqrt[3])*x + (81 + 54*Sqrt[3])*x^2 - 36
*x^3 + (18 + 3*Sqrt[3])*x^4 - 18*Sqrt[3]*x^5 + (9 - 12*Sqrt[3])*x^6 + 3*Sq
rt[3]*x^8 + 2*Sqrt[3]*x^9 + x^10)/(3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)^3,
x]
```

output
```
-((Sqrt[3]*(-3 - 48*Sqrt[3] + (48 + Sqrt[3])*x + (48 + Sqrt[3])*x^2 + (1 +
 16*Sqrt[3])*x^3))/((48 + Sqrt[3])*(3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.066$, Rules used = {2019, 2527, 25, 2527, 27, 2527, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}+2\sqrt{3}x^9+3\sqrt{3}x^8+\left(9-12\sqrt{3}\right)x^6-18\sqrt{3}x^5+\left(18+3\sqrt{3}\right)x^4-36x^3+\left(81+54\sqrt{3}\right)x^2+\left(-108-54\right.}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^3}$$

$$\Big\downarrow \ 2019$$

$$\int \frac{x^6+2\sqrt{3}x^5+2\sqrt{3}x^4+\left(3-3\sqrt{3}\right)x^2+\left(-18-6\sqrt{3}\right)x+9}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx$$

$$\Big\downarrow \ 2527$$

$$-\int -\frac{2\sqrt{3}x^5+3\sqrt{3}x^4-12x^3+3\left(1+2\sqrt{3}\right)x^2-6\left(3+\sqrt{3}\right)x+9}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \ 25$$

$$\int \frac{2\sqrt{3}x^5+3\sqrt{3}x^4-12x^3+3\left(1+2\sqrt{3}\right)x^2-6\left(3+\sqrt{3}\right)x+9}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \ 2527$$

$$-\frac{1}{2}\int -\frac{6\left(\sqrt{3}x^4-4x^3+x^2-2\sqrt{3}x+3\right)}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{\sqrt{3}x^2}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{\sqrt{3}x^4-4x^3+x^2-2\sqrt{3}x+3}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{\sqrt{3}x^2}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \text{2527}$$

$$3\left(-\frac{1}{3}\int -\frac{6\left(-2x^3-\sqrt{3}x+3\right)}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{x}{\sqrt{3}\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)}\right)-\frac{\sqrt{3}x^2}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$3\left(2\int \frac{-2x^3-\sqrt{3}x+3}{\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)^2}dx-\frac{x}{\sqrt{3}\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)}\right)-\frac{\sqrt{3}x^2}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

$$\Big\downarrow \text{2021}$$

$$-\frac{\sqrt{3}x^2}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}+3\left(\frac{1}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}-\frac{x}{\sqrt{3}\left(x^4+\sqrt{3}x^2-6x+3\sqrt{3}\right)}\right)-\frac{x^3}{x^4+\sqrt{3}x^2-6x+3\sqrt{3}}$$

input
```
Int[(27*Sqrt[3] + (-108 - 54*Sqrt[3])*x + (81 + 54*Sqrt[3])*x^2 - 36*x^3 +
 (18 + 3*Sqrt[3])*x^4 - 18*Sqrt[3]*x^5 + (9 - 12*Sqrt[3])*x^6 + 3*Sqrt[3]*
x^8 + 2*Sqrt[3]*x^9 + x^10)/(3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)^3,x]
```

output
```
-((Sqrt[3]*x^2)/(3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)) - x^3/(3*Sqrt[3] -
6*x + Sqrt[3]*x^2 + x^4) + 3*((3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)^(-1) -
 x/(Sqrt[3]*(3*Sqrt[3] - 6*x + Sqrt[3]*x^2 + x^4)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2527
```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Simp[1/((m + n*p + 1)*Coeff[Qn, x, n])   Int[ExpandToSum[(m
+ n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
 (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 <
0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

method	result
default	$\frac{3-x^3-\sqrt{3}\,x^2-\sqrt{3}\,x}{3\sqrt{3}-6x+\sqrt{3}\,x^2+x^4}$
risch	$\frac{3-x^3-\sqrt{3}\,x^2-\sqrt{3}\,x}{3\sqrt{3}-6x+\sqrt{3}\,x^2+x^4}$
norman	$\frac{12x^4+\left(9\sqrt{3}+3\right)x^3+\left(9+3\sqrt{3}\right)x^2-9x-x^7-\sqrt{3}\,x^6-9\sqrt{3}}{x^8-12x^5-3x^4+18x^2-27}$
parallelrisch	$-\frac{-243+3\sqrt{3}\,x^8-9x^8+18\sqrt{3}\,x^6+36x^7+36\sqrt{3}\,x^5+18x^6-45\sqrt{3}\,x^4+108x^5-189x^4-162\sqrt{3}\,x^2+324\sqrt{3}\,x-162x^2-243\sqrt{3}+6}{36\left(3\sqrt{3}-6x+\sqrt{3}\,x^2+x^4\right)^2}$
orering	$-\frac{\left(\sqrt{3}\,x^2+x^3+\sqrt{3}\,x-3\right)\left(27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}\,x^5+\left(9-12\sqrt{3}\right)x^6+3\sqrt{3}\,x}{\left(2\sqrt{3}\,x^5+x^6+2\sqrt{3}\,x^4-3\sqrt{3}\,x^2-6\sqrt{3}\,x+3x^2-18x+9\right)\left(3\sqrt{3}-6x+\sqrt{3}\,x^2+x^4\right)^2}$
gosper	$-\frac{\left(\sqrt{3}\,x^2+x^3+\sqrt{3}\,x-3\right)\left(2\sqrt{3}\,x^9+x^{10}+3\sqrt{3}\,x^8-12\sqrt{3}\,x^6-18\sqrt{3}\,x^5+9x^6+3\sqrt{3}\,x^4+18x^4+54\sqrt{3}\,x^2-36x^3-54\sqrt{3}\,x+81x^2+}{\left(3\sqrt{3}-6x+\sqrt{3}\,x^2+x^4\right)^2\left(2\sqrt{3}\,x^5+x^6+2\sqrt{3}\,x^4-3\sqrt{3}\,x^2-6\sqrt{3}\,x+3x^2-18x+9\right)}$

input
```
int((27*3^(1/2)+(-108-54*3^(1/2))*x+(81+54*3^(1/2))*x^2-36*x^3+(18+3*3^(1/
2))*x^4-18*3^(1/2)*x^5+(9-12*3^(1/2))*x^6+3*3^(1/2)*x^8+2*3^(1/2)*x^9+x^10
)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(3-x^3-3^(1/2)*x^2-3^(1/2)*x)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int \frac{27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}x^5+\left(9-12\sqrt{3}\right)x^6+3}{\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)^3}$$
$$= -\frac{x^7-12\,x^4-3\,x^3-9\,x^2+\sqrt{3}(x^6-9\,x^3-3\,x^2+9)+9\,x}{x^8-12\,x^5-3\,x^4+18\,x^2-27}$$

input
```
integrate((27*3^(1/2)+(-108-54*3^(1/2))*x+(81+54*3^(1/2))*x^2-36*x^3+(18+3
*3^(1/2))*x^4-18*3^(1/2)*x^5+(9-12*3^(1/2))*x^6+3*3^(1/2)*x^8+2*3^(1/2)*x^
9+x^10)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)^3,x, algorithm="fricas")
```

output
```
-(x^7 - 12*x^4 - 3*x^3 - 9*x^2 + sqrt(3)*(x^6 - 9*x^3 - 3*x^2 + 9) + 9*x)/
(x^8 - 12*x^5 - 3*x^4 + 18*x^2 - 27)
```

Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{27\sqrt{3} + \left(-108 - 54\sqrt{3}\right)x + \left(81 + 54\sqrt{3}\right)x^2 - 36x^3 + \left(18 + 3\sqrt{3}\right)x^4 - 18\sqrt{3}x^5 + \left(9 - 12\sqrt{3}\right)x^6 + 3}{\left(3\sqrt{3} - 6x + \sqrt{3}x^2 + x^4\right)^3}$$

= Too large to display

input
```
integrate((27*3**(1/2)+(-108-54*3**(1/2))*x+(81+54*3**(1/2))*x**2-36*x**3+
(18+3*3**(1/2))*x**4-18*3**(1/2)*x**5+(9-12*3**(1/2))*x**6+3*3**(1/2)*x**8
+2*3**(1/2)*x**9+x**10)/(3*3**(1/2)-6*x+3**(1/2)*x**2+x**4)**3,x)
```

output
```
(x**3*(-678294144927038056133041780314489485975271078177439587284258533294
51533875000069116985192494203642812857385753447140092464961556141116566711
97681710907833502781709019037507064564527536661349019115988897128139077833
20714303886030118173679517022473558004986697459794753688116114199152354721
99064681001631231195136189064244329590049426855748475873966104529335810635
57658273384974029313353114350264334369683152888942025016104607115192887613
36930324458873232236129596630289421651816222495320810360274130004583167949
16206633918173416421404011049558631649998907177865914068718540566260649978
71070544572277775554313736576030764331994693102467114578978953073405637783
06663691562712679943811177797763395992825986699561780980283081208084704315
71884268343228508285890897093651529652469386445912881115529525807028495086
49987248227630762388203851367003038996918843431109903929504095227129837527
83668181701323167922445874921113227546202369309277639434023075988083717221
83550627895756494629530878593008596026406997633957133150820735578044799908
03634054609875825410473903019980757706584665217166263999588373907335406797
18216238308186080776915263322387209187775288215146150631078838204675064893
85023953734270731517927168759131039282346009137763090288142080719757721477
74322240211468646864830286679235908674523684677029756128760460115617193599
05012967481866373350326268250721 - 391613307163372452875517724015331761023
50111299355506110502700765374451714356467082586072440795222432283843799...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.79

$$\int \frac{27\sqrt{3} + \left(-108 - 54\sqrt{3}\right)x + \left(81 + 54\sqrt{3}\right)x^2 - 36x^3 + \left(18 + 3\sqrt{3}\right)x^4 - 18\sqrt{3}x^5 + \left(9 - 12\sqrt{3}\right)x^6 + 3}{\left(3\sqrt{3} - 6x + \sqrt{3}x^2 + x^4\right)^3}$$
$$= -\frac{x^7\left(32\sqrt{3} + 769\right) + x^6\left(769\sqrt{3} + 96\right) + 2x^5\left(769\sqrt{3} + 96\right) - 6x^4\left(32\sqrt{3} + 769\right) - 3x^3\left(737\sqrt{3} - 6\right.}{x^8\left(32\sqrt{3} + 769\right) + 2x^6\left(769\sqrt{3} + 96\right) - 12x^5\left(32\sqrt{3} + 769\right) + 3x^4\left(1570\sqrt{3} + 961\right) - 12x^3\left(769\sqrt{3}\right.}$$

input
```
integrate((27*3^(1/2)+(-108-54*3^(1/2))*x+(81+54*3^(1/2))*x^2-36*x^3+(18+3
*3^(1/2))*x^4-18*3^(1/2)*x^5+(9-12*3^(1/2))*x^6+3*3^(1/2)*x^8+2*3^(1/2)*x^
9+x^10)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)^3,x, algorithm="maxima")
```

output
```
-(x^7*(32*sqrt(3) + 769) + x^6*(769*sqrt(3) + 96) + 2*x^5*(769*sqrt(3) + 9
6) - 6*x^4*(32*sqrt(3) + 769) - 3*x^3*(737*sqrt(3) - 673) - 9*x^2*(737*sqr
t(3) - 673) + 27*x*(32*sqrt(3) + 769) - 6921*sqrt(3) - 864)/(x^8*(32*sqrt(
3) + 769) + 2*x^6*(769*sqrt(3) + 96) - 12*x^5*(32*sqrt(3) + 769) + 3*x^4*(
1570*sqrt(3) + 961) - 12*x^3*(769*sqrt(3) + 96) + 54*x^2*(32*sqrt(3) + 769
) - 36*x*(769*sqrt(3) + 96) + 864*sqrt(3) + 20763)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{27\sqrt{3} + \left(-108 - 54\sqrt{3}\right)x + \left(81 + 54\sqrt{3}\right)x^2 - 36x^3 + \left(18 + 3\sqrt{3}\right)x^4 - 18\sqrt{3}x^5 + \left(9 - 12\sqrt{3}\right)x^6 + 3}{\left(3\sqrt{3} - 6x + \sqrt{3}x^2 + x^4\right)^3}$$
= Exception raised: TypeError

input
```
integrate((27*3^(1/2)+(-108-54*3^(1/2))*x+(81+54*3^(1/2))*x^2-36*x^3+(18+3
*3^(1/2))*x^4-18*3^(1/2)*x^5+(9-12*3^(1/2))*x^6+3*3^(1/2)*x^8+2*3^(1/2)*x^
9+x^10)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1084716759416832,13101256450146816]:[1,0,-3]%%},[4]%%%
}+%%%{%%{
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}x^5+\left(9-12\sqrt{3}\right)x^6+3\sqrt{3}x^8+2\sqrt{3}x^9+x^{10}}{\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)^3}
= -\frac{x^3+\sqrt{3}\,x^2+\sqrt{3}\,x-3}{x^4+\sqrt{3}\,x^2-6\,x+3\,\sqrt{3}}$$

input
```
int((x^4*(3*3^(1/2) + 18) - x^6*(12*3^(1/2) - 9) + x^2*(54*3^(1/2) + 81) +
 27*3^(1/2) - 18*3^(1/2)*x^5 + 3*3^(1/2)*x^8 + 2*3^(1/2)*x^9 - 36*x^3 + x^
10 - x*(54*3^(1/2) + 108))/(3*3^(1/2) - 6*x + 3^(1/2)*x^2 + x^4)^3,x)
```

output
```
-(3^(1/2)*x + 3^(1/2)*x^2 + x^3 - 3)/(3*3^(1/2) - 6*x + 3^(1/2)*x^2 + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{27\sqrt{3}+\left(-108-54\sqrt{3}\right)x+\left(81+54\sqrt{3}\right)x^2-36x^3+\left(18+3\sqrt{3}\right)x^4-18\sqrt{3}x^5+\left(9-12\sqrt{3}\right)x^6+3\sqrt{3}x^8+2\sqrt{3}x^9+x^{10}}{\left(3\sqrt{3}-6x+\sqrt{3}x^2+x^4\right)^3}
= \frac{-\sqrt{3}\,x^6+9\sqrt{3}\,x^3+3\sqrt{3}\,x^2-9\sqrt{3}+4x^8-x^7-48x^5+3x^3+81x^2-9x-108}{x^8-12x^5-3x^4+18x^2-27}$$

input
```
int((27*3^(1/2)+(-108-54*3^(1/2))*x+(81+54*3^(1/2))*x^2-36*x^3+(18+3*3^(1/
2))*x^4-18*3^(1/2)*x^5+(9-12*3^(1/2))*x^6+3*3^(1/2)*x^8+2*3^(1/2)*x^9+x^10
)/(3*3^(1/2)-6*x+3^(1/2)*x^2+x^4)^3,x)
```

output

```
( - sqrt(3)*x**6 + 9*sqrt(3)*x**3 + 3*sqrt(3)*x**2 - 9*sqrt(3) + 4*x**8 -
x**7 - 48*x**5 + 3*x**3 + 81*x**2 - 9*x - 108)/(x**8 - 12*x**5 - 3*x**4 +
18*x**2 - 27)
```

3.41 $\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3} dx$

Optimal result

Integrand size = 46, antiderivative size = 20

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3} dx = \frac{x}{\sqrt{2}-3x^2-x^4}$$

output
```
x/(2^(1/2)-3*x^2-x^4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 1 in optimal.

Time = 0.22 (sec) , antiderivative size = 288, normalized size of antiderivative = 14.40

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3} dx =$$
$$-\frac{\left(\sqrt{2}-3x^2-x^4\right)^3\left(210x\,\mathrm{AppellF1}\left(\frac{1}{2},3,3,\frac{3}{2},\frac{2x^2}{-3+\sqrt{9+4\sqrt{2}}},-\frac{2x^2}{3+\sqrt{9+4\sqrt{2}}}\right)+21\left(-9+2\sqrt{2}\right)x^5\,\mathrm{AppellF1}\right.}{}$$

input
```
Integrate[(2 - 9*x^4 + 2*Sqrt[2]*x^4 - 12*x^6 - 3*x^8)/(Sqrt[2] - 3*x^2 -
x^4)^3,x]
```

output
```
-1/105*((Sqrt[2] - 3*x^2 - x^4)^3*(210*x*AppellF1[1/2, 3, 3, 3/2, (2*x^2)/
(-3 + Sqrt[9 + 4*Sqrt[2]]), (-2*x^2)/(3 + Sqrt[9 + 4*Sqrt[2]])] + 21*(-9 +
 2*Sqrt[2])*x^5*AppellF1[5/2, 3, 3, 7/2, (2*x^2)/(-3 + Sqrt[9 + 4*Sqrt[2]]
), (-2*x^2)/(3 + Sqrt[9 + 4*Sqrt[2]])] - 5*(36*x^7*AppellF1[7/2, 3, 3, 9/2
, (2*x^2)/(-3 + Sqrt[9 + 4*Sqrt[2]]), (-2*x^2)/(3 + Sqrt[9 + 4*Sqrt[2]])]
+ 7*x^9*AppellF1[9/2, 3, 3, 11/2, (2*x^2)/(-3 + Sqrt[9 + 4*Sqrt[2]]), (-2*
x^2)/(3 + Sqrt[9 + 4*Sqrt[2]])])))/(-2 + 3*Sqrt[2]*x^2 + Sqrt[2]*x^4)^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{6, 2019, 2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-3x^8-12x^6+2\sqrt{2}x^4-9x^4+2}{\left(-x^4-3x^2+\sqrt{2}\right)^3}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{-3x^8-12x^6+\left(2\sqrt{2}-9\right)x^4+2}{\left(-x^4-3x^2+\sqrt{2}\right)^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{3x^4+3x^2+\sqrt{2}}{\left(-x^4-3x^2+\sqrt{2}\right)^2}dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{x}{-x^4-3x^2+\sqrt{2}}$$

input
```
Int[(2 - 9*x^4 + 2*Sqrt[2]*x^4 - 12*x^6 - 3*x^8)/(Sqrt[2] - 3*x^2 - x^4)^3
,x]
```

output
```
x/(Sqrt[2] - 3*x^2 - x^4)
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{x}{-\sqrt{2}+3x^2+x^4}$	20
parallelrisch	$-\frac{x^5+3x^3-\sqrt{2}\,x}{\left(\sqrt{2}-3x^2-x^4\right)^2}$	34
norman	$\frac{-3x^3-x^5-\sqrt{2}\,x}{x^8+6x^6+9x^4-2}$	36
default	$\frac{\left(-113-72\sqrt{2}\right)x}{\left(1+2\sqrt{2}\right)^5\left(x^2-\sqrt{2}+1\right)}-\frac{\left(-113-72\sqrt{2}\right)x}{\left(1+2\sqrt{2}\right)^5\left(x^2+\sqrt{2}+2\right)}$	61
gosper	$\frac{\left(x^2+\sqrt{2}+2\right)\left(-x^2+\sqrt{2}-1\right)x\left(2-9x^4+2\sqrt{2}\,x^4-12x^6-3x^8\right)}{\left(3x^4+3x^2+\sqrt{2}\right)\left(\sqrt{2}-3x^2-x^4\right)^3}$	78
orering	$\frac{\left(x^2+\sqrt{2}+2\right)\left(-x^2+\sqrt{2}-1\right)x\left(2-9x^4+2\sqrt{2}\,x^4-12x^6-3x^8\right)}{\left(3x^4+3x^2+\sqrt{2}\right)\left(\sqrt{2}-3x^2-x^4\right)^3}$	78

input
```
int((2-9*x^4+2*2^(1/2)*x^4-12*x^6-3*x^8)/(2^(1/2)-3*x^2-x^4)^3,x,method=_R
ETURNVERBOSE)
```

output
```
-x/(-2^(1/2)+3*x^2+x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3}\,dx = -\frac{x^5+3\,x^3+\sqrt{2}x}{x^8+6\,x^6+9\,x^4-2}$$

input
```
integrate((2-9*x^4+2*2^(1/2)*x^4-12*x^6-3*x^8)/(2^(1/2)-3*x^2-x^4)^3,x, al
gorithm="fricas")
```

output
```
-(x^5 + 3*x^3 + sqrt(2)*x)/(x^8 + 6*x^6 + 9*x^4 - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3}\,dx$$

$$= \frac{}{x^4\cdot\left(22008094116350812143001749908067955820471359033951143400093732637078466984606818067\right.}$$

input
```
integrate((2-9*x**4+2*2**(1/2)*x**4-12*x**6-3*x**8)/(2**(1/2)-3*x**2-x**4)
**3,x)
```

output
```
x*(-1556207259066341771535971142273921112871404354245607770965160208380551
0351009805161079225555597665966164772904 2809*sqrt(2) - 2200809411635081214
3001749908067955820471359033951143400093732637078466984606818067617278537 6
41904898657048550 6912)/(x**4*(22008094116350812143001749908067955820471359
03395114340009373263707846698460681806761727853764190489865704855 06912 + 1
5562072590663417715359711422739211128714043542456077709651602083805510351 0
0980516107922555559766596616477290428 09*sqrt(2)) + x**2*(6602428234905243 6
429005249724203867461414077101853430200281197911235400953820454202851835 61
29257146959711456520736 + 46686217771990253146079134268217633386142130627 3
682331289548062514165310530294154832376766667929978984943187128427*sqrt(2)
) - 3112414518132683543071942284547842225742808708491215541930320416761102
070201961032215845111119533193232954580856 18 - 22008094116350812143001749 9
080679558204713590339511434000937326370784669846068180676172785376419048 98
6570485506912*sqrt(2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.55

$$\int \frac{2 - 9x^4 + 2\sqrt{2}x^4 - 12x^6 - 3x^8}{\left(\sqrt{2} - 3x^2 - x^4\right)^3}\,dx = -\frac{x^5\left(72\sqrt{2} + 113\right) + 3\,x^3\left(72\sqrt{2} + 113\right) - x\left(113\sqrt{2} + 144\right)}{x^8\left(72\sqrt{2} + 113\right) + 6\,x^6\left(72\sqrt{2} + 113\right) + x^4\left(422\sqrt{2} + 729\right) - 6\,x^2\left(113\sqrt{2} + 144\right) + 144\sqrt{2} + 226}$$

input
```
integrate((2-9*x^4+2*2^(1/2)*x^4-12*x^6-3*x^8)/(2^(1/2)-3*x^2-x^4)^3,x, al
gorithm="maxima")
```

output
```
-(x^5*(72*sqrt(2) + 113) + 3*x^3*(72*sqrt(2) + 113) - x*(113*sqrt(2) + 144
))/(x^8*(72*sqrt(2) + 113) + 6*x^6*(72*sqrt(2) + 113) + x^4*(422*sqrt(2) +
 729) - 6*x^2*(113*sqrt(2) + 144) + 144*sqrt(2) + 226)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3}\,dx = -\frac{x}{x^4+3\,x^2-\sqrt{2}}$$

input
```
integrate((2-9*x^4+2*2^(1/2)*x^4-12*x^6-3*x^8)/(2^(1/2)-3*x^2-x^4)^3,x, al
gorithm="giac")
```

output
```
-x/(x^4 + 3*x^2 - sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3}\,dx = -\frac{x}{x^4+3\,x^2-\sqrt{2}}$$

input
```
int((9*x^4 - 2*2^(1/2)*x^4 + 12*x^6 + 3*x^8 - 2)/(3*x^2 - 2^(1/2) + x^4)^3
,x)
```

output
```
-x/(3*x^2 - 2^(1/2) + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{2-9x^4+2\sqrt{2}x^4-12x^6-3x^8}{\left(\sqrt{2}-3x^2-x^4\right)^3}\,dx = \frac{x\left(-\sqrt{2}-x^4-3x^2\right)}{x^8+6x^6+9x^4-2}$$

input
```
int((2-9*x^4+2*2^(1/2)*x^4-12*x^6-3*x^8)/(2^(1/2)-3*x^2-x^4)^3,x)
```

output
```
(x*( - sqrt(2) - x**4 - 3*x**2))/(x**8 + 6*x**6 + 9*x**4 - 2)
```

3.42 $\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^2+\left(-51201+34026\sqrt{2}\right)x^4+\left(-63\right.}{\left(\sqrt{2}-3x^2\right.}$

Optimal result

Integrand size = 93, antiderivative size = 79

$$\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^2+\left(-51201+34026\sqrt{2}\right)x^4+\left(-63033+41310\sqrt{2}\right)x^6+\left(-\right.}{\left(\sqrt{2}-3x^2-x^4\right)^3}$$
$$=\frac{343\left(-9+4\sqrt{2}\right)x\left(-44-81\sqrt{2}+21\left(9+\sqrt{2}\right)x^2+14\left(9+\sqrt{2}\right)x^4+21x^6\right)}{\left(9+4\sqrt{2}\right)^2\left(-\sqrt{2}+3x^2+x^4\right)^2}$$

output
```
343*(-9+4*2^(1/2))*x*(-44-81*2^(1/2)+21*(9+2^(1/2))*x^2+14*(9+2^(1/2))*x^4
+21*x^6)/(9+4*2^(1/2))^2/(-2^(1/2)+3*x^2+x^4)^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^2+\left(-51201+34026\sqrt{2}\right)x^4+\left(-63033+41310\sqrt{2}\right)x^6+\left(-\right.}{\left(\sqrt{2}-3x^2-x^4\right)^3}$$
$$=\frac{343\left(-9+4\sqrt{2}\right)x\left(-44-81\sqrt{2}+21\left(9+\sqrt{2}\right)x^2+14\left(9+\sqrt{2}\right)x^4+21x^6\right)}{\left(9+4\sqrt{2}\right)^2\left(-\sqrt{2}+3x^2+x^4\right)^2}$$

input
```
Integrate[(23038 - 15444*Sqrt[2] + (10530 - 5562*Sqrt[2])*x^2 + (-51201 +
34026*Sqrt[2])*x^4 + (-63033 + 41310*Sqrt[2])*x^6 + (-29811 + 20142*Sqrt[2
])*x^8 + (-4779 + 3300*Sqrt[2])*x^10)/(Sqrt[2] - 3*x^2 - x^4)^3,x]
```

output
```
(343*(-9 + 4*Sqrt[2])*x*(-44 - 81*Sqrt[2] + 21*(9 + Sqrt[2])*x^2 + 14*(9 +
 Sqrt[2])*x^4 + 21*x^6))/((9 + 4*Sqrt[2])^2*(-Sqrt[2] + 3*x^2 + x^4)^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2206, 27, 2204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(3300\sqrt{2}-4779\right)x^{10}+\left(20142\sqrt{2}-29811\right)x^8+\left(41310\sqrt{2}-63033\right)x^6+\left(34026\sqrt{2}-51201\right)x^4+\left(10530-\right.}{\left(-x^4-3x^2+\sqrt{2}\right)^3}$$

$$\Big\downarrow \text{2206}$$

$$\frac{686\sqrt{2}\left(9-4\sqrt{2}\right)x}{\left(9+4\sqrt{2}\right)\left(-x^4-3x^2+\sqrt{2}\right)^2}-\frac{\int -\frac{196\left(-3\left(144-113\sqrt{2}\right)x^6-6\left(206-195\sqrt{2}\right)x^4-3\left(166-297\sqrt{2}\right)x^2+2\left(729-422\sqrt{2}\right)\right)}{\left(-x^4-3x^2+\sqrt{2}\right)^2}dx}{4\sqrt{2}\left(9+4\sqrt{2}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{49\int \frac{-3\left(144-113\sqrt{2}\right)x^6-6\left(206-195\sqrt{2}\right)x^4-3\left(166-297\sqrt{2}\right)x^2+2\left(729-422\sqrt{2}\right)}{\left(-x^4-3x^2+\sqrt{2}\right)^2}dx}{\sqrt{2}\left(9+4\sqrt{2}\right)}+\frac{686\sqrt{2}\left(9-4\sqrt{2}\right)x}{\left(9+4\sqrt{2}\right)\left(-x^4-3x^2+\sqrt{2}\right)^2}$$

$$\Big\downarrow \text{2204}$$

$$\frac{686\sqrt{2}(9-4\sqrt{2})\,x}{(9+4\sqrt{2})\left(-x^4-3x^2+\sqrt{2}\right)^2}-\frac{49x\left(3(144-113\sqrt{2})\,x^2-729\sqrt{2}+844\right)}{\sqrt{2}\,(9+4\sqrt{2})\left(-x^4-3x^2+\sqrt{2}\right)}$$

input
```
Int[(23038 - 15444*Sqrt[2] + (10530 - 5562*Sqrt[2])*x^2 + (-51201 + 34026*
Sqrt[2])*x^4 + (-63033 + 41310*Sqrt[2])*x^6 + (-29811 + 20142*Sqrt[2])*x^8
 + (-4779 + 3300*Sqrt[2])*x^10)/(Sqrt[2] - 3*x^2 - x^4)^3,x]
```

output
```
(686*Sqrt[2]*(9 - 4*Sqrt[2])*x)/((9 + 4*Sqrt[2])*(Sqrt[2] - 3*x^2 - x^4)^2
) - (49*x*(844 - 729*Sqrt[2] + 3*(144 - 113*Sqrt[2])*x^2))/(Sqrt[2]*(9 + 4
*Sqrt[2])*(Sqrt[2] - 3*x^2 - x^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\left(-4779+3300\sqrt{2}\right)x^{7}+\left(-24274+16614\sqrt{2}\right)x^{5}+\left(-36411+24921\sqrt{2}\right)x^{3}+\left(11519\sqrt{2}-15444\right)x}{\left(-\sqrt{2}+3x^{2}+x^{4}\right)^{2}}$
parallelrisch	$\frac{3300\sqrt{2}\,x^{7}-4779x^{7}+16614\sqrt{2}\,x^{5}-24274x^{5}+24921\sqrt{2}\,x^{3}-36411x^{3}+11519\sqrt{2}\,x-15444x}{\left(\sqrt{2}-3x^{2}-x^{4}\right)^{2}}$
default	$\frac{343\left(-183\sqrt{2}-66\right)x^{3}+343\left(356-131\sqrt{2}\right)x}{\left(1+2\sqrt{2}\right)^{5}\left(x^{2}-\sqrt{2}+1\right)^{2}}+\frac{343\left(-111\sqrt{2}+213\right)x^{3}+343\left(260-23\sqrt{2}\right)x}{\left(1+2\sqrt{2}\right)^{5}\left(x^{2}+\sqrt{2}+2\right)^{2}}$
norman	$\frac{\left(-346320+233349\sqrt{2}\right)x^{9}+\left(-211866+144747\sqrt{2}\right)x^{11}+\left(-130869+81537\sqrt{2}\right)x^{7}+\left(-52948+36414\sqrt{2}\right)x^{13}+\left(-30888+23\right.}{\left(x^{8}+6x^{6}+9x^{4}-2\right)^{2}}$
orering	$\frac{\left(21x^{6}+14\sqrt{2}\,x^{4}+126x^{4}+21\sqrt{2}\,x^{2}+189x^{2}-81\sqrt{2}-44\right)x\left(-x^{2}+\sqrt{2}-1\right)\left(x^{2}+\sqrt{2}+2\right)\left(23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)\right.}{\left(21x^{10}+42\sqrt{2}\,x^{8}+189x^{8}+210x^{6}\sqrt{2}+567x^{6}+126\sqrt{2}\,x^{4}+399x^{4}-1\right.}$
gosper	$\frac{\left(x^{2}+\sqrt{2}+2\right)\left(-x^{2}+\sqrt{2}-1\right)x\left(21x^{6}+14\sqrt{2}\,x^{4}+126x^{4}+21\sqrt{2}\,x^{2}+189x^{2}-81\sqrt{2}-44\right)\left(3300\sqrt{2}\,x^{10}-4779x^{10}+20142\sqrt{2}\,x^{8}-\right.}{\left(21x^{10}+42\sqrt{2}\,x^{8}+189x^{8}+210x^{6}\sqrt{2}+567x^{6}+126\sqrt{2}\,x^{4}+399x^{4}-162\right.}$

input
```
int((23038-15444*2^(1/2)+(10530-5562*2^(1/2))*x^2+(-51201+34026*2^(1/2))*x
^4+(-63033+41310*2^(1/2))*x^6+(-29811+20142*2^(1/2))*x^8+(-4779+3300*2^(1/
2))*x^10)/(2^(1/2)-3*x^2-x^4)^3,x,method=_RETURNVERBOSE)
```

output
```
((-4779+3300*2^(1/2))*x^7+(-24274+16614*2^(1/2))*x^5+(-36411+24921*2^(1/2)
)*x^3+(11519*2^(1/2)-15444)*x)/(-2^(1/2)+3*x^2+x^4)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.54

$$\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^{2}+\left(-51201+34026\sqrt{2}\right)x^{4}+\left(-63033+41310\sqrt{2}\right)x^{6}+\left(-\right.}{\left(\sqrt{2}-3x^{2}-x^{4}\right)^{3}}$$
$$= -\frac{4779\,x^{15}+52948\,x^{13}+211866\,x^{11}+346320\,x^{9}+130869\,x^{7}-157584\,x^{5}-65406\,x^{3}-\sqrt{2}(3300\,x^{15}+}{x^{16}+12\,x^{14}+54\,x^{12}+108\,x^{10}+}$$

input
```
integrate((23038-15444*2^(1/2)+(10530-5562*2^(1/2))*x^2+(-51201+34026*2^(1
/2))*x^4+(-63033+41310*2^(1/2))*x^6+(-29811+20142*2^(1/2))*x^8+(-4779+3300
*2^(1/2))*x^10)/(2^(1/2)-3*x^2-x^4)^3,x, algorithm="fricas")
```

output
```
-(4779*x^15 + 52948*x^13 + 211866*x^11 + 346320*x^9 + 130869*x^7 - 157584*
x^5 - 65406*x^3 - sqrt(2)*(3300*x^15 + 36414*x^13 + 144747*x^11 + 233349*x
^9 + 81537*x^7 - 112455*x^5 - 42822*x^3 + 23038*x) + 30888*x)/(x^16 + 12*x
^14 + 54*x^12 + 108*x^10 + 77*x^8 - 24*x^6 - 36*x^4 + 4)
```

Sympy [A] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int \frac{23038 - 15444\sqrt{2} + \left(10530 - 5562\sqrt{2}\right)x^2 + \left(-51201 + 34026\sqrt{2}\right)x^4 + \left(-63033 + 41310\sqrt{2}\right)x^6 + \left(-\right.}{\left(\sqrt{2} - 3x^2 - x^4\right)^3}$$

$$= \frac{x^7 \cdot \left(57624 - 64827\sqrt{2}\right) + x^5 \cdot \left(259308 - 350546\sqrt{2}\right) + x^3 \cdot \left(388962 - 525819\sqrt{2}\right) + x\left(379358 - 8643\right.}{x^8 \cdot \left(144 + 113\sqrt{2}\right) + x^6 \cdot \left(864 + 678\sqrt{2}\right) + x^4 \cdot \left(844 + 729\sqrt{2}\right) + x^2\left(-1356 - 864\sqrt{2}\right) + 288 + 226}$$

input
```
integrate((23038-15444*2**(1/2)+(10530-5562*2**(1/2))*x**2+(-51201+34026*2
**(1/2))*x**4+(-63033+41310*2**(1/2))*x**6+(-29811+20142*2**(1/2))*x**8+(-
4779+3300*2**(1/2))*x**10)/(2**(1/2)-3*x**2-x**4)**3,x)
```

output
```
(x**7*(57624 - 64827*sqrt(2)) + x**5*(259308 - 350546*sqrt(2)) + x**3*(388
962 - 525819*sqrt(2)) + x*(379358 - 86436*sqrt(2)))/(x**8*(144 + 113*sqrt(
2)) + x**6*(864 + 678*sqrt(2)) + x**4*(844 + 729*sqrt(2)) + x**2*(-1356 -
864*sqrt(2)) + 288 + 226*sqrt(2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{23038 - 15444\sqrt{2} + \left(10530 - 5562\sqrt{2}\right)x^2 + \left(-51201 + 34026\sqrt{2}\right)x^4 + \left(-63033 + 41310\sqrt{2}\right)x^6 + (-}{\left(\sqrt{2} - 3x^2 - x^4\right)^3}$$

$$= \frac{2401\left(3x^7\left(4\sqrt{2} - 9\right) + 2x^5\left(27\sqrt{2} - 73\right) + 3x^3\left(27\sqrt{2} - 73\right) + x\left(79\sqrt{2} - 36\right)\right)}{x^8\left(72\sqrt{2} + 113\right) + 6x^6\left(72\sqrt{2} + 113\right) + x^4\left(422\sqrt{2} + 729\right) - 6x^2\left(113\sqrt{2} + 144\right) + 144\sqrt{2} + 226}$$

input
```
integrate((23038-15444*2^(1/2)+(10530-5562*2^(1/2))*x^2+(-51201+34026*2^(1
/2))*x^4+(-63033+41310*2^(1/2))*x^6+(-29811+20142*2^(1/2))*x^8+(-4779+3300
*2^(1/2))*x^10)/(2^(1/2)-3*x^2-x^4)^3,x, algorithm="maxima")
```

output
```
2401*(3*x^7*(4*sqrt(2) - 9) + 2*x^5*(27*sqrt(2) - 73) + 3*x^3*(27*sqrt(2)
- 73) + x*(79*sqrt(2) - 36))/(x^8*(72*sqrt(2) + 113) + 6*x^6*(72*sqrt(2) +
 113) + x^4*(422*sqrt(2) + 729) - 6*x^2*(113*sqrt(2) + 144) + 144*sqrt(2)
+ 226)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{23038 - 15444\sqrt{2} + \left(10530 - 5562\sqrt{2}\right)x^2 + \left(-51201 + 34026\sqrt{2}\right)x^4 + \left(-63033 + 41310\sqrt{2}\right)x^6 + (-}{\left(\sqrt{2} - 3x^2 - x^4\right)^3}$$

$$= -\frac{16807\left(21x^7\left(6779728593562346495625258905846910628021672046753605663369490841\sqrt{2} + 95879\right.\right.}{}$$

input
```
integrate((23038-15444*2^(1/2)+(10530-5562*2^(1/2))*x^2+(-51201+34026*2^(1
/2))*x^4+(-63033+41310*2^(1/2))*x^6+(-29811+20142*2^(1/2))*x^8+(-4779+3300
*2^(1/2))*x^10)/(2^(1/2)-3*x^2-x^4)^3,x, algorithm="giac")
```

output
```
-16807*(21*x^7*(677972859356234649562525890584691062802167204675360566336949
0841*sqrt(2) + 9587984126224539477133762937848928007964996456787765843151
815992) + 14*x^5*(7060554146828565793776109309047112366016004487757021681
3477233561*sqrt(2) + 998513143231455482854543842523341733277283122045971039
15105325610) + 21*x^3*(7060554146828565793776109309047112366016004487757021
6813477233561*sqrt(2) + 99851314323145548285454384252334173327728312204597
103915105325610) - 2*x*(537467386170465471727673094911513618139059141528
483841241777346178*sqrt(2) + 7600936668554899346425887560062761770449853578
3637290728226870945))/((x^4 + 3*x^2 - sqrt(2))^2*(213468901883918113923781
766686479494392000196729450362492145965009 13*sqrt(2) + 30189061618912853
6774496539528565456983359178585208434475537 22725456))
```

Mupad [B] (verification not implemented)

Time = 10.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^2+\left(-51201+34026\sqrt{2}\right)x^4+\left(-63033+41310\sqrt{2}\right)x^6+\left(-\right.}{\left(\sqrt{2}-3x^2-x^4\right)^3}$$

$$=\frac{\left(3300\sqrt{2}-4779\right)x^7+\left(16614\sqrt{2}-24274\right)x^5+\left(24921\sqrt{2}-36411\right)x^3+\left(11519\sqrt{2}-15444\right)x}{x^8+6x^6+\left(9-2\sqrt{2}\right)x^4-6\sqrt{2}x^2+2}$$

input
```
int(-(x^10*(3300*2^(1/2) - 4779) - x^2*(5562*2^(1/2) - 10530) + x^8*(20142
*2^(1/2) - 29811) + x^4*(34026*2^(1/2) - 51201) + x^6*(41310*2^(1/2) - 630
33) - 15444*2^(1/2) + 23038)/(3*x^2 - 2^(1/2) + x^4)^3,x)
```

output
```
(x^7*(3300*2^(1/2) - 4779) + x^5*(16614*2^(1/2) - 24274) + x^3*(24921*2^(1
/2) - 36411) + x*(11519*2^(1/2) - 15444))/(6*x^6 - 6*2^(1/2)*x^2 - x^4*(2*
2^(1/2) - 9) + x^8 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{23038-15444\sqrt{2}+\left(10530-5562\sqrt{2}\right)x^2+\left(-51201+34026\sqrt{2}\right)x^4+\left(-63033+41310\sqrt{2}\right)x^6+\left(-\right.}{\left(\sqrt{2}-3x^2-x^4\right)^3}$$

$$=\frac{x\left(3300\sqrt{2}\,x^{14}+36414\sqrt{2}\,x^{12}+144747\sqrt{2}\,x^{10}+233349\sqrt{2}\,x^8+81537\sqrt{2}\,x^6-112455\sqrt{2}\,x^4-42822\sqrt{\right.}}{x^{16}+12x^{14}+54x^{12}+10}$$

input
```
int((23038-15444*2^(1/2)+(10530-5562*2^(1/2))*x^2+(-51201+34026*2^(1/2))*x
^4+(-63033+41310*2^(1/2))*x^6+(-29811+20142*2^(1/2))*x^8+(-4779+3300*2^(1/
2))*x^10)/(2^(1/2)-3*x^2-x^4)^3,x)
```

output
```
(x*(3300*sqrt(2)*x**14 + 36414*sqrt(2)*x**12 + 144747*sqrt(2)*x**10 + 2333
49*sqrt(2)*x**8 + 81537*sqrt(2)*x**6 - 112455*sqrt(2)*x**4 - 42822*sqrt(2)
*x**2 + 23038*sqrt(2) - 4779*x**14 - 52948*x**12 - 211866*x**10 - 346320*x
**8 - 130869*x**6 + 157584*x**4 + 65406*x**2 - 30888))/(x**16 + 12*x**14 +
 54*x**12 + 108*x**10 + 77*x**8 - 24*x**6 - 36*x**4 + 4)
```

3.43 $\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$

Optimal result

Integrand size = 89, antiderivative size = 36

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx = -\frac{x^2}{2\left(-8+9\sqrt{2}\right)\left(\sqrt{2}-3x^4-x^8\right)}$$

output
```
-1/2*x^2/(-8+9*2^(1/2))/(2^(1/2)-3*x^4-x^8)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx = -\frac{x^2}{2\left(-8+9\sqrt{2}\right)\left(\sqrt{2}-3x^4-x^8\right)}$$

input
```
Integrate[(2*x - 9*x^9 + 2*Sqrt[2]*x^9 - 12*x^13 - 3*x^17)/((Sqrt[2] - 3*x
^4 - x^8)^2*(-18 + 8*Sqrt[2] - 24*x^4 + 27*Sqrt[2]*x^4 - 8*x^8 + 9*Sqrt[2]
*x^8)),x]
```

output
```
-1/2*x^2/((-8 + 9*Sqrt[2])*(Sqrt[2] - 3*x^4 - x^8))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-3x^{17}-12x^{13}+2\sqrt{2}x^9-9x^9+2x}{\left(-x^8-3x^4+\sqrt{2}\right)^2\left(9\sqrt{2}x^8-8x^8+27\sqrt{2}x^4-24x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{-3x^{17}-12x^{13}+2\sqrt{2}x^9-9x^9+2x}{\left(-x^8-3x^4+\sqrt{2}\right)^2\left(9\sqrt{2}x^8-8x^8+\left(27\sqrt{2}-24\right)x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{-3x^{17}-12x^{13}+2\sqrt{2}x^9-9x^9+2x}{\left(-x^8-3x^4+\sqrt{2}\right)^2\left(\left(9\sqrt{2}-8\right)x^8+\left(27\sqrt{2}-24\right)x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{-3x^{17}-12x^{13}+\left(2\sqrt{2}-9\right)x^9+2x}{\left(-x^8-3x^4+\sqrt{2}\right)^2\left(\left(9\sqrt{2}-8\right)x^8+\left(27\sqrt{2}-24\right)x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{3x^9+3x^5+\sqrt{2}x}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(9\sqrt{2}-8\right)x^8+\left(27\sqrt{2}-24\right)x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 2028$$

$$\int \frac{x\left(3x^8+3x^4+\sqrt{2}\right)}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(9\sqrt{2}-8\right)x^8+\left(27\sqrt{2}-24\right)x^4+8\sqrt{2}-18\right)}dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int -\frac{3x^8+3x^4+\sqrt{2}}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(8-9\sqrt{2}\right)x^8+3\left(8-9\sqrt{2}\right)x^4+2\left(9-4\sqrt{2}\right)\right)}dx^2$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{2}\int \frac{3x^8+3x^4+\sqrt{2}}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(8-9\sqrt{2}\right)x^8+3\left(8-9\sqrt{2}\right)x^4+2\left(9-4\sqrt{2}\right)\right)}dx^2$$

$$\Big\downarrow \; 7279$$

$$-\frac{1}{2}\int \left(\frac{3x^8}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(8-9\sqrt{2}\right)x^8+3\left(8-9\sqrt{2}\right)x^4+2\left(9-4\sqrt{2}\right)\right)}+\frac{3}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(8-9\sqrt{2}\right)x\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\sqrt{2}\int \frac{1}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(8-9\sqrt{2}\right)x^8+3\left(8-9\sqrt{2}\right)x^4+2\left(9-4\sqrt{2}\right)\right)}dx^2-3\int \frac{}{\left(-x^8-3x^4+\sqrt{2}\right)\left(\left(\right.\right.}\right.$$

input
```
Int[(2*x - 9*x^9 + 2*Sqrt[2]*x^9 - 12*x^13 - 3*x^17)/((Sqrt[2] - 3*x^4 - x
^8)^2*(-18 + 8*Sqrt[2] - 24*x^4 + 27*Sqrt[2]*x^4 - 8*x^8 + 9*Sqrt[2]*x^8))
,x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\left(\frac{2}{49}+\frac{9\sqrt{2}}{196}\right)x^2}{-\sqrt{2}+3x^4+x^8}$	28
norman	$\frac{\left(\frac{2}{49}+\frac{9\sqrt{2}}{196}\right)x^{10}+\left(\frac{6}{49}+\frac{27\sqrt{2}}{196}\right)x^6+\left(\frac{9}{98}+\frac{2\sqrt{2}}{49}\right)x^2}{x^{16}+6x^{12}+9x^8-2}$	53
parallelrisch	$\frac{9\sqrt{2}\,x^{10}+8x^{10}+27x^6\sqrt{2}+24x^6-8\sqrt{2}\,x^2-18x^2}{196\left(\sqrt{2}-3x^4-x^8\right)^2}$	59
default	$\frac{\left(-113-72\sqrt{2}\right)x^2}{2\left(-8+9\sqrt{2}\right)\left(1+2\sqrt{2}\right)^5\left(x^4+\sqrt{2}+2\right)}-\frac{\left(-113-72\sqrt{2}\right)x^2}{2\left(-8+9\sqrt{2}\right)\left(1+2\sqrt{2}\right)^5\left(x^4-\sqrt{2}+1\right)}$	84
gosper	$\frac{\left(x^4+\sqrt{2}+2\right)\left(-x^4+\sqrt{2}-1\right)x^2\left(-3x^{16}-12x^{12}+2\sqrt{2}\,x^8-9x^8+2\right)}{2\left(3x^8+3x^4+\sqrt{2}\right)\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}\,x^4-8x^8+9\sqrt{2}\,x^8\right)}$	116
orering	$\frac{x\left(x^4+\sqrt{2}+2\right)\left(-x^4+\sqrt{2}-1\right)\left(2x-9x^9+2\sqrt{2}\,x^9-12x^{13}-3x^{17}\right)}{2\left(3x^8+3x^4+\sqrt{2}\right)\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}\,x^4-8x^8+9\sqrt{2}\,x^8\right)}$	116

input
```
int((2*x-9*x^9+2*2^(1/2)*x^9-12*x^13-3*x^17)/(2^(1/2)-3*x^4-x^8)^2/(-18+8*
2^(1/2)-24*x^4+27*2^(1/2)*x^4-8*x^8+9*2^(1/2)*x^8),x,method=_RETURNVERBOSE
)
```

output
```
(2/49+9/196*2^(1/2))*x^2/(-2^(1/2)+3*x^4+x^8)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$$
$$=\frac{8\,x^{10}+24\,x^6+18\,x^2+\sqrt{2}(9\,x^{10}+27\,x^6+8\,x^2)}{196\,(x^{16}+6\,x^{12}+9\,x^8-2)}$$

input
```
integrate((2*x-9*x^9+2*2^(1/2)*x^9-12*x^13-3*x^17)/(2^(1/2)-3*x^4-x^8)^2/(
-18+8*2^(1/2)-24*x^4+27*2^(1/2)*x^4-8*x^8+9*2^(1/2)*x^8),x, algorithm="fri
cas")
```

output
```
1/196*(8*x^10 + 24*x^6 + 18*x^2 + sqrt(2)*(9*x^10 + 27*x^6 + 8*x^2))/(x^16
 + 6*x^12 + 9*x^8 - 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx=\text{Timed out}$$

input
```
integrate((2*x-9*x**9+2*2**(1/2)*x**9-12*x**13-3*x**17)/(2**(1/2)-3*x**4-x
**8)**2/(-18+8*2**(1/2)-24*x**4+27*2**(1/2)*x**4-8*x**8+9*2**(1/2)*x**8),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input
```
integrate((2*x-9*x^9+2*2^(1/2)*x^9-12*x^13-3*x^17)/(2^(1/2)-3*x^4-x^8)^2/(
-18+8*2^(1/2)-24*x^4+27*2^(1/2)*x^4-8*x^8+9*2^(1/2)*x^8),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$$
$$=\frac{x^2\left(2370846461804\sqrt{2}+3352883803641\right)}{98\left(x^8+3x^4-\sqrt{2}\right)\left(228758827313\sqrt{2}+323513589456\right)}$$

input
```
integrate((2*x-9*x^9+2*2^(1/2)*x^9-12*x^13-3*x^17)/(2^(1/2)-3*x^4-x^8)^2/(
-18+8*2^(1/2)-24*x^4+27*2^(1/2)*x^4-8*x^8+9*2^(1/2)*x^8),x, algorithm="gia
c")
```

output
```
1/98*x^2*(2370846461804*sqrt(2) + 3352883803641)/((x^8 + 3*x^4 - sqrt(2))*
(228758827313*sqrt(2) + 323513589456))
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$$
$$=\frac{x^2\left(\frac{9\sqrt{2}}{196}+\frac{2}{49}\right)}{x^8+3x^4-\sqrt{2}}$$

input
```
int(-(9*x^9 - 2*2^(1/2)*x^9 - 2*x + 12*x^13 + 3*x^17)/((3*x^4 - 2^(1/2) +
x^8)^2*(8*2^(1/2) + 27*2^(1/2)*x^4 + 9*2^(1/2)*x^8 - 24*x^4 - 8*x^8 - 18))
,x)
```

output
```
(x^2*((9*2^(1/2))/196 + 2/49))/(3*x^4 - 2^(1/2) + x^8)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{2x-9x^9+2\sqrt{2}x^9-12x^{13}-3x^{17}}{\left(\sqrt{2}-3x^4-x^8\right)^2\left(-18+8\sqrt{2}-24x^4+27\sqrt{2}x^4-8x^8+9\sqrt{2}x^8\right)}\,dx$$
$$=\frac{x^2\left(9\sqrt{2}\,x^8+27\sqrt{2}\,x^4+8\sqrt{2}+8x^8+24x^4+18\right)}{196x^{16}+1176x^{12}+1764x^8-392}$$

input
```
int((2*x-9*x^9+2*2^(1/2)*x^9-12*x^13-3*x^17)/(2^(1/2)-3*x^4-x^8)^2/(-18+8*
2^(1/2)-24*x^4+27*2^(1/2)*x^4-8*x^8+9*2^(1/2)*x^8),x)
```

output
```
(x**2*(9*sqrt(2)*x**8 + 27*sqrt(2)*x**4 + 8*sqrt(2) + 8*x**8 + 24*x**4 + 1
8))/(196*(x**16 + 6*x**12 + 9*x**8 - 2))
```

3.44 $\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49}{(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2)^2(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x}$

Optimal result

Integrand size = 119, antiderivative size = 58

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx = -\frac{\left(8-3\sqrt{7}\right)\left(3+\sqrt{7}+\sqrt{7}x\right)}{7\left(2\left(8-3\sqrt{7}\right)+2\left(7+3\sqrt{7}\right)x+7x^2\right)}$$

output
```
-1/7*(8-3*7^(1/2))*(3+7^(1/2)+7^(1/2)*x)/(16-6*7^(1/2)+2*(7+3*7^(1/2))*x+7
*x^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx$$
$$= -\frac{3832073521103764228193740275217176667475959453642040802037565598915+1448387648936598}{7\,(360490498929830235838}$$

input
```
Integrate[(-500 + 192*Sqrt[7] + 952*x + 360*Sqrt[7]*x + 672*x^2 + 252*Sqrt
[7]*x^2 + 196*x^3 + 84*Sqrt[7]*x^3 + 49*x^4)/((16 - 6*Sqrt[7] + 14*x + 6*S
qrt[7]*x + 7*x^2)^2*(2*Sqrt[7] + 630*x + 238*Sqrt[7]*x + 147*x^2 + 56*Sqrt
[7]*x^2)),x]
```

output
```
-1/7*(383207352110376422819374027521717666747595945364204080203756598915
+ 1448387648936598037907455646451256190184263094401177986262766639167*Sqrt
[7] + (1795812989971104599350193324478071660768904403465226048627570115051
 + 678753510377553209614515650246368335569018350058938251136665161288*Sqrt
[7])*x)/((36049049892983023583045990401 + 13625260145284667680742546880*Sq
rt[7])^2*(261152656 + 98706426*Sqrt[7] + (62169690254 + 23497934214*Sqrt[7
])*x + 7*(2081028097 + 786554688*Sqrt[7])*x^2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.084$, Rules used = {6, 6, 6, 6, 6, 6, 2019, 2126, 2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+252\sqrt{7}x^2+672x^2+360\sqrt{7}x+952x+192\sqrt{7}-500}{\left(7x^2+6\sqrt{7}x+14x-6\sqrt{7}+16\right)^2\left(56\sqrt{7}x^2+147x^2+238\sqrt{7}x+630x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+252\sqrt{7}x^2+672x^2+360\sqrt{7}x+952x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(56\sqrt{7}x^2+147x^2+238\sqrt{7}x+630x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+252\sqrt{7}x^2+672x^2+360\sqrt{7}x+952x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(56\sqrt{7}x^2+147x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+252\sqrt{7}x^2+672x^2+360\sqrt{7}x+952x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(\left(147+56\sqrt{7}\right)x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow 6$$

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+252\sqrt{7}x^2+672x^2+\left(952+360\sqrt{7}\right)x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(\left(147+56\sqrt{7}\right)x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow 6$$

$$\int \frac{49x^4+84\sqrt{7}x^3+196x^3+\left(672+252\sqrt{7}\right)x^2+\left(952+360\sqrt{7}\right)x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(\left(147+56\sqrt{7}\right)x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow 6$$

$$\int \frac{49x^4+\left(196+84\sqrt{7}\right)x^3+\left(672+252\sqrt{7}\right)x^2+\left(952+360\sqrt{7}\right)x+192\sqrt{7}-500}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)^2\left(\left(147+56\sqrt{7}\right)x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow 2019$$

$$\int \frac{7x^2+\left(14+6\sqrt{7}\right)x+18\sqrt{7}+16}{\left(7x^2+\left(14+6\sqrt{7}\right)x-6\sqrt{7}+16\right)\left(\left(147+56\sqrt{7}\right)x^2+\left(630+238\sqrt{7}\right)x+2\sqrt{7}\right)}dx$$

$$\Big\downarrow 2126$$

$$-\frac{1}{7}\left(21-8\sqrt{7}\right)\int \frac{7x^2+2\left(7+3\sqrt{7}\right)x+2\left(8+9\sqrt{7}\right)}{\left(7x^2+2\left(7+3\sqrt{7}\right)x+2\left(8-3\sqrt{7}\right)\right)^2}dx$$

$$\Big\downarrow 2191$$

$$-\frac{1}{7}\left(21-8\sqrt{7}\right)\left(-\frac{\int 0dx}{336\sqrt{7}}-\frac{\sqrt{7}x+\sqrt{7}+3}{\sqrt{7}\left(7x^2+2\left(7+3\sqrt{7}\right)x+2\left(8-3\sqrt{7}\right)\right)}\right)$$

$$\Big\downarrow 24$$

$$\frac{\left(21-8\sqrt{7}\right)\left(\sqrt{7}x+\sqrt{7}+3\right)}{7\sqrt{7}\left(7x^2+2\left(7+3\sqrt{7}\right)x+2\left(8-3\sqrt{7}\right)\right)}$$

input
```
Int[(-500 + 192*Sqrt[7] + 952*x + 360*Sqrt[7]*x + 672*x^2 + 252*Sqrt[7]*x^
2 + 196*x^3 + 84*Sqrt[7]*x^3 + 49*x^4)/((16 - 6*Sqrt[7] + 14*x + 6*Sqrt[7]
*x + 7*x^2)^2*(2*Sqrt[7] + 630*x + 238*Sqrt[7]*x + 147*x^2 + 56*Sqrt[7]*x^
2)),x]
```

output
```
((21 - 8*Sqrt[7])*(3 + Sqrt[7] + Sqrt[7]*x))/(7*Sqrt[7]*(2*(8 - 3*Sqrt[7])
 + 2*(7 + 3*Sqrt[7])*x + 7*x^2))
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2126
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(c/f)^p   Int[Px*(d + e*x + f*x^2)^(p +
 q), x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && PolyQ[Px, x] && EqQ[c*
d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !Inte
gerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\left(\frac{3}{7}-\frac{8\sqrt{7}}{49}\right)x+\frac{\sqrt{7}}{49}-\frac{3}{49}}{x^{2}+\frac{6\sqrt{7}\,x}{7}+2x+\frac{16}{7}-\frac{6\sqrt{7}}{7}}$	39
default	$\frac{-\frac{x}{7}-\frac{3\sqrt{7}}{49}-\frac{1}{7}}{\left(8\sqrt{7}+21\right)\left(x^{2}+\frac{6\sqrt{7}\,x}{7}+2x+\frac{16}{7}-\frac{6\sqrt{7}}{7}\right)}$	42
norman	$\frac{\left(192+\frac{506\sqrt{7}}{7}\right)x+\left(123-21\sqrt{7}\right)x^{2}+\left(63+6\sqrt{7}\right)x^{3}+\left(\frac{21}{2}+\frac{7\sqrt{7}}{2}\right)x^{4}}{49x^{4}+196x^{3}+168x^{2}+952x+4}$	67
parallelrisch	$\frac{-90-56\sqrt{7}\,x^{3}+21\sqrt{7}\,x^{2}+147x^{3}-258\sqrt{7}\,x-63x^{2}+34\sqrt{7}+672x}{7\left(16-6\sqrt{7}+14x+6\sqrt{7}\,x+7x^{2}\right)^{2}}$	68
gosper	$-\frac{\left(-500+192\sqrt{7}+952x+360\sqrt{7}\,x+672x^{2}+252\sqrt{7}\,x^{2}+196x^{3}+84\sqrt{7}\,x^{3}+49x^{4}\right)\left(7+3\sqrt{7}+7x\right)}{7\left(2\sqrt{7}+630x+238\sqrt{7}\,x+147x^{2}+56\sqrt{7}\,x^{2}\right)\left(16-6\sqrt{7}+14x+6\sqrt{7}\,x+7x^{2}\right)\left(6\sqrt{7}\,x+7x^{2}+18\sqrt{7}+14x+16\right)}$	136
orering	$-\frac{\left(-500+192\sqrt{7}+952x+360\sqrt{7}\,x+672x^{2}+252\sqrt{7}\,x^{2}+196x^{3}+84\sqrt{7}\,x^{3}+49x^{4}\right)\left(7+3\sqrt{7}+7x\right)}{7\left(2\sqrt{7}+630x+238\sqrt{7}\,x+147x^{2}+56\sqrt{7}\,x^{2}\right)\left(16-6\sqrt{7}+14x+6\sqrt{7}\,x+7x^{2}\right)\left(6\sqrt{7}\,x+7x^{2}+18\sqrt{7}+14x+16\right)}$	136

input
```
int((-500+192*7^(1/2)+952*x+360*7^(1/2)*x+672*x^2+252*7^(1/2)*x^2+196*x^3+
84*7^(1/2)*x^3+49*x^4)/(16-6*7^(1/2)+14*x+6*7^(1/2)*x+7*x^2)^2/(2*7^(1/2)+
630*x+238*7^(1/2)*x+147*x^2+56*7^(1/2)*x^2),x,method=_RETURNVERBOSE)
```

output
```
((3/7-8/49*7^(1/2))*x+1/49*7^(1/2)-3/49)/(x^2+6/7*7^(1/2)*x+2*x+16/7-6/7*7
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx$$
$$=\frac{147\,x^{3}+609\,x^{2}-\sqrt{7}(56\,x^{3}+231\,x^{2}-30\,x+2)-84\,x-6}{7\,(49\,x^{4}+196\,x^{3}+168\,x^{2}+952\,x+4)}$$

input
```
integrate((-500+192*7^(1/2)+952*x+360*7^(1/2)*x+672*x^2+252*7^(1/2)*x^2+19
6*x^3+84*7^(1/2)*x^3+49*x^4)/(16-6*7^(1/2)+14*x+6*7^(1/2)*x+7*x^2)^2/(2*7^
(1/2)+630*x+238*7^(1/2)*x+147*x^2+56*7^(1/2)*x^2),x, algorithm="fricas")
```

output
```
1/7*(147*x^3 + 609*x^2 - sqrt(7)*(56*x^3 + 231*x^2 - 30*x + 2) - 84*x - 6)
/(49*x^4 + 196*x^3 + 168*x^2 + 952*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx$$
$$= \frac{x\left(-282008183944355983\right.}{x^2\cdot\left(220227365899240983767039280866098904703691612654261779032+832381202943168774735173\right.}$$

input
```
integrate((-500+192*7**(1/2)+952*x+360*7**(1/2)*x+672*x**2+252*7**(1/2)*x*
*2+196*x**3+84*7**(1/2)*x**3+49*x**4)/(16-6*7**(1/2)+14*x+6*7**(1/2)*x+7*x
**2)**2/(2*7**(1/2)+630*x+238*7**(1/2)*x+147*x**2+56*7**(1/2)*x**2),x)
```

output
```
(x*(-2820081839443559835193968096972409623372929099954 18119*sqrt(7) - 7461
2352240172406957439273105292821406444076146978147 2) - 15921480742347920201
3258316014465110107631949145603582 9 - 601775407830809156194136551577067339
7934818077681816 07*sqrt(7))/(x**2*(220227365899240983767039280866098904703
6916126542617790 32 + 832381202943168774735173239644584014511907404116342 25
925*sqrt(7)) + x*(939883453564383232375182505518948218114527667778328913 61
4 + 35524255421655459817592546009985872121983143452692140530 6*sqrt(7)) + 3
948114575220983769271555335761373472722100739935853666 + 14922470448034481
39148785462105856428128881522939562944*sqrt(7))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate((-500+192*7^(1/2)+952*x+360*7^(1/2)*x+672*x^2+252*7^(1/2)*x^2+19
6*x^3+84*7^(1/2)*x^3+49*x^4)/(16-6*7^(1/2)+14*x+6*7^(1/2)*x+7*x^2)^2/(2*7^
(1/2)+630*x+238*7^(1/2)*x+147*x^2+56*7^(1/2)*x^2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((-500+192*7^(1/2)+952*x+360*7^(1/2)*x+672*x^2+252*7^(1/2)*x^2+19
6*x^3+84*7^(1/2)*x^3+49*x^4)/(16-6*7^(1/2)+14*x+6*7^(1/2)*x+7*x^2)^2/(2*7^
(1/2)+630*x+238*7^(1/2)*x+147*x^2+56*7^(1/2)*x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[-168,0]:[1,0,-7]%%},[2]%%%}+%%%{%%{[-336,-1008]:[1,0,-
7]%%},[1]
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx = -\frac{\frac{x\left(8\sqrt{7}-21\right)}{7}-\frac{\sqrt{7}}{7}+\frac{3}{7}}{7\,x^2+\left(6\sqrt{7}+14\right)\,x-6\sqrt{7}+16}$$

input
```
int((952*x + 360*7^(1/2)*x + 192*7^(1/2) + 252*7^(1/2)*x^2 + 84*7^(1/2)*x^
3 + 672*x^2 + 196*x^3 + 49*x^4 - 500)/((14*x + 6*7^(1/2)*x - 6*7^(1/2) + 7
*x^2 + 16)^2*(630*x + 238*7^(1/2)*x + 2*7^(1/2) + 56*7^(1/2)*x^2 + 147*x^2
)),x)
```

output
```
-((x*(8*7^(1/2) - 21))/7 - 7^(1/2)/7 + 3/7)/(7*x^2 - 6*7^(1/2) + x*(6*7^(1
/2) + 14) + 16)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{-500+192\sqrt{7}+952x+360\sqrt{7}x+672x^2+252\sqrt{7}x^2+196x^3+84\sqrt{7}x^3+49x^4}{\left(16-6\sqrt{7}+14x+6\sqrt{7}x+7x^2\right)^2\left(2\sqrt{7}+630x+238\sqrt{7}x+147x^2+56\sqrt{7}x^2\right)}\,dx = \frac{392\sqrt{7}\,x^4-5124\sqrt{7}\,x^2+8456\sqrt{7}\,x-24\sqrt{7}-1029x^4+13524x^2-22344x-252}{9604x^4+38416x^3+32928x^2+186592x+784}$$

input
```
int((-500+192*7^(1/2)+952*x+360*7^(1/2)*x+672*x^2+252*7^(1/2)*x^2+196*x^3+
84*7^(1/2)*x^3+49*x^4)/(16-6*7^(1/2)+14*x+6*7^(1/2)*x+7*x^2)^2/(2*7^(1/2)+
630*x+238*7^(1/2)*x+147*x^2+56*7^(1/2)*x^2),x)
```

output
```
(392*sqrt(7)*x**4 - 5124*sqrt(7)*x**2 + 8456*sqrt(7)*x - 24*sqrt(7) - 1029
*x**4 + 13524*x**2 - 22344*x - 252)/(196*(49*x**4 + 196*x**3 + 168*x**2 +
952*x + 4))
```

3.45 $\int \frac{199290375x^3+21907179x^{11}-10200897x^{19}+464373x^{27}-8127x^{35}+49x^{43}}{(243-63x^8+x^{16})^3}$

Optimal result

Integrand size = 44, antiderivative size = 37

$$\int \frac{199290375x^3+21907179x^{11}-10200897x^{19}+464373x^{27}-8127x^{35}+49x^{43}}{(243-63x^8+x^{16})^3}\,dx = \frac{820125x^4-182574x^{12}+5796x^{20}-49x^{28}}{4\left(243-63x^8+x^{16}\right)^2}$$

output
```
1/4*(-49*x^28+5796*x^20-182574*x^12+820125*x^4)/(x^16-63*x^8+243)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{199290375x^3+21907179x^{11}-10200897x^{19}+464373x^{27}-8127x^{35}+49x^{43}}{(243-63x^8+x^{16})^3}\,dx = \frac{x^4(820125-182574x^8+5796x^{16}-49x^{24})}{4\left(243-63x^8+x^{16}\right)^2}$$

input
```
Integrate[(199290375*x^3 + 21907179*x^11 - 10200897*x^19 + 464373*x^27 - 8
127*x^35 + 49*x^43)/(243 - 63*x^8 + x^16)^3,x]
```

output
```
(x^4*(820125 - 182574*x^8 + 5796*x^16 - 49*x^24))/(4*(243 - 63*x^8 + x^16)
^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2311, 9, 2292, 2206, 27, 2204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{49x^{43}-8127x^{35}+464373x^{27}-10200897x^{19}+21907179x^{11}+199290375x^3}{(x^{16}-63x^8+243)^3}dx$$

$$\Big\downarrow \text{2311}$$

$$\int \frac{x(49x^{42}-8127x^{34}+464373x^{26}-10200897x^{18}+21907179x^{10}+199290375x^2)}{(x^{16}-63x^8+243)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3(49x^{40}-8127x^{32}+464373x^{24}-10200897x^{16}+21907179x^8+199290375)}{(x^{16}-63x^8+243)^3}dx$$

$$\Big\downarrow \text{2292}$$

$$\frac{1}{4}\int \frac{49x^{40}-8127x^{32}+464373x^{24}-10200897x^{16}+21907179x^8+199290375}{(x^{16}-63x^8+243)^3}dx^4$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{161838x^4}{(x^{16}-63x^8+243)^2}-\frac{\int -\frac{20391588(7x^{24}-720x^{16}+19278x^8+94041)}{(x^{16}-63x^8+243)^2}dx^4}{2913084}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(7\int \frac{7x^{24}-720x^{16}+19278x^8+94041}{(x^{16}-63x^8+243)^2}dx^4+\frac{161838x^4}{(x^{16}-63x^8+243)^2}\right)$$

$$\Big\downarrow \text{2204}$$

$$\frac{1}{4}\left(\frac{7(387-7x^8)\,x^4}{x^{16}-63x^8+243}+\frac{161838x^4}{(x^{16}-63x^8+243)^2}\right)$$

input
```
Int[(199290375*x^3 + 21907179*x^11 - 10200897*x^19 + 464373*x^27 - 8127*x^
35 + 49*x^43)/(243 - 63*x^8 + x^16)^3,x]
```

output
```
((161838*x^4)/(243 - 63*x^8 + x^16)^2 + (7*x^4*(387 - 7*x^8))/(243 - 63*x^
8 + x^16))/4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2292
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Sy
mbol] :> With[{g = GCD[m + 1, n]}, Simp[1/g   Subst[Int[x^((m + 1)/g - 1)*(
Px /. x -> x^(1/g))*(a + b*x^(n/g) + c*x^(2*(n/g)))^p, x], x, x^g], x] /; N
eQ[g, 1]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Px, x^n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2311
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :>
Int[x*PolynomialQuotient[Pq, x, x]*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{
a, b, c, n, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0],
0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	siz
gosper	$-\frac{\left(49 x^{24}-5796 x^{16}+182574 x^{8}-820125\right) x^{4}}{4 \left(x^{16}-63 x^{8}+243\right)^{2}}$	35
risch	$\frac{\frac{820125}{4} x^{4}-\frac{91287}{2} x^{12}+1449 x^{20}-\frac{49}{4} x^{28}}{\left(x^{16}-63 x^{8}+243\right)^{2}}$	35
default	$\frac{-49 x^{28}+5796 x^{20}-182574 x^{12}+820125 x^{4}}{4 \left(x^{16}-63 x^{8}+243\right)^{2}}$	36
parallelrisch	$\frac{-49 x^{28}+5796 x^{20}-182574 x^{12}+820125 x^{4}}{4 \left(x^{16}-63 x^{8}+243\right)^{2}}$	36
orering	$-\frac{x \left(49 x^{24}-5796 x^{16}+182574 x^{8}-820125\right) \left(49 x^{43}-8127 x^{35}+464373 x^{27}-10200897 x^{19}+21907179 x^{11}+199290375 x^{3}\right)}{4 \left(x^{16}-63 x^{8}+243\right)^{2} \left(49 x^{40}-8127 x^{32}+464373 x^{24}-10200897 x^{16}+21907179 x^{8}+199290375\right)}$	93

input
```
int((49*x^43-8127*x^35+464373*x^27-10200897*x^19+21907179*x^11+199290375*x
^3)/(x^16-63*x^8+243)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(49*x^24-5796*x^16+182574*x^8-820125)*x^4/(x^16-63*x^8+243)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3} dx = -\frac{49\,x^{28} - 5796\,x^{20} + 182574\,x^{12} - 820125\,x^4}{4\,(x^{32} - 126\,x^{24} + 4455\,x^{16} - 30618\,x^8 + 59049)}$$

input
```
integrate((49*x^43-8127*x^35+464373*x^27-10200897*x^19+21907179*x^11+19929
0375*x^3)/(x^16-63*x^8+243)^3,x, algorithm="fricas")
```

output
```
-1/4*(49*x^28 - 5796*x^20 + 182574*x^12 - 820125*x^4)/(x^32 - 126*x^24 + 4
455*x^16 - 30618*x^8 + 59049)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3} dx = \frac{-49x^{28} + 5796x^{20} - 182574x^{12} + 820125x^4}{4x^{32} - 504x^{24} + 17820x^{16} - 122472x^8 + 236196}$$

input
```
integrate((49*x**43-8127*x**35+464373*x**27-10200897*x**19+21907179*x**11+
199290375*x**3)/(x**16-63*x**8+243)**3,x)
```

output
```
(-49*x**28 + 5796*x**20 - 182574*x**12 + 820125*x**4)/(4*x**32 - 504*x**24
 + 17820*x**16 - 122472*x**8 + 236196)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3}\,dx$$
$$= -\frac{49\,x^{28} - 5796\,x^{20} + 182574\,x^{12} - 820125\,x^4}{4\,(x^{32} - 126\,x^{24} + 4455\,x^{16} - 30618\,x^8 + 59049)}$$

input
```
integrate((49*x^43-8127*x^35+464373*x^27-10200897*x^19+21907179*x^11+19929
0375*x^3)/(x^16-63*x^8+243)^3,x, algorithm="maxima")
```

output
```
-1/4*(49*x^28 - 5796*x^20 + 182574*x^12 - 820125*x^4)/(x^32 - 126*x^24 + 4
455*x^16 - 30618*x^8 + 59049)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3}\,dx$$
$$= -\frac{49\,x^{28} - 5796\,x^{20} + 182574\,x^{12} - 820125\,x^4}{4\,(x^{16} - 63\,x^8 + 243)^2}$$

input
```
integrate((49*x^43-8127*x^35+464373*x^27-10200897*x^19+21907179*x^11+19929
0375*x^3)/(x^16-63*x^8+243)^3,x, algorithm="giac")
```

output
```
-1/4*(49*x^28 - 5796*x^20 + 182574*x^12 - 820125*x^4)/(x^16 - 63*x^8 + 243
)^2
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3}\,dx = -\frac{x^4\,(49\,x^{24} - 5796\,x^{16} + 182574\,x^8 - 820125)}{4\,(x^{16} - 63\,x^8 + 243)^2}$$

input
```
int((199290375*x^3 + 21907179*x^11 - 10200897*x^19 + 464373*x^27 - 8127*x^
35 + 49*x^43)/(x^16 - 63*x^8 + 243)^3,x)
```

output
```
-(x^4*(182574*x^8 - 5796*x^16 + 49*x^24 - 820125))/(4*(x^16 - 63*x^8 + 243
)^2)
```

Reduce [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{199290375x^3 + 21907179x^{11} - 10200897x^{19} + 464373x^{27} - 8127x^{35} + 49x^{43}}{(243 - 63x^8 + x^{16})^3}\,dx = \frac{x^4(-49x^{24} + 5796x^{16} - 182574x^8 + 820125)}{4x^{32} - 504x^{24} + 17820x^{16} - 122472x^8 + 236196}$$

input
```
int((49*x^43-8127*x^35+464373*x^27-10200897*x^19+21907179*x^11+199290375*x
^3)/(x^16-63*x^8+243)^3,x)
```

output
```
(x**4*( - 49*x**24 + 5796*x**16 - 182574*x**8 + 820125))/(4*(x**32 - 126*x
**24 + 4455*x**16 - 30618*x**8 + 59049))
```

3.46 $\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 136

$$\begin{aligned}\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx &= \frac{-566-720x^4-190x^8-9x^{12}}{636\left(460+936x^4+376x^8+36x^{12}+x^{16}\right)} \\ &\quad -\frac{2791\text{arctanh}\left(\frac{1}{18}\left(8\sqrt{6}+\sqrt{6}x^4\right)\right)}{202248\sqrt{6}}+\frac{2827\text{arctanh}\left(\frac{1}{18}\left(10\sqrt{6}+\sqrt{6}x^4\right)\right)}{202248\sqrt{6}} \\ &\quad -\frac{153\log\left(10+16x^4+x^8\right)}{44944}+\frac{153\log\left(46+20x^4+x^8\right)}{44944}\end{aligned}$$

output
```
(-9*x^12-190*x^8-720*x^4-566)/(636*x^16+22896*x^12+239136*x^8+595296*x^4+2
92560)-2791/1213488*arctanh(4/9*6^(1/2)+1/18*6^(1/2)*x^4)*6^(1/2)+2827/121
3488*arctanh(5/9*6^(1/2)+1/18*6^(1/2)*x^4)*6^(1/2)-153/44944*ln(x^8+16*x^4
+10)+153/44944*ln(x^8+20*x^4+46)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05

$$\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx$$
$$= \frac{-\frac{3816(566+720x^4+190x^8+9x^{12})}{460+936x^4+376x^8+36x^{12}+x^{16}} + (8262-2827\sqrt{6})\log(-10+3\sqrt{6}-x^4) + (-8262+2791\sqrt{6})\log(-8+3}{2426976}$$

input
```
Integrate[(624*x^3 + 144*x^7 + 24*x^11)/(-460 - 936*x^4 - 376*x^8 - 36*x^1
2 - x^16)^2,x]
```

output
```
((-3816*(566 + 720*x^4 + 190*x^8 + 9*x^12))/(460 + 936*x^4 + 376*x^8 + 36*
x^12 + x^16) + (8262 - 2827*Sqrt[6])*Log[-10 + 3*Sqrt[6] - x^4] + (-8262 +
 2791*Sqrt[6])*Log[-8 + 3*Sqrt[6] - x^4] - (8262 + 2791*Sqrt[6])*Log[8 + 3
*Sqrt[6] + x^4] + (8262 + 2827*Sqrt[6])*Log[10 + 3*Sqrt[6] + x^4])/2426976
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2028, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{24x^{11}+144x^7+624x^3}{(-x^{16}-36x^{12}-376x^8-936x^4-460)^2}\,dx$$
$$\Big\downarrow \text{2028}$$
$$\int \frac{x^3(24x^8+144x^4+624)}{(-x^{16}-36x^{12}-376x^8-936x^4-460)^2}dx$$
$$\Big\downarrow \text{2460}$$
$$\int \left(-\frac{9(17x^4+13)x^3}{5618(x^8+16x^4+10)}+\frac{9(17x^4+81)x^3}{5618(x^8+20x^4+46)}-\frac{3(7x^4-4)x^3}{53(x^8+16x^4+10)^2}-\frac{3(5x^4+2)x^3}{53(x^8+20x^4+46)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(102-41\sqrt{6})\log\left(x^4-3\sqrt{6}+8\right)}{89888}-\frac{5\log\left(x^4-3\sqrt{6}+8\right)}{3816\sqrt{6}}+\frac{(306-89\sqrt{6})\log\left(x^4-3\sqrt{6}+10\right)}{89888}-\frac{\log\left(x^4-3\sqrt{6}+10\right)}{954\sqrt{6}}-\frac{3(102+41\sqrt{6})\log\left(x^4+3\sqrt{6}+8\right)}{89888}+\frac{5\log\left(x^4+3\sqrt{6}+8\right)}{3816\sqrt{6}}+\frac{(306+89\sqrt{6})\log\left(x^4+3\sqrt{6}+10\right)}{89888}+\frac{\log\left(x^4+3\sqrt{6}+10\right)}{954\sqrt{6}}-\frac{8x^4+35}{1272\left(x^8+20x^4+46\right)}-\frac{10x^4+17}{1272\left(x^8+16x^4+10\right)}$$

input
```
Int[(624*x^3 + 144*x^7 + 24*x^11)/(-460 - 936*x^4 - 376*x^8 - 36*x^12 - x^
16)^2,x]
```

output
```
-1/1272*(17 + 10*x^4)/(10 + 16*x^4 + x^8) - (35 + 8*x^4)/(1272*(46 + 20*x^
4 + x^8)) - (5*Log[8 - 3*Sqrt[6] + x^4])/(3816*Sqrt[6]) - (3*(102 - 41*Sqr
t[6])*Log[8 - 3*Sqrt[6] + x^4])/89888 - Log[10 - 3*Sqrt[6] + x^4]/(954*Sqr
t[6]) + ((306 - 89*Sqrt[6])*Log[10 - 3*Sqrt[6] + x^4])/89888 + (5*Log[8 +
3*Sqrt[6] + x^4])/(3816*Sqrt[6]) - (3*(102 + 41*Sqrt[6])*Log[8 + 3*Sqrt[6]
 + x^4])/89888 + Log[10 + 3*Sqrt[6] + x^4]/(954*Sqrt[6]) + ((306 + 89*Sqrt
[6])*Log[10 + 3*Sqrt[6] + x^4])/89888
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result
default	$\frac{-\frac{x^4}{159}-\frac{35}{1272}}{x^8+20x^4+46}+\frac{153\ln(x^8+20x^4+46)}{44944}+\frac{2827\sqrt{6}\,\operatorname{arctanh}\left(\frac{\left(2x^4+20\right)\sqrt{6}}{36}\right)}{1213488}-\frac{3\left(\frac{530x^4}{9}+\frac{901}{9}\right)}{22472(x^8+16x^4+10)}-\frac{153\ln(x^8+16x^4+10)}{44944}$
risch	$\frac{-\frac{283}{318}-\frac{60}{53}x^4-\frac{95}{318}x^8-\frac{3}{212}x^{12}}{x^{16}+36x^{12}+376x^8+936x^4+460}-\frac{153\ln\left(x^4+8-3\sqrt{6}\right)}{44944}+\frac{2791\ln\left(x^4+8-3\sqrt{6}\right)\sqrt{6}}{2426976}-\frac{153\ln\left(x^4+8+3\sqrt{6}\right)}{44944}-\frac{2791\ln\left(x^4+8\right.}{2426\ldots}$

input
```
int((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
3/22472*(-424/9*x^4-1855/9)/(x^8+20*x^4+46)+153/44944*ln(x^8+20*x^4+46)+28
27/1213488*6^(1/2)*arctanh(1/36*(2*x^4+20)*6^(1/2))-3/22472*(530/9*x^4+901
/9)/(x^8+16*x^4+10)-153/44944*ln(x^8+16*x^4+10)-2791/1213488*6^(1/2)*arcta
nh(1/36*(2*x^4+16)*6^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(108) = 216.

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

$$\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx =$$
$$-\frac{34344\,x^{12}+725040\,x^8+2747520\,x^4-2827\,\sqrt{6}(x^{16}+36\,x^{12}+376\,x^8+936\,x^4+460)\log\left(\frac{x^8+20\,x^4+6\ldots}{x^8+2\ldots}\right.}{\ldots}$$

input
```
integrate((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,
x, algorithm="fricas")
```

output
```
-1/2426976*(34344*x^12 + 725040*x^8 + 2747520*x^4 - 2827*sqrt(6)*(x^16 + 3
6*x^12 + 376*x^8 + 936*x^4 + 460)*log((x^8 + 20*x^4 + 6*sqrt(6)*(x^4 + 10)
 + 154)/(x^8 + 20*x^4 + 46)) - 2791*sqrt(6)*(x^16 + 36*x^12 + 376*x^8 + 93
6*x^4 + 460)*log((x^8 + 16*x^4 - 6*sqrt(6)*(x^4 + 8) + 118)/(x^8 + 16*x^4
+ 10)) - 8262*(x^16 + 36*x^12 + 376*x^8 + 936*x^4 + 460)*log(x^8 + 20*x^4
+ 46) + 8262*(x^16 + 36*x^12 + 376*x^8 + 936*x^4 + 460)*log(x^8 + 16*x^4 +
 10) + 2159856)/(x^16 + 36*x^12 + 376*x^8 + 936*x^4 + 460)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(128) = 256$.

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.18

$$\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx$$

$$= \frac{-9x^{12}-190x^8-720x^4-566}{636x^{16}+22896x^{12}+239136x^8+595296x^4+292560} + \left(-\frac{153}{44944} + \frac{2791\sqrt{6}}{2426976}\right)\log\left(x^4 - \frac{1569947793173238153 43}{347707278551091640 50} - \frac{289353574307315080028 16\left(-\frac{153}{44944}+\frac{2791\sqrt{6}}{2426976}\right)^3}{687684977950025} + \cdots\right.$$

$$+ \left(\frac{153}{44944} - \frac{2827\sqrt{6}}{2426976}\right)\log\left(x^4 - \frac{17043567126518911\sqrt{6}}{8199676419080100} - \frac{289353574307315080028 16\left(\frac{153}{44944}-\frac{2827\sqrt{6}}{2426976}\right)^3}{687684977950025} + \frac{262\cdots}{\cdots}\right.$$

$$+ \left(-\frac{153}{44944} - \frac{2791\sqrt{6}}{2426976}\right)\log\left(x^4 - \frac{1549415193319901\sqrt{6}}{755040071552700} - \frac{1569947793173238153 43}{347707278551091640 50} - \frac{28935357430731508002 8\cdots}{687684977\cdots}\right.$$

$$+ \left(\frac{2827\sqrt{6}}{2426976} + \frac{153}{44944}\right)\log\left(x^4 - \frac{289353574307315080028 16\left(\frac{2827\sqrt{6}}{2426976}+\frac{153}{44944}\right)^3}{687684977950025} + \frac{17043567126518911\sqrt{6}}{8199676419080100} + \frac{265447\cdots}{34770\cdots}\right.$$

input
```
integrate((24*x**11+144*x**7+624*x**3)/(-x**16-36*x**12-376*x**8-936*x**4-
460)**2,x)
```

output
```
(-9*x**12 - 190*x**8 - 720*x**4 - 566)/(636*x**16 + 22896*x**12 + 239136*x
**8 + 595296*x**4 + 292560) + (-153/44944 + 2791*sqrt(6)/2426976)*log(x**4
 - 156994779317323815343/34770727855109164050 - 289353574307315080028l6*(-
153/44944 + 2791*sqrt(6)/2426976)**3/687684977950025 + 262251655755486967 2
96*(-153/44944 + 2791*sqrt(6)/2426976)**2/687684977950025 + 15494151933199
01*sqrt(6)/755040071552700) + (153/44944 - 2827*sqrt(6)/2426976)*log(x**4
- 17043567126518911*sqrt(6)/8199676419080100 - 28935357430731508002816*(15
3/44944 - 2827*sqrt(6)/2426976)**3/687684977950025 + 26225165575548696729 6
*(153/44944 - 2827*sqrt(6)/2426976)**2/687684977950025 + 26544707548057391
2703/34770727855109164050) + (-153/44944 - 2791*sqrt(6)/2426976)*log(x**4
- 1549415193319901*sqrt(6)/755040071552700 - 156994779317323815343/3477072
7855109164050 - 28935357430731508002816*(-153/44944 - 2791*sqrt(6)/2426976
)**3/687684977950025 + 262251655755486967296*(-153/44944 - 2791*sqrt(6)/24
26976)**2/687684977950025) + (2827*sqrt(6)/2426976 + 153/44944)*log(x**4 -
 28935357430731508002816*(2827*sqrt(6)/2426976 + 153/44944)**3/68768497795
0025 + 17043567126518911*sqrt(6)/8199676419080100 + 265447075480573912703/
34770727855109164050 + 262251655755486967296*(2827*sqrt(6)/2426976 + 153/4
4944)**2/687684977950025)
```

Maxima [F]

$$\int \frac{624x^3 + 144x^7 + 24x^{11}}{(-460 - 936x^4 - 376x^8 - 36x^{12} - x^{16})^2}\,dx = \int \frac{24\,(x^{11} + 6\,x^7 + 26\,x^3)}{(x^{16} + 36\,x^{12} + 376\,x^8 + 936\,x^4 + 460)^2}\,dx$$

input
```
integrate((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,
x, algorithm="maxima")
```

output
```
-1/636*(9*x^12 + 190*x^8 + 720*x^4 + 566)/(x^16 + 36*x^12 + 376*x^8 + 936*
x^4 + 460) + 1/16854*integrate((459*x^7 + 1763*x^3)/(x^8 + 20*x^4 + 46), x
) - 1/16854*integrate((459*x^7 + 881*x^3)/(x^8 + 16*x^4 + 10), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueUnable to find common minimal polynomial Er
ror: Bad
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{624x^3+144x^7+24x^{11}}{(-460-936x^4-376x^8-36x^{12}-x^{16})^2}\,dx = \ln\left(x^4-3\sqrt{6}+8\right)\left(\frac{2791\sqrt{6}}{2426976}-\frac{153}{44944}\right)-\frac{\frac{3x^{12}}{212}+\frac{95x^8}{318}+\frac{60x^4}{53}+\frac{283}{318}}{x^{16}+36x^{12}+376x^8+936x^4+460}$$
$$-\ln\left(x^4+3\sqrt{6}+8\right)\left(\frac{2791\sqrt{6}}{2426976}+\frac{153}{44944}\right)$$
$$-\ln\left(x^4-3\sqrt{6}+10\right)\left(\frac{2827\sqrt{6}}{2426976}-\frac{153}{44944}\right)$$
$$+\ln\left(x^4+3\sqrt{6}+10\right)\left(\frac{2827\sqrt{6}}{2426976}+\frac{153}{44944}\right)$$

input
```
int((624*x^3 + 144*x^7 + 24*x^11)/(936*x^4 + 376*x^8 + 36*x^12 + x^16 + 46
0)^2,x)
```

output
```
log(x^4 - 3*6^(1/2) + 8)*((2791*6^(1/2))/2426976 - 153/44944) - ((60*x^4)/
53 + (95*x^8)/318 + (3*x^12)/212 + 283/318)/(936*x^4 + 376*x^8 + 36*x^12 +
 x^16 + 460) - log(3*6^(1/2) + x^4 + 8)*((2791*6^(1/2))/2426976 + 153/4494
4) - log(x^4 - 3*6^(1/2) + 10)*((2827*6^(1/2))/2426976 - 153/44944) + log(
3*6^(1/2) + x^4 + 10)*((2827*6^(1/2))/2426976 + 153/44944)
```

Reduce [F]

$$\int \frac{624x^3 + 144x^7 + 24x^{11}}{(-460 - 936x^4 - 376x^8 - 36x^{12} - x^{16})^2}\,dx$$
$$= \int \frac{24x^{11} + 144x^7 + 624x^3}{(-x^{16} - 36x^{12} - 376x^8 - 936x^4 - 460)^2}dx$$

input
```
int((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,x)
```

output
```
int((24*x^11+144*x^7+624*x^3)/(-x^16-36*x^12-376*x^8-936*x^4-460)^2,x)
```

3.47 $\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$

Optimal result

Integrand size = 86, antiderivative size = 41

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = \frac{1}{4}\log\left(3-2\sqrt{2}-2x+x^2\right)-\frac{1}{4}\log\left(3-2\sqrt{2}+2x+x^2\right)$$

output
```
1/4*ln(3-2*2^(1/2)-2*x+x^2)-1/4*ln(3-2*2^(1/2)+2*x+x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.39

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = -\frac{(3-2\sqrt{2}+x^2)^2\left(-17+12\sqrt{2}+\left(-2+4\sqrt{2}\right)x^2-x^4\right)\left(\log\left(-3+2\sqrt{2}-2x-x^2\right)-\log\left(-3+2\sqrt{2}\right.\right.}{4\left(-577+408\sqrt{2}+8\left(-41+29\sqrt{2}\right)x^2+\left(-78+56\sqrt{2}\right)x^4+8\left(-1+\sqrt{2}\right)x^6-x^8\right)}$$

input
```
Integrate[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt
[2] + 328*x^2 - 232*Sqrt[2]*x^2 + 78*x^4 - 56*Sqrt[2]*x^4 + 8*x^6 - 8*Sqrt
[2]*x^6 + x^8),x]
```

output
```
-1/4*((3 - 2*Sqrt[2] + x^2)^2*(-17 + 12*Sqrt[2] + (-2 + 4*Sqrt[2])*x^2 - x
^4)*(Log[-3 + 2*Sqrt[2] - 2*x - x^2] - Log[-3 + 2*Sqrt[2] + 2*x - x^2]))/(
-577 + 408*Sqrt[2] + 8*(-41 + 29*Sqrt[2])*x^2 + (-78 + 56*Sqrt[2])*x^4 + 8
*(-1 + Sqrt[2])*x^6 - x^8)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.86 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {6, 6, 6, 2019, 2019, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4-232\sqrt{2}x^2+328x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8+\left(8-8\sqrt{2}\right)x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)\left(x^2+2\sqrt{2}-3\right)}{x^6+\left(5-6\sqrt{2}\right)x^4+\left(39-28\sqrt{2}\right)x^2-70\sqrt{2}+99}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{2}\int \left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+\frac{1}{2}\int \left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt[2] +
328*x^2 - 232*Sqrt[2]*x^2 + 78*x^4 - 56*Sqrt[2]*x^4 + 8*x^6 - 8*Sqrt[2]*x^
6 + x^8),x]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34
risch	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34
parallelrisch	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-232*2^
(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*ln(3-2*2^(1/2)-2*x+x^2)-1/4*ln(3-2*2^(1/2)+2*x+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{(3-2\sqrt{2}+x^2)^2\,(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = -\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(3-2\sqrt{2}+x^2)^2\,(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((3-2*2**(1/2)+x**2)**2*(-3+2*2**(1/2)+x**2)/(577-408*2**(1/2)+32
8*x**2-232*2**(1/2)*x**2+78*x**4-56*2**(1/2)*x**4+8*x**6-8*x**6*2**(1/2)+x
**8),x)
```

output
```
Exception raised: PolynomialError >> 1/(-48933191211425560206189241747804 7
2498117708482611714912381696*_t**4 + 34600991330696983980044763592797029 30
052248019321310378430976*sqrt(2)*_t**4 - 15976923948457567091783895111318 4
628965915778476
```

Maxima [F]

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$=\int \frac{(x^2+2\sqrt{2}-3)(x^2-2\sqrt{2}+3)^2}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4-232\sqrt{2}x^2+328x^2-408\sqrt{2}+577}\,dx$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="maxima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)*(x^2 - 2*sqrt(2) + 3)^2/(x^8 - 8*sqrt(2)*x
^6 + 8*x^6 - 56*sqrt(2)*x^4 + 78*x^4 - 232*sqrt(2)*x^2 + 328*x^2 - 408*sqr
t(2) + 577), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$=-\frac{1}{4}\log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}x^2-4x^2-28\right)}\right)}{2}$$

input
```
int(((x^2 - 2*2^(1/2) + 3)^2*(2*2^(1/2) + x^2 - 3))/(328*x^2 - 232*2^(1/2)
*x^2 - 56*2^(1/2)*x^4 - 8*2^(1/2)*x^6 - 408*2^(1/2) + 78*x^4 + 8*x^6 + x^8
 + 577),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-232*2^
(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.48 $\int \frac{\sqrt{5}(-24-2\sqrt{21})+\sqrt{5}(-105+15\sqrt{21})x^2}{-16+(-48\sqrt{5}+4\sqrt{105})x+(50+60\sqrt{21})x^2+(-210\sqrt{5}-30\sqrt{105})x^3+5$

Optimal result

Integrand size = 93, antiderivative size = 191

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$

$$= -\frac{1}{2}\sqrt{5}\text{RootSum}\Big[16+48\sqrt{5}\#1-4\sqrt{105}\#1-50\#1^2-60\sqrt{21}\#1^2+210\sqrt{5}\#1^3 + 30\sqrt{105}\#1^3$$

$$-525\#1^4\&, \frac{-24\log(x-\#1)-2\sqrt{21}\log(x-\#1)-105\log(x-\#1)\#1^2+15\sqrt{21}\log(x-\#1)\#1^2}{24\sqrt{5}-2\sqrt{105}-50\#1-60\sqrt{21}\#1+315\sqrt{5}\#1^2+45\sqrt{105}\#1^2-1050\#1^3}\&$$

output
```
-1/2*5^(1/2)*RootSum(_Z1 -> 16+48*5^(1/2)*_Z1-4*105^(1/2)*_Z1-50*_Z1^2-60*
21^(1/2)*_Z1^2+210*5^(1/2)*_Z1^3+30*105^(1/2)*_Z1^3-525*_Z1^4,_Z1 -> (-24*
ln(x-_Z1)-2*21^(1/2)*ln(x-_Z1)-105*ln(x-_Z1)*_Z1^2+15*21^(1/2)*ln(x-_Z1)*_
Z1^2)/(24*5^(1/2)-2*105^(1/2)-50*_Z1-60*21^(1/2)*_Z1+315*5^(1/2)*_Z1^2+45*
105^(1/2)*_Z1^2-1050*_Z1^3))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$=-\frac{1}{2}\sqrt{5}\text{RootSum}\Bigg[16+48\sqrt{5}\#1-4\sqrt{105}\#1-50\#1^2-60\sqrt{21}\#1^2+210\sqrt{5}\#1^3+30\sqrt{105}\#1^3-525\#1^4\&,\frac{-24\log(x-\#1)-2\sqrt{21}\log(x-\#1)-105\log(x-\#1)\#1^2+15\sqrt{21}\log(x-\#1)\#1^2}{24\sqrt{5}-2\sqrt{105}-50\#1-60\sqrt{21}\#1+315\sqrt{5}\#1^2+45\sqrt{105}\#1^2-1050\#1^3}\&\Bigg]$$

input
```
Integrate[(Sqrt[5]*(-24 - 2*Sqrt[21]) + Sqrt[5]*(-105 + 15*Sqrt[21])*x^2)/
(-16 + (-48*Sqrt[5] + 4*Sqrt[105])*x + (50 + 60*Sqrt[21])*x^2 + (-210*Sqrt
[5] - 30*Sqrt[105])*x^3 + 525*x^4),x]
```

output
```
-1/2*(Sqrt[5]*RootSum[16 + 48*Sqrt[5]*#1 - 4*Sqrt[105]*#1 - 50*#1^2 - 60*S
qrt[21]*#1^2 + 210*Sqrt[5]*#1^3 + 30*Sqrt[105]*#1^3 - 525*#1^4 & , (-24*Lo
g[x - #1] - 2*Sqrt[21]*Log[x - #1] - 105*Log[x - #1]*#1^2 + 15*Sqrt[21]*Lo
g[x - #1]*#1^2)/(24*Sqrt[5] - 2*Sqrt[105] - 50*#1 - 60*Sqrt[21]*#1 + 315*S
qrt[5]*#1^2 + 45*Sqrt[105]*#1^2 - 1050*#1^3) & ])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{5}\left(15\sqrt{21}-105\right)x^2+\sqrt{5}\left(-24-2\sqrt{21}\right)}{525x^4+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+\left(50+60\sqrt{21}\right)x^2+\left(4\sqrt{105}-48\sqrt{5}\right)x-16}\,dx$$
$$\Big\downarrow \text{7292}$$
$$\int \frac{-\sqrt{5}\left(15\sqrt{21}-105\right)x^2-\sqrt{5}\left(-24-2\sqrt{21}\right)}{-525x^4+30\sqrt{35}\left(\sqrt{3}+\sqrt{7}\right)x^3-10\left(5+6\sqrt{21}\right)x^2+4\sqrt{15}\left(4\sqrt{3}-\sqrt{7}\right)x+16}\,dx$$
$$\Big\downarrow \text{7292}$$

$$\int \frac{-\sqrt{5}(15\sqrt{21}-105)\,x^2-\sqrt{5}(-24-2\sqrt{21})}{-525x^4+30\sqrt{35}\,(\sqrt{3}+\sqrt{7})\,x^3-10\,(5+6\sqrt{21})\,x^2+4\sqrt{5}\,(12-\sqrt{21})\,x+16}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{15\sqrt{5}(7-\sqrt{21})\,x^2}{-525x^4+30\sqrt{35}\,(\sqrt{3}+\sqrt{7})\,x^3-10\,(5+6\sqrt{21})\,x^2+4\sqrt{5}\,(12-\sqrt{21})\,x+16}+\frac{}{-525x^4+30\sqrt{35}\,(\sqrt{3}+\sqrt{}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\sqrt{15}\left(4\sqrt{3}+\sqrt{7}\right)\int \frac{1}{-525x^4+30\sqrt{35}\,(\sqrt{3}+\sqrt{7})\,x^3-10\,(5+6\sqrt{21})\,x^2+4\sqrt{5}\,(12-\sqrt{21})\,x+16}dx-$$
$$15\sqrt{35}\left(\sqrt{3}-\sqrt{7}\right)\int \frac{x^2}{-525x^4+30\sqrt{35}\,(\sqrt{3}+\sqrt{7})\,x^3-10\,(5+6\sqrt{21})\,x^2+4\sqrt{5}\,(12-\sqrt{21})\,x+16}dx$$

input
```
Int[(Sqrt[5]*(-24 - 2*Sqrt[21]) + Sqrt[5]*(-105 + 15*Sqrt[21])*x^2)/(-16 +
 (-48*Sqrt[5] + 4*Sqrt[105])*x + (50 + 60*Sqrt[21])*x^2 + (-210*Sqrt[5] -
30*Sqrt[105])*x^3 + 525*x^4),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.27

method	result	size
default	$\sqrt{5}\left(\frac{\sqrt{21}\,\sqrt{7}\,\arctan\left(\frac{\left(-2\sqrt{5}\,\sqrt{21}+70x\right)\sqrt{7}}{70}\right)}{35}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-6\sqrt{5}+30x\right)\sqrt{3}}{30}\right)}{5}\right)$	52
risch	$\frac{\sqrt{15}\,\ln\left(\sqrt{5}\,\sqrt{15}-3\sqrt{5}+15x\right)}{10}-\frac{\sqrt{15}\,\ln\left(-\sqrt{5}\,\sqrt{15}-3\sqrt{5}+15x\right)}{10}+\frac{\sqrt{15}\,\arctan\left(\frac{\left(-3\sqrt{5}\,\sqrt{21}+105x\right)\sqrt{5}\,\sqrt{21}\,\sqrt{15}}{1575}\right)}{5}$	76

input
```
int((5^(1/2)*(-24-2*21^(1/2))+5^(1/2)*(-105+15*21^(1/2))*x^2)/(-16+(-48*5^
(1/2)+4*105^(1/2))*x+(50+60*21^(1/2))*x^2+(-210*5^(1/2)-30*105^(1/2))*x^3+
525*x^4),x,method=_RETURNVERBOSE)
```

output
```
5^(1/2)*(1/35*21^(1/2)*7^(1/2)*arctan(1/70*(-2*5^(1/2)*21^(1/2)+70*x)*7^(1
/2))+1/5*3^(1/2)*arctanh(1/30*(-6*5^(1/2)+30*x)*3^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((5^(1/2)*(-24-2*21^(1/2))+5^(1/2)*(-105+15*21^(1/2))*x^2)/(-16+(
-48*5^(1/2)+4*105^(1/2))*x+(50+60*21^(1/2))*x^2+(-210*5^(1/2)-30*105^(1/2)
)*x^3+525*x^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((5**(1/2)*(-24-2*21**(1/2))+5**(1/2)*(-105+15*21**(1/2))*x**2)/(
-16+(-48*5**(1/2)+4*105**(1/2))*x+(50+60*21**(1/2))*x**2+(-210*5**(1/2)-30
*105**(1/2))*x**3+525*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(-11312*_t**4 + 1680*sqrt(21)*_t**4
 - 3360*_t**3 + 288*sqrt(21)*_t**3 - 600*sqrt(21)*_t**2 + 2568*_t**2 - 72*
sqrt(21)*_t + 360*_t - 207 + 45*sqrt(21)) contains an element of the set o
f generators.
```

Maxima [F]

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$=\int \frac{15\,\sqrt{5}x^2\left(\sqrt{21}-7\right)-2\,\sqrt{5}\left(\sqrt{21}+12\right)}{525\,x^4-30\,x^3\left(\sqrt{105}+7\,\sqrt{5}\right)+10\,x^2\left(6\,\sqrt{21}+5\right)+4\,x\left(\sqrt{105}-12\,\sqrt{5}\right)-16}\,dx$$

input
```
integrate((5^(1/2)*(-24-2*21^(1/2))+5^(1/2)*(-105+15*21^(1/2))*x^2)/(-16+(
-48*5^(1/2)+4*105^(1/2))*x+(50+60*21^(1/2))*x^2+(-210*5^(1/2)-30*105^(1/2)
)*x^3+525*x^4),x, algorithm="maxima")
```

output
```
integrate((15*sqrt(5)*x^2*(sqrt(21) - 7) - 2*sqrt(5)*(sqrt(21) + 12))/(525
*x^4 - 30*x^3*(sqrt(105) + 7*sqrt(5)) + 10*x^2*(6*sqrt(21) + 5) + 4*x*(sqr
t(105) - 12*sqrt(5)) - 16), x)
```

Giac [B] (verification not implemented)

Default grade assigned because unable to parse optimal

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$=\frac{1}{35}\,\sqrt{105}\sqrt{7}\arctan\left(\frac{1}{35}\,\sqrt{7}\left(35\,x-\sqrt{105}\right)\right)-\frac{1}{10}\,\sqrt{5}\sqrt{3}\log\left(\frac{\left|3000\,x-600\,\sqrt{5}-1000\,\sqrt{3}\right|}{\left|3000\,x-600\,\sqrt{5}+1000\,\sqrt{3}\right|}\right)$$

input
```
integrate((5^(1/2)*(-24-2*21^(1/2))+5^(1/2)*(-105+15*21^(1/2))*x^2)/(-16+(
-48*5^(1/2)+4*105^(1/2))*x+(50+60*21^(1/2))*x^2+(-210*5^(1/2)-30*105^(1/2)
)*x^3+525*x^4),x, algorithm="giac")
```

output
```
1/35*sqrt(105)*sqrt(7)*arctan(1/35*sqrt(7)*(35*x - sqrt(105))) - 1/10*sqrt
(5)*sqrt(3)*log(abs(3000*x - 600*sqrt(5) - 1000*sqrt(3))/abs(3000*x - 600*
sqrt(5) + 1000*sqrt(3)))
```

Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 1659, normalized size of antiderivative = 8.69

$$\int \frac{\sqrt{5}(-24-2\sqrt{21})+\sqrt{5}(-105+15\sqrt{21})\,x^2}{-16+(-48\sqrt{5}+4\sqrt{105})\,x+(50+60\sqrt{21})\,x^2+(-210\sqrt{5}-30\sqrt{105})\,x^3+525x^4}\,dx$$
$= \text{Too large to display}$

input
```
int((5^(1/2)*(2*21^(1/2) + 24) - 5^(1/2)*x^2*(15*21^(1/2) - 105))/(x^3*(21
0*5^(1/2) + 30*105^(1/2)) - x^2*(60*21^(1/2) + 50) + x*(48*5^(1/2) - 4*105
^(1/2)) - 525*x^4 + 16),x)
```

output
```
symsum(log((4232*5^(1/2)*root(4126651200*5^(1/2)*21^(1/2)*105^(1/2)*z^4 +
11534040000*5^(1/2)*105^(1/2)*z^4 - 42151200000*21^(1/2)*z^4 - 63105572600
0*z^4 - 368373600*5^(1/2)*21^(1/2)*105^(1/2)*z^2 + 768474000*5^(1/2)*105^(
1/2)*z^2 - 3842370000*21^(1/2)*z^2 + 38679228000*z^2 + 1508220000*5^(1/2)*
105^(1/2)*z - 7541100000*21^(1/2)*z - 20468700*5^(1/2)*21^(1/2)*105^(1/2)
- 103950000*5^(1/2)*105^(1/2) + 170572500*21^(1/2) + 6598753875, z, k))/64
3125 - (304*x)/6125 + (5192*105^(1/2)*root(4126651200*5^(1/2)*21^(1/2)*105
^(1/2)*z^4 + 11534040000*5^(1/2)*105^(1/2)*z^4 - 42151200000*21^(1/2)*z^4
- 631055726000*z^4 - 368373600*5^(1/2)*21^(1/2)*105^(1/2)*z^2 + 768474000*
5^(1/2)*105^(1/2)*z^2 - 3842370000*21^(1/2)*z^2 + 38679228000*z^2 + 150822
0000*5^(1/2)*105^(1/2)*z - 7541100000*21^(1/2)*z - 20468700*5^(1/2)*21^(1/
2)*105^(1/2) - 103950000*5^(1/2)*105^(1/2) + 170572500*21^(1/2) + 65987538
75, z, k))/1929375 - (11824*root(4126651200*5^(1/2)*21^(1/2)*105^(1/2)*z^4
 + 11534040000*5^(1/2)*105^(1/2)*z^4 - 42151200000*21^(1/2)*z^4 - 63105572
6000*z^4 - 368373600*5^(1/2)*21^(1/2)*105^(1/2)*z^2 + 768474000*5^(1/2)*10
5^(1/2)*z^2 - 3842370000*21^(1/2)*z^2 + 38679228000*z^2 + 1508220000*5^(1/
2)*105^(1/2)*z - 7541100000*21^(1/2)*z - 20468700*5^(1/2)*21^(1/2)*105^(1/
2) - 103950000*5^(1/2)*105^(1/2) + 170572500*21^(1/2) + 6598753875, z, k)*
x)/77175 + (304*21^(1/2)*x)/25725 + (908*5^(1/2))/643125 - (428*105^(1/2))
/1929375 - (28624*5^(1/2)*root(4126651200*5^(1/2)*21^(1/2)*105^(1/2)*z^...
```

Reduce [B] (verification not implemented)

Time = 191.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{5}\left(-24-2\sqrt{21}\right)+\sqrt{5}\left(-105+15\sqrt{21}\right)x^2}{-16+\left(-48\sqrt{5}+4\sqrt{105}\right)x+\left(50+60\sqrt{21}\right)x^2+\left(-210\sqrt{5}-30\sqrt{105}\right)x^3+525x^4}\,dx$$
$$=\frac{\sqrt{15}\left(-4atan\left(\frac{\sqrt{105}-35x}{5\sqrt{7}}\right)-2atanh\left(\frac{15x}{3\sqrt{5}-5\sqrt{3}}\right)-2\log\left(\sqrt{15}\,x-\sqrt{5}-\sqrt{3}\right)+\log\left(2\sqrt{15}+15x^2-8\right)\right)}{20}$$

input
```
int((5^(1/2)*(-24-2*21^(1/2))+5^(1/2)*(-105+15*21^(1/2))*x^2)/(-16+(-48*5^
(1/2)+4*105^(1/2))*x+(50+60*21^(1/2))*x^2+(-210*5^(1/2)-30*105^(1/2))*x^3+
525*x^4),x)
```

output
```
(sqrt(15)*( - 4*atan((sqrt(105) - 35*x)/(5*sqrt(7))) - 2*atanh((15*x)/(3*s
qrt(5) - 5*sqrt(3))) - 2*log(sqrt(15)*x - sqrt(5) - sqrt(3)) + log(2*sqrt(
15) + 15*x**2 - 8)))/20
```

3.49 $\int \frac{5000\sqrt{5}(1000\sqrt{3}-750\sqrt{5})x+5000\sqrt{5}(-525\ldots}{-1937500+500000\sqrt{15}+(3125000-625000\sqrt{15})x^2+(-88750000-406\ldots}$

Optimal result

Integrand size = 115, antiderivative size = 84

$$\int \frac{5000\sqrt{5}(1000\sqrt{3}-750\sqrt{5})\,x+5000\sqrt{5}(-5250\sqrt{3}-3150\sqrt{5})\,x^3+65625\ldots}{-1937500+500000\sqrt{15}+(3125000-625000\sqrt{15})\,x^2+(-88750000-4062500\sqrt{15})\,x^4+(1640625\ldots} = -\sqrt[4]{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{2}{\sqrt{15}}}-\frac{7}{2}\sqrt[4]{\frac{5}{3}}x^2\right)-\sqrt[4]{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{2}{\sqrt{15}}}-\frac{1}{2}3^{3/4}\sqrt[4]{5}x^2\right)$$

output
```
1/5*3^(1/4)*5^(3/4)*arctanh(-1/30*(450+120*15^(1/2))^(1/2)+7/6*5^(1/4)*3^(
3/4)*x^2)+1/5*3^(1/4)*5^(3/4)*arctanh(-1/30*(450+120*15^(1/2))^(1/2)+1/2*5
^(1/4)*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.31

$$\int \frac{5000\sqrt{5}(1000\sqrt{3}-750\sqrt{5})x+5000\sqrt{5}(-5250\sqrt{3}-3150\sqrt{5})x^3+65625000\sqrt{15}x^5}{-1937500+500000\sqrt{15}+(3125000-625000\sqrt{15})x^2+(-88750000-4062500\sqrt{15})x^4+(164062500+32812500\sqrt{15})x^6-172265625x^8}\,dx$$
$$=\text{RootSum}\Big[-124+32\sqrt{15}+200\#1^2-40\sqrt{15}\#1^2-5680\#1^4-260\sqrt{15}\#1^4+10500\#1^6+2100\sqrt{15}\#1^6-11025\#1^8\&, \frac{-30\log(x-\#1)+8\sqrt{15}\log(x-\#1)-126\log(x-\#1)\#1^2-42\sqrt{15}\log(x-\#1)\#1^2+105\sqrt{15}\log(x-\#1)\#1^4}{10-2\sqrt{15}-568\#1^2-26\sqrt{15}\#1^2+1575\#1^4+315\sqrt{15}\#1^4-2205\#1^6}\&\Big]$$

input
```
Integrate[(5000*Sqrt[5]*(1000*Sqrt[3] - 750*Sqrt[5])*x + 5000*Sqrt[5]*(-52
50*Sqrt[3] - 3150*Sqrt[5])*x^3 + 65625000*Sqrt[15]*x^5)/(-1937500 + 500000
*Sqrt[15] + (3125000 - 625000*Sqrt[15])*x^2 + (-88750000 - 4062500*Sqrt[15
])*x^4 + (164062500 + 32812500*Sqrt[15])*x^6 - 172265625*x^8),x]
```

output
```
RootSum[-124 + 32*Sqrt[15] + 200*#1^2 - 40*Sqrt[15]*#1^2 - 5680*#1^4 - 260
*Sqrt[15]*#1^4 + 10500*#1^6 + 2100*Sqrt[15]*#1^6 - 11025*#1^8 & , (-30*Log
[x - #1] + 8*Sqrt[15]*Log[x - #1] - 126*Log[x - #1]*#1^2 - 42*Sqrt[15]*Log
[x - #1]*#1^2 + 105*Sqrt[15]*Log[x - #1]*#1^4)/(10 - 2*Sqrt[15] - 568*#1^2
 - 26*Sqrt[15]*#1^2 + 1575*#1^4 + 315*Sqrt[15]*#1^4 - 2205*#1^6) & ]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 291 vs. 2(84) = 168.

Time = 3.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2028, 7266, 27, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{65625000\sqrt{15}x^5+5000\sqrt{5}(-5250\sqrt{3}-3150\sqrt{5})x^3+5000\sqrt{5}(1000\sqrt{3}-750\sqrt{5})x}{-172265625x^8+(164062500+32812500\sqrt{15})x^6+(-88750000-4062500\sqrt{15})x^4+(3125000-625000\sqrt{15})x^2 \ldots}\,dx$$
$$\Big\downarrow \text{2028}$$

$$\int \frac{x\left(65625000\sqrt{15}x^4+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^2+5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{\right.\right.}{-172265625x^8+\left(164062500+32812500\sqrt{15}\right)x^6+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(3125000-625000\sqrt{1\right.}$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int \frac{40\left(-105\sqrt{15}x^4+42\left(3+\sqrt{15}\right)x^2+2\left(15-4\sqrt{15}\right)\right)}{11025x^8-2100\left(5+\sqrt{15}\right)x^6+20\left(284+13\sqrt{15}\right)x^4-40\left(5-\sqrt{15}\right)x^2+4\left(31-8\sqrt{15}\right)}dx^2$$

$$\Big\downarrow \; 27$$

$$20\int \frac{-105\sqrt{15}x^4+42\left(3+\sqrt{15}\right)x^2+2\left(15-4\sqrt{15}\right)}{11025x^8-2100\left(5+\sqrt{15}\right)x^6+20\left(284+13\sqrt{15}\right)x^4-40\left(5-\sqrt{15}\right)x^2+4\left(31-8\sqrt{15}\right)}dx^2$$

$$\Big\downarrow \; 2492$$

$$\frac{4\int \left(\frac{2205\sqrt[4]{15}\left(\sqrt{3}+\sqrt{5}-5\sqrt[4]{15}\right)}{2\left(105x^4-2\left(25-4\sqrt[4]{3}5^{3/4}+5\sqrt{15}\right)x^2+2\left(4-\sqrt{15}\right)\right)}-\frac{2205\sqrt[4]{15}\left(\sqrt{3}+\sqrt{5}+5\sqrt[4]{15}\right)}{2\left(105x^4-2\left(25+4\sqrt[4]{3}5^{3/4}+5\sqrt{15}\right)x^2+2\left(4-\sqrt{15}\right)\right)}\right)dx^2}{2205}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(\frac{441\sqrt[4]{3}5^{3/4}\left(\sqrt{3}+\sqrt{5}-5\sqrt[4]{15}\right)\text{arctanh}\left(\frac{-21\sqrt{5}x^2-4\sqrt[4]{15}+5\sqrt{5}+5\sqrt{3}}{2\sqrt{8-10\ 3^{3/4}\sqrt[4]{5}-10\sqrt[4]{3}5^{3/4}+27\sqrt{15}}}\right)}{4\sqrt{8-10\ 3^{3/4}\sqrt[4]{5}-10\sqrt[4]{3}5^{3/4}+27\sqrt{15}}}-\frac{441\sqrt[4]{3}5^{3/4}\left(\sqrt{3}+\sqrt{5}+5\sqrt[4]{15}\right)\text{arctanh}\left(\frac{\cdots}{2\sqrt{8+\cdots}}\right.}{4\sqrt{8+10\ 3^{3/4}\sqrt[4]{5}+10\sqrt[4]{3}5\cdots}}\right.}{2205}$$

input

```
Int[(5000*Sqrt[5]*(1000*Sqrt[3] - 750*Sqrt[5])*x + 5000*Sqrt[5]*(-5250*Sqr
t[3] - 3150*Sqrt[5])*x^3 + 65625000*Sqrt[15]*x^5)/(-1937500 + 500000*Sqrt[
15] + (3125000 - 625000*Sqrt[15])*x^2 + (-88750000 - 4062500*Sqrt[15])*x^4
 + (164062500 + 32812500*Sqrt[15])*x^6 - 172265625*x^8),x]
```

output
```
(4*((441*3^(1/4)*5^(3/4)*(Sqrt[3] + Sqrt[5] - 5*15^(1/4))*ArcTanh[(5*Sqrt[
3] + 5*Sqrt[5] - 4*15^(1/4) - 21*Sqrt[5]*x^2)/(2*Sqrt[8 - 10*3^(3/4)*5^(1/
4) - 10*3^(1/4)*5^(3/4) + 27*Sqrt[15]])])/(4*Sqrt[8 - 10*3^(3/4)*5^(1/4) -
 10*3^(1/4)*5^(3/4) + 27*Sqrt[15]]) - (441*3^(1/4)*5^(3/4)*(Sqrt[3] + Sqrt
[5] + 5*15^(1/4))*ArcTanh[(5*Sqrt[3] + 5*Sqrt[5] + 4*15^(1/4) - 21*Sqrt[5]
*x^2)/(2*Sqrt[8 + 10*3^(3/4)*5^(1/4) + 10*3^(1/4)*5^(3/4) + 27*Sqrt[15]])]
)/(4*Sqrt[8 + 10*3^(3/4)*5^(1/4) + 10*3^(1/4)*5^(3/4) + 27*Sqrt[15]])))/22
05
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{15^{\frac{1}{4}}\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(90x^{2}-6\sqrt{15}-30\right)\sqrt{5}\,15^{\frac{3}{4}}}{900}\right)}{5}+\frac{15^{\frac{1}{4}}\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(490x^{2}-14\sqrt{15}-70\right)\sqrt{5}\,15^{\frac{3}{4}}}{2100}\right)}{5}$	60

input
```
int((5000*5^(1/2)*(1000*3^(1/2)-750*5^(1/2))*x+5000*5^(1/2)*(-5250*3^(1/2)
-3150*5^(1/2))*x^3+65625000*15^(1/2)*x^5)/(-1937500+500000*15^(1/2)+(31250
00-625000*15^(1/2))*x^2+(-88750000-4062500*15^(1/2))*x^4+(164062500+328125
00*15^(1/2))*x^6-172265625*x^8),x,method=_RETURNVERBOSE)
```

output
```
1/5*15^(1/4)*5^(1/2)*arctanh(1/900*(90*x^2-6*15^(1/2)-30)*5^(1/2)*15^(3/4)
)+1/5*15^(1/4)*5^(1/2)*arctanh(1/2100*(490*x^2-14*15^(1/2)-70)*5^(1/2)*15^
(3/4))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+656250}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(16406250\right.}$$
$= \text{Timed out}$

input
```
integrate((5000*5^(1/2)*(1000*3^(1/2)-750*5^(1/2))*x+5000*5^(1/2)*(-5250*3
^(1/2)-3150*5^(1/2))*x^3+65625000*15^(1/2)*x^5)/(-1937500+500000*15^(1/2)+
(3125000-625000*15^(1/2))*x^2+(-88750000-4062500*15^(1/2))*x^4+(164062500+
32812500*15^(1/2))*x^6-172265625*x^8),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+656250}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(16406250\right.}$$
$= \text{Timed out}$

input
```
integrate((5000*5**(1/2)*(1000*3**(1/2)-750*5**(1/2))*x+5000*5**(1/2)*(-52
50*3**(1/2)-3150*5**(1/2))*x**3+65625000*15**(1/2)*x**5)/(-1937500+500000*
15**(1/2)+(3125000-625000*15**(1/2))*x**2+(-88750000-4062500*15**(1/2))*x*
*4+(164062500+32812500*15**(1/2))*x**6-172265625*x**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+656250}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(16406250\right.}$$
$$= \int -\frac{8\left(525\sqrt{15}x^5-42\sqrt{5}x^3\left(3\sqrt{5}+5\sqrt{3}\right)-10\sqrt{5}x\left(3\sqrt{5}-4\sqrt{3}\right)\right)}{11025x^8-2100x^6\left(\sqrt{15}+5\right)+20x^4\left(13\sqrt{15}+284\right)+40x^2\left(\sqrt{15}-5\right)-32\sqrt{15}+124}\,dx$$

input
```
integrate((5000*5^(1/2)*(1000*3^(1/2)-750*5^(1/2))*x+5000*5^(1/2)*(-5250*3
^(1/2)-3150*5^(1/2))*x^3+65625000*15^(1/2)*x^5)/(-1937500+500000*15^(1/2)+
(3125000-625000*15^(1/2))*x^2+(-88750000-4062500*15^(1/2))*x^4+(164062500+
32812500*15^(1/2))*x^6-172265625*x^8),x, algorithm="maxima")
```

output
```
-8*integrate((525*sqrt(15)*x^5 - 42*sqrt(5)*x^3*(3*sqrt(5) + 5*sqrt(3)) -
10*sqrt(5)*x*(3*sqrt(5) - 4*sqrt(3)))/(11025*x^8 - 2100*x^6*(sqrt(15) + 5)
 + 20*x^4*(13*sqrt(15) + 284) + 40*x^2*(sqrt(15) - 5) - 32*sqrt(15) + 124)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+65625000\sqrt{15}x^5}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(164062500+32812500\sqrt{15}\right)x^6-172265625x^8}\,dx$$
= Exception raised: TypeError

input
```
integrate((5000*5^(1/2)*(1000*3^(1/2)-750*5^(1/2))*x+5000*5^(1/2)*(-5250*3
^(1/2)-3150*5^(1/2))*x^3+65625000*15^(1/2)*x^5)/(-1937500+500000*15^(1/2)+
(3125000-625000*15^(1/2))*x^2+(-88750000-4062500*15^(1/2))*x^4+(164062500+
32812500*15^(1/2))*x^6-172265625*x^8),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:   ***   Warning: increasing stack s
ize to 4096000.Francis algorithm failure for[-1.0,0.0,infinity,infinity,in
finity]pr
```

Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 1086, normalized size of antiderivative = 12.93

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+65625000\sqrt{15}x^5}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(164062500+32812500\sqrt{15}\right)x^6-172265625x^8}\,dx$$
= Too large to display

input
```
int(-(65625000*15^(1/2)*x^5 + 5000*5^(1/2)*x*(1000*3^(1/2) - 750*5^(1/2))
- 5000*5^(1/2)*x^3*(5250*3^(1/2) + 3150*5^(1/2)))/(x^2*(625000*15^(1/2) -
3125000) + x^4*(4062500*15^(1/2) + 88750000) - 500000*15^(1/2) - x^6*(3281
2500*15^(1/2) + 164062500) + 172265625*x^8 + 1937500),x)
```

output
```
symsum(log(14413269276*root(18530304000*15^(1/2)*z^4 - 71793623040*z^4 + 7
179362304*15^(1/2)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 2692260864
, z, k) - 3705932916*15^(1/2)*root(18530304000*15^(1/2)*z^4 - 71793623040*
z^4 + 7179362304*15^(1/2)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 269
2260864, z, k) - 150361596*15^(1/2) - 24252197760*15^(1/2)*root(1853030400
0*15^(1/2)*z^4 - 71793623040*z^4 + 7179362304*15^(1/2)*z^2 - 27795456000*z
^2 + 694886400*15^(1/2) - 2692260864, z, k)^2 - 38435384736*15^(1/2)*root(
18530304000*15^(1/2)*z^4 - 71793623040*z^4 + 7179362304*15^(1/2)*z^2 - 277
95456000*z^2 + 694886400*15^(1/2) - 2692260864, z, k)^3 - 238264183984*15^
(1/2)*root(18530304000*15^(1/2)*z^4 - 71793623040*z^4 + 7179362304*15^(1/2
)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 2692260864, z, k)^4 - 98824
877760*15^(1/2)*root(18530304000*15^(1/2)*z^4 - 71793623040*z^4 + 71793623
04*15^(1/2)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 2692260864, z, k)
^5 - 604371210240*15^(1/2)*root(18530304000*15^(1/2)*z^4 - 71793623040*z^4
 + 7179362304*15^(1/2)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 269226
0864, z, k)^6 + 117295652025*root(18530304000*15^(1/2)*z^4 - 71793623040*z
^4 + 7179362304*15^(1/2)*z^2 - 27795456000*z^2 + 694886400*15^(1/2) - 2692
260864, z, k)*x^2 - 3456415935*15^(1/2)*x^2 + 93859916874*root(18530304000
*15^(1/2)*z^4 - 71793623040*z^4 + 7179362304*15^(1/2)*z^2 - 27795456000*z^
2 + 694886400*15^(1/2) - 2692260864, z, k)^2 + 148237316640*root(185303...
```

Reduce [F]

$$\int \frac{5000\sqrt{5}\left(1000\sqrt{3}-750\sqrt{5}\right)x+5000\sqrt{5}\left(-5250\sqrt{3}-3150\sqrt{5}\right)x^3+656250}{-1937500+500000\sqrt{15}+\left(3125000-625000\sqrt{15}\right)x^2+\left(-88750000-4062500\sqrt{15}\right)x^4+\left(16406250\right.}$$
= Too large to display

input
```
int((5000*5^(1/2)*(1000*3^(1/2)-750*5^(1/2))*x+5000*5^(1/2)*(-5250*3^(1/2)
-3150*5^(1/2))*x^3+65625000*15^(1/2)*x^5)/(-1937500+500000*15^(1/2)+(31250
00-625000*15^(1/2))*x^2+(-88750000-4062500*15^(1/2))*x^4+(164062500+328125
00*15^(1/2))*x^6-172265625*x^8),x)
```

output

```
40*( - 1157625*sqrt(15)*int(x**13/(121550625*x**16 - 231525000*x**14 + 169
344000*x**12 - 107310000*x**10 + 40702600*x**8 - 7204000*x**6 + 1674240*x*
*4 - 11200*x**2 + 16),x) + 1565550*sqrt(15)*int(x**11/(121550625*x**16 - 2
31525000*x**14 + 169344000*x**12 - 107310000*x**10 + 40702600*x**8 - 72040
00*x**6 + 1674240*x**4 - 11200*x**2 + 16),x) - 861000*sqrt(15)*int(x**9/(1
21550625*x**16 - 231525000*x**14 + 169344000*x**12 - 107310000*x**10 + 407
02600*x**8 - 7204000*x**6 + 1674240*x**4 - 11200*x**2 + 16),x) + 373800*sq
rt(15)*int(x**7/(121550625*x**16 - 231525000*x**14 + 169344000*x**12 - 107
310000*x**10 + 40702600*x**8 - 7204000*x**6 + 1674240*x**4 - 11200*x**2 +
16),x) - 79700*sqrt(15)*int(x**5/(121550625*x**16 - 231525000*x**14 + 1693
44000*x**12 - 107310000*x**10 + 40702600*x**8 - 7204000*x**6 + 1674240*x**
4 - 11200*x**2 + 16),x) + 9640*sqrt(15)*int(x**3/(121550625*x**16 - 231525
000*x**14 + 169344000*x**12 - 107310000*x**10 + 40702600*x**8 - 7204000*x*
*6 + 1674240*x**4 - 11200*x**2 + 16),x) - 32*sqrt(15)*int(x/(121550625*x**
16 - 231525000*x**14 + 169344000*x**12 - 107310000*x**10 + 40702600*x**8 -
 7204000*x**6 + 1674240*x**4 - 11200*x**2 + 16),x) - 1918350*int(x**11/(12
1550625*x**16 - 231525000*x**14 + 169344000*x**12 - 107310000*x**10 + 4070
2600*x**8 - 7204000*x**6 + 1674240*x**4 - 11200*x**2 + 16),x) + 740250*int
(x**9/(121550625*x**16 - 231525000*x**14 + 169344000*x**12 - 107310000*x**
10 + 40702600*x**8 - 7204000*x**6 + 1674240*x**4 - 11200*x**2 + 16),x) ...
```

3.50 $\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 117

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx = -\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{7}x\right)}{8\ 3^{3/8}5^{5/8}}-\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{11}x\right)}{4\ 3^{3/8}5^{5/8}} - \frac{\operatorname{arctanh}\left(\sqrt[8]{\frac{5}{3}}\sqrt{7}x\right)}{8\ 3^{3/8}5^{5/8}}-\frac{\operatorname{arctanh}\left(\sqrt[8]{\frac{5}{3}}\sqrt{11}x\right)}{4\ 3^{3/8}5^{5/8}}$$

output
```
-1/120*arctan(1/3*5^(1/8)*3^(7/8)*7^(1/2)*x)*3^(5/8)*5^(3/8)-1/60*arctan(1
/3*5^(1/8)*3^(7/8)*11^(1/2)*x)*3^(5/8)*5^(3/8)-1/120*arctanh(1/3*5^(1/8)*3
^(7/8)*7^(1/2)*x)*3^(5/8)*5^(3/8)-1/60*arctanh(1/3*5^(1/8)*3^(7/8)*11^(1/2
)*x)*3^(5/8)*5^(3/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx=\frac{1}{160}\text{RootSum}\Big[3\sqrt{15}-2550\#1^4$$
$$+29645\sqrt{15}\#1^8\&,\frac{-3\sqrt{7}\log(x-\#1)-6\sqrt{11}\log(x-\#1)+121\sqrt{105}\log(x-\#1)\#1^4+98\sqrt{165}\log(}{-255\#1^3+5929\sqrt{15}\#1^7}$$

input
```
Integrate[(3*Sqrt[7] + 6*Sqrt[11] + (-121*Sqrt[105] - 98*Sqrt[165])*x^4)/(
-12*Sqrt[15] + 10200*x^4 - 118580*Sqrt[15]*x^8),x]
```

output
```
RootSum[3*Sqrt[15] - 2550*#1^4 + 29645*Sqrt[15]*#1^8 & , (-3*Sqrt[7]*Log[x
 - #1] - 6*Sqrt[11]*Log[x - #1] + 121*Sqrt[105]*Log[x - #1]*#1^4 + 98*Sqrt
[165]*Log[x - #1]*#1^4)/(-255*#1^3 + 5929*Sqrt[15]*#1^7) & ]/160
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1752, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-121\sqrt{105}-98\sqrt{165}\right)x^4+6\sqrt{11}+3\sqrt{7}}{-118580\sqrt{15}x^8+10200x^4-12\sqrt{15}}dx$$
$$\Big\downarrow \text{1752}$$
$$-98\sqrt{165}\int \frac{1}{2940-118580\sqrt{15}x^4}dx-121\sqrt{105}\int \frac{1}{7260-118580\sqrt{15}x^4}dx$$
$$\Big\downarrow \text{756}$$

$$-121\sqrt{105}\left(\frac{\int \frac{1}{\sqrt[4]{3}-7\sqrt[4]{5}x^2}dx}{4840\ 3^{3/4}}+\frac{\int \frac{1}{7\sqrt[4]{5}x^2+\sqrt[4]{3}}dx}{4840\ 3^{3/4}}\right)-98\sqrt{165}\left(\frac{\int \frac{1}{\sqrt[4]{3}-11\sqrt[4]{5}x^2}dx}{1960\ 3^{3/4}}+\frac{\int \frac{1}{11\sqrt[4]{5}x^2+\sqrt[4]{3}}dx}{1960\ 3^{3/4}}\right)$$

$$\Big\downarrow \text{216}$$

$$-98\sqrt{165}\left(\frac{\int \frac{1}{\sqrt[4]{3}-11\sqrt[4]{5}x^2}dx}{1960\ 3^{3/4}}+\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{11}x\right)}{1960\ 3^{7/8}\sqrt[8]{5}\sqrt{11}}\right)-121\sqrt{105}\left(\frac{\int \frac{1}{\sqrt[4]{3}-7\sqrt[4]{5}x^2}dx}{4840\ 3^{3/4}}+\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{7}x\right)}{4840\ 3^{7/8}\sqrt[8]{5}\sqrt{7}}\right)$$

$$\Big\downarrow \text{219}$$

$$-121\sqrt{105}\left(\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{7}x\right)}{4840\ 3^{7/8}\sqrt[8]{5}\sqrt{7}}+\frac{\text{arctanh}\left(\sqrt[8]{\frac{5}{3}}\sqrt{7}x\right)}{4840\ 3^{7/8}\sqrt[8]{5}\sqrt{7}}\right)-98\sqrt{165}\left(\frac{\arctan\left(\sqrt[8]{\frac{5}{3}}\sqrt{11}x\right)}{1960\ 3^{7/8}\sqrt[8]{5}\sqrt{11}}+\frac{\text{arctanh}\left(\sqrt[8]{\frac{5}{3}}\sqrt{11}x\right)}{1960\ 3^{7/8}\sqrt[8]{5}\sqrt{11}}\right)$$

input
```
Int[(3*Sqrt[7] + 6*Sqrt[11] + (-121*Sqrt[105] - 98*Sqrt[165])*x^4)/(-12*Sq
rt[15] + 10200*x^4 - 118580*Sqrt[15]*x^8),x]
```

output
```
-121*Sqrt[105]*(ArcTan[(5/3)^(1/8)*Sqrt[7]*x]/(4840*3^(7/8)*5^(1/8)*Sqrt[7
]) + ArcTanh[(5/3)^(1/8)*Sqrt[7]*x]/(4840*3^(7/8)*5^(1/8)*Sqrt[7])) - 98*S
qrt[165]*(ArcTan[(5/3)^(1/8)*Sqrt[11]*x]/(1960*3^(7/8)*5^(1/8)*Sqrt[11]) +
 ArcTanh[(5/3)^(1/8)*Sqrt[11]*x]/(1960*3^(7/8)*5^(1/8)*Sqrt[11]))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(85) = 170$.

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.64

method	result
default	$-\frac{\left(-49\sqrt{7}\,\sqrt{15}-98\sqrt{11}\,\sqrt{15}+121\sqrt{105}+98\sqrt{165}\right)\sqrt{35}\,\sqrt{\sqrt{5}\,15^{\frac{1}{4}}}\left(\ln\left(\frac{x+\frac{\sqrt{35}\,\sqrt{\sqrt{5}\,15^{\frac{1}{4}}}}{35}}{x-\frac{\sqrt{35}\,\sqrt{\sqrt{5}\,15^{\frac{1}{4}}}}{35}}\right)+2\arctan\left(\frac{x\sqrt{35}}{\sqrt{\sqrt{5}\,15^{\frac{1}{4}}}}\right)\right)}{604800}+\frac{\left(-121\sqrt{\right.}}{}$

input
```
int((3*7^(1/2)+6*11^(1/2)+(-121*105^(1/2)-98*165^(1/2))*x^4)/(-12*15^(1/2)
+10200*x^4-118580*15^(1/2)*x^8),x,method=_RETURNVERBOSE)
```

output
```
-1/604800*(-49*7^(1/2)*15^(1/2)-98*11^(1/2)*15^(1/2)+121*105^(1/2)+98*165^
(1/2))*35^(1/2)*(5^(1/2)*15^(1/4))^(1/2)*(ln((x+1/35*35^(1/2)*(5^(1/2)*15^
(1/4))^(1/2))/(x-1/35*35^(1/2)*(5^(1/2)*15^(1/4))^(1/2)))+2*arctan(x*35^(1
/2)/(5^(1/2)*15^(1/4))^(1/2)))+1/950400*(-121*7^(1/2)*15^(1/2)-242*11^(1/2
)*15^(1/2)+121*105^(1/2)+98*165^(1/2))*55^(1/2)*(5^(1/2)*15^(1/4))^(1/2)*(
ln((x+1/55*55^(1/2)*(5^(1/2)*15^(1/4))^(1/2))/(x-1/55*55^(1/2)*(5^(1/2)*15
^(1/4))^(1/2)))+2*arctan(x*55^(1/2)/(5^(1/2)*15^(1/4))^(1/2)))
```

Fricas [F]

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx = \int \frac{x^4\left(98\sqrt{165}+121\sqrt{105}\right)-6\sqrt{11}-3\sqrt{7}}{4\left(29645\sqrt{15}x^8-2550x^4+3\sqrt{15}\right)}\,dx$$

input
```
integrate((3*7^(1/2)+6*11^(1/2)+(-121*105^(1/2)-98*165^(1/2))*x^4)/(-12*15
^(1/2)+10200*x^4-118580*15^(1/2)*x^8),x, algorithm="fricas")
```

output
```
0
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((3*7**(1/2)+6*11**(1/2)+(-121*105**(1/2)-98*165**(1/2))*x**4)/(-
12*15**(1/2)+10200*x**4-118580*15**(1/2)*x**8),x)
```

output
```
Exception raised: PolynomialError >> 1/(88766196443113618114494782100594040
03892090960648942070894847733907374495278002081072424698386397863110524278
65545878613259174768870299313417544282422774969138741079283023024302083445
868952121128899
```

Maxima [F]

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx$$
$$= \int \frac{x^4\left(98\sqrt{165}+121\sqrt{105}\right)-6\sqrt{11}-3\sqrt{7}}{4\left(29645\sqrt{15}x^8-2550x^4+3\sqrt{15}\right)}\,dx$$

input
```
integrate((3*7^(1/2)+6*11^(1/2)+(-121*105^(1/2)-98*165^(1/2))*x^4)/(-12*15
^(1/2)+10200*x^4-118580*15^(1/2)*x^8),x, algorithm="maxima")
```

output
```
1/4*integrate((x^4*(98*sqrt(165) + 121*sqrt(105)) - 6*sqrt(11) - 3*sqrt(7)
)/(29645*sqrt(15)*x^8 - 2550*x^4 + 3*sqrt(15)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((3*7^(1/2)+6*11^(1/2)+(-121*105^(1/2)-98*165^(1/2))*x^4)/(-12*15
^(1/2)+10200*x^4-118580*15^(1/2)*x^8),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1274,0,-1480290,0,427747950,0,-19021101750,0]:[1,0,-11
40,0,3163
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 1889, normalized size of antiderivative = 16.15

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx = \text{Too large to display}$$

input
```
int(-(3*7^(1/2) - x^4*(121*105^(1/2) + 98*165^(1/2)) + 6*11^(1/2))/(12*15^
(1/2) + 118580*15^(1/2)*x^8 - 10200*x^4),x)
```

output
```
atan((((39715937915904*x)/176832909307587547962569058918847O5078125 + ((17
*15^(1/2))/147456000 - (5^(1/2)*888446500935303168^(1/2))/5349660268953600
)^(1/4)*((940369969152*7^(1/2)*15^(1/2))/3608834883828317305358552222833361
328125 + (117546246144*11^(1/2)*15^(1/2))/14614290025420458509303228009822
0703125 - ((4092730840461606912*15^(1/2)*x)/707331637230350191850276235675
388203125 + ((34665798542819328*7^(1/2))/288706790706265384428684177826689
0625 + (69331597085638656*11^(1/2))/116914320203363668074425824078576562 5)
*((17*15^(1/2))/147456000 - (5^(1/2)*888446500935303168^(1/2))/53496602689
53600)^(1/4))*((17*15^(1/2))/147456000 - (5^(1/2)*888446500935303168^(1/2)
)/5349660268953600)^(3/4)))*((17*15^(1/2))/147456000 - (5^(1/2)*8884465009
35303168^(1/2))/5349660268953600)^(1/4)*1i + ((39715937915904*x)/176832909
307587547962569058918847O5078125 - ((17*15^(1/2))/147456000 - (5^(1/2)*888
446500935303168^(1/2))/5349660268953600)^(1/4)*((940369969152*7^(1/2)*15^(
1/2))/3608834883828317305358552222833361328125 + (117546246144*11^(1/2)*15^
(1/2))/146142900254204585093032280098220703125 + ((4092730840461606912*15^
(1/2)*x)/707331637230350191850276235675388203125 - ((34665798542819328*7^(
1/2))/2887067907062653844286841778266890625 + (69331597085638656*11^(1/2))
/116914320203363668074425824078576562 5)*((17*15^(1/2))/147456000 - (5^(1/2
)*888446500935303168^(1/2))/5349660268953600)^(1/4))*((17*15^(1/2))/147456
000 - (5^(1/2)*888446500935303168^(1/2))/5349660268953600)^(3/4)))*((17...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{3\sqrt{7}+6\sqrt{11}+\left(-121\sqrt{105}-98\sqrt{165}\right)x^4}{-12\sqrt{15}+10200x^4-118580\sqrt{15}x^8}\,dx$$
$$=\frac{\sqrt{3}\,15^{\frac{1}{8}}5^{\frac{1}{4}}\left(-4atan\left(\frac{11\,5^{\frac{1}{4}}x15^{\frac{7}{8}}}{15\sqrt{11}}\right)-2atan\left(\frac{7\,5^{\frac{1}{4}}x15^{\frac{7}{8}}}{15\sqrt{7}}\right)+2\log\left(-15^{\frac{1}{8}}\sqrt{11}+11\,5^{\frac{1}{4}}x\right)+\log\left(-15^{\frac{1}{8}}\sqrt{7}+7\,5^{\frac{1}{4}}x\right.\right.}{240}$$

input
```
int((3*7^(1/2)+6*11^(1/2)+(-121*105^(1/2)-98*165^(1/2))*x^4)/(-12*15^(1/2)
+10200*x^4-118580*15^(1/2)*x^8),x)
```

output
```
(sqrt(3)*15**(1/8)*5**(1/4)*( - 4*atan((11*5**(1/4)*x)/(15**(1/8)*sqrt(11)
)) - 2*atan((7*5**(1/4)*x)/(15**(1/8)*sqrt(7))) + 2*log( - 15**(1/8)*sqrt(
11) + 11*5**(1/4)*x) + log( - 15**(1/8)*sqrt(7) + 7*5**(1/4)*x) - 2*log(15
**(1/8)*sqrt(11) + 11*5**(1/4)*x) - log(15**(1/8)*sqrt(7) + 7*5**(1/4)*x))
)/240
```

3.51 $\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}$

Optimal result

Integrand size = 105, antiderivative size = 75

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx = \frac{\operatorname{arctanh}\left(\sqrt[4]{\frac{5}{3}}\sqrt{11}x\right)}{8\sqrt[4]{3}5^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}+\frac{2}{\sqrt{15}}}-\frac{7}{2}\sqrt[4]{\frac{5}{3}}x^2\right)}{16\sqrt[4]{3}5^{3/4}}$$

output
```
1/120*arctanh(1/3*5^(1/4)*3^(3/4)*11^(1/2)*x)*3^(3/4)*5^(1/4)+1/240*arctan
h(-1/30*(450+120*15^(1/2))^(1/2)+7/6*5^(1/4)*3^(3/4)*x^2)*3^(3/4)*5^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.77

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx$$
$$=-\frac{1}{80}\text{RootSum}\Big[-30+8\sqrt{15}-650\#1^2+40\sqrt{15}\#1^2+3850\#1^4+1015\sqrt{15}\#1^4$$
$$-13475\#1^6\&,\frac{8\sqrt{11}\log(x-\#1)-2\sqrt{165}\log(x-\#1)-14\sqrt{15}\log(x-\#1)\#1-70\sqrt{11}\log(x-\#1)}{130\#1-8\sqrt{15}\#1-1540\#1^3}$$

input
```
Integrate[(8*Sqrt[11] - 2*Sqrt[165] - 14*Sqrt[15]*x + (-70*Sqrt[11] - 14*S
qrt[165])*x^2 + 770*x^3 + 245*Sqrt[11]*x^4)/(8*(-30 + 8*Sqrt[15]) + 8*(-65
0 + 40*Sqrt[15])*x^2 + 8*(3850 + 1015*Sqrt[15])*x^4 - 107800*x^6),x]
```

output
```
-1/80*RootSum[-30 + 8*Sqrt[15] - 650*#1^2 + 40*Sqrt[15]*#1^2 + 3850*#1^4 +
 1015*Sqrt[15]*#1^4 - 13475*#1^6 & , (8*Sqrt[11]*Log[x - #1] - 2*Sqrt[165]
*Log[x - #1] - 14*Sqrt[15]*Log[x - #1]*#1 - 70*Sqrt[11]*Log[x - #1]*#1^2 -
 14*Sqrt[165]*Log[x - #1]*#1^2 + 770*Log[x - #1]*#1^3 + 245*Sqrt[11]*Log[x
 - #1]*#1^4)/(130*#1 - 8*Sqrt[15]*#1 - 1540*#1^3 - 406*Sqrt[15]*#1^3 + 808
5*#1^5) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{245\sqrt{11}x^4+770x^3+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2-14\sqrt{15}x-2\sqrt{165}+8\sqrt{11}}{-107800x^6+8\left(3850+1015\sqrt{15}\right)x^4+8\left(40\sqrt{15}-650\right)x^2+8\left(8\sqrt{15}-30\right)}dx$$
$$\Big\downarrow \text{ 7292}$$
$$\int \frac{-245\sqrt{11}x^4-770x^3+14\sqrt{55}\left(\sqrt{3}+\sqrt{5}\right)x^2+14\sqrt{15}x-2\sqrt{11}\left(4-\sqrt{15}\right)}{8\left(13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)\right)}dx$$
$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int -\frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4+10\left(65-4\sqrt{15}\right)x^2+2\left(15-4\sqrt{15}\right)}dx$$

$\Big\downarrow$ 25

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4+10\left(65-4\sqrt{15}\right)x^2+2\left(15-4\sqrt{15}\right)}dx$$

$\Big\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\Big\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\Big\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\Big\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\Big\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\Big\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}\left(\sqrt{3}+\sqrt{5}\right)x^2-14\sqrt{15}x+2\sqrt{11}\left(4-\sqrt{15}\right)}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}\left(\sqrt{3}+\sqrt{5}\right)x^2-14\sqrt{15}x+2\sqrt{11}\left(4-\sqrt{15}\right)}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}\left(\sqrt{3}+\sqrt{5}\right)x^2-14\sqrt{15}x+2\sqrt{11}\left(4-\sqrt{15}\right)}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}\left(\sqrt{3}+\sqrt{5}\right)x^2-14\sqrt{15}x+2\sqrt{11}\left(4-\sqrt{15}\right)}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$\downarrow$ 7239

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$\downarrow$ 7293

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$$\Big\downarrow \text{7239}$$

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$$\Big\downarrow \text{7239}$$

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$$\Big\downarrow \text{7239}$$

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{8}\int \left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

$$\Big\downarrow \text{7239}$$

$$-\frac{1}{8}\int \frac{245\sqrt{11}x^4+770x^3-14\sqrt{55}(\sqrt{3}+\sqrt{5})\,x^2-14\sqrt{15}x+2\sqrt{11}(4-\sqrt{15})}{13475x^6-35\left(110+29\sqrt{15}\right)x^4-10\left(-65+4\sqrt{15}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{8}\int\left(\frac{245\sqrt{11}x^4}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)x^4+650\left(1-\frac{4\sqrt{\frac{3}{5}}}{13}\right)x^2+30\left(1-\frac{4}{\sqrt{15}}\right)}+\frac{}{13475x^6-3850\left(1+\frac{29\sqrt{\frac{3}{5}}}{22}\right)}\right.$$

input
```
Int[(8*Sqrt[11] - 2*Sqrt[165] - 14*Sqrt[15]*x + (-70*Sqrt[11] - 14*Sqrt[16
5])*x^2 + 770*x^3 + 245*Sqrt[11]*x^4)/(8*(-30 + 8*Sqrt[15]) + 8*(-650 + 40
*Sqrt[15])*x^2 + 8*(3850 + 1015*Sqrt[15])*x^4 - 107800*x^6),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{11}\,\sqrt{55}\,15^{\frac{3}{4}}\operatorname{arctanh}\left(\frac{x\sqrt{55}\,15^{\frac{3}{4}}}{15}\right)}{6600}+\frac{\sqrt{5}\,15^{\frac{3}{4}}\operatorname{arctanh}\left(\frac{\left(490x^{2}-14\sqrt{15}-70\right)\sqrt{5}\,15^{\frac{3}{4}}}{2100}\right)}{1200}$	52

input
```
int((8*11^(1/2)-2*165^(1/2)-14*15^(1/2)*x+(-70*11^(1/2)-14*165^(1/2))*x^2+
770*x^3+245*11^(1/2)*x^4)/(-240+64*15^(1/2)+8*(-650+40*15^(1/2))*x^2+8*(38
50+1015*15^(1/2))*x^4-107800*x^6),x,method=_RETURNVERBOSE)
```

output
```
1/6600*11^(1/2)*55^(1/2)*15^(3/4)*arctanh(1/15*x*55^(1/2)*15^(3/4))+1/1200
*5^(1/2)*15^(3/4)*arctanh(1/2100*(490*x^2-14*15^(1/2)-70)*5^(1/2)*15^(3/4)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{8\sqrt{11} - 2\sqrt{165} - 14\sqrt{15}x + \left(-70\sqrt{11} - 14\sqrt{165}\right)x^2 + 770x^3 + 245\sqrt{11}x^4}{8\left(-30 + 8\sqrt{15}\right) + 8\left(-650 + 40\sqrt{15}\right)x^2 + 8\left(3850 + 1015\sqrt{15}\right)x^4 - 107800x^6}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((8*11^(1/2)-2*165^(1/2)-14*15^(1/2)*x+(-70*11^(1/2)-14*165^(1/2)
)*x^2+770*x^3+245*11^(1/2)*x^4)/(-240+64*15^(1/2)+8*(-650+40*15^(1/2))*x^2
+8*(3850+1015*15^(1/2))*x^4-107800*x^6),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{8\sqrt{11} - 2\sqrt{165} - 14\sqrt{15}x + \left(-70\sqrt{11} - 14\sqrt{165}\right)x^2 + 770x^3 + 245\sqrt{11}x^4}{8\left(-30 + 8\sqrt{15}\right) + 8\left(-650 + 40\sqrt{15}\right)x^2 + 8\left(3850 + 1015\sqrt{15}\right)x^4 - 107800x^6}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((8*11**(1/2)-2*165**(1/2)-14*15**(1/2)*x+(-70*11**(1/2)-14*165**
(1/2))*x**2+770*x**3+245*11**(1/2)*x**4)/(-240+64*15**(1/2)+8*(-650+40*15*
*(1/2))*x**2+8*(3850+1015*15**(1/2))*x**4-107800*x**6),x)
```

output
```
Exception raised: PolynomialError >> 1/(8026406129722253778490365429016317
86571585667051597111057095382995817099486707204483301622083173133484870882
5122193224499200000000000000000000000000000000*_t**48 + 2072409151354094
443219300735040
```

Maxima [F]

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx$$
$$=\int -\frac{245\sqrt{11}x^4+770\,x^3-14\,x^2\left(\sqrt{165}+5\sqrt{11}\right)-14\sqrt{15}x-2\sqrt{165}+8\sqrt{11}}{8\left(13475\,x^6-35\,x^4\left(29\sqrt{15}+110\right)-10\,x^2\left(4\sqrt{15}-65\right)-8\sqrt{15}+30\right)}\,dx$$

input
```
integrate((8*11^(1/2)-2*165^(1/2)-14*15^(1/2)*x+(-70*11^(1/2)-14*165^(1/2)
)*x^2+770*x^3+245*11^(1/2)*x^4)/(-240+64*15^(1/2)+8*(-650+40*15^(1/2))*x^2
+8*(3850+1015*15^(1/2))*x^4-107800*x^6),x, algorithm="maxima")
```

output
```
-1/8*integrate((245*sqrt(11)*x^4 + 770*x^3 - 14*x^2*(sqrt(165) + 5*sqrt(11
)) - 14*sqrt(15)*x - 2*sqrt(165) + 8*sqrt(11))/(13475*x^6 - 35*x^4*(29*sqr
t(15) + 110) - 10*x^2*(4*sqrt(15) - 65) - 8*sqrt(15) + 30), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx$$
$$=\text{Timed out}$$

input
```
integrate((8*11^(1/2)-2*165^(1/2)-14*15^(1/2)*x+(-70*11^(1/2)-14*165^(1/2)
)*x^2+770*x^3+245*11^(1/2)*x^4)/(-240+64*15^(1/2)+8*(-650+40*15^(1/2))*x^2
+8*(3850+1015*15^(1/2))*x^4-107800*x^6),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 53.74 (sec) , antiderivative size = 850, normalized size of antiderivative = 11.33

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx$$
$$= \text{Too large to display}$$

input
```
int(-(x^2*(70*11^(1/2) + 14*165^(1/2)) + 14*15^(1/2)*x - 8*11^(1/2) + 2*16
5^(1/2) - 245*11^(1/2)*x^4 - 770*x^3)/(8*x^2*(40*15^(1/2) - 650) + 8*x^4*(
1015*15^(1/2) + 3850) + 64*15^(1/2) - 107800*x^6 - 240),x)
```

output
```
symsum(log((1087204857*x)/125820422412127520000000 - (76374500551*root(31
15770387240868380672000000000*15^(1/2)*z^6 - 1205708599628486148096000000
000*z^6 + 94195984345975480320000000*15^(1/2)*z^4 - 365129342254789263360
0000000*z^4 + 71314324659138528000000*15^(1/2)*z^2 - 27596479788860004000
000*z^2 + 15970185062997687 50*15^(1/2) - 6190479571105775000, z, k)*x)/110
0928696106115800000000 - (63923093*15^(1/2)*x)/571911010964216000000000 - (4
718059*11^(1/2))/45492921326699000000000 + (692479*165^(1/2))/2599595504382
800000000 - (4154874*11^(1/2)*root(3115770387240868380672000000000*15^(1/2
)*z^6 - 1205708599628486148096000000000*z^6 + 94195984345975480320000000
*15^(1/2)*z^4 - 3651293422547892633600000000*z^4 + 71314324659138528000000
*15^(1/2)*z^2 - 27596479788860004000000*z^2 + 1597018506299768750*15^(1/2
) - 6190479571105775000, z, k)^2)/101546699389953125 - (57975508992*11^(1/
2)*root(3115770387240868380672000000000*15^(1/2)*z^6 - 120570859962848614 8
096000000000*z^6 + 94195984345975480320000000*15^(1/2)*z^4 - 36512934225
47892633600000000*z^4 + 71314324659138528000000*15^(1/2)*z^2 - 27596479788
8600040000000*z^2 + 1597018506299768750*15^(1/2) - 6190479571105775000, z,
 k)^4)/142165379145934375 + (37744472*165^(1/2)*root(311577038724086838067
2000000000*15^(1/2)*z^6 - 1205708599628486148096000000000*z^6 + 941959843
459754803200000000*15^(1/2)*z^4 - 3651293422547892633600000000*z^4 + 71314
324659138528000000*15^(1/2)*z^2 - 27596479788860004000000*z^2 + 159701...
```

Reduce [F]

$$\int \frac{8\sqrt{11}-2\sqrt{165}-14\sqrt{15}x+\left(-70\sqrt{11}-14\sqrt{165}\right)x^2+770x^3+245\sqrt{11}x^4}{8\left(-30+8\sqrt{15}\right)+8\left(-650+40\sqrt{15}\right)x^2+8\left(3850+1015\sqrt{15}\right)x^4-107800x^6}\,dx$$
$$= \frac{\sqrt{3}\,15^{\frac{1}{4}}\log\left(5^{\frac{1}{4}}\sqrt{11}\,x+3^{\frac{1}{4}}\right)}{240}-\frac{\sqrt{3}\,15^{\frac{1}{4}}\log\left(5^{\frac{1}{4}}\sqrt{11}\,x-3^{\frac{1}{4}}\right)}{240}$$
$$+\frac{3807265\sqrt{15}\left(\int \frac{x^5}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{821}$$
$$+\frac{3948159\sqrt{15}\left(\int \frac{x^3}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{1642}$$
$$+\frac{311141\sqrt{15}\left(\int \frac{x}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{11494}$$
$$+\frac{9361\sqrt{15}\log\left(5^{\frac{1}{4}}\sqrt{11}\,x+3^{\frac{1}{4}}\right)}{1379280}+\frac{9361\sqrt{15}\log\left(5^{\frac{1}{4}}\sqrt{11}\,x-3^{\frac{1}{4}}\right)}{1379280}$$
$$-\frac{121\sqrt{15}\log(60025x^8-34300x^6+5880x^4-1960x^2+4)}{65680}$$
$$+\frac{803\sqrt{15}\log\left(11\sqrt{5}\,x^2+\sqrt{3}\right)}{1379280}$$
$$+\frac{170769655\left(\int \frac{x^5}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{3284}$$
$$+\frac{4626185\left(\int \frac{x^3}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{1642}$$
$$+\frac{816850\left(\int \frac{x}{36315125x^{12}-20751500x^{10}+3377325x^8-1082900x^6-15220x^4+5880x^2-12}dx\right)}{5747}$$
$$+\frac{1089\log\left(5^{\frac{1}{4}}\sqrt{11}\,x+3^{\frac{1}{4}}\right)}{22988}+\frac{1089\log\left(5^{\frac{1}{4}}\sqrt{11}\,x-3^{\frac{1}{4}}\right)}{22988}$$
$$-\frac{121\log(60025x^8-34300x^6+5880x^4-1960x^2+4)}{13136}-\frac{121\log\left(11\sqrt{5}\,x^2+\sqrt{3}\right)}{11494}$$

input
```
int((8*11^(1/2)-2*165^(1/2)-14*15^(1/2)*x+(-70*11^(1/2)-14*165^(1/2))*x^2+
770*x^3+245*11^(1/2)*x^4)/(-240+64*15^(1/2)+8*(-650+40*15^(1/2))*x^2+8*(38
50+1015*15^(1/2))*x^4-107800*x^6),x)
```

output

```
(5747*sqrt(3)*15**(1/4)*log(5**(1/4)*sqrt(11)*x + 3**(1/4)) - 5747*sqrt(3)
*15**(1/4)*log(5**(1/4)*sqrt(11)*x - 3**(1/4)) + 6396205200*sqrt(15)*int(x
**5/(36315125*x**12 - 20751500*x**10 + 3377325*x**8 - 1082900*x**6 - 15220
*x**4 + 5880*x**2 - 12),x) + 3316453560*sqrt(15)*int(x**3/(36315125*x**12
- 20751500*x**10 + 3377325*x**8 - 1082900*x**6 - 15220*x**4 + 5880*x**2 -
12),x) + 37336920*sqrt(15)*int(x/(36315125*x**12 - 20751500*x**10 + 337732
5*x**8 - 1082900*x**6 - 15220*x**4 + 5880*x**2 - 12),x) + 9361*sqrt(15)*lo
g(5**(1/4)*sqrt(11)*x + 3**(1/4)) + 9361*sqrt(15)*log(5**(1/4)*sqrt(11)*x
- 3**(1/4)) - 2541*sqrt(15)*log(60025*x**8 - 34300*x**6 + 5880*x**4 - 1960
*x**2 + 4) + 803*sqrt(15)*log(11*sqrt(5)*x**2 + sqrt(3)) + 71723255100*int
(x**5/(36315125*x**12 - 20751500*x**10 + 3377325*x**8 - 1082900*x**6 - 152
20*x**4 + 5880*x**2 - 12),x) + 3885995400*int(x**3/(36315125*x**12 - 20751
500*x**10 + 3377325*x**8 - 1082900*x**6 - 15220*x**4 + 5880*x**2 - 12),x)
+ 196044000*int(x/(36315125*x**12 - 20751500*x**10 + 3377325*x**8 - 108290
0*x**6 - 15220*x**4 + 5880*x**2 - 12),x) + 65340*log(5**(1/4)*sqrt(11)*x +
 3**(1/4)) + 65340*log(5**(1/4)*sqrt(11)*x - 3**(1/4)) - 12705*log(60025*x
**8 - 34300*x**6 + 5880*x**4 - 1960*x**2 + 4) - 14520*log(11*sqrt(5)*x**2
+ sqrt(3)))/1379280
```

3.52 $\int \frac{(-20\sqrt{3}+3\sqrt{5})x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}(70-90\sqrt{15})x^2+4\sqrt{3}(140-25\sqrt{15})x^4+4\sqrt{3}(-150-150\sqrt{1$

Optimal result

Integrand size = 116, antiderivative size = 204

$$\int \frac{(-20\sqrt{3}+3\sqrt{5})\,x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\,(70-90\sqrt{15})\,x^2+4\sqrt{3}\,(140-25\sqrt{15})\,x^4+4\sqrt{3}\,(-150-150\sqrt{15})\,x^6+1800\sqrt{3}x^8}\,dx$$
$$= -\frac{1}{80}\text{RootSum}\Big[2+70\#1^2-90\sqrt{15}\#1^2+140\#1^4-25\sqrt{15}\#1^4-150\#1^6-150\sqrt{15}\#1^6$$
$$+450\#1^8\&,\frac{20\sqrt{3}\log(x-\#1)-3\sqrt{5}\log(x-\#1)-36\sqrt{5}\log(x-\#1)\#1^2+75\sqrt{3}\log(x-\#1)\#1^4}{7\sqrt{3}-27\sqrt{5}+28\sqrt{3}\#1^2-15\sqrt{5}\#1^2-45\sqrt{3}\#1^4-135\sqrt{5}\#1^4+180\sqrt{3}\#1^6}\&$$

output
```
-1/80*RootSum(_Z1 -> 2+70*_Z1^2-90*15^(1/2)*_Z1^2+140*_Z1^4-25*15^(1/2)*_Z
1^4-150*_Z1^6-150*15^(1/2)*_Z1^6+450*_Z1^8,_Z1 -> (20*3^(1/2)*ln(x-_Z1)-3*
5^(1/2)*ln(x-_Z1)-36*5^(1/2)*ln(x-_Z1)*_Z1^2+75*3^(1/2)*ln(x-_Z1)*_Z1^4)/(
7*3^(1/2)-27*5^(1/2)+28*3^(1/2)*_Z1^2-15*5^(1/2)*_Z1^2-45*3^(1/2)*_Z1^4-13
5*5^(1/2)*_Z1^4+180*3^(1/2)*_Z1^6))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.57

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
$$= -\frac{\left(20\sqrt{3}-3\sqrt{5}-36\sqrt{5}x^2+75\sqrt{3}x^4\right)\text{RootSum}\Big[2\sqrt{3}+70\sqrt{3}\#1^2-270\sqrt{5}\#1^2+140\sqrt{3}\#1^4-75\sqrt{5}}{80\,(-}$$

input
```
Integrate[((-20*Sqrt[3] + 3*Sqrt[5])*x + 36*Sqrt[5]*x^3 - 75*Sqrt[3]*x^5)/
(8*Sqrt[3] + 4*Sqrt[3]*(70 - 90*Sqrt[15])*x^2 + 4*Sqrt[3]*(140 - 25*Sqrt[1
5])*x^4 + 4*Sqrt[3]*(-150 - 150*Sqrt[15])*x^6 + 1800*Sqrt[3]*x^8),x]
```

output
```
-1/80*((20*Sqrt[3] - 3*Sqrt[5] - 36*Sqrt[5]*x^2 + 75*Sqrt[3]*x^4)*RootSum[
2*Sqrt[3] + 70*Sqrt[3]*#1^2 - 270*Sqrt[5]*#1^2 + 140*Sqrt[3]*#1^4 - 75*Sqr
t[5]*#1^4 - 150*Sqrt[3]*#1^6 - 450*Sqrt[5]*#1^6 + 450*Sqrt[3]*#1^8 & , (-7
5*Log[x - #1] + 23*Sqrt[15]*Log[x - #1] - 180*Log[x - #1]*#1^2 + 36*Sqrt[1
5]*Log[x - #1]*#1^2 - 225*Log[x - #1]*#1^4 + 75*Sqrt[15]*Log[x - #1]*#1^4)
/(7*Sqrt[3] - 27*Sqrt[5] + 28*Sqrt[3]*#1^2 - 15*Sqrt[5]*#1^2 - 45*Sqrt[3]*
#1^4 - 135*Sqrt[5]*#1^4 + 180*Sqrt[3]*#1^6) & ])/(-75 + 23*Sqrt[15] + 36*(
-5 + Sqrt[15])*x^2 + 75*(-3 + Sqrt[15])*x^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-75\sqrt{3}x^5+36\sqrt{5}x^3+\left(3\sqrt{5}-20\sqrt{3}\right)x}{1800\sqrt{3}x^8+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+8\sqrt{3}}dx$$
$$\Big\downarrow \text{ 2028}$$
$$\int \frac{x\left(-75\sqrt{3}x^4+36\sqrt{5}x^2+3\sqrt{5}-20\sqrt{3}\right)}{1800\sqrt{3}x^8+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+8\sqrt{3}}dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int -\frac{75\sqrt{3}x^4-36\sqrt{5}x^2-3\sqrt{5}+20\sqrt{3}}{4\left(450\sqrt{3}x^8-150\sqrt{3}\left(1+\sqrt{15}\right)x^6+5\sqrt{3}\left(28-5\sqrt{15}\right)x^4+10\sqrt{3}\left(7-9\sqrt{15}\right)x^2+2\sqrt{3}\right)}dx^2$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{8}\int \frac{75\sqrt{3}x^4-36\sqrt{5}x^2-3\sqrt{5}+20\sqrt{3}}{450\sqrt{3}x^8-150\sqrt{3}\left(1+\sqrt{15}\right)x^6+5\sqrt{3}\left(28-5\sqrt{15}\right)x^4+10\sqrt{3}\left(7-9\sqrt{15}\right)x^2+2\sqrt{3}}dx^2$$

$$\Big\downarrow \; 7293$$

$$-\frac{1}{8}\int \left(\frac{75x^4}{450x^8-150\left(1+\sqrt{15}\right)x^6+5\left(28-5\sqrt{15}\right)x^4+10\left(7-9\sqrt{15}\right)x^2+2}+\frac{}{-450x^8+150\left(1+\sqrt{15}\right)x^6-}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{8}\left(\left(20-\sqrt{15}\right)\int \frac{1}{-450x^8+150\left(1+\sqrt{15}\right)x^6-5\left(28-5\sqrt{15}\right)x^4-10\left(7-9\sqrt{15}\right)x^2-2}dx^2-12\sqrt{15}\int \frac{}{-45}\right.$$

input
```
Int[((-20*Sqrt[3] + 3*Sqrt[5])*x + 36*Sqrt[5]*x^3 - 75*Sqrt[3]*x^5)/(8*Sqr
t[3] + 4*Sqrt[3]*(70 - 90*Sqrt[15])*x^2 + 4*Sqrt[3]*(140 - 25*Sqrt[15])*x^
4 + 4*Sqrt[3]*(-150 - 150*Sqrt[15])*x^6 + 1800*Sqrt[3]*x^8),x]
```

output
```
$Aborted
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(11) = 22.

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\sqrt{3}\left(\frac{\sqrt{3}\,\sqrt{5}\,15^{\frac{3}{4}}\,\operatorname{arctanh}\left(\frac{\left(20x^{2}-2\sqrt{15}-10\right)\sqrt{5}\,15^{\frac{3}{4}}}{300}\right)}{300}-\frac{\sqrt{3}\,\sqrt{5}\,15^{\frac{3}{4}}\,\arctan\left(\frac{\left(90x^{2}-6\sqrt{15}+30\right)\sqrt{5}\,15^{\frac{3}{4}}}{900}\right)}{300}\right)}{12}$	71

input
```
int(((-20*3^(1/2)+3*5^(1/2))*x+36*5^(1/2)*x^3-75*3^(1/2)*x^5)/(8*3^(1/2)+4
*3^(1/2)*(70-90*15^(1/2))*x^2+4*3^(1/2)*(140-25*15^(1/2))*x^4+4*3^(1/2)*(-
150-150*15^(1/2))*x^6+1800*3^(1/2)*x^8),x,method=_RETURNVERBOSE)
```

output
```
1/12*3^(1/2)*(1/300*3^(1/2)*5^(1/2)*15^(3/4)*arctanh(1/300*(20*x^2-2*15^(1
/2)-10)*5^(1/2)*15^(3/4))-1/300*3^(1/2)*5^(1/2)*15^(3/4)*arctan(1/900*(90*
x^2-6*15^(1/2)+30)*5^(1/2)*15^(3/4)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^{3}-75\sqrt{3}x^{5}}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^{2}+4\sqrt{3}\left(140-25\sqrt{15}\right)x^{4}+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^{6}+1800\sqrt{3}x^{8}}\,dx$$
$$=\text{Timed out}$$

input
```
integrate(((-20*3^(1/2)+3*5^(1/2))*x+36*5^(1/2)*x^3-75*3^(1/2)*x^5)/(8*3^(
1/2)+4*3^(1/2)*(70-90*15^(1/2))*x^2+4*3^(1/2)*(140-25*15^(1/2))*x^4+4*3^(1
/2)*(-150-150*15^(1/2))*x^6+1800*3^(1/2)*x^8),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
$$= \text{Timed out}$$

input
```
integrate(((-20*3**(1/2)+3*5**(1/2))*x+36*5**(1/2)*x**3-75*3**(1/2)*x**5)/
(8*3**(1/2)+4*3**(1/2)*(70-90*15**(1/2))*x**2+4*3**(1/2)*(140-25*15**(1/2)
)*x**4+4*3**(1/2)*(-150-150*15**(1/2))*x**6+1800*3**(1/2)*x**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
$$= \int -\frac{75\sqrt{3}x^5-36\sqrt{5}x^3-x\left(3\sqrt{5}-20\sqrt{3}\right)}{4\left(450\sqrt{3}x^8-150\sqrt{3}x^6\left(\sqrt{15}+1\right)-5\sqrt{3}x^4\left(5\sqrt{15}-28\right)-10\sqrt{3}x^2\left(9\sqrt{15}-7\right)+2\sqrt{3}\right)}\,dx$$

input
```
integrate(((-20*3^(1/2)+3*5^(1/2))*x+36*5^(1/2)*x^3-75*3^(1/2)*x^5)/(8*3^(
1/2)+4*3^(1/2)*(70-90*15^(1/2))*x^2+4*3^(1/2)*(140-25*15^(1/2))*x^4+4*3^(1
/2)*(-150-150*15^(1/2))*x^6+1800*3^(1/2)*x^8),x, algorithm="maxima")
```

output
```
-1/4*integrate((75*sqrt(3)*x^5 - 36*sqrt(5)*x^3 - x*(3*sqrt(5) - 20*sqrt(3
)))/(450*sqrt(3)*x^8 - 150*sqrt(3)*x^6*(sqrt(15) + 1) - 5*sqrt(3)*x^4*(5*s
qrt(15) - 28) - 10*sqrt(3)*x^2*(9*sqrt(15) - 7) + 2*sqrt(3)), x)
```

Giac [F]

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
$$=\int -\frac{75\sqrt{3}x^5-36\sqrt{5}x^3-x\left(3\sqrt{5}-20\sqrt{3}\right)}{4\left(450\sqrt{3}x^8-150\sqrt{3}x^6\left(\sqrt{15}+1\right)-5\sqrt{3}x^4\left(5\sqrt{15}-28\right)-10\sqrt{3}x^2\left(9\sqrt{15}-7\right)+2\sqrt{3}\right)}\,dx$$

input
```
integrate(((-20*3^(1/2)+3*5^(1/2))*x+36*5^(1/2)*x^3-75*3^(1/2)*x^5)/(8*3^(
1/2)+4*3^(1/2)*(70-90*15^(1/2))*x^2+4*3^(1/2)*(140-25*15^(1/2))*x^4+4*3^(1
/2)*(-150-150*15^(1/2))*x^6+1800*3^(1/2)*x^8),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.73

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int((75*3^(1/2)*x^5 - 36*5^(1/2)*x^3 + x*(20*3^(1/2) - 3*5^(1/2)))/(4*3^(1
/2)*x^2*(90*15^(1/2) - 70) - 1800*3^(1/2)*x^8 - 8*3^(1/2) + 4*3^(1/2)*x^4*
(25*15^(1/2) - 140) + 4*3^(1/2)*x^6*(150*15^(1/2) + 150)),x)
```

output
```
symsum(log((519438023*15^(1/2)*root(164974559232000000*15^(1/2)*z^4 - 1750
227232358400000*z^4 - 419552000*15^(1/2) + 4451058025, z, k))/637729200000
000000 - (13405639*root(164974559232000000*15^(1/2)*z^4 - 1750227232358400
000*z^4 - 419552000*15^(1/2) + 4451058025, z, k))/25509168000000000 + (336
71*15^(1/2))/170061120000000000 + (123517*15^(1/2)*root(164974559232000000
*15^(1/2)*z^4 - 1750227232358400000*z^4 - 419552000*15^(1/2) + 4451058025,
 z, k)^2)/29893556250000 + (97241057*15^(1/2)*root(164974559232000000*15^(
1/2)*z^4 - 1750227232358400000*z^4 - 419552000*15^(1/2) + 4451058025, z, k
)^3)/2491129687500 - (395459293*15^(1/2)*root(164974559232000000*15^(1/2)*
z^4 - 1750227232358400000*z^4 - 419552000*15^(1/2) + 4451058025, z, k)^4)/
3736694531250 + (15882735488*15^(1/2)*root(164974559232000000*15^(1/2)*z^4
 - 1750227232358400000*z^4 - 419552000*15^(1/2) + 4451058025, z, k)^5)/415
18828125 + (164204969984*15^(1/2)*root(164974559232000000*15^(1/2)*z^4 - 1
750227232358400000*z^4 - 419552000*15^(1/2) + 4451058025, z, k)^6)/2491129
6875 - (185540082571*root(164974559232000000*15^(1/2)*z^4 - 17502272323584
00000*z^4 - 419552000*15^(1/2) + 4451058025, z, k)*x^2)/306110016000000000
 - (5307890617*15^(1/2)*x^2)/8162933760000000000 + (114841366009*root(1649
74559232000000*15^(1/2)*z^4 - 1750227232358400000*z^4 - 419552000*15^(1/2)
 + 4451058025, z, k)^2)/191318760000000000 + (6880723*root(164974559232000
000*15^(1/2)*z^4 - 1750227232358400000*z^4 - 419552000*15^(1/2) + 44510...
```

Reduce [F]

$$\int \frac{\left(-20\sqrt{3}+3\sqrt{5}\right)x+36\sqrt{5}x^3-75\sqrt{3}x^5}{8\sqrt{3}+4\sqrt{3}\left(70-90\sqrt{15}\right)x^2+4\sqrt{3}\left(140-25\sqrt{15}\right)x^4+4\sqrt{3}\left(-150-150\sqrt{15}\right)x^6+1800\sqrt{3}x^8}\,dx$$
= Too large to display

input
```
int(((-20*3^(1/2)+3*5^(1/2))*x+36*5^(1/2)*x^3-75*3^(1/2)*x^5)/(8*3^(1/2)+4
*3^(1/2)*(70-90*15^(1/2))*x^2+4*3^(1/2)*(140-25*15^(1/2))*x^4+4*3^(1/2)*(-
150-150*15^(1/2))*x^6+1800*3^(1/2)*x^8),x)
```

output
```
( - 5850*sqrt(15)*int(x**11/(202500*x**16 - 135000*x**14 - 189000*x**12 -
91500*x**10 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) -
3225*sqrt(15)*int(x**9/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500
*x**10 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) - 8220*
sqrt(15)*int(x**7/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**1
0 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) + 480*sqrt(1
5)*int(x**5/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**10 - 41
3975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) - 1706*sqrt(15)*in
t(x**3/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**10 - 413975*
x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) + 2*sqrt(15)*int(x/(202
500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**10 - 413975*x**8 - 4850
0*x**6 - 116040*x**4 + 280*x**2 + 4),x) - 33750*int(x**13/(202500*x**16 -
135000*x**14 - 189000*x**12 - 91500*x**10 - 413975*x**8 - 48500*x**6 - 116
040*x**4 + 280*x**2 + 4),x) + 11250*int(x**11/(202500*x**16 - 135000*x**14
 - 189000*x**12 - 91500*x**10 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 2
80*x**2 + 4),x) + 7500*int(x**9/(202500*x**16 - 135000*x**14 - 189000*x**1
2 - 91500*x**10 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x
) + 4500*int(x**7/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**1
0 - 413975*x**8 - 48500*x**6 - 116040*x**4 + 280*x**2 + 4),x) + 13625*int(
x**5/(202500*x**16 - 135000*x**14 - 189000*x**12 - 91500*x**10 - 413975...
```

3.53 $\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx$

Optimal result

Integrand size = 87, antiderivative size = 71

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx = -\frac{1}{8}\sqrt[4]{3}\arctan\left(\sqrt{-\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{1}{2}3^{3/4}x^2\right)-\frac{1}{8}\sqrt[4]{3}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{x^2}{\sqrt[4]{3}}\right)$$

output
```
1/8*3^(1/4)*arctan(-1/6*(-18+12*3^(1/2))^(1/2)+1/2*3^(3/4)*x^2)+1/8*3^(1/4
)*arctanh(-1/6*(18+12*3^(1/2))^(1/2)+1/3*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.51

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx = \frac{1}{8}\text{RootSum}\left[2+10\#1^2-30\sqrt{3}\#1^2+50\#1^4-5\sqrt{3}\#1^4-6\#1^6-30\sqrt{3}\#1^6+18\#1^8\&,\frac{15\log(x-\#1)-2\sqrt{3}\log(x-\#1)+12\sqrt{3}\log(x-\#1)\#1^2+3\sqrt{3}\log(x-\#1)\#1^4}{-5+15\sqrt{3}-50\#1^2+5\sqrt{3}\#1^2+9\#1^4+45\sqrt{3}\#1^4-36\#1^6}\&\right]$$

input
```
Integrate[(15*x - 2*Sqrt[3]*x + 12*Sqrt[3]*x^3 + 3*Sqrt[3]*x^5)/(-4 - 20*x
^2 + 60*Sqrt[3]*x^2 - 100*x^4 + 10*Sqrt[3]*x^4 + 12*x^6 + 60*Sqrt[3]*x^6 -
 36*x^8),x]
```

output
```
RootSum[2 + 10*#1^2 - 30*Sqrt[3]*#1^2 + 50*#1^4 - 5*Sqrt[3]*#1^4 - 6*#1^6
- 30*Sqrt[3]*#1^6 + 18*#1^8 & , (15*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] +
12*Sqrt[3]*Log[x - #1]*#1^2 + 3*Sqrt[3]*Log[x - #1]*#1^4)/(-5 + 15*Sqrt[3]
 - 50*#1^2 + 5*Sqrt[3]*#1^2 + 9*#1^4 + 45*Sqrt[3]*#1^4 - 36*#1^6) & ]/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3\sqrt{3}x^5+12\sqrt{3}x^3-2\sqrt{3}x+15x}{-36x^8+60\sqrt{3}x^6+12x^6+10\sqrt{3}x^4-100x^4+60\sqrt{3}x^2-20x^2-4}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{3\sqrt{3}x^5+12\sqrt{3}x^3+\left(15-2\sqrt{3}\right)x}{-36x^8+60\sqrt{3}x^6+12x^6+10\sqrt{3}x^4-100x^4+60\sqrt{3}x^2-20x^2-4}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{3\sqrt{3}x^5+12\sqrt{3}x^3+\left(15-2\sqrt{3}\right)x}{-36x^8+60\sqrt{3}x^6+12x^6+10\sqrt{3}x^4-100x^4+\left(60\sqrt{3}-20\right)x^2-4}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{3\sqrt{3}x^5+12\sqrt{3}x^3+\left(15-2\sqrt{3}\right)x}{-36x^8+60\sqrt{3}x^6+12x^6+\left(10\sqrt{3}-100\right)x^4+\left(60\sqrt{3}-20\right)x^2-4}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{3\sqrt{3}x^5+12\sqrt{3}x^3+\left(15-2\sqrt{3}\right)x}{-36x^8+\left(12+60\sqrt{3}\right)x^6+\left(10\sqrt{3}-100\right)x^4+\left(60\sqrt{3}-20\right)x^2-4}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x\left(3\sqrt{3}x^4+12\sqrt{3}x^2-2\sqrt{3}+15\right)}{-36x^8+\left(12+60\sqrt{3}\right)x^6+\left(10\sqrt{3}-100\right)x^4+\left(60\sqrt{3}-20\right)x^2-4}dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int -\frac{3\sqrt{3}x^4+12\sqrt{3}x^2-2\sqrt{3}+15}{2\left(18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2\right)}dx^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{3\sqrt{3}x^4+12\sqrt{3}x^2-2\sqrt{3}+15}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2}dx^2$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{4}\int \left(\frac{3\sqrt{3}x^4}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2}+\frac{12\sqrt{3}x^2}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\left(15-2\sqrt{3}\right)\int \frac{1}{-18x^8+6\left(1+5\sqrt{3}\right)x^6-5\left(10-\sqrt{3}\right)x^4-10\left(1-3\sqrt{3}\right)x^2-2}dx^2-12\sqrt{3}\int \frac{}{18x^8-6\left(1+\right.}\right.$$

input
```
Int[(15*x - 2*Sqrt[3]*x + 12*Sqrt[3]*x^3 + 3*Sqrt[3]*x^5)/(-4 - 20*x^2 + 6
0*Sqrt[3]*x^2 - 100*x^4 + 10*Sqrt[3]*x^4 + 12*x^6 + 60*Sqrt[3]*x^6 - 36*x^
8),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{3^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\left(4x^2-2\sqrt{3}-2\right)3^{\frac{3}{4}}}{12}\right)}{8}+\frac{3^{\frac{1}{4}}\arctan\left(\frac{\left(18x^2-6\sqrt{3}+6\right)3^{\frac{3}{4}}}{36}\right)}{8}$	48

input
```
int((15*x-2*3^(1/2)*x+12*3^(1/2)*x^3+3*3^(1/2)*x^5)/(-4-20*x^2+60*3^(1/2)*
x^2-100*x^4+10*3^(1/2)*x^4+12*x^6+60*3^(1/2)*x^6-36*x^8),x,method=_RETURNV
ERBOSE)
```

output
```
1/8*3^(1/4)*arctanh(1/12*(4*x^2-2*3^(1/2)-2)*3^(3/4))+1/8*3^(1/4)*arctan(1
/36*(18*x^2-6*3^(1/2)+6)*3^(3/4))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.46

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx$$
$$= -\frac{1}{8}\cdot 3^{\frac{1}{4}}\arctan\left(-\frac{1}{12}\cdot 3^{\frac{3}{4}}\left(5\,x^2-2\,\sqrt{3}\right)-\frac{1}{12}\cdot 3^{\frac{3}{4}}\left(x^2+2\right)\right)$$
$$+\frac{1}{16}\cdot 3^{\frac{1}{4}}\log\left(36\,x^2-\sqrt{3}\left(5\,\sqrt{3}+3\right)-15\,\sqrt{3}+36\cdot 3^{\frac{1}{4}}-3\right)$$
$$-\frac{1}{16}\cdot 3^{\frac{1}{4}}\log\left(36\,x^2-\sqrt{3}\left(5\,\sqrt{3}+3\right)-15\,\sqrt{3}-36\cdot 3^{\frac{1}{4}}-3\right)$$

input
```
integrate((15*x-2*3^(1/2)*x+12*3^(1/2)*x^3+3*3^(1/2)*x^5)/(-4-20*x^2+60*3^
(1/2)*x^2-100*x^4+10*3^(1/2)*x^4+12*x^6+60*x^6*3^(1/2)-36*x^8),x, algorith
m="fricas")
```

output
```
-1/8*3^(1/4)*arctan(-1/12*3^(3/4)*(5*x^2 - 2*sqrt(3)) - 1/12*3^(3/4)*(x^2
+ 2)) + 1/16*3^(1/4)*log(36*x^2 - sqrt(3)*(5*sqrt(3) + 3) - 15*sqrt(3) + 3
6*3^(1/4) - 3) - 1/16*3^(1/4)*log(36*x^2 - sqrt(3)*(5*sqrt(3) + 3) - 15*sq
rt(3) - 36*3^(1/4) - 3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx = \text{Timed out}$$

input
```
integrate((15*x-2*3**(1/2)*x+12*3**(1/2)*x**3+3*3**(1/2)*x**5)/(-4-20*x**2
+60*3**(1/2)*x**2-100*x**4+10*3**(1/2)*x**4+12*x**6+60*x**6*3**(1/2)-36*x*
*8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{15x - 2\sqrt{3}x + 12\sqrt{3}x^3 + 3\sqrt{3}x^5}{-4 - 20x^2 + 60\sqrt{3}x^2 - 100x^4 + 10\sqrt{3}x^4 + 12x^6 + 60\sqrt{3}x^6 - 36x^8}\,dx$$
$$= \int -\frac{3\,\sqrt{3}x^5 + 12\,\sqrt{3}x^3 - 2\,\sqrt{3}x + 15\,x}{2\left(18\,x^8 - 30\,\sqrt{3}x^6 - 6\,x^6 - 5\,\sqrt{3}x^4 + 50\,x^4 - 30\,\sqrt{3}x^2 + 10\,x^2 + 2\right)}\,dx$$

input
```
integrate((15*x-2*3^(1/2)*x+12*3^(1/2)*x^3+3*3^(1/2)*x^5)/(-4-20*x^2+60*3^
(1/2)*x^2-100*x^4+10*3^(1/2)*x^4+12*x^6+60*x^6*3^(1/2)-36*x^8),x, algorith
m="maxima")
```

output
```
-1/2*integrate((3*sqrt(3)*x^5 + 12*sqrt(3)*x^3 - 2*sqrt(3)*x + 15*x)/(18*x
^8 - 30*sqrt(3)*x^6 - 6*x^6 - 5*sqrt(3)*x^4 + 50*x^4 - 30*sqrt(3)*x^2 + 10
*x^2 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{15x - 2\sqrt{3}x + 12\sqrt{3}x^3 + 3\sqrt{3}x^5}{-4 - 20x^2 + 60\sqrt{3}x^2 - 100x^4 + 10\sqrt{3}x^4 + 12x^6 + 60\sqrt{3}x^6 - 36x^8}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((15*x-2*3^(1/2)*x+12*3^(1/2)*x^3+3*3^(1/2)*x^5)/(-4-20*x^2+60*3^
(1/2)*x^2-100*x^4+10*3^(1/2)*x^4+12*x^6+60*x^6*3^(1/2)-36*x^8),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueUnable to find common minimal polynomial Er
ror: Bad
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 761, normalized size of antiderivative = 10.72

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int((15*x - 2*3^(1/2)*x + 12*3^(1/2)*x^3 + 3*3^(1/2)*x^5)/(60*3^(1/2)*x^2
+ 10*3^(1/2)*x^4 + 60*3^(1/2)*x^6 - 20*x^2 - 100*x^4 + 12*x^6 - 36*x^8 - 4
),x)
```

output
```
symsum(log(913779360*root(110045429760*3^(1/2)*z^4 - 797206315008*z^4 - 50
37480*3^(1/2) + 36493209, z, k) + 2494825632*3^(1/2)*root(110045429760*3^(
1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 36493209, z, k) + 29231280
*3^(1/2) + 15800090880*3^(1/2)*root(110045429760*3^(1/2)*z^4 - 79720631500
8*z^4 - 5037480*3^(1/2) + 36493209, z, k)^2 + 1595164114944*3^(1/2)*root(1
10045429760*3^(1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 36493209, z
, k)^3 + 6872364728320*3^(1/2)*root(110045429760*3^(1/2)*z^4 - 79720631500
8*z^4 - 5037480*3^(1/2) + 36493209, z, k)^4 + 208043658706944*3^(1/2)*root
(110045429760*3^(1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 36493209,
 z, k)^5 + 514188032081920*3^(1/2)*root(110045429760*3^(1/2)*z^4 - 7972063
15008*z^4 - 5037480*3^(1/2) + 36493209, z, k)^6 - 173691869310*root(110045
429760*3^(1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 36493209, z, k)*
x^2 - 2446451235*3^(1/2)*x^2 + 142704702816*root(110045429760*3^(1/2)*z^4
- 797206315008*z^4 - 5037480*3^(1/2) + 36493209, z, k)^2 + 766721802240*ro
ot(110045429760*3^(1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 3649320
9, z, k)^3 + 38013718364160*root(110045429760*3^(1/2)*z^4 - 797206315008*z
^4 - 5037480*3^(1/2) + 36493209, z, k)^4 + 180885808742400*root(1100454297
60*3^(1/2)*z^4 - 797206315008*z^4 - 5037480*3^(1/2) + 36493209, z, k)^5 +
1892922901397504*root(110045429760*3^(1/2)*z^4 - 797206315008*z^4 - 503748
0*3^(1/2) + 36493209, z, k)^6 - 1303067223*x^2 - 465671581680*root(1100...
```

Reduce [F]

$$\int \frac{15x-2\sqrt{3}x+12\sqrt{3}x^3+3\sqrt{3}x^5}{-4-20x^2+60\sqrt{3}x^2-100x^4+10\sqrt{3}x^4+12x^6+60\sqrt{3}x^6-36x^8}\,dx = -27\sqrt{3}\left(\int \frac{x^{13}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -99\sqrt{3}\left(\int \frac{x^{11}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -21\sqrt{3}\left(\int \frac{x^{9}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -546\sqrt{3}\left(\int \frac{x^{7}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -\frac{101\sqrt{3}\left(\int \frac{x^{5}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)}{2} -227\sqrt{3}\left(\int \frac{x^{3}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) +2\sqrt{3}\left(\int \frac{x}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -135\left(\int \frac{x^{11}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -\frac{1395\left(\int \frac{x^{9}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)}{2} -90\left(\int \frac{x^{7}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -900\left(\int \frac{x^{5}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) +15\left(\int \frac{x^{3}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right) -15\left(\int \frac{x}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$

input
```
int((15*x-2*3^(1/2)*x+12*3^(1/2)*x^3+3*3^(1/2)*x^5)/(-4-20*x^2+60*3^(1/2)*
x^2-100*x^4+10*3^(1/2)*x^4+12*x^6+60*x^6*3^(1/2)-36*x^8),x)
```

output

```
( - 54*sqrt(3)*int(x**13/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 -
 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 198*sqrt(3)*int(x**11
/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2
400*x**4 + 40*x**2 + 4),x) - 42*sqrt(3)*int(x**9/(324*x**16 - 216*x**14 -
864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x)
 - 1092*sqrt(3)*int(x**7/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 -
 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 101*sqrt(3)*int(x**5/
(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 24
00*x**4 + 40*x**2 + 4),x) - 454*sqrt(3)*int(x**3/(324*x**16 - 216*x**14 -
864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x)
 + 4*sqrt(3)*int(x/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*
x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 270*int(x**11/(324*x**16 -
216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*
x**2 + 4),x) - 1395*int(x**9/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**
10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 180*int(x**7/(324
*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x
**4 + 40*x**2 + 4),x) - 1800*int(x**5/(324*x**16 - 216*x**14 - 864*x**12 -
 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) + 30*int(x
**3/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6
- 2400*x**4 + 40*x**2 + 4),x) - 30*int(x/(324*x**16 - 216*x**14 - 864*x...
```

3.54 $\int \frac{12(-420+1999\sqrt{3})x}{(-1999+140\sqrt{3})(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4)}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 34

$$\int \frac{12(-420+1999\sqrt{3})\,x}{(-1999+140\sqrt{3})\,(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4)}\,dx = -\sqrt[4]{3}\arctan\left(\sqrt{-\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{1}{2}3^{3/4}x^2\right)$$

output
```
3^(1/4)*arctan(-1/6*(-18+12*3^(1/2))^(1/2)+1/2*3^(3/4)*x^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{12(-420+1999\sqrt{3})\,x}{(-1999+140\sqrt{3})\,(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4)}\,dx = -\frac{(-420+1999\sqrt{3})\arctan\left(\frac{1-\sqrt{3}+3x^2}{2\sqrt[4]{3}}\right)}{\sqrt[4]{3}\,(-1999+140\sqrt{3})}$$

input
```
Integrate[(12*(-420 + 1999*Sqrt[3])*x)/((-1999 + 140*Sqrt[3])*(-4 - 2*Sqrt
[3] - 6*x^2 + 6*Sqrt[3]*x^2 - 9*x^4)),x]
```

output
```
-(((-420 + 1999*Sqrt[3])*ArcTan[(1 - Sqrt[3] + 3*x^2)/(2*3^(1/4))])/(3^(1/
4)*(-1999 + 140*Sqrt[3])))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6, 27, 25, 1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{12\left(1999\sqrt{3}-420\right)x}{\left(140\sqrt{3}-1999\right)\left(-9x^4+6\sqrt{3}x^2-6x^2-2\sqrt{3}-4\right)}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{12\left(1999\sqrt{3}-420\right)x}{\left(140\sqrt{3}-1999\right)\left(-9x^4+\left(6\sqrt{3}-6\right)x^2-2\sqrt{3}-4\right)}dx$$

$$\Big\downarrow \ 27$$

$$-12\sqrt{3}\int -\frac{x}{9x^4+6\left(1-\sqrt{3}\right)x^2+2\left(2+\sqrt{3}\right)}dx$$

$$\Big\downarrow \ 25$$

$$12\sqrt{3}\int \frac{x}{9x^4+6\left(1-\sqrt{3}\right)x^2+2\left(2+\sqrt{3}\right)}dx$$

$$\Big\downarrow \ 1432$$

$$6\sqrt{3}\int \frac{1}{9x^4+6\left(1-\sqrt{3}\right)x^2+2\left(2+\sqrt{3}\right)}dx^2$$

$$\Big\downarrow \ 1083$$

$$-12\sqrt{3}\int \frac{1}{-x^4-144\sqrt{3}}d\left(18x^2+6\left(1-\sqrt{3}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\sqrt[4]{3}\arctan\left(\frac{18x^2+6(1-\sqrt{3})}{12\sqrt[4]{3}}\right)$$

input
```
Int[(12*(-420 + 1999*Sqrt[3])*x)/((-1999 + 140*Sqrt[3])*(-4 - 2*Sqrt[3] -
6*x^2 + 6*Sqrt[3]*x^2 - 9*x^4)),x]
```

output
```
3^(1/4)*ArcTan[(6*(1 - Sqrt[3]) + 18*x^2)/(12*3^(1/4))]
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{\left(-420+1999\sqrt{3}\right) 3^{\frac{3}{4}} \arctan\left(\frac{\left(18x^{2}-6\sqrt{3}+6\right) 3^{\frac{3}{4}}}{36}\right)}{3\left(-1999+140\sqrt{3}\right)}$	40

input
```
int(12*(-420+1999*3^(1/2))*x/(-1999+140*3^(1/2))/(-4-2*3^(1/2)-6*x^2+6*3^(
1/2)*x^2-9*x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-420+1999*3^(1/2))/(-1999+140*3^(1/2))*3^(3/4)*arctan(1/36*(18*x^2-6
*3^(1/2)+6)*3^(3/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{12\left(-420+1999\sqrt{3}\right) x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4\right)} dx = 3^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 3^{\frac{1}{4}}\left(\sqrt{3}(3\,x^2+1)-3\right)\right)$$

input
```
integrate(12*(-420+1999*3^(1/2))*x/(-1999+140*3^(1/2))/(-4-2*3^(1/2)-6*x^2
+6*3^(1/2)*x^2-9*x^4),x, algorithm="fricas")
```

output
```
3^(1/4)*arctan(1/6*3^(1/4)*(sqrt(3)*(3*x^2 + 1) - 3))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{12\left(-420+1999\sqrt{3}\right)x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4\right)}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate(12*(-420+1999*3**(1/2))*x/(-1999+140*3**(1/2))/(-4-2*3**(1/2)-6*
x**2+6*3**(1/2)*x**2-9*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(-17313583079757718563650236225520
06976*_t**2 + 9871436368547438042606199717148418888*sqrt(3)*_t**2 - 45428740
0851412562586425867754 96*_t + 27842854251208551842166254171016*sqrt(3)*_t
- 3639190287298
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{12\left(-420+1999\sqrt{3}\right)x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4\right)}\,dx = -\frac{3^{\frac{3}{4}}\left(1999\sqrt{3}-420\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\left(3x^2-\sqrt{3}+1\right)\right)}{3\left(140\sqrt{3}-1999\right)}$$

input
```
integrate(12*(-420+1999*3^(1/2))*x/(-1999+140*3^(1/2))/(-4-2*3^(1/2)-6*x^2
+6*3^(1/2)*x^2-9*x^4),x, algorithm="maxima")
```

output
```
-1/3*3^(3/4)*(1999*sqrt(3) - 420)*arctan(1/6*3^(3/4)*(3*x^2 - sqrt(3) + 1)
)/(140*sqrt(3) - 1999)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{12\left(-420+1999\sqrt{3}\right)x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4\right)}\,dx$$
$$= -\frac{3^{\frac{3}{4}}\left(1999\sqrt{3}-420\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\left(3x^2-\sqrt{3}+1\right)\right)}{3\left(140\sqrt{3}-1999\right)}$$

input
```
integrate(12*(-420+1999*3^(1/2))*x/(-1999+140*3^(1/2))/(-4-2*3^(1/2)-6*x^2
+6*3^(1/2)*x^2-9*x^4),x, algorithm="giac")
```

output
```
-1/3*3^(3/4)*(1999*sqrt(3) - 420)*arctan(1/6*3^(3/4)*(3*x^2 - sqrt(3) + 1)
)/(140*sqrt(3) - 1999)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{12\left(-420+1999\sqrt{3}\right)x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}x^2-9x^4\right)}\,dx$$
$$= -\frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{32x^2}{\sqrt{-4\sqrt{3}}\left(\frac{32\sqrt{3}}{9}+\frac{8\sqrt{3}x^2}{3}-8x^2+\frac{16}{3}\right)}\right)}{\sqrt{-4\sqrt{3}}}$$

input
```
int(-(12*x*(1999*3^(1/2) - 420))/((140*3^(1/2) - 1999)*(2*3^(1/2) - 6*3^(1
/2)*x^2 + 6*x^2 + 9*x^4 + 4)),x)
```

output
```
-(2*3^(1/2)*atanh((32*x^2)/((-4*3^(1/2))^(1/2)*((32*3^(1/2))/9 + (8*3^(1/2
)*x^2)/3 - 8*x^2 + 16/3))))/(-4*3^(1/2))^(1/2)
```

Reduce [F]

$$\int \frac{12\left(-420+1999\sqrt{3}\right)x}{\left(-1999+140\sqrt{3}\right)\left(-4-2\sqrt{3}-6x^2+6\sqrt{3}\,x^2-9x^4\right)}\,dx$$
$$= 108\sqrt{3}\left(\int \frac{x^5}{81x^8+108x^6+120x^2+4}dx\right) + 72\sqrt{3}\left(\int \frac{x^3}{81x^8+108x^6+120x^2+4}dx\right) + 48\sqrt{3}\left(\int \frac{x}{81x^8+108x^6+120x^2+4}dx\right) + 216\left(\int \frac{x^3}{81x^8+108x^6+120x^2+4}dx\right) - 72\left(\int \frac{x}{81x^8+108x^6+120x^2+4}dx\right)$$

input
```
int(12*(-420+1999*3^(1/2))*x/(-1999+140*3^(1/2))/(-4-2*3^(1/2)-6*x^2+6*3^(
1/2)*x^2-9*x^4),x)
```

output
```
12*(9*sqrt(3)*int(x**5/(81*x**8 + 108*x**6 + 120*x**2 + 4),x) + 6*sqrt(3)*
int(x**3/(81*x**8 + 108*x**6 + 120*x**2 + 4),x) + 4*sqrt(3)*int(x/(81*x**8
 + 108*x**6 + 120*x**2 + 4),x) + 18*int(x**3/(81*x**8 + 108*x**6 + 120*x**
2 + 4),x) - 6*int(x/(81*x**8 + 108*x**6 + 120*x**2 + 4),x))
```

3.55 $\int \frac{8(15-2\sqrt{3})x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})x^2+(-100+10\sqrt{3})x^4+(12+60\sqrt{3})x^6-36x^8}\,dx$

Optimal result

Integrand size = 82, antiderivative size = 67

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx = -\sqrt[4]{3}\arctan\left(\sqrt{-\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{1}{2}3^{3/4}x^2\right)-\sqrt[4]{3}\,\mathrm{arctanh}\left(\sqrt{\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{x^2}{\sqrt[4]{3}}\right)$$

output
```
3^(1/4)*arctan(-1/6*(-18+12*3^(1/2))^(1/2)+1/2*3^(3/4)*x^2)+3^(1/4)*arctan
h(-1/6*(18+12*3^(1/2))^(1/2)+1/3*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.60

$$\int \frac{8(15-2\sqrt{3})x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})x^2+(-100+10\sqrt{3})x^4+(12+60\sqrt{3})x^6-36x^8}\,dx$$
$$= \text{RootSum}\Big[2+10\#1^2-30\sqrt{3}\#1^2+50\#1^4-5\sqrt{3}\#1^4-6\#1^6-30\sqrt{3}\#1^6 +18\#1^8\&, \frac{15\log(x-\#1)-2\sqrt{3}\log(x-\#1)+12\sqrt{3}\log(x-\#1)\#1^2+3\sqrt{3}\log(x-\#1)\#1^4}{-5+15\sqrt{3}-50\#1^2+5\sqrt{3}\#1^2+9\#1^4+45\sqrt{3}\#1^4-36\#1^6}\&\Big]$$

input
```
Integrate[(8*(15 - 2*Sqrt[3])*x + 96*Sqrt[3]*x^3 + 24*Sqrt[3]*x^5)/(-4 + (
-20 + 60*Sqrt[3])*x^2 + (-100 + 10*Sqrt[3])*x^4 + (12 + 60*Sqrt[3])*x^6 -
36*x^8),x]
```

output
```
RootSum[2 + 10*#1^2 - 30*Sqrt[3]*#1^2 + 50*#1^4 - 5*Sqrt[3]*#1^4 - 6*#1^6
- 30*Sqrt[3]*#1^6 + 18*#1^8 & , (15*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] +
12*Sqrt[3]*Log[x - #1]*#1^2 + 3*Sqrt[3]*Log[x - #1]*#1^4)/(-5 + 15*Sqrt[3]
 - 50*#1^2 + 5*Sqrt[3]*#1^2 + 9*#1^4 + 45*Sqrt[3]*#1^4 - 36*#1^6) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{24\sqrt{3}x^5+96\sqrt{3}x^3+8(15-2\sqrt{3})x}{-36x^8+(12+60\sqrt{3})x^6+(10\sqrt{3}-100)x^4+(60\sqrt{3}-20)x^2-4}\,dx$$
$$\Big\downarrow \text{2028}$$
$$\int \frac{x(24\sqrt{3}x^4+96\sqrt{3}x^2+8(15-2\sqrt{3}))}{-36x^8+(12+60\sqrt{3})x^6+(10\sqrt{3}-100)x^4+(60\sqrt{3}-20)x^2-4}\,dx$$
$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int -\frac{4\left(3\sqrt{3}x^4+12\sqrt{3}x^2-2\sqrt{3}+15\right)}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2}dx^2$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{3\sqrt{3}x^4+12\sqrt{3}x^2-2\sqrt{3}+15}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2}dx^2$$

$$\Big\downarrow \text{7293}$$

$$-2\int \left(\frac{3\sqrt{3}x^4}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)x^4+10\left(1-3\sqrt{3}\right)x^2+2}+\frac{12\sqrt{3}x^2}{18x^8-6\left(1+5\sqrt{3}\right)x^6+5\left(10-\sqrt{3}\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\left(\left(15-2\sqrt{3}\right)\int \frac{1}{-18x^8+6\left(1+5\sqrt{3}\right)x^6-5\left(10-\sqrt{3}\right)x^4-10\left(1-3\sqrt{3}\right)x^2-2}dx^2\right)+12\sqrt{3}\int \frac{}{18x^8}\right.$$

input
```
Int[(8*(15 - 2*Sqrt[3])*x + 96*Sqrt[3]*x^3 + 24*Sqrt[3]*x^5)/(-4 + (-20 +
60*Sqrt[3])*x^2 + (-100 + 10*Sqrt[3])*x^4 + (12 + 60*Sqrt[3])*x^6 - 36*x^8
),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

method	result	size
default	$3^{\frac{1}{4}}\operatorname{arctanh}\left(\frac{\left(4x^2-2\sqrt{3}-2\right)3^{\frac{3}{4}}}{12}\right)+3^{\frac{1}{4}}\arctan\left(\frac{\left(18x^2-6\sqrt{3}+6\right)3^{\frac{3}{4}}}{36}\right)$	46

input
```
int((8*(15-2*3^(1/2))*x+96*3^(1/2)*x^3+24*3^(1/2)*x^5)/(-4+(-20+60*3^(1/2)
)*x^2+(-100+10*3^(1/2))*x^4+(12+60*3^(1/2))*x^6-36*x^8),x,method=_RETURNVE
RBOSE)
```

output
```
3^(1/4)*arctanh(1/12*(4*x^2-2*3^(1/2)-2)*3^(3/4))+3^(1/4)*arctan(1/36*(18*
x^2-6*3^(1/2)+6)*3^(3/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx$$
$$= -3^{\frac{1}{4}}\arctan\left(-\frac{1}{12}\cdot 3^{\frac{3}{4}}\left(5\,x^2-2\,\sqrt{3}\right)-\frac{1}{12}\cdot 3^{\frac{3}{4}}\left(x^2+2\right)\right)$$
$$+\frac{1}{2}\cdot 3^{\frac{1}{4}}\log\left(36\,x^2-\sqrt{3}\left(5\,\sqrt{3}+3\right)-15\,\sqrt{3}+36\cdot 3^{\frac{1}{4}}-3\right)$$
$$-\frac{1}{2}\cdot 3^{\frac{1}{4}}\log\left(36\,x^2-\sqrt{3}\left(5\,\sqrt{3}+3\right)-15\,\sqrt{3}-36\cdot 3^{\frac{1}{4}}-3\right)$$

input
```
integrate((8*(15-2*3^(1/2))*x+96*3^(1/2)*x^3+24*3^(1/2)*x^5)/(-4+(-20+60*3
^(1/2))*x^2+(-100+10*3^(1/2))*x^4+(12+60*3^(1/2))*x^6-36*x^8),x, algorithm
="fricas")
```

output
```
-3^(1/4)*arctan(-1/12*3^(3/4)*(5*x^2 - 2*sqrt(3)) - 1/12*3^(3/4)*(x^2 + 2)
) + 1/2*3^(1/4)*log(36*x^2 - sqrt(3)*(5*sqrt(3) + 3) - 15*sqrt(3) + 36*3^(
1/4) - 3) - 1/2*3^(1/4)*log(36*x^2 - sqrt(3)*(5*sqrt(3) + 3) - 15*sqrt(3)
- 36*3^(1/4) - 3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx$$
$$= \text{Timed out}$$

input
```
integrate((8*(15-2*3**(1/2))*x+96*3**(1/2)*x**3+24*3**(1/2)*x**5)/(-4+(-20
+60*3**(1/2))*x**2+(-100+10*3**(1/2))*x**4+(12+60*3**(1/2))*x**6-36*x**8),
x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx$$
$$=\int -\frac{4\left(3\sqrt{3}x^5+12\sqrt{3}x^3-x\left(2\sqrt{3}-15\right)\right)}{18\,x^8-6\,x^6\left(5\sqrt{3}+1\right)-5\,x^4\left(\sqrt{3}-10\right)-10\,x^2\left(3\sqrt{3}-1\right)+2}\,dx$$

input
```
integrate((8*(15-2*3^(1/2))*x+96*3^(1/2)*x^3+24*3^(1/2)*x^5)/(-4+(-20+60*3
^(1/2))*x^2+(-100+10*3^(1/2))*x^4+(12+60*3^(1/2))*x^6-36*x^8),x, algorithm
="maxima")
```

output
```
-4*integrate((3*sqrt(3)*x^5 + 12*sqrt(3)*x^3 - x*(2*sqrt(3) - 15))/(18*x^8
 - 6*x^6*(5*sqrt(3) + 1) - 5*x^4*(sqrt(3) - 10) - 10*x^2*(3*sqrt(3) - 1) +
 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx$$
$$=\text{Exception raised: TypeError}$$

input
```
integrate((8*(15-2*3^(1/2))*x+96*3^(1/2)*x^3+24*3^(1/2)*x^5)/(-4+(-20+60*3
^(1/2))*x^2+(-100+10*3^(1/2))*x^4+(12+60*3^(1/2))*x^6-36*x^8),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueUnable to find common minimal polynomial Er
ror: Bad
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 761, normalized size of antiderivative = 11.36

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+\left(-20+60\sqrt{3}\right)x^2+\left(-100+10\sqrt{3}\right)x^4+\left(12+60\sqrt{3}\right)x^6-36x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int((96*3^(1/2)*x^3 + 24*3^(1/2)*x^5 - 8*x*(2*3^(1/2) - 15))/(x^6*(60*3^(1
/2) + 12) + x^2*(60*3^(1/2) - 20) + x^4*(10*3^(1/2) - 100) - 36*x^8 - 4),x
)
```

output
```
symsum(log(456889680*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3
^(1/2) + 36493209, z, k) + 1247412816*3^(1/2)*root(26866560*3^(1/2)*z^4 -
194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k) + 116925120*3^(1/2) + 98
7505680*3^(1/2)*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2
) + 36493209, z, k)^2 + 12462219648*3^(1/2)*root(26866560*3^(1/2)*z^4 - 19
4630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)^3 + 6711293680*3^(1/2)*roo
t(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)
^4 + 25395954432*3^(1/2)*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 50374
80*3^(1/2) + 36493209, z, k)^5 + 7845886720*3^(1/2)*root(26866560*3^(1/2)*
z^4 - 194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)^6 - 86845934655*ro
ot(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k
)*x^2 - 9785804940*3^(1/2)*x^2 + 8919043926*root(26866560*3^(1/2)*z^4 - 19
4630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)^2 + 5990014080*root(268665
60*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)^3 + 371
22771840*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2) + 364
93209, z, k)^4 + 22080787200*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5
037480*3^(1/2) + 36493209, z, k)^5 + 28883711264*root(26866560*3^(1/2)*z^4
 - 194630448*z^4 - 5037480*3^(1/2) + 36493209, z, k)^6 - 5212268892*x^2 -
29104473855*root(26866560*3^(1/2)*z^4 - 194630448*z^4 - 5037480*3^(1/2) +
36493209, z, k)^2*x^2 - 540228895440*root(26866560*3^(1/2)*z^4 - 194630...
```

Reduce [F]

$$\int \frac{8(15-2\sqrt{3})\,x+96\sqrt{3}x^3+24\sqrt{3}x^5}{-4+(-20+60\sqrt{3})\,x^2+(-100+10\sqrt{3})\,x^4+(12+60\sqrt{3})\,x^6-36x^8}\,dx =$$

$$-216\sqrt{3}\left(\int \frac{x^{13}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-792\sqrt{3}\left(\int \frac{x^{11}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-168\sqrt{3}\left(\int \frac{x^{9}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-4368\sqrt{3}\left(\int \frac{x^{7}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-404\sqrt{3}\left(\int \frac{x^{5}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-1816\sqrt{3}\left(\int \frac{x^{3}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$+16\sqrt{3}\left(\int \frac{x}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-1080\left(\int \frac{x^{11}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-5580\left(\int \frac{x^{9}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-720\left(\int \frac{x^{7}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-7200\left(\int \frac{x^{5}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$+120\left(\int \frac{x^{3}}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$
$$-120\left(\int \frac{x}{324x^{16}-216x^{14}-864x^{12}-1140x^{10}-3023x^8+76x^6-2400x^4+40x^2+4}dx\right)$$

input
```
int((8*(15-2*3^(1/2))*x+96*3^(1/2)*x^3+24*3^(1/2)*x^5)/(-4+(-20+60*3^(1/2)
)*x^2+(-100+10*3^(1/2))*x^4+(12+60*3^(1/2))*x^6-36*x^8),x)
```

output

```
4*( - 54*sqrt(3)*int(x**13/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10
 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 198*sqrt(3)*int(x**
11/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 -
 2400*x**4 + 40*x**2 + 4),x) - 42*sqrt(3)*int(x**9/(324*x**16 - 216*x**14
- 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),
x) - 1092*sqrt(3)*int(x**7/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10
 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 101*sqrt(3)*int(x**
5/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 -
2400*x**4 + 40*x**2 + 4),x) - 454*sqrt(3)*int(x**3/(324*x**16 - 216*x**14
- 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),
x) + 4*sqrt(3)*int(x/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 302
3*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 270*int(x**11/(324*x**16
- 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 4
0*x**2 + 4),x) - 1395*int(x**9/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x
**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) - 180*int(x**7/(3
24*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400
*x**4 + 40*x**2 + 4),x) - 1800*int(x**5/(324*x**16 - 216*x**14 - 864*x**12
 - 1140*x**10 - 3023*x**8 + 76*x**6 - 2400*x**4 + 40*x**2 + 4),x) + 30*int
(x**3/(324*x**16 - 216*x**14 - 864*x**12 - 1140*x**10 - 3023*x**8 + 76*x**
6 - 2400*x**4 + 40*x**2 + 4),x) - 30*int(x/(324*x**16 - 216*x**14 - 864...
```

3.56

$$\int \frac{-80\left(-3+2\sqrt{3}\right)x-80\left(3-2\sqrt{3}\right)x^3-80\left(90+\right.}{14-8\sqrt{3}+\left(-28+20\sqrt{3}\right)x^2+\left(-1110-418\sqrt{3}\right)x^4+\left(296-268\sqrt{3}\right)x^6+\left(\right.}$$

Optimal result

Integrand size = 171, antiderivative size = 194

$$\int \frac{-80\left(-3+2\sqrt{3}\right)x-80\left(3-2\sqrt{3}\right)x^3-80\left(90+30\sqrt{3}\right)x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+\left(-28+20\sqrt{3}\right)x^2+\left(-1110-418\sqrt{3}\right)x^4+\left(296-268\sqrt{3}\right)x^6+\left(434-2401\sqrt{3}\right)x^8+\left(2\right.}$$
$$= \sqrt[8]{3}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(-6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}+\frac{x^2}{\sqrt[8]{3}}\right)$$
$$-\sqrt[8]{3}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(-6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}-\frac{1}{2}3^{7/8}x^2\right)$$
$$-\sqrt[8]{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}-\frac{x^2}{\sqrt[8]{3}}\right)$$
$$+\sqrt[8]{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}+\frac{1}{2}3^{7/8}x^2\right)$$

output
```
3^(1/8)*arctan(1/6*(-18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/3*x^2*3^(7/8))+3^
(1/8)*arctan(-1/6*(-18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/2*x^2*3^(7/8))+3^(
1/8)*arctanh(-1/6*(18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/3*x^2*3^(7/8))+3^(1
/8)*arctanh(1/6*(18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/2*x^2*3^(7/8))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.94

$$\int \frac{-80(-3+2\sqrt{3})\,x-80(3-2\sqrt{3})\,x^3-80(90+30\sqrt{3})\,x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})\,x^2+(-1110-418\sqrt{3})\,x^4+(296-268\sqrt{3})\,x^6+(434-2401\sqrt{3})\,x^8+(20}$$

$$= 10\text{RootSum}\Big[-14+8\sqrt{3}+28\#1^2-20\sqrt{3}\#1^2+1110\#1^4+418\sqrt{3}\#1^4-296\#1^6 +268\sqrt{3}\#1^6-434\#1^8+2401\sqrt{3}\#1^8-204\#1^{10}+828\sqrt{3}\#1^{10}+324\#1^{12} +468\sqrt{3}\#1^{12}+432\#1^{14} -648\#1^{16}\&, \frac{-3\log(x-\#1)+2\sqrt{3}\log(x-\#1)+3\log(x-\#1)\#1^2-2\sqrt{3}\log(x-\#1)\#1^2+90\log}{7-5\sqrt{3}+555\#1^2+209\sqrt{3}\#1^2-222\#1^4+201\sqrt{3}\#1^4-434\#1}$$

input
```
Integrate[(-80*(-3 + 2*Sqrt[3])*x - 80*(3 - 2*Sqrt[3])*x^3 - 80*(90 + 30*S
qrt[3])*x^5 - 960*Sqrt[3]*x^7 - 7440*Sqrt[3]*x^9 - 1440*Sqrt[3]*x^11)/(14
- 8*Sqrt[3] + (-28 + 20*Sqrt[3])*x^2 + (-1110 - 418*Sqrt[3])*x^4 + (296 -
268*Sqrt[3])*x^6 + (434 - 2401*Sqrt[3])*x^8 + (204 - 828*Sqrt[3])*x^10 + (
-324 - 468*Sqrt[3])*x^12 - 432*x^14 + 648*x^16),x]
```

output
```
10*RootSum[-14 + 8*Sqrt[3] + 28*#1^2 - 20*Sqrt[3]*#1^2 + 1110*#1^4 + 418*S
qrt[3]*#1^4 - 296*#1^6 + 268*Sqrt[3]*#1^6 - 434*#1^8 + 2401*Sqrt[3]*#1^8 -
 204*#1^10 + 828*Sqrt[3]*#1^10 + 324*#1^12 + 468*Sqrt[3]*#1^12 + 432*#1^14
 - 648*#1^16 & , (-3*Log[x - #1] + 2*Sqrt[3]*Log[x - #1] + 3*Log[x - #1]*#
1^2 - 2*Sqrt[3]*Log[x - #1]*#1^2 + 90*Log[x - #1]*#1^4 + 30*Sqrt[3]*Log[x
- #1]*#1^4 + 12*Sqrt[3]*Log[x - #1]*#1^6 + 93*Sqrt[3]*Log[x - #1]*#1^8 + 1
8*Sqrt[3]*Log[x - #1]*#1^10)/(7 - 5*Sqrt[3] + 555*#1^2 + 209*Sqrt[3]*#1^2
- 222*#1^4 + 201*Sqrt[3]*#1^4 - 434*#1^6 + 2401*Sqrt[3]*#1^6 - 255*#1^8 +
1035*Sqrt[3]*#1^8 + 486*#1^10 + 702*Sqrt[3]*#1^10 + 756*#1^12 - 1296*#1^14
) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-1440\sqrt{3}x^{11}-7440\sqrt{3}x^9-960\sqrt{3}x^7-80\left(90+30\sqrt{3}\right)x^5-80\left(3-2\sqrt{3}\right)x^3}{648x^{16}-432x^{14}+\left(-324-468\sqrt{3}\right)x^{12}+\left(204-828\sqrt{3}\right)x^{10}+\left(434-2401\sqrt{3}\right)x^8+\left(296-268\sqrt{3}\right)x^6+(}$$

$$\Big\downarrow \; 7292$$

$$\int \frac{80x\left(-18\sqrt{3}x^{10}-93\sqrt{3}x^8-12\sqrt{3}x^6-90\left(1+\frac{1}{\sqrt{3}}\right)x^4-3\left(1-\frac{2}{\sqrt{3}}\right)x^2}{648x^{16}-432x^{14}+\left(-324-468\sqrt{3}\right)x^{12}+\left(204-828\sqrt{3}\right)x^{10}+\left(434-2401\sqrt{3}\right)x^8+\left(296-268\sqrt{3}\right)x^6+(}$$

$$\Big\downarrow \; 27$$

$$80\int \frac{x\left(-18\sqrt{3}x^{10}-93\sqrt{3}x^8-12\sqrt{3}x^6-30\left(3+\sqrt{3}\right)x^4-\left(3-2\sqrt{3}\right)x^2}{648x^{16}-432x^{14}-36\left(9+13\sqrt{3}\right)x^{12}+12\left(17-69\sqrt{3}\right)x^{10}+7\left(62-343\sqrt{3}\right)x^8+4\left(74-67\sqrt{3}\right)x^6-2(}$$

$$\Big\downarrow \; 7266$$

$$40\int \frac{-18\sqrt{3}x^{10}-93\sqrt{3}x^8-12\sqrt{3}x^6-30\left(3+\sqrt{3}\right)x^4-\left(3-2\sqrt{3}\right)x^2-}{648x^{16}-432x^{14}-36\left(9+13\sqrt{3}\right)x^{12}+12\left(17-69\sqrt{3}\right)x^{10}+7\left(62-343\sqrt{3}\right)x^8+4\left(74-67\sqrt{3}\right)x^6-2(}$$

$$\Big\downarrow \; 7292$$

$$40\int \frac{-18\sqrt{3}x^{10}-93\sqrt{3}x^8-12\sqrt{3}x^6-30\left(3+\sqrt{3}\right)x^4-\left(3-2\sqrt{3}\right)x^2+}{648x^{16}-432x^{14}-36\left(9+13\sqrt{3}\right)x^{12}+12\left(17-69\sqrt{3}\right)x^{10}+7\left(62-343\sqrt{3}\right)x^8+4\left(74-67\sqrt{3}\right)x^6-2(}$$

$$\Big\downarrow \; 7293$$

$$40\int \left(\frac{18\sqrt{3}x^{10}}{-648x^{16}+432x^{14}+324\left(1+\frac{13}{3\sqrt{3}}\right)x^{12}-204\left(1-\frac{69\sqrt{3}}{17}\right)x^{10}-434\left(1-\frac{343\sqrt{3}}{62}\right)x^8-296\left(1-\frac{67\sqrt{3}}{74}\right)x^6}\right.$$

$$\Big\downarrow \; 2009$$

$$40\left(12\sqrt{3}\int \frac{x^6}{-648x^{16}+432x^{14}+324\left(1+\frac{13}{3\sqrt{3}}\right)x^{12}-204\left(1-\frac{69\sqrt{3}}{17}\right)x^{10}-434\left(1-\frac{343\sqrt{3}}{62}\right)x^8-296\left(1-\frac{67\sqrt{}}{74}\right.}\right.$$

input
```
Int[(-80*(-3 + 2*Sqrt[3])*x - 80*(3 - 2*Sqrt[3])*x^3 - 80*(90 + 30*Sqrt[3]
)*x^5 - 960*Sqrt[3]*x^7 - 7440*Sqrt[3]*x^9 - 1440*Sqrt[3]*x^11)/(14 - 8*Sq
rt[3] + (-28 + 20*Sqrt[3])*x^2 + (-1110 - 418*Sqrt[3])*x^4 + (296 - 268*Sq
rt[3])*x^6 + (434 - 2401*Sqrt[3])*x^8 + (204 - 828*Sqrt[3])*x^10 + (-324 -
 468*Sqrt[3])*x^12 - 432*x^14 + 648*x^16),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

method	result
default	$2\sqrt{3}\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-16\_Z^3+\left(-4\sqrt{3}+12\right)\_Z^2+\left(-12\sqrt{3}-4\right)\_Z+2-\sqrt{3}\right)}\frac{\left(1+2\_R\right)\ln\left(x^2-\_R\right)}{-8\_R^3+12\_R^2-6\_R+1+\sqrt{3}\left(3+2\_\right)}\right)$

input
```
int((-80*(-3+2*3^(1/2))*x-80*(3-2*3^(1/2))*x^3-80*(90+30*3^(1/2))*x^5-960*
3^(1/2)*x^7-7440*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(-28+20*3^(1
/2))*x^2+(-1110-418*3^(1/2))*x^4+(296-268*3^(1/2))*x^6+(434-2401*3^(1/2))*
x^8+(204-828*3^(1/2))*x^10+(-324-468*3^(1/2))*x^12-432*x^14+648*x^16),x,me
thod=_RETURNVERBOSE)
```

output
```
2*3^(1/2)*sum((1+2*_R)/(-8*_R^3+12*_R^2-6*_R+1+3^(1/2)*(3+2*_R))*ln(x^2-_R
),_R=RootOf(8*_Z^4-16*_Z^3+(-4*3^(1/2)+12)*_Z^2+(-12*3^(1/2)-4)*_Z+2-3^(1/
2)))-2*3^(1/2)*sum((3*_R-1)/(-27*_R^3-27*_R^2-9*_R-1+3*3^(1/2)*(-1+_R))*ln
(x^2-_R),_R=RootOf(81*_Z^4+108*_Z^3+(-18*3^(1/2)+54)*_Z^2+(36*3^(1/2)+12)*
_Z-2*3^(1/2)+4))
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int \frac{-80(-3+2\sqrt{3})\,x-80(3-2\sqrt{3})\,x^3-80(90+30\sqrt{3})\,x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})\,x^2+(-1110-418\sqrt{3})\,x^4+(296-268\sqrt{3})\,x^6+(434-2401\sqrt{3})\,x^8+(20}$$

$$= 3^{\frac{1}{8}}\arctan\left(\frac{1}{30}\cdot 3^{\frac{7}{8}}(6\,x^2-1)+\frac{1}{10}\cdot 3^{\frac{1}{8}}\right)$$
$$+3^{\frac{1}{8}}\arctan\left(\frac{1}{30}\cdot 3^{\frac{3}{8}}\left(162\,x^6-126\,x^4-576\,x^2+\sqrt{3}(90\,x^6-66\,x^4-293\,x^2+1)\right)\right.$$
$$\left.-\frac{1}{10}\cdot 3^{\frac{1}{8}}\left(72\,x^6-54\,x^4-240\,x^2+6\sqrt{3}(6\,x^6-5\,x^4-24\,x^2)+1\right)\right)$$
$$+\frac{1}{2}\cdot 3^{\frac{1}{8}}\log\left(6\,x^4+10\cdot 3^{\frac{1}{8}}x^2-x^2-3^{\frac{1}{4}}(x^2-2)-\sqrt{3}-1\right)$$
$$-\frac{1}{2}\cdot 3^{\frac{1}{8}}\log\left(6\,x^4-10\cdot 3^{\frac{1}{8}}x^2-x^2-3^{\frac{1}{4}}(x^2-2)-\sqrt{3}-1\right)$$

input
```
integrate((-80*(-3+2*3^(1/2))*x-80*(3-2*3^(1/2))*x^3-80*(90+30*3^(1/2))*x^
5-960*3^(1/2)*x^7-7440*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(-28+2
0*3^(1/2))*x^2+(-1110-418*3^(1/2))*x^4+(296-268*3^(1/2))*x^6+(434-2401*3^(
1/2))*x^8+(204-828*3^(1/2))*x^10+(-324-468*3^(1/2))*x^12-432*x^14+648*x^16
),x, algorithm="fricas")
```

output
```
3^(1/8)*arctan(1/30*3^(7/8)*(6*x^2 - 1) + 1/10*3^(1/8)) + 3^(1/8)*arctan(1
/30*3^(3/8)*(162*x^6 - 126*x^4 - 576*x^2 + sqrt(3)*(90*x^6 - 66*x^4 - 293*
x^2 + 1)) - 1/10*3^(1/8)*(72*x^6 - 54*x^4 - 240*x^2 + 6*sqrt(3)*(6*x^6 - 5
*x^4 - 24*x^2) + 1)) + 1/2*3^(1/8)*log(6*x^4 + 10*3^(1/8)*x^2 - x^2 - 3^(1
/4)*(x^2 - 2) - sqrt(3) - 1) - 1/2*3^(1/8)*log(6*x^4 - 10*3^(1/8)*x^2 - x^
2 - 3^(1/4)*(x^2 - 2) - sqrt(3) - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-80(-3+2\sqrt{3})\,x-80(3-2\sqrt{3})\,x^3-80(90+30\sqrt{3})\,x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})\,x^2+(-1110-418\sqrt{3})\,x^4+(296-268\sqrt{3})\,x^6+(434-2401\sqrt{3})\,x^8+(2} = \text{Timed out}$$

input
```
integrate((-80*(-3+2*3**(1/2))*x-80*(3-2*3**(1/2))*x**3-80*(90+30*3**(1/2)
)*x**5-960*3**(1/2)*x**7-7440*3**(1/2)*x**9-1440*3**(1/2)*x**11)/(14-8*3**
(1/2)+(-28+20*3**(1/2))*x**2+(-1110-418*3**(1/2))*x**4+(296-268*3**(1/2))*
x**6+(434-2401*3**(1/2))*x**8+(204-828*3**(1/2))*x**10+(-324-468*3**(1/2))
*x**12-432*x**14+648*x**16),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{-80(-3+2\sqrt{3})\,x-80(3-2\sqrt{3})\,x^3-80(90+30\sqrt{3})\,x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})\,x^2+(-1110-418\sqrt{3})\,x^4+(296-268\sqrt{3})\,x^6+(434-2401\sqrt{3})\,x^8+(2}$$
$$= \int -\frac{80\,(18\,\sqrt{3}x^{11}+93\,\sqrt{3}x^9+12\,\sqrt{3}x^7+30\,x^5(\sqrt{3}+3)-x^3(2\,\sqrt{3}}{648\,x^{16}-432\,x^{14}-36\,x^{12}(13\,\sqrt{3}+9)-12\,x^{10}(69\,\sqrt{3}-17)-7\,x^8(343\,\sqrt{3}-62)-4\,x^6(67\,\sqrt{3}-}$$

input
```
integrate((-80*(-3+2*3^(1/2))*x-80*(3-2*3^(1/2))*x^3-80*(90+30*3^(1/2))*x^
5-960*3^(1/2)*x^7-7440*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(-28+2
0*3^(1/2))*x^2+(-1110-418*3^(1/2))*x^4+(296-268*3^(1/2))*x^6+(434-2401*3^(
1/2))*x^8+(204-828*3^(1/2))*x^10+(-324-468*3^(1/2))*x^12-432*x^14+648*x^16
),x, algorithm="maxima")
```

output
```
-80*integrate((18*sqrt(3)*x^11 + 93*sqrt(3)*x^9 + 12*sqrt(3)*x^7 + 30*x^5*
(sqrt(3) + 3) - x^3*(2*sqrt(3) - 3) + x*(2*sqrt(3) - 3))/(648*x^16 - 432*x
^14 - 36*x^12*(13*sqrt(3) + 9) - 12*x^10*(69*sqrt(3) - 17) - 7*x^8*(343*sq
rt(3) - 62) - 4*x^6*(67*sqrt(3) - 74) - 2*x^4*(209*sqrt(3) + 555) + 4*x^2*
(5*sqrt(3) - 7) - 8*sqrt(3) + 14), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-80(-3+2\sqrt{3})x-80(3-2\sqrt{3})x^3-80(90+30\sqrt{3})x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})x^2+(-1110-418\sqrt{3})x^4+(296-268\sqrt{3})x^6+(434-2401\sqrt{3})x^8+(20}$$
= Exception raised: TypeError

input
```
integrate((-80*(-3+2*3^(1/2))*x-80*(3-2*3^(1/2))*x^3-80*(90+30*3^(1/2))*x^
5-960*3^(1/2)*x^7-7440*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(-28+2
0*3^(1/2))*x^2+(-1110-418*3^(1/2))*x^4+(296-268*3^(1/2))*x^6+(434-2401*3^(
1/2))*x^8+(204-828*3^(1/2))*x^10+(-324-468*3^(1/2))*x^12-432*x^14+648*x^16
),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[infin
ity,infinity,infinity,infinity,infinity]proot error [undef,undef,undef,und
ef,undef]
```

Mupad [B] (verification not implemented)

Time = 20.62 (sec) , antiderivative size = 841, normalized size of antiderivative = 4.34

$$\int \frac{-80(-3+2\sqrt{3})x-80(3-2\sqrt{3})x^3-80(90+30\sqrt{3})x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})x^2+(-1110-418\sqrt{3})x^4+(296-268\sqrt{3})x^6+(434-2401\sqrt{3})x^8+(20}$$
= Too large to display

input
```
int((80*x^5*(30*3^(1/2) + 90) - 80*x^3*(2*3^(1/2) - 3) + 960*3^(1/2)*x^7 +
 7440*3^(1/2)*x^9 + 1440*3^(1/2)*x^11 + 80*x*(2*3^(1/2) - 3))/(x^6*(268*3^
(1/2) - 296) - x^2*(20*3^(1/2) - 28) + x^12*(468*3^(1/2) + 324) + x^10*(82
8*3^(1/2) - 204) + x^4*(418*3^(1/2) + 1110) + x^8*(2401*3^(1/2) - 434) + 8
*3^(1/2) + 432*x^14 - 648*x^16 - 14),x)
```

output
```
symsum(log(2441392483387725377009512982635682733148193359375/5340065077318
915416139886903649811 2 - root(32173507920061259683061760000000000*3^(1/2)*z
^8 - 602726352556278420203650560000000000*z^8 + 75340794069534802525456320000
00000*3^(1/2)*z^4 - 120650654700229723811481600000000000*z^4 + 3770332959382
178869108800000000*3^(1/2) - 7063199444018887736761530000000000, z, k)*(roo
t(32173507920061259683061760000000000*3^(1/2)*z^8 - 602726352556278420203650
5600000000*z^8 + 7534079406953480252545632000000000*3^(1/2)*z^4 - 12065065
470022972381148160000000000*z^4 + 37703329593821788691088000000000*3^(1/2)
- 7063199444018887736761530000000000, z, k)*((2552211521493444779558910207336
52787446811767578125*3^(1/2))/5340065077318915416139886903649811 2 - x^2*(
(6362576388018852111306272669458741480994398193359375*3^(1/2))/42720520618
551323329119095229198489 6 - 17184368189833613490039096968670222987810710693
359375/12816156185565396998735728568759546 88) + root(3217350792006125968 3
061760000000000*3^(1/2)*z^8 - 602726352556278420203650560000000000*z^8 + 7534
079406953480252545632000000000*3^(1/2)*z^4 - 1206506547002297238114816000000
0000*z^4 + 37703329593821788691088000000000*3^(1/2) - 706319944401888773 67
615300000000, z, k)*(root(32173507920061259683061760000000000*3^(1/2)*z^8 -
 602726352556278420203650560000000000*z^8 + 7534079406953480252545632000000000
0*3^(1/2)*z^4 - 120650654700229723811481600000000000*z^4 + 37703329593821788
691088000000000*3^(1/2) - 706319944401888773676153000000000, z, k)*((137...
```

Reduce [F]

$$\int \frac{-80(-3+2\sqrt{3})\,x-80(3-2\sqrt{3})\,x^3-80(90+30\sqrt{3})\,x^5-960\sqrt{3}x^7}{14-8\sqrt{3}+(-28+20\sqrt{3})\,x^2+(-1110-418\sqrt{3})\,x^4+(296-268\sqrt{3})\,x^6+(434-2401\sqrt{3})\,x^8+(2}$$
= too large to display

input
```
int((-80*(-3+2*3^(1/2))*x-80*(3-2*3^(1/2))*x^3-80*(90+30*3^(1/2))*x^5-960*
3^(1/2)*x^7-7440*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(-28+20*3^(1
/2))*x^2+(-1110-418*3^(1/2))*x^4+(296-268*3^(1/2))*x^6+(434-2401*3^(1/2))*
x^8+(204-828*3^(1/2))*x^10+(-324-468*3^(1/2))*x^12-432*x^14+648*x^16),x)
```

output

```
80*( - 11664*sqrt(3)*int(x**27/(419904*x**32 - 559872*x**30 - 233280*x**28
 + 544320*x**26 - 165888*x**24 - 2448576*x**22 - 10732680*x**20 - 11772696
*x**18 - 18728831*x**16 - 6071176*x**14 - 7056644*x**12 - 1099480*x**10 +
620416*x**8 + 107744*x**6 - 51560*x**4 + 176*x**2 + 4),x) - 52488*sqrt(3)*
int(x**25/(419904*x**32 - 559872*x**30 - 233280*x**28 + 544320*x**26 - 165
888*x**24 - 2448576*x**22 - 10732680*x**20 - 11772696*x**18 - 18728831*x**
16 - 6071176*x**14 - 7056644*x**12 - 1099480*x**10 + 620416*x**8 + 107744*
x**6 - 51560*x**4 + 176*x**2 + 4),x) + 38232*sqrt(3)*int(x**23/(419904*x**
32 - 559872*x**30 - 233280*x**28 + 544320*x**26 - 165888*x**24 - 2448576*x
**22 - 10732680*x**20 - 11772696*x**18 - 18728831*x**16 - 6071176*x**14 -
7056644*x**12 - 1099480*x**10 + 620416*x**8 + 107744*x**6 - 51560*x**4 + 1
76*x**2 + 4),x) + 12204*sqrt(3)*int(x**21/(419904*x**32 - 559872*x**30 - 2
33280*x**28 + 544320*x**26 - 165888*x**24 - 2448576*x**22 - 10732680*x**20
 - 11772696*x**18 - 18728831*x**16 - 6071176*x**14 - 7056644*x**12 - 10994
80*x**10 + 620416*x**8 + 107744*x**6 - 51560*x**4 + 176*x**2 + 4),x) - 864
0*sqrt(3)*int(x**19/(419904*x**32 - 559872*x**30 - 233280*x**28 + 544320*x
**26 - 165888*x**24 - 2448576*x**22 - 10732680*x**20 - 11772696*x**18 - 18
728831*x**16 - 6071176*x**14 - 7056644*x**12 - 1099480*x**10 + 620416*x**8
 + 107744*x**6 - 51560*x**4 + 176*x**2 + 4),x) - 82698*sqrt(3)*int(x**17/(
419904*x**32 - 559872*x**30 - 233280*x**28 + 544320*x**26 - 165888*x**2...
```

3.57 $\int \frac{-16(3-2\sqrt{3})x-16(15-10\sqrt{3})x^3-16(9}{14-8\sqrt{3}+(140-100\sqrt{3})x^2+(1578+158\sqrt{3})x^4+(440-820\sqrt{3})x^6+(16}$

Optimal result

Integrand size = 171, antiderivative size = 177

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
$$= \sqrt[8]{3}\arctan\left(\frac{1}{\sqrt[8]{3}}-3^{3/8}x\right) + \sqrt[8]{3}\arctan\left(\frac{1}{\sqrt[8]{3}}+3^{3/8}x\right)+\sqrt[8]{3}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(-6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}+\frac{x^2}{\sqrt[8]{3}}\right) + \sqrt[8]{3}\operatorname{arctanh}\left(\frac{1}{\sqrt[8]{3}}-3^{3/8}x\right)+\sqrt[8]{3}\operatorname{arctanh}\left(\frac{1}{\sqrt[8]{3}}+3^{3/8}x\right)-\sqrt[8]{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{\frac{1}{3}\left(6+\sqrt{6\left(3+2\sqrt{3}\right)}\right)}-\frac{}{}$$

output
```
-3^(1/8)*arctan(-1/3*3^(7/8)+3^(3/8)*x)+3^(1/8)*arctan(1/3*3^(7/8)+3^(3/8)
*x)+3^(1/8)*arctan(1/6*(-18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/3*x^2*3^(7/8)
)-3^(1/8)*arctanh(-1/3*3^(7/8)+3^(3/8)*x)+3^(1/8)*arctanh(1/3*3^(7/8)+3^(3
/8)*x)+3^(1/8)*arctanh(-1/6*(18+3*(18+12*3^(1/2))^(1/2))^(1/2)+1/3*x^2*3^(
7/8))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.13

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$

$$= 2\text{RootSum}\Big[-14+8\sqrt{3}-140\#1^2+100\sqrt{3}\#1^2-1578\#1^4-158\sqrt{3}\#1^4-440\#1^6 +820\sqrt{3}\#1^6-1682\#1^8-1343\sqrt{3}\#1^8+2580\#1^{10}+540\sqrt{3}\#1^{10}-3132\#1^{12} +468\sqrt{3}\#1^{12}+2160\#1^{14} -648\#1^{16}\&,\frac{3\log(x-\#1)-2\sqrt{3}\log(x-\#1)+15\log(x-\#1)\#1^2-10\sqrt{3}\log(x-\#1)\#1^2+90\log}{-35+25\sqrt{3}-789\#1^2-79\sqrt{3}\#1^2-330\#1^4+615\sqrt{3}\#1^4-1682\#}$$

input
```
Integrate[(-16*(3 - 2*Sqrt[3])*x - 16*(15 - 10*Sqrt[3])*x^3 - 16*(90 + 30*
Sqrt[3])*x^5 + 960*Sqrt[3]*x^7 + 240*Sqrt[3]*x^9 - 1440*Sqrt[3]*x^11)/(14
- 8*Sqrt[3] + (140 - 100*Sqrt[3])*x^2 + (1578 + 158*Sqrt[3])*x^4 + (440 -
820*Sqrt[3])*x^6 + (1682 + 1343*Sqrt[3])*x^8 + (-2580 - 540*Sqrt[3])*x^10
+ (3132 - 468*Sqrt[3])*x^12 - 2160*x^14 + 648*x^16),x]
```

output
```
2*RootSum[-14 + 8*Sqrt[3] - 140*#1^2 + 100*Sqrt[3]*#1^2 - 1578*#1^4 - 158*
Sqrt[3]*#1^4 - 440*#1^6 + 820*Sqrt[3]*#1^6 - 1682*#1^8 - 1343*Sqrt[3]*#1^8
 + 2580*#1^10 + 540*Sqrt[3]*#1^10 - 3132*#1^12 + 468*Sqrt[3]*#1^12 + 2160*
#1^14 - 648*#1^16 & , (3*Log[x - #1] - 2*Sqrt[3]*Log[x - #1] + 15*Log[x -
#1]*#1^2 - 10*Sqrt[3]*Log[x - #1]*#1^2 + 90*Log[x - #1]*#1^4 + 30*Sqrt[3]*
Log[x - #1]*#1^4 - 60*Sqrt[3]*Log[x - #1]*#1^6 - 15*Sqrt[3]*Log[x - #1]*#1
^8 + 90*Sqrt[3]*Log[x - #1]*#1^10)/(-35 + 25*Sqrt[3] - 789*#1^2 - 79*Sqrt[
3]*#1^2 - 330*#1^4 + 615*Sqrt[3]*#1^4 - 1682*#1^6 - 1343*Sqrt[3]*#1^6 + 32
25*#1^8 + 675*Sqrt[3]*#1^8 - 4698*#1^10 + 702*Sqrt[3]*#1^10 + 3780*#1^12 -
 1296*#1^14) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-1440\sqrt{3}x^{11}+240\sqrt{3}x^9+960\sqrt{3}x^7-16\left(90+30\sqrt{3}\right)x^5-16\left(15-10\sqrt{3}\right.}{648x^{16}-2160x^{14}+\left(3132-468\sqrt{3}\right)x^{12}+\left(-2580-540\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+\left(440-820\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{16x\left(-90\sqrt{3}x^{10}+15\sqrt{3}x^8+60\sqrt{3}x^6-90\left(1+\frac{1}{\sqrt{3}}\right)x^4-15\left(1-\frac{2}{\sqrt{3}}\right.\right.}{648x^{16}-2160x^{14}+\left(3132-468\sqrt{3}\right)x^{12}+\left(-2580-540\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+\left(440-820\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{27}$$

$$16\int -\frac{x\left(90\sqrt{3}x^{10}-15\sqrt{3}x^8-60\sqrt{3}x^6+30\left(3+\sqrt{3}\right)x^4+5\left(3-2\sqrt{3}\right)\right.}{648x^{16}-2160x^{14}+36\left(87-13\sqrt{3}\right)x^{12}-60\left(43+9\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+20\left(22-41\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{25}$$

$$-16\int \frac{x\left(90\sqrt{3}x^{10}-15\sqrt{3}x^8-60\sqrt{3}x^6+30\left(3+\sqrt{3}\right)x^4+5\left(3-2\sqrt{3}\right)\right.}{648x^{16}-2160x^{14}+36\left(87-13\sqrt{3}\right)x^{12}-60\left(43+9\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+20\left(22-41\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{7266}$$

$$-8\int \frac{90\sqrt{3}x^{10}-15\sqrt{3}x^8-60\sqrt{3}x^6+30\left(3+\sqrt{3}\right)x^4+5\left(3-2\sqrt{3}\right)x}{648x^{16}-2160x^{14}+36\left(87-13\sqrt{3}\right)x^{12}-60\left(43+9\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+20\left(22-41\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{7292}$$

$$-8\int \frac{90\sqrt{3}x^{10}-15\sqrt{3}x^8-60\sqrt{3}x^6+30\left(3+\sqrt{3}\right)x^4+5\left(3-2\sqrt{3}\right)x^2}{648x^{16}-2160x^{14}+36\left(87-13\sqrt{3}\right)x^{12}-60\left(43+9\sqrt{3}\right)x^{10}+\left(1682+1343\sqrt{3}\right)x^8+20\left(22-41\sqrt{3}\right)x^6}$$

$$\Big\downarrow \text{7293}$$

$$-8\int \left(\frac{90\sqrt{3}x^{10}}{648x^{16}-2160x^{14}+3132\left(1-\frac{13}{29\sqrt{3}}\right)x^{12}-2580\left(1+\frac{9\sqrt{3}}{43}\right)x^{10}+1682\left(1+\frac{1343\sqrt{3}}{1682}\right)x^8+440\left(1-\frac{41\sqrt{3}}{22}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-8\left(60\sqrt{3}\int \frac{x^6}{-648x^{16}+2160x^{14}-3132\left(1-\frac{13}{29\sqrt{3}}\right)x^{12}+2580\left(1+\frac{9\sqrt{3}}{43}\right)x^{10}-1682\left(1+\frac{1343\sqrt{3}}{1682}\right)x^8-440\left(1\right.}\right.$$

input

```
Int[(-16*(3 - 2*Sqrt[3])*x - 16*(15 - 10*Sqrt[3])*x^3 - 16*(90 + 30*Sqrt[3
])*x^5 + 960*Sqrt[3]*x^7 + 240*Sqrt[3]*x^9 - 1440*Sqrt[3]*x^11)/(14 - 8*Sq
rt[3] + (140 - 100*Sqrt[3])*x^2 + (1578 + 158*Sqrt[3])*x^4 + (440 - 820*Sq
rt[3])*x^6 + (1682 + 1343*Sqrt[3])*x^8 + (-2580 - 540*Sqrt[3])*x^10 + (313
2 - 468*Sqrt[3])*x^12 - 2160*x^14 + 648*x^16),x]
```

output

```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98

method	result
default	$2\sqrt{3}\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-16\_Z^3+\left(-4\sqrt{3}+12\right)\_Z^2+\left(-12\sqrt{3}-4\right)\_Z+2-\sqrt{3}\right)}\frac{\left(1+2\_R\right)\ln\left(x^2-\_R\right)}{-8\_R^3+12\_R^2-6\_R+1+\sqrt{3}\left(3+2\_R\right.}\right.$

input

```
int((-16*(3-2*3^(1/2))*x-16*(15-10*3^(1/2))*x^3-16*(90+30*3^(1/2))*x^5+960
*3^(1/2)*x^7+240*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(140-100*3^(
1/2))*x^2+(1578+158*3^(1/2))*x^4+(440-820*3^(1/2))*x^6+(1682+1343*3^(1/2))
*x^8+(-2580-540*3^(1/2))*x^10+(3132-468*3^(1/2))*x^12-2160*x^14+648*x^16),
x,method=_RETURNVERBOSE)
```

output

```
2*3^(1/2)*sum((1+2*_R)/(-8*_R^3+12*_R^2-6*_R+1+3^(1/2)*(3+2*_R))*ln(x^2-_R
),_R=RootOf(8*_Z^4-16*_Z^3+(-4*3^(1/2)+12)*_Z^2+(-12*3^(1/2)-4)*_Z+2-3^(1/
2)))-2*3^(1/2)*sum((3*_R+1)/(-27*_R^3+27*_R^2-9*_R+1+3*3^(1/2)*(1+_R))*ln(
x^2-_R),_R=RootOf(81*_Z^4-108*_Z^3+(-18*3^(1/2)+54)*_Z^2+(-36*3^(1/2)-12)*
_Z-2*3^(1/2)+4))
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$

$$\begin{aligned}
&= 3^{\frac{1}{8}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{7}{8}}(6\,x^2-5)+\frac{5}{2}\cdot 3^{\frac{1}{8}}\right) \\
&\quad - 3^{\frac{1}{8}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{8}}\left(162\,x^6-630\,x^4+576\,x^2+\sqrt{3}(90\,x^6-330\,x^4+319\,x^2-5)\right)\right. \\
&\qquad\qquad \left. -\frac{1}{2}\cdot 3^{\frac{1}{8}}\left(72\,x^6-270\,x^4+240\,x^2+6\,\sqrt{3}(6\,x^6-25\,x^4+24\,x^2)-5\right)\right) \\
&\quad + \frac{1}{2}\cdot 3^{\frac{1}{8}}\log\left(6\,x^4+2\cdot 3^{\frac{1}{8}}x^2-5\,x^2-3^{\frac{1}{4}}(5\,x^2+2)+\sqrt{3}+1\right) \\
&\quad - \frac{1}{2}\cdot 3^{\frac{1}{8}}\log\left(6\,x^4-2\cdot 3^{\frac{1}{8}}x^2-5\,x^2-3^{\frac{1}{4}}(5\,x^2+2)+\sqrt{3}+1\right)
\end{aligned}$$

input
```
integrate((-16*(3-2*3^(1/2))*x-16*(15-10*3^(1/2))*x^3-16*(90+30*3^(1/2))*x
^5+960*3^(1/2)*x^7+240*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(140-1
00*3^(1/2))*x^2+(1578+158*3^(1/2))*x^4+(440-820*3^(1/2))*x^6+(1682+1343*3^
(1/2))*x^8+(-2580-540*3^(1/2))*x^10+(3132-468*3^(1/2))*x^12-2160*x^14+648*
x^16),x, algorithm="fricas")
```

output
```
3^(1/8)*arctan(1/6*3^(7/8)*(6*x^2 - 5) + 5/2*3^(1/8)) - 3^(1/8)*arctan(1/6
*3^(3/8)*(162*x^6 - 630*x^4 + 576*x^2 + sqrt(3)*(90*x^6 - 330*x^4 + 319*x^
2 - 5)) - 1/2*3^(1/8)*(72*x^6 - 270*x^4 + 240*x^2 + 6*sqrt(3)*(6*x^6 - 25*
x^4 + 24*x^2) - 5)) + 1/2*3^(1/8)*log(6*x^4 + 2*3^(1/8)*x^2 - 5*x^2 - 3^(1
/4)*(5*x^2 + 2) + sqrt(3) + 1) - 1/2*3^(1/8)*log(6*x^4 - 2*3^(1/8)*x^2 - 5
*x^2 - 3^(1/4)*(5*x^2 + 2) + sqrt(3) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
$$=\text{Timed out}$$

input
```
integrate((-16*(3-2*3**(1/2))*x-16*(15-10*3**(1/2))*x**3-16*(90+30*3**(1/2
))*x**5+960*3**(1/2)*x**7+240*3**(1/2)*x**9-1440*3**(1/2)*x**11)/(14-8*3**
(1/2)+(140-100*3**(1/2))*x**2+(1578+158*3**(1/2))*x**4+(440-820*3**(1/2))*
x**6+(1682+1343*3**(1/2))*x**8+(-2580-540*3**(1/2))*x**10+(3132-468*3**(1/
2))*x**12-2160*x**14+648*x**16),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
$$=\int -\frac{16\left(90\sqrt{3}x^{11}-15\sqrt{3}x^9-60\sqrt{3}x^7+30\,x^5\left(\sqrt{3}+3\right)-5\,x^3\left(2\right.\right.}{648\,x^{16}-2160\,x^{14}-36\,x^{12}\left(13\sqrt{3}-87\right)-60\,x^{10}\left(9\sqrt{3}+43\right)+x^8\left(1343\sqrt{3}+1682\right)-20\,x^6\left(41\sqrt{\right.}}$$

input
```
integrate((-16*(3-2*3^(1/2))*x-16*(15-10*3^(1/2))*x^3-16*(90+30*3^(1/2))*x
^5+960*3^(1/2)*x^7+240*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(140-1
00*3^(1/2))*x^2+(1578+158*3^(1/2))*x^4+(440-820*3^(1/2))*x^6+(1682+1343*3^
(1/2))*x^8+(-2580-540*3^(1/2))*x^10+(3132-468*3^(1/2))*x^12-2160*x^14+648*
x^16),x, algorithm="maxima")
```

output
```
-16*integrate((90*sqrt(3)*x^11 - 15*sqrt(3)*x^9 - 60*sqrt(3)*x^7 + 30*x^5*
(sqrt(3) + 3) - 5*x^3*(2*sqrt(3) - 3) - x*(2*sqrt(3) - 3))/(648*x^16 - 216
0*x^14 - 36*x^12*(13*sqrt(3) - 87) - 60*x^10*(9*sqrt(3) + 43) + x^8*(1343*
sqrt(3) + 1682) - 20*x^6*(41*sqrt(3) - 22) + 2*x^4*(79*sqrt(3) + 789) - 20
*x^2*(5*sqrt(3) - 7) - 8*sqrt(3) + 14), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
= Exception raised: TypeError

input
```
integrate((-16*(3-2*3^(1/2))*x-16*(15-10*3^(1/2))*x^3-16*(90+30*3^(1/2))*x
^5+960*3^(1/2)*x^7+240*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(140-1
00*3^(1/2))*x^2+(1578+158*3^(1/2))*x^4+(440-820*3^(1/2))*x^6+(1682+1343*3^
(1/2))*x^8+(-2580-540*3^(1/2))*x^10+(3132-468*3^(1/2))*x^12-2160*x^14+648*
x^16),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[infin
ity,0.0,infinity,infinity,infinity]proot error [undef,0.0,undef,undef,unde
f]proot e
```

Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 837, normalized size of antiderivative = 4.73

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
= Too large to display

input
```
int(-(16*x^3*(10*3^(1/2) - 15) - 16*x^5*(30*3^(1/2) + 90) + 960*3^(1/2)*x^
7 + 240*3^(1/2)*x^9 - 1440*3^(1/2)*x^11 + 16*x*(2*3^(1/2) - 3))/(x^2*(100*
3^(1/2) - 140) + x^6*(820*3^(1/2) - 440) - x^4*(158*3^(1/2) + 1578) - x^8*
(1343*3^(1/2) + 1682) + x^10*(540*3^(1/2) + 2580) + x^12*(468*3^(1/2) - 31
32) + 8*3^(1/2) + 2160*x^14 - 648*x^16 - 14),x)
```

output
```
symsum(log(root(185290476904455051041625341952*3^(1/2)*z^8 + 4542967087543
22101063029817344*z^8 - 567870885942902626328787271 68*3^(1/2)*z^4 - 694839
2883917064414060950 3232*z^4 + 217137277622408262939404 6976*3^(1/2) + 53237
8955571471212183238 0672, z, k)*(x^2*((116452365523047784926105240954688839
1307053779667*3^(1/2))/3204039046391349249683932142189 88672 + 134082915512
7388744734478240447187948901339593923/21360260309275661664559547614599 2448
) + root(185290476904455051041625341952*3^(1/2)*z^8 + 4542967087543221010 6
3029817344*z^8 - 56787088594290262632878727168*3^(1/2)*z^4 - 6948392883917
0644140609503232*z^4 + 2171372776224082629394046976*3^(1/2) + 532378955571
4712121832380672, z, k)*((14814514368105567951390204291701229791620991467*
3^(1/2))/534006507731891541613988690364 98112 - x^2*((137301582705476336508
19936627817610377868308256655*3^(1/2))/42720520618551323329119095229198489
6 + 71331809161667252793264507457982328197470692629905/1281615618556539699
87357285687595468 8) + root(185290476904455051041625341952*3^(1/2)*z^8 + 45
42967087543221010 63029817344*z^8 - 56787088594290262632878727168*3^(1/2)*z
^4 - 69483928839170644140609503232*z^4 + 2171372776224082629394046976*3^(1
/2) + 5323789555714712121832380672, z, k)*((490612725533053311932446343021
52037598418395*3^(1/2))/200252440399459328105245758886867 92 - root(1852904
76904455051041625341952*3^(1/2)*z^8 + 454296708754322101063029817344*z^8 -
 56787088594290262632878727168*3^(1/2)*z^4 - 69483928839170644140609503...
```

Reduce [F]

$$\int \frac{-16(3-2\sqrt{3})\,x-16(15-10\sqrt{3})\,x^3-16(90+30\sqrt{3})\,x^5+960\sqrt{3}}{14-8\sqrt{3}+(140-100\sqrt{3})\,x^2+(1578+158\sqrt{3})\,x^4+(440-820\sqrt{3})\,x^6+(1682+1343\sqrt{3})\,x^8+(-}$$
$= \text{too large to display}$

input
```
int((-16*(3-2*3^(1/2))*x-16*(15-10*3^(1/2))*x^3-16*(90+30*3^(1/2))*x^5+960
*3^(1/2)*x^7+240*3^(1/2)*x^9-1440*3^(1/2)*x^11)/(14-8*3^(1/2)+(140-100*3^(
1/2))*x^2+(1578+158*3^(1/2))*x^4+(440-820*3^(1/2))*x^6+(1682+1343*3^(1/2))
*x^8+(-2580-540*3^(1/2))*x^10+(3132-468*3^(1/2))*x^12-2160*x^14+648*x^16),
x)
```

output
```
16*( - 58320*sqrt(3)*int(x**27/(419904*x**32 - 2799360*x**30 + 8724672*x**
28 - 16873920*x**26 + 22477824*x**24 - 24373440*x**22 + 20233080*x**20 - 1
0509720*x**18 + 2232577*x**16 + 992840*x**14 + 1230460*x**12 + 3344600*x**
10 + 2157952*x**8 + 509600*x**6 + 41368*x**4 - 880*x**2 + 4),x) + 204120*s
qrt(3)*int(x**25/(419904*x**32 - 2799360*x**30 + 8724672*x**28 - 16873920*
x**26 + 22477824*x**24 - 24373440*x**22 + 20233080*x**20 - 10509720*x**18
+ 2232577*x**16 + 992840*x**14 + 1230460*x**12 + 3344600*x**10 + 2157952*x
**8 + 509600*x**6 + 41368*x**4 - 880*x**2 + 4),x) - 275400*sqrt(3)*int(x**
23/(419904*x**32 - 2799360*x**30 + 8724672*x**28 - 16873920*x**26 + 224778
24*x**24 - 24373440*x**22 + 20233080*x**20 - 10509720*x**18 + 2232577*x**1
6 + 992840*x**14 + 1230460*x**12 + 3344600*x**10 + 2157952*x**8 + 509600*x
**6 + 41368*x**4 - 880*x**2 + 4),x) + 130140*sqrt(3)*int(x**21/(419904*x**
32 - 2799360*x**30 + 8724672*x**28 - 16873920*x**26 + 22477824*x**24 - 243
73440*x**22 + 20233080*x**20 - 10509720*x**18 + 2232577*x**16 + 992840*x**
14 + 1230460*x**12 + 3344600*x**10 + 2157952*x**8 + 509600*x**6 + 41368*x*
*4 - 880*x**2 + 4),x) + 69120*sqrt(3)*int(x**19/(419904*x**32 - 2799360*x*
*30 + 8724672*x**28 - 16873920*x**26 + 22477824*x**24 - 24373440*x**22 + 2
0233080*x**20 - 10509720*x**18 + 2232577*x**16 + 992840*x**14 + 1230460*x*
*12 + 3344600*x**10 + 2157952*x**8 + 509600*x**6 + 41368*x**4 - 880*x**2 +
 4),x) - 325554*sqrt(3)*int(x**17/(419904*x**32 - 2799360*x**30 + 87246...
```

3.58 $\int \frac{8(3-2\sqrt{3})x+24\sqrt{3}x^5}{-28+16\sqrt{3}+(-20+20\sqrt{3})x^2+(-148-22\sqrt{3})x^4+(60+60\sqrt{3})x^6-36x$

Optimal result

Integrand size = 79, antiderivative size = 74

$$\int \frac{8\left(3-2\sqrt{3}\right)x+24\sqrt{3}x^5}{-28+16\sqrt{3}+\left(-20+20\sqrt{3}\right)x^2+\left(-148-22\sqrt{3}\right)x^4+\left(60+60\sqrt{3}\right)x^6-36x^8}\,dx$$
$$=\sqrt[4]{3}\text{arctanh}\left(\frac{1}{\sqrt[4]{3}}-\sqrt[4]{3}x\right)+\sqrt[4]{3}\text{arctanh}\left(\frac{1}{\sqrt[4]{3}}+\sqrt[4]{3}x\right)$$
$$-\sqrt[4]{3}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{1}{\sqrt{3}}}-\frac{x^2}{\sqrt[4]{3}}\right)$$

output
```
-3^(1/4)*arctanh(-1/3*3^(3/4)+3^(1/4)*x)+3^(1/4)*arctanh(1/3*3^(3/4)+3^(1/
4)*x)+3^(1/4)*arctanh(-1/6*(18+12*3^(1/2))^(1/2)+1/3*3^(3/4)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.20

$$\int \frac{8(3-2\sqrt{3})\,x+24\sqrt{3}x^5}{-28+16\sqrt{3}+(-20+20\sqrt{3})\,x^2+(-148-22\sqrt{3})\,x^4+(60+60\sqrt{3})\,x^6-36x^8}\,dx$$
$$=\text{RootSum}\left[-14+8\sqrt{3}-10\#1^2+10\sqrt{3}\#1^2-74\#1^4-11\sqrt{3}\#1^4+30\#1^6+30\sqrt{3}\#1^6 -18\#1^8\&,\frac{3\log(x-\#1)-2\sqrt{3}\log(x-\#1)+3\sqrt{3}\log(x-\#1)\#1^4}{-5+5\sqrt{3}-74\#1^2-11\sqrt{3}\#1^2+45\#1^4+45\sqrt{3}\#1^4-36\#1^6}\&\right]$$

input
```
Integrate[(8*(3 - 2*Sqrt[3])*x + 24*Sqrt[3]*x^5)/(-28 + 16*Sqrt[3] + (-20
+ 20*Sqrt[3])*x^2 + (-148 - 22*Sqrt[3])*x^4 + (60 + 60*Sqrt[3])*x^6 - 36*x
^8),x]
```

output
```
RootSum[-14 + 8*Sqrt[3] - 10*#1^2 + 10*Sqrt[3]*#1^2 - 74*#1^4 - 11*Sqrt[3]
*#1^4 + 30*#1^6 + 30*Sqrt[3]*#1^6 - 18*#1^8 & , (3*Log[x - #1] - 2*Sqrt[3]
*Log[x - #1] + 3*Sqrt[3]*Log[x - #1]*#1^4)/(-5 + 5*Sqrt[3] - 74*#1^2 - 11*
Sqrt[3]*#1^2 + 45*#1^4 + 45*Sqrt[3]*#1^4 - 36*#1^6) & ]
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.063, Rules used = {2027, 7266, 27, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{24\sqrt{3}x^5+8(3-2\sqrt{3})\,x}{-36x^8+(60+60\sqrt{3})\,x^6+(-148-22\sqrt{3})\,x^4+(20\sqrt{3}-20)\,x^2+16\sqrt{3}-28}dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x(24\sqrt{3}x^4+8(3-2\sqrt{3}))}{-36x^8+(60+60\sqrt{3})\,x^6+(-148-22\sqrt{3})\,x^4+(20\sqrt{3}-20)\,x^2+16\sqrt{3}-28}dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int -\frac{4(3\sqrt{3}x^4-2\sqrt{3}+3)}{18x^8-30\left(1+\sqrt{3}\right)x^6+\left(74+11\sqrt{3}\right)x^4+10\left(1-\sqrt{3}\right)x^2+2\left(7-4\sqrt{3}\right)}dx^2$$

$$\Big\downarrow \; 27$$

$$-2\int \frac{3\sqrt{3}x^4-2\sqrt{3}+3}{18x^8-30\left(1+\sqrt{3}\right)x^6+\left(74+11\sqrt{3}\right)x^4+10\left(1-\sqrt{3}\right)x^2+2\left(7-4\sqrt{3}\right)}dx^2$$

$$\Big\downarrow \; 2492$$

$$-\frac{1}{9}\int \left(\frac{3\ 3^{3/4}(-12\sqrt{3}x^2-2\ 3^{3/4}+5\sqrt{3}+15)}{2\left(6x^4-\left(5-2\sqrt[4]{3}+5\sqrt{3}\right)x^2+2\left(2-\sqrt{3}\right)\right)}-\frac{3\ 3^{3/4}(-12\sqrt{3}x^2+2\ 3^{3/4}+5\sqrt{3}+15)}{2\left(6x^4-\left(5+2\sqrt[4]{3}+5\sqrt{3}\right)x^2+2\left(2-\sqrt{3}\right)\right)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9}\left(\frac{9}{2}\sqrt[4]{3}\log\left(6x^4-\left(5-2\sqrt[4]{3}+5\sqrt{3}\right)x^2+2\left(2-\sqrt{3}\right)\right)-\frac{9}{2}\sqrt[4]{3}\log\left(6x^4-\left(5+2\sqrt[4]{3}+5\sqrt{3}\right)x^2+2\left(2-\sqrt{3}\right)\right)\right)$$

input
```
Int[(8*(3 - 2*Sqrt[3])*x + 24*Sqrt[3]*x^5)/(-28 + 16*Sqrt[3] + (-20 + 20*S
qrt[3])*x^2 + (-148 - 22*Sqrt[3])*x^4 + (60 + 60*Sqrt[3])*x^6 - 36*x^8),x]
```

output
```
((9*3^(1/4)*Log[2*(2 - Sqrt[3]) - (5 - 2*3^(1/4) + 5*Sqrt[3])*x^2 + 6*x^4]
)/2 - (9*3^(1/4)*Log[2*(2 - Sqrt[3]) - (5 + 2*3^(1/4) + 5*Sqrt[3])*x^2 + 6
*x^4])/2)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$3^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{\left(4x^2-2\sqrt{3}-2\right)3^{\frac{3}{4}}}{12}\right)-3^{\frac{1}{4}} \operatorname{arctanh}\left(\frac{\left(18x^2-6-6\sqrt{3}\right)3^{\frac{3}{4}}}{36}\right)$	47

input
```
int((8*(3-2*3^(1/2))*x+24*3^(1/2)*x^5)/(-28+16*3^(1/2)+(-20+20*3^(1/2))*x^
2+(-148-22*3^(1/2))*x^4+(60+60*3^(1/2))*x^6-36*x^8),x,method=_RETURNVERBOS
E)
```

output
```
3^(1/4)*arctanh(1/12*(4*x^2-2*3^(1/2)-2)*3^(3/4))-3^(1/4)*arctanh(1/36*(18
*x^2-6-6*3^(1/2))*3^(3/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.70

$$\int \frac{8(3-2\sqrt{3})x+24\sqrt{3}x^5}{-28+16\sqrt{3}+(-20+20\sqrt{3})x^2+(-148-22\sqrt{3})x^4+(60+60\sqrt{3})x^6-36x^8}\,dx = \frac{1}{2} \cdot 3^{\frac{1}{4}} \log\left(\frac{324\,x^{16}-1080\,x^{14}+864\,x^{12}-1740\,x^{10}+3085\,x^8-20\,x^6+2496\,x^4-200\,x^2+8\sqrt{3}(9\,x^{12}-\ldots}{324\,x^{16}-\ldots}\right.$$

input
```
integrate((8*(3-2*3^(1/2))*x+24*3^(1/2)*x^5)/(-28+16*3^(1/2)+(-20+20*3^(1/
2))*x^2+(-148-22*3^(1/2))*x^4+(60+60*3^(1/2))*x^6-36*x^8),x, algorithm="fr
icas")
```

output
```
1/2*3^(1/4)*log((324*x^16 - 1080*x^14 + 864*x^12 - 1740*x^10 + 3085*x^8 -
20*x^6 + 2496*x^4 - 200*x^2 + 8*sqrt(3)*(9*x^12 - 15*x^10 + 37*x^8 + 5*x^6
 + 7*x^4) + 2*3^(1/4)*(108*x^14 - 270*x^12 + 216*x^10 - 445*x^8 + 246*x^6
- 210*x^4 + 8*x^2 + sqrt(3)*(90*x^12 - 102*x^10 - 75*x^8 - 244*x^6 - 90*x^
4 + 4*x^2)) + 4)/(324*x^16 - 1080*x^14 + 864*x^12 - 2100*x^10 + 3217*x^8 -
 140*x^6 + 2400*x^4 - 200*x^2 + 4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{8(3-2\sqrt{3})x+24\sqrt{3}x^5}{-28+16\sqrt{3}+(-20+20\sqrt{3})x^2+(-148-22\sqrt{3})x^4+(60+60\sqrt{3})x^6-36x^8}\,dx = \text{Timed out}$$

input
```
integrate((8*(3-2*3**(1/2))*x+24*3**(1/2)*x**5)/(-28+16*3**(1/2)+(-20+20*3
**(1/2))*x**2+(-148-22*3**(1/2))*x**4+(60+60*3**(1/2))*x**6-36*x**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{8\left(3-2\sqrt{3}\right)x+24\sqrt{3}x^5}{-28+16\sqrt{3}+\left(-20+20\sqrt{3}\right)x^2+\left(-148-22\sqrt{3}\right)x^4+\left(60+60\sqrt{3}\right)x^6-36x^8}\,dx$$
$$=\int -\frac{4\left(3\sqrt{3}x^5-x\left(2\sqrt{3}-3\right)\right)}{18\,x^8-30\,x^6\left(\sqrt{3}+1\right)+x^4\left(11\sqrt{3}+74\right)-10\,x^2\left(\sqrt{3}-1\right)-8\sqrt{3}+14}\,dx$$

input
```
integrate((8*(3-2*3^(1/2))*x+24*3^(1/2)*x^5)/(-28+16*3^(1/2)+(-20+20*3^(1/
2))*x^2+(-148-22*3^(1/2))*x^4+(60+60*3^(1/2))*x^6-36*x^8),x, algorithm="ma
xima")
```

output
```
-4*integrate((3*sqrt(3)*x^5 - x*(2*sqrt(3) - 3))/(18*x^8 - 30*x^6*(sqrt(3)
 + 1) + x^4*(11*sqrt(3) + 74) - 10*x^2*(sqrt(3) - 1) - 8*sqrt(3) + 14), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{8\left(3-2\sqrt{3}\right)x+24\sqrt{3}x^5}{-28+16\sqrt{3}+\left(-20+20\sqrt{3}\right)x^2+\left(-148-22\sqrt{3}\right)x^4+\left(60+60\sqrt{3}\right)x^6-36x^8}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((8*(3-2*3^(1/2))*x+24*3^(1/2)*x^5)/(-28+16*3^(1/2)+(-20+20*3^(1/
2))*x^2+(-148-22*3^(1/2))*x^4+(60+60*3^(1/2))*x^6-36*x^8),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument ValueUnable to find common minimal polynomial Er
ror: Bad
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 1086, normalized size of antiderivative = 14.68

$$\int \frac{8\left(3-2\sqrt{3}\right)x+24\sqrt{3}x^5}{-28+16\sqrt{3}+\left(-20+20\sqrt{3}\right)x^2+\left(-148-22\sqrt{3}\right)x^4+\left(60+60\sqrt{3}\right)x^6-36x^8}\,dx$$
$= \text{Too large to display}$

input
```
int((24*3^(1/2)*x^5 - 8*x*(2*3^(1/2) - 3))/(x^2*(20*3^(1/2) - 20) + x^6*(6
0*3^(1/2) + 60) - x^4*(22*3^(1/2) + 148) + 16*3^(1/2) - 36*x^8 - 28),x)
```

output
```
symsum(log(87551280*root(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/
2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 475209, z, k) - 50276880*3^(1/2)*r
oot(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2
+ 271800*3^(1/2) - 475209, z, k) - 1944000*3^(1/2) - 329319432*3^(1/2)*roo
t(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2 +
271800*3^(1/2) - 475209, z, k)^2 - 233470080*3^(1/2)*root(1449600*3^(1/2)*
z^4 - 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 4
75209, z, k)^3 - 1548784624*3^(1/2)*root(1449600*3^(1/2)*z^4 - 2534448*z^4
 + 1267224*3^(1/2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 475209, z, k)^4 -
268143360*3^(1/2)*root(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/2)
*z^2 - 2174400*z^2 + 271800*3^(1/2) - 475209, z, k)^5 - 1708447104*3^(1/2)
*root(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^
2 + 271800*3^(1/2) - 475209, z, k)^6 - 125503551*root(1449600*3^(1/2)*z^4
- 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 47520
9, z, k)*x^2 + 8650260*3^(1/2)*x^2 + 592458234*root(1449600*3^(1/2)*z^4 -
2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 475209,
 z, k)^2 + 402215040*root(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1
/2)*z^2 - 2174400*z^2 + 271800*3^(1/2) - 475209, z, k)^3 + 2580641856*root
(1449600*3^(1/2)*z^4 - 2534448*z^4 + 1267224*3^(1/2)*z^2 - 2174400*z^2 + 2
71800*3^(1/2) - 475209, z, k)^4 + 466940160*root(1449600*3^(1/2)*z^4 - ...
```

Reduce [F]

$$\int \frac{8\left(3-2\sqrt{3}\right)x+24\sqrt{3}\,x^5}{-28+16\sqrt{3}+\left(-20+20\sqrt{3}\right)x^2+\left(-148-22\sqrt{3}\right)x^4+\left(60+60\sqrt{3}\right)x^6-36x^8}\,dx$$

$$= -216\sqrt{3}\left(\int \frac{x^{13}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+360\sqrt{3}\left(\int \frac{x^{11}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$-744\sqrt{3}\left(\int \frac{x^{9}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$-720\sqrt{3}\left(\int \frac{x^{7}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+556\sqrt{3}\left(\int \frac{x^{5}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$-40\sqrt{3}\left(\int \frac{x^{3}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+16\sqrt{3}\left(\int \frac{x}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$-1080\left(\int \frac{x^{11}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+180\left(\int \frac{x^{9}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+720\left(\int \frac{x^{7}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$-1440\left(\int \frac{x^{5}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+120\left(\int \frac{x^{3}}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$
$$+24\left(\int \frac{x}{324x^{16}-1080x^{14}+864x^{12}-2100x^{10}+3217x^8-140x^6+2400x^4-200x^2+4}dx\right)$$

input
```
int((8*(3-2*3^(1/2))*x+24*3^(1/2)*x^5)/(-28+16*3^(1/2)+(-20+20*3^(1/2))*x^
2+(-148-22*3^(1/2))*x^4+(60+60*3^(1/2))*x^6-36*x^8),x)
```

output

```
4*( - 54*sqrt(3)*int(x**13/(324*x**16 - 1080*x**14 + 864*x**12 - 2100*x**1
0 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) + 90*sqrt(3)*int(x
**11/(324*x**16 - 1080*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140*x*
*6 + 2400*x**4 - 200*x**2 + 4),x) - 186*sqrt(3)*int(x**9/(324*x**16 - 1080
*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x
**2 + 4),x) - 180*sqrt(3)*int(x**7/(324*x**16 - 1080*x**14 + 864*x**12 - 2
100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) + 139*sqrt
(3)*int(x**5/(324*x**16 - 1080*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8
- 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) - 10*sqrt(3)*int(x**3/(324*x**16
 - 1080*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4
- 200*x**2 + 4),x) + 4*sqrt(3)*int(x/(324*x**16 - 1080*x**14 + 864*x**12 -
 2100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) - 270*in
t(x**11/(324*x**16 - 1080*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140
*x**6 + 2400*x**4 - 200*x**2 + 4),x) + 45*int(x**9/(324*x**16 - 1080*x**14
 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 +
4),x) + 180*int(x**7/(324*x**16 - 1080*x**14 + 864*x**12 - 2100*x**10 + 32
17*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) - 360*int(x**5/(324*x**1
6 - 1080*x**14 + 864*x**12 - 2100*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4
 - 200*x**2 + 4),x) + 30*int(x**3/(324*x**16 - 1080*x**14 + 864*x**12 - 21
00*x**10 + 3217*x**8 - 140*x**6 + 2400*x**4 - 200*x**2 + 4),x) + 6*int(...
```

3.59 $\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$

Optimal result

Integrand size = 68, antiderivative size = 124

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})x-4\sqrt{3}x^2-8x^3+4x^4}\,dx = -\sqrt{1+\sqrt{3}+\sqrt{1+2\sqrt{3}}}\operatorname{arctanh}\left(\sqrt{1+\sqrt{3}+\sqrt{1+2\sqrt{3}}}-\sqrt{2}x\right) - \sqrt{1+\sqrt{3}-\sqrt{1+2\sqrt{3}}}\operatorname{arctanh}\left(\sqrt{1+\sqrt{3}-\sqrt{1+2\sqrt{3}}}+\sqrt{2}x\right)$$

output
```
(1+3^(1/2)+(1+2*3^(1/2))^(1/2))^(1/2)*arctanh(-(1+3^(1/2)+(1+2*3^(1/2))^(1
/2))^(1/2)+x*2^(1/2))-(1+3^(1/2)-(1+2*3^(1/2))^(1/2))^(1/2)*arctanh((1+3^(
1/2)-(1+2*3^(1/2))^(1/2))^(1/2)+x*2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$= -\frac{1}{2}\text{RootSum}\Big[-1+\sqrt{3}-2\#1-2\sqrt{3}\#1+2\sqrt{3}\#1^2+4\#1^3 -2\#1^4\&,\frac{-\log(x-\#1)-\sqrt{3}\log(x-\#1)+4\sqrt{3}\log(x-\#1)\#1+2\log(x-\#1)\#1^2}{1+\sqrt{3}-2\sqrt{3}\#1-6\#1^2+4\#1^3}\&\Big]$$

input
```
Integrate[(-2*(-1 - Sqrt[3]) - 8*Sqrt[3]*x - 4*x^2)/(2 - 2*Sqrt[3] + (4 +
4*Sqrt[3])*x - 4*Sqrt[3]*x^2 - 8*x^3 + 4*x^4),x]
```

output
```
-1/2*RootSum[-1 + Sqrt[3] - 2*#1 - 2*Sqrt[3]*#1 + 2*Sqrt[3]*#1^2 + 4*#1^3
- 2*#1^4 & , (-Log[x - #1] - Sqrt[3]*Log[x - #1] + 4*Sqrt[3]*Log[x - #1]*#
1 + 2*Log[x - #1]*#1^2)/(1 + Sqrt[3] - 2*Sqrt[3]*#1 - 6*#1^2 + 4*#1^3) & ]
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2459, 2202, 27, 27, 1432, 1083, 219, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-4x^2-8\sqrt{3}x-2(-1-\sqrt{3})}{4x^4-8x^3-4\sqrt{3}x^2+(4+4\sqrt{3})x-2\sqrt{3}+2}\,dx$$
$$\Big\downarrow \text{2459}$$
$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-4(1+2\sqrt{3})\left(x-\frac{1}{2}\right)-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)+\int -\frac{4\left(1+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)-4\left(1+2\sqrt{3}\right)\int \frac{4\left(x-\frac{1}{2}\right)}{16\left(x-\frac{1}{2}\right)^4-8\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2-4\sqrt{3}+13}d\left(x-\frac{1}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)-16\left(1+2\sqrt{3}\right)\int \frac{x-\frac{1}{2}}{16\left(x-\frac{1}{2}\right)^4-8\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2-4\sqrt{3}+13}d\left(x-\frac{1}{2}\right)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)-8\left(1+2\sqrt{3}\right)\int \frac{1}{16\left(x-\frac{1}{2}\right)^4-8\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2-4\sqrt{3}+13}d\left(x-\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{1083}$$

$$16\left(1+2\sqrt{3}\right)\int \frac{1}{512\left(1+2\sqrt{3}\right)-\left(x-\frac{1}{2}\right)^4}d\left(32\left(x-\frac{1}{2}\right)^2-8\left(3+2\sqrt{3}\right)\right)+\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{-4\left(x-\frac{1}{2}\right)^2-2\sqrt{3}+1}{4\left(x-\frac{1}{2}\right)^4-2\left(3+2\sqrt{3}\right)\left(x-\frac{1}{2}\right)^2+\frac{1}{4}\left(13-4\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)+\sqrt{\frac{1}{2}\left(1+2\sqrt{3}\right)}\text{arctanh}\left(\frac{32\left(x-\frac{1}{2}\right)^2-8\left(3+2\sqrt{3}\right)}{16\sqrt{2\left(1+2\sqrt{3}\right)}}\right)$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\sqrt{1+2\sqrt{3}}\left(x-\frac{1}{2}\right)+\frac{1}{4}\left(-1+2\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)-\frac{1}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2+\sqrt{1+2\sqrt{3}}\left(x-\frac{1}{2}\right)+\frac{1}{4}\left(-1+2\sqrt{3}\right)}d\left(x-\frac{1}{2}\right)+\sqrt{\frac{1}{2}\left(1+2\sqrt{3}\right)}\text{arctanh}\left(\frac{32\left(x-\frac{1}{2}\right)^2-8\left(3+2\sqrt{3}\right)}{16\sqrt{2\left(1+2\sqrt{3}\right)}}\right)$$

$$\Big\downarrow \color{blue}{1081}$$

$$-\frac{1}{2}\int \left(-\frac{\sqrt{2}}{2\left(x-\frac{1}{2}\right)-\sqrt{1+2\sqrt{3}}+\sqrt{2}}-\frac{\sqrt{2}}{-2\left(x-\frac{1}{2}\right)+\sqrt{1+2\sqrt{3}}+\sqrt{2}}\right)d\left(x-\frac{1}{2}\right)-\frac{1}{2}\int \left(-\frac{\sqrt{2}}{2\left(x-\frac{1}{2}\right)+\sqrt{1+2\sqrt{3}}+\sqrt{2}}-\frac{\sqrt{2}}{-2\left(x-\frac{1}{2}\right)-\sqrt{1+2\sqrt{3}}+\sqrt{2}}\right)d\left(x-\frac{1}{2}\right)+\sqrt{\frac{1}{2}\left(1+2\sqrt{3}\right)}\text{arctanh}\left(\frac{32\left(x-\frac{1}{2}\right)^2-8\left(3+2\sqrt{3}\right)}{16\sqrt{2\left(1+2\sqrt{3}\right)}}\right)$$

$$\Big\downarrow \color{blue}{2009}$$

$$\sqrt{\frac{1}{2}\left(1+2\sqrt{3}\right)}\text{arctanh}\left(\frac{32\left(x-\frac{1}{2}\right)^2-8\left(3+2\sqrt{3}\right)}{16\sqrt{2\left(1+2\sqrt{3}\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(2\left(x-\frac{1}{2}\right)-\sqrt{1+2\sqrt{3}}+\sqrt{2}\right)}{\sqrt{2}}-\frac{\log\left(-2\left(x-\frac{1}{2}\right)+\sqrt{1+2\sqrt{3}}+\sqrt{2}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(2\left(x-\frac{1}{2}\right)+\sqrt{1+2\sqrt{3}}+\sqrt{2}\right)}{\sqrt{2}}-\frac{\log\left(-2\left(x-\frac{1}{2}\right)-\sqrt{1+2\sqrt{3}}+\sqrt{2}\right)}{\sqrt{2}}\right)$$

input

```
Int[(-2*(-1 - Sqrt[3]) - 8*Sqrt[3]*x - 4*x^2)/(2 - 2*Sqrt[3] + (4 + 4*Sqrt
[3])*x - 4*Sqrt[3]*x^2 - 8*x^3 + 4*x^4),x]
```

output

```
Sqrt[(1 + 2*Sqrt[3])/2]*ArcTanh[(-8*(3 + 2*Sqrt[3]) + 32*(-1/2 + x)^2)/(16
*Sqrt[2*(1 + 2*Sqrt[3])])] + (-(Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[3]] - 2*(-1/
2 + x)]/Sqrt[2]) + Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[3]] + 2*(-1/2 + x)]/Sqrt[
2])/2 + (-(Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[3]] - 2*(-1/2 + x)]/Sqrt[2]) + Lo
g[Sqrt[2] + Sqrt[1 + 2*Sqrt[3]] + 2*(-1/2 + x)]/Sqrt[2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-4\_Z^3-2\sqrt{3}\,\_Z^2+\left(2\sqrt{3}+2\right)\_Z-\sqrt{3}+1\right)}\frac{\left(-2\_R^2+1+\sqrt{3}\left(1-4\_R\right)\right)\ln\left(x-\_R\right)}{-4\_R^3+6\_R^2-1+\sqrt{3}\left(-1+2\_R\right)}\right)}{2}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-4\_Z^3-2\,\mathrm{RootOf}\left(\_Z^2-3,\mathrm{index}=1\right)\_Z^2+\left(2\,\mathrm{RootOf}\left(\_Z^2-3,\mathrm{index}=1\right)+2\right)\_Z+1-\mathrm{RootOf}\left(\_Z^2-3,\mathrm{index}=1\right)\right)}\frac{(2\_R}{-4\_}\right.}{2}$

input
```
int((2+2*3^(1/2)-8*3^(1/2)*x-4*x^2)/(2-2*3^(1/2)+(4+4*3^(1/2))*x-4*3^(1/2)
*x^2-8*x^3+4*x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sum((-2*_R^2+1+3^(1/2)*(1-4*_R))/(-4*_R^3+6*_R^2-1+3^(1/2)*(-1+2*_R))
*ln(x-_R),_R=RootOf(2*_Z^4-4*_Z^3-2*3^(1/2)*_Z^2+(2*3^(1/2)+2)*_Z-3^(1/2)+
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(88) = 176$.

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.35

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})\,x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$= \frac{1}{2}\sqrt{\sqrt{3}+\sqrt{2\sqrt{3}+1}+1}\log\left(\left(\sqrt{3}\sqrt{2\sqrt{3}+1}-\sqrt{3}\right)\sqrt{\sqrt{3}+\sqrt{2\sqrt{3}+1}+1}+6x-3\sqrt{2\sqrt{3}+1}-3\right)$$
$$-\frac{1}{2}\sqrt{\sqrt{3}+\sqrt{2\sqrt{3}+1}+1}\log\left(-\left(\sqrt{3}\sqrt{2\sqrt{3}+1}-\sqrt{3}\right)\sqrt{\sqrt{3}+\sqrt{2\sqrt{3}+1}+1}+6x-3\sqrt{2\sqrt{3}+1}-3\right)$$
$$-\frac{1}{2}\sqrt{\sqrt{3}-\sqrt{2\sqrt{3}+1}+1}\log\left(\left(\sqrt{3}\sqrt{2\sqrt{3}+1}+\sqrt{3}\right)\sqrt{\sqrt{3}-\sqrt{2\sqrt{3}+1}+1}+6x+3\sqrt{2\sqrt{3}+1}-3\right)$$
$$+\frac{1}{2}\sqrt{\sqrt{3}-\sqrt{2\sqrt{3}+1}+1}\log\left(-\left(\sqrt{3}\sqrt{2\sqrt{3}+1}+\sqrt{3}\right)\sqrt{\sqrt{3}-\sqrt{2\sqrt{3}+1}+1}+6x+3\sqrt{2\sqrt{3}+1}-3\right)$$

input
```
integrate((2+2*3^(1/2)-8*3^(1/2)*x-4*x^2)/(2-2*3^(1/2)+(4+4*3^(1/2))*x-4*3
^(1/2)*x^2-8*x^3+4*x^4),x, algorithm="fricas")
```

output
```
1/2*sqrt(sqrt(3) + sqrt(2*sqrt(3) + 1) + 1)*log((sqrt(3)*sqrt(2*sqrt(3) +
1) - sqrt(3))*sqrt(sqrt(3) + sqrt(2*sqrt(3) + 1) + 1) + 6*x - 3*sqrt(2*sqr
t(3) + 1) - 3) - 1/2*sqrt(sqrt(3) + sqrt(2*sqrt(3) + 1) + 1)*log(-(sqrt(3)
*sqrt(2*sqrt(3) + 1) - sqrt(3))*sqrt(sqrt(3) + sqrt(2*sqrt(3) + 1) + 1) +
6*x - 3*sqrt(2*sqrt(3) + 1) - 3) - 1/2*sqrt(sqrt(3) - sqrt(2*sqrt(3) + 1)
+ 1)*log((sqrt(3)*sqrt(2*sqrt(3) + 1) + sqrt(3))*sqrt(sqrt(3) - sqrt(2*sqr
t(3) + 1) + 1) + 6*x + 3*sqrt(2*sqrt(3) + 1) - 3) + 1/2*sqrt(sqrt(3) - sqr
t(2*sqrt(3) + 1) + 1)*log(-(sqrt(3)*sqrt(2*sqrt(3) + 1) + sqrt(3))*sqrt(sq
rt(3) - sqrt(2*sqrt(3) + 1) + 1) + 6*x + 3*sqrt(2*sqrt(3) + 1) - 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})\,x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((2+2*3**(1/2)-8*3**(1/2)*x-4*x**2)/(2-2*3**(1/2)+(4+4*3**(1/2))*
x-4*3**(1/2)*x**2-8*x**3+4*x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(192*sqrt(3)*_t**4 + 336*_t**4 - 48
0*_t**3 - 256*sqrt(3)*_t**3 + 48*sqrt(3)*_t**2 + 184*_t**2 + 8*_t + 16*sqr
t(3)*_t + 1) contains an element of the set of generators.
```

Maxima [F]

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})\,x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$= \int -\frac{2\,x^2+4\,\sqrt{3}x-\sqrt{3}-1}{2\,x^4-4\,x^3-2\,\sqrt{3}x^2+2\,x\left(\sqrt{3}+1\right)-\sqrt{3}+1}\,dx$$

input
```
integrate((2+2*3^(1/2)-8*3^(1/2)*x-4*x^2)/(2-2*3^(1/2)+(4+4*3^(1/2))*x-4*3
^(1/2)*x^2-8*x^3+4*x^4),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 + 4*sqrt(3)*x - sqrt(3) - 1)/(2*x^4 - 4*x^3 - 2*sqrt(3)*
x^2 + 2*x*(sqrt(3) + 1) - sqrt(3) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(88) = 176$.

Time = 0.17 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.58

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+\left(4+4\sqrt{3}\right)x-4\sqrt{3}x^2-8x^3+4x^4}\,dx = \text{Too large to display}$$

input
```
integrate((2+2*3^(1/2)-8*3^(1/2)*x-4*x^2)/(2-2*3^(1/2)+(4+4*3^(1/2))*x-4*3
^(1/2)*x^2-8*x^3+4*x^4),x, algorithm="giac")
```

output
```
1/2*(((sqrt(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) - 1)^2 - 4*sqrt(3)*(sqr
t(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) - 1) - 2*sqrt(3) - 2)*log(x + 1/2
*sqrt(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) - 1/2)/((sqrt(2*sqrt(3) + 2*s
qrt(4*sqrt(3) + 2) + 3) - 1)^3/(sqrt(3) - 1) + 3*(sqrt(2*sqrt(3) + 2*sqrt(
4*sqrt(3) + 2) + 3) - 1)^2/(sqrt(3) - 1) - 2*sqrt(3)*(sqrt(2*sqrt(3) + 2*s
qrt(4*sqrt(3) + 2) + 3) - 1)/(sqrt(3) - 1) - 2*(sqrt(3) + 1)/(sqrt(3) - 1)
) - ((sqrt(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) + 1)^2 + 4*sqrt(3)*(sqrt
(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) + 1) - 2*sqrt(3) - 2)*log(x - 1/2*
sqrt(2*sqrt(3) + 2*sqrt(4*sqrt(3) + 2) + 3) - 1/2)/((sqrt(2*sqrt(3) + 2*sq
rt(4*sqrt(3) + 2) + 3) + 1)^3/(sqrt(3) - 1) - 3*(sqrt(2*sqrt(3) + 2*sqrt(4
*sqrt(3) + 2) + 3) + 1)^2/(sqrt(3) - 1) - 2*sqrt(3)*(sqrt(2*sqrt(3) + 2*sq
rt(4*sqrt(3) + 2) + 3) + 1)/(sqrt(3) - 1) + 2*(sqrt(3) + 1)/(sqrt(3) - 1))
 + ((sqrt(2*sqrt(3) - 2*sqrt(4*sqrt(3) + 2) + 3) - 1)^2 - 4*sqrt(3)*(sqrt(
2*sqrt(3) - 2*sqrt(4*sqrt(3) + 2) + 3) - 1) - 2*sqrt(3) - 2)*log(x + 1/2*s
qrt(2*sqrt(3) - 2*sqrt(4*sqrt(3) + 2) + 3) - 1/2)/((sqrt(2*sqrt(3) - 2*sqr
t(4*sqrt(3) + 2) + 3) - 1)^3/(sqrt(3) - 1) + 3*(sqrt(2*sqrt(3) - 2*sqrt(4*
sqrt(3) + 2) + 3) - 1)^2/(sqrt(3) - 1) - 2*sqrt(3)*(sqrt(2*sqrt(3) - 2*sqr
t(4*sqrt(3) + 2) + 3) - 1)/(sqrt(3) - 1) - 2*(sqrt(3) + 1)/(sqrt(3) - 1))
- ((sqrt(2*sqrt(3) - 2*sqrt(4*sqrt(3) + 2) + 3) + 1)^2 + 4*sqrt(3)*(sqrt(2
*sqrt(3) - 2*sqrt(4*sqrt(3) + 2) + 3) + 1) - 2*sqrt(3) - 2)*log(x - 1/2...
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+\left(4+4\sqrt{3}\right)x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$=\sum_{k=1}^{4}\ln\left(\frac{15\sqrt{3}}{2}+\mathrm{root}\left(z^4-\frac{\sqrt{3}\,z^2}{2}-\frac{z^2}{2}+\frac{3}{16},z,k\right)\left(\sqrt{3}-\mathrm{root}\left(z^4-\frac{\sqrt{3}\,z^2}{2}-\frac{z^2}{2}+\frac{3}{16},z,k\right)\left(16\sqrt{3}+\mathrm{root}\left(z^4-\frac{\sqrt{3}\,z^2}{2}-\frac{z^2}{2}+\frac{3}{16},z,k\right)\left(32\sqrt{3}-x\left(64\sqrt{3}+120\right.\right.\right.\right.\right.$$
$$\left.\left.-x\left(46\sqrt{3}+78\right)+28\right)-x\left(26\sqrt{3}+24\right)+12\right)\mathrm{root}\left(z^4-\frac{\sqrt{3}\,z^2}{2}-\frac{z^2}{2}+\frac{3}{16},z,k\right)$$

input
```
int((8*3^(1/2)*x - 2*3^(1/2) + 4*x^2 - 2)/(2*3^(1/2) + 4*3^(1/2)*x^2 + 8*x
^3 - 4*x^4 - x*(4*3^(1/2) + 4) - 2),x)
```

output
```
symsum(log((15*3^(1/2))/2 + root(z^4 - (3^(1/2)*z^2)/2 - z^2/2 + 3/16, z,
k)*(3^(1/2) - root(z^4 - (3^(1/2)*z^2)/2 - z^2/2 + 3/16, z, k)*(16*3^(1/2)
 + root(z^4 - (3^(1/2)*z^2)/2 - z^2/2 + 3/16, z, k)*(32*3^(1/2) - x*(64*3^
(1/2) + 120) + 60) - x*(32*3^(1/2) + 104) + 74) - x*(46*3^(1/2) + 78) + 28
) - x*(26*3^(1/2) + 24) + 12)*root(z^4 - (3^(1/2)*z^2)/2 - z^2/2 + 3/16, z
, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{-2(-1-\sqrt{3})-8\sqrt{3}x-4x^2}{2-2\sqrt{3}+(4+4\sqrt{3})x-4\sqrt{3}x^2-8x^3+4x^4}\,dx$$
$$= -4\sqrt{3}\left(\int \frac{x^5}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+7\sqrt{3}\left(\int \frac{x^4}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$-4\sqrt{3}\left(\int \frac{x^2}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$-2\sqrt{3}\left(\int \frac{x}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+\sqrt{3}\left(\int \frac{1}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$-2\left(\int \frac{x^6}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+4\left(\int \frac{x^5}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+\int \frac{x^4}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx$$
$$-16\left(\int \frac{x^3}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+14\left(\int \frac{x^2}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$-8\left(\int \frac{x}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$
$$+2\left(\int \frac{1}{2x^8-8x^7+8x^6+4x^5-12x^4+8x^3-10x^2+8x-1}dx\right)$$

input
```
int((2+2*3^(1/2)-8*3^(1/2)*x-4*x^2)/(2-2*3^(1/2)+(4+4*3^(1/2))*x-4*3^(1/2)
*x^2-8*x^3+4*x^4),x)
```

output

```
 - 4*sqrt(3)*int(x**5/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**
3 - 10*x**2 + 8*x - 1),x) + 7*sqrt(3)*int(x**4/(2*x**8 - 8*x**7 + 8*x**6 +
 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) - 4*sqrt(3)*int(x**2/(2
*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x
) - 2*sqrt(3)*int(x/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3
- 10*x**2 + 8*x - 1),x) + sqrt(3)*int(1/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5
 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) - 2*int(x**6/(2*x**8 - 8*x**7
+ 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) + 4*int(x**5/
(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1)
,x) + int(x**4/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*
x**2 + 8*x - 1),x) - 16*int(x**3/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x
**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) + 14*int(x**2/(2*x**8 - 8*x**7 + 8*x*
*6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) - 8*int(x/(2*x**8 -
 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x - 1),x) + 2*i
nt(1/(2*x**8 - 8*x**7 + 8*x**6 + 4*x**5 - 12*x**4 + 8*x**3 - 10*x**2 + 8*x
 - 1),x)
```

3.60 $\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 101

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\operatorname{arctanh}\left(\frac{1}{2}\left(\sqrt{7}-\sqrt{\frac{1}{2}\left(5-\sqrt{21}\right)}x\right)\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{21}\right)}\operatorname{arctanh}\left(\frac{1}{2}\left(\sqrt{7}-\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)\right)$$

output
```
1/2*(1/2*7^(1/2)+1/2*3^(1/2))*arctanh(1/2*7^(1/2)-1/2*(1/2*7^(1/2)-1/2*3^(
1/2))*x)-1/2*(1/2*7^(1/2)-1/2*3^(1/2))*arctanh(-1/2*7^(1/2)+1/2*(1/2*7^(1/
2)+1/2*3^(1/2))*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = \frac{1}{2}\text{RootSum}\left[9-42\#1+43\#1^2-14\#1^3+\#1^4\&,\frac{6\log(x-\#1)-14\log(x-\#1)\#1+5\log(x-\#1)\#1^2}{-21+43\#1-21\#1^2+2\#1^3}\&\right]$$

input
```
Integrate[(6 - 14*x + 5*x^2)/(9 - 42*x + 43*x^2 - 14*x^3 + x^4),x]
```

output
```
RootSum[9 - 42*#1 + 43*#1^2 - 14*#1^3 + #1^4 & , (6*Log[x - #1] - 14*Log[x
 - #1]*#1 + 5*Log[x - #1]*#1^2)/(-21 + 43*#1 - 21*#1^2 + 2*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2-14x+6}{x^4-14x^3+43x^2-42x+9}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{4-\sqrt{3}x}{4\left(x^2-\left(7-2\sqrt{3}\right)x+3\right)}+\frac{\sqrt{3}x+4}{4\left(x^2-\left(7+2\sqrt{3}\right)x+3\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{7}\text{arctanh}\left(\frac{-2x-2\sqrt{3}+7}{\sqrt{7\left(7-4\sqrt{3}\right)}}\right)+\frac{1}{4}\sqrt{7}\text{arctanh}\left(\frac{-2x+2\sqrt{3}+7}{\sqrt{7\left(7+4\sqrt{3}\right)}}\right)-\frac{1}{8}\sqrt{3}\log\left(x^2-\left(7-2\sqrt{3}\right)x+3\right)+\frac{1}{8}\sqrt{3}\log\left(x^2-\left(7+2\sqrt{3}\right)x+3\right)$$

input

```
Int[(6 - 14*x + 5*x^2)/(9 - 42*x + 43*x^2 - 14*x^3 + x^4),x]
```

output

```
(Sqrt[7]*ArcTanh[(7 - 2*Sqrt[3] - 2*x)/Sqrt[7*(7 - 4*Sqrt[3])]])/4 + (Sqrt
[7]*ArcTanh[(7 + 2*Sqrt[3] - 2*x)/Sqrt[7*(7 + 4*Sqrt[3])]])/4 - (Sqrt[3]*L
og[3 - (7 - 2*Sqrt[3])*x + x^2])/8 + (Sqrt[3]*Log[3 - (7 + 2*Sqrt[3])*x +
x^2])/8
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492

```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-14\_Z^3+43\_Z^2-42\_Z+9\right)}\frac{\left(5\_R^2-14\_R+6\right)\ln\left(x-\_R\right)}{2\_R^3-21\_R^2+43\_R-21}\right)}{2}$	59
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-14\_Z^3+43\_Z^2-42\_Z+9\right)}\frac{\left(5\_R^2-14\_R+6\right)\ln\left(x-\_R\right)}{2\_R^3-21\_R^2+43\_R-21}\right)}{2}$	59

input
```
int((5*x^2-14*x+6)/(x^4-14*x^3+43*x^2-42*x+9),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((5*_R^2-14*_R+6)/(2*_R^3-21*_R^2+43*_R-21)*ln(x-_R),_R=RootOf(_Z^4
-14*_Z^3+43*_Z^2-42*_Z+9))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(57) = 114$.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = -\frac{1}{4}\sqrt{-\frac{1}{2}\sqrt{21}+\frac{5}{2}}\log\left(2\,x+\sqrt{21}+4\sqrt{-\frac{1}{2}\sqrt{21}+\frac{5}{2}}-7\right)+\frac{1}{4}\sqrt{-\frac{1}{2}\sqrt{21}+\frac{5}{2}}\log\left(2\,x+\sqrt{21}-4\sqrt{-\frac{1}{2}\sqrt{21}+\frac{5}{2}}-7\right)-\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}\log\left(2\,x-\sqrt{21}+4\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}-7\right)+\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}\log\left(2\,x-\sqrt{21}-4\sqrt{\frac{1}{2}\sqrt{21}+\frac{5}{2}}-7\right)$$

input
```
integrate((5*x^2-14*x+6)/(x^4-14*x^3+43*x^2-42*x+9),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-1/2*sqrt(21) + 5/2)*log(2*x + sqrt(21) + 4*sqrt(-1/2*sqrt(21) +
 5/2) - 7) + 1/4*sqrt(-1/2*sqrt(21) + 5/2)*log(2*x + sqrt(21) - 4*sqrt(-1/
2*sqrt(21) + 5/2) - 7) - 1/4*sqrt(1/2*sqrt(21) + 5/2)*log(2*x - sqrt(21) +
 4*sqrt(1/2*sqrt(21) + 5/2) - 7) + 1/4*sqrt(1/2*sqrt(21) + 5/2)*log(2*x -
sqrt(21) - 4*sqrt(1/2*sqrt(21) + 5/2) - 7)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = \text{RootSum}\left(256t^4-80t^2+1,\left(t\mapsto t\log\left(-16t^2-8t+x-1\right)\right)\right)$$

input
```
integrate((5*x**2-14*x+6)/(x**4-14*x**3+43*x**2-42*x+9),x)
```

output
```
RootSum(256*_t**4 - 80*_t**2 + 1, Lambda(_t, _t*log(-16*_t**2 - 8*_t + x -
 1)))
```

Maxima [F]

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = \int \frac{5\,x^2-14\,x+6}{x^4-14\,x^3+43\,x^2-42\,x+9}\,dx$$

input
```
integrate((5*x^2-14*x+6)/(x^4-14*x^3+43*x^2-42*x+9),x, algorithm="maxima")
```

output
```
integrate((5*x^2 - 14*x + 6)/(x^4 - 14*x^3 + 43*x^2 - 42*x + 9), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.25

$$\begin{aligned}\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx &= -0.114212562937000\log\left(x-0.295011649026000\right)\\ &\quad-0.547225264829000\log\left(x-1.41348572884000\right)\\ &\quad+0.114212562937000\log\left(x-2.12241265602000\right)\\ &\quad+0.547225264829000\log\left(x-10.1690899661000\right)\end{aligned}$$

input
```
integrate((5*x^2-14*x+6)/(x^4-14*x^3+43*x^2-42*x+9),x, algorithm="giac")
```

output
```
-0.114212562937000*log(x - 0.295011649026000) - 0.547225264829000*log(x -
1.41348572884000) + 0.114212562937000*log(x - 2.12241265602000) + 0.547225
264829000*log(x - 10.1690899661000)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.62

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = \operatorname{atanh}\left(\frac{3087\sqrt{3}}{3528\,x-882\sqrt{3}\sqrt{7}+672\sqrt{3}\sqrt{7}\,x-5292} + \frac{1512\sqrt{7}}{3528\,x-882\sqrt{3}\sqrt{7}+672\sqrt{3}\sqrt{7}\,x-5292} - \frac{4263\sqrt{3}\,x}{2\left(3528\,x-882\sqrt{3}\sqrt{7}+672\sqrt{3}\sqrt{7}\,x-5292\right)} - \frac{2205\sqrt{7}\,x}{2\left(3528\,x-882\sqrt{3}\sqrt{7}+672\sqrt{3}\sqrt{7}\,x-5292\right)}\right)\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{7}}{4}\right) + \operatorname{atanh}\left(\frac{3087\sqrt{3}}{3528\,x+882\sqrt{3}\sqrt{7}-672\sqrt{3}\sqrt{7}\,x-5292} - \frac{1512\sqrt{7}}{3528\,x+882\sqrt{3}\sqrt{7}-672\sqrt{3}\sqrt{7}\,x-5292} - \frac{4263\sqrt{3}\,x}{2\left(3528\,x+882\sqrt{3}\sqrt{7}-672\sqrt{3}\sqrt{7}\,x-5292\right)} + \frac{2205\sqrt{7}\,x}{2\left(3528\,x+882\sqrt{3}\sqrt{7}-672\sqrt{3}\sqrt{7}\,x-5292\right)}\right)\left(\frac{\sqrt{3}}{4}-\frac{\sqrt{7}}{4}\right)$$

input
```
int((5*x^2 - 14*x + 6)/(43*x^2 - 42*x - 14*x^3 + x^4 + 9),x)
```

output
```
atanh((3087*3^(1/2))/(3528*x - 882*3^(1/2)*7^(1/2) + 672*3^(1/2)*7^(1/2)*x
 - 5292) + (1512*7^(1/2))/(3528*x - 882*3^(1/2)*7^(1/2) + 672*3^(1/2)*7^(1
/2)*x - 5292) - (4263*3^(1/2)*x)/(2*(3528*x - 882*3^(1/2)*7^(1/2) + 672*3^
(1/2)*7^(1/2)*x - 5292)) - (2205*7^(1/2)*x)/(2*(3528*x - 882*3^(1/2)*7^(1/
2) + 672*3^(1/2)*7^(1/2)*x - 5292)))*(3^(1/2)/4 + 7^(1/2)/4) + atanh((3087
*3^(1/2))/(3528*x + 882*3^(1/2)*7^(1/2) - 672*3^(1/2)*7^(1/2)*x - 5292) -
(1512*7^(1/2))/(3528*x + 882*3^(1/2)*7^(1/2) - 672*3^(1/2)*7^(1/2)*x - 529
2) - (4263*3^(1/2)*x)/(2*(3528*x + 882*3^(1/2)*7^(1/2) - 672*3^(1/2)*7^(1/
2)*x - 5292)) + (2205*7^(1/2)*x)/(2*(3528*x + 882*3^(1/2)*7^(1/2) - 672*3^
(1/2)*7^(1/2)*x - 5292)))*(3^(1/2)/4 - 7^(1/2)/4)
```

Reduce [F]

$$\int \frac{6-14x+5x^2}{9-42x+43x^2-14x^3+x^4}\,dx = 5\left(\int \frac{x^2}{x^4-14x^3+43x^2-42x+9}dx\right) - 14\left(\int \frac{x}{x^4-14x^3+43x^2-42x+9}dx\right) + 6\left(\int \frac{1}{x^4-14x^3+43x^2-42x+9}dx\right)$$

input
```
int((5*x^2-14*x+6)/(x^4-14*x^3+43*x^2-42*x+9),x)
```

output
```
5*int(x**2/(x**4 - 14*x**3 + 43*x**2 - 42*x + 9),x) - 14*int(x/(x**4 - 14*
x**3 + 43*x**2 - 42*x + 9),x) + 6*int(1/(x**4 - 14*x**3 + 43*x**2 - 42*x +
 9),x)
```

3.61 $\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 108

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = -\frac{1}{4}\arctan\left(2-\sqrt{3}+2x^2\right)-\frac{1}{4}\arctan\left(2+\sqrt{3}+2x^2\right) + \frac{1}{8}\sqrt{3}\log\left(2-\sqrt{3}+2x^2-\sqrt{3}x^2+x^4\right) - \frac{1}{8}\sqrt{3}\log\left(2+\sqrt{3}+2x^2+\sqrt{3}x^2+x^4\right)$$

output
```
-1/4*arctan(2-3^(1/2)+2*x^2)-1/4*arctan(2+3^(1/2)+2*x^2)+1/8*3^(1/2)*ln(2-
3^(1/2)+2*x^2-3^(1/2)*x^2+x^4)-1/8*3^(1/2)*ln(2+3^(1/2)+2*x^2+3^(1/2)*x^2+
x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = \frac{1}{4}\text{RootSum}\left[1+2\#1^2+5\#1^4+4\#1^6+\#1^8\&,\frac{-\log(x-\#1)+2\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{1+5\#1^2+6\#1^4+2\#1^6}\&\right]$$

input
```
Integrate[(x*(-1 + 2*x^2 + x^4))/(1 + 2*x^2 + 5*x^4 + 4*x^6 + x^8),x]
```

output
```
RootSum[1 + 2*#1^2 + 5*#1^4 + 4*#1^6 + #1^8 & , (-Log[x - #1] + 2*Log[x -
#1]*#1^2 + Log[x - #1]*#1^4)/(1 + 5*#1^2 + 6*#1^4 + 2*#1^6) & ]/4
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {7266, 25, 2459, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^4+2x^2-1)}{x^8+4x^6+5x^4+2x^2+1}\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int -\frac{-x^4-2x^2+1}{x^8+4x^6+5x^4+2x^2+1}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{-x^4-2x^2+1}{x^8+4x^6+5x^4+2x^2+1}dx^2$$

$$\Big\downarrow \text{2459}$$

$$-\frac{1}{2}\int \frac{2-x^4}{x^8-x^4+1}d(x^2+1)$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2\sqrt{3}-3(x^2+1)}{x^4-\sqrt{3}(x^2+1)+1}d(x^2+1)}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}\left(\sqrt{3}(x^2+1)+2\right)}{x^4+\sqrt{3}(x^2+1)+1}d(x^2+1)}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2\sqrt{3}-3(x^2+1)}{x^4-\sqrt{3}(x^2+1)+1}d(x^2+1)}{2\sqrt{3}}-\frac{1}{2}\int \frac{\sqrt{3}(x^2+1)+2}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)-\frac{1}{2}\sqrt{3}\int \frac{2(x^2+1)+\sqrt{3}}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)\right)-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}(x^2+1)+}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)-\frac{1}{2}\sqrt{3}\int \frac{2(x^2+1)+\sqrt{3}}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)\right)-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}(x^2+1)+}\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{-x^4-1}d\left(2(x^2+1)+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{2(x^2+1)+\sqrt{3}}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)\right)-\frac{\frac{3}{2}\int \frac{\sqrt{3}-2(x^2+1)}{x^4-\sqrt{3}(x^2+1)+1}d(x^2+1}{}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\sqrt{3}\int \frac{2(x^2+1)+\sqrt{3}}{x^4+\sqrt{3}\,(x^2+1)+1}d(x^2+1)-\arctan\left(2(x^2+1)+\sqrt{3}\right)\right)-\frac{\frac{3}{2}\int \frac{\sqrt{3}-2(x^2+1)}{x^4-\sqrt{3}(x^2+1)+1}d(x^2+1)-}{2\sqrt{}}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\arctan\left(2(x^2+1)+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(x^4+\sqrt{3}(x^2+1)+1\right)\right)-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2(x^2+1)\right)-\frac{3}{2}\log}{2\sqrt{3}}\right.$$

input
```
Int[(x*(-1 + 2*x^2 + x^4))/(1 + 2*x^2 + 5*x^4 + 4*x^6 + x^8),x]
```

output
```
(-1/2*(-(Sqrt[3]*ArcTan[Sqrt[3] - 2*(1 + x^2)]) - (3*Log[1 + x^4 - Sqrt[3]
*(1 + x^2)])/2)/Sqrt[3] + (-ArcTan[Sqrt[3] + 2*(1 + x^2)] - (Sqrt[3]*Log[1
 + x^4 + Sqrt[3]*(1 + x^2)])/2)/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_R\ln\left(x^2-\_R+1\right)\right)}{4}$	28
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+4\_Z^3+5\_Z^2+2\_Z+1\right)}\frac{\left(\_R^2+2\_R-1\right)\ln\left(x^2-\_R\right)}{2\_R^3+6\_R^2+5\_R+1}\right)}{4}$	59

input
```
int(x*(x^4+2*x^2-1)/(x^8+4*x^6+5*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(x^2-_R+1),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = -\frac{1}{8}\sqrt{3}\log\left(x^4+2\,x^2+\sqrt{3}(x^2+1)+2\right) + \frac{1}{8}\sqrt{3}\log\left(x^4+2\,x^2-\sqrt{3}(x^2+1)+2\right) - \frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}+2\right) + \frac{1}{4}\arctan\left(-2\,x^2+\sqrt{3}-2\right)$$

input
```
integrate(x*(x^4+2*x^2-1)/(x^8+4*x^6+5*x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
-1/8*sqrt(3)*log(x^4 + 2*x^2 + sqrt(3)*(x^2 + 1) + 2) + 1/8*sqrt(3)*log(x^
4 + 2*x^2 - sqrt(3)*(x^2 + 1) + 2) - 1/4*arctan(2*x^2 + sqrt(3) + 2) + 1/4
*arctan(-2*x^2 + sqrt(3) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = \frac{\sqrt{3}\log\left(x^4+x^2\cdot\left(2-\sqrt{3}\right)-\sqrt{3}+2\right)}{8} - \frac{\sqrt{3}\log\left(x^4+x^2\left(\sqrt{3}+2\right)+\sqrt{3}+2\right)}{8} - \frac{\operatorname{atan}\left(2x^2-\sqrt{3}+2\right)}{4} - \frac{\operatorname{atan}\left(2x^2+\sqrt{3}+2\right)}{4}$$

input
```
integrate(x*(x**4+2*x**2-1)/(x**8+4*x**6+5*x**4+2*x**2+1),x)
```

output
```
sqrt(3)*log(x**4 + x**2*(2 - sqrt(3)) - sqrt(3) + 2)/8 - sqrt(3)*log(x**4
+ x**2*(sqrt(3) + 2) + sqrt(3) + 2)/8 - atan(2*x**2 - sqrt(3) + 2)/4 - ata
n(2*x**2 + sqrt(3) + 2)/4
```

Maxima [F]

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = \int \frac{(x^4+2\,x^2-1)x}{x^8+4\,x^6+5\,x^4+2\,x^2+1}\,dx$$

input
```
integrate(x*(x^4+2*x^2-1)/(x^8+4*x^6+5*x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^4 + 2*x^2 - 1)*x/(x^8 + 4*x^6 + 5*x^4 + 2*x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(x^4+2*x^2-1)/(x^8+4*x^6+5*x^4+2*x^2+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.86

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = -\frac{\mathrm{atan}\left(\frac{27\,x^2}{54\,x^2-\sqrt{3}\,27\mathrm{i}+27}-\frac{\sqrt{3}\,x^2\,27\mathrm{i}}{54\,x^2-\sqrt{3}\,27\mathrm{i}+27}\right)}{4} - \frac{\mathrm{atan}\left(\frac{27\,x^2}{54\,x^2+\sqrt{3}\,27\mathrm{i}+27}+\frac{\sqrt{3}\,x^2\,27\mathrm{i}}{54\,x^2+\sqrt{3}\,27\mathrm{i}+27}\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{27\,x^2}{54\,x^2-\sqrt{3}\,27\mathrm{i}+27}-\frac{\sqrt{3}\,x^2\,27\mathrm{i}}{54\,x^2-\sqrt{3}\,27\mathrm{i}+27}\right)\,1\mathrm{i}}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{27\,x^2}{54\,x^2+\sqrt{3}\,27\mathrm{i}+27}+\frac{\sqrt{3}\,x^2\,27\mathrm{i}}{54\,x^2+\sqrt{3}\,27\mathrm{i}+27}\right)\,1\mathrm{i}}{4}$$

input
```
int((x*(2*x^2 + x^4 - 1))/(2*x^2 + 5*x^4 + 4*x^6 + x^8 + 1),x)
```

output
```
(3^(1/2)*atan((27*x^2)/(54*x^2 - 3^(1/2)*27i + 27) - (3^(1/2)*x^2*27i)/(54
*x^2 - 3^(1/2)*27i + 27))*1i)/4 - atan((27*x^2)/(3^(1/2)*27i + 54*x^2 + 27
) + (3^(1/2)*x^2*27i)/(3^(1/2)*27i + 54*x^2 + 27))/4 - atan((27*x^2)/(54*x
^2 - 3^(1/2)*27i + 27) - (3^(1/2)*x^2*27i)/(54*x^2 - 3^(1/2)*27i + 27))/4
- (3^(1/2)*atan((27*x^2)/(3^(1/2)*27i + 54*x^2 + 27) + (3^(1/2)*x^2*27i)/(
3^(1/2)*27i + 54*x^2 + 27))*1i)/4
```

Reduce [F]

$$\int \frac{x(-1+2x^2+x^4)}{1+2x^2+5x^4+4x^6+x^8}\,dx = \frac{\sqrt{3}\log\left(-\sqrt{3}\,x^2-\sqrt{3}+x^4+2x^2+2\right)}{12} - \frac{\sqrt{3}\log\left(-\sqrt{\sqrt{6}-\sqrt{3}+\sqrt{2}-2}\,x+\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+x^2\right)}{12} - \frac{\sqrt{3}\log\left(\sqrt{\sqrt{6}-\sqrt{3}+\sqrt{2}-2}\,x+\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+x^2\right)}{12} - \left(\int \frac{x}{x^8+4x^6+5x^4+2x^2+1}dx\right)$$

input
```
int(x*(x^4+2*x^2-1)/(x^8+4*x^6+5*x^4+2*x^2+1),x)
```

output
```
(sqrt(3)*log( - sqrt(3)*x**2 - sqrt(3) + x**4 + 2*x**2 + 2) - sqrt(3)*log(
( - 2*sqrt(sqrt(6) - sqrt(3) + sqrt(2) - 2)*x + sqrt(6) + sqrt(2) + 2*x**2
)/2) - sqrt(3)*log((2*sqrt(sqrt(6) - sqrt(3) + sqrt(2) - 2)*x + sqrt(6) +
sqrt(2) + 2*x**2)/2) - 12*int(x/(x**8 + 4*x**6 + 5*x**4 + 2*x**2 + 1),x))/
12
```

3.62 $\int \frac{1}{2-4x+6x^2-4x^3+x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4} dx = -\frac{\arctan\left(1+\sqrt{2}-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\arctan\left(1-\sqrt{2}+\sqrt{2}x\right)}{2\sqrt{2}} - \frac{\log\left(2+\sqrt{2}-2x-\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(2-\sqrt{2}-2x+\sqrt{2}x+x^2\right)}{4\sqrt{2}}$$

output
```
1/4*arctan(-1-2^(1/2)+x*2^(1/2))*2^(1/2)+1/4*arctan(1-2^(1/2)+x*2^(1/2))*2
^(1/2)-1/8*ln(2+2^(1/2)-2*x-x*2^(1/2)+x^2)*2^(1/2)+1/8*ln(2-2^(1/2)-2*x+x*
2^(1/2)+x^2)*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4} dx = \frac{1}{4}\text{RootSum}\left[2-4\#1+6\#1^2-4\#1^3+\#1^4\&, \frac{\log(x-\#1)}{-1+3\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[(2 - 4*x + 6*x^2 - 4*x^3 + x^4)^(-1),x]
```

output
```
RootSum[2 - 4*#1 + 6*#1^2 - 4*#1^3 + #1^4 & , Log[x - #1]/(-1 + 3*#1 - 3*#
1^2 + #1^3) & ]/4
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {2458, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-4x^3+6x^2-4x+2}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{(x-1)^4+1}d(x-1)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\int \frac{1-(x-1)^2}{(x-1)^4+1}d(x-1)+\frac{1}{2}\int \frac{(x-1)^2+1}{(x-1)^4+1}d(x-1)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{(x-1)^2-\sqrt{2}(x-1)+1}d(x-1)+\frac{1}{2}\int \frac{1}{(x-1)^2+\sqrt{2}(x-1)+1}d(x-1)\right)+\frac{1}{2}\int \frac{1-(x-1)^2}{(x-1)^4+1}d(x-1)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}(x-1)\right)^2-1}d\left(1-\sqrt{2}(x-1)\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}(x-1)+1\right)^2-1}d\left(\sqrt{2}(x-1)+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-(x-1)^2}{(x-1)^4+1}d(x-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{1-(x-1)^2}{(x-1)^4+1}d(x-1)$$

$\big\downarrow$ 1479

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2(x-1)}{(x-1)^2-\sqrt{2}(x-1)+1}d(x-1)}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}(x-1)+1\right)}{(x-1)^2+\sqrt{2}(x-1)+1}d(x-1)}{2\sqrt{2}}\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2(x-1)}{(x-1)^2-\sqrt{2}(x-1)+1}d(x-1)}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}(x-1)+1\right)}{(x-1)^2+\sqrt{2}(x-1)+1}d(x-1)}{2\sqrt{2}}\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2(x-1)}{(x-1)^2-\sqrt{2}(x-1)+1}d(x-1)}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}(x-1)+1}{(x-1)^2+\sqrt{2}(x-1)+1}d(x-1)\right)$$

$\big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\log\left((x-1)^2+\sqrt{2}(x-1)+1\right)}{2\sqrt{2}}-\frac{\log\left((x-1)^2-\sqrt{2}(x-1)+1\right)}{2\sqrt{2}}\right)$$

input
```
Int[(2 - 4*x + 6*x^2 - 4*x^3 + x^4)^(-1),x]
```

output
```
(-1/2*Log[1 - Sqrt[2]*(-1 + x) + (-1 + x)^2]/Sqrt[2] + Log[1 + Sqrt[2]*(-1
 + x) + (-1 + x)^2]/(2*Sqrt[2]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+6\_Z^2-4\_Z+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3-3\_R^2+3\_R-1}\right)}{4}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+6\_Z^2-4\_Z+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3-3\_R^2+3\_R-1}\right)}{4}$	47

input
```
int(1/(x^4-4*x^3+6*x^2-4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3-3*_R^2+3*_R-1)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+6*_Z^2-4*_Z+
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x-1)+1\right) + \frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x-1)-1\right) + \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}(x-1)-2\,x+2\right) - \frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}(x-1)-2\,x+2\right)$$

input
```
integrate(1/(x^4-4*x^3+6*x^2-4*x+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*(x - 1) + 1) + 1/4*sqrt(2)*arctan(sqrt(2)*(x -
1) - 1) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*(x - 1) - 2*x + 2) - 1/8*sqrt(2)*l
og(x^2 - sqrt(2)*(x - 1) - 2*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2+x\left(-2-\sqrt{2}\right)+\sqrt{2}+2\right)}{8} + \frac{\sqrt{2}\log\left(x^2+x\left(-2+\sqrt{2}\right)-\sqrt{2}+2\right)}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(-\sqrt{2}x+1+\sqrt{2}\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-\sqrt{2}+1\right)}{4}$$

input
```
integrate(1/(x**4-4*x**3+6*x**2-4*x+2),x)
```

output
```
-sqrt(2)*log(x**2 + x*(-2 - sqrt(2)) + sqrt(2) + 2)/8 + sqrt(2)*log(x**2 +
 x*(-2 + sqrt(2)) - sqrt(2) + 2)/8 - sqrt(2)*atan(-sqrt(2)*x + 1 + sqrt(2)
)/4 + sqrt(2)*atan(sqrt(2)*x - sqrt(2) + 1)/4
```

Maxima [F]

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx = \int \frac{1}{x^4-4\,x^3+6\,x^2-4\,x+2}\,dx$$

input
```
integrate(1/(x^4-4*x^3+6*x^2-4*x+2),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - 4*x^3 + 6*x^2 - 4*x + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx = \frac{1}{16}\sqrt{2}\Big(\pi+4\,\arctan\left(\sqrt{2}x-\sqrt{2}-1\right)\Big) - \frac{1}{16}\sqrt{2}\Big(\pi+4\,\arctan\left(-\sqrt{2}x+\sqrt{2}-1\right)\Big) + \frac{1}{8}\sqrt{2}\log\left(\left(x+\sqrt{2}-1\right)^2+(x-1)^2\right) - \frac{1}{8}\sqrt{2}\log\left(\left(x-\sqrt{2}-1\right)^2+(x-1)^2\right)$$

input
```
integrate(1/(x^4-4*x^3+6*x^2-4*x+2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(pi + 4*arctan(sqrt(2)*x - sqrt(2) - 1)) - 1/16*sqrt(2)*(pi +
 4*arctan(-sqrt(2)*x + sqrt(2) - 1)) + 1/8*sqrt(2)*log((x + sqrt(2) - 1)^2
 + (x - 1)^2) - 1/8*sqrt(2)*log((x - sqrt(2) - 1)^2 + (x - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.43

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(6*x^2 - 4*x - 4*x^3 + x^4 + 2),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2) - 2^(1/2)*(1/2 - 1i/2))*(1/4 + 1i/4) +
 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2) - 2^(1/2)*(1/2 + 1i/2))*(1/4 - 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{1}{2-4x+6x^2-4x^3+x^4}\,dx$$
$$= \frac{\sqrt{2}\left(-2atan\left(\frac{\sqrt{2}-2x+2}{\sqrt{2}}\right)+2atan\left(\frac{\sqrt{2}+2x-2}{\sqrt{2}}\right)-\log\left(-\sqrt{2}\,x+\sqrt{2}+x^2-2x+2\right)+\log\left(\sqrt{2}\,x-\sqrt{2}+x^2\right.}{8}$$

input
```
int(1/(x^4-4*x^3+6*x^2-4*x+2),x)
```

output
```
(sqrt(2)*( - 2*atan((sqrt(2) - 2*x + 2)/sqrt(2)) + 2*atan((sqrt(2) + 2*x -
 2)/sqrt(2)) - log( - sqrt(2)*x + sqrt(2) + x**2 - 2*x + 2) + log(sqrt(2)*
x - sqrt(2) + x**2 - 2*x + 2)))/8
```

3.63 $\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8} \, dx$

Optimal result

Integrand size = 53, antiderivative size = 17

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8} \, dx = 2\arctan(x)-2\arctan\left(x^2\right)+2\arctan\left(x^3\right)$$

output
```
2*arctan(x)-2*arctan(x^2)+2*arctan(x^3)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8} \, dx = 2\left(-\frac{1}{2}\arctan\left(\frac{x(1-x+x^3)}{-1+x^2-x^3}\right)+\frac{1}{2}\arctan\left(\frac{x(1-x+x^3)}{1-x^2+x^3}\right)\right)$$

input
```
Integrate[(2 - 4*x + 2*x^2 + 4*x^3 + 2*x^4 - 4*x^5 + 2*x^6)/(1 - x^2 + 2*x
^4 - x^6 + x^8),x]
```

output
```
2*(-1/2*ArcTan[(x*(1 - x + x^3))/(-1 + x^2 - x^3)] + ArcTan[(x*(1 - x + x^
3))/(1 - x^2 + x^3)]/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^6-4x^5+2x^4+4x^3+2x^2-4x+2}{x^8-x^6+2x^4-x^2+1}dx$$
$$\Big\downarrow \text{2460}$$
$$\int \left(\frac{2(x^2+1)}{x^4-x^2+1}-\frac{4x}{x^4+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-2\arctan\left(x^2\right)-2\arctan\left(\sqrt{3}-2x\right)+2\arctan\left(2x+\sqrt{3}\right)$$

input
```
Int[(2 - 4*x + 2*x^2 + 4*x^3 + 2*x^4 - 4*x^5 + 2*x^6)/(1 - x^2 + 2*x^4 - x
^6 + x^8),x]
```

output
```
-2*ArcTan[Sqrt[3] - 2*x] - 2*ArcTan[x^2] + 2*ArcTan[Sqrt[3] + 2*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

method	result	size
risch	$2\arctan(1+x)+2\arctan(x^5+x^4-x^2+x+1)$	25
default	$-2\arctan(x^2)+2\arctan(2x+\sqrt{3})+2\arctan(2x-\sqrt{3})$	30
parallelrisch	$i\ln(x^2-i)-i\ln(x^2+i)-i\ln(x^2-ix-1)+i\ln(x^2+ix-1)$	48

input
```
int((2*x^6-4*x^5+2*x^4+4*x^3+2*x^2-4*x+2)/(x^8-x^6+2*x^4-x^2+1),x,method=_
RETURNVERBOSE)
```

output
```
2*arctan(1+x)+2*arctan(x^5+x^4-x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx$$
$$= 2\arctan\left(x^5+x^4-x^2+x+1\right)+2\arctan(x+1)$$

input
```
integrate((2*x^6-4*x^5+2*x^4+4*x^3+2*x^2-4*x+2)/(x^8-x^6+2*x^4-x^2+1),x, a
lgorithm="fricas")
```

output
```
2*arctan(x^5 + x^4 - x^2 + x + 1) + 2*arctan(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx$$
$$= 2\,\mathrm{atan}\,(x+1)+2\,\mathrm{atan}\,\left(x^5+x^4-x^2+x+1\right)$$

input
```
integrate((2*x**6-4*x**5+2*x**4+4*x**3+2*x**2-4*x+2)/(x**8-x**6+2*x**4-x**
2+1),x)
```

output
```
2*atan(x + 1) + 2*atan(x**5 + x**4 - x**2 + x + 1)
```

Maxima [F]

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx$$
$$= \int \frac{2\,(x^6-2\,x^5+x^4+2\,x^3+x^2-2\,x+1)}{x^8-x^6+2\,x^4-x^2+1}\,dx$$

input
```
integrate((2*x^6-4*x^5+2*x^4+4*x^3+2*x^2-4*x+2)/(x^8-x^6+2*x^4-x^2+1),x, a
lgorithm="maxima")
```

output
```
2*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 2*arctan(1/2*sqrt(2)*(2*x - sqrt(2
))) + 2*integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.24

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx = 2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - 2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + 2\arctan\left(2x+\sqrt{3}\right) + 2\arctan\left(2x-\sqrt{3}\right)$$

input
```
integrate((2*x^6-4*x^5+2*x^4+4*x^3+2*x^2-4*x+2)/(x^8-x^6+2*x^4-x^2+1),x, a
lgorithm="giac")
```

output
```
2*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 2*arctan(1/2*sqrt(2)*(2*x - sqrt(2
))) + 2*arctan(2*x + sqrt(3)) + 2*arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx = 2\,\mathrm{atan}\left(x^5+x^4-x^2+x+1\right) + 2\,\mathrm{atan}(x+1)$$

input
```
int((2*x^2 - 4*x + 4*x^3 + 2*x^4 - 4*x^5 + 2*x^6 + 2)/(2*x^4 - x^2 - x^6 +
 x^8 + 1),x)
```

output
```
2*atan(x - x^2 + x^4 + x^5 + 1) + 2*atan(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{2-4x+2x^2+4x^3+2x^4-4x^5+2x^6}{1-x^2+2x^4-x^6+x^8}\,dx$$
$$= -2atan\left(\sqrt{3}-2x\right)+2atan\left(\sqrt{3}+2x\right)+2atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)+2atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)$$

input
```
int((2*x^6-4*x^5+2*x^4+4*x^3+2*x^2-4*x+2)/(x^8-x^6+2*x^4-x^2+1),x)
```

output
```
2*( - atan(sqrt(3) - 2*x) + atan(sqrt(3) + 2*x) + atan((sqrt(2) - 2*x)/sqr
t(2)) + atan((sqrt(2) + 2*x)/sqrt(2)))
```

3.64 $\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 158

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = -2\sqrt{2}\arctan\left(1-\sqrt{2}x\right)+2\sqrt{2}\arctan\left(1+\sqrt{2}x\right)+\sqrt{2}\arctan\left(1-\sqrt{2}x^2\right)-\sqrt{2}\arctan\left(1+\sqrt{2}x^2\right)-\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)+\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)+\frac{\log\left(1-\sqrt{2}x^2+x^4\right)}{\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x^2+x^4\right)}{\sqrt{2}}$$

output
```
2*2^(1/2)*arctan(-1+x*2^(1/2))+2*2^(1/2)*arctan(1+x*2^(1/2))-2^(1/2)*arcta
n(-1+x^2*2^(1/2))-2^(1/2)*arctan(1+x^2*2^(1/2))-2^(1/2)*ln(1-x*2^(1/2)+x^2
)+2^(1/2)*ln(1+x*2^(1/2)+x^2)+1/2*ln(1-x^2*2^(1/2)+x^4)*2^(1/2)-1/2*ln(1+x
^2*2^(1/2)+x^4)*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx$$
$$= \frac{-4\arctan\left(1-\sqrt{2}x\right)+4\arctan\left(1+\sqrt{2}x\right)+2\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)+2\arctan\left(\cot\left(\frac{\pi}{8}\right)-x\,c\right.}{}$$

input
```
Integrate[(8 - 8*x - 8*x^5 + 8*x^8)/(1 + x^4 + x^8 + x^12),x]
```

output
```
(-4*ArcTan[1 - Sqrt[2]*x] + 4*ArcTan[1 + Sqrt[2]*x] + 2*ArcTan[(x + Cos[Pi
/8])*Csc[Pi/8]] + 2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8]] + 2*ArcTan[Sec[Pi/8]*(
x + Sin[Pi/8])] - 2*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]] - 2*Log[1 - Sqrt[2]*x
+ x^2] + 2*Log[1 + Sqrt[2]*x + x^2] + Log[1 + x^2 - 2*x*Cos[Pi/8]] + Log[1
 + x^2 + 2*x*Cos[Pi/8]] - Log[1 + x^2 - 2*x*Sin[Pi/8]] - Log[1 + x^2 + 2*x
*Sin[Pi/8]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^8-8x^5-8x+8}{x^{12}+x^8+x^4+1}\,dx$$
$$\Big\downarrow \; 2460$$
$$\int \left(\frac{8}{x^4+1}-\frac{8x}{x^8+1}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\sqrt{2}\arctan\left(1-\sqrt{2}x^2\right)-\sqrt{2}\arctan\left(\sqrt{2}x^2+1\right)-2\sqrt{2}\arctan\left(1-\sqrt{2}x\right)+2\sqrt{2}\arctan\left(\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{\log\left(x^4-\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}x^2+1\right)}{\sqrt{2}}$$

input
```
Int[(8 - 8*x - 8*x^5 + 8*x^8)/(1 + x^4 + x^8 + x^12),x]
```

output
```
-2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + Sqrt[
2]*ArcTan[1 - Sqrt[2]*x^2] - Sqrt[2]*ArcTan[1 + Sqrt[2]*x^2] - Sqrt[2]*Log
[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x + x^2] + Log[1 - Sqrt[2]
*x^2 + x^4]/Sqrt[2] - Log[1 + Sqrt[2]*x^2 + x^4]/Sqrt[2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.24

method	result
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(x+\_R\right)\right)+\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\_R\ln\left(x^2-\_R\right)\right)$
default	$\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,x+x^2}{1-\sqrt{2}\,x+x^2}\right)+2\arctan\left(1+\sqrt{2}\,x\right)+2\arctan\left(-1+\sqrt{2}\,x\right)\right)-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,x^2+x^4}{1-\sqrt{2}\,x^2+x^4}\right)+2\arctan\right.}{}$

input
```
int((8*x^8-8*x^5-8*x+8)/(x^12+x^8+x^4+1),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(x+_R),_R=RootOf(_Z^4+1))+sum(_R*ln(x^2-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = -\sqrt{2}\arctan\left(\sqrt{2}x^2+1\right)-\sqrt{2}\arctan\left(\sqrt{2}x^2-1\right)+2\sqrt{2}\arctan\left(\sqrt{2}x+1\right)+2\sqrt{2}\arctan\left(\sqrt{2}x-1\right)-\frac{1}{2}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)+\frac{1}{2}\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((8*x^8-8*x^5-8*x+8)/(x^12+x^8+x^4+1),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*x^2 + 1) - sqrt(2)*arctan(sqrt(2)*x^2 - 1) + 2*sqr
t(2)*arctan(sqrt(2)*x + 1) + 2*sqrt(2)*arctan(sqrt(2)*x - 1) - 1/2*sqrt(2)
*log(x^4 + sqrt(2)*x^2 + 1) + 1/2*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1) + sqr
t(2)*log(x^2 + sqrt(2)*x + 1) - sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = -\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)}{2}-\frac{\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)}{2}+2\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)+2\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)-\sqrt{2}\operatorname{atan}\left(\sqrt{2}x^2-1\right)-\sqrt{2}\operatorname{atan}\left(\sqrt{2}x^2+1\right)$$

input
```
integrate((8*x**8-8*x**5-8*x+8)/(x**12+x**8+x**4+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1) + sqrt(2)*log(x**2 + sqrt(2)*x + 1) + s
qrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/2 - sqrt(2)*log(x**4 + sqrt(2)*x**2 +
1)/2 + 2*sqrt(2)*atan(sqrt(2)*x - 1) + 2*sqrt(2)*atan(sqrt(2)*x + 1) - sqr
t(2)*atan(sqrt(2)*x**2 - 1) - sqrt(2)*atan(sqrt(2)*x**2 + 1)
```

Maxima [F]

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx=\int \frac{8\,(x^8-x^5-x+1)}{x^{12}+x^8+x^4+1}\,dx$$

input
```
integrate((8*x^8-8*x^5-8*x+8)/(x^12+x^8+x^4+1),x, algorithm="maxima")
```

output
```
2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 2*sqrt(2)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2))) + sqrt(2)*log(x^2 + sqrt(2)*x + 1) - sqrt(2)*log(x^2 -
 sqrt(2)*x + 1) - 8*integrate(x/(x^8 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(126) = 252$.

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.69

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = 2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + 2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \sqrt{2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) - \sqrt{2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \sqrt{2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) - \sqrt{2}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) + \sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{2}\sqrt{2}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) + \frac{1}{2}\sqrt{2}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right) - \frac{1}{2}\sqrt{2}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) - \frac{1}{2}\sqrt{2}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

input
```
integrate((8*x^8-8*x^5-8*x+8)/(x^12+x^8+x^4+1),x, algorithm="giac")
```

output
```
2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 2*sqrt(2)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2))) + sqrt(2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(
2) + 2)) - sqrt(2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) +
sqrt(2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - sqrt(2)*arc
tan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + sqrt(2)*log(x^2 + sqrt
(2)*x + 1) - sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/2*sqrt(2)*log(x^2 + x*sq
rt(sqrt(2) + 2) + 1) + 1/2*sqrt(2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/
2*sqrt(2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/2*sqrt(2)*log(x^2 - x*sq
rt(-sqrt(2) + 2) + 1)
```

Mupad [B] (verification not implemented)

Time = 9.95 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.44

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(2+2\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)(2-2\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)(-1-\mathrm{i}) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)(-1+1\mathrm{i})$$

input

```
int(-(8*x + 8*x^5 - 8*x^8 - 8)/(x^4 + x^8 + x^12 + 1),x)
```

output

```
2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(2 + 2i) + 2^(1/2)*atan(2^(1/2)*x*(1/
2 + 1i/2))*(2 - 2i) - 2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1 + 1i) - 2^
(1/2)*atan(2^(1/2)*x^2*(1/2 + 1i/2))*(1 - 1i)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.69

$$\int \frac{8-8x-8x^5+8x^8}{1+x^4+x^8+x^{12}}\,dx = \sqrt{\sqrt{2}+2}\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2x}{\sqrt{\sqrt{2}+2}}\right) + \sqrt{\sqrt{2}+2}\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2x}{\sqrt{\sqrt{2}+2}}\right) + \sqrt{\sqrt{2}+2}\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}-2x}{\sqrt{-\sqrt{2}+2}}\right) + \sqrt{\sqrt{2}+2}\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}+2x}{\sqrt{-\sqrt{2}+2}}\right) - 2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) - \frac{\sqrt{2}\log\left(-\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{2} + \frac{\sqrt{2}\log\left(-\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{2} - \sqrt{2}\log\left(-\sqrt{2}\,x+x^2+1\right) - \frac{\sqrt{2}\log\left(\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{2} + \frac{\sqrt{2}\log\left(\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{2} + \sqrt{2}\log\left(\sqrt{2}\,x+x^2+1\right)$$

input
```
int((8*x^8-8*x^5-8*x+8)/(x^12+x^8+x^4+1),x)
```

output

```
(2*sqrt(sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) - 2*x
)/sqrt(sqrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((sqrt
( - sqrt(2) + 2) + 2*x)/sqrt(sqrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*sqrt( - s
qrt(2) + 2)*atan((sqrt(sqrt(2) + 2) - 2*x)/sqrt( - sqrt(2) + 2)) + 2*sqrt(
sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*x)/sqrt( - s
qrt(2) + 2)) - 4*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2)) + 4*sqrt(2)*atan((s
qrt(2) + 2*x)/sqrt(2)) - sqrt(2)*log( - sqrt( - sqrt(2) + 2)*x + x**2 + 1)
 + sqrt(2)*log( - sqrt(sqrt(2) + 2)*x + x**2 + 1) - 2*sqrt(2)*log( - sqrt(
2)*x + x**2 + 1) - sqrt(2)*log(sqrt( - sqrt(2) + 2)*x + x**2 + 1) + sqrt(2
)*log(sqrt(sqrt(2) + 2)*x + x**2 + 1) + 2*sqrt(2)*log(sqrt(2)*x + x**2 + 1
))/2
```

3.65 $\int \frac{16x^3-248x^{11}+80x^{15}-392x^{19}-80x^{23}+24x^{27}}{1+10x^4+26x^8+40x^{12}+71x^{16}+40x^{20}+26x^{24}+10x^{28}+x^{32}}\,dx$

Optimal result

Integrand size = 74, antiderivative size = 141

$$\begin{aligned}
&\int \frac{16x^3-248x^{11}+80x^{15}-392x^{19}-80x^{23}+24x^{27}}{1+10x^4+26x^8+40x^{12}+71x^{16}+40x^{20}+26x^{24}+10x^{28}+x^{32}}\,dx \\
&= \left(-1-\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(-1+\sqrt{5}\right)x^4\right)+\left(1-\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(1+\sqrt{5}\right)x^4\right) \\
&\quad-\sqrt{5}\text{arctanh}\left(\frac{1}{5}\left(3\sqrt{5}+2\sqrt{5}x^4\right)\right)-\sqrt{5}\text{arctanh}\left(\frac{1}{15}\left(7\sqrt{5}+2\sqrt{5}x^4\right)\right) \\
&\quad+\frac{1}{2}\log\left(1+3x^4+x^8\right)-\frac{1}{2}\log\left(1+7x^4+x^8\right)
\end{aligned}$$

output
```
(-5^(1/2)-1)*arctan(1/2*(5^(1/2)-1)*x^4)+(-5^(1/2)+1)*arctan(1/2*(5^(1/2)+
1)*x^4)-5^(1/2)*arctanh(3/5*5^(1/2)+2/5*5^(1/2)*x^4)-5^(1/2)*arctanh(7/15*
5^(1/2)+2/15*5^(1/2)*x^4)+1/2*ln(x^8+3*x^4+1)-1/2*ln(x^8+7*x^4+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07

$$\int \frac{16x^3-248x^{11}+80x^{15}-392x^{19}-80x^{23}+24x^{27}}{1+10x^4+26x^8+40x^{12}+71x^{16}+40x^{20}+26x^{24}+10x^{28}+x^{32}}\,dx$$
$$=\frac{1}{2}\left(\left(1+\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\left(-1+\sqrt{5}\right)\log\left(-7+3\sqrt{5}-2x^4\right)\right.$$
$$-\left(-1+\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)-\left(1+\sqrt{5}\right)\log\left(7+3\sqrt{5}+2x^4\right)$$
$$\left.-2\text{RootSum}\left[1+3\#1^8+\#1^{16}\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1^8}{3\#1^4+2\#1^{12}}\&\right]\right)$$

input
```
Integrate[(16*x^3 - 248*x^11 + 80*x^15 - 392*x^19 - 80*x^23 + 24*x^27)/(1
+ 10*x^4 + 26*x^8 + 40*x^12 + 71*x^16 + 40*x^20 + 26*x^24 + 10*x^28 + x^32
),x]
```

output
```
((1 + Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (-1 + Sqrt[5])*Log[-7 + 3*Sqrt[
5] - 2*x^4] - (-1 + Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4] - (1 + Sqrt[5])*Log[
7 + 3*Sqrt[5] + 2*x^4] - 2*RootSum[1 + 3*#1^8 + #1^16 & , (2*Log[x - #1] +
 3*Log[x - #1]*#1^8)/(3*#1^4 + 2*#1^12) & ])/2
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{24x^{27}-80x^{23}-392x^{19}+80x^{15}-248x^{11}+16x^3}{x^{32}+10x^{28}+26x^{24}+40x^{20}+71x^{16}+40x^{12}+26x^8+10x^4+1}\,dx$$
$$\Big\downarrow \; 2460$$

$$\int \left(-\frac{8(3x^8+2)\,x^3}{x^{16}+3x^8+1}+\frac{4(x^4+4)\,x^3}{x^8+3x^4+1}-\frac{4(x^4-4)\,x^3}{x^8+7x^4+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\sqrt{2\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^4\right)-\sqrt{2\left(3-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^4\right)-\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x^4-3\sqrt{5}+7\right)+\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x^4+3\sqrt{5}+7\right)$$

input
```
Int[(16*x^3 - 248*x^11 + 80*x^15 - 392*x^19 - 80*x^23 + 24*x^27)/(1 + 10*x
^4 + 26*x^8 + 40*x^12 + 71*x^16 + 40*x^20 + 26*x^24 + 10*x^28 + x^32),x]
```

output
```
-(Sqrt[2*(3 + Sqrt[5])]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^4]) - Sqrt[2*(3 - S
qrt[5])]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^4] - ((1 - Sqrt[5])*Log[7 - 3*Sqrt
[5] + 2*x^4])/2 + ((1 + Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/2 + ((1 - Sqrt[
5])*Log[3 + Sqrt[5] + 2*x^4])/2 - ((1 + Sqrt[5])*Log[7 + 3*Sqrt[5] + 2*x^4
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
default	$\frac{\ln(x^8+3x^4+1)}{2}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)-\frac{\ln(x^8+7x^4+1)}{2}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4+7)\sqrt{5}}{15}\right)-\frac{4\sqrt{5}\left(5+3\sqrt{5}\right)}{5\left(2+\right.}$
risch	$\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4+3\_Z^2+1)}\_R\ln\left(x^4-\_R\right)\right)+\frac{\ln\left(2x^4-\sqrt{5}+3\right)}{2}+\frac{\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{2}+\frac{\ln\left(2x^4+\sqrt{5}+3\right)}{2}-$

input
```
int((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28+26*x^
24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^8+3*x^4+1)-5^(1/2)*arctanh(1/5*(2*x^4+3)*5^(1/2))-1/2*ln(x^8+7*x^
4+1)-5^(1/2)*arctanh(1/15*(2*x^4+7)*5^(1/2))-4/5*5^(1/2)*(5+3*5^(1/2))/(2+
2*5^(1/2))*arctan(4*x^4/(2+2*5^(1/2)))-4/5*(3*5^(1/2)-5)*5^(1/2)/(-2+2*5^(
1/2))*arctan(4*x^4/(-2+2*5^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{16x^3-248x^{11}+80x^{15}-392x^{19}-80x^{23}+24x^{27}}{1+10x^4+26x^8+40x^{12}+71x^{16}+40x^{20}+26x^{24}+10x^{28}+x^{32}}\,dx &= -\left(\sqrt{5}-1\right)\arctan\left(\frac{1}{2}\sqrt{5}x^4+\frac{1}{2}x^4\right)-\left(\sqrt{5}+1\right)\arctan\left(\frac{1}{2}\sqrt{5}x^4-\frac{1}{2}x^4\right)\\&\quad+\frac{1}{2}\sqrt{5}\log\left(\frac{2x^8+14x^4-3\sqrt{5}(2x^4+7)+47}{x^8+7x^4+1}\right)\\&\quad+\frac{1}{2}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)\\&\quad-\frac{1}{2}\log\left(x^8+7x^4+1\right)+\frac{1}{2}\log\left(x^8+3x^4+1\right)\end{aligned}$$

input
```
integrate((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28
+26*x^24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x, algorithm="fricas")
```

output
```
-(sqrt(5) - 1)*arctan(1/2*sqrt(5)*x^4 + 1/2*x^4) - (sqrt(5) + 1)*arctan(1/
2*sqrt(5)*x^4 - 1/2*x^4) + 1/2*sqrt(5)*log((2*x^8 + 14*x^4 - 3*sqrt(5)*(2*
x^4 + 7) + 47)/(x^8 + 7*x^4 + 1)) + 1/2*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(
5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1)) - 1/2*log(x^8 + 7*x^4 + 1) + 1/2*lo
g(x^8 + 3*x^4 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(126) = 252$.

Time = 0.39 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.24

$$\int \frac{16x^3 - 248x^{11} + 80x^{15} - 392x^{19} - 80x^{23} + 24x^{27}}{1 + 10x^4 + 26x^8 + 40x^{12} + 71x^{16} + 40x^{20} + 26x^{24} + 10x^{28} + x^{32}}\,dx = \left(\frac{1}{2}\right.$$

$$\left.-\frac{\sqrt{5}}{2}\right)\log\left(x^4 - \frac{1}{12} - \frac{\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^6}{6} + \frac{\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^7}{6} + \frac{\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^4}{6} - \frac{4\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^3}{3} + \frac{11\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^2}{6} + \frac{3\sqrt{5}}{4}\right.$$

$$+\left(-\frac{1}{2}\right.$$

$$\left.+\frac{\sqrt{5}}{2}\right)\log\left(x^4 - \frac{3\sqrt{5}}{4} - \frac{4\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^3}{3} - \frac{\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^6}{6} + \frac{\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^7}{6} + \frac{\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^4}{6} + \frac{11\left(-\frac{1}{2}+\right.}{6}\right.$$

$$+\left(\frac{1}{2}\right.$$

$$\left.+\frac{\sqrt{5}}{2}\right)\log\left(x^4 - \frac{4\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^3}{3} - \frac{\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^6}{6} - \frac{3\sqrt{5}}{4} - \frac{1}{12} + \frac{\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^4}{6} + \frac{11\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)^2}{6} + \frac{\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{6}\right.$$

$$+\left(-\frac{\sqrt{5}}{2}\right.$$

$$\left.-\frac{1}{2}\right)\log\left(x^4 + \frac{\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^7}{6} - \frac{\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^6}{6} + \frac{\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^4}{6} + \frac{17}{12} + \frac{3\sqrt{5}}{4} + \frac{11\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^2}{6} - 4\left(-\right.\right.$$

$$+\operatorname{RootSum}\left(t^4 + 3t^2 + 1, \left(t \mapsto t\log\left(\frac{t^7}{6} - \frac{t^6}{6} + \frac{t^4}{6} - \frac{4t^3}{3} + \frac{11t^2}{6} - \frac{3t}{2} + x^4 + \frac{2}{3}\right)\right)\right)$$

input
```
integrate((24*x**27-80*x**23-392*x**19+80*x**15-248*x**11+16*x**3)/(x**32+
10*x**28+26*x**24+40*x**20+71*x**16+40*x**12+26*x**8+10*x**4+1),x)
```

output
```
(1/2 - sqrt(5)/2)*log(x**4 - 1/12 - (1/2 - sqrt(5)/2)**6/6 + (1/2 - sqrt(5
)/2)**7/6 + (1/2 - sqrt(5)/2)**4/6 - 4*(1/2 - sqrt(5)/2)**3/3 + 11*(1/2 -
sqrt(5)/2)**2/6 + 3*sqrt(5)/4) + (-1/2 + sqrt(5)/2)*log(x**4 - 3*sqrt(5)/4
 - 4*(-1/2 + sqrt(5)/2)**3/3 - (-1/2 + sqrt(5)/2)**6/6 + (-1/2 + sqrt(5)/2
)**7/6 + (-1/2 + sqrt(5)/2)**4/6 + 11*(-1/2 + sqrt(5)/2)**2/6 + 17/12) + (
1/2 + sqrt(5)/2)*log(x**4 - 4*(1/2 + sqrt(5)/2)**3/3 - (1/2 + sqrt(5)/2)**
6/6 - 3*sqrt(5)/4 - 1/12 + (1/2 + sqrt(5)/2)**4/6 + 11*(1/2 + sqrt(5)/2)**
2/6 + (1/2 + sqrt(5)/2)**7/6) + (-sqrt(5)/2 - 1/2)*log(x**4 + (-sqrt(5)/2
- 1/2)**7/6 - (-sqrt(5)/2 - 1/2)**6/6 + (-sqrt(5)/2 - 1/2)**4/6 + 17/12 +
3*sqrt(5)/4 + 11*(-sqrt(5)/2 - 1/2)**2/6 - 4*(-sqrt(5)/2 - 1/2)**3/3) + Ro
otSum(_t**4 + 3*_t**2 + 1, Lambda(_t, _t*log(_t**7/6 - _t**6/6 + _t**4/6 -
 4*_t**3/3 + 11*_t**2/6 - 3*_t/2 + x**4 + 2/3)))
```

Maxima [F]

$$\int \frac{16x^3 - 248x^{11} + 80x^{15} - 392x^{19} - 80x^{23} + 24x^{27}}{1 + 10x^4 + 26x^8 + 40x^{12} + 71x^{16} + 40x^{20} + 26x^{24} + 10x^{28} + x^{32}}\,dx$$
$$= \int \frac{8\,(3\,x^{27} - 10\,x^{23} - 49\,x^{19} + 10\,x^{15} - 31\,x^{11} + 2\,x^3)}{x^{32} + 10\,x^{28} + 26\,x^{24} + 40\,x^{20} + 71\,x^{16} + 40\,x^{12} + 26\,x^8 + 10\,x^4 + 1}\,dx$$

input
```
integrate((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28
+26*x^24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x, algorithm="maxima")
```

output
```
8*integrate((3*x^27 - 10*x^23 - 49*x^19 + 10*x^15 - 31*x^11 + 2*x^3)/(x^32
 + 10*x^28 + 26*x^24 + 40*x^20 + 71*x^16 + 40*x^12 + 26*x^8 + 10*x^4 + 1),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{16x^3 - 248x^{11} + 80x^{15} - 392x^{19} - 80x^{23} + 24x^{27}}{1 + 10x^4 + 26x^8 + 40x^{12} + 71x^{16} + 40x^{20} + 26x^{24} + 10x^{28} + x^{32}} \, dx$$
$= \text{Exception raised: TypeError}$

input
```
integrate((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28
+26*x^24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int(sage0,sageVARx)  Error: Bad Arg
ument Type
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 2153, normalized size of antiderivative = 15.27

$$\int \frac{16x^3 - 248x^{11} + 80x^{15} - 392x^{19} - 80x^{23} + 24x^{27}}{1 + 10x^4 + 26x^8 + 40x^{12} + 71x^{16} + 40x^{20} + 26x^{24} + 10x^{28} + x^{32}} \, dx$$
$= \text{Too large to display}$

input
```
int((16*x^3 - 248*x^11 + 80*x^15 - 392*x^19 - 80*x^23 + 24*x^27)/(10*x^4 +
 26*x^8 + 40*x^12 + 71*x^16 + 40*x^20 + 26*x^24 + 10*x^28 + x^32 + 1),x)
```

output

```
atan(((- 5^(1/2)/2 - 3/2)^(1/2)*117779131845107704926873936971012505600000
00000000000000000000000000000000000000000i)/(51058337120893890751916603222
89090560000000000000000000000000000000000000000000000*5^(1/2) - 5267248564
920505865498027973043814400000000000000000000000000000000000000000000000*5
^(1/2)*x^4 + 1177791318451077049268739369710125056000000000000000000000000
0000000000000000000000*x^4 - 114170233933457692052694155217233510400000000
00000000000000000000000000000000000000) - (5^(1/2)*(- 5^(1/2)/2 - 3/2)^(1/
2)*52672485649205058654980279730438144000000000000000000000000000000000000
0000000000i)/(510583371208938907519166032228909056000000000000000000000000
00000000000000000000000*5^(1/2) - 5267248564920505865498027973043814400000
000000000000000000000000000000000000000000*5^(1/2)*x^4 + 11777913184510770
492687393697101250560000000000000000000000000000000000000000000000*x^4 - 1
14170233933457692052694155217233510400000000000000000000000000000000000000
00000000) + (x^4*(- 5^(1/2)/2 - 3/2)^(1/2)*2989011937024212649588327408830
7752960000000000000000000000000000000000000000000000i)/(510583371208938907
5191660322289090560000000000000000000000000000000000000000000000*5^(1/2) -
 52672485649205058654980279730438144000000000000000000000000000000000000000
000000000*5^(1/2)*x^4 + 11777913184510770492687393697101250560000000000000
00000000000000000000000000000000000*x^4 - 1141702339334576920526941552172335
1040000000000000000000000000000000000000000000000) - (5^(1/2)*x^4*(- 5^...
```

Reduce [F]

$$\int \frac{16x^3 - 248x^{11} + 80x^{15} - 392x^{19} - 80x^{23} + 24x^{27}}{1 + 10x^4 + 26x^8 + 40x^{12} + 71x^{16} + 40x^{20} + 26x^{24} + 10x^{28} + x^{32}}\,dx = \int \frac{24x^{27} - 80x^{23} - 392x^{19} + 80x^{15} - 248x^{11} + 16x^3}{x^{32} + 10x^{28} + 26x^{24} + 40x^{20} + 71x^{16} + 40x^{12} + 26x^8 + 10x^4 + 1}dx$$

input

```
int((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28+26*x^
24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x)
```

output

```
int((24*x^27-80*x^23-392*x^19+80*x^15-248*x^11+16*x^3)/(x^32+10*x^28+26*x^
24+40*x^20+71*x^16+40*x^12+26*x^8+10*x^4+1),x)
```

3.66 $\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$

Optimal result

Integrand size = 81, antiderivative size = 181

$$\begin{aligned}
&\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx \\
&= -\sqrt{7+\sqrt{41}}\arctan\left(\sqrt{\frac{1}{2}\left(7-\sqrt{41}\right)}x^4\right) \\
&\quad -\sqrt{7-\sqrt{41}}\arctan\left(\sqrt{\frac{1}{2}\left(7+\sqrt{41}\right)}x^4\right) \\
&\quad -\sqrt{7+\sqrt{41}}\text{arctanh}\left(\frac{\sqrt{29-\sqrt{41}}}{4}+\frac{1}{2}\sqrt{\frac{1}{2}\left(7-\sqrt{41}\right)}x^4\right) \\
&\quad -\sqrt{7-\sqrt{41}}\text{arctanh}\left(\frac{\sqrt{29+\sqrt{41}}}{4}+\frac{1}{2}\sqrt{\frac{1}{2}\left(7+\sqrt{41}\right)}x^4\right)
\end{aligned}$$

output
```
-(7+41^(1/2))^(1/2)*arctan(1/2*(14-2*41^(1/2))^(1/2)*x^4)-(7-41^(1/2))^(1/
2)*arctan(1/2*(14+2*41^(1/2))^(1/2)*x^4)-(7+41^(1/2))^(1/2)*arctanh(1/4*(2
9-41^(1/2))^(1/2)+1/4*(14-2*41^(1/2))^(1/2)*x^4)-(7-41^(1/2))^(1/2)*arctan
h(1/4*(29+41^(1/2))^(1/2)+1/4*(14+2*41^(1/2))^(1/2)*x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.86

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx = 8\left(-\frac{1}{8}\text{RootSum}\left[1+7\#1^8+2\#1^{16}\&,\frac{2\log(x-\#1)+7\log(x-\#1)\#1^8}{7\#1^4+4\#1^{12}}\&\right] + \frac{1}{4}\text{RootSum}\left[16+76\#1^4+105\#1^8+44\#1^{12} + 4\#1^{16}\&,\frac{13\log(x-\#1)+36\log(x-\#1)\#1^4+14\log(x-\#1)\#1^8}{38+105\#1^4+66\#1^8+8\#1^{12}}\&\right]\right)$$

input
```
Integrate[(-48*x^3 - 640*x^7 - 896*x^11 - 928*x^15 - 3960*x^19 - 1312*x^23
 + 224*x^27)/(16 + 76*x^4 + 217*x^8 + 576*x^12 + 771*x^16 + 460*x^20 + 238
*x^24 + 88*x^28 + 8*x^32),x]
```

output
```
8*(-1/8*RootSum[1 + 7*#1^8 + 2*#1^16 & , (2*Log[x - #1] + 7*Log[x - #1]*#1
^8)/(7*#1^4 + 4*#1^12) & ] + RootSum[16 + 76*#1^4 + 105*#1^8 + 44*#1^12 +
4*#1^16 & , (13*Log[x - #1] + 36*Log[x - #1]*#1^4 + 14*Log[x - #1]*#1^8)/(
38 + 105*#1^4 + 66*#1^8 + 8*#1^12) & ]/4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{224x^{27}-1312x^{23}-3960x^{19}-928x^{15}-896x^{11}-640x^7-48x^3}{8x^{32}+88x^{28}+238x^{24}+460x^{20}+771x^{16}+576x^{12}+217x^8+76x^4+16}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{16x^3(14x^8+36x^4+13)}{4x^{16}+44x^{12}+105x^8+76x^4+16}-\frac{8x^3(7x^8+2)}{2x^{16}+7x^8+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$52\text{Subst}\left(\int \frac{1}{4x^4+44x^3+105x^2+76x+16}dx,x,x^4\right)+144\text{Subst}\left(\int \frac{x}{4x^4+44x^3+105x^2+76x+16}dx,x,x^4\right)+56\text{Subst}\left(\int \frac{x^2}{4x^4+44x^3+105x^2+76x+16}dx,x,x^4\right)-\sqrt{7-\sqrt{41}}\arctan\left(\frac{2x^4}{\sqrt{7-\sqrt{41}}}\right)-\sqrt{7+\sqrt{41}}\arctan\left(\frac{2x^4}{\sqrt{7+\sqrt{41}}}\right)$$

input
```
Int[(-48*x^3 - 640*x^7 - 896*x^11 - 928*x^15 - 3960*x^19 - 1312*x^23 + 224
*x^27)/(16 + 76*x^4 + 217*x^8 + 576*x^12 + 771*x^16 + 460*x^20 + 238*x^24
+ 88*x^28 + 8*x^32),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

method	result
risch	$\left(\sum_{\_R=\text{RootOf}(2\_Z^4-7\_Z^2+1)}\_R\ln\left(x^4+\_R^2-2\_R+1\right)\right)+\left(\sum_{\_R=\text{RootOf}(2\_Z^4+7\_Z^2+1)}\_R\ln\left(x^4\right.\right.$
default	$-\frac{\left(41+7\sqrt{41}\right)\sqrt{41}\,\arctan\left(\frac{2x^4}{\sqrt{7+\sqrt{41}}}\right)}{41\sqrt{7+\sqrt{41}}}-\frac{\left(-41+7\sqrt{41}\right)\sqrt{41}\,\arctan\left(\frac{2x^4}{\sqrt{7-\sqrt{41}}}\right)}{41\sqrt{7-\sqrt{41}}}+2\left(\sum_{\_R=\text{RootOf}(4\_Z^4+44\_Z^3+105\_Z^2+}\right.$

input
```
int((224*x^27-1312*x^23-3960*x^19-928*x^15-896*x^11-640*x^7-48*x^3)/(8*x^3
2+88*x^28+238*x^24+460*x^20+771*x^16+576*x^12+217*x^8+76*x^4+16),x,method=
_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^4+_R^2-2*_R+1),_R=RootOf(2*_Z^4-7*_Z^2+1))+sum(_R*ln(x^4-_R),_
R=RootOf(2*_Z^4+7*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$$
$$= \sqrt{\sqrt{41}+7}\arctan\left(\frac{1}{4}\left(\sqrt{41}x^4-7\,x^4\right)\sqrt{\sqrt{41}+7}\right) - \sqrt{-\sqrt{41}+7}\arctan\left(\frac{1}{4}\left(\sqrt{41}x^4+7\,x^4\right)\sqrt{-\sqrt{41}+7}\right) - \frac{1}{2}\sqrt{\sqrt{41}+7}\log\left(4\,x^4+\sqrt{41}+4\sqrt{\sqrt{41}+7}+11\right) + \frac{1}{2}\sqrt{\sqrt{41}+7}\log\left(4\,x^4+\sqrt{41}-4\sqrt{\sqrt{41}+7}+11\right) - \frac{1}{2}\sqrt{-\sqrt{41}+7}\log\left(4\,x^4-\sqrt{41}+4\sqrt{-\sqrt{41}+7}+11\right) + \frac{1}{2}\sqrt{-\sqrt{41}+7}\log\left(4\,x^4-\sqrt{41}-4\sqrt{-\sqrt{41}+7}+11\right)$$

input
```
integrate((224*x^27-1312*x^23-3960*x^19-928*x^15-896*x^11-640*x^7-48*x^3)/
(8*x^32+88*x^28+238*x^24+460*x^20+771*x^16+576*x^12+217*x^8+76*x^4+16),x,
algorithm="fricas")
```

output
```
sqrt(sqrt(41) + 7)*arctan(1/4*(sqrt(41)*x^4 - 7*x^4)*sqrt(sqrt(41) + 7)) -
 sqrt(-sqrt(41) + 7)*arctan(1/4*(sqrt(41)*x^4 + 7*x^4)*sqrt(-sqrt(41) + 7)
) - 1/2*sqrt(sqrt(41) + 7)*log(4*x^4 + sqrt(41) + 4*sqrt(sqrt(41) + 7) + 1
1) + 1/2*sqrt(sqrt(41) + 7)*log(4*x^4 + sqrt(41) - 4*sqrt(sqrt(41) + 7) +
11) - 1/2*sqrt(-sqrt(41) + 7)*log(4*x^4 - sqrt(41) + 4*sqrt(-sqrt(41) + 7)
 + 11) + 1/2*sqrt(-sqrt(41) + 7)*log(4*x^4 - sqrt(41) - 4*sqrt(-sqrt(41) +
 7) + 11)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$$
$$=\operatorname{RootSum}\left(2t^4-7t^2+1,\left(t\mapsto t\log\left(\frac{2t^7}{7}-\frac{2t^6}{7}+\frac{t^4}{7}-\frac{47t^3}{14}+\frac{27t^2}{7}-\frac{3t}{2}+x^4+\frac{4}{7}\right)\right)\right)$$
$$+\operatorname{RootSum}\left(2t^4+7t^2+1,\left(t\mapsto t\log\left(\frac{2t^7}{7}-\frac{2t^6}{7}+\frac{t^4}{7}-\frac{47t^3}{14}+\frac{27t^2}{7}-\frac{3t}{2}+x^4+\frac{4}{7}\right)\right)\right)$$

input
```
integrate((224*x**27-1312*x**23-3960*x**19-928*x**15-896*x**11-640*x**7-48
*x**3)/(8*x**32+88*x**28+238*x**24+460*x**20+771*x**16+576*x**12+217*x**8+
76*x**4+16),x)
```

output
```
RootSum(2*_t**4 - 7*_t**2 + 1, Lambda(_t, _t*log(2*_t**7/7 - 2*_t**6/7 + _
t**4/7 - 47*_t**3/14 + 27*_t**2/7 - 3*_t/2 + x**4 + 4/7))) + RootSum(2*_t*
*4 + 7*_t**2 + 1, Lambda(_t, _t*log(2*_t**7/7 - 2*_t**6/7 + _t**4/7 - 47*_
t**3/14 + 27*_t**2/7 - 3*_t/2 + x**4 + 4/7)))
```

Maxima [F]

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$$
$$=\int \frac{8\,(28\,x^{27}-164\,x^{23}-495\,x^{19}-116\,x^{15}-112\,x^{11}-80\,x^7-6\,x^3)}{8\,x^{32}+88\,x^{28}+238\,x^{24}+460\,x^{20}+771\,x^{16}+576\,x^{12}+217\,x^8+76\,x^4+16}\,dx$$

input
```
integrate((224*x^27-1312*x^23-3960*x^19-928*x^15-896*x^11-640*x^7-48*x^3)/
(8*x^32+88*x^28+238*x^24+460*x^20+771*x^16+576*x^12+217*x^8+76*x^4+16),x,
algorithm="maxima")
```

output
```
8*integrate((28*x^27 - 164*x^23 - 495*x^19 - 116*x^15 - 112*x^11 - 80*x^7
- 6*x^3)/(8*x^32 + 88*x^28 + 238*x^24 + 460*x^20 + 771*x^16 + 576*x^12 + 2
17*x^8 + 76*x^4 + 16), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$$
$$=\frac{1}{41}\left(7\,x^8\sqrt{-82\,\sqrt{41}+574}+2\,\sqrt{-82\,\sqrt{41}+574}\right)\arctan\left(\frac{2\,x^4}{\sqrt{\sqrt{41}+7}}\right)$$
$$-\frac{1}{41}\left(7\,x^8\sqrt{82\,\sqrt{41}+574}+2\,\sqrt{82\,\sqrt{41}+574}\right)\arctan\left(\frac{x^4}{\sqrt{-\frac{1}{4}\,\sqrt{41}+\frac{7}{4}}}\right)$$

input
```
integrate((224*x^27-1312*x^23-3960*x^19-928*x^15-896*x^11-640*x^7-48*x^3)/
(8*x^32+88*x^28+238*x^24+460*x^20+771*x^16+576*x^12+217*x^8+76*x^4+16),x,
algorithm="giac")
```

output
```
1/41*(7*x^8*sqrt(-82*sqrt(41) + 574) + 2*sqrt(-82*sqrt(41) + 574))*arctan(
2*x^4/sqrt(sqrt(41) + 7)) - 1/41*(7*x^8*sqrt(82*sqrt(41) + 574) + 2*sqrt(8
2*sqrt(41) + 574))*arctan(x^4/sqrt(-1/4*sqrt(41) + 7/4))
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.48

$$\int \frac{-48x^3-640x^7-896x^{11}-928x^{15}-3960x^{19}-1312x^{23}+224x^{27}}{16+76x^4+217x^8+576x^{12}+771x^{16}+460x^{20}+238x^{24}+88x^{28}+8x^{32}}\,dx$$
$$=\text{Too large to display}$$

input
```
int(-(48*x^3 + 640*x^7 + 896*x^11 + 928*x^15 + 3960*x^19 + 1312*x^23 - 224
*x^27)/(76*x^4 + 217*x^8 + 576*x^12 + 771*x^16 + 460*x^20 + 238*x^24 + 88*
x^28 + 8*x^32 + 16),x)
```

output

```
atan(((- 41^(1/2) - 7)^(1/2)*400088822273805431939150701133361978714566596
82194924814904907552130859880405890234375i)/(128*((24471814588661885266627
76646848037686590845300635962861999156225153644617329736752734375*41^(1/2)
)/8192 - (6248337367708426852640351508190625035908461961705194913276183738
721025221516073828125*41^(1/2)*x^4)/64 + (40008882227380543193915070113336
197871456659682194924814904907552130859880405890234375*x^4)/64 - 156696051
87166514361043137847068345981278008800225210012892253244081958050600228389
453125/8192)) + (x^4*(- 41^(1/2) - 7)^(1/2)*105010838061839665060237903725
123983509585359463825473716105588969936567832360402794140625i)/(16384*((24
47181458866188526662776646848037686590845300635962861999156225153644617329
736752734375*41^(1/2))/8192 - (6248337367708426852640351508190625035908461
961705194913276183738721025221516073828125*41^(1/2)*x^4)/64 + (40008882227
38054319391507011333619787145665968219492481490490755213085988040589023437
5*x^4)/64 - 15669605187166514361043137847068345981278008800225210012892253
244081958050600228389453125/8192)) - (41^(1/2)*(- 41^(1/2) - 7)^(1/2)*6248
33736770842685264035150819062503590846196170519491327618373872102522151607
3828125i)/(128*((244718145886618852666277664684803768659084530063596286199
9156225153644617329736752734375*41^(1/2))/8192 - (624833736770842685264035
1508190625035908461961705194913276183738721025221516073828125*41^(1/2)*x^4
)/64 + (400088822273805431939150701133361978714566596821949248149049075...
```

Reduce [F]

$$\int \frac{-48x^3 - 640x^7 - 896x^{11} - 928x^{15} - 3960x^{19} - 1312x^{23} + 224x^{27}}{16 + 76x^4 + 217x^8 + 576x^{12} + 771x^{16} + 460x^{20} + 238x^{24} + 88x^{28} + 8x^{32}}\,dx$$
$$= 224\left(\int \frac{x^{27}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-1312\left(\int \frac{x^{23}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-3960\left(\int \frac{x^{19}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-928\left(\int \frac{x^{15}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-896\left(\int \frac{x^{11}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-640\left(\int \frac{x^{7}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$
$$-48\left(\int \frac{x^{3}}{8x^{32} + 88x^{28} + 238x^{24} + 460x^{20} + 771x^{16} + 576x^{12} + 217x^8 + 76x^4 + 16}dx\right)$$

input
```
int((224*x^27-1312*x^23-3960*x^19-928*x^15-896*x^11-640*x^7-48*x^3)/(8*x^3
2+88*x^28+238*x^24+460*x^20+771*x^16+576*x^12+217*x^8+76*x^4+16),x)
```

output
```
8*(28*int(x**27/(8*x**32 + 88*x**28 + 238*x**24 + 460*x**20 + 771*x**16 +
576*x**12 + 217*x**8 + 76*x**4 + 16),x) - 164*int(x**23/(8*x**32 + 88*x**2
8 + 238*x**24 + 460*x**20 + 771*x**16 + 576*x**12 + 217*x**8 + 76*x**4 + 1
6),x) - 495*int(x**19/(8*x**32 + 88*x**28 + 238*x**24 + 460*x**20 + 771*x*
*16 + 576*x**12 + 217*x**8 + 76*x**4 + 16),x) - 116*int(x**15/(8*x**32 + 8
8*x**28 + 238*x**24 + 460*x**20 + 771*x**16 + 576*x**12 + 217*x**8 + 76*x*
*4 + 16),x) - 112*int(x**11/(8*x**32 + 88*x**28 + 238*x**24 + 460*x**20 +
771*x**16 + 576*x**12 + 217*x**8 + 76*x**4 + 16),x) - 80*int(x**7/(8*x**32
 + 88*x**28 + 238*x**24 + 460*x**20 + 771*x**16 + 576*x**12 + 217*x**8 + 7
6*x**4 + 16),x) - 6*int(x**3/(8*x**32 + 88*x**28 + 238*x**24 + 460*x**20 +
 771*x**16 + 576*x**12 + 217*x**8 + 76*x**4 + 16),x))
```

3.67 $\int \frac{-14x+5x^5}{7-5x^4+63x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{-14x+5x^5}{7-5x^4+63x^8}\,dx = \frac{1}{12}\sqrt{\frac{37}{7}}\arctan\left(\frac{1}{37}\left(-\sqrt{1739}-6\sqrt{259}x^2\right)\right) + \frac{1}{12}\sqrt{\frac{37}{7}}\arctan\left(\frac{1}{37}\left(\sqrt{1739}-6\sqrt{259}x^2\right)\right) + \frac{1}{24}\sqrt{\frac{47}{7}}\log\left(7-\sqrt{329}x^2+21x^4\right) - \frac{1}{24}\sqrt{\frac{47}{7}}\log\left(7+\sqrt{329}x^2+21x^4\right)$$

output
```
-1/84*259^(1/2)*arctan(1/37*1739^(1/2)+6/37*259^(1/2)*x^2)-1/84*259^(1/2)*
arctan(-1/37*1739^(1/2)+6/37*259^(1/2)*x^2)+1/168*329^(1/2)*ln(7-329^(1/2)
*x^2+21*x^4)-1/168*329^(1/2)*ln(7+329^(1/2)*x^2+21*x^4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{-14x + 5x^5}{7 - 5x^4 + 63x^8}\,dx = \frac{\left(1739i + 5\sqrt{1739}\right)\arctan\left(\frac{3x^2}{\sqrt{\frac{1}{14}\left(-5-i\sqrt{1739}\right)}}\right)}{6\sqrt{24346\left(-5 - i\sqrt{1739}\right)}} + \frac{\left(-1739i + 5\sqrt{1739}\right)\arctan\left(\frac{3x^2}{\sqrt{\frac{1}{14}\left(-5+i\sqrt{1739}\right)}}\right)}{6\sqrt{24346\left(-5 + i\sqrt{1739}\right)}}$$

input
```
Integrate[(-14*x + 5*x^5)/(7 - 5*x^4 + 63*x^8),x]
```

output
```
((1739*I + 5*Sqrt[1739])*ArcTan[(3*x^2)/Sqrt[(-5 - I*Sqrt[1739])/14]])/(6*
Sqrt[24346*(-5 - I*Sqrt[1739])]) + ((-1739*I + 5*Sqrt[1739])*ArcTan[(3*x^2
)/Sqrt[(-5 + I*Sqrt[1739])/14]])/(6*Sqrt[24346*(-5 + I*Sqrt[1739])])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2027, 1814, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^5 - 14x}{63x^8 - 5x^4 + 7}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x\left(5x^4 - 14\right)}{63x^8 - 5x^4 + 7}dx$$

$$\Big\downarrow \text{1814}$$

$$\frac{1}{2}\int -\frac{14-5x^4}{63x^8-5x^4+7}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{14-5x^4}{63x^8-5x^4+7}dx^2$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{2}\left(-\frac{\int \frac{7\left(2\sqrt{329}-47x^2\right)}{21x^4-\sqrt{329}x^2+7}dx^2}{2\sqrt{329}}-\frac{\int \frac{7\left(47x^2+2\sqrt{329}\right)}{21x^4+\sqrt{329}x^2+7}dx^2}{2\sqrt{329}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\int \frac{2\sqrt{329}-47x^2}{21x^4-\sqrt{329}x^2+7}dx^2-\frac{1}{2}\sqrt{\frac{7}{47}}\int \frac{47x^2+2\sqrt{329}}{21x^4+\sqrt{329}x^2+7}dx^2\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{37}{6}\sqrt{\frac{47}{7}}\int \frac{1}{21x^4-\sqrt{329}x^2+7}dx^2-\frac{47}{42}\int -\frac{\sqrt{329}-42x^2}{21x^4-\sqrt{329}x^2+7}dx^2\right)-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{37}{6}\sqrt{\frac{47}{7}}\int \frac{}{21x\ldots}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{37}{6}\sqrt{\frac{47}{7}}\int \frac{1}{21x^4-\sqrt{329}x^2+7}dx^2+\frac{47}{42}\int \frac{\sqrt{329}-42x^2}{21x^4-\sqrt{329}x^2+7}dx^2\right)-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{37}{6}\sqrt{\frac{47}{7}}\int \frac{}{21x^4\ldots}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{47}{42}\int \frac{\sqrt{329}-42x^2}{21x^4-\sqrt{329}x^2+7}dx^2-\frac{37}{3}\sqrt{\frac{47}{7}}\int \frac{1}{-x^4-259}d\left(42x^2-\sqrt{329}\right)\right)-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{47}{42}\int \frac{42x\ldots}{21x^4+\ldots}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{47}{42}\int \frac{\sqrt{329}-42x^2}{21x^4-\sqrt{329}x^2+7}dx^2+\frac{1}{21}\sqrt{1739}\arctan\left(\frac{42x^2-\sqrt{329}}{\sqrt{259}}\right)\right)-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{47}{42}\int \frac{42x^2+\ldots}{21x^4+\sqrt{32\ldots}}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{1}{21}\sqrt{1739}\arctan\left(\frac{42x^2-\sqrt{329}}{\sqrt{259}}\right)-\frac{47}{42}\log\left(21x^4-\sqrt{329}x^2+7\right)\right)-\frac{1}{2}\sqrt{\frac{7}{47}}\left(\frac{1}{21}\sqrt{1739}\arctan\left(\right.\right.\right.$$

input
```
Int[(-14*x + 5*x^5)/(7 - 5*x^4 + 63*x^8),x]
```

output
```
(-1/2*(Sqrt[7/47]*((Sqrt[1739]*ArcTan[(-Sqrt[329] + 42*x^2)/Sqrt[259]])/21
 - (47*Log[7 - Sqrt[329]*x^2 + 21*x^4])/42)) - (Sqrt[7/47]*((Sqrt[1739]*Ar
cTan[(Sqrt[329] + 42*x^2)/Sqrt[259]])/21 + (47*Log[7 + Sqrt[329]*x^2 + 21*
x^4])/42))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1814
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Sub
st[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^
p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(63\_Z^{4}-5\_Z^{2}+7\right)}\_R\ln\left(x^{2}-\_R\right)\right)}{4}$
default	$-\frac{\sqrt{47}\,\sqrt{7}\,\ln\left(21x^{4}+\sqrt{47}\,\sqrt{7}\,x^{2}+7\right)}{168}-\frac{\sqrt{259}\,\arctan\left(\frac{\left(42x^{2}+\sqrt{47}\,\sqrt{7}\right)\sqrt{259}}{259}\right)}{84}+\frac{\sqrt{47}\,\sqrt{7}\,\ln\left(21x^{4}-\sqrt{47}\,\sqrt{7}\,x^{2}+7\right)}{168}-\sqrt{259}$ ar

input
```
int((5*x^5-14*x)/(63*x^8-5*x^4+7),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(x^2-_R),_R=RootOf(63*_Z^4-5*_Z^2+7))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int \frac{-14x+5x^5}{7-5x^4+63x^8}\,dx = -\frac{1}{12}\sqrt{\frac{37}{7}}\arctan\left(\frac{42}{37}\sqrt{\frac{37}{7}}x^2+\frac{7}{37}\sqrt{\frac{47}{7}}\sqrt{\frac{37}{7}}\right) + \frac{1}{12}\sqrt{\frac{37}{7}}\arctan\left(-\frac{42}{37}\sqrt{\frac{37}{7}}x^2+\frac{7}{37}\sqrt{\frac{47}{7}}\sqrt{\frac{37}{7}}\right) - \frac{1}{24}\sqrt{\frac{47}{7}}\log\left(3\,x^4+\sqrt{\frac{47}{7}}x^2+1\right) + \frac{1}{24}\sqrt{\frac{47}{7}}\log\left(3\,x^4-\sqrt{\frac{47}{7}}x^2+1\right)$$

input
```
integrate((5*x^5-14*x)/(63*x^8-5*x^4+7),x, algorithm="fricas")
```

output
```
-1/12*sqrt(37/7)*arctan(42/37*sqrt(37/7)*x^2 + 7/37*sqrt(47/7)*sqrt(37/7))
 + 1/12*sqrt(37/7)*arctan(-42/37*sqrt(37/7)*x^2 + 7/37*sqrt(47/7)*sqrt(37/
7)) - 1/24*sqrt(47/7)*log(3*x^4 + sqrt(47/7)*x^2 + 1) + 1/24*sqrt(47/7)*lo
g(3*x^4 - sqrt(47/7)*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{-14x+5x^5}{7-5x^4+63x^8}\,dx = \frac{\sqrt{329}\log\left(x^4-\frac{\sqrt{329}x^2}{21}+\frac{1}{3}\right)}{168} - \frac{\sqrt{329}\log\left(x^4+\frac{\sqrt{329}x^2}{21}+\frac{1}{3}\right)}{168} - \frac{\sqrt{259}\,\mathrm{atan}\left(\frac{6\sqrt{259}x^2}{37}-\frac{\sqrt{1739}}{37}\right)}{84} - \frac{\sqrt{259}\,\mathrm{atan}\left(\frac{6\sqrt{259}x^2}{37}+\frac{\sqrt{1739}}{37}\right)}{84}$$

input
```
integrate((5*x**5-14*x)/(63*x**8-5*x**4+7),x)
```

output
```
sqrt(329)*log(x**4 - sqrt(329)*x**2/21 + 1/3)/168 - sqrt(329)*log(x**4 + s
qrt(329)*x**2/21 + 1/3)/168 - sqrt(259)*atan(6*sqrt(259)*x**2/37 - sqrt(17
39)/37)/84 - sqrt(259)*atan(6*sqrt(259)*x**2/37 + sqrt(1739)/37)/84
```

Maxima [F]

$$\int \frac{-14x + 5x^5}{7 - 5x^4 + 63x^8}\,dx = \int \frac{5\,x^5 - 14\,x}{63\,x^8 - 5\,x^4 + 7}\,dx$$

input
```
integrate((5*x^5-14*x)/(63*x^8-5*x^4+7),x, algorithm="maxima")
```

output
```
integrate((5*x^5 - 14*x)/(63*x^8 - 5*x^4 + 7), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{-14x + 5x^5}{7 - 5x^4 + 63x^8}\,dx \\
&= \frac{1}{1036}\left(5\sqrt{259}x^4 - 14\sqrt{259}\right)\arctan\left(\frac{3}{259}\sqrt{777}\left(\frac{1}{9}\right)^{\frac{3}{4}}\left(42\,x^2 + \sqrt{987}\left(\frac{1}{9}\right)^{\frac{1}{4}}\right)\right) \\
&\quad + \frac{1}{1036}\left(5\sqrt{259}x^4 - 14\sqrt{259}\right)\arctan\left(\frac{3}{259}\sqrt{777}\left(\frac{1}{9}\right)^{\frac{3}{4}}\left(42\,x^2 - \sqrt{987}\left(\frac{1}{9}\right)^{\frac{1}{4}}\right)\right) \\
&\quad + \frac{1}{2632}\left(5\sqrt{329}x^4 - 14\sqrt{329}\right)\log\left(x^4 + \frac{1}{21}\sqrt{987}\left(\frac{1}{9}\right)^{\frac{1}{4}}x^2 + \frac{1}{3}\right) \\
&\quad - \frac{1}{2632}\left(5\sqrt{329}x^4 - 14\sqrt{329}\right)\log\left(x^4 - \frac{1}{21}\sqrt{987}\left(\frac{1}{9}\right)^{\frac{1}{4}}x^2 + \frac{1}{3}\right)
\end{aligned}$$

input
```
integrate((5*x^5-14*x)/(63*x^8-5*x^4+7),x, algorithm="giac")
```

output
```
1/1036*(5*sqrt(259)*x^4 - 14*sqrt(259))*arctan(3/259*sqrt(777)*(1/9)^(3/4)
*(42*x^2 + sqrt(987)*(1/9)^(1/4))) + 1/1036*(5*sqrt(259)*x^4 - 14*sqrt(259
))*arctan(3/259*sqrt(777)*(1/9)^(3/4)*(42*x^2 - sqrt(987)*(1/9)^(1/4))) +
1/2632*(5*sqrt(329)*x^4 - 14*sqrt(329))*log(x^4 + 1/21*sqrt(987)*(1/9)^(1/
4)*x^2 + 1/3) - 1/2632*(5*sqrt(329)*x^4 - 14*sqrt(329))*log(x^4 - 1/21*sqr
t(987)*(1/9)^(1/4)*x^2 + 1/3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{-14x+5x^5}{7-5x^4+63x^8}\,dx = \operatorname{atan}\left(\frac{136732606894\sqrt{259}\,x^2}{2233402022407689\left(-\frac{26294732095}{319057431772527}+\frac{\sqrt{259}\,\sqrt{329}\,5258946419i}{2233402022407689}\right)} - \frac{\sqrt{329}\,x^2\,84143142704i}{2233402022407689\left(-\frac{26294732095}{319057431772527}+\frac{\sqrt{259}\,\sqrt{329}\,5258946419i}{2233402022407689}\right)}\right)\left(\frac{\sqrt{259}}{84} - \frac{\sqrt{329}\,1i}{84}\right) - \operatorname{atan}\left(\frac{136732606894\sqrt{259}\,x^2}{2233402022407689\left(\frac{26294732095}{319057431772527}+\frac{\sqrt{259}\,\sqrt{329}\,5258946419i}{2233402022407689}\right)} + \frac{\sqrt{329}\,x^2\,84143142704i}{2233402022407689\left(\frac{26294732095}{319057431772527}+\frac{\sqrt{259}\,\sqrt{329}\,5258946419i}{2233402022407689}\right)}\right)\left(\frac{\sqrt{259}}{84} + \frac{\sqrt{329}\,1i}{84}\right)$$

input
```
int(-(14*x - 5*x^5)/(63*x^8 - 5*x^4 + 7),x)
```

output
```
atan((136732606894*259^(1/2)*x^2)/(2233402022407689*((259^(1/2)*329^(1/2)*
5258946419i)/2233402022407689 - 26294732095/319057431772527)) - (329^(1/2)
*x^2*84143142704i)/(2233402022407689*((259^(1/2)*329^(1/2)*5258946419i)/22
33402022407689 - 26294732095/319057431772527)))*(259^(1/2)/84 - (329^(1/2)
*1i)/84) - atan((136732606894*259^(1/2)*x^2)/(2233402022407689*((259^(1/2)
*329^(1/2)*5258946419i)/2233402022407689 + 26294732095/319057431772527)) +
 (329^(1/2)*x^2*84143142704i)/(2233402022407689*((259^(1/2)*329^(1/2)*5258
946419i)/2233402022407689 + 26294732095/319057431772527)))*(259^(1/2)/84 +
 (329^(1/2)*1i)/84)
```

Reduce [F]

$$\int \frac{-14x + 5x^5}{7 - 5x^4 + 63x^8}\,dx = \int \frac{5x^5 - 14x}{63x^8 - 5x^4 + 7}dx$$

input
```
int((5*x^5-14*x)/(63*x^8-5*x^4+7),x)
```

output
```
int((5*x^5-14*x)/(63*x^8-5*x^4+7),x)
```

3.68 $\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 99

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \frac{\arctan\left(\sqrt{-\frac{1}{2}+\frac{47}{4\sqrt{210}}}+\frac{\sqrt[4]{\frac{21}{5}}x^4}{2^{3/4}}\right)}{1344\ 5^{3/4}\sqrt[4]{42}}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{2}+\frac{47}{4\sqrt{210}}}+\frac{\sqrt[4]{\frac{21}{5}}x^4}{2^{3/4}}\right)}{1344\ 5^{3/4}\sqrt[4]{42}}$$

output
```
1/282240*arctan(1/420*(-88200+9870*210^(1/2))^(1/2)+1/10*21^(1/4)*5^(3/4)*
x^4*2^(1/4))*5^(1/4)*42^(3/4)+1/282240*arctanh(1/420*(88200+9870*210^(1/2)
)^(1/2)+1/10*21^(1/4)*5^(3/4)*x^4*2^(1/4))*5^(1/4)*42^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \frac{1}{672}\text{RootSum}\left[1369+9576\#1^4+10164\#1^8+7056\#1^{12} + 1764\#1^{16}\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{57+121\#1^4+126\#1^8+42\#1^{12}}\&\right]$$

input
```
Integrate[(-x^3 + x^7)/(1369 + 9576*x^4 + 10164*x^8 + 7056*x^12 + 1764*x^1
6),x]
```

output
```
RootSum[1369 + 9576*#1^4 + 10164*#1^8 + 7056*#1^12 + 1764*#1^16 & , (-Log[
x - #1] + Log[x - #1]*#1^4)/(57 + 121*#1^4 + 126*#1^8 + 42*#1^12) & ]/672
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7-x^3}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^3(x^4-1)}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{4}\int -\frac{1-x^4}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx^4$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{4}\int \frac{1-x^4}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx^4$$

$$\Big\downarrow \text{ 7293}$$

$$-\frac{1}{4}\int\left(\frac{1}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}-\frac{x^4}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\int\frac{x^4}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx^4-\int\frac{1}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}d\right.$$

input
```
Int[(-x^3 + x^7)/(1369 + 9576*x^4 + 10164*x^8 + 7056*x^12 + 1764*x^16),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5250\_Z^4-1\right)}\_R\ln\left(x^4+25\_R^2+10\_R+1\right)\right)}{2688}$	30
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1764\_Z^4+7056\_Z^3+10164\_Z^2+9576\_Z+1369\right)}\frac{\left(-1+\_R\right)\ln\left(x^4-\_R\right)}{42\_R^3+126\_R^2+121\_R+57}\right)}{672}$	56

input
```
int((x^7-x^3)/(1764*x^16+7056*x^12+10164*x^8+9576*x^4+1369),x,method=_RETU
RNVERBOSE)
```

output
```
1/2688*sum(_R*ln(x^4+25*_R^2+10*_R+1),_R=RootOf(5250*_Z^4-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = -\frac{1}{7056000}\cdot 5250^{\frac{3}{4}}\arctan\left(-\frac{1}{10}\cdot 5250^{\frac{1}{4}}\left(x^4+1\right)+\frac{1}{2100}\cdot 5250^{\frac{3}{4}}\right) + \frac{1}{14112000}\cdot 5250^{\frac{3}{4}}\log\left(1050\,x^4+2\cdot 5250^{\frac{3}{4}}+25\,\sqrt{210}+1050\right) - \frac{1}{14112000}\cdot 5250^{\frac{3}{4}}\log\left(1050\,x^4-2\cdot 5250^{\frac{3}{4}}+25\,\sqrt{210}+1050\right)$$

input
```
integrate((x^7-x^3)/(1764*x^16+7056*x^12+10164*x^8+9576*x^4+1369),x, algor
ithm="fricas")
```

output
```
-1/7056000*5250^(3/4)*arctan(-1/10*5250^(1/4)*(x^4 + 1) + 1/2100*5250^(3/4
)) + 1/14112000*5250^(3/4)*log(1050*x^4 + 2*5250^(3/4) + 25*sqrt(210) + 10
50) - 1/14112000*5250^(3/4)*log(1050*x^4 - 2*5250^(3/4) + 25*sqrt(210) + 1
050)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \frac{\sqrt[4]{370440}\log\left(x^4+\frac{\sqrt{210}}{42}+1+\frac{\sqrt[4]{370440}}{21}\right)}{564480} - \frac{\sqrt[4]{370440}\log\left(x^4-\frac{\sqrt[4]{370440}}{21}+\frac{\sqrt{210}}{42}+1\right)}{564480} + \frac{\sqrt[4]{370440}\,\mathrm{atan}\left(\frac{\sqrt[4]{5250}x^4}{10}-\frac{42^{\frac{3}{4}}\cdot\sqrt[4]{5}}{84}+\frac{\sqrt[4]{5250}}{10}\right)}{282240}$$

input
```
integrate((x**7-x**3)/(1764*x**16+7056*x**12+10164*x**8+9576*x**4+1369),x)
```

output
```
370440**(1/4)*log(x**4 + sqrt(210)/42 + 1 + 370440**(1/4)/21)/564480 - 370
440**(1/4)*log(x**4 - 370440**(1/4)/21 + sqrt(210)/42 + 1)/564480 + 370440
**(1/4)*atan(5250**(1/4)*x**4/10 - 42**(3/4)*5**(1/4)/84 + 5250**(1/4)/10)
/282240
```

Maxima [F]

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \int \frac{x^7-x^3}{1764\,x^{16}+7056\,x^{12}+10164\,x^8+9576\,x^4+1369}\,dx$$

input
```
integrate((x^7-x^3)/(1764*x^16+7056*x^12+10164*x^8+9576*x^4+1369),x, algor
ithm="maxima")
```

output
```
integrate((x^7 - x^3)/(1764*x^16 + 7056*x^12 + 10164*x^8 + 9576*x^4 + 1369
), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = 0$$

input
```
integrate((x^7-x^3)/(1764*x^16+7056*x^12+10164*x^8+9576*x^4+1369),x, algor
ithm="giac")
```

output
```
0
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.24

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \text{Too large to display}$$

input
```
int(-(x^3 - x^7)/(9576*x^4 + 10164*x^8 + 7056*x^12 + 1764*x^16 + 1369),x)
```

output
```
- (5^(1/4)*42^(3/4)*atan((391566802103725373*5^(1/4)*42^(3/4))/(1993144122
393116452399887706990731448784854292889600O*((1332495735760493477*5^(1/2)*
42^(1/2))/398628824478623290479977541398146289756970858577920O0 - (1102569
11397727229*x^4)/387394387248419135549054947908791340871691796480O0 + (156
5860509261188820 1*5^(1/2)*42^(1/2)*x^4)/7972576489572465809599550827962925
795139417171558400O - 386409047289802934 87/7972576489572465809599550827962
925795139417171558400O)) - (1134243848406542453*5^(3/4)*42^(1/4))/(1993144
1223931164523998877069907314487848542928896000*((1332495735760493477*5^(1/
2)*42^(1/2))/3986288244786232904799775413981462897569708585779200O - (1102
5691139772722 9*x^4)/3873943872484191355490549479087913408716917964800O + (
15658605092611888201*5^(1/2)*42^(1/2)*x^4)/7972576489572465809599550827962
9257951394171715584000 - 38640904728980293487/797257648957246580959955082 7
9629257951394171715584000)) + (4599050569894883867*5^(1/4)*42^(3/4)*x^4)/(
3986288244786232904799775413981462897569708585779200O*((133249573576049347
7*5^(1/2)*42^(1/2))/3986288244786232904799775413981462897569708585779200O
- (1102569113977272 29*x^4)/38739438724841913554905494790879134087169179648
000 + (15658605092611888201*5^(1/2)*42^(1/2)*x^4)/79725764895724658095995 5
0827962925795139417171558400O - 38640904728980293487/797257648957246580959
9550827962925795139417171558400O)) - (1586671692801201 1*5^(3/4)*42^(1/4)*x
^4)/(47455812437931344104759231118826939256782245068800*((1332495735760...
```

Reduce [F]

$$\int \frac{-x^3+x^7}{1369+9576x^4+10164x^8+7056x^{12}+1764x^{16}}\,dx = \int \frac{x^7}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx - \left(\int \frac{x^3}{1764x^{16}+7056x^{12}+10164x^8+9576x^4+1369}dx\right)$$

input
```
int((x^7-x^3)/(1764*x^16+7056*x^12+10164*x^8+9576*x^4+1369),x)
```

output
```
int(x**7/(1764*x**16 + 7056*x**12 + 10164*x**8 + 9576*x**4 + 1369),x) - in
t(x**3/(1764*x**16 + 7056*x**12 + 10164*x**8 + 9576*x**4 + 1369),x)
```

3.69 $\int \frac{24x-2304x^3+4992x^5+2304x^7+1728x^9+3072x^{11}+1536x^{13}-3072x^{15}+}{-3-624x^4-212x^8+640x^{12}+240x^{16}+256x^{20}+64x^{24}}$

Optimal result

Integrand size = 79, antiderivative size = 145

$$\int \frac{24x-2304x^3+4992x^5+2304x^7+1728x^9+3072x^{11}+1536x^{13}-3072x^{15}+384x^{17}}{-3-624x^4-212x^8+640x^{12}+240x^{16}+256x^{20}+64x^{24}}\,dx$$
$$= -\sqrt{2}\sqrt[4]{3}\arctan\left(\frac{\sqrt{2}x^2}{\sqrt[4]{3}}\right)-\sqrt{2}\sqrt[4]{3}\arctan\left(\sqrt{-\frac{1}{2}+\frac{7}{8\sqrt{3}}}+\frac{x^4}{\sqrt{2}\sqrt[4]{3}}\right)$$
$$-\sqrt{2}\sqrt[4]{3}\text{arctanh}\left(\frac{\sqrt{2}x^2}{\sqrt[4]{3}}\right)-\sqrt{2}\sqrt[4]{3}\text{arctanh}\left(\sqrt{\frac{1}{2}+\frac{7}{8\sqrt{3}}}+\frac{x^4}{\sqrt{2}\sqrt[4]{3}}\right)$$

output
```
-2^(1/2)*3^(1/4)*arctan(1/3*2^(1/2)*x^2*3^(3/4))-2^(1/2)*3^(1/4)*arctan(1/
12*(-72+42*3^(1/2))^(1/2)+1/6*x^4*2^(1/2)*3^(3/4))-2^(1/2)*3^(1/4)*arctanh
(1/3*2^(1/2)*x^2*3^(3/4))-2^(1/2)*3^(1/4)*arctanh(1/12*(72+42*3^(1/2))^(1/
2)+1/6*x^4*2^(1/2)*3^(3/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.23

$$\int \frac{24x-2304x^3+4992x^5+2304x^7+1728x^9+3072x^{11}+1536x^{13}-3072x^{15}+384x^{17}}{-3-624x^4-212x^8+640x^{12}+240x^{16}+256x^{20}+64x^{24}}\,dx$$
$$=\frac{1}{2}\left(\sqrt{2}\sqrt[4]{3}\left(2\arctan\left(1-\frac{2^{3/4}x}{\sqrt[8]{3}}\right)+2\arctan\left(1+\frac{2^{3/4}x}{\sqrt[8]{3}}\right)+\log\left(3-\sqrt[4]{2}3^{7/8}x\right)+\log\left(3+\sqrt[4]{2}3^{7/8}x\right)-\log\left(3\right.\right.\right.$$

input
```
Integrate[(24*x - 2304*x^3 + 4992*x^5 + 2304*x^7 + 1728*x^9 + 3072*x^11 +
1536*x^13 - 3072*x^15 + 384*x^17)/(-3 - 624*x^4 - 212*x^8 + 640*x^12 + 240
*x^16 + 256*x^20 + 64*x^24),x]
```

output
```
(Sqrt[2]*3^(1/4)*(2*ArcTan[1 - (2^(3/4)*x)/3^(1/8)] + 2*ArcTan[1 + (2^(3/4
)*x)/3^(1/8)] + Log[3 - 2^(1/4)*3^(7/8)*x] + Log[3 + 2^(1/4)*3^(7/8)*x] -
Log[3 + Sqrt[2]*3^(3/4)*x^2]) - 24*RootSum[1 + 208*#1^4 + 72*#1^8 + 64*#1^
12 + 16*#1^16 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(13 + 9*#1^4 + 12*#1^8
 + 4*#1^12) & ])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{384x^{17}-3072x^{15}+1536x^{13}+3072x^{11}+1728x^9+2304x^7+4992x^5-2304x^3+24x}{64x^{24}+256x^{20}+240x^{16}+640x^{12}-212x^8-624x^4-3}\,dx$$
$$\Big\downarrow \text{ 2460}$$
$$\int \left(\frac{24x}{4x^8-3}-\frac{768x^3(x^4-1)}{16x^{16}+64x^{12}+72x^8+208x^4+1}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-192\text{Subst}\left(\int \frac{1}{-16x^4-64x^3-72x^2-208x-1}dx,x,x^4\right)-192\text{Subst}\left(\int \frac{x}{16x^4+64x^3+72x^2+208x+1}dx,x,x^4\right)-\sqrt{2}\sqrt[4]{3}\arctan\left(\frac{\sqrt{2}x^2}{\sqrt[4]{3}}\right)-\sqrt{2}\sqrt[4]{3}\text{arctanh}\left(\frac{\sqrt{2}x^2}{\sqrt[4]{3}}\right)$$

input
```
Int[(24*x - 2304*x^3 + 4992*x^5 + 2304*x^7 + 1728*x^9 + 3072*x^11 + 1536*x
^13 - 3072*x^15 + 384*x^17)/(-3 - 624*x^4 - 212*x^8 + 640*x^12 + 240*x^16
+ 256*x^20 + 64*x^24),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

method	result
risch	$3\left(\sum_{\_R=\mathrm{RootOf}(108\_Z^4-1)} \_R\ln\left(x^6-3\_R\, x^4+\left(9\_R^2-6\_R+1\right)x^2-27\_R^3+18\_R^2-3\_R\right.\right.$
default	$-12\left(\sum_{\_R=\mathrm{RootOf}(16\_Z^4+64\_Z^3+72\_Z^2+208\_Z+1)} \frac{(-1+\_R)\ln(x^4-\_R)}{4\_R^3+12\_R^2+9\_R+13}\right)-\frac{\sqrt{2}\,3^{\frac{1}{4}}\left(\ln\left(\frac{x^2+\frac{\sqrt{2}\,3^{\frac{1}{4}}}{2}}{x^2-\frac{\sqrt{2}\,3^{\frac{1}{4}}}{2}}\right)+2\,\text{arcta}\right.}{2}$

input
```
int((384*x^17-3072*x^15+1536*x^13+3072*x^11+1728*x^9+2304*x^7+4992*x^5-230
4*x^3+24*x)/(64*x^24+256*x^20+240*x^16+640*x^12-212*x^8-624*x^4-3),x,metho
d=_RETURNVERBOSE)
```

output
```
3*sum(_R*ln(x^6-3*_R*x^4+(9*_R^2-6*_R+1)*x^2-27*_R^3+18*_R^2-3*_R),_R=Root
Of(108*_Z^4-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(105) = 210$.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.46

$$\int \frac{24x - 2304x^3 + 4992x^5 + 2304x^7 + 1728x^9 + 3072x^{11} + 1536x^{13} - 3072x^{15} + 384x^{17}}{-3 - 624x^4 - 212x^8 + 640x^{12} + 240x^{16} + 256x^{20} + 64x^{24}} dx$$
$$= -2\left(\frac{3}{4}\right)^{\frac{1}{4}} \arctan\left(\frac{1}{24}\left(\frac{3}{4}\right)^{\frac{3}{4}}(8x^{10} - 4x^8 + 16x^6 - 4x^4 + 14x^2 + 21) + \frac{1}{16}\left(\frac{3}{4}\right)^{\frac{1}{4}}(4x^8 - 8x^6 + 12x^4 + 8x^2 - 5)\right)$$
$$+2\left(\frac{3}{4}\right)^{\frac{1}{4}} \arctan\left(\frac{1}{12}\left(\frac{3}{4}\right)^{\frac{3}{4}}(4x^6 - 10x^4 + 8x^2 - 5) + \frac{1}{8}\left(\frac{3}{4}\right)^{\frac{1}{4}}(2x^4 - 6x^2 + 11)\right)$$
$$-2\left(\frac{3}{4}\right)^{\frac{1}{4}} \arctan\left(\frac{4}{3}\left(\frac{3}{4}\right)^{\frac{3}{4}}(x^2 - 2)\right)$$
$$-\left(\frac{3}{4}\right)^{\frac{1}{4}} \log\left(x^6 + x^2 + \frac{1}{2}\sqrt{3}(x^2 + 2) + \left(\frac{3}{4}\right)^{\frac{1}{4}}(x^4 + 2x^2 + 1) + \left(\frac{3}{4}\right)^{\frac{3}{4}}\right)$$
$$+\left(\frac{3}{4}\right)^{\frac{1}{4}} \log\left(x^6 + x^2 + \frac{1}{2}\sqrt{3}(x^2 + 2) - \left(\frac{3}{4}\right)^{\frac{1}{4}}(x^4 + 2x^2 + 1) - \left(\frac{3}{4}\right)^{\frac{3}{4}}\right)$$

input

```
integrate((384*x^17-3072*x^15+1536*x^13+3072*x^11+1728*x^9+2304*x^7+4992*x
^5-2304*x^3+24*x)/(64*x^24+256*x^20+240*x^16+640*x^12-212*x^8-624*x^4-3),x
, algorithm="fricas")
```

output

```
-2*(3/4)^(1/4)*arctan(1/24*(3/4)^(3/4)*(8*x^10 - 4*x^8 + 16*x^6 - 4*x^4 +
14*x^2 + 21) + 1/16*(3/4)^(1/4)*(4*x^8 - 8*x^6 + 12*x^4 + 8*x^2 - 5)) + 2*
(3/4)^(1/4)*arctan(1/12*(3/4)^(3/4)*(4*x^6 - 10*x^4 + 8*x^2 - 5) + 1/8*(3/
4)^(1/4)*(2*x^4 - 6*x^2 + 11)) - 2*(3/4)^(1/4)*arctan(4/3*(3/4)^(3/4)*(x^2
 - 2)) - (3/4)^(1/4)*log(x^6 + x^2 + 1/2*sqrt(3)*(x^2 + 2) + (3/4)^(1/4)*(
x^4 + 2*x^2 + 1) + (3/4)^(3/4)) + (3/4)^(1/4)*log(x^6 + x^2 + 1/2*sqrt(3)*
(x^2 + 2) - (3/4)^(1/4)*(x^4 + 2*x^2 + 1) - (3/4)^(3/4))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(136) = 272$.

Time = 0.38 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.26

$$\int \frac{24x - 2304x^3 + 4992x^5 + 2304x^7 + 1728x^9 + 3072x^{11} + 1536x^{13} - 3072x^{15} + 384x^{17}}{-3 - 624x^4 - 212x^8 + 640x^{12} + 240x^{16} + 256x^{20} + 64x^{24}}\,dx$$
$$= \frac{\sqrt{2}\cdot\sqrt[4]{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x^2}{3} - \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}}{3}\right) + 2\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x^6}{12} + x^4\left(-\frac{5\sqrt{2}\cdot 3^{\frac{3}{4}}}{24} + \frac{\sqrt{2}\cdot\sqrt[4]{3}}{8}\right) + x^2\left(-\frac{3\sqrt{2}\cdot\sqrt[4]{3}}{8} + \sqrt{}\right.\right.\right.}{}$$
$$+ \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(x^6 - \frac{\sqrt{2}\cdot\sqrt[4]{3}x^4}{2} + x^2\left(-\sqrt{2}\cdot\sqrt[4]{3} + \frac{\sqrt{3}}{2} + 1\right) - \frac{\sqrt{2}\cdot\sqrt[4]{3}}{2} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}}{4} + \sqrt{3}\right)}{2}$$
$$- \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(x^6 + \frac{\sqrt{2}\cdot\sqrt[4]{3}x^4}{2} + x^2\left(\frac{\sqrt{3}}{2} + 1 + \sqrt{2}\cdot\sqrt[4]{3}\right) + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}}{4} + \frac{\sqrt{2}\cdot\sqrt[4]{3}}{2} + \sqrt{3}\right)}{2}$$

input
```
integrate((384*x**17-3072*x**15+1536*x**13+3072*x**11+1728*x**9+2304*x**7+
4992*x**5-2304*x**3+24*x)/(64*x**24+256*x**20+240*x**16+640*x**12-212*x**8
-624*x**4-3),x)
```

output
```
sqrt(2)*3**(1/4)*(-2*atan(sqrt(2)*3**(3/4)*x**2/3 - 2*sqrt(2)*3**(3/4)/3)
+ 2*atan(sqrt(2)*3**(3/4)*x**6/12 + x**4*(-5*sqrt(2)*3**(3/4)/24 + sqrt(2)
*3**(1/4)/8) + x**2*(-3*sqrt(2)*3**(1/4)/8 + sqrt(2)*3**(3/4)/6) - 5*sqrt(
2)*3**(3/4)/48 + 11*sqrt(2)*3**(1/4)/16) + 2*atan(-sqrt(2)*3**(3/4)*x**10/
12 + x**8*(-sqrt(2)*3**(1/4)/8 + sqrt(2)*3**(3/4)/24) + x**6*(-sqrt(2)*3**
(3/4)/6 + sqrt(2)*3**(1/4)/4) + x**4*(-3*sqrt(2)*3**(1/4)/8 + sqrt(2)*3**(
3/4)/24) + x**2*(-7*sqrt(2)*3**(3/4)/48 - sqrt(2)*3**(1/4)/4) - 7*sqrt(2)*
3**(3/4)/32 + 5*sqrt(2)*3**(1/4)/32))/2 + sqrt(2)*3**(1/4)*log(x**6 - sqrt
(2)*3**(1/4)*x**4/2 + x**2*(-sqrt(2)*3**(1/4) + sqrt(3)/2 + 1) - sqrt(2)*3
**(1/4)/2 - sqrt(2)*3**(3/4)/4 + sqrt(3))/2 - sqrt(2)*3**(1/4)*log(x**6 +
sqrt(2)*3**(1/4)*x**4/2 + x**2*(sqrt(3)/2 + 1 + sqrt(2)*3**(1/4)) + sqrt(2
)*3**(3/4)/4 + sqrt(2)*3**(1/4)/2 + sqrt(3))/2
```

Maxima [F]

$$\int \frac{24x-2304x^3+4992x^5+2304x^7+1728x^9+3072x^{11}+1536x^{13}-3072x^{15}+384x^{17}}{-3-624x^4-212x^8+640x^{12}+240x^{16}+256x^{20}+64x^{24}}\,dx$$
$$=\int \frac{24\,(16\,x^{17}-128\,x^{15}+64\,x^{13}+128\,x^{11}+72\,x^9+96\,x^7+208\,x^5-96\,x^3+x)}{64\,x^{24}+256\,x^{20}+240\,x^{16}+640\,x^{12}-212\,x^8-624\,x^4-3}\,dx$$

input
```
integrate((384*x^17-3072*x^15+1536*x^13+3072*x^11+1728*x^9+2304*x^7+4992*x
^5-2304*x^3+24*x)/(64*x^24+256*x^20+240*x^16+640*x^12-212*x^8-624*x^4-3),x
, algorithm="maxima")
```

output
```
24*integrate((16*x^17 - 128*x^15 + 64*x^13 + 128*x^11 + 72*x^9 + 96*x^7 +
208*x^5 - 96*x^3 + x)/(64*x^24 + 256*x^20 + 240*x^16 + 640*x^12 - 212*x^8
- 624*x^4 - 3), x)
```

Giac [F]

$$\int \frac{24x-2304x^3+4992x^5+2304x^7+1728x^9+3072x^{11}+1536x^{13}-3072x^{15}+384x^{17}}{-3-624x^4-212x^8+640x^{12}+240x^{16}+256x^{20}+64x^{24}}\,dx$$
$$=\int \frac{24\,(16\,x^{17}-128\,x^{15}+64\,x^{13}+128\,x^{11}+72\,x^9+96\,x^7+208\,x^5-96\,x^3+x)}{64\,x^{24}+256\,x^{20}+240\,x^{16}+640\,x^{12}-212\,x^8-624\,x^4-3}\,dx$$

input
```
integrate((384*x^17-3072*x^15+1536*x^13+3072*x^11+1728*x^9+2304*x^7+4992*x
^5-2304*x^3+24*x)/(64*x^24+256*x^20+240*x^16+640*x^12-212*x^8-624*x^4-3),x
, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 951, normalized size of antiderivative = 6.56

$$\int \frac{24x - 2304x^3 + 4992x^5 + 2304x^7 + 1728x^9 + 3072x^{11} + 1536x^{13} - 3072x^{15} + 384x^{17}}{-3 - 624x^4 - 212x^8 + 640x^{12} + 240x^{16} + 256x^{20} + 64x^{24}}\, dx$$
= Too large to display

input
```
int((24*x - 2304*x^3 + 4992*x^5 + 2304*x^7 + 1728*x^9 + 3072*x^11 + 1536*x
^13 - 3072*x^15 + 384*x^17)/(640*x^12 - 212*x^8 - 624*x^4 + 240*x^16 + 256
*x^20 + 64*x^24 - 3),x)
```

output
```
2^(1/2)*3^(1/4)*atan((2738433225476105601148099171872214949560320*2^(1/2)*
3^(1/4))/(2942638004399783889911021122852329546055 68*3^(1/2) - 3162070319
804927630841030047989339143536640*3^(1/2)*x^2 + 611052768165094902395261 20
2093074533702434816*3^(1/2)*x^4 - 6566189314277520175612528538433890221597
65504*3^(1/2)*x^6 + 5476866450952211202296198343744429899120640*x^2 - 1058
374440567580622429902121601418936058904576*x^4 + 113729735044448542972022 1
1607814987687772 81536*x^6 - 509679853196692333903536321086367013326 0288) -
 (158103515990246381542051502399466957176832 0*2^(1/2)*3^(3/4))/(2942638004
3997838899911021122852329546055 68*3^(1/2) - 316207031980492763084103004798
9339143536640*3^(1/2)*x^2 + 611052768165094902395261202093074533702434816*
3^(1/2)*x^4 - 656618931427752017561252853843389022159765504*3^(1/2)*x^6 +
5476866450952211202296198343744429899120640*x^2 - 105837444056758062242990
2121601418936058904576*x^4 + 113729735044448542972022116078149876877728153
6*x^6 - 509679853196692333903536321086367013326 0288) - (294263800439978388
9991102112285232954605568*2^(1/2)*3^(1/4)*x^2)/(2942638004399783889911021
1228523295460 5568*3^(1/2) - 3162070319804927630841030047989339143536640*3^
(1/2)*x^2 + 611052768165094902395261202093074533702434816*3^(1/2)*x^4 - 65
6618931427752017561252853843389022159765504*3^(1/2)*x^6 + 547686645095221 1
2022961983437444298991206 40*x^2 - 1058374440567580622429902121601418936058
904576*x^4 + 1137297350444485429720221160781498768777281536*x^6 - 50967...
```

Reduce [F]

$$\int \frac{24x - 2304x^3 + 4992x^5 + 2304x^7 + 1728x^9 + 3072x^{11} + 1536x^{13} - 3072x^{15} + 384x^{17}}{-3 - 624x^4 - 212x^8 + 640x^{12} + 240x^{16} + 256x^{20} + 64x^{24}}\,dx$$
$$= \sqrt{2}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(2^{\frac{1}{4}}3^{\frac{1}{8}} - 2x\right)2^{\frac{3}{4}}3^{\frac{7}{8}}}{6}\right) + \sqrt{2}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(2^{\frac{1}{4}}3^{\frac{1}{8}} + 2x\right)2^{\frac{3}{4}}3^{\frac{7}{8}}}{6}\right)$$
$$- \frac{\sqrt{2}\,3^{\frac{1}{4}}\log\left(3^{\frac{1}{4}} + \sqrt{2}\,x^2\right)}{2} + \frac{\sqrt{2}\,3^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right)}{2}$$
$$+ \frac{\sqrt{2}\,3^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right)}{2} - 4\sqrt{3}\log\left(3^{\frac{1}{4}} + \sqrt{2}\,x^2\right)$$
$$- 4\sqrt{3}\log\left(-2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right) + 4\sqrt{3}\log\left(-\sqrt{2}\,2^{\frac{1}{4}}3^{\frac{1}{8}}x + 3^{\frac{1}{4}} + \sqrt{2}\,x^2\right)$$
$$- 4\sqrt{3}\log\left(2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right) + 4\sqrt{3}\log\left(\sqrt{2}\,2^{\frac{1}{4}}3^{\frac{1}{8}}x + 3^{\frac{1}{4}} + \sqrt{2}\,x^2\right)$$
$$- 19968\left(\int \frac{x^{11}}{64x^{24} + 256x^{20} + 240x^{16} + 640x^{12} - 212x^8 - 624x^4 - 3}dx\right)$$
$$+ 36864\left(\int \frac{x^{7}}{64x^{24} + 256x^{20} + 240x^{16} + 640x^{12} - 212x^8 - 624x^4 - 3}dx\right)$$
$$- 9600\left(\int \frac{x^{3}}{64x^{24} + 256x^{20} + 240x^{16} + 640x^{12} - 212x^8 - 624x^4 - 3}dx\right)$$
$$+ 6\log\left(3^{\frac{1}{4}} + \sqrt{2}\,x^2\right) + 6\log\left(-2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right)$$
$$+ 6\log\left(-\sqrt{2}\,2^{\frac{1}{4}}3^{\frac{1}{8}}x + 3^{\frac{1}{4}} + \sqrt{2}\,x^2\right) + 6\log\left(2^{\frac{1}{4}}3^{\frac{1}{8}} + \sqrt{2}\,x\right)$$
$$+ 6\log\left(\sqrt{2}\,2^{\frac{1}{4}}3^{\frac{1}{8}}x + 3^{\frac{1}{4}} + \sqrt{2}\,x^2\right) - 3\log\left(16x^{16} + 64x^{12} + 72x^8 + 208x^4 + 1\right)$$

input
```
int((384*x^17-3072*x^15+1536*x^13+3072*x^11+1728*x^9+2304*x^7+4992*x^5-230
4*x^3+24*x)/(64*x^24+256*x^20+240*x^16+640*x^12-212*x^8-624*x^4-3),x)
```

output

```
(2*sqrt(2)*3**(1/4)*atan((2**(1/4)*3**(1/8) - 2*x)/(2**(1/4)*3**(1/8))) +
2*sqrt(2)*3**(1/4)*atan((2**(1/4)*3**(1/8) + 2*x)/(2**(1/4)*3**(1/8))) - s
qrt(2)*3**(1/4)*log(3**(1/4) + sqrt(2)*x**2) + sqrt(2)*3**(1/4)*log( - 2**
(1/4)*3**(1/8) + sqrt(2)*x) + sqrt(2)*3**(1/4)*log(2**(1/4)*3**(1/8) + sqr
t(2)*x) - 8*sqrt(3)*log(3**(1/4) + sqrt(2)*x**2) - 8*sqrt(3)*log( - 2**(1/
4)*3**(1/8) + sqrt(2)*x) + 8*sqrt(3)*log( - sqrt(2)*2**(1/4)*3**(1/8)*x +
3**(1/4) + sqrt(2)*x**2) - 8*sqrt(3)*log(2**(1/4)*3**(1/8) + sqrt(2)*x) +
8*sqrt(3)*log(sqrt(2)*2**(1/4)*3**(1/8)*x + 3**(1/4) + sqrt(2)*x**2) - 399
36*int(x**11/(64*x**24 + 256*x**20 + 240*x**16 + 640*x**12 - 212*x**8 - 62
4*x**4 - 3),x) + 73728*int(x**7/(64*x**24 + 256*x**20 + 240*x**16 + 640*x*
*12 - 212*x**8 - 624*x**4 - 3),x) - 19200*int(x**3/(64*x**24 + 256*x**20 +
 240*x**16 + 640*x**12 - 212*x**8 - 624*x**4 - 3),x) + 12*log(3**(1/4) + s
qrt(2)*x**2) + 12*log( - 2**(1/4)*3**(1/8) + sqrt(2)*x) + 12*log( - sqrt(2
)*2**(1/4)*3**(1/8)*x + 3**(1/4) + sqrt(2)*x**2) + 12*log(2**(1/4)*3**(1/8
) + sqrt(2)*x) + 12*log(sqrt(2)*2**(1/4)*3**(1/8)*x + 3**(1/4) + sqrt(2)*x
**2) - 6*log(16*x**16 + 64*x**12 + 72*x**8 + 208*x**4 + 1))/2
```

3.70 $\int \frac{-1+x^2}{9+5x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{-1+x^2}{9+5x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1}{11}\left(-\sqrt{11}-2\sqrt{11}x\right)\right)}{3\sqrt{11}} + \frac{\arctan\left(\frac{1}{11}\left(-\sqrt{11}+2\sqrt{11}x\right)\right)}{3\sqrt{11}} + \frac{1}{3}\log\left(3-x+x^2\right) - \frac{1}{3}\log\left(3+x+x^2\right)$$

output

```
1/33*arctan(1/11*11^(1/2)+2/11*11^(1/2)*x)*11^(1/2)+1/33*arctan(-1/11*11^(
1/2)+2/11*11^(1/2)*x)*11^(1/2)+1/3*ln(x^2-x+3)-1/3*ln(x^2+x+3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{(7i+\sqrt{11})\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{22\left(5-i\sqrt{11}\right)}} + \frac{(-7i+\sqrt{11})\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{22\left(5+i\sqrt{11}\right)}}$$

input
```
Integrate[(-1 + x^2)/(9 + 5*x^2 + x^4),x]
```

output
```
((7*I + Sqrt[11])*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[22*(5 - I*Sqrt[
11])] + ((-7*I + Sqrt[11])*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[22*(5
+ I*Sqrt[11])]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1483, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^4+5x^2+9}\,dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{6}\int -\frac{1-4x}{x^2-x+3}dx+\frac{1}{6}\int -\frac{4x+1}{x^2+x+3}dx$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{6}\int \frac{1-4x}{x^2-x+3}dx-\frac{1}{6}\int \frac{4x+1}{x^2+x+3}dx$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{6}\left(\int \frac{1}{x^2-x+3}dx+2\int -\frac{1-2x}{x^2-x+3}dx\right)+\frac{1}{6}\left(\int \frac{1}{x^2+x+3}dx-2\int \frac{2x+1}{x^2+x+3}dx\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{6}\left(\int \frac{1}{x^2-x+3}dx-2\int \frac{1-2x}{x^2-x+3}dx\right)+\frac{1}{6}\left(\int \frac{1}{x^2+x+3}dx-2\int \frac{2x+1}{x^2+x+3}dx\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{6}\left(-2\int \frac{1-2x}{x^2-x+3}dx-2\int \frac{1}{-(2x-1)^2-11}d(2x-1)\right)+\frac{1}{6}\left(-2\int \frac{2x+1}{x^2+x+3}dx-2\int \frac{1}{-(2x+1)^2-11}d(2x+1)\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{6}\left(\frac{2\arctan\left(\frac{2x-1}{\sqrt{11}}\right)}{\sqrt{11}}-2\int \frac{1-2x}{x^2-x+3}dx\right)+\frac{1}{6}\left(\frac{2\arctan\left(\frac{2x+1}{\sqrt{11}}\right)}{\sqrt{11}}-2\int \frac{2x+1}{x^2+x+3}dx\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{6}\left(\frac{2\arctan\left(\frac{2x-1}{\sqrt{11}}\right)}{\sqrt{11}}+2\log\left(x^2-x+3\right)\right)+\frac{1}{6}\left(\frac{2\arctan\left(\frac{2x+1}{\sqrt{11}}\right)}{\sqrt{11}}-2\log\left(x^2+x+3\right)\right)$$

input
```
Int[(-1 + x^2)/(9 + 5*x^2 + x^4),x]
```

output
```
((2*ArcTan[(-1 + 2*x)/Sqrt[11]])/Sqrt[11] + 2*Log[3 - x + x^2])/6 + ((2*Ar
cTan[(1 + 2*x)/Sqrt[11]])/Sqrt[11] - 2*Log[3 + x + x^2])/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2-x+3)}{3}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{33}-\frac{\ln(x^2+x+3)}{3}+\frac{\sqrt{11}\,\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)}{33}$	54
risch	$-\frac{\ln(4x^2+4x+12)}{3}+\frac{\sqrt{11}\,\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)}{33}+\frac{\ln(4x^2-4x+12)}{3}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{33}$	60

input
```
int((x^2-1)/(x^4+5*x^2+9),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^2-x+3)+1/33*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))-1/3*ln(x^2+x+3
)+1/33*11^(1/2)*arctan(1/11*(1+2*x)*11^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{1}{33}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right) + \frac{1}{33}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right) - \frac{1}{3}\log\left(x^2+x+3\right) + \frac{1}{3}\log\left(x^2-x+3\right)$$

input
```
integrate((x^2-1)/(x^4+5*x^2+9),x, algorithm="fricas")
```

output
```
1/33*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/33*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) - 1/3*log(x^2 + x + 3) + 1/3*log(x^2 - x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{\log\left(x^2-x+3\right)}{3} - \frac{\log\left(x^2+x+3\right)}{3} + \frac{\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11}-\frac{\sqrt{11}}{11}\right)}{33} + \frac{\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{\sqrt{11}}{11}\right)}{33}$$

input
```
integrate((x**2-1)/(x**4+5*x**2+9),x)
```

output
```
log(x**2 - x + 3)/3 - log(x**2 + x + 3)/3 + sqrt(11)*atan(2*sqrt(11)*x/11
- sqrt(11)/11)/33 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/33
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{1}{33}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+1)\right) + \frac{1}{33}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x-1)\right) - \frac{1}{3}\,\log\left(x^2+x+3\right) + \frac{1}{3}\,\log\left(x^2-x+3\right)$$

input
```
integrate((x^2-1)/(x^4+5*x^2+9),x, algorithm="maxima")
```

output
```
1/33*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/33*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) - 1/3*log(x^2 + x + 3) + 1/3*log(x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{1}{33}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x+1)\right) + \frac{1}{33}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x-1)\right) - \frac{1}{3}\,\log\left(x^2+x+3\right) + \frac{1}{3}\,\log\left(x^2-x+3\right)$$

input
```
integrate((x^2-1)/(x^4+5*x^2+9),x, algorithm="giac")
```

output
```
1/33*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/33*sqrt(11)*arctan(1/11*
sqrt(11)*(2*x - 1)) - 1/3*log(x^2 + x + 3) + 1/3*log(x^2 - x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \operatorname{atan}\left(\frac{x\,136\mathrm{i}}{27\left(-\frac{184}{9}+\frac{\sqrt{11}\,8\mathrm{i}}{9}\right)}-\frac{88\sqrt{11}\,x}{27\left(-\frac{184}{9}+\frac{\sqrt{11}\,8\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{33}-\frac{2}{3}\mathrm{i}\right) + \operatorname{atan}\left(\frac{x\,136\mathrm{i}}{27\left(\frac{184}{9}+\frac{\sqrt{11}\,8\mathrm{i}}{9}\right)}+\frac{88\sqrt{11}\,x}{27\left(\frac{184}{9}+\frac{\sqrt{11}\,8\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{33}+\frac{2}{3}\mathrm{i}\right)$$

input
```
int((x^2 - 1)/(5*x^2 + x^4 + 9),x)
```

output
```
atan((x*136i)/(27*((11^(1/2)*8i)/9 - 184/9)) - (88*11^(1/2)*x)/(27*((11^(1
/2)*8i)/9 - 184/9)))*(11^(1/2)/33 - 2i/3) + atan((x*136i)/(27*((11^(1/2)*8
i)/9 + 184/9)) + (88*11^(1/2)*x)/(27*((11^(1/2)*8i)/9 + 184/9)))*(11^(1/2)
/33 + 2i/3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{-1+x^2}{9+5x^2+x^4}\,dx = \frac{\sqrt{11}\,atan\left(\frac{2x-1}{\sqrt{11}}\right)}{33}+\frac{\sqrt{11}\,atan\left(\frac{2x+1}{\sqrt{11}}\right)}{33} + \frac{\log(x^2-x+3)}{3}-\frac{\log(x^2+x+3)}{3}$$

input
```
int((x^2-1)/(x^4+5*x^2+9),x)
```

output
```
(sqrt(11)*atan((2*x - 1)/sqrt(11)) + sqrt(11)*atan((2*x + 1)/sqrt(11)) + 1
1*log(x**2 - x + 3) - 11*log(x**2 + x + 3))/33
```

3.71 $\int \frac{136x^3+1092x^7+3136x^{11}+508x^{15}-192x^{19}+20x^{23}}{-25-211x^4-424x^8-3x^{12}+48x^{16}-11x^{20}+x^{24}}\,dx$

Optimal result

Integrand size = 64, antiderivative size = 153

$$\begin{aligned}
&\int \frac{136x^3+1092x^7+3136x^{11}+508x^{15}-192x^{19}+20x^{23}}{-25-211x^4-424x^8-3x^{12}+48x^{16}-11x^{20}+x^{24}}\,dx \\
&= \sqrt{2\left(5+\sqrt{29}\right)}\arctan\left(\frac{1}{2}\sqrt{-2+\sqrt{29}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(-5+\sqrt{29}\right)}x^4\right) \\
&\quad -\sqrt{29}\text{arctanh}\left(\frac{1}{29}\left(-5\sqrt{29}+2\sqrt{29}x^4\right)\right) \\
&\quad -\sqrt{2\left(-5+\sqrt{29}\right)}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{29}}}{2}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{29}\right)}x^4\right) \\
&\quad +\frac{5}{2}\log\left(-1-5x^4+x^8\right)
\end{aligned}$$

output
```
-(10+2*29^(1/2))^(1/2)*arctan(-1/2*(-2+29^(1/2))^(1/2)+1/4*(-10+2*29^(1/2)
)^(1/2)*x^4)-29^(1/2)*arctanh(-5/29*29^(1/2)+2/29*29^(1/2)*x^4)-(-10+2*29^
(1/2))^(1/2)*arctanh(1/2*(2+29^(1/2))^(1/2)+1/4*(10+2*29^(1/2))^(1/2)*x^4)
+5/2*ln(x^8-5*x^4-1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{136x^3+1092x^7+3136x^{11}+508x^{15}-192x^{19}+20x^{23}}{-25-211x^4-424x^8-3x^{12}+48x^{16}-11x^{20}+x^{24}}\,dx = \frac{1}{2}\left(\left(5+\sqrt{29}\right)\log\left(5+\sqrt{29}-2x^4\right)-\left(-5+\sqrt{29}\right)\log\left(-5+\sqrt{29}+2x^4\right) - 4\text{RootSum}\left[25+86\#1^4+19\#1^8-6\#1^{12}+\#1^{16}\&,\frac{-4\log(x-\#1)+14\log(x-\#1)\#1^4+5\log(x-\#1)\#1^8}{43+19\#1^4-9\#1^8+2\#1^{12}}\&\right]\right)$$

input
```
Integrate[(136*x^3 + 1092*x^7 + 3136*x^11 + 508*x^15 - 192*x^19 + 20*x^23)
/(-25 - 211*x^4 - 424*x^8 - 3*x^12 + 48*x^16 - 11*x^20 + x^24),x]
```

output
```
((5 + Sqrt[29])*Log[5 + Sqrt[29] - 2*x^4] - (-5 + Sqrt[29])*Log[-5 + Sqrt[
29] + 2*x^4] - 4*RootSum[25 + 86*#1^4 + 19*#1^8 - 6*#1^12 + #1^16 & , (-4*
Log[x - #1] + 14*Log[x - #1]*#1^4 + 5*Log[x - #1]*#1^8)/(43 + 19*#1^4 - 9*
#1^8 + 2*#1^12) & ])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{20x^{23}-192x^{19}+508x^{15}+3136x^{11}+1092x^7+136x^3}{x^{24}-11x^{20}+48x^{16}-3x^{12}-424x^8-211x^4-25}\,dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{4x^3(5x^4+2)}{x^8-5x^4-1}-\frac{16x^3(5x^8+14x^4-4)}{x^{16}-6x^{12}+19x^8+86x^4+25}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$16\text{Subst}\left(\int \frac{1}{x^4-6x^3+19x^2+86x+25}dx,x,x^4\right)-$$
$$56\text{Subst}\left(\int \frac{x}{x^4-6x^3+19x^2+86x+25}dx,x,x^4\right)-$$
$$20\text{Subst}\left(\int \frac{x^2}{x^4-6x^3+19x^2+86x+25}dx,x,x^4\right)+\frac{1}{2}\left(5-\sqrt{29}\right)\log\left(-2x^4-\sqrt{29}+5\right)+$$
$$\frac{1}{2}\left(5+\sqrt{29}\right)\log\left(-2x^4+\sqrt{29}+5\right)$$

input
```
Int[(136*x^3 + 1092*x^7 + 3136*x^11 + 508*x^15 - 192*x^19 + 20*x^23)/(-25
- 211*x^4 - 424*x^8 - 3*x^12 + 48*x^16 - 11*x^20 + x^24),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.60

method	result
risch	$\frac{5\ln\left(2x^4-\sqrt{29}-5\right)}{2}+\frac{\ln\left(2x^4-\sqrt{29}-5\right)\sqrt{29}}{2}+\frac{5\ln\left(2x^4+\sqrt{29}-5\right)}{2}-\frac{\ln\left(2x^4+\sqrt{29}-5\right)\sqrt{29}}{2}+\Bigg(\sum_{\_R=\text{RootOf}(\_Z^4+5\_Z^2-}$
default	$\frac{5\ln\left(x^8-5x^4-1\right)}{2}-\sqrt{29}\,\text{arctanh}\left(\frac{(2x^4-5)\sqrt{29}}{29}\right)+2\Bigg(\sum_{\_R=\text{RootOf}(\_Z^4-6\_Z^3+19\_Z^2+86\_Z+25)}\frac{\left(-5\_R^2-14\_\right.}{2\_R^3-9\_}$

input
```
int((20*x^23-192*x^19+508*x^15+3136*x^11+1092*x^7+136*x^3)/(x^24-11*x^20+4
8*x^16-3*x^12-424*x^8-211*x^4-25),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(2*x^4-29^(1/2)-5)+1/2*ln(2*x^4-29^(1/2)-5)*29^(1/2)+5/2*ln(2*x^4+29
^(1/2)-5)-1/2*ln(2*x^4+29^(1/2)-5)*29^(1/2)+sum(_R*ln(x^4+_R^2-2*_R+1),_R=
RootOf(_Z^4+5*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

$$\int \frac{136x^3+1092x^7+3136x^{11}+508x^{15}-192x^{19}+20x^{23}}{-25-211x^4-424x^8-3x^{12}+48x^{16}-11x^{20}+x^{24}}\,dx$$
$$= -2\sqrt{\frac{1}{2}\sqrt{29}+\frac{5}{2}}\arctan\left(-\frac{1}{4}\left(5x^4-\sqrt{29}(x^4+1)+7\right)\sqrt{\frac{1}{2}\sqrt{29}+\frac{5}{2}}\right)$$
$$-\sqrt{\frac{1}{2}\sqrt{29}-\frac{5}{2}}\log\left(2x^4+\sqrt{29}+4\sqrt{\frac{1}{2}\sqrt{29}-\frac{5}{2}}-3\right)$$
$$+\sqrt{\frac{1}{2}\sqrt{29}-\frac{5}{2}}\log\left(2x^4+\sqrt{29}-4\sqrt{\frac{1}{2}\sqrt{29}-\frac{5}{2}}-3\right)$$
$$+\frac{1}{2}\sqrt{29}\log\left(\frac{2x^8-10x^4-\sqrt{29}(2x^4-5)+27}{x^8-5x^4-1}\right)+\frac{5}{2}\log\left(x^8-5x^4-1\right)$$

input
```
integrate((20*x^23-192*x^19+508*x^15+3136*x^11+1092*x^7+136*x^3)/(x^24-11*
x^20+48*x^16-3*x^12-424*x^8-211*x^4-25),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2*sqrt(29) + 5/2)*arctan(-1/4*(5*x^4 - sqrt(29)*(x^4 + 1) + 7)*s
qrt(1/2*sqrt(29) + 5/2)) - sqrt(1/2*sqrt(29) - 5/2)*log(2*x^4 + sqrt(29) +
 4*sqrt(1/2*sqrt(29) - 5/2) - 3) + sqrt(1/2*sqrt(29) - 5/2)*log(2*x^4 + sq
rt(29) - 4*sqrt(1/2*sqrt(29) - 5/2) - 3) + 1/2*sqrt(29)*log((2*x^8 - 10*x^
4 - sqrt(29)*(2*x^4 - 5) + 27)/(x^8 - 5*x^4 - 1)) + 5/2*log(x^8 - 5*x^4 -
1)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.49

$$\int \frac{136x^3+1092x^7+3136x^{11}+508x^{15}-192x^{19}+20x^{23}}{-25-211x^4-424x^8-3x^{12}+48x^{16}-11x^{20}+x^{24}}\,dx = \left(\frac{5}{2}-\frac{\sqrt{29}}{2}\right)\log\left(x^4-\frac{166}{35}-\frac{2\left(\frac{5}{2}-\frac{\sqrt{29}}{2}\right)^5}{7}+\frac{51\left(\frac{5}{2}-\frac{\sqrt{29}}{2}\right)^4}{35}-\frac{10\left(\frac{5}{2}-\frac{\sqrt{29}}{2}\right)^3}{7}+\frac{58\left(\frac{5}{2}-\frac{\sqrt{29}}{2}\right)^2}{7}+\frac{6\sqrt{29}}{7}\right)+\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)\log\left(x^4-\frac{2\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)^5}{7}-\frac{10\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)^3}{7}-\frac{166}{35}-\frac{6\sqrt{29}}{7}+\frac{58\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)^2}{7}+\frac{51\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)^4}{35}\right)+\operatorname{RootSum}\left(t^4+5t^2-1,\left(t\mapsto t\log\left(-\frac{2t^5}{7}+\frac{51t^4}{35}-\frac{10t^3}{7}+\frac{58t^2}{7}-\frac{12t}{7}+x^4-\frac{16}{35}\right)\right)\right)$$

input
```
integrate((20*x**23-192*x**19+508*x**15+3136*x**11+1092*x**7+136*x**3)/(x*
*24-11*x**20+48*x**16-3*x**12-424*x**8-211*x**4-25),x)
```

output
```
(5/2 - sqrt(29)/2)*log(x**4 - 166/35 - 2*(5/2 - sqrt(29)/2)**5/7 + 51*(5/2
 - sqrt(29)/2)**4/35 - 10*(5/2 - sqrt(29)/2)**3/7 + 58*(5/2 - sqrt(29)/2)*
*2/7 + 6*sqrt(29)/7) + (5/2 + sqrt(29)/2)*log(x**4 - 2*(5/2 + sqrt(29)/2)*
*5/7 - 10*(5/2 + sqrt(29)/2)**3/7 - 166/35 - 6*sqrt(29)/7 + 58*(5/2 + sqrt
(29)/2)**2/7 + 51*(5/2 + sqrt(29)/2)**4/35) + RootSum(_t**4 + 5*_t**2 - 1,
 Lambda(_t, _t*log(-2*_t**5/7 + 51*_t**4/35 - 10*_t**3/7 + 58*_t**2/7 - 12
*_t/7 + x**4 - 16/35)))
```

Maxima [F]

$$\int \frac{136x^3 + 1092x^7 + 3136x^{11} + 508x^{15} - 192x^{19} + 20x^{23}}{-25 - 211x^4 - 424x^8 - 3x^{12} + 48x^{16} - 11x^{20} + x^{24}}\,dx = \int \frac{4\,(5\,x^{23} - 48\,x^{19} + 127\,x^{15} + 784\,x^{11} + 273\,x^7 + 34\,x^3)}{x^{24} - 11\,x^{20} + 48\,x^{16} - 3\,x^{12} - 424\,x^8 - 211\,x^4 - 25}\,dx$$

input
```
integrate((20*x^23-192*x^19+508*x^15+3136*x^11+1092*x^7+136*x^3)/(x^24-11*
x^20+48*x^16-3*x^12-424*x^8-211*x^4-25),x, algorithm="maxima")
```

output
```
4*integrate((5*x^23 - 48*x^19 + 127*x^15 + 784*x^11 + 273*x^7 + 34*x^3)/(x
^24 - 11*x^20 + 48*x^16 - 3*x^12 - 424*x^8 - 211*x^4 - 25), x)
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{136x^3 + 1092x^7 + 3136x^{11} + 508x^{15} - 192x^{19} + 20x^{23}}{-25 - 211x^4 - 424x^8 - 3x^{12} + 48x^{16} - 11x^{20} + x^{24}}\,dx = \frac{1}{2}\,\sqrt{29}\log\left(\frac{|2\,x^4 - \sqrt{29} - 5|}{|2\,x^4 + \sqrt{29} - 5|}\right) + \frac{5}{2}\,\log\left(|x^8 - 5\,x^4 - 1|\right)$$

input
```
integrate((20*x^23-192*x^19+508*x^15+3136*x^11+1092*x^7+136*x^3)/(x^24-11*
x^20+48*x^16-3*x^12-424*x^8-211*x^4-25),x, algorithm="giac")
```

output
```
1/2*sqrt(29)*log(abs(2*x^4 - sqrt(29) - 5)/abs(2*x^4 + sqrt(29) - 5)) + 5/
2*log(abs(x^8 - 5*x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.51

$$\int \frac{136x^3 + 1092x^7 + 3136x^{11} + 508x^{15} - 192x^{19} + 20x^{23}}{-25 - 211x^4 - 424x^8 - 3x^{12} + 48x^{16} - 11x^{20} + x^{24}}\,dx = \text{Too large to display}$$

input
```
int(-(136*x^3 + 1092*x^7 + 3136*x^11 + 508*x^15 - 192*x^19 + 20*x^23)/(211
*x^4 + 424*x^8 + 3*x^12 - 48*x^16 + 11*x^20 - x^24 + 25),x)
```

output
```
log(x^4 - 29^(1/2)/2 - 5/2)*(29^(1/2)/2 + 5/2) - log(29^(1/2)/2 + x^4 - 5/
2)*(29^(1/2)/2 - 5/2) + 2^(1/2)*atan((2^(1/2)*(- 29^(1/2) - 5)^(1/2)*89447
86416240580834974380694522926529073777921316462966865920000000000000000000
000i)/(14578380223168233495652943178402657493761522045746070054502400000000
000000000000000*29^(1/2) + 46295538123172172900694516550692966436466410249
18408984002560000000000000000000000*29^(1/2)*x^4 - 24930910262826055278263
661804971438378100028050743841657978880000000000000000000000*x^4 - 7850698
012283122556254995428654618979229489991299578221035520000000000000000000000
0) + (2^(1/2)*x^4*(- 29^(1/2) - 5)^(1/2)*28405329961047274805148902283896 1
08710549615273584907234836480000000000000000000000i)/(14578380223168233495
65294317840265749376152204574607005450240000000000000000000000*29^(1/2) +
46295538123172172900694516550692966436466410249184089840025600000000000000
00000000*29^(1/2)*x^4 - 24930910262826055278263661804971438378100028050743
841657978880000000000000000000000*x^4 - 7850698012283122556254995428654618
979229489991299578221035520000000000000000000000) - (2^(1/2)*29^(1/2)*(- 2
9^(1/2) - 5)^(1/2)*166100513848529442412968266701615136959386707231152751 5
095040000000000000000000000i)/(1457838022316823349565294317840265749376152
20457460700545024000000000000000000000000*29^(1/2) + 46295538123172172900694
51655069296643646641024918408984002560000000000000000000000*29^(1/2)*x^4 -
 2493091026282605527826366180497143837810002805074384165797888000000000...
```

Reduce [F]

$$\int \frac{136x^3 + 1092x^7 + 3136x^{11} + 508x^{15} - 192x^{19} + 20x^{23}}{-25 - 211x^4 - 424x^8 - 3x^{12} + 48x^{16} - 11x^{20} + x^{24}}\,dx$$
$$= -\frac{53\sqrt{29}\log\left(-\left(\sqrt{29}-5\right)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x + \sqrt{\sqrt{29}-5} + \sqrt{2}\,x^2\right)}{522}$$
$$+ \frac{53\sqrt{29}\log\left(-\left(\sqrt{29}+5\right)^{\frac{1}{4}}2^{\frac{1}{4}} + \sqrt{2}\,x\right)}{522} + \frac{53\sqrt{29}\log\left(\sqrt{\sqrt{29}+5} + \sqrt{2}\,x^2\right)}{522}$$
$$- \frac{53\sqrt{29}\log\left(\left(\sqrt{29}-5\right)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x + \sqrt{\sqrt{29}-5} + \sqrt{2}\,x^2\right)}{522}$$
$$+ \frac{53\sqrt{29}\log\left(\left(\sqrt{29}+5\right)^{\frac{1}{4}}2^{\frac{1}{4}} + \sqrt{2}\,x\right)}{522}$$
$$+ \frac{37520\left(\int \frac{x^{11}}{x^{24}-11x^{20}+48x^{16}-3x^{12}-424x^8-211x^4-25}dx\right)}{9}$$
$$+ \frac{4240\left(\int \frac{x^{7}}{x^{24}-11x^{20}+48x^{16}-3x^{12}-424x^8-211x^4-25}dx\right)}{3}$$
$$- \frac{880\left(\int \frac{x^{3}}{x^{24}-11x^{20}+48x^{16}-3x^{12}-424x^8-211x^4-25}dx\right)}{3}$$
$$+ \frac{19\log(x^{16} - 6x^{12} + 19x^8 + 86x^4 + 25)}{9}$$
$$- \frac{31\log\left(-\left(\sqrt{29}-5\right)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x + \sqrt{\sqrt{29}-5} + \sqrt{2}\,x^2\right)}{18}$$
$$- \frac{31\log\left(-\left(\sqrt{29}+5\right)^{\frac{1}{4}}2^{\frac{1}{4}} + \sqrt{2}\,x\right)}{18} - \frac{31\log\left(\sqrt{\sqrt{29}+5} + \sqrt{2}\,x^2\right)}{18}$$
$$- \frac{31\log\left(\left(\sqrt{29}-5\right)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x + \sqrt{\sqrt{29}-5} + \sqrt{2}\,x^2\right)}{18}$$
$$- \frac{31\log\left(\left(\sqrt{29}+5\right)^{\frac{1}{4}}2^{\frac{1}{4}} + \sqrt{2}\,x\right)}{18}$$

input
```
int((20*x^23-192*x^19+508*x^15+3136*x^11+1092*x^7+136*x^3)/(x^24-11*x^20+4
8*x^16-3*x^12-424*x^8-211*x^4-25),x)
```

output
```
( - 53*sqrt(29)*log( - (sqrt(29) - 5)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(sqr
t(29) - 5) + sqrt(2)*x**2) + 53*sqrt(29)*log( - (sqrt(29) + 5)**(1/4)*2**(
1/4) + sqrt(2)*x) + 53*sqrt(29)*log(sqrt(sqrt(29) + 5) + sqrt(2)*x**2) - 5
3*sqrt(29)*log((sqrt(29) - 5)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(sqrt(29) -
5) + sqrt(2)*x**2) + 53*sqrt(29)*log((sqrt(29) + 5)**(1/4)*2**(1/4) + sqrt
(2)*x) + 2176160*int(x**11/(x**24 - 11*x**20 + 48*x**16 - 3*x**12 - 424*x*
*8 - 211*x**4 - 25),x) + 737760*int(x**7/(x**24 - 11*x**20 + 48*x**16 - 3*
x**12 - 424*x**8 - 211*x**4 - 25),x) - 153120*int(x**3/(x**24 - 11*x**20 +
 48*x**16 - 3*x**12 - 424*x**8 - 211*x**4 - 25),x) + 1102*log(x**16 - 6*x*
*12 + 19*x**8 + 86*x**4 + 25) - 899*log( - (sqrt(29) - 5)**(1/4)*sqrt(2)*2
**(1/4)*x + sqrt(sqrt(29) - 5) + sqrt(2)*x**2) - 899*log( - (sqrt(29) + 5)
**(1/4)*2**(1/4) + sqrt(2)*x) - 899*log(sqrt(sqrt(29) + 5) + sqrt(2)*x**2)
 - 899*log((sqrt(29) - 5)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(sqrt(29) - 5) +
 sqrt(2)*x**2) - 899*log((sqrt(29) + 5)**(1/4)*2**(1/4) + sqrt(2)*x))/522
```

3.72 $\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx$

Optimal result

Integrand size = 69, antiderivative size = 151

$$\begin{aligned}
&\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx \\
&= -2\sqrt{-1+\sqrt{2}}\arctan\left(\sqrt{1+\sqrt{2}}x^2\right) \\
&\quad + 2\sqrt{1+\sqrt{2}}\arctan\left(\sqrt{-\frac{1}{2}+\frac{1}{\sqrt{2}}}-\frac{1}{2}\sqrt{-1+\sqrt{2}}x^2\right) \\
&\quad - 2\sqrt{1+\sqrt{2}}\,\mathrm{arctanh}\left(\sqrt{-1+\sqrt{2}}x^2\right) \\
&\quad - 2\sqrt{-1+\sqrt{2}}\,\mathrm{arctanh}\left(\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}+\frac{1}{2}\sqrt{1+\sqrt{2}}x^2\right)
\end{aligned}$$

output
```
-2*(2^(1/2)-1)^(1/2)*arctan((1+2^(1/2))^(1/2)*x^2)-2*(1+2^(1/2))^(1/2)*arc
tan(-1/2*(-2+2*2^(1/2))^(1/2)+1/2*(2^(1/2)-1)^(1/2)*x^2)-2*(1+2^(1/2))^(1/
2)*arctanh((2^(1/2)-1)^(1/2)*x^2)-2*(2^(1/2)-1)^(1/2)*arctanh(1/2*(2+2*2^(
1/2))^(1/2)+1/2*(1+2^(1/2))^(1/2)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx = -8\left(-\frac{1}{8}\text{RootSum}\left[-1-2\#1^4+\#1^8\&,\frac{\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^2+\#1^6}\&\right] + \frac{1}{4}\text{RootSum}\left[4+32\#1^2+4\#1^4 + \#1^8\&,\frac{-2\log(x-\#1)+4\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{8+2\#1^2+\#1^6}\&\right]\right)$$

input
```
Integrate[(320*x^3 + 16*x^5 + 384*x^7 + 104*x^9 - 64*x^11 - 8*x^13)/(-4 -
32*x^2 - 12*x^4 - 64*x^6 - 5*x^8 + 32*x^10 + 2*x^12 + x^16),x]
```

output
```
-8*(-1/8*RootSum[-1 - 2*#1^4 + #1^8 & , (Log[x - #1] + Log[x - #1]*#1^4)/(
-#1^2 + #1^6) & ] + RootSum[4 + 32*#1^2 + 4*#1^4 + #1^8 & , (-2*Log[x - #1
] + 4*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(8 + 2*#1^2 + #1^6) & ]/4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-8x^{13}-64x^{11}+104x^9+384x^7+16x^5+320x^3}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}\,dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{8x(-x^4-1)}{-x^8+2x^4+1}-\frac{16x(x^4+4x^2-2)}{x^8+4x^4+32x^2+4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$16\text{Subst}\left(\int \frac{1}{x^4+4x^2+32x+4}dx,x,x^2\right)-32\text{Subst}\left(\int \frac{x}{x^4+4x^2+32x+4}dx,x,x^2\right)-8\text{Subst}\left(\int \frac{x^2}{x^4+4x^2+32x+4}dx,x,x^2\right)-2\sqrt{\sqrt{2}-1}\arctan\left(\frac{x^2}{\sqrt{\sqrt{2}-1}}\right)-2\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{x^2}{\sqrt{1+\sqrt{2}}}\right)$$

input
```
Int[(320*x^3 + 16*x^5 + 384*x^7 + 104*x^9 - 64*x^11 - 8*x^13)/(-4 - 32*x^2
 - 12*x^4 - 64*x^6 - 5*x^8 + 32*x^10 + 2*x^12 + x^16),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

method	result
risch	$\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\_R\ln\left(x^2-\_R\right)\right)+\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2-1\right)}\_R\ln\left(\_R^2+x^2-2\_R\right.\right.$
default	$2\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+4\_Z^2+32\_Z+4\right)}\frac{\left(-\_R^2-4\_R+2\right)\ln\left(x^2-\_R\right)}{\_R^3+2\_R+8}\right)-\frac{\left(2+\sqrt{2}\right)\sqrt{2}\,\text{arctanh}\left(\frac{x^2}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\left(\sqrt{2}-2\right.}{}$

input
```
int((-8*x^13-64*x^11+104*x^9+384*x^7+16*x^5+320*x^3)/(x^16+2*x^12+32*x^10-
5*x^8-64*x^6-12*x^4-32*x^2-4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(x^2-_R),_R=RootOf(_Z^4-2*_Z^2-1))+sum(_R*ln(_R^2+x^2-2*_R+1),_R=
RootOf(_Z^4+2*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx$$
$$= -2\sqrt{\sqrt{2}+1}\arctan\left(-\frac{1}{2}\left(x^2-\sqrt{2}(x^2+1)+2\right)\sqrt{\sqrt{2}+1}\right)$$
$$-2\sqrt{\sqrt{2}-1}\arctan\left(\left(\sqrt{2}x^2+x^2\right)\sqrt{\sqrt{2}-1}\right)$$
$$-\sqrt{\sqrt{2}-1}\log\left(x^2+\sqrt{2}+2\sqrt{\sqrt{2}-1}\right)+\sqrt{\sqrt{2}-1}\log\left(x^2+\sqrt{2}-2\sqrt{\sqrt{2}-1}\right)$$
$$-\sqrt{\sqrt{2}+1}\log\left(x^2+\sqrt{\sqrt{2}+1}\right)+\sqrt{\sqrt{2}+1}\log\left(x^2-\sqrt{\sqrt{2}+1}\right)$$

input
```
integrate((-8*x^13-64*x^11+104*x^9+384*x^7+16*x^5+320*x^3)/(x^16+2*x^12+32
*x^10-5*x^8-64*x^6-12*x^4-32*x^2-4),x, algorithm="fricas")
```

output
```
-2*sqrt(sqrt(2) + 1)*arctan(-1/2*(x^2 - sqrt(2)*(x^2 + 1) + 2)*sqrt(sqrt(2
) + 1)) - 2*sqrt(sqrt(2) - 1)*arctan((sqrt(2)*x^2 + x^2)*sqrt(sqrt(2) - 1)
) - sqrt(sqrt(2) - 1)*log(x^2 + sqrt(2) + 2*sqrt(sqrt(2) - 1)) + sqrt(sqrt
(2) - 1)*log(x^2 + sqrt(2) - 2*sqrt(sqrt(2) - 1)) - sqrt(sqrt(2) + 1)*log(
x^2 + sqrt(sqrt(2) + 1)) + sqrt(sqrt(2) + 1)*log(x^2 - sqrt(sqrt(2) + 1))
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72

$$\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx =$$
$$-\operatorname{RootSum}\left(t^4-2t^2-1,\left(t\mapsto t\log\left(-\frac{t^7}{4}-\frac{t^6}{4}-\frac{t^4}{4}+\frac{5t^3}{4}+\frac{7t^2}{4}+\frac{3t}{2}+x^2+\frac{3}{4}\right)\right)\right)$$
$$-\operatorname{RootSum}\left(t^4+2t^2-1,\left(t\mapsto t\log\left(-\frac{t^7}{4}-\frac{t^6}{4}-\frac{t^4}{4}+\frac{5t^3}{4}+\frac{7t^2}{4}+\frac{3t}{2}+x^2+\frac{3}{4}\right)\right)\right)$$

input
```
integrate((-8*x**13-64*x**11+104*x**9+384*x**7+16*x**5+320*x**3)/(x**16+2*
x**12+32*x**10-5*x**8-64*x**6-12*x**4-32*x**2-4),x)
```

output
```
-RootSum(_t**4 - 2*_t**2 - 1, Lambda(_t, _t*log(-_t**7/4 - _t**6/4 - _t**4
/4 + 5*_t**3/4 + 7*_t**2/4 + 3*_t/2 + x**2 + 3/4))) - RootSum(_t**4 + 2*_t
**2 - 1, Lambda(_t, _t*log(-_t**7/4 - _t**6/4 - _t**4/4 + 5*_t**3/4 + 7*_t
**2/4 + 3*_t/2 + x**2 + 3/4)))
```

Maxima [F]

$$\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx = \int -\frac{8\,(x^{13}+8\,x^{11}-13\,x^9-48\,x^7-2\,x^5-40\,x^3)}{x^{16}+2\,x^{12}+32\,x^{10}-5\,x^8-64\,x^6-12\,x^4-32\,x^2-4}\,dx$$

input
```
integrate((-8*x^13-64*x^11+104*x^9+384*x^7+16*x^5+320*x^3)/(x^16+2*x^12+32
*x^10-5*x^8-64*x^6-12*x^4-32*x^2-4),x, algorithm="maxima")
```

output
```
-8*integrate((x^13 + 8*x^11 - 13*x^9 - 48*x^7 - 2*x^5 - 40*x^3)/(x^16 + 2*
x^12 + 32*x^10 - 5*x^8 - 64*x^6 - 12*x^4 - 32*x^2 - 4), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx = -\left(x^4\sqrt{2\,\sqrt{2}+2}+\sqrt{2\,\sqrt{2}+2}\right)\arctan\left(\frac{x^2}{\sqrt{\sqrt{2}-1}}\right) - \frac{1}{2}\left(x^4\sqrt{2\,\sqrt{2}-2}+\sqrt{2\,\sqrt{2}-2}\right)\log\left(x^2+\sqrt{\sqrt{2}+1}\right) + \frac{1}{2}\left(x^4\sqrt{2\,\sqrt{2}-2}+\sqrt{2\,\sqrt{2}-2}\right)\log\left(\left|x^2-\sqrt{\sqrt{2}+1}\right|\right)$$

input
```
integrate((-8*x^13-64*x^11+104*x^9+384*x^7+16*x^5+320*x^3)/(x^16+2*x^12+32
*x^10-5*x^8-64*x^6-12*x^4-32*x^2-4),x, algorithm="giac")
```

output
```
-(x^4*sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) + 2))*arctan(x^2/sqrt(sqrt(2) -
 1)) - 1/2*(x^4*sqrt(2*sqrt(2) - 2) + sqrt(2*sqrt(2) - 2))*log(x^2 + sqrt(
sqrt(2) + 1)) + 1/2*(x^4*sqrt(2*sqrt(2) - 2) + sqrt(2*sqrt(2) - 2))*log(ab
s(x^2 - sqrt(sqrt(2) + 1)))
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 629, normalized size of antiderivative = 4.17

$$\int \frac{320x^3 + 16x^5 + 384x^7 + 104x^9 - 64x^{11} - 8x^{13}}{-4 - 32x^2 - 12x^4 - 64x^6 - 5x^8 + 32x^{10} + 2x^{12} + x^{16}} \, dx = \text{Too large to display}$$

input
```
int(-(320*x^3 + 16*x^5 + 384*x^7 + 104*x^9 - 64*x^11 - 8*x^13)/(32*x^2 + 1
2*x^4 + 64*x^6 + 5*x^8 - 32*x^10 - 2*x^12 - x^16 + 4),x)
```

output
```
atan(((- 2^(1/2) - 1)^(1/2)*841579333379105903449187258897975096522047488i
)/(382994240663054868450414598110619467534827520*2^(1/2) + 301510773435543
48572505772910968817987669196 80*2^(1/2)*x^2 - 426400624961629749397688472 6
739467309770866688*x^2 - 541635645052411448624667998445701723706097664) -
(2^(1/2)*(- 2^(1/2) - 1)^(1/2)*595086452071602043963558838309115181616070 6
56i)/(382994240663054868450414598110619467534827520*2^(1/2) + 301510773435
5434857250577291096881798766919680*2^(1/2)*x^2 - 426400624961629749397688 4
726739467309770866688*x^2 - 541635645052411448624667998445701723706097664)
 + (x^2*(- 2^(1/2) - 1)^(1/2)*662530192078247175846471342050287877914991 00
16i)/(382994240663054868450414598110619467534827520*2^(1/2) + 301510773435
5434857250577291096881798766919680*2^(1/2)*x^2 - 426400624961629749397688 4
726739467309770866688*x^2 - 541635645052411448624667998445701723706097664)
 - (2^(1/2)*x^2*(- 2^(1/2) - 1)^(1/2)*468479591630585044570147835618845485
8031890432i)/(382994240663054868450414598110619467534827520*2^(1/2) + 3015
10773435543485725057729109688179876691968 0*2^(1/2)*x^2 - 42640062496162974
939768847267394673097708666 88*x^2 - 541635645052411448624667998445701723 70
6097664))*(- 2^(1/2) - 1)^(1/2)*2i - atan(((1 - 2^(1/2))^(1/2)*39140865051
70584608431264366553680838656 0000i)/(381784114077275246696218284381943785 0
6240000*2^(1/2) - 798582211758236224518371194311165345792 0000*2^(1/2)*x^2
- 391408650517058460843126436655368083865600 00*x^2 + 745219368492329640...
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{320x^3+16x^5+384x^7+104x^9-64x^{11}-8x^{13}}{-4-32x^2-12x^4-64x^6-5x^8+32x^{10}+2x^{12}+x^{16}}\,dx \\
&= -8\left(\int \frac{x^{13}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right) \\
&\quad - 64\left(\int \frac{x^{11}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right) \\
&\quad + 104\left(\int \frac{x^{9}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right) \\
&\quad + 384\left(\int \frac{x^{7}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right) \\
&\quad + 16\left(\int \frac{x^{5}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right) \\
&\quad + 320\left(\int \frac{x^{3}}{x^{16}+2x^{12}+32x^{10}-5x^8-64x^6-12x^4-32x^2-4}dx\right)
\end{aligned}
$$

input

```
int((-8*x^13-64*x^11+104*x^9+384*x^7+16*x^5+320*x^3)/(x^16+2*x^12+32*x^10-
5*x^8-64*x^6-12*x^4-32*x^2-4),x)
```

output

```
8*( - int(x**13/(x**16 + 2*x**12 + 32*x**10 - 5*x**8 - 64*x**6 - 12*x**4 -
 32*x**2 - 4),x) - 8*int(x**11/(x**16 + 2*x**12 + 32*x**10 - 5*x**8 - 64*x
**6 - 12*x**4 - 32*x**2 - 4),x) + 13*int(x**9/(x**16 + 2*x**12 + 32*x**10
- 5*x**8 - 64*x**6 - 12*x**4 - 32*x**2 - 4),x) + 48*int(x**7/(x**16 + 2*x*
*12 + 32*x**10 - 5*x**8 - 64*x**6 - 12*x**4 - 32*x**2 - 4),x) + 2*int(x**5
/(x**16 + 2*x**12 + 32*x**10 - 5*x**8 - 64*x**6 - 12*x**4 - 32*x**2 - 4),x
) + 40*int(x**3/(x**16 + 2*x**12 + 32*x**10 - 5*x**8 - 64*x**6 - 12*x**4 -
 32*x**2 - 4),x))
```

3.73 $\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 97

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8} \, dx = \frac{1}{8}\sqrt{1+\sqrt{2}}\arctan\left(\sqrt{-\frac{1}{2}+\frac{1}{\sqrt{2}}}-\frac{1}{2}\sqrt{-1+\sqrt{2}}x^2\right) - \frac{1}{8}\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}+\frac{1}{2}\sqrt{1+\sqrt{2}}x^2\right)$$

output
```
-1/8*(1+2^(1/2))^(1/2)*arctan(-1/2*(-2+2*2^(1/2))^(1/2)+1/2*(2^(1/2)-1)^(1
/2)*x^2)-1/8*(2^(1/2)-1)^(1/2)*arctanh(1/2*(2+2*2^(1/2))^(1/2)+1/2*(1+2^(1
/2))^(1/2)*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8} \, dx = -\frac{1}{8}\text{RootSum}\left[4+32\#1+4\#1^2+\#1^4\&,\frac{-2\log(x^2-\#1)+4\log(x^2-\#1)\#1+\log(x^2-\#1)\#1^2}{8+2\#1+\#1^3}\&\right]$$

input
```
Integrate[(2*x - 4*x^3 - x^5)/(4 + 32*x^2 + 4*x^4 + x^8),x]
```

output
```
-1/8*RootSum[4 + 32*#1 + 4*#1^2 + #1^4 & , (-2*Log[x^2 - #1] + 4*Log[x^2 -
 #1]*#1 + Log[x^2 - #1]*#1^2)/(8 + 2*#1 + #1^3) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^5-4x^3+2x}{x^8+4x^4+32x^2+4}\,dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x(-x^4-4x^2+2)}{x^8+4x^4+32x^2+4}dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{2}\int \frac{-x^4-4x^2+2}{x^8+4x^4+32x^2+4}dx^2$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{1}{2}\int \left(-\frac{x^4}{x^8+4x^4+32x^2+4}-\frac{4x^2}{x^8+4x^4+32x^2+4}+\frac{2}{x^8+4x^4+32x^2+4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^8+4x^4+32x^2+4}dx^2-4\int \frac{x^2}{x^8+4x^4+32x^2+4}dx^2-\int \frac{x^4}{x^8+4x^4+32x^2+4}dx^2\right)$$

input
```
Int[(2*x - 4*x^3 - x^5)/(4 + 32*x^2 + 4*x^4 + x^8),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2\_Z^{2}-1\right)}\_R\ln\left(\_R^{2}+x^{2}-2\_R+1\right)\right)}{16}$	31
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+4\_Z^{2}+32\_Z+4\right)}\frac{\left(-\_R^{2}-4\_R+2\right)\ln\left(x^{2}-\_R\right)}{\_R^{3}+2\_R+8}\right)}{8}$	49

input
```
int((-x^5-4*x^3+2*x)/(x^8+4*x^4+32*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(_R^2+x^2-2*_R+1),_R=RootOf(_Z^4+2*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx = -\frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(-\frac{1}{2}\left(x^2-\sqrt{2}(x^2+1)+2\right)\sqrt{\sqrt{2}+1}\right) - \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(x^2+\sqrt{2}+2\sqrt{\sqrt{2}-1}\right) + \frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(x^2+\sqrt{2}-2\sqrt{\sqrt{2}-1}\right)$$

input
```
integrate((-x^5-4*x^3+2*x)/(x^8+4*x^4+32*x^2+4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) + 1)*arctan(-1/2*(x^2 - sqrt(2)*(x^2 + 1) + 2)*sqrt(sqrt
(2) + 1)) - 1/16*sqrt(sqrt(2) - 1)*log(x^2 + sqrt(2) + 2*sqrt(sqrt(2) - 1)
) + 1/16*sqrt(sqrt(2) - 1)*log(x^2 + sqrt(2) - 2*sqrt(sqrt(2) - 1))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.30

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx = -\operatorname{RootSum}\left(65536t^4+512t^2-1,\left(t\mapsto t\log\left(256t^2+32t+x^2+1\right)\right)\right)$$

input
```
integrate((-x**5-4*x**3+2*x)/(x**8+4*x**4+32*x**2+4),x)
```

output
```
-RootSum(65536*_t**4 + 512*_t**2 - 1, Lambda(_t, _t*log(256*_t**2 + 32*_t
+ x**2 + 1)))
```

Maxima [F]

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx = \int -\frac{x^5+4\,x^3-2\,x}{x^8+4\,x^4+32\,x^2+4}\,dx$$

input
```
integrate((-x^5-4*x^3+2*x)/(x^8+4*x^4+32*x^2+4),x, algorithm="maxima")
```

output
```
-integrate((x^5 + 4*x^3 - 2*x)/(x^8 + 4*x^4 + 32*x^2 + 4), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx = 0$$

input
```
integrate((-x^5-4*x^3+2*x)/(x^8+4*x^4+32*x^2+4),x, algorithm="giac")
```

output
```
0
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{114688\sqrt{\sqrt{2}-1}}{58368\sqrt{2}+421376\sqrt{2}\,x^2+594944\,x^2+74752}+\frac{77824\sqrt{2}\sqrt{\sqrt{2}-1}}{58368\sqrt{2}+421376\sqrt{2}\,x^2+594944\,x^2+74752}+\frac{920576\,x^2\sqrt{\sqrt{2}-1}}{58368\sqrt{2}+421376\sqrt{2}\,x^2+594944\,x^2}\right.}{8}$$
$$-\frac{\operatorname{atanh}\left(\frac{114688\sqrt{-\sqrt{2}-1}}{58368\sqrt{2}+421376\sqrt{2}\,x^2-594944\,x^2-74752}-\frac{77824\sqrt{2}\sqrt{-\sqrt{2}-1}}{58368\sqrt{2}+421376\sqrt{2}\,x^2-594944\,x^2-74752}+\frac{920576\,x^2\sqrt{-\sqrt{2}-}}{58368\sqrt{2}+421376\sqrt{2}\,x^2-5949}\right.}{8}$$

input
```
int(-(4*x^3 - 2*x + x^5)/(32*x^2 + 4*x^4 + x^8 + 4),x)
```

output
```
(atanh((114688*(2^(1/2) - 1)^(1/2))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 +
594944*x^2 + 74752) + (77824*2^(1/2)*(2^(1/2) - 1)^(1/2))/(58368*2^(1/2) +
 421376*2^(1/2)*x^2 + 594944*x^2 + 74752) + (920576*x^2*(2^(1/2) - 1)^(1/2
))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 + 594944*x^2 + 74752) + (652288*2^(
1/2)*x^2*(2^(1/2) - 1)^(1/2))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 + 594944
*x^2 + 74752))*(2^(1/2) - 1)^(1/2))/8 - (atanh((114688*(- 2^(1/2) - 1)^(1/
2))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 - 594944*x^2 - 74752) - (77824*2^(
1/2)*(- 2^(1/2) - 1)^(1/2))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 - 594944*x
^2 - 74752) + (920576*x^2*(- 2^(1/2) - 1)^(1/2))/(58368*2^(1/2) + 421376*2
^(1/2)*x^2 - 594944*x^2 - 74752) - (652288*2^(1/2)*x^2*(- 2^(1/2) - 1)^(1/
2))/(58368*2^(1/2) + 421376*2^(1/2)*x^2 - 594944*x^2 - 74752))*(- 2^(1/2)
- 1)^(1/2))/8
```

Reduce [F]

$$\int \frac{2x-4x^3-x^5}{4+32x^2+4x^4+x^8}\,dx = -\left(\int \frac{x^5}{x^8+4x^4+32x^2+4}dx\right) - 4\left(\int \frac{x^3}{x^8+4x^4+32x^2+4}dx\right) + 2\left(\int \frac{x}{x^8+4x^4+32x^2+4}dx\right)$$

input
```
int((-x^5-4*x^3+2*x)/(x^8+4*x^4+32*x^2+4),x)
```

output
```
- int(x**5/(x**8 + 4*x**4 + 32*x**2 + 4),x) - 4*int(x**3/(x**8 + 4*x**4 +
32*x**2 + 4),x) + 2*int(x/(x**8 + 4*x**4 + 32*x**2 + 4),x)
```

3.74 $\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8} dx$

Optimal result

Integrand size = 30, antiderivative size = 167

$$\begin{aligned}\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx &= \frac{1}{8}\sqrt{-1+\sqrt{2}}\arctan\left(\sqrt{1+\sqrt{2}}-\sqrt{1+\sqrt{2}}x\right) \\ &\quad+\frac{1}{8}\sqrt{-1+\sqrt{2}}\arctan\left(\sqrt{1+\sqrt{2}}+\sqrt{1+\sqrt{2}}x\right) \\ &\quad+\frac{1}{8}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}-\sqrt{-1+\sqrt{2}}x\right) \\ &\quad+\frac{1}{8}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}+\sqrt{-1+\sqrt{2}}x\right)\end{aligned}$$

output
```
-1/8*(2^(1/2)-1)^(1/2)*arctan(-(1+2^(1/2))^(1/2)+(1+2^(1/2))^(1/2)*x)+1/8*
(2^(1/2)-1)^(1/2)*arctan((1+2^(1/2))^(1/2)+(1+2^(1/2))^(1/2)*x)-1/8*(1+2^(
1/2))^(1/2)*arctanh(-(2^(1/2)-1)^(1/2)+(2^(1/2)-1)^(1/2)*x)+1/8*(1+2^(1/2)
)^(1/2)*arctanh((2^(1/2)-1)^(1/2)+(2^(1/2)-1)^(1/2)*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx = \frac{1}{16}\text{RootSum}\left[-2+4\#1^2-4\#1^3+\#1^4\&,\frac{2\log(x-\#1)-2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{2\#1-3\#1^2+\#1^3}\&\right] - \frac{1}{16}\text{RootSum}\left[-2+4\#1^2+4\#1^3+\#1^4\&,\frac{2\log(x-\#1)+2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{2\#1+3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[(2*x + x^5)/(4 - 16*x^2 + 12*x^4 - 8*x^6 + x^8),x]
```

output
```
RootSum[-2 + 4*#1^2 - 4*#1^3 + #1^4 & , (2*Log[x - #1] - 2*Log[x - #1]*#1
+ Log[x - #1]*#1^2)/(2*#1 - 3*#1^2 + #1^3) & ]/16 - RootSum[-2 + 4*#1^2 +
4*#1^3 + #1^4 & , (2*Log[x - #1] + 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(2
*#1 + 3*#1^2 + #1^3) & ]/16
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2027, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+2x}{x^8-8x^6+12x^4-16x^2+4}\,dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x(x^4+2)}{x^8-8x^6+12x^4-16x^2+4}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{-x^2-2x-2}{4(x^4+4x^3+4x^2-2)}+\frac{x^2-2x+2}{4(x^4-4x^3+4x^2-2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\sqrt{\sqrt{2}-1}\arctan\left(\frac{1-x}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x+1}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{1-x}{\sqrt{1+\sqrt{2}}}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\text{arctanh}\left(\frac{x+1}{\sqrt{1+\sqrt{2}}}\right)$$

input

```
Int[(2*x + x^5)/(4 - 16*x^2 + 12*x^4 - 8*x^6 + x^8),x]
```

output

```
(Sqrt[-1 + Sqrt[2]]*ArcTan[(1 - x)/Sqrt[-1 + Sqrt[2]]])/8 + (Sqrt[-1 + Sqr
t[2]]*ArcTan[(1 + x)/Sqrt[-1 + Sqrt[2]]])/8 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(
1 - x)/Sqrt[1 + Sqrt[2]]])/8 + (Sqrt[1 + Sqrt[2]]*ArcTanh[(1 + x)/Sqrt[1 +
 Sqrt[2]]])/8
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027

```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2462

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /; !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-2\_Z^{2}-1\right)}\_R\ln\left(-\_R^{2}+x^{2}-2\_R-1\right)\right)}{16}$	33
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-8\_Z^{3}+12\_Z^{2}-16\_Z+4\right)}\frac{\left(\_R^{2}+2\right)\ln\left(x^{2}-\_R\right)}{\_R^{3}-6\_R^{2}+6\_R-4}\right)}{8}$	54

input
```
int((x^5+2*x)/(x^8-8*x^6+12*x^4-16*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/16*sum(_R*ln(-_R^2+x^2-2*_R-1),_R=RootOf(_Z^4-2*_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

$$\begin{aligned}\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx &= -\frac{1}{8}\sqrt{\sqrt{2}-1}\arctan\left(\frac{1}{2}\left(x^2+\sqrt{2}(x^2-1)\right)\sqrt{\sqrt{2}-1}\right)\\ &\quad-\frac{1}{16}\sqrt{\sqrt{2}+1}\log\left(x^2-\sqrt{2}+2\sqrt{\sqrt{2}+1}-2\right)\\ &\quad+\frac{1}{16}\sqrt{\sqrt{2}+1}\log\left(x^2-\sqrt{2}-2\sqrt{\sqrt{2}+1}-2\right)\end{aligned}$$

input
```
integrate((x^5+2*x)/(x^8-8*x^6+12*x^4-16*x^2+4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) - 1)*arctan(1/2*(x^2 + sqrt(2)*(x^2 - 1))*sqrt(sqrt(2) -
 1)) - 1/16*sqrt(sqrt(2) + 1)*log(x^2 - sqrt(2) + 2*sqrt(sqrt(2) + 1) - 2)
 + 1/16*sqrt(sqrt(2) + 1)*log(x^2 - sqrt(2) - 2*sqrt(sqrt(2) + 1) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx = -\sqrt{\frac{1}{256}+\frac{\sqrt{2}}{256}}\log\left(x^2-2-\sqrt{2}+32\sqrt{\frac{1}{256}+\frac{\sqrt{2}}{256}}\right) + \sqrt{\frac{1}{256}+\frac{\sqrt{2}}{256}}\log\left(x^2-32\sqrt{\frac{1}{256}+\frac{\sqrt{2}}{256}}-2-\sqrt{2}\right) - 2\sqrt{-\frac{1}{256}+\frac{\sqrt{2}}{256}}\,\mathrm{atan}\left(\frac{x^2}{2\sqrt{-1+\sqrt{2}}}-\frac{1}{\sqrt{-1+\sqrt{2}}}+\frac{\sqrt{2}}{2\sqrt{-1+\sqrt{2}}}\right)$$

input
```
integrate((x**5+2*x)/(x**8-8*x**6+12*x**4-16*x**2+4),x)
```

output
```
-sqrt(1/256 + sqrt(2)/256)*log(x**2 - 2 - sqrt(2) + 32*sqrt(1/256 + sqrt(2
)/256)) + sqrt(1/256 + sqrt(2)/256)*log(x**2 - 32*sqrt(1/256 + sqrt(2)/256
) - 2 - sqrt(2)) - 2*sqrt(-1/256 + sqrt(2)/256)*atan(x**2/(2*sqrt(-1 + sqr
t(2))) - 1/sqrt(-1 + sqrt(2)) + sqrt(2)/(2*sqrt(-1 + sqrt(2))))
```

Maxima [F]

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx = \int \frac{x^5+2\,x}{x^8-8\,x^6+12\,x^4-16\,x^2+4}\,dx$$

input
```
integrate((x^5+2*x)/(x^8-8*x^6+12*x^4-16*x^2+4),x, algorithm="maxima")
```

output
```
integrate((x^5 + 2*x)/(x^8 - 8*x^6 + 12*x^4 - 16*x^2 + 4), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx=0$$

input
```
integrate((x^5+2*x)/(x^8-8*x^6+12*x^4-16*x^2+4),x, algorithm="giac")
```

output
```
0
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.83

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx = \frac{\operatorname{atanh}\left(\frac{32768\sqrt{1-\sqrt{2}}}{33792\sqrt{2}-111104\sqrt{2}\,x^2+155648\,x^2-48128}-\frac{20480\sqrt{2}\sqrt{1-\sqrt{2}}}{33792\sqrt{2}-111104\sqrt{2}\,x^2+155648\,x^2-48128}-\frac{101376\,x^2\sqrt{1-\sqrt{2}}}{33792\sqrt{2}-111104\sqrt{2}\,x^2+155648\,x^2 \cdots}\right.}{8} - \frac{\operatorname{atanh}\left(\frac{32768\sqrt{\sqrt{2}+1}}{33792\sqrt{2}-111104\sqrt{2}\,x^2-155648\,x^2+48128}+\frac{20480\sqrt{2}\sqrt{\sqrt{2}+1}}{33792\sqrt{2}-111104\sqrt{2}\,x^2-155648\,x^2+48128}-\frac{101376\,x^2\sqrt{\sqrt{2}+1}}{33792\sqrt{2}-111104\sqrt{2}\,x^2-1556 \cdots}\right.}{8}$$

input
```
int((2*x + x^5)/(12*x^4 - 16*x^2 - 8*x^6 + x^8 + 4),x)
```

output
```
(atanh((32768*(1 - 2^(1/2))^(1/2))/(33792*2^(1/2) - 111104*2^(1/2)*x^2 + 1
55648*x^2 - 48128) - (20480*2^(1/2)*(1 - 2^(1/2))^(1/2))/(33792*2^(1/2) -
111104*2^(1/2)*x^2 + 155648*x^2 - 48128) - (101376*x^2*(1 - 2^(1/2))^(1/2)
)/(33792*2^(1/2) - 111104*2^(1/2)*x^2 + 155648*x^2 - 48128) + (70656*2^(1/
2)*x^2*(1 - 2^(1/2))^(1/2))/(33792*2^(1/2) - 111104*2^(1/2)*x^2 + 155648*x
^2 - 48128))*(1 - 2^(1/2))^(1/2))/8 - (atanh((32768*(2^(1/2) + 1)^(1/2))/(
33792*2^(1/2) - 111104*2^(1/2)*x^2 - 155648*x^2 + 48128) + (20480*2^(1/2)*
(2^(1/2) + 1)^(1/2))/(33792*2^(1/2) - 111104*2^(1/2)*x^2 - 155648*x^2 + 48
128) - (101376*x^2*(2^(1/2) + 1)^(1/2))/(33792*2^(1/2) - 111104*2^(1/2)*x^
2 - 155648*x^2 + 48128) - (70656*2^(1/2)*x^2*(2^(1/2) + 1)^(1/2))/(33792*2
^(1/2) - 111104*2^(1/2)*x^2 - 155648*x^2 + 48128))*(2^(1/2) + 1)^(1/2))/8
```

Reduce [F]

$$\int \frac{2x+x^5}{4-16x^2+12x^4-8x^6+x^8}\,dx = \int \frac{x^5}{x^8-8x^6+12x^4-16x^2+4}dx + 2\left(\int \frac{x}{x^8-8x^6+12x^4-16x^2+4}dx\right)$$

input
```
int((x^5+2*x)/(x^8-8*x^6+12*x^4-16*x^2+4),x)
```

output
```
int(x**5/(x**8 - 8*x**6 + 12*x**4 - 16*x**2 + 4),x) + 2*int(x/(x**8 - 8*x*
*6 + 12*x**4 - 16*x**2 + 4),x)
```

3.75 $\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 186

$$\begin{aligned}\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx\\ =2\arctan(1-x)+2\arctan(1+x)-\sqrt{2}\arctan\left(1-\sqrt{2}-\sqrt{2}x\right)\\ +\sqrt{2}\arctan\left(1+\sqrt{2}-\sqrt{2}x\right)-\sqrt{2}\arctan\left(1-\sqrt{2}+\sqrt{2}x\right)\\ +\sqrt{2}\arctan\left(1+\sqrt{2}+\sqrt{2}x\right)-2\log(x)+\log\left(-4+x^2\right)\\ +\frac{\log\left(6+4\sqrt{2}-2x^2-2\sqrt{2}x^2+x^4\right)}{\sqrt{2}}-\frac{\log\left(6-4\sqrt{2}-2x^2+2\sqrt{2}x^2+x^4\right)}{\sqrt{2}}\end{aligned}$$

output
```
-2*arctan(-1+x)+2*arctan(1+x)+2^(1/2)*arctan(-1+2^(1/2)+x*2^(1/2))-arctan(
-1-2^(1/2)+x*2^(1/2))*2^(1/2)-arctan(1-2^(1/2)+x*2^(1/2))*2^(1/2)+2^(1/2)*
arctan(1+2^(1/2)+x*2^(1/2))-2*ln(x)+ln(x^2-4)+1/2*ln(6+4*2^(1/2)-2*x^2-2*x
^2*2^(1/2)+x^4)*2^(1/2)-1/2*ln(6-4*2^(1/2)-2*x^2+2*x^2*2^(1/2)+x^4)*2^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

$$\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx$$
$$= 2\arctan(1-x)+2\arctan(1+x)+\log(2-x)-2\log(x)+\log(2+x) - \text{RootSum}\left[2-4\#1+6\#1^2-4\#1^3+\#1^4\&,\frac{\log(x-\#1)}{-1+3\#1-3\#1^2+\#1^3}\&\right] + \text{RootSum}\left[2+4\#1+6\#1^2+4\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+3\#1+3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[(128 + 896*x^2 + 896*x^4 + 128*x^6)/(-64*x - 112*x^3 - 112*x^5 +
 68*x^7 - 56*x^9 + 28*x^11 - 8*x^13 + x^15),x]
```

output
```
2*ArcTan[1 - x] + 2*ArcTan[1 + x] + Log[2 - x] - 2*Log[x] + Log[2 + x] - R
ootSum[2 - 4*#1 + 6*#1^2 - 4*#1^3 + #1^4 & , Log[x - #1]/(-1 + 3*#1 - 3*#1
^2 + #1^3) & ] + RootSum[2 + 4*#1 + 6*#1^2 + 4*#1^3 + #1^4 & , Log[x - #1]
/(1 + 3*#1 + 3*#1^2 + #1^3) & ]
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2026, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{128x^6+896x^4+896x^2+128}{x^{15}-8x^{13}+28x^{11}-56x^9+68x^7-112x^5-112x^3-64x}\,dx$$
$$\Big\downarrow \ 2026$$
$$\int \frac{128x^6+896x^4+896x^2+128}{x\left(x^{14}-8x^{12}+28x^{10}-56x^8+68x^6-112x^4-112x^2-64\right)}\,dx$$
$$\Big\downarrow \ 2460$$

$$\int \left(-\frac{2}{x^2-2x+2}+\frac{2}{x^2+2x+2}-\frac{4}{x^4-4x^3+6x^2-4x+2}+\frac{4}{x^4+4x^3+6x^2+4x+2}-\frac{2}{x}+\frac{1}{x+2}+\frac{1}{x-2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{2}\arctan\left(1-\sqrt{2}(1-x)\right)+\sqrt{2}\arctan\left(\sqrt{2}(1-x)+1\right)+2\arctan(1-x)+2\arctan(x+1)-\sqrt{2}\arctan\left(1-\sqrt{2}(x+1)\right)+\sqrt{2}\arctan\left(\sqrt{2}(x+1)+1\right)-\frac{\log\left((x-1)^2-\sqrt{2}(1-x)+1\right)}{\sqrt{2}}+\frac{\log\left((x-1)^2+\sqrt{2}(1-x)+1\right)}{\sqrt{2}}+\log(2-x)-2\log(x)+\log(x+2)-\frac{\log\left((x+1)^2-\sqrt{2}(x+1)+1\right)}{\sqrt{2}}+\frac{\log\left((x+1)^2+\sqrt{2}(x+1)+1\right)}{\sqrt{2}}$$

input
```
Int[(128 + 896*x^2 + 896*x^4 + 128*x^6)/(-64*x - 112*x^3 - 112*x^5 + 68*x^
7 - 56*x^9 + 28*x^11 - 8*x^13 + x^15),x]
```

output
```
-(Sqrt[2]*ArcTan[1 - Sqrt[2]*(1 - x)]) + Sqrt[2]*ArcTan[1 + Sqrt[2]*(1 - x
)] + 2*ArcTan[1 - x] + 2*ArcTan[1 + x] - Sqrt[2]*ArcTan[1 - Sqrt[2]*(1 + x
)] + Sqrt[2]*ArcTan[1 + Sqrt[2]*(1 + x)] - Log[1 - Sqrt[2]*(1 - x) + (-1 +
 x)^2]/Sqrt[2] + Log[1 + Sqrt[2]*(1 - x) + (-1 + x)^2]/Sqrt[2] + Log[2 - x
] - 2*Log[x] + Log[2 + x] - Log[1 - Sqrt[2]*(1 + x) + (1 + x)^2]/Sqrt[2] +
 Log[1 + Sqrt[2]*(1 + x) + (1 + x)^2]/Sqrt[2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.24

method	result
risch	$-2\ln(x)+\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(-\_R^2+x^2-2\_R-1\right)\right)+\ln(x^2-4)-2\arctan\left(\frac{x^2}{2}\right)$
default	$\ln(x-2)+2\arctan(1+x)-2\ln(x)+\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+4\_Z^3+6\_Z^2+4\_Z+2)}\frac{\ln(x-\_R)}{\_R^3+3\_R^2+3\_R+1}\right.$

input
```
int((128*x^6+896*x^4+896*x^2+128)/(x^15-8*x^13+28*x^11-56*x^9+68*x^7-112*x
^5-112*x^3-64*x),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(x)+sum(_R*ln(-_R^2+x^2-2*_R-1),_R=RootOf(_Z^4+1))+ln(x^2-4)-2*arctan
(1/2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.59

$$\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx$$
$$= -\sqrt{2}\arctan\left(\sqrt{2}x^2+x^2+1\right)-\sqrt{2}\arctan\left(\sqrt{2}x^2-x^2-1\right)$$
$$-\frac{1}{2}\sqrt{2}\log\left(x^4-2\,x^2+2\,\sqrt{2}(x^2-2)+6\right)+\frac{1}{2}\sqrt{2}\log\left(x^4-2\,x^2-2\,\sqrt{2}(x^2-2)+6\right)$$
$$-2\arctan\left(\frac{1}{2}\,x^2\right)+\log\left(x^2-4\right)-2\log\left(x\right)$$

input
```
integrate((128*x^6+896*x^4+896*x^2+128)/(x^15-8*x^13+28*x^11-56*x^9+68*x^7
-112*x^5-112*x^3-64*x),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(sqrt(2)*x^2 + x^2 + 1) - sqrt(2)*arctan(sqrt(2)*x^2 - x^2
- 1) - 1/2*sqrt(2)*log(x^4 - 2*x^2 + 2*sqrt(2)*(x^2 - 2) + 6) + 1/2*sqrt(2
)*log(x^4 - 2*x^2 - 2*sqrt(2)*(x^2 - 2) + 6) - 2*arctan(1/2*x^2) + log(x^2
 - 4) - 2*log(x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.27

$$\int \frac{128 + 896x^2 + 896x^4 + 128x^6}{-64x - 112x^3 - 112x^5 + 68x^7 - 56x^9 + 28x^{11} - 8x^{13} + x^{15}} \, dx = -2\log(x) + \log(x^2 - 4) + i\log(x^2 - 2i) - i\log(x^2 + 2i) + \operatorname{RootSum}\left(t^4 + 1, \left(t \mapsto t\log\left(-t^2 - 2t + x^2 - 1\right)\right)\right)$$

input
```
integrate((128*x**6+896*x**4+896*x**2+128)/(x**15-8*x**13+28*x**11-56*x**9
+68*x**7-112*x**5-112*x**3-64*x),x)
```

output
```
-2*log(x) + log(x**2 - 4) + I*log(x**2 - 2*I) - I*log(x**2 + 2*I) + RootSu
m(_t**4 + 1, Lambda(_t, _t*log(-_t**2 - 2*_t + x**2 - 1)))
```

Maxima [F]

$$\int \frac{128 + 896x^2 + 896x^4 + 128x^6}{-64x - 112x^3 - 112x^5 + 68x^7 - 56x^9 + 28x^{11} - 8x^{13} + x^{15}} \, dx = \int \frac{128\,(x^6 + 7\,x^4 + 7\,x^2 + 1)}{x^{15} - 8\,x^{13} + 28\,x^{11} - 56\,x^9 + 68\,x^7 - 112\,x^5 - 112\,x^3 - 64\,x} \, dx$$

input
```
integrate((128*x^6+896*x^4+896*x^2+128)/(x^15-8*x^13+28*x^11-56*x^9+68*x^7
-112*x^5-112*x^3-64*x),x, algorithm="maxima")
```

output
```
2*arctan(x + 1) - 2*arctan(x - 1) + 4*integrate(1/(x^4 + 4*x^3 + 6*x^2 + 4
*x + 2), x) - 4*integrate(1/(x^4 - 4*x^3 + 6*x^2 - 4*x + 2), x) + log(x +
2) + log(x - 2) - 2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.12

$$\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx = -2\arctan\left(\frac{1}{2}\,x^2\right)-\log\left(x^2\right)+\log\left(\left|x^2-4\right|\right)$$

input
```
integrate((128*x^6+896*x^4+896*x^2+128)/(x^15-8*x^13+28*x^11-56*x^9+68*x^7
-112*x^5-112*x^3-64*x),x, algorithm="giac")
```

output
```
-2*arctan(1/2*x^2) - log(x^2) + log(abs(x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx$$

$$= 2\,\mathrm{atan}\Bigg(\frac{187197827062294009568444137017003579801600000}{1861\,\left(2669509464018326614341299852165706153984\,x^2+2255097167320371451051024985271\cdots\right.}$$

$$-\frac{15721}{3722}\Bigg)+\ln\left(x^2-4\right)-2\,\ln\left(x\right)$$

$$+\sqrt{2}\,\mathrm{atan}\Bigg(\frac{\sqrt{2}\,(3355740312792431282642475\cdots}{x^2\,\left(173059045693899167869323889017475741777920000-58486988538473719350589843\cdots\right.}$$

$$+\frac{\sqrt{2}\,x^2\,(-41356569179401369608088805152146\cdots}{x^2\,\left(173059045693899167869323889017475741777920000-58486988538473719350589843481476831\cdots\right.}$$

$$-\mathrm{i}\Bigg)$$

$$+\sqrt{2}\,\mathrm{atan}\Bigg(\frac{\sqrt{2}\,(3355740312792431282642475\cdots}{x^2\,\left(173059045693899167869323889017475741777920000+58486988538473719350589843\cdots\right.}$$

$$+\frac{\sqrt{2}\,x^2\,(-41356569179401369608088805152146\cdots}{x^2\,\left(173059045693899167869323889017475741777920000+58486988538473719350589843481476831\cdots\right.}$$

$$+1\mathrm{i}\Bigg)$$

input
```
int(-(896*x^2 + 896*x^4 + 128*x^6 + 128)/(64*x + 112*x^3 + 112*x^5 - 68*x^
7 + 56*x^9 - 28*x^11 + 8*x^13 - x^15),x)
```

output
```
2*atan(187197827062294009568444137017003579801600000/(1861*(26695094640183
2661434129985216570615398 4*x^2 + 225509716732037145105102498527120185098 24
)) - 15721/3722) + log(x^2 - 4) - 2*log(x) + 2^(1/2)*atan((2^(1/2)*(335574
031279243128264247567502040838963 20000 - 678182418460690123114268334088648
02570240000i))/(x^2*(173059045693899167869323889017475741777920000 - 58486
988538473719350589843481476831641600000i) + (959095505402348625970600656 81
222968279040000 + 474572508995768228658845320355907423436800 00i)) - (2^(1/
2)*x^2*(413565691794013696080888051521464723046400 00 + 122371223206902505 3
0039809393794129854464 0000i))/(x^2*(1730590456938991678693238890174757417 7
7920000 - 584869885384737193505898434814768316416 00000i) + (95909550540234
8625970600656812229682790400 00 + 474572508995768228658845320355907423436 80
000i)))*(1 - 1i) + 2^(1/2)*atan((2^(1/2)*(335574031279243128264247567502 04
083896320000 + 678182418460690123114268334088648025702400 00i))/(x^2*(17305
904569389916786932388901747574177792 0000 + 584869885384737193505898434814 7
6831641600000i) + (959095505402348625970600656812229682790400 00 - 47457250
8995768228658845320355907423436800 00i)) - (2^(1/2)*x^2*(413565691794013696
080888051521464723046400 00 - 12237122320690250530039809393794129854464 0000
i))/(x^2*(173059045693899167869323889017475741777920000 + 5848698853847371
935058984348147683164160000 0i) + (959095505402348625970600656812229682790 4
0000 - 474572508995768228658845320355907423436800 00i)))*(1 + 1i)
```

Reduce [F]

$$\int \frac{128+896x^2+896x^4+128x^6}{-64x-112x^3-112x^5+68x^7-56x^9+28x^{11}-8x^{13}+x^{15}}\,dx$$
$$= \frac{12atan(x-1)}{7}-\frac{12atan(x+1)}{7}$$
$$+\frac{11520\left(\int \frac{x^3}{x^{14}-8x^{12}+28x^{10}-56x^8+68x^6-112x^4-112x^2-64}dx\right)}{7}$$
$$+\frac{11904\left(\int \frac{x}{x^{14}-8x^{12}+28x^{10}-56x^8+68x^6-112x^4-112x^2-64}dx\right)}{7}$$
$$+\frac{4224\left(\int \frac{1}{x^{15}-8x^{13}+28x^{11}-56x^9+68x^7-112x^5-112x^3-64x}dx\right)}{7}$$
$$-\frac{4\log(x^8-4x^6+8x^4+8x^2+4)}{7}-\frac{4\log(x^2-2x+2)}{7}$$
$$-\frac{4\log(x^2+2x+2)}{7}-\frac{2\log(x-2)}{7}-\frac{2\log(x+2)}{7}+\frac{52\log(x)}{7}$$

input
```
int((128*x^6+896*x^4+896*x^2+128)/(x^15-8*x^13+28*x^11-56*x^9+68*x^7-112*x
^5-112*x^3-64*x),x)
```

output
```
(2*(6*atan(x - 1) - 6*atan(x + 1) + 5760*int(x**3/(x**14 - 8*x**12 + 28*x*
*10 - 56*x**8 + 68*x**6 - 112*x**4 - 112*x**2 - 64),x) + 5952*int(x/(x**14
 - 8*x**12 + 28*x**10 - 56*x**8 + 68*x**6 - 112*x**4 - 112*x**2 - 64),x) +
 2112*int(1/(x**15 - 8*x**13 + 28*x**11 - 56*x**9 + 68*x**7 - 112*x**5 - 1
12*x**3 - 64*x),x) - 2*log(x**8 - 4*x**6 + 8*x**4 + 8*x**2 + 4) - 2*log(x*
*2 - 2*x + 2) - 2*log(x**2 + 2*x + 2) - log(x - 2) - log(x + 2) + 26*log(x
)))/7
```

3.76 $\int \frac{x^2}{2-4x+2x^2+x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = -\frac{1}{3}\arctan\left(1+\sqrt{2}-2x-\sqrt{2}x\right) - \frac{1}{3}\arctan\left(1-\sqrt{2}-2x+\sqrt{2}x\right) + \frac{\log\left(2-\sqrt{2}-\sqrt{2}x+x^2\right)}{6\sqrt{2}} - \frac{\log\left(2+\sqrt{2}+\sqrt{2}x+x^2\right)}{6\sqrt{2}}$$

output

```
1/3*arctan(x*2^(1/2)-2^(1/2)+2*x-1)-1/3*arctan(1-2^(1/2)-2*x+x*2^(1/2))+1/
12*ln(2-2^(1/2)-x*2^(1/2)+x^2)*2^(1/2)-1/12*ln(2+2^(1/2)+x*2^(1/2)+x^2)*2^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \frac{1}{4}\text{RootSum}\left[2-4\#1+2\#1^2+\#1^4\&,\frac{\log(x-\#1)\#1^2}{-1+\#1+\#1^3}\&\right]$$

input
```
Integrate[x^2/(2 - 4*x + 2*x^2 + x^4),x]
```

output
```
RootSum[2 - 4*#1 + 2*#1^2 + #1^4 & , (Log[x - #1]*#1^2)/(-1 + #1 + #1^3) &
 ]/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+2x^2-4x+2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^2}{x^4+2x^2-4x+2}dx$$

input
```
Int[x^2/(2 - 4*x + 2*x^2 + x^4),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2-4\_Z+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3+\_R-1}\right)}{4}$	38
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2-4\_Z+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3+\_R-1}\right)}{4}$	38

input
```
int(x^2/(x^4+2*x^2-4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(_R^3+_R-1)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2-4*_Z+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = -\frac{1}{12}\sqrt{2}\log\left(x^2+\sqrt{2}(x+1)+2\right) + \frac{1}{12}\sqrt{2}\log\left(x^2-\sqrt{2}(x+1)+2\right) + \frac{1}{3}\arctan\left(\sqrt{2}(x-1)+2x-1\right) - \frac{1}{3}\arctan\left(\sqrt{2}(x-1)-2x+1\right)$$

input
```
integrate(x^2/(x^4+2*x^2-4*x+2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*log(x^2 + sqrt(2)*(x + 1) + 2) + 1/12*sqrt(2)*log(x^2 - sqrt
(2)*(x + 1) + 2) + 1/3*arctan(sqrt(2)*(x - 1) + 2*x - 1) - 1/3*arctan(sqrt
(2)*(x - 1) - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x-\sqrt{2}+2\right)}{12} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+\sqrt{2}+2\right)}{12} - \frac{\operatorname{atan}\left(\frac{2x}{-2+\sqrt{2}}-\frac{\sqrt{2}}{-2+\sqrt{2}}\right)}{3} + \frac{\operatorname{atan}\left(\frac{2x}{\sqrt{2}+2}+\frac{\sqrt{2}}{\sqrt{2}+2}\right)}{3}$$

input
```
integrate(x**2/(x**4+2*x**2-4*x+2),x)
```

output
```
sqrt(2)*log(x**2 - sqrt(2)*x - sqrt(2) + 2)/12 - sqrt(2)*log(x**2 + sqrt(2
)*x + sqrt(2) + 2)/12 - atan(2*x/(-2 + sqrt(2)) - sqrt(2)/(-2 + sqrt(2)))/
3 + atan(2*x/(sqrt(2) + 2) + sqrt(2)/(sqrt(2) + 2))/3
```

Maxima [F]

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \int \frac{x^2}{x^4+2\,x^2-4\,x+2}\,dx$$

input
```
integrate(x^2/(x^4+2*x^2-4*x+2),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^4 + 2*x^2 - 4*x + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \frac{1}{6}\pi - \frac{1}{12}\sqrt{2}\log\left(\left(x+\sqrt{2}+1\right)^2+(x-1)^2\right) + \frac{1}{12}\sqrt{2}\log\left(\left(x-\sqrt{2}+1\right)^2+(x-1)^2\right) + \frac{1}{3}\arctan\left(x\left(\sqrt{2}+2\right)-\sqrt{2}-1\right) + \frac{1}{3}\arctan\left(-x\left(\sqrt{2}-2\right)+\sqrt{2}-1\right)$$

input
```
integrate(x^2/(x^4+2*x^2-4*x+2),x, algorithm="giac")
```

output
```
1/6*pi - 1/12*sqrt(2)*log((x + sqrt(2) + 1)^2 + (x - 1)^2) + 1/12*sqrt(2)*
log((x - sqrt(2) + 1)^2 + (x - 1)^2) + 1/3*arctan(x*(sqrt(2) + 2) - sqrt(2
) - 1) + 1/3*arctan(-x*(sqrt(2) - 2) + sqrt(2) - 1)
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.44

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \text{Too large to display}$$

input
```
int(x^2/(2*x^2 - 4*x + x^4 + 2),x)
```

output
```
atan(((2^(1/2)/12 - 1i/6)*((2^(1/2)/12 - 1i/6)*((2^(1/2)/12 - 1i/6)*(512*x
 - 448) - 32*x + 48) - 8)*1i + (2^(1/2)/12 - 1i/6)*((2^(1/2)/12 - 1i/6)*(3
2*x + (2^(1/2)/12 - 1i/6)*(512*x - 448) - 48) - 8)*1i)/(8*x + (2^(1/2)/12
- 1i/6)*((2^(1/2)/12 - 1i/6)*((2^(1/2)/12 - 1i/6)*(512*x - 448) - 32*x + 4
8) - 8) - (2^(1/2)/12 - 1i/6)*((2^(1/2)/12 - 1i/6)*(32*x + (2^(1/2)/12 - 1
i/6)*(512*x - 448) - 48) - 8) - 4))*((2^(1/2)*1i)/6 + 1/3) + atan(((2^(1/2
)/12 + 1i/6)*((2^(1/2)/12 + 1i/6)*((2^(1/2)/12 + 1i/6)*(512*x - 448) - 32*
x + 48) - 8)*1i + (2^(1/2)/12 + 1i/6)*((2^(1/2)/12 + 1i/6)*(32*x + (2^(1/2
)/12 + 1i/6)*(512*x - 448) - 48) - 8)*1i)/(8*x + (2^(1/2)/12 + 1i/6)*((2^(
1/2)/12 + 1i/6)*((2^(1/2)/12 + 1i/6)*(512*x - 448) - 32*x + 48) - 8) - (2^
(1/2)/12 + 1i/6)*((2^(1/2)/12 + 1i/6)*(32*x + (2^(1/2)/12 + 1i/6)*(512*x -
 448) - 48) - 8) - 4))*((2^(1/2)*1i)/6 - 1/3)
```

Reduce [F]

$$\int \frac{x^2}{2-4x+2x^2+x^4}\,dx = \int \frac{x^2}{x^4+2x^2-4x+2}dx$$

input
```
int(x^2/(x^4+2*x^2-4*x+2),x)
```

output
```
int(x**2/(x**4 + 2*x**2 - 4*x + 2),x)
```

3.77 $\int \frac{-21504x^3-3072x^{11}-63744x^{19}-3840x^{27}}{-1024-192x^8-8688x^{16}-1632x^{24}+72x^{32}-12x^{40}+x^{48}}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 327

$$\begin{aligned}
&\int \frac{-21504x^3-3072x^{11}-63744x^{19}-3840x^{27}}{-1024-192x^8-8688x^{16}-1632x^{24}+72x^{32}-12x^{40}+x^{48}}\,dx\\
&= -2\arctan\left(1-x^2\right)-2\arctan\left(1+x^2\right)+\frac{\arctan\left(1-\sqrt{2}-\sqrt{2}x^2\right)}{\sqrt{2}}\\
&\quad-\frac{\arctan\left(1+\sqrt{2}-\sqrt{2}x^2\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}-2x^2-\sqrt{2}x^2\right)}{\sqrt{2}}\\
&\quad+\frac{\arctan\left(1-\sqrt{2}+2x^2-\sqrt{2}x^2\right)}{\sqrt{2}}+\frac{\arctan\left(1-\sqrt{2}+\sqrt{2}x^2\right)}{\sqrt{2}}\\
&\quad-\frac{\arctan\left(1+\sqrt{2}+\sqrt{2}x^2\right)}{\sqrt{2}}+\frac{\arctan\left(1-\sqrt{2}-2x^2+\sqrt{2}x^2\right)}{\sqrt{2}}\\
&\quad-\frac{\arctan\left(1+\sqrt{2}+2x^2+\sqrt{2}x^2\right)}{\sqrt{2}}+\operatorname{arctanh}\left(\frac{x^4}{4}\right)\\
&\quad-\frac{\log\left(4-8\sqrt{2}x^4+16x^8-4\sqrt{2}x^{12}+x^{16}\right)}{2\sqrt{2}}\\
&\quad+\frac{\log\left(4+8\sqrt{2}x^4+16x^8+4\sqrt{2}x^{12}+x^{16}\right)}{2\sqrt{2}}
\end{aligned}$$

output
```
2*arctan(x^2-1)-2*arctan(x^2+1)-1/2*arctan(-1+2^(1/2)+x^2*2^(1/2))*2^(1/2)
+1/2*arctan(-1-2^(1/2)+x^2*2^(1/2))*2^(1/2)+1/2*arctan(x^2*2^(1/2)+2*x^2-2
^(1/2)-1)*2^(1/2)-1/2*arctan(x^2*2^(1/2)-2*x^2+2^(1/2)-1)*2^(1/2)+1/2*arct
an(1-2^(1/2)+x^2*2^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)+x^2*2^(1/2))*2^(1/2
)+1/2*arctan(1-2^(1/2)-2*x^2+x^2*2^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)+2*x
^2+x^2*2^(1/2))*2^(1/2)+arctanh(1/4*x^4)-1/4*ln(4-8*2^(1/2)*x^4+16*x^8-4*2
^(1/2)*x^12+x^16)*2^(1/2)+1/4*ln(4+8*2^(1/2)*x^4+16*x^8+4*2^(1/2)*x^12+x^1
6)*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.87

$$\int \frac{-21504x^3-3072x^{11}-63744x^{19}-3840x^{27}}{-1024-192x^8-8688x^{16}-1632x^{24}+72x^{32}-12x^{40}+x^{48}}\,dx$$
$$= \frac{1}{2}\left(-4\arctan\left(1-x^2\right)-4\arctan\left(1+x^2\right)-\log\left(2-x^2\right)-\log\left(2+x^2\right)+\log\left(2-2x+x^2\right)+\log\left(2+2x+x^2\right)-\text{RootSum}\left[2-4\#1^2+2\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-1+\#1^2+\#1^6}\&\right]+\text{RootSum}\left[2+4\#1^2+2\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{1+\#1^2+\#1^6}\&\right]+\text{RootSum}\left[2-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log(x-\#1)}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]-\text{RootSum}\left[2+4\#1^2+6\#1^4+4\#1^6+\#1^8\&,\frac{\log(x-\#1)}{1+3\#1^2+3\#1^4+\#1^6}\&\right]\right)$$

input
```
Integrate[(-21504*x^3 - 3072*x^11 - 63744*x^19 - 3840*x^27)/(-1024 - 192*x
^8 - 8688*x^16 - 1632*x^24 + 72*x^32 - 12*x^40 + x^48),x]
```

output
```
(-4*ArcTan[1 - x^2] - 4*ArcTan[1 + x^2] - Log[2 - x^2] - Log[2 + x^2] + Lo
g[2 - 2*x + x^2] + Log[2 + 2*x + x^2] - RootSum[2 - 4*#1^2 + 2*#1^4 + #1^8
 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1 + #1^2 + #1^6) & ] + RootSum[2
+ 4*#1^2 + 2*#1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(1 + #1^2 +
 #1^6) & ] + RootSum[2 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , Log[x - #1]/(
-1 + 3*#1^2 - 3*#1^4 + #1^6) & ] - RootSum[2 + 4*#1^2 + 6*#1^4 + 4*#1^6 +
#1^8 & , Log[x - #1]/(1 + 3*#1^2 + 3*#1^4 + #1^6) & ])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-3840x^{27}-63744x^{19}-3072x^{11}-21504x^3}{x^{48}-12x^{40}+72x^{32}-1632x^{24}-8688x^{16}-192x^8-1024}\,dx$$

$$\Big\downarrow \text{ 2029}$$

$$\int \frac{x^3\left(-3840x^{24}-63744x^{16}-3072x^8-21504\right)}{x^{48}-12x^{40}+72x^{32}-1632x^{24}-8688x^{16}-192x^8-1024}dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{x-1}{x^2-2x+2}-\frac{x}{x^2-2}-\frac{x}{x^2+2}+\frac{x+1}{x^2+2x+2}+\frac{4x}{x^4-2x^2+2}-\frac{4x}{x^4+2x^2+2}-\frac{4x\left(x^4-1\right)}{x^8+2x^4-4x^2+2}+\frac{}{x^8}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-2\text{Subst}\left(\int \frac{x^2}{x^4+2x^2-4x+2}dx,x,x^2\right)+2\text{Subst}\left(\int \frac{x^2}{x^4+2x^2+4x+2}dx,x,x^2\right)+\frac{2}{3}\arctan\left(-\frac{2}{x^2}-\sqrt{2}+1\right)+\frac{2}{3}\arctan\left(-\frac{2}{x^2}+\sqrt{2}+1\right)+\frac{2}{3}\arctan\left(\frac{2}{x^2}-\sqrt{2}+1\right)+\frac{2}{3}\arctan\left(\frac{2}{x^2}+\sqrt{2}+1\right)-2\arctan\left(1-x^2\right)-2\arctan\left(x^2+1\right)+\frac{\arctan\left(1-\sqrt{2}(1-x^2)\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}(1-x^2)+1\right)}{\sqrt{2}}+\frac{\arctan\left(1-\sqrt{2}(x^2+1)\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}(x^2+1)+1\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}(1-x^2)}{x^4}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}(x^2+1)}{x^4}\right)}{\sqrt{2}}-\frac{1}{2}\log\left(2-x^2\right)-\frac{1}{2}\log\left(x^2+2\right)+\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(x^2+2x+2\right)+\frac{\log\left(\left(x^2-1\right)^2-\sqrt{2}(1-x^2)+1\right)}{2\sqrt{2}}-\frac{\log\left(\left(x^2-1\right)^2+\sqrt{2}(1-x^2)+1\right)}{2\sqrt{2}}+\frac{\log\left(\left(x^2+1\right)^2-\sqrt{2}(x^2+1)+1\right)}{2\sqrt{2}}-\frac{\log\left(\left(x^2+1\right)^2+\sqrt{2}(x^2+1)+1\right)}{2\sqrt{2}}+\frac{\log\left(\frac{-\sqrt{2}x^4+2x^4-2\sqrt{2}x^2+2x^2+2}{x^4}\right)}{6\sqrt{2}}+\frac{\log\left(\frac{-\sqrt{2}x^4+2x^4+2\sqrt{2}x^2-2x^2+2}{x^4}\right)}{6\sqrt{2}}-\frac{\log\left(\frac{\left(2+\sqrt{2}\right)x^4-2\left(1+\sqrt{2}\right)x^2+2}{x^4}\right)}{6\sqrt{2}}-\frac{\log\left(\frac{\left(2+\sqrt{2}\right)x^4+2\left(1+\sqrt{2}\right)x^2+2}{x^4}\right)}{6\sqrt{2}}$$

input
```
Int[(-21504*x^3 - 3072*x^11 - 63744*x^19 - 3840*x^27)/(-1024 - 192*x^8 - 8
688*x^16 - 1632*x^24 + 72*x^32 - 12*x^40 + x^48),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.17

method	result
risch	$-\frac{\ln(x^4-4)}{2}+2\arctan\left(\frac{x^4}{2}\right)+\frac{\ln(x^4+4)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(x^8+4\_Rx^4+2\_R^2\right)\right)}{2}$
default	$2\arctan\left(\frac{x^4}{2}\right)+2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4-4\_Z^3+8\_Z^2+8\_Z+4)}\frac{(1+\_R)\ln(x^4-\_R)}{\_R^3-3\_R^2+4\_R+2}\right)-\frac{\ln(x^4-4)}{2}+\frac{\ln(x^4+4)}{2}$

input
```
int((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^32-1632
*x^24-8688*x^16-192*x^8-1024),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^4-4)+2*arctan(1/2*x^4)+1/2*ln(x^4+4)+1/2*sum(_R*ln(x^8+4*_R*x^4+
2*_R^2),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.52

$$\int \frac{-21504x^3-3072x^{11}-63744x^{19}-3840x^{27}}{-1024-192x^8-8688x^{16}-1632x^{24}+72x^{32}-12x^{40}+x^{48}}\,dx$$
$$=\frac{1}{2}\sqrt{2}\arctan\left(\frac{7}{6}x^8+\frac{1}{6}\sqrt{2}(x^{12}+14x^4)+\frac{4}{3}\right)$$
$$+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{7}{6}x^8+\frac{1}{6}\sqrt{2}(x^{12}+14x^4)-\frac{4}{3}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}x^4+\frac{3}{4}\right)$$
$$+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}x^4-\frac{3}{4}\right)+\frac{1}{4}\sqrt{2}\log\left(x^{16}+16x^8+4\sqrt{2}(x^{12}+2x^4)+4\right)$$
$$-\frac{1}{4}\sqrt{2}\log\left(x^{16}+16x^8-4\sqrt{2}(x^{12}+2x^4)+4\right)$$
$$+2\arctan\left(\frac{1}{2}x^4\right)+\frac{1}{2}\log\left(x^4+4\right)-\frac{1}{2}\log\left(x^4-4\right)$$

input
```
integrate((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^3
2-1632*x^24-8688*x^16-192*x^8-1024),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(7/6*x^8 + 1/6*sqrt(2)*(x^12 + 14*x^4) + 4/3) + 1/2*sqrt
(2)*arctan(-7/6*x^8 + 1/6*sqrt(2)*(x^12 + 14*x^4) - 4/3) + 1/2*sqrt(2)*arc
tan(1/4*sqrt(2)*x^4 + 3/4) + 1/2*sqrt(2)*arctan(1/4*sqrt(2)*x^4 - 3/4) + 1
/4*sqrt(2)*log(x^16 + 16*x^8 + 4*sqrt(2)*(x^12 + 2*x^4) + 4) - 1/4*sqrt(2)
*log(x^16 + 16*x^8 - 4*sqrt(2)*(x^12 + 2*x^4) + 4) + 2*arctan(1/2*x^4) + 1
/2*log(x^4 + 4) - 1/2*log(x^4 - 4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.69

$$\int \frac{-21504x^3-3072x^{11}-63744x^{19}-3840x^{27}}{-1024-192x^8-8688x^{16}-1632x^{24}+72x^{32}-12x^{40}+x^{48}}\,dx = -\frac{\sqrt{2}\left(-2\operatorname{atan}\left(\frac{\sqrt{2}x^4}{4}-\frac{3}{4}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}x^{12}}{6}-\frac{7x^8}{6}+\frac{7\sqrt{2}x^4}{3}-\frac{4}{3}\right)\right)}{4} - \frac{\sqrt{2}\left(-2\operatorname{atan}\left(\frac{\sqrt{2}x^4}{4}+\frac{3}{4}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}x^{12}}{6}+\frac{7x^8}{6}+\frac{7\sqrt{2}x^4}{3}+\frac{4}{3}\right)\right)}{4} - \frac{\log(x^4-4)}{2}+\frac{\log(x^4+4)}{2}-\frac{\sqrt{2}\log\left(x^{16}-4\sqrt{2}x^{12}+16x^8-8\sqrt{2}x^4+4\right)}{4} + \frac{\sqrt{2}\log\left(x^{16}+4\sqrt{2}x^{12}+16x^8+8\sqrt{2}x^4+4\right)}{4}+2\operatorname{atan}\left(\frac{x^4}{2}\right)$$

input
```
integrate((-3840*x**27-63744*x**19-3072*x**11-21504*x**3)/(x**48-12*x**40+
72*x**32-1632*x**24-8688*x**16-192*x**8-1024),x)
```

output
```
-sqrt(2)*(-2*atan(sqrt(2)*x**4/4 - 3/4) - 2*atan(sqrt(2)*x**12/6 - 7*x**8/
6 + 7*sqrt(2)*x**4/3 - 4/3))/4 - sqrt(2)*(-2*atan(sqrt(2)*x**4/4 + 3/4) -
2*atan(sqrt(2)*x**12/6 + 7*x**8/6 + 7*sqrt(2)*x**4/3 + 4/3))/4 - log(x**4
- 4)/2 + log(x**4 + 4)/2 - sqrt(2)*log(x**16 - 4*sqrt(2)*x**12 + 16*x**8 -
 8*sqrt(2)*x**4 + 4)/4 + sqrt(2)*log(x**16 + 4*sqrt(2)*x**12 + 16*x**8 + 8
*sqrt(2)*x**4 + 4)/4 + 2*atan(x**4/2)
```

Maxima [F]

$$\int \frac{-21504x^3 - 3072x^{11} - 63744x^{19} - 3840x^{27}}{-1024 - 192x^8 - 8688x^{16} - 1632x^{24} + 72x^{32} - 12x^{40} + x^{48}}\,dx$$
$$= \int -\frac{768\,(5\,x^{27} + 83\,x^{19} + 4\,x^{11} + 28\,x^3)}{x^{48} - 12\,x^{40} + 72\,x^{32} - 1632\,x^{24} - 8688\,x^{16} - 192\,x^8 - 1024}\,dx$$

input
```
integrate((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^3
2-1632*x^24-8688*x^16-192*x^8-1024),x, algorithm="maxima")
```

output
```
4*integrate((x^5 - x)/(x^8 + 2*x^4 + 4*x^2 + 2), x) - 4*integrate((x^5 - x
)/(x^8 + 2*x^4 - 4*x^2 + 2), x) - 4*integrate(x/(x^8 + 4*x^6 + 6*x^4 + 4*x
^2 + 2), x) + 4*integrate(x/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 2), x) - 4*inte
grate(x/(x^4 + 2*x^2 + 2), x) + 4*integrate(x/(x^4 - 2*x^2 + 2), x) + 1/2*
log(x^2 + 2*x + 2) + 1/2*log(x^2 - 2*x + 2) - 1/2*log(x^2 + 2) - 1/2*log(x
^2 - 2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{-21504x^3 - 3072x^{11} - 63744x^{19} - 3840x^{27}}{-1024 - 192x^8 - 8688x^{16} - 1632x^{24} + 72x^{32} - 12x^{40} + x^{48}}\,dx = \text{Timed out}$$

input
```
integrate((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^3
2-1632*x^24-8688*x^16-192*x^8-1024),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.25

$$\int \frac{-21504x^3 - 3072x^{11} - 63744x^{19} - 3840x^{27}}{-1024 - 192x^8 - 8688x^{16} - 1632x^{24} + 72x^{32} - 12x^{40} + x^{48}}\,dx = \text{Too large to display}$$

input
```
int((21504*x^3 + 3072*x^11 + 63744*x^19 + 3840*x^27)/(192*x^8 + 8688*x^16
+ 1632*x^24 - 72*x^32 + 12*x^40 - x^48 + 1024),x)
```

output
```
2*atan(x^4/2) - atan((x^4*1i)/4)*1i - 2^(1/2)*atan((2^(1/2)*x^4*(764886904
45726928928043971784974348792916843261154049524815686201788737984099646078
28394633776665441993199104289579518086625046676785965701785495403705254975
311367263415128406680534025128252141595525120000000000000000000 + 40898633798
79310808240510565117378645620131517107279988428287019008178399655663770556
23104992286116205196899694450685882227368471498014546349028880705248228162
11180533360229681678688376601795780988108800000000000000000000i))/(x^8*(293468
31061130009252612269359037033812279539608056712352274639097967630495164070
94596156420924881651011292025308521594227224682847941527791318946052689434
314233121329187787843712100715697107929923584000000000000000000000 - 88975141
61733455211409716533450140584178882022520312410133204002926738496885752093
18040895963451069985307526836268164816087840490451455059573801649163193173
422562302519776359628166296867018140874178560000000000000000000i) - (17795028
32346691042281943306690028116835776404504062482026640800585347699377150418
63608179192690213997061505367253632963217568098090291011914760329832638634
684512460503955271925633259373403628174835712000000000000000000 + 5869366212
22600185052245387180740676245590792161134247045492781959352609903281418919
23128418497633020225840506170431884544493656958830555826378921053788686284
662426583755756874242014313942158598471680000000000000000000i)))*(1/2 + 1i/
2) - 2^(1/2)*atan((2^(1/2)*x^4*(7648869044572692892804397178497434879 29...
```

Reduce [F]

$$\int \frac{-21504x^3 - 3072x^{11} - 63744x^{19} - 3840x^{27}}{-1024 - 192x^8 - 8688x^{16} - 1632x^{24} + 72x^{32} - 12x^{40} + x^{48}}\,dx$$
$$= \int \frac{-3840x^{27} - 63744x^{19} - 3072x^{11} - 21504x^3}{x^{48} - 12x^{40} + 72x^{32} - 1632x^{24} - 8688x^{16} - 192x^8 - 1024}dx$$

input
```
int((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^32-1632
*x^24-8688*x^16-192*x^8-1024),x)
```

output
```
int((-3840*x^27-63744*x^19-3072*x^11-21504*x^3)/(x^48-12*x^40+72*x^32-1632
*x^24-8688*x^16-192*x^8-1024),x)
```

3.78 $\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$

Optimal result

Integrand size = 80, antiderivative size = 484

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$$
$$= -\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x^2\right)$$
$$-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x^2\right)$$
$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x+\frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^3\right)$$
$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{3}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x-\sqrt{-\frac{11}{8}+\frac{5\sqrt{5}}{8}}x^3+\sqrt{-2+\sqrt{5}}x^5\right.$$
$$\left.+\frac{1}{2}\sqrt{-2+\sqrt{5}}x^7\right)$$
$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^2\right)$$
$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^2\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\log\left(-1+\sqrt{5}-2\sqrt{2\left(-1+\sqrt{5}\right)}x+2x^2+2x^4\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\log\left(-1+\sqrt{5}+2\sqrt{2\left(-1+\sqrt{5}\right)}x+2x^2+2x^4\right)$$

output
```
1/2*(-2+2*5^(1/2))^(1/2)*arctan(-1/2*(2+2*5^(1/2))^(1/2)*x+1/2*(2+2*5^(1/2
))^(1/2)*x^2)-1/2*(-2+2*5^(1/2))^(1/2)*arctan(1/2*(2+2*5^(1/2))^(1/2)*x+1/
2*(2+2*5^(1/2))^(1/2)*x^2)-1/2*(2+2*5^(1/2))^(1/2)*arctan(1/4*(-2+2*5^(1/2
))^(1/2)*x+1/4*(-2+2*5^(1/2))^(1/2)*x^3)-1/2*(2+2*5^(1/2))^(1/2)*arctan(3/
4*(-2+2*5^(1/2))^(1/2)*x-1/4*(-22+10*5^(1/2))^(1/2)*x^3+(-2+5^(1/2))^(1/2)
*x^5+1/2*(-2+5^(1/2))^(1/2)*x^7)+1/2*(2+2*5^(1/2))^(1/2)*arctanh(-1/2*(-2+
2*5^(1/2))^(1/2)*x+1/2*(-2+2*5^(1/2))^(1/2)*x^2)-1/2*(2+2*5^(1/2))^(1/2)*a
rctanh(1/2*(-2+2*5^(1/2))^(1/2)*x+1/2*(-2+2*5^(1/2))^(1/2)*x^2)+1/4*(-2+2*
5^(1/2))^(1/2)*ln(-1+5^(1/2)-2*(-2+2*5^(1/2))^(1/2)*x+2*x^2+2*x^4)-1/4*(-2
+2*5^(1/2))^(1/2)*ln(-1+5^(1/2)+2*(-2+2*5^(1/2))^(1/2)*x+2*x^2+2*x^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.91

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$$
$$= \frac{1}{2}\Bigg(-\text{RootSum}\Big[-1-\#1^2+2\#1^3-4\#1^5+6\#1^6-4\#1^7+\#1^8\&,\frac{2\log(x-\#1)+\log(x-\#1)\#1^2-2\log(x-\#1)\#1^3+\log(x-\#1)\#1^4}{\#1-\#1^2-2\#1^3+6\#1^4-6\#1^5+2\#1^6}\&\Big]$$
$$+\text{RootSum}\Big[-1-\#1^2-2\#1^3+4\#1^5+6\#1^6+4\#1^7+\#1^8\&,\frac{2\log(x-\#1)+\log(x-\#1)\#1^2+2\log(x-\#1)\#1^3+\log(x-\#1)\#1^4}{-\#1-\#1^2+2\#1^3+6\#1^4+6\#1^5+2\#1^6}\&\Big]$$
$$-\text{RootSum}\Big[1-2\#1^2+\#1^4+16\#1^6+2\#1^8+2\#1^{10}+4\#1^{12}+4\#1^{14}+\#1^{16}\&,\frac{2\log(x-\#1)-3\log(x-\#1)\#1^2-2\log(x-\#1)\#1^4-\log(x-\#1)\#1^6+20\log(x-\#1)}{-\#1+\#1^3+24\#1^5+4\#1^7+5\#1^9+12\#1^{11}+14\#1}$$

input
```
Integrate[(-8 + 24*x^4 - 272*x^8 - 252*x^12 + 244*x^16 - 296*x^20 - 16*x^2
4 - 12*x^28)/(1 - 2*x^4 + 69*x^8 - 236*x^12 - 34*x^16 - 114*x^20 + 4*x^24
- 8*x^28 + x^32),x]
```

output
```
(-RootSum[-1 - #1^2 + 2*#1^3 - 4*#1^5 + 6*#1^6 - 4*#1^7 + #1^8 & , (2*Log[
x - #1] + Log[x - #1]*#1^2 - 2*Log[x - #1]*#1^3 + Log[x - #1]*#1^4)/(#1 -
#1^2 - 2*#1^3 + 6*#1^4 - 6*#1^5 + 2*#1^6) & ] + RootSum[-1 - #1^2 - 2*#1^3
 + 4*#1^5 + 6*#1^6 + 4*#1^7 + #1^8 & , (2*Log[x - #1] + Log[x - #1]*#1^2 +
 2*Log[x - #1]*#1^3 + Log[x - #1]*#1^4)/(-#1 - #1^2 + 2*#1^3 + 6*#1^4 + 6*
#1^5 + 2*#1^6) & ] - RootSum[1 - 2*#1^2 + #1^4 + 16*#1^6 + 2*#1^8 + 2*#1^1
0 + 4*#1^12 + 4*#1^14 + #1^16 & , (2*Log[x - #1] - 3*Log[x - #1]*#1^2 - 2*
Log[x - #1]*#1^4 - Log[x - #1]*#1^6 + 20*Log[x - #1]*#1^8 + 7*Log[x - #1]*
#1^10 + 3*Log[x - #1]*#1^12)/(-#1 + #1^3 + 24*#1^5 + 4*#1^7 + 5*#1^9 + 12*
#1^11 + 14*#1^13 + 4*#1^15) & ])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-12x^{28}-16x^{24}-296x^{20}+244x^{16}-252x^{12}-272x^8+24x^4-8}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}\,dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{-2x^5+5x^4-4x^3+x^2-4x+2}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}+\frac{2x^5+5x^4+4x^3+x^2+4x+2}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}-\frac{2(3x^{12}+7x^{10}+20x}{x^{16}+4x^{14}+4x^{12}+2x^{10}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\int \frac{1}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx-4\int \frac{x}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx+\int \frac{x^2}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx-4\int \frac{x^3}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx+5\int \frac{x^4}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx-2\int \frac{x^5}{x^8-4x^7+6x^6-4x^5+2x^3-x^2-1}dx+2\int \frac{1}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx+4\int \frac{x}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx+\int \frac{x^2}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx+4\int \frac{x^3}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx+5\int \frac{x^4}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx+2\int \frac{x^5}{x^8+4x^7+6x^6+4x^5-2x^3-x^2-1}dx-4\int \frac{1}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx+6\int \frac{x^2}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx+4\int \frac{x^4}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx+2\int \frac{x^6}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx-40\int \frac{x^8}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx-14\int \frac{x^{10}}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx-6\int \frac{x^{12}}{x^{16}+4x^{14}+4x^{12}+2x^{10}+2x^8+16x^6+x^4-2x^2+1}dx$$

input
```
Int[(-8 + 24*x^4 - 272*x^8 - 252*x^12 + 244*x^16 - 296*x^20 - 16*x^24 - 12
*x^28)/(1 - 2*x^4 + 69*x^8 - 236*x^12 - 34*x^16 - 114*x^20 + 4*x^24 - 8*x^
28 + x^32),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(x^4-\_R^2+2\_Rx-x^2\right)\right)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(x^4+\_R^2-2\_Rx+x\right.\right.}{2}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(x^2-\_R+x\right)\right)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_R\ln\left(x^2+\_R-x\right)\right)}{2}+\frac{\left(\_R=\mathrm{RootO}\right.}{}$

input
```
int((-12*x^28-16*x^24-296*x^20+244*x^16-252*x^12-272*x^8+24*x^4-8)/(x^32-8
*x^28+4*x^24-114*x^20-34*x^16-236*x^12+69*x^8-2*x^4+1),x,method=_RETURNVER
BOSE)
```

output
```
1/2*sum(_R*ln(x^4-_R^2+2*_R*x-x^2),_R=RootOf(_Z^4-_Z^2-1))+1/2*sum(_R*ln(x
^4+_R^2-2*_R*x+x^2),_R=RootOf(_Z^4+_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.75

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$$
$$= \sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\arctan\left(-\frac{1}{4}\left(3\,x^7+6\,x^5+4\,x^3-\sqrt{5}(x^7+2\,x^5+2\,x^3-3\,x)-3\,x\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\arctan\left(-\frac{1}{4}\left(x^3-\sqrt{5}(x^3+x)+x\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\arctan\left(\frac{1}{4}\left(3\,x^7-6\,x^5+4\,x^3+\sqrt{5}(x^7-2\,x^5+2\,x^3+3\,x)+3\,x\right)\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)$$
$$+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\arctan\left(\frac{1}{4}\left(x^3+\sqrt{5}(x^3-x)-x\right)\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\log\left(2\,x^4+2\,x^2+4\,x\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\sqrt{5}-1\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\log\left(2\,x^4+2\,x^2-4\,x\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\sqrt{5}-1\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\log\left(2\,x^4-2\,x^2+4\,x\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}-\sqrt{5}-1\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\log\left(2\,x^4-2\,x^2-4\,x\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}-\sqrt{5}-1\right)$$

input
```
integrate((-12*x^28-16*x^24-296*x^20+244*x^16-252*x^12-272*x^8+24*x^4-8)/(
x^32-8*x^28+4*x^24-114*x^20-34*x^16-236*x^12+69*x^8-2*x^4+1),x, algorithm=
"fricas")
```

output
```
sqrt(1/2*sqrt(5) + 1/2)*arctan(-1/4*(3*x^7 + 6*x^5 + 4*x^3 - sqrt(5)*(x^7
+ 2*x^5 + 2*x^3 - 3*x) - 3*x)*sqrt(1/2*sqrt(5) + 1/2)) - sqrt(1/2*sqrt(5)
+ 1/2)*arctan(-1/4*(x^3 - sqrt(5)*(x^3 + x) + x)*sqrt(1/2*sqrt(5) + 1/2))
- sqrt(1/2*sqrt(5) - 1/2)*arctan(1/4*(3*x^7 - 6*x^5 + 4*x^3 + sqrt(5)*(x^7
 - 2*x^5 + 2*x^3 + 3*x) + 3*x)*sqrt(1/2*sqrt(5) - 1/2)) + sqrt(1/2*sqrt(5)
 - 1/2)*arctan(1/4*(x^3 + sqrt(5)*(x^3 - x) - x)*sqrt(1/2*sqrt(5) - 1/2))
- 1/2*sqrt(1/2*sqrt(5) - 1/2)*log(2*x^4 + 2*x^2 + 4*x*sqrt(1/2*sqrt(5) - 1
/2) + sqrt(5) - 1) + 1/2*sqrt(1/2*sqrt(5) - 1/2)*log(2*x^4 + 2*x^2 - 4*x*s
qrt(1/2*sqrt(5) - 1/2) + sqrt(5) - 1) + 1/2*sqrt(1/2*sqrt(5) + 1/2)*log(2*
x^4 - 2*x^2 + 4*x*sqrt(1/2*sqrt(5) + 1/2) - sqrt(5) - 1) - 1/2*sqrt(1/2*sq
rt(5) + 1/2)*log(2*x^4 - 2*x^2 - 4*x*sqrt(1/2*sqrt(5) + 1/2) - sqrt(5) - 1
)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.21

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx = -\operatorname{RootSum}\left(16t^4-4t^2-1,\left(t\mapsto t\log\left(-16t^4+x^4+x^2\left(-64t^6+8t^2\right)+x\left(-256t^7+32t^3\right)+1\right)\right)\right) - \operatorname{RootSum}\left(16t^4+4t^2-1,\left(t\mapsto t\log\left(-16t^4+x^4+x^2\left(-64t^6+8t^2\right)+x\left(-256t^7+32t^3\right)+1\right)\right)\right)$$

input
```
integrate((-12*x**28-16*x**24-296*x**20+244*x**16-252*x**12-272*x**8+24*x*
*4-8)/(x**32-8*x**28+4*x**24-114*x**20-34*x**16-236*x**12+69*x**8-2*x**4+1
),x)
```

output
```
-RootSum(16*_t**4 - 4*_t**2 - 1, Lambda(_t, _t*log(-16*_t**4 + x**4 + x**2
*(-64*_t**6 + 8*_t**2) + x*(-256*_t**7 + 32*_t**3) + 1))) - RootSum(16*_t*
*4 + 4*_t**2 - 1, Lambda(_t, _t*log(-16*_t**4 + x**4 + x**2*(-64*_t**6 + 8
*_t**2) + x*(-256*_t**7 + 32*_t**3) + 1)))
```

Maxima [F]

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$$
$$=\int -\frac{4\,(3\,x^{28}+4\,x^{24}+74\,x^{20}-61\,x^{16}+63\,x^{12}+68\,x^{8}-6\,x^{4}+2)}{x^{32}-8\,x^{28}+4\,x^{24}-114\,x^{20}-34\,x^{16}-236\,x^{12}+69\,x^{8}-2\,x^{4}+1}\,dx$$

input
```
integrate((-12*x^28-16*x^24-296*x^20+244*x^16-252*x^12-272*x^8+24*x^4-8)/(
x^32-8*x^28+4*x^24-114*x^20-34*x^16-236*x^12+69*x^8-2*x^4+1),x, algorithm=
"maxima")
```

output
```
-4*integrate((3*x^28 + 4*x^24 + 74*x^20 - 61*x^16 + 63*x^12 + 68*x^8 - 6*x
^4 + 2)/(x^32 - 8*x^28 + 4*x^24 - 114*x^20 - 34*x^16 - 236*x^12 + 69*x^8 -
 2*x^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.73

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx =$$

$$-\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\left(\arctan\left(\frac{x^3+x}{\sqrt{2\sqrt{5}+2}}\right)-\arctan\left(-\frac{2\,x^7+4\,x^5-x^3(\sqrt{5}-1)+3\,x(\sqrt{5}+1)}{4\,\sqrt{\sqrt{5}+2}}\right)\right)$$
$$-\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\arctan\left(\frac{2\,(x^2+x)}{\sqrt{2\sqrt{5}-2}}\right)-\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\arctan\left(-\frac{2\,(x^2-x)}{\sqrt{2\sqrt{5}-2}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\log\left(\left|2344\,x^4+2344\,x^2+2344\,x\sqrt{2\sqrt{5}-2}+1172\sqrt{5}-1172\right|\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\log\left(\left|2344\,x^4+2344\,x^2-2344\,x\sqrt{2\sqrt{5}-2}+1172\sqrt{5}-1172\right|\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\log\left(\left|104\,x^2+104\,x+52\sqrt{2\sqrt{5}+2}\right|\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\log\left(\left|104\,x^2+104\,x-52\sqrt{2\sqrt{5}+2}\right|\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\log\left(\left|104\,x^2-104\,x+52\sqrt{2\sqrt{5}+2}\right|\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\log\left(\left|104\,x^2-104\,x-52\sqrt{2\sqrt{5}+2}\right|\right)$$

input
```
integrate((-12*x^28-16*x^24-296*x^20+244*x^16-252*x^12-272*x^8+24*x^4-8)/(
x^32-8*x^28+4*x^24-114*x^20-34*x^16-236*x^12+69*x^8-2*x^4+1),x, algorithm=
"giac")
```

output
```
-sqrt(1/2)*sqrt(sqrt(5) + 1)*(arctan((x^3 + x)/sqrt(2*sqrt(5) + 2)) - arct
an(-1/4*(2*x^7 + 4*x^5 - x^3*(sqrt(5) - 1) + 3*x*(sqrt(5) + 1))/sqrt(sqrt(
5) + 2))) - sqrt(1/2)*sqrt(sqrt(5) - 1)*arctan(2*(x^2 + x)/sqrt(2*sqrt(5)
- 2)) - sqrt(1/2)*sqrt(sqrt(5) - 1)*arctan(-2*(x^2 - x)/sqrt(2*sqrt(5) - 2
)) - 1/2*sqrt(1/2)*sqrt(sqrt(5) - 1)*log(abs(2344*x^4 + 2344*x^2 + 2344*x*
sqrt(2*sqrt(5) - 2) + 1172*sqrt(5) - 1172)) + 1/2*sqrt(1/2)*sqrt(sqrt(5) -
 1)*log(abs(2344*x^4 + 2344*x^2 - 2344*x*sqrt(2*sqrt(5) - 2) + 1172*sqrt(5
) - 1172)) - 1/2*sqrt(1/2)*sqrt(sqrt(5) + 1)*log(abs(104*x^2 + 104*x + 52*
sqrt(2*sqrt(5) + 2))) + 1/2*sqrt(1/2)*sqrt(sqrt(5) + 1)*log(abs(104*x^2 +
104*x - 52*sqrt(2*sqrt(5) + 2))) + 1/2*sqrt(1/2)*sqrt(sqrt(5) + 1)*log(abs
(104*x^2 - 104*x + 52*sqrt(2*sqrt(5) + 2))) - 1/2*sqrt(1/2)*sqrt(sqrt(5) +
 1)*log(abs(104*x^2 - 104*x - 52*sqrt(2*sqrt(5) + 2)))
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.99

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx = \text{Too large to display}$$

input
```
int((272*x^8 - 24*x^4 + 252*x^12 - 244*x^16 + 296*x^20 + 16*x^24 + 12*x^28
 + 8)/(2*x^4 - 69*x^8 + 236*x^12 + 34*x^16 + 114*x^20 - 4*x^24 + 8*x^28 -
x^32 - 1),x)
```

output

```
(2^(1/2)*atan((2^(1/2)*x*(5^(1/2) - 1)^(1/2)*161154740338306427663880912961
83347357627609590253855994778226853596903348633600000000000000000i)/(4454173
68797007436283527902825556339892841344322745233246803542196855368581120000
0000000000*5^(1/2) + 720712665789049404071753323967222055977078270379895133
2984215600965979597701120000000000000000*5^(1/2)*x^2 + 72071266578904940407
175332396722205597707827037989513298421560096597959770112000000000000000*5^(
1/2)*x^4 + 1611547403383064276638809129618334735762760959025385599477822685
35969033486336000000000000000*x^2 + 16115474033830642766388091296183347357
62760959025385599477822685359690334863360000000000000000*x^4 + 99600796278109
13718599787451088877206131519643704503272162765973510382682112000000000000
0000) + (2^(1/2)*5^(1/2)*x*(5^(1/2) - 1)^(1/2)*720712665789049404071753323
96722205597707827037989513298421560096597959770112000000000000000i)/(445417
36879700743628352790282555633989284134432274523324680354219685536858112000
00000000000*5^(1/2) + 72071266578904940407175332396722205597707827037989513
298421560096597959770112000000000000000*5^(1/2)*x^2 + 72071266578904940407
175332396722205597707827037989513298421560096597959770112000000000000000*5^
(1/2)*x^4 + 161154740338306427663880912961833473576276095902538559947782268
535969033486336000000000000000*x^2 + 16115474033830642766388091296183347357
62760959025385599477822685359690334863360000000000000000*x^4 + 996007962781
0913718599787451088877206131519643704503272162765973510382682112000000...
```

Reduce [F]

$$\int \frac{-8+24x^4-272x^8-252x^{12}+244x^{16}-296x^{20}-16x^{24}-12x^{28}}{1-2x^4+69x^8-236x^{12}-34x^{16}-114x^{20}+4x^{24}-8x^{28}+x^{32}}\,dx$$
$$= -12\left(\int \frac{x^{28}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$-16\left(\int \frac{x^{24}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$-296\left(\int \frac{x^{20}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$+244\left(\int \frac{x^{16}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$-252\left(\int \frac{x^{12}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$-272\left(\int \frac{x^{8}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$+24\left(\int \frac{x^{4}}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$
$$-8\left(\int \frac{1}{x^{32}-8x^{28}+4x^{24}-114x^{20}-34x^{16}-236x^{12}+69x^8-2x^4+1}dx\right)$$

input
```
int((-12*x^28-16*x^24-296*x^20+244*x^16-252*x^12-272*x^8+24*x^4-8)/(x^32-8
*x^28+4*x^24-114*x^20-34*x^16-236*x^12+69*x^8-2*x^4+1),x)
```

output
```
4*( - 3*int(x**28/(x**32 - 8*x**28 + 4*x**24 - 114*x**20 - 34*x**16 - 236*
x**12 + 69*x**8 - 2*x**4 + 1),x) - 4*int(x**24/(x**32 - 8*x**28 + 4*x**24
- 114*x**20 - 34*x**16 - 236*x**12 + 69*x**8 - 2*x**4 + 1),x) - 74*int(x**
20/(x**32 - 8*x**28 + 4*x**24 - 114*x**20 - 34*x**16 - 236*x**12 + 69*x**8
 - 2*x**4 + 1),x) + 61*int(x**16/(x**32 - 8*x**28 + 4*x**24 - 114*x**20 -
34*x**16 - 236*x**12 + 69*x**8 - 2*x**4 + 1),x) - 63*int(x**12/(x**32 - 8*
x**28 + 4*x**24 - 114*x**20 - 34*x**16 - 236*x**12 + 69*x**8 - 2*x**4 + 1)
,x) - 68*int(x**8/(x**32 - 8*x**28 + 4*x**24 - 114*x**20 - 34*x**16 - 236*
x**12 + 69*x**8 - 2*x**4 + 1),x) + 6*int(x**4/(x**32 - 8*x**28 + 4*x**24 -
 114*x**20 - 34*x**16 - 236*x**12 + 69*x**8 - 2*x**4 + 1),x) - 2*int(1/(x*
*32 - 8*x**28 + 4*x**24 - 114*x**20 - 34*x**16 - 236*x**12 + 69*x**8 - 2*x
**4 + 1),x))
```

3.79 $\int \frac{x}{1+x^8} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 93

$$\int \frac{x}{1+x^8} \, dx = -\frac{\arctan\left(1-\sqrt{2}x^2\right)}{4\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}x^2\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}x^2+x^4\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x^2+x^4\right)}{8\sqrt{2}}$$

output
```
1/8*2^(1/2)*arctan(-1+x^2*2^(1/2))+1/8*2^(1/2)*arctan(1+x^2*2^(1/2))-1/16*
ln(1-x^2*2^(1/2)+x^4)*2^(1/2)+1/16*ln(1+x^2*2^(1/2)+x^4)*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int \frac{x}{1+x^8} \, dx = -\frac{2\arctan\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\cot\left(\frac{\pi}{8}\right) - x\csc\left(\frac{\pi}{8}\right)\right) + 2\arctan\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right) -}{}$$

input
```
Integrate[x/(1 + x^8),x]
```

output
```
-1/8*(2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]] + 2*ArcTan[Cot[Pi/8] - x*Csc[Pi/
8]] + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])] - 2*ArcTan[x*Sec[Pi/8] - Tan[Pi/
8]] + Log[1 + x^2 - 2*x*Cos[Pi/8]] + Log[1 + x^2 + 2*x*Cos[Pi/8]] - Log[1
+ x^2 - 2*x*Sin[Pi/8]] - Log[1 + x^2 + 2*x*Sin[Pi/8]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {807, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^8+1} dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^8+1}dx^2$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8+1}dx^2+\frac{1}{2}\int \frac{x^4+1}{x^8+1}dx^2\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}x^2+1}dx^2+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}x^2+1}dx^2\right)+\frac{1}{2}\int \frac{1-x^4}{x^8+1}dx^2\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x^4-1}d\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^4-1}d\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-x^4}{x^8+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x^4}{x^8+1}dx^2+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2x^2}{x^4-\sqrt{2}x^2+1}dx^2}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x^2+1\right)}{x^4+\sqrt{2}x^2+1}dx^2}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}\right)\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x^2}{x^4-\sqrt{2}x^2+1}dx^2}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x^2+1\right)}{x^4+\sqrt{2}x^2+1}dx^2}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}\right)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2x^2}{x^4-\sqrt{2}x^2+1}dx^2}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x^2+1}{x^4+\sqrt{2}x^2+1}dx^2\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}\right)\right)$$

$\big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}x^2+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x^2\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^4+\sqrt{2}x^2+1\right)}{2\sqrt{2}}-\frac{\log\left(x^4-\sqrt{2}x^2+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[x/(1 + x^8),x]
```

output
```
((-(ArcTan[1 - Sqrt[2]*x^2]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x^2]/Sqrt[2])/2
+ (-1/2*Log[1 - Sqrt[2]*x^2 + x^4]/Sqrt[2] + Log[1 + Sqrt[2]*x^2 + x^4]/(2
*Sqrt[2]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)} \_R \ln\left(x^2+\_R\right)\right)}{8}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,x^2+x^4}{1-\sqrt{2}\,x^2+x^4}\right)+2\arctan\left(1+\sqrt{2}\,x^2\right)+2\arctan\left(-1+\sqrt{2}\,x^2\right)\right)}{16}$
meijerg	$-\frac{x^2\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^8\right)^{\frac{1}{4}}+\sqrt{x^8}\right)}{16\left(x^8\right)^{\frac{1}{4}}}+\frac{x^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^8\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^8\right)^{\frac{1}{4}}}\right)}{8\left(x^8\right)^{\frac{1}{4}}}+\frac{x^2\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^8\right)^{\frac{1}{4}}+\sqrt{x^8}\right)}{16\left(x^8\right)^{\frac{1}{4}}}+\frac{x^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(\ldots}{2+\sqrt{2}\ldots}\right)}{8\left(x^8\right)^{\frac{1}{4}}}$

input
```
int(x/(x^8+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(x^2+_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+x^8}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\sqrt{2}x^2+1\right)+\frac{1}{8}\sqrt{2}\arctan\left(\sqrt{2}x^2-1\right) + \frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)$$

input
```
integrate(x/(x^8+1),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*arctan(sqrt(2)*x^2 + 1) + 1/8*sqrt(2)*arctan(sqrt(2)*x^2 - 1)
+ 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/16*sqrt(2)*log(x^4 - sqrt(2)
*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x}{1+x^8} \, dx = -\frac{\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)}{16} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x^2-1\right)}{8} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x^2+1\right)}{8}$$

input
```
integrate(x/(x**8+1),x)
```

output
```
-sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 + sqrt(2)*log(x**4 + sqrt(2)*x**2
 + 1)/16 + sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 + sqrt(2)*atan(sqrt(2)*x**2 +
1)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x}{1+x^8} \, dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right) - \frac{1}{16}\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)$$

input
```
integrate(x/(x^8+1),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x^2 - sqrt(2))) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/
16*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x}{1+x^8}\,dx = \frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x^2+\sqrt{2}\right)\right)+\frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x^2-\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)-\frac{1}{16}\,\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)$$

input
```
integrate(x/(x^8+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x^2 - sqrt(2))) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/
16*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \frac{x}{1+x^8}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

input
```
int(x/(x^8 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)
*x^2*(1/2 + 1i/2))*(1/8 - 1i/8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.27

$$\int \frac{x}{1+x^8}\,dx = -\frac{\sqrt{\sqrt{2}+2}\,\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2x}{\sqrt{\sqrt{2}+2}}\right)}{8} - \frac{\sqrt{\sqrt{2}+2}\,\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2x}{\sqrt{\sqrt{2}+2}}\right)}{8} - \frac{\sqrt{\sqrt{2}+2}\,\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}-2x}{\sqrt{-\sqrt{2}+2}}\right)}{8} - \frac{\sqrt{\sqrt{2}+2}\,\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}+2x}{\sqrt{-\sqrt{2}+2}}\right)}{8} + \frac{\sqrt{2}\log\left(-\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{16} - \frac{\sqrt{2}\log\left(-\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{16} + \frac{\sqrt{2}\log\left(\sqrt{-\sqrt{2}+2}\,x+x^2+1\right)}{16} - \frac{\sqrt{2}\log\left(\sqrt{\sqrt{2}+2}\,x+x^2+1\right)}{16}$$

input
```
int(x/(x^8+1),x)
```

output
```
( - 2*sqrt(sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) -
2*x)/sqrt(sqrt(2) + 2)) - 2*sqrt(sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((s
qrt( - sqrt(2) + 2) + 2*x)/sqrt(sqrt(2) + 2)) - 2*sqrt(sqrt(2) + 2)*sqrt(
- sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) - 2*x)/sqrt( - sqrt(2) + 2)) - 2*sq
rt(sqrt(2) + 2)*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*x)/sqrt(
- sqrt(2) + 2)) + sqrt(2)*log( - sqrt( - sqrt(2) + 2)*x + x**2 + 1) - sqrt
(2)*log( - sqrt(sqrt(2) + 2)*x + x**2 + 1) + sqrt(2)*log(sqrt( - sqrt(2) +
 2)*x + x**2 + 1) - sqrt(2)*log(sqrt(sqrt(2) + 2)*x + x**2 + 1))/16
```

3.80 $\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = -\frac{41+24\sqrt{2}}{64\ 2^{3/4}\left(\sqrt{3}+2\sqrt[4]{2}x\right)^2}+\frac{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^2}{64\ 2^{3/4}} - \frac{\left(3+4\sqrt{2}\right)\log\left(\sqrt{3}+2\sqrt[4]{2}x\right)}{16\ 2^{3/4}}$$

output
```
-1/128*(41+24*2^(1/2))*2^(1/4)/(3^(1/2)+2*2^(1/4)*x)^2+1/128*(3^(1/2)+2*2^
(1/4)*x)^2*2^(1/4)-1/32*(3+4*2^(1/2))*ln(3^(1/2)+2*2^(1/4)*x)*2^(1/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = \frac{1}{128}\left(\frac{8\sqrt{3}\left(2833+1968\sqrt{2}\right)x}{3936+2833\sqrt{2}}+4\ 2^{3/4}x^2 - \frac{2^{3/4}\left(210617+148680\sqrt{2}\right)}{\left(3936+2833\sqrt{2}\right)\left(\sqrt{3}+2\sqrt[4]{2}x\right)^2} - \frac{4\ 2^{3/4}\left(24243+17236\sqrt{2}\right)\log\left(\sqrt{3}+2\sqrt[4]{2}x\right)}{3936+2833\sqrt{2}}\right)$$

input
```
Integrate[(-2^(1/4) + Sqrt[3]*x + 2^(1/4)*x^2)^2/(Sqrt[3] + 2*2^(1/4)*x)^3
,x]
```

output
```
((8*Sqrt[3]*(2833 + 1968*Sqrt[2])*x)/(3936 + 2833*Sqrt[2]) + 4*2^(3/4)*x^2
 - (2^(3/4)*(210617 + 148680*Sqrt[2]))/((3936 + 2833*Sqrt[2])*(Sqrt[3] + 2
*2^(1/4)*x)^2) - (4*2^(3/4)*(24243 + 17236*Sqrt[2])*Log[Sqrt[3] + 2*2^(1/4
)*x])/(3936 + 2833*Sqrt[2]))/128
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt[4]{2}x^2+\sqrt{3}x-\sqrt[4]{2}\right)^2}{\left(2\sqrt[4]{2}x+\sqrt{3}\right)^3}dx$$

$$\Big\downarrow \text{ 1107}$$

$$\int \left(\frac{2\sqrt[4]{2}x+\sqrt{3}}{16\sqrt{2}}+\frac{-8-3\sqrt{2}}{16\left(2\sqrt[4]{2}x+\sqrt{3}\right)}+\frac{48+41\sqrt{2}}{32\left(2\sqrt[4]{2}x+\sqrt{3}\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(2\sqrt[4]{2}x+\sqrt{3}\right)^2}{64\ 2^{3/4}}-\frac{41+24\sqrt{2}}{64\ 2^{3/4}\left(2\sqrt[4]{2}x+\sqrt{3}\right)^2}-\frac{\left(3+4\sqrt{2}\right)\log\left(2\sqrt[4]{2}x+\sqrt{3}\right)}{16\ 2^{3/4}}$$

input
```
Int[(-2^(1/4) + Sqrt[3]*x + 2^(1/4)*x^2)^2/(Sqrt[3] + 2*2^(1/4)*x)^3,x]
```

output
```
-1/64*(41 + 24*Sqrt[2])/(2^(3/4)*(Sqrt[3] + 2*2^(1/4)*x)^2) + (Sqrt[3] + 2
*2^(1/4)*x)^2/(64*2^(3/4)) - ((3 + 4*Sqrt[2])*Log[Sqrt[3] + 2*2^(1/4)*x])/
(16*2^(3/4))
```

Defintions of rubi rules used

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

method	result
default	$\frac{2^{\frac{3}{4}} x^{2}}{32}+\frac{\sqrt{2}\, \sqrt{3}\, x}{32}-\frac{2^{\frac{1}{4}} \left(41 \sqrt{2}+48\right)}{64 \left(4 x+\sqrt{3}\, 2^{\frac{3}{4}}\right)^{2}}-\frac{2^{\frac{1}{4}} \left(3+4 \sqrt{2}\right) \ln \left(4 x+\sqrt{3}\, 2^{\frac{3}{4}}\right)}{32}$
risch	$\frac{\sqrt{2}\, \sqrt{3}\, x}{32}+\frac{2^{\frac{3}{4}} x^{2}}{32}-\frac{41\, 2^{\frac{3}{4}}}{1024 \left(x+\frac{\sqrt{3}\, 2^{\frac{3}{4}}}{4}\right)^{2}}-\frac{3\, 2^{\frac{1}{4}}}{64 \left(x+\frac{\sqrt{3}\, 2^{\frac{3}{4}}}{4}\right)^{2}}-\frac{\ln \left(4 x+\sqrt{3}\, 2^{\frac{3}{4}}\right) 2^{\frac{3}{4}}}{8}-\frac{3 \ln \left(4 x+\sqrt{3}\, 2^{\frac{3}{4}}\right) 2^{\frac{1}{4}}}{32}$
meijerg	$\frac{3\, 2^{\frac{1}{4}} \left(-\frac{2 x\, 2^{\frac{1}{4}} \sqrt{3}\, \left(-\frac{40\, 2^{\frac{3}{4}} \sqrt{3}\, x^{3}}{9}+\frac{80 \sqrt{2}\, x^{2}}{3}+60\, 2^{\frac{1}{4}} \sqrt{3}\, x+60\right)}{15 \left(1+\frac{2\, 2^{\frac{1}{4}} \sqrt{3}\, x}{3}\right)^{2}}+12 \ln \left(1+\frac{2\, 2^{\frac{1}{4}} \sqrt{3}\, x}{3}\right)\right)}{128}+\frac{2^{\frac{1}{4}} \left(\sqrt{3}+\sqrt{3+4 \sqrt{2}}\right) \sqrt{3}\, \left(\frac{x \left(\frac{16 \sqrt{2}\, x^{2}}{3}+12\, 2^{\frac{1}{4}} \cdots\right)}{3 \left(1+\frac{2\, 2^{\frac{1}{4}} \cdots}{}\right)}\cdots\right)}{64}$

input
```
int((-2^(1/4)+3^(1/2)*x+2^(1/4)*x^2)^2/(3^(1/2)+2*2^(1/4)*x)^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/32*2^(3/4)*x^2+1/32*2^(1/2)*3^(1/2)*x-1/64*2^(1/4)*(41*2^(1/2)+48)/(4*x+
3^(1/2)*2^(3/4))^2-1/32*2^(1/4)*(3+4*2^(1/2))*ln(4*x+3^(1/2)*2^(3/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(65) = 130.

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.69

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = -\frac{4\left(4\cdot 2^{\frac{3}{4}}(1024\,x^8-576\,x^4+81)+3\cdot 2^{\frac{1}{4}}(1024\,x^8-576\,x^4+81)\right)\log\left(\sqrt{3}2^{\frac{3}{4}}+4\,x\right)-8\cdot 2^{\frac{3}{4}}(512\,x^{10}\cdots}{\cdots}$$

input
```
integrate((-2^(1/4)+3^(1/2)*x+x^2*2^(1/4))^2/(3^(1/2)+2*2^(1/4)*x)^3,x, al
gorithm="fricas")
```

output
```
-1/128*(4*(4*2^(3/4)*(1024*x^8 - 576*x^4 + 81) + 3*2^(1/4)*(1024*x^8 - 576
*x^4 + 81))*log(sqrt(3)*2^(3/4) + 4*x) - 8*2^(3/4)*(512*x^10 - 944*x^6 - 8
64*x^4 - 513*x^2 - 81) - 8*sqrt(3)*(768*x^5 + 984*x^3 + sqrt(2)*(512*x^9 +
 368*x^5 + 576*x^3 + 225*x) + 216*x) + 3*2^(1/4)*(2048*x^6 + 3936*x^4 + 17
28*x^2 + 369))/(1024*x^8 - 576*x^4 + 81)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = \frac{2^{\frac{3}{4}}x^2}{32}+\frac{\sqrt{6}x}{32}+\frac{\sqrt[4]{2}(-4\sqrt{2}-3)\log\left(16\cdot 2^{\frac{3}{4}}x+8\sqrt{6}\right)}{32} + \frac{-328\cdot 2^{\frac{3}{4}}-384\cdot\sqrt[4]{2}}{8192x^2+4096\cdot 2^{\frac{3}{4}}\sqrt{3}x+3072\sqrt{2}}$$

input
```
integrate((-2**(1/4)+3**(1/2)*x+x**2*2**(1/4))**2/(3**(1/2)+2*2**(1/4)*x)*
*3,x)
```

output
```
2**(3/4)*x**2/32 + sqrt(6)*x/32 + 2**(1/4)*(-4*sqrt(2) - 3)*log(16*2**(3/4
)*x + 8*sqrt(6))/32 + (-328*2**(3/4) - 384*2**(1/4))/(8192*x**2 + 4096*2**
(3/4)*sqrt(3)*x + 3072*sqrt(2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = -\frac{1}{32}\cdot 2^{\frac{1}{4}}\left(4\sqrt{2}+3\right)\log\left(2\cdot 2^{\frac{1}{4}}x+\sqrt{3}\right) + \frac{1}{32}\sqrt{2}\left(2^{\frac{1}{4}}x^2+\sqrt{3}x\right) - \frac{24\sqrt{2}+41}{64\left(8\cdot 2^{\frac{1}{4}}x^2+8\sqrt{3}x+3\cdot 2^{\frac{3}{4}}\right)}$$

input
```
integrate((-2^(1/4)+3^(1/2)*x+x^2*2^(1/4))^2/(3^(1/2)+2*2^(1/4)*x)^3,x, al
gorithm="maxima")
```

output
```
-1/32*2^(1/4)*(4*sqrt(2) + 3)*log(2*2^(1/4)*x + sqrt(3)) + 1/32*sqrt(2)*(2
^(1/4)*x^2 + sqrt(3)*x) - 1/64*(24*sqrt(2) + 41)/(8*2^(1/4)*x^2 + 8*sqrt(3
)*x + 3*2^(3/4))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate((-2^(1/4)+3^(1/2)*x+x^2*2^(1/4))^2/(3^(1/2)+2*2^(1/4)*x)^3,x, al
gorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: (roo
tof([[31074142760670943287241187834324561826724616361239789583728 64,0,-362
514019804215706356094811396004428261346120684446472009154 56,0,144735628146
4232130181365104903
```

Mupad [B] (verification not implemented)

Time = 11.72 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.19

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx=x\left(\frac{\sqrt{2}\sqrt{3}}{8}-\frac{3\sqrt{6}}{32}\right)+\frac{2^{3/4}x^2}{32}+\left(\sum_{k=1}^{3}\ln\left(\frac{9\,2^{1/4}\sqrt{3}}{1024}+\frac{27\,2^{3/4}\sqrt{3}}{8192}+\frac{207\,2^{1/4}\sqrt{6}}{32768}+\frac{23\,2^{3/4}\sqrt{6}}{8192}+\mathrm{root}\left(127401984\,2^{3/4}\sqrt{3}\sqrt{6}\,z^3-7644119\right.\right.\right.$$

input
```
int((3^(1/2)*x - 2^(1/4) + 2^(1/4)*x^2)^2/(2*2^(1/4)*x + 3^(1/2))^3,x)
```

output
```
x*((2^(1/2)*3^(1/2))/8 - (3*6^(1/2))/32) + (2^(3/4)*x^2)/32 + symsum(log((
9*2^(1/4)*3^(1/2))/1024 + (27*2^(3/4)*3^(1/2))/8192 + (207*2^(1/4)*6^(1/2)
)/32768 + (23*2^(3/4)*6^(1/2))/8192 + root(127401984*2^(3/4)*3^(1/2)*6^(1/
2)*z^3 - 764411904*2^(1/4)*z^3 + 31850496*2^(1/2)*3^(1/2)*6^(1/2)*z^2 + 23
887872*3^(1/2)*6^(1/2)*z^2 - 71663616*2^(1/2)*z^2 - 191102976*z^2 + 597196
8*2^(1/4)*3^(1/2)*6^(1/2)*z - 17915904*2^(3/4)*z - 127800*2^(1/2)*3^(1/2)*
6^(1/2) + 260352*3^(1/2)*6^(1/2) - 186336*2^(1/2) + 1609268, z, k)*((27*2^
(1/2)*3^(1/2))/128 - (9*2^(1/2)*6^(1/2))/128 + x*((9*2^(1/4))/32 + (15*2^(
3/4))/512 + (9*2^(1/4)*3^(1/2)*6^(1/2))/128) + root(127401984*2^(3/4)*3^(1
/2)*6^(1/2)*z^3 - 764411904*2^(1/4)*z^3 + 31850496*2^(1/2)*3^(1/2)*6^(1/2)
*z^2 + 23887872*3^(1/2)*6^(1/2)*z^2 - 71663616*2^(1/2)*z^2 - 191102976*z^2
 + 5971968*2^(1/4)*3^(1/2)*6^(1/2)*z - 17915904*2^(3/4)*z - 127800*2^(1/2)
*3^(1/2)*6^(1/2) + 260352*3^(1/2)*6^(1/2) - 186336*2^(1/2) + 1609268, z, k
)*((27*2^(1/4)*3^(1/2))/16 - (27*2^(3/4)*6^(1/2))/32) + (9*3^(1/2))/32 - (
93*6^(1/2))/1024) + x*((3*3^(1/2)*6^(1/2))/512 + (23*2^(1/2))/2048 - (27*2
^(1/2)*3^(1/2)*6^(1/2))/4096 + 639/8192))*root(127401984*2^(3/4)*3^(1/2)*6
^(1/2)*z^3 - 764411904*2^(1/4)*z^3 + 31850496*2^(1/2)*3^(1/2)*6^(1/2)*z^2
+ 23887872*3^(1/2)*6^(1/2)*z^2 - 71663616*2^(1/2)*z^2 - 191102976*z^2 + 59
71968*2^(1/4)*3^(1/2)*6^(1/2)*z - 17915904*2^(3/4)*z - 127800*2^(1/2)*3^(1
/2)*6^(1/2) + 260352*3^(1/2)*6^(1/2) - 186336*2^(1/2) + 1609268, z, k),...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1488, normalized size of antiderivative = 16.00

$$\int \frac{\left(-\sqrt[4]{2}+\sqrt{3}x+\sqrt[4]{2}x^2\right)^2}{\left(\sqrt{3}+2\sqrt[4]{2}x\right)^3}\,dx = \text{Too large to display}$$

input
```
int((-2^(1/4)+3^(1/2)*x+x^2*2^(1/4))^2/(3^(1/2)+2*2^(1/4)*x)^3,x)
```

output
```
(4026531840*sqrt(6)*x**21 - 503316480*sqrt(6)*x**17 + 4529848320*sqrt(6)*x
**15 + 283115520*sqrt(6)*x**13 - 3822059520*sqrt(6)*x**11 - 895795200*sqrt
(6)*x**9 + 1074954240*sqrt(6)*x**7 + 355518720*sqrt(6)*x**5 - 100776960*sq
rt(6)*x**3 - 39366000*sqrt(6)*x + 6039797760*sqrt(3)*x**17 + 7738490880*sq
rt(3)*x**15 - 3397386240*sqrt(3)*x**13 - 6529351680*sqrt(3)*x**11 + 183638
0160*sqrt(3)*x**7 + 268738560*sqrt(3)*x**5 - 172160640*sqrt(3)*x**3 - 3779
1360*sqrt(3)*x - 2516582400*sqrt(2)*2**(3/4)*log(128*sqrt(2)*x**6 + 108*sq
rt(2)*x**2 - 288*x**4 - 27)*x**20 + 3538944000*sqrt(2)*2**(3/4)*log(128*sq
rt(2)*x**6 + 108*sqrt(2)*x**2 - 288*x**4 - 27)*x**16 - 1990656000*sqrt(2)*
2**(3/4)*log(128*sqrt(2)*x**6 + 108*sqrt(2)*x**2 - 288*x**4 - 27)*x**12 +
559872000*sqrt(2)*2**(3/4)*log(128*sqrt(2)*x**6 + 108*sqrt(2)*x**2 - 288*x
**4 - 27)*x**8 - 78732000*sqrt(2)*2**(3/4)*log(128*sqrt(2)*x**6 + 108*sqrt
(2)*x**2 - 288*x**4 - 27)*x**4 + 4428675*sqrt(2)*2**(3/4)*log(128*sqrt(2)*
x**6 + 108*sqrt(2)*x**2 - 288*x**4 - 27) - 2063597568*sqrt(2)*2**(3/4)*x**
20 - 2548039680*sqrt(2)*2**(3/4)*x**14 + 1088225280*sqrt(2)*2**(3/4)*x**12
 + 2149908480*sqrt(2)*2**(3/4)*x**10 - 459095040*sqrt(2)*2**(3/4)*x**8 - 6
04661760*sqrt(2)*2**(3/4)*x**6 + 64560240*sqrt(2)*2**(3/4)*x**4 + 56687040
*sqrt(2)*2**(3/4)*x**2 - 2421009*sqrt(2)*2**(3/4) + 4026531840*2**(3/4)*x*
*22 - 2415919104*2**(3/4)*x**20 - 10821304320*2**(3/4)*x**18 + 7537950720*
2**(3/4)*x**14 - 2120048640*2**(3/4)*x**10 + 537477120*2**(3/4)*x**8 + ...
```

3.81 $\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 203

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4} \, dx = -\sqrt{-\frac{450831}{36736}+\frac{48859}{2296\sqrt{2}}} \arctan\left(\sqrt{-\frac{87}{287}+\frac{104\sqrt{2}}{287}} + \sqrt{\frac{204}{287}+\frac{152\sqrt{2}}{287}}x\right) + \sqrt{\frac{450831}{36736}+\frac{48859}{2296\sqrt{2}}} \operatorname{arctanh}\left(\sqrt{\frac{87}{287}+\frac{104\sqrt{2}}{287}} + \sqrt{-\frac{204}{287}+\frac{152\sqrt{2}}{287}}x\right) - \frac{33\log\left(-2+2\sqrt{2}+5x-3\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{33\log\left(-2-2\sqrt{2}+5x+3\sqrt{2}x+x^2\right)}{16\sqrt{2}} + \frac{5}{4}\log\left(-4+4x+3x^2+10x^3+x^4\right)$$

output
```
-1/4592*(-258776994+224360528*2^(1/2))^(1/2)*arctan(1/287*(-24969+29848*2^
(1/2))^(1/2)+2/287*(14637+10906*2^(1/2))^(1/2)*x)+1/4592*(258776994+224360
528*2^(1/2))^(1/2)*arctanh(1/287*(24969+29848*2^(1/2))^(1/2)+2/287*(-14637
+10906*2^(1/2))^(1/2)*x)-33/32*ln(-2+2*2^(1/2)+5*x-3*x*2^(1/2)+x^2)*2^(1/2
)+33/32*ln(-2-2*2^(1/2)+5*x+3*x*2^(1/2)+x^2)*2^(1/2)+5/4*ln(x^4+10*x^3+3*x
^2+4*x-4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = \frac{1}{2}\text{RootSum}\Big[-4+4\#1+3\#1^2+10\#1^3 +\#1^4\&,\frac{6\log(x-\#1)-9\log(x-\#1)\#1-3\log(x-\#1)\#1^2+5\log(x-\#1)\#1^3}{2+3\#1+15\#1^2+2\#1^3}\&\Big]$$

input
```
Integrate[(-6 + 9*x + 3*x^2 - 5*x^3)/(4 - 4*x - 3*x^2 - 10*x^3 - x^4),x]
```

output
```
RootSum[-4 + 4*#1 + 3*#1^2 + 10*#1^3 + #1^4 & , (6*Log[x - #1] - 9*Log[x -
 #1]*#1 - 3*Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(2 + 3*#1 + 15*#1^2 + 2
*#1^3) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-5x^3+3x^2+9x-6}{-x^4-10x^3-3x^2-4x+4}dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{5}{4}\log\left(-x^4-10x^3-3x^2-4x+4\right)-\frac{1}{4}\int \frac{2\left(-81x^2-33x+2\right)}{-x^4-10x^3-3x^2-4x+4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}\log\left(-x^4-10x^3-3x^2-4x+4\right)-\frac{1}{2}\int\frac{-81x^2-33x+2}{-x^4-10x^3-3x^2-4x+4}dx$$

$$\Big\downarrow \text{7293}$$

$$\frac{5}{4}\log\left(-x^4-10x^3-3x^2-4x+4\right)-\frac{1}{2}\int\left(\frac{81x^2}{x^4+10x^3+3x^2+4x-4}+\frac{33x}{x^4+10x^3+3x^2+4x-4}-\frac{2}{x^4+10x^3+3x^2+4x-4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\int\frac{1}{x^4+10x^3+3x^2+4x-4}dx-33\int\frac{x}{x^4+10x^3+3x^2+4x-4}dx-81\int\frac{x^2}{x^4+10x^3+3x^2+4x-4}dx\right)+\frac{5}{4}\log\left(-x^4-10x^3-3x^2-4x+4\right)$$

input
```
Int[(-6 + 9*x + 3*x^2 - 5*x^3)/(4 - 4*x - 3*x^2 - 10*x^3 - x^4),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+10\_Z^3+3\_Z^2+4\_Z-4\right)}\frac{\left(5\_R^3-3\_R^2-9\_R+6\right)\ln\left(x-\_R\right)}{2\_R^3+15\_R^2+3\_R+2}\right)}{2}$	64
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+10\_Z^3+3\_Z^2+4\_Z-4\right)}\frac{\left(5\_R^3-3\_R^2-9\_R+6\right)\ln\left(x-\_R\right)}{2\_R^3+15\_R^2+3\_R+2}\right)}{2}$	64

input
```
int((-5*x^3+3*x^2+9*x-6)/(-x^4-10*x^3-3*x^2-4*x+4),x,method=_RETURNVERBOSE
)
```

output
```
1/2*sum((5*_R^3-3*_R^2-9*_R+6)/(2*_R^3+15*_R^2+3*_R+2)*ln(x-_R),_R=RootOf(
_Z^4+10*_Z^3+3*_Z^2+4*_Z-4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = -\frac{1}{32}\left(33\sqrt{2}-40\right)\log\left(x^2-\sqrt{2}(3x-2)+5x-2\right) + \frac{1}{32}\left(33\sqrt{2}+2\sqrt{\frac{195436}{287}\sqrt{2}+\frac{450831}{574}}+40\right)\log\left(\left(2463\sqrt{2}+1138\right)\sqrt{\frac{195436}{287}\sqrt{2}+\frac{450831}{574}} + 37762x+56643\sqrt{2}+94405\right) + \frac{1}{32}\left(33\sqrt{2}-2\sqrt{\frac{195436}{287}\sqrt{2}+\frac{450831}{574}}+40\right)\log\left(-\left(2463\sqrt{2}+1138\right)\sqrt{\frac{195436}{287}\sqrt{2}+\frac{450831}{574}} + 37762x+56643\sqrt{2}+94405\right) - \frac{1}{8}\sqrt{\frac{195436}{287}\sqrt{2}-\frac{450831}{574}}\arctan\left(\frac{1}{18881}\left(\sqrt{2}(574x+85)+900x+528\right)\sqrt{\frac{195436}{287}\sqrt{2}-\frac{450831}{574}}\right)$$

input
```
integrate((-5*x^3+3*x^2+9*x-6)/(-x^4-10*x^3-3*x^2-4*x+4),x, algorithm="fri
cas")
```

output
```
-1/32*(33*sqrt(2) - 40)*log(x^2 - sqrt(2)*(3*x - 2) + 5*x - 2) + 1/32*(33*
sqrt(2) + 2*sqrt(195436/287*sqrt(2) + 450831/574) + 40)*log((2463*sqrt(2)
+ 1138)*sqrt(195436/287*sqrt(2) + 450831/574) + 37762*x + 56643*sqrt(2) +
94405) + 1/32*(33*sqrt(2) - 2*sqrt(195436/287*sqrt(2) + 450831/574) + 40)*
log(-(2463*sqrt(2) + 1138)*sqrt(195436/287*sqrt(2) + 450831/574) + 37762*x
 + 56643*sqrt(2) + 94405) - 1/8*sqrt(195436/287*sqrt(2) - 450831/574)*arct
an(1/18881*(sqrt(2)*(574*x + 85) + 900*x + 528)*sqrt(195436/287*sqrt(2) -
450831/574))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx$$
$$= \operatorname{RootSum}\left(293888t^4-1469440t^3-298296t^2-1111648t+109543, \left(t \mapsto t\log\left(\frac{9017217237504t^3}{39142099388053}-\frac{2\ldots}{3\ldots}\right.\right.\right.$$

input
```
integrate((-5*x**3+3*x**2+9*x-6)/(-x**4-10*x**3-3*x**2-4*x+4),x)
```

output
```
RootSum(293888*_t**4 - 1469440*_t**3 - 298296*_t**2 - 1111648*_t + 109543,
 Lambda(_t, _t*log(9017217237504*_t**3/39142099388053 - 25638855795840*_t*
*2/39142099388053 - 44500448358308*_t/39142099388053 + x - 14955523031473/
39142099388053)))
```

Maxima [F]

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = \int \frac{5\,x^3-3\,x^2-9\,x+6}{x^4+10\,x^3+3\,x^2+4\,x-4}\,dx$$

input
```
integrate((-5*x^3+3*x^2+9*x-6)/(-x^4-10*x^3-3*x^2-4*x+4),x, algorithm="max
ima")
```

output
```
integrate((5*x^3 - 3*x^2 - 9*x + 6)/(x^4 + 10*x^3 + 3*x^2 + 4*x - 4), x)
```

Giac [F]

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = \int \frac{5\,x^3-3\,x^2-9\,x+6}{x^4+10\,x^3+3\,x^2+4\,x-4}\,dx$$

input
```
integrate((-5*x^3+3*x^2+9*x-6)/(-x^4-10*x^3-3*x^2-4*x+4),x, algorithm="gia
c")
```

output
```
integrate((5*x^3 - 3*x^2 - 9*x + 6)/(x^4 + 10*x^3 + 3*x^2 + 4*x - 4), x)
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = \sum_{k=1}^{4} \ln\left(-9012\,\mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)+73457\,x + \mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)x\,145550 + \mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)^2 x\,51000 - \mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)^3 x\,15216 - 180592\,\mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)^2 + 34464\,\mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)^3 - 41042\right)\mathrm{root}\left(z^4-5\,z^3-\frac{37287\,z^2}{36736}-\frac{34739\,z}{9184}+\frac{15649}{41984},z,k\right)$$

input
```
int(-(9*x + 3*x^2 - 5*x^3 - 6)/(4*x + 3*x^2 + 10*x^3 + x^4 - 4),x)
```

output
```
symsum(log(73457*x - 9012*root(z^4 - 5*z^3 - (37287*z^2)/36736 - (34739*z)
/9184 + 15649/41984, z, k) + 145550*root(z^4 - 5*z^3 - (37287*z^2)/36736 -
 (34739*z)/9184 + 15649/41984, z, k)*x + 51000*root(z^4 - 5*z^3 - (37287*z
^2)/36736 - (34739*z)/9184 + 15649/41984, z, k)^2*x - 15216*root(z^4 - 5*z
^3 - (37287*z^2)/36736 - (34739*z)/9184 + 15649/41984, z, k)^3*x - 180592*
root(z^4 - 5*z^3 - (37287*z^2)/36736 - (34739*z)/9184 + 15649/41984, z, k)
^2 + 34464*root(z^4 - 5*z^3 - (37287*z^2)/36736 - (34739*z)/9184 + 15649/4
1984, z, k)^3 - 41042)*root(z^4 - 5*z^3 - (37287*z^2)/36736 - (34739*z)/91
84 + 15649/41984, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{-6+9x+3x^2-5x^3}{4-4x-3x^2-10x^3-x^4}\,dx = -\frac{81\left(\int \frac{x^2}{x^4+10x^3+3x^2+4x-4}dx\right)}{2} - \frac{33\left(\int \frac{x}{x^4+10x^3+3x^2+4x-4}dx\right)}{2} + \int \frac{1}{x^4+10x^3+3x^2+4x-4}dx + \frac{5\log(x^4+10x^3+3x^2+4x-4)}{4}$$

input
```
int((-5*x^3+3*x^2+9*x-6)/(-x^4-10*x^3-3*x^2-4*x+4),x)
```

output
```
( - 162*int(x**2/(x**4 + 10*x**3 + 3*x**2 + 4*x - 4),x) - 66*int(x/(x**4 +
 10*x**3 + 3*x**2 + 4*x - 4),x) + 4*int(1/(x**4 + 10*x**3 + 3*x**2 + 4*x -
 4),x) + 5*log(x**4 + 10*x**3 + 3*x**2 + 4*x - 4))/4
```

3.82 $\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 58

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = \frac{1}{2}\left(\left(2+\sqrt{7}\right)\log\left(1+\left(2+\sqrt{7}\right)x^2-x^3\right) - \left(-2+\sqrt{7}\right)\log\left(-1+\left(-2+\sqrt{7}\right)x^2+x^3\right)\right)$$

output
```
1/2*(2+7^(1/2))*ln(1+(2+7^(1/2))*x^2-x^3)-1/2*(-2+7^(1/2))*ln(-1+(-2+7^(1/
2))*x^2+x^3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = \frac{1}{2}\left(\left(2+\sqrt{7}\right)\log\left(1+\left(2+\sqrt{7}\right)x^2-x^3\right) - \left(-2+\sqrt{7}\right)\log\left(-1+\left(-2+\sqrt{7}\right)x^2+x^3\right)\right)$$

input
```
Integrate[(22*x - 6*x^2 - 12*x^3 - 13*x^4 + 6*x^5)/(1 + 4*x^2 - 2*x^3 - 3*
x^4 - 4*x^5 + x^6),x]
```

output
```
((2 + Sqrt[7])*Log[1 + (2 + Sqrt[7])*x^2 - x^3] - (-2 + Sqrt[7])*Log[-1 +
(-2 + Sqrt[7])*x^2 + x^3])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^5-13x^4-12x^3-6x^2+22x}{x^6-4x^5-3x^4-2x^3+4x^2+1}\,dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{6}\int \frac{42\left(x^4+2x\right)}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx+\log \left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

$$\Big\downarrow \text{ 27}$$

$$7\int \frac{x^4+2x}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx+\log \left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

$$\Big\downarrow \text{ 2027}$$

$$7\int \frac{x\left(x^3+2\right)}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx+\log \left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

$$\Big\downarrow \text{ 7293}$$

$$7\int \left(\frac{x^4}{x^6-4x^5-3x^4-2x^3+4x^2+1}+\frac{2x}{x^6-4x^5-3x^4-2x^3+4x^2+1}\right)dx+\log \left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

$$\Big\downarrow \text{ 2009}$$

$$7\left(2\int \frac{x}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx+\int \frac{x^4}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx\right)+\log \left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

input
```
Int[(22*x - 6*x^2 - 12*x^3 - 13*x^4 + 6*x^5)/(1 + 4*x^2 - 2*x^3 - 3*x^4 -
4*x^5 + x^6),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

method	result
default	$\left(1+\frac{\sqrt{7}}{2}\right)\ln\left(x^{3}+\left(-\sqrt{7}-2\right)x^{2}-1\right)+\left(1-\frac{\sqrt{7}}{2}\right)\ln\left(-1+\left(-2+\sqrt{7}\right)x^{2}+x^{3}\right)$
risch	$\ln\left(x^{3}+\left(-\sqrt{7}-2\right)x^{2}-1\right)+\frac{\ln\left(x^{3}+\left(-\sqrt{7}-2\right)x^{2}-1\right)\sqrt{7}}{2}+\ln\left(-1+\left(-2+\sqrt{7}\right)x^{2}+x^{3}\right)-\frac{\ln\left(-1+\left(\right.\right.}{}$

input
```
int((6*x^5-13*x^4-12*x^3-6*x^2+22*x)/(x^6-4*x^5-3*x^4-2*x^3+4*x^2+1),x,met
hod=_RETURNVERBOSE)
```

output
```
(1+1/2*7^(1/2))*ln(x^3+(-7^(1/2)-2)*x^2-1)+(1-1/2*7^(1/2))*ln(-1+(-2+7^(1/
2))*x^2+x^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\begin{aligned}&\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx\\&=\frac{1}{2}\sqrt{7}\log\left(\frac{x^6-4\,x^5+11\,x^4-2\,x^3+4\,x^2-2\,\sqrt{7}(x^5-2\,x^4-x^2)+1}{x^6-4\,x^5-3\,x^4-2\,x^3+4\,x^2+1}\right)\\&\quad+\log\left(x^6-4\,x^5-3\,x^4-2\,x^3+4\,x^2+1\right)\end{aligned}$$

input
```
integrate((6*x^5-13*x^4-12*x^3-6*x^2+22*x)/(x^6-4*x^5-3*x^4-2*x^3+4*x^2+1)
,x, algorithm="fricas")
```

output
```
1/2*sqrt(7)*log((x^6 - 4*x^5 + 11*x^4 - 2*x^3 + 4*x^2 - 2*sqrt(7)*(x^5 - 2
*x^4 - x^2) + 1)/(x^6 - 4*x^5 - 3*x^4 - 2*x^3 + 4*x^2 + 1)) + log(x^6 - 4*
x^5 - 3*x^4 - 2*x^3 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = \left(1-\frac{\sqrt{7}}{2}\right)\log\left(x^3-2x^2\cdot\left(1-\frac{\sqrt{7}}{2}\right)-1\right) + \left(1+\frac{\sqrt{7}}{2}\right)\log\left(x^3-2x^2\cdot\left(1+\frac{\sqrt{7}}{2}\right)-1\right)$$

input
```
integrate((6*x**5-13*x**4-12*x**3-6*x**2+22*x)/(x**6-4*x**5-3*x**4-2*x**3+
4*x**2+1),x)
```

output
```
(1 - sqrt(7)/2)*log(x**3 - 2*x**2*(1 - sqrt(7)/2) - 1) + (1 + sqrt(7)/2)*l
og(x**3 - 2*x**2*(1 + sqrt(7)/2) - 1)
```

Maxima [F]

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = \int \frac{6\,x^5-13\,x^4-12\,x^3-6\,x^2+22\,x}{x^6-4\,x^5-3\,x^4-2\,x^3+4\,x^2+1}\,dx$$

input
```
integrate((6*x^5-13*x^4-12*x^3-6*x^2+22*x)/(x^6-4*x^5-3*x^4-2*x^3+4*x^2+1)
,x, algorithm="maxima")
```

output
```
integrate((6*x^5 - 13*x^4 - 12*x^3 - 6*x^2 + 22*x)/(x^6 - 4*x^5 - 3*x^4 -
2*x^3 + 4*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = -\frac{1}{2}\sqrt{7}\log\left(\left|x^3+\sqrt{7}x^2-2\,x^2-1\right|\right) + \frac{1}{2}\sqrt{7}\log\left(\left|x^3-\sqrt{7}x^2-2\,x^2-1\right|\right) + \log\left(\left|x^6-4\,x^5-3\,x^4-2\,x^3+4\,x^2+1\right|\right)$$

input
```
integrate((6*x^5-13*x^4-12*x^3-6*x^2+22*x)/(x^6-4*x^5-3*x^4-2*x^3+4*x^2+1)
,x, algorithm="giac")
```

output
```
-1/2*sqrt(7)*log(abs(x^3 + sqrt(7)*x^2 - 2*x^2 - 1)) + 1/2*sqrt(7)*log(abs
(x^3 - sqrt(7)*x^2 - 2*x^2 - 1)) + log(abs(x^6 - 4*x^5 - 3*x^4 - 2*x^3 + 4
*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = \ln\left(\sqrt{7}\,x^2-2\,x^2+x^3-1\right) + \ln\left(x^3-2\,x^2-\sqrt{7}\,x^2-1\right) - \frac{\sqrt{7}\,\ln\left(\sqrt{7}\,x^2-2\,x^2+x^3-1\right)}{2} + \frac{\sqrt{7}\,\ln\left(x^3-2\,x^2-\sqrt{7}\,x^2-1\right)}{2}$$

input
```
int(-(6*x^2 - 22*x + 12*x^3 + 13*x^4 - 6*x^5)/(4*x^2 - 2*x^3 - 3*x^4 - 4*x
^5 + x^6 + 1),x)
```

output
```
log(7^(1/2)*x^2 - 2*x^2 + x^3 - 1) + log(x^3 - 2*x^2 - 7^(1/2)*x^2 - 1) -
(7^(1/2)*log(7^(1/2)*x^2 - 2*x^2 + x^3 - 1))/2 + (7^(1/2)*log(x^3 - 2*x^2
- 7^(1/2)*x^2 - 1))/2
```

Reduce [F]

$$\int \frac{22x-6x^2-12x^3-13x^4+6x^5}{1+4x^2-2x^3-3x^4-4x^5+x^6}\,dx = 7\left(\int \frac{x^4}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx\right) + 14\left(\int \frac{x}{x^6-4x^5-3x^4-2x^3+4x^2+1}dx\right) + \log\left(x^6-4x^5-3x^4-2x^3+4x^2+1\right)$$

input
```
int((6*x^5-13*x^4-12*x^3-6*x^2+22*x)/(x^6-4*x^5-3*x^4-2*x^3+4*x^2+1),x)
```

output
```
7*int(x**4/(x**6 - 4*x**5 - 3*x**4 - 2*x**3 + 4*x**2 + 1),x) + 14*int(x/(x
**6 - 4*x**5 - 3*x**4 - 2*x**3 + 4*x**2 + 1),x) + log(x**6 - 4*x**5 - 3*x*
*4 - 2*x**3 + 4*x**2 + 1)
```

3.83 $\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$

Optimal result

Integrand size = 86, antiderivative size = 41

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = \frac{1}{4}\log\left(3-2\sqrt{2}-2x+x^2\right)-\frac{1}{4}\log\left(3-2\sqrt{2}+2x+x^2\right)$$

output
```
1/4*ln(3-2*2^(1/2)-2*x+x^2)-1/4*ln(3-2*2^(1/2)+2*x+x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.39

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = -\frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-17+12\sqrt{2}+\left(-2+4\sqrt{2}\right)x^2-x^4\right)\left(\log\left(-3+2\sqrt{2}-2x-x^2\right)-\log(-3+2\sqrt{2}\right)}{4\left(-577+408\sqrt{2}+8\left(-41+29\sqrt{2}\right)x^2+\left(-78+56\sqrt{2}\right)x^4+8\left(-1+\sqrt{2}\right)x^6-x^8\right)}$$

input
```
Integrate[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt
[2] + 328*x^2 - 232*Sqrt[2]*x^2 + 78*x^4 - 56*Sqrt[2]*x^4 + 8*x^6 - 8*Sqrt
[2]*x^6 + x^8),x]
```

output
```
-1/4*((3 - 2*Sqrt[2] + x^2)^2*(-17 + 12*Sqrt[2] + (-2 + 4*Sqrt[2])*x^2 - x
^4)*(Log[-3 + 2*Sqrt[2] - 2*x - x^2] - Log[-3 + 2*Sqrt[2] + 2*x - x^2]))/(
-577 + 408*Sqrt[2] + 8*(-41 + 29*Sqrt[2])*x^2 + (-78 + 56*Sqrt[2])*x^4 + 8
*(-1 + Sqrt[2])*x^6 - x^8)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.85 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {6, 6, 6, 2019, 2019, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4-232\sqrt{2}x^2+328x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\sqrt{2}x^6+8x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8+\left(8-8\sqrt{2}\right)x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)\left(x^2+2\sqrt{2}-3\right)}{x^6+\left(5-6\sqrt{2}\right)x^4+\left(39-28\sqrt{2}\right)x^2-70\sqrt{2}+99}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{ 1081}$$

$$\frac{1}{2}\int \left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+ \frac{1}{2}\int \left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+ \frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt[2] +
328*x^2 - 232*Sqrt[2]*x^2 + 78*x^4 - 56*Sqrt[2]*x^4 + 8*x^6 - 8*Sqrt[2]*x^
6 + x^8),x]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34
risch	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34
parallelrisch	$\frac{\ln\left(3-2\sqrt{2}-2x+x^2\right)}{4}-\frac{\ln\left(3-2\sqrt{2}+2x+x^2\right)}{4}$	34

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-232*2^
(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*ln(3-2*2^(1/2)-2*x+x^2)-1/4*ln(3-2*2^(1/2)+2*x+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$= -\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((3-2*2**(1/2)+x**2)**2*(-3+2*2**(1/2)+x**2)/(577-408*2**(1/2)+32
8*x**2-232*2**(1/2)*x**2+78*x**4-56*2**(1/2)*x**4+8*x**6-8*x**6*2**(1/2)+x
**8),x)
```

output
```
Exception raised: PolynomialError >> 1/(-489331912114255602061892417478047
2498117708482611714912381696*_t**4 + 346009913306969839800447635927970293O
0522480193213103784309 76*sqrt(2)*_t**4 - 159769239484575670917838951113184
628965915778476
```

Maxima [F]

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2 \left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$= \int \frac{\left(x^2+2\sqrt{2}-3\right)\left(x^2-2\sqrt{2}+3\right)^2}{x^8-8\sqrt{2}x^6+8x^6-56\sqrt{2}x^4+78x^4-232\sqrt{2}x^2+328x^2-408\sqrt{2}+577}\,dx$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="maxima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)*(x^2 - 2*sqrt(2) + 3)^2/(x^8 - 8*sqrt(2)*x
^6 + 8*x^6 - 56*sqrt(2)*x^4 + 78*x^4 - 232*sqrt(2)*x^2 + 328*x^2 - 408*sqr
t(2) + 577), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2 \left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx$$
$$= -\frac{1}{4}\log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-
232*2^(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x, algorith
m="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}x^2-4x^2-28\right)}\right)}{2}$$

input
```
int(((x^2 - 2*2^(1/2) + 3)^2*(2*2^(1/2) + x^2 - 3))/(328*x^2 - 232*2^(1/2)
*x^2 - 56*2^(1/2)*x^4 - 8*2^(1/2)*x^6 - 408*2^(1/2) + 78*x^4 + 8*x^6 + x^8
 + 577),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\begin{aligned}&\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+328x^2-232\sqrt{2}x^2+78x^4-56\sqrt{2}x^4+8x^6-8\sqrt{2}x^6+x^8}\,dx\\&=6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right)\\&\quad+4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right)\\&\quad-2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)\\&\quad+\int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx-\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right)\\&\quad+27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right)-3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)\end{aligned}$$

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+328*x^2-232*2^
(1/2)*x^2+78*x^4-56*2^(1/2)*x^4+8*x^6-8*x^6*2^(1/2)+x^8),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.84 $\int \frac{79-64x}{6+24x+12x^2-24x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \frac{79-64x}{6+24x+12x^2-24x^3} \, dx = \frac{95\text{arctanh}\left(\frac{1}{3}\left(-\sqrt{3}+2\sqrt{3}x\right)\right)}{3\sqrt{3}} - \frac{37}{2}\log(1+2x) + \frac{37}{4}\log\left(-1-2x+2x^2\right)$$

output
```
95/9*arctanh(-1/3*3^(1/2)+2/3*x*3^(1/2))*3^(1/2)-37/2*ln(1+2*x)+37/4*ln(2*
x^2-2*x-1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{79-64x}{6+24x+12x^2-24x^3} \, dx = \frac{1}{36}\left(\left(333-190\sqrt{3}\right)\log\left(1+\sqrt{3}-2x\right) - 666\log(1+2x) + \left(333+190\sqrt{3}\right)\log\left(-1+\sqrt{3}+2x\right)\right)$$

input
```
Integrate[(79 - 64*x)/(6 + 24*x + 12*x^2 - 24*x^3),x]
```

output
```
((333 - 190*Sqrt[3])*Log[1 + Sqrt[3] - 2*x] - 666*Log[1 + 2*x] + (333 + 19
0*Sqrt[3])*Log[-1 + Sqrt[3] + 2*x])/36
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{79-64x}{-24x^3+12x^2+24x+6}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{222x-301}{6(2x^2-2x-1)}-\frac{37}{2x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{36}\left(333+190\sqrt{3}\right)\log\left(-2x-\sqrt{3}+1\right)+\frac{1}{36}\left(333-190\sqrt{3}\right)\log\left(-2x+\sqrt{3}+1\right)-\frac{37}{2}\log(2x+1)$$

input
```
Int[(79 - 64*x)/(6 + 24*x + 12*x^2 - 24*x^3),x]
```

output
```
((333 + 190*Sqrt[3])*Log[1 - Sqrt[3] - 2*x])/36 + ((333 - 190*Sqrt[3])*Log
[1 + Sqrt[3] - 2*x])/36 - (37*Log[1 + 2*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

method	result
default	$\frac{37\ln\left(2x^{2}-2x-1\right)}{4}+\frac{95\sqrt{3}\,\operatorname{arctanh}\left(\frac{(4x-2)\sqrt{3}}{6}\right)}{9}-\frac{37\ln(1+2x)}{2}$
risch	$-\frac{37\ln(1+2x)}{2}+\frac{37\ln\left(380x-190+190\sqrt{3}\right)}{4}+\frac{95\ln\left(380x-190+190\sqrt{3}\right)\sqrt{3}}{18}+\frac{37\ln\left(380x-190-190\sqrt{3}\right)}{4}-\frac{95\ln\left(380x-19\right.}{1}$

input
```
int((79-64*x)/(-24*x^3+12*x^2+24*x+6),x,method=_RETURNVERBOSE)
```

output
```
37/4*ln(2*x^2-2*x-1)+95/9*3^(1/2)*arctanh(1/6*(4*x-2)*3^(1/2))-37/2*ln(1+2
*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx = \frac{95}{18}\sqrt{3}\log\left(\frac{2\,x^2+\sqrt{3}(2\,x-1)-2\,x+2}{2\,x^2-2\,x-1}\right) + \frac{37}{4}\log\left(2\,x^2-2\,x-1\right)-\frac{37}{2}\log\left(2\,x+1\right)$$

input
```
integrate((79-64*x)/(-24*x^3+12*x^2+24*x+6),x, algorithm="fricas")
```

output
```
95/18*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x - 1) - 2*x + 2)/(2*x^2 - 2*x - 1))
 + 37/4*log(2*x^2 - 2*x - 1) - 37/2*log(2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73

$$\begin{aligned}\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx &= -\frac{37\log\left(x+\frac{1}{2}\right)}{2} \\ &\quad+\left(\frac{95\sqrt{3}}{18}+\frac{37}{4}\right)\log\left(x-\frac{157101607}{112695460}-\frac{9571\sqrt{3}}{593134}+\frac{148932\left(\frac{95\sqrt{3}}{18}+\frac{37}{4}\right)^2}{28173865}\right) \\ &\quad+\left(\frac{37}{4}-\frac{95\sqrt{3}}{18}\right)\log\left(x-\frac{157101607}{112695460}+\frac{148932\left(\frac{37}{4}-\frac{95\sqrt{3}}{18}\right)^2}{28173865}+\frac{9571\sqrt{3}}{593134}\right)\end{aligned}$$

input
```
integrate((79-64*x)/(-24*x**3+12*x**2+24*x+6),x)
```

output
```
-37*log(x + 1/2)/2 + (95*sqrt(3)/18 + 37/4)*log(x - 157101607/112695460 -
9571*sqrt(3)/593134 + 148932*(95*sqrt(3)/18 + 37/4)**2/28173865) + (37/4 -
 95*sqrt(3)/18)*log(x - 157101607/112695460 + 148932*(37/4 - 95*sqrt(3)/18
)**2/28173865 + 9571*sqrt(3)/593134)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx = -\frac{95}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}-1}{2x+\sqrt{3}-1}\right) \\ +\frac{37}{4}\log\left(2x^2-2x-1\right)-\frac{37}{2}\log\left(2x+1\right)\end{aligned}$$

input
```
integrate((79-64*x)/(-24*x^3+12*x^2+24*x+6),x, algorithm="maxima")
```

output
```
-95/18*sqrt(3)*log((2*x - sqrt(3) - 1)/(2*x + sqrt(3) - 1)) + 37/4*log(2*x
^2 - 2*x - 1) - 37/2*log(2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx = -\frac{95}{18}\sqrt{3}\log\left(\frac{|4\,x-2\,\sqrt{3}-2|}{|4\,x+2\,\sqrt{3}-2|}\right) + \frac{37}{4}\log\left(|2\,x^2-2\,x-1|\right) - \frac{37}{2}\log\left(|2\,x+1|\right)$$

input
```
integrate((79-64*x)/(-24*x^3+12*x^2+24*x+6),x, algorithm="giac")
```

output
```
-95/18*sqrt(3)*log(abs(4*x - 2*sqrt(3) - 2)/abs(4*x + 2*sqrt(3) - 2)) + 37
/4*log(abs(2*x^2 - 2*x - 1)) - 37/2*log(abs(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx = \ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{95\,\sqrt{3}}{18}+\frac{37}{4}\right) - \ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{95\,\sqrt{3}}{18}-\frac{37}{4}\right) - \frac{37\,\ln\left(x+\frac{1}{2}\right)}{2}$$

input
```
int(-(64*x - 79)/(24*x + 12*x^2 - 24*x^3 + 6),x)
```

output
```
log(x + 3^(1/2)/2 - 1/2)*((95*3^(1/2))/18 + 37/4) - log(x - 3^(1/2)/2 - 1/
2)*((95*3^(1/2))/18 - 37/4) - (37*log(x + 1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{79-64x}{6+24x+12x^2-24x^3}\,dx = -\frac{95\sqrt{3}\log\left(-\sqrt{3}+2x-1\right)}{18} + \frac{95\sqrt{3}\log\left(\sqrt{3}+2x-1\right)}{18} + \frac{37\log\left(-\sqrt{3}+2x-1\right)}{4} + \frac{37\log\left(\sqrt{3}+2x-1\right)}{4} - \frac{37\log(2x+1)}{2}$$

input
```
int((79-64*x)/(-24*x^3+12*x^2+24*x+6),x)
```

output
```
( - 190*sqrt(3)*log( - sqrt(3) + 2*x - 1) + 190*sqrt(3)*log(sqrt(3) + 2*x
- 1) + 333*log( - sqrt(3) + 2*x - 1) + 333*log(sqrt(3) + 2*x - 1) - 666*lo
g(2*x + 1))/36
```

3.85 $\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = -\frac{3759}{31}\sqrt{3}\arctan\left(\frac{1}{3}\left(-\sqrt{3}+2\sqrt{3}x\right)\right) + \frac{27129}{217}\log(-6+x) - \frac{1922}{21}\log(1+x) - \frac{3115}{186}\log\left(1-x+x^2\right)$$

output
```
-3759/31*3^(1/2)*arctan(-1/3*3^(1/2)+2/3*x*3^(1/2))+27129/217*ln(-6+x)-192
2/21*ln(1+x)-3115/186*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = \frac{-157878\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+162774\log(6-x)-119164\log(1+x)-21805\log\left(1-x+x^2\right)}{1302}$$

input
```
Integrate[(1665 + 386*x + 643*x^2)/(-6 + x - 6*x^3 + x^4),x]
```

output
```
(-157878*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 162774*Log[6 - x] - 119164*L
og[1 + x] - 21805*Log[1 - x + x^2])/1302
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{643x^2+386x+1665}{x^4-6x^3+x-6}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(-\frac{7(445x+2194)}{93(x^2-x+1)}+\frac{27129}{217(x-6)}-\frac{1922}{21(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3759}{31}\sqrt{3}\arctan\left(\frac{1-2x}{\sqrt{3}}\right)-\frac{3115}{186}\log\left(x^2-x+1\right)+\frac{27129}{217}\log(6-x)-\frac{1922}{21}\log(x+1)$$

input
```
Int[(1665 + 386*x + 643*x^2)/(-6 + x - 6*x^3 + x^4),x]
```

output
```
(3759*Sqrt[3]*ArcTan[(1 - 2*x)/Sqrt[3]])/31 + (27129*Log[6 - x])/217 - (19
22*Log[1 + x])/21 - (3115*Log[1 - x + x^2])/186
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{3115 \ln\left(x^{2}-x+1\right)}{186}-\frac{3759 \sqrt{3}\, \arctan\left(\frac{(2 x-1) \sqrt{3}}{3}\right)}{31}-\frac{1922 \ln(1+x)}{21}+\frac{27129 \ln(-6+x)}{217}$	41
risch	$\frac{27129 \ln(-6+x)}{217}-\frac{3115 \ln\left(2595321 x^{2}-2595321 x+2595321\right)}{186}-\frac{3759 \sqrt{3}\, \arctan\left(\frac{2\left(1611 x-\frac{1611}{2}\right) \sqrt{3}}{4833}\right)}{31}-\frac{1922 \ln(1+x)}{21}$	43

input
```
int((643*x^2+386*x+1665)/(x^4-6*x^3+x-6),x,method=_RETURNVERBOSE)
```

output
```
-3115/186*ln(x^2-x+1)-3759/31*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1922/21*
ln(1+x)+27129/217*ln(-6+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = -\frac{3759}{31}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3115}{186}\log\left(x^2-x+1\right) - \frac{1922}{21}\log\left(x+1\right) + \frac{27129}{217}\log\left(x-6\right)$$

input
```
integrate((643*x^2+386*x+1665)/(x^4-6*x^3+x-6),x, algorithm="fricas")
```

output
```
-3759/31*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3115/186*log(x^2 - x + 1)
 - 1922/21*log(x + 1) + 27129/217*log(x - 6)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1665 + 386x + 643x^2}{-6 + x - 6x^3 + x^4}\,dx = \frac{27129\log(x-6)}{217} - \frac{1922\log(x+1)}{21} - \frac{3115\log(x^2-x+1)}{186} - \frac{3759\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{31}$$

input
```
integrate((643*x**2+386*x+1665)/(x**4-6*x**3+x-6),x)
```

output
```
27129*log(x - 6)/217 - 1922*log(x + 1)/21 - 3115*log(x**2 - x + 1)/186 - 3
759*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/31
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1665 + 386x + 643x^2}{-6 + x - 6x^3 + x^4}\,dx = -\frac{3759}{31}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3115}{186}\log\left(x^2-x+1\right) - \frac{1922}{21}\log(x+1) + \frac{27129}{217}\log(x-6)$$

input
```
integrate((643*x^2+386*x+1665)/(x^4-6*x^3+x-6),x, algorithm="maxima")
```

output
```
-3759/31*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3115/186*log(x^2 - x + 1)
 - 1922/21*log(x + 1) + 27129/217*log(x - 6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = -\frac{3759}{31}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3115}{186}\log\left(x^2-x+1\right) - \frac{1922}{21}\log\left(|x+1|\right) + \frac{27129}{217}\log\left(|x-6|\right)$$

input
```
integrate((643*x^2+386*x+1665)/(x^4-6*x^3+x-6),x, algorithm="giac")
```

output
```
-3759/31*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3115/186*log(x^2 - x + 1)
 - 1922/21*log(abs(x + 1)) + 27129/217*log(abs(x - 6))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = \frac{27129\ln(x-6)}{217} - \frac{1922\ln(x+1)}{21} + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3115}{186}+\frac{\sqrt{3}\,3759\mathrm{i}}{62}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3115}{186}+\frac{\sqrt{3}\,3759\mathrm{i}}{62}\right)$$

input
```
int((386*x + 643*x^2 + 1665)/(x - 6*x^3 + x^4 - 6),x)
```

output
```
(27129*log(x - 6))/217 - (1922*log(x + 1))/21 + log(x - (3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*3759i)/62 - 3115/186) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*3759i)/62 + 3115/186)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{1665+386x+643x^2}{-6+x-6x^3+x^4}\,dx = -\frac{3759\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{31} - \frac{3115\log(x^2-x+1)}{186} + \frac{27129\log(x-6)}{217} - \frac{1922\log(x+1)}{21}$$

input
```
int((643*x^2+386*x+1665)/(x^4-6*x^3+x-6),x)
```

output
```
( - 157878*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 21805*log(x**2 - x + 1) + 162
774*log(x - 6) - 119164*log(x + 1))/1302
```

3.86 $\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 174

$$\begin{aligned}\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = &-\sqrt{\frac{3167+186\sqrt{309}}{3399}}\arctan\left(\sqrt{-\frac{93}{11}+\frac{6\sqrt{309}}{11}}\right.\\ &\left.-\sqrt{-\frac{82}{11}+\frac{6\sqrt{309}}{11}}x\right)\\ &+\sqrt{\frac{-3167+186\sqrt{309}}{3399}}\text{arctanh}\left(\sqrt{\frac{93}{11}+\frac{6\sqrt{309}}{11}}\right.\\ &\left.+\sqrt{\frac{82}{11}+\frac{6\sqrt{309}}{11}}x\right)\\ &-\frac{3}{2}\sqrt{\frac{3}{103}}\log\left(10+3x-\sqrt{309}x+10x^2\right)\\ &+\frac{3}{2}\sqrt{\frac{3}{103}}\log\left(10+3x+\sqrt{309}x+10x^2\right)\end{aligned}$$

output
```
1/3399*(10764633+632214*309^(1/2))^(1/2)*arctan(-1/11*(-1023+66*309^(1/2))
^(1/2)+1/11*(-902+66*309^(1/2))^(1/2)*x)+1/3399*(-10764633+632214*309^(1/2
))^(1/2)*arctanh(1/11*(1023+66*309^(1/2))^(1/2)+1/11*(902+66*309^(1/2))^(1
/2)*x)-3/206*309^(1/2)*ln(10+3*x-309^(1/2)*x+10*x^2)+3/206*309^(1/2)*ln(10
+3*x+309^(1/2)*x+10*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = \text{RootSum}\left[5+3\#1-5\#1^2+3\#1^3 + 5\#1^4\&, \frac{9\log(x-\#1)+10\log(x-\#1)\#1}{3-10\#1+9\#1^2+20\#1^3}\&\right]$$

input
```
Integrate[(-9 - 10*x)/(-5 - 3*x + 5*x^2 - 3*x^3 - 5*x^4),x]
```

output
```
RootSum[5 + 3*#1 - 5*#1^2 + 3*#1^3 + 5*#1^4 & , (9*Log[x - #1] + 10*Log[x
- #1]*#1)/(3 - 10*#1 + 9*#1^2 + 20*#1^3) & ]
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-10x-9}{-5x^4-3x^3+5x^2-3x-5}dx$$

$$\Big\downarrow \text{2492}$$

$$-\frac{1}{5}\int\left(\frac{5\left(-90x-9\sqrt{309}+73\right)}{\sqrt{309}\left(10x^2+\left(3+\sqrt{309}\right)x+10\right)}-\frac{5\left(-90x+9\sqrt{309}+73\right)}{\sqrt{309}\left(10x^2+\left(3-\sqrt{309}\right)x+10\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}\left(5\sqrt{\frac{3167+186\sqrt{309}}{3399}}\arctan\left(\frac{20x-\sqrt{309}+3}{\sqrt{2\left(41+3\sqrt{309}\right)}}\right)+5\sqrt{\frac{186\sqrt{309}-3167}{3399}}\operatorname{arctanh}\left(\frac{20x+\sqrt{309}+3}{\sqrt{2\left(3\sqrt{309}-41\right)}}\right)-\right.$$

input
```
Int[(-9 - 10*x)/(-5 - 3*x + 5*x^2 - 3*x^3 - 5*x^4),x]
```

output
```
(5*Sqrt[(3167 + 186*Sqrt[309])/3399]*ArcTan[(3 - Sqrt[309] + 20*x)/Sqrt[2*
(41 + 3*Sqrt[309])]] + 5*Sqrt[(-3167 + 186*Sqrt[309])/3399]*ArcTanh[(3 + S
qrt[309] + 20*x)/Sqrt[2*(-41 + 3*Sqrt[309])]] - (15*Sqrt[3/103]*Log[10 + (
3 - Sqrt[309])*x + 10*x^2])/2 + (15*Sqrt[3/103]*Log[10 + (3 + Sqrt[309])*x
 + 10*x^2])/2)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.31

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(5\_Z^4+3\_Z^3-5\_Z^2+3\_Z+5)} \frac{(9+10\_R)\ln(x-\_R)}{20\_R^3+9\_R^2-10\_R+3}$
default	$-\frac{3\sqrt{309}\,\ln\left(10+3x-\sqrt{309}\,x+10x^2\right)}{206}-\frac{2\left(-\frac{9\sqrt{309}\,(-\sqrt{309}+3)}{2}-73\sqrt{309}-2781\right)\arctan\left(\frac{3-\sqrt{309}+20x}{\sqrt{82+6\sqrt{309}}}\right)}{309\sqrt{82+6\sqrt{309}}}+\frac{3\sqrt{309}\,\ln\left(10+3x+\right.}{206}$

input
```
int((-9-10*x)/(-5*x^4-3*x^3+5*x^2-3*x-5),x,method=_RETURNVERBOSE)
```

output
```
sum((9+10*_R)/(20*_R^3+9*_R^2-10*_R+3)*ln(x-_R),_R=RootOf(5*_Z^4+3*_Z^3-5*
_Z^2+3*_Z+5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx$$
$$= \frac{1}{2}\left(3\sqrt{\frac{3}{103}}+\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}-\frac{3167}{3399}}\right)\log\left(103\left(25\sqrt{\frac{3}{103}}+9\right)\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}-\frac{3167}{3399}}+980\,x+5047\sqrt{\frac{3}{103}}+147\right)$$
$$+\frac{1}{2}\left(3\sqrt{\frac{3}{103}}-\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}-\frac{3167}{3399}}\right)\log\left(-103\left(25\sqrt{\frac{3}{103}}+9\right)\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}-\frac{3167}{3399}}+980\,x+5047\sqrt{\frac{3}{103}}+147\right)$$
$$+\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}+\frac{3167}{3399}}\arctan\left(\frac{103}{245}\left(\sqrt{\frac{3}{103}}(173\,x+165)-27\,x-30\right)\sqrt{\frac{62}{11}\sqrt{\frac{3}{103}}+\frac{3167}{3399}}\right)$$
$$-\frac{3}{2}\sqrt{\frac{3}{103}}\log\left(10\,x^2-103\sqrt{\frac{3}{103}}x+3\,x+10\right)$$

input
```
integrate((-9-10*x)/(-5*x^4-3*x^3+5*x^2-3*x-5),x, algorithm="fricas")
```

output
```
1/2*(3*sqrt(3/103) + sqrt(62/11*sqrt(3/103) - 3167/3399))*log(103*(25*sqrt
(3/103) + 9)*sqrt(62/11*sqrt(3/103) - 3167/3399) + 980*x + 5047*sqrt(3/103
) + 147) + 1/2*(3*sqrt(3/103) - sqrt(62/11*sqrt(3/103) - 3167/3399))*log(-
103*(25*sqrt(3/103) + 9)*sqrt(62/11*sqrt(3/103) - 3167/3399) + 980*x + 504
7*sqrt(3/103) + 147) + sqrt(62/11*sqrt(3/103) + 3167/3399)*arctan(103/245*
(sqrt(3/103)*(173*x + 165) - 27*x - 30)*sqrt(62/11*sqrt(3/103) + 3167/3399
)) - 3/2*sqrt(3/103)*log(10*x^2 - 103*sqrt(3/103)*x + 3*x + 10)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.24

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx$$
$$= \text{RootSum}\left(1050291t^4+351642t^2-258633t+32825,\left(t\mapsto t\log\left(\frac{116699t^3}{1164485}+\frac{1985016t^2}{1164485}+\frac{6621767t}{3493455}\right.\right.\right.$$

input
```
integrate((-9-10*x)/(-5*x**4-3*x**3+5*x**2-3*x-5),x)
```

output
```
RootSum(1050291*_t**4 + 351642*_t**2 - 258633*_t + 32825, Lambda(_t, _t*lo
g(116699*_t**3/1164485 + 1985016*_t**2/1164485 + 6621767*_t/3493455 + x +
485416/1164485)))
```

Maxima [F]

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = \int \frac{10\,x+9}{5\,x^4+3\,x^3-5\,x^2+3\,x+5}\,dx$$

input
```
integrate((-9-10*x)/(-5*x^4-3*x^3+5*x^2-3*x-5),x, algorithm="maxima")
```

output
```
integrate((10*x + 9)/(5*x^4 + 3*x^3 - 5*x^2 + 3*x + 5), x)
```

Giac [F]

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = \int \frac{10\,x+9}{5\,x^4+3\,x^3-5\,x^2+3\,x+5}\,dx$$

input
```
integrate((-9-10*x)/(-5*x^4-3*x^3+5*x^2-3*x-5),x, algorithm="giac")
```

output
```
integrate((10*x + 9)/(5*x^4 + 3*x^3 - 5*x^2 + 3*x + 5), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.84

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = \sum_{k=1}^{4}\ln\left(-\frac{501\,\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)}{25}+8\,x-\frac{\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)\,x\,452}{25}-\frac{\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)^2\,x\,9579}{625}-\frac{\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)^3\,x\,32136}{625}-\frac{927\,\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)^2}{125}-\frac{6489\,\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)^3}{125}+\frac{36}{5}\right)\,\mathrm{root}\left(z^4+\frac{1138\,z^2}{3399}-\frac{279\,z}{1133}+\frac{32825}{1050291},z,k\right)$$

input
```
int((10*x + 9)/(3*x - 5*x^2 + 3*x^3 + 5*x^4 + 5),x)
```

output
```
symsum(log(8*x - (501*root(z^4 + (1138*z^2)/3399 - (279*z)/1133 + 32825/10
50291, z, k))/25 - (452*root(z^4 + (1138*z^2)/3399 - (279*z)/1133 + 32825/
1050291, z, k)*x)/25 - (9579*root(z^4 + (1138*z^2)/3399 - (279*z)/1133 + 3
2825/1050291, z, k)^2*x)/625 - (32136*root(z^4 + (1138*z^2)/3399 - (279*z)
/1133 + 32825/1050291, z, k)^3*x)/625 - (927*root(z^4 + (1138*z^2)/3399 -
(279*z)/1133 + 32825/1050291, z, k)^2)/125 - (6489*root(z^4 + (1138*z^2)/3
399 - (279*z)/1133 + 32825/1050291, z, k)^3)/125 + 36/5)*root(z^4 + (1138*
z^2)/3399 - (279*z)/1133 + 32825/1050291, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{-9-10x}{-5-3x+5x^2-3x^3-5x^4}\,dx = 10\left(\int \frac{x}{5x^4+3x^3-5x^2+3x+5}dx\right) + 9\left(\int \frac{1}{5x^4+3x^3-5x^2+3x+5}dx\right)$$

input
```
int((-9-10*x)/(-5*x^4-3*x^3+5*x^2-3*x-5),x)
```

output
```
10*int(x/(5*x**4 + 3*x**3 - 5*x**2 + 3*x + 5),x) + 9*int(1/(5*x**4 + 3*x**
3 - 5*x**2 + 3*x + 5),x)
```

3.87 $\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx = -\sqrt{\frac{88}{95}+\frac{56}{5\sqrt{19}}}\arctan\left(\sqrt{\frac{1}{5}\left(3+\sqrt{19}\right)}x\right) + \sqrt{-\frac{88}{95}+\frac{56}{5\sqrt{19}}}\text{arctanh}\left(\sqrt{\frac{1}{5}\left(-3+\sqrt{19}\right)}x\right) + \frac{9\text{arctanh}\left(\frac{1}{19}\left(-3\sqrt{19}+2\sqrt{19}x^2\right)\right)}{2\sqrt{19}}$$

output

```
-2/95*(2090+1330*19^(1/2))^(1/2)*arctan(1/5*(15+5*19^(1/2))^(1/2)*x)+2/95*
(-2090+1330*19^(1/2))^(1/2)*arctanh(1/5*(-15+5*19^(1/2))^(1/2)*x)+9/38*arc
tanh(-3/19*19^(1/2)+2/19*19^(1/2)*x^2)*19^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.20

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx = \frac{1}{76}\left(8\left(-19+\sqrt{19}\right)\sqrt{\frac{2}{-3+\sqrt{19}}}\arctan\left(\sqrt{\frac{2}{-3+\sqrt{19}}}x\right)+8\sqrt{\frac{2}{3+\sqrt{19}}}\left(19+\sqrt{19}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{19}}}x\right)+9\sqrt{19}\left(\log\left(3-\sqrt{19}-2x^2\right)-\log\left(3+\sqrt{19}-2x^2\right)\right)\right)$$

input
```
Integrate[(8 - 9*x - 8*x^2)/(-5 - 6*x^2 + 2*x^4),x]
```

output
```
(8*(-19 + Sqrt[19])*Sqrt[2/(-3 + Sqrt[19])]*ArcTan[Sqrt[2/(-3 + Sqrt[19])]
*x] + 8*Sqrt[2/(3 + Sqrt[19])]*(19 + Sqrt[19])*ArcTanh[Sqrt[2/(3 + Sqrt[19
])]*x] + 9*Sqrt[19]*(Log[3 - Sqrt[19] - 2*x^2] - Log[3 + Sqrt[19] - 2*x^2]
))/76
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2202, 27, 25, 1432, 1081, 1480, 216, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-8x^2-9x+8}{2x^4-6x^2-5}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int -\frac{9x}{2x^4-6x^2-5}dx+\int \frac{8-8x^2}{2x^4-6x^2-5}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{8-8x^2}{2x^4-6x^2-5}dx - 9\int -\frac{x}{-2x^4+6x^2+5}dx$$

$$\Big\downarrow \text{25}$$

$$9\int \frac{x}{-2x^4+6x^2+5}dx + \int \frac{8-8x^2}{2x^4-6x^2-5}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{9}{2}\int \frac{1}{-2x^4+6x^2+5}dx^2 + \int \frac{8-8x^2}{2x^4-6x^2-5}dx$$

$$\Big\downarrow \text{1081}$$

$$\int \frac{8-8x^2}{2x^4-6x^2-5}dx - 9\int \left(\frac{1}{2\sqrt{19}\left(-2x^2-\sqrt{19}+3\right)} - \frac{1}{2\sqrt{19}\left(-2x^2+\sqrt{19}+3\right)}\right)dx^2$$

$$\Big\downarrow \text{1480}$$

$$-\frac{4}{19}\left(19+\sqrt{19}\right)\int \frac{1}{2x^2-\sqrt{19}-3}dx - \frac{4}{19}\left(19-\sqrt{19}\right)\int \frac{1}{2x^2+\sqrt{19}-3}dx - 9\int \left(\frac{1}{2\sqrt{19}\left(-2x^2-\sqrt{19}+3\right)} - \frac{1}{2\sqrt{19}\left(-2x^2+\sqrt{19}+3\right)}\right)dx^2$$

$$\Big\downarrow \text{216}$$

$$-\frac{4}{19}\left(19+\sqrt{19}\right)\int \frac{1}{2x^2-\sqrt{19}-3}dx - 9\int \left(\frac{1}{2\sqrt{19}\left(-2x^2-\sqrt{19}+3\right)} - \frac{1}{2\sqrt{19}\left(-2x^2+\sqrt{19}+3\right)}\right)dx^2 - \frac{2}{19}\left(19-\sqrt{19}\right)\sqrt{\frac{2}{\sqrt{19}-3}}\arctan\left(\sqrt{\frac{2}{\sqrt{19}-3}}x\right)$$

$$\Big\downarrow \text{220}$$

$$-9\int \left(\frac{1}{2\sqrt{19}\left(-2x^2-\sqrt{19}+3\right)} - \frac{1}{2\sqrt{19}\left(-2x^2+\sqrt{19}+3\right)}\right)dx^2 - \frac{2}{19}\left(19-\sqrt{19}\right)\sqrt{\frac{2}{\sqrt{19}-3}}\arctan\left(\sqrt{\frac{2}{\sqrt{19}-3}}x\right) + \frac{2}{19}\sqrt{\frac{2}{3+\sqrt{19}}}\left(19+\sqrt{19}\right)\text{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{19}}}x\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{19}\left(19-\sqrt{19}\right)\sqrt{\frac{2}{\sqrt{19}-3}}\arctan\left(\sqrt{\frac{2}{\sqrt{19}-3}}x\right)+\frac{2}{19}\sqrt{\frac{2}{3+\sqrt{19}}}\left(19+\sqrt{19}\right)\operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{19}}}x\right)-9\left(\frac{\log\left(-2x^2+\sqrt{19}+3\right)}{4\sqrt{19}}-\frac{\log\left(-2x^2-\sqrt{19}+3\right)}{4\sqrt{19}}\right)$$

input
```
Int[(8 - 9*x - 8*x^2)/(-5 - 6*x^2 + 2*x^4),x]
```

output
```
(-2*(19 - Sqrt[19])*Sqrt[2/(-3 + Sqrt[19])]*ArcTan[Sqrt[2/(-3 + Sqrt[19])]
*x])/19 + (2*Sqrt[2/(3 + Sqrt[19])]*(19 + Sqrt[19])*ArcTanh[Sqrt[2/(3 + Sq
rt[19])]*x])/19 - 9*(-1/4*Log[3 - Sqrt[19] - 2*x^2]/Sqrt[19] + Log[3 + Sqr
t[19] - 2*x^2]/(4*Sqrt[19]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-6\_Z^2-5\right)} \frac{\left(-8\_R^2-9\_R+8\right)\ln\left(x-\_R\right)}{2\_R^3-3\_R}\right)}{4}$
default	$-\frac{9\sqrt{19}\,\ln\left(2x^2-\sqrt{19}-3\right)}{76}+\frac{\left(4\sqrt{19}+76\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{19}}}\right)}{19\sqrt{6+2\sqrt{19}}}+\frac{9\sqrt{19}\,\ln\left(2x^2-3+\sqrt{19}\right)}{76}+\frac{\left(4\sqrt{19}-76\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{19}-6}}\right)}{19\sqrt{2\sqrt{19}-6}}$

input
```
int((-8*x^2-9*x+8)/(2*x^4-6*x^2-5),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((-8*_R^2-9*_R+8)/(2*_R^3-3*_R)*ln(x-_R),_R=RootOf(2*_Z^4-6*_Z^2-5)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx$$
$$= -\frac{1}{76}\left(9\sqrt{19}-76\sqrt{\frac{14}{95}\sqrt{19}-\frac{22}{95}}\right)\log\left(\left(8\sqrt{19}+19\right)\sqrt{\frac{14}{95}\sqrt{19}-\frac{22}{95}}+18\,x\right)$$
$$-\frac{1}{76}\left(9\sqrt{19}+76\sqrt{\frac{14}{95}\sqrt{19}-\frac{22}{95}}\right)\log\left(-\left(8\sqrt{19}+19\right)\sqrt{\frac{14}{95}\sqrt{19}-\frac{22}{95}}+18\,x\right)$$
$$-2\sqrt{\frac{14}{95}\sqrt{19}+\frac{22}{95}}\arctan\left(\frac{1}{18}\left(\sqrt{19}x+19\,x\right)\sqrt{\frac{14}{95}\sqrt{19}+\frac{22}{95}}\right)$$
$$+\frac{9}{76}\sqrt{19}\log\left(2\,x^2+\sqrt{19}-3\right)$$

input
```
integrate((-8*x^2-9*x+8)/(2*x^4-6*x^2-5),x, algorithm="fricas")
```

output
```
-1/76*(9*sqrt(19) - 76*sqrt(14/95*sqrt(19) - 22/95))*log((8*sqrt(19) + 19)
*sqrt(14/95*sqrt(19) - 22/95) + 18*x) - 1/76*(9*sqrt(19) + 76*sqrt(14/95*s
qrt(19) - 22/95))*log(-(8*sqrt(19) + 19)*sqrt(14/95*sqrt(19) - 22/95) + 18
*x) - 2*sqrt(14/95*sqrt(19) + 22/95)*arctan(1/18*(sqrt(19)*x + 19*x)*sqrt(
14/95*sqrt(19) + 22/95)) + 9/76*sqrt(19)*log(2*x^2 + sqrt(19) - 3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx =$$
$$-\operatorname{RootSum}\left(462080t^4-32224t^2-612864t-76059,\left(t\mapsto t\log\left(-\frac{9393220t^3}{4904289}-\frac{373160t^2}{544921}+\frac{625499}{1961715}\right.\right.\right.$$

input
```
integrate((-8*x**2-9*x+8)/(2*x**4-6*x**2-5),x)
```

output
```
-RootSum(462080*_t**4 - 32224*_t**2 - 612864*_t - 76059, Lambda(_t, _t*log
(-9393220*_t**3/4904289 - 373160*_t**2/544921 + 625499*_t/19617156 + x + 2
102419/1089842)))
```

Maxima [F]

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx = \int -\frac{8\,x^2+9\,x-8}{2\,x^4-6\,x^2-5}\,dx$$

input
```
integrate((-8*x^2-9*x+8)/(2*x^4-6*x^2-5),x, algorithm="maxima")
```

output
```
-integrate((8*x^2 + 9*x - 8)/(2*x^4 - 6*x^2 - 5), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx \\
&= \frac{1}{380}\left(32\sqrt{19}\sqrt{2\sqrt{19}+6}-45\sqrt{19}-76\sqrt{2\sqrt{19}+6}\right)\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{19}+\frac{3}{2}}\right|\right) \\
&\quad -\frac{1}{380}\left(32\sqrt{19}\sqrt{2\sqrt{19}+6}+45\sqrt{19}-76\sqrt{2\sqrt{19}+6}\right)\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{19}+\frac{3}{2}}\right|\right) \\
&\quad -\frac{2}{95}\sqrt{1330\sqrt{19}+2090}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{19}-\frac{3}{2}}}\right)+\frac{9}{76}\sqrt{19}\log\left(x^2+\frac{1}{2}\sqrt{19}-\frac{3}{2}\right)
\end{aligned}$$

input
```
integrate((-8*x^2-9*x+8)/(2*x^4-6*x^2-5),x, algorithm="giac")
```

output
```
1/380*(32*sqrt(19)*sqrt(2*sqrt(19) + 6) - 45*sqrt(19) - 76*sqrt(2*sqrt(19)
 + 6))*log(abs(x + sqrt(1/2*sqrt(19) + 3/2))) - 1/380*(32*sqrt(19)*sqrt(2*
sqrt(19) + 6) + 45*sqrt(19) - 76*sqrt(2*sqrt(19) + 6))*log(abs(x - sqrt(1/
2*sqrt(19) + 3/2))) - 2/95*sqrt(1330*sqrt(19) + 2090)*arctan(x/sqrt(1/2*sq
rt(19) - 3/2)) + 9/76*sqrt(19)*log(x^2 + 1/2*sqrt(19) - 3/2)
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.19

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx = \sum_{k=1}^{4} \ln\left(576\,\mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)-\frac{153\,x}{8} - \frac{\mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)\,x\,883}{2} + \mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)^2\,x\,342 + \mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)^3\,x\,456 + 304\,\mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)^2 - 207\right)\,\mathrm{root}\left(z^4-\frac{53\,z^2}{760}+\frac{126\,z}{95}-\frac{76059}{462080},z,k\right)$$

input
```
int((9*x + 8*x^2 - 8)/(6*x^2 - 2*x^4 + 5),x)
```

output
```
symsum(log(576*root(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z, k)
- (153*x)/8 - (883*root(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z,
 k)*x)/2 + 342*root(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z, k)^
2*x + 456*root(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z, k)^3*x +
 304*root(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z, k)^2 - 207)*r
oot(z^4 - (53*z^2)/760 + (126*z)/95 - 76059/462080, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.69

$$\int \frac{8-9x-8x^2}{-5-6x^2+2x^4}\,dx = -\frac{16\sqrt{\sqrt{19}-3}\,\sqrt{38}\,atan\left(\frac{2x}{\sqrt{\sqrt{19}-3}\,\sqrt{2}}\right)}{95} - \frac{2\sqrt{\sqrt{19}-3}\,\sqrt{2}\,atan\left(\frac{2x}{\sqrt{\sqrt{19}-3}\,\sqrt{2}}\right)}{5} - \frac{8\sqrt{\sqrt{19}+3}\,\sqrt{38}\log\left(-\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{95} + \frac{8\sqrt{\sqrt{19}+3}\,\sqrt{38}\log\left(\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{95} + \frac{\sqrt{\sqrt{19}+3}\,\sqrt{2}\log\left(-\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{5} - \frac{\sqrt{\sqrt{19}+3}\,\sqrt{2}\log\left(\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{5} - \frac{9\sqrt{19}\log\left(-\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{76} - \frac{9\sqrt{19}\log\left(\sqrt{\sqrt{19}+3}+\sqrt{2}\,x\right)}{76} + \frac{9\sqrt{19}\log\left(\sqrt{19}+2x^2-3\right)}{76}$$

input
```
int((-8*x^2-9*x+8)/(2*x^4-6*x^2-5),x)
```

output
```
( - 64*sqrt(sqrt(19) - 3)*sqrt(38)*atan((2*x)/(sqrt(sqrt(19) - 3)*sqrt(2))
) - 152*sqrt(sqrt(19) - 3)*sqrt(2)*atan((2*x)/(sqrt(sqrt(19) - 3)*sqrt(2))
) - 32*sqrt(sqrt(19) + 3)*sqrt(38)*log( - sqrt(sqrt(19) + 3) + sqrt(2)*x)
+ 32*sqrt(sqrt(19) + 3)*sqrt(38)*log(sqrt(sqrt(19) + 3) + sqrt(2)*x) + 76*
sqrt(sqrt(19) + 3)*sqrt(2)*log( - sqrt(sqrt(19) + 3) + sqrt(2)*x) - 76*sqr
t(sqrt(19) + 3)*sqrt(2)*log(sqrt(sqrt(19) + 3) + sqrt(2)*x) - 45*sqrt(19)*
log( - sqrt(sqrt(19) + 3) + sqrt(2)*x) - 45*sqrt(19)*log(sqrt(sqrt(19) + 3
) + sqrt(2)*x) + 45*sqrt(19)*log(sqrt(19) + 2*x**2 - 3))/380
```

3.88 $\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx$

Optimal result

Integrand size = 73, antiderivative size = 147

$$\begin{aligned}
&\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx \\
&= \frac{1}{24}\Big(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+3\left(-1+\sqrt{5}\right)x^2-2x^6\right) \\
&\quad +\left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+3\left(1+\sqrt{5}\right)x^2-2x^6\right) \\
&\quad -\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+3\left(-1+\sqrt{5}\right)x^2+2x^6\right) \\
&\quad -\left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+3\left(1+\sqrt{5}\right)x^2+2x^6\right)\Big)
\end{aligned}$$

output
```
1/24*(5^(1/2)+1)*ln(1-5^(1/2)+3*(5^(1/2)-1)*x^2-2*x^6)+1/24*(5^(1/2)-1)*ln
(-1-5^(1/2)+3*(5^(1/2)+1)*x^2-2*x^6)-1/24*(5^(1/2)+1)*ln(1-5^(1/2)+3*(5^(1
/2)-1)*x^2+2*x^6)-1/24*(5^(1/2)-1)*ln(-1-5^(1/2)+3*(5^(1/2)+1)*x^2+2*x^6)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx$$
$$=\frac{1}{24}\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+3\left(-1+\sqrt{5}\right)x^2-2x^6\right)+\left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+3\left(1+\sqrt{5}\right)x^2-2x^6\right)-\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+3\left(-1+\sqrt{5}\right)x^2+2x^6\right)-\left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+3\left(1+\sqrt{5}\right)x^2+2x^6\right)\right)$$

input
```
Integrate[(x^5*(3 - 24*x^2 + 63*x^4 - 54*x^6 - 2*x^12 + 4*x^14))/(1 - 12*x
^2 + 54*x^4 - 108*x^6 + 81*x^8 - 3*x^12 + 18*x^14 - 27*x^16 + x^24),x]
```

output
```
((1 + Sqrt[5])*Log[1 - Sqrt[5] + 3*(-1 + Sqrt[5])*x^2 - 2*x^6] + (-1 + Sqr
t[5])*Log[-1 - Sqrt[5] + 3*(1 + Sqrt[5])*x^2 - 2*x^6] - (1 + Sqrt[5])*Log[
1 - Sqrt[5] + 3*(-1 + Sqrt[5])*x^2 + 2*x^6] - (-1 + Sqrt[5])*Log[-1 - Sqrt
[5] + 3*(1 + Sqrt[5])*x^2 + 2*x^6])/24
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(4x^{14}-2x^{12}-54x^6+63x^4-24x^2+3\right)}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}\,dx$$
$$\Big\downarrow \text{ 2460}$$
$$\int \left(\frac{x\left(-x^{10}-7x^6+3x^4+3x^2-1\right)}{2\left(-x^{12}-3x^8+x^6+9x^4-6x^2+1\right)}-\frac{x\left(x^{10}-7x^6+3x^4-3x^2+1\right)}{2\left(x^{12}-3x^8+x^6-9x^4+6x^2-1\right)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$
\begin{aligned}
&-\frac{5}{8}\text{Subst}\left(\int \frac{x^2}{x^6-3x^4+x^3-9x^2+6x-1}dx,x,x^2\right)+ \\
&\frac{5}{4}\text{Subst}\left(\int \frac{x^3}{x^6-3x^4+x^3-9x^2+6x-1}dx,x,x^2\right)- \\
&\frac{5}{8}\text{Subst}\left(\int \frac{x^2}{x^6+3x^4-x^3-9x^2+6x-1}dx,x,x^2\right)+ \\
&\frac{5}{4}\text{Subst}\left(\int \frac{x^3}{x^6+3x^4-x^3-9x^2+6x-1}dx,x,x^2\right)+ \\
&\frac{1}{24}\log\left(-x^{12}-3x^8+x^6+9x^4-6x^2+1\right)-\frac{1}{24}\log\left(x^{12}-3x^8+x^6-9x^4+6x^2-1\right)
\end{aligned}
$$

input
```
Int[(x^5*(3 - 24*x^2 + 63*x^4 - 54*x^6 - 2*x^12 + 4*x^14))/(1 - 12*x^2 + 5
4*x^4 - 108*x^6 + 81*x^8 - 3*x^12 + 18*x^14 - 27*x^16 + x^24),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\frac{\sqrt{5}}{6}-\frac{1}{6}\right)\ln\left(2x^6+\left(-3-3\sqrt{5}\right)x^2+\sqrt{5}+1\right)}{4}+\frac{\left(-\frac{1}{6}-\frac{\sqrt{5}}{6}\right)\ln\left(2x^6+\left(3\sqrt{5}-3\right)x^2-\sqrt{5}+1\right)}{4}+\frac{\left(\frac{1}{6}+\frac{\sqrt{5}}{6}\right)\ln\left(2x^6+\left(3-3\sqrt{5}\right)x^2+\sqrt{\right.}}{4}$
risch	$\frac{\ln\left(2x^6+\left(3-3\sqrt{5}\right)x^2+\sqrt{5}-1\right)}{24}+\frac{\ln\left(2x^6+\left(3-3\sqrt{5}\right)x^2+\sqrt{5}-1\right)\sqrt{5}}{24}+\frac{\ln\left(2x^6+\left(3\sqrt{5}+3\right)x^2-\sqrt{5}-1\right)}{24}-\frac{\ln\left(2x^6+\left(3\sqrt{5}+3\right)x\right.}{24}$

input
```
int(x^5*(4*x^14-2*x^12-54*x^6+63*x^4-24*x^2+3)/(x^24-27*x^16+18*x^14-3*x^1
2+81*x^8-108*x^6+54*x^4-12*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(1/6*5^(1/2)-1/6)*ln(2*x^6+(-3-3*5^(1/2))*x^2+5^(1/2)+1)+1/4*(-1/6-1/6
*5^(1/2))*ln(2*x^6+(3*5^(1/2)-3)*x^2-5^(1/2)+1)+1/4*(1/6+1/6*5^(1/2))*ln(2
*x^6+(3-3*5^(1/2))*x^2+5^(1/2)-1)+1/4*(1/6-1/6*5^(1/2))*ln(2*x^6+(3*5^(1/2
)+3)*x^2-5^(1/2)-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.56

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx$$
$$= \frac{1}{24}\sqrt{5}\log\left(\frac{2\,x^{12}+6\,x^8-2\,x^6+27\,x^4-18\,x^2-\sqrt{5}(6\,x^8-2\,x^6+9\,x^4-6\,x^2+1)+3}{x^{12}+3\,x^8-x^6-9\,x^4+6\,x^2-1}\right)$$
$$+\frac{1}{24}\sqrt{5}\log\left(\frac{2\,x^{12}-6\,x^8+2\,x^6+27\,x^4-18\,x^2-\sqrt{5}(6\,x^8-2\,x^6-9\,x^4+6\,x^2-1)+3}{x^{12}-3\,x^8+x^6-9\,x^4+6\,x^2-1}\right)$$
$$+\frac{1}{24}\log\left(x^{12}+3\,x^8-x^6-9\,x^4+6\,x^2-1\right)-\frac{1}{24}\log\left(x^{12}-3\,x^8+x^6-9\,x^4+6\,x^2-1\right)$$

input
```
integrate(x^5*(4*x^14-2*x^12-54*x^6+63*x^4-24*x^2+3)/(x^24-27*x^16+18*x^14
-3*x^12+81*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="fricas")
```

output
```
1/24*sqrt(5)*log((2*x^12 + 6*x^8 - 2*x^6 + 27*x^4 - 18*x^2 - sqrt(5)*(6*x^
8 - 2*x^6 + 9*x^4 - 6*x^2 + 1) + 3)/(x^12 + 3*x^8 - x^6 - 9*x^4 + 6*x^2 -
1)) + 1/24*sqrt(5)*log((2*x^12 - 6*x^8 + 2*x^6 + 27*x^4 - 18*x^2 - sqrt(5)
*(6*x^8 - 2*x^6 - 9*x^4 + 6*x^2 - 1) + 3)/(x^12 - 3*x^8 + x^6 - 9*x^4 + 6*
x^2 - 1)) + 1/24*log(x^12 + 3*x^8 - x^6 - 9*x^4 + 6*x^2 - 1) - 1/24*log(x^
12 - 3*x^8 + x^6 - 9*x^4 + 6*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.92

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx = \left(\frac{1}{24} - \frac{\sqrt{5}}{24}\right)\log\left(x^6+x^2\left(-\frac{9}{2}+5184\left(\frac{1}{24}-\frac{\sqrt{5}}{24}\right)^3+\frac{9\sqrt{5}}{2}\right)-\frac{3\sqrt{5}}{2}-1728\left(\frac{1}{24}-\frac{\sqrt{5}}{24}\right)^3+\frac{3}{2}\right) + \left(\frac{1}{24} + \frac{\sqrt{5}}{24}\right)\log\left(x^6+x^2\left(-\frac{9\sqrt{5}}{2}-\frac{9}{2}+5184\left(\frac{1}{24}+\frac{\sqrt{5}}{24}\right)^3\right)-1728\left(\frac{1}{24}+\frac{\sqrt{5}}{24}\right)^3+\frac{3}{2}+\frac{3\sqrt{5}}{2}\right) + \left(-\frac{1}{24} + \frac{\sqrt{5}}{24}\right)\log\left(x^6+x^2\left(-\frac{9\sqrt{5}}{2}+5184\left(-\frac{1}{24}+\frac{\sqrt{5}}{24}\right)^3+\frac{9}{2}\right)-\frac{3}{2}-1728\left(-\frac{1}{24}+\frac{\sqrt{5}}{24}\right)^3+\frac{3\sqrt{5}}{2}\right) + \left(-\frac{\sqrt{5}}{24} - \frac{1}{24}\right)\log\left(x^6+x^2\cdot\left(5184\left(-\frac{\sqrt{5}}{24}-\frac{1}{24}\right)^3+\frac{9}{2}+\frac{9\sqrt{5}}{2}\right)-\frac{3\sqrt{5}}{2}-\frac{3}{2}-1728\left(-\frac{\sqrt{5}}{24}-\frac{1}{24}\right)^3\right)$$

input
```
integrate(x**5*(4*x**14-2*x**12-54*x**6+63*x**4-24*x**2+3)/(x**24-27*x**16
+18*x**14-3*x**12+81*x**8-108*x**6+54*x**4-12*x**2+1),x)
```

output
```
(1/24 - sqrt(5)/24)*log(x**6 + x**2*(-9/2 + 5184*(1/24 - sqrt(5)/24)**3 +
9*sqrt(5)/2) - 3*sqrt(5)/2 - 1728*(1/24 - sqrt(5)/24)**3 + 3/2) + (1/24 +
sqrt(5)/24)*log(x**6 + x**2*(-9*sqrt(5)/2 - 9/2 + 5184*(1/24 + sqrt(5)/24)
**3) - 1728*(1/24 + sqrt(5)/24)**3 + 3/2 + 3*sqrt(5)/2) + (-1/24 + sqrt(5)
/24)*log(x**6 + x**2*(-9*sqrt(5)/2 + 5184*(-1/24 + sqrt(5)/24)**3 + 9/2) -
 3/2 - 1728*(-1/24 + sqrt(5)/24)**3 + 3*sqrt(5)/2) + (-sqrt(5)/24 - 1/24)*
log(x**6 + x**2*(5184*(-sqrt(5)/24 - 1/24)**3 + 9/2 + 9*sqrt(5)/2) - 3*sqr
t(5)/2 - 3/2 - 1728*(-sqrt(5)/24 - 1/24)**3)
```

Maxima [F]

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx = \int \frac{(4\,x^{14}-2\,x^{12}-54\,x^6+63\,x^4-24\,x^2+3)x^5}{x^{24}-27\,x^{16}+18\,x^{14}-3\,x^{12}+81\,x^8-108\,x^6+54\,x^4-12\,x^2+1}\,dx$$

input
```
integrate(x^5*(4*x^14-2*x^12-54*x^6+63*x^4-24*x^2+3)/(x^24-27*x^16+18*x^14
-3*x^12+81*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="maxima")
```

output
```
integrate((4*x^14 - 2*x^12 - 54*x^6 + 63*x^4 - 24*x^2 + 3)*x^5/(x^24 - 27*
x^16 + 18*x^14 - 3*x^12 + 81*x^8 - 108*x^6 + 54*x^4 - 12*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx = \frac{1}{24}\log\left(\left|x^{12}+3\,x^8-x^6-9\,x^4+6\,x^2-1\right|\right) - \frac{1}{24}\log\left(\left|x^{12}-3\,x^8+x^6-9\,x^4+6\,x^2-1\right|\right)$$

input
```
integrate(x^5*(4*x^14-2*x^12-54*x^6+63*x^4-24*x^2+3)/(x^24-27*x^16+18*x^14
-3*x^12+81*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="giac")
```

output
```
1/24*log(abs(x^12 + 3*x^8 - x^6 - 9*x^4 + 6*x^2 - 1)) - 1/24*log(abs(x^12
- 3*x^8 + x^6 - 9*x^4 + 6*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 12.32 (sec) , antiderivative size = 1389, normalized size of antiderivative = 9.45

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx$$
$$= \text{Too large to display}$$

input
```
int(-(x^5*(24*x^2 - 63*x^4 + 54*x^6 + 2*x^12 - 4*x^14 - 3))/(54*x^4 - 12*x
^2 - 108*x^6 + 81*x^8 - 3*x^12 + 18*x^14 - 27*x^16 + x^24 + 1),x)
```

output
```
(atan((x^2*263824405954560000000i)/(201166109540352000000000*x^4 - 134110739
6935680000000*x^2 + 293138228838400000000*x^6 - 879414686515200000000*x^8
+ 469021166141440000000*x^12 + 223517899489280000000) - (x^4*39573660893184
0000000i)/(201166109540352000000000*x^4 - 1341107396935680000000*x^2 + 29313
8228838400000000*x^6 - 879414686515200000000*x^8 + 469021166141440000000*x^
12 + 223517899489280000000) - (x^6*883078914375680000000i)/(201166109540352
0000000*x^4 - 1341107396935680000000*x^2 + 293138228838400000000*x^6 - 879
414686515200000000*x^8 + 469021166141440000000*x^12 + 223517899489280000000
) + (x^8*264923674312704000000i)/(201166109540352000000000*x^4 - 13411073969
35680000000*x^2 + 293138228838400000000*x^6 - 879414686515200000000*x^8 +
469021166141440000000*x^12 + 223517899489280000000) + (x^12*439707343257600
00000i)/(201166109540352000000000*x^4 - 1341107396935680000000*x^2 + 2931382
28838400000000*x^6 - 879414686515200000000*x^8 + 469021166141440000000*x^12
 + 223517899489280000000) + (5^(1/2)*498334989025280000000i)/(2011661095403
520000000*x^4 - 1341107396935680000000*x^2 + 293138228838400000000*x^6 - 8
79414686515200000000*x^8 + 469021166141440000000*x^12 + 2235178994892800000
00) - 439707343257600000000i/(201166109540352000000000*x^4 - 1341107396935680
000000*x^2 + 293138228838400000000*x^6 - 879414686515200000000*x^8 + 46902
116614144000000*x^12 + 223517899489280000000) - (5^(1/2)*x^2*2990009934151
68000000i)/(201166109540352000000000*x^4 - 1341107396935680000000*x^2 + 2...
```

Reduce [F]

$$\int \frac{x^5(3-24x^2+63x^4-54x^6-2x^{12}+4x^{14})}{1-12x^2+54x^4-108x^6+81x^8-3x^{12}+18x^{14}-27x^{16}+x^{24}}\,dx$$
$$= 4\left(\int \frac{x^{19}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-2\left(\int \frac{x^{17}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-54\left(\int \frac{x^{11}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$+63\left(\int \frac{x^{9}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-24\left(\int \frac{x^{7}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$+3\left(\int \frac{x^{5}}{x^{24}-27x^{16}+18x^{14}-3x^{12}+81x^8-108x^6+54x^4-12x^2+1}dx\right)$$

input
```
int(x^5*(4*x^14-2*x^12-54*x^6+63*x^4-24*x^2+3)/(x^24-27*x^16+18*x^14-3*x^1
2+81*x^8-108*x^6+54*x^4-12*x^2+1),x)
```

output
```
4*int(x**19/(x**24 - 27*x**16 + 18*x**14 - 3*x**12 + 81*x**8 - 108*x**6 +
54*x**4 - 12*x**2 + 1),x) - 2*int(x**17/(x**24 - 27*x**16 + 18*x**14 - 3*x
**12 + 81*x**8 - 108*x**6 + 54*x**4 - 12*x**2 + 1),x) - 54*int(x**11/(x**2
4 - 27*x**16 + 18*x**14 - 3*x**12 + 81*x**8 - 108*x**6 + 54*x**4 - 12*x**2
 + 1),x) + 63*int(x**9/(x**24 - 27*x**16 + 18*x**14 - 3*x**12 + 81*x**8 -
108*x**6 + 54*x**4 - 12*x**2 + 1),x) - 24*int(x**7/(x**24 - 27*x**16 + 18*
x**14 - 3*x**12 + 81*x**8 - 108*x**6 + 54*x**4 - 12*x**2 + 1),x) + 3*int(x
**5/(x**24 - 27*x**16 + 18*x**14 - 3*x**12 + 81*x**8 - 108*x**6 + 54*x**4
- 12*x**2 + 1),x)
```

3.89 $\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx$

Optimal result

Integrand size = 68, antiderivative size = 212

$$\begin{aligned}
&\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx \\
&= \frac{1}{4}\arctan\left(\frac{1}{9}\left(2-9\sqrt{3}+6x^2\right)\right)+\frac{1}{4}\arctan\left(\frac{1}{9}\left(2+9\sqrt{3}+6x^2\right)\right) \\
&+\frac{1}{4}\arctan\left(18+\sqrt{3}-54x^2-2x^4-18\sqrt{3}x^4-6x^6\right) \\
&+\frac{1}{4}\arctan\left(18-\sqrt{3}-54x^2-2x^4+18\sqrt{3}x^4-6x^6\right) \\
&+\frac{1}{8}\sqrt{3}\log\left(1-6x^2+9x^4+\sqrt{3}x^4-3\sqrt{3}x^6+x^8\right) \\
&-\frac{1}{8}\sqrt{3}\log\left(1-6x^2+9x^4-\sqrt{3}x^4+3\sqrt{3}x^6+x^8\right)
\end{aligned}$$

output

```
1/4*arctan(2/9-3^(1/2)+2/3*x^2)+1/4*arctan(2/9+3^(1/2)+2/3*x^2)-1/4*arctan
(-18-3^(1/2)+54*x^2+2*x^4+18*3^(1/2)*x^4+6*x^6)-1/4*arctan(-18+3^(1/2)+54*
x^2+2*x^4-18*3^(1/2)*x^4+6*x^6)+1/8*3^(1/2)*ln(1-6*x^2+9*x^4+3^(1/2)*x^4-3
*x^6*3^(1/2)+x^8)-1/8*3^(1/2)*ln(1-6*x^2+9*x^4-3^(1/2)*x^4+3*x^6*3^(1/2)+x
^8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx$$
$$= \frac{1}{4}\text{RootSum}\Big[1-12\#1^2+54\#1^4-108\#1^6+80\#1^8+6\#1^{10}-9\#1^{12}$$
$$+\#1^{16}\&, \frac{2\log(x-\#1)\#1^2-15\log(x-\#1)\#1^4+36\log(x-\#1)\#1^6-27\log(x-\#1)\#1^8-4\log(}{-6+54\#1^2-162\#1^4+160\#1^6+15\#1^8-27\#1^{10}+4\#1}$$

input
```
Integrate[(x^3*(2 - 15*x^2 + 36*x^4 - 27*x^6 - 4*x^8 + 6*x^10))/(1 - 12*x^
2 + 54*x^4 - 108*x^6 + 80*x^8 + 6*x^10 - 9*x^12 + x^16),x]
```

output
```
RootSum[1 - 12*#1^2 + 54*#1^4 - 108*#1^6 + 80*#1^8 + 6*#1^10 - 9*#1^12 + #
1^16 & , (2*Log[x - #1]*#1^2 - 15*Log[x - #1]*#1^4 + 36*Log[x - #1]*#1^6 -
 27*Log[x - #1]*#1^8 - 4*Log[x - #1]*#1^10 + 6*Log[x - #1]*#1^12)/(-6 + 54
*#1^2 - 162*#1^4 + 160*#1^6 + 15*#1^8 - 27*#1^10 + 4*#1^14) & ]/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(6x^{10}-4x^8-27x^6+36x^4-15x^2+2)}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}\,dx$$
$$\Big\downarrow \text{7283}$$
$$\frac{1}{2}\int \frac{x^2(6x^{10}-4x^8-27x^6+36x^4-15x^2+2)}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx^2$$
$$\Big\downarrow \text{7293}$$
$$\frac{1}{2}\int \left(\frac{6x^{12}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}-\frac{4x^{10}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\int \frac{x^2}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx^2-15\int \frac{x^4}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+5}\right.$$

input
```
Int[(x^3*(2 - 15*x^2 + 36*x^4 - 27*x^6 - 4*x^8 + 6*x^10))/(1 - 12*x^2 + 54
*x^4 - 108*x^6 + 80*x^8 + 6*x^10 - 9*x^12 + x^16),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.20

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_R\ln\left(x^4+\left(3\_R^3-3\_R\right)x^2-\_R^3+\_R\right)\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_R\ln\left(x^4+\left(3\_R^3-3\_R\right)x^2-\_R^3+\_R\right)\right)}{4}$	43

input
```
int(x^3*(6*x^10-4*x^8-27*x^6+36*x^4-15*x^2+2)/(x^16-9*x^12+6*x^10+80*x^8-1
08*x^6+54*x^4-12*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(x^4+(3*_R^3-3*_R)*x^2-_R^3+_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

$$\begin{aligned}
&\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx \\
&= -\frac{1}{8}\sqrt{3}\log\left(x^8+9\,x^4-6\,x^2+\sqrt{3}(3\,x^6-x^4)+1\right) \\
&\quad+\frac{1}{8}\sqrt{3}\log\left(x^8+9\,x^4-6\,x^2-\sqrt{3}(3\,x^6-x^4)+1\right) \\
&\quad-\frac{1}{4}\arctan\left(6\,x^6+2\,x^4+54\,x^2+\sqrt{3}(18\,x^4-1)-18\right) \\
&\quad+\frac{1}{4}\arctan\left(-6\,x^6-2\,x^4-54\,x^2+\sqrt{3}(18\,x^4-1)+18\right) \\
&\quad+\frac{1}{4}\arctan\left(\frac{2}{3}\,x^2+\sqrt{3}+\frac{2}{9}\right)-\frac{1}{4}\arctan\left(-\frac{2}{3}\,x^2+\sqrt{3}-\frac{2}{9}\right)
\end{aligned}$$

input
```
integrate(x^3*(6*x^10-4*x^8-27*x^6+36*x^4-15*x^2+2)/(x^16-9*x^12+6*x^10+80
*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="fricas")
```

output
```
-1/8*sqrt(3)*log(x^8 + 9*x^4 - 6*x^2 + sqrt(3)*(3*x^6 - x^4) + 1) + 1/8*sq
rt(3)*log(x^8 + 9*x^4 - 6*x^2 - sqrt(3)*(3*x^6 - x^4) + 1) - 1/4*arctan(6*
x^6 + 2*x^4 + 54*x^2 + sqrt(3)*(18*x^4 - 1) - 18) + 1/4*arctan(-6*x^6 - 2*
x^4 - 54*x^2 + sqrt(3)*(18*x^4 - 1) + 18) + 1/4*arctan(2/3*x^2 + sqrt(3) +
 2/9) - 1/4*arctan(-2/3*x^2 + sqrt(3) - 2/9)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx \\
&= \frac{\sqrt{3}\log\left(x^8-3\sqrt{3}x^6+x^4(\sqrt{3}+9)-6x^2+1\right)}{8} \\
&\quad-\frac{\sqrt{3}\log\left(x^8+3\sqrt{3}x^6+x^4\cdot(9-\sqrt{3})-6x^2+1\right)}{8}+\frac{\operatorname{atan}\left(\frac{2x^2}{3}+\frac{2}{9}+\sqrt{3}\right)}{4} \\
&\quad+\frac{\operatorname{atan}\left(\frac{2x^2}{3}-\sqrt{3}+\frac{2}{9}\right)}{4}+\frac{\operatorname{atan}\left(-6x^6+x^4(-18\sqrt{3}-2)-54x^2+\sqrt{3}+18\right)}{4} \\
&\quad-\frac{\operatorname{atan}\left(6x^6-x^4(-2+18\sqrt{3})+54x^2-18+\sqrt{3}\right)}{4}
\end{aligned}$$

input
```
integrate(x**3*(6*x**10-4*x**8-27*x**6+36*x**4-15*x**2+2)/(x**16-9*x**12+6
*x**10+80*x**8-108*x**6+54*x**4-12*x**2+1),x)
```

output
```
sqrt(3)*log(x**8 - 3*sqrt(3)*x**6 + x**4*(sqrt(3) + 9) - 6*x**2 + 1)/8 - s
qrt(3)*log(x**8 + 3*sqrt(3)*x**6 + x**4*(9 - sqrt(3)) - 6*x**2 + 1)/8 + at
an(2*x**2/3 + 2/9 + sqrt(3))/4 + atan(2*x**2/3 - sqrt(3) + 2/9)/4 + atan(-
6*x**6 + x**4*(-18*sqrt(3) - 2) - 54*x**2 + sqrt(3) + 18)/4 - atan(6*x**6
- x**4*(-2 + 18*sqrt(3)) + 54*x**2 - 18 + sqrt(3))/4
```

Maxima [F]

$$\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx = \int \frac{(6\,x^{10}-4\,x^8-27\,x^6+36\,x^4-15\,x^2+2)x^3}{x^{16}-9\,x^{12}+6\,x^{10}+80\,x^8-108\,x^6+54\,x^4-12\,x^2+1}\,dx$$

input
```
integrate(x^3*(6*x^10-4*x^8-27*x^6+36*x^4-15*x^2+2)/(x^16-9*x^12+6*x^10+80
*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="maxima")
```

output
```
integrate((6*x^10 - 4*x^8 - 27*x^6 + 36*x^4 - 15*x^2 + 2)*x^3/(x^16 - 9*x^
12 + 6*x^10 + 80*x^8 - 108*x^6 + 54*x^4 - 12*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx$$
$$= -\frac{1}{8}\sqrt{3}\log\left(\left(2x^4+3\sqrt{3}x^2-\sqrt{3}\right)^2+\left(3x^2-1\right)^2\right)$$
$$+\frac{1}{8}\sqrt{3}\log\left(\left(2x^4-3\sqrt{3}x^2+\sqrt{3}\right)^2+\left(3x^2-1\right)^2\right)$$
$$-\frac{1}{4}\arctan\left(6x^6+2x^4\left(9\sqrt{3}+1\right)+54x^2-\sqrt{3}-18\right)$$
$$-\frac{1}{4}\arctan\left(6x^6-2x^4\left(9\sqrt{3}-1\right)+54x^2+\sqrt{3}-18\right)$$
$$+\frac{1}{4}\arctan\left(\frac{2}{3}x^2+\sqrt{3}+\frac{2}{9}\right)+\frac{1}{4}\arctan\left(\frac{2}{3}x^2-\sqrt{3}+\frac{2}{9}\right)$$

input
```
integrate(x^3*(6*x^10-4*x^8-27*x^6+36*x^4-15*x^2+2)/(x^16-9*x^12+6*x^10+80
*x^8-108*x^6+54*x^4-12*x^2+1),x, algorithm="giac")
```

output
```
-1/8*sqrt(3)*log((2*x^4 + 3*sqrt(3)*x^2 - sqrt(3))^2 + (3*x^2 - 1)^2) + 1/
8*sqrt(3)*log((2*x^4 - 3*sqrt(3)*x^2 + sqrt(3))^2 + (3*x^2 - 1)^2) - 1/4*a
rctan(6*x^6 + 2*x^4*(9*sqrt(3) + 1) + 54*x^2 - sqrt(3) - 18) - 1/4*arctan(
6*x^6 - 2*x^4*(9*sqrt(3) - 1) + 54*x^2 + sqrt(3) - 18) + 1/4*arctan(2/3*x^
2 + sqrt(3) + 2/9) + 1/4*arctan(2/3*x^2 - sqrt(3) + 2/9)
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 1081, normalized size of antiderivative = 5.10

$$\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx = \text{Too large to display}$$

input
```
int(-(x^3*(15*x^2 - 36*x^4 + 27*x^6 + 4*x^8 - 6*x^10 - 2))/(54*x^4 - 12*x^
2 - 108*x^6 + 80*x^8 + 6*x^10 - 9*x^12 + x^16 + 1),x)
```

output
```
atan(85712904/(3^(1/2)*x^2*621355131i - 3^(1/2)*207118377i + 3^(1/2)*x^4*1
22611968i - 1890235035*x^2 + 539261712*x^4 + 630078345) - (257138712*x^2)/
(3^(1/2)*x^2*621355131i - 3^(1/2)*207118377i + 3^(1/2)*x^4*122611968i - 18
90235035*x^2 + 539261712*x^4 + 630078345) - (4361607*x^4)/(3^(1/2)*x^2*621
355131i - 3^(1/2)*207118377i + 3^(1/2)*x^4*122611968i - 1890235035*x^2 + 5
39261712*x^4 + 630078345) + (3^(1/2)*330936840i)/(3^(1/2)*x^2*621355131i -
 3^(1/2)*207118377i + 3^(1/2)*x^4*122611968i - 1890235035*x^2 + 539261712*
x^4 + 630078345) - (3^(1/2)*x^2*992810520i)/(3^(1/2)*x^2*621355131i - 3^(1
/2)*207118377i + 3^(1/2)*x^4*122611968i - 1890235035*x^2 + 539261712*x^4 +
 630078345) + (3^(1/2)*x^4*418598361i)/(3^(1/2)*x^2*621355131i - 3^(1/2)*2
07118377i + 3^(1/2)*x^4*122611968i - 1890235035*x^2 + 539261712*x^4 + 6300
78345))/4 - atan((257138712*x^2)/(3^(1/2)*207118377i - 3^(1/2)*x^2*6213551
31i - 3^(1/2)*x^4*122611968i - 1890235035*x^2 + 539261712*x^4 + 630078345)
 - 85712904/(3^(1/2)*207118377i - 3^(1/2)*x^2*621355131i - 3^(1/2)*x^4*122
611968i - 1890235035*x^2 + 539261712*x^4 + 630078345) + (4361607*x^4)/(3^(
1/2)*207118377i - 3^(1/2)*x^2*621355131i - 3^(1/2)*x^4*122611968i - 189023
5035*x^2 + 539261712*x^4 + 630078345) + (3^(1/2)*330936840i)/(3^(1/2)*2071
18377i - 3^(1/2)*x^2*621355131i - 3^(1/2)*x^4*122611968i - 1890235035*x^2
+ 539261712*x^4 + 630078345) - (3^(1/2)*x^2*992810520i)/(3^(1/2)*207118377
i - 3^(1/2)*x^2*621355131i - 3^(1/2)*x^4*122611968i - 1890235035*x^2 + ...
```

Reduce [F]

$$\int \frac{x^3(2-15x^2+36x^4-27x^6-4x^8+6x^{10})}{1-12x^2+54x^4-108x^6+80x^8+6x^{10}-9x^{12}+x^{16}}\,dx$$
$$= 6\left(\int \frac{x^{13}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-4\left(\int \frac{x^{11}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-27\left(\int \frac{x^{9}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$+36\left(\int \frac{x^{7}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$-15\left(\int \frac{x^{5}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$
$$+2\left(\int \frac{x^{3}}{x^{16}-9x^{12}+6x^{10}+80x^8-108x^6+54x^4-12x^2+1}dx\right)$$

input
```
int(x^3*(6*x^10-4*x^8-27*x^6+36*x^4-15*x^2+2)/(x^16-9*x^12+6*x^10+80*x^8-1
08*x^6+54*x^4-12*x^2+1),x)
```

output
```
6*int(x**13/(x**16 - 9*x**12 + 6*x**10 + 80*x**8 - 108*x**6 + 54*x**4 - 12
*x**2 + 1),x) - 4*int(x**11/(x**16 - 9*x**12 + 6*x**10 + 80*x**8 - 108*x**
6 + 54*x**4 - 12*x**2 + 1),x) - 27*int(x**9/(x**16 - 9*x**12 + 6*x**10 + 8
0*x**8 - 108*x**6 + 54*x**4 - 12*x**2 + 1),x) + 36*int(x**7/(x**16 - 9*x**
12 + 6*x**10 + 80*x**8 - 108*x**6 + 54*x**4 - 12*x**2 + 1),x) - 15*int(x**
5/(x**16 - 9*x**12 + 6*x**10 + 80*x**8 - 108*x**6 + 54*x**4 - 12*x**2 + 1)
,x) + 2*int(x**3/(x**16 - 9*x**12 + 6*x**10 + 80*x**8 - 108*x**6 + 54*x**4
 - 12*x**2 + 1),x)
```

3.90 $\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-2}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1}$

Optimal result

Integrand size = 98, antiderivative size = 291

$$\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}}$$
$$= \frac{1}{8}\arctan\left(\frac{1}{3}\left(-3\sqrt{7}+4x\right)\right)+\frac{1}{8}\arctan\left(\frac{1}{3}\left(3\sqrt{7}+4x\right)\right)$$
$$-\frac{1}{8}\arctan\left(\frac{1}{9}\left(8-9\sqrt{7}+12x\right)\right)-\frac{1}{8}\arctan\left(\frac{1}{9}\left(8+9\sqrt{7}+12x\right)\right)$$
$$-\frac{1}{8}\arctan\left(\frac{1}{2}\left(18+2\sqrt{7}-27x-4x^2-9\sqrt{7}x^2-6x^3\right)\right)$$
$$-\frac{1}{8}\arctan\left(\frac{1}{2}\left(18-2\sqrt{7}-27x-4x^2+9\sqrt{7}x^2-6x^3\right)\right)$$
$$+\frac{1}{16}\sqrt{7}\log\left(36-108x-12\sqrt{7}x+89x^2+18\sqrt{7}x^2+18x^3-27x^4-4\sqrt{7}x^4+4x^6\right)$$
$$-\frac{1}{16}\sqrt{7}\log\left(36-108x+12\sqrt{7}x+89x^2-18\sqrt{7}x^2+18x^3-27x^4+4\sqrt{7}x^4+4x^6\right)$$

output

```
1/8*arctan(-7^(1/2)+4/3*x)+1/8*arctan(7^(1/2)+4/3*x)-1/8*arctan(8/9-7^(1/2
)+4/3*x)-1/8*arctan(8/9+7^(1/2)+4/3*x)+1/8*arctan(-9-7^(1/2)+27/2*x+2*x^2+
9/2*7^(1/2)*x^2+3*x^3)+1/8*arctan(-9+7^(1/2)+27/2*x+2*x^2-9/2*7^(1/2)*x^2+
3*x^3)+1/16*7^(1/2)*ln(36-108*x-12*7^(1/2)*x+89*x^2+18*7^(1/2)*x^2+18*x^3-
27*x^4-4*x^4*7^(1/2)+4*x^6)-1/16*7^(1/2)*ln(36-108*x+12*7^(1/2)*x+89*x^2-1
8*7^(1/2)*x^2+18*x^3-27*x^4+4*x^4*7^(1/2)+4*x^6)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.86

$$\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}}$$
$$= \frac{\arctan\left(\frac{2x}{3\sqrt{\frac{1}{2}i\left(3i+\sqrt{7}\right)}}\right)}{\sqrt{-6+2i\sqrt{7}}} - \frac{\operatorname{arctanh}\left(\frac{2x}{3\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{6+2i\sqrt{7}}}$$
$$- \frac{1}{8}\text{RootSum}\Big[16-96\#1+216\#1^2-216\#1^3+69\#1^4+36\#1^5-27\#1^6$$
$$+4\#1^8\&, \frac{48\log(x-\#1)\#1-180\log(x-\#1)\#1^2+216\log(x-\#1)\#1^3-81\log(x-\#1)\#1^4-32}{-48+216\#1-324\#1^2+138\#1^3+90\#1^4-81\#1^5+16}$$

input
```
Integrate[(-324 + 972*x - 633*x^2 - 252*x^3 + 324*x^4 + 33*x^5 + 108*x^6 -
 216*x^7 + 32*x^9)/(1296 - 7776*x + 17064*x^2 - 14904*x^3 - 179*x^4 + 8364
*x^5 - 3186*x^6 - 1836*x^7 + 1329*x^8 + 144*x^9 - 216*x^10 + 16*x^12),x]
```

output
```
ArcTan[(2*x)/(3*Sqrt[(I/2)*(3*I + Sqrt[7])])]/Sqrt[-6 + (2*I)*Sqrt[7]] - A
rcTanh[(2*x)/(3*Sqrt[(3 + I*Sqrt[7])/2])]/Sqrt[6 + (2*I)*Sqrt[7]] - RootSu
m[16 - 96*#1 + 216*#1^2 - 216*#1^3 + 69*#1^4 + 36*#1^5 - 27*#1^6 + 4*#1^8
& , (48*Log[x - #1]*#1 - 180*Log[x - #1]*#1^2 + 216*Log[x - #1]*#1^3 - 81*
Log[x - #1]*#1^4 - 32*Log[x - #1]*#1^5 + 24*Log[x - #1]*#1^6)/(-48 + 216*#
1 - 324*#1^2 + 138*#1^3 + 90*#1^4 - 81*#1^5 + 16*#1^7) & ]/8
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{32x^9-216x^7+108x^6+33x^5+324x^4-252x^3-633x^2+972x-324}{16x^{12}-216x^{10}+144x^9+1329x^8-1836x^7-3186x^6+8364x^5-179x^4-14904x^3+17064x^2-7776x+12}$$
$$\Big\downarrow \; 2462$$

$$\int \left(\frac{3(8x^2-27)}{4(4x^4-27x^2+81)}-\frac{x(24x^5-32x^4-81x^3+216x^2-180x+48)}{4(4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-12\int \frac{x}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx+ 45\int \frac{x^2}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx- 54\int \frac{x^3}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx+ \frac{81}{4}\int \frac{x^4}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx+ 8\int \frac{x^5}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx- 6\int \frac{x^6}{4x^8-27x^6+36x^5+69x^4-216x^3+216x^2-96x+16}dx-\frac{1}{8}\arctan\left(\sqrt{7}-\frac{4x}{3}\right)+ \frac{1}{8}\arctan\left(\frac{4x}{3}+\sqrt{7}\right)+\frac{1}{16}\sqrt{7}\log\left(2x^2-3\sqrt{7}x+9\right)-\frac{1}{16}\sqrt{7}\log\left(2x^2+3\sqrt{7}x+9\right)$$

input
```
Int[(-324 + 972*x - 633*x^2 - 252*x^3 + 324*x^4 + 33*x^5 + 108*x^6 - 216*x
^7 + 32*x^9)/(1296 - 7776*x + 17064*x^2 - 14904*x^3 - 179*x^4 + 8364*x^5 -
 3186*x^6 - 1836*x^7 + 1329*x^8 + 144*x^9 - 216*x^10 + 16*x^12),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-3\_Z^2+1\right)}\_R\ln\left(4x^3+\left(16\_R^3+36\_R^2-12\_R-27\right)x-24\_R^2+18\right)\right)}{4}$
default	$\frac{\sqrt{7}\,\ln\left(2x^2-3\sqrt{7}\,x+9\right)}{16}+\frac{\arctan\left(-\sqrt{7}+\frac{4x}{3}\right)}{8}-\frac{\sqrt{7}\,\ln\left(2x^2+3\sqrt{7}\,x+9\right)}{16}+\frac{\arctan\left(\sqrt{7}+\frac{4x}{3}\right)}{8}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4\_Z^4-3\_Z^2+\right.}\right.}{}$

input
```
int((32*x^9-216*x^7+108*x^6+33*x^5+324*x^4-252*x^3-633*x^2+972*x-324)/(16*
x^12-216*x^10+144*x^9+1329*x^8-1836*x^7-3186*x^6+8364*x^5-179*x^4-14904*x^
3+17064*x^2-7776*x+1296),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(4*x^3+(16*_R^3+36*_R^2-12*_R-27)*x-24*_R^2+18),_R=RootOf(4*_
Z^4-3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}} \\
&= -\frac{1}{16}\sqrt{7}\log\left(4x^6-27x^4+18x^3+89x^2+2\sqrt{7}(2x^4-9x^2+6x)-108x+36\right) \\
&\quad+\frac{1}{16}\sqrt{7}\log\left(4x^6-27x^4+18x^3+89x^2-2\sqrt{7}(2x^4-9x^2+6x)-108x+36\right) \\
&\quad+\frac{1}{8}\arctan\left(\frac{734210}{958431}x^5+\frac{152964}{319477}x^4-\frac{3793665}{638954}x^3-\frac{45872}{319477}x^2\right. \\
&\quad+\frac{1}{958431}\sqrt{7}(146916x^5+68808x^4-1698677x^3-131112x^2+6488874x-3023445) \\
&\quad\left.+\frac{41819189}{1916862}x-\frac{3597777}{319477}\right) \\
&\quad-\frac{1}{8}\arctan\left(-\frac{734210}{958431}x^5-\frac{152964}{319477}x^4+\frac{3793665}{638954}x^3+\frac{45872}{319477}x^2\right. \\
&\quad+\frac{1}{958431}\sqrt{7}(146916x^5+68808x^4-1698677x^3-131112x^2+6488874x-3023445) \\
&\quad\left.-\frac{41819189}{1916862}x+\frac{3597777}{319477}\right)-\frac{1}{8}\arctan\left(\frac{32}{551}x^2\right. \\
&\quad\left.+\frac{1}{167284151}\sqrt{7}(21859272x^2+15419184x-57758867)+\frac{24192}{303601}x+\frac{57564432}{167284151}\right) \\
&\quad+\frac{1}{8}\arctan\left(-\frac{32}{551}x^2+\frac{1}{167284151}\sqrt{7}(21859272x^2+15419184x-57758867)\right. \\
&\quad\left.-\frac{24192}{303601}x-\frac{57564432}{167284151}\right)
\end{aligned}$$

input
```
integrate((32*x^9-216*x^7+108*x^6+33*x^5+324*x^4-252*x^3-633*x^2+972*x-324
)/(16*x^12-216*x^10+144*x^9+1329*x^8-1836*x^7-3186*x^6+8364*x^5-179*x^4-14
904*x^3+17064*x^2-7776*x+1296),x, algorithm="fricas")
```

output
```
-1/16*sqrt(7)*log(4*x^6 - 27*x^4 + 18*x^3 + 89*x^2 + 2*sqrt(7)*(2*x^4 - 9*
x^2 + 6*x) - 108*x + 36) + 1/16*sqrt(7)*log(4*x^6 - 27*x^4 + 18*x^3 + 89*x
^2 - 2*sqrt(7)*(2*x^4 - 9*x^2 + 6*x) - 108*x + 36) + 1/8*arctan(734210/958
431*x^5 + 152964/319477*x^4 - 3793665/638954*x^3 - 45872/319477*x^2 + 1/95
8431*sqrt(7)*(146916*x^5 + 68808*x^4 - 1698677*x^3 - 131112*x^2 + 6488874*
x - 3023445) + 41819189/1916862*x - 3597777/319477) - 1/8*arctan(-734210/9
58431*x^5 - 152964/319477*x^4 + 3793665/638954*x^3 + 45872/319477*x^2 + 1/
958431*sqrt(7)*(146916*x^5 + 68808*x^4 - 1698677*x^3 - 131112*x^2 + 648887
4*x - 3023445) - 41819189/1916862*x + 3597777/319477) - 1/8*arctan(32/551*
x^2 + 1/167284151*sqrt(7)*(21859272*x^2 + 15419184*x - 57758867) + 24192/3
03601*x + 57564432/167284151) + 1/8*arctan(-32/551*x^2 + 1/167284151*sqrt(
7)*(21859272*x^2 + 15419184*x - 57758867) - 24192/303601*x - 57564432/1672
84151)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(260) = 520$.

Time = 0.81 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.18

$$\int \frac{-324 + 972x - 633x^2 - 252x^3 + 324x^4 + 33x^5 + 108x^6 - 216x^7 + 32x^9}{1296 - 7776x + 17064x^2 - 14904x^3 - 179x^4 + 8364x^5 - 3186x^6 - 1836x^7 + 1329x^8 + 144x^9 - 216x^{10}}$$
= Too large to display

input
```
integrate((32*x**9-216*x**7+108*x**6+33*x**5+324*x**4-252*x**3-633*x**2+97
2*x-324)/(16*x**12-216*x**10+144*x**9+1329*x**8-1836*x**7-3186*x**6+8364*x
**5-179*x**4-14904*x**3+17064*x**2-7776*x+1296),x)
```

output
```
sqrt(7)*log(x**6 + x**4*(-27/4 - sqrt(7)) + 9*x**3/2 + x**2*(9*sqrt(7)/2 +
 89/4) + x*(-27 - 3*sqrt(7)) + 9)/16 - sqrt(7)*log(x**6 + x**4*(-27/4 + sq
rt(7)) + 9*x**3/2 + x**2*(89/4 - 9*sqrt(7)/2) + x*(-27 + 3*sqrt(7)) + 9)/1
6 + atan(x**2*(44280*sqrt(7)/(-376177 + 83952*sqrt(7)) - 54944/(-376177 +
83952*sqrt(7))) + x*(9003712080*sqrt(7)/(-123787337137 + 30197125134*sqrt(
7)) - 9619833888/(-123787337137 + 30197125134*sqrt(7))) + 133745022/(-1875
77335 + 28630458*sqrt(7)) - 74617651*sqrt(7)/(-187577335 + 28630458*sqrt(7
)))/8 + atan(x**2*(54944/(83952*sqrt(7) + 376177) + 44280*sqrt(7)/(83952*s
qrt(7) + 376177)) + x*(9619833888/(30197125134*sqrt(7) + 123787337137) + 9
003712080*sqrt(7)/(30197125134*sqrt(7) + 123787337137)) - 74617651*sqrt(7)
/(28630458*sqrt(7) + 187577335) - 133745022/(28630458*sqrt(7) + 187577335)
)/8 + atan(x**5*(13736/(-3210 + 15093*sqrt(7)) + 11070*sqrt(7)/(-3210 + 15
093*sqrt(7))) + x**4*(27472*sqrt(7)/(10494*sqrt(7) + 312673) + 154980/(104
94*sqrt(7) + 312673)) + x**3*(-79688899*sqrt(7)/(-15153168 + 17945118*sqrt
(7)) - 132666462/(-15153168 + 17945118*sqrt(7))) + x**2*(-1500618680/(3500
7088 + 1561829283*sqrt(7)) - 229043664*sqrt(7)/(35007088 + 1561829283*sqrt
(7))) + x*(183771619031726/(-3419774020020 + 5451936535422*sqrt(7)) + 9578
9159022429*sqrt(7)/(-3419774020020 + 5451936535422*sqrt(7))) - 25714393*sq
rt(7)/(-2525528 + 2990853*sqrt(7)) - 37603017/(-2525528 + 2990853*sqrt(7))
)/8 + atan(x**5*(-13736/(3210 + 15093*sqrt(7)) + 11070*sqrt(7)/(3210 + ...
```

Maxima [F]

$$\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}$$
$$=\int \frac{32\,x^9-216\,x^7+108\,x^6+33\,x^5+324\,x^4-252\,x^3-633\,x^2+972\,x-324}{16\,x^{12}-216\,x^{10}+144\,x^9+1329\,x^8-1836\,x^7-3186\,x^6+8364\,x^5-179\,x^4-14904\,x^3+17064\,x^2-$$

input
```
integrate((32*x^9-216*x^7+108*x^6+33*x^5+324*x^4-252*x^3-633*x^2+972*x-324
)/(16*x^12-216*x^10+144*x^9+1329*x^8-1836*x^7-3186*x^6+8364*x^5-179*x^4-14
904*x^3+17064*x^2-7776*x+1296),x, algorithm="maxima")
```

output
```
integrate((32*x^9 - 216*x^7 + 108*x^6 + 33*x^5 + 324*x^4 - 252*x^3 - 633*x
^2 + 972*x - 324)/(16*x^12 - 216*x^10 + 144*x^9 + 1329*x^8 - 1836*x^7 - 31
86*x^6 + 8364*x^5 - 179*x^4 - 14904*x^3 + 17064*x^2 - 7776*x + 1296), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.81

$$
\begin{aligned}
&\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}} \\
&= \frac{1}{16}\sqrt{7}\log\left(\left(4\,x^2+3\,\sqrt{7}x-2\,\sqrt{7}\right)^2+(3\,x-2)^2\right) \\
&\quad -\frac{1}{16}\sqrt{7}\log\left(\left(4\,x^2-3\,\sqrt{7}x+2\,\sqrt{7}\right)^2+(3\,x-2)^2\right) \\
&\quad -\frac{1}{16}\sqrt{7}\log\left(x^2+\frac{3}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{9}{2}\right)+\frac{1}{16}\sqrt{7}\log\left(x^2-\frac{3}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{9}{2}\right) \\
&\quad +\frac{1}{8}\arctan\left(3\,x^3+\frac{1}{2}\,x^2\left(9\,\sqrt{7}+4\right)+\frac{27}{2}\,x-\sqrt{7}-9\right) \\
&\quad +\frac{1}{8}\arctan\left(3\,x^3-\frac{1}{2}\,x^2\left(9\,\sqrt{7}-4\right)+\frac{27}{2}\,x+\sqrt{7}-9\right) \\
&\quad +\frac{1}{8}\arctan\left(\frac{2}{3}\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+3\,\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) \\
&\quad +\frac{1}{8}\arctan\left(\frac{2}{3}\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-3\,\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) \\
&\quad -\frac{1}{8}\arctan\left(\frac{4}{3}\,x+\sqrt{7}+\frac{8}{9}\right)-\frac{1}{8}\arctan\left(\frac{4}{3}\,x-\sqrt{7}+\frac{8}{9}\right)
\end{aligned}
$$

input

```
integrate((32*x^9-216*x^7+108*x^6+33*x^5+324*x^4-252*x^3-633*x^2+972*x-324
)/(16*x^12-216*x^10+144*x^9+1329*x^8-1836*x^7-3186*x^6+8364*x^5-179*x^4-14
904*x^3+17064*x^2-7776*x+1296),x, algorithm="giac")
```

output

```
1/16*sqrt(7)*log((4*x^2 + 3*sqrt(7)*x - 2*sqrt(7))^2 + (3*x - 2)^2) - 1/16
*sqrt(7)*log((4*x^2 - 3*sqrt(7)*x + 2*sqrt(7))^2 + (3*x - 2)^2) - 1/16*sqr
t(7)*log(x^2 + 3/2*sqrt(14)*(1/4)^(1/4)*x + 9/2) + 1/16*sqrt(7)*log(x^2 -
3/2*sqrt(14)*(1/4)^(1/4)*x + 9/2) + 1/8*arctan(3*x^3 + 1/2*x^2*(9*sqrt(7)
+ 4) + 27/2*x - sqrt(7) - 9) + 1/8*arctan(3*x^3 - 1/2*x^2*(9*sqrt(7) - 4)
+ 27/2*x + sqrt(7) - 9) + 1/8*arctan(2/3*sqrt(2)*(1/4)^(3/4)*(4*x + 3*sqrt
(14)*(1/4)^(1/4))) + 1/8*arctan(2/3*sqrt(2)*(1/4)^(3/4)*(4*x - 3*sqrt(14)*
(1/4)^(1/4))) - 1/8*arctan(4/3*x + sqrt(7) + 8/9) - 1/8*arctan(4/3*x - sqr
t(7) + 8/9)
```

Mupad [B] (verification not implemented)

Time = 11.65 (sec) , antiderivative size = 969, normalized size of antiderivative = 3.33

$$\int \frac{-324+972x-633x^2-252x^3+324x^4+33x^5+108x^6-216x^7+32x^9}{1296-7776x+17064x^2-14904x^3-179x^4+8364x^5-3186x^6-1836x^7+1329x^8+144x^9-216x^{10}}$$
= Too large to display

input
```
int(-(972*x - 633*x^2 - 252*x^3 + 324*x^4 + 33*x^5 + 108*x^6 - 216*x^7 + 3
2*x^9 - 324)/(7776*x - 17064*x^2 + 14904*x^3 + 179*x^4 - 8364*x^5 + 3186*x
^6 + 1836*x^7 - 1329*x^8 - 144*x^9 + 216*x^10 - 16*x^12 - 1296),x)
```

output
```
atan((7^(1/2)*213884473604285216343651705i)/(274877906944*((7^(1/2)*820603
5754570468182777237575i)/1099511627776 - (7^(1/2)*x*1980737862748863783082
20367i)/2199023255552 - (1014692513515722083835177808 5*x)/2199023255552 +
(7^(1/2)*x^3*82054722586311054945007719 3i)/2199023255552 + (27336547637054
0132942968227*x^3)/2199023255552 + 246147262880752284186729188 7/1099511627
776)) - (208051845175814789263929626 7*x)/(1099511627776*((7^(1/2)*82060357
5457046818277723757i)/1099511627776 - (7^(1/2)*x*198073786274886378308222 0
367i)/2199023255552 - (1014692513515722083835177808 5*x)/2199023255552 + (7
^(1/2)*x^3*820547225863110549450077193i)/2199023255552 + (2733654763705401
32942968227*x^3)/2199023255552 + 2461472628807522841867291887/109951162777
6)) + 574689121320992720156864883/(274877906944*((7^(1/2)*8206035754570468
18277723757i)/1099511627776 - (7^(1/2)*x*198073786274886378308222 0367i)/21
99023255552 - (1014692513515722083835177808 5*x)/2199023255552 + (7^(1/2)*x
^3*820547225863110549450077193i)/2199023255552 + (2733654763705401329429 68
227*x^3)/2199023255552 + 2461472628807522841867291887/1099511627776)) + (3
781267478883027434417211 9*x^3)/(549755813888*((7^(1/2)*8206035754570468182
77723757i)/1099511627776 - (7^(1/2)*x*198073786274886378308222 0367i)/21990
23255552 - (1014692513515722083835177808 5*x)/2199023255552 + (7^(1/2)*x^3*
820547225863110549450077193i)/2199023255552 + (273365476370540132942968227
*x^3)/2199023255552 + 2461472628807522841867291887/1099511627776)) - (7...
```

Reduce [F]

$$\int \frac{-324 + 972x - 633x^2 - 252x^3 + 324x^4 + 33x^5 + 108x^6 - 216x^7 + 32x^9}{1296 - 7776x + 17064x^2 - 14904x^3 - 179x^4 + 8364x^5 - 3186x^6 - 1836x^7 + 1329x^8 + 144x^9 - 216x^{10}}$$

$$= 32\left(\int \frac{x^9}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x^2 -}\right.$$

$$-216\left(\int \frac{x^7}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$+108\left(\int \frac{x^6}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$+33\left(\int \frac{x^5}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x^2}\right.$$

$$+324\left(\int \frac{x^4}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$-252\left(\int \frac{x^3}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$-633\left(\int \frac{x^2}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$+972\left(\int \frac{x}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

$$-324\left(\int \frac{1}{16x^{12} - 216x^{10} + 144x^9 + 1329x^8 - 1836x^7 - 3186x^6 + 8364x^5 - 179x^4 - 14904x^3 + 17064x}\right.$$

input
```
int((32*x^9-216*x^7+108*x^6+33*x^5+324*x^4-252*x^3-633*x^2+972*x-324)/(16*
x^12-216*x^10+144*x^9+1329*x^8-1836*x^7-3186*x^6+8364*x^5-179*x^4-14904*x^
3+17064*x^2-7776*x+1296),x)
```

output

```
32*int(x**9/(16*x**12 - 216*x**10 + 144*x**9 + 1329*x**8 - 1836*x**7 - 318
6*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064*x**2 - 7776*x + 1296),x
) - 216*int(x**7/(16*x**12 - 216*x**10 + 144*x**9 + 1329*x**8 - 1836*x**7
- 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064*x**2 - 7776*x + 12
96),x) + 108*int(x**6/(16*x**12 - 216*x**10 + 144*x**9 + 1329*x**8 - 1836*
x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064*x**2 - 7776*x
 + 1296),x) + 33*int(x**5/(16*x**12 - 216*x**10 + 144*x**9 + 1329*x**8 - 1
836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064*x**2 - 77
76*x + 1296),x) + 324*int(x**4/(16*x**12 - 216*x**10 + 144*x**9 + 1329*x**
8 - 1836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064*x**2
 - 7776*x + 1296),x) - 252*int(x**3/(16*x**12 - 216*x**10 + 144*x**9 + 132
9*x**8 - 1836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 + 17064
*x**2 - 7776*x + 1296),x) - 633*int(x**2/(16*x**12 - 216*x**10 + 144*x**9
+ 1329*x**8 - 1836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3 +
17064*x**2 - 7776*x + 1296),x) + 972*int(x/(16*x**12 - 216*x**10 + 144*x**
9 + 1329*x**8 - 1836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**3
+ 17064*x**2 - 7776*x + 1296),x) - 324*int(1/(16*x**12 - 216*x**10 + 144*x
**9 + 1329*x**8 - 1836*x**7 - 3186*x**6 + 8364*x**5 - 179*x**4 - 14904*x**
3 + 17064*x**2 - 7776*x + 1296),x)
```

3.91 $\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx$

Optimal result

Integrand size = 101, antiderivative size = 125

$$\begin{aligned}
&\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx\\
&=\arctan\left(\frac{1}{9}\left(6+\sqrt{3}x+12x^2\right)\right)\\
&\quad+\arctan\left(\frac{813+827\sqrt{3}x+399x^2+1437\sqrt{3}x^3+2952x^4+1008\sqrt{3}x^5}{1764}\right)\\
&\quad+\arctan\left(\frac{1}{188}\left(-306+368\sqrt{3}x+483x^2-543\sqrt{3}x^3-349x^4+911\sqrt{3}x^5+1656x^6\right.\right.\\
&\qquad\left.\left.+336\sqrt{3}x^7\right)\right)
\end{aligned}$$

output
```
arctan(2/3+1/9*x*3^(1/2)+4/3*x^2)+arctan(271/588+827/1764*x*3^(1/2)+19/84*
x^2+479/588*3^(1/2)*x^3+82/49*x^4+4/7*3^(1/2)*x^5)+arctan(-153/94+92/47*x*
3^(1/2)+483/188*x^2-543/188*3^(1/2)*x^3-349/188*x^4+911/188*3^(1/2)*x^5+41
4/47*x^6+84/47*3^(1/2)*x^7)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx$$
$$= -\frac{1}{2}i\log\left(2i-(2+2i)\sqrt{3}x-3ix^2+3\sqrt{3}x^3+4x^4\right)$$
$$+\frac{1}{2}i\log\left(-2i-(2-2i)\sqrt{3}x+3ix^2+3\sqrt{3}x^3+4x^4\right)$$

input
```
Integrate[(4*Sqrt[3] - 12*Sqrt[3]*x^2 + 4*x^3 + 33*Sqrt[3]*x^4 + 24*x^5)/(
4 - 8*Sqrt[3]*x + 12*x^2 + 12*Sqrt[3]*x^3 - 27*x^4 - 16*Sqrt[3]*x^5 + 27*x
^6 + 24*Sqrt[3]*x^7 + 16*x^8),x]
```

output
```
(-1/2*I)*Log[2*I - (2 + 2*I)*Sqrt[3]*x - (3*I)*x^2 + 3*Sqrt[3]*x^3 + 4*x^4
] + (I/2)*Log[-2*I - (2 - 2*I)*Sqrt[3]*x + (3*I)*x^2 + 3*Sqrt[3]*x^3 + 4*x
^4]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{24x^5+33\sqrt{3}x^4+4x^3-12\sqrt{3}x^2+4\sqrt{3}}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{24x^5}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}+\frac{}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}\right.$$
$$\Big\downarrow \text{2009}$$

$$4\sqrt{3}\int \frac{1}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}dx - 12\sqrt{3}\int \frac{x^2}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}dx + 4\int \frac{x^3}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}dx + 33\sqrt{3}\int \frac{x^4}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}dx + 24\int \frac{x^5}{16x^8+24\sqrt{3}x^7+27x^6-16\sqrt{3}x^5-27x^4+12\sqrt{3}x^3+12x^2-8\sqrt{3}x+4}dx$$

input
```
Int[(4*Sqrt[3] - 12*Sqrt[3]*x^2 + 4*x^3 + 33*Sqrt[3]*x^4 + 24*x^5)/(4 - 8*
Sqrt[3]*x + 12*x^2 + 12*Sqrt[3]*x^3 - 27*x^4 - 16*Sqrt[3]*x^5 + 27*x^6 + 2
4*Sqrt[3]*x^7 + 16*x^8),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

method	result
parallelrisch	$-\frac{i \ln\left(\frac{3\sqrt{3}\,x^3}{4}+x^4-\frac{i\sqrt{3}\,x}{2}-\frac{3ix^2}{4}-\frac{\sqrt{3}\,x}{2}+\frac{i}{2}\right)}{2}+\frac{i \ln\left(\frac{3\sqrt{3}\,x^3}{4}+x^4+\frac{i\sqrt{3}\,x}{2}+\frac{3ix^2}{4}-\frac{\sqrt{3}\,x}{2}-\frac{i}{2}\right)}{2}$
risch	$\arctan\left(\frac{2}{3}+\frac{\sqrt{3}\,x}{9}+\frac{4x^2}{3}\right)+\arctan\left(\frac{271}{588}+\frac{827\sqrt{3}\,x}{1764}+\frac{19x^2}{84}+\frac{479\sqrt{3}\,x^3}{588}+\frac{82x^4}{49}+\frac{4\sqrt{3}\,x^5}{7}\right)+\text{arcta}$
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(4-8\sqrt{3}\,\_Z+12\_Z^2+12\sqrt{3}\,\_Z^3-27\_Z^4-16\sqrt{3}\,\_Z^5+27\_Z^6+24\sqrt{3}\,\_Z^7+16\_Z^8\right)} \frac{\left(24\_R^5+4\_R^3+\sqrt{3}\left(3\right.\right.}{64\_R^7+81\_R^5-54\_R^3+12\_}\right.}{2}$

input
```
int((4*3^(1/2)-12*3^(1/2)*x^2+4*x^3+33*3^(1/2)*x^4+24*x^5)/(4-8*3^(1/2)*x+
12*x^2+12*3^(1/2)*x^3-27*x^4-16*3^(1/2)*x^5+27*x^6+24*3^(1/2)*x^7+16*x^8),
x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*ln(3/4*3^(1/2)*x^3+x^4-1/2*I*3^(1/2)*x-3/4*I*x^2-1/2*3^(1/2)*x+1/2*
I)+1/2*I*ln(3/4*3^(1/2)*x^3+x^4+1/2*I*3^(1/2)*x+3/4*I*x^2-1/2*3^(1/2)*x-1/
2*I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx = \arctan\left(\frac{414}{47}\,x^6-\frac{349}{188}\,x^4+\frac{483}{188}\,x^2+\frac{1}{188}\,\sqrt{3}(336\,x^7+911\,x^5-543\,x^3+368\,x)-\frac{153}{94}\right)+\arctan\left(\frac{82}{49}\,x^4+\frac{19}{84}\,x^2+\frac{1}{1764}\,\sqrt{3}(1008\,x^5+1437\,x^3+827\,x)+\frac{271}{588}\right)+\arctan\left(\frac{4}{3}\,x^2+\frac{1}{9}\,\sqrt{3}x+\frac{2}{3}\right)$$

input
```
integrate((4*3^(1/2)-12*3^(1/2)*x^2+4*x^3+33*3^(1/2)*x^4+24*x^5)/(4-8*3^(1
/2)*x+12*x^2+12*3^(1/2)*x^3-27*x^4-16*3^(1/2)*x^5+27*x^6+24*3^(1/2)*x^7+16
*x^8),x, algorithm="fricas")
```

output
```
arctan(414/47*x^6 - 349/188*x^4 + 483/188*x^2 + 1/188*sqrt(3)*(336*x^7 + 9
11*x^5 - 543*x^3 + 368*x) - 153/94) + arctan(82/49*x^4 + 19/84*x^2 + 1/176
4*sqrt(3)*(1008*x^5 + 1437*x^3 + 827*x) + 271/588) + arctan(4/3*x^2 + 1/9*
sqrt(3)*x + 2/3)
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.14

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx$$
$$= \operatorname{atan}\left(\frac{4x^2}{3}+\frac{\sqrt{3}x}{9}+\frac{2}{3}\right) + \operatorname{atan}\left(\frac{4\sqrt{3}x^5}{7}+\frac{82x^4}{49}+\frac{479\sqrt{3}x^3}{588}+\frac{19x^2}{84}+\frac{827\sqrt{3}x}{1764}+\frac{271}{588}\right) + \operatorname{atan}\left(\frac{84\sqrt{3}x^7}{47}+\frac{414x^6}{47}+\frac{911\sqrt{3}x^5}{188}-\frac{349x^4}{188}-\frac{543\sqrt{3}x^3}{188}+\frac{483x^2}{188}+\frac{92\sqrt{3}x}{47}-\frac{153}{94}\right)$$

input
```
integrate((4*3**(1/2)-12*3**(1/2)*x**2+4*x**3+33*3**(1/2)*x**4+24*x**5)/(4
-8*3**(1/2)*x+12*x**2+12*3**(1/2)*x**3-27*x**4-16*3**(1/2)*x**5+27*x**6+24
*3**(1/2)*x**7+16*x**8),x)
```

output
```
atan(4*x**2/3 + sqrt(3)*x/9 + 2/3) + atan(4*sqrt(3)*x**5/7 + 82*x**4/49 +
479*sqrt(3)*x**3/588 + 19*x**2/84 + 827*sqrt(3)*x/1764 + 271/588) + atan(8
4*sqrt(3)*x**7/47 + 414*x**6/47 + 911*sqrt(3)*x**5/188 - 349*x**4/188 - 54
3*sqrt(3)*x**3/188 + 483*x**2/188 + 92*sqrt(3)*x/47 - 153/94)
```

Maxima [F]

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx = \int \frac{24\,x^5+33\,\sqrt{3}x^4+4\,x^3-12\,\sqrt{3}x^2+4\,\sqrt{3}}{16\,x^8+24\,\sqrt{3}x^7+27\,x^6-16\,\sqrt{3}x^5-27\,x^4+12\,\sqrt{3}x^3+12\,x^2-8\,\sqrt{3}x+4}\,dx$$

input
```
integrate((4*3^(1/2)-12*3^(1/2)*x^2+4*x^3+33*3^(1/2)*x^4+24*x^5)/(4-8*3^(1
/2)*x+12*x^2+12*3^(1/2)*x^3-27*x^4-16*3^(1/2)*x^5+27*x^6+24*3^(1/2)*x^7+16
*x^8),x, algorithm="maxima")
```

output
```
integrate((24*x^5 + 33*sqrt(3)*x^4 + 4*x^3 - 12*sqrt(3)*x^2 + 4*sqrt(3))/(
16*x^8 + 24*sqrt(3)*x^7 + 27*x^6 - 16*sqrt(3)*x^5 - 27*x^4 + 12*sqrt(3)*x^
3 + 12*x^2 - 8*sqrt(3)*x + 4), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx$$
$$= -\arctan\left(-\frac{84}{47}\sqrt{3}x^7-\frac{414}{47}x^6-\frac{911}{188}\sqrt{3}x^5+\frac{349}{188}x^4+\frac{543}{188}\sqrt{3}x^3-\frac{483}{188}x^2-\frac{92}{47}\sqrt{3}x+\frac{153}{94}\right)+\arctan\left(\frac{1}{1764}\sqrt{3}\left(1008\,x^5+984\,\sqrt{3}x^4+1437\,x^3+133\,\sqrt{3}x^2+827\,x+271\,\sqrt{3}\right)\right)+\arctan\left(\frac{1}{9}\sqrt{3}\left(4\,\sqrt{3}x^2+x+2\,\sqrt{3}\right)\right)$$

input
```
integrate((4*3^(1/2)-12*3^(1/2)*x^2+4*x^3+33*3^(1/2)*x^4+24*x^5)/(4-8*3^(1
/2)*x+12*x^2+12*3^(1/2)*x^3-27*x^4-16*3^(1/2)*x^5+27*x^6+24*3^(1/2)*x^7+16
*x^8),x, algorithm="giac")
```

output
```
-arctan(-84/47*sqrt(3)*x^7 - 414/47*x^6 - 911/188*sqrt(3)*x^5 + 349/188*x^
4 + 543/188*sqrt(3)*x^3 - 483/188*x^2 - 92/47*sqrt(3)*x + 153/94) + arctan
(1/1764*sqrt(3)*(1008*x^5 + 984*sqrt(3)*x^4 + 1437*x^3 + 133*sqrt(3)*x^2 +
 827*x + 271*sqrt(3))) + arctan(1/9*sqrt(3)*(4*sqrt(3)*x^2 + x + 2*sqrt(3)
))
```

Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx$$

$$=\sum_{k=1}^{8}\ln\left(\frac{\left(4\,\mathrm{root}\left(z^8+z^6+\frac{3z^4}{8}+\frac{z^2}{16}+\frac{1}{256},z,k\right)^2+1\right)^3\left(210727320\,x-125336697\sqrt{3}\,\mathrm{root}\left(z^8+z^6+\right.\right.}{42949}\right.$$

$$+z^6+\frac{3\,z^4}{8}+\frac{z^2}{16}+\frac{1}{256},z,k\right)$$

input
```
int((4*3^(1/2) - 12*3^(1/2)*x^2 + 33*3^(1/2)*x^4 + 4*x^3 + 24*x^5)/(12*3^(
1/2)*x^3 - 8*3^(1/2)*x - 16*3^(1/2)*x^5 + 24*3^(1/2)*x^7 + 12*x^2 - 27*x^4
 + 27*x^6 + 16*x^8 + 4),x)
```

output
```
symsum(log((343*(4*root(z^8 + z^6 + (3*z^4)/8 + z^2/16 + 1/256, z, k)^2 +
1)^3*(210727320*x - 125336697*3^(1/2)*root(z^8 + z^6 + (3*z^4)/8 + z^2/16
+ 1/256, z, k) + 614708847*root(z^8 + z^6 + (3*z^4)/8 + z^2/16 + 1/256, z,
 k)*x - 135190001*3^(1/2)))/4294967296)*root(z^8 + z^6 + (3*z^4)/8 + z^2/1
6 + 1/256, z, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{4\sqrt{3}-12\sqrt{3}x^2+4x^3+33\sqrt{3}x^4+24x^5}{4-8\sqrt{3}x+12x^2+12\sqrt{3}x^3-27x^4-16\sqrt{3}x^5+27x^6+24\sqrt{3}x^7+16x^8}\,dx =$$

$$-48\sqrt{3}\left(\int \frac{x^{12}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+987\sqrt{3}\left(\int \frac{x^{10}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$-1375\sqrt{3}\left(\int \frac{x^{8}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+972\sqrt{3}\left(\int \frac{x^{6}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$-88\sqrt{3}\left(\int \frac{x^{4}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+16\sqrt{3}\left(\int \frac{1}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+384\left(\int \frac{x^{13}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$-1664\left(\int \frac{x^{11}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+1908\left(\int \frac{x^{9}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$-1872\left(\int \frac{x^{7}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+1560\left(\int \frac{x^{5}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$-416\left(\int \frac{x^{3}}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$
$$+96\left(\int \frac{x}{256x^{16}-864x^{14}+2169x^{12}-3570x^{10}+3809x^8-1632x^6+504x^4-96x^2+16}dx\right)$$

input
```
int((4*3^(1/2)-12*3^(1/2)*x^2+4*x^3+33*3^(1/2)*x^4+24*x^5)/(4-8*3^(1/2)*x+
12*x^2+12*3^(1/2)*x^3-27*x^4-16*3^(1/2)*x^5+27*x^6+24*3^(1/2)*x^7+16*x^8),
x)
```

output

```
 - 48*sqrt(3)*int(x**12/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x**10 +
 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x) + 987*sqrt(3)*int(x**
10/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**
6 + 504*x**4 - 96*x**2 + 16),x) - 1375*sqrt(3)*int(x**8/(256*x**16 - 864*x
**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**
2 + 16),x) + 972*sqrt(3)*int(x**6/(256*x**16 - 864*x**14 + 2169*x**12 - 35
70*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x) - 88*sqrt(3
)*int(x**4/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 -
1632*x**6 + 504*x**4 - 96*x**2 + 16),x) + 16*sqrt(3)*int(1/(256*x**16 - 86
4*x**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*
x**2 + 16),x) + 384*int(x**13/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x
**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x) - 1664*int(x**1
1/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**6
 + 504*x**4 - 96*x**2 + 16),x) + 1908*int(x**9/(256*x**16 - 864*x**14 + 21
69*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x
) - 1872*int(x**7/(256*x**16 - 864*x**14 + 2169*x**12 - 3570*x**10 + 3809*
x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x) + 1560*int(x**5/(256*x**16
- 864*x**14 + 2169*x**12 - 3570*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 -
 96*x**2 + 16),x) - 416*int(x**3/(256*x**16 - 864*x**14 + 2169*x**12 - 357
0*x**10 + 3809*x**8 - 1632*x**6 + 504*x**4 - 96*x**2 + 16),x) + 96*int(...
```

3.92 $\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$

Optimal result

Integrand size = 84, antiderivative size = 404

$$
\begin{aligned}
&\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx \\
&= -\frac{1}{15}\sqrt{5-2\sqrt{5}}\operatorname{arctanh}\left(\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}x\right) \\
&\quad+\sqrt{\frac{1}{45}+\frac{2}{45\sqrt{5}}}\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}x\right)-\frac{\operatorname{arctanh}(\sqrt{5-2\sqrt{5}}x^2)}{6\sqrt{5}} \\
&\quad+\frac{1}{30}\sqrt{5-2\sqrt{5}}\log\left(1-\sqrt{5}-2\sqrt{5-2\sqrt{5}}x+2x^2\right) \\
&\quad-\frac{1}{30}\sqrt{5-2\sqrt{5}}\log\left(1-\sqrt{5}+2\sqrt{5-2\sqrt{5}}x+2x^2\right) \\
&\quad+\sqrt{\frac{1}{180}+\frac{1}{90\sqrt{5}}}\log\left(1+\sqrt{5}-2\sqrt{5+2\sqrt{5}}x+2x^2\right) \\
&\quad-\sqrt{\frac{1}{180}+\frac{1}{90\sqrt{5}}}\log\left(1+\sqrt{5}+2\sqrt{5+2\sqrt{5}}x+2x^2\right) \\
&\quad+\frac{\log\left(3+\sqrt{5}-8x^2-2\sqrt{5}x^2+2x^4\right)}{12\sqrt{5}}-\frac{\log\left(3-\sqrt{5}-8x^2+2\sqrt{5}x^2+2x^4\right)}{12\sqrt{5}} \\
&\quad+\frac{1}{12}\log\left(1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}\right)
\end{aligned}
$$

output
```
-1/15*(5-2*5^(1/2))^(1/2)*arctanh(1/2*(10-2*5^(1/2))^(1/2)*x)+1/15*(5+2*5^
(1/2))^(1/2)*arctanh(1/2*(10+2*5^(1/2))^(1/2)*x)+1/30*arctanh(-5^(1/2)+2*5
^(1/2)*x^2)*5^(1/2)+1/30*(5-2*5^(1/2))^(1/2)*ln(1-5^(1/2)-2*(5-2*5^(1/2))^
(1/2)*x+2*x^2)-1/30*(5-2*5^(1/2))^(1/2)*ln(1-5^(1/2)+2*(5-2*5^(1/2))^(1/2)
*x+2*x^2)+1/30*(5+2*5^(1/2))^(1/2)*ln(1+5^(1/2)-2*(5+2*5^(1/2))^(1/2)*x+2*
x^2)-1/30*(5+2*5^(1/2))^(1/2)*ln(1+5^(1/2)+2*(5+2*5^(1/2))^(1/2)*x+2*x^2)+
1/60*ln(3+5^(1/2)-8*x^2-2*5^(1/2)*x^2+2*x^4)*5^(1/2)-1/60*ln(3-5^(1/2)-8*x
^2+2*5^(1/2)*x^2+2*x^4)*5^(1/2)+1/12*ln(5*x^12-45*x^10+111*x^8-113*x^6+54*
x^4-12*x^2+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.21

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$$
$$= \text{RootSum}\big[1-120\#1+4500\#1^2-54000\#1^3+162000\#1^4\&,\log\left(3-9x^2-x^3-150\#1+450x^2\#1+1800\#1^2-5400x^2\#1^2-5400\#1^3+16200x^2\#1^3\right)\#1\&\big]$$

input
```
Integrate[(x*(-2 + x + 18*x^2 - 7*x^3 - 56*x^4 + 15*x^5 + 72*x^6 - 12*x^7
- 36*x^8 + 3*x^9 + 5*x^10))/(1 - 12*x^2 + 54*x^4 - 113*x^6 + 111*x^8 - 45*
x^10 + 5*x^12),x]
```

output
```
RootSum[1 - 120*#1 + 4500*#1^2 - 54000*#1^3 + 162000*#1^4 & , Log[3 - 9*x^
2 - x^3 - 150*#1 + 450*x^2*#1 + 1800*#1^2 - 5400*x^2*#1^2 - 5400*#1^3 + 16
200*x^2*#1^3]*#1 & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(5x^{10}+3x^9-36x^8-12x^7+72x^6+15x^5-56x^4-7x^3+18x^2+x-2)}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{5x^3-x^2-3x+1}{3(5x^4-5x^2+1)}+\frac{2x^7+2x^6-11x^5-7x^4+13x^3+6x^2-3x-1}{3(x^8-8x^6+14x^4-7x^2+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}\text{Subst}\left(\int \frac{1}{x^4-8x^3+14x^2-7x+1}dx,x,x^2\right)-\frac{1}{6}\text{Subst}\left(\int \frac{x}{x^4-8x^3+14x^2-7x+1}dx,x,x^2\right)+\frac{1}{6}\text{Subst}\left(\int \frac{x^2}{x^4-8x^3+14x^2-7x+1}dx,x,x^2\right)-\frac{1}{3}\int \frac{1}{x^8-8x^6+14x^4-7x^2+1}dx+2\int \frac{x^2}{x^8-8x^6+14x^4-7x^2+1}dx-\frac{7}{3}\int \frac{x^4}{x^8-8x^6+14x^4-7x^2+1}dx+\frac{2}{3}\int \frac{x^6}{x^8-8x^6+14x^4-7x^2+1}dx-\frac{1}{15}\sqrt{5-2\sqrt{5}}\text{arctanh}\left(\sqrt{\frac{10}{5+\sqrt{5}}}x\right)+\frac{1}{15}\sqrt{5+2\sqrt{5}}\text{arctanh}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}x\right)+\frac{1}{60}\left(5+\sqrt{5}\right)\log\left(-10x^2-\sqrt{5}+5\right)+\frac{1}{60}\left(5-\sqrt{5}\right)\log\left(-10x^2+\sqrt{5}+5\right)+\frac{1}{12}\log\left(x^8-8x^6+14x^4-7x^2+1\right)$$

input
```
Int[(x*(-2 + x + 18*x^2 - 7*x^3 - 56*x^4 + 15*x^5 + 72*x^6 - 12*x^7 - 36*x
^8 + 3*x^9 + 5*x^10))/(1 - 12*x^2 + 54*x^4 - 113*x^6 + 111*x^8 - 45*x^10 +
 5*x^12),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.17

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(125\_Z^4-250\_Z^3+125\_Z^2-20\_Z+1\right)}\_R\ln\left(x^3+\left(-75\_R^3+150\_R^2-75\_R+9\right)x^2+25\_R^3-50\_R^2+25\_\right.\right.}{6}$
default	$\frac{\left(5+\sqrt{5}\right)\ln\left(10x^2+\sqrt{5}-5\right)}{60}-\frac{\left(-\sqrt{5}-1\right)\operatorname{arctanh}\left(\frac{10x}{\sqrt{50-10\sqrt{5}}}\right)}{3\sqrt{50-10\sqrt{5}}}-\frac{\left(\sqrt{5}-5\right)\ln\left(10x^2-\sqrt{5}-5\right)}{60}+\frac{\left(-\sqrt{5}+1\right)\operatorname{arctanh}\left(\frac{10x}{\sqrt{50+10}}\right.}{3\sqrt{50+10\sqrt{5}}}$

input
```
int(x*(5*x^10+3*x^9-36*x^8-12*x^7+72*x^6+15*x^5-56*x^4-7*x^3+18*x^2+x-2)/(
5*x^12-45*x^10+111*x^8-113*x^6+54*x^4-12*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R*ln(x^3+(-75*_R^3+150*_R^2-75*_R+9)*x^2+25*_R^3-50*_R^2+25*_R-3)
,_R=RootOf(125*_Z^4-250*_Z^3+125*_Z^2-20*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.54

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$$
$$= \frac{1}{60}\left(\sqrt{5}-2\sqrt{2\sqrt{5}+5}+5\right)\log\left(10\,x^3+\left(15\,x^2-\sqrt{5}(3\,x^2-1)-5\right)\sqrt{2\sqrt{5}+5}\right)$$
$$+\frac{1}{60}\left(\sqrt{5}+2\sqrt{2\sqrt{5}+5}+5\right)\log\left(10\,x^3 - \left(15\,x^2-\sqrt{5}(3\,x^2-1)-5\right)\sqrt{2\sqrt{5}+5}\right)$$
$$-\frac{1}{60}\left(\sqrt{5}+2\sqrt{-2\sqrt{5}+5}-5\right)\log\left(10\,x^3 + \left(15\,x^2+\sqrt{5}(3\,x^2-1)-5\right)\sqrt{-2\sqrt{5}+5}\right)$$
$$-\frac{1}{60}\left(\sqrt{5}-2\sqrt{-2\sqrt{5}+5}-5\right)\log\left(10\,x^3 - \left(15\,x^2+\sqrt{5}(3\,x^2-1)-5\right)\sqrt{-2\sqrt{5}+5}\right)$$

input
```
integrate(x*(5*x^10+3*x^9-36*x^8-12*x^7+72*x^6+15*x^5-56*x^4-7*x^3+18*x^2+
x-2)/(5*x^12-45*x^10+111*x^8-113*x^6+54*x^4-12*x^2+1),x, algorithm="fricas
")
```

output
```
1/60*(sqrt(5) - 2*sqrt(2*sqrt(5) + 5) + 5)*log(10*x^3 + (15*x^2 - sqrt(5)*
(3*x^2 - 1) - 5)*sqrt(2*sqrt(5) + 5)) + 1/60*(sqrt(5) + 2*sqrt(2*sqrt(5) +
 5) + 5)*log(10*x^3 - (15*x^2 - sqrt(5)*(3*x^2 - 1) - 5)*sqrt(2*sqrt(5) +
5)) - 1/60*(sqrt(5) + 2*sqrt(-2*sqrt(5) + 5) - 5)*log(10*x^3 + (15*x^2 + s
qrt(5)*(3*x^2 - 1) - 5)*sqrt(-2*sqrt(5) + 5)) - 1/60*(sqrt(5) - 2*sqrt(-2*
sqrt(5) + 5) - 5)*log(10*x^3 - (15*x^2 + sqrt(5)*(3*x^2 - 1) - 5)*sqrt(-2*
sqrt(5) + 5))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.15

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$$
$$= \operatorname{RootSum}\left(162000t^4-54000t^3+4500t^2-120t+1,\left(t\mapsto t\log\left(5400t^3-1800t^2+150t+x^3+x^2(-162\right.\right.\right.$$

input
```
integrate(x*(5*x**10+3*x**9-36*x**8-12*x**7+72*x**6+15*x**5-56*x**4-7*x**3
+18*x**2+x-2)/(5*x**12-45*x**10+111*x**8-113*x**6+54*x**4-12*x**2+1),x)
```

output
```
RootSum(162000*_t**4 - 54000*_t**3 + 4500*_t**2 - 120*_t + 1, Lambda(_t, _
t*log(5400*_t**3 - 1800*_t**2 + 150*_t + x**3 + x**2*(-16200*_t**3 + 5400*
_t**2 - 450*_t + 9) - 3)))
```

Maxima [F]

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$$
$$= \int \frac{(5\,x^{10}+3\,x^9-36\,x^8-12\,x^7+72\,x^6+15\,x^5-56\,x^4-7\,x^3+18\,x^2+x-2)x}{5\,x^{12}-45\,x^{10}+111\,x^8-113\,x^6+54\,x^4-12\,x^2+1}\,dx$$

input
```
integrate(x*(5*x^10+3*x^9-36*x^8-12*x^7+72*x^6+15*x^5-56*x^4-7*x^3+18*x^2+
x-2)/(5*x^12-45*x^10+111*x^8-113*x^6+54*x^4-12*x^2+1),x, algorithm="maxima
")
```

output
```
integrate((5*x^10 + 3*x^9 - 36*x^8 - 12*x^7 + 72*x^6 + 15*x^5 - 56*x^4 - 7
*x^3 + 18*x^2 + x - 2)*x/(5*x^12 - 45*x^10 + 111*x^8 - 113*x^6 + 54*x^4 -
12*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx = \int \frac{(5\,x^{10}+3\,x^9-36\,x^8-12\,x^7+72\,x^6+15\,x^5-56\,x^4-7\,x^3+18\,x^2+x-2)x}{5\,x^{12}-45\,x^{10}+111\,x^8-113\,x^6+54\,x^4-12\,x^2+1}\,dx$$

input
```
integrate(x*(5*x^10+3*x^9-36*x^8-12*x^7+72*x^6+15*x^5-56*x^4-7*x^3+18*x^2+
x-2)/(5*x^12-45*x^10+111*x^8-113*x^6+54*x^4-12*x^2+1),x, algorithm="giac")
```

output
```
integrate((5*x^10 + 3*x^9 - 36*x^8 - 12*x^7 + 72*x^6 + 15*x^5 - 56*x^4 - 7
*x^3 + 18*x^2 + x - 2)*x/(5*x^12 - 45*x^10 + 111*x^8 - 113*x^6 + 54*x^4 -
12*x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.83

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx = \text{Too large to display}$$

input
```
int((x*(x + 18*x^2 - 7*x^3 - 56*x^4 + 15*x^5 + 72*x^6 - 12*x^7 - 36*x^8 +
3*x^9 + 5*x^10 - 2))/(54*x^4 - 12*x^2 - 113*x^6 + 111*x^8 - 45*x^10 + 5*x^
12 + 1),x)
```

output
```
symsum(log((146752739407872000000*x + 3914915958685440000*root(z^4 - 120*z
^3 + 4500*z^2 - 54000*z + 162000, z, k)^2 - 877535262010152000*root(z^4 -
120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^3 + 98732772668808000*root(z^
4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^4 - 6106360017121200*root
(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^5 + 213667660002240*ro
ot(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^6 - 4206592440936*ro
ot(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^7 - 183131208*root(z
^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^9 - 14828663965694400000
*x^2*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^2 + 333318757
2573816000*x^2*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^3 -
 111817024500716640000*x^3*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 16200
0, z, k)^2 - 376069644198360000*x^2*root(z^4 - 120*z^3 + 4500*z^2 - 54000*
z + 162000, z, k)^4 + 17011847867603976000*x^3*root(z^4 - 120*z^3 + 4500*z
^2 - 54000*z + 162000, z, k)^3 + 23321109901309200*x^2*root(z^4 - 120*z^3
+ 4500*z^2 - 54000*z + 162000, z, k)^5 - 1489935486584556000*x^3*root(z^4
- 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^4 - 818119549632960*x^2*roo
t(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^6 + 77798117717812800
*x^3*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^5 + 443376954
04968*x^3*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^7 + 7064
06168*x^2*root(z^4 - 120*z^3 + 4500*z^2 - 54000*z + 162000, z, k)^9 + 1...
```

Reduce [F]

$$\int \frac{x(-2+x+18x^2-7x^3-56x^4+15x^5+72x^6-12x^7-36x^8+3x^9+5x^{10})}{1-12x^2+54x^4-113x^6+111x^8-45x^{10}+5x^{12}}\,dx$$

$$= \frac{3\sqrt{\sqrt{5}-5}\,\sqrt{10}\,atan\left(\frac{10x}{\sqrt{\sqrt{5}-5}\,\sqrt{10}}\right)}{5} + \frac{33\sqrt{\sqrt{5}-5}\,\sqrt{2}\,atan\left(\frac{10x}{\sqrt{\sqrt{5}-5}\,\sqrt{10}}\right)}{10}$$
$$- \frac{3\sqrt{\sqrt{5}+5}\,\sqrt{10}\log\left(-\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{10}$$
$$+ \frac{3\sqrt{\sqrt{5}+5}\,\sqrt{10}\log\left(\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{10}$$
$$+ \frac{33\sqrt{\sqrt{5}+5}\,\sqrt{2}\log\left(-\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{20}$$
$$- \frac{33\sqrt{\sqrt{5}+5}\,\sqrt{2}\log\left(\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{20} - \frac{7\sqrt{5}\log\left(-\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{120}$$
$$- \frac{7\sqrt{5}\log\left(\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{120} + \frac{7\sqrt{5}\log(\sqrt{5}+10x^2-5)}{120}$$
$$+ 69\left(\int \frac{x^6}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)$$
$$+ \frac{73\left(\int \frac{x^5}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)}{24}$$
$$- 154\left(\int \frac{x^4}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)$$
$$- \frac{53\left(\int \frac{x^3}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)}{24}$$
$$+ 82\left(\int \frac{x^2}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)$$
$$+ \frac{3\left(\int \frac{x}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)}{8}$$
$$- 12\left(\int \frac{1}{5x^{12}-45x^{10}+111x^8-113x^6+54x^4-12x^2+1}dx\right)$$
$$+ \frac{17\log(x^8-8x^6+14x^4-7x^2+1)}{144} + \frac{\log\left(-\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{72}$$
$$+ \frac{\log\left(\sqrt{\sqrt{5}+5}+\sqrt{10}\,x\right)}{72} + \frac{\log(\sqrt{5}+10x^2-5)}{72}$$

input
```
int(x*(5*x^10+3*x^9-36*x^8-12*x^7+72*x^6+15*x^5-56*x^4-7*x^3+18*x^2+x-2)/(
5*x^12-45*x^10+111*x^8-113*x^6+54*x^4-12*x^2+1),x)
```

output
```
(432*sqrt(sqrt(5) - 5)*sqrt(10)*atan((10*x)/(sqrt(sqrt(5) - 5)*sqrt(10)))
+ 2376*sqrt(sqrt(5) - 5)*sqrt(2)*atan((10*x)/(sqrt(sqrt(5) - 5)*sqrt(10)))
 - 216*sqrt(sqrt(5) + 5)*sqrt(10)*log( - sqrt(sqrt(5) + 5) + sqrt(10)*x) +
 216*sqrt(sqrt(5) + 5)*sqrt(10)*log(sqrt(sqrt(5) + 5) + sqrt(10)*x) + 1188
*sqrt(sqrt(5) + 5)*sqrt(2)*log( - sqrt(sqrt(5) + 5) + sqrt(10)*x) - 1188*s
qrt(sqrt(5) + 5)*sqrt(2)*log(sqrt(sqrt(5) + 5) + sqrt(10)*x) - 42*sqrt(5)*
log( - sqrt(sqrt(5) + 5) + sqrt(10)*x) - 42*sqrt(5)*log(sqrt(sqrt(5) + 5)
+ sqrt(10)*x) + 42*sqrt(5)*log(sqrt(5) + 10*x**2 - 5) + 49680*int(x**6/(5*
x**12 - 45*x**10 + 111*x**8 - 113*x**6 + 54*x**4 - 12*x**2 + 1),x) + 2190*
int(x**5/(5*x**12 - 45*x**10 + 111*x**8 - 113*x**6 + 54*x**4 - 12*x**2 + 1
),x) - 110880*int(x**4/(5*x**12 - 45*x**10 + 111*x**8 - 113*x**6 + 54*x**4
 - 12*x**2 + 1),x) - 1590*int(x**3/(5*x**12 - 45*x**10 + 111*x**8 - 113*x*
*6 + 54*x**4 - 12*x**2 + 1),x) + 59040*int(x**2/(5*x**12 - 45*x**10 + 111*
x**8 - 113*x**6 + 54*x**4 - 12*x**2 + 1),x) + 270*int(x/(5*x**12 - 45*x**1
0 + 111*x**8 - 113*x**6 + 54*x**4 - 12*x**2 + 1),x) - 8640*int(1/(5*x**12
- 45*x**10 + 111*x**8 - 113*x**6 + 54*x**4 - 12*x**2 + 1),x) + 85*log(x**8
 - 8*x**6 + 14*x**4 - 7*x**2 + 1) + 10*log( - sqrt(sqrt(5) + 5) + sqrt(10)
*x) + 10*log(sqrt(sqrt(5) + 5) + sqrt(10)*x) + 10*log(sqrt(5) + 10*x**2 -
5))/720
```

3.93 $\int \frac{x}{2+4x+5x^2+2x^3+x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4} \, dx = -\frac{\arctan\left(\frac{1+2x}{\sqrt{7-4\sqrt{2}}}\right)}{2\sqrt{2\left(7-4\sqrt{2}\right)}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{7+4\sqrt{2}}}\right)}{2\sqrt{2\left(7+4\sqrt{2}\right)}} - \frac{\operatorname{arctanh}\left(\frac{7+4\left(\frac{1}{2}+x\right)^2}{4\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
-1/2*arctan((1+2*x)/(7-4*2^(1/2))^(1/2))/(14-8*2^(1/2))^(1/2)+1/2*arctan((
1+2*x)/(7+4*2^(1/2))^(1/2))/(14+8*2^(1/2))^(1/2)-1/4*arctanh(1/8*(7+4*(1/2
+x)^2)*2^(1/2))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4} \, dx = \frac{1}{2}\text{RootSum}\left[2+4\#1+5\#1^2+2\#1^3+\#1^4\&, \frac{\log(x-\#1)\#1}{2+5\#1+3\#1^2+2\#1^3}\&\right]$$

input
```
Integrate[x/(2 + 4*x + 5*x^2 + 2*x^3 + x^4),x]
```

output
```
RootSum[2 + 4*#1 + 5*#1^2 + 2*#1^3 + #1^4 & , (Log[x - #1]*#1)/(2 + 5*#1 +
 3*#1^2 + 2*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2459, 2202, 27, 1406, 216, 1432, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+2x^3+5x^2+4x+2}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2202}$$

$$\int -\frac{1}{2\left(\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}\right)}d\left(x+\frac{1}{2}\right)+\int \frac{x+\frac{1}{2}}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+\frac{1}{2}}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)-\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{1406}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{1}{4}\left(7+4\sqrt{2}\right)}d\left(x+\frac{1}{2}\right)}{2\sqrt{2}}-\frac{\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{1}{4}\left(7-4\sqrt{2}\right)}d\left(x+\frac{1}{2}\right)}{2\sqrt{2}}\right)+\int \frac{x+\frac{1}{2}}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x+\frac{1}{2}}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7+4\sqrt{2}}}\right)}{\sqrt{2\left(7+4\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7-4\sqrt{2}}}\right)}{\sqrt{2\left(7-4\sqrt{2}\right)}}\right)$$

$\downarrow$ 1432

$$\frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^4+\frac{7}{2}\left(x+\frac{1}{2}\right)^2+\frac{17}{16}}d\left(x+\frac{1}{2}\right)^2+\frac{1}{2}\left(\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7+4\sqrt{2}}}\right)}{\sqrt{2\left(7+4\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7-4\sqrt{2}}}\right)}{\sqrt{2\left(7-4\sqrt{2}\right)}}\right)$$

$\downarrow$ 1081

$$\frac{1}{2}\int \left(\frac{\sqrt{2}}{4\left(x+\frac{1}{2}\right)^2-4\sqrt{2}+7}-\frac{\sqrt{2}}{4\left(x+\frac{1}{2}\right)^2+4\sqrt{2}+7}\right)d\left(x+\frac{1}{2}\right)^2+ \frac{1}{2}\left(\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7+4\sqrt{2}}}\right)}{\sqrt{2\left(7+4\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7-4\sqrt{2}}}\right)}{\sqrt{2\left(7-4\sqrt{2}\right)}}\right)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7+4\sqrt{2}}}\right)}{\sqrt{2\left(7+4\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{7-4\sqrt{2}}}\right)}{\sqrt{2\left(7-4\sqrt{2}\right)}}\right)+ \frac{1}{2}\left(\frac{\log\left(4\left(x+\frac{1}{2}\right)^2-4\sqrt{2}+7\right)}{2\sqrt{2}}-\frac{\log\left(4\left(x+\frac{1}{2}\right)^2+4\sqrt{2}+7\right)}{2\sqrt{2}}\right)$$

input
```
Int[x/(2 + 4*x + 5*x^2 + 2*x^3 + x^4),x]
```

output
```
(-(ArcTan[(2*(1/2 + x))/Sqrt[7 - 4*Sqrt[2]]]/Sqrt[2*(7 - 4*Sqrt[2])]) + Ar
cTan[(2*(1/2 + x))/Sqrt[7 + 4*Sqrt[2]]]/Sqrt[2*(7 + 4*Sqrt[2])])/2 + (Log[
7 - 4*Sqrt[2] + 4*(1/2 + x)^2]/(2*Sqrt[2]) - Log[7 + 4*Sqrt[2] + 4*(1/2 +
x)^2]/(2*Sqrt[2]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] && !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2\_Z^3+5\_Z^2+4\_Z+2\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^3+3\_R^2+5\_R+2}\right)}{2}$	50
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+2\_Z^3+5\_Z^2+4\_Z+2\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^3+3\_R^2+5\_R+2}\right)}{2}$	50

input
```
int(x/(x^4+2*x^3+5*x^2+4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R/(2*_R^3+3*_R^2+5*_R+2)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^3+5*_Z^2+4*
_Z+2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx$$
$$= -\frac{1}{2}\sqrt{\frac{2}{17}\sqrt{2}+\frac{7}{34}}\arctan\left(\sqrt{2}(2x+1)\sqrt{\frac{2}{17}\sqrt{2}+\frac{7}{34}}\right)$$
$$+\frac{1}{2}\sqrt{-\frac{2}{17}\sqrt{2}+\frac{7}{34}}\arctan\left(\sqrt{2}(2x+1)\sqrt{-\frac{2}{17}\sqrt{2}+\frac{7}{34}}\right)$$
$$-\frac{1}{8}\sqrt{2}\log\left(x^2+x+\sqrt{2}+2\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+x-\sqrt{2}+2\right)$$

input
```
integrate(x/(x^4+2*x^3+5*x^2+4*x+2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2/17*sqrt(2) + 7/34)*arctan(sqrt(2)*(2*x + 1)*sqrt(2/17*sqrt(2)
+ 7/34)) + 1/2*sqrt(-2/17*sqrt(2) + 7/34)*arctan(sqrt(2)*(2*x + 1)*sqrt(-2
/17*sqrt(2) + 7/34)) - 1/8*sqrt(2)*log(x^2 + x + sqrt(2) + 2) + 1/8*sqrt(2
)*log(x^2 + x - sqrt(2) + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. 2(88) = 176.

Time = 0.44 (sec) , antiderivative size = 1216, normalized size of antiderivative = 11.16

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(x**4+2*x**3+5*x**2+4*x+2),x)
```

output
```
sqrt(2)*log(x**2 + x*(-2243*sqrt(2)/208 - 19*sqrt(297 - 68*sqrt(2))/52 + 2
49/26 + 123*sqrt(2)*sqrt(297 - 68*sqrt(2))/208) - 104443*sqrt(297 - 68*sqr
t(2))/10816 - 451653*sqrt(2)/5408 + 10053*sqrt(2)*sqrt(297 - 68*sqrt(2))/2
704 + 1958011/10816)/8 - sqrt(2)*log(x**2 + x*(-123*sqrt(2)*sqrt(68*sqrt(2
) + 297)/208 - 19*sqrt(68*sqrt(2) + 297)/52 + 249/26 + 2243*sqrt(2)/208) -
 104443*sqrt(68*sqrt(2) + 297)/10816 - 10053*sqrt(2)*sqrt(68*sqrt(2) + 297
)/2704 + 451653*sqrt(2)/5408 + 1958011/10816)/8 + 2*sqrt(41/544 - sqrt(68*
sqrt(2) + 297)/272)*atan(416*sqrt(17)*x/(135*sqrt(2)*sqrt(41 - 2*sqrt(68*s
qrt(2) + 297)) + 41*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))*sqrt(68*sq
rt(2) + 297) + 1292*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))) - 123*sqrt(34)*sq
rt(68*sqrt(2) + 297)/(135*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297)) + 41
*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))*sqrt(68*sqrt(2) + 297) + 1292
*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))) - 76*sqrt(17)*sqrt(68*sqrt(2) + 297)
/(135*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297)) + 41*sqrt(2)*sqrt(41 - 2
*sqrt(68*sqrt(2) + 297))*sqrt(68*sqrt(2) + 297) + 1292*sqrt(41 - 2*sqrt(68
*sqrt(2) + 297))) + 1992*sqrt(17)/(135*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2)
 + 297)) + 41*sqrt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))*sqrt(68*sqrt(2)
+ 297) + 1292*sqrt(41 - 2*sqrt(68*sqrt(2) + 297))) + 2243*sqrt(34)/(135*sq
rt(2)*sqrt(41 - 2*sqrt(68*sqrt(2) + 297)) + 41*sqrt(2)*sqrt(41 - 2*sqrt(68
*sqrt(2) + 297))*sqrt(68*sqrt(2) + 297) + 1292*sqrt(41 - 2*sqrt(68*sqrt...
```

Maxima [F]

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx = \int \frac{x}{x^4+2\,x^3+5\,x^2+4\,x+2}\,dx$$

input
```
integrate(x/(x^4+2*x^3+5*x^2+4*x+2),x, algorithm="maxima")
```

output
```
integrate(x/(x^4 + 2*x^3 + 5*x^2 + 4*x + 2), x)
```

Giac [F]

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx = \int \frac{x}{x^4+2\,x^3+5\,x^2+4\,x+2}\,dx$$

input
```
integrate(x/(x^4+2*x^3+5*x^2+4*x+2),x, algorithm="giac")
```

output
```
integrate(x/(x^4 + 2*x^3 + 5*x^2 + 4*x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx = \sum_{k=1}^{4} \ln\left(x - \mathrm{root}\left(z^4 - \frac{5\,z^2}{136} + \frac{z}{136} + \frac{1}{544}, z, k\right)\left(-6\,x + \mathrm{root}\left(z^4 - \frac{5\,z^2}{136} + \frac{z}{136} + \frac{1}{544}, z, k\right)\left(32\,x + \mathrm{root}\left(z^4 - \frac{5\,z^2}{136} + \frac{z}{136} + \frac{1}{544}, z, k\right)(224\,x + 112) + 32\right) + 4\right)\right)\mathrm{root}\left(z^4 - \frac{5\,z^2}{136} + \frac{z}{136} + \frac{1}{544}, z, k\right)$$

input
```
int(x/(4*x + 5*x^2 + 2*x^3 + x^4 + 2),x)
```

output
```
symsum(log(x - root(z^4 - (5*z^2)/136 + z/136 + 1/544, z, k)*(root(z^4 - (
5*z^2)/136 + z/136 + 1/544, z, k)*(32*x + root(z^4 - (5*z^2)/136 + z/136 +
 1/544, z, k)*(224*x + 112) + 32) - 6*x + 4))*root(z^4 - (5*z^2)/136 + z/1
36 + 1/544, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.82

$$\int \frac{x}{2+4x+5x^2+2x^3+x^4}\,dx = \frac{7\sqrt{4\sqrt{2}+7}\,\sqrt{2}\,atan\left(\frac{2x+1}{\sqrt{4\sqrt{2}+7}}\right)}{68} - \frac{2\sqrt{4\sqrt{2}+7}\,atan\left(\frac{2x+1}{\sqrt{4\sqrt{2}+7}}\right)}{17} + \frac{7\sqrt{4\sqrt{2}-7}\,\sqrt{2}\log\left(-\sqrt{4\sqrt{2}-7}+2x+1\right)}{136} - \frac{7\sqrt{4\sqrt{2}-7}\,\sqrt{2}\log\left(\sqrt{4\sqrt{2}-7}+2x+1\right)}{136} + \frac{\sqrt{4\sqrt{2}-7}\log\left(-\sqrt{4\sqrt{2}-7}+2x+1\right)}{17} - \frac{\sqrt{4\sqrt{2}-7}\log\left(\sqrt{4\sqrt{2}-7}+2x+1\right)}{17} + \frac{\sqrt{2}\log\left(-\sqrt{4\sqrt{2}-7}+2x+1\right)}{8} + \frac{\sqrt{2}\log\left(\sqrt{4\sqrt{2}-7}+2x+1\right)}{8} - \frac{\sqrt{2}\log\left(\sqrt{2}+x^2+x+2\right)}{8}$$

input
```
int(x/(x^4+2*x^3+5*x^2+4*x+2),x)
```

output
```
(14*sqrt(4*sqrt(2) + 7)*sqrt(2)*atan((2*x + 1)/sqrt(4*sqrt(2) + 7)) - 16*s
qrt(4*sqrt(2) + 7)*atan((2*x + 1)/sqrt(4*sqrt(2) + 7)) + 7*sqrt(4*sqrt(2)
- 7)*sqrt(2)*log( - sqrt(4*sqrt(2) - 7) + 2*x + 1) - 7*sqrt(4*sqrt(2) - 7)
*sqrt(2)*log(sqrt(4*sqrt(2) - 7) + 2*x + 1) + 8*sqrt(4*sqrt(2) - 7)*log( -
 sqrt(4*sqrt(2) - 7) + 2*x + 1) - 8*sqrt(4*sqrt(2) - 7)*log(sqrt(4*sqrt(2)
 - 7) + 2*x + 1) + 17*sqrt(2)*log( - sqrt(4*sqrt(2) - 7) + 2*x + 1) + 17*s
qrt(2)*log(sqrt(4*sqrt(2) - 7) + 2*x + 1) - 17*sqrt(2)*log(sqrt(2) + x**2
+ x + 2))/136
```

3.94 $\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}$

Optimal result

Integrand size = 95, antiderivative size = 231

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= \frac{1}{2}\left(1+\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\left(3x+x^2\right)\right) + \frac{1}{2}\left(-1+\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(1+\sqrt{5}\right)\left(3x+x^2\right)\right) + \frac{1}{2}\left(1+\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(4x-2\sqrt{5}x+x^2-\sqrt{5}x^2-9x^3+9\sqrt{5}x^3-6x^4+6\sqrt{5}x^4-x^5+\sqrt{5}x^5\right)\right) + \frac{1}{2}\left(-1+\sqrt{5}\right)\arctan\left(\frac{1}{2}\left(-4x-2\sqrt{5}x-x^2-\sqrt{5}x^2+9x^3+9\sqrt{5}x^3+6x^4+6\sqrt{5}x^4+x^5+\sqrt{5}x^5\right)\right)$$

output
```
1/2*(5^(1/2)+1)*arctan(1/2*(5^(1/2)-1)*(x^2+3*x))+1/2*(5^(1/2)-1)*arctan(1
/2*(5^(1/2)+1)*(x^2+3*x))+1/2*(5^(1/2)+1)*arctan(2*x-x*5^(1/2)+1/2*x^2-1/2
*5^(1/2)*x^2-9/2*x^3+9/2*5^(1/2)*x^3-3*x^4+3*5^(1/2)*x^4-1/2*x^5+1/2*5^(1/
2)*x^5)+1/2*(5^(1/2)-1)*arctan(-2*x-x*5^(1/2)-1/2*x^2-1/2*5^(1/2)*x^2+9/2*
x^3+9/2*5^(1/2)*x^3+3*x^4+3*5^(1/2)*x^4+1/2*x^5+1/2*5^(1/2)*x^5)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$=\frac{1}{2}\text{RootSum}\Big[1-9\#1^2-4\#1^3+37\#1^4+30\#1^5-75\#1^6-90\#1^7+48\#1^8+104\#1^9+54\#1^{10}+12\#1^{11}+\#1^{12}\&,\frac{3\log(x-\#1)-7\log(x-\#1)\#1^2-21\log(x-\#1)\#1^4-32\log(x-\#1)\#1^5+72\log(x-}{-9\#1-6\#1^2+74\#1^3+75\#1^4-225\#1^5-315\#1^6+192\#}$$

input
```
Integrate[(3 - 7*x^2 - 21*x^4 - 32*x^5 + 72*x^6 + 108*x^7 + 45*x^8 + 6*x^9
)/(1 - 9*x^2 - 4*x^3 + 37*x^4 + 30*x^5 - 75*x^6 - 90*x^7 + 48*x^8 + 104*x^
9 + 54*x^10 + 12*x^11 + x^12),x]
```

output
```
RootSum[1 - 9*#1^2 - 4*#1^3 + 37*#1^4 + 30*#1^5 - 75*#1^6 - 90*#1^7 + 48*#
1^8 + 104*#1^9 + 54*#1^10 + 12*#1^11 + #1^12 & , (3*Log[x - #1] - 7*Log[x
- #1]*#1^2 - 21*Log[x - #1]*#1^4 - 32*Log[x - #1]*#1^5 + 72*Log[x - #1]*#1
^6 + 108*Log[x - #1]*#1^7 + 45*Log[x - #1]*#1^8 + 6*Log[x - #1]*#1^9)/(-9*
#1 - 6*#1^2 + 74*#1^3 + 75*#1^4 - 225*#1^5 - 315*#1^6 + 192*#1^7 + 468*#1^
8 + 270*#1^9 + 66*#1^10 + 6*#1^11) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^9+45x^8+108x^7+72x^6-32x^5-21x^4-7x^2+3}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}\,dx$$
$$\Big\downarrow \ 7293$$
$$\int \Big(\frac{6x^9}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}+\frac{}{x^{12}+12x^{11}+54x^{10}}$$
$$\Big\downarrow \ 2009$$

$$3\int \frac{1}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx-7\int \frac{x^2}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx-21\int \frac{x^4}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx-32\int \frac{x^5}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+72\int \frac{x^6}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+108\int \frac{x^7}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+45\int \frac{x^8}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+6\int \frac{x^9}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx$$

input
```
Int[(3 - 7*x^2 - 21*x^4 - 32*x^5 + 72*x^6 + 108*x^7 + 45*x^8 + 6*x^9)/(1 -
 9*x^2 - 4*x^3 + 37*x^4 + 30*x^5 - 75*x^6 - 90*x^7 + 48*x^8 + 104*x^9 + 54
*x^10 + 12*x^11 + x^12),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\_R\ln\left(x^3+\_Rx+3x^2-1\right)\right)}{2}$	33
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\_R\ln\left(x^3+\_Rx+3x^2-1\right)\right)}{2}$	33

input
```
int((6*x^9+45*x^8+108*x^7+72*x^6-32*x^5-21*x^4-7*x^2+3)/(x^12+12*x^11+54*x
^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
1/2*sum(_R*ln(x^3+_R*x+3*x^2-1),_R=RootOf(_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.77

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx = \frac{1}{2}\left(\sqrt{5}-1\right)\arctan\left(\frac{1}{2}\,x^5+3\,x^4+\frac{9}{2}\,x^3-\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{5}(x^5+6\,x^4+9\,x^3-x^2-2\,x)-2\,x\right)+\frac{1}{2}\left(\sqrt{5}+1\right)\arctan\left(-\frac{1}{2}\,x^5-3\,x^4-\frac{9}{2}\,x^3+\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{5}(x^5+6\,x^4+9\,x^3-x^2-2\,x)+2\,x\right)+\frac{1}{2}\left(\sqrt{5}-1\right)\arctan\left(\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{5}(x^2+3\,x)+\frac{3}{2}\,x\right)+\frac{1}{2}\left(\sqrt{5}+1\right)\arctan\left(-\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{5}(x^2+3\,x)-\frac{3}{2}\,x\right)$$

input
```
integrate((6*x^9+45*x^8+108*x^7+72*x^6-32*x^5-21*x^4-7*x^2+3)/(x^12+12*x^1
1+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1),x, alg
orithm="fricas")
```

output
```
1/2*(sqrt(5) - 1)*arctan(1/2*x^5 + 3*x^4 + 9/2*x^3 - 1/2*x^2 + 1/2*sqrt(5)
*(x^5 + 6*x^4 + 9*x^3 - x^2 - 2*x) - 2*x) + 1/2*(sqrt(5) + 1)*arctan(-1/2*
x^5 - 3*x^4 - 9/2*x^3 + 1/2*x^2 + 1/2*sqrt(5)*(x^5 + 6*x^4 + 9*x^3 - x^2 -
 2*x) + 2*x) + 1/2*(sqrt(5) - 1)*arctan(1/2*x^2 + 1/2*sqrt(5)*(x^2 + 3*x)
+ 3/2*x) + 1/2*(sqrt(5) + 1)*arctan(-1/2*x^2 + 1/2*sqrt(5)*(x^2 + 3*x) - 3
/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= \left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right)\left(2\,\mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{5}}+\frac{6x}{-1+\sqrt{5}}\right)\right.$$
$$\left.+2\,\mathrm{atan}\left(\frac{2x^5}{-1+\sqrt{5}}+\frac{12x^4}{-1+\sqrt{5}}+\frac{18x^3}{-1+\sqrt{5}}-\frac{2x^2}{-1+\sqrt{5}}+x\left(-\frac{3}{-1+\sqrt{5}}-\frac{\sqrt{5}}{-1+\sqrt{5}}\right)\right)\right)$$
$$+\left(\frac{1}{4}+\frac{\sqrt{5}}{4}\right)\left(2\,\mathrm{atan}\left(\frac{2x^2}{1+\sqrt{5}}+\frac{6x}{1+\sqrt{5}}\right)\right.$$
$$\left.+2\,\mathrm{atan}\left(\frac{2x^5}{1+\sqrt{5}}+\frac{12x^4}{1+\sqrt{5}}+\frac{18x^3}{1+\sqrt{5}}-\frac{2x^2}{1+\sqrt{5}}+x\left(-\frac{3}{1+\sqrt{5}}+\frac{\sqrt{5}}{1+\sqrt{5}}\right)\right)\right)$$

input
```
integrate((6*x**9+45*x**8+108*x**7+72*x**6-32*x**5-21*x**4-7*x**2+3)/(x**1
2+12*x**11+54*x**10+104*x**9+48*x**8-90*x**7-75*x**6+30*x**5+37*x**4-4*x**
3-9*x**2+1),x)
```

output
```
(-1/4 + sqrt(5)/4)*(2*atan(2*x**2/(-1 + sqrt(5)) + 6*x/(-1 + sqrt(5))) + 2
*atan(2*x**5/(-1 + sqrt(5)) + 12*x**4/(-1 + sqrt(5)) + 18*x**3/(-1 + sqrt(
5)) - 2*x**2/(-1 + sqrt(5)) + x*(-3/(-1 + sqrt(5)) - sqrt(5)/(-1 + sqrt(5)
)))) + (1/4 + sqrt(5)/4)*(2*atan(2*x**2/(1 + sqrt(5)) + 6*x/(1 + sqrt(5)))
 + 2*atan(2*x**5/(1 + sqrt(5)) + 12*x**4/(1 + sqrt(5)) + 18*x**3/(1 + sqrt
(5)) - 2*x**2/(1 + sqrt(5)) + x*(-3/(1 + sqrt(5)) + sqrt(5)/(1 + sqrt(5)))
))
```

Maxima [F]

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= \int \frac{6\,x^9+45\,x^8+108\,x^7+72\,x^6-32\,x^5-21\,x^4-7\,x^2+3}{x^{12}+12\,x^{11}+54\,x^{10}+104\,x^9+48\,x^8-90\,x^7-75\,x^6+30\,x^5+37\,x^4-4\,x^3-9\,x^2+1}\,dx$$

input
```
integrate((6*x^9+45*x^8+108*x^7+72*x^6-32*x^5-21*x^4-7*x^2+3)/(x^12+12*x^1
1+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1),x, alg
orithm="maxima")
```

output
```
integrate((6*x^9 + 45*x^8 + 108*x^7 + 72*x^6 - 32*x^5 - 21*x^4 - 7*x^2 + 3
)/(x^12 + 12*x^11 + 54*x^10 + 104*x^9 + 48*x^8 - 90*x^7 - 75*x^6 + 30*x^5
+ 37*x^4 - 4*x^3 - 9*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.68

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$=\frac{1}{2}\left(\sqrt{5}-1\right)\left(\arctan\left(\frac{1}{2}x^5\left(\sqrt{5}+1\right)+3x^4\left(\sqrt{5}+1\right)+\frac{9}{2}x^3\left(\sqrt{5}+1\right)-\frac{1}{2}x^2\left(\sqrt{5}+1\right)-x\left(\sqrt{5}+2\right)\right.\right.$$
$$-\frac{1}{2}\left(\sqrt{5}+1\right)\left(\arctan\left(-\frac{1}{2}x^5\left(\sqrt{5}-1\right)-3x^4\left(\sqrt{5}-1\right)-\frac{9}{2}x^3\left(\sqrt{5}-1\right)+\frac{1}{2}x^2\left(\sqrt{5}-1\right)+x\left(\sqrt{5}-\right.\right.\right.$$

input
```
integrate((6*x^9+45*x^8+108*x^7+72*x^6-32*x^5-21*x^4-7*x^2+3)/(x^12+12*x^1
1+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1),x, alg
orithm="giac")
```

output
```
1/2*(sqrt(5) - 1)*(arctan(1/2*x^5*(sqrt(5) + 1) + 3*x^4*(sqrt(5) + 1) + 9/
2*x^3*(sqrt(5) + 1) - 1/2*x^2*(sqrt(5) + 1) - x*(sqrt(5) + 2)) + arctan(1/
2*x^2*(sqrt(5) + 1) + 3/2*x*(sqrt(5) + 1))) - 1/2*(sqrt(5) + 1)*(arctan(-1
/2*x^5*(sqrt(5) - 1) - 3*x^4*(sqrt(5) - 1) - 9/2*x^3*(sqrt(5) - 1) + 1/2*x
^2*(sqrt(5) - 1) + x*(sqrt(5) - 2)) + arctan(-1/2*x^2*(sqrt(5) - 1) - 3/2*
x*(sqrt(5) - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.15

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
= Too large to display

input
```
int((72*x^6 - 21*x^4 - 32*x^5 - 7*x^2 + 108*x^7 + 45*x^8 + 6*x^9 + 3)/(37*
x^4 - 4*x^3 - 9*x^2 + 30*x^5 - 75*x^6 - 90*x^7 + 48*x^8 + 104*x^9 + 54*x^1
0 + 12*x^11 + x^12 + 1),x)
```

output
```
2*atanh((12107306250*(- 5^(1/2)/8 - 3/8)^(1/2))/(2127206250*x - 1145137500
*5^(1/2)*x + 782583750*5^(1/2) - 2347751250*5^(1/2)*x^2 - 782583750*5^(1/2
)*x^3 + 4264406250*x^2 + 1421468750*x^3 - 1421468750) - (36321918750*x^2*(
- 5^(1/2)/8 - 3/8)^(1/2))/(2127206250*x - 1145137500*5^(1/2)*x + 782583750
*5^(1/2) - 2347751250*5^(1/2)*x^2 - 782583750*5^(1/2)*x^3 + 4264406250*x^2
 + 1421468750*x^3 - 1421468750) - (12107306250*x^3*(- 5^(1/2)/8 - 3/8)^(1/
2))/(2127206250*x - 1145137500*5^(1/2)*x + 782583750*5^(1/2) - 2347751250*
5^(1/2)*x^2 - 782583750*5^(1/2)*x^3 + 4264406250*x^2 + 1421468750*x^3 - 14
21468750) - (5562618750*5^(1/2)*(- 5^(1/2)/8 - 3/8)^(1/2))/(2127206250*x -
 1145137500*5^(1/2)*x + 782583750*5^(1/2) - 2347751250*5^(1/2)*x^2 - 78258
3750*5^(1/2)*x^3 + 4264406250*x^2 + 1421468750*x^3 - 1421468750) + (284293
7500*x*(- 5^(1/2)/8 - 3/8)^(1/2))/(2127206250*x - 1145137500*5^(1/2)*x + 7
82583750*5^(1/2) - 2347751250*5^(1/2)*x^2 - 782583750*5^(1/2)*x^3 + 426440
6250*x^2 + 1421468750*x^3 - 1421468750) - (1565167500*5^(1/2)*x*(- 5^(1/2)
/8 - 3/8)^(1/2))/(2127206250*x - 1145137500*5^(1/2)*x + 782583750*5^(1/2)
- 2347751250*5^(1/2)*x^2 - 782583750*5^(1/2)*x^3 + 4264406250*x^2 + 142146
8750*x^3 - 1421468750) + (16687856250*5^(1/2)*x^2*(- 5^(1/2)/8 - 3/8)^(1/2
))/(2127206250*x - 1145137500*5^(1/2)*x + 782583750*5^(1/2) - 2347751250*5
^(1/2)*x^2 - 782583750*5^(1/2)*x^3 + 4264406250*x^2 + 1421468750*x^3 - 142
1468750) + (5562618750*5^(1/2)*x^3*(- 5^(1/2)/8 - 3/8)^(1/2))/(21272062...
```

Reduce [F]

$$\int \frac{3-7x^2-21x^4-32x^5+72x^6+108x^7+45x^8+6x^9}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$=6\left(\int \frac{x^9}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+45\left(\int \frac{x^8}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+108\left(\int \frac{x^7}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+72\left(\int \frac{x^6}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$-32\left(\int \frac{x^5}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$-21\left(\int \frac{x^4}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$-7\left(\int \frac{x^2}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+3\left(\int \frac{1}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$

input
```
int((6*x^9+45*x^8+108*x^7+72*x^6-32*x^5-21*x^4-7*x^2+3)/(x^12+12*x^11+54*x
^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1),x)
```

output

```
6*int(x**9/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 7
5*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) + 45*int(x**8/(x**12
+ 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 +
 37*x**4 - 4*x**3 - 9*x**2 + 1),x) + 108*int(x**7/(x**12 + 12*x**11 + 54*x
**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3
 - 9*x**2 + 1),x) + 72*int(x**6/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 +
48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x)
- 32*int(x**5/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7
- 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) - 21*int(x**4/(x**
12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**
5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) - 7*int(x**2/(x**12 + 12*x**11 + 54*
x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**
3 - 9*x**2 + 1),x) + 3*int(1/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*
x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x)
```

3.95 $\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}}$

Optimal result

Integrand size = 113, antiderivative size = 364

$$
\begin{aligned}
&\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx \\
&= \frac{\arctan\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\left(3x+x^2\right)\right)}{\sqrt{5}} + \frac{\arctan\left(\frac{1}{2}\left(1+\sqrt{5}\right)\left(3x+x^2\right)\right)}{\sqrt{5}} \\
&\quad + \frac{\arctan\left(\frac{1}{2}\left(4x-2\sqrt{5}x+x^2-\sqrt{5}x^2-9x^3+9\sqrt{5}x^3-6x^4+6\sqrt{5}x^4-x^5+\sqrt{5}x^5\right)\right)}{\sqrt{5}} \\
&\quad + \frac{\arctan\left(\frac{1}{2}\left(-4x-2\sqrt{5}x-x^2-\sqrt{5}x^2+9x^3+9\sqrt{5}x^3+6x^4+6\sqrt{5}x^4+x^5+\sqrt{5}x^5\right)\right)}{\sqrt{5}} \\
&\quad + \frac{\log\left(2-9x^2-\sqrt{5}x^2-4x^3+18x^4+12x^5+2x^6\right)}{4\sqrt{5}} \\
&\quad - \frac{\log\left(2-9x^2+\sqrt{5}x^2-4x^3+18x^4+12x^5+2x^6\right)}{4\sqrt{5}} \\
&\quad + \frac{1}{4}\log\left(1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}\right)
\end{aligned}
$$

output
```
1/5*arctan(1/2*(5^(1/2)-1)*(x^2+3*x))*5^(1/2)+1/5*arctan(1/2*(5^(1/2)+1)*(
x^2+3*x))*5^(1/2)+1/5*arctan(2*x-x*5^(1/2)+1/2*x^2-1/2*5^(1/2)*x^2-9/2*x^3
+9/2*5^(1/2)*x^3-3*x^4+3*5^(1/2)*x^4-1/2*x^5+1/2*5^(1/2)*x^5)*5^(1/2)+1/5*
arctan(-2*x-x*5^(1/2)-1/2*x^2-1/2*5^(1/2)*x^2+9/2*x^3+9/2*5^(1/2)*x^3+3*x^
4+3*5^(1/2)*x^4+1/2*x^5+1/2*5^(1/2)*x^5)*5^(1/2)+1/20*ln(2-9*x^2-5^(1/2)*x
^2-4*x^3+18*x^4+12*x^5+2*x^6)*5^(1/2)-1/20*ln(2-9*x^2+5^(1/2)*x^2-4*x^3+18
*x^4+12*x^5+2*x^6)*5^(1/2)+1/4*ln(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x
^7-75*x^6+30*x^5+37*x^4-4*x^3-9*x^2+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.80

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= \frac{1}{2}\text{RootSum}\Big[1-9\#1^2-4\#1^3+37\#1^4+30\#1^5-75\#1^6-90\#1^7+48\#1^8 + 104\#1^9+54\#1^{10}+12\#1^{11} + \#1^{12}\&, \frac{\log(x-\#1)-5\log(x-\#1)\#1-5\log(x-\#1)\#1^2+37\log(x-\#1)\#1^3+31\log(x-\#1)}{-9\#1-6\#1^2+}$$

input
```
Integrate[(1 - 5*x - 5*x^2 + 37*x^3 + 31*x^4 - 118*x^5 - 129*x^6 + 133*x^7
 + 249*x^8 + 137*x^9 + 33*x^10 + 3*x^11)/(1 - 9*x^2 - 4*x^3 + 37*x^4 + 30*
x^5 - 75*x^6 - 90*x^7 + 48*x^8 + 104*x^9 + 54*x^10 + 12*x^11 + x^12),x]
```

output
```
RootSum[1 - 9*#1^2 - 4*#1^3 + 37*#1^4 + 30*#1^5 - 75*#1^6 - 90*#1^7 + 48*#
1^8 + 104*#1^9 + 54*#1^10 + 12*#1^11 + #1^12 & , (Log[x - #1] - 5*Log[x -
#1]*#1 - 5*Log[x - #1]*#1^2 + 37*Log[x - #1]*#1^3 + 31*Log[x - #1]*#1^4 -
118*Log[x - #1]*#1^5 - 129*Log[x - #1]*#1^6 + 133*Log[x - #1]*#1^7 + 249*L
og[x - #1]*#1^8 + 137*Log[x - #1]*#1^9 + 33*Log[x - #1]*#1^10 + 3*Log[x -
#1]*#1^11)/(-9*#1 - 6*#1^2 + 74*#1^3 + 75*#1^4 - 225*#1^5 - 315*#1^6 + 192
*#1^7 + 468*#1^8 + 270*#1^9 + 66*#1^10 + 6*#1^11) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^{11}+33x^{10}+137x^9+249x^8+133x^7-129x^6-118x^5+31x^4+37x^3-5x^2-5x+1}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}\,dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{1}{12}\int \frac{6(4x^9+30x^8+74x^7+57x^6-11x^5-13x^4-4x^2-x+2)}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+\frac{1}{4}\log\left(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{4x^9+30x^8+74x^7+57x^6-11x^5-13x^4-4x^2-x+2}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx+\frac{1}{4}\log\left(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1\right)$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}\int \left(\frac{4x^9}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}+\frac{}{x^{12}+12x^{11}+54x}\right.$$
$$\frac{1}{4}\log\left(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx-\int \frac{}{x^{12}+12x^{11}}\right.$$
$$\frac{1}{4}\log\left(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1\right)$$

input
```
Int[(1 - 5*x - 5*x^2 + 37*x^3 + 31*x^4 - 118*x^5 - 129*x^6 + 133*x^7 + 249
*x^8 + 137*x^9 + 33*x^10 + 3*x^11)/(1 - 9*x^2 - 4*x^3 + 37*x^4 + 30*x^5 -
75*x^6 - 90*x^7 + 48*x^8 + 104*x^9 + 54*x^10 + 12*x^11 + x^12),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}(5\_Z^4-10\_Z^3+9\_Z^2-4\_Z+1)}\_R\ln\left(x^3+3x^2+\left(5\_R^3-10\_R^2+9\_R-3\right)x-1\right)}{2}$	57
risch	$\frac{\sum_{\_R=\operatorname{RootOf}(5\_Z^4-10\_Z^3+9\_Z^2-4\_Z+1)}\_R\ln\left(x^3+3x^2+\left(5\_R^3-10\_R^2+9\_R-3\right)x-1\right)}{2}$	57

input
```
int((3*x^11+33*x^10+137*x^9+249*x^8+133*x^7-129*x^6-118*x^5+31*x^4+37*x^3-
5*x^2-5*x+1)/(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*
x^4-4*x^3-9*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R*ln(x^3+3*x^2+(5*_R^3-10*_R^2+9*_R-3)*x-1),_R=RootOf(5*_Z^4-10*_
Z^3+9*_Z^2-4*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.70

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= -\frac{1}{20}\left(\sqrt{5}-5\right)\log\left(2x^6+12x^5+18x^4-4x^3+\sqrt{5}x^2-9x^2+2\right)$$
$$+\frac{1}{20}\left(\sqrt{5}+5\right)\log\left(2x^6+12x^5+18x^4-4x^3-\sqrt{5}x^2-9x^2+2\right)$$
$$+\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}x^5+3x^4+\frac{9}{2}x^3-\frac{1}{2}x^2+\frac{1}{2}\sqrt{5}(x^5+6x^4+9x^3-x^2-2x)-2x\right)$$
$$+\frac{1}{5}\sqrt{5}\arctan\left(-\frac{1}{2}x^5-3x^4-\frac{9}{2}x^3+\frac{1}{2}x^2+\frac{1}{2}\sqrt{5}(x^5+6x^4+9x^3-x^2-2x)+2x\right)$$
$$+\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}x^2+\frac{1}{2}\sqrt{5}(x^2+3x)+\frac{3}{2}x\right)$$
$$+\frac{1}{5}\sqrt{5}\arctan\left(-\frac{1}{2}x^2+\frac{1}{2}\sqrt{5}(x^2+3x)-\frac{3}{2}x\right)$$

input
```
integrate((3*x^11+33*x^10+137*x^9+249*x^8+133*x^7-129*x^6-118*x^5+31*x^4+3
7*x^3-5*x^2-5*x+1)/(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x
^5+37*x^4-4*x^3-9*x^2+1),x, algorithm="fricas")
```

output
```
-1/20*(sqrt(5) - 5)*log(2*x^6 + 12*x^5 + 18*x^4 - 4*x^3 + sqrt(5)*x^2 - 9*
x^2 + 2) + 1/20*(sqrt(5) + 5)*log(2*x^6 + 12*x^5 + 18*x^4 - 4*x^3 - sqrt(5
)*x^2 - 9*x^2 + 2) + 1/5*sqrt(5)*arctan(1/2*x^5 + 3*x^4 + 9/2*x^3 - 1/2*x^
2 + 1/2*sqrt(5)*(x^5 + 6*x^4 + 9*x^3 - x^2 - 2*x) - 2*x) + 1/5*sqrt(5)*arc
tan(-1/2*x^5 - 3*x^4 - 9/2*x^3 + 1/2*x^2 + 1/2*sqrt(5)*(x^5 + 6*x^4 + 9*x^
3 - x^2 - 2*x) + 2*x) + 1/5*sqrt(5)*arctan(1/2*x^2 + 1/2*sqrt(5)*(x^2 + 3*
x) + 3/2*x) + 1/5*sqrt(5)*arctan(-1/2*x^2 + 1/2*sqrt(5)*(x^2 + 3*x) - 3/2*
x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.84

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= \frac{\sqrt{5}\cdot\left(2\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}+\frac{6x}{-1+\sqrt{5}}\right)+2\operatorname{atan}\left(\frac{2x^5}{-1+\sqrt{5}}+\frac{12x^4}{-1+\sqrt{5}}+\frac{18x^3}{-1+\sqrt{5}}-\frac{2x^2}{-1+\sqrt{5}}+x\left(-\frac{3}{-1+\sqrt{5}}-\frac{\sqrt{5}}{-1+\sqrt{5}}\right)\right)\right)}{10}$$
$$+\frac{\sqrt{5}\cdot\left(2\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}+\frac{6x}{1+\sqrt{5}}\right)+2\operatorname{atan}\left(\frac{2x^5}{1+\sqrt{5}}+\frac{12x^4}{1+\sqrt{5}}+\frac{18x^3}{1+\sqrt{5}}-\frac{2x^2}{1+\sqrt{5}}+x\left(-\frac{3}{1+\sqrt{5}}+\frac{\sqrt{5}}{1+\sqrt{5}}\right)\right)\right)}{10}$$
$$+\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x^6+6x^5+9x^4-2x^3+x^2\left(-\frac{9}{2}-\frac{\sqrt{5}}{2}\right)+1\right)$$
$$+\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x^6+6x^5+9x^4-2x^3+x^2\left(-\frac{9}{2}+\frac{\sqrt{5}}{2}\right)+1\right)$$

input
```
integrate((3*x**11+33*x**10+137*x**9+249*x**8+133*x**7-129*x**6-118*x**5+3
1*x**4+37*x**3-5*x**2-5*x+1)/(x**12+12*x**11+54*x**10+104*x**9+48*x**8-90*
x**7-75*x**6+30*x**5+37*x**4-4*x**3-9*x**2+1),x)
```

output
```
sqrt(5)*(2*atan(2*x**2/(-1 + sqrt(5)) + 6*x/(-1 + sqrt(5))) + 2*atan(2*x**
5/(-1 + sqrt(5)) + 12*x**4/(-1 + sqrt(5)) + 18*x**3/(-1 + sqrt(5)) - 2*x**
2/(-1 + sqrt(5)) + x*(-3/(-1 + sqrt(5)) - sqrt(5)/(-1 + sqrt(5)))))/10 + s
qrt(5)*(2*atan(2*x**2/(1 + sqrt(5)) + 6*x/(1 + sqrt(5))) + 2*atan(2*x**5/(
1 + sqrt(5)) + 12*x**4/(1 + sqrt(5)) + 18*x**3/(1 + sqrt(5)) - 2*x**2/(1 +
 sqrt(5)) + x*(-3/(1 + sqrt(5)) + sqrt(5)/(1 + sqrt(5)))))/10 + (sqrt(5)/2
0 + 1/4)*log(x**6 + 6*x**5 + 9*x**4 - 2*x**3 + x**2*(-9/2 - sqrt(5)/2) + 1
) + (1/4 - sqrt(5)/20)*log(x**6 + 6*x**5 + 9*x**4 - 2*x**3 + x**2*(-9/2 +
sqrt(5)/2) + 1)
```

Maxima [F]

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$=\int \frac{3\,x^{11}+33\,x^{10}+137\,x^{9}+249\,x^{8}+133\,x^{7}-129\,x^{6}-118\,x^{5}+31\,x^{4}+37\,x^{3}-5\,x^{2}-5\,x+1}{x^{12}+12\,x^{11}+54\,x^{10}+104\,x^{9}+48\,x^{8}-90\,x^{7}-75\,x^{6}+30\,x^{5}+37\,x^{4}-4\,x^{3}-9\,x^{2}+1}\,dx$$

input
```
integrate((3*x^11+33*x^10+137*x^9+249*x^8+133*x^7-129*x^6-118*x^5+31*x^4+3
7*x^3-5*x^2-5*x+1)/(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x
^5+37*x^4-4*x^3-9*x^2+1),x, algorithm="maxima")
```

output
```
integrate((3*x^11 + 33*x^10 + 137*x^9 + 249*x^8 + 133*x^7 - 129*x^6 - 118*
x^5 + 31*x^4 + 37*x^3 - 5*x^2 - 5*x + 1)/(x^12 + 12*x^11 + 54*x^10 + 104*x
^9 + 48*x^8 - 90*x^7 - 75*x^6 + 30*x^5 + 37*x^4 - 4*x^3 - 9*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.75

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= -\frac{1}{5}\sqrt{5}\left(\arctan\left(-\frac{1}{2}x^5\left(\sqrt{5}+1\right)-3x^4\left(\sqrt{5}+1\right)-\frac{9}{2}x^3\left(\sqrt{5}+1\right)+\frac{1}{2}x^2\left(\sqrt{5}+1\right)+x\left(\sqrt{5}+2\right)\right)\right.$$
$$+\frac{1}{5}\sqrt{5}\left(\arctan\left(\frac{1}{2}x^5\left(\sqrt{5}-1\right)+3x^4\left(\sqrt{5}-1\right)+\frac{9}{2}x^3\left(\sqrt{5}-1\right)-\frac{1}{2}x^2\left(\sqrt{5}-1\right)-x\left(\sqrt{5}-2\right)\right)+\right.$$
$$-\frac{1}{20}\sqrt{5}\log\left(4\left(x^3+3x^2-1\right)^2+\left(\sqrt{5}x+x\right)^2\right)$$
$$+\frac{1}{20}\sqrt{5}\log\left(4\left(x^3+3x^2-1\right)^2+\left(\sqrt{5}x-x\right)^2\right)+\frac{1}{4}\log\left(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1\right)$$

input
```
integrate((3*x^11+33*x^10+137*x^9+249*x^8+133*x^7-129*x^6-118*x^5+31*x^4+3
7*x^3-5*x^2-5*x+1)/(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x
^5+37*x^4-4*x^3-9*x^2+1),x, algorithm="giac")
```

output
```
-1/5*sqrt(5)*(arctan(-1/2*x^5*(sqrt(5) + 1) - 3*x^4*(sqrt(5) + 1) - 9/2*x^
3*(sqrt(5) + 1) + 1/2*x^2*(sqrt(5) + 1) + x*(sqrt(5) + 2)) + arctan(-1/2*x
^2*(sqrt(5) + 1) - 3/2*x*(sqrt(5) + 1))) + 1/5*sqrt(5)*(arctan(1/2*x^5*(sq
rt(5) - 1) + 3*x^4*(sqrt(5) - 1) + 9/2*x^3*(sqrt(5) - 1) - 1/2*x^2*(sqrt(5
) - 1) - x*(sqrt(5) - 2)) + arctan(1/2*x^2*(sqrt(5) - 1) + 3/2*x*(sqrt(5)
- 1))) - 1/20*sqrt(5)*log(4*(x^3 + 3*x^2 - 1)^2 + (sqrt(5)*x + x)^2) + 1/2
0*sqrt(5)*log(4*(x^3 + 3*x^2 - 1)^2 + (sqrt(5)*x - x)^2) + 1/4*log(x^12 +
12*x^11 + 54*x^10 + 104*x^9 + 48*x^8 - 90*x^7 - 75*x^6 + 30*x^5 + 37*x^4 -
 4*x^3 - 9*x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.36

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$
$$= -\ln\left(3\,x^2+x^3-1-\frac{x\,1\mathrm{i}}{2}-\frac{\sqrt{5}\,x\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{4}+\sqrt{5}\left(\frac{1}{20}+\frac{1}{10}\mathrm{i}\right)\right)$$
$$+\ln\left(3\,x^2+x^3-1-\frac{x\,1\mathrm{i}}{2}+\frac{\sqrt{5}\,x\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{4}+\sqrt{5}\left(\frac{1}{20}+\frac{1}{10}\mathrm{i}\right)\right)$$
$$+\ln\left(3\,x^2+x^3-1+\frac{x\,1\mathrm{i}}{2}-\frac{\sqrt{5}\,x\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{4}+\sqrt{5}\left(\frac{1}{20}-\frac{1}{10}\mathrm{i}\right)\right)$$
$$-\ln\left(3\,x^2+x^3-1+\frac{x\,1\mathrm{i}}{2}+\frac{\sqrt{5}\,x\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{4}+\sqrt{5}\left(\frac{1}{20}-\frac{1}{10}\mathrm{i}\right)\right)$$

input
```
int((37*x^3 - 5*x^2 - 5*x + 31*x^4 - 118*x^5 - 129*x^6 + 133*x^7 + 249*x^8
 + 137*x^9 + 33*x^10 + 3*x^11 + 1)/(37*x^4 - 4*x^3 - 9*x^2 + 30*x^5 - 75*x
^6 - 90*x^7 + 48*x^8 + 104*x^9 + 54*x^10 + 12*x^11 + x^12 + 1),x)
```

output
```
log((5^(1/2)*x*1i)/2 - (x*1i)/2 + 3*x^2 + x^3 - 1)*(5^(1/2)*(1/20 + 1i/10)
 + 1/4) - log(3*x^2 - (5^(1/2)*x*1i)/2 - (x*1i)/2 + x^3 - 1)*(5^(1/2)*(1/2
0 + 1i/10) - 1/4) + log((x*1i)/2 - (5^(1/2)*x*1i)/2 + 3*x^2 + x^3 - 1)*(5^
(1/2)*(1/20 - 1i/10) + 1/4) - log((x*1i)/2 + (5^(1/2)*x*1i)/2 + 3*x^2 + x^
3 - 1)*(5^(1/2)*(1/20 - 1i/10) - 1/4)
```

Reduce [F]

$$\int \frac{1-5x-5x^2+37x^3+31x^4-118x^5-129x^6+133x^7+249x^8+137x^9+33x^{10}+3x^{11}}{1-9x^2-4x^3+37x^4+30x^5-75x^6-90x^7+48x^8+104x^9+54x^{10}+12x^{11}+x^{12}}\,dx$$

$$=2\left(\int \frac{x^9}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+15\left(\int \frac{x^8}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+37\left(\int \frac{x^7}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$+\frac{57\left(\int \frac{x^6}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)}{2}$$
$$-\frac{11\left(\int \frac{x^5}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)}{2}$$
$$-\frac{13\left(\int \frac{x^4}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)}{2}$$
$$-2\left(\int \frac{x^2}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)$$
$$-\frac{\left(\int \frac{x}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx\right)}{2}$$
$$+\int \frac{1}{x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1}dx$$
$$+\frac{\log(x^{12}+12x^{11}+54x^{10}+104x^9+48x^8-90x^7-75x^6+30x^5+37x^4-4x^3-9x^2+1)}{4}$$

input
```
int((3*x^11+33*x^10+137*x^9+249*x^8+133*x^7-129*x^6-118*x^5+31*x^4+37*x^3-
5*x^2-5*x+1)/(x^12+12*x^11+54*x^10+104*x^9+48*x^8-90*x^7-75*x^6+30*x^5+37*
x^4-4*x^3-9*x^2+1),x)
```

output
```
(8*int(x**9/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 -
75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) + 60*int(x**8/(x**12
 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5
+ 37*x**4 - 4*x**3 - 9*x**2 + 1),x) + 148*int(x**7/(x**12 + 12*x**11 + 54*
x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**
3 - 9*x**2 + 1),x) + 114*int(x**6/(x**12 + 12*x**11 + 54*x**10 + 104*x**9
+ 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x
) - 22*int(x**5/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**
7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) - 26*int(x**4/(x
**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x
**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) - 8*int(x**2/(x**12 + 12*x**11 + 5
4*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x
**3 - 9*x**2 + 1),x) - 2*int(x/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 4
8*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) +
 4*int(1/(x**12 + 12*x**11 + 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*
x**6 + 30*x**5 + 37*x**4 - 4*x**3 - 9*x**2 + 1),x) + log(x**12 + 12*x**11
+ 54*x**10 + 104*x**9 + 48*x**8 - 90*x**7 - 75*x**6 + 30*x**5 + 37*x**4 -
4*x**3 - 9*x**2 + 1))/4
```

3.96 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right) + 2\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

output
```
2*11^(1/2)*arctan(1/55*(7-40*x)*11^(1/2))-2*11^(1/2)*arctan(1/66*(800*x^3-
40*x^2+30*x+57)*11^(1/2))+2*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \frac{1}{2}\text{RootSum}\Big[9+24\#1-12\#1^2+80\#1^3 +320\#1^4\&, \frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\Big]$$

input
```
Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
 320*x^4),x]
```

output
```
RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 14
4*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1
+ 30*#1^2 + 160*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2525, 27, 2502}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2560x^3-400x^2-576x-84}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{\int -\frac{56320(20x^2+12x+3)}{320x^4+80x^3-12x^2+24x+9}dx}{1280}+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

$$\Big\downarrow \text{27}$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\int \frac{20x^2+12x+3}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \text{2502}$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\left(\frac{\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}\right)$$

input
```
Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4),x]
```

output
```
-44*(-1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 4
0*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])) + 2*Log[9 + 24*x - 12*x^2 + 8
0*x^3 + 320*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2502
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result
default	$4\left(\frac{1}{2}+\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+\left(10i\sqrt{11}+\right.\right.$
risch	$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\,\arctan\left(-\frac{20x^2\sqrt{11}}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\frac{40\right.$

input
```
int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_R
ETURNVERBOSE)
```

output
```
4*(1/2+1/4*I*11^(1/2))*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(
1/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx &= -2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)\\ &\quad -2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)+2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)\end{aligned}$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="fricas")
```

output
```
-2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(
11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx &= \sqrt{11}\left(-2\,\mathrm{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)\right.\\ &\quad \left.-2\,\mathrm{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)\\ &\quad +2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)\end{aligned}$$

input
```
integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),
x)
```

output
```
sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x
**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(
x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

Maxima [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \int \frac{4\,(640\,x^3-100\,x^2-144\,x-21)}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="maxima")
```

output
```
4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 +
24*x + 9), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}(40\,x-7)\right)\right) + 2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="giac")
```

output
```
-2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan
(-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\,\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right) - 2\sqrt{11}\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right) - 2\sqrt{11}\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)$$

input
```
int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
 9),x)
```

output
```
2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*
x)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2
))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

Reduce [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -880\left(\int \frac{x^2}{320x^4+80x^3-12x^2+24x+9}dx\right) - 528\left(\int \frac{x}{320x^4+80x^3-12x^2+24x+9}dx\right) - 132\left(\int \frac{1}{320x^4+80x^3-12x^2+24x+9}dx\right) + 2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

input
```
int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

output
```
2*( - 440*int(x**2/(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9),x) - 264*int(
x/(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9),x) - 66*int(1/(320*x**4 + 80*x
**3 - 12*x**2 + 24*x + 9),x) + log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9
))
```

3.97 $\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx$

Optimal result

Integrand size = 63, antiderivative size = 250

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx = \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x\right) - \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}+\sqrt{2+\sqrt{5}}x+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^2+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x^3\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\log\left(\frac{1}{2}\left(2-\sqrt{2\left(-1+\sqrt{5}\right)}\right)+\frac{1}{2}\left(2-\sqrt{2\left(-1+\sqrt{5}\right)}\right)x+x^2\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\log\left(\frac{1}{2}\left(2+\sqrt{2\left(-1+\sqrt{5}\right)}\right)+\frac{1}{2}\left(2+\sqrt{2\left(-1+\sqrt{5}\right)}\right)x+x^2\right)$$

output
```
1/2*(2+2*5^(1/2))^(1/2)*arctan(1/2*(-2+2*5^(1/2))^(1/2)*x)-1/2*(2+2*5^(1/2
))^(1/2)*arctan(1/2*(2+2*5^(1/2))^(1/2)+(2+5^(1/2))^(1/2)*x+1/2*(-2+2*5^(1
/2))^(1/2)*x^2+1/2*(-2+2*5^(1/2))^(1/2)*x^3)-1/4*(-2+2*5^(1/2))^(1/2)*ln(1
-1/2*(-2+2*5^(1/2))^(1/2)+1/2*(2-(-2+2*5^(1/2))^(1/2))*x+x^2)+1/4*(-2+2*5^
(1/2))^(1/2)*ln(1+1/2*(-2+2*5^(1/2))^(1/2)+1/2*(2+(-2+2*5^(1/2))^(1/2))*x+
x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx$$
$$= \frac{1}{2}\text{RootSum}\Big[1+4\#1+12\#1^2+22\#1^3+26\#1^4+20\#1^5+11\#1^6+4\#1^7$$
$$+\#1^8\&, \frac{-2\log(x-\#1)\#1-5\log(x-\#1)\#1^2+5\log(x-\#1)\#1^4+4\log(x-\#1)\#1^5+\log(x-\#}{2+12\#1+33\#1^2+52\#1^3+50\#1^4+33\#1^5+14\#1^6+4\#1^7}$$

input
```
Integrate[(-2*x - 5*x^2 + 5*x^4 + 4*x^5 + x^6)/(1 + 4*x + 12*x^2 + 22*x^3
+ 26*x^4 + 20*x^5 + 11*x^6 + 4*x^7 + x^8),x]
```

output
```
RootSum[1 + 4*#1 + 12*#1^2 + 22*#1^3 + 26*#1^4 + 20*#1^5 + 11*#1^6 + 4*#1^
7 + #1^8 & , (-2*Log[x - #1]*#1 - 5*Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^4
+ 4*Log[x - #1]*#1^5 + Log[x - #1]*#1^6)/(2 + 12*#1 + 33*#1^2 + 52*#1^3 +
50*#1^4 + 33*#1^5 + 14*#1^6 + 4*#1^7) & ]/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+4x^5+5x^4-5x^2-2x}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}\,dx$$
$$\Big\downarrow \text{7292}$$

$$\int \frac{x(x^5+4x^4+5x^3-5x-2)}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{x^6}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}+\frac{4x^5}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+\right.$$

$$\Big\downarrow \; 2009$$

$$-2\int \frac{x}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx-5\int \frac{x^2}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx+5\int \frac{x^4}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx+4\int \frac{x^5}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx+\int \frac{x^6}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx$$

input
```
Int[(-2*x - 5*x^2 + 5*x^4 + 4*x^5 + x^6)/(1 + 4*x + 12*x^2 + 22*x^3 + 26*x
^4 + 20*x^5 + 11*x^6 + 4*x^7 + x^8),x]
```

output
```
$Aborted
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(x^2+\left(1+\_R\right)x+1+\_R\right)\right)}{2}$	29
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)}\_R\ln\left(x^2+\left(1+\_R\right)x+1+\_R\right)\right)}{2}$	29

input
```
int((x^6+4*x^5+5*x^4-5*x^2-2*x)/(x^8+4*x^7+11*x^6+20*x^5+26*x^4+22*x^3+12*
x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R*ln(x^2+(1+_R)*x+1+_R),_R=RootOf(_Z^4+_Z^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx$$
$$= -\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\arctan\left(-\frac{1}{2}\left(x^3+x^2-\sqrt{5}(x^3+x^2+x)-x-2\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\arctan\left(\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\log\left(x^2+(x+1)\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+x+1\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\log\left(x^2-(x+1)\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+x+1\right)$$

input
```
integrate((x^6+4*x^5+5*x^4-5*x^2-2*x)/(x^8+4*x^7+11*x^6+20*x^5+26*x^4+22*x
^3+12*x^2+4*x+1),x, algorithm="fricas")
```

output
```
-sqrt(1/2*sqrt(5) + 1/2)*arctan(-1/2*(x^3 + x^2 - sqrt(5)*(x^3 + x^2 + x)
- x - 2)*sqrt(1/2*sqrt(5) + 1/2)) + sqrt(1/2*sqrt(5) + 1/2)*arctan(1/2*(sq
rt(5)*x - x)*sqrt(1/2*sqrt(5) + 1/2)) + 1/2*sqrt(1/2*sqrt(5) - 1/2)*log(x^
2 + (x + 1)*sqrt(1/2*sqrt(5) - 1/2) + x + 1) - 1/2*sqrt(1/2*sqrt(5) - 1/2)
*log(x^2 - (x + 1)*sqrt(1/2*sqrt(5) - 1/2) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.12

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx = \text{RootSum}\left(16t^4+4t^2-1,\left(t\mapsto t\log\left(2t+x^2+x(2t+1)+1\right)\right)\right)$$

input
```
integrate((x**6+4*x**5+5*x**4-5*x**2-2*x)/(x**8+4*x**7+11*x**6+20*x**5+26*
x**4+22*x**3+12*x**2+4*x+1),x)
```

output
```
RootSum(16*_t**4 + 4*_t**2 - 1, Lambda(_t, _t*log(2*_t + x**2 + x*(2*_t +
1) + 1)))
```

Maxima [F]

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx = \int \frac{x^6+4\,x^5+5\,x^4-5\,x^2-2\,x}{x^8+4\,x^7+11\,x^6+20\,x^5+26\,x^4+22\,x^3+12\,x^2+4\,x+1}\,dx$$

input
```
integrate((x^6+4*x^5+5*x^4-5*x^2-2*x)/(x^8+4*x^7+11*x^6+20*x^5+26*x^4+22*x
^3+12*x^2+4*x+1),x, algorithm="maxima")
```

output
```
integrate((x^6 + 4*x^5 + 5*x^4 - 5*x^2 - 2*x)/(x^8 + 4*x^7 + 11*x^6 + 20*x
^5 + 26*x^4 + 22*x^3 + 12*x^2 + 4*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx$$
$$=\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+1}\left(\arctan\left(-\frac{1}{2}\,x^3\sqrt{2\,\sqrt{5}-2}-\frac{1}{2}\,x^2\sqrt{2\,\sqrt{5}-2}-x\sqrt{\sqrt{5}+2}-\frac{1}{2}\,\sqrt{2\,\sqrt{5}+2}\right)+\arctan\left(\right.\right.$$
$$+\frac{1}{2}\,\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\log\left(232\,x^2+116\,x\sqrt{2\,\sqrt{5}-2}+232\,x+116\,\sqrt{2\,\sqrt{5}-2}+232\right)$$
$$-\frac{1}{2}\,\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-1}\log\left(232\,x^2-116\,x\sqrt{2\,\sqrt{5}-2}+232\,x-116\,\sqrt{2\,\sqrt{5}-2}+232\right)$$

input
```
integrate((x^6+4*x^5+5*x^4-5*x^2-2*x)/(x^8+4*x^7+11*x^6+20*x^5+26*x^4+22*x
^3+12*x^2+4*x+1),x, algorithm="giac")
```

output
```
sqrt(1/2)*sqrt(sqrt(5) + 1)*(arctan(-1/2*x^3*sqrt(2*sqrt(5) - 2) - 1/2*x^2
*sqrt(2*sqrt(5) - 2) - x*sqrt(sqrt(5) + 2) - 1/2*sqrt(2*sqrt(5) + 2)) + ar
ctan(1/2*x*sqrt(2*sqrt(5) - 2))) + 1/2*sqrt(1/2)*sqrt(sqrt(5) - 1)*log(232
*x^2 + 116*x*sqrt(2*sqrt(5) - 2) + 232*x + 116*sqrt(2*sqrt(5) - 2) + 232)
- 1/2*sqrt(1/2)*sqrt(sqrt(5) - 1)*log(232*x^2 - 116*x*sqrt(2*sqrt(5) - 2)
+ 232*x - 116*sqrt(2*sqrt(5) - 2) + 232)
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.37

$$\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx$$
$$=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{750\,\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{4500\,x-3750\,\sqrt{5}\,x-3750\,\sqrt{5}-6500\,\sqrt{5}\,x^2+12250\,x^2+4500}+\frac{750\,\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}}{4500\,x-3750\,\sqrt{5}\,x-3750\,\sqrt{5}-6500\,\sqrt{5}\,x^2+12250\,x^2+4500}-\right.}{}$$
$$+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{750\,\sqrt{2}\,\sqrt{\sqrt{5}-1}}{4500\,x+3750\,\sqrt{5}\,x+3750\,\sqrt{5}+6500\,\sqrt{5}\,x^2+12250\,x^2+4500}-\frac{5375\,\sqrt{2}\,x^2\,\sqrt{\sqrt{5}-1}}{4500\,x+3750\,\sqrt{5}\,x+3750\,\sqrt{5}+6500\,\sqrt{5}\,x^2+12250\,x^2+4500}+\right.}{}$$

input
```
int((5*x^4 - 5*x^2 - 2*x + 4*x^5 + x^6)/(4*x + 12*x^2 + 22*x^3 + 26*x^4 +
20*x^5 + 11*x^6 + 4*x^7 + x^8 + 1),x)
```

output

```
(2^(1/2)*atanh((750*2^(1/2)*(- 5^(1/2) - 1)^(1/2))/(4500*x - 3750*5^(1/2)*
x - 3750*5^(1/2) - 6500*5^(1/2)*x^2 + 12250*x^2 + 4500) + (750*2^(1/2)*x*(
- 5^(1/2) - 1)^(1/2))/(4500*x - 3750*5^(1/2)*x - 3750*5^(1/2) - 6500*5^(1/
2)*x^2 + 12250*x^2 + 4500) - (5375*2^(1/2)*x^2*(- 5^(1/2) - 1)^(1/2))/(450
0*x - 3750*5^(1/2)*x - 3750*5^(1/2) - 6500*5^(1/2)*x^2 + 12250*x^2 + 4500)
 - (625*2^(1/2)*5^(1/2)*(- 5^(1/2) - 1)^(1/2))/(4500*x - 3750*5^(1/2)*x -
3750*5^(1/2) - 6500*5^(1/2)*x^2 + 12250*x^2 + 4500) - (625*2^(1/2)*5^(1/2)
*x*(- 5^(1/2) - 1)^(1/2))/(4500*x - 3750*5^(1/2)*x - 3750*5^(1/2) - 6500*5
^(1/2)*x^2 + 12250*x^2 + 4500) + (2625*2^(1/2)*5^(1/2)*x^2*(- 5^(1/2) - 1)
^(1/2))/(4500*x - 3750*5^(1/2)*x - 3750*5^(1/2) - 6500*5^(1/2)*x^2 + 12250
*x^2 + 4500))*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*atanh((750*2^(1/2)*(5^(1
/2) - 1)^(1/2))/(4500*x + 3750*5^(1/2)*x + 3750*5^(1/2) + 6500*5^(1/2)*x^2
 + 12250*x^2 + 4500) - (5375*2^(1/2)*x^2*(5^(1/2) - 1)^(1/2))/(4500*x + 37
50*5^(1/2)*x + 3750*5^(1/2) + 6500*5^(1/2)*x^2 + 12250*x^2 + 4500) + (625*
2^(1/2)*5^(1/2)*(5^(1/2) - 1)^(1/2))/(4500*x + 3750*5^(1/2)*x + 3750*5^(1/
2) + 6500*5^(1/2)*x^2 + 12250*x^2 + 4500) + (750*2^(1/2)*x*(5^(1/2) - 1)^(
1/2))/(4500*x + 3750*5^(1/2)*x + 3750*5^(1/2) + 6500*5^(1/2)*x^2 + 12250*x
^2 + 4500) + (625*2^(1/2)*5^(1/2)*x*(5^(1/2) - 1)^(1/2))/(4500*x + 3750*5^
(1/2)*x + 3750*5^(1/2) + 6500*5^(1/2)*x^2 + 12250*x^2 + 4500) - (2625*2^(1
/2)*5^(1/2)*x^2*(5^(1/2) - 1)^(1/2))/(4500*x + 3750*5^(1/2)*x + 3750*5^...
```

Reduce [F]

$$\begin{aligned}
&\int \frac{-2x-5x^2+5x^4+4x^5+x^6}{1+4x+12x^2+22x^3+26x^4+20x^5+11x^6+4x^7+x^8}\,dx \\
&= \int \frac{x^6}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx \\
&\quad + 4\left(\int \frac{x^5}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx\right) \\
&\quad + 5\left(\int \frac{x^4}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx\right) \\
&\quad - 5\left(\int \frac{x^2}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx\right) \\
&\quad - 2\left(\int \frac{x}{x^8+4x^7+11x^6+20x^5+26x^4+22x^3+12x^2+4x+1}dx\right)
\end{aligned}$$

input
```
int((x^6+4*x^5+5*x^4-5*x^2-2*x)/(x^8+4*x^7+11*x^6+20*x^5+26*x^4+22*x^3+12*
x^2+4*x+1),x)
```

output
```
int(x**6/(x**8 + 4*x**7 + 11*x**6 + 20*x**5 + 26*x**4 + 22*x**3 + 12*x**2
+ 4*x + 1),x) + 4*int(x**5/(x**8 + 4*x**7 + 11*x**6 + 20*x**5 + 26*x**4 +
22*x**3 + 12*x**2 + 4*x + 1),x) + 5*int(x**4/(x**8 + 4*x**7 + 11*x**6 + 20
*x**5 + 26*x**4 + 22*x**3 + 12*x**2 + 4*x + 1),x) - 5*int(x**2/(x**8 + 4*x
**7 + 11*x**6 + 20*x**5 + 26*x**4 + 22*x**3 + 12*x**2 + 4*x + 1),x) - 2*in
t(x/(x**8 + 4*x**7 + 11*x**6 + 20*x**5 + 26*x**4 + 22*x**3 + 12*x**2 + 4*x
 + 1),x)
```

3.98 $\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 111

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx = -2^{3/4}\sqrt{-1+\sqrt{1+2\sqrt{3}}}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(-1+\sqrt{1+2\sqrt{3}}\right)}}\right) - 2^{3/4}\sqrt{1+\sqrt{1+2\sqrt{3}}}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(1+\sqrt{1+2\sqrt{3}}\right)}}\right)$$

output
```
-2^(3/4)*(-1+(1+2*3^(1/2))^(1/2))^(1/2)*arctan(2^(1/4)*x/(-3+3*(1+2*3^(1/2
))^(1/2))^(1/2))-2^(3/4)*(1+(1+2*3^(1/2))^(1/2))^(1/2)*arctanh(2^(1/4)*x/(
3+3*(1+2*3^(1/2))^(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.56

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx = -\frac{2^{3/4}\left(\frac{\left(6+\sqrt{3}-\sqrt{3+6\sqrt{3}}\right)\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(-1+\sqrt{1+2\sqrt{3}}\right)}}\right)}{\sqrt{\left(1+2\sqrt{3}\right)\left(-1+\sqrt{1+2\sqrt{3}}\right)}}+\frac{\left(6+\sqrt{3}+\sqrt{3+6\sqrt{3}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(1+\sqrt{1+2\sqrt{3}}\right)}}\right)}{\sqrt{\left(1+2\sqrt{3}\right)\left(1+\sqrt{1+2\sqrt{3}}\right)}}\right)}{\sqrt{3}}$$

input
```
Integrate[(-36 - 2*Sqrt[6]*x^2)/(9*Sqrt[3] + 3*Sqrt[2]*x^2 - x^4),x]
```

output
```
-((2^(3/4)*(((6 + Sqrt[3] - Sqrt[3 + 6*Sqrt[3]])*ArcTan[(2^(1/4)*x)/Sqrt[3
*(-1 + Sqrt[1 + 2*Sqrt[3]])]])/Sqrt[(1 + 2*Sqrt[3])*(-1 + Sqrt[1 + 2*Sqrt[
3]])] + ((6 + Sqrt[3] + Sqrt[3 + 6*Sqrt[3]])*ArcTanh[(2^(1/4)*x)/Sqrt[3*(1
 + Sqrt[1 + 2*Sqrt[3]])]])/Sqrt[(1 + 2*Sqrt[3])*(1 + Sqrt[1 + 2*Sqrt[3]])]
))/Sqrt[3])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1480, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2\sqrt{6}x^2-36}{-x^4+3\sqrt{2}x^2+9\sqrt{3}}\,dx$$

$$\Big\downarrow \; 1480$$

$$-\sqrt{6}\left(1-\sqrt{1+2\sqrt{3}}\right)\int \frac{1}{\frac{3\left(1-\sqrt{1+2\sqrt{3}}\right)}{\sqrt{2}}-x^2}dx-$$
$$\sqrt{6}\left(1+\sqrt{1+2\sqrt{3}}\right)\int \frac{1}{\frac{3\left(1+\sqrt{1+2\sqrt{3}}\right)}{\sqrt{2}}-x^2}dx$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2^{3/4}\left(1-\sqrt{1+2\sqrt{3}}\right)\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(\sqrt{1+2\sqrt{3}}-1\right)}}\right)}{\sqrt{\sqrt{1+2\sqrt{3}}-1}}-$$
$$\sqrt{6}\left(1+\sqrt{1+2\sqrt{3}}\right)\int \frac{1}{\frac{3\left(1+\sqrt{1+2\sqrt{3}}\right)}{\sqrt{2}}-x^2}dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2^{3/4}\left(1-\sqrt{1+2\sqrt{3}}\right)\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(\sqrt{1+2\sqrt{3}}-1\right)}}\right)}{\sqrt{\sqrt{1+2\sqrt{3}}-1}}-$$
$$2^{3/4}\sqrt{1+\sqrt{1+2\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt[4]{2}x}{\sqrt{3\left(1+\sqrt{1+2\sqrt{3}}\right)}}\right)$$

input
```
Int[(-36 - 2*Sqrt[6]*x^2)/(9*Sqrt[3] + 3*Sqrt[2]*x^2 - x^4),x]
```

output
```
(2^(3/4)*(1 - Sqrt[1 + 2*Sqrt[3]])*ArcTan[(2^(1/4)*x)/Sqrt[3*(-1 + Sqrt[1
+ 2*Sqrt[3]])]])/Sqrt[-1 + Sqrt[1 + 2*Sqrt[3]]] - 2^(3/4)*Sqrt[1 + Sqrt[1
+ 2*Sqrt[3]]]*ArcTanh[(2^(1/4)*x)/Sqrt[3*(1 + Sqrt[1 + 2*Sqrt[3]])]]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

method	result
risch	$\sqrt{3}\,\sqrt{2}\left(\sum_{\_R=\mathrm{RootOf}\left(-9\,\mathrm{RootOf}\left(\_Z^2-3,\mathrm{index}=1\right)-3\,\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)\_Z^2+\_Z^4\right)}\frac{\left(-\_R^2-3\sqrt{3}\,\sqrt{2}\right)\ln\left(x-\_R\right)}{3\sqrt{2}\,\_R-2\,\_R^3}\right.$
default	$-\frac{2\left(\sqrt{2}\,\sqrt{6}-\sqrt{6}\,\sqrt{2+4\sqrt{3}}+12\right)\arctan\left(\frac{2x}{\sqrt{-6\sqrt{2}+6\sqrt{2+4\sqrt{3}}}}\right)}{\sqrt{2+4\sqrt{3}}\,\sqrt{-6\sqrt{2}+6\sqrt{2+4\sqrt{3}}}}+\frac{2\left(-\sqrt{2}\,\sqrt{6}-\sqrt{6}\,\sqrt{2+4\sqrt{3}}-12\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{6\sqrt{2}+6\sqrt{2+4\sqrt{3}}}}\right)}{\sqrt{2+4\sqrt{3}}\,\sqrt{6\sqrt{2}+6\sqrt{2+4\sqrt{3}}}}$

input
```
int((-36-2*6^(1/2)*x^2)/(9*3^(1/2)+3*2^(1/2)*x^2-x^4),x,method=_RETURNVERB
OSE)
```

output
```
3^(1/2)*2^(1/2)*sum((-_R^2-3*3^(1/2)*2^(1/2))/(3*2^(1/2)*_R-2*_R^3)*ln(x-_
R),_R=RootOf(-9*RootOf(_Z^2-3,index=1)-3*RootOf(_Z^2-2,index=1)*_Z^2+_Z^4)
)
```

Fricas [F]

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx=\int \frac{2\left(\sqrt{6}x^2+18\right)}{x^4-3\sqrt{2}x^2-9\sqrt{3}}\,dx$$

input
```
integrate((-36-2*6^(1/2)*x^2)/(9*3^(1/2)+3*2^(1/2)*x^2-x^4),x, algorithm="
fricas")
```

output
```
0
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx=\text{Exception raised: PolynomialError}$$

input
```
integrate((-36-2*6**(1/2)*x**2)/(9*3**(1/2)+3*2**(1/2)*x**2-x**4),x)
```

output
```
Exception raised: PolynomialError >> 1/(2*_t**4 - 2*sqrt(2)*_t**2 + 1) con
tains an element of the set of generators.
```

Maxima [F]

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx=\int \frac{2\left(\sqrt{6}x^2+18\right)}{x^4-3\sqrt{2}x^2-9\sqrt{3}}\,dx$$

input
```
integrate((-36-2*6^(1/2)*x^2)/(9*3^(1/2)+3*2^(1/2)*x^2-x^4),x, algorithm="
maxima")
```

output
```
2*integrate((sqrt(6)*x^2 + 18)/(x^4 - 3*sqrt(2)*x^2 - 9*sqrt(3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-36-2*6^(1/2)*x^2)/(9*3^(1/2)+3*2^(1/2)*x^2-x^4),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:   ***   Warning: increasing stack s
ize to 4096000.  ***   Warning: increasing stack size to 8192000.  ***   W
arning: i
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 2068, normalized size of antiderivative = 18.63

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx = \text{Too large to display}$$

input
```
int(-(2*6^(1/2)*x^2 + 36)/(9*3^(1/2) + 3*2^(1/2)*x^2 - x^4),x)
```

output
```
- atan(((x*(864*2^(1/2)*6^(1/2) + 864*3^(1/2) + 6048) + ((48*3^(1/2)*6^(1/
2) - 132*2^(1/2)*3^(1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/
2) + 36*((4*3^(1/2) + 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2)*(5184*3^(1
/2) - x*(864*2^(1/2)*3^(1/2) + 432*2^(1/2))*((48*3^(1/2)*6^(1/2) - 132*2^(
1/2)*3^(1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/2) + 36*((4*
3^(1/2) + 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2) + 2592))*((48*3^(1/2)*
6^(1/2) - 132*2^(1/2)*3^(1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3
)^(1/2) + 36*((4*3^(1/2) + 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2)*1i +
(x*(864*2^(1/2)*6^(1/2) + 864*3^(1/2) + 6048) - ((48*3^(1/2)*6^(1/2) - 132
*2^(1/2)*3^(1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/2) + 36*
((4*3^(1/2) + 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2)*(5184*3^(1/2) + x*
(864*2^(1/2)*3^(1/2) + 432*2^(1/2))*((48*3^(1/2)*6^(1/2) - 132*2^(1/2)*3^(
1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/2) + 36*((4*3^(1/2)
+ 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2) + 2592))*((48*3^(1/2)*6^(1/2)
- 132*2^(1/2)*3^(1/2) + 288*6^(1/2) + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/2)
+ 36*((4*3^(1/2) + 2)^3)^(1/2))/(6*(52*3^(1/2) + 48)))^(1/2)*1i)/(5184*2^(
1/2) - 864*3^(1/2)*6^(1/2) + 5184*6^(1/2) - (x*(864*2^(1/2)*6^(1/2) + 864*
3^(1/2) + 6048) + ((48*3^(1/2)*6^(1/2) - 132*2^(1/2)*3^(1/2) + 288*6^(1/2)
 + 6*3^(1/2)*((4*3^(1/2) + 2)^3)^(1/2) + 36*((4*3^(1/2) + 2)^3)^(1/2))/(6*
(52*3^(1/2) + 48)))^(1/2)*(5184*3^(1/2) - x*(864*2^(1/2)*3^(1/2) + 432*...
```

Reduce [F]

$$\int \frac{-36-2\sqrt{6}x^2}{9\sqrt{3}+3\sqrt{2}x^2-x^4}\,dx = 2\sqrt{6}\left(\int \frac{x^{14}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) - 36\sqrt{6}\left(\int \frac{x^{10}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 1458\sqrt{6}\left(\int \frac{x^{6}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 12\sqrt{3}\left(\int \frac{x^{12}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 108\sqrt{3}\left(\int \frac{x^{8}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 8748\sqrt{3}\left(\int \frac{x^{4}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) - 78732\sqrt{3}\left(\int \frac{1}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 162\sqrt{2}\left(\int \frac{x^{10}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) - 972\sqrt{2}\left(\int \frac{x^{6}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 13122\sqrt{2}\left(\int \frac{x^{2}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) + 36\left(\int \frac{x^{12}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right) - 8748\left(\int \frac{x^{4}}{x^{16}-36x^{12}-162x^8-8748x^4+59049}dx\right)$$

input

```
int((-36-2*6^(1/2)*x^2)/(9*3^(1/2)+3*2^(1/2)*x^2-x^4),x)
```

output
```
2*(sqrt(6)*int(x**14/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 59049),x)
- 18*sqrt(6)*int(x**10/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 59049),x
) + 729*sqrt(6)*int(x**6/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 59049)
,x) + 6*sqrt(3)*int(x**12/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 59049
),x) + 54*sqrt(3)*int(x**8/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 5904
9),x) + 4374*sqrt(3)*int(x**4/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 5
9049),x) - 39366*sqrt(3)*int(1/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 +
59049),x) + 81*sqrt(2)*int(x**10/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4
+ 59049),x) - 486*sqrt(2)*int(x**6/(x**16 - 36*x**12 - 162*x**8 - 8748*x**
4 + 59049),x) + 6561*sqrt(2)*int(x**2/(x**16 - 36*x**12 - 162*x**8 - 8748*
x**4 + 59049),x) + 18*int(x**12/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 +
 59049),x) - 4374*int(x**4/(x**16 - 36*x**12 - 162*x**8 - 8748*x**4 + 5904
9),x))
```

3.99 $\int \frac{264\sqrt{6}x-528\sqrt{3}x^3+24\sqrt{2}x^5-96x^7+60\sqrt{2}x^9-24x^{11}}{99+6\sqrt{3}x^4-6\sqrt{6}x^6+(-4+3\sqrt{3})x^8+8\sqrt{2}x^{10}-12x^{12}+4\sqrt{2}x^{14}-x^{16}}\,dx$

Optimal result

Integrand size = 117, antiderivative size = 139

$$\int \frac{264\sqrt{6}x-528\sqrt{3}x^3+24\sqrt{2}x^5-96x^7+60\sqrt{2}x^9-24x^{11}}{99+6\sqrt{3}x^4-6\sqrt{6}x^6+\left(-4+3\sqrt{3}\right)x^8+8\sqrt{2}x^{10}-12x^{12}+4\sqrt{2}x^{14}-x^{16}}\,dx = 2^{3/4}\sqrt[4]{25-\sqrt{141}}\arctan\left(\frac{\sqrt[4]{2\left(25+\sqrt{141}\right)}x^2}{\sqrt{33}}-\frac{\sqrt[4]{\frac{1}{2}\left(25+\sqrt{141}\right)}x^4}{\sqrt{33}}\right) + 2^{3/4}\sqrt[4]{25+\sqrt{141}}\operatorname{arctanh}\left(\frac{\sqrt[4]{2\left(25-\sqrt{141}\right)}x^2}{\sqrt{33}}-\frac{\sqrt[4]{\frac{1}{2}\left(25-\sqrt{141}\right)}x^4}{\sqrt{33}}\right)$$

output
```
-2^(3/4)*(25-141^(1/2))^(1/4)*arctan(-1/33*(50+2*141^(1/2))^(1/4)*x^2*33^(
1/2)+1/33*(25/2+1/2*141^(1/2))^(1/4)*x^4*33^(1/2))-2^(3/4)*(25+141^(1/2))^
(1/4)*arctanh(-1/33*(50-2*141^(1/2))^(1/4)*x^2*33^(1/2)+1/33*(25/2-1/2*141
^(1/2))^(1/4)*x^4*33^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.81

$$\int \frac{264\sqrt{6}x - 528\sqrt{3}x^3 + 24\sqrt{2}x^5 - 96x^7 + 60\sqrt{2}x^9 - 24x^{11}}{99 + 6\sqrt{3}x^4 - 6\sqrt{6}x^6 + \left(-4 + 3\sqrt{3}\right)x^8 + 8\sqrt{2}x^{10} - 12x^{12} + 4\sqrt{2}x^{14} - x^{16}}\,dx$$

$$= 3\text{RootSum}\Big[99 + 6\sqrt{3}\#1^4 - 6\sqrt{6}\#1^6 - 4\#1^8 + 3\sqrt{3}\#1^8 + 8\sqrt{2}\#1^{10} - 12\#1^{12} + 4\sqrt{2}\#1^{14} - \#1^{16}\&, \frac{-22\sqrt{6}\log(x - \#1) + 44\sqrt{3}\log(x - \#1)\#1^2 - 2\sqrt{2}\log(x - \#1)\#1^4 + 8\log(x - \#1)\#1^6 -}{-6\sqrt{3}\#1^2 + 9\sqrt{6}\#1^4 + 8\#1^6 - 6\sqrt{3}\#1^6 - 20\sqrt{2}\#1^8 + 36\#1^{10} - 1}$$

input
```
Integrate[(264*Sqrt[6]*x - 528*Sqrt[3]*x^3 + 24*Sqrt[2]*x^5 - 96*x^7 + 60*
Sqrt[2]*x^9 - 24*x^11)/(99 + 6*Sqrt[3]*x^4 - 6*Sqrt[6]*x^6 + (-4 + 3*Sqrt[
3])*x^8 + 8*Sqrt[2]*x^10 - 12*x^12 + 4*Sqrt[2]*x^14 - x^16),x]
```

output
```
3*RootSum[99 + 6*Sqrt[3]*#1^4 - 6*Sqrt[6]*#1^6 - 4*#1^8 + 3*Sqrt[3]*#1^8 +
 8*Sqrt[2]*#1^10 - 12*#1^12 + 4*Sqrt[2]*#1^14 - #1^16 & , (-22*Sqrt[6]*Log
[x - #1] + 44*Sqrt[3]*Log[x - #1]*#1^2 - 2*Sqrt[2]*Log[x - #1]*#1^4 + 8*Lo
g[x - #1]*#1^6 - 5*Sqrt[2]*Log[x - #1]*#1^8 + 2*Log[x - #1]*#1^10)/(-6*Sqr
t[3]*#1^2 + 9*Sqrt[6]*#1^4 + 8*#1^6 - 6*Sqrt[3]*#1^6 - 20*Sqrt[2]*#1^8 + 3
6*#1^10 - 14*Sqrt[2]*#1^12 + 4*#1^14) & ]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-24x^{11} + 60\sqrt{2}x^9 - 96x^7 + 24\sqrt{2}x^5 - 528\sqrt{3}x^3 + 264\sqrt{6}x}{-x^{16} + 4\sqrt{2}x^{14} - 12x^{12} + 8\sqrt{2}x^{10} + \left(3\sqrt{3} - 4\right)x^8 - 6\sqrt{6}x^6 + 6\sqrt{3}x^4 + 99}\,dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{12x\left(-2x^{10} + 5\sqrt{2}x^8 - 8x^6 + 2\sqrt{2}x^4 - 44\sqrt{3}x^2 + 22\sqrt{6}\right)}{-x^{16} + 4\sqrt{2}x^{14} - 12x^{12} + 8\sqrt{2}x^{10} + \left(3\sqrt{3} - 4\right)x^8 - 6\sqrt{6}x^6 + 6\sqrt{3}x^4 + 99}\,dx$$

$$\Big\downarrow \; 27$$

$$12\int \frac{x\left(-2x^{10}+5\sqrt{2}x^8-8x^6+2\sqrt{2}x^4-44\sqrt{3}x^2+22\sqrt{6}\right)}{-x^{16}+4\sqrt{2}x^{14}-12x^{12}+8\sqrt{2}x^{10}-\left(4-3\sqrt{3}\right)x^8-6\sqrt{6}x^6+6\sqrt{3}x^4+99}dx$$

$$\Big\downarrow \; 7266$$

$$6\int \frac{-2x^{10}+5\sqrt{2}x^8-8x^6+2\sqrt{2}x^4-44\sqrt{3}x^2+22\sqrt{6}}{-x^{16}+4\sqrt{2}x^{14}-12x^{12}+8\sqrt{2}x^{10}-\left(4-3\sqrt{3}\right)x^8-6\sqrt{6}x^6+6\sqrt{3}x^4+99}dx^2$$

$$\Big\downarrow \; 7293$$

$$6\int \left(\frac{2x^{10}}{x^{16}-4\sqrt{2}x^{14}+12x^{12}-8\sqrt{2}x^{10}+4\left(1-\frac{3\sqrt{3}}{4}\right)x^8+6\sqrt{6}x^6-6\sqrt{3}x^4-99}+\frac{}{-x^{16}+4\sqrt{2}x^{14}-12x^{12}+}\right.$$

$$\Big\downarrow \; 2009$$

$$6\left(22\sqrt{6}\int \frac{1}{-x^{16}+4\sqrt{2}x^{14}-12x^{12}+8\sqrt{2}x^{10}-4\left(1-\frac{3\sqrt{3}}{4}\right)x^8-6\sqrt{6}x^6+6\sqrt{3}x^4+99}dx^2+2\sqrt{2}\int \frac{}{-x^{16}+4}\right.$$

input
```
Int[(264*Sqrt[6]*x - 528*Sqrt[3]*x^3 + 24*Sqrt[2]*x^5 - 96*x^7 + 60*Sqrt[2
]*x^9 - 24*x^11)/(99 + 6*Sqrt[3]*x^4 - 6*Sqrt[6]*x^6 + (-4 + 3*Sqrt[3])*x^
8 + 8*Sqrt[2]*x^10 - 12*x^12 + 4*Sqrt[2]*x^14 - x^16),x]
```

output
```
$Aborted
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*x^9-24
*x^11)/(99+6*3^(1/2)*x^4-6*6^(1/2)*x^6+(-4+3*3^(1/2))*x^8+8*2^(1/2)*x^10-1
2*x^12+4*2^(1/2)*x^14-x^16),x)
```

output
```
int((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*x^9-24
*x^11)/(99+6*3^(1/2)*x^4-6*6^(1/2)*x^6+(-4+3*3^(1/2))*x^8+8*2^(1/2)*x^10-1
2*x^12+4*2^(1/2)*x^14-x^16),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{264\sqrt{6}x - 528\sqrt{3}x^3 + 24\sqrt{2}x^5 - 96x^7 + 60\sqrt{2}x^9 - 24x^{11}}{99 + 6\sqrt{3}x^4 - 6\sqrt{6}x^6 + (-4 + 3\sqrt{3})\,x^8 + 8\sqrt{2}x^{10} - 12x^{12} + 4\sqrt{2}x^{14} - x^{16}}\,dx = \text{Timed out}$$

input
```
integrate((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*
x^9-24*x^11)/(99+6*3^(1/2)*x^4-6*x^6*6^(1/2)+(-4+3*3^(1/2))*x^8+8*2^(1/2)*
x^10-12*x^12+4*2^(1/2)*x^14-x^16),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{264\sqrt{6}x - 528\sqrt{3}x^3 + 24\sqrt{2}x^5 - 96x^7 + 60\sqrt{2}x^9 - 24x^{11}}{99 + 6\sqrt{3}x^4 - 6\sqrt{6}x^6 + (-4 + 3\sqrt{3})\,x^8 + 8\sqrt{2}x^{10} - 12x^{12} + 4\sqrt{2}x^{14} - x^{16}}\,dx = \text{Timed out}$$

input
```
integrate((264*6**(1/2)*x-528*3**(1/2)*x**3+24*2**(1/2)*x**5-96*x**7+60*2*
*(1/2)*x**9-24*x**11)/(99+6*3**(1/2)*x**4-6*x**6*6**(1/2)+(-4+3*3**(1/2))*
x**8+8*2**(1/2)*x**10-12*x**12+4*2**(1/2)*x**14-x**16),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{264\sqrt{6}x-528\sqrt{3}x^3+24\sqrt{2}x^5-96x^7+60\sqrt{2}x^9-24x^{11}}{99+6\sqrt{3}x^4-6\sqrt{6}x^6+\left(-4+3\sqrt{3}\right)x^8+8\sqrt{2}x^{10}-12x^{12}+4\sqrt{2}x^{14}-x^{16}}\,dx$$
$$=\int \frac{12\left(2x^{11}-5\sqrt{2}x^9+8x^7-2\sqrt{2}x^5+44\sqrt{3}x^3-22\sqrt{6}x\right)}{x^{16}-4\sqrt{2}x^{14}+12x^{12}-8\sqrt{2}x^{10}-x^8\left(3\sqrt{3}-4\right)+6\sqrt{6}x^6-6\sqrt{3}x^4-99}\,dx$$

input
```
integrate((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*
x^9-24*x^11)/(99+6*3^(1/2)*x^4-6*x^6*6^(1/2)+(-4+3*3^(1/2))*x^8+8*2^(1/2)*
x^10-12*x^12+4*2^(1/2)*x^14-x^16),x, algorithm="maxima")
```

output
```
12*integrate((2*x^11 - 5*sqrt(2)*x^9 + 8*x^7 - 2*sqrt(2)*x^5 + 44*sqrt(3)*
x^3 - 22*sqrt(6)*x)/(x^16 - 4*sqrt(2)*x^14 + 12*x^12 - 8*sqrt(2)*x^10 - x^
8*(3*sqrt(3) - 4) + 6*sqrt(6)*x^6 - 6*sqrt(3)*x^4 - 99), x)
```

Giac [F]

$$\int \frac{264\sqrt{6}x-528\sqrt{3}x^3+24\sqrt{2}x^5-96x^7+60\sqrt{2}x^9-24x^{11}}{99+6\sqrt{3}x^4-6\sqrt{6}x^6+\left(-4+3\sqrt{3}\right)x^8+8\sqrt{2}x^{10}-12x^{12}+4\sqrt{2}x^{14}-x^{16}}\,dx$$
$$=\int \frac{12\left(2x^{11}-5\sqrt{2}x^9+8x^7-2\sqrt{2}x^5+44\sqrt{3}x^3-22\sqrt{6}x\right)}{x^{16}-4\sqrt{2}x^{14}+12x^{12}-8\sqrt{2}x^{10}-x^8\left(3\sqrt{3}-4\right)+6\sqrt{6}x^6-6\sqrt{3}x^4-99}\,dx$$

input
```
integrate((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*
x^9-24*x^11)/(99+6*3^(1/2)*x^4-6*x^6*6^(1/2)+(-4+3*3^(1/2))*x^8+8*2^(1/2)*
x^10-12*x^12+4*2^(1/2)*x^14-x^16),x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{264\sqrt{6}x - 528\sqrt{3}x^3 + 24\sqrt{2}x^5 - 96x^7 + 60\sqrt{2}x^9 - 24x^{11}}{99 + 6\sqrt{3}x^4 - 6\sqrt{6}x^6 + \left(-4 + 3\sqrt{3}\right)x^8 + 8\sqrt{2}x^{10} - 12x^{12} + 4\sqrt{2}x^{14} - x^{16}}\,dx$$
$$= \text{Hanged}$$

input
```
int((264*6^(1/2)*x - 528*3^(1/2)*x^3 + 24*2^(1/2)*x^5 + 60*2^(1/2)*x^9 - 9
6*x^7 - 24*x^11)/(x^8*(3*3^(1/2) - 4) + 6*3^(1/2)*x^4 + 8*2^(1/2)*x^10 - 6
*6^(1/2)*x^6 + 4*2^(1/2)*x^14 - 12*x^12 - x^16 + 99),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{264\sqrt{6}x - 528\sqrt{3}x^3 + 24\sqrt{2}x^5 - 96x^7 + 60\sqrt{2}x^9 - 24x^{11}}{99 + 6\sqrt{3}x^4 - 6\sqrt{6}x^6 + \left(-4 + 3\sqrt{3}\right)x^8 + 8\sqrt{2}x^{10} - 12x^{12} + 4\sqrt{2}x^{14} - x^{16}}\,dx$$
$$= \int \frac{264\sqrt{6}\,x - 528\sqrt{3}\,x^3 + 24\sqrt{2}\,x^5 - 96x^7 + 60\sqrt{2}\,x^9 - 24x^{11}}{99 + 6\sqrt{3}\,x^4 - 6x^6\sqrt{6} + \left(-4 + 3\sqrt{3}\right)x^8 + 8\sqrt{2}\,x^{10} - 12x^{12} + 4\sqrt{2}\,x^{14} - x^{16}}dx$$

input
```
int((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*x^9-24
*x^11)/(99+6*3^(1/2)*x^4-6*x^6*6^(1/2)+(-4+3*3^(1/2))*x^8+8*2^(1/2)*x^10-1
2*x^12+4*2^(1/2)*x^14-x^16),x)
```

output
```
int((264*6^(1/2)*x-528*3^(1/2)*x^3+24*2^(1/2)*x^5-96*x^7+60*2^(1/2)*x^9-24
*x^11)/(99+6*3^(1/2)*x^4-6*x^6*6^(1/2)+(-4+3*3^(1/2))*x^8+8*2^(1/2)*x^10-1
2*x^12+4*2^(1/2)*x^14-x^16),x)
```

3.100 $\int \frac{1}{b+ax^4} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 195

$$\int \frac{1}{b+ax^4} \, dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}-\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\arctan\left(\frac{\sqrt[4]{b}+\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

output
```
-1/4*arctan((b^(1/4)-2^(1/2)*a^(1/4)*x)/b^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)+1
/4*arctan((b^(1/4)+2^(1/2)*a^(1/4)*x)/b^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)-1/8
*ln(b^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)*x+a^(1/2)*x^2)*2^(1/2)/a^(1/4)/b^(3/4)
+1/8*ln(b^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*x+a^(1/2)*x^2)*2^(1/2)/a^(1/4)/b^(
3/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.69

$$\int \frac{1}{b+ax^4}\,dx$$
$$= \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)-\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2\right)+\log\left(\sqrt{b}+\sqrt{2}\right.}{4\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

input
```
Integrate[(b + a*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a^(1/4
)*x)/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2] + L
og[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(1/4)*
b^(3/4))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^4+b}\,dx$$
$$\Big\downarrow \text{755}$$
$$\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\int \frac{\sqrt{a}x^2+\sqrt{b}}{ax^4+b}dx}{2\sqrt{b}}$$
$$\Big\downarrow \text{1476}$$
$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 217$$

$$\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 1479$$

$$\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}}{x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}$$

input
```
Int[(b + a*x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + Ar
cTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b
]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2]/(Sqrt[2]
*a^(1/4)*b^(1/4)) + Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.14

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^4+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4a}$	27
default	$\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}{x^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}-1\right)\right)}{8b}$	102

input
```
int(1/(a*x^4+b),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int \frac{1}{b+ax^4}\,dx = \frac{1}{4}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(a*x^4+b),x, algorithm="fricas")
```

output
```
1/4*(-1/(a*b^3))^(1/4)*log(b*(-1/(a*b^3))^(1/4) + x) + 1/4*I*(-1/(a*b^3))^
(1/4)*log(I*b*(-1/(a*b^3))^(1/4) + x) - 1/4*I*(-1/(a*b^3))^(1/4)*log(-I*b*
(-1/(a*b^3))^(1/4) + x) - 1/4*(-1/(a*b^3))^(1/4)*log(-b*(-1/(a*b^3))^(1/4)
 + x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.10

$$\int \frac{1}{b+ax^4}\,dx = \operatorname{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(4tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**4+b),x)
```

output
```
RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(4*_t*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.87

$$\int \frac{1}{b+ax^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\log\left(\sqrt{a}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{b}\right)}{8\,a^{\frac{1}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{a}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{b}\right)}{8\,a^{\frac{1}{4}}b^{\frac{3}{4}}}$$

input
```
integrate(1/(a*x^4+b),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(a)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(a)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*log(sqrt(a)*x^2 + sqrt(2)*a^
(1/4)*b^(1/4)*x + sqrt(b))/(a^(1/4)*b^(3/4)) - 1/8*sqrt(2)*log(sqrt(a)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(b))/(a^(1/4)*b^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \frac{1}{b+ax^4}\,dx = \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,ab} + \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,ab} + \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{8\,ab} - \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{8\,ab}$$

input
```
integrate(1/(a*x^4+b),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(b/a)^(1/4))/(
b/a)^(1/4))/(a*b) + 1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(b/a)^(1/4))/(b/a)^(1/4))/(a*b) + 1/8*sqrt(2)*(a^3*b)^(1/4)*log(x^2
+ sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/(a*b) - 1/8*sqrt(2)*(a^3*b)^(1/4)*log
(x^2 - sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.17

$$\int \frac{1}{b+ax^4}\,dx = \frac{\operatorname{atan}\left(\frac{(-a)^{1/4}x}{b^{1/4}}\right)+\operatorname{atanh}\left(\frac{(-a)^{1/4}x}{b^{1/4}}\right)}{2\,(-a)^{1/4}\,b^{3/4}}$$

input
```
int(1/(b + a*x^4),x)
```

output
```
(atan(((-a)^(1/4)*x)/b^(1/4)) + atanh(((-a)^(1/4)*x)/b^(1/4)))/(2*(-a)^(1/
4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int \frac{1}{b+ax^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\,x^2+\sqrt{b}\right)+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{\right.\right.}{8b^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int(1/(a*x^4+b),x)
```

output
```
(b**(1/4)*a**(3/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
a)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(a)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) - log( - b**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a)*x**2 + sqrt(b)) + log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)*
x**2 + sqrt(b))))/(8*a*b)
```

3.101 $\int \frac{1}{b+ax^5}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 417

$$\int \frac{1}{b+ax^5}\,dx = -\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\sqrt[5]{b}-\sqrt{2+\frac{2}{\sqrt{5}}}\sqrt[5]{a}x}{\sqrt[5]{b}}\right)}{5\sqrt[5]{a}b^{4/5}} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\sqrt[5]{b}+\sqrt{\frac{2}{5}\left(5-\sqrt{5}\right)}\sqrt[5]{a}x}{\sqrt[5]{b}}\right)}{5\sqrt[5]{a}b^{4/5}} + \frac{\log\left(\sqrt[5]{b}+\sqrt[5]{a}x\right)}{5\sqrt[5]{a}b^{4/5}} - \frac{\log\left(\frac{2b^{2/5}-\sqrt[5]{a}\sqrt[5]{b}x-\sqrt{5}\sqrt[5]{a}\sqrt[5]{b}x+2a^{2/5}x^2}{a^{2/5}}\right)}{4\sqrt{5}\sqrt[5]{a}b^{4/5}} + \frac{\log\left(\frac{2b^{2/5}-\sqrt[5]{a}\sqrt[5]{b}x+\sqrt{5}\sqrt[5]{a}\sqrt[5]{b}x+2a^{2/5}x^2}{a^{2/5}}\right)}{4\sqrt{5}\sqrt[5]{a}b^{4/5}} - \frac{\log\left(b^{4/5}-\sqrt[5]{a}b^{3/5}x+a^{2/5}b^{2/5}x^2-a^{3/5}\sqrt[5]{b}x^3+a^{4/5}x^4\right)}{20\sqrt[5]{a}b^{4/5}}$$

output
```
-1/10*(10-2*5^(1/2))^(1/2)*arctan((1/5*(25+10*5^(1/2))^(1/2)*b^(1/5)-1/5*(
50+10*5^(1/2))^(1/2)*a^(1/5)*x)/b^(1/5))/a^(1/5)/b^(4/5)+1/10*(10+2*5^(1/2
))^(1/2)*arctan((1/5*(25-10*5^(1/2))^(1/2)*b^(1/5)+1/5*(50-10*5^(1/2))^(1/
2)*a^(1/5)*x)/b^(1/5))/a^(1/5)/b^(4/5)+1/5*ln(b^(1/5)+a^(1/5)*x)/a^(1/5)/b
^(4/5)-1/20*ln((2*b^(2/5)-a^(1/5)*b^(1/5)*x-5^(1/2)*a^(1/5)*b^(1/5)*x+2*a^
(2/5)*x^2)/a^(2/5))*5^(1/2)/a^(1/5)/b^(4/5)+1/20*ln((2*b^(2/5)-a^(1/5)*b^(
1/5)*x+5^(1/2)*a^(1/5)*b^(1/5)*x+2*a^(2/5)*x^2)/a^(2/5))*5^(1/2)/a^(1/5)/b
^(4/5)-1/20*ln(b^(4/5)-a^(1/5)*b^(3/5)*x+a^(2/5)*b^(2/5)*x^2-a^(3/5)*b^(1/
5)*x^3+a^(4/5)*x^4)/a^(1/5)/b^(4/5)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.75

$$\int \frac{1}{b+ax^5}\,dx = \frac{2\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\arctan\left(\frac{-\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{b}+\sqrt[5]{a}x}{\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\sqrt[5]{b}}\right)}{5\sqrt[5]{a}b^{4/5}} + \frac{2\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\arctan\left(\frac{-\frac{1}{4}\left(1+\sqrt{5}\right)\sqrt[5]{b}+\sqrt[5]{a}x}{\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\sqrt[5]{b}}\right)}{5\sqrt[5]{a}b^{4/5}} + \frac{\log\left(\sqrt[5]{b}+\sqrt[5]{a}x\right)}{5\sqrt[5]{a}b^{4/5}} - \frac{\left(1-\sqrt{5}\right)\log\left(b^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+a^{2/5}x^2\right)}{20\sqrt[5]{a}b^{4/5}} - \frac{\left(1+\sqrt{5}\right)\log\left(b^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+a^{2/5}x^2\right)}{20\sqrt[5]{a}b^{4/5}}$$

input
```
Integrate[(b + a*x^5)^(-1),x]
```

output
```
(2*Sqrt[5/8 + Sqrt[5]/8]*ArcTan[(-1/4*((1 - Sqrt[5])*b^(1/5)) + a^(1/5)*x)
/(Sqrt[5/8 + Sqrt[5]/8]*b^(1/5))])/(5*a^(1/5)*b^(4/5)) + (2*Sqrt[5/8 - Sqr
t[5]/8]*ArcTan[(-1/4*((1 + Sqrt[5])*b^(1/5)) + a^(1/5)*x)/(Sqrt[5/8 - Sqrt
[5]/8]*b^(1/5))])/(5*a^(1/5)*b^(4/5)) + Log[b^(1/5) + a^(1/5)*x]/(5*a^(1/5
)*b^(4/5)) - ((1 - Sqrt[5])*Log[b^(2/5) - ((1 - Sqrt[5])*a^(1/5)*b^(1/5)*x
)/2 + a^(2/5)*x^2])/(20*a^(1/5)*b^(4/5)) - ((1 + Sqrt[5])*Log[b^(2/5) - ((
1 + Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + a^(2/5)*x^2])/(20*a^(1/5)*b^(4/5))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.76, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {751, 16, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^5+b}dx$$

$$\Big\downarrow \text{751}$$

$$\frac{2\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{2\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}x+\sqrt[5]{b}}dx}{5b^{4/5}}$$

$$\Big\downarrow \text{16}$$

$$\frac{2\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{2\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-\frac{\left(1-\sqrt{5}\right)\int -\frac{\sqrt[5]{a}\left(\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-\frac{\left(1+\sqrt{5}\right)\int -\frac{\sqrt[5]{a}\left(\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\left(1-\sqrt{5}\right)\int \frac{\sqrt[5]{a}\left(\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{a}\left(\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-(5+\sqrt{5})\sqrt[5]{b}\int \frac{1}{-\left(4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}\right)^2-2\left(5+\sqrt{5}\right)a^{2/5}b^{2/5}}d\left(4a^{2/5}\right.}{5b^{4/5}}$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-(5-\sqrt{5})\sqrt[5]{b}\int \frac{1}{-\left(4a^{2/5}x-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}\right)^2-2\left(5-\sqrt{5}\right)a^{2/5}b^{2/5}}d\left(4a^{2/5}\right.}{5b^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$\downarrow$ 217

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}}{5b^{4/5}}+$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}}{5b^{4/5}}+$$

$$\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}{4\sqrt[5]{a}}}{5b^{4/5}}+$$

$$\frac{\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}-\frac{\left(1+\sqrt{5}\right)\log\left(2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}{4\sqrt[5]{a}}}{5b^{4/5}}+$$

$$\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}$$

input

```
Int[(b + a*x^5)^(-1),x]
```

output
```
Log[b^(1/5) + a^(1/5)*x]/(5*a^(1/5)*b^(4/5)) + ((Sqrt[(5 + Sqrt[5])/2]*Arc
Tan[(-((1 - Sqrt[5])*a^(1/5)*b^(1/5)) + 4*a^(2/5)*x)/(Sqrt[2*(5 + Sqrt[5])
]*a^(1/5)*b^(1/5))])/a^(1/5) - ((1 - Sqrt[5])*Log[2*b^(2/5) - (1 - Sqrt[5]
)*a^(1/5)*b^(1/5)*x + 2*a^(2/5)*x^2])/(4*a^(1/5)))/(5*b^(4/5)) + ((Sqrt[(5
 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*a^(1/5)*b^(1/5)) + 4*a^(2/5)*x)/(Sq
rt[2*(5 - Sqrt[5])]*a^(1/5)*b^(1/5))])/a^(1/5) - ((1 + Sqrt[5])*Log[2*b^(2
/5) - (1 + Sqrt[5])*a^(1/5)*b^(1/5)*x + 2*a^(2/5)*x^2])/(4*a^(1/5)))/(5*b^
(4/5))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 751
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k -
1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n)
  Int[1/(r + s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

method	result	size
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^5 a+b\right)} \frac{\ln\left(x-\_R\right)}{\_R^4}}{5a}$	27
default	Expression too large to display	895

input
```
int(1/(a*x^5+b),x,method=_RETURNVERBOSE)
```

output
```
1/5/a*sum(1/_R^4*ln(x-_R),_R=RootOf(_Z^5*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.04 (sec) , antiderivative size = 1130235, normalized size of antiderivative = 2710.40

$$\int \frac{1}{b+ax^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x^5+b),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.05

$$\int \frac{1}{b+ax^5}\,dx = \operatorname{RootSum}\left(3125t^5ab^4-1,\left(t\mapsto t\log\left(5tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**5+b),x)
```

output
```
RootSum(3125*_t**5*a*b**4 - 1, Lambda(_t, _t*log(5*_t*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.57

$$\int \frac{1}{b+ax^5}\,dx = \frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4\,a^{\frac{2}{5}}x+a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}-1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}\right)}{5\,a^{\frac{1}{5}}b^{\frac{4}{5}}\sqrt{2\sqrt{5}+10}} + \frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4\,a^{\frac{2}{5}}x-a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}+1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}\right)}{5\,a^{\frac{1}{5}}b^{\frac{4}{5}}\sqrt{-2\sqrt{5}+10}} - \frac{\left(\sqrt{5}+3\right)\log\left(2\,a^{\frac{2}{5}}x^2-a^{\frac{1}{5}}b^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2\,b^{\frac{2}{5}}\right)}{10\,a^{\frac{1}{5}}b^{\frac{4}{5}}\left(\sqrt{5}+1\right)} - \frac{\left(\sqrt{5}-3\right)\log\left(2\,a^{\frac{2}{5}}x^2+a^{\frac{1}{5}}b^{\frac{1}{5}}x\left(\sqrt{5}-1\right)+2\,b^{\frac{2}{5}}\right)}{10\,a^{\frac{1}{5}}b^{\frac{4}{5}}\left(\sqrt{5}-1\right)} + \frac{\log\left(a^{\frac{1}{5}}x+b^{\frac{1}{5}}\right)}{5\,a^{\frac{1}{5}}b^{\frac{4}{5}}}$$

input
```
integrate(1/(a*x^5+b),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*a^(2/5)*x + a^(1/5)*b^(1/5)*(sqrt(5) -
 1))/(a^(1/5)*b^(1/5)*sqrt(2*sqrt(5) + 10)))/(a^(1/5)*b^(4/5)*sqrt(2*sqrt(
5) + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*a^(2/5)*x - a^(1/5)*b^(1/5
)*(sqrt(5) + 1))/(a^(1/5)*b^(1/5)*sqrt(-2*sqrt(5) + 10)))/(a^(1/5)*b^(4/5)
*sqrt(-2*sqrt(5) + 10)) - 1/10*(sqrt(5) + 3)*log(2*a^(2/5)*x^2 - a^(1/5)*b
^(1/5)*x*(sqrt(5) + 1) + 2*b^(2/5))/(a^(1/5)*b^(4/5)*(sqrt(5) + 1)) - 1/10
*(sqrt(5) - 3)*log(2*a^(2/5)*x^2 + a^(1/5)*b^(1/5)*x*(sqrt(5) - 1) + 2*b^(
2/5))/(a^(1/5)*b^(4/5)*(sqrt(5) - 1)) + 1/5*log(a^(1/5)*x + b^(1/5))/(a^(1
/5)*b^(4/5))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \frac{1}{b+ax^5}\,dx = &-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{5}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right|\right)}{5\,b}\\ &+\frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\left(\sqrt{5}-1\right)\left(-\frac{b}{a}\right)^{\frac{1}{5}}-4\,x}{\sqrt{2\sqrt{5}+10}\left(-\frac{b}{a}\right)^{\frac{1}{5}}}\right)}{10\,ab}\\ &+\frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\left(\sqrt{5}+1\right)\left(-\frac{b}{a}\right)^{\frac{1}{5}}+4\,x}{\sqrt{-2\sqrt{5}+10}\left(-\frac{b}{a}\right)^{\frac{1}{5}}}\right)}{10\,ab}\\ &+\frac{\left(-a^4b\right)^{\frac{1}{5}}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}\left(-\frac{b}{a}\right)^{\frac{1}{5}}+\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right)+\left(-\frac{b}{a}\right)^{\frac{2}{5}}\right)}{5\,ab\left(\sqrt{5}-1\right)}\\ &-\frac{\left(-a^4b\right)^{\frac{1}{5}}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}\left(-\frac{b}{a}\right)^{\frac{1}{5}}-\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right)+\left(-\frac{b}{a}\right)^{\frac{2}{5}}\right)}{5\,ab\left(\sqrt{5}+1\right)}\end{aligned}$$

input
```
integrate(1/(a*x^5+b),x, algorithm="giac")
```

output
```
-1/5*(-b/a)^(1/5)*log(abs(x - (-b/a)^(1/5)))/b + 1/10*(-a^4*b)^(1/5)*sqrt(
2*sqrt(5) + 10)*arctan(-((sqrt(5) - 1)*(-b/a)^(1/5) - 4*x)/(sqrt(2*sqrt(5)
 + 10)*(-b/a)^(1/5)))/(a*b) + 1/10*(-a^4*b)^(1/5)*sqrt(-2*sqrt(5) + 10)*ar
ctan(((sqrt(5) + 1)*(-b/a)^(1/5) + 4*x)/(sqrt(-2*sqrt(5) + 10)*(-b/a)^(1/5
)))/(a*b) + 1/5*(-a^4*b)^(1/5)*log(x^2 + 1/2*x*(sqrt(5)*(-b/a)^(1/5) + (-b
/a)^(1/5)) + (-b/a)^(2/5))/(a*b*(sqrt(5) - 1)) - 1/5*(-a^4*b)^(1/5)*log(x^
2 - 1/2*x*(sqrt(5)*(-b/a)^(1/5) - (-b/a)^(1/5)) + (-b/a)^(2/5))/(a*b*(sqrt
(5) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.56

$$\int \frac{1}{b+ax^5}\,dx = \frac{\ln\left(a^{1/5}\,x+b^{1/5}\right)}{5\,a^{1/5}\,b^{4/5}} - \frac{\ln\left(5\,a^4\,x-\frac{5\,a^{19/5}\,b^{1/5}\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}-\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^{1/5}\,b^{4/5}} - \frac{\ln\left(5\,a^4\,x-\frac{5\,a^{19/5}\,b^{1/5}\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{4}\right)\,\left(\sqrt{-2\,\sqrt{5}-10}-\sqrt{5}+1\right)}{20\,a^{1/5}\,b^{4/5}} + \frac{\ln\left(5\,a^4\,x+\frac{5\,a^{19/5}\,b^{1/5}\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{4}\right)\,\left(\sqrt{5}+\sqrt{-2\,\sqrt{5}-10}-1\right)}{20\,a^{1/5}\,b^{4/5}} - \frac{\ln\left(5\,a^4\,x-\frac{5\,a^{19/5}\,b^{1/5}\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{4}\right)\,\left(\sqrt{5}+\sqrt{2\,\sqrt{5}-10}+1\right)}{20\,a^{1/5}\,b^{4/5}}$$

input
```
int(1/(b + a*x^5),x)
```

output
```
log(a^(1/5)*x + b^(1/5))/(5*a^(1/5)*b^(4/5)) - (log(5*a^4*x - (5*a^(19/5)*
b^(1/5)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) -
 10)^(1/2) + 1))/(20*a^(1/5)*b^(4/5)) - (log(5*a^4*x - (5*a^(19/5)*b^(1/5)
*((- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) -
5^(1/2) + 1))/(20*a^(1/5)*b^(4/5)) + (log(5*a^4*x + (5*a^(19/5)*b^(1/5)*(5
^(1/2) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(
1/2) - 1))/(20*a^(1/5)*b^(4/5)) - (log(5*a^4*x - (5*a^(19/5)*b^(1/5)*(5^(1
/2) + (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) +
1))/(20*a^(1/5)*b^(4/5))
```

Reduce [F]

$$\int \frac{1}{b+ax^5}\,dx = \int \frac{1}{a\,x^5+b}dx$$

input
```
int(1/(a*x^5+b),x)
```

output
```
int(1/(a*x**5 + b),x)
```

3.102 $\int \frac{1}{b+ax^6}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 215

$$\int \frac{1}{b+ax^6}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6\sqrt[6]{a}b^{5/6}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{b}+2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6\sqrt[6]{a}b^{5/6}} - \frac{\log\left(\sqrt[3]{b}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}} + \frac{\log\left(\sqrt[3]{b}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}}$$

output
```
1/3*arctan(a^(1/6)*x/b^(1/6))/a^(1/6)/b^(5/6)-1/6*arctan((3^(1/2)*b^(1/6)-
2*a^(1/6)*x)/b^(1/6))/a^(1/6)/b^(5/6)+1/6*arctan((3^(1/2)*b^(1/6)+2*a^(1/6
)*x)/b^(1/6))/a^(1/6)/b^(5/6)-1/12*ln(b^(1/3)-3^(1/2)*a^(1/6)*b^(1/6)*x+a^
(1/3)*x^2)*3^(1/2)/a^(1/6)/b^(5/6)+1/12*ln(b^(1/3)+3^(1/2)*a^(1/6)*b^(1/6)
*x+a^(1/3)*x^2)*3^(1/2)/a^(1/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

$$\int \frac{1}{b+ax^6}\,dx = \frac{4\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)-2\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)+2\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)-\sqrt{3}\log\left(\sqrt[3]{b}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)+\sqrt{3}\log\left(\sqrt[3]{b}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)}{12\sqrt[6]{a}b^{5/6}}$$

input
```
Integrate[(b + a*x^6)^(-1),x]
```

output
```
(4*ArcTan[(a^(1/6)*x)/b^(1/6)] - 2*ArcTan[Sqrt[3] - (2*a^(1/6)*x)/b^(1/6)]
 + 2*ArcTan[Sqrt[3] + (2*a^(1/6)*x)/b^(1/6)] - Sqrt[3]*Log[b^(1/3) - Sqrt[
3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2] + Sqrt[3]*Log[b^(1/3) + Sqrt[3]*a^(1/6
)*b^(1/6)*x + a^(1/3)*x^2])/(12*a^(1/6)*b^(5/6))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^6+b}\,dx$$

$$\Big\downarrow \text{753}$$

$$\frac{\int \frac{1}{\sqrt[3]{a}x^2+\sqrt[3]{b}}dx}{3b^{2/3}}+\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{2\left(\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{2\left(\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3b^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt[3]{a}x^2+\sqrt[3]{b}}dx}{3b^{2/3}}+\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\sqrt{3}\int -\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}\right)}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}\right)}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)}{\sqrt{3}\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\int \frac{1}{-\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)^2-\frac{1}{3}}d\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)}{\sqrt{3}\sqrt[6]{a}}}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)\right)}{\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)\right)}{\sqrt[6]{a}}}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

$$\Big\downarrow \ 1103$$

$$\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)\right)}{\sqrt[6]{a}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)\right)}{\sqrt[6]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}$$

input
```
Int[(b + a*x^6)^(-1),x]
```

output
```
ArcTan[(a^(1/6)*x)/b^(1/6)]/(3*a^(1/6)*b^(5/6)) + (-(ArcTan[Sqrt[3]*(1 - (
2*a^(1/6)*x)/(Sqrt[3]*b^(1/6)))]/a^(1/6)) - (Sqrt[3]*Log[b^(1/3) - Sqrt[3]
*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(2*a^(1/6)))/(6*b^(5/6)) + (ArcTan[Sqrt
[3]*(1 + (2*a^(1/6)*x)/(Sqrt[3]*b^(1/6)))]/a^(1/6) + (Sqrt[3]*Log[b^(1/3)
+ Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(2*a^(1/6)))/(6*b^(5/6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^6 a+b\right)} \frac{\ln\left(x-\_R\right)}{\_R^5}}{6a}$
default	$\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}} \arctan\left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3b} - \frac{\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}} \ln\left(x^2-\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}} x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12b} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{6}} \arctan\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b} + \frac{\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}} \ln\left(x^2+\sqrt{3}\left(\right.\right.}{12b}$

input
```
int(1/(a*x^6+b),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*sum(1/_R^5*ln(x-_R),_R=RootOf(_Z^6*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{1}{b+ax^6}\,dx = {} & \frac{1}{12}\left(\sqrt{-3}+1\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}b+b\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right) \\
& -\frac{1}{12}\left(\sqrt{-3}+1\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}b+b\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right) \\
& +\frac{1}{12}\left(\sqrt{-3}-1\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}b-b\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right) \\
& -\frac{1}{12}\left(\sqrt{-3}-1\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}b-b\right)\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right) \\
& +\frac{1}{6}\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(b\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right) \\
& -\frac{1}{6}\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\log\left(-b\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+x\right)
\end{aligned}
$$

input
```
integrate(1/(a*x^6+b),x, algorithm="fricas")
```

output
```
1/12*(sqrt(-3) + 1)*(-1/(a*b^5))^(1/6)*log(1/2*(sqrt(-3)*b + b)*(-1/(a*b^5
))^(1/6) + x) - 1/12*(sqrt(-3) + 1)*(-1/(a*b^5))^(1/6)*log(-1/2*(sqrt(-3)*
b + b)*(-1/(a*b^5))^(1/6) + x) + 1/12*(sqrt(-3) - 1)*(-1/(a*b^5))^(1/6)*lo
g(1/2*(sqrt(-3)*b - b)*(-1/(a*b^5))^(1/6) + x) - 1/12*(sqrt(-3) - 1)*(-1/(
a*b^5))^(1/6)*log(-1/2*(sqrt(-3)*b - b)*(-1/(a*b^5))^(1/6) + x) + 1/6*(-1/
(a*b^5))^(1/6)*log(b*(-1/(a*b^5))^(1/6) + x) - 1/6*(-1/(a*b^5))^(1/6)*log(
-b*(-1/(a*b^5))^(1/6) + x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.09

$$\int \frac{1}{b+ax^6}\,dx = \operatorname{RootSum}\left(46656t^6ab^5+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**6+b),x)
```

output
```
RootSum(46656*_t**6*a*b**5 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.86

$$\int \frac{1}{b+ax^6}\,dx = \frac{\sqrt{3}\log\left(a^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{12\,a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(a^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{12\,a^{\frac{1}{6}}b^{\frac{5}{6}}} + \frac{\arctan\left(\frac{a^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{3\,b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{\arctan\left(\frac{2\,a^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{\arctan\left(\frac{2\,a^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}$$

input
```
integrate(1/(a*x^6+b),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*log(a^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + b^(1/3))/(a^(1/
6)*b^(5/6)) - 1/12*sqrt(3)*log(a^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + b
^(1/3))/(a^(1/6)*b^(5/6)) + 1/3*arctan(a^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(b
^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arctan((2*a^(1/3)*x + sqrt(3)*a^(1/6)*
b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arct
an((2*a^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)
*sqrt(a^(1/3)*b^(1/3)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{1}{b+ax^6}\,dx = \frac{\sqrt{3}(a^5b)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,ab} - \frac{\sqrt{3}(a^5b)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,ab} + \frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{6\,ab} + \frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{6\,ab} + \frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3\,ab}$$

input
```
integrate(1/(a*x^6+b),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 + sqrt(3)*x*(b/a)^(1/6) + (b/a)^(1/3))/
(a*b) - 1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 - sqrt(3)*x*(b/a)^(1/6) + (b/a)
^(1/3))/(a*b) + 1/6*(a^5*b)^(1/6)*arctan((2*x + sqrt(3)*(b/a)^(1/6))/(b/a)
^(1/6))/(a*b) + 1/6*(a^5*b)^(1/6)*arctan((2*x - sqrt(3)*(b/a)^(1/6))/(b/a)
^(1/6))/(a*b) + 1/3*(a^5*b)^(1/6)*arctan(x/(b/a)^(1/6))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02

$$\int \frac{1}{b+ax^6}\,dx = \frac{\operatorname{atanh}\left(\frac{(-a)^{1/6}\,x}{b^{1/6}}\right)}{3\,(-a)^{1/6}\,b^{5/6}} - \frac{\operatorname{atan}\left(\frac{(-a)^{29/6}\,x\,1\mathrm{i}}{b^{5/6}\,\left(\frac{(-a)^{14/3}}{b^{2/3}}+\frac{\sqrt{3}\,(-a)^{14/3}\,1\mathrm{i}}{b^{2/3}}\right)}+\frac{\sqrt{3}\,(-a)^{29/6}\,x}{b^{5/6}\,\left(\frac{(-a)^{14/3}}{b^{2/3}}+\frac{\sqrt{3}\,(-a)^{14/3}\,1\mathrm{i}}{b^{2/3}}\right)}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{6\,(-a)^{1/6}\,b^{5/6}} + \frac{\operatorname{atan}\left(\frac{(-a)^{29/6}\,x\,1\mathrm{i}}{b^{5/6}\,\left(\frac{(-a)^{14/3}}{b^{2/3}}-\frac{\sqrt{3}\,(-a)^{14/3}\,1\mathrm{i}}{b^{2/3}}\right)}-\frac{\sqrt{3}\,(-a)^{29/6}\,x}{b^{5/6}\,\left(\frac{(-a)^{14/3}}{b^{2/3}}-\frac{\sqrt{3}\,(-a)^{14/3}\,1\mathrm{i}}{b^{2/3}}\right)}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{6\,(-a)^{1/6}\,b^{5/6}}$$

input
```
int(1/(b + a*x^6),x)
```

output
```
atanh(((-a)^(1/6)*x)/b^(1/6))/(3*(-a)^(1/6)*b^(5/6)) - (atan(((-a)^(29/6)*
x*1i)/(b^(5/6)*((-a)^(14/3)/b^(2/3) + (3^(1/2)*(-a)^(14/3)*1i)/b^(2/3))) +
 (3^(1/2)*(-a)^(29/6)*x)/(b^(5/6)*((-a)^(14/3)/b^(2/3) + (3^(1/2)*(-a)^(14
/3)*1i)/b^(2/3))))*(3^(1/2)*1i - 1)*1i)/(6*(-a)^(1/6)*b^(5/6)) + (atan(((-
a)^(29/6)*x*1i)/(b^(5/6)*((-a)^(14/3)/b^(2/3) - (3^(1/2)*(-a)^(14/3)*1i)/b
^(2/3))) - (3^(1/2)*(-a)^(29/6)*x)/(b^(5/6)*((-a)^(14/3)/b^(2/3) - (3^(1/2
)*(-a)^(14/3)*1i)/b^(2/3))))*(3^(1/2)*1i + 1)*1i)/(6*(-a)^(1/6)*b^(5/6))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{1}{b+ax^6}\,dx = \frac{-2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2a^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2a^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+4atan\left(\frac{a^{\frac{1}{6}}x}{b^{\frac{1}{6}}}\right)-\sqrt{3}\log\left(-b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}\,x+a^{\frac{1}{3}}x^2+b^{\frac{1}{3}}\right)+\sqrt{}}{12b^{\frac{5}{6}}a^{\frac{1}{6}}}$$

input
```
int(1/(a*x^6+b),x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*a**(1/3)*x)/(
b**(1/6)*a**(1/6))) + 2*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*a**(1/3)*x)/(b
**(1/6)*a**(1/6))) + 4*atan((a**(1/3)*x)/(b**(1/6)*a**(1/6))) - sqrt(3)*lo
g( - b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3)*x**2 + b**(1/3)) + sqrt(3)*log
(b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3)*x**2 + b**(1/3))))/(12*a**(1/3)*b)
```

3.103 $\int \frac{1}{b+ax^8} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 557

$$\int \frac{1}{b+ax^8}\,dx = -\frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{-\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}-\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8\sqrt[8]{a}b^{7/8}} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{-\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}+\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8\sqrt[8]{a}b^{7/8}} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}-\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8\sqrt[8]{a}b^{7/8}} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}+\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8\sqrt[8]{a}b^{7/8}} - \frac{\sqrt{2-\sqrt{2}}\log\left(\sqrt[4]{b}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16\sqrt[8]{a}b^{7/8}} + \frac{\sqrt{2-\sqrt{2}}\log\left(\sqrt[4]{b}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16\sqrt[8]{a}b^{7/8}} - \frac{\sqrt{2+\sqrt{2}}\log\left(\sqrt[4]{b}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16\sqrt[8]{a}b^{7/8}} + \frac{\sqrt{2+\sqrt{2}}\log\left(\sqrt[4]{b}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16\sqrt[8]{a}b^{7/8}}$$

output
```
-1/8*(2+2^(1/2))^(1/2)*arctan((-b^(1/8)+b^(1/8)*2^(1/2)-(4-2*2^(1/2))^(1/2
)*a^(1/8)*x)/b^(1/8))/a^(1/8)/b^(7/8)+1/8*(2+2^(1/2))^(1/2)*arctan((-b^(1/
8)+b^(1/8)*2^(1/2)+(4-2*2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/a^(1/8)/b^(7/8)
-1/8*(2-2^(1/2))^(1/2)*arctan((b^(1/8)+b^(1/8)*2^(1/2)-(4+2*2^(1/2))^(1/2)
*a^(1/8)*x)/b^(1/8))/a^(1/8)/b^(7/8)+1/8*(2-2^(1/2))^(1/2)*arctan((b^(1/8)
+b^(1/8)*2^(1/2)+(4+2*2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/a^(1/8)/b^(7/8)-1
/16*(2-2^(1/2))^(1/2)*ln(b^(1/4)-(2-2^(1/2))^(1/2)*a^(1/8)*b^(1/8)*x+a^(1/
4)*x^2)/a^(1/8)/b^(7/8)+1/16*(2-2^(1/2))^(1/2)*ln(b^(1/4)+(2-2^(1/2))^(1/2
)*a^(1/8)*b^(1/8)*x+a^(1/4)*x^2)/a^(1/8)/b^(7/8)-1/16*(2+2^(1/2))^(1/2)*ln
(b^(1/4)-(2+2^(1/2))^(1/2)*a^(1/8)*b^(1/8)*x+a^(1/4)*x^2)/a^(1/8)/b^(7/8)+
1/16*(2+2^(1/2))^(1/2)*ln(b^(1/4)+(2+2^(1/2))^(1/2)*a^(1/8)*b^(1/8)*x+a^(1
/4)*x^2)/a^(1/8)/b^(7/8)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.58

$$\int \frac{1}{b+ax^8}\,dx = \frac{2\arctan\left(\frac{\sqrt[8]{a}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{b}}-\tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right)+2\arctan\left(\frac{\sqrt[8]{a}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{b}}+\tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)\log\left(\sqrt[4]{b}\right.}{}$$

input
```
Integrate[(b + a*x^8)^(-1),x]
```

output
```
(2*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) - Tan[Pi/8]]*Cos[Pi/8] + 2*ArcTan[
(a^(1/8)*x*Sec[Pi/8])/b^(1/8) + Tan[Pi/8]]*Cos[Pi/8] - Cos[Pi/8]*Log[b^(1/
4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + Cos[Pi/8]*Log[b^(1/4)
+ a^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - 2*ArcTan[Cot[Pi/8] - (a^(
1/8)*x*Csc[Pi/8])/b^(1/8)]*Sin[Pi/8] + 2*ArcTan[Cot[Pi/8] + (a^(1/8)*x*Csc
[Pi/8])/b^(1/8)]*Sin[Pi/8] - Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)
*x*Sin[Pi/8]]*Sin[Pi/8] + Log[b^(1/4) + a^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*
Sin[Pi/8]]*Sin[Pi/8])/(8*a^(1/8)*b^(7/8))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.54, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^8+b}dx$$

$$\Big\downarrow \text{758}$$

$$\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{b}}+\frac{\int \frac{1}{\sqrt[4]{-a}x^2+\sqrt[4]{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{-\frac{\int -\frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}\right)}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\frac{\int \frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}\right)}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\frac{\int \frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int \frac{\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}+\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}x+\sqrt[4]{-a}x^2+\sqrt[4]{b}\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}x+\sqrt[4]{-a}x^2+\sqrt[4]{b}\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}$$

input
```
Int[(b + a*x^8)^(-1),x]
```

output
```
(ArcTan[((-a)^(1/8)*x)/b^(1/8)]/(2*(-a)^(1/8)*b^(3/8)) + ArcTanh[((-a)^(1/
8)*x)/b^(1/8)]/(2*(-a)^(1/8)*b^(3/8)))/(2*Sqrt[b]) + ((-(ArcTan[1 - (Sqrt[
2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8))) + ArcTan[1 + (Sqrt
[2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8)))/(2*b^(1/4)) + (-1
/2*Log[b^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-a)^(1/4)*x^2]/(Sqrt[2]*(
-a)^(1/8)*b^(1/8)) + Log[b^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-a)^(1/
4)*x^2]/(2*Sqrt[2]*(-a)^(1/8)*b^(1/8)))/(2*b^(1/4)))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}}{8a}$	27
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}}{8a}$	27

input
```
int(1/(a*x^8+b),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.45

$$\int \frac{1}{b+ax^8}\,dx = \left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) - \left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) + \left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) - \left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) + \frac{1}{8}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) + \frac{1}{8}i\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(i\,b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) - \frac{1}{8}i\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(-i\,b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right) - \frac{1}{8}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\log\left(-b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+x\right)$$

input
```
integrate(1/(a*x^8+b),x, algorithm="fricas")
```

output
```
(1/16*I + 1/16)*sqrt(2)*(-1/(a*b^7))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*b*(-1
/(a*b^7))^(1/8) + x) - (1/16*I - 1/16)*sqrt(2)*(-1/(a*b^7))^(1/8)*log(-(1/
2*I - 1/2)*sqrt(2)*b*(-1/(a*b^7))^(1/8) + x) + (1/16*I - 1/16)*sqrt(2)*(-1
/(a*b^7))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*b*(-1/(a*b^7))^(1/8) + x) - (1/1
6*I + 1/16)*sqrt(2)*(-1/(a*b^7))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*b*(-1/(a
*b^7))^(1/8) + x) + 1/8*(-1/(a*b^7))^(1/8)*log(b*(-1/(a*b^7))^(1/8) + x) +
 1/8*I*(-1/(a*b^7))^(1/8)*log(I*b*(-1/(a*b^7))^(1/8) + x) - 1/8*I*(-1/(a*b
^7))^(1/8)*log(-I*b*(-1/(a*b^7))^(1/8) + x) - 1/8*(-1/(a*b^7))^(1/8)*log(-
b*(-1/(a*b^7))^(1/8) + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.04

$$\int \frac{1}{b+ax^8}\,dx = \operatorname{RootSum}\left(16777216t^8ab^7+1,\left(t\mapsto t\log\left(8tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**8+b),x)
```

output
```
RootSum(16777216*_t**8*a*b**7 + 1, Lambda(_t, _t*log(8*_t*b + x)))
```

Maxima [F]

$$\int \frac{1}{b+ax^8}\,dx = \int \frac{1}{ax^8+b}\,dx$$

input
```
integrate(1/(a*x^8+b),x, algorithm="maxima")
```

output
```
integrate(1/(a*x^8 + b), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.78

$$\int \frac{1}{b+ax^8}\,dx = \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2\,x+\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2\,x-\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2\,x+\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2\,x-\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b\sqrt{-2\,\sqrt{2}+4}} - \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b\sqrt{2\,\sqrt{2}+4}} - \frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b\sqrt{2\,\sqrt{2}+4}}$$

input
```
integrate(1/(a*x^8+b),x, algorithm="giac")
```

output
```
1/4*(b/a)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqrt(2
) + 2)*(b/a)^(1/8)))/(b*sqrt(-2*sqrt(2) + 4)) + 1/4*(b/a)^(1/8)*arctan((2*
x - sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqrt(2) + 2)*(b/a)^(1/8)))/(b*sq
rt(-2*sqrt(2) + 4)) + 1/4*(b/a)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(b/a
)^(1/8))/(sqrt(-sqrt(2) + 2)*(b/a)^(1/8)))/(b*sqrt(2*sqrt(2) + 4)) + 1/4*(
b/a)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(-sqrt(2) + 2
)*(b/a)^(1/8)))/(b*sqrt(2*sqrt(2) + 4)) + 1/8*(b/a)^(1/8)*log(x^2 + x*sqrt
(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(b*sqrt(-2*sqrt(2) + 4)) - 1/8*(b
/a)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(b*sqrt
(-2*sqrt(2) + 4)) + 1/8*(b/a)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(b/a)^(
1/8) + (b/a)^(1/4))/(b*sqrt(2*sqrt(2) + 4)) - 1/8*(b/a)^(1/8)*log(x^2 - x*
sqrt(-sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(b*sqrt(2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.20

$$\int \frac{1}{b+ax^8}\,dx = \frac{\operatorname{atan}\left(\frac{(-a)^{1/8}\,x}{b^{1/8}}\right)}{4\,(-a)^{1/8}\,b^{7/8}} - \frac{\operatorname{atan}\left(\frac{(-a)^{1/8}\,x\,1\mathrm{i}}{b^{1/8}}\right)\,1\mathrm{i}}{4\,(-a)^{1/8}\,b^{7/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,(-a)^{1/8}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{b^{1/8}}\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{(-a)^{1/8}\,b^{7/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,(-a)^{1/8}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{b^{1/8}}\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{(-a)^{1/8}\,b^{7/8}}$$

input
```
int(1/(b + a*x^8),x)
```

output
```
atan(((-a)^(1/8)*x)/b^(1/8))/(4*(-a)^(1/8)*b^(7/8)) - (atan(((-a)^(1/8)*x*
1i)/b^(1/8))*1i)/(4*(-a)^(1/8)*b^(7/8)) + (2^(1/2)*atan((2^(1/2)*(-a)^(1/8
)*x*(1/2 - 1i/2))/b^(1/8))*(1/8 + 1i/8))/((-a)^(1/8)*b^(7/8)) + (2^(1/2)*a
tan((2^(1/2)*(-a)^(1/8)*x*(1/2 + 1i/2))/b^(1/8))*(1/8 - 1i/8))/((-a)^(1/8)
*b^(7/8))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.58

$$\int \frac{1}{b+ax^8}\,dx = \frac{-2\sqrt{\sqrt{2}+2}\,atan\left(\frac{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2a^{\frac{1}{4}}x}{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\,atan\left(\frac{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}+2a^{\frac{1}{4}}x}{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)-2\sqrt{-\sqrt{2}+2}\,atan\left(\frac{b^{\frac{1}{8}}a^{\frac{1}{8}}\cdots}{b^{\frac{1}{8}}\cdots}\right)\cdots}{\cdots}$$

input
```
int(1/(a*x^8+b),x)
```

output

```
(b**(1/8)*a**(7/8)*( - 2*sqrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) + 2*s
qrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*a**(1/4)
*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt( - sqrt(2) + 2)*atan((
b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2))) + 2*sqrt( - sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt(
sqrt(2) + 2) + 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) - s
qrt( - sqrt(2) + 2)*log( - b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*x + a**(
1/4)*x**2 + b**(1/4)) + sqrt( - sqrt(2) + 2)*log(b**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4)) - sqrt(sqrt(2) + 2)*log( - b**
(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4)) + sqrt(sqrt
(2) + 2)*log(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1
/4))))/(16*a*b)
```

3.104 $\int \frac{1}{b+ax^{12}} dx$

Optimal result

Integrand size = 9, antiderivative size = 831

$$\int \frac{1}{b+ax^{12}}\,dx = -\frac{\arctan\left(\frac{\sqrt[12]{b}-\sqrt{2}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{6\sqrt{2}\sqrt[12]{a}b^{11/12}}+\frac{\arctan\left(\frac{\sqrt[12]{b}+\sqrt{2}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{6\sqrt{2}\sqrt[12]{a}b^{11/12}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{b}-2\sqrt{2-\sqrt{3}}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{12\sqrt[12]{a}b^{11/12}}$$
$$+\frac{\sqrt{2+\sqrt{3}}\arctan\left(\frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{b}+2\sqrt{2-\sqrt{3}}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{12\sqrt[12]{a}b^{11/12}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sqrt[12]{b}+\sqrt{3}\sqrt[12]{b}-2\sqrt{2+\sqrt{3}}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{12\sqrt[12]{a}b^{11/12}}$$
$$+\frac{\sqrt{2-\sqrt{3}}\arctan\left(\frac{2\sqrt[12]{b}+\sqrt{3}\sqrt[12]{b}+2\sqrt{2+\sqrt{3}}\sqrt[12]{a}x}{\sqrt[12]{b}}\right)}{12\sqrt[12]{a}b^{11/12}}$$
$$-\frac{\log\left(\sqrt[6]{b}-\sqrt{2}\sqrt[12]{a}\sqrt[12]{b}x+\sqrt[6]{a}x^2\right)}{12\sqrt{2}\sqrt[12]{a}b^{11/12}}$$
$$+\frac{\log\left(\sqrt[6]{b}+\sqrt{2}\sqrt[12]{a}\sqrt[12]{b}x+\sqrt[6]{a}x^2\right)}{12\sqrt{2}\sqrt[12]{a}b^{11/12}}$$
$$-\frac{\log\left(\sqrt[3]{b}-\sqrt{2}\sqrt[12]{a}\sqrt[4]{b}x+\sqrt[6]{a}\sqrt[6]{b}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[12]{b}x^3+\sqrt[3]{a}x^4\right)}{24\sqrt{2}\sqrt[12]{a}b^{11/12}}$$
$$+\frac{\log\left(\sqrt[3]{b}+\sqrt{2}\sqrt[12]{a}\sqrt[4]{b}x+\sqrt[6]{a}\sqrt[6]{b}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[12]{b}x^3+\sqrt[3]{a}x^4\right)}{24\sqrt{2}\sqrt[12]{a}b^{11/12}}$$
$$-\frac{\log\left(\sqrt[3]{b}-\sqrt{6}\sqrt[12]{a}\sqrt[4]{b}x+3\sqrt[6]{a}\sqrt[6]{b}x^2-\sqrt{6}\sqrt[4]{a}\sqrt[12]{b}x^3+\sqrt[3]{a}x^4\right)}{8\sqrt{6}\sqrt[12]{a}b^{11/12}}$$
$$+\frac{\log\left(\sqrt[3]{b}+\sqrt{6}\sqrt[12]{a}\sqrt[4]{b}x+3\sqrt[6]{a}\sqrt[6]{b}x^2+\sqrt{6}\sqrt[4]{a}\sqrt[12]{b}x^3+\sqrt[3]{a}x^4\right)}{8\sqrt{6}\sqrt[12]{a}b^{11/12}}$$

output
```
-1/12*arctan((b^(1/12)-2^(1/2)*a^(1/12)*x)/b^(1/12))*2^(1/2)/a^(1/12)/b^(1
1/12)+1/12*arctan((b^(1/12)+2^(1/2)*a^(1/12)*x)/b^(1/12))*2^(1/2)/a^(1/12)
/b^(11/12)-1/12*(1/2*6^(1/2)+1/2*2^(1/2))*arctan((2*b^(1/12)-3^(1/2)*b^(1/
12)-2*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/12)*x)/b^(1/12))/a^(1/12)/b^(11/12)+1
/12*(1/2*6^(1/2)+1/2*2^(1/2))*arctan((2*b^(1/12)-3^(1/2)*b^(1/12)+2*(1/2*6
^(1/2)-1/2*2^(1/2))*a^(1/12)*x)/b^(1/12))/a^(1/12)/b^(11/12)-1/12*(1/2*6^(
1/2)-1/2*2^(1/2))*arctan((2*b^(1/12)+3^(1/2)*b^(1/12)-2*(1/2*6^(1/2)+1/2*2
^(1/2))*a^(1/12)*x)/b^(1/12))/a^(1/12)/b^(11/12)+1/12*(1/2*6^(1/2)-1/2*2^(
1/2))*arctan((2*b^(1/12)+3^(1/2)*b^(1/12)+2*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1
/12)*x)/b^(1/12))/a^(1/12)/b^(11/12)-1/24*ln(b^(1/6)-2^(1/2)*a^(1/12)*b^(1
/12)*x+a^(1/6)*x^2)*2^(1/2)/a^(1/12)/b^(11/12)+1/24*ln(b^(1/6)+2^(1/2)*a^(
1/12)*b^(1/12)*x+a^(1/6)*x^2)*2^(1/2)/a^(1/12)/b^(11/12)-1/48*ln(b^(1/3)-2
^(1/2)*a^(1/12)*b^(1/4)*x+a^(1/6)*b^(1/6)*x^2-2^(1/2)*a^(1/4)*b^(1/12)*x^3
+a^(1/3)*x^4)*2^(1/2)/a^(1/12)/b^(11/12)+1/48*ln(b^(1/3)+2^(1/2)*a^(1/12)*
b^(1/4)*x+a^(1/6)*b^(1/6)*x^2+2^(1/2)*a^(1/4)*b^(1/12)*x^3+a^(1/3)*x^4)*2^
(1/2)/a^(1/12)/b^(11/12)-1/48*ln(b^(1/3)-6^(1/2)*a^(1/12)*b^(1/4)*x+3*a^(1
/6)*b^(1/6)*x^2-6^(1/2)*a^(1/4)*b^(1/12)*x^3+a^(1/3)*x^4)*6^(1/2)/a^(1/12)
/b^(11/12)+1/48*ln(b^(1/3)+6^(1/2)*a^(1/12)*b^(1/4)*x+3*a^(1/6)*b^(1/6)*x^
2+6^(1/2)*a^(1/4)*b^(1/12)*x^3+a^(1/3)*x^4)*6^(1/2)/a^(1/12)/b^(11/12)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 778, normalized size of antiderivative = 0.94

$$\int \frac{1}{b+ax^{12}}\,dx$$

$$= \frac{-2\left(1+\sqrt{3}\right)\arctan\left(\frac{1-\sqrt{3}-\frac{2\sqrt{2}\sqrt[12]{a}x}{\sqrt[12]{b}}}{1+\sqrt{3}}\right)+2\left(-1+\sqrt{3}\right)\arctan\left(\frac{1+\sqrt{3}-\frac{2\sqrt{2}\sqrt[12]{a}x}{\sqrt[12]{b}}}{1-\sqrt{3}}\right)-4\arctan\left(1-\frac{\sqrt{2}\sqrt[12]{a}x}{\sqrt[12]{b}}\right.}{}$$

input
```
Integrate[(b + a*x^12)^(-1),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTan[(1 - Sqrt[3] - (2*Sqrt[2]*a^(1/12)*x)/b^(1/12))/(
1 + Sqrt[3])] + 2*(-1 + Sqrt[3])*ArcTan[(1 + Sqrt[3] - (2*Sqrt[2]*a^(1/12)
*x)/b^(1/12))/(1 - Sqrt[3])] - 4*ArcTan[1 - (Sqrt[2]*a^(1/12)*x)/b^(1/12)]
 + 4*ArcTan[1 + (Sqrt[2]*a^(1/12)*x)/b^(1/12)] + 2*ArcTan[(1 - Sqrt[3] + (
2*Sqrt[2]*a^(1/12)*x)/b^(1/12))/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(1 - Sqr
t[3] + (2*Sqrt[2]*a^(1/12)*x)/b^(1/12))/(1 + Sqrt[3])] + 2*ArcTan[(1 + Sqr
t[3] + (2*Sqrt[2]*a^(1/12)*x)/b^(1/12))/(1 - Sqrt[3])] - 2*Sqrt[3]*ArcTan[
(1 + Sqrt[3] + (2*Sqrt[2]*a^(1/12)*x)/b^(1/12))/(1 - Sqrt[3])] - 2*Log[b^(
1/6) - Sqrt[2]*a^(1/12)*b^(1/12)*x + a^(1/6)*x^2] + 2*Log[b^(1/6) + Sqrt[2
]*a^(1/12)*b^(1/12)*x + a^(1/6)*x^2] + Log[2*b^(1/6) - Sqrt[2]*(-1 + Sqrt[
3])*a^(1/12)*b^(1/12)*x + 2*a^(1/6)*x^2] - Sqrt[3]*Log[2*b^(1/6) - Sqrt[2]
*(-1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a^(1/6)*x^2] - Log[2*b^(1/6) + Sqr
t[2]*(-1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a^(1/6)*x^2] + Sqrt[3]*Log[2*b
^(1/6) + Sqrt[2]*(-1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a^(1/6)*x^2] - Log
[2*b^(1/6) - Sqrt[2]*(1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a^(1/6)*x^2] -
Sqrt[3]*Log[2*b^(1/6) - Sqrt[2]*(1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a^(1
/6)*x^2] + Log[2*b^(1/6) + Sqrt[2]*(1 + Sqrt[3])*a^(1/12)*b^(1/12)*x + 2*a
^(1/6)*x^2] + Sqrt[3]*Log[2*b^(1/6) + Sqrt[2]*(1 + Sqrt[3])*a^(1/12)*b^(1/
12)*x + 2*a^(1/6)*x^2])/(24*Sqrt[2]*a^(1/12)*b^(11/12))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.56, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.444, Rules used = {758, 753, 27, 218, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^{12}+b}dx$$

$$\Big\downarrow \text{758}$$

$$\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^6}dx}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{-a}x^6+\sqrt{b}}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{753}$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{2\left(\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}\right)}dx}{3b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{2\left(\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}\right)}dx}{3b^{5/12}}+\frac{\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt[6]{b}}dx}{3\sqrt[3]{b}}}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^6}dx}{2\sqrt{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt[6]{b}}dx}{3\sqrt[3]{b}}}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^6}dx}{2\sqrt{b}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\arctan\left(\frac{\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{3\sqrt[12]{-a}b^{5/12}}}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^6}dx}{2\sqrt{b}}$$

$$\Big\downarrow \; 754$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\arctan\left(\frac{\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{3\sqrt[12]{-a}b^{5/12}}}{2\sqrt{b}}+\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt[12]{-a}x}{2\left(\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}\right)}dx}{3b^{5/12}}+\frac{\int \frac{\sqrt[12]{-a}x+2\sqrt[12]{b}}{2\left(\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}\right)}dx}{3b^{5/12}}+\frac{\int \frac{1}{\sqrt[6]{b}-\sqrt[6]{-a}x^2}dx}{3\sqrt[3]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\arctan\left(\frac{\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{3\sqrt[12]{-a}b^{5/12}}}{2\sqrt{b}}+\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{1}{\sqrt[6]{b}-\sqrt[6]{-a}x^2}dx}{3\sqrt[3]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt{3}\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt{3}\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\arctan\left(\frac{\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{3\sqrt[12]{-a}b^{5/12}}}{2\sqrt{b}}+$$
$$\frac{\frac{\int \frac{2\sqrt[12]{b}-\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\int \frac{\sqrt[12]{-a}x+2\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\text{arctanh}\left(\frac{\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{3\sqrt[12]{-a}b^{5/12}}}{2\sqrt{b}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{\sqrt{3}\int -\frac{\sqrt[12]{-a}\left(\sqrt{3}\sqrt[12]{b}-2\sqrt[12]{-a}x\right)}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\cdots}{6b^{5/12}}\cdots}{2\sqrt{b}}$$
$$\frac{\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{\int -\frac{\sqrt[12]{-a}\left(\sqrt[12]{b}-2\sqrt[12]{-a}x\right)}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\cdots}{6b^{5/12}}\cdots}{2\sqrt{b}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[12]{-a}\left(\sqrt{3}\sqrt[12]{b}-2\sqrt[12]{-a}x\right)}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\cdots}{6b^{5/12}}\cdots}{2\sqrt{b}}$$
$$\frac{\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{\int \frac{\sqrt[12]{-a}\left(\sqrt[12]{b}-2\sqrt[12]{-a}x\right)}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\cdots}{6b^{5/12}}\cdots}{2\sqrt{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\frac{1}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\cdots}{6b^{5/\cdots}}}{2\sqrt{b}}$$

$$\frac{\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{1}{2}\int \frac{\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx}{6b^{5/12}}+\frac{\frac{3}{2}\sqrt[12]{b}\int \frac{1}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{1}{2}\int \frac{\cdots}{\sqrt[6]{-a}\cdots}}{6b^{5/12}}}{2\sqrt{b}}$$

$\downarrow$ 1082

$$\frac{\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt{3}\sqrt[12]{b}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt{3}\sqrt[12]{b}}\right)}{\sqrt{3}\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[12]{-a}x+\sqrt{3}\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{\int \cdots}{\cdots}}{6b^{5/12}}}{2\sqrt{b}}$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}\right)}{\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\int \frac{2\sqrt[12]{-a}x+\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[12]{-a}\cdots}{\sqrt[12]{b}}\cdots\right)\cdots}\cdots}{\cdots}}{6b^{5/12}}}{2\sqrt{b}}$$

$\downarrow$ 217

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}}{\sqrt{3}}\right)}{\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\int \frac{2\sqrt[12]{-a}x+\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}+1}{\sqrt{3}}\right)}{\sqrt[12]{-a}}}{6b^{5/12}}+\cdots}{2\sqrt{b}}$$

$$\frac{\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[12]{b}-2\sqrt[12]{-a}x}{\sqrt[6]{-a}x^2-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt{3}\sqrt[12]{b}}\right)\right)}{\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[12]{-a}x+\sqrt{3}\sqrt[12]{b}}{\sqrt[6]{-a}x^2+\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{b}}dx+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[12]{-\cdots}}{\sqrt{3}\sqrt[12]{\cdots}}\cdots\right)\right)}{\sqrt[12]{-a}\cdots}}{6b^{5/12}}}{2\sqrt{b}}$$

$\downarrow$ 1103

$$\frac{\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}}{\sqrt{3}}\right)}{\sqrt[12]{-a}}-\frac{\log\left(-\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{-a}x^2+\sqrt[6]{b}\right)}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[12]{-a}x}{\sqrt[12]{b}}+1}{\sqrt{3}}\right)}{\sqrt[12]{-a}}+\frac{\log\left(\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{-a}x^2+\sqrt[6]{b}\right)}{2\sqrt[12]{-a}}}{6b^{5/12}}+}{2\sqrt{b}}$$

$$\frac{\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[12]{-a}x}{\sqrt{3}\sqrt[12]{b}}\right)\right)}{\sqrt[12]{-a}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{-a}x^2+\sqrt[6]{b}\right)}{2\sqrt[12]{-a}}}{6b^{5/12}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[12]{-a}x}{\sqrt{3}\sqrt[12]{b}}+1\right)\right)}{\sqrt[12]{-a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[12]{-a}\sqrt[12]{b}x+\sqrt[6]{-}\right.}{2\sqrt[12]{-a}}}{6b^{5/12}}}{2\sqrt{b}}$$

input
```
Int[(b + a*x^12)^(-1),x]
```

output
```
(ArcTanh[((-a)^(1/12)*x)/b^(1/12)]/(3*(-a)^(1/12)*b^(5/12)) + (-((Sqrt[3]*
ArcTan[(1 - (2*(-a)^(1/12)*x)/b^(1/12))/Sqrt[3]])/(-a)^(1/12)) - Log[b^(1/
6) - (-a)^(1/12)*b^(1/12)*x + (-a)^(1/6)*x^2]/(2*(-a)^(1/12)))/(6*b^(5/12)
) + ((Sqrt[3]*ArcTan[(1 + (2*(-a)^(1/12)*x)/b^(1/12))/Sqrt[3]])/(-a)^(1/12
) + Log[b^(1/6) + (-a)^(1/12)*b^(1/12)*x + (-a)^(1/6)*x^2]/(2*(-a)^(1/12))
)/(6*b^(5/12)))/(2*Sqrt[b]) + (ArcTan[((-a)^(1/12)*x)/b^(1/12)]/(3*(-a)^(1
/12)*b^(5/12)) + (-(ArcTan[Sqrt[3]*(1 - (2*(-a)^(1/12)*x)/(Sqrt[3]*b^(1/12
)))]/(-a)^(1/12)) - (Sqrt[3]*Log[b^(1/6) - Sqrt[3]*(-a)^(1/12)*b^(1/12)*x
+ (-a)^(1/6)*x^2])/(2*(-a)^(1/12)))/(6*b^(5/12)) + (ArcTan[Sqrt[3]*(1 + (2
*(-a)^(1/12)*x)/(Sqrt[3]*b^(1/12)))]/(-a)^(1/12) + (Sqrt[3]*Log[b^(1/6) +
Sqrt[3]*(-a)^(1/12)*b^(1/12)*x + (-a)^(1/6)*x^2])/(2*(-a)^(1/12)))/(6*b^(5
/12)))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{12}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{11}}}{12a}$	27
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{12}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{11}}}{12a}$	27

input
```
int(1/(a*x^12+b),x,method=_RETURNVERBOSE)
```

output
```
1/12/a*sum(1/_R^11*ln(x-_R),_R=RootOf(_Z^12*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.77

$$\int \frac{1}{b+ax^{12}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x^12+b),x, algorithm="fricas")
```

output
```
1/12*(1/2*I*sqrt(3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/12)*log(b*(1/2*I*sqrt(3)
 + 1/2)^(3/2)*(-1/(a*b^11))^(1/12) + x) - 1/12*(1/2*I*sqrt(3) + 1/2)^(3/2)
*(-1/(a*b^11))^(1/12)*log(-b*(1/2*I*sqrt(3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/
12) + x) + 1/24*(-I*sqrt(3) - 1)*(-1/(a*b^11))^(1/12)*log(1/2*b*(-I*sqrt(3
) - 1)*(-1/(a*b^11))^(1/12) + x) - 1/24*(-I*sqrt(3) - 1)*(-1/(a*b^11))^(1/
12)*log(-1/2*b*(-I*sqrt(3) - 1)*(-1/(a*b^11))^(1/12) + x) + 1/12*((1/2*I*s
qrt(3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/12) - sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(
a*b^11))^(1/12))*log(b*(1/2*I*sqrt(3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/12) -
b*sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(a*b^11))^(1/12) + x) - 1/12*((1/2*I*sqrt(
3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/12) - sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(a*b^
11))^(1/12))*log(-b*(1/2*I*sqrt(3) + 1/2)^(3/2)*(-1/(a*b^11))^(1/12) + b*s
qrt(1/2*I*sqrt(3) + 1/2)*(-1/(a*b^11))^(1/12) + x) + 1/24*((-I*sqrt(3) - 1
)*(-1/(a*b^11))^(1/12) + 2*(-1/(a*b^11))^(1/12))*log(1/2*b*(-I*sqrt(3) - 1
)*(-1/(a*b^11))^(1/12) + b*(-1/(a*b^11))^(1/12) + x) - 1/24*((-I*sqrt(3) -
 1)*(-1/(a*b^11))^(1/12) + 2*(-1/(a*b^11))^(1/12))*log(-1/2*b*(-I*sqrt(3)
- 1)*(-1/(a*b^11))^(1/12) - b*(-1/(a*b^11))^(1/12) + x) + 1/12*sqrt(1/2*I*
sqrt(3) + 1/2)*(-1/(a*b^11))^(1/12)*log(b*sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(a
*b^11))^(1/12) + x) - 1/12*sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(a*b^11))^(1/12)*
log(-b*sqrt(1/2*I*sqrt(3) + 1/2)*(-1/(a*b^11))^(1/12) + x) + 1/12*(-1/(a*b
^11))^(1/12)*log(b*(-1/(a*b^11))^(1/12) + x) - 1/12*(-1/(a*b^11))^(1/12...
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.02

$$\int \frac{1}{b+ax^{12}}\,dx = \text{RootSum}\left(8916100448256t^{12}ab^{11}+1,\left(t\mapsto t\log\left(12tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**12+b),x)
```

output
```
RootSum(8916100448256*_t**12*a*b**11 + 1, Lambda(_t, _t*log(12*_t*b + x)))
```

Maxima [F]

$$\int \frac{1}{b+ax^{12}}\,dx=\int \frac{1}{ax^{12}+b}\,dx$$

input
```
integrate(1/(a*x^12+b),x, algorithm="maxima")
```

output
```
integrate(1/(a*x^12 + b), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.78

$$\int \frac{1}{b+ax^{12}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a*x^12+b),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + sqrt(2))*(b/a)^(1/12)*arctan(((sqrt(6) - sqrt(2))*(b/a)^(1
/12) + 4*x)/((sqrt(6) + sqrt(2))*(b/a)^(1/12)))/b + 1/24*(sqrt(6) + sqrt(2
))*(b/a)^(1/12)*arctan(-((sqrt(6) - sqrt(2))*(b/a)^(1/12) - 4*x)/((sqrt(6)
 + sqrt(2))*(b/a)^(1/12)))/b + 1/12*sqrt(2)*(b/a)^(1/12)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(b/a)^(1/12))/(b/a)^(1/12))/b + 1/12*sqrt(2)*(b/a)^(1/12)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(b/a)^(1/12))/(b/a)^(1/12))/b + 1/24*sq
rt(2)*(b/a)^(1/12)*log(x^2 + sqrt(2)*x*(b/a)^(1/12) + (b/a)^(1/6))/b - 1/2
4*sqrt(2)*(b/a)^(1/12)*log(x^2 - sqrt(2)*x*(b/a)^(1/12) + (b/a)^(1/6))/b +
 1/6*(b/a)^(1/12)*arctan(((sqrt(6) + sqrt(2))*(b/a)^(1/12) + 4*x)/((sqrt(6
) - sqrt(2))*(b/a)^(1/12)))/(b*(sqrt(6) + sqrt(2))) + 1/6*(b/a)^(1/12)*arc
tan(-((sqrt(6) + sqrt(2))*(b/a)^(1/12) - 4*x)/((sqrt(6) - sqrt(2))*(b/a)^(
1/12)))/(b*(sqrt(6) + sqrt(2))) + 1/12*(b/a)^(1/12)*log(x^2 + 1/2*x*(sqrt(
6)*(b/a)^(1/12) + sqrt(2)*(b/a)^(1/12)) + (b/a)^(1/6))/(b*(sqrt(6) - sqrt(
2))) - 1/12*(b/a)^(1/12)*log(x^2 - 1/2*x*(sqrt(6)*(b/a)^(1/12) + sqrt(2)*(
b/a)^(1/12)) + (b/a)^(1/6))/(b*(sqrt(6) - sqrt(2))) + 1/12*(b/a)^(1/12)*lo
g(x^2 + 1/2*x*(sqrt(6)*(b/a)^(1/12) - sqrt(2)*(b/a)^(1/12)) + (b/a)^(1/6))
/(b*(sqrt(6) + sqrt(2))) - 1/12*(b/a)^(1/12)*log(x^2 - 1/2*x*(sqrt(6)*(b/a
)^(1/12) - sqrt(2)*(b/a)^(1/12)) + (b/a)^(1/6))/(b*(sqrt(6) + sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.52

$$\int \frac{1}{b+ax^{12}}\,dx = \text{Too large to display}$$

input
```
int(1/(b + a*x^12),x)
```

output
```
atan(((-a)^(1/12)*x)/b^(1/12))/(6*(-a)^(1/12)*b^(11/12)) - (atan(((-a)^(1/
12)*x*1i)/b^(1/12))*1i)/(6*(-a)^(1/12)*b^(11/12)) - (atan(((-a)^(131/12)*x
*1i)/(b^(11/12)*((-a)^(65/6)/b^(5/6) + (3^(1/2)*(-a)^(65/6)*1i)/b^(5/6)))
+ (3^(1/2)*(-a)^(131/12)*x)/(b^(11/12)*((-a)^(65/6)/b^(5/6) + (3^(1/2)*(-a
)^(65/6)*1i)/b^(5/6))))*(3^(1/2)*1i - 1)*1i)/(12*(-a)^(1/12)*b^(11/12)) +
(atan(((-a)^(131/12)*x*1i)/(b^(11/12)*((-a)^(65/6)/b^(5/6) - (3^(1/2)*(-a)
^(65/6)*1i)/b^(5/6))) - (3^(1/2)*(-a)^(131/12)*x)/(b^(11/12)*((-a)^(65/6)/
b^(5/6) - (3^(1/2)*(-a)^(65/6)*1i)/b^(5/6))))*(3^(1/2)*1i + 1)*1i)/(12*(-a
)^(1/12)*b^(11/12)) - (atan(((-a)^(131/12)*x)/(b^(11/12)*((-a)^(65/6)/b^(5
/6) - (3^(1/2)*(-a)^(65/6)*1i)/b^(5/6))) + (3^(1/2)*(-a)^(131/12)*x*1i)/(b
^(11/12)*((-a)^(65/6)/b^(5/6) - (3^(1/2)*(-a)^(65/6)*1i)/b^(5/6))))*(3^(1/
2) - 1i)*1i)/(12*(-a)^(1/12)*b^(11/12)) + (atan(((-a)^(131/12)*x)/(b^(11/1
2)*((-a)^(65/6)/b^(5/6) + (3^(1/2)*(-a)^(65/6)*1i)/b^(5/6))) - (3^(1/2)*(-
a)^(131/12)*x*1i)/(b^(11/12)*((-a)^(65/6)/b^(5/6) + (3^(1/2)*(-a)^(65/6)*1
i)/b^(5/6))))*(3^(1/2) + 1i)*1i)/(12*(-a)^(1/12)*b^(11/12))
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 649, normalized size of antiderivative = 0.78

$$\int \frac{1}{b+ax^{12}}\,dx = \text{Too large to display}$$

input
```
int(1/(a*x^12+b),x)
```

output
```
(b**(1/12)*a**(1/4)*( - 4*sqrt( - sqrt(3) + 2)*atan((b**(1/12)*a**(1/12)*s
qrt(6) + b**(1/12)*a**(1/12)*sqrt(2) - 4*a**(1/6)*x)/(2*b**(1/12)*a**(1/12
)*sqrt( - sqrt(3) + 2))) + 4*sqrt( - sqrt(3) + 2)*atan((b**(1/12)*a**(1/12
)*sqrt(6) + b**(1/12)*a**(1/12)*sqrt(2) + 4*a**(1/6)*x)/(2*b**(1/12)*a**(1
/12)*sqrt( - sqrt(3) + 2))) - 4*sqrt(2)*atan((b**(1/12)*a**(1/12)*sqrt(2)
 - 2*a**(1/6)*x)/(b**(1/12)*a**(1/12)*sqrt(2))) + 4*sqrt(2)*atan((b**(1/12)
*a**(1/12)*sqrt(2) + 2*a**(1/6)*x)/(b**(1/12)*a**(1/12)*sqrt(2))) - 2*sqrt
(6)*atan((2*b**(1/12)*a**(1/12)*sqrt( - sqrt(3) + 2) - 4*a**(1/6)*x)/(b**(
1/12)*a**(1/12)*sqrt(6) + b**(1/12)*a**(1/12)*sqrt(2))) - 2*sqrt(2)*atan((
2*b**(1/12)*a**(1/12)*sqrt( - sqrt(3) + 2) - 4*a**(1/6)*x)/(b**(1/12)*a**(
1/12)*sqrt(6) + b**(1/12)*a**(1/12)*sqrt(2))) + 2*sqrt(6)*atan((2*b**(1/12
)*a**(1/12)*sqrt( - sqrt(3) + 2) + 4*a**(1/6)*x)/(b**(1/12)*a**(1/12)*sqrt
(6) + b**(1/12)*a**(1/12)*sqrt(2))) + 2*sqrt(2)*atan((2*b**(1/12)*a**(1/12
)*sqrt( - sqrt(3) + 2) + 4*a**(1/6)*x)/(b**(1/12)*a**(1/12)*sqrt(6) + b**(
1/12)*a**(1/12)*sqrt(2))) - 2*sqrt( - sqrt(3) + 2)*log( - b**(1/12)*a**(1/
12)*sqrt( - sqrt(3) + 2)*x + a**(1/6)*x**2 + b**(1/6)) + 2*sqrt( - sqrt(3)
 + 2)*log(b**(1/12)*a**(1/12)*sqrt( - sqrt(3) + 2)*x + a**(1/6)*x**2 + b**
(1/6)) - sqrt(6)*log(( - b**(1/12)*a**(1/12)*sqrt(6)*x - b**(1/12)*a**(1/1
2)*sqrt(2)*x + 2*a**(1/6)*x**2 + 2*b**(1/6))/2) + sqrt(6)*log((b**(1/12)*a
**(1/12)*sqrt(6)*x + b**(1/12)*a**(1/12)*sqrt(2)*x + 2*a**(1/6)*x**2 + ...
```

3.105 $\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx = \frac{-7887a^9x^2+22a^6x^5+931a^3x^8-104x^{11}}{202500a^{15}(6a^6+a^3x^3-x^6)^2} + \frac{671\arctan\left(\frac{\sqrt{3}a+2\sqrt[6]{3}x}{3a}\right)}{253125\ 3^{5/6}a^{16}} + \frac{89\arctan\left(\frac{\sqrt{3}a-2^{2/3}\sqrt{3}x}{3a}\right)}{28125\sqrt[3]{2}\sqrt{3}a^{16}} + \frac{89\log\left(\sqrt[3]{2}a+x\right)}{84375\sqrt[3]{2}a^{16}} + \frac{671\log\left(-\sqrt[3]{3}a+x\right)}{759375\sqrt[3]{3}a^{16}} - \frac{89\log\left(2^{2/3}a^2-\sqrt[3]{2}ax+x^2\right)}{168750\sqrt[3]{2}a^{16}} - \frac{671\log\left(3^{2/3}a^2+\sqrt[3]{3}ax+x^2\right)}{1518750\sqrt[3]{3}a^{16}}$$

output

```
1/202500*(-7887*a^9*x^2+22*a^6*x^5+931*a^3*x^8-104*x^11)/a^15/(6*a^6+a^3*x
^3-x^6)^2+671/759375*arctan(1/3*(3^(1/2)*a+2*3^(1/6)*x)/a)*3^(1/6)/a^16+89
/168750*arctan(1/3*(3^(1/2)*a-2^(2/3)*x*3^(1/2))/a)*2^(2/3)*3^(1/2)/a^16+8
9/168750*ln(2^(1/3)*a+x)*2^(2/3)/a^16+671/2278125*ln(-3^(1/3)*a+x)*3^(2/3)
/a^16-89/337500*ln(2^(2/3)*a^2-2^(1/3)*a*x+x^2)*2^(2/3)/a^16-671/4556250*l
n(3^(2/3)*a^2+3^(1/3)*a*x+x^2)*3^(2/3)/a^16
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95

$$\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx = \frac{\frac{10125a^7(-13a^3x^2+x^5)}{(6a^6+a^3x^3-x^6)^2}+\frac{45a(827a^3x^2-104x^5)}{-6a^6-a^3x^3+x^6}-4806\,2^{2/3}\sqrt{3}\arctan\left(\frac{-a+2^{2/3}x}{\sqrt{3}a}\right)+8052\sqrt[6]{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2x}{3^{5/6}a}\right)}{}$$

input

```
Integrate[x/((-3*a^3 + x^3)^3*(2*a^3 + x^3)^3),x]
```

output

```
((10125*a^7*(-13*a^3*x^2 + x^5))/(6*a^6 + a^3*x^3 - x^6)^2 + (45*a*(827*a^
3*x^2 - 104*x^5))/(-6*a^6 - a^3*x^3 + x^6) - 4806*2^(2/3)*Sqrt[3]*ArcTan[(
-a + 2^(2/3)*x)/(Sqrt[3]*a)] + 8052*3^(1/6)*ArcTan[1/Sqrt[3] + (2*x)/(3^(5
/6)*a)] + 4806*2^(2/3)*Log[2*a + 2^(2/3)*x] + 2684*3^(2/3)*Log[3*a - 3^(2/
3)*x] - 2403*2^(2/3)*Log[2*a^2 - 2^(2/3)*a*x + 2^(1/3)*x^2] - 1342*3^(2/3)
*Log[3*a^2 + 3^(2/3)*a*x + 3^(1/3)*x^2])/(9112500*a^16)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {972, 27, 1049, 27, 1049, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^3-3a^3)^3(2a^3+x^3)^3}\,dx$$

$$\Big\downarrow \text{972}$$

$$\frac{\int -\frac{2x(13a^3+5x^3)}{(3a^3-x^3)^2(2a^3+x^3)^3}dx}{90a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{x(13a^3+5x^3)}{(3a^3-x^3)^2(2a^3+x^3)^3}dx}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{1049}$$

$$-\frac{\frac{\int \frac{a^3x(83a^3+196x^3)}{(3a^3-x^3)(2a^3+x^3)^3}dx}{45a^6}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{x(83a^3+196x^3)}{(3a^3-x^3)(2a^3+x^3)^3}dx}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{1049}$$

$$-\frac{\frac{-\frac{\int -\frac{12a^3x(362a^3+103x^3)}{(3a^3-x^3)(2a^3+x^3)^2}dx}{60a^6}-\frac{103x^2}{20a^3(2a^3+x^3)^2}}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\int \frac{x(362a^3+103x^3)}{(3a^3-x^3)(2a^3+x^3)^2}dx}{5a^3}-\frac{103x^2}{20a^3(2a^3+x^3)^2}}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{1049}$$

$$-\frac{\frac{\frac{\frac{26x^2}{5a^3(2a^3+x^3)}-\frac{\int -\frac{6a^3x(749a^3-26x^3)}{(3a^3-x^3)(2a^3+x^3)}dx}{30a^6}}{5a^3}-\frac{103x^2}{20a^3(2a^3+x^3)^2}}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6(3a^3-x^3)^2(2a^3+x^3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\frac{\int \frac{x\left(749a^3-26x^3\right)}{\left(3a^3-x^3\right)\left(2a^3+x^3\right)}dx}{5a^3}+\frac{26x^2}{5a^3\left(2a^3+x^3\right)}}{5a^3}-\frac{103x^2}{20a^3\left(2a^3+x^3\right)^2}}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6\left(3a^3-x^3\right)^2\left(2a^3+x^3\right)^2}$$

$$\Big\downarrow \; 1054$$

$$-\frac{\frac{\frac{\frac{\int \left(\frac{801x}{5\left(2a^3+x^3\right)}-\frac{671x}{5\left(x^3-3a^3\right)}\right)dx}{5a^3}+\frac{26x^2}{5a^3\left(2a^3+x^3\right)}}{5a^3}-\frac{103x^2}{20a^3\left(2a^3+x^3\right)^2}}{45a^3}+\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}}{45a^6}-\frac{x^2}{90a^6\left(3a^3-x^3\right)^2\left(2a^3+x^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{x^2}{90a^6\left(3a^3-x^3\right)^2\left(2a^3+x^3\right)^2}-\frac{\frac{28x^2}{45a^3(3a^3-x^3)(2a^3+x^3)^2}+\frac{\frac{\frac{26x^2}{5a^3\left(2a^3+x^3\right)}+\frac{\frac{267\log\left(2^{2/3}a^2-\sqrt[3]{2}ax+x^2\right)}{10\sqrt[3]{2}a}+\frac{671\log\left(3^{2/3}a^2+\sqrt[3]{3}ax+x^2\right)}{30\sqrt[3]{3}a}-\frac{267\sqrt{3}\arctan\left(\frac{a-2^{2/3}x}{\sqrt{3}a}\right)}{5\sqrt[3]{2}a}-\frac{671\arctan\left(\cdots\right)}{5\ 3^{5/\cdots}}\cdots}{5a^3}}{5a^3}\cdots}{45a^3}}{45a^6}$$

input

```
Int[x/((-3*a^3 + x^3)^3*(2*a^3 + x^3)^3),x]
```

output

```
-1/90*x^2/(a^6*(3*a^3 - x^3)^2*(2*a^3 + x^3)^2) - ((28*x^2)/(45*a^3*(3*a^3
 - x^3)*(2*a^3 + x^3)^2) + ((-103*x^2)/(20*a^3*(2*a^3 + x^3)^2) + ((26*x^2
)/(5*a^3*(2*a^3 + x^3)) + ((-267*Sqrt[3]*ArcTan[(a - 2^(2/3)*x)/(Sqrt[3]*a
)])/(5*2^(1/3)*a) - (671*ArcTan[1/Sqrt[3] + (2*x)/(3^(5/6)*a)])/(5*3^(5/6)
*a) - (671*Log[3^(1/3)*a - x])/(15*3^(1/3)*a) - (267*Log[2^(1/3)*a + x])/(
5*2^(1/3)*a) + (267*Log[2^(2/3)*a^2 - 2^(1/3)*a*x + x^2])/(10*2^(1/3)*a) +
 (671*Log[3^(2/3)*a^2 + 3^(1/3)*a*x + x^2])/(30*3^(1/3)*a))/(5*a^3))/(5*a^
3))/(45*a^3))/(45*a^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.62

method	result
risch	$\frac{-\frac{26x^{11}}{50625a^{15}}+\frac{931x^{8}}{202500a^{12}}+\frac{11x^{5}}{101250a^{9}}-\frac{2629x^{2}}{67500a^{6}}}{(3a^{3}-x^{3})^{2}(2a^{3}+x^{3})^{2}}+\frac{89\left(\sum_{\_R=\mathrm{RootOf}\left(2a^{48}\_Z^{3}-1\right)}\_R\ln\left(\left(2022624591\_R^{3}a^{48}-542540405\right)x+102\right.\right.}{84375}$
default	$-\frac{\frac{\frac{445}{54}a^{3}x^{2}-\frac{185}{81}x^{5}}{\left(3a^{3}-x^{3}\right)^{2}}-\frac{671\,3^{\frac{2}{3}}\ln\left(x-3^{\frac{1}{3}}\left(a^{3}\right)^{\frac{1}{3}}\right)}{729\left(a^{3}\right)^{\frac{1}{3}}}+\frac{671\,3^{\frac{2}{3}}\ln\left(x^{2}+3^{\frac{1}{3}}\left(a^{3}\right)^{\frac{1}{3}}x+3^{\frac{2}{3}}\left(a^{3}\right)^{\frac{2}{3}}\right)}{1458\left(a^{3}\right)^{\frac{1}{3}}}-\frac{671\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3\left(a^{3}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{243\left(a^{3}\right)^{\frac{1}{3}}}}{3125a^{15}}-\frac{\frac{355}{36}a^{3}\cdots}{(2a\cdots}$

input
```
int(x/(-3*a^3+x^3)^3/(2*a^3+x^3)^3,x,method=_RETURNVERBOSE)
```

output
```
36*(-13/911250/a^15*x^11+931/7290000/a^12*x^8+11/3645000/a^9*x^5-2629/2430
000/a^6*x^2)/(3*a^3-x^3)^2/(2*a^3+x^3)^2+89/84375*sum(_R*ln((2022624591*_R
^3*a^48-542540405)*x+1027844802*a^81*_R^5+423621380*a^33*_R^2),_R=RootOf(2
*_Z^3*a^48-1))+671/759375*sum(_R*ln((5083075283*_R^3*a^48-2319387615)*x+18
12670266*a^81*_R^5+1270864140*a^33*_R^2),_R=RootOf(3*_Z^3*a^48-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.56

$$\int \frac{x}{(-3a^3+x^3)^3\,(2a^3+x^3)^3}\,dx =$$

$$-\frac{354915\,a^{10}x^{2}-990\,a^{7}x^{5}-41895\,a^{4}x^{8}+4680\,ax^{11}-28836\cdot 2^{\frac{1}{6}}\sqrt{\frac{1}{6}}(36\,a^{12}+12\,a^{9}x^{3}-11\,a^{6}x^{6}-2\,a^{3}\cdots}{\cdots}$$

input
```
integrate(x/(-3*a^3+x^3)^3/(2*a^3+x^3)^3,x, algorithm="fricas")
```

output
```
-1/9112500*(354915*a^10*x^2 - 990*a^7*x^5 - 41895*a^4*x^8 + 4680*a*x^11 -
28836*2^(1/6)*sqrt(1/6)*(36*a^12 + 12*a^9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 + x
^12)*arctan(2^(1/6)*sqrt(1/6)*(2^(1/3)*a - 2*x)/a) + 1342*3^(2/3)*(36*a^12
 + 12*a^9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 + x^12)*log(3^(2/3)*a^2 + 3^(1/3)*a
*x + x^2) + 2403*2^(2/3)*(36*a^12 + 12*a^9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 +
x^12)*log(2^(2/3)*a^2 - 2^(1/3)*a*x + x^2) - 2684*3^(2/3)*(36*a^12 + 12*a^
9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 + x^12)*log(-3^(1/3)*a + x) - 4806*2^(2/3)*
(36*a^12 + 12*a^9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 + x^12)*log(2^(1/3)*a + x)
- 8052*3^(1/6)*(36*a^12 + 12*a^9*x^3 - 11*a^6*x^6 - 2*a^3*x^9 + x^12)*arct
an(1/3*3^(1/6)*(3^(1/3)*a + 2*x)/a))/(36*a^28 + 12*a^25*x^3 - 11*a^22*x^6
- 2*a^19*x^9 + a^16*x^12)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.51

$$\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx$$
$$= \frac{-7887a^9x^2+22a^6x^5+931a^3x^8-104x^{11}}{7290000a^{27}+2430000a^{24}x^3-2227500a^{21}x^6-405000a^{18}x^9+202500a^{15}x^{12}} + \frac{\operatorname{RootSum}\left(1201354980468750t^3-704969,\left(t\mapsto t\log\left(\frac{528457385286173611879348754882812 50t^5a}{3343617664699171541}-\frac{250002181672}{3343617}\right.\right.\right.}{}$$

input
```
integrate(x/(-3*a**3+x**3)**3/(2*a**3+x**3)**3,x)
```

output
```
(-7887*a**9*x**2 + 22*a**6*x**5 + 931*a**3*x**8 - 104*x**11)/(7290000*a**2
7 + 2430000*a**24*x**3 - 2227500*a**21*x**6 - 405000*a**18*x**9 + 202500*a
**15*x**12) + (RootSum(1201354980468750*_t**3 - 704969, Lambda(_t, _t*log(
528457385286173611879348754882812 50*_t**5*a/3343617664699171541 - 25000218
16726978590820 3125*_t**2*a/3343617664699171541 + x))) + RootSum(1313681671
142578125*_t**3 - 302111711, Lambda(_t, _t*log(528457385286173611879348754
88281250*_t**5*a/3343617664699171541 - 2500021816726978590820 3125*_t**2*a/
3343617664699171541 + x))))/a**16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.85

$$\int \frac{x}{\left(-3a^3+x^3\right)^3\left(2a^3+x^3\right)^3}\,dx$$
$$= -\frac{7887\,a^9x^2-22\,a^6x^5-931\,a^3x^8+104\,x^{11}}{202500\,(36\,a^{27}+12\,a^{24}x^3-11\,a^{21}x^6-2\,a^{18}x^9+a^{15}x^{12})} - \frac{89\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a-2\,x\right)}{6\,a}\right)}{168750\,a^{16}} - \frac{671\cdot 3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}a^2+3^{\frac{1}{3}}ax+x^2\right)}{4556250\,a^{16}} - \frac{89\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^2-2^{\frac{1}{3}}ax+x^2\right)}{337500\,a^{16}} + \frac{671\cdot 3^{\frac{2}{3}}\log\left(-3^{\frac{1}{3}}a+x\right)}{2278125\,a^{16}} + \frac{89\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{1}{3}}a+x\right)}{168750\,a^{16}} + \frac{671\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(3^{\frac{1}{3}}a+2\,x\right)}{3\,a}\right)}{759375\,a^{16}}$$

input
```
integrate(x/(-3*a^3+x^3)^3/(2*a^3+x^3)^3,x, algorithm="maxima")
```

output
```
-1/202500*(7887*a^9*x^2 - 22*a^6*x^5 - 931*a^3*x^8 + 104*x^11)/(36*a^27 +
12*a^24*x^3 - 11*a^21*x^6 - 2*a^18*x^9 + a^15*x^12) - 89/168750*sqrt(3)*2^
(2/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a - 2*x)/a)/a^16 - 671/4556250*
3^(2/3)*log(3^(2/3)*a^2 + 3^(1/3)*a*x + x^2)/a^16 - 89/337500*2^(2/3)*log(
2^(2/3)*a^2 - 2^(1/3)*a*x + x^2)/a^16 + 671/2278125*3^(2/3)*log(-3^(1/3)*a
 + x)/a^16 + 89/168750*2^(2/3)*log(2^(1/3)*a + x)/a^16 + 671/759375*3^(1/6
)*arctan(1/3*3^(1/6)*(3^(1/3)*a + 2*x)/a)/a^16
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx$$
$$= \frac{89\sqrt{3}2^{\frac{2}{3}}\left(\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}(-a^3)^{\frac{1}{3}}\right)}{6(-a^3)^{\frac{1}{3}}}\right)}{168750\,a^{16}}$$
$$-\frac{89\cdot 2^{\frac{2}{3}}\left(\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)^2\log\left(x^2+2^{\frac{1}{3}}(-a^3)^{\frac{1}{3}}x+2^{\frac{2}{3}}(-a^3)^{\frac{2}{3}}\right)}{337500\,a^{16}}$$
$$-\frac{671\cdot 3^{\frac{2}{3}}\log\left(x^2+3^{\frac{1}{3}}(a^3)^{\frac{1}{3}}x+3^{\frac{2}{3}}(a^3)^{\frac{2}{3}}\right)}{4556250\,a^{16}}+\frac{671\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(2x+3^{\frac{1}{3}}(a^3)^{\frac{1}{3}}\right)}{3(a^3)^{\frac{1}{3}}}\right)}{759375\,a^{16}}$$
$$+\frac{89\cdot 2^{\frac{2}{3}}(-a^3)^{\frac{2}{3}}\log\left(\left|x-2^{\frac{1}{3}}(-a^3)^{\frac{1}{3}}\right|\right)}{168750\,a^{18}}+\frac{671\cdot 3^{\frac{2}{3}}(a^3)^{\frac{2}{3}}\log\left(\left|x-3^{\frac{1}{3}}(a^3)^{\frac{1}{3}}\right|\right)}{2278125\,a^{18}}$$
$$-\frac{7887\,a^9x^2-22\,a^6x^5-931\,a^3x^8+104\,x^{11}}{202500\,(6\,a^6+a^3x^3-x^6)^2a^{15}}$$

input
```
integrate(x/(-3*a^3+x^3)^3/(2*a^3+x^3)^3,x, algorithm="giac")
```

output
```
89/168750*sqrt(3)*2^(2/3)*(1/2*I*sqrt(3) + 1/2)^2*arctan(1/6*sqrt(3)*2^(2/
3)*(2*x + 2^(1/3)*(-a^3)^(1/3))/(-a^3)^(1/3))/a^16 - 89/337500*2^(2/3)*(1/
2*I*sqrt(3) + 1/2)^2*log(x^2 + 2^(1/3)*(-a^3)^(1/3)*x + 2^(2/3)*(-a^3)^(2/
3))/a^16 - 671/4556250*3^(2/3)*log(x^2 + 3^(1/3)*(a^3)^(1/3)*x + 3^(2/3)*(
a^3)^(2/3))/a^16 + 671/759375*3^(1/6)*arctan(1/3*3^(1/6)*(2*x + 3^(1/3)*(a
^3)^(1/3))/(a^3)^(1/3))/a^16 + 89/168750*2^(2/3)*(-a^3)^(2/3)*log(abs(x -
2^(1/3)*(-a^3)^(1/3)))/a^18 + 671/2278125*3^(2/3)*(a^3)^(2/3)*log(abs(x -
3^(1/3)*(a^3)^(1/3)))/a^18 - 1/202500*(7887*a^9*x^2 - 22*a^6*x^5 - 931*a^3
*x^8 + 104*x^11)/((6*a^6 + a^3*x^3 - x^6)^2*a^15)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.26

$$\int \frac{x}{\left(-3a^3+x^3\right)^3\left(2a^3+x^3\right)^3}\,dx$$

$$= \frac{89\,1201354980468750^{2/3}\ \ln\left(x+2^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{1201354980468750}$$

$$- \frac{\frac{2629\,x^2}{67500\,a^6}-\frac{11\,x^5}{101250\,a^9}-\frac{931\,x^8}{202500\,a^{12}}+\frac{26\,x^{11}}{50625\,a^{15}}}{36\,a^{12}+12\,a^9\,x^3-11\,a^6\,x^6-2\,a^3\,x^9+x^{12}}$$

$$+ \frac{671\,1313681671142578125^{2/3}\ \ln\left(x-3^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{1313681671142578125}$$

$$+ \frac{89\,1201354980468750^{2/3}\ \ln\left(x-\frac{2^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}}{2}-\frac{2^{1/3}\,\sqrt{3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\,1\mathrm{i}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{2402709960937500}$$

$$- \frac{89\,1201354980468750^{2/3}\ \ln\left(x-\frac{2^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\,1\mathrm{i}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{2402709960937500}$$

$$- \frac{671\,1313681671142578125^{2/3}\ \ln\left(x+\frac{3^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}}{2}-\frac{3^{5/6}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\,1\mathrm{i}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{2627363342285156250}$$

$$+ \frac{671\,1313681671142578125^{2/3}\ \ln\left(x+\frac{3^{1/3}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}}{2}+\frac{3^{5/6}\,a^{81}\,\left(\frac{1}{a^{48}}\right)^{5/3}\,1\mathrm{i}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(\frac{1}{a^{48}}\right)^{1/3}}{2627363342285156250}$$

input
```
int(-x/((2*a^3 + x^3)^3*(3*a^3 - x^3)^3),x)
```

output
```
(89*1201354980468750^(2/3)*log(x + 2^(1/3)*a^81*(1/a^48)^(5/3))*(1/a^48)^(
1/3))/1201354980468750 - ((2629*x^2)/(67500*a^6) - (11*x^5)/(101250*a^9) -
 (931*x^8)/(202500*a^12) + (26*x^11)/(50625*a^15))/(36*a^12 + x^12 - 2*a^3
*x^9 - 11*a^6*x^6 + 12*a^9*x^3) + (671*131368167114257812 5^(2/3)*log(x - 3
^(1/3)*a^81*(1/a^48)^(5/3))*(1/a^48)^(1/3))/1313681671142578125 + (89*1201
354980468750^(2/3)*log(x - (2^(1/3)*a^81*(1/a^48)^(5/3))/2 - (2^(1/3)*3^(1
/2)*a^81*(1/a^48)^(5/3)*1i)/2)*(3^(1/2)*1i - 1)*(1/a^48)^(1/3))/2402709960
937500 - (89*1201354980468750^(2/3)*log(x - (2^(1/3)*a^81*(1/a^48)^(5/3))/
2 + (2^(1/3)*3^(1/2)*a^81*(1/a^48)^(5/3)*1i)/2)*(3^(1/2)*1i + 1)*(1/a^48)^
(1/3))/2402709960937500 - (671*1313681671142578125^(2/3)*log(x + (3^(1/3)*
a^81*(1/a^48)^(5/3))/2 - (3^(5/6)*a^81*(1/a^48)^(5/3)*1i)/2)*(3^(1/2)*1i +
 1)*(1/a^48)^(1/3))/2627363342285156250 + (671*1313681671142578125^(2/3)*l
og(x + (3^(1/3)*a^81*(1/a^48)^(5/3))/2 + (3^(5/6)*a^81*(1/a^48)^(5/3)*1i)/
2)*(3^(1/2)*1i - 1)*(1/a^48)^(1/3))/2627363342285156250
```

Reduce [F]

$$\int \frac{x}{(-3a^3+x^3)^3(2a^3+x^3)^3}\,dx = -\left(\int \frac{x}{216a^{18}+108a^{15}x^3-90a^{12}x^6-35a^9x^9+15a^6x^{12}+3a^3x^{15}-x^{18}}dx\right)$$

input
```
int(x/(-3*a^3+x^3)^3/(2*a^3+x^3)^3,x)
```

output
```
 - int(x/(216*a**18 + 108*a**15*x**3 - 90*a**12*x**6 - 35*a**9*x**9 + 15*a
**6*x**12 + 3*a**3*x**15 - x**18),x)
```

3.106 $\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx = \frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b(a^4+2b^4)} + \frac{\arctan\left(\frac{a-\sqrt{2}x}{a}\right)}{2\sqrt{2}a(a^4+2b^4)} - \frac{\arctan\left(\frac{a+\sqrt{2}x}{a}\right)}{2\sqrt{2}a(a^4+2b^4)} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b(a^4+2b^4)} - \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a(a^4+2b^4)} + \frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a(a^4+2b^4)}$$

output
```
1/4*arctan(1/2*x*2^(3/4)/b)*2^(3/4)/b/(a^4+2*b^4)+1/4*arctan((a-x*2^(1/2))
/a)*2^(1/2)/a/(a^4+2*b^4)-1/4*arctan((a+x*2^(1/2))/a)*2^(1/2)/a/(a^4+2*b^4
)-1/4*arctanh(1/2*x*2^(3/4)/b)*2^(3/4)/b/(a^4+2*b^4)-1/8*ln(a^2-2^(1/2)*a*
x+x^2)*2^(1/2)/a/(a^4+2*b^4)+1/8*ln(a^2+2^(1/2)*a*x+x^2)*2^(1/2)/a/(a^4+2*
b^4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx$$
$$= \frac{2\sqrt[4]{2}a\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)+2b\arctan\left(1-\frac{\sqrt{2}x}{a}\right)-2b\arctan\left(1+\frac{\sqrt{2}x}{a}\right)+\sqrt[4]{2}a\log\left(2b-2^{3/4}x\right)-\sqrt[4]{2}a\log\left(\right.}{4\sqrt{2}ab\left(a^4+2b^4\right)}$$

input
```
Integrate[x^2/((a^4 + x^4)*(-2*b^4 + x^4)),x]
```

output
```
(2*2^(1/4)*a*ArcTan[x/(2^(1/4)*b)] + 2*b*ArcTan[1 - (Sqrt[2]*x)/a] - 2*b*A
rcTan[1 + (Sqrt[2]*x)/a] + 2^(1/4)*a*Log[2*b - 2^(3/4)*x] - 2^(1/4)*a*Log[
2*b + 2^(3/4)*x] - b*Log[a^2 - Sqrt[2]*a*x + x^2] + b*Log[a^2 + Sqrt[2]*a*
x + x^2])/(4*Sqrt[2]*a*b*(a^4 + 2*b^4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {982, 25, 826, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^4+x^4)(x^4-2b^4)}dx$$

$$\Big\downarrow \; 982$$

$$\frac{\int -\frac{x^2}{2b^4-x^4}dx}{a^4+2b^4}-\frac{\int \frac{x^2}{a^4+x^4}dx}{a^4+2b^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{x^2}{2b^4-x^4}dx}{a^4+2b^4}-\frac{\int \frac{x^2}{a^4+x^4}dx}{a^4+2b^4}$$

$$\Big\downarrow \; 826$$

$$-\frac{\int \frac{x^2}{2b^4-x^4}dx}{a^4+2b^4}-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}$$

$$\Big\downarrow \text{827}$$

$$-\frac{\frac{1}{2}\int \frac{1}{\sqrt{2}b^2-x^2}dx-\frac{1}{2}\int \frac{1}{\sqrt{2}b^2+x^2}dx}{a^4+2b^4}-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{1}{2}\int \frac{1}{\sqrt{2}b^2-x^2}dx-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}dx-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}-\frac{\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{a^2-\sqrt{2}xa+x^2}dx+\frac{1}{2}\int \frac{1}{a^2+\sqrt{2}xa+x^2}dx\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}-\frac{\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{a}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{a}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}-\frac{\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)-\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}dx}{a^4+2b^4}-\frac{\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}+\frac{\int -\frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4+2b^4}-\frac{\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{\sqrt{2}\left(a+\sqrt{2}x\right)}{a^2+\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4+2b^4}-\frac{\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}a-2x}{a^2-\sqrt{2}xa+x^2}dx}{2\sqrt{2}a}-\frac{\int \frac{a+\sqrt{2}x}{a^2+\sqrt{2}xa+x^2}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4+2b^4}-\frac{\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}b}\right)}{2\sqrt[4]{2}b}}{a^4+2b^4}-\frac{\frac{1}{2}\left(\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{2\sqrt{2}a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{\sqrt{2}a}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{a}\right)}{\sqrt{2}a}\right)}{a^4+2b^4}$$

input
```
Int[x^2/((a^4 + x^4)*(-2*b^4 + x^4)),x]
```

output
```
-((-1/2*ArcTan[x/(2^(1/4)*b)]/(2^(1/4)*b) + ArcTanh[x/(2^(1/4)*b)]/(2*2^(1
/4)*b))/(a^4 + 2*b^4)) - ((-(ArcTan[1 - (Sqrt[2]*x)/a]/(Sqrt[2]*a)) + ArcT
an[1 + (Sqrt[2]*x)/a]/(Sqrt[2]*a))/2 + (Log[a^2 - Sqrt[2]*a*x + x^2]/(2*Sq
rt[2]*a) - Log[a^2 + Sqrt[2]*a*x + x^2]/(2*Sqrt[2]*a))/2)/(a^4 + 2*b^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\arctan\left(\frac{x}{\sqrt{\sqrt{2}\,\sqrt{b^4}}}\right)-\ln\left(\frac{x+\sqrt{\sqrt{2}\,\sqrt{b^4}}}{x-\sqrt{\sqrt{2}\,\sqrt{b^4}}}\right)}{4(a^4+2b^4)\sqrt{\sqrt{2}\,\sqrt{b^4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}{x^2+\left(a^4\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{a^4}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(a^4\right)^{\frac{1}{4}}}-1\right)\right)}{8(a^4+2b^4)\left(a^4\right)^{\frac{1}{4}}}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-1+\left(2b^4a^{16}+16b^8a^{12}+48b^{12}a^8+64b^{16}a^4+32b^{20}\right)\_Z^4\right)}\_R\ln\left(\left(\left(a^{24}+8a^{20}b^4+28a^{16}b^8+64a^{12}b^{12}+112a^8b^{16}+128a^4b\right.\right.\right.}{4}$

input
```
int(x^2/(a^4+x^4)/(-2*b^4+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a^4+2*b^4)/(2^(1/2)*(b^4)^(1/2))^(1/2)*(2*arctan(x/(2^(1/2)*(b^4)^(1/
2))^(1/2))-ln((x+(2^(1/2)*(b^4)^(1/2))^(1/2))/(x-(2^(1/2)*(b^4)^(1/2))^(1/
2))))-1/8/(a^4+2*b^4)/(a^4)^(1/4)*2^(1/2)*(ln((x^2-(a^4)^(1/4)*x*2^(1/2)+(
a^4)^(1/2))/(x^2+(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2)))+2*arctan(2^(1/2)/(a^4
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a^4)^(1/4)*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a^4+x^4)\,(-2b^4+x^4)}\,dx$$
$$=\frac{2\cdot 2^{\frac{3}{4}}a\arctan\left(\frac{2^{\frac{3}{4}}x}{2\,b}\right)-2^{\frac{3}{4}}a\log\left(2^{\frac{1}{4}}b+x\right)+2^{\frac{3}{4}}a\log\left(-2^{\frac{1}{4}}b+x\right)-2\sqrt{2}b\arctan\left(\frac{\sqrt{2}x+a}{a}\right)-2\sqrt{2}b\arcta}{8\,(a^5b+2\,ab^5)}$$

input
```
integrate(x^2/(a^4+x^4)/(-2*b^4+x^4),x, algorithm="fricas")
```

output
```
1/8*(2*2^(3/4)*a*arctan(1/2*2^(3/4)*x/b) - 2^(3/4)*a*log(2^(1/4)*b + x) +
2^(3/4)*a*log(-2^(1/4)*b + x) - 2*sqrt(2)*b*arctan((sqrt(2)*x + a)/a) - 2*
sqrt(2)*b*arctan((sqrt(2)*x - a)/a) + sqrt(2)*b*log(sqrt(2)*a*x + a^2 + x^
2) - sqrt(2)*b*log(-sqrt(2)*a*x + a^2 + x^2))/(a^5*b + 2*a*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(182) = 364$.

Time = 4.62 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^{20}+2048a^{16}b^4+6144a^{12}b^8+8192a^8b^{12}+4096a^4b^{16}\right)+1,\left(t\mapsto t\log\left(32768t^7a^{32}b^4\right.\right.\right.$$
$$+\operatorname{RootSum}\left(t^4\cdot\left(512a^{16}b^4+4096a^{12}b^8+12288a^8b^{12}+16384a^4b^{16}+8192b^{20}\right)-1,\left(t\mapsto t\log\left(32768t^7a\right.\right.\right.$$

input
```
integrate(x**2/(a**4+x**4)/(-2*b**4+x**4),x)
```

output
```
RootSum(_t**4*(256*a**20 + 2048*a**16*b**4 + 6144*a**12*b**8 + 8192*a**8*b
**12 + 4096*a**4*b**16) + 1, Lambda(_t, _t*log(32768*_t**7*a**32*b**4 + 32
7680*_t**7*a**28*b**8 + 1179648*_t**7*a**24*b**12 + 1310720*_t**7*a**20*b*
*16 - 2621440*_t**7*a**16*b**20 - 9437184*_t**7*a**12*b**24 - 10485760*_t*
*7*a**8*b**28 - 4194304*_t**7*a**4*b**32 - 64*_t**3*a**16 - 256*_t**3*a**1
2*b**4 - 512*_t**3*a**8*b**8 - 1024*_t**3*a**4*b**12 - 1024*_t**3*b**16 +
x))) + RootSum(_t**4*(512*a**16*b**4 + 4096*a**12*b**8 + 12288*a**8*b**12
+ 16384*a**4*b**16 + 8192*b**20) - 1, Lambda(_t, _t*log(32768*_t**7*a**32*
b**4 + 327680*_t**7*a**28*b**8 + 1179648*_t**7*a**24*b**12 + 1310720*_t**7
*a**20*b**16 - 2621440*_t**7*a**16*b**20 - 9437184*_t**7*a**12*b**24 - 104
85760*_t**7*a**8*b**28 - 4194304*_t**7*a**4*b**32 - 64*_t**3*a**16 - 256*_
t**3*a**12*b**4 - 512*_t**3*a**8*b**8 - 1024*_t**3*a**4*b**12 - 1024*_t**3
*b**16 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx = \frac{\frac{2\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x}{2b}\right)}{b}+\frac{2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{1}{4}}b-x}{2^{\frac{1}{4}}b+x}\right)}{b}}{8\left(a^4+2b^4\right)}$$
$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a+2x\right)}{2a}\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a-2x\right)}{2a}\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}ax+a^2+x^2\right)}{a}+\frac{\sqrt{2}\log\left(-\sqrt{2}ax+a^2+x^2\right)}{a}}{8\left(a^4+2b^4\right)}$$

input
```
integrate(x^2/(a^4+x^4)/(-2*b^4+x^4),x, algorithm="maxima")
```

output
```
1/8*(2*2^(3/4)*arctan(1/2*2^(3/4)*x/b)/b + 2^(3/4)*log(-(2^(1/4)*b - x)/(2
^(1/4)*b + x))/b)/(a^4 + 2*b^4) - 1/8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(
2)*a + 2*x)/a)/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a - 2*x)/a)/a -
sqrt(2)*log(sqrt(2)*a*x + a^2 + x^2)/a + sqrt(2)*log(-sqrt(2)*a*x + a^2 +
x^2)/a)/(a^4 + 2*b^4)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.30

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx = -\frac{a^2|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{2\left(\sqrt{2}a^8+2\sqrt{2}a^4b^4\right)} - \frac{a^2|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{2\left(\sqrt{2}a^8+2\sqrt{2}a^4b^4\right)} + \frac{a^2|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{4\left(\sqrt{2}a^8+2\sqrt{2}a^4b^4\right)} - \frac{a^2|a|\log\left(-\sqrt{2}x|a|+x^2+|a|^2\right)}{4\left(\sqrt{2}a^8+2\sqrt{2}a^4b^4\right)} + \frac{\sqrt{2}\log\left(\left|x-2^{\frac{1}{4}}(b^4)^{\frac{1}{4}}\right|\right)}{4\left(2^{\frac{3}{4}}a^4b+2\cdot 2^{\frac{3}{4}}b^5\right)} + \frac{\arctan\left(\frac{2^{\frac{3}{4}}x}{2(b^4)^{\frac{1}{4}}}\right)}{2\left(2^{\frac{1}{4}}a^4b+2\cdot 2^{\frac{1}{4}}b^5\right)} - \frac{\log\left(\left|x+2^{\frac{1}{4}}(b^4)^{\frac{1}{4}}\right|\right)}{4\left(2^{\frac{1}{4}}a^4b+2\cdot 2^{\frac{1}{4}}b^5\right)}$$

input
```
integrate(x^2/(a^4+x^4)/(-2*b^4+x^4),x, algorithm="giac")
```

output
```
-1/2*a^2*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/(sqrt(2)
*a^8 + 2*sqrt(2)*a^4*b^4) - 1/2*a^2*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*ab
s(a) - 2*x)/abs(a))/(sqrt(2)*a^8 + 2*sqrt(2)*a^4*b^4) + 1/4*a^2*abs(a)*log
(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/(sqrt(2)*a^8 + 2*sqrt(2)*a^4*b^4) - 1/
4*a^2*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/(sqrt(2)*a^8 + 2*sqrt
(2)*a^4*b^4) + 1/4*sqrt(2)*log(abs(x - 2^(1/4)*(b^4)^(1/4)))/(2^(3/4)*a^4*
b + 2*2^(3/4)*b^5) + 1/2*arctan(1/2*2^(3/4)*x/(b^4)^(1/4))/(2^(1/4)*a^4*b
+ 2*2^(1/4)*b^5) - 1/4*log(abs(x + 2^(1/4)*(b^4)^(1/4)))/(2^(1/4)*a^4*b +
2*2^(1/4)*b^5)
```

Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 3743, normalized size of antiderivative = 16.78

$$\int \frac{x^2}{(a^4+x^4)(-2b^4+x^4)}\,dx = \text{Too large to display}$$

input
```
int(-x^2/((2*b^4 - x^4)*(a^4 + x^4)),x)
```

output
```
atan((((-1/(256*a^20 + 4096*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 204
8*a^16*b^4))^(3/4)*(4096*a^16*b^12 - 24576*a^8*b^20 - 8192*a^12*b^16 - 163
84*a^4*b^24 + 3072*a^20*b^8 + 512*a^24*b^4 + x*(-1/(256*a^20 + 4096*a^4*b^
16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 2048*a^16*b^4))^(1/4)*(131072*a^4*b^2
8 + 262144*a^8*b^24 + 229376*a^12*b^20 + 131072*a^16*b^16 + 57344*a^20*b^1
2 + 16384*a^24*b^8 + 2048*a^28*b^4)) - x*(16*a^4*b^8 - 8*a^8*b^4))*(-1/(25
6*a^20 + 4096*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 2048*a^16*b^4))^(
1/4)*1i + ((-1/(256*a^20 + 4096*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 +
 2048*a^16*b^4))^(3/4)*(16384*a^4*b^24 + 24576*a^8*b^20 + 8192*a^12*b^16 -
 4096*a^16*b^12 - 3072*a^20*b^8 - 512*a^24*b^4 + x*(-1/(256*a^20 + 4096*a^
4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 2048*a^16*b^4))^(1/4)*(131072*a^4
*b^28 + 262144*a^8*b^24 + 229376*a^12*b^20 + 131072*a^16*b^16 + 57344*a^20
*b^12 + 16384*a^24*b^8 + 2048*a^28*b^4)) - x*(16*a^4*b^8 - 8*a^8*b^4))*(-1
/(256*a^20 + 4096*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 2048*a^16*b^4
))^(1/4)*1i)/(((-1/(256*a^20 + 4096*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b
^8 + 2048*a^16*b^4))^(3/4)*(4096*a^16*b^12 - 24576*a^8*b^20 - 8192*a^12*b^
16 - 16384*a^4*b^24 + 3072*a^20*b^8 + 512*a^24*b^4 + x*(-1/(256*a^20 + 409
6*a^4*b^16 + 8192*a^8*b^12 + 6144*a^12*b^8 + 2048*a^16*b^4))^(1/4)*(131072
*a^4*b^28 + 262144*a^8*b^24 + 229376*a^12*b^20 + 131072*a^16*b^16 + 57344*
a^20*b^12 + 16384*a^24*b^8 + 2048*a^28*b^4)) - x*(16*a^4*b^8 - 8*a^8*b^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a^4+x^4)\left(-2b^4+x^4\right)}\,dx = \frac{\sqrt{2}\left(2atan\left(\frac{\sqrt{2}\,a-2x}{\sqrt{2}\,a}\right)b-2atan\left(\frac{\sqrt{2}\,a+2x}{\sqrt{2}\,a}\right)b+2\,2^{\frac{1}{4}}atan\left(\frac{x2^{\frac{3}{4}}}{2b}\right)a+2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}b-x\right)a-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}b+x\right)a}{8ab\left(a^4+2b^4\right)}$$

input
```
int(x^2/(a^4+x^4)/(-2*b^4+x^4),x)
```

output
```
(sqrt(2)*(2*atan((sqrt(2)*a - 2*x)/(sqrt(2)*a))*b - 2*atan((sqrt(2)*a + 2*
x)/(sqrt(2)*a))*b + 2*2**(1/4)*atan(x/(2**(1/4)*b))*a + 2**(1/4)*log(2**(1
/4)*b - x)*a - 2**(1/4)*log(2**(1/4)*b + x)*a - log( - sqrt(2)*a*x + a**2
+ x**2)*b + log(sqrt(2)*a*x + a**2 + x**2)*b))/(8*a*b*(a**4 + 2*b**4))
```

3.107 $\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\begin{aligned}\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx &= \frac{(-b+c)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(b+c)}-\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}-\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}a^{3/4}(b+c)}\\
&\quad+\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}+\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}a^{3/4}(b+c)}+\frac{(b-c)\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(b+c)}\\
&\quad+\frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2\right)}{2\sqrt{2}a^{3/4}(b+c)}\\
&\quad-\frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2\right)}{2\sqrt{2}a^{3/4}(b+c)}\end{aligned}$$

output
```
1/2*(-b+c)*arctan(a^(1/4)*x/c^(1/4))/a^(3/4)/c^(1/4)/(b+c)-1/2*b^(3/4)*arc
tan((b^(1/4)-2^(1/2)*a^(1/4)*x)/b^(1/4))*2^(1/2)/a^(3/4)/(b+c)+1/2*b^(3/4)
*arctan((b^(1/4)+2^(1/2)*a^(1/4)*x)/b^(1/4))*2^(1/2)/a^(3/4)/(b+c)+1/2*(b-
c)*arctanh(a^(1/4)*x/c^(1/4))/a^(3/4)/c^(1/4)/(b+c)+1/4*b^(3/4)*ln(b^(1/2)
-2^(1/2)*a^(1/4)*b^(1/4)*x+a^(1/2)*x^2)*2^(1/2)/a^(3/4)/(b+c)-1/4*b^(3/4)*
ln(b^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)*x+a^(1/2)*x^2)*2^(1/2)/a^(3/4)/(b+c)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx = \frac{-2(b-c)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)-2\sqrt{2}b^{3/4}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)+2\sqrt{2}b^{3/4}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)-b\,\mathrm{l}}{}$$

input
```
Integrate[(x^2*(-b + a*x^4))/((b + a*x^4)*(-c + a*x^4)),x]
```

output
```
(-2*(b - c)*ArcTan[(a^(1/4)*x)/c^(1/4)] - 2*Sqrt[2]*b^(3/4)*c^(1/4)*ArcTan
[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)] + 2*Sqrt[2]*b^(3/4)*c^(1/4)*ArcTan[1 + (
Sqrt[2]*a^(1/4)*x)/b^(1/4)] - b*Log[c^(1/4) - a^(1/4)*x] + c*Log[c^(1/4) -
 a^(1/4)*x] + b*Log[c^(1/4) + a^(1/4)*x] - c*Log[c^(1/4) + a^(1/4)*x] + Sq
rt[2]*b^(3/4)*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^
2] - Sqrt[2]*b^(3/4)*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[a]*x^2])/(4*a^(3/4)*c^(1/4)*(b + c))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(ax^4-b)}{(ax^4+b)(ax^4-c)}\,dx$$

$$\Big\downarrow \text{1054}$$

$$\int \left(\frac{x^2(b-c)}{(b+c)(c-ax^4)}+\frac{2bx^2}{(b+c)(ax^4+b)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}a^{3/4}(b+c)}+\frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}a^{3/4}(b+c)}-\frac{(b-c)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(b+c)}+\frac{(b-c)\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(b+c)}+\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}a^{3/4}(b+c)}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}a^{3/4}(b+c)}$$

input
```
Int[(x^2*(-b + a*x^4))/((b + a*x^4)*(-c + a*x^4)),x]
```

output
```
-1/2*((b - c)*ArcTan[(a^(1/4)*x)/c^(1/4)])/(a^(3/4)*c^(1/4)*(b + c)) - (b^
(3/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)])/(Sqrt[2]*a^(3/4)*(b + c)) +
 (b^(3/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)])/(Sqrt[2]*a^(3/4)*(b + c
)) + ((b - c)*ArcTanh[(a^(1/4)*x)/c^(1/4)])/(2*a^(3/4)*c^(1/4)*(b + c)) +
(b^(3/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2])/(2*Sqrt[2
]*a^(3/4)*(b + c)) - (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[a]*x^2])/(2*Sqrt[2]*a^(3/4)*(b + c))
```

Defintions of rubi rules used

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.60

method	result
default	$-\frac{(b-c)\left(2\arctan\left(\frac{x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{c}{a}\right)^{\frac{1}{4}}}{x-\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)\right)}{4(b+c)a\left(\frac{c}{a}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}{x^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}-\right.\right.}{4(b+c)a\left(\frac{b}{a}\right)^{\frac{1}{4}}}$
risch	$\left(\sum_{\_R=\text{RootOf}\left(\left(a^3b^4c+4a^3b^3c^2+6a^3b^2c^3+4a^3b\,c^4+a^3c^5\right)\_Z^4-b^4+4b^3c-6b^2c^2+4b\,c^3-c^4\right)}\_R\ln\left(\left(\left(-a^4b^6-2a^4b^5c-3a^4b^4c^2-12a^4b^3\right.\right.\right.\right.$

input
```
int(x^2*(a*x^4-b)/(a*x^4+b)/(a*x^4-c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b-c)/(b+c)/a/(c/a)^(1/4)*(2*arctan(x/(c/a)^(1/4))-ln((x+(c/a)^(1/4))
/(x-(c/a)^(1/4))))+1/4*b/(b+c)/a/(b/a)^(1/4)*2^(1/2)*(ln((x^2-(b/a)^(1/4)*
x*2^(1/2)+(b/a)^(1/2))/(x^2+(b/a)^(1/4)*x*2^(1/2)+(b/a)^(1/2)))+2*arctan(2
^(1/2)/(b/a)^(1/4)*x+1)+2*arctan(2^(1/2)/(b/a)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1503, normalized size of antiderivative = 5.29

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)\,(-c+ax^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a*x^4-b)/(a*x^4+b)/(a*x^4-c),x, algorithm="fricas")
```

output
```
1/2*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))
^(1/4)*log(b^2*x + (a^2*b^3 + 3*a^2*b^2*c + 3*a^2*b*c^2 + a^2*c^3)*(-b^3/(
a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))^(3/4)) - 1
/2*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))^
(1/4)*log(b^2*x - (a^2*b^3 + 3*a^2*b^2*c + 3*a^2*b*c^2 + a^2*c^3)*(-b^3/(a
^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))^(3/4)) + 1/
2*I*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))
^(1/4)*log(b^2*x - (I*a^2*b^3 + 3*I*a^2*b^2*c + 3*I*a^2*b*c^2 + I*a^2*c^3)
*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^3*c^4))^(3
/4)) - 1/2*I*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 +
a^3*c^4))^(1/4)*log(b^2*x - (-I*a^2*b^3 - 3*I*a^2*b^2*c - 3*I*a^2*b*c^2 -
I*a^2*c^3)*(-b^3/(a^3*b^4 + 4*a^3*b^3*c + 6*a^3*b^2*c^2 + 4*a^3*b*c^3 + a^
3*c^4))^(3/4)) - 1/4*((b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)/(a^3*b^4
*c + 4*a^3*b^3*c^2 + 6*a^3*b^2*c^3 + 4*a^3*b*c^4 + a^3*c^5))^(1/4)*log(-(b
^3 - 3*b^2*c + 3*b*c^2 - c^3)*x + (a^2*b^3*c + 3*a^2*b^2*c^2 + 3*a^2*b*c^3
 + a^2*c^4)*((b^4 - 4*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)/(a^3*b^4*c + 4*a^
3*b^3*c^2 + 6*a^3*b^2*c^3 + 4*a^3*b*c^4 + a^3*c^5))^(3/4)) + 1/4*((b^4 - 4
*b^3*c + 6*b^2*c^2 - 4*b*c^3 + c^4)/(a^3*b^4*c + 4*a^3*b^3*c^2 + 6*a^3*b^2
*c^3 + 4*a^3*b*c^4 + a^3*c^5))^(1/4)*log(-(b^3 - 3*b^2*c + 3*b*c^2 - c^3)*
x - (a^2*b^3*c + 3*a^2*b^2*c^2 + 3*a^2*b*c^3 + a^2*c^4)*((b^4 - 4*b^3*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a*x**4-b)/(a*x**4+b)/(a*x**4-c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.94

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx$$
$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}-\frac{\sqrt{2}\log\left(\sqrt{a}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{b}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{a}x^2-\sqrt{\cdots}\right.}{a^{\frac{3}{4}}b\cdots}\right.}{4\,(b+c)}$$
$$-\frac{(b-c)\left(\frac{2\arctan\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\log\left(\frac{\sqrt{a}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{a}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,(b+c)}$$

input
```
integrate(x^2*(a*x^4-b)/(a*x^4+b)/(a*x^4-c),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(a)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(a)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) - sqrt(2)*log(sqrt(a)*x^2 + sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(b))/(a^(3/4)*b^(1/4)) + sqrt(2)*log(sqrt(a)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(b))/(a^(3/4)*b^(1/4)))/(b + c) - 1/4*(b - c
)*(2*arctan(sqrt(a)*x/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c)
)) + log((sqrt(a)*x - sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*x + sqrt(sqrt(a)*sqr
t(c))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))))/(b + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(203) = 406$.

Time = 0.17 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.67

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx = -\frac{(b-c)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}b+\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}c\right)} - \frac{(b-c)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{c}{a}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}b+\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}c\right)} + \frac{(b-c)\log\left(x^2+\sqrt{2}x\left(-\frac{c}{a}\right)^{\frac{1}{4}}+\sqrt{-\frac{c}{a}}\right)}{4\left(\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}b+\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}c\right)} - \frac{(b-c)\log\left(x^2-\sqrt{2}x\left(-\frac{c}{a}\right)^{\frac{1}{4}}+\sqrt{-\frac{c}{a}}\right)}{4\left(\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}b+\sqrt{2}\left(-a^3c\right)^{\frac{1}{4}}c\right)} + \frac{\left(a^3b\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b+\sqrt{2}a^3c} + \frac{\left(a^3b\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b+\sqrt{2}a^3c} - \frac{\left(a^3b\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{2\left(\sqrt{2}a^3b+\sqrt{2}a^3c\right)} + \frac{\left(a^3b\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{2\left(\sqrt{2}a^3b+\sqrt{2}a^3c\right)}$$

input
```
integrate(x^2*(a*x^4-b)/(a*x^4+b)/(a*x^4-c),x, algorithm="giac")
```

output
```
-1/2*(b - c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))
/(sqrt(2)*(-a^3*c)^(1/4)*b + sqrt(2)*(-a^3*c)^(1/4)*c) - 1/2*(b - c)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))/(sqrt(2)*(-a^3*c)
^(1/4)*b + sqrt(2)*(-a^3*c)^(1/4)*c) + 1/4*(b - c)*log(x^2 + sqrt(2)*x*(-c
/a)^(1/4) + sqrt(-c/a))/(sqrt(2)*(-a^3*c)^(1/4)*b + sqrt(2)*(-a^3*c)^(1/4)
*c) - 1/4*(b - c)*log(x^2 - sqrt(2)*x*(-c/a)^(1/4) + sqrt(-c/a))/(sqrt(2)*
(-a^3*c)^(1/4)*b + sqrt(2)*(-a^3*c)^(1/4)*c) + (a^3*b)^(3/4)*arctan(1/2*sq
rt(2)*(2*x + sqrt(2)*(b/a)^(1/4))/(b/a)^(1/4))/(sqrt(2)*a^3*b + sqrt(2)*a^
3*c) + (a^3*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(b/a)^(1/4))/(b/a)^
(1/4))/(sqrt(2)*a^3*b + sqrt(2)*a^3*c) - 1/2*(a^3*b)^(3/4)*log(x^2 + sqrt(
2)*x*(b/a)^(1/4) + sqrt(b/a))/(sqrt(2)*a^3*b + sqrt(2)*a^3*c) + 1/2*(a^3*b
)^(3/4)*log(x^2 - sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/(sqrt(2)*a^3*b + sqrt
(2)*a^3*c)
```

Mupad [B] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 5162, normalized size of antiderivative = 18.18

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx = \text{Too large to display}$$

input
```
int((x^2*(b - a*x^4))/((b + a*x^4)*(c - a*x^4)),x)
```

output
```
atan((((b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)/(256*a^3*c^5 + 1024*a^
3*b*c^4 + 256*a^3*b^4*c + 1536*a^3*b^2*c^3 + 1024*a^3*b^3*c^2))^(3/4)*(x*(
(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)/(256*a^3*c^5 + 1024*a^3*b*c^4
+ 256*a^3*b^4*c + 1536*a^3*b^2*c^3 + 1024*a^3*b^3*c^2))^(1/4)*(1024*a^14*b
^9*c + 5120*a^14*b^3*c^7 + 18432*a^14*b^4*c^6 + 23552*a^14*b^5*c^5 + 12288
*a^14*b^6*c^4 + 3072*a^14*b^7*c^3 + 2048*a^14*b^8*c^2) - 256*a^13*b^9*c +
256*a^13*b^3*c^7 + 2048*a^13*b^4*c^6 + 5376*a^13*b^5*c^5 + 6144*a^13*b^6*c
^4 + 2816*a^13*b^7*c^3) + x*(64*a^11*b^8*c - 16*a^11*b^3*c^6 + 64*a^11*b^4
*c^5 - 96*a^11*b^5*c^4 + 128*a^11*b^6*c^3 - 144*a^11*b^7*c^2))*((b^4 - 4*b
^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)/(256*a^3*c^5 + 1024*a^3*b*c^4 + 256*a^3*
b^4*c + 1536*a^3*b^2*c^3 + 1024*a^3*b^3*c^2))^(1/4)*1i - (((b^4 - 4*b^3*c
- 4*b*c^3 + c^4 + 6*b^2*c^2)/(256*a^3*c^5 + 1024*a^3*b*c^4 + 256*a^3*b^4*c
 + 1536*a^3*b^2*c^3 + 1024*a^3*b^3*c^2))^(3/4)*(256*a^13*b^3*c^7 - x*((b^4
 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)/(256*a^3*c^5 + 1024*a^3*b*c^4 + 25
6*a^3*b^4*c + 1536*a^3*b^2*c^3 + 1024*a^3*b^3*c^2))^(1/4)*(1024*a^14*b^9*c
 + 5120*a^14*b^3*c^7 + 18432*a^14*b^4*c^6 + 23552*a^14*b^5*c^5 + 12288*a^1
4*b^6*c^4 + 3072*a^14*b^7*c^3 + 2048*a^14*b^8*c^2) - 256*a^13*b^9*c + 2048
*a^13*b^4*c^6 + 5376*a^13*b^5*c^5 + 6144*a^13*b^6*c^4 + 2816*a^13*b^7*c^3)
 - x*(64*a^11*b^8*c - 16*a^11*b^3*c^6 + 64*a^11*b^4*c^5 - 96*a^11*b^5*c^4
+ 128*a^11*b^6*c^3 - 144*a^11*b^7*c^2))*((b^4 - 4*b^3*c - 4*b*c^3 + c^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.83

$$\int \frac{x^2(-b+ax^4)}{(b+ax^4)(-c+ax^4)}\,dx$$
$$= \frac{-2b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c+2b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2c^{\frac{3}{4}}\,atan\left(\frac{\sqrt{a}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b+2c^{\frac{7}{4}}\,atan\left(\frac{\sqrt{a}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)\cdots}{\cdots}$$

input
```
int(x^2*(a*x^4-b)/(a*x^4+b)/(a*x^4-c),x)
```

output
```
(a**(1/4)*( - 2*b**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
a)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c + 2*b**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(a)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*c**(3/4)
*atan((sqrt(a)*x)/(c**(1/4)*a**(1/4)))*b + 2*c**(3/4)*atan((sqrt(a)*x)/(c*
*(1/4)*a**(1/4)))*c + b**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x
+ sqrt(a)*x**2 + sqrt(b))*c - b**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a)*x**2 + sqrt(b))*c + c**(3/4)*log(a**(1/4)*x + c**(1/4))*b -
 c**(3/4)*log(a**(1/4)*x + c**(1/4))*c - c**(3/4)*log(a**(1/4)*x - c**(1/4
))*b + c**(3/4)*log(a**(1/4)*x - c**(1/4))*c))/(4*a*c*(b + c))
```

3.108 $\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)(d+bx^4)}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 503

$$\begin{aligned}
&\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)(d+bx^4)}\,dx \\
&= \frac{\left(-af+c\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(bc+ad)} \\
&\quad - \frac{\left(bf+d\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\arctan\left(\frac{\sqrt[4]{d}-\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{d}(bc+ad)} \\
&\quad + \frac{\left(bf+d\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\arctan\left(\frac{\sqrt[4]{d}+\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{d}(bc+ad)} \\
&\quad + \frac{\left(af-c\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2a^{3/4}\sqrt[4]{c}(bc+ad)} \\
&\quad + \frac{\left(bf+d\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}\sqrt[4]{d}(bc+ad)} \\
&\quad - \frac{\left(bf+d\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)\right)\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}\sqrt[4]{d}(bc+ad)}
\end{aligned}$$

output
```
1/2*(-a*f+c*(A/(B+C)/F/H+D/F/H+G/H))*arctan(a^(1/4)*x/c^(1/4))/a^(3/4)/c^(
1/4)/(a*d+b*c)-1/4*(b*f+d*(A/(B+C)/F/H+D/F/H+G/H))*arctan((d^(1/4)-2^(1/2)
*b^(1/4)*x)/d^(1/4))*2^(1/2)/b^(3/4)/d^(1/4)/(a*d+b*c)+1/4*(b*f+d*(A/(B+C)
/F/H+D/F/H+G/H))*arctan((d^(1/4)+2^(1/2)*b^(1/4)*x)/d^(1/4))*2^(1/2)/b^(3/
4)/d^(1/4)/(a*d+b*c)+1/2*(a*f-c*(A/(B+C)/F/H+D/F/H+G/H))*arctanh(a^(1/4)*x
/c^(1/4))/a^(3/4)/c^(1/4)/(a*d+b*c)+1/8*(b*f+d*(A/(B+C)/F/H+D/F/H+G/H))*ln
(d^(1/2)-2^(1/2)*b^(1/4)*d^(1/4)*x+b^(1/2)*x^2)*2^(1/2)/b^(3/4)/d^(1/4)/(a
*d+b*c)-1/8*(b*f+d*(A/(B+C)/F/H+D/F/H+G/H))*ln(d^(1/2)+2^(1/2)*b^(1/4)*d^(
1/4)*x+b^(1/2)*x^2)*2^(1/2)/b^(3/4)/d^(1/4)/(a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.85

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)\,(d+bx^4)}\,dx$$

$$=\frac{\frac{4(Ac+(B+C)(c(D+FG)-afFH))\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{a^{3/4}\sqrt[4]{c}}-\frac{2\sqrt{2}(Ad+(B+C)(d(D+FG)+bfFH))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\sqrt{2}(Ad+(B+C)(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^2*(-f + (A/((B + C)*F*H) + D/(F*H) + G/H)*x^4))/((-c + a*x^4)
*(d + b*x^4)),x]
```

output
```
((4*(A*c + (B + C)*(c*(D + F*G) - a*f*F*H))*ArcTan[(a^(1/4)*x)/c^(1/4)])/(
a^(3/4)*c^(1/4)) - (2*Sqrt[2]*(A*d + (B + C)*(d*(D + F*G) + b*f*F*H))*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/d^(1/4)])/(b^(3/4)*d^(1/4)) + (2*Sqrt[2]*(A*d +
 (B + C)*(d*(D + F*G) + b*f*F*H))*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/d^(1/4)])
/(b^(3/4)*d^(1/4)) + (2*(A*c + (B + C)*(c*(D + F*G) - a*f*F*H))*Log[c^(1/4
) - a^(1/4)*x])/(a^(3/4)*c^(1/4)) - (2*(A*c + (B + C)*(c*(D + F*G) - a*f*F
*H))*Log[c^(1/4) + a^(1/4)*x])/(a^(3/4)*c^(1/4)) + (Sqrt[2]*(A*d + (B + C)
*(d*(D + F*G) + b*f*F*H))*Log[Sqrt[d] - Sqrt[2]*b^(1/4)*d^(1/4)*x + Sqrt[b
]*x^2])/(b^(3/4)*d^(1/4)) - (Sqrt[2]*(A*d + (B + C)*(d*(D + F*G) + b*f*F*H
))*Log[Sqrt[d] + Sqrt[2]*b^(1/4)*d^(1/4)*x + Sqrt[b]*x^2])/(b^(3/4)*d^(1/4
)))/(8*(B + C)*(b*c + a*d)*F*H)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x^4\left(\frac{A}{FH(B+C)}+\frac{D}{FH}+\frac{G}{H}\right)-f\right)}{(ax^4-c)\,(bx^4+d)}dx$$

$$\Big\downarrow \text{1054}$$

$$\int \left(\frac{x^2(-(B+C)(c(D+FG)-afFH)-Ac)}{FH(B+C)\,(c-ax^4)\,(ad+bc)}+\frac{x^2(Ad+(B+C)(bfFH+d(D+FG)))}{FH(B+C)\,(bx^4+d)\,(ad+bc)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)((B+C)(c(D+FG)-afFH)+Ac)}{2a^{3/4}\sqrt[4]{c}FH(B+C)(ad+bc)}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)((B+C)(c(D+FG)-afFH)+Ac)}{2a^{3/4}\sqrt[4]{c}FH(B+C)(ad+bc)}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)(Ad+(B+C)(bfFH+d(D+FG)))}{2\sqrt{2}b^{3/4}\sqrt[4]{d}FH(B+C)(ad+bc)}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)(Ad+(B+C)(bfFH+d(D+FG)))}{2\sqrt{2}b^{3/4}\sqrt[4]{d}FH(B+C)(ad+bc)}+\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2+\sqrt{d}\right)(Ad+(B+C)(bfFH+d(D+FG)))}{4\sqrt{2}b^{3/4}\sqrt[4]{d}FH(B+C)(ad+bc)}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2+\sqrt{d}\right)(Ad+(B+C)(bfFH+d(D+FG)))}{4\sqrt{2}b^{3/4}\sqrt[4]{d}FH(B+C)(ad+bc)}$$

input
```
Int[(x^2*(-f + (A/((B + C)*F*H) + D/(F*H) + G/H)*x^4))/((-c + a*x^4)*(d +
b*x^4)),x]
```

output
```
((A*c + (B + C)*(c*(D + F*G) - a*f*F*H))*ArcTan[(a^(1/4)*x)/c^(1/4)])/(2*a
^(3/4)*c^(1/4)*(B + C)*(b*c + a*d)*F*H) - ((A*d + (B + C)*(d*(D + F*G) + b
*f*F*H))*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*b^(3/4)*(B +
C)*d^(1/4)*(b*c + a*d)*F*H) + ((A*d + (B + C)*(d*(D + F*G) + b*f*F*H))*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/d^(1/4)])/(2*Sqrt[2]*b^(3/4)*(B + C)*d^(1/4)*(
b*c + a*d)*F*H) - ((A*c + (B + C)*(c*(D + F*G) - a*f*F*H))*ArcTanh[(a^(1/4
)*x)/c^(1/4)])/(2*a^(3/4)*c^(1/4)*(B + C)*(b*c + a*d)*F*H) + ((A*d + (B +
C)*(d*(D + F*G) + b*f*F*H))*Log[Sqrt[d] - Sqrt[2]*b^(1/4)*d^(1/4)*x + Sqrt
[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(B + C)*d^(1/4)*(b*c + a*d)*F*H) - ((A*d + (B
 + C)*(d*(D + F*G) + b*f*F*H))*Log[Sqrt[d] + Sqrt[2]*b^(1/4)*d^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(B + C)*d^(1/4)*(b*c + a*d)*F*H)
```

Defintions of rubi rules used

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.50

method	result
default	$\frac{\frac{(-BFHaf-CFHaf+BFGc+CFGc+BDc+CDc+Ac)\left(2\arctan\left(\frac{x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{c}{a}\right)^{\frac{1}{4}}}{x-\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)\right)}{4(ad+bc)a\left(\frac{c}{a}\right)^{\frac{1}{4}}}+\frac{(BFHbf+CFHbf+BFGd+CFGd+BDd+CD}{}}{(B+C)FH}$

input
```
int(x^2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x^4)/(a*x^4-c)/(b*x^4+d),x,method=_RET
URNVERBOSE)
```

output
```
1/(B+C)/F/H*(1/4*(-B*F*H*a*f-C*F*H*a*f+B*F*G*c+C*F*G*c+B*D*c+C*D*c+A*c)/(a
*d+b*c)/a/(c/a)^(1/4)*(2*arctan(x/(c/a)^(1/4))-ln((x+(c/a)^(1/4))/(x-(c/a)
^(1/4))))+1/8*(B*F*H*b*f+C*F*H*b*f+B*F*G*d+C*F*G*d+B*D*d+C*D*d+A*d)/(a*d+b
*c)/b/(d/b)^(1/4)*2^(1/2)*(ln((x^2-(d/b)^(1/4)*x*2^(1/2)+(d/b)^(1/2))/(x^2
+(d/b)^(1/4)*x*2^(1/2)+(d/b)^(1/2)))+2*arctan(2^(1/2)/(d/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(d/b)^(1/4)*x-1)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)(d+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x^4)/(a*x^4-c)/(b*x^4+d),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)(d+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x**4)/(a*x**4-c)/(b*x**4+d),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.67

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)\,(d+bx^4)}\,dx$$

$$= \frac{((B+C)FHbf+((B+C)FG+(B+C)D+A)d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{b}\sqrt{d}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{d}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\right.}{2\sqrt{\sqrt{}}}\right.}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{d}}}\right.}{8\left((B+C)FHbc+(B+C)FHad\right)}$$

$$-\frac{((B+C)FHaf-((B+C)FG+(B+C)D+A)c)\left(\frac{2\arctan\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\log\left(\frac{\sqrt{a}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{a}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\left((B+C)FHbc+(B+C)FHad\right)}$$

input
```
integrate(x^2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x^4)/(a*x^4-c)/(b*x^4+d),x, algo
rithm="maxima")
```

output
```
1/8*((B + C)*F*H*b*f + ((B + C)*F*G + (B + C)*D + A)*d)*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*b^(1/4)*d^(1/4))/sqrt(sqrt(b)*sqrt(d)))
/(sqrt(b)*sqrt(sqrt(b)*sqrt(d))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*b^(1/4)*d^(1/4))/sqrt(sqrt(b)*sqrt(d)))/(sqrt(b)*sqrt(sqrt(b)
*sqrt(d))) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*b^(1/4)*d^(1/4)*x + sqrt(d)
)/(b^(3/4)*d^(1/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*b^(1/4)*d^(1/4)*x
+ sqrt(d))/(b^(3/4)*d^(1/4)))/((B + C)*F*H*b*c + (B + C)*F*H*a*d) - 1/4*((
B + C)*F*H*a*f - ((B + C)*F*G + (B + C)*D + A)*c)*(2*arctan(sqrt(a)*x/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + log((sqrt(a)*x - sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*x + sqrt(sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(c))))/((B + C)*F*H*b*c + (B + C)*F*H*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(417) = 834$.

Time = 0.18 (sec) , antiderivative size = 1229, normalized size of antiderivative = 2.44

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)(d+bx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x^4)/(a*x^4-c)/(b*x^4+d),x, algo
rithm="giac")
```

output
```
-1/2*(B*F*H*a*f + C*F*H*a*f - B*F*G*c - C*F*G*c - B*D*c - C*D*c - A*c)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))/(sqrt(2)*(-a^3*
c)^(1/4)*B*F*H*b*c + sqrt(2)*(-a^3*c)^(1/4)*C*F*H*b*c + sqrt(2)*(-a^3*c)^(
1/4)*B*F*H*a*d + sqrt(2)*(-a^3*c)^(1/4)*C*F*H*a*d) - 1/2*(B*F*H*a*f + C*F*
H*a*f - B*F*G*c - C*F*G*c - B*D*c - C*D*c - A*c)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))/(sqrt(2)*(-a^3*c)^(1/4)*B*F*H*b*c + s
qrt(2)*(-a^3*c)^(1/4)*C*F*H*b*c + sqrt(2)*(-a^3*c)^(1/4)*B*F*H*a*d + sqrt(
2)*(-a^3*c)^(1/4)*C*F*H*a*d) + 1/2*((b^3*d)^(3/4)*B*F*H*b*f + (b^3*d)^(3/4
)*C*F*H*b*f + (b^3*d)^(3/4)*B*F*G*d + (b^3*d)^(3/4)*C*F*G*d + (b^3*d)^(3/4
)*B*D*d + (b^3*d)^(3/4)*C*D*d + (b^3*d)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(d/b)^(1/4))/(d/b)^(1/4))/(sqrt(2)*B*F*H*b^4*c*d + sqrt(2)*C*F*
H*b^4*c*d + sqrt(2)*B*F*H*a*b^3*d^2 + sqrt(2)*C*F*H*a*b^3*d^2) + 1/2*((b^3
*d)^(3/4)*B*F*H*b*f + (b^3*d)^(3/4)*C*F*H*b*f + (b^3*d)^(3/4)*B*F*G*d + (b
^3*d)^(3/4)*C*F*G*d + (b^3*d)^(3/4)*B*D*d + (b^3*d)^(3/4)*C*D*d + (b^3*d)^
(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(d/b)^(1/4))/(d/b)^(1/4))/(sq
rt(2)*B*F*H*b^4*c*d + sqrt(2)*C*F*H*b^4*c*d + sqrt(2)*B*F*H*a*b^3*d^2 + sq
rt(2)*C*F*H*a*b^3*d^2) + 1/4*(B*F*H*a*f + C*F*H*a*f - B*F*G*c - C*F*G*c -
B*D*c - C*D*c - A*c)*log(x^2 + sqrt(2)*x*(-c/a)^(1/4) + sqrt(-c/a))/(sqrt(
2)*(-a^3*c)^(1/4)*B*F*H*b*c + sqrt(2)*(-a^3*c)^(1/4)*C*F*H*b*c + sqrt(2)*(
-a^3*c)^(1/4)*B*F*H*a*d + sqrt(2)*(-a^3*c)^(1/4)*C*F*H*a*d) - 1/4*(B*F*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)\,(d+bx^4)}\,dx=\int \frac{x^2\,\left(f-x^4\,\left(\frac{G}{H}+\frac{D}{F\,H}+\frac{A}{F\,H\,(B+C)}\right)\right)}{(c-a\,x^4)\,\left(b\,x^4+d\right)}\,dx$$

input
```
int((x^2*(f - x^4*(G/H + D/(F*H) + A/(F*H*(B + C)))))/((c - a*x^4)*(d + b*
x^4)),x)
```

output
```
int((x^2*(f - x^4*(G/H + D/(F*H) + A/(F*H*(B + C)))))/((c - a*x^4)*(d + b*
x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1623, normalized size of antiderivative = 3.23

$$\int \frac{x^2\left(-f+\left(\frac{A}{(B+C)FH}+\frac{D}{FH}+\frac{G}{H}\right)x^4\right)}{(-c+ax^4)\,(d+bx^4)}\,dx=\text{Too large to display}$$

input
```
int(x^2*(-f+(A/(B+C)/F/H+D/F/H+G/H)*x^4)/(a*x^4-c)/(b*x^4+d),x)
```

output
```
( - 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a**2*c*d - 2*d**(3/4)*b**(1/4)*sqrt(2)*a
tan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))
*a*b**2*c*f**2*h - 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqr
t(2) - 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a*b*c**2*f**2*h - 2*d**(3
/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(d**(1
/4)*b**(1/4)*sqrt(2)))*a*b*c*d**2 - 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(
1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a*b*c*d*
f*g - 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt
(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a*c**2*d**2 - 2*d**(3/4)*b**(1/4)*sqrt
(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt
(2)))*a*c**2*d*f*g + 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*s
qrt(2) + 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a**2*c*d + 2*d**(3/4)*b
**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(d**(1/4)*b
**(1/4)*sqrt(2)))*a*b**2*c*f**2*h + 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(
1/4)*b**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a*b*c**2
*f**2*h + 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) + 2*
sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a*b*c*d**2 + 2*d**(3/4)*b**(1/4)*s
qrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*s
qrt(2)))*a*b*c*d*f*g + 2*d**(3/4)*b**(1/4)*sqrt(2)*atan((d**(1/4)*b**(1...
```

3.109 $\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 345

$$\begin{aligned}\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx &= \frac{x}{4(bc+ad)(c-ax^4)} + \frac{(3bc-ad)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{8\sqrt[4]{a}c^{3/4}(bc+ad)^2} \\ &\quad + \frac{b^{3/4}\sqrt[4]{d}\arctan\left(\frac{\sqrt[4]{d}-\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}(bc+ad)^2} \\ &\quad - \frac{b^{3/4}\sqrt[4]{d}\arctan\left(\frac{\sqrt[4]{d}+\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{2\sqrt{2}(bc+ad)^2} \\ &\quad + \frac{(3bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{8\sqrt[4]{a}c^{3/4}(bc+ad)^2} \\ &\quad + \frac{b^{3/4}\sqrt[4]{d}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2\right)}{4\sqrt{2}(bc+ad)^2} \\ &\quad - \frac{b^{3/4}\sqrt[4]{d}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2\right)}{4\sqrt{2}(bc+ad)^2}\end{aligned}$$

output
```
1/4*x/(a*d+b*c)/(-a*x^4+c)+1/8*(-a*d+3*b*c)*arctan(a^(1/4)*x/c^(1/4))/a^(1
/4)/c^(3/4)/(a*d+b*c)^2+1/4*b^(3/4)*d^(1/4)*arctan((d^(1/4)-2^(1/2)*b^(1/4
)*x)/d^(1/4))*2^(1/2)/(a*d+b*c)^2-1/4*b^(3/4)*d^(1/4)*arctan((d^(1/4)+2^(1
/2)*b^(1/4)*x)/d^(1/4))*2^(1/2)/(a*d+b*c)^2+1/8*(-a*d+3*b*c)*arctanh(a^(1/
4)*x/c^(1/4))/a^(1/4)/c^(3/4)/(a*d+b*c)^2+1/8*b^(3/4)*d^(1/4)*ln(d^(1/2)-2
^(1/2)*b^(1/4)*d^(1/4)*x+b^(1/2)*x^2)*2^(1/2)/(a*d+b*c)^2-1/8*b^(3/4)*d^(1
/4)*ln(d^(1/2)+2^(1/2)*b^(1/4)*d^(1/4)*x+b^(1/2)*x^2)*2^(1/2)/(a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx$$

$$= \frac{\frac{4(bc+ad)x}{c-ax^4} - \frac{2(-3bc+ad)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{\sqrt[4]{a}c^{3/4}} + 4\sqrt{2}b^{3/4}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right) - 4\sqrt{2}b^{3/4}\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right.}{}$$

input
```
Integrate[x^4/((-c + a*x^4)^2*(d + b*x^4)),x]
```

output
```
((4*(b*c + a*d)*x)/(c - a*x^4) - (2*(-3*b*c + a*d)*ArcTan[(a^(1/4)*x)/c^(1
/4)])/(a^(1/4)*c^(3/4)) + 4*Sqrt[2]*b^(3/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/d^(1/4)] - 4*Sqrt[2]*b^(3/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/d^(1/4)] + ((-3*b*c + a*d)*Log[c^(1/4) - a^(1/4)*x])/(a^(1/4)*c^(3/4))
+ ((3*b*c - a*d)*Log[c^(1/4) + a^(1/4)*x])/(a^(1/4)*c^(3/4)) + 2*Sqrt[2]*b
^(3/4)*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*b^(1/4)*d^(1/4)*x + Sqrt[b]*x^2] - 2*
Sqrt[2]*b^(3/4)*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*b^(1/4)*d^(1/4)*x + Sqrt[b]*
x^2])/(16*(b*c + a*d)^2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {971, 25, 1020, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^4-c)^2(bx^4+d)}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{\int -\frac{d-3bx^4}{(c-ax^4)(bx^4+d)}dx}{4(ad+bc)}+\frac{x}{4(c-ax^4)(ad+bc)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\int \frac{d-3bx^4}{(c-ax^4)(bx^4+d)}dx}{4(ad+bc)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\frac{4bd\int \frac{1}{bx^4+d}dx}{ad+bc}-\frac{(3bc-ad)\int \frac{1}{c-ax^4}dx}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \text{755}$$

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int \frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{d}}{bx^4+d}dx}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\int \frac{1}{c-ax^4}dx}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \text{756}$$

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int \frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{d}}{bx^4+d}dx}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\sqrt{a}x^2}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{a}x^2+\sqrt{c}}dx}{2\sqrt{c}}\right)}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int\frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{d}}{bx^4+d}dx}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\int\frac{1}{\sqrt{c}-\sqrt{a}x^2}dx}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}\right)}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \; 221$$

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int\frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{d}}{bx^4+d}dx}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}\right)}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \; 1476$$

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{d}}+\frac{\int\frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}\right)}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \; 1082$$

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int\frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}\right)}{ad+bc}}{4(ad+bc)}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\int\frac{\sqrt{d}-\sqrt{b}x^2}{bx^4+d}dx}{2\sqrt{d}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}\right)}{ad+bc}}{4(ad+bc)}$$

$\Big\downarrow$ 1479

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{d}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{d}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan}{2\sqrt[4]{}}\right.}{}}{4(ad+bc)}$$

$\Big\downarrow$ 25

$$\frac{x}{4\left(c-ax^4\right)(ad+bc)}-\frac{\frac{4bd\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{d}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{d}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt[4]{a}c^{3}}\right.}{}}{4(ad+bc)}$$

$\Big\downarrow$ 27

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\frac{4bd\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{d}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt{b}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{d}}{x^2+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{b}}+\frac{\sqrt{d}}{\sqrt{b}}}dx}{2\sqrt{b}\sqrt[4]{d}}}{2\sqrt{d}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{c}}\right)}{2\sqrt[4]{a}c^{3/4}}+\frac{\text{arctan}}{2\sqrt[4]{}}\right.}{ad+bc}}{4(ad+bc)}$$

$\downarrow$ 1103

$$\frac{x}{4(c-ax^4)(ad+bc)}-\frac{\frac{4bd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{d}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2+\sqrt{d}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}x+\sqrt{b}x^2+\sqrt{d}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{ad+bc}-\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt[4]{a}c^{3/}}\right.}{}}{4(ad+bc)}$$

input
```
Int[x^4/((-c + a*x^4)^2*(d + b*x^4)),x]
```

output
```
x/(4*(b*c + a*d)*(c - a*x^4)) - (-((((3*b*c - a*d)*(ArcTan[(a^(1/4)*x)/c^(1
/4)]/(2*a^(1/4)*c^(3/4)) + ArcTanh[(a^(1/4)*x)/c^(1/4)]/(2*a^(1/4)*c^(3/4)
)))/(b*c + a*d)) + (4*b*d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/d^(1/4)]/(Sqr
t[2]*b^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/d^(1/4)]/(Sqrt[2]*
b^(1/4)*d^(1/4)))/(2*Sqrt[d]) + (-1/2*Log[Sqrt[d] - Sqrt[2]*b^(1/4)*d^(1/4
)*x + Sqrt[b]*x^2]/(Sqrt[2]*b^(1/4)*d^(1/4)) + Log[Sqrt[d] + Sqrt[2]*b^(1/
4)*d^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*b^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(b*
c + a*d))/(4*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.58

method	result
default	$\frac{\frac{\left(-\frac{ad}{4}-\frac{bc}{4}\right)x}{a\,x^4-c}-\frac{(ad-3bc)\left(\frac{c}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{c}{a}\right)^{\frac{1}{4}}}{x-\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}\right)\right)}{16c}}{(ad+bc)^2}-\frac{b\left(\frac{d}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{b}}}{x^2-\left(\frac{d}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{8(ad+bc)^2}$
risch	Expression too large to display

input
```
int(x^4/(a*x^4-c)^2/(b*x^4+d),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d+b*c)^2*((-1/4*a*d-1/4*b*c)*x/(a*x^4-c)-1/16*(a*d-3*b*c)*(c/a)^(1/4)
/c*(ln((x+(c/a)^(1/4))/(x-(c/a)^(1/4)))+2*arctan(x/(c/a)^(1/4))))-1/8*b/(a
*d+b*c)^2*(d/b)^(1/4)*2^(1/2)*(ln((x^2+(d/b)^(1/4)*x*2^(1/2)+(d/b)^(1/2))/
(x^2-(d/b)^(1/4)*x*2^(1/2)+(d/b)^(1/2)))+2*arctan(2^(1/2)/(d/b)^(1/4)*x+1)
+2*arctan(2^(1/2)/(d/b)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 2801, normalized size of antiderivative = 8.12

$$\int \frac{x^4}{(-c+ax^4)^2\,(d+bx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(a*x^4-c)^2/(b*x^4+d),x, algorithm="fricas")
```

output
```
-1/16*(((a*b*c + a^2*d)*x^4 - b*c^2 - a*c*d)*((81*b^4*c^4 - 108*a*b^3*c^3*
d + 54*a^2*b^2*c^2*d^2 - 12*a^3*b*c*d^3 + a^4*d^4)/(a*b^8*c^11 + 8*a^2*b^7
*c^10*d + 28*a^3*b^6*c^9*d^2 + 56*a^4*b^5*c^8*d^3 + 70*a^5*b^4*c^7*d^4 + 5
6*a^6*b^3*c^6*d^5 + 28*a^7*b^2*c^5*d^6 + 8*a^8*b*c^4*d^7 + a^9*c^3*d^8))^(
1/4)*log(-(3*b*c - a*d)*x + (b^2*c^3 + 2*a*b*c^2*d + a^2*c*d^2)*((81*b^4*c
^4 - 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 12*a^3*b*c*d^3 + a^4*d^4)/(a*b
^8*c^11 + 8*a^2*b^7*c^10*d + 28*a^3*b^6*c^9*d^2 + 56*a^4*b^5*c^8*d^3 + 70*
a^5*b^4*c^7*d^4 + 56*a^6*b^3*c^6*d^5 + 28*a^7*b^2*c^5*d^6 + 8*a^8*b*c^4*d^
7 + a^9*c^3*d^8))^(1/4)) - ((a*b*c + a^2*d)*x^4 - b*c^2 - a*c*d)*((81*b^4*
c^4 - 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 12*a^3*b*c*d^3 + a^4*d^4)/(a*
b^8*c^11 + 8*a^2*b^7*c^10*d + 28*a^3*b^6*c^9*d^2 + 56*a^4*b^5*c^8*d^3 + 70
*a^5*b^4*c^7*d^4 + 56*a^6*b^3*c^6*d^5 + 28*a^7*b^2*c^5*d^6 + 8*a^8*b*c^4*d
^7 + a^9*c^3*d^8))^(1/4)*log(-(3*b*c - a*d)*x - (b^2*c^3 + 2*a*b*c^2*d + a
^2*c*d^2)*((81*b^4*c^4 - 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 12*a^3*b*c
*d^3 + a^4*d^4)/(a*b^8*c^11 + 8*a^2*b^7*c^10*d + 28*a^3*b^6*c^9*d^2 + 56*a
^4*b^5*c^8*d^3 + 70*a^5*b^4*c^7*d^4 + 56*a^6*b^3*c^6*d^5 + 28*a^7*b^2*c^5*
d^6 + 8*a^8*b*c^4*d^7 + a^9*c^3*d^8))^(1/4)) + (-I*(a*b*c + a^2*d)*x^4 + I
*b*c^2 + I*a*c*d)*((81*b^4*c^4 - 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 12
*a^3*b*c*d^3 + a^4*d^4)/(a*b^8*c^11 + 8*a^2*b^7*c^10*d + 28*a^3*b^6*c^9*d^
2 + 56*a^4*b^5*c^8*d^3 + 70*a^5*b^4*c^7*d^4 + 56*a^6*b^3*c^6*d^5 + 28*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(a*x**4-c)**2/(b*x**4+d),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx = -\frac{\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{b}\sqrt{d}}}\right)}{\sqrt{\sqrt{b}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{b}\sqrt{d}}}\right)}{\sqrt{\sqrt{b}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}b^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{d}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{\frac{1}{4}}}{\cdots}\right.}{8\left(b^2c^2+2abcd+a^2d^2\right)\cdots} - \frac{x}{4\left((abc+a^2d)x^4-bc^2-acd\right)} + \frac{\frac{2(3bc-ad)\arctan\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{(3bc-ad)\log\left(\frac{\sqrt{a}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{a}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{16\left(b^2c^2+2abcd+a^2d^2\right)}$$

input
```
integrate(x^4/(a*x^4-c)^2/(b*x^4+d),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*b^(1/4)*
d^(1/4))/sqrt(sqrt(b)*sqrt(d)))/sqrt(sqrt(b)*sqrt(d)) + 2*sqrt(2)*sqrt(d)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*b^(1/4)*d^(1/4))/sqrt(sqrt(b)*sq
rt(d)))/sqrt(sqrt(b)*sqrt(d)) + sqrt(2)*d^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*
b^(1/4)*d^(1/4)*x + sqrt(d))/b^(1/4) - sqrt(2)*d^(1/4)*log(sqrt(b)*x^2 - s
qrt(2)*b^(1/4)*d^(1/4)*x + sqrt(d))/b^(1/4))*b/(b^2*c^2 + 2*a*b*c*d + a^2*
d^2) - 1/4*x/((a*b*c + a^2*d)*x^4 - b*c^2 - a*c*d) + 1/16*(2*(3*b*c - a*d)
*arctan(sqrt(a)*x/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) -
 (3*b*c - a*d)*log((sqrt(a)*x - sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*x + sqrt(s
qrt(a)*sqrt(c))))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)))/(b^2*c^2 + 2*a*b*c*d +
a^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(258) = 516.

Time = 0.14 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.94

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(a*x^4-c)^2/(b*x^4+d),x, algorithm="giac")
```

output
```
1/8*(3*(-a^3*c)^(1/4)*b*c - (-a^3*c)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))/(sqrt(2)*a*b^2*c^3 + 2*sqrt(2)*a^2*b*c
^2*d + sqrt(2)*a^3*c*d^2) + 1/8*(3*(-a^3*c)^(1/4)*b*c - (-a^3*c)^(1/4)*a*d
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-c/a)^(1/4))/(-c/a)^(1/4))/(sqrt(2)*a
*b^2*c^3 + 2*sqrt(2)*a^2*b*c^2*d + sqrt(2)*a^3*c*d^2) + 1/16*(3*(-a^3*c)^(
1/4)*b*c - (-a^3*c)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(-c/a)^(1/4) + sqrt(-c/
a))/(sqrt(2)*a*b^2*c^3 + 2*sqrt(2)*a^2*b*c^2*d + sqrt(2)*a^3*c*d^2) - 1/16
*(3*(-a^3*c)^(1/4)*b*c - (-a^3*c)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(-c/a)^(1
/4) + sqrt(-c/a))/(sqrt(2)*a*b^2*c^3 + 2*sqrt(2)*a^2*b*c^2*d + sqrt(2)*a^3
*c*d^2) - 1/2*(b^3*d)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(d/b)^(1/4))
/(d/b)^(1/4))/(sqrt(2)*b^2*c^2 + 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/
2*(b^3*d)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(d/b)^(1/4))/(d/b)^(1/4)
)/(sqrt(2)*b^2*c^2 + 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/4*(b^3*d)^(1
/4)*log(x^2 + sqrt(2)*x*(d/b)^(1/4) + sqrt(d/b))/(sqrt(2)*b^2*c^2 + 2*sqrt
(2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/4*(b^3*d)^(1/4)*log(x^2 - sqrt(2)*x*(d/
b)^(1/4) + sqrt(d/b))/(sqrt(2)*b^2*c^2 + 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d
^2) - 1/4*x/((a*x^4 - c)*(b*c + a*d))
```

Mupad [B] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 20619, normalized size of antiderivative = 59.77

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((c - a*x^4)^2*(d + b*x^4)),x)
```

output
```
2*atan((((((((a^6*b^6*d^5)/16 + (51*a^5*b^7*c*d^4)/16 + (81*a^3*b^9*c^3*d^2
)/16 - (189*a^4*b^8*c^2*d^3)/16)*1i)/(a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*d +
3*a^2*b*c*d^2) - ((((a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 - 108*a*b^3
*c^3*d - 12*a^3*b*c*d^3)/(65536*a*b^8*c^11 + 65536*a^9*c^3*d^8 + 524288*a^
2*b^7*c^10*d + 524288*a^8*b*c^4*d^7 + 1835008*a^3*b^6*c^9*d^2 + 3670016*a^
4*b^5*c^8*d^3 + 4587520*a^5*b^4*c^7*d^4 + 3670016*a^6*b^3*c^6*d^5 + 183500
8*a^7*b^2*c^5*d^6))^(1/4)*(4096*a^4*b^13*c^10*d^2 - 1024*a^13*b^4*c*d^11 +
 31744*a^5*b^12*c^9*d^3 + 106496*a^6*b^11*c^8*d^4 + 200704*a^7*b^10*c^7*d^
5 + 229376*a^8*b^9*c^6*d^6 + 157696*a^9*b^8*c^5*d^7 + 57344*a^10*b^7*c^4*d
^8 + 4096*a^11*b^6*c^3*d^9 - 4096*a^12*b^5*c^2*d^10))/(a^3*d^3 + b^3*c^3 +
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2) - (x*(65536*a^4*b^15*c^11*d^2 - 8192*a^14*
b^5*c*d^12 - 4096*a^15*b^4*d^13 + 487424*a^5*b^14*c^10*d^3 + 1564672*a^6*b
^13*c^9*d^4 + 2830336*a^7*b^12*c^8*d^5 + 3178496*a^8*b^11*c^7*d^6 + 235110
4*a^9*b^10*c^6*d^7 + 1261568*a^10*b^9*c^5*d^8 + 581632*a^11*b^8*c^4*d^9 +
229376*a^12*b^7*c^3*d^10 + 45056*a^13*b^6*c^2*d^11)*1i)/(64*(a^6*d^6 + b^6
*c^6 + 15*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 + 6*a*
b^5*c^5*d + 6*a^5*b*c*d^5)))*((a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 -
 108*a*b^3*c^3*d - 12*a^3*b*c*d^3)/(65536*a*b^8*c^11 + 65536*a^9*c^3*d^8 +
 524288*a^2*b^7*c^10*d + 524288*a^8*b*c^4*d^7 + 1835008*a^3*b^6*c^9*d^2 +
3670016*a^4*b^5*c^8*d^3 + 4587520*a^5*b^4*c^7*d^4 + 3670016*a^6*b^3*c^6...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.90

$$\int \frac{x^4}{(-c+ax^4)^2(d+bx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a*x^4-c)^2/(b*x^4+d),x)
```

output

```
(4*d**(1/4)*b**(3/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(d**(1/4)*b**(1/4)*sqrt(2)))*a**2*c*x**4 - 4*d**(1/4)*b**(3/4)*sqrt(2)*a
tan((d**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))
*a*c**2 - 4*d**(1/4)*b**(3/4)*sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) + 2*
sqrt(b)*x)/(d**(1/4)*b**(1/4)*sqrt(2)))*a**2*c*x**4 + 4*d**(1/4)*b**(3/4)*
sqrt(2)*atan((d**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(d**(1/4)*b**(1/4)*
sqrt(2)))*a*c**2 - 2*c**(1/4)*a**(3/4)*atan((sqrt(a)*x)/(c**(1/4)*a**(1/4)
))*a**2*d*x**4 + 6*c**(1/4)*a**(3/4)*atan((sqrt(a)*x)/(c**(1/4)*a**(1/4)))
*a*b*c*x**4 + 2*c**(1/4)*a**(3/4)*atan((sqrt(a)*x)/(c**(1/4)*a**(1/4)))*a*
c*d - 6*c**(1/4)*a**(3/4)*atan((sqrt(a)*x)/(c**(1/4)*a**(1/4)))*b*c**2 - c
**(1/4)*a**(3/4)*log(a**(1/4)*x + c**(1/4))*a**2*d*x**4 + 3*c**(1/4)*a**(3
/4)*log(a**(1/4)*x + c**(1/4))*a*b*c*x**4 + c**(1/4)*a**(3/4)*log(a**(1/4)
*x + c**(1/4))*a*c*d - 3*c**(1/4)*a**(3/4)*log(a**(1/4)*x + c**(1/4))*b*c*
*2 + c**(1/4)*a**(3/4)*log(a**(1/4)*x - c**(1/4))*a**2*d*x**4 - 3*c**(1/4)
*a**(3/4)*log(a**(1/4)*x - c**(1/4))*a*b*c*x**4 - c**(1/4)*a**(3/4)*log(a*
*(1/4)*x - c**(1/4))*a*c*d + 3*c**(1/4)*a**(3/4)*log(a**(1/4)*x - c**(1/4)
)*b*c**2 + 2*d**(1/4)*b**(3/4)*sqrt(2)*log( - d**(1/4)*b**(1/4)*sqrt(2)*x
+ sqrt(b)*x**2 + sqrt(d))*a**2*c*x**4 - 2*d**(1/4)*b**(3/4)*sqrt(2)*log( -
 d**(1/4)*b**(1/4)*sqrt(2)*x + sqrt(b)*x**2 + sqrt(d))*a*c**2 - 2*d**(1/4)
*b**(3/4)*sqrt(2)*log(d**(1/4)*b**(1/4)*sqrt(2)*x + sqrt(b)*x**2 + sqrt...
```

3.110 $\int \frac{1}{x^2(b+ax^8)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 565

$$\int \frac{1}{x^2(b+ax^8)}\,dx = -\frac{1}{bx} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{-\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}-\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8b^{9/8}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{-\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}+\sqrt{2\left(2-\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8b^{9/8}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}-\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8b^{9/8}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{\sqrt[8]{b}+\sqrt{2}\sqrt[8]{b}+\sqrt{2\left(2+\sqrt{2}\right)}\sqrt[8]{a}x}{\sqrt[8]{b}}\right)}{8b^{9/8}} - \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\log\left(\sqrt[4]{b}-\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16b^{9/8}} + \frac{\sqrt{2-\sqrt{2}}\sqrt[8]{a}\log\left(\sqrt[4]{b}+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16b^{9/8}} - \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\log\left(\sqrt[4]{b}-\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16b^{9/8}} + \frac{\sqrt{2+\sqrt{2}}\sqrt[8]{a}\log\left(\sqrt[4]{b}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\sqrt[8]{b}x+\sqrt[4]{a}x^2\right)}{16b^{9/8}}$$

output
```
-1/b/x+1/8*(2+2^(1/2))^(1/2)*a^(1/8)*arctan((-b^(1/8)+b^(1/8)*2^(1/2)-(4-2
*2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/b^(9/8)-1/8*(2+2^(1/2))^(1/2)*a^(1/8)*
arctan((-b^(1/8)+b^(1/8)*2^(1/2)+(4-2*2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/b
^(9/8)+1/8*(2-2^(1/2))^(1/2)*a^(1/8)*arctan((b^(1/8)+b^(1/8)*2^(1/2)-(4+2*
2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/b^(9/8)-1/8*(2-2^(1/2))^(1/2)*a^(1/8)*a
rctan((b^(1/8)+b^(1/8)*2^(1/2)+(4+2*2^(1/2))^(1/2)*a^(1/8)*x)/b^(1/8))/b^(
9/8)-1/16*(2-2^(1/2))^(1/2)*a^(1/8)*ln(b^(1/4)-(2-2^(1/2))^(1/2)*a^(1/8)*b
^(1/8)*x+a^(1/4)*x^2)/b^(9/8)+1/16*(2-2^(1/2))^(1/2)*a^(1/8)*ln(b^(1/4)+(2
-2^(1/2))^(1/2)*a^(1/8)*b^(1/8)*x+a^(1/4)*x^2)/b^(9/8)-1/16*(2+2^(1/2))^(1
/2)*a^(1/8)*ln(b^(1/4)-(2+2^(1/2))^(1/2)*a^(1/8)*b^(1/8)*x+a^(1/4)*x^2)/b^
(9/8)+1/16*(2+2^(1/2))^(1/2)*a^(1/8)*ln(b^(1/4)+(2+2^(1/2))^(1/2)*a^(1/8)*
b^(1/8)*x+a^(1/4)*x^2)/b^(9/8)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2 (b + ax^8)} \, dx =$$
$$-\frac{8\sqrt[8]{b} + 2\sqrt[8]{a}x \arctan\left(\frac{\sqrt[8]{a}x\sec(\frac{\pi}{8})}{\sqrt[8]{b}} - \tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right) + 2\sqrt[8]{a}x \arctan\left(\frac{\sqrt[8]{a}x\sec(\frac{\pi}{8})}{\sqrt[8]{b}} + \tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right)}{}$$

input
```
Integrate[1/(x^2*(b + a*x^8)),x]
```

output
```
-1/8*(8*b^(1/8) + 2*a^(1/8)*x*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) - Tan[P
i/8]]*Cos[Pi/8] + 2*a^(1/8)*x*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) + Tan[P
i/8]]*Cos[Pi/8] + a^(1/8)*x*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8
)*b^(1/8)*x*Cos[Pi/8]] - a^(1/8)*x*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 + 2
*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - 2*a^(1/8)*x*ArcTan[Cot[Pi/8] - (a^(1/8)*x*
Csc[Pi/8])/b^(1/8)]*Sin[Pi/8] + 2*a^(1/8)*x*ArcTan[Cot[Pi/8] + (a^(1/8)*x*
Csc[Pi/8])/b^(1/8)]*Sin[Pi/8] + a^(1/8)*x*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^
(1/8)*b^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8] - a^(1/8)*x*Log[b^(1/4) + a^(1/4)*x^2
 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8])/(b^(9/8)*x)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.58, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {847, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^8+b)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{a \int \frac{x^6}{ax^8+b}dx}{b}-\frac{1}{bx}$$

$$\Big\downarrow \text{830}$$

$$-\frac{a \left(\frac{\int \frac{x^2}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{-a}}-\frac{\int \frac{x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$$\Big\downarrow \text{826}$$

$$-\frac{a \left(\frac{\int \frac{x^2}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}-\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$$\Big\downarrow \text{827}$$

$$-\frac{a \left(\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{-a}}-\frac{\int \frac{1}{\sqrt[4]{-a}x^2+\sqrt[4]{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}-\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a\left(\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{-a}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}-\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$\downarrow$ 221

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}-\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$\downarrow$ 1476

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}}{2\sqrt[4]{-a}}-\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$\downarrow$ 1082

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$\downarrow$ 217

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}-\frac{\int\frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$$\Big\downarrow \; 1479$$

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}-\frac{-\frac{\int -\frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\cdots}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\cdots\right)}{b}\cdots$$

$$\frac{1}{bx}$$

$$\Big\downarrow \; 25$$

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}-\frac{\frac{\int \frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int \frac{\sqrt{2}\left(\cdots\right)}{x^2+\frac{\sqrt{2}\cdots}{\cdots}}\cdots}{2\sqrt{2}\cdots}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\cdots\right)}{b}\cdots$$

$$\frac{1}{bx}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}-\frac{\int\frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int\frac{\sqrt{2}\cdots}{x^2+\frac{\sqrt{2}\cdots}{\cdots}}}{2\cdots}}{2\sqrt[4]{-a}\cdots}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

$\Big\downarrow$ 1103

$$-\frac{a\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}-\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2(-a)^{3/8}\sqrt[8]{b}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{-a}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}x+\sqrt[4]{-a}x^2+\sqrt[4]{b}\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\right)}{b}-\frac{1}{bx}$$

input
```
Int[1/(x^2*(b + a*x^8)),x]
```

output
```
-(1/(b*x)) - (a*((-1/2*ArcTan[((-a)^(1/8)*x)/b^(1/8)]/((-a)^(3/8)*b^(1/8))
 + ArcTanh[((-a)^(1/8)*x)/b^(1/8)]/(2*(-a)^(3/8)*b^(1/8)))/(2*Sqrt[-a]) -
((-(ArcTan[1 - (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8)
)) + ArcTan[1 + (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8
)))/(2*(-a)^(1/4)) - (-1/2*Log[b^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-
a)^(1/4)*x^2]/(Sqrt[2]*(-a)^(1/8)*b^(1/8)) + Log[b^(1/4) + Sqrt[2]*(-a)^(1
/8)*b^(1/8)*x + (-a)^(1/4)*x^2]/(2*Sqrt[2]*(-a)^(1/8)*b^(1/8)))/(2*(-a)^(1
/4)))/(2*Sqrt[-a])))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.06

method	result	size
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}a+b\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{8b}-\frac{1}{bx}$	36
risch	$-\frac{1}{bx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{9}\_Z^{8}+a\right)}\_R\ln\left(\left(9\_R^{8}b^{9}+8a\right)x+b^{8}\_R^{7}\right)\right)}{8}$	50

input
```
int(1/x^2/(a*x^8+b),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^8*a+b))-1/b/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2(b+ax^8)}\,dx =$$
$$-\frac{-(i-1)\sqrt{2}bx\left(-\frac{a}{b^9}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}b^8\left(-\frac{a}{b^9}\right)^{\frac{7}{8}}+ax\right)+(i+1)\sqrt{2}bx\left(-\frac{a}{b^9}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}b^8\right.}{}$$

input
```
integrate(1/x^2/(a*x^8+b),x, algorithm="fricas")
```

output
```
-1/16*(-(I - 1)*sqrt(2)*b*x*(-a/b^9)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*b^8*(
-a/b^9)^(7/8) + a*x) + (I + 1)*sqrt(2)*b*x*(-a/b^9)^(1/8)*log(-(1/2*I - 1/
2)*sqrt(2)*b^8*(-a/b^9)^(7/8) + a*x) - (I + 1)*sqrt(2)*b*x*(-a/b^9)^(1/8)*
log((1/2*I - 1/2)*sqrt(2)*b^8*(-a/b^9)^(7/8) + a*x) + (I - 1)*sqrt(2)*b*x*
(-a/b^9)^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*b^8*(-a/b^9)^(7/8) + a*x) + 2*b*
x*(-a/b^9)^(1/8)*log(b^8*(-a/b^9)^(7/8) + a*x) - 2*I*b*x*(-a/b^9)^(1/8)*lo
g(I*b^8*(-a/b^9)^(7/8) + a*x) + 2*I*b*x*(-a/b^9)^(1/8)*log(-I*b^8*(-a/b^9)
^(7/8) + a*x) - 2*b*x*(-a/b^9)^(1/8)*log(-b^8*(-a/b^9)^(7/8) + a*x) + 16)/
(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\left(b+ax^8\right)}\,dx = \operatorname{RootSum}\left(16777216t^8b^9+a,\left(t\mapsto t\log\left(-\frac{2097152t^7b^8}{a}+x\right)\right)\right) - \frac{1}{bx}$$

input
```
integrate(1/x**2/(a*x**8+b),x)
```

output
```
RootSum(16777216*_t**8*b**9 + a, Lambda(_t, _t*log(-2097152*_t**7*b**8/a +
 x))) - 1/(b*x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(b+ax^8\right)}\,dx = \int \frac{1}{(ax^8+b)x^2}\,dx$$

input
```
integrate(1/x^2/(a*x^8+b),x, algorithm="maxima")
```

output
```
-a*integrate(x^6/(a*x^8 + b), x)/b - 1/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2 (b+ax^8)}\,dx = -\frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\arctan\left(\frac{2\,x+\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b^2\sqrt{-2\,\sqrt{2}+4}} - \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\arctan\left(\frac{2\,x-\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b^2\sqrt{-2\,\sqrt{2}+4}} - \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\arctan\left(\frac{2\,x+\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b^2\sqrt{2\,\sqrt{2}+4}} - \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\arctan\left(\frac{2\,x-\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4\,b^2\sqrt{2\,\sqrt{2}+4}} + \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b^2\sqrt{-2\,\sqrt{2}+4}} - \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b^2\sqrt{-2\,\sqrt{2}+4}} + \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b^2\sqrt{2\,\sqrt{2}+4}} - \frac{a\left(\frac{b}{a}\right)^{\frac{7}{8}}\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8\,b^2\sqrt{2\,\sqrt{2}+4}} - \frac{1}{bx}$$

input
```
integrate(1/x^2/(a*x^8+b),x, algorithm="giac")
```

output
```
-1/4*a*(b/a)^(7/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqr
t(2) + 2)*(b/a)^(1/8)))/(b^2*sqrt(-2*sqrt(2) + 4)) - 1/4*a*(b/a)^(7/8)*arc
tan((2*x - sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqrt(2) + 2)*(b/a)^(1/8))
)/(b^2*sqrt(-2*sqrt(2) + 4)) - 1/4*a*(b/a)^(7/8)*arctan((2*x + sqrt(sqrt(2
) + 2)*(b/a)^(1/8))/(sqrt(-sqrt(2) + 2)*(b/a)^(1/8)))/(b^2*sqrt(2*sqrt(2)
+ 4)) - 1/4*a*(b/a)^(7/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(b/a)^(1/8))/(sq
rt(-sqrt(2) + 2)*(b/a)^(1/8)))/(b^2*sqrt(2*sqrt(2) + 4)) + 1/8*a*(b/a)^(7/
8)*log(x^2 + x*sqrt(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(b^2*sqrt(-2*s
qrt(2) + 4)) - 1/8*a*(b/a)^(7/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(b/a)^(1/8)
 + (b/a)^(1/4))/(b^2*sqrt(-2*sqrt(2) + 4)) + 1/8*a*(b/a)^(7/8)*log(x^2 + x
*sqrt(-sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(b^2*sqrt(2*sqrt(2) + 4)) -
 1/8*a*(b/a)^(7/8)*log(x^2 - x*sqrt(-sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4
))/(b^2*sqrt(2*sqrt(2) + 4)) - 1/(b*x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2 (b+ax^8)} \, dx = -\frac{1}{b\,x} - \frac{(-a)^{1/8}\,\mathrm{atan}\left(\frac{(-a)^{1/8}\,x}{b^{1/8}}\right)}{4\,b^{9/8}} - \frac{(-a)^{1/8}\,\mathrm{atan}\left(\frac{(-a)^{1/8}\,x\,1\mathrm{i}}{b^{1/8}}\right)\,1\mathrm{i}}{4\,b^{9/8}} + \frac{\sqrt{2}\,(-a)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-a)^{1/8}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{b^{1/8}}\right)\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{b^{9/8}} + \frac{\sqrt{2}\,(-a)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-a)^{1/8}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{b^{1/8}}\right)\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{b^{9/8}}$$

input
```
int(1/(x^2*(b + a*x^8)),x)
```

output
```
- 1/(b*x) - ((-a)^(1/8)*atan(((-a)^(1/8)*x)/b^(1/8)))/(4*b^(9/8)) - ((-a)^
(1/8)*atan(((-a)^(1/8)*x*1i)/b^(1/8))*1i)/(4*b^(9/8)) - (2^(1/2)*(-a)^(1/8
)*atan((2^(1/2)*(-a)^(1/8)*x*(1/2 - 1i/2))/b^(1/8))*(1/8 - 1i/8))/b^(9/8)
- (2^(1/2)*(-a)^(1/8)*atan((2^(1/2)*(-a)^(1/8)*x*(1/2 + 1i/2))/b^(1/8))*(1
/8 + 1i/8))/b^(9/8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2 (b + ax^8)} \, dx$$

$$= \frac{2b^{\frac{7}{8}} a^{\frac{1}{8}} \sqrt{\sqrt{2}+2}\, atan\left(\frac{b^{\frac{1}{8}} a^{\frac{1}{8}} \sqrt{-\sqrt{2}+2} - 2a^{\frac{1}{4}} x}{b^{\frac{1}{8}} a^{\frac{1}{8}} \sqrt{\sqrt{2}+2}}\right) x - 2b^{\frac{7}{8}} a^{\frac{1}{8}} \sqrt{\sqrt{2}+2}\, atan\left(\frac{b^{\frac{1}{8}} a^{\frac{1}{8}} \sqrt{-\sqrt{2}+2} + 2a^{\frac{1}{4}} x}{b^{\frac{1}{8}} a^{\frac{1}{8}} \sqrt{\sqrt{2}+2}}\right) x + 2b^{\frac{7}{8}} a^{\frac{1}{8}} \sqrt{-\sqrt{}}$$

input
```
int(1/x^2/(a*x^8+b),x)
```

output
```
(2*b**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( - sqr
t(2) + 2) - 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*x - 2*b**
(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( - sqrt(2) +
 2) + 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*x + 2*b**(7/8)*
a**(1/8)*sqrt( - sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) -
2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*x - 2*b**(7/8)*a**
(1/8)*sqrt( - sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*a
**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*x - b**(7/8)*a**(1/8)
*sqrt( - sqrt(2) + 2)*log( - b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*x + a*
*(1/4)*x**2 + b**(1/4))*x + b**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*log(b**
(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4))*x - b**(
7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*log( - b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)
*x + a**(1/4)*x**2 + b**(1/4))*x + b**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*log
(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4))*x - 16*
b)/(16*b**2*x)
```

3.111 $\int \frac{1}{\left(-bx^2+ax^8\right)^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 232

$$\begin{aligned}\int \frac{1}{\left(-bx^2+ax^8\right)^4} dx &= \frac{-1296b^4-32400ab^3x^6+114475a^2b^2x^{12}-123500a^3bx^{18}+43225a^4x^{24}}{9072b^5x^7\left(b-ax^6\right)^3} \\ &\quad - \frac{6175a^{7/6}\arctan\left(\frac{-\sqrt[3]{b}+\sqrt[3]{a}x^2}{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}\right)}{2592\sqrt{3}b^{31/6}} + \frac{6175a^{7/6}\text{arctanh}\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3888b^{31/6}} \\ &\quad - \frac{6175a^{7/6}\log\left(\sqrt[3]{b}-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)}{15552b^{31/6}} + \frac{6175a^{7/6}\log\left(\sqrt[3]{b}+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2\right)}{15552b^{31/6}}\end{aligned}$$

output
```
1/9072*(43225*a^4*x^24-123500*a^3*b*x^18+114475*a^2*b^2*x^12-32400*a*b^3*x
^6-1296*b^4)/b^5/x^7/(-a*x^6+b)^3-6175/7776*a^(7/6)*arctan(1/3*(-b^(1/3)+a
^(1/3)*x^2)*3^(1/2)/a^(1/6)/b^(1/6)/x)*3^(1/2)/b^(31/6)+6175/3888*a^(7/6)*
arctanh(a^(1/6)*x/b^(1/6))/b^(31/6)-6175/15552*a^(7/6)*ln(b^(1/3)-a^(1/6)*
b^(1/6)*x+a^(1/3)*x^2)/b^(31/6)+6175/15552*a^(7/6)*ln(b^(1/3)+a^(1/6)*b^(1
/6)*x+a^(1/3)*x^2)/b^(31/6)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.22

$$\int \frac{1}{(-bx^2+ax^8)^4}\,dx$$

$$= \frac{-\frac{15552b^{7/6}}{x^7}-\frac{435456a\sqrt[6]{b}}{x}+\frac{24696a^2b^{7/6}x^5}{(b-ax^6)^2}+\frac{83244a^2\sqrt[6]{b}x^5}{b-ax^6}-\frac{6048a^2b^{13/6}x^5}{(-b+ax^6)^3}+86450\sqrt{3}a^{7/6}\arctan\left(\frac{1-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}}{\sqrt{3}}\right)-}{}$$

input
```
Integrate[(-(b*x^2) + a*x^8)^(-4),x]
```

output
```
((-15552*b^(7/6))/x^7 - (435456*a*b^(1/6))/x + (24696*a^2*b^(7/6)*x^5)/(b
- a*x^6)^2 + (83244*a^2*b^(1/6)*x^5)/(b - a*x^6) - (6048*a^2*b^(13/6)*x^5)
/(-b + a*x^6)^3 + 86450*Sqrt[3]*a^(7/6)*ArcTan[(1 - (2*a^(1/6)*x)/b^(1/6))
/Sqrt[3]] - 86450*Sqrt[3]*a^(7/6)*ArcTan[(1 + (2*a^(1/6)*x)/b^(1/6))/Sqrt[
3]] - 86450*a^(7/6)*Log[b^(1/6) - a^(1/6)*x] + 86450*a^(7/6)*Log[b^(1/6) +
 a^(1/6)*x] - 43225*a^(7/6)*Log[b^(1/3) - a^(1/6)*b^(1/6)*x + a^(1/3)*x^2]
 + 43225*a^(7/6)*Log[b^(1/3) + a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(108864*b
^(31/6))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.35, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {2026, 819, 25, 819, 819, 847, 847, 825, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^8-bx^2)^4}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^8(ax^6-b)^4}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{1}{18bx^7\left(b-ax^6\right)^3}-\frac{25\int -\frac{1}{x^8(b-ax^6)^3}dx}{18b}$$

$$\Big\downarrow \text{25}$$

$$\frac{25\int \frac{1}{x^8(b-ax^6)^3}dx}{18b}+\frac{1}{18bx^7\left(b-ax^6\right)^3}$$

$$\Big\downarrow \text{819}$$

$$\frac{25\left(\frac{19\int \frac{1}{x^8\left(b-ax^6\right)^2}dx}{12b}+\frac{1}{12bx^7(b-ax^6)^2}\right)}{18b}+\frac{1}{18bx^7\left(b-ax^6\right)^3}$$

$$\Big\downarrow \text{819}$$

$$\frac{25\left(\frac{19\left(\frac{13\int \frac{1}{x^8\left(b-ax^6\right)}dx}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}\right)}{12b}+\frac{1}{12bx^7(b-ax^6)^2}\right)}{18b}+\frac{1}{18bx^7\left(b-ax^6\right)^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{25\left(\frac{19\left(\frac{13\left(\frac{a\int \frac{1}{x^2\left(b-ax^6\right)}dx}{b}-\frac{1}{7bx^7}\right)}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}\right)}{12b}+\frac{1}{12bx^7(b-ax^6)^2}\right)}{18b}+\frac{1}{18bx^7\left(b-ax^6\right)^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{25\left(\frac{19\left(\frac{13\left(\frac{a\left(\frac{a\int\frac{x^4}{b-ax^6}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{7bx^7}\right)}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}\right)}{12b}+\frac{1}{12bx^7\left(b-ax^6\right)^2}\right)}{18b}+\frac{1}{18bx^7\left(b-ax^6\right)^3}$$

$$\Big\downarrow \text{825}$$

$$25\left(\frac{19\left(\frac{13\left(\frac{a\left(\frac{a\left(\frac{\int\frac{1}{\sqrt[3]{b}-\sqrt[3]{a}x^2}dx}{3a^{2/3}}+\frac{\int-\frac{\sqrt[6]{a}x+\sqrt[6]{b}}{2\left(\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3a^{2/3}\sqrt[6]{b}}+\frac{\int-\frac{\sqrt[6]{b}-\sqrt[6]{a}x}{2\left(\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3a^{2/3}\sqrt[6]{b}}\right)}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{7bx^7}\right)}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}}{12b}\right.$$

↓ 27

$$25\left(\frac{19\left(\frac{13\left(\frac{a\left(\frac{a\left(\frac{\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{a}x^2}dx}{3a^{2/3}}-\frac{\int \frac{\sqrt[6]{a}x+\sqrt[6]{b}}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}-\frac{\int \frac{\sqrt[6]{b}-\sqrt[6]{a}x}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}\right)}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{7bx^7}\right)}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}\right)}{12b}+\frac{\cdots}{12bx^7}\right.$$

↓ 221

$$\left(\frac{25\left(\frac{19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\int\frac{\sqrt[6]{a}x+\sqrt[6]{b}}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}-\frac{\int\frac{\sqrt[6]{b}-\sqrt[6]{a}x}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3a^{5/6}\sqrt[6]{b}}\right)}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{7bx^7}\right)}{6b}+\frac{1}{6bx^7\left(b-ax^6\right)}\right)}{12b}+\frac{}{1\cdots}\right.$$

↓ 1142

$$19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\int-\frac{\sqrt[6]{a}\left(\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6a^{2/3}\sqrt[6]{b}}-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\int\frac{\sqrt[6]{a}\left(2\sqrt[6]{a}\cdots\right.}{\sqrt[3]{a}x^2+\sqrt[6]{a}\cdots}}{2\sqrt[6]{\cdots}}}{6a^{2/3}\sqrt[6]{b}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{6b}\cdots\right.$$

↓ 25

$$19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\int\frac{\sqrt[6]{a}\left(\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6a^{2/3}\sqrt[6]{b}}-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\int\frac{\sqrt[6]{a}\left(2\sqrt[6]{a}x\cdots\right)}{\sqrt[3]{a}x^2+\sqrt[6]{a}\cdots}}{2\sqrt[6]{a}\cdots}}{6a^{2/3}\sqrt[6]{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{6b}\cdots\right.$$

↓ 27

$$25\left(\frac{19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{1}{2}\int\frac{\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}-\frac{\frac{3}{2}\sqrt[6]{b}\int\frac{1}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{1}{2}\int\frac{2\sqrt[6]{a}\cdots}{\sqrt[3]{a}x^2+\sqrt[6]{\cdots}}\cdots}{6a^{2/3}\sqrt[6]{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{6b}\cdots\right)}{12b}\cdots\right.$$

↓ 1082

$$19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{\sqrt[6]{a}}-\frac{1}{2}\int\frac{\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}-\frac{-\frac{1}{2}\int\frac{2\sqrt[6]{a}x+\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}+\cdots\right.}\cdots}{\cdots}}{6a^{2/3}\sqrt[6]{b}}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{6b}\cdots\right.$$

↓ 217

$$19\left(13\left(a\left(a\left(-\frac{-\frac{1}{2}\int\frac{\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}}{\sqrt{3}}\right)}{\sqrt[6]{a}}}{6a^{2/3}\sqrt[6]{b}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}+1}{\sqrt{3}}\right)}{\sqrt[6]{a}}-\frac{1}{2}\int\frac{2\sqrt[6]{a}x+\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6a^{2/3}\sqrt[6]{b}}+\frac{\mathrm{arc}\cdots}{\cdots}\right.\right.\right.\right.$$

$$\cdots\frac{\cdots}{b}\cdots\frac{\cdots}{b}\cdots\frac{\cdots}{6b}\cdots$$

↓ 1103

$$19\left(\frac{13\left(\frac{a\left(\frac{a\left(-\frac{\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}}{\sqrt{3}}\right)}{\sqrt[6]{a}}}{6a^{2/3}\sqrt[6]{b}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}+1}{\sqrt{3}}\right)}{\sqrt[6]{a}}-\frac{\log\left(\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}}{6a^{2/3}\sqrt[6]{b}}+\frac{\text{arct}\cdots}{\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{6b}\cdots\right.$$

input
```
Int[(-(b*x^2) + a*x^8)^(-4),x]
```

output
```
1/(18*b*x^7*(b - a*x^6)^3) + (25*(1/(12*b*x^7*(b - a*x^6)^2) + (19*(1/(6*b
*x^7*(b - a*x^6)) + (13*(-1/7*1/(b*x^7) + (a*(-(1/(b*x)) + (a*(ArcTanh[(a^
(1/6)*x)/b^(1/6)]/(3*a^(5/6)*b^(1/6)) - (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/6)
*x)/b^(1/6))/Sqrt[3]])/a^(1/6)) + Log[b^(1/3) - a^(1/6)*b^(1/6)*x + a^(1/3
)*x^2]/(2*a^(1/6)))/(6*a^(2/3)*b^(1/6)) - ((Sqrt[3]*ArcTan[(1 + (2*a^(1/6)
*x)/b^(1/6))/Sqrt[3]])/a^(1/6) - Log[b^(1/3) + a^(1/6)*b^(1/6)*x + a^(1/3)
*x^2]/(2*a^(1/6)))/(6*a^(2/3)*b^(1/6))))/b))/b))/(6*b)))/(12*b)))/(18*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{25a\,x^{6}}{7b^{2}}-\frac{114475a^{2}x^{12}}{9072b^{3}}+\frac{30875a^{3}x^{18}}{2268b^{4}}-\frac{6175a^{4}x^{24}}{1296b^{5}}+\frac{1}{7b}}{x^{7}(a\,x^{6}-b)^{3}}+\frac{6175 (\sum_{\_R=\mathrm{RootOf}(b^{31}\_Z^{6}-a^{7})}\_R\ln((-7\_R^{6}b^{31}+6a^{7})x-a\,b^{26}\_R^{5}}{7776}$
default	$-\frac{a^{2} (\frac{\frac{1357}{1296}b^{2}x^{5}-\frac{569}{324}ab\,x^{11}+\frac{991}{1296}a^{2}x^{17}}{(a\,x^{6}-b)^{3}}+\frac{6175\ln(-x+(\frac{b}{a})^{\frac{1}{6}})}{7776a(\frac{b}{a})^{\frac{1}{6}}}+\frac{6175(\frac{b}{a})^{\frac{5}{6}}\ln((\frac{b}{a})^{\frac{1}{6}}x-x^{2}-(\frac{b}{a})^{\frac{1}{3}})}{15552b}+\frac{6175\sqrt{3}\arctan(-\frac{\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3(\frac{b}{a})^{\frac{1}{6}}})}{7776a(\frac{b}{a})^{\frac{1}{6}}}}{b^{5}}$

input
```
int(1/(a*x^8-b*x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
(25/7*a/b^2*x^6-114475/9072/b^3*a^2*x^12+30875/2268*a^3/b^4*x^18-6175/1296
*a^4/b^5*x^24+1/7/b)/x^7/(a*x^6-b)^3+6175/7776*sum(_R*ln((-7*_R^6*b^31+6*a
^7)*x-a*b^26*_R^5),_R=RootOf(_Z^6*b^31-a^7))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(176) = 352.

Time = 0.08 (sec) , antiderivative size = 728, normalized size of antiderivative = 3.14

$$\int \frac{1}{(-bx^2+ax^8)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*x^8-b*x^2)^4,x, algorithm="fricas")
```

output
```
-1/108864*(518700*a^4*x^24 - 1482000*a^3*b*x^18 + 1373700*a^2*b^2*x^12 - 3
88800*a*b^3*x^6 - 15552*b^4 - 86450*(a^3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b
^7*x^13 - b^8*x^7)*(a^7/b^31)^(1/6)*log(897810767584960937 5*b^26*(a^7/b^31
)^(5/6) + 897810767584960937 5*a^6*x) + 86450*(a^3*b^5*x^25 - 3*a^2*b^6*x^1
9 + 3*a*b^7*x^13 - b^8*x^7)*(a^7/b^31)^(1/6)*log(-897810767584960937 5*b^26
*(a^7/b^31)^(5/6) + 897810767584960937 5*a^6*x) - 43225*(a^3*b^5*x^25 - 3*a
^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*x^7 - sqrt(-3)*(a^3*b^5*x^25 - 3*a^2*b^6*
x^19 + 3*a*b^7*x^13 - b^8*x^7))*(a^7/b^31)^(1/6)*log(897810767584960937 5*a
^6*x + 897810767584960937 5/2*(sqrt(-3)*b^26 + b^26)*(a^7/b^31)^(5/6)) + 43
225*(a^3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*x^7 - sqrt(-3)*(a^
3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*x^7))*(a^7/b^31)^(1/6)*lo
g(897810767584960937 5*a^6*x - 897810767584960937 5/2*(sqrt(-3)*b^26 + b^26)
*(a^7/b^31)^(5/6)) + 43225*(a^3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 -
 b^8*x^7 + sqrt(-3)*(a^3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*x^
7))*(a^7/b^31)^(1/6)*log(897810767584960937 5*a^6*x + 897810767584960937 5/2
*(sqrt(-3)*b^26 - b^26)*(a^7/b^31)^(5/6)) - 43225*(a^3*b^5*x^25 - 3*a^2*b^
6*x^19 + 3*a*b^7*x^13 - b^8*x^7 + sqrt(-3)*(a^3*b^5*x^25 - 3*a^2*b^6*x^19
+ 3*a*b^7*x^13 - b^8*x^7))*(a^7/b^31)^(1/6)*log(897810767584960937 5*a^6*x
- 897810767584960937 5/2*(sqrt(-3)*b^26 - b^26)*(a^7/b^31)^(5/6)))/(a^3*b^5
*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.51

$$\int \frac{1}{\left(-bx^2+ax^8\right)^4}\,dx$$
$$= \operatorname{RootSum}\left(221073919720733357899776t^6b^{31}-554398148983713378906 25a^7,\left(t\mapsto t\log\left(\frac{28430288029\ldots}{897810767\ldots}\right.\right.\right.$$
$$+\frac{-43225a^4x^{24}+123500a^3bx^{18}-114475a^2b^2x^{12}+32400ab^3x^6+1296b^4}{9072a^3b^5x^{25}-27216a^2b^6x^{19}+27216ab^7x^{13}-9072b^8x^7}$$

input
```
integrate(1/(a*x**8-b*x**2)**4,x)
```

output
```
RootSum(22107391972073335789976*_t**6*b**31 - 55439814898371337890625*a**
7, Lambda(_t, _t*log(28430288029929701376*_t**5*b**26/(8978107675849609375
*a**6) + x))) + (-43225*a**4*x**24 + 123500*a**3*b*x**18 - 114475*a**2*b**
2*x**12 + 32400*a*b**3*x**6 + 1296*b**4)/(9072*a**3*b**5*x**25 - 27216*a**
2*b**6*x**19 + 27216*a*b**7*x**13 - 9072*b**8*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(-bx^2+ax^8\right)^4}\,dx$$
$$= -\frac{43225\,a^4x^{24}-123500\,a^3bx^{18}+114475\,a^2b^2x^{12}-32400\,ab^3x^6-1296\,b^4}{9072\,\left(a^3b^5x^{25}-3\,a^2b^6x^{19}+3\,ab^7x^{13}-b^8x^7\right)}$$
$$-\frac{6175\,a^2\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+x\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}+\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{2}{3}}\right)}{a\left(\frac{\sqrt{b}}{\sqrt{a}}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\right.\right.}{a\left(\right.}\right.}{15552\,b^5}$$

input
```
integrate(1/(a*x^8-b*x^2)^4,x, algorithm="maxima")
```

output
```
-1/9072*(43225*a^4*x^24 - 123500*a^3*b*x^18 + 114475*a^2*b^2*x^12 - 32400*
a*b^3*x^6 - 1296*b^4)/(a^3*b^5*x^25 - 3*a^2*b^6*x^19 + 3*a*b^7*x^13 - b^8*
x^7) - 6175/15552*a^2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (sqrt(b)/sqrt(a
))^(1/3))/(sqrt(b)/sqrt(a))^(1/3))/(a*(sqrt(b)/sqrt(a))^(1/3)) + 2*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (sqrt(b)/sqrt(a))^(1/3))/(sqrt(b)/sqrt(a))^(1/3
))/(a*(sqrt(b)/sqrt(a))^(1/3)) - log(x^2 + x*(sqrt(b)/sqrt(a))^(1/3) + (sq
rt(b)/sqrt(a))^(2/3))/(a*(sqrt(b)/sqrt(a))^(1/3)) + log(x^2 - x*(sqrt(b)/s
qrt(a))^(1/3) + (sqrt(b)/sqrt(a))^(2/3))/(a*(sqrt(b)/sqrt(a))^(1/3)) - 2*l
og(x + (sqrt(b)/sqrt(a))^(1/3))/(a*(sqrt(b)/sqrt(a))^(1/3)) + 2*log(x - (s
qrt(b)/sqrt(a))^(1/3))/(a*(sqrt(b)/sqrt(a))^(1/3)))/b^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.14

$$\int \frac{1}{(-bx^2+ax^8)^4}\,dx = \frac{6175\,a^2\left(-\frac{b}{a}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(-\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3888\,b^6} - \frac{6175\,\sqrt{3}(-a^5b)^{\frac{5}{6}}\log\left(x^2+\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{6}}+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{15552\,a^3b^6} + \frac{6175\,\sqrt{3}(-a^5b)^{\frac{5}{6}}\log\left(x^2-\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{6}}+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{15552\,a^3b^6} - \frac{991\,a^4x^{17}-2276\,a^3bx^{11}+1357\,a^2b^2x^5}{1296\,(ax^6-b)^3b^5} + \frac{6175\,(-a^5b)^{\frac{5}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{6}}}{\left(-\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{7776\,a^3b^6} + \frac{6175\,(-a^5b)^{\frac{5}{6}}\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{6}}}{\left(-\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{7776\,a^3b^6} - \frac{28\,ax^6+b}{7\,b^5x^7}$$

input

```
integrate(1/(a*x^8-b*x^2)^4,x, algorithm="giac")
```

output

```
6175/3888*a^2*(-b/a)^(5/6)*arctan(x/(-b/a)^(1/6))/b^6 - 6175/15552*sqrt(3)
*(-a^5*b)^(5/6)*log(x^2 + sqrt(3)*x*(-b/a)^(1/6) + (-b/a)^(1/3))/(a^3*b^6)
 + 6175/15552*sqrt(3)*(-a^5*b)^(5/6)*log(x^2 - sqrt(3)*x*(-b/a)^(1/6) + (-
b/a)^(1/3))/(a^3*b^6) - 1/1296*(991*a^4*x^17 - 2276*a^3*b*x^11 + 1357*a^2*
b^2*x^5)/((a*x^6 - b)^3*b^5) + 6175/7776*(-a^5*b)^(5/6)*arctan((2*x + sqrt
(3)*(-b/a)^(1/6))/(-b/a)^(1/6))/(a^3*b^6) + 6175/7776*(-a^5*b)^(5/6)*arcta
n((2*x - sqrt(3)*(-b/a)^(1/6))/(-b/a)^(1/6))/(a^3*b^6) - 1/7*(28*a*x^6 + b
)/(b^5*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-bx^2+ax^8)^4}\,dx$$
$$= \frac{\frac{1}{7b}+\frac{25\,a\,x^6}{7\,b^2}-\frac{114475\,a^2\,x^{12}}{9072\,b^3}+\frac{30875\,a^3\,x^{18}}{2268\,b^4}-\frac{6175\,a^4\,x^{24}}{1296\,b^5}}{a^3\,x^{25}-3\,a^2\,b\,x^{19}+3\,a\,b^2\,x^{13}-b^3\,x^7}-\frac{a^{7/6}\,\mathrm{atan}\left(\frac{a^{1/6}\,x\,1\mathrm{i}}{b^{1/6}}\right)\,6175\mathrm{i}}{3888\,b^{31/6}}$$
$$-\frac{a^{7/6}\,\mathrm{atan}\left(\frac{a^{23/2}\,b^{45/2}\,x\,506564570290921895001600000000000\mathrm{i}}{253282285145460947500800000000000\,a^{34/3}\,b^{68/3}-\sqrt{3}\,a^{34/3}\,b^{68/3}\,253282285145460947500800000000000\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3888\,b^{31/6}}$$
$$+\frac{a^{7/6}\,\mathrm{atan}\left(\frac{a^{23/2}\,b^{45/2}\,x\,506564570290921895001600000000000\mathrm{i}}{253282285145460947500800000000000\,a^{34/3}\,b^{68/3}+\sqrt{3}\,a^{34/3}\,b^{68/3}\,253282285145460947500800000000000\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3888\,b^{31/6}}$$

input
```
int(1/(a*x^8 - b*x^2)^4,x)
```

output
```
(1/(7*b) + (25*a*x^6)/(7*b^2) - (114475*a^2*x^12)/(9072*b^3) + (30875*a^3*
x^18)/(2268*b^4) - (6175*a^4*x^24)/(1296*b^5))/(a^3*x^25 - b^3*x^7 + 3*a*b
^2*x^13 - 3*a^2*b*x^19) - (a^(7/6)*atan((a^(1/6)*x*1i)/b^(1/6))*6175i)/(38
88*b^(31/6)) - (a^(7/6)*atan((a^(23/2)*b^(45/2)*x*506564570290921895001600
00000000i)/(253282285145460947500800000000000*a^(34/3)*b^(68/3) - 3^(1/2)*a
^(34/3)*b^(68/3)*253282285145460947500800000000000i))*((3^(1/2)*1i)/2 + 1/2
)*6175i)/(3888*b^(31/6)) + (a^(7/6)*atan((a^(23/2)*b^(45/2)*x*506564570290
921895001600000000000i)/(253282285145460947500800000000000*a^(34/3)*b^(68/3)
 + 3^(1/2)*a^(34/3)*b^(68/3)*253282285145460947500800000000000i))*((3^(1/2)
*1i)/2 - 1/2)*6175i)/(3888*b^(31/6))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.69

$$\int \frac{1}{(-bx^2+ax^8)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a*x^8-b*x^2)^4,x)
```

output
```
(86450*b**(1/6)*a**(1/6)*sqrt(3)*atan((b**(1/6)*a**(1/6) - 2*a**(1/3)*x)/(
b**(1/6)*a**(1/6)*sqrt(3)))*a**4*x**25 - 259350*b**(1/6)*a**(1/6)*sqrt(3)*
atan((b**(1/6)*a**(1/6) - 2*a**(1/3)*x)/(b**(1/6)*a**(1/6)*sqrt(3)))*a**3*
b*x**19 + 259350*b**(1/6)*a**(1/6)*sqrt(3)*atan((b**(1/6)*a**(1/6) - 2*a**
(1/3)*x)/(b**(1/6)*a**(1/6)*sqrt(3)))*a**2*b**2*x**13 - 86450*b**(1/6)*a**
(1/6)*sqrt(3)*atan((b**(1/6)*a**(1/6) - 2*a**(1/3)*x)/(b**(1/6)*a**(1/6)*s
qrt(3)))*a*b**3*x**7 - 86450*b**(1/6)*a**(1/6)*sqrt(3)*atan((b**(1/6)*a**(
1/6) + 2*a**(1/3)*x)/(b**(1/6)*a**(1/6)*sqrt(3)))*a**4*x**25 + 259350*b**(
1/6)*a**(1/6)*sqrt(3)*atan((b**(1/6)*a**(1/6) + 2*a**(1/3)*x)/(b**(1/6)*a*
*(1/6)*sqrt(3)))*a**3*b*x**19 - 259350*b**(1/6)*a**(1/6)*sqrt(3)*atan((b**
(1/6)*a**(1/6) + 2*a**(1/3)*x)/(b**(1/6)*a**(1/6)*sqrt(3)))*a**2*b**2*x**1
3 + 86450*b**(1/6)*a**(1/6)*sqrt(3)*atan((b**(1/6)*a**(1/6) + 2*a**(1/3)*x
)/(b**(1/6)*a**(1/6)*sqrt(3)))*a*b**3*x**7 - 43225*b**(1/6)*a**(1/6)*log(
- b**(1/6)*a**(1/6)*x + a**(1/3)*x**2 + b**(1/3))*a**4*x**25 + 129675*b**(
1/6)*a**(1/6)*log( - b**(1/6)*a**(1/6)*x + a**(1/3)*x**2 + b**(1/3))*a**3*
b*x**19 - 129675*b**(1/6)*a**(1/6)*log( - b**(1/6)*a**(1/6)*x + a**(1/3)*x
**2 + b**(1/3))*a**2*b**2*x**13 + 43225*b**(1/6)*a**(1/6)*log( - b**(1/6)*
a**(1/6)*x + a**(1/3)*x**2 + b**(1/3))*a*b**3*x**7 - 86450*b**(1/6)*a**(1/
6)*log( - b**(1/6)*a**(1/6) + a**(1/3)*x)*a**4*x**25 + 259350*b**(1/6)*a**
(1/6)*log( - b**(1/6)*a**(1/6) + a**(1/3)*x)*a**3*b*x**19 - 259350*b**(...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
          finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file